

Mathematical Analysis of Algorithms

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What is algorithm analysis?

**Why do we analyze
algorithms?**

Algorithm analysis is the prediction and comparison of algorithm performance.

Algorithm analysis lets us choose or design the best (or good enough) algorithm for a given problem.

**How do we mathematically
analyze algorithms?**

**Give a step-by-step
procedure.**

How to Analyze an Algorithm:

- ① Define a reasonable **model of computation (cost model)**.
 - ① What are the **basic operations**?
 - ② How much does each basic operation cost?
- ② Model the algorithm's cost with a function $T(n)$.
 - ① How many basic operations are performed for an input of size n ?
- ③ Simplify $T(n)$.
- ④ Classify $T(n)$'s growth rate.
 - ① How quickly does $T(n)$ grow?
- ⑤ Interpret $T(n)$'s growth rate.
 - ① How suitable is the algorithm for my problem?

Defining a Model of Computation

```
int foo(int n) {  
    int A = new int[n];  
  
    A[0] = 1;  
    for (int i = 1; i < n; i++)  
        A[i] = A[i - 1] * (i + 1);  
  
    int sum = 0;  
    for (int i = 0; i < n; i++)  
        for (int j = 0; j < i; j++)  
            sum += A[j];  
  
    delete [] A;  
    return sum;  
}
```

List as many basic operations as you can think of!

Defining a Model of Computation

```
int foo(int n) {  
    int* A = new int[n];  
  
    A[0] = 1;  
    for (int i = 1; i < n; i++)  
        A[i] = A[i - 1] * (i + 1);  
  
    int sum = 0;  
    for (int i = 0; i < n; i++)  
        for (int j = 0; j < i; j++)  
            sum += A[j];  
  
    delete[] A;  
    return sum;  
}
```

Basic Operations:

- ① Additions
- ② Multiplications
- ③ Comparisons
- ④ Branches
- ⑤ Local variables
- ⑥ Memory allocations
- ⑦ Allocated memory
- ⑧ Loads
- ⑨ Stores
- ⑩ Assignments
- ⑪ ...

Which basic operations are most reasonable? Why?

Modeling the Algorithm's Cost

```
int foo(int n) {  
    int* A = new int[n];  
  
    A[0] = 1;  
    for (int i = 1; i < n; i++)    // Additions?  
        A[i] = A[i - 1] * (i + 1); // Additions?  
  
    int sum = 0;  
    for (int i = 0; i < n; i++)    // Additions?  
        for (int j = 0; j < i; j++) // Additions?  
            sum += A[j];           // Additions?  
  
    delete [] A;  
    return sum;  
}
```

How many additions? $T(n) = ?$

Modeling the Algorithm's Cost

```
int foo(int n) {  
    int* A = new int[n];  
  
    A[0] = 1;  
    for (int i = 1; i < n; i++)           //  $\sum_{i=1}^{n-1} 1$  additions  
        A[i] = A[i - 1] * (i + 1);       //  $\sum_{i=1}^{n-1} 2$  additions  
  
    int sum = 0;  
    for (int i = 0; i < n; i++)           //  $\sum_{i=0}^{n-1} 1$  additions  
        for (int j = 0; j < i; j++)       //  $\sum_{i=0}^{n-1} \sum_{j=0}^i 1$  additions  
            sum += A[j];                  //  $\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$  additions  
  
    delete[] A;  
    return sum;  
}
```

$$T(n) = \sum_{i=1}^{n-1} 1 + \sum_{i=1}^{n-1} 2 + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Modeling the Algorithm's Cost

```
int foo(int n) {  
    int* A = new int[n];  
  
    A[0] = 1;  
    for (int i = 1; i < n; i++)  
        A[i] = A[i - 1] * (i + 1); // Multiplications?  
  
    int sum = 0;  
    for (int i = 0; i < n; i++)  
        for (int j = 0; j < i; j++)  
            sum += A[j];  
  
    delete [] A;  
    return sum;  
}
```

