Question 5

$y = \frac{(x-1)^{7/2}(x+1)^{1/2}}{x^2+2}$, Taking the natural log of both sides yields

$ln(y) = ln((x-1)^{7/2}) + ln((x+1)^{1/2}) - ln(x^2+2)$

$ln(y) = \frac{7}{2}ln(x-1) + \frac{1}{2}ln(x+1) - ln(x^2+2)$, Differentiating with respect to x yields

$\frac{d}{dx}[ln(y)] = \frac{d}{dx}[\frac{7}{2}ln(x-1) + \frac{1}{2}ln(x+1) - ln(x^2+2)]$

$\frac{1}{y}(\frac{dy}{dx}) = \frac{7}{2}(\frac{1}{x-1}) + \frac{1}{2}(\frac{1}{x+1}) - \frac{1}{x^2+2}(2x)$

$\frac{dy}{dx} = y[\frac{7}{2(x-1)} + \frac{1}{2(x+1)} - \frac{2x}{x^2+2}]$

Note: $y = \frac{(x-1)^{7/2}(x+1)^{1/2}}{x^2+2}$

When x = 2, $y = \frac{(2-1)^{7/2}(2+1)^{1/2}}{2^2+2}$

$y = \frac{1(3)^{1/2}}{6}$, $y = \frac{1\sqrt{3}}{6}$, $y = \frac{\sqrt{3}}{6}$

Therefore $\frac{dy}{dx}\Big|_{x=2} = \frac{\sqrt{3}}{6}[\frac{7}{2(2-1)} + \frac{1}{2(2+1)} - \frac{2(2)}{2^2+2}]$

$= \frac{\sqrt{3}}{6}[\frac{7}{2(1)} + \frac{1}{2(3)} - \frac{4}{4+2}]$

$= \frac{\sqrt{3}}{6}[\frac{7}{2} + \frac{1}{6} - \frac{4}{6}]$

$= \frac{\sqrt{3}}{6}[\frac{7}{2} - \frac{3}{6}]$

$= \frac{\sqrt{3}}{6}[\frac{7}{2} - \frac{1}{2}]$

$= \frac{\sqrt{3}}{6}[\frac{6}{2}]$

$= \frac{\sqrt{3}}{2}$, QED