FOLGEN & REIHEN

① a) 7 mal

b) Papierdicke 0.1mm

$0.4\,mm$; $0.8\,mm$; $1.6\,mm$; $3.2\,mm$

c) $h_k = 0.1\,mm \cdot 2^k$

d) $h_{15} = 3.28m$, $h_{20} = 104.86\,m$, $h_{42} = 439'804.651\,km$

② 2 4 6 8 10 Folge der geraden Zahlen

1 3 5 7 9 Folge der ungeraden Zahlen

1 4 9 16 25 Folge der Quadratzahlen

③ a) $a_1 = -2$, $a_2 = 1$, $a_3 = 4$, $a_4 = 7$, $a_5 = 10$, $a_6 = 13$

$a_{100} = 295$, $a_{101} = 298$

b) $b_1 = \frac{1}{2}$, $b_2 = \frac{2}{3}$, $b_3 = \frac{3}{4}$, $b_4 = \frac{4}{5}$, $b_5 = \frac{5}{6}$, $b_6 = \frac{6}{7}$

$b_{100} = \frac{100}{101}$, $b_{101} = \frac{101}{102}$

c) $c_1 = 2$, $c_2 = \frac{9}{4}$, $c_3 = \frac{64}{27}$, $c_4 = 2.4414$,

$c_5 = 2.48832$, $c_6 = 2.5216$

$c_{100} = 2.704813829$, $c_{101} = 2.704945977$

d) $d_1 = 1$, $d_2 = 0$, $d_3 = -1$, $d_4 = 0$, $d_5 = 1$, $d_6 = 0$

$d_{100} = 0$, $d_{101} = 1$

④ a) $a_k = 5k - 2$

b) $b_k = 2^k - 1$

c) $c_k = (-1)^k \cdot k^2$

d) $d_k = n \cdot (n+1)$

⑤ a) $a_k = k^3$

b) $b_k = k! = k \cdot (k-1) \cdot (k-2) \cdot \ldots \cdot 2 \cdot 1$

c) $c_k = \sin\left(\frac{\pi}{2}(k-1)\right)$

d) Folge der Primzahlen

e) Pi mit Nachkommastellen

f) $a_k = 2 + \frac{1}{2 + a_{k-1}}$, $a_1 = 2$

⑥ a) $a_k = 4k - 11$ $\quad a_k = a_{k-1} + 4$, $a_1 = -7$

b) $b_k = \frac{1}{2^k}$ $\quad b_k = b_{k-1} \cdot \frac{1}{2}$, $b_1 = \frac{1}{2}$

c) $c_k = 10a_{k-1} + k$ $\quad c_1 = 10$

d) $a_{k-1} = (k-1) \cdot 2^{k-1} = k \cdot 2^{k-1} - 2^{k-1}$

$a_k = \frac{k}{k-1} \cdot 2a_{k-1} = \frac{2k}{k-1} a_{k-1}$, $a_1 = 2$

e)

3	4	5
180°	360°	540°

$W_n = W_{n-1} + 180°$ $\quad W_3 = 180°$

f) 

0 1 3 6 10 15 ...

$S_n = S_{n-1} + (n-1)$ $\quad S_1 = 0$

g) $x_1 = a$ $\quad y_1 = \frac{q}{x_1}$

$x_2 = \frac{y_1 + x_1}{2}$ $\quad y_2 = \frac{q}{x_2}$

$x_3 = \frac{y_2 + x_2}{2}$ ...

$x_k = \frac{x_{k-1}^2 + q}{x_{k-1}}$ $\quad x_1 = a$ (Näherungswert)

(7) $S_1 = 1 \quad S_3 = 6 \;, \; S_{100} = (1+100)\cdot\frac{100}{2} = 5050$

$S_n = (1+n)\cdot\frac{n}{2}$

(8) $S_1 = \frac{1}{2} \;, \; S_2 = \frac{2}{3} \;, \; S_3 = \frac{3}{4} \;, \; S_n = \frac{n}{n+1}$

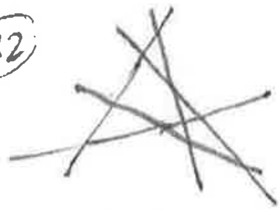
(9) $S_1 = 1 \;, \; S_2 = \frac{3}{2} \;, \; S_3 = \frac{7}{4} \;, \; S_4 = \frac{15}{8} \;, \quad S_n = \frac{2^n - 1}{2^{n-1}}$

(10) $\lim\limits_{n\to\infty} \frac{n}{n+1} = \underline{\underline{1}} \quad , \quad \lim\limits_{n\to\infty} \frac{2^n - 1}{2^{n-1}} = \lim\limits_{n\to\infty} \frac{2^n}{2^{n-1}} = \underline{\underline{2}}$

