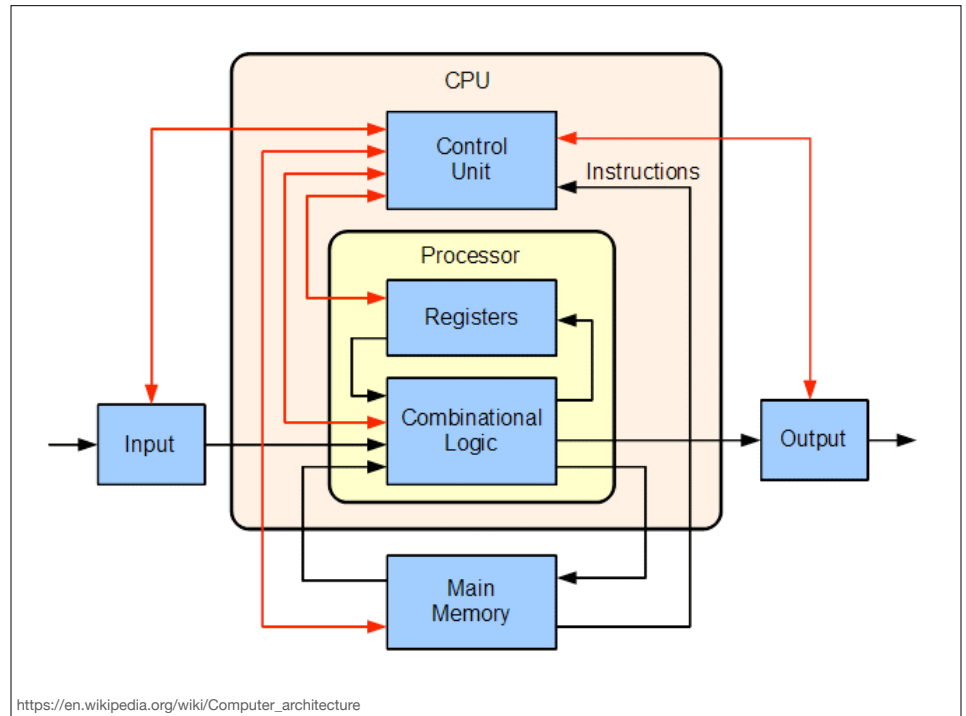


CSC 411

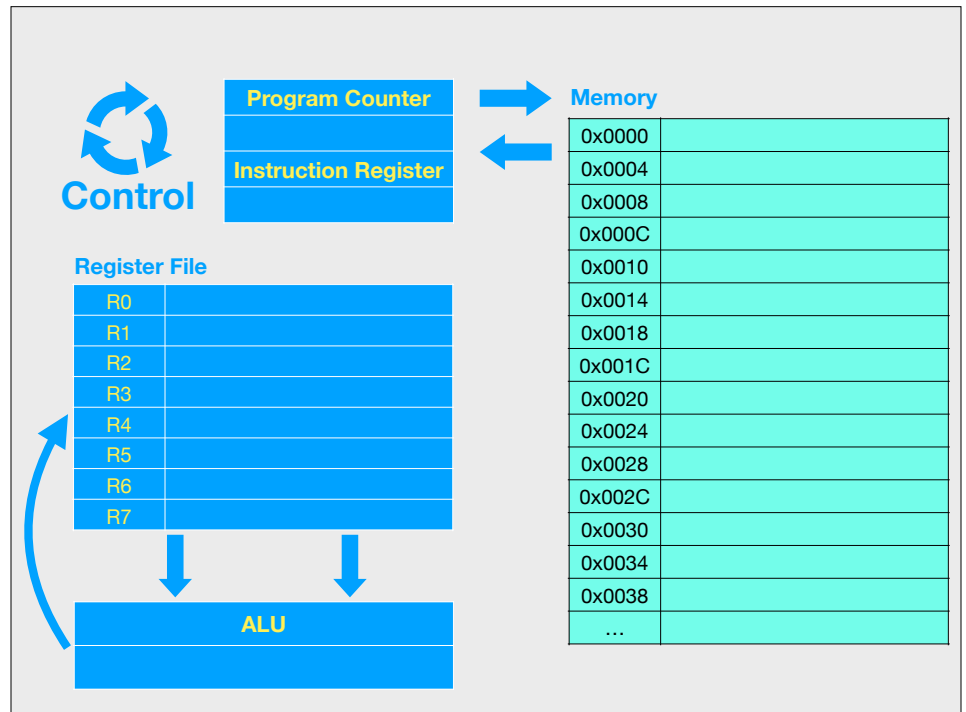
Computer Organization (Spring 2022)

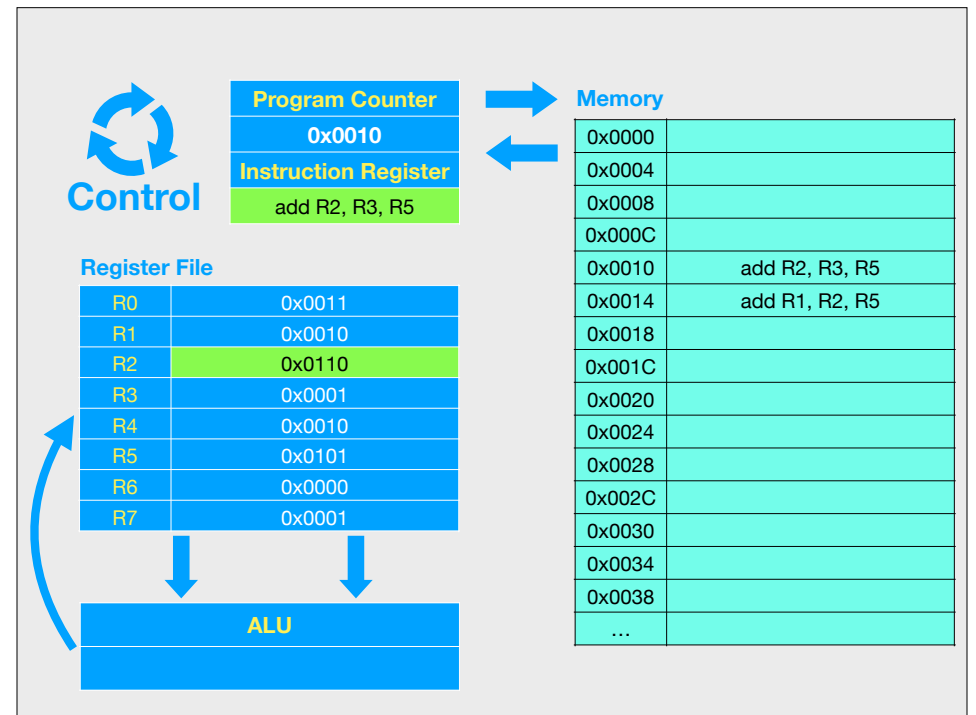
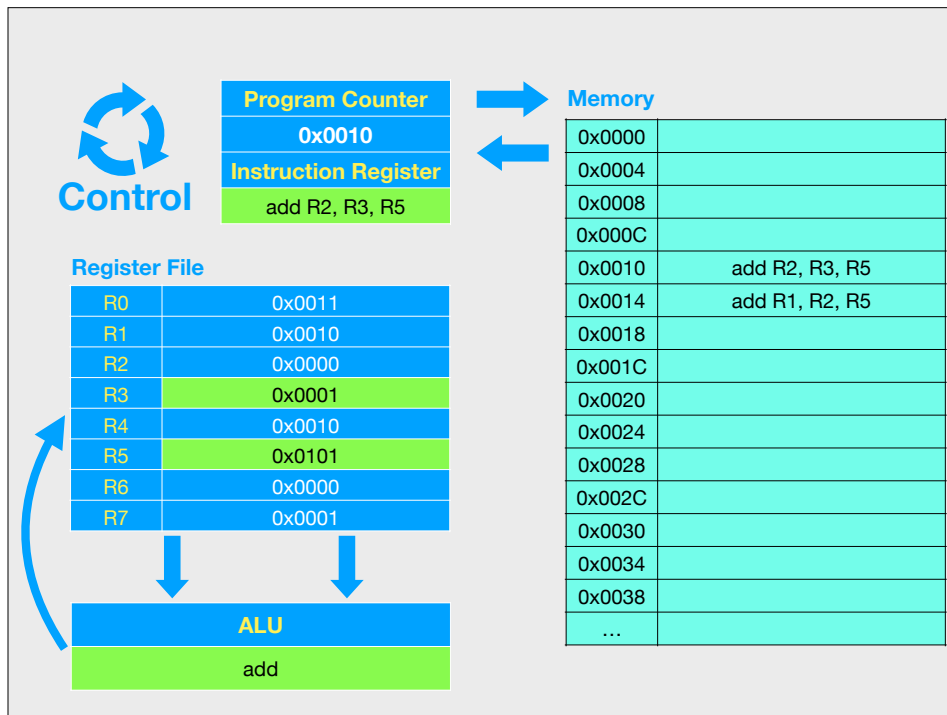
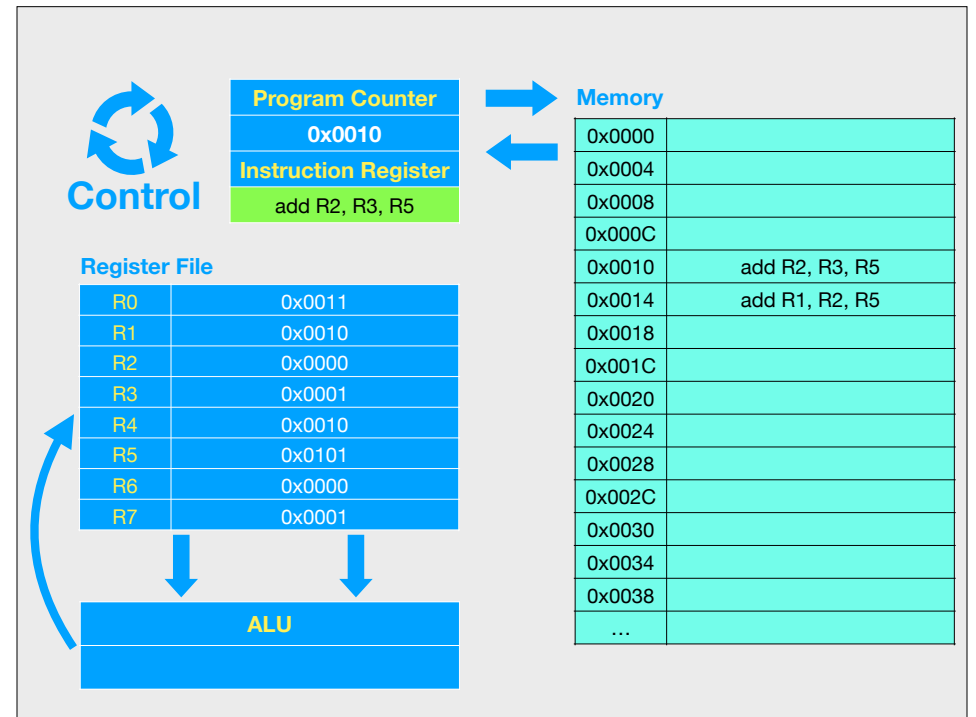
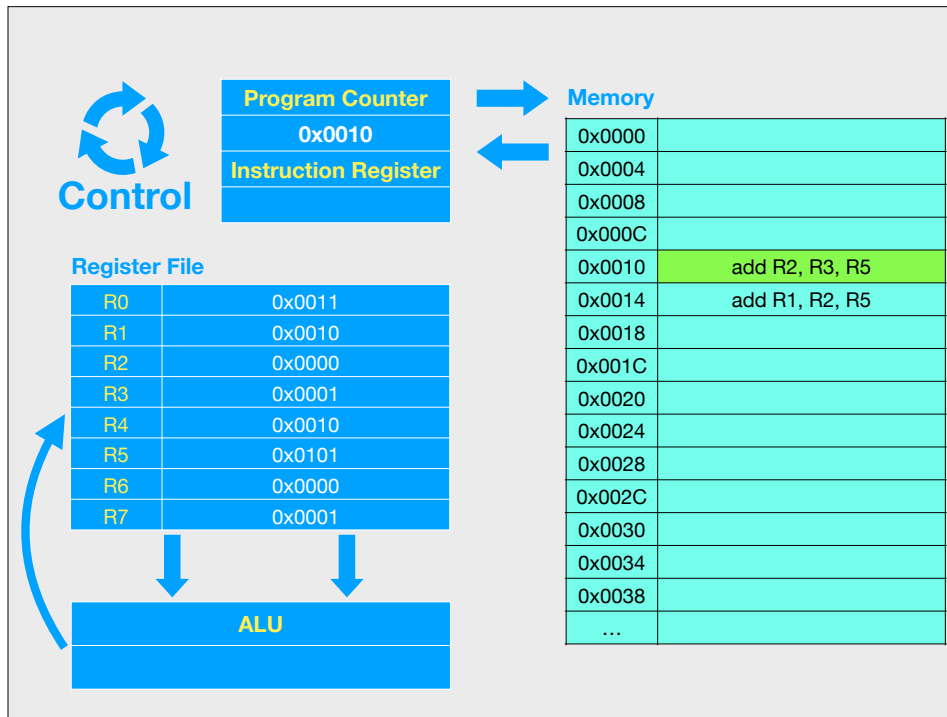
Lecture 6: Executing Instructions and Integer Representation