How many multiplications? $T(n) = ?$

Modeling the Algorithm's Cost

```
int foo(int n) {  
    int* A = new int[n];  
  
    A[0] = 1;  
    for (int i = 1; i < n; i++)  
        A[i] = A[i - 1] * (i + 1); //  $\sum_{i=1}^{n-1} 1$  multiplications  
  
    int sum = 0;  
    for (int i = 0; i < n; i++)  
        for (int j = 0; j < i; j++)  
            sum += A[j];  
  
    delete[] A;  
    return sum;  
}
```

Multiplications: $T(n) = \sum_{i=1}^{n-1} 1$

Modeling the Algorithm's Cost

```
int foo(int n) {  
    int* A = new int[n];  
  
    A[0] = 1;  
    for (int i = 1; i < n; i++) // Comparisons?  
        A[i] = A[i - 1] * (i + 1);  
  
    int sum = 0;  
    for (int i = 0; i < n; i++) // Comparisons?  
        for (int j = 0; j < i; j++) // Comparisons?  
            sum += A[j];  
  
    delete [] A;  
    return sum;  
}
```

How many comparisons? $T(n) = ?$

Modeling the Algorithm's Cost

```
int foo(int n) {  
    int* A = new int[n];  
  
    A[0] = 1;  
    for (int i = 1; i < n; i++) //  $\sum_{i=1}^n 1$  comparisons  
        A[i] = A[i - 1] * (i + 1);  
  
    int sum = 0;  
    for (int i = 0; i < n; i++) //  $\sum_{i=0}^n 1$  comparisons  
        for (int j = 0; j < i; j++) //  $\sum_{i=0}^{n-1} \sum_{j=0}^i 1$  comparisons  
            sum += A[j];  
  
    delete[] A;  
    return sum;  
}
```

Comparisons: $T(n) = \sum_{i=1}^n 1 + \sum_{i=0}^n 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1$

Modeling the Algorithm's Cost

```
int foo(int n) {  
    int* A = new int[n];  
  
    A[0] = 1; // Memory accesses?  
    for (int i = 1; i < n; i++)  
        A[i] = A[i - 1] * (i + 1); // Memory accesses?  
  
    int sum = 0;  
    for (int i = 0; i < n; i++)  
        for (int j = 0; j < i; j++)  
            sum += A[j]; // Memory accesses?  
  
    delete [] A;  
    return sum;  
}
```

How many memory accesses (indexing)? $T(n) = ?$

Modeling the Algorithm's Cost

```
int foo(int n) {  
    int* A = new int[n];  
  
    A[0] = 1; // 1 memory accesses  
    for (int i = 1; i < n; i++)  
        A[i] = A[i - 1] * (i + 1); //  $\sum_{i=1}^{n-1} 2$  memory accesses  
  
    int sum = 0;  
    for (int i = 0; i < n; i++)  
        for (int j = 0; j < i; j++)  
            sum += A[j]; //  $\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$  memory accesses  
  
    delete[] A;  
    return sum;  
}
```

Memory Accesses: $T(n) = 1 + \sum_{i=1}^{n-1} 2 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$

Modeling the Algorithm's Cost

```
int foo(int n) {  
    int* A = new int[n];           // Assignments?  
  
    A[0] = 1;                       // Assignments?  
    for (int i = 1; i < n; i++)     // Assignments?  
        A[i] = A[i - 1] * (i + 1); // Assignments?  
  
    int sum = 0;                    // Assignments?  
    for (int i = 0; i < n; i++)     // Assignments?  
        for (int j = 0; j < i; j++) // Assignments?  
            sum += A[j];           // Assignments?  
  
    delete [] A;  
    return sum;  
}
```