(11) $2^2 - 1 = 3 \qquad 2^3 - 1 = 7 \qquad 2^5 - 1 = 31 \qquad 2^7 - 1 = 127$ prim

Falsch $\quad 2^{11} - 1 = 23 \cdot 89$

(12)



$A(0) = 0 \qquad A(3) = 3$

$A(1) = 0 \qquad A(4) = 6$

$A(2) = 1 \qquad A(5) = 10$

$A(n) = (1 + (n-1))\cdot\frac{(n-1)}{2}$

$= n \cdot \frac{(n-1)}{2}$

(13) $S_n = (1+n)\cdot\frac{n}{2} \qquad S_1 = 1 \quad S_2 = 3 \quad S_3 = 6 \; \ldots$

Verankerung: $n = 1 \qquad S_1 = (1+1)\cdot\frac{1}{2} = 1$ ✓

Schritt: $\quad S_{n+1} = (n+2)\cdot\frac{n+1}{2} = \frac{1}{2}(n+1)(n+2)$

$S_{n+1} = S_n + a_{n+1} = (1+n)\cdot\frac{n}{2} + n + 1 = \frac{n^2}{2} + \frac{3}{2}n + 1$

$= \frac{1}{2}(n^2 + 3n + 2) = \frac{1}{2}(n+1)(n+2)$ ✓

(14) $S_n = \frac{n(n+1)(2n+1)}{6} \qquad a_n = (n)^2$

Verankerung: $\quad S_1 = \frac{1\cdot 2\cdot 3}{6} = 1$ ✓

Schritt: $\quad S_{n+1} = \frac{(n+1)(n+2)(2n+3)}{6} = \frac{(n^2+3n+2)(2n+3)}{6} = \frac{2n^3+9n^2+13n+6}{6}$

$S_{n+1} = S_n + a_{n+1} = \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{(n^2+n)(2n+1)}{6} + \frac{6n^2+12n+6}{6}$

$= \frac{2n^3+3n^2+n+6n^2+12n+6}{6} = \frac{2n^3+9n^2+13n+6}{6}$ ✓

b) $s_1 = \frac{1}{2}$ $s_2 = \frac{2}{3}$ $s_3 = \frac{3}{4}$ $s_n = \frac{n}{n+1}$ $a_n = \frac{1}{n(n+1)}$

Verankerung: $s_1 = \frac{1}{1+1} = \frac{1}{2}$ ✓

Schritt: $s_{n+1} = \frac{n+1}{n+2}$

$$s_{n+1} = s_n + a_{n+1} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n(n+2)}{(n+1)(n+2)} + \frac{1}{(n+1)(n+2)}$$

$$= \frac{n^2+2n+1}{(n+1)(n+2)} = \frac{(n+1)^2}{(n+1)(n+2)} = \frac{n+1}{n+2} ✓$$

c) $s_1 = 5$ $s_2 = 22$ $s_3 = 51$ $a_n = 12n - 7$

$s_n = n(6n-1)$

Verankerung: $s_1 = 1 \cdot (6 \cdot 1 - 1) = 1 \cdot 5 = 5$

Schritt: $s_{n+1} = (n+1)(6n+5) = 6n^2 + 11n + 5$

$$s_{n+1} = s_n + a_{n+1} = n(6n-1) + (12n+5) = 6n^2 - n + 12n + 5$$

$$= 6n^2 + 11n + 5 ✓$$

d) $s_1 = 1$ $s_2 = 9$ $s_3 = 36$ $a_n = n^3$ $s_n = \left(\frac{n(n+1)}{2}\right)^2$

Verankerung: $s_1 = \left(\frac{1 \cdot 2}{2}\right)^2 = 1$ ✓

Schritt: $s_{n+1} = \left(\frac{(n+1)(n+2)}{2}\right)^2 = \left(\frac{n^2+3n+2}{2}\right)^2 = \frac{n^4+6n^3+13n^2+12n+4}{4}$

$$s_{n+1} = s_n + a_{n+1} = \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3$$

$$= \frac{n^2(n+1)^2}{4} + n^3 + 3n^2 + 3n + 1 = \frac{n^2(n^2+2n+1)}{4} + \frac{4n^3+12n^2+12n+4}{4}$$

$$= \frac{n^4+6n^3+13n^2+12n+4}{4} ✓$$

15 Sei $n^3 + 5n$ durch 3 teilbar

Verankerung: $1^3 + 5 \cdot 1 = 6$ ✓

Schritt: $(n+1)^3 + 5(n+1) = n^3 + 3n^2 + 3n + 1 + 5n + 5$

$$= n^3 + 5n + 3n^2 + 3n + 6 = \underbrace{n^3+5n}_{\text{durch 3 teilbar nach Voraussetzung}} + \underbrace{3(n^2+n+2)}_{\text{klar durch 3 teilbar}}$$

ist durch 3 teilbar.