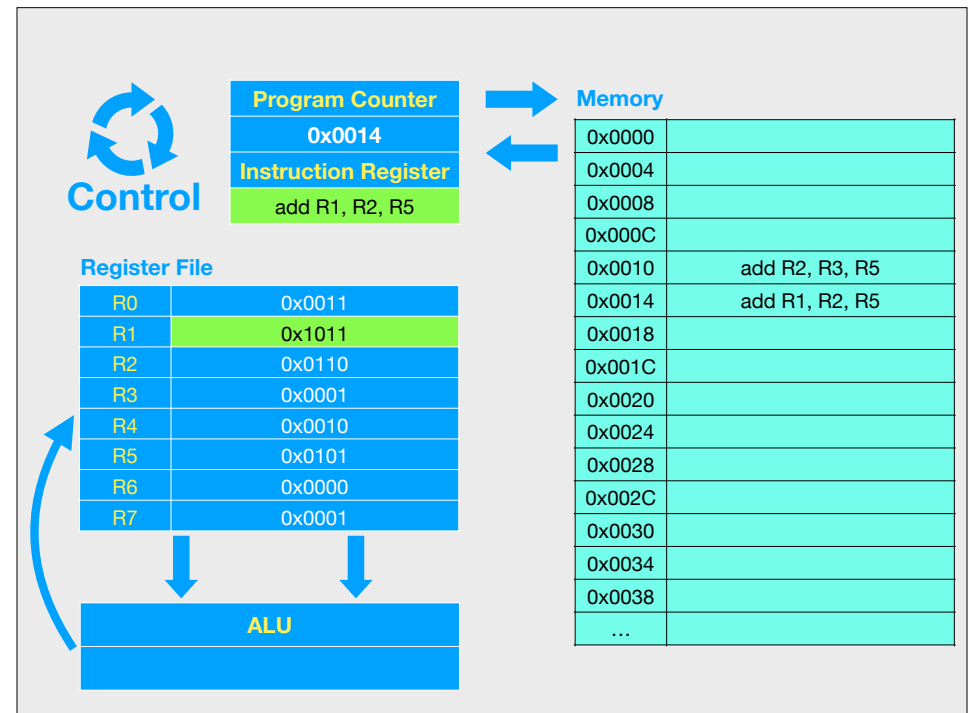
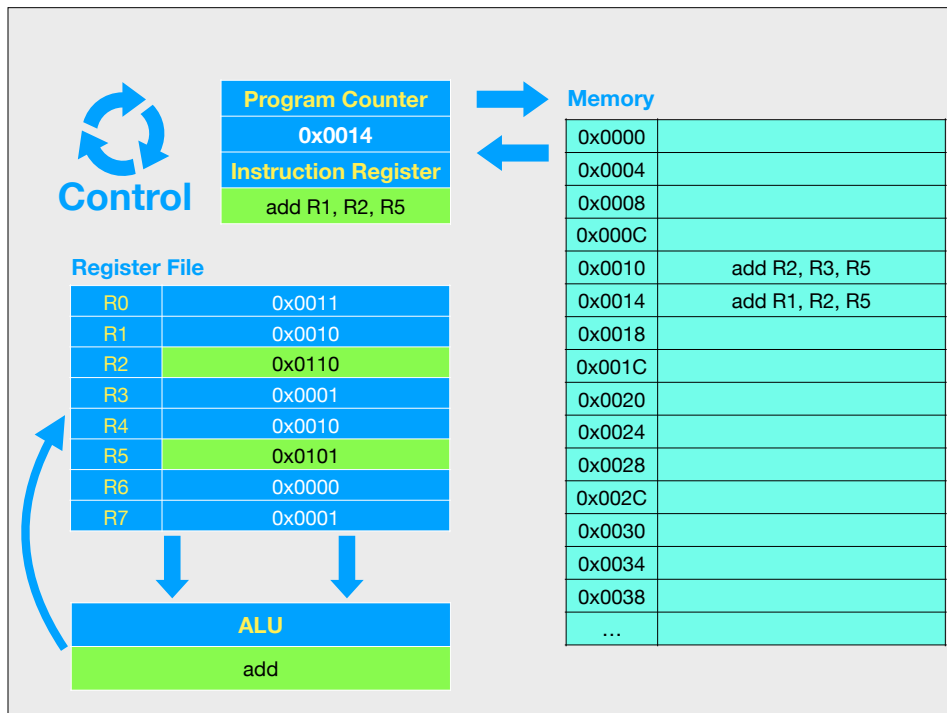
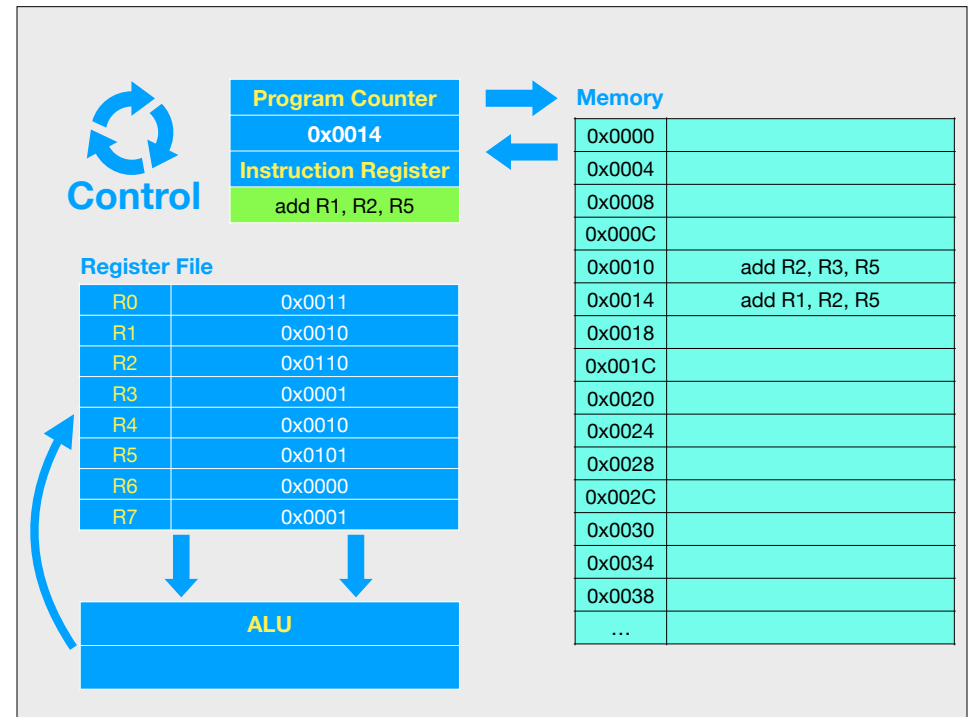
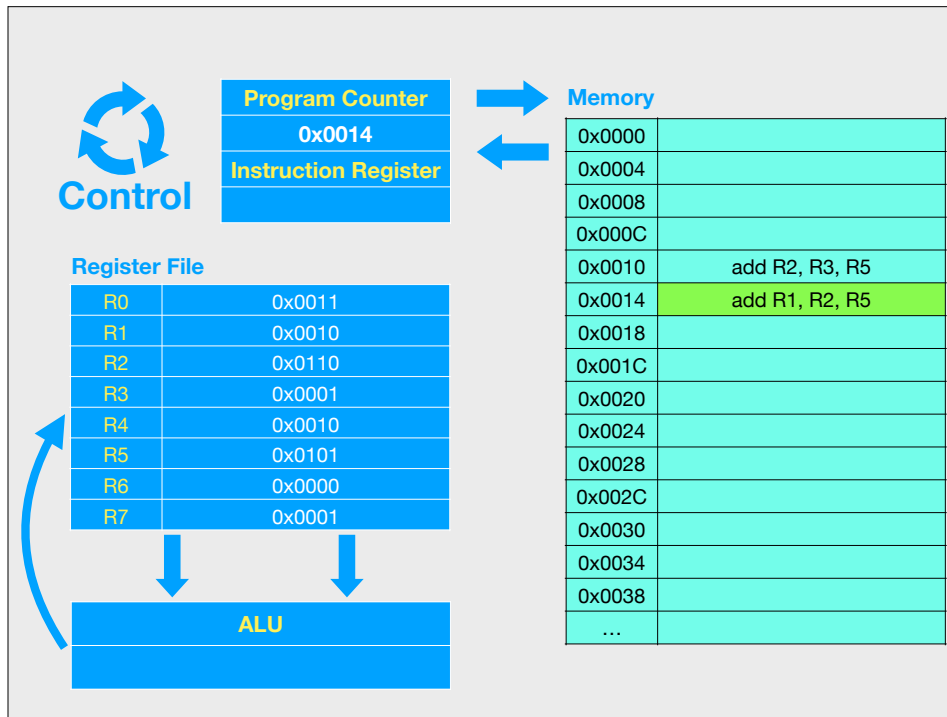
Prof. Marco Alvarez, University of Rhode Island



