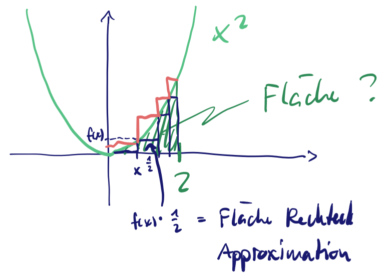
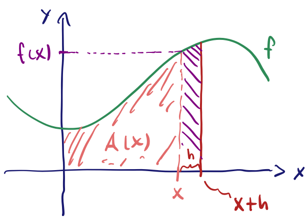
INFINITESIMALRECHNUNG

Problem : Berechne

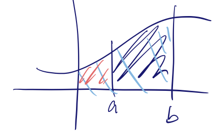


Idee :



Bem 1 :

$$A(b) - A(a) = \int_a^b f(x)\,dx$$



$$\rightarrow \quad A(x+h) \approx A(x) + f(x)\cdot h$$

Umso genauer, je näher die Säulenbreite h bei 0 ist :

$$\rightarrow \quad \lim_{h\to 0} A(x+h) = \lim_{h\to 0} A(x) + f(x)\cdot h \qquad /-A(x)$$

$$\lim_{h\to 0} A(x+h) - A(x) = f(x)\cdot h \qquad /:h$$

$$\lim_{h\to 0} \frac{A(x+h) - A(x)}{h} = f(x)$$

$$\boxed{A'(x) = f(x)}$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

Wenn du die Fläche unterhalb einer Begrenzung, gegeben durch den Graphen von $f(x)$, berechnen willst, dann suchst du eine Funktion A mit

$$\frac{dA(x)}{dx} = f(x)$$

Man schreibt $\quad A(x) =: \int_0^x f(x)\,dx$

Bemerkung: $$\frac{dA}{dx} = f(x) \qquad / \cdot dx$$

$$dA = f(x)\, dx \qquad / \int$$

$$\int dA = \int f(x)\, dx$$

$$A = \int f(x)\, dx$$

$$\int dA = \int 1 \cdot dA = A$$

Integrationskonstante

Bem 2: $$\int x^2\, dx = \frac{1}{3}x^3 + C$$

zu einer Funktion gibt es unendlich viele Stammfunktionen.

Sie unterscheiden sich durch eine additive Konstante.

Bem 3: $$\int x^2\, da = a x^2 + C$$

$x^2 a$

Zurück zum Anfangsbeispiel:

Untere Approximation: $0 \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} + 1^2 \cdot \frac{1}{2} + \left(\frac{3}{2}\right)^2 \cdot \frac{1}{2} = \frac{14}{8} = \underline{\frac{7}{4}} = 1.75$

Obere Approximation: $\left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} + 1^2 \cdot \frac{1}{2} + \left(\frac{3}{2}\right)^2 \cdot \frac{1}{2} + 2^2 \cdot \frac{1}{2} = \underline{\underline{\frac{15}{4}}} = 3.75$

Nun exakt: $$\int_0^2 x^2\, dx = \frac{1}{3}x^3 + C \Big|_0^2 = \frac{1}{3} \cdot 2^3 + C - \left(\frac{1}{3} \cdot 0^3 + C\right)$$

$$= \frac{8}{3} \approx 2.67$$

Übungen:

a) $\int x\, dx = \frac{1}{2}x^2 + C$

b) $\int dx = x + C$

c) $\int_1^3 x^3\, dx = \frac{1}{4}x^4 \Big|_1^3 = \frac{1}{4} \cdot 3^4 - \frac{1}{4} \cdot 1^4$

d) $\int \sin(t)\, dt = -\cos(t) + C$

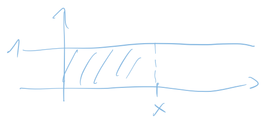
e) $\int e^x\, dx = e^x + C$

f) $\int \cos(kt)\, dt = \frac{1}{k}\sin(kt) + C$

g) $\int e^{i\omega t}\, dt = -\frac{i}{\omega} e^{i\omega t} + C$

h) $\int \frac{1}{x}\, dx = \ln(x) + C$

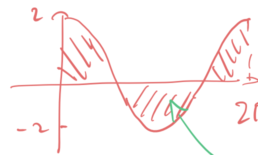
i) $\int \sqrt{x}\, dx = \frac{2}{3}x^{\frac{3}{2}} + C$