(16) $\frac{a_{k-1}+a_{k+1}}{2} = \frac{a_k - d + a_k + d}{2} = \frac{2a_k}{2} = a_k$ ✓

(17) $a_1 = 81 \quad a_n = 1020$

$1020 = 81 + (n-1)\cdot 3$

$939 = (n-1)\cdot 3$

$314 = n$

$S_{314} = (81+1020)\cdot \frac{314}{2} = \underline{\underline{172'857}}$

(18) $a_1 = 5 \quad d = 10$

$a_{13} = 5 + (13-1)\cdot 10 = \underline{\underline{125m}}$

$S_{13} = (5+125)\cdot \frac{13}{2} = \underline{\underline{845m}}$

c) $1805 = (5 + a_n)\cdot \frac{n}{2}$

$a_n = 5 + (n-1)\cdot 10$

$\Rightarrow \quad 1805 = (5 + 5 + (n-1)\cdot 10)\cdot \frac{n}{2}$

$1805 = 10n \cdot \frac{n}{2}$

$1805 = 5n^2$

$361 = n^2$

$\pm 19 = n \quad \rightarrow \underline{\underline{19}}$ Sekunden

(19) $\sqrt{a_{k-1}\cdot a_{k+1}} = \sqrt{\frac{a_k}{q}\cdot a_k \cdot q} = \sqrt{a_k^2} = a_k$ ✓

(20) $a_1 = -2 \quad q = -2$

$4096 = (-2)\cdot(-2)^{n-1}$

$4096 = 2^n$

$\lg 4096 = \log 2^n$

$n = \frac{\lg 4096}{\lg 2} = 12$

$S_{12} = (-2)\cdot \frac{(-2)^{12}-1}{(-2)-1} = \underline{\underline{2'730}}$

(21) $n = 10 \qquad a_1 = 1 \qquad a_{10} = 2$

$$2 = 1 \cdot q^9$$

$$\sqrt[9]{2} = q \approx 1.08 \qquad \underline{\underline{a_k = \left(\sqrt[9]{2}\right)^{k-1}}}$$

$$S_{10} = 1 \cdot \frac{\left(\sqrt[9]{2}\right)^{10} - 1}{\sqrt[9]{2} - 1} = \underline{\underline{14.49}}$$

(22) $a_1 = 4 \qquad d = 1$

$$S_n = (a_1 + a_n) \cdot \frac{n}{2} \qquad a_n = a_1 + (n-1) \cdot d = 3 + n$$

$$S_n = (4 + 3 + n) \cdot \frac{n}{2} = 60$$

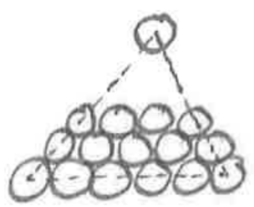
$$\frac{7}{2}n + \frac{1}{2}n^2 = 60$$

$$\frac{1}{2}n^2 + \frac{7}{2}n - 60 = 0$$

$$n^2 + 7n - 120 = 0$$

$$(n+15)(n-8) = 0 \qquad \Rightarrow n_1 = -15 \ , \ n_2 = \underline{\underline{8}} \text{ Schichten}$$

$a_8 = 4 + 7 = \underline{\underline{11}}$ Rohre in der untersten Schicht



$$s = 10d = 2m$$

$$H = \frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3}\ m$$

$$s = 3d = 0.6m$$

$$h = \frac{\sqrt{3}}{2} \cdot 0.6 = \frac{3\sqrt{3}}{10}\ m$$

Höhe : $H - h + d = \sqrt{3} - \frac{3\sqrt{3}}{10} + 0.2 \approx \underline{\underline{1.41m}}$

(23) $a_1 = 0.9458 = q$

$$a_3 = a_1 \cdot q^2 = q^3 = 0.846$$

Es verliert $\underline{\underline{15.394\%}}$

(24) 6M 5M 4M 3M 2M 1M

1 100

$\cdot q$ oder $:q$

einfacher: $a_1 = 100$ $a_6 = 1$

$$1 = 100 \cdot q^5$$

$$\frac{1}{100} = q^5$$

$$\sqrt[5]{\frac{1}{100}} = q \approx \underline{\underline{0.3981}}$$

$$a_{-1.6} = 100 \cdot q^{-2.6} \approx \underline{\underline{1096}} \quad \sim 10^3$$

