① Verwende Tertia-Trigonometrie im ebenen Dreieck.

② Siehe Unterricht

③ a) $\begin{pmatrix} -6 \\ 8 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 12 \\ 4 \end{pmatrix} \qquad \varphi = \cos^{-1}\left(\frac{\vec{v} \cdot \vec{w}}{|\vec{v}|\,|\vec{w}|}\right)$

$$\vec{v} \cdot \vec{w} = (-6)\cdot(-3) + 8\cdot 12 + 0\cdot 4 = 18 + 96 = 114$$

$$|\vec{v}| = \sqrt{6^2+8^2} = 10 \quad , \quad |\vec{w}| = \sqrt{3^2+12^2+4^2} = 13$$

$$\rightarrow \quad \varphi = \cos^{-1}\left(\frac{114}{10 \cdot 13}\right) \approx$$

b) $\begin{pmatrix} -3 \\ 12 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \rightarrow \quad \varphi = \cos^{-1}\left(\frac{-3}{13}\right) \approx$

④ Es gilt $\vec{v} \cdot \vec{w} = 0 \iff \vec{v} \perp \vec{w}$

$$\vec{e_x} \cdot \vec{e_y} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 1\cdot 0 + 0 \cdot 1 + 0 \cdot 0 = 0 \quad \checkmark \qquad ----$$

⑤ Für Nerds und Hauptfachmathematiker

⑥ $\vec{u} = \begin{pmatrix} 7 \\ u_y \\ u_z \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$

$$\vec{u} \cdot \vec{v} = 7 - 3u_y + 5u_z = 0 \qquad (1)$$

$$\vec{u} \cdot \vec{w} = 14 + u_y - 4u_z = 0 \qquad (2) \quad \rightarrow \quad u_y = 4u_z - 14$$

$$\rightarrow \quad 7 - 3(4u_z - 14) + 5u_z = 0$$

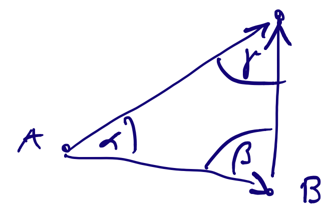
$$49 - 7u_z = 0$$

$$u_z = 7 \quad \rightarrow \quad u_y = 14$$

⑦ Man findet $\vec{u} \cdot \vec{w} = 0 \quad \checkmark$

c

⑧ $A = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \quad B = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 7 \\ -19 \\ -1 \end{pmatrix}$

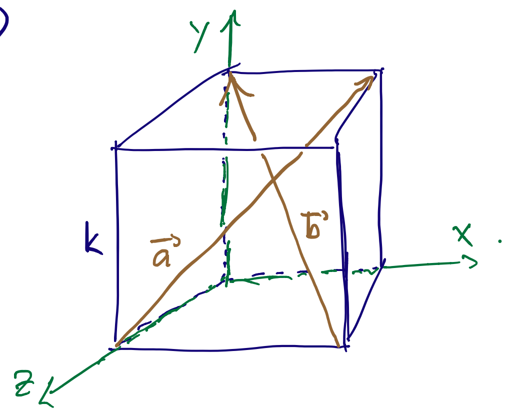


$\vec{AB} = \begin{pmatrix} -5 \\ -1 \\ 4 \end{pmatrix}, \quad \vec{AC} = \begin{pmatrix} 5 \\ -20 \\ 2 \end{pmatrix}, \quad \vec{BC} = \begin{pmatrix} 10 \\ -19 \\ -2 \end{pmatrix}$

$$\rightarrow \alpha = \cos^{-1}\left(\frac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}|\cdot|\vec{AC}|}\right) = \cos^{-1}\left(\frac{3}{\sqrt{42}\cdot\sqrt{429}}\right) =$$

$$\beta = \cos^{-1}\left(\frac{-\vec{AB}\cdot\vec{BC}}{|\vec{AB}|\cdot|\vec{BC}|}\right) = \quad , \quad \gamma = \cos^{-1}\left(\frac{-\vec{AC}\cdot(-\vec{BC})}{|\vec{AC}|\cdot|\vec{BC}|}\right)$$

