NOTIZEN ZUR REPETITION FÜR DIE GF MATH-MATUR

GERADEN

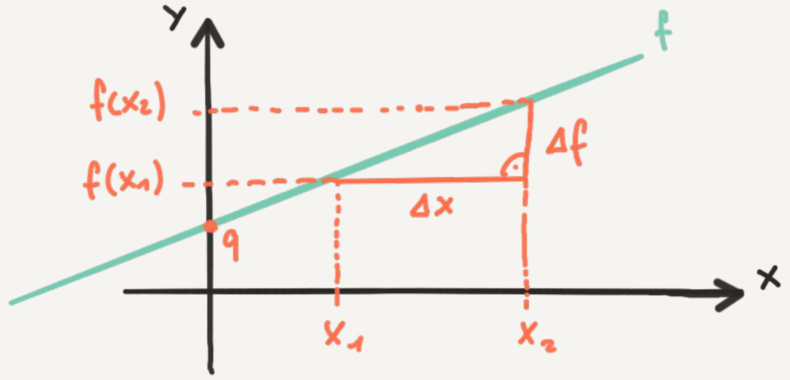
$$f(x) = mx + q$$

Steigung : $m = \frac{\Delta f}{\Delta x} = \tan(\alpha)$

y - Achsenabschnitt : $q = f(0)$

Differenzenquotient : $\frac{\Delta f}{\Delta x}$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



Gerade durch 2 Punkte, finde f ?

z.B. $(-1|1)$, $(3|-2)$

$\rightarrow \quad m = \frac{-2-1}{3-(-1)} = \frac{-3}{4} = -\frac{3}{4}$

$\rightarrow \quad f(x) = -\frac{3}{4}x + q$

$1 = -\frac{3}{4} \cdot (-1) + q$

$\frac{1}{4} = q \qquad \rightarrow \quad f(x) = -\frac{3}{4}x + \frac{1}{4}$

PARABEL

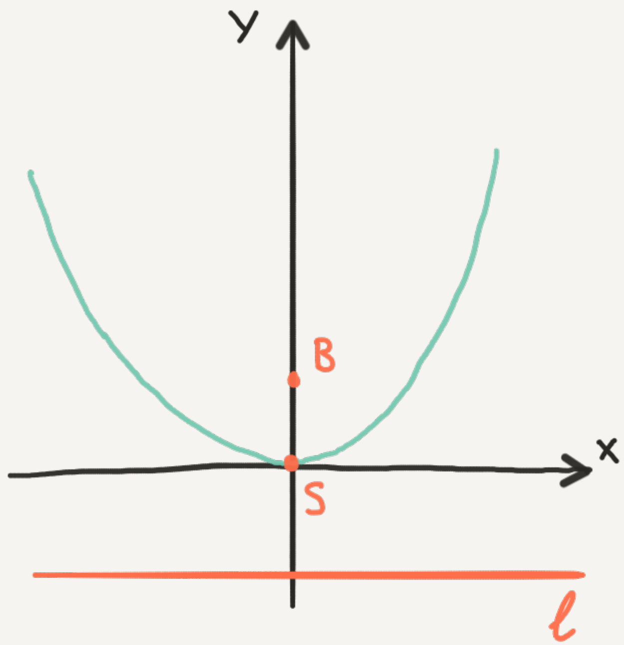
$$f(x) = ax^2 + bx + c \qquad \text{(Normalform)}$$
$$= a(x-u)^2 + v \qquad \text{(Scheitelform)}$$

Krümmungsmass : a

Scheitelpunkt : $(u|v)$; $x_s = -\frac{b}{2a}$

Nullstellenformel : $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Abstand Brennpunkt – Scheitel : $p = \frac{1}{4a}$



geg : $f(x) = x^2 - x - 6$, finde Scheitelpunkt & Nullstellen

$\rightarrow \quad x_s = -\frac{b}{2a} = -\frac{-1}{2 \cdot 1} = \frac{1}{2}$

$y_s = f(x_s) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 6 = -6\frac{1}{4} \quad \Rightarrow \quad S\left(\frac{1}{2} \middle| -6\frac{1}{4}\right)$

