Question 1

Prelude

Note: $\cos(2x) = \cos^2(x) - \sin^2(x)$
Therefore, $\cos(2x) = \cos^2(x) - (1 - \cos^2(x))$
Thus, $\cos(2x) = \cos^2(x) - 1 + \cos^2(x))$
Therfore, $\cos(2x) = 2\cos^2(x) - 1$

Solution

$\cos(x) + \cos(3x) = 0$
$\cos(x) + \cos(2x + x) = 0$
$\cos(x) + \cos(2x)\cos(x) - \sin(2x)\sin(x) = 0$
$\cos(x) + [2\cos^2(x) - 1]\cos(x) - 2\sin(x)\cos(x)\sin(x) = 0$
$\cos(x) + 2\cos^3(x) - \cos(x) - 2\sin^2(x)\cos(x) = 0$
$2\cos^3(x) - 2(1 - \cos^2(x))\cos(x) = 0$
$2\cos^3(x) + (-2 + 2\cos^2(x))\cos(x) = 0$
$2\cos^3(x) - 2\cos(x) + 2\cos^3(x) = 0$
$4\cos^3(x) - 2\cos(x) = 0$
$2\cos(x)[2\cos^2(x) - 1] = 0$
$2\cos(x)[\cos(2x)] = 0$

Therefore $\cos(x) = 0$ or $\cos(2x) = 0$

The rest of the solution is contained within the next few images.