CSC 411

Computer Organization (Spring 2022)
Lecture 2: Number Systems, Bitwise Operations

Number Systems

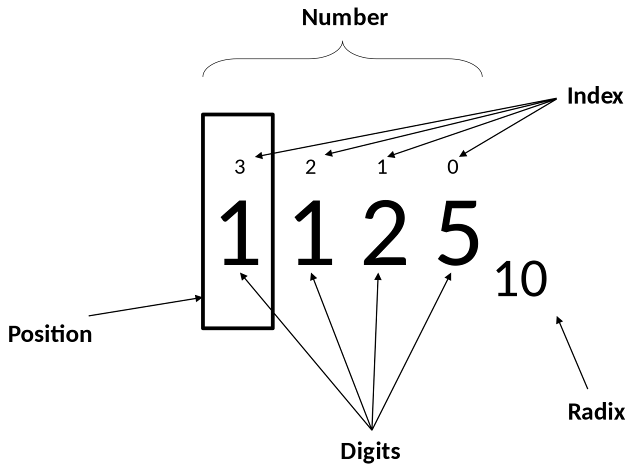
Number systems

- A way to represent numbers
  - numbers are expressed in a certain **base**
- Why study number systems in **CS**?
  - to understand data representation
- Examples of number systems
  - binary
  - decimal
  - octal
  - hexadecimal

Number Systems

System	Base	Digits
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F

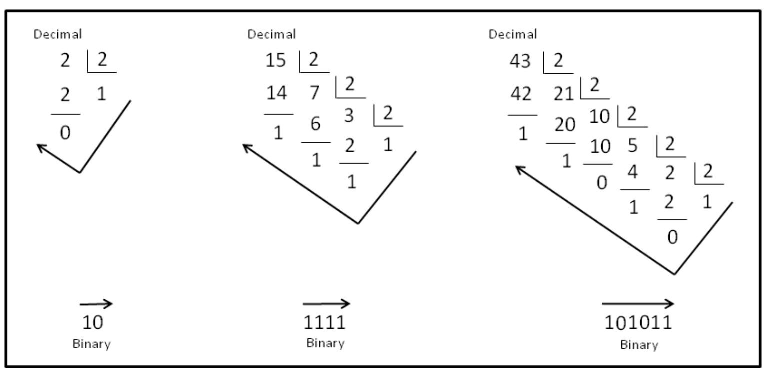
Positional notation



https://en.wikipedia.org/wiki/Positional_notation

Conversions to decimal

Conversions from decimal



https://en.wikiversity.org/wiki/Numeral_systems

Examples

Binary to hexadecimal

Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Bin	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Dec	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Oct	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17

Humans think in **base 10.** Computers think in **base 2.**
Humans use **base 16** to easily manipulate data in **base 2.**

Color codes

	Name	Hex (RGB)
	White	#FFFFFF
	Silver	#C0C0C0
	Gray	#808080
	Black	#000000
	Red	#FF0000
	Maroon	#800000
	Yellow	#FFFF00
	Olive	#808000
	Lime	#00FF00
	Green	#008000
	Aqua	#00FFFF
	Teal	#008080
	Blue	#0000FF
	Navy	#000080
	Fuchsia	#FF00FF
	Purple	#800080

Integer literals in C/C++

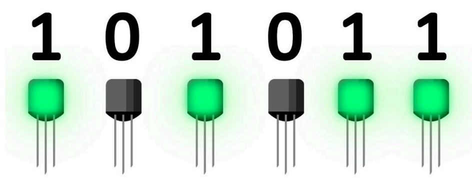
```
int d = 42;
int o = 052;
int x = 0x2a;
int X = 0X2A;
int b = 0b101010; // C++14
```

- **decimal-literal** is a non-zero decimal digit (1, 2, 3, 4, 5, 6, 7, 8, 9), followed by zero or more decimal digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
- **octal-literal** is the digit zero (0) followed by zero or more octal digits (0, 1, 2, 3, 4, 5, 6, 7)
- **hex-literal** is the character sequence `0x` or the character sequence `0X` followed by one or more hexadecimal digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, A, b, B, c, C, d, D, e, E, f, F)
- **binary-literal** is the character sequence `0b` or the character sequence `0B` followed by one or more binary digits (0, 1)

Bits and Bytes

Bits and computers

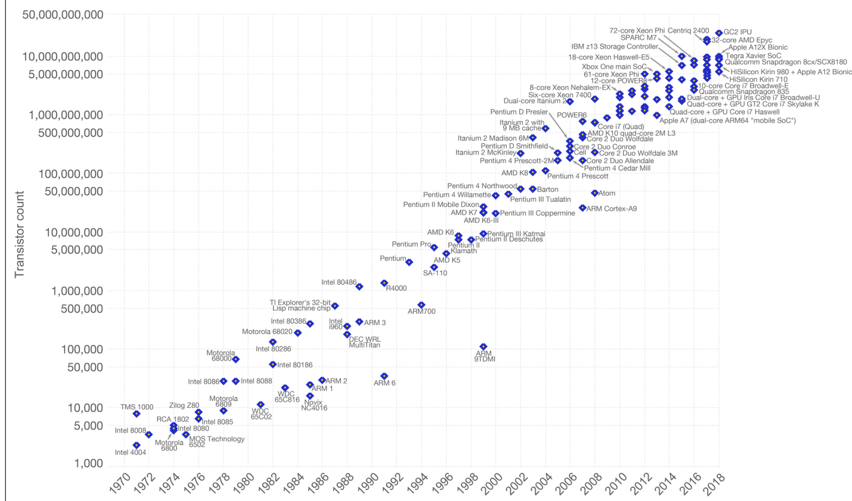
- A bit can only have two values (states)
  - easy to embed into physical devices
- Transistor
  - processors have billions of transistors
  - transistors can be switched on and off



Moore's Law – The number of transistors on integrated circuit chips (1971-2018)

Our World in Data

Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important as other aspects of technological progress – such as processing speed or the price of electronic products – are linked to Moore's law.