Scheitelform wäre $f(x) = \left(x - \frac{1}{2}\right)^2 - 6\frac{1}{4}$

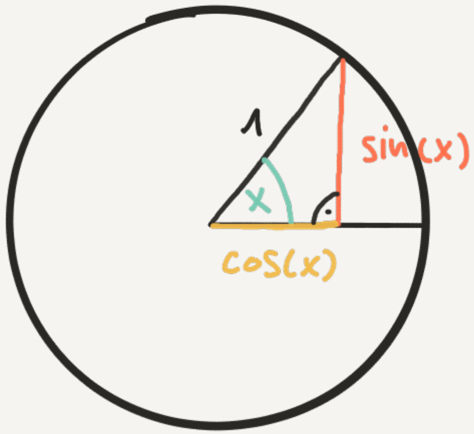
$0 = x^2 - x - 6$

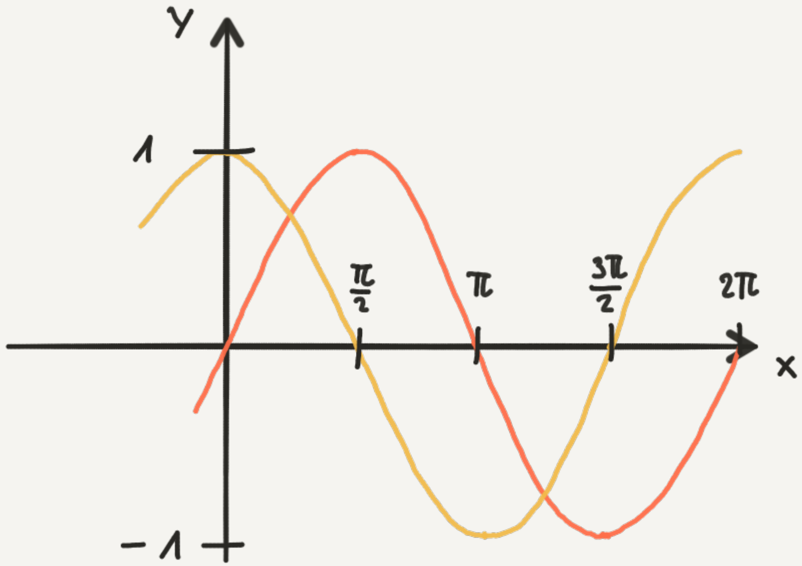
$$x_{1|2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1} = \frac{1 \pm \sqrt{25}}{2}$$

$$= \frac{1 \pm 5}{2} \quad \Rightarrow \quad x_1 = 3 \ , \ x_2 = -2$$

SCHWINGUNG

$f(x) = \sin(x)$





$A \sin(f(t-\varphi)) + m$

Gugel Hopf, AHa, Geht Auch!

Bogenmass: $360° \sim 2\pi$

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\sin^2(x) + \cos^2(x) = 1 \quad \forall x \in \mathbb{R}$$

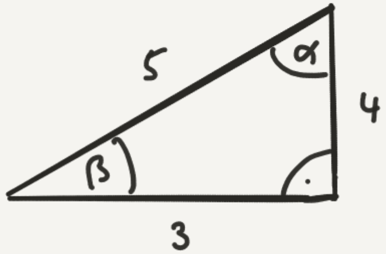
<u>Allgemeines Dreieck</u>:

Sinussatz: $\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)}$

Cosinussatz: $c^2 = a^2 + b^2 - 2ab \cdot \cos(\gamma)$

Rechtwinkliges Dreieck mit $a = 3$, $b = 4$, $c = 5$.