$$a_{24} = 1 \cdot q^{18} \approx 6.3 \cdot 10^{-8} \quad \sim 10^{-7}$$

$\Big\} \cdot 10^{10}$ $\underline{\underline{\text{heller}}}$

(25) $\frac{15}{10}$ DIN $\sim$ $\frac{1}{60}$ s

$+x \cdot \frac{3}{10}$ $\cdot \left(\frac{1}{2}\right)^4$

$\frac{27}{10}$ DIN $\sim$ $\underline{\underline{\frac{1}{1042}}}$ s

(26) a b c AF $\rightarrow$ a a+d a+2d

$$a + a+d + a+2d = 3$$

$$3a + 3d = 3$$

$$a+d = 1$$

b c a GF $\rightarrow$ a+d $q(a+d)$ $q^2(a+d)$

1 q q^2

$$1 + q^2 + q^2 = 3$$

$$q^2 + q - 2 = 0$$

$$(q+2)(q+1) = 0 \Rightarrow q_1 = -2 \;, \; q_2 = 1$$

also entweder $\underline{\underline{a = b = c = 1}}$ oder $\underline{\underline{a = 4 \quad b = 1 \quad c = -2}}$

(27) a) $a_1 = 1 \quad q = 2 \quad n = 64$

$$S_{64} = 1 \cdot \frac{2^{64}-1}{2-1} = 2^{64} - 1 = \underline{\underline{1.8 \cdot 10^{19}}} \text{ Körner}$$

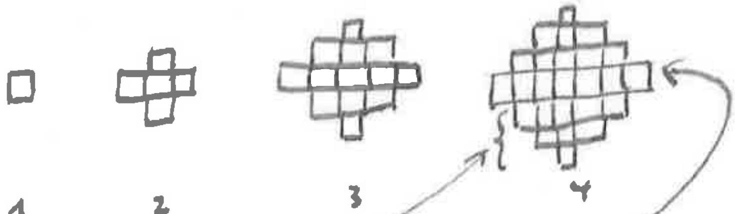
b) $10\,t = 10^7 g \longrightarrow \cdot 20 = 2 \cdot 10^8$ Körner pro Wagen

Zuglänge: $\dfrac{1.8 \cdot 10^{19}}{2 \cdot 10^8} \cdot 10\,m = 9.2 \cdot 10^{11}\,m \approx \underline{\underline{922 \text{ Mio km}}}$

c) $V = \pi r^2 h = \pi \cdot 10^{-8}\,m^3$

Fläche CH: $41'285\ km^2 = G \qquad h = \dfrac{V \cdot \text{Körner}}{G} = \underline{\underline{14\,m}}$

(28)



$$A(n) = 2 \cdot (n-1)^2 + (2n-1)$$
$$= 2(n^2 - 2n + 1) + 2n - 1 = 2n^2 - 4n + 2 + 2n - 1$$
$$= 2n^2 - 2n + 1 = n^2 + n^2 - 2n + 1 = \underline{\underline{n^2 + (n-1)^2}}$$

(29) $a_1 = 1 \qquad a_{13} = 2$

$2 = 1 \cdot q^{12}$

$2 = q^{12}$

$\sqrt[12]{2} = q \approx 1.06 \qquad \longrightarrow$ vgl. Tabelle 1 p. 8

(30) a) $a_{10} = 5.1 \qquad a_{11} = 5.\overline{09} \qquad a_{1000} = 5.001 \qquad a_{1001} = 5.00099\ldots$

$a_{1000000} = 5.000001 \qquad a_{1000001} = 5.0000009\ldots$

$\rightarrow$ konvergent gegen 5

b) $b_{10} = 300$ $\quad b_{11} = -\frac{3000}{301}$ $\quad , b_{10^3} = 3$ $\quad b_{10^3+1} = -\frac{3000}{1001}$

$b_{10^6} = 0.003$ $\quad b_{10^6+1} = -\frac{3000}{1000001}$ $\quad \rightarrow$ konvergent gegen 0

c) $c_{10} = 10^3$ $\quad c_{11} = 11^3$ $\quad c_{10^3} = 10^9$ $\quad c_{10^3+1} = (10^3+1)^3$

$c_{10^6} = 10^{18}$ $\quad c_{10^6+1} = (10^6+1)^3$ $\quad \rightarrow$ divergent

d) $d_{10} = 2$ $\quad d_{11} = 0$ $\quad d_{10^3} = 2$ $\quad d_{10^3+1} = 0$ $\quad d_{10^6} = 2$ $\quad d_{10^6+1} = 0$

(31) $|a_k - 2| < \frac{1}{100}$

$|2 + (-1)^k \cdot \frac{4}{k} - 2| < \frac{1}{100}$

$|(-1)^k \cdot \frac{4}{k}| < \frac{1}{100}$

$\frac{4}{k} < \frac{1}{100}$

$400 < k$, d.h. $N(\frac{1}{100}) = 400$

ab $k = 401$ sind die Glieder in der ε-Umgebung.