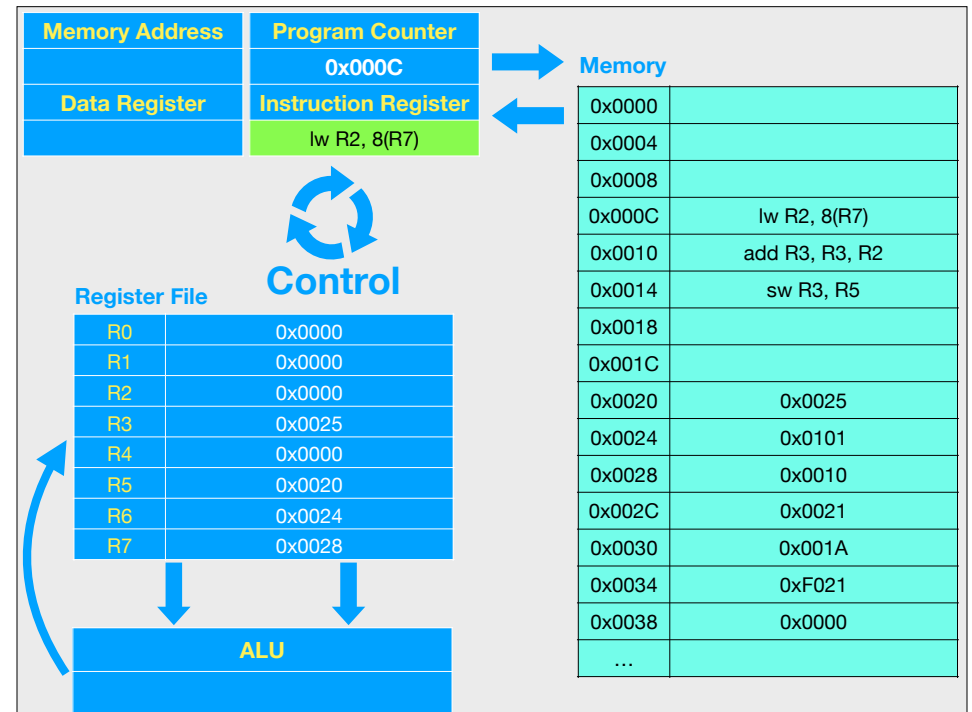
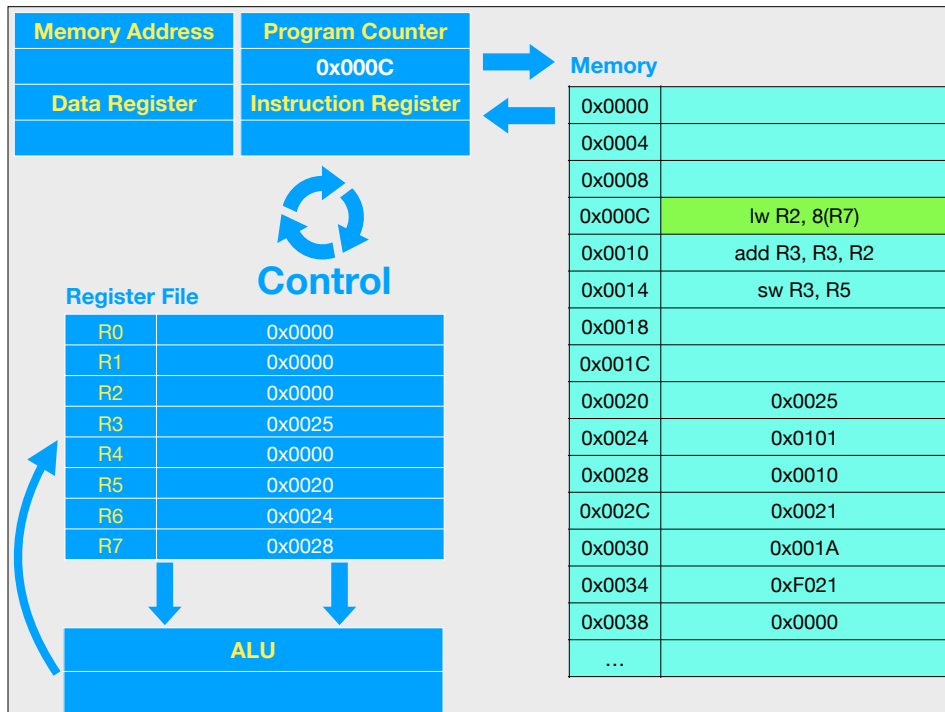
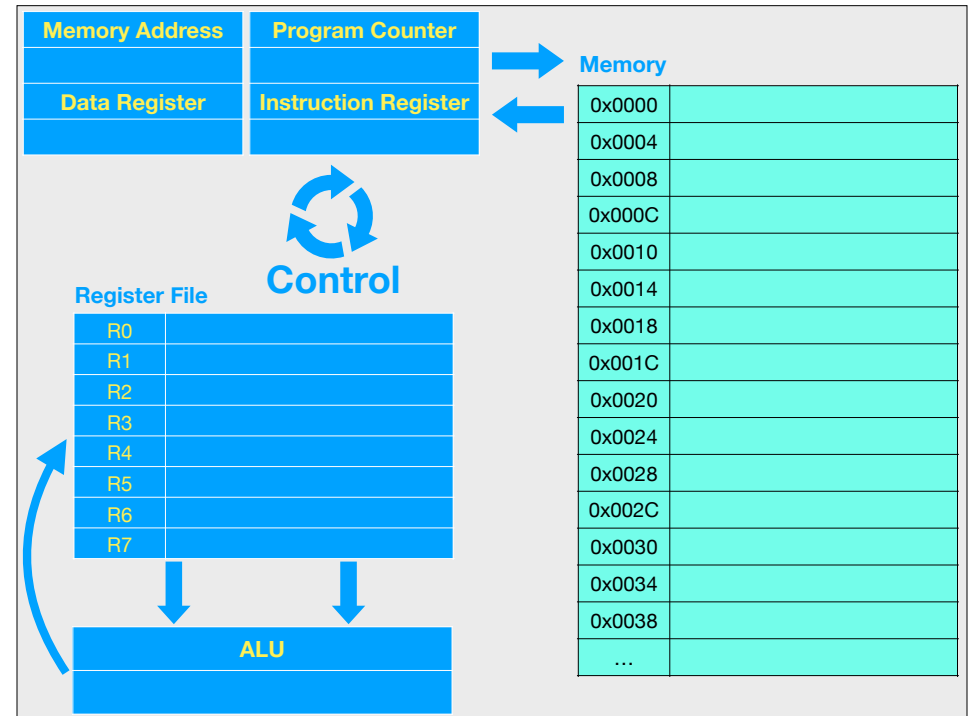
Executing instructions

