Question 4

$f(x) = x^{ln(x)}$, Taking the natural log of both sides yields

$ln(f(x)) = ln(x^{ln(x)})$, Applying log rules yields

$ln(f(x)) = (ln(x))(ln(x))$

$ln(f(x)) = (ln(x))^2$, Differentiating with respect to x yields

$\frac{d}{dx}(ln(f(x))) = \frac{d}{dx}((ln(x))^2)$

$\frac{1}{f(x)}(f'(x)) = 2ln(x) * [\frac{1}{x}]$

$f'(x) = \frac{f(x)*2ln(x)}{x}$, Note: $f(x) = x^{ln(x)}$

$f'(x) = \frac{x^{ln(x)}*2ln(x)}{x}$

$f'(x) = \frac{2ln(x)[x^{ln(x)}]}{x}$

Therefore $f'(e^3) = \frac{2ln(e^3)[(e^3)^{ln(e^3)}]}{e^3}$

$$= \frac{2(3)[(e^3)^3]}{e^3}$$

$$= \frac{6(e^9)}{e^3}$$

$$= 6e^6, \text{ QED}$$