(32) a) $\lim_{k \to \infty} a_k = 0$

$|a_k - 0| < \frac{1}{1000}$

$|\frac{5}{k+1}| < \frac{1}{1000}$

$\frac{5}{k+1} < \frac{1}{1000}$

$\frac{5000}{k+1} < 1$

$5000 < k+1$

$4999 < k$

$N(10^{-3}) = \underline{4999}$

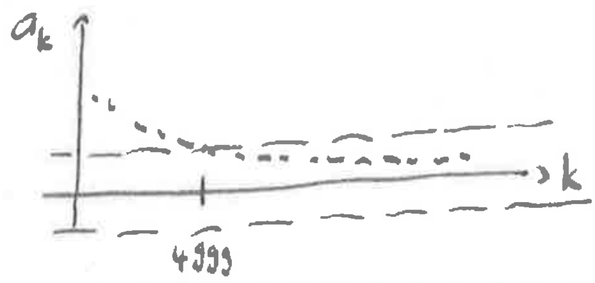
$|a_k - 0| < \varepsilon$

$\frac{5}{k+1} < \varepsilon$

$\frac{5}{\varepsilon} < k+1$

$\frac{5}{\varepsilon} - 1 < k$

$N(\varepsilon) = \underline{\underline{\frac{5}{\varepsilon} - 1}}$



b) $\lim\limits_{k \to \infty} b_k = \frac{7}{2}$

$\left| b_k - \frac{7}{2} \right| < \varepsilon$

$\left| \frac{7k+8}{2k-3} - \frac{7}{2} \right| < \varepsilon$

$\left| \frac{7k+8}{2k-3} - \frac{7}{2} < \varepsilon \right. \qquad (k \geq 2)$

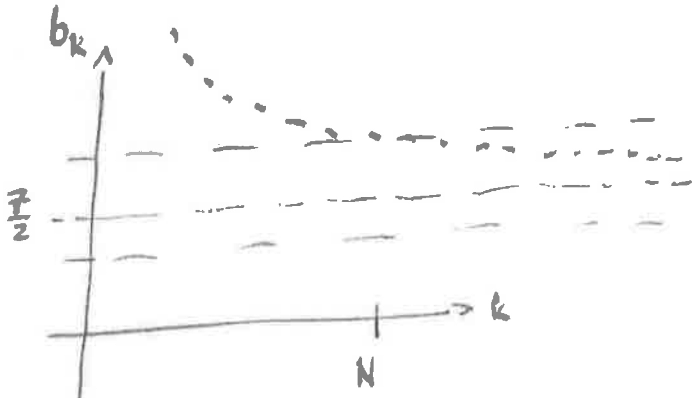
$\frac{7k+8}{2k-3} < \varepsilon + \frac{7}{2}$

$7k+8 < \left(\varepsilon + \frac{7}{2}\right)(2k-3)$

$\cancel{7k} + 8 < 2k\varepsilon - 3\varepsilon + \cancel{7k} - \frac{21}{2}$

$3\varepsilon + \frac{21}{2} < 2k\varepsilon$

$\frac{3\varepsilon + \frac{21}{2}}{2\varepsilon} < k$



$N(\varepsilon) = \frac{3\varepsilon + 10.5}{2\varepsilon}$

c) $\lim\limits_{k \to \infty} x_k = \infty$ $\rightarrow$ divergent

d) $\lim\limits_{k \to \infty} u_k = -2$

$|u_k - (-2)| < \varepsilon$

$\left| \frac{3-2k^2}{k^2} + 2 \right| < \varepsilon$

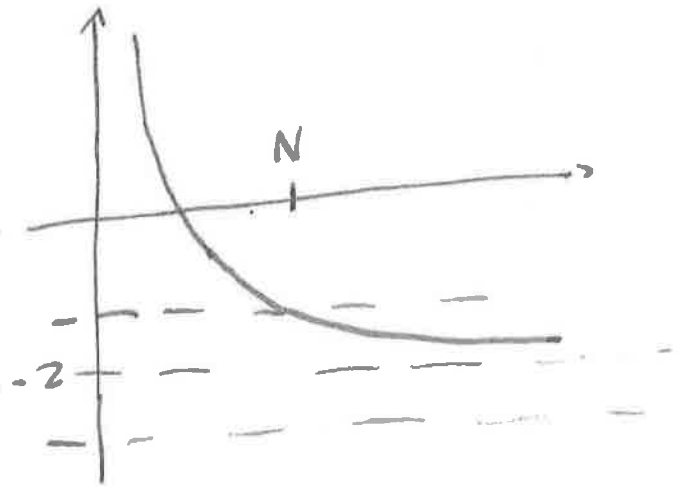
$\frac{3-2k^2}{k^2} + 2 < \varepsilon$

$\frac{3-2k^2}{k^2} < \varepsilon - 2$

$3 - \cancel{2k^2} < \varepsilon k^2 - \cancel{2k^2}$

$\frac{3}{\varepsilon} < k^2$

$\pm\sqrt{\frac{3}{\varepsilon}} < k$



$N(\varepsilon) = \sqrt{\frac{3}{\varepsilon}}$

(33) a) höchstens endlich viele (bis zur Stichzahl N)

b) höchstens einen ... Gäbe es zwei, g_1, g_2, dann könnte man $\varepsilon = \ldots\ldots$