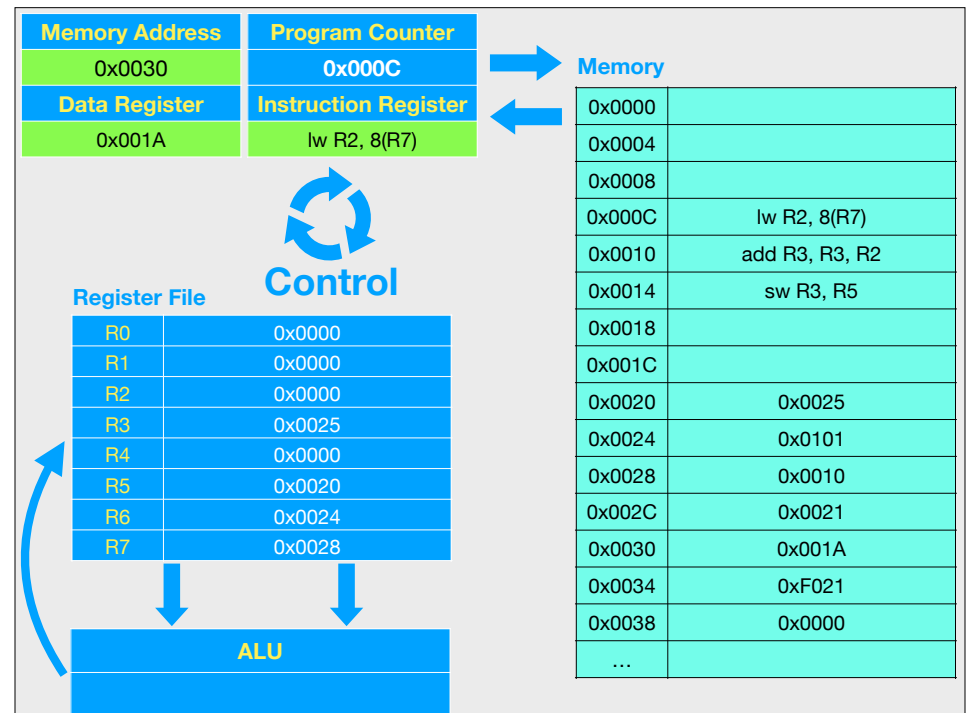
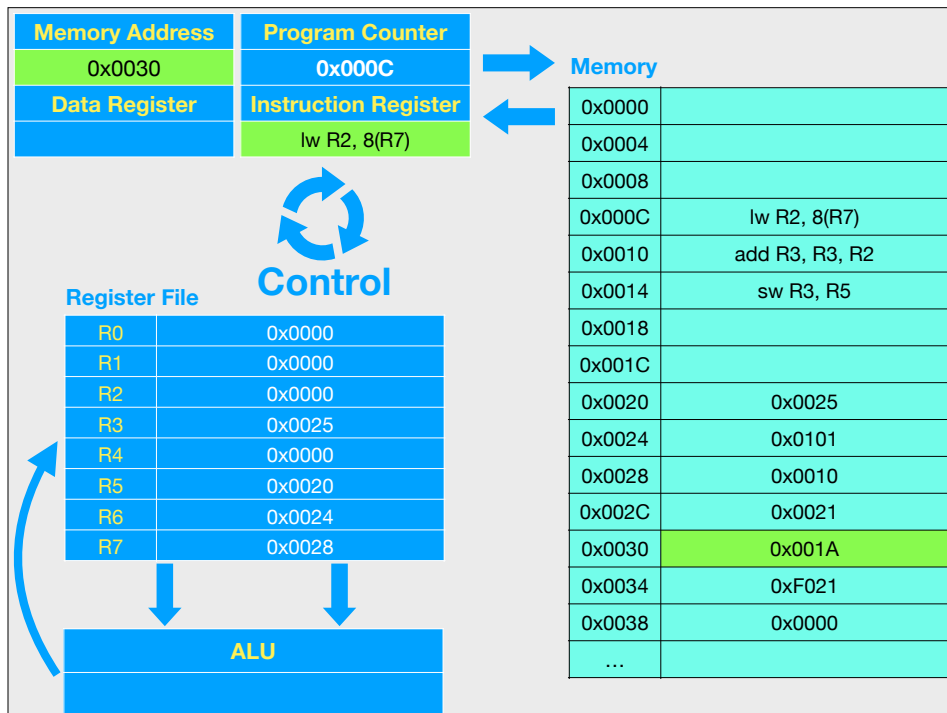
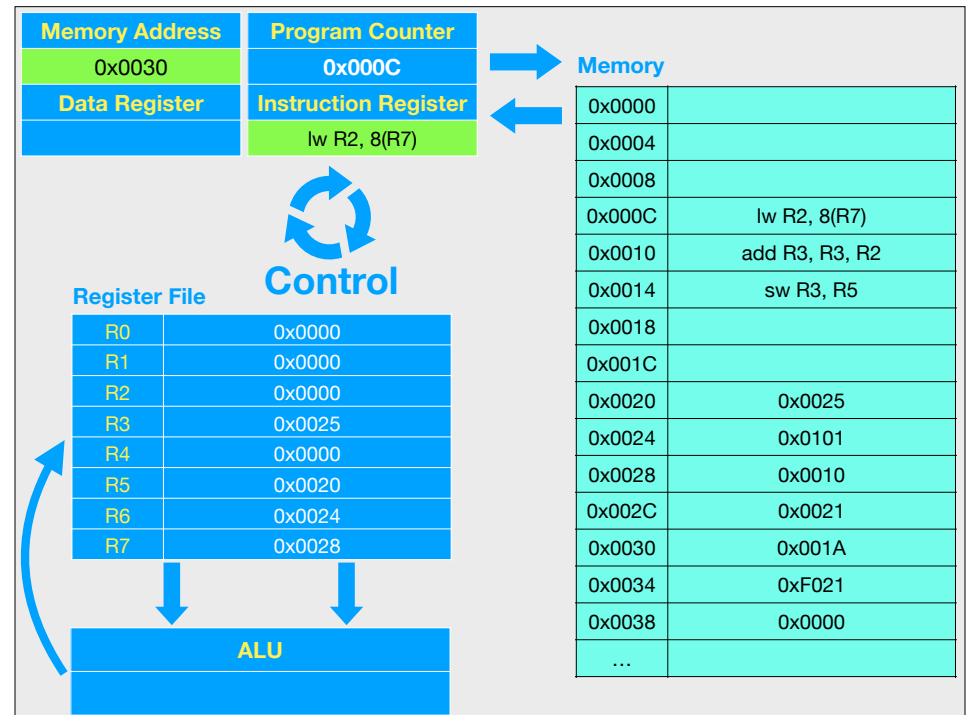
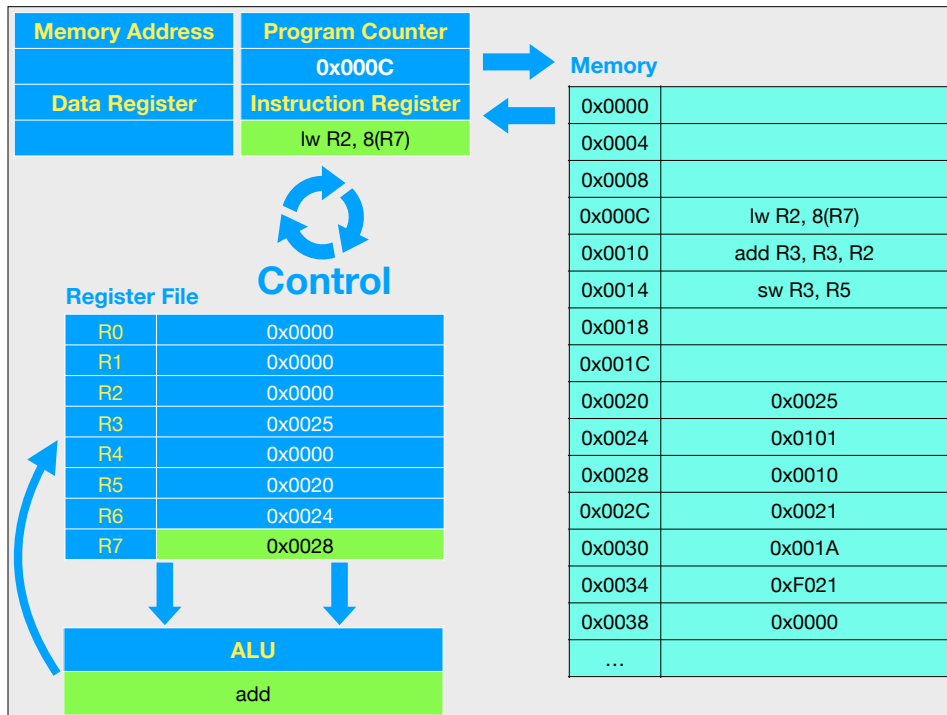