$\int dx = \int 1 \, dx = x$

j) $\int_0^{2\pi} 2\sin(x)\,dx = -2\cos(x)\Big|_0^{2\pi} = -2+2$

$= 0$



unterhalb x-Achse ist Flächenwert negativ

* Löse die Gleichung: $\dot{N}(t) = k \cdot N(t)$

$\left[\; \dot{} \;\hat{=}\; \frac{d}{dt} \qquad ' \;\hat{=}\; \frac{d}{dx} \; \right]$

$$\frac{dN}{dt} = k \cdot N$$
$$dN = k \cdot N \cdot dt$$
$$\frac{1}{N} dN = k\,dt$$
$$\int \frac{1}{N} dN = \int k\,dt$$
$$\ln(N) = kt + C$$
$$N = e^{kt+C} = N_0 e^{kt}$$

Beispiele: $\dot{N}(t) = k \cdot N(t) \qquad N(t) = ?$

$$\frac{dN}{dt} = k \cdot N \qquad / \cdot dt$$
$$dN = k \cdot N \cdot dt \qquad / :N$$
$$\frac{1}{N} dN = k \cdot dt \qquad / \int$$
$$\int \frac{1}{N} dN = \int k \cdot dt$$
$$\ln(N) + C_1 = k \cdot t + C_2 \qquad / - C_1$$
$$\ln(N) = k \cdot t + C \qquad (C := C_2 - C_1) \qquad / e^{()}$$
$$e^{\ln(N)} = e^{k \cdot t + C}$$
$$N(t) = e^{k \cdot t} \cdot e^{C}$$

Bem: $N(0) = e^{k \cdot 0} \cdot e^{C} = e^{C} =: N_0$

$N(t) = N_0 \cdot e^{kt}$

Variante 2:

$$\frac{1}{N}\,dN = k \cdot dt \qquad / \int$$

$$\int_{N_0}^{N} \frac{1}{\tilde N}\, d\tilde N = \int_{t_0}^{t} k\, d\tilde t$$

$$\ln(\tilde N)\Big|_{N_0}^{N} = k\tilde t\,\Big|_{t_0}^{t}$$

$$\ln(N) - \ln(N_0) = k\cdot t - kt_0 \qquad / \text{ TU}$$

$$\ln\left(\frac{N}{N_0}\right) = k(t - t_0) \qquad / e^{(\,)}$$

$$\frac{N}{N_0} = e^{k(t-t_0)} \qquad / \cdot N_0$$

$$N = N_0 e^{k(t-t_0)}$$

Beispiel 2:

$$\int a\, dt = at + v_0 \quad =: v(t)$$

$$\int (at + v_0)\, dt = \tfrac{1}{2}at^2 + v_0 t + s_0 \quad =: s(t)$$

$$\frac{ds}{dt} = v(t)\,, \quad \frac{dv}{dt} = a(t)$$

(gleichmässig beschleunigte Bewegung)

Beispiel 3: Kaffee wird kalt

$$\rightarrow \quad \dot T(t) = \lambda \cdot (T(t) - T_u) \qquad\qquad T(t) = ?$$

T: Temperatur zur Zeit t

t: Zeit

T_u: Umgebungstemperatur

λ: Proportionalitätskonstante

$$\rightarrow \quad \frac{dT}{dt} = \lambda(T - T_u) \qquad / : (T - T_u) \cdot dt$$

$$\frac{1}{T - T_u}\, dT = \lambda\, dt \qquad / \int$$

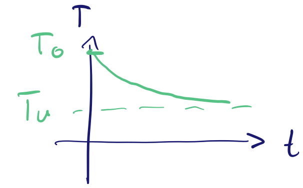
$$\ln(T - T_u) = \lambda t + \tilde C \qquad / e^{(\,)}$$

