Question 1

a)

To determine if

$$\sum_{n=0}^{\infty} ne^{-n^2}$$

converges, I will use the integral test.

That is, I will find

$$\int_0^{\infty} ne^{-n^2} dn$$

and if this converges/diverges, then the sum converges/diverges respectively.

Let $u = n^2$, therefore $du = 2n\ dn$, therefore $\frac{1}{2}du = n\ dn$

Therefore when $n = 0$, $u = 0^2 = 0$, and when $n = \infty$, $u = n^2 = \infty$

Therefore

$$\int_0^{\infty} ne^{-n^2} dn$$

$$= \lim_{t\to\infty} \int_0^t \frac{1}{2} e^{-u} du$$

$$= \lim_{t\to\infty} \frac{-e^{-u}}{2}\bigg|_0^t$$

$$= \lim_{t\to\infty} \frac{-e^{-t}}{2} - \frac{-e^{-0}}{2}$$

$= \frac{0}{2} + \frac{1}{2} = \frac{1}{2}$, therefore since the integral converges and since all the terms are positive, the sum converges absolutely.