CSC 411

Computer Organization (Spring 2022)
Lecture 2: Number Systems, Bitwise Operations

Number Systems