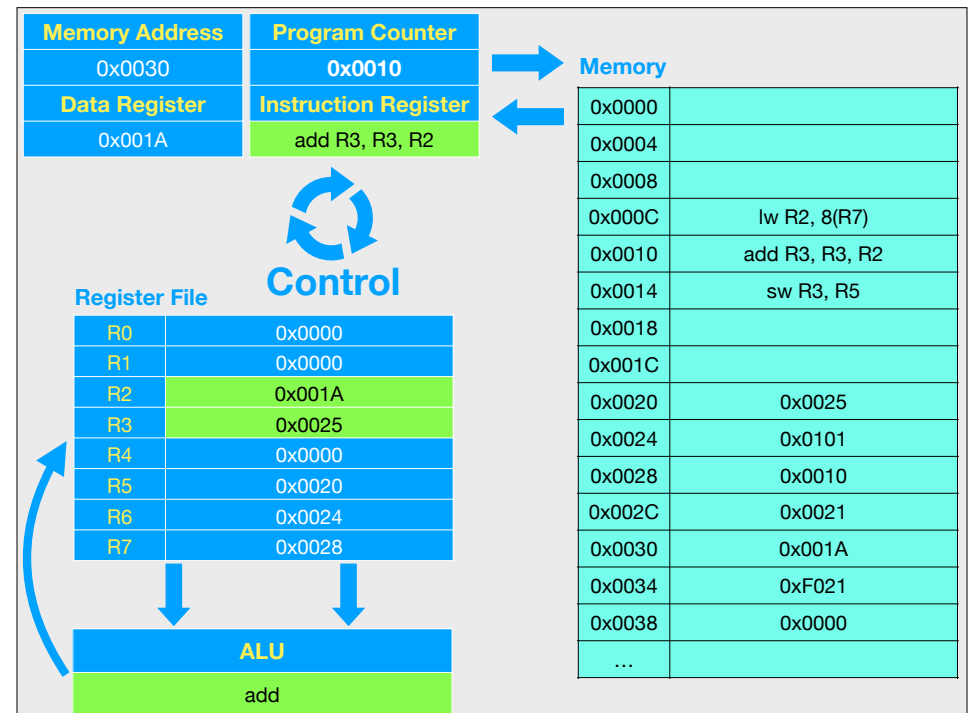
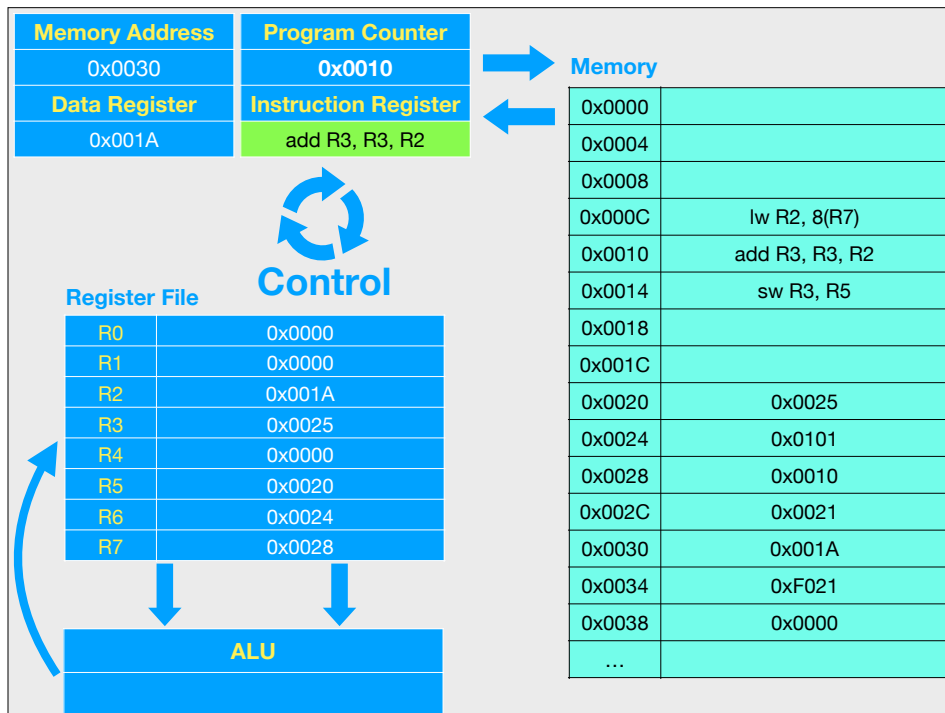
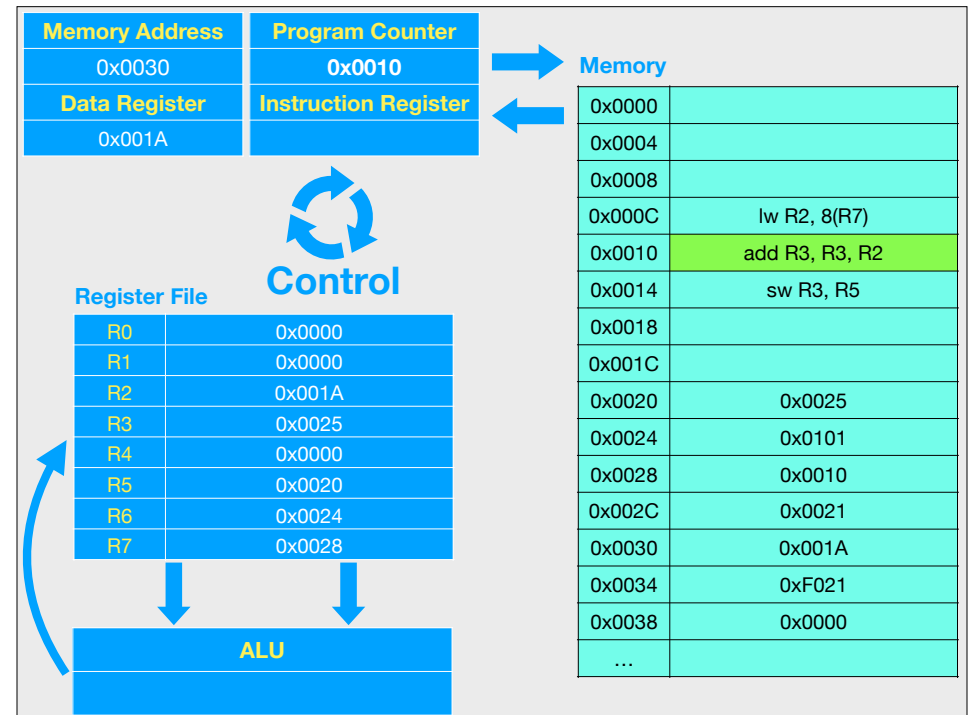
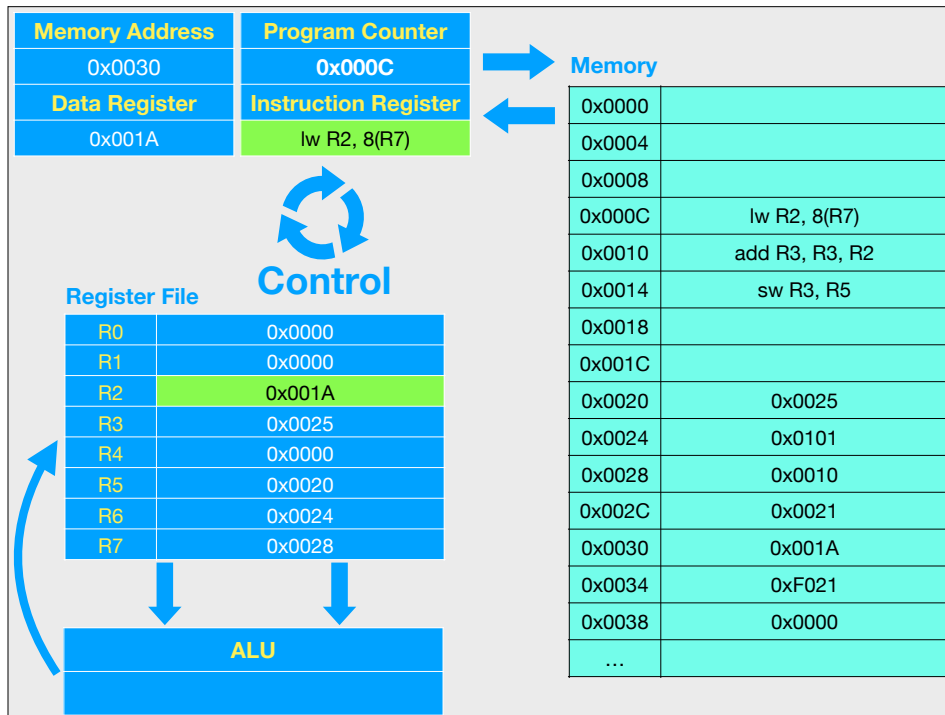