(34) monoton fallend : $a_k = -k$

monoton wachsend : $b_k = k$

beschränkt : $c_k = 1$, $d_k = \sin(k)$

(35) Eine konvergente Folge muss beschränkt sein.

Sei $\langle a_k \rangle$ konvergent $\Rightarrow \forall \varepsilon > 0 \ \exists N(\varepsilon)$, so dass $|a_k - g| < \varepsilon$ $\forall k > N(\varepsilon)$. Damit ist $|a_k - g| \leq |a_k + g| \leq |a_k| + |g| = M$ eine obere Schranke, falls $a_k, g > 0$. In den andern Fällen passt man einfach das Vorzeichen an. $\square$

Ist eine Folge monoton und beschränkt, dann ist sie konvergent.

(36) Der Beweis ist etwas anspruchsvoll.

(36) Allgemein gilt, dass Mathematiklehrer monoton und beschränkt sind. Daraus folgt unmittelbar mit Üb (35), dass sie konvergent sind. $\square$

REIHEN

(37) $\frac{263}{315}$ $\frac{\pi}{4} = 0.78539816\ldots$ 6.1 %

0.499964... $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ 0.7 %

0.884703 0.8849558.. 0.02 %

(38) a) $\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{12}$ ...

$s_1 = \frac{1}{2}$ $s_2 = \frac{2}{3}$ $s_3 = \frac{3}{4}$ $s_k = \frac{k}{k+1}$

$\lim\limits_{k\to\infty} s_k = 1$

$\left| \frac{k}{k+1} - 1 \right| < \varepsilon$

$1 - \frac{k}{k+1} < 0.001$

$0.999 < \frac{k}{k+1}$

$0.999k + 0.999 < k$

$0.999 < 0.001k$

$999 < k$ $N(10^{-3}) = 1000$

b) 1. $s_1 = 1$ $s_2 = 3$ $s_3 = 11$ ...

2. $s_1 = -1$ $s_2 = -5$ $s_3 = -37$ ...

$s_1 = 1$ $s_2 = -1$ $s_3 = 3$ $s_4 = -5$ ....

divergent

46 a) $\frac{1}{3}$ $\frac{11}{15}$ $\frac{122}{105}$ $\frac{506}{315}$

nicht beschränkt $\rightarrow$ divergiert, weil $\lim\limits_{k\to\infty} \frac{k}{2k+1} = \frac{1}{2}$

b) $$s_k = \sum_{k=0}^{\infty} \frac{(-1)^k + (\frac{1}{4})^k + \ldots}{(2k)!}$$

$\rightarrow$ ev. konvergent ?! $\rightarrow$ divergiert, weil $\lim\limits_{k\to\infty} a_k = 1$

c) $s_k = k \cdot a + (k \cdot \frac{(k-1)}{2})d$ $\rightarrow$ divergent, für $a \neq 0$ oder $d \neq 0$

(41) a) $q = \frac{2}{3}$ $\quad s = \frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = \underline{\underline{3}}$

b) $q = -\frac{1}{3}$ $\quad s = \frac{2}{1-(-\frac{1}{3})} = \frac{2}{\frac{4}{3}} = \underline{\underline{\frac{3}{2}}}$

c) $q = -\frac{99}{100}$ $\quad s = \frac{1}{1-(-\frac{99}{100})} = \frac{1}{\frac{199}{100}} = \underline{\underline{\frac{100}{199}}}$

d) $q = \frac{8}{9}$ $\quad s = \frac{\cancel{1}\frac{3}{2}}{1-\frac{8}{9}} = \frac{\frac{3}{2}\cancel{1}}{\frac{1}{9}} = \underline{\underline{\cancel{4}}}\ \frac{27}{2}$