Finde Winkel



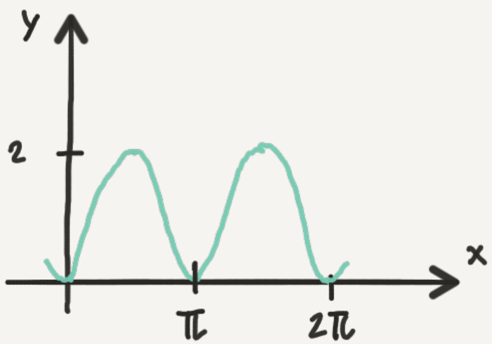
$$\sin(\alpha) = \frac{3}{5}$$

$$\alpha = \arcsin\left(\frac{3}{5}\right)$$

$$\sin(\beta) = \frac{4}{5}$$

$$\beta = \arcsin\left(\frac{4}{5}\right)$$

Finde f



1. Mittellage ist 1
2. Amplitude ist 1
3. Frequenz ist 2
4. Phase mit $-\cos$ $\longrightarrow$ $f(x) = -\cos(2x) + 1$

POTENZEN

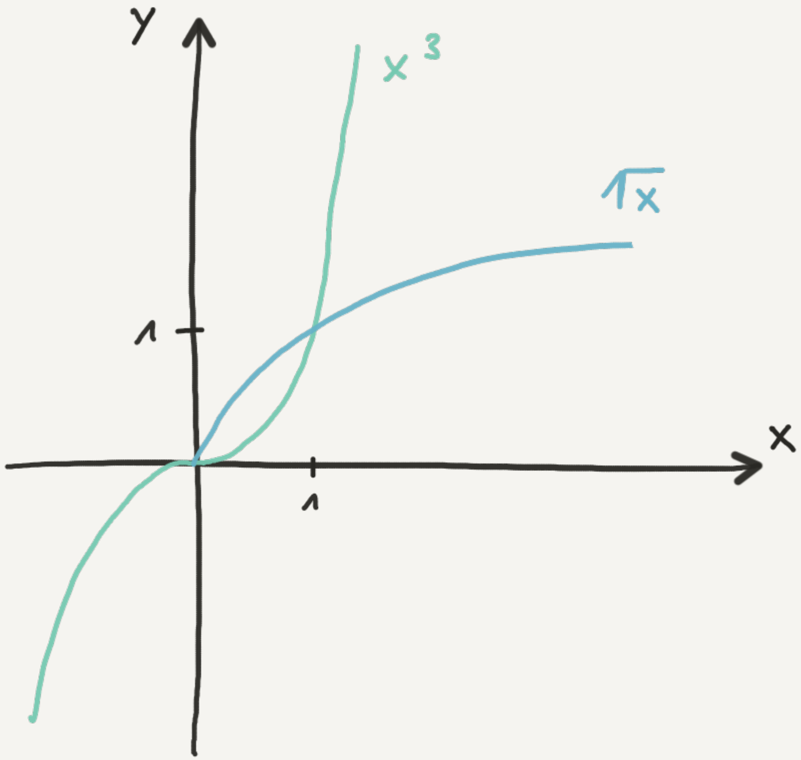
Regeln: $a^m \cdot a^n = a^{m+n}$

$(a^n)^m = a^{n \cdot m}$

$a^0 = 1$

$a^{-n} = \frac{1}{a^n}$

$\sqrt[n]{a} = a^{\frac{1}{n}}$



Schreibe um

$$\sqrt[3]{2\cdot x^5} = (2x^5)^{\frac{1}{3}} = 2^{\frac{1}{3}}\cdot x^{5/3}$$

$$\frac{ax^2+vx-L}{3x} = \frac{1}{3}(ax^2+vx-L)\cdot x^{-1}$$

$$\frac{5}{x} - \sqrt{\frac{x^2}{3-x}} = 5x^{-1} - \left(x^2(3-x)^{-1}\right)^{\frac{1}{2}} = 5x^{-1} - x(3-x)^{-\frac{1}{2}}$$

