Question 3

Question:

Find the equation of the line which is tangent to the curve $xsin(xy-y^2)=x^2-1$ at the point(1,1)

Answer:

$xsin(xy-y^2)=x^2-1$, Differentiating with respect to x yields

$\frac{d}{dx}(xsin(xy-y^2))=\frac{d}{dx}(x^2-1)$

$(1)sin(xy-y^2)+xcos(xy-y^2)[(1)y+x(\frac{dy}{dx})-2y(\frac{dy}{dx})]=2x$, by Chain Rule

$sin(xy-y^2)+(y+(x-2y)(\frac{dy}{dx}))(xcos(xy-y^2))=2x$

$(x-2y)(\frac{dy}{dx})(xcos(xy-y^2))+xycos(xy-y^2)+sin(xy-y^2)=2x$

$(\frac{dy}{dx})(x^2cos(xy-y^2)-2xycos(xy-y^2))=2x-xycos(xy-y^2)-sin(xy-y^2)$

$\frac{dy}{dx}=\frac{2x-xycos(xy-y^2)-sin(xy-y^2)}{x^2cos(xy-y^2)-2xycos(xy-y^2)}$

To find the slope of the tangent line we need to evaluate $\frac{dy}{dx}$ at (1,1)

$$\left.\frac{dy}{dx}\right|_{(1,1)}=\frac{2(1)-(1)(1)cos((1)(1)-(1)^2)-sin((1)(1)-(1)^2)}{(1)^2cos((1)(1)-(1)^2)-2(1)(1)cos((1)(1)-(1)^2)}$$

$$=\frac{2-cos(1-1)-sin(1-1)}{(1)cos(1-1)-2cos(1-1)}$$

$$=\frac{2-cos(0)-sin(0)}{cos(0)-2cos(0)}$$

$$=\frac{2-1-0}{1-2(1)}$$

$$=\frac{1}{1-2}$$

$$=\frac{1}{-1}$$

= -1