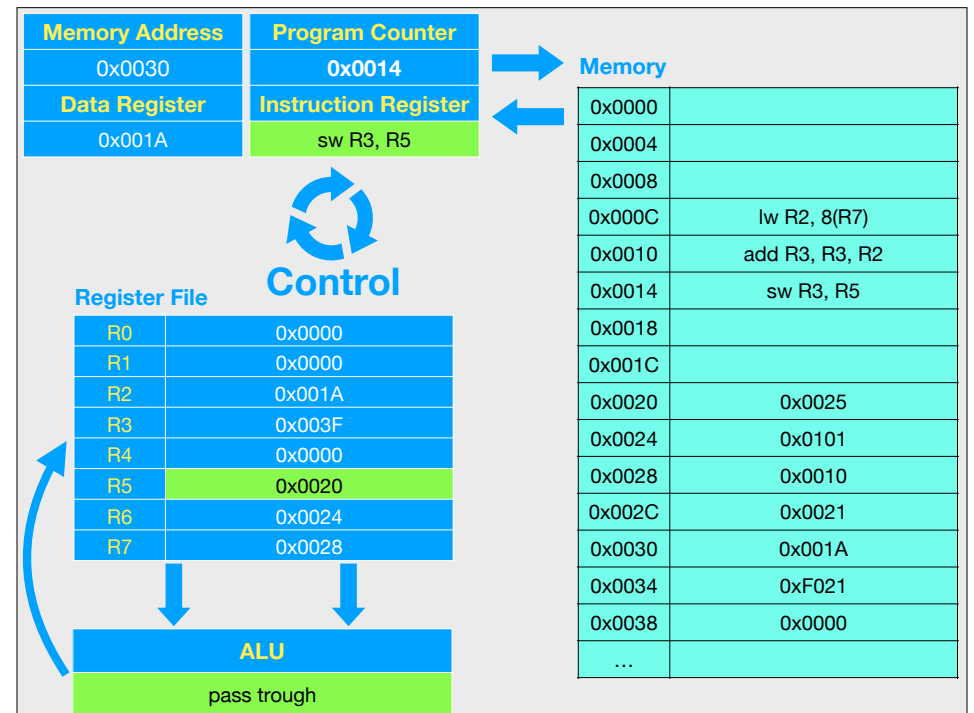
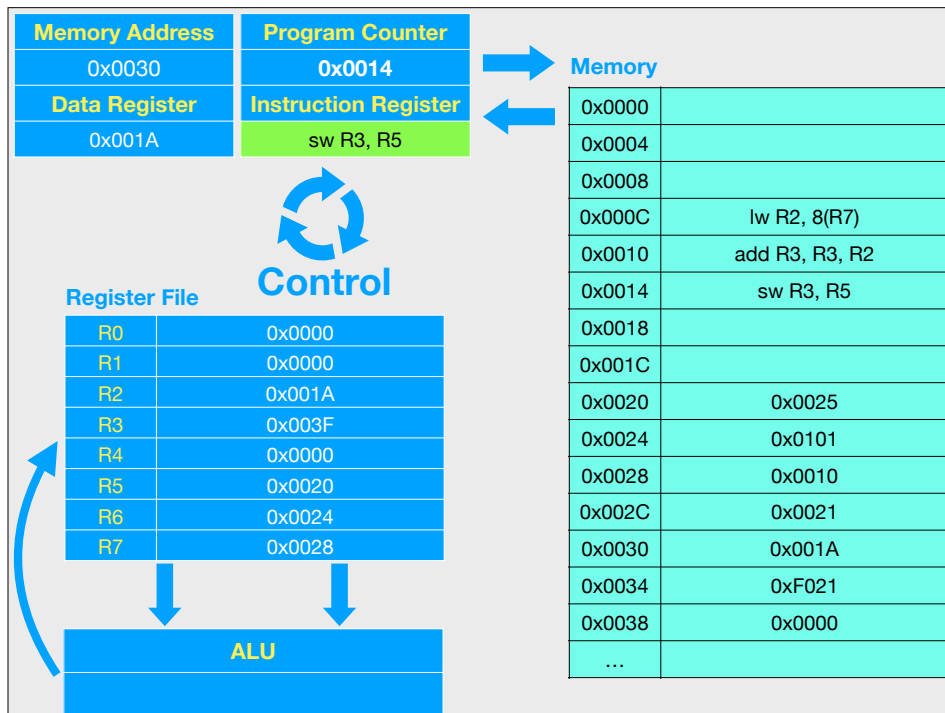
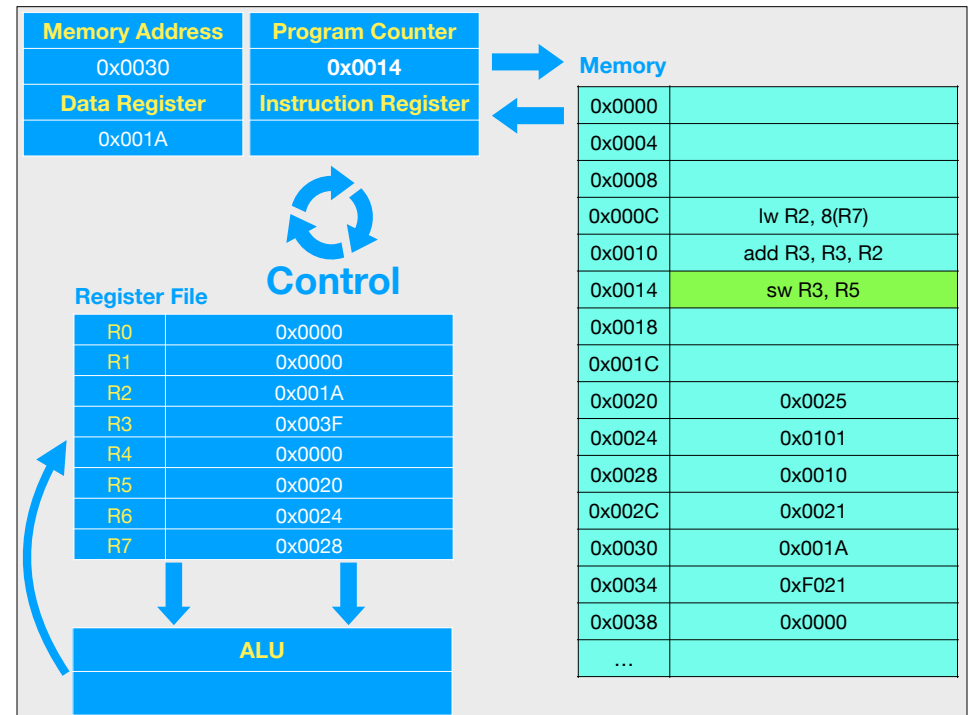
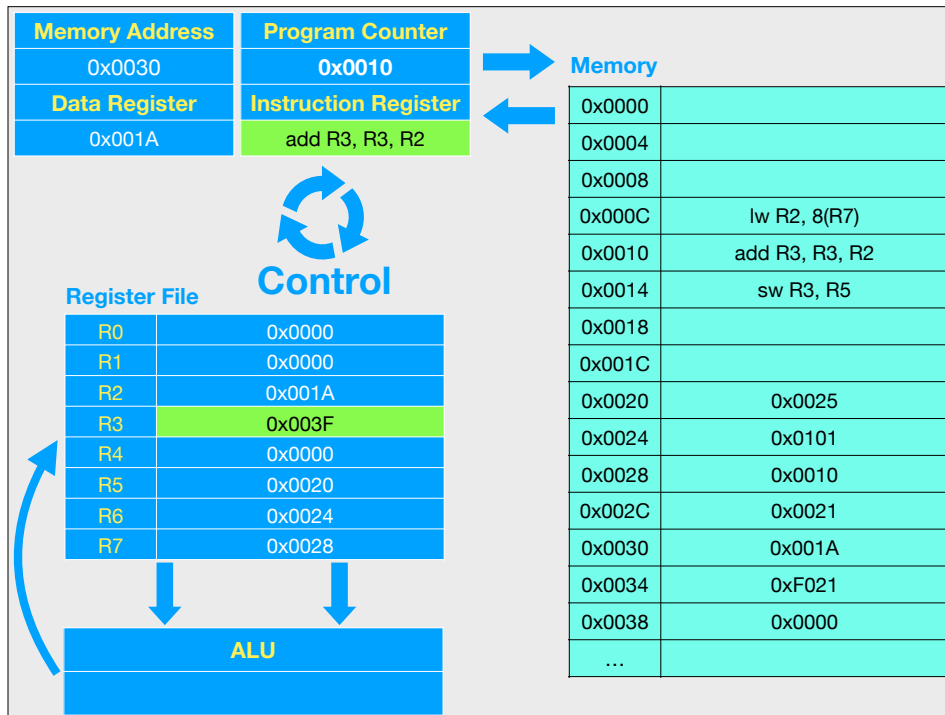