Löse nach x

$$3x^{-1} - x^{\frac{1}{2}} = 0$$

$$\frac{3}{x} - \sqrt{x} = 0$$

$$\frac{3}{x} = \sqrt{x}$$

$$\frac{9}{x^2} = x$$

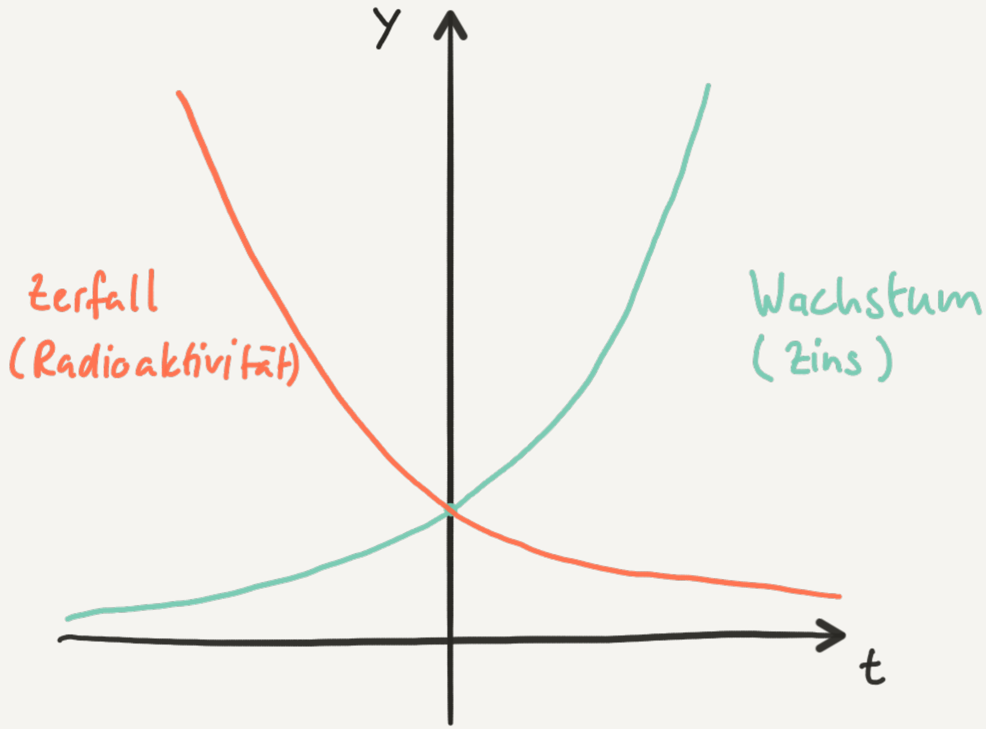
$$9 = x^3 \longrightarrow x = \sqrt[3]{9}$$

EXPONENTIELL

$f(t) = a \cdot b^t$

Startwert : a

Wachstums-/Zerfallsfaktor : b



Euler'sche Zahl : e

$e \approx 2.71$

$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

$N(t) = N_0 \cdot e^{-\lambda \cdot t}$

$(N(0) = N_0 \cdot e^{-\lambda \cdot 0} = N_0 \cdot e^0 = N_0)$

geg.: Zerfall mit Halbwertszeit 3 Tage und Menge zu Beginn 2 g. Finde $N(t)$

$$N(t) = 2 \cdot \left(\tfrac{1}{2}\right)^{\frac{t}{3}} = 2 \cdot \sqrt[3]{\tfrac{1}{2}}^{\,t}$$

Schreibe als e-Funktion $N_0 \cdot e^{ct}$

$$\rightarrow \quad 2 \cdot e^{ct} = 2 \cdot \sqrt[3]{\tfrac{1}{2}}^{\,t}$$

$$\Rightarrow \quad e^{c} = \sqrt[3]{\tfrac{1}{2}}$$

$$c = \ln\left(\sqrt[3]{\tfrac{1}{2}}\right) = \tfrac{1}{3} \ln\left(\tfrac{1}{2}\right) = -\tfrac{1}{3} \ln(2)$$

