

Question 2

Let $u = x^2$, therefore $du = 2x \, dx$, therefore $\frac{1}{2}du = x \, dx$

Therefore $(u)(\frac{1}{2} \, du) = (x^2)(x \, dx)$, therefore $\frac{1}{2} \, u \, du = x^3 \, dx$

Therefore $\int x^3 \cos(x^2) dx = \frac{1}{2} \int u \cos(u) du$

Let $v = u$, and $dw = \cos(u) \, du$, therefore $dv = du$, and $w = \sin(u)$

Therefore $\int u \cos(u) du = \int v dw = vw - \int w dv$ (By integration by parts)

Therefore $\int u \cos(u) du = u \sin(u) - \int \sin(u) du = u \sin(u) - (-\cos(u)) + C_1$

$= u \sin(u) + \cos(u) + C_1 = x^2 \sin(x^2) + \cos(x^2) + C_1$

Therefore $\frac{1}{2} \int u \cos(u) du = \frac{1}{2} (x^2 \sin(x^2) + \cos(x^2) + C_1)$

$= \frac{1}{2} x^2 \sin(x^2) + \frac{1}{2} \cos(x^2) + \frac{1}{2} C_1 = \frac{1}{2} x^2 \sin(x^2) + \frac{1}{2} \cos(x^2) + C$

Therefore $\int x^3 \cos(x^2) dx = \frac{1}{2} x^2 \sin(x^2) + \frac{1}{2} \cos(x^2) + C$, QED