Lab-6

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2017 13118

Supreeth

(6.1) Yes, x is periodic

The period of x is 2π.

(6.2) (a) The Given Impulse Response is $h[n]$.

input signal is $e^{j\omega_0 n}$.

$$y[n] = h[n] * e^{j\omega_0 n}.$$

$$= \sum_{\tau=-\infty}^{\infty} h[\tau] \times e^{(n-\tau)j\omega_0}.$$

$$= e^{j\omega_0 n} \sum_{\tau=-\infty}^{\infty} h(\tau) e^{-j\omega_0 \tau}.$$

$$= e^{j\omega_0 n} \quad H(e^{j\omega_0}) = H(e^{j\omega_0}).x[n].$$

$$\text{Output} = y[n] = x[n] \cdot H(e^{j\omega_0}).$$

(b) $y[n] = x[n] * h[n]$.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k].$$

Now applying discrete time fourier transform.

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k] h[n-k] \right) e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} [x[k] \left(\sum_{n=-\infty}^{\infty} h[n-k] e^{-j\omega[n-k]} \right] e^{-j\omega k}$$

(replace n-k by some τ, we get)

$$= \sum_{k=-\infty}^{\infty} x[k] e^{-j\omega k} (\underbrace{H(e^{j\omega})}_{\text{from def.}}) = H(e^{j\omega}) \sum_{k=-\infty}^{\infty} x[k] e^{-j\omega k}$$

$$= H(e^{j\omega}) X(e^{j\omega}).$$

$$\boxed{Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega})}$$

$\therefore$ Hence Proved.

6.3. ②

ⓐ The given statement may be or may not be true.
We can prove this clearly the following counter example.

$$y[n] = x[n] + 1 \quad \text{--- ①}$$

Now $y[n-1] = x[n-1] + 1$ [sub. $[n-1]$ instead of n]

sub. ① & ②.

$$y[n] - y[n-1] = x[n] - x[n-1].$$

$$y[n] = \sum_{l=0}^{L} a_l x[n-l] + \sum_{m=1}^{M} b_M y[n-m].$$

Given system $y[n] = x[n] + 1$. This is not linear. so it is "not necessary" that given system is an LTI system.

ⓑ Now, assuming that the given difference eqn. corresponds to an LTI system, we take DTFT on Both sides, we get.

$$y[n] = \sum_{m=0}^{M} b_M x[n-m] - \sum_{l=1}^{L} a_l y[n-l].$$

$$\sum_{l=0}^{L} a_l y[n-l] = \sum_{m=0}^{M} b_M x[n-m].$$

$$\sum_{l=0}^{L} a_l Y(e^{j\omega}) e^{j\omega l} = \sum_{M=0}^{M} b_M X(e^{j\omega}) e^{j\omega m}$$

$$Y(e^{j\omega}) \left[\sum_{l=0}^{L} a_x e^{j\omega l}\right] = X(e^{j\omega}) \left[\sum_{m=0}^{M} b_m e^{j\omega m}\right]$$

$$\boxed{H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{m=0}^{M} b_M e^{j\omega m}}{\sum_{l=0}^{L} a_l e^{j\omega l}}}$$

ⓒ Pre computation by using given formula (computed/given above in (b)) will give

$$H(e^{j\omega}) = \frac{1}{1 - 0.9 e^{j\omega}}.$$

(d) The computation by using the given formula (3)
(computed/given above), will give in
in (b).

$$H(e^{j\omega}) = \frac{1}{1+0.9e^{-j\omega}}.$$

(e) The nature of $y[n] = x[n] + 0.9y[n-1]$ is low pass filter.

The nature of $y[n] = x[n] - 0.9[n-1]$ is high pass filter.

Impulse response of $y[n] = x[n] + 0.9y[n-1]$.

(f) Impulse response

$$h[n] = \begin{cases} 0 & n<0 \\ (0.9)^n & n \geq 0 \end{cases}$$

Impulse response of $y[n] = x[n] - 0.9y[n-1]$

$$h[n] = \begin{cases} 0 & n<0 \\ (-0.9)^n & n \geq 0 \end{cases}$$