(42) $q = \frac{p^3}{27}$ $\quad$ Es muss $-1 < \frac{p^3}{27} < 1$

1.: $-1 < \frac{p^3}{27}$
$-27 < p^3$
$-3 < p$

2.: $\frac{p^3}{27} < 1$
$p^3 < 27$
$p < 3$

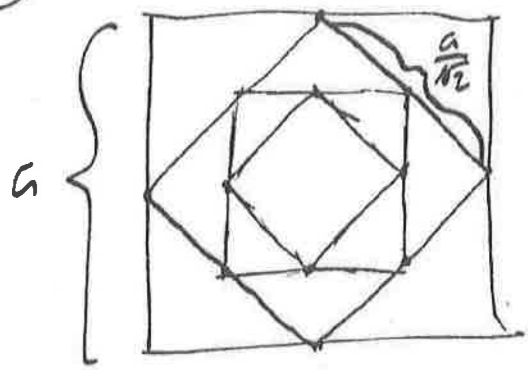
$\rightarrow \underline{\underline{-3 < p < 3}}$

(43) $0.4 \quad 0.04 \quad 0.004 \quad \ldots$

$a_1 = 0.4 \quad q = \frac{1}{10}$

$S = \frac{0.4}{1-\frac{1}{10}} = \frac{\frac{4}{10}}{\frac{9}{10}} = \underline{\underline{\frac{4}{9}}}$

(44)



$F = a^2 + \frac{a^2}{2} + \frac{a^2}{4} + \ldots$

$= \frac{a^2}{1-\frac{1}{2}} = \underline{\underline{2a^2}}$

$U = 4a + 2\sqrt{2}a + 2a + \ldots$

$= \cancel{\frac{4a}{1-\frac{1}{\sqrt{2}}}}$ divergiert nicht, da $q = \frac{1}{\sqrt{2}}$

$$U = 4a + 4\cdot\frac{a}{\sqrt{2}} + 4\cdot\frac{a}{2} + \dots$$

$$= 4a + 2\sqrt{2}a + 2a + \dots \qquad q = \frac{1}{\sqrt{2}}$$

$$= \frac{4a}{1-\frac{1}{\sqrt{2}}} = \frac{4a}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{4\sqrt{2}a}{\sqrt{2}-1} = \frac{4\sqrt{2}a(\sqrt{2}+1)}{1}$$

$$= \underline{\underline{8a + 4\sqrt{2}a}}$$

(45) $s = \frac{a_1}{1-q}$

a) $a_1 = 1 \quad q = x \qquad -1 < x < 1$

b) $a_1 = 1 \quad q = x^2 \qquad -1 < x^2 < 1 \qquad 0 < x < 1$

(46) Die Reihe der unendlich vielen Zeitintervalle ist endlich.

→ Achilles wird die Schildkröte ein- und überholen.

(47) $a_1 \quad a_1 q \quad a_1 q^2 \quad \dots$

$s = \frac{a_1}{1-q} = 1 \qquad a_1 = 1 - q$

$\sqrt{a_1} \quad \sqrt{a_1 q} \quad \sqrt{a_1 q^2}$

$s = \frac{a_1}{1-\sqrt{q}} = 2 \qquad a_1 = 2 - 2\sqrt{q}$

$\Rightarrow 1 - q = 2 - 2\sqrt{q} \qquad 0 = q^2 - 2q + 1$

$2\sqrt{q} - \cancel{\sqrt{q}} = 1 \qquad 0 = (q-1)^2$

$2\sqrt{q} = 1 + q \qquad q = 1$

$4q = 1 + 2q + q^2 \qquad a_1 = 0$

(47)

$$a_1 \quad a_1 q \quad a_1 q^2 \quad \dots$$

$$s = \frac{a_1}{1-q} = 1 \quad \Leftrightarrow \quad a_1 = 1-q$$

$$\sqrt{a_1} \quad \sqrt{a_1 q} \quad \sqrt{a_1 q^2} \quad \dots$$

$$\tilde{s} = \frac{\sqrt{a_1}}{1-\sqrt{q}} = 2 \quad \Leftrightarrow \quad \sqrt{a_1} = 2 - 2\sqrt{q}$$

$$a_1 = 4 - 8\sqrt{q} + 4q$$

$$\Rightarrow \quad 1-q = 4 - 8\sqrt{q} + 4q$$

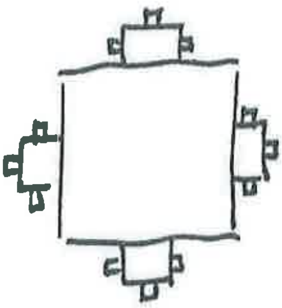
$$8\sqrt{q} = 3 + 5q$$

$$64q = 9 + 30q + 25q^2$$

$$0 = 25q^2 - 34q + 9 \xrightarrow{TR} q_1 = 1 \text{ (geht nicht)}, \quad q_2 = 0.36$$

$$\Rightarrow \quad q = \underline{\underline{0.36}} \quad \text{und} \quad a_1 = 1-q = \underline{\underline{0.64}}$$