Finde Zinseszinsformel: Startkapital K_0, Zinssatz p, Laufzeit t

$t=0$ K_0, $t=1$ $K_1 = K_0(1+p)$, $t=2$ $K_2 = K_1(1+p) = K_0(1+p)^2$

$$K(t) = K_0 \cdot (1+p)^t$$

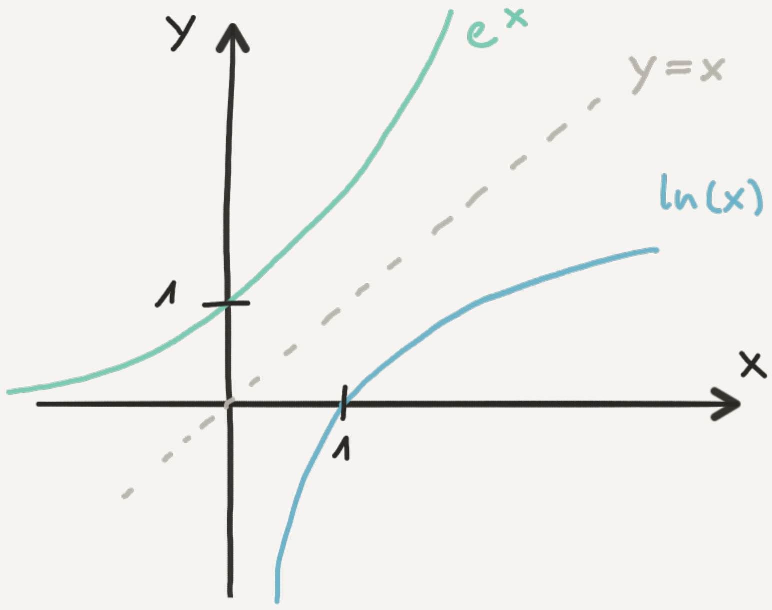
LOGARITHMEN

$$f(x) = \ln(x)$$

$$y = \ln(x) \quad :\Longleftrightarrow \quad e^y = x$$

$$y = \log_b(x) \quad :\Longleftrightarrow \quad b^y = x$$

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)} = \frac{\ln(x)}{\ln(b)}$$



Inversfunktion der Exponentialfunktion:

$$\ln(e^x) = x$$

Bestimme für einen Zerfall mit Halbwertszeit $T_{1/2}$ den Faktor λ in

$N(t) = N_0 \cdot e^{-\lambda t}$

$$\rightarrow \quad N(T_{1/2}) = \frac{N_0}{2} = N_0 \cdot e^{-\lambda \cdot T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda \cdot T_{1/2}}$$

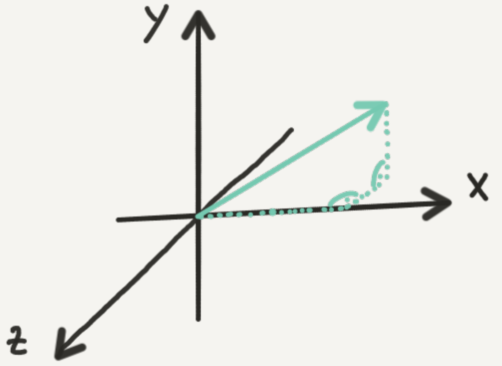
$$\ln\left(\frac{1}{2}\right) = -\lambda \cdot T_{1/2}$$

$$\frac{\ln\left(\frac{1}{2}\right)}{T_{1/2}} = -\lambda$$

$$\Rightarrow \quad \lambda = \frac{\ln(2)}{T_{1/2}}$$

VEKTOREN

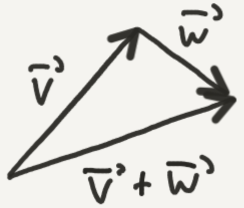
3D



$$\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

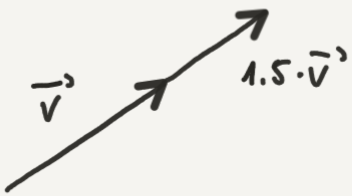
Länge : $|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Addition :



$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} + \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} = \begin{pmatrix} v_x + w_x \\ v_y + w_y \\ v_z + w_z \end{pmatrix}$$

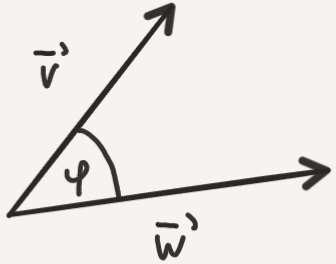
S-Multiplikation :



$$1.5 \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} 1.5\, v_x \\ 1.5\, v_y \\ 1.5\, v_z \end{pmatrix}$$

$\vec{v}, \vec{w}$ kollinear : $\vec{v} = \lambda \cdot \vec{w}$

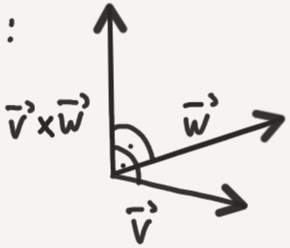
Skalarprodukt:



$$\vec{v}\cdot\vec{w} = v\cdot w\cdot\cos(\varphi) = v_x w_x + v_y w_y + v_z w_z$$

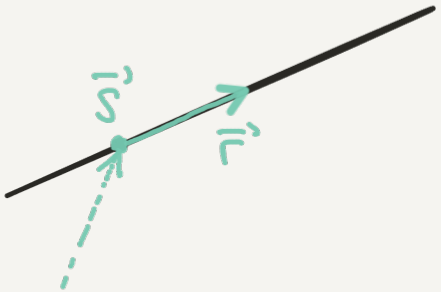
$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$ (Betrag/Länge)

