Rechne mit i

a) $i^2 = \underline{\underline{-1}}$ (Def) b) $i^4 = (i^2)^2 = (-1)^2 = \underline{\underline{1}}$ c) $i^5 = i^4 \cdot i = 1 \cdot i = \underline{\underline{i}}$ d) $(-i)^2 = (-1)^2 \cdot i^2 = \underline{\underline{-1}}$ e) $-i^2 = -(-1) = \underline{\underline{1}}$ f) $-i^4 = \underline{\underline{-1}}$

Re und Im

a) $\mathrm{Re}(-1+4i) = \underline{\underline{-1}}$, $\mathrm{Im}(-1+4i) = \underline{\underline{4}}$

b) $\mathrm{Re}(2-5i) = \underline{\underline{2}}$, $\mathrm{Im}(2-5i) = \underline{\underline{-5}}$

c) $\mathrm{Re}(\frac{3}{4}+7i) = \underline{\underline{\frac{3}{4}}}$, $\mathrm{Im}(\frac{3}{4}+7i) = \underline{\underline{7}}$

d) $\mathrm{Re}(\sqrt{17}+\frac{5}{6}i) = \underline{\underline{\sqrt{17}}}$, $\mathrm{Im}(\sqrt{17}+\frac{5}{6}i) = \underline{\underline{\frac{5}{6}}}$

e) $\mathrm{Re}(0+\sqrt{15}i) = \underline{\underline{0}}$, $\mathrm{Im}(0+\sqrt{15}i) = \underline{\underline{\sqrt{15}}}$

f) $\mathrm{Re}(-\frac{1}{6}+0 \cdot i) = \underline{\underline{-\frac{1}{6}}}$, $\mathrm{Im}(-\frac{1}{6}+0 \cdot i) = \underline{\underline{0}}$

g) $\mathrm{Re}(0+1 \cdot i) = \underline{\underline{0}}$, $\mathrm{Im}(0+1 \cdot i) = \underline{\underline{1}}$

h) $\mathrm{Re}(0+0 \cdot i) = \underline{\underline{0}}$, $\mathrm{Im}(0+0 \cdot i) = \underline{\underline{0}}$

Re plus Im

$z = a+ib$ mit $a,b \in \mathbb{R}$

$\Rightarrow \mathrm{Re}(z) = \mathrm{Re}(a+ib) = a$ und $\mathrm{Im}(z) = \mathrm{Im}(a+ib) = b$

$\rightarrow a+ib \overset{?}{=} a+b \quad (a,b \in \mathbb{R}) \quad \Rightarrow b = 0 \quad \Rightarrow a = a$, d.h. $z = \underline{\underline{a}} \in \mathbb{R}$ beliebig

Re von Im

$\mathrm{Re}(\mathrm{Im}(z)) = \mathrm{Re}(\mathrm{Im}(a+ib)) = \mathrm{Re}(b) = b = 0$, d.h. $z = \underline{\underline{a}} \in \mathbb{R}$ beliebig

Zahlenmengen

$\in \mathbb{C}$ immer richtig ... ich wähle jeweils minimale Menge

a) $2 \in \mathbb{N}$, b) $-\sqrt{3} \in \mathbb{R}$, c) $3+\frac{1}{2}i \in \mathbb{C}$, d) $0 \in \mathbb{Z}$, e) $4i \in \mathbb{C}$, f) $i^8 = (i^2)^4 = (-1)^4 = 1 \in \mathbb{N}$

Subtrahiere

$(3+i)-(1-2i) = 3+i-1+2i = \underline{\underline{2+3i}}$

Addieren und subtrahieren

a) $4+3i + 2+i = \underline{\underline{6+4i}}$

b) $\frac{1}{4}+2i + \frac{1}{5}-i = \underline{\underline{\frac{9}{20}+i}}$

c) $\sqrt{15}+3i - 2+i - 4i = \underline{\underline{\sqrt{15}-2}}$

d) $4+3i - 2-3i = \underline{\underline{2}}$

e) $\mathrm{Re}(-2+i+2+3i) = \mathrm{Re}(4i) = \underline{\underline{0}}$

f) $\mathrm{Im}(7-4-3i-5+4i) = \mathrm{Im}(-2+i) = \underline{\underline{1}}$

mission impossible

Seien $b_1 i$ und $b_2 i$ mit $b_1, b_2 \in \mathbb{R}$ rein imaginär. $\rightarrow b_1 i + b_2 i \overset{?}{=} 1 \Leftrightarrow (b_1+b_2) \cdot i = 1 \Leftrightarrow \overset{\in \mathbb{C}\setminus\mathbb{R}}{i} = \overbrace{\frac{1}{b_1+b_2}}^{\in \mathbb{R}}$ W!