How many assignments? $T(n) = ?$

Modeling the Algorithm's Cost

```
int foo(int n) {  
    int* A = new int[n];           // 1 assignments  
  
    A[0] = 1;                       // 1 assignments  
    for (int i = 1; i < n; i++)     //  $\sum_{i=1}^{n-1} 1$  assignments  
        A[i] = A[i - 1] * (i + 1); //  $\sum_{i=1}^{n-1} 1$  assignments  
  
    int sum = 0;                    // 1 assignments  
    for (int i = 0; i < n; i++)     //  $\sum_{i=0}^{n-1} 1$  assignments  
        for (int j = 0; j < i; j++) //  $\sum_{i=0}^{n-1} \sum_{j=0}^i 1$  assignments  
            sum += A[j];           //  $\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$  assignments  
  
    delete[] A;  
    return sum;  
}
```

$$T(n) = 3 + \sum_{i=1}^n 1 + \sum_{i=1}^{n-1} 1 + \sum_{i=0}^n 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplifying the Cost Function

Addition Cost Function

$$T(n) = \sum_{i=1}^{n-1} 1 + \sum_{i=1}^{n-1} 2 + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

Apply (1) with $m = n - 1$:

$$\sum_{i=1}^{n-1} 1 = n - 1$$

Identities:

$$\textcircled{1} \sum_{i=1}^m 1 = m$$

$$\textcircled{2} \sum_{i=1}^m i = \frac{m(m+1)}{2}$$

$$\textcircled{3} \sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i) \text{ where } c \text{ is a constant and } f \text{ is a function}$$

$$\textcircled{4} \sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$$

where f and g are functions

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n - 1) + \sum_{i=1}^{n-1} 2 + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

$$\sum_{i=1}^{n-1} 2 = ?$$

**What identity
should we use?**

Identities:

① $\sum_{i=1}^m 1 = m$

② $\sum_{i=1}^m i = \frac{m(m+1)}{2}$

③ $\sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i)$ where c is a **constant** and f is a function

④ $\sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$
where f and g are functions

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n-1) + \sum_{i=1}^{n-1} 2 + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

Apply (3) with $a = 1$,
 $b = n-1$, $c = 2$ and $f(i) = 1$

$$\sum_{i=1}^{n-1} 2 = \sum_{i=1}^{n-1} 2 \cdot 1 = 2 \sum_{i=1}^{n-1} 1$$

**What identity
should we use?**

Identities:

① $\sum_{i=1}^m 1 = m$

② $\sum_{i=1}^m i = \frac{m(m+1)}{2}$

③ $\sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i)$ where c is a **constant** and f is a function

④ $\sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$
where f and g are functions

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n-1) + \sum_{i=1}^{n-1} 2 + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

Apply (3) with $a = 1$,
 $b = n-1$, $c = 2$ and $f(i) = 1$

$$\sum_{i=1}^{n-1} 2 = \sum_{i=1}^{n-1} 2 \cdot 1 = 2 \sum_{i=1}^{n-1} 1$$

Apply (1) with $m = n-1$

$$2 \sum_{i=1}^{n-1} 1 = 2(n-1)$$

Identities:

① $\sum_{i=1}^m 1 = m$

② $\sum_{i=1}^m i = \frac{m(m+1)}{2}$

③ $\sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i)$ where c is a **constant** and f is a function

④ $\sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$
where f and g are functions

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n-1) + 2(n-1) + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

$$\sum_{i=0}^{n-1} 1 = ?$$

**What identity
should we use?**

Identities:

① $\sum_{i=1}^m 1 = m$

② $\sum_{i=1}^m i = \frac{m(m+1)}{2}$

③ $\sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i)$ where c is a **constant** and f is a function

④ $\sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$
where f and g are functions

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n-1) + 2(n-1) + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

Apply (1) with $m = n - 1$

$$\sum_{i=0}^{n-1} 1 = 1 + \sum_{i=1}^{n-1} 1 = (n-1) + 1 = n$$

Identities:

$$\textcircled{1} \sum_{i=1}^m 1 = m$$

$$\textcircled{2} \sum_{i=1}^m i = \frac{m(m+1)}{2}$$

$$\textcircled{3} \sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i) \text{ where } c \text{ is a constant and } f \text{ is a function}$$

$$\textcircled{4} \sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$$

where f and g are functions

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n-1) + 2(n-1) + n + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

$$\sum_{i=0}^{n-1} \sum_{j=0}^i 1 = ?$$

**What identity
should we use?**

Identities:

① $\sum_{i=1}^m 1 = m$

② $\sum_{i=1}^m i = \frac{m(m+1)}{2}$

③ $\sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i)$ where c is a **constant** and f is a function

④ $\sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$
where f and g are functions

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n-1) + 2(n-1) + n + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

Apply (1) with $m = i$

$$\begin{aligned} \sum_{i=0}^{n-1} \sum_{j=0}^i 1 &= \sum_{i=0}^{n-1} \left(1 + \sum_{j=1}^i 1 \right) \\ &= \sum_{i=0}^{n-1} (i+1) \end{aligned}$$

**What identity
should we use?**

Identities:

① $\sum_{i=1}^m 1 = m$

② $\sum_{i=1}^m i = \frac{m(m+1)}{2}$

③ $\sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i)$ where c is a **constant** and f is a function

④ $\sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$
where f and g are functions

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n-1) + 2(n-1) + n + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

Apply (4) with $a = 0$,
 $b = n - 1$, $f(i) = i$ and
 $g(i) = 1$

$$\sum_{i=0}^{n-1} (i+1) = \sum_{i=0}^{n-1} i + \sum_{i=0}^{n-1} 1$$

**What identities
should we use?**

Identities:

① $\sum_{i=1}^m 1 = m$

② $\sum_{i=1}^m i = \frac{m(m+1)}{2}$

③ $\sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i)$ where c is a
constant and f is a function

④ $\sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$
where f and g are functions

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n-1) + 2(n-1) + n + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

Apply (1) with $m = n - 1$

$$\begin{aligned} \sum_{i=0}^{n-1} 1 &= 1 + \sum_{i=1}^{n-1} 1 \\ &= 1 + (n-1) = n \end{aligned}$$

so

$$\sum_{i=0}^{n-1} i + \sum_{i=0}^{n-1} 1 = n + \sum_{i=0}^{n-1} i$$

Identities:

$$\textcircled{1} \sum_{i=1}^m 1 = m$$

$$\textcircled{2} \sum_{i=1}^m i = \frac{m(m+1)}{2}$$

$$\textcircled{3} \sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i) \text{ where } c \text{ is a constant and } f \text{ is a function}$$

$$\textcircled{4} \sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i) \\ \text{where } f \text{ and } g \text{ are functions}$$

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n-1) + 2(n-1) + n + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

Apply (2) with $m = n - 1$

$$\begin{aligned} \sum_{i=0}^{n-1} i &= 0 + \sum_{i=1}^{n-1} i \\ &= \frac{(n-1)((n-1) + 1)}{2} \end{aligned}$$

so

$$n + \sum_{i=0}^{n-1} i = n + \frac{n(n-1)}{2}$$

Identities:

- ① $\sum_{i=1}^m 1 = m$
- ② $\sum_{i=1}^m i = \frac{m(m+1)}{2}$
- ③ $\sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i)$ where c is a **constant** and f is a function
- ④ $\sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$
where f and g are functions

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n-1) + 2(n-1) + n + \left(n + \frac{n(n-1)}{2}\right) + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

$$\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1 = ?$$

**What is the
simplified form?**

Identities:

$$\textcircled{1} \sum_{i=1}^m 1 = m$$

$$\textcircled{2} \sum_{i=1}^m i = \frac{m(m+1)}{2}$$

$$\textcircled{3} \sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i) \text{ where } c \text{ is a constant and } f \text{ is a function}$$

$$\textcircled{4} \sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$$

where f and g are functions

Simplifying the Cost Function

Addition Cost Function

$$T(n) = (n-1) + 2(n-1) + n + \left(n + \frac{n(n-1)}{2}\right) + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Simplification:

Apply (1) then (2)

$$\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1 = \sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$$

Identities:

$$\textcircled{1} \sum_{i=1}^m 1 = m$$

$$\textcircled{2} \sum_{i=1}^m i = \frac{m(m+1)}{2}$$

$$\textcircled{3} \sum_{i=a}^b cf(i) = c \sum_{i=a}^b f(i) \text{ where } c \text{ is a constant and } f \text{ is a function}$$

$$\textcircled{4} \sum_{i=a}^b (f(i) + g(i)) = \sum_{i=a}^b f(i) + \sum_{i=a}^b g(i)$$

where f and g are functions

Simplifying the Cost Function

Simplification:

We have that

$$\begin{aligned}T(n) &= (n-1) + 2(n-1) + n + \left(n + \frac{n(n-1)}{2}\right) + \frac{n(n-1)}{2} \\&= n-1 + 2n-2 + n + n + n(n-1) \\&= n^2 + 4n - 3\end{aligned}$$

so the final answer is

$$T(n) = n^2 + 4n - 3$$

Simplifying the Cost Function

Multiplication Cost Function

$$T(n) = \sum_{i=1}^{n-1} 1$$

Comparisons Cost Function

$$T(n) = \sum_{i=1}^n 1 + \sum_{i=0}^n 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1$$

Memory Accesses Cost Function

$$T(n) = 1 + \sum_{i=1}^{n-1} 2 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Assignments Cost Function

$$T(n) = 3 + \sum_{i=1}^{n-1} 1 + \sum_{i=1}^{n-1} 1 + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1$$

Try to simplify the other cost functions!

Simplifying the Cost Function

Multiplication Cost Function

$$\begin{aligned} T(n) &= \sum_{i=1}^{n-1} 1 \\ &= n - 1 \end{aligned}$$

Simplifying the Cost Function

Comparisons Cost Function

$$\begin{aligned}T(n) &= \sum_{i=1}^n 1 + \sum_{i=0}^n 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 \\&= n + (n + 1) + \frac{n(n + 1)}{2} \\&= \frac{1}{2}n^2 + \frac{5}{2}n + 1\end{aligned}$$

Simplifying the Cost Function

Memory Accesses Cost Function

$$\begin{aligned}T(n) &= 1 + \sum_{i=1}^{n-1} 2 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1 \\&= 1 + 2(n-1) + \frac{n(n-1)}{2} \\&= \frac{1}{2}n^2 + \frac{3}{2}n - 1\end{aligned}$$

Simplifying the Cost Function

Assignments Cost Function

$$\begin{aligned}T(n) &= 3 + \sum_{i=1}^{n-1} 1 + \sum_{i=1}^{n-1} 1 + \sum_{i=0}^{n-1} 1 + \sum_{i=0}^{n-1} \sum_{j=0}^i 1 + \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} 1 \\&= 3 + (n-1) + (n-1) + n + \frac{n(n+1)}{2} + \frac{n(n-1)}{2} \\&= n^2 + 3n + 1\end{aligned}$$

Pitfall: Cannot represent loops with non-one increments using standard summations!

```
int baz(int n) {  
    int total = 0;  
    for (int i = 0; i < n; i += 2)  
        total += i * i;  
    return total;  
}
```

Solution: Compute number of operations for small values of n , then guess and check the formula OR learn Knuth's summation notation (advanced)!

Extra Practice

```
int bar(int n) {  
    int* A = new int[n];  
  
    for (int i = 0; i < n; i++)  
        A[i] = i + 1;  
  
    int result = 0;  
    for (int k = 0; k < n * n; k++)  
        for (int j = 0; j <= k; j++)  
            for (int i = j; i < n; i++)  
                result += A[i];  
  
    for (int t = 0; t < 7; t++)  
        result += A[0];  
  
    delete[] A;  
    return result;  
}
```

**Choose a model of
computation and
analyze this algorithm!**