$\lambda t + \tilde C$

$$T - T_u = e^{\dots} \qquad / + T_u$$

$$T(t) = T_u + T_0 e^{\lambda t}$$

$\rightarrow$ für $\lambda < 0$ (Zerfall) ist $\lim\limits_{t \to \infty} T(t) = T_u$



Traktrix: $\quad y(x) = a \cdot \ln\left(\frac{a + \sqrt{a^2 - x^2}}{x}\right) - \sqrt{a^2 - x^2}$
(Skript p. 14)

$$\rightarrow y'(x) = a \cdot \frac{1}{\frac{a + \sqrt{a^2 - x^2}}{x}} \cdot \frac{\frac{1}{2}(a^2 - x^2)^{-\frac{1}{2}} \cdot (-2x) \cdot x - (a + \sqrt{a^2 - x^2}) \cdot 1}{x^2}$$

Bsp Kettenregel $\left(e^{\sin(x^2)}\right)' = e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x$

$$- \frac{1}{2}(a^2 - x^2)^{-\frac{1}{2}} \cdot (-2x)$$

$y' = -\frac{u^{\frac{1}{2}}}{x}$ möchte man

$$= \frac{ax}{a + u^{1/2}} \cdot \frac{-\frac{1}{u^{1/2}} \cdot x^2 - a - u^{\frac{1}{2}}}{x^2} + \frac{x}{u^{1/2}} \qquad (u := a^2 - x^2)$$

$$= \frac{ax}{a + u^{1/2}} \cdot \left(-\frac{1}{u^{1/2}} - \frac{a + u^{1/2}}{x^2}\right) + \frac{x}{u^{1/2}}$$

$$= \frac{-ax}{u^{1/2}(a + u^{1/2})} - \frac{ax(a + u^{1/2})}{(a + u^{1/2})x^2} + \frac{x(a + u^{1/2})}{u^{1/2}(a + u^{1/2})}$$

$$= -\frac{a\cancel{(a + u^{1/2})}}{x\cancel{(a + u^{1/2})}} + \frac{x^{2}\cancel{u^{1/2}}}{\cancel{u^{1/2}}(a + u^{1/2})x}$$

$$= -\frac{a}{x} + \frac{x^2}{(a + u^{1/2})x} = -\frac{a(a + u^{1/2}) - x^2}{x(a + u^{1/2})}$$

$$= -\frac{a(a + u^{1/2}) - a^2 + u}{x(a + u^{1/2})} = -\frac{\cancel{a^2} + au^{1/2} \cancel{- a^2} + u}{x(a + u^{1/2})}$$

$$= -\frac{u^{1/2}(a + u^{1/2})}{x(a + u^{1/2})} = -\frac{u^{1/2}}{x} = -\frac{\sqrt{a^2 - x^2}}{x} \qquad \square$$

Üb 16 $\quad y' = (x^2 + 1) \cdot (y - 2)$

$$\frac{dy}{dx} = (x^2 + 1)(y - 2) \qquad / \cdot dx$$

$$dy = (x^2 + 1)(y - 2)\, dx \qquad / : (y - 2)$$

$$\frac{1}{y-2}\,dy = (x^2+1)\,dx \qquad /\int$$

$$\ln(y-2) = \tfrac{1}{3}x^3 + x + \hat{c} \qquad /e^{(\,)}$$

$$y-2 = e^{\frac{1}{3}x^3 + x + \hat{c}} \qquad /+2$$

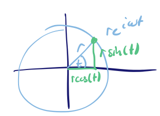
$$y(x) = 2 + y_0 e^{\frac{1}{3}x^3 + x}$$

Hook'sche Feder (proportional)

Remind: $(\sin(x))' = \cos(x)$

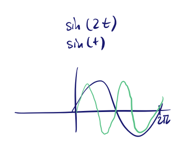
$(\cos(x))' = -\sin(x)$

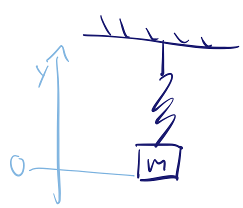
$$\left(e^{i\omega t}\right)^{\cdot} = i\omega e^{i\omega t}$$



$$\left(i\omega e^{i\omega t}\right)^{\cdot} = (i\omega)^2 e^{i\omega t} = -\omega^2 e^{i\omega t}$$