Vektorprodukt:

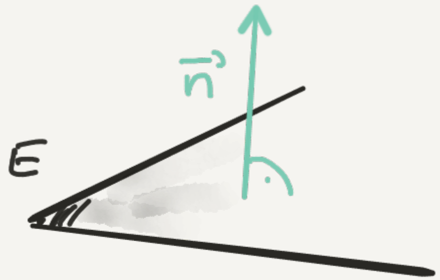


$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \times \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} = \begin{pmatrix} v_y w_z - v_z w_y \\ v_z w_x - v_x w_z \\ v_x w_y - v_y w_x \end{pmatrix}$$ (rechte Hand)

Gerade: $g: \vec{s} + t\cdot\vec{r}$



Ebene: $E: Ax + By + Cz + D = 0$, $\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \vec{n}$



Schneide Gerade $g: \begin{pmatrix}1\\2\\3\end{pmatrix} + t\cdot\begin{pmatrix}-2\\1\\0\end{pmatrix}$ mit Ebene E geg durch die Punkte $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$.

Koordinatenform E: $\begin{pmatrix}-1\\1\\0\end{pmatrix} \times \begin{pmatrix}-1\\0\\1\end{pmatrix} = \begin{pmatrix}1-0\\0-(-1)\\0-(-1)\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow 1\cdot x + 1\cdot y + 1\cdot z + D = 0$

$\longrightarrow 1\cdot 1 + 1\cdot 0 + 1\cdot 0 + D = 0 \longrightarrow D = -1 \Rightarrow E: x + y + z - 1 = 0$

Jetzt $g \cap E$: $1\cdot(1-2t) + 1\cdot(2+t) + 1\cdot(3+0\cdot t) - 1 = 0$

$$5 - t = 0$$
$$t = 5$$

$$g(5) = \begin{pmatrix}1\\2\\3\end{pmatrix} + 5\cdot\begin{pmatrix}-2\\1\\0\end{pmatrix} = \begin{pmatrix}-9\\7\\3\end{pmatrix} = S$$

Schnittwinkel: $\begin{pmatrix}-2\\1\\0\end{pmatrix}\cdot\begin{pmatrix}1\\1\\1\end{pmatrix} = \left|\begin{pmatrix}-2\\1\\0\end{pmatrix}\right| \cdot \left|\begin{pmatrix}1\\1\\1\end{pmatrix}\right| \cdot \cos(\varphi) \longrightarrow -1 = \sqrt{5}\cdot\sqrt{3}\cdot\cos(\varphi)$

$$\varphi = \cos^{-1}\left(-\frac{1}{\sqrt{15}}\right) \longrightarrow \alpha = 90^\circ - \varphi$$

FOLGEN

$a_1 \quad a_2 \quad a_3 \quad \ldots$

$a_k = 2k$ (explizit)

$a_k = a_{k-1} + a_{k-2}$, $a_1 = a_2 = 1$ (rekursiv) (Fibonacci)

AF : $a_{k+1} = a_k + d$

$$a_k = a_1 + (k-1) \cdot d$$

$$S_k = k \cdot \frac{a_1 + a_k}{2}$$

GF: $a_{k+1} = a_k \cdot q$

$$a_k = a_1 \cdot q^{k-1}$$

$$S_k = a_1 \cdot \frac{1 - q^k}{1 - q}$$

$$-1 < q < 1 \quad : \quad \lim_{k \to \infty} S_k = \frac{a_1}{1 - q}$$

$-3 \quad -1 \quad -\frac{1}{3} \quad -\frac{1}{9} - \ldots = \ ?$

$\rightarrow$ GF: $q = \frac{-1}{-3} = \frac{1}{3} \longrightarrow s = \frac{-3}{1-\frac{1}{3}} = -\frac{3}{\frac{2}{3}} = -\frac{9}{2} = -4.5$

Bis zu welchem Summand muss ich aufsummieren, um -4.4 zu erhalten?