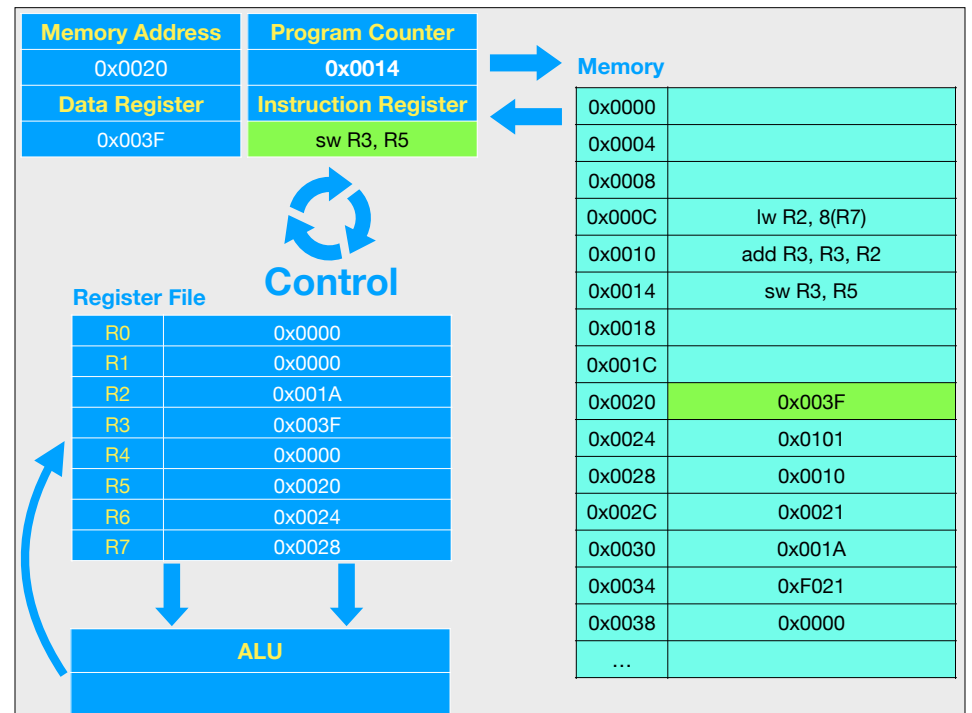
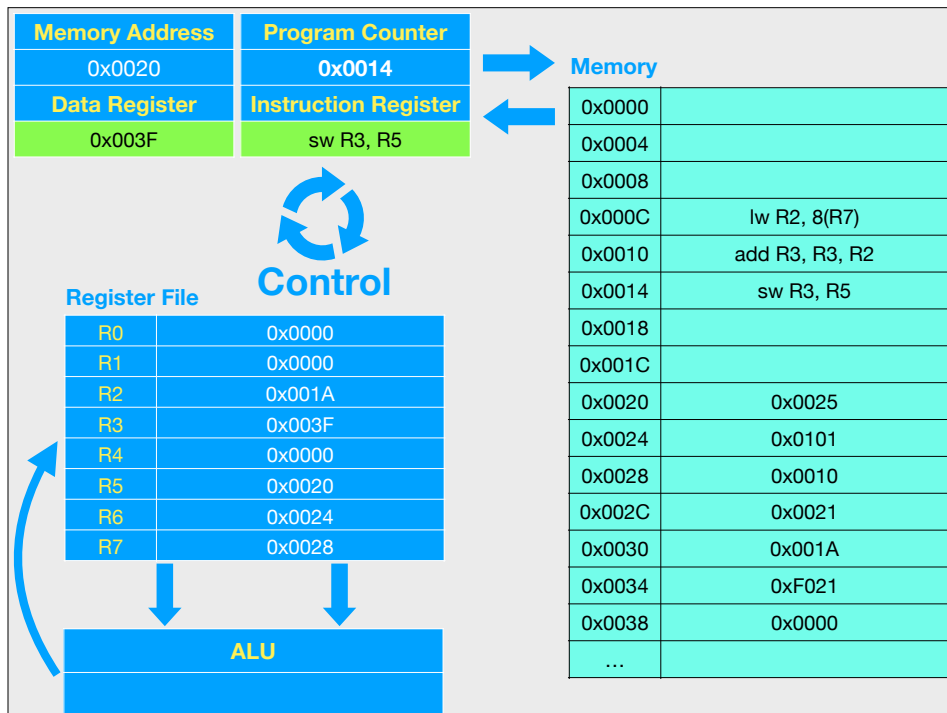
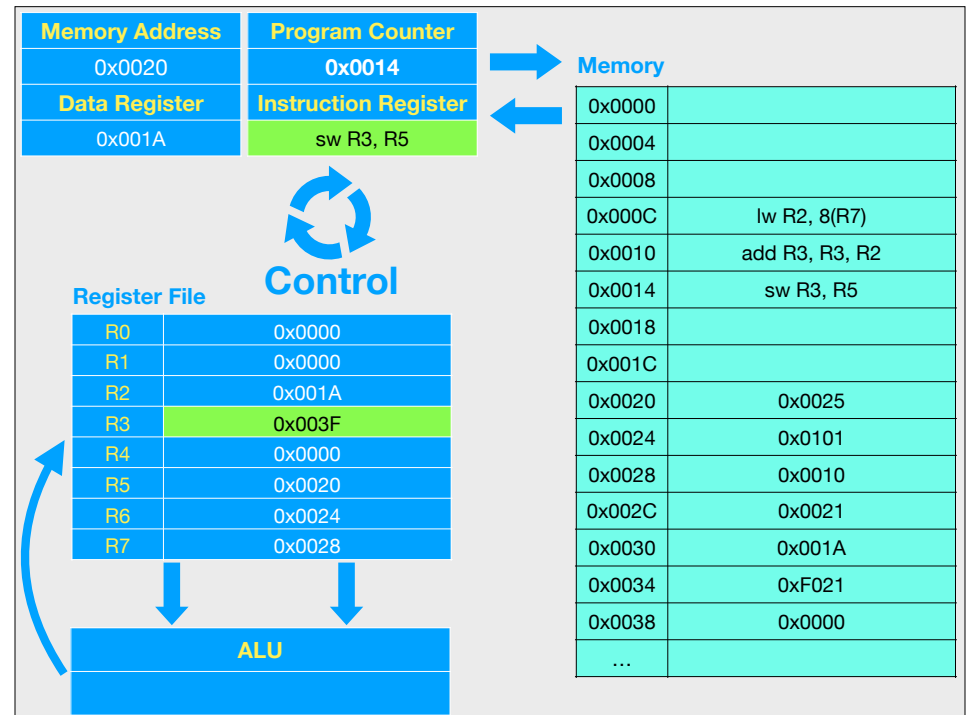
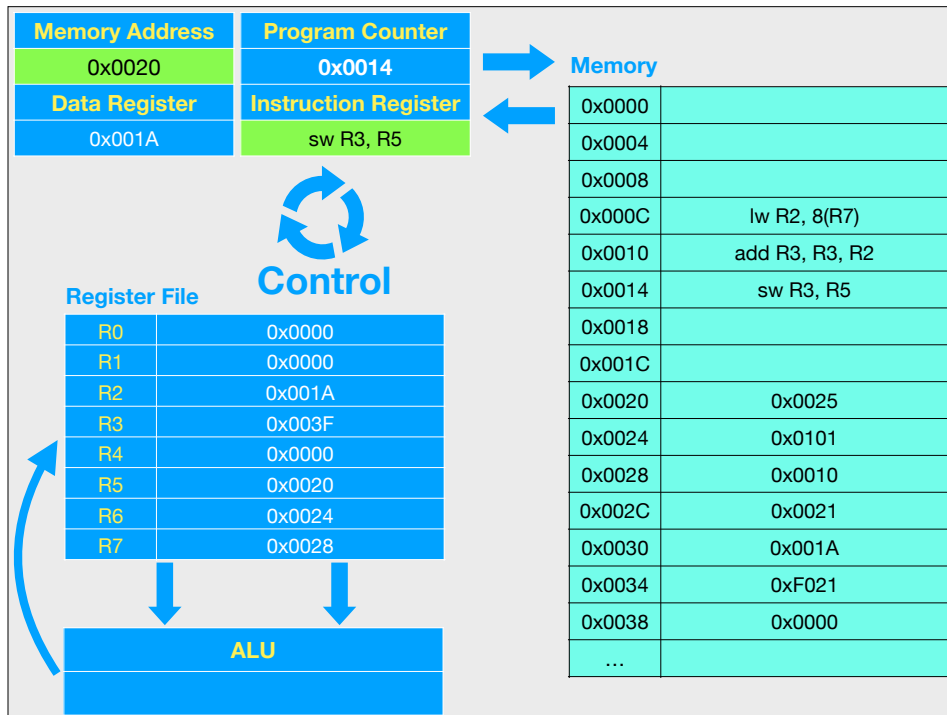
Executing memory instructions











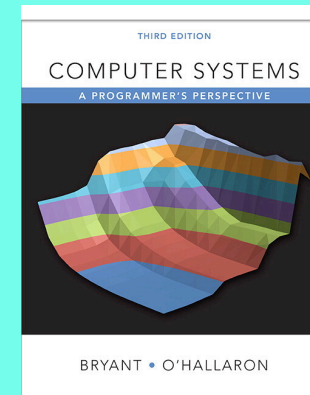
Representing integers

Disclaimer

The following slides are from:

Computer Systems (Bryant and O'Hallaron)

A Programmer's Perspective



Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - Summary
- Representations in memory, pointers, strings
- Summary

Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

```
short int x = 15213;  
short int y = -15213;
```

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

- C does not mandate using two's complement
 - But, most machines do, and we will assume so
- C short 2 bytes long

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
y	-15213	C4 93	11000100 10010011

- Sign Bit
 - For 2's complement, most significant bit indicates sign
 - 0 for nonnegative
 - 1 for negative

Two-complement: Simple Example

$$10 = \begin{matrix} & -16 & 8 & 4 & 2 & 1 \\ 0 & 1 & 0 & 1 & 0 & \end{matrix} \quad 8+2 = 10$$

$$-10 = \begin{matrix} & -16 & 8 & 4 & 2 & 1 \\ 1 & 0 & 1 & 1 & 0 & \end{matrix} \quad -16+4+2 = -10$$

Two-complement Encoding Example (Cont.)

x = 15213: 00111011 01101101
y = -15213: 11000100 10010011

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum	15213		-15213	

Numeric Ranges

■ Unsigned Values

- $UMin = 0$
000...0
- $UMax = 2^w - 1$
111...1

■ Two's Complement Values

- $TMin = -2^{w-1}$
100...0
- $TMax = 2^{w-1} - 1$
011...1
- Minus 1
111...1

Values for $W = 16$

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

Values for Different Word Sizes

	W			
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

■ Observations

- $|TMin| = TMax + 1$
 - Asymmetric range
- $UMax = 2 * TMax + 1$
- Question: $abs(TMin)$?

■ C Programming

- `#include <limits.h>`
- Declares constants, e.g.,
 - `ULONG_MAX`
 - `LONG_MAX`
 - `LONG_MIN`
- Values platform specific

Unsigned & Signed Numeric Values

X	B2U(X)	B2T(X)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

■ Equivalence

- Same encodings for nonnegative values

■ Uniqueness

- Every bit pattern represents unique integer value
- Each representable integer has unique bit encoding

■ ⇒ Can Invert Mappings

- $U2B(x) = B2U^{-1}(x)$
 - Bit pattern for unsigned integer
- $T2B(x) = B2T^{-1}(x)$
 - Bit pattern for two's comp integer

Today: Bits, Bytes, and Integers

■ Representing information as bits

■ Bit-level manipulations

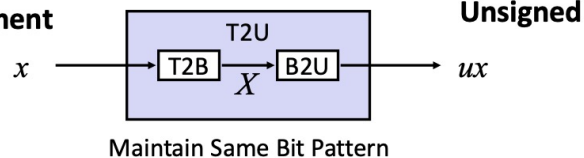
■ Integers

- Representation: unsigned and signed
- Conversion, casting
- Expanding, truncating
- Addition, negation, multiplication, shifting
- Summary

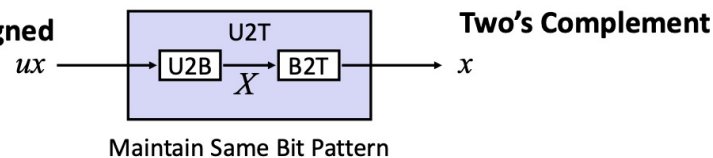
■ Representations in memory, pointers, strings

Mapping Between Signed & Unsigned

Two's Complement



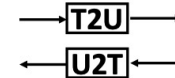
Unsigned



- Mappings between unsigned and two's complement numbers:
Keep bit representations and reinterpret

Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15



Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

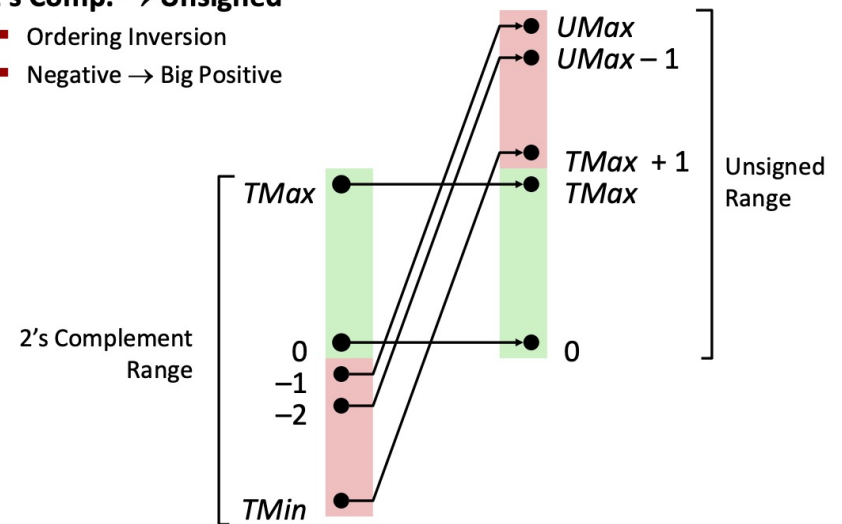
↔ =

↔ +/- 16

Conversion Visualized

2's Comp. → Unsigned

- Ordering Inversion
- Negative → Big Positive



Signed vs. Unsigned in C

Constants

- By default are considered to be signed
- Unsigned if have "U" as suffix, or if too big to be signed
`0U, 2147483648`
- Watch out! A leading minus sign is not part of the constant!
`-2147483648 == 2147483648` on 32-bit machines (why?)

Casting

- Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```
- Implicit casting also occurs via assignments and procedure calls

```
tx = ux;           int fun(unsigned u);
uy = ty;           uy = fun(tx);
```

Casting Surprises

Expression Evaluation

- If there is a mix of unsigned and signed in single expression, **signed values implicitly cast to unsigned**
- Including comparison operations `<`, `>`, `==`, `<=`, `>=`
- Examples for $W = 32$: **$TMIN = -2,147,483,648$, $TMAX = 2,147,483,647$**

Constant ₁	Constant ₂	Relation	Evaluation
0	0U	==	unsigned
-1	0	<	signed
-1	0U	>	unsigned
2147483647	-2147483647-1	>	signed
2147483647U	-2147483647-1	<	unsigned
-1	-2	>	signed
(unsigned)-1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int) 2147483648U	>	signed

Summary

Casting Signed \leftrightarrow Unsigned: Basic Rules

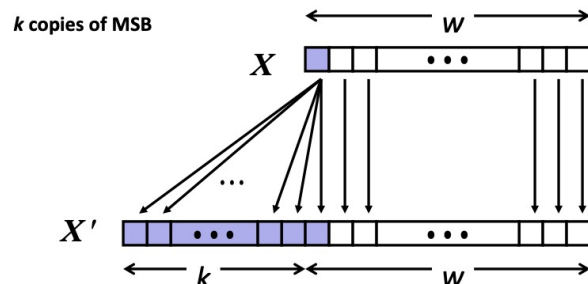
- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w
- Expression containing signed and unsigned int
 - `int` is cast to `unsigned`!!

Today: Bits, Bytes, and Integers

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Sign Extension

- Task:
 - Given w -bit signed integer x
 - Convert it to $w+k$ -bit integer with same value
- Rule:
 - Make k copies of sign bit:
 - $X' = \underbrace{X_{w-1}, \dots, X_{w-1}}_{k \text{ copies of MSB}}, X_{w-1}, X_{w-2}, \dots, X_0$



Sign Extension: Simple Example

Positive number

	-16	8	4	2	1	
10 =	0	1	0	1	0	
	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

Negative number

	-16	8	4	2	1	
-10 =	1	0	1	1	0	
	-32	16	8	4	2	1
-10 =	1	1	0	1	1	0

Larger Sign Extension Example

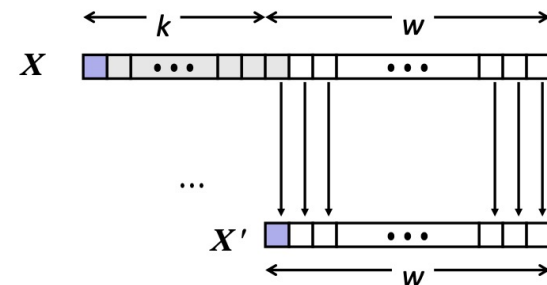
```
short int x = 15213;
int      ix = (int) x;
short int y = -15213;
int      iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Converting from smaller to larger integer data type
- C automatically performs sign extension

Truncation

- **Task:**
 - Given $k+w$ -bit signed or unsigned integer X
 - Convert it to w -bit integer X' with same value for "small enough" X
- **Rule:**
 - Drop top k bits:
 - $X' = x_{w-1}, x_{w-2}, \dots, x_0$



Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

$$2 \bmod 16 = 2$$

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

$$-6 \bmod 16 = 26U \bmod 16 = 10U = -6$$

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

$$10 \bmod 16 = 10U \bmod 16 = 10U = -6$$

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

$$-10 \bmod 16 = 22U \bmod 16 = 6U = 6$$

Summary: Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
 - Unsigned: zeros added
 - Signed: sign extension
 - Both yield expected result
- **Truncating (e.g., unsigned to unsigned short)**
 - Unsigned/signed: bits are truncated
 - Result reinterpreted
 - Unsigned: mod operation
 - Signed: similar to mod
 - For small (in magnitude) numbers yields expected behavior