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$





Federkraft: $F_F = -k \cdot y$

$(D), k$: Federkonstante

y: Auslenkung aus der Ruhelage

Newton 2: $F = m \cdot a$

$\rightarrow \quad m \cdot a = -k \cdot y$

$m \cdot \ddot{y} = -k\,y$

y: Ort

$\dot{y}$: Geschwindigkeit

$\ddot{y}$: Beschleunigung

$\rightarrow \quad \ddot{y} = -\frac{k}{m} \cdot y \qquad y(t)\,?$

Ansatz: $y(t) = y_0 e^{i\omega t}$ $\qquad y_0 e^{i\omega t} = y_0\cos(\omega t) + i y_0 \sin(\omega t)$

$\dot{y}(t) = y_0 i\omega e^{i\omega t}$

$\ddot{y}(t) = -y_0\omega^2 e^{i\omega t}$

$$\rightarrow \quad \cancel{-}\cancel{y_0}\,\omega^2 \cancel{e^{i\omega t}} = \cancel{-}\frac{k}{m}\cdot \cancel{y_0 e^{i\omega t}}$$

$$\omega^2 = \frac{k}{m}$$

$$\rightarrow \quad \omega_0 = \pm\sqrt{\frac{k}{m}} \qquad \rightarrow \quad \boxed{y(t) = y_0 e^{i\sqrt{\frac{k}{m}}t}}$$

Eigenfrequenz

Case Study Vermeer

Inhomogene Differentialgleichung 1. Ordnung:

$$\underbrace{\overbrace{N'(t) = \lambda N(t)}^{\text{homogene DGL}} + R(t)}_{\text{inhomogene DGL}}$$

Störterm

N: Menge Pb

R: Menge Ra

Wegen Halbwertszeiten ^{210}Pb, ^{226}Ra $\rightarrow$ $N'(t) = \lambda N(t) + R$

Satz: Die Lösung der inhomogenen Differentialgleichung

$$y'(x) = \lambda \cdot y(x) + s(x)$$

lässt sich additive zusammensetzen mit der allgemeinen Lösung der zugehörigen homogenen Gleichung

$$y_h'(x) = \lambda \cdot y_h(x)$$

und einer partikulären Lösung der inhomogenen Gleichung

$$y_p'(x) = \lambda \cdot y_p(x) + s(x).$$

Dies gilt auch $\lambda = \lambda(x)$.

$$\boxed{y = y_h + y_p}$$

Beweis: Betrachte die Lösung $y = y_h + y_p$. In der Tat:

$$y' = y_h' + y_p' = \lambda(x) \cdot y_h(x) + \lambda(x) y_p(x) + s(x)$$

$$= \lambda(x) \left(y_h(x) + y_p(x) \right) + s(x) = \lambda(x) \cdot y(x) + s(x)$$

$\square$

Beispiel: $y'(x) = \overset{\lambda(x)}{x} \cdot y(x) + \overset{s(x)}{x}$

Löse zuerst die zugehörige homogene Gleichung:

$\rightarrow \quad y' = x \cdot y \qquad$ (Separation der Variablen)

$$\frac{dy}{dx} = x \cdot y$$

$$\frac{1}{y} \cdot dy = x \cdot dx$$

$$\int \frac{1}{y} \cdot dy = \int x \cdot dx$$

$$\ln(y) = \frac{1}{2}x^2 + C \qquad / e^{(\,)}$$

$$y_h(x) = e^{\frac{1}{2}x^2 + C} = \underline{\underline{y_0 e^{\frac{1}{2}x^2}}}$$

Nun suchen wir eine partikuläre Lösung der inhomogenen Gleichung:

$\rightarrow$ Methode Variation der Konstanten

Annahme: / Ansatz: $\quad y(x) = y_0(x) \cdot e^{\frac{1}{2}x^2}$

$$\rightarrow \quad y'(x) = y_0'(x) \cdot e^{\frac{1}{2}x^2} + y_0(x) \cdot e^{\frac{1}{2}x^2} \cdot (x)$$

einsetzen: $\quad y_0'(x) e^{\frac{1}{2}x^2} + \cancel{y_0(x)\, x e^{\frac{1}{2}x^2}} = \cancel{x \cdot y_0(x) e^{\frac{1}{2}x^2}} + x$

$$y_0'(x)\, e^{\frac{1}{2}x^2} = x$$

$: e^{\frac{1}{2}x^2} \hat{=} \cdot e^{-\frac{1}{2}x^2}$

$$y_0'(x) = x e^{-\frac{1}{2}x^2}$$

$$\int y_0'(x)\, dx = \int x\, e^{-\frac{1}{2}x^2} dx$$

$$y_0(x) = -e^{-\frac{1}{2}x^2}$$

Test $\rightarrow$ $-e^{-\frac{1}{2}x^2} \cdot (-x)$

eingesetzt in Ansatz: $y_p(x) = -e^{-\frac{1}{2}x^2} \cdot e^{\frac{1}{2}x^2} = \underline{\underline{-1}}$

D.h. die allgemeine Lösung der inhomogenen Gleichung ist:

$$y(x) = \underline{\underline{y_0 e^{\frac{1}{2}x^2} - 1}}$$

→ Check: $y'(x) = y_0 x e^{\frac{1}{2}x^2} \rightarrow y_0 x e^{\frac{1}{2}x^2} = (y_0 e^{\frac{1}{2}x^2} - 1) \cdot x + x$ ✓

Üb: $\dot{N} = \lambda N + R$

→ homogen: $\dot{N} = \lambda N$

$$\frac{dN}{dt} = \lambda N$$

$$\frac{1}{N} dN = \lambda\, dt$$

$$\int \frac{1}{N} dN = \int \lambda\, dt$$

$$\ln(N) = \lambda t + c$$

$$N(t) = \underline{N_0 e^{\lambda t}}$$

→ Variation: $N(t) = N_0(t)\, e^{\lambda t}$

$$\rightarrow N'(t) = \dot{N}_0(t)\, e^{\lambda t} + N_0(t)\, \lambda e^{\lambda t}$$

$$\rightarrow \dot{N}_0 e^{\lambda t} + \cancel{N_0 \lambda e^{\lambda t}} = \cancel{\lambda N_0 e^{\lambda t}} + R$$

$$\dot N_0 e^{\lambda t} = R$$
$$\dot N_0 = R e^{-\lambda t}$$
$$N_0 = \int R e^{-\lambda t}\, dt$$
$$N_0 = -\frac{R}{\lambda} e^{-\lambda t}$$

$$\rightarrow \quad N_p(t) = -\frac{R}{\lambda} e^{-\lambda t} \cdot e^{\lambda t} = \underline{\underline{-\frac{R}{\lambda}}}$$

$$\Rightarrow \quad \underline{\underline{N(t) = N_0 e^{\lambda t} - \frac{R}{\lambda}}}$$

(20) $y' - y \tan(x) = -\sin(x)$

$$y' = \tan(x) \cdot y - \sin(x)$$

→ homogen : $\frac{dy}{dx} = \tan(x) \cdot y$

$$\frac{1}{y}\, dy = \tan(x)\, dx \quad // \int$$

$$\ln(y) = \int \tan(x)\, dx \quad ?$$

$\tan(x) = \frac{\sin(x)}{\cos(x)}$ Versuch : $(\ln(\cos(x)))' = \frac{1}{\cos(x)} \cdot (-\sin(x))$

$$\rightarrow \quad \ln(y) = -\ln(\cos(x)) + C$$

$$y(x) = e^{-\ln(\cos(x)) + C} = e^{\ln(\cos(x)^{-1}) + C} = y_0 \cdot \frac{1}{\cos(x)} =: y_h$$