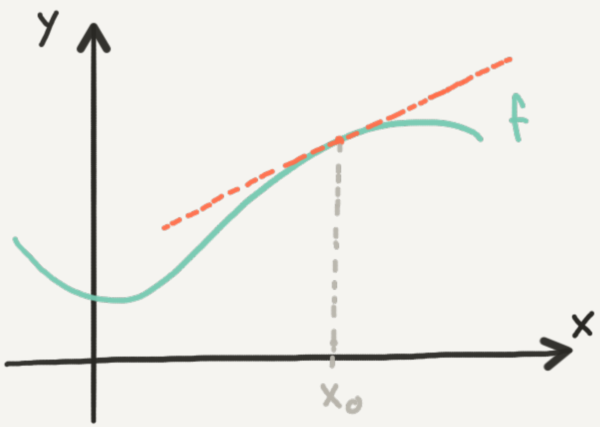
$\rightarrow s_n = a_1 \cdot \frac{1-q^n}{1-q} = -3 \cdot \frac{1-\left(\frac{1}{3}\right)^n}{\frac{2}{3}} = -\frac{9}{2}\left(1-\left(\frac{1}{3}\right)^n\right)$

$$\Rightarrow -4.4 = -4.5\left(1-\left(\frac{1}{3}\right)^n\right)$$

$$\frac{4.4}{4.5} = 1-\left(\frac{1}{3}\right)^n$$

$$\left(\frac{1}{3}\right)^n = 1-\frac{4.4}{4.5} \longrightarrow n = \log_{\frac{1}{3}}\left(1-\frac{4.4}{4.5}\right) = \frac{\ln\left(1-\frac{4.4}{4.5}\right)}{\ln\left(\frac{1}{3}\right)}$$

ABLEITUNG



Steigung: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =: f'(x)$

$(f(x)+c)' = f'(x)$

$(c \cdot f(x))' = c \cdot f'(x)$

$(f(x) + g(x))' = f'(x) + g'(x)$

$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$(f(g(x)))' = f'(g(x)) \cdot g'(x)$

$(x^n)' = n \cdot x^{n-1}$

$(\sin(x))' = \cos(x)$

$(\cos(x))' = -\sin(x)$

$(e^x)' = e^x$!

$(\ln(x))' = \frac{1}{x}$

Nullstellen, Extrema, Wendestellen von $f(x) = e^{-0.5(x-2)^2}$

NS : keine, $0 = e^{-0.5(x-2)^2}$ / ln nicht def. , $e^x > 0 \quad \forall x \in \mathbb{R}$

Max : $f'(x) = e^{-0.5(x-2)^2} \cdot (-(x-2)) = (2-x)e^{-0.5(x-2)^2} \stackrel{!}{=} 0$

$\rightarrow x-2=0 \rightarrow x=2$

$f''(x) = (-1)e^{-0.5(x-2)^2} + (2-x)e^{-0.5(x-2)^2} \cdot (2-x) = e^{-0.5(x-2)^2} \cdot (-1+(2-x)^2)$

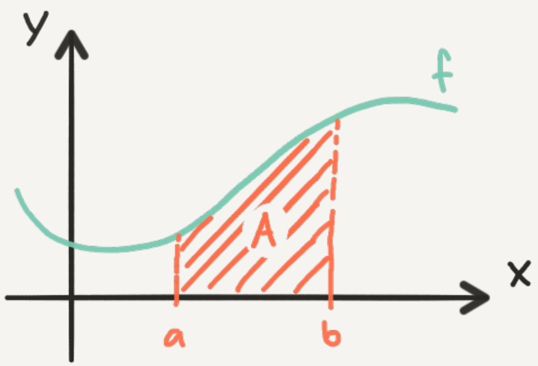
$f''(2) = e^0 \cdot (-1) = -1 \rightarrow$ Max bei $(2|1)$

WS : $\rightarrow -1+(2-x)^2 \stackrel{!}{=} 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0 \quad \longrightarrow \quad x_1 = 1 , \; x_2 = 3$

INTEGRAL



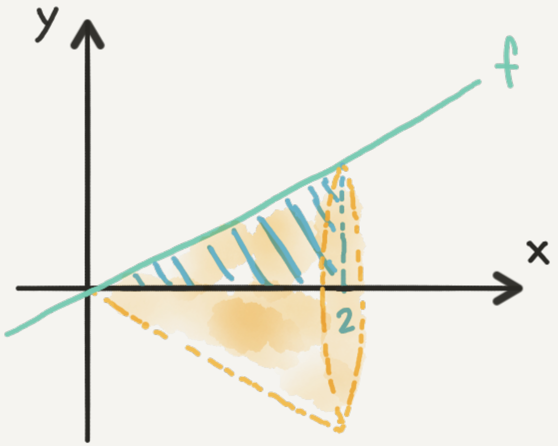
Stammfunktion : $\int f(x)\,dx = F(x) + c$

$$A = \int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

wobei $F'(x) = f(x)$

Rotationsvolumen : $V = \pi \int_a^b [f(x)]^2\,dx$
(um x-Achse)