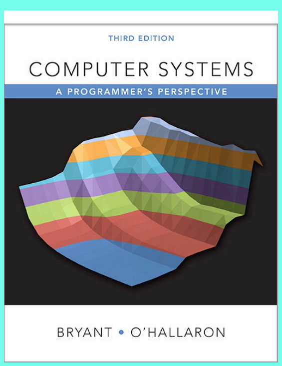
Data source: Wikipedia (https://en.wikipedia.org/wiki/Transistor_count)
The data visualization is available at OurWorldinData.org. There you find more visualizations and research on this topic.

Disclaimer

The following slides are from:

Computer Systems (Bryant and O'Hallaron)

A Programmer's Perspective



Everything is bits

- **Each bit is 0 or 1**
- **By encoding/interpreting sets of bits in various ways**
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- **Why bits? Electronic Implementation**
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires

An Amazing & Successful Abstraction.

(which we won't dig into in 213)

Encoding Byte Values

- **Byte = 8 bits**
  - Binary 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Hexadecimal 00_{16} to FF_{16}
    - Base 16 number representation
    - Use characters '0' to '9' and 'A' to 'F'
    - Write $FA1D37B_{16}$ in C as
      - 0xFA1D37B
      - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

15213: $\underbrace{0011}_{3}\ \underbrace{1011}_{B}\ \underbrace{0110}_{6}\ \underbrace{1101}_{D}$

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1
`short`	2	2	2
`int`	4	4	4
`long`	4	8	8
`float`	4	4	4
`double`	8	8	8
pointer	4	8	8

Boolean Algebra

- **Developed by George Boole in 19th Century**
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0

And

- **A&B = 1 when both A=1 and B=1**

&	0	1
0	0	0
1	0	1

Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

Not

- **~A = 1 when A=0**

~	
0	1
1	0

Exclusive-Or (Xor)

- **A^B = 1 when either A=1 or B=1, but not both**

^	0	1
0	0	1
1	1	0

General Boolean Algebras

- **Operate on Bit Vectors**
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

- **All of the Properties of Boolean Algebra Apply**

Example: Representing & Manipulating Sets

- **Representation**
  - Width w bit vector represents subsets of {0, …, w–1}
  - $a_j = 1$ if $j \in A$
    - 01101001 { 0, 3, 5, 6 }
    - *76543210*
    - 01010101 { 0, 2, 4, 6 }
    - *76543210*
- **Operations**
  - & Intersection 01000001 { 0, 6 }
  - | Union 01111101 { 0, 2, 3, 4, 5, 6 }
  - ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
  - ~ Complement 10101010 { 1, 3, 5, 7 }

Bit-Level Operations in C

- **Operations &, |, ~, ^ Available in C**
  - Apply to any "integral" data type
    - long, int, short, char, unsigned
  - View arguments as bit vectors
  - Arguments applied bit-wise
- **Examples (Char data type)**
  - ~0x41 →
  - ~0x00 →
  - 0x69 & 0x55 →
  - 0x69 | 0x55 →

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
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- **Operations &, |, ~, ^ Available in C**
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  - View arguments as bit vectors
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- **Examples (Char data type)**
  - ~0x41 → 0xBE
    - ~$0100\ 0001_2$ → $1011\ 1110_2$
  - ~0x00 → 0xFF
    - ~$0000\ 0000_2$ → $1111\ 1111_2$
  - 0x69 & 0x55 → 0x41
    - $0110\ 1001_2$ & $0101\ 0101_2$ → $0100\ 0001_2$
  - 0x69 | 0x55 → 0x7D
    - $0110\ 1001_2$ | $0101\ 0101_2$ → $0111\ 1101_2$

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Contrast: Logic Operations in C

- **Contrast to Bit-Level Operators**
  - **Logic Operations: &&, ||, !**
    - View 0 as "False"
    - Anything nonzero as "True"
    - Always return 0 or 1
    - Early termination
- **Examples (char data type)**
  - !0x41 → 0x00
  - !0x00 → 0x01
  - !!0x41→ 0x01
  - 0x69 && 0x55 → 0x01
  - 0x69 || 0x55 → 0x01
  - p && *p (avoids null pointer access)

Watch out for && vs. & (and || vs. |)…
Super common C programming pitfall!

Shift Operations

- **Left Shift: `x << y`**
  - Shift bit-vector **x** left **y** positions
    - – Throw away extra bits on left
    - Fill with 0's on right
- **Right Shift: `x >> y`**
  - Shift bit-vector **x** right **y** positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- **Undefined Behavior**
  - Shift amount < 0 or ≥ word size

Argument x	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument x	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000