→ Variation : $y(x) = y_0(x) \cdot \frac{1}{\cos(x)}$

$$\Rightarrow \quad y'(x) = y_0'(x) \cdot \frac{1}{\cos(x)} + y_0(x) \cdot (-1) \cos(x)^{-2} \cdot (-\sin(x))$$

$$\Rightarrow \quad y_0'(x) \cdot \frac{1}{\cos(x)} + \cancel{y_0(x) \cdot \frac{\sin(x)}{\cos(x)^2}} = \cancel{\tan(x) \cdot y_0(x) \cdot \frac{1}{\cos(x)}} - \sin(x)$$

$$y_0'(x) \cdot \frac{1}{\cos(x)} = -\sin(x)$$

$$y_0'(x) = -\sin(x)\cos(x)$$

$$\int y_0'(x)\, dx = -\int \sin(x)\cos(x)\, dx \quad ?$$

Versuch : $(\sin^2(x))' = 2\sin(x) \cdot \cos(x)$ $\quad (\frac{1}{2}\cos^2(x))' = -\sin(x)\cos(x)$

$$\Rightarrow \quad y_0(x) = -\frac{1}{2}\sin^2(x)$$

$$\Rightarrow \quad y_p = -\frac{1}{2}\sin^2(x)\cdot\frac{1}{\cos(x)} = -\frac{1}{2}\tan(x)\cdot\sin(x)$$

$\frac{1}{2}\cos^2(x)\cdot\frac{1}{\cos(x)}$

$= \frac{1}{2}\cos(x)$

$$\Rightarrow \quad y(x) = \underline{\underline{\frac{y_0}{\cos(x)} - \frac{1}{2}\tan(x)\sin(x)}} \quad + \frac{1}{2}\cos(x)$$

beide Varianten möglich

Check: $y'(x) = -\frac{y_0\sin(x)}{\cos^2(x)} - \frac{1}{2}\sin(x) = -y_0\frac{\tan(x)}{\cos(x)} - \frac{1}{2}\sin(x)$

$\tan(x)\cdot y - \sin(x) = \tan(x)\cdot\left(\frac{y_0}{\cos(x)} + \frac{1}{2}\cos(x)\right) - \sin(x) = y_0\frac{\tan(x)}{\cos(x)} + \frac{1}{2}\sin(x)$

$- \sin(x)$ ✓

(21) $y' + \frac{y}{x} = x^2 + 4$

$y' = -\frac{1}{x}y + x^2 + 4$

$\rightarrow$ homogen: $\frac{dy}{dx} = -\frac{1}{x}y$

$$\frac{1}{y}dy = -\frac{1}{x}dx$$

$$\ln(y) = -\ln(x) + C = \ln\left(\frac{1}{x}\right) + C$$

$$y(x) = \underline{\underline{y_0\cdot\frac{1}{x}}} =: y_h$$

entweder $-\ln(x) = \ln(x^{-1}) = \ln\left(\frac{1}{x}\right)$

oder $e^{-\ln(x)} = \frac{1}{e^{\ln(x)}} = \frac{1}{x}$

$\rightarrow$ inhomogen: $y(x) = y_0(x)\cdot\frac{1}{x} \quad \Rightarrow \quad y'(x) = y_0'(x)\cdot\frac{1}{x} + y_0(x)\cdot\left(-\frac{1}{x^2}\right)$

$$\rightarrow \quad y_0'\cdot\frac{1}{x} \cancel{- \frac{1}{x^2}y_0} = \cancel{-\frac{1}{x}\cdot y_0\cdot\frac{1}{x}} + x^2 + 4$$

$$y_0' = x^3 + 4x$$

$$y_0 = \int(x^3 + 4x)\cdot dx = \frac{1}{4}x^4 + 2x^2$$

$$\rightarrow \quad y_p(x) = \left(\frac{1}{4}x^4 + 2x^2\right)\cdot\frac{1}{x} = \frac{1}{4}x^3 + 2x$$

$$\Rightarrow \quad y(x) = \underline{\underline{y_0\cdot\frac{1}{x} + \frac{1}{4}x^3 + 2x}}$$

Check: $y'(x) = -y_0 \cdot \frac{1}{x^2} + \frac{3}{4}x^2 + 2$

$$-\frac{1}{x}y(x) + x^2 + 4 = -y_0 \cdot \frac{1}{x^2} - \frac{1}{4}x^2 - 2 + x^2 + 4 = -y_0 \frac{1}{x^2} + \frac{3}{4}x^2 + 2 \checkmark$$

Logistische Funktion (Wale)

