# Question 1

## a)

$f(u) = tan(u),$ $\qquad$ $f(0) = tan(0) = 0$

$f'(u) = \frac{d}{du}(tan(u)) = sec^2(u),$ $\qquad$ $f'(0) = sec^2(0) = 1^2 = 1$

$$f''(u) = \frac{d}{du}(sec^2(u)) = 2sec(u) * \frac{d}{du}(sec(u)) = 2sec(u) * sec(u)tan(u)$$

$$= 2sec^2(u)tan(u) = 2(tan^2(u) + 1)tan(u) = 2(tan^3(u) + tan(u))$$

$$f''(0) = 2(tan^3(0) + tan(0)) = 2(0 + 0) = 2(0) = 0$$

$$f^{(3)}(u) = \frac{d}{du}(2(tan^3(u) + tan(u))) = 2(\frac{d}{du}(tan^3(u) + tan(u)))$$

$$= 2(3tan^2(u) * \frac{d}{du}(tan(u)) + sec^2(u)) = 2(3tan^2(u) * sec^2(u) + sec^2(u))$$

$$= 6tan^2(u)sec^2(u) + 2sec^2(u) = 6tan^2(u)[tan^2(u) + 1] + 2sec^2(u)$$

$$= 6tan^4(u) + 6tan^2(u) + 2sec^2(u) = 6tan^4(u) + 6(sec^2(u) - 1) + 2sec^2(u)$$

$$= 6tan^4(u) + 6sec^2(u) - 6 + 2sec^2(u) = 6tan^4(u) + 8sec^2(u) - 6$$

$$f^{(3)}(0) = 6tan^4(0) + 8sec^2(0) - 6 = 6(0) + 8(1) - 6 = 0 + 8 - 6 = 2$$

Note: Since $f'(u) = sec^2(u)$ , and $f''(u) = \frac{d}{du}(f'(u)) = 2(tan^3(u) + tan(u))$

Therefore $\frac{d}{du}(sec^2(u)) = 2(tan^3(u) + tan(u))$

$f^{(4)}(u) = \frac{d}{du}(6tan^4(u) + 8sec^2(u) - 6) = 6\frac{d}{du}(tan^4(u)) + 8\frac{d}{du}(sec^2(u))$

$= 6(4tan^3(u) * \frac{d}{du}(tan(u))) + 8(2(tan^3(u) + tan(u)))$

$= 24tan^3(u) * sec^2(u) + 16(tan^3(u) + tan(u))$

$= 24tan^3(u)sec^2(u) + 16tan^3(u) + 16tan(u)$

$= 8tan^3(u)[3sec^2(u)+2]+16tan(u) = 8tan^3(u)[3(tan^2(u)+1)+2]+16tan(u)$

$= 8tan^3(u)[3tan^2(u)+3+2]+16tan(u) = 8tan^3(u)[3tan^2(u)+5]+16tan(u)$

$= 24tan^5(u) + 40tan^3(u) + 16tan(u)$

$f^{(4)}(0) = 24tan^5(0) + 40tan^3(0) + 16tan(0) = 24(0)^5 + 40(0)^5 + 16(0)^5$

$= 24(0) + 40(0) + 16(0) = 0 + 0 + 0 = 0$

$f^{(5)}(u) = \frac{d}{du}(24tan^5(u) + 40tan^3(u) + 16tan(u))$

$= 24\frac{d}{du}(tan^5(u)) + 40\frac{d}{du}(tan^3(u)) + 16\frac{d}{du}(tan(u))$

$= 24(5tan^4(u) * \frac{d}{du}(tan(u))) + 40(3tan^2(u) * \frac{d}{du}(tan(u))) + 16sec^2(u)$

$= 120tan^4(u) * sec^2(u) + 120tan^2(u) * sec^2(u) + 16sec^2(u)$

$= 120tan^4(u)*(tan^2(u)+1)+120tan^2(u)*(tan^2(u)+1)+16*(tan^2(u)+1)$

$= 120tan^6(u) + 120tan^4(u) + 120tan^4(u) + 120tan^2(u) + 16tan^2(u) + 16$

$= 120tan^6(u) + 240tan^4(u) + 136tan^2(u) + 16$

$|f^{(5)}(x)| \leq k$, with $x \in [0, \frac{\pi}{4}]$

Note: $|f^{(5)}(x)| = |120tan^6(x) + 240tan^4(x) + 136tan^2(x) + 16|$, which is increasing on the interval $[0, \frac{\pi}{4}]$

Therefore the max value of $f^{(5)}(x)$ on this interval is when x = $\frac{\pi}{4}$

Subsituting this in yields, $f^{(5)}(\frac{\pi}{4}) = 120tan^6(\frac{\pi}{4}){+}240tan^4(\frac{\pi}{4}){+}136tan^2(\frac{\pi}{4})$

$+16 = 120(1)^6 + 240(1)^4 + 136(1)^2 + 16 = 120 + 240 + 136 + 16 = 512$

Therefore, we can take k = 512, since $512 \geq f^{(5)}(x)$ on this interval.

Therefore the error term $\frac{k(x-a)^5}{5!} = \frac{512(x-0)^5}{5!} = \frac{512x^5}{120} = \frac{64x^5}{15}$

Therefore the 4th order Maclaurin polynomial for $f(u) = tan(u)$ is

$y = f(0) + f'(0)u + \frac{f''(0)}{2!}u^2 + \frac{f^{(3)}(0)}{3!}u^3 + \frac{f^{(4)}(0)}{4!}u^4 \pm R$

$= 0 + 1u + \frac{0}{2}u^2 + \frac{2}{6}u^3 + \frac{0}{24}u^4 \pm R$

$= u + \frac{u^3}{3} \pm R$ , where $R = \frac{64u^5}{15}$