Bestimme für $f(x) = \frac{1}{2}x$ die Fläche und das Rotationsvolumen über $[0, 2]$.



$$A = \int_0^2 \frac{1}{2}x\,dx = \frac{1}{4}x^2 \Big|_0^2 = \frac{1}{4}\cdot 2^2 - \frac{1}{4}\cdot 0^2 = 1$$

$$\text{RotV} = \pi \int_0^2 \left[\frac{1}{2}x\right]^2 dx = \pi \int_0^2 \frac{1}{4}x^2\,dx = \pi \left[\frac{1}{12}x^3 \Big|_0^2\right]$$

$$= \pi \left[\frac{1}{12}\cdot 2^3 - \frac{1}{12}\cdot 0^3\right] = \pi \cdot \frac{8}{12} = \frac{2\pi}{3}$$

(vgl. Dreiecksfläche und Kegelvolumen)

STOCHASTIK

Laplace: $\frac{g}{m}$

Gegenwahrscheinlichkeit: $P(\bar{A}) = 1 - P(A)$

"und" ~ $\cdot$, "oder" ~ $+$ (Baum)

Fakultät: $k! = k \cdot (k-1) \cdot \ldots \cdot 2 \cdot 1$

Binomialkoeffizient: $\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$

Binomialverteilung:

"genau": $\binom{n}{k} \cdot p^k \cdot q^{n-k}$ = binompdf(n, p, k)

"höchstens": $\sum_{i=0}^{k} \binom{n}{i} \cdot p^i \cdot q^{n-i}$ = binomcdf(n, p, k)

Erwartungswert: $\Sigma p_i \cdot X_i$

$E = n \cdot p$ (binomial)

Standardabweichung: $\sqrt{\Sigma (E_i - X_i)^2 \cdot p_i}$

$\sigma = \sqrt{npq}$ (binomial)

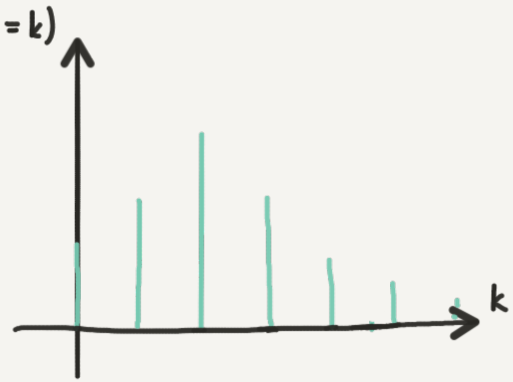
2 x nacheinander "6" würfeln : $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

5 Karten aus 52 , Chance auf Paar ?

$$1 \cdot \frac{3}{51} \cdot \frac{48}{50} \cdot \frac{44}{46} \cdot \frac{40}{45} \cdot \binom{5}{2} = 0.42$$

Einsatz für faires Spiel mit P(Erfolg) = 0.3 , Teilnahme 10.–

$$\begin{aligned} E(x) = -10 \cdot (1-0.3) + x \cdot 0.3 &= 0 \\ 0.3x &= 10(1-0.3) \\ 0.3x &= 7 \\ x &= \frac{7}{0.3} \end{aligned}$$



In 3 Würfen genau/höchstens/mindestens eine "6" :

$$\binom{3}{1} \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^2 = \text{binompdf}(3, \tfrac{1}{6}, 1)$$

$$\binom{3}{0} \cdot \left(\frac{1}{6}\right)^0 \cdot \left(\frac{5}{6}\right)^3 + \binom{3}{1} \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^2 = \text{binomcdf}(3, \tfrac{1}{6}, 1)$$

$$\binom{3}{1} \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^2 + \binom{3}{2} \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right) + \binom{3}{3} \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^0 = 1 - \text{binomcdf}(3, \tfrac{1}{6}, 0)$$