$\rightarrow \quad \dot{p} = b \cdot p - d \cdot p = (b-d) \cdot p = \beta \cdot p$

p : Populationsgrösse

d : death rate

b : birth rate

$\rightarrow \quad \dot{p} = \beta \cdot p$ (exponentiell)

$\rightarrow \quad \dot{p} = \beta \cdot p - S \cdot p^2$ (logistische Gleichung)

S : Stressfaktor

$\rightarrow \quad \dot{p} = p(\beta - Sp)$

$$\frac{dp}{dt} = p(\beta - Sp)$$

$$\frac{1}{p(\beta - Sp)} \cdot dp = 1 \cdot dt \quad / \int$$

$$\int_{p_0}^{p} \frac{1}{p(\beta - Sp)} \cdot dp = \int_{t_0}^{t} 1 \cdot dt$$

$$= t \Big|_{t_0}^{t} = t - t_0$$

$\rightarrow \quad \left(\ln(\beta p - Sp^2)\right)' = \frac{1}{\beta p - Sp^2} \cdot (-Sp + \beta)$

$\rightarrow \quad \int \tan(x)\,dx\,? \quad \rightarrow \quad \int \frac{\sin(x)}{\cos(x)}\,dx \quad \rightarrow \quad \left(\ln(\cos(x))\right)' = \frac{1}{\cos(x)} \cdot (-\sin(x))$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln(f(x)) + C$$

$$\frac{1}{x^2 - x - 6} = \frac{1}{(x-3)(x+2)} = \frac{a_1}{(x-3)} + \frac{a_2}{(x+2)}$$

Partialbruchzerlegung

$$\Rightarrow \quad 1 = a_1(x+2) + a_2(x-3)$$

$$1 = (a_1 + a_2)x + (2a_1 - 3a_2)$$

$$\Rightarrow \quad a_1 + a_2 = 0 \qquad \Leftrightarrow \qquad a_2 = -a_1$$

$$2a_1 - 3a_2 = 1 \qquad\qquad 2a_1 + 3a_1 = 1$$

$$a_1 = \frac{1}{5}$$

$$\rightarrow \quad a_2 = -\frac{1}{5}$$

$$\rightarrow \quad \frac{1}{(x-3)(x+2)} = \frac{1/5}{x-3} - \frac{1/5}{x+2}$$

$$\int_{p_0}^{p} \frac{1}{p(\beta - Sp)} \cdot dp \quad \text{mit Partialbruchzerlegung}$$

$$\frac{1}{p \cdot (\beta - Sp)} = \frac{a_1}{p} + \frac{a_2}{\beta - Sp}$$

$$1 = a_1(\beta - Sp) + a_2 p$$

$$1 = (a_2 - Sa_1) \cdot p + a_1 \beta$$

$$\Rightarrow \quad a_2 - Sa_1 = 0 \qquad a_1 \beta = 1 \quad \Leftrightarrow \quad a_1 = \frac{1}{\beta}$$

$$a_2 = S \cdot \frac{1}{\beta}$$

$$\Rightarrow \quad \frac{1}{p \cdot (\beta - Sp)} = \frac{1}{\beta p} + \frac{S}{\beta^2 - S\beta p}$$

$$\Rightarrow \quad \int_{p_0}^{p} \frac{1}{p(\beta - Sp)} \cdot dp = \int_{p_0}^{p} \left(\frac{1}{\beta p} + \frac{S}{\beta^2 - S\beta p} \right) dp =$$

$$= \frac{1}{\beta} \int_{p_0}^{p} \frac{1}{p}\, dp + \frac{S}{\beta} \int_{p_0}^{p} \frac{1}{\beta - Sp}\, dp = \frac{1}{\beta} \cdot \ln(p) \Big|_{p_0}^{p} + \frac{\cancel{S}}{\beta} \cdot \left(-\frac{1}{\cancel{S}}\right) \ln(\beta - Sp) \Big|_{p_0}^{p}$$

$$= \frac{1}{\beta}\left(\ln(p) - \ln(p_0)\right) - \frac{1}{\beta}\left(\ln(\beta - Sp) - \ln(\beta - Sp_0)\right)$$

$$= \frac{1}{\beta}\left(\ln\left(\frac{p}{p_0}\right) + \ln\left(\frac{\beta - Sp_0}{\beta - Sp}\right)\right) = \frac{1}{\beta} \ln\left(\frac{p(\beta - Sp_0)}{p_0(\beta - Sp)} \right)$$

$$\Rightarrow \quad \ln\left(\frac{p(\beta - Sp_0)}{p_0(\beta - Sp)} \right) = \beta(t - t_0) \qquad p(t)\ ?$$