⑨



$\vec{a} = \begin{pmatrix} k \\ k \\ -k \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} -k \\ k \\ -k \end{pmatrix}$

$\vec{a}\cdot\vec{b} = -k^2 + k^2 + k^2 = k^2$

$|\vec{a}| = |\vec{b}| = \sqrt{k^2+k^2+k^2} = \sqrt{3k^2} = \sqrt{3}\,k$

$\varphi = \cos^{-1}\left(\frac{k^2}{\sqrt{3}k\cdot\sqrt{3}k}\right) = \cos^{-1}\left(\frac{1}{3}\right) =$

⑩ $\vec{v}$? $\quad \cos(60^\circ) = \frac{1}{2} = \frac{\vec{e_x}\cdot\vec{v}}{1\cdot|\vec{v}|} = \frac{v_x}{|\vec{v}|}$

$\cos(45^\circ) = \frac{\sqrt{2}}{2} = \frac{\vec{e_y}\cdot\vec{v}}{1\cdot|\vec{v}|} = \frac{v_y}{|\vec{v}|}$

Wähle $|\vec{v}| = 1 \Rightarrow v_x = \frac{1}{2}, \; v_y = \frac{\sqrt{2}}{2}$.

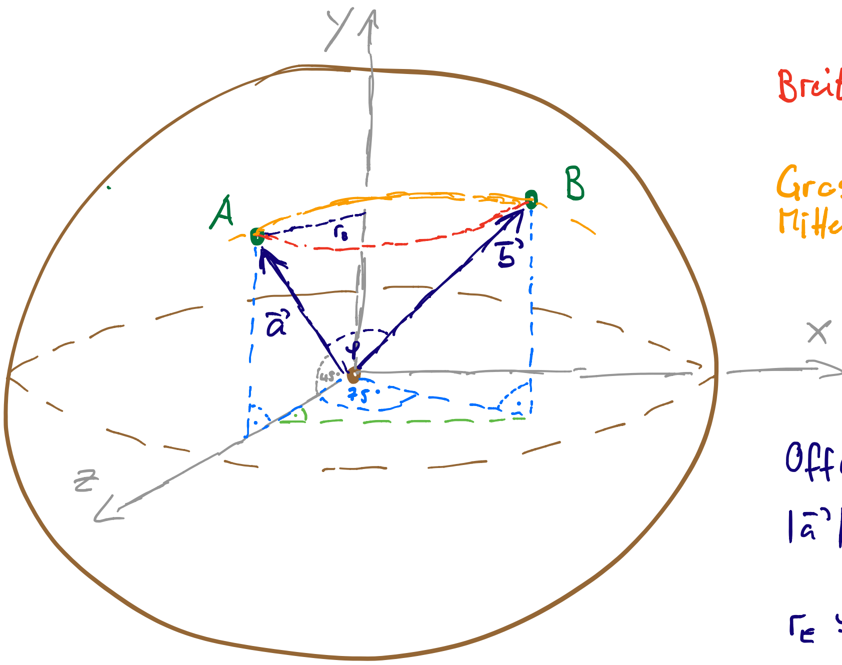
Es muss $\sqrt{v_x^2 + v_y^2 + v_z^2} = 1$, nach Wahl von $|\vec{v}|$.

Somit $\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 + v_z^2} = 1 \rightarrow v_z^2 = \frac{1}{4} \rightarrow v_z = \pm\frac{1}{2}$

$\rightarrow \gamma = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$

Bem.: Der Schritt $|\vec{v}| = 1$ zu setzen ist nicht zwingend; aber praktisch.

