

Question 1

b)

$$\begin{aligned} & \int_0^{\frac{1}{4}} \frac{\tan(\pi t)}{t} dt \\ &= \int_0^{\frac{1}{4}} \frac{\pi \tan(\pi t)}{\pi t} dt \end{aligned}$$

Let $u = \pi t$, therefore $du = \pi dt$, therefore $dt = \frac{du}{\pi}$

Therefore the bounds for $t = 0$, and $t = \frac{1}{4}$ change to

$u = \pi(0) = 0$, and $u = \pi \frac{1}{4} = \frac{\pi}{4}$

Therefore $\frac{\pi \tan(\pi t)}{\pi t} dt = \frac{\pi \tan(u)}{u} \frac{du}{\pi} = \frac{\tan(u)}{u} du$

$$\begin{aligned} & \int_0^{\frac{1}{4}} \frac{\pi \tan(\pi t)}{\pi t} dt \\ &= \int_0^{\frac{\pi}{4}} \frac{\tan(u)}{u} du \\ &= \int_0^{\frac{\pi}{4}} \frac{u + \frac{u^3}{3} \pm \frac{64u^5}{15}}{u} du \\ &= \int_0^{\frac{\pi}{4}} \left(1 + \frac{u^2}{3} \pm \frac{64u^4}{15}\right) du \\ &= \left(u + \frac{u^3}{9} \pm \frac{64u^5}{75}\right) \Big|_0^{\frac{\pi}{4}} \\ &= \frac{\pi}{4} + \frac{1}{9} \left(\frac{\pi}{4}\right)^3 \pm \frac{64}{75} \left(\frac{\pi}{4}\right)^3 \end{aligned}$$

$$\doteq 0.839 \pm 0.255$$