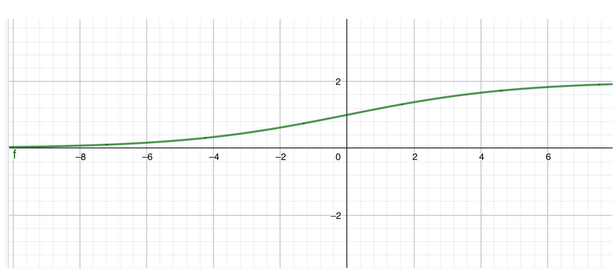
$$\Rightarrow \quad \frac{p(\beta - Sp_0)}{p_0(\beta - Sp)} = e^{\beta(t - t_0)}$$

$$p(\beta - Sp_0) = e^{\beta(t - t_0)} \cdot p_0(\beta - Sp)$$

$$p\beta - Spp_0 + Spp_0 e^{\beta(t - t_0)} = p_0 \beta e^{\beta(t - t_0)}$$

$$p\left(\beta - Sp_0 + Sp_0 e^{\beta(t - t_0)}\right) = p_0 \beta e^{\beta(t - t_0)}$$

$$\Rightarrow \quad p(t) = \frac{p_0 \beta e^{\beta(t - t_0)}}{\beta - Sp_0\left(1 - e^{\beta(t - t_0)}\right)}$$



Fourier-Reihen - Vorbereitung

a) $\int_{-\pi}^{\pi} \sin(t)\,dt$, $\int_{-\pi}^{\pi} \cos(t)\,dt$

b) $\int_{-\pi}^{\pi} \sin^2(t)\,dt$, $\int_{-\pi}^{\pi} \cos^2(t)\,dt$

c) $\int_{-\pi}^{\pi} \sin(t)\cos(t)\,dt$

Hint zu b) $\to$ partielle Integration:

$$(f\cdot g)' = f'\cdot g + f\cdot g' \qquad / \int$$

$$f\cdot g = \int f'\cdot g + \int f\cdot g'$$

$$\boxed{\int f\cdot g' = f\cdot g - \int f'\cdot g}$$

$$\int x\cdot\sin(x) = x(-\cos(x)) - \int(-\cos(x))$$

a) $\int_{-\pi}^{\pi} \sin(t)\,dt = -\cos(t)\Big|_{-\pi}^{\pi} = -\cos(\pi) + \cos(-\pi) = 1 - 1 = \underline{\underline{0}}$

$\int_{-\pi}^{\pi} \cos(t)\,dt = \sin(t)\Big|_{-\pi}^{\pi} = \sin(\pi) - \sin(-\pi) = 0 - 0 = \underline{\underline{0}}$

b) $\int_{-\pi}^{\pi} \underbrace{\sin(t)}_{f}\cdot\underbrace{\sin(t)}_{g'}\,dt = \underbrace{\sin(t)}_{f}\underbrace{(-\cos(t))}_{g}\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \underbrace{\cos(t)}_{f'}\underbrace{(-\cos(t))}_{g}\,dt$

$$= -\sin(t)\cos(t)\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi}(1-\sin^2(t))\,dt$$

$\Rightarrow 2\int_{-\pi}^{\pi}\sin^2(t)\,dt = -\sin(t)\cos(t)\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi} 1\,dt$ $\quad + \,|_{-\pi}^{\pi}$

$$= -\underbrace{\sin(\pi)}_{0}\cos(\pi) + \underbrace{\sin(-\pi)}_{0}\cos(-\pi) + (\pi + \pi) = 2\pi$$

$\Rightarrow \int_{-\pi}^{\pi}\sin^2(t)\,dt = \underline{\underline{\pi}}$