(11)



Breitenkreis 45°

Grosskreis mit Mittelpunkt M_E

Offensichtlich

$$|\vec{a}| = r_E = |\vec{b}|$$

$$r_E \simeq 6370 \text{ km}$$

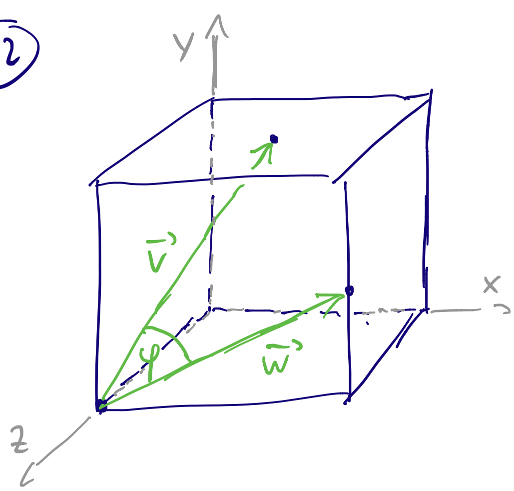
Breitenkreis : $\quad r_B = \dfrac{r_E}{\sqrt{2}} \quad \rightarrow \quad \ell_B = 2\pi r_B \cdot \dfrac{75}{360} =$

Grosskreis : $\quad \vec{a} = \begin{pmatrix} 0 \\ r_E/\sqrt{2} \\ r_E/\sqrt{2} \end{pmatrix} \quad , \quad \vec{b} = \begin{pmatrix} r_E/\sqrt{2} \cdot \sin(75°) \\ r_E/\sqrt{2} \\ r_E/\sqrt{2} \cdot \cos(75°) \end{pmatrix}$

guck

$$\varphi = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{r_E^{\,2}}\right) \quad \rightarrow \quad \ell_G = 2\pi r_E \cdot \frac{\varphi}{360}$$

(12)



$$\vec{v} = \begin{pmatrix} k/2 \\ k \\ -k/2 \end{pmatrix} \qquad \vec{w} = \begin{pmatrix} k \\ k/2 \\ 0 \end{pmatrix}$$

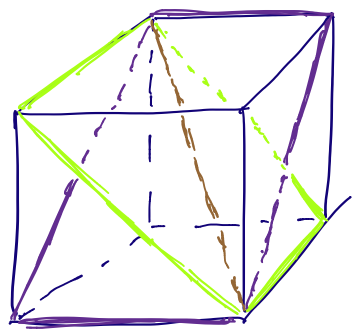
$$\vec{v}\cdot\vec{w} = \frac{k^2}{2} + \frac{k^2}{2} + 0 = k^2$$

$$|\vec{v}| = \sqrt{\frac{k^2}{4} + k^2 + \frac{k^2}{4}} = \sqrt{\frac{3k^2}{2}}$$

$$|\vec{w}| = \sqrt{k^2 + \frac{k^2}{4}} = \sqrt{\frac{5k^2}{4}}$$

$$\rightarrow \varphi = \cos^{-1}\left(\frac{k^2}{\sqrt{\frac{3k^2}{2}\cdot\frac{5k^2}{4}}}\right) = \cos^{-1}\left(\sqrt{\frac{8}{15}}\right)$$

(13)



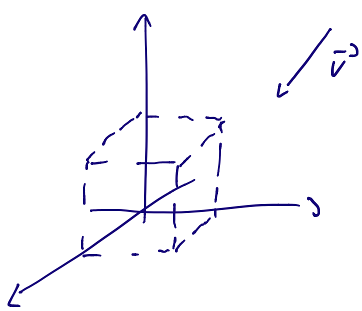
Idee: Winkel der beiden Flächen bleibt gleich, wenn ich je Vektoren senkrecht auf den Flächen vergleiche. (sog. Normalenvektoren)

Sei Koordsys wie üblich "hinten, unten, links".

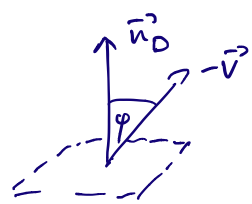
$$\rightarrow \quad \vec{n}_{grün} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{n}_{vigo} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\text{Somit} \quad \varphi = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

(14) $H(\varphi) = k \cdot L \cdot \cos(\varphi) \qquad \vec{v} = \begin{pmatrix} -3 \\ -2 \\ -1 \end{pmatrix}$



z.B. für "Deckel":



$$\vec{n}_D = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \varphi = \cos^{-1}\left(\frac{2}{\sqrt{14}}\right)$$

$$\rightarrow H(\varphi) = 0.5 \cdot 1 \cdot \frac{2}{\sqrt{14}} = \quad \ldots$$