multiplication rule

$v = 1+i$, $w = 4i$, $z = 2-5i$

a) $(1+i) \cdot (2-5i) = 2-5i+2i-5i^2 = 2-3i+5 = \underline{\underline{7-3i}}$

b) $(1+i) \cdot (4i-(2-5i)) = (1+i) \cdot (-2+9i) = -2+9i-2i-9 = \underline{\underline{-11+7i}}$

c) $\mathrm{Re}((1+i) \cdot 4i \cdot (2-5i)) = \mathrm{Re}((4i-4)(2-5i)) = \mathrm{Re}(8i+20-8+20i) = \mathrm{Re}(12+28i) = \underline{\underline{12}}$

d) $\mathrm{Im}(1+i + 4i(2-5i)) = \mathrm{Im}(1+i+8i+20) = \mathrm{Im}(21+9i) = \underline{\underline{9}}$

swap

$(a+ib) \cdot (-i) = -b - ia = -(b+ia)$ $\rightarrow$ nein, es ist eine $-90°$ Drehung

Produkt

Seien $z = a+ib$ und $w = c+id$, $a,b,c,d \in \mathbb{R}$

$\rightarrow \mathrm{Re}((a+ib) \cdot (c+id)) = \mathrm{Re}(ac + iad + ibc - bd) = ac - bd$ und

$\mathrm{Re}(a+ib) \cdot \mathrm{Re}(c+id) = a \cdot c$, d.h. i.A. gilt die Gleichung nicht. $\square$

konjugiert komplex

$(5+3i) \cdot (5-3i) = 25+9 = \underline{\underline{34}}$ (dritter Binom)

rechne

a) $\overline{\frac{1}{2}+4i} = \underline{\underline{\frac{1}{2}-4i}}$, b) $\overline{3i} = \underline{\underline{-3i}}$ c) $\overline{3i+\frac{1}{2}+4i} = \overline{\frac{1}{2}+7i} = \underline{\underline{\frac{1}{2}-7i}}$ d) $\overline{(3i)^2-\frac{1}{2}-4i} = \overline{-9\frac{1}{2}-4i} = \underline{\underline{-9\frac{1}{2}+4i}}$

z bar

a) $(a+ib)(a-ib) = a^2+b^2 = \mathrm{Re}(a+ib)^2 + \mathrm{Im}(a+ib)^2 = \mathrm{Re}(z)^2 + \mathrm{Im}(z)^2$ $\square$

b) $a+ib + a-ib = 2a = 2 \cdot \mathrm{Re}(a+ib) = 2 \cdot \mathrm{Re}(z)$

c) $\overline{\overline{a+ib}} = \overline{a-ib} = a+ib = z$

Achsen

$z + \bar{z} = a+ib + a-ib = 2a \overset{!}{=} 0 \Leftrightarrow a = 0$, d.h. $x \in i \cdot \mathbb{R}$ und $a+ib - a+ib = i \cdot 2b \overset{!}{=} 0 \Leftrightarrow b = 0$, d.h. $x \in \mathbb{R}$

Division

a) $\frac{3+2i}{7-i} = \frac{(3+2i)(7+i)}{(7-i)(7+i)} = \frac{21+3i+14i-2}{49+1} = \underline{\underline{\frac{19}{50} + \frac{17}{50}i}}$

b) $\frac{i}{-4-4i} = \frac{-i}{4+4i} = \frac{-i(4-4i)}{16+16} = \frac{-4-4i}{32} = \underline{-\frac{1}{8} - \frac{1}{8}i}$

c) $\frac{1}{i} = \frac{i}{-1} = \underline{\underline{-i}}$

d) $\frac{3+4i}{-i} = \frac{i(3+4i)}{1} = \underline{\underline{-4+3i}}$

Die vier Grundrechenarten

a) $(6+4i)^2 - 5i\cdot(3-2i) = 36+48i-16-15i-10 = \underline{\underline{10+33i}}$

b) $\frac{6+4i-(3-2i)}{6+4i+3-2i} = \frac{3+6i}{9+2i} = \frac{(3+6i)(9-2i)}{81+4} = \frac{39+48i}{85} = \underline{\underline{\frac{39}{85} + \frac{48}{85}i}}$

c) $Re\Big(\big((6+4i)(3-2i)\cdot(6+4i+5i)\big)\Big) = Re\big(26(6+9i)\big) = 26\cdot 6 = \underline{\underline{156}}$

d) $\overline{\frac{5i(3-2i)}{6+4i}} = \overline{\frac{10+15i}{6+4i}} = \overline{\frac{(10+15i)(6-4i)}{52}} = \overline{\frac{120+50i}{52}} = \underline{\underline{\frac{30}{13} - \frac{25}{26}i}}$

QuadrGl komplex

a) $z^2 = -4 \iff z = \pm\sqrt{-4} = \underline{\underline{\pm 2i}}$

b) $z^2 + 3 = 0 \iff z^2 = -3 \iff z = \pm\sqrt{-3} = \underline{\underline{\pm\sqrt{3}\,i}}$

c) $6z^2 = 15 \iff z^2 = \frac{15}{6} = \frac{5}{2} \iff z = \underline{\underline{\pm\sqrt{\frac{5}{2}}}}$

d) $z^3 = -8z \implies z_1 = \underline{\underline{0}} \implies z^2 = -8 \iff z = \pm\sqrt{-8} = \underline{\underline{\pm 2\sqrt{2}\,i}}$

ergänzt

$z^2 - 6z + 13 = 0 \iff (z-3)^2 + 4 = 0 \iff (z-3)^2 = -4 \iff z-3 = \pm\sqrt{-4} \iff z = \underline{\underline{3 \pm 2i}}$

Repe

$az^2 + bz + c = 0 \overset{a\neq 0}{\iff} z^2 + \frac{b}{a}z + \frac{c}{a} = 0 \iff \left(z+\frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0 \iff \left(z+\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} \iff \left(z+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$\iff z + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}} \iff z = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\square$

zauberformelhaft

$z^2 + \frac{1}{2}z + 3 = 0 \to z_{1/2} = \frac{-\frac{1}{2}\pm\sqrt{\frac{1}{4}-12}}{2} = \frac{-\frac{1}{2}\pm\sqrt{-\frac{47}{4}}}{2} = \underline{\underline{-\frac{1}{4} \pm \frac{\sqrt{47}}{4}i}}$

erweiterte QuadrGl

a) $z^2 - 4z + 20 = 0 \to z_{1/2} = \frac{4\pm\sqrt{16-80}}{2} = \frac{4\pm\sqrt{-64}}{2} = \frac{4\pm 8i}{2} = \underline{\underline{2\pm 4i}}$

b) $-\frac{1}{5}z^2 + \frac{4}{5}z + 9 = 0 \to z_{1/2} = \frac{-\frac{4}{5}\pm\sqrt{\frac{16}{25}+\frac{36}{5}}}{-\frac{2}{5}} = \frac{-\frac{4}{5}\pm\sqrt{\frac{196}{25}}}{-2/5} = \frac{-\frac{4}{5}\pm\frac{14}{5}}{-2/5} = \begin{cases} \frac{10/5}{-2/5} = \underline{\underline{-5}} \\ \frac{-18/5}{-2/5} = \underline{\underline{9}} \end{cases}$

c) $2z^2 - z + 1 = 0 \to z_{1/2} = \frac{1\pm\sqrt{1-8}}{4} = \frac{1\pm\sqrt{-7}}{4} = \underline{\underline{\frac{1\pm\sqrt{7}i}{4}}}$

netter Zusammenhang

$z_1\cdot z_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}\cdot\frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{b^2-(b^2-4ac)}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a} \in \mathbb{R}$ $\square$

umgekehrt

$z^2 - 2z + a = 0 \to z_{1/2} = \frac{2\pm\sqrt{4-4a^2}}{2} = 1\pm\sqrt{1-a^2} \implies 1-a^2 = -1 \iff a^2 = 2 \to a = \underline{\underline{\pm\sqrt{2}}} \to z_2 = \underline{\underline{1-i}}$

und komplex

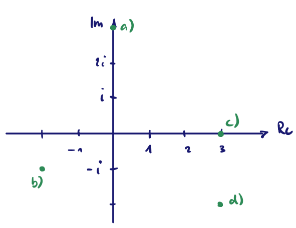
$z^2 - z + \frac{1}{4} - \frac{1}{2}i = 0 \iff z_{1/2} = \frac{1\pm\sqrt{1-4(\frac{1}{4}-\frac{1}{2}i)}}{2} = \frac{1\pm\sqrt{2i}}{2}$

Was ist $\sqrt{2i}$? $\sqrt{2i} = a+ib \implies 2i = a^2-b^2+2ab\,i \implies \begin{matrix} a^2-b^2 = 0 \\ 2 = 2ab \end{matrix} \iff b = \frac{1}{a} \iff a^2 - \frac{1}{a^2} = 0 \iff a^4 - 1 = 0 \iff a = \pm 1$

$\implies b = \pm 1$

$\to \sqrt{2i} = 1+i$ oder $-1-i \implies \frac{1\pm(1+i)}{2} = \underline{\underline{\frac{2+i}{2}}}$ bzw. $\underline{\underline{\frac{-i}{2}}}$

Argand Diagramm



oder Gauss'sche Zahlenebene

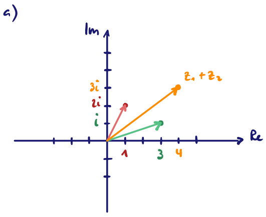
$u = 3$, $v = -3+i$, $w = -2i$

minus und bar

$-z$ ist eine Punktspiegelung von z am Ursprung. $\bar{z}$ ist eine Achsenspiegelung von z an der reellen Achse.

geometrisch addieren

a)



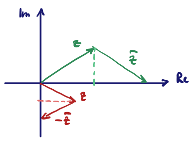
b) Ortsvektoren aneinander hängen

Geometrie

+ : 

− : 

S-Multiplikation



Permanenz

alle Rechenregeln gelten ✓

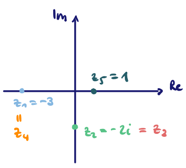
abs

a) $|-\pi| = \pi$ b) $|i| = 1$ c) $|1.5+2i| = \sqrt{1.5^2+2^2} = \sqrt{6.25} = 2.5$ d) $|-3-4i| = \sqrt{9+16} = \sqrt{25} = 5$

Pythagoras

$|\sqrt{2}+\sqrt{2}i| = \sqrt{\sqrt{2}^2+\sqrt{2}^2} = \sqrt{4} = 2$ und $\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = 45° = \frac{\pi}{4}$

Zeige, dass



polar

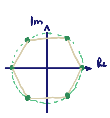
$3 = 3 \cdot e^0$, $3i = 3e^{i\frac{\pi}{2}}$, $-3 = 3e^{i\pi}$, $-3i = 3e^{i\frac{3}{2}\pi}$

Argument

$z = 4 \cdot e^{i\frac{4\pi}{3}}$, $|z| = 4$, $\arg(z) = \frac{4}{3}\pi$, $\arg(-z) = \frac{2}{3}\pi$, $\arg(\bar{z}) = \frac{2}{3}\pi$

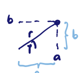
Pfeilchen

$z_n = 1 \cdot e^{i \cdot \frac{\pi}{3} n}$



Basics

$z = a + ib = re^{i\varphi}$



$\cos(\varphi) = \frac{a}{r}$, $\sin(\varphi) = \frac{b}{r}$ $\Rightarrow$ $z = a + ib = r\cos(\varphi) + i \cdot r\sin(\varphi)$

Normalform

a) $1 \cdot e^i \rightarrow r = 1$, $\varphi = 1$ $\rightarrow z = \cos(1) + i \cdot \sin(1)$

b) $6 \cdot e^{i \cdot \frac{5}{3}\pi} \rightarrow z = 6\cos(\frac{1}{3}\pi) - i \cdot 6\sin(\frac{1}{3}\pi)$

c) $\sqrt{2}e^{-i\frac{5}{4}\pi} = \sqrt{2}e^{i\frac{3}{4}\pi} \rightarrow z = -\sqrt{2}\cos(\frac{1}{4}\pi) + i \cdot \sqrt{2}\sin(\frac{1}{4}\pi)$

d) einzeln umrechnen und dann subtrahieren

Basics once again

$z = \sqrt{3} + i \rightarrow r = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{4} = 2$, $\varphi = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ $\rightarrow z = \underline{\underline{2e^{i\frac{\pi}{6}}}}$

Polarform

check mit Wolfram Alpha <u>und nochmal</u>

juhuu

Siehe Satz, brauche dazu Potenzgesetze

de Moivre

$1+i = \sqrt{2}e^{i\frac{\pi}{4}}$ und $1-i = \sqrt{2}e^{-i\frac{\pi}{4}}$

$\Rightarrow (1+i)^6 = \sqrt{2}^6 e^{i\frac{6}{4}\pi} = 8e^{i\frac{3}{2}\pi}$ und $(1-i)^6 = 8e^{-i\frac{6}{4}\pi} = 8e^{i\frac{\pi}{2}}$ $\Rightarrow (1+i)^6 + (1-i)^6 = \underline{\underline{0}}$

Einheit

$z^{-1} = \frac{1}{r}e^{-i\varphi}$

Rechnen

check z.B. mit Wolfram Alpha

Abstände

$|z-1| = |z-3| \Leftrightarrow |a+ib-1| = |a+ib-3| \Leftrightarrow |(a-1)+ib| = |(a-3)+ib| \overset{()^2}{\Leftrightarrow} (a-1)^2 + \cancel{b^2} = (a-3)^2 + \cancel{b^2} \Leftrightarrow -2a+1 = -6a+9 \Leftrightarrow 4a = 8$

$\Leftrightarrow a=2$ $\rightarrow$ Parallele zu Im durch Re 2.