(48)



a) $a^2 + 4\left(\frac{a}{3}\right)^2 + 12\left(\frac{a}{9}\right)^2 + 36\left(\frac{a}{27}\right)^2$

$$= a^2\left(1 + \frac{4}{9} + \frac{12}{81} + \frac{36}{3^6} + \dots\right)$$

$$= a^2\left(1 + \frac{4}{3^2} + \frac{4\cdot 3}{3^4} + \frac{4\cdot 3^2}{3^6} + \frac{4\cdot 3^3}{3^8} + \dots\right)$$

$$\Rightarrow a^2 + 4a^2\left(\frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \dots\right)$$

$$A = a^2 + 4a^2 \cdot \frac{\frac{1}{9}}{1-\frac{1}{3}} = a^2 + 4a^2 \cdot \frac{1}{6} = a^2 + \frac{2}{3}a^2 = \underline{\underline{\frac{5}{3}a^2}}$$

b) $4a + \left(4a + 8 \cdot \frac{a}{3}\right) + \left(4a + 48 \cdot \frac{a}{9}\right) + \ldots$

u ist divergent / unendlich

(49) $s = 1 + 2p + 3p^2 + 4p^3 + \ldots$

$ps = p + 2p^2 + 3p^3 + 4p^4 + \ldots$

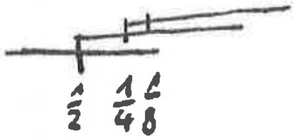
$s - ps = 1 - p + 2p - 2p^2 + 3p^2 - 3p^3 + 4p^3 \ldots$

$= 1 + p + p^2 + p^3 + \ldots$

$\Rightarrow s - ps = \frac{1}{1-p}$

$s(1-p) = \frac{1}{1-p} \quad \Rightarrow \quad s = \frac{1}{(1-p)^2}$

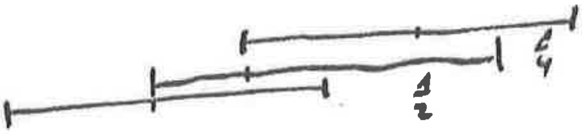
(50) a) $u_1 = \frac{1}{2} \quad u_2 = \frac{3}{4} \quad u_3 = \frac{7}{8} \quad u_k = \frac{2^k - 1}{2^k}$



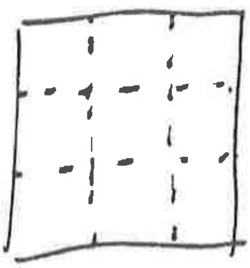
$\lim_{u_k \to \infty} u_k = 1$

falsch

b) $S_{52} = a_1 \cdot \frac{1 - q^{52}}{1 - q}$



(51)



$F = 1 - \frac{1}{9} - \frac{8}{81} - \frac{64}{3^6} - \frac{8^3}{3^8}$

$= 1 - \left(\frac{8^0}{3^2} - \frac{8^1}{3^4} - \frac{8^2}{3^6} - \frac{8^3}{3^8} - \ldots\right)$

$\Rightarrow q = \frac{8}{9}$

$F = 1 - \frac{\frac{1}{9}}{1 - \frac{8}{9}} = 1 - 1 = 0\,!$

52 $q = \frac{\frac{1}{3}}{1} = \frac{1}{3}$ $\quad a_k = a_{k-1} \cdot \frac{1}{3}$

b) $s_1 = 1 \cdot \frac{1-\left(\frac{1}{3}\right)^1}{1-\frac{1}{3}} = 1$

$s_2 = \frac{4}{3} = 1.\overline{3}$

$s_3 = \frac{13}{9} = 1.\overline{4}$ $\quad S_{100} = 1 \cdot \frac{1-\left(\frac{1}{3}\right)^{100}}{1-\frac{1}{3}} \approx \underline{\underline{1.5}}$

$s_4 = \frac{40}{27} = 1.\overline{481}$

c) $s = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \underline{\underline{\frac{3}{2}}} = 1.5$

d) $s - s_n < 10^{-6}$

$$1.5 - 1 \cdot \frac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}} < 10^{-6}$$

$$1.5 - 1.5\left(1-\left(\tfrac{1}{3}\right)^n\right) < 10^{-6}$$

$$\left(\tfrac{1}{3}\right)^n < 10^{-6}$$

$$n > \log_{\frac{1}{3}} 10^{-6} = \frac{\ln 10^{-6}}{\ln \frac{1}{3}} = 12.58\ldots$$

d.h. ab dem Index $\underline{\underline{n = 13}}$.

Test: $S_{13} = 1 \cdot \frac{1-\left(\frac{1}{3}\right)^{13}}{1-\frac{1}{3}} = 1.499999059$ ✓