Question 6

Let $y = (1 + sin(\frac{3}{x}))^x$, Taking the natural log of both sides yields

$$ln(y) = xln((1 + sin(\frac{3}{x}))$$

$\lim_{x\to\infty} ln(y) = \lim_{x\to\infty} xln((1 + sin(\frac{3}{x}))$

$= \lim_{x\to\infty} \frac{ln(1+sin(\frac{3}{x}))}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{ln(1+sin(3x^{-1}))}{x^{-1}}$

$= \lim_{x\to\infty} \frac{\frac{1}{1+sin(\frac{3}{x})}cos(\frac{3}{x})(-3x^{-2})}{-1x^{-2}}$

$= \lim_{x\to\infty} \frac{3cos(\frac{3}{x})}{1+sin(\frac{3}{x})}$

$= \frac{3cos(0)}{1+sin(0)} = \frac{3(1)}{1+0} = \frac{3}{1} = 3$

Finally if $ln(y) \to 3$, then we must have $y \to e^3$

That is $\lim_{x\to\infty}(1 + sin(\frac{3}{x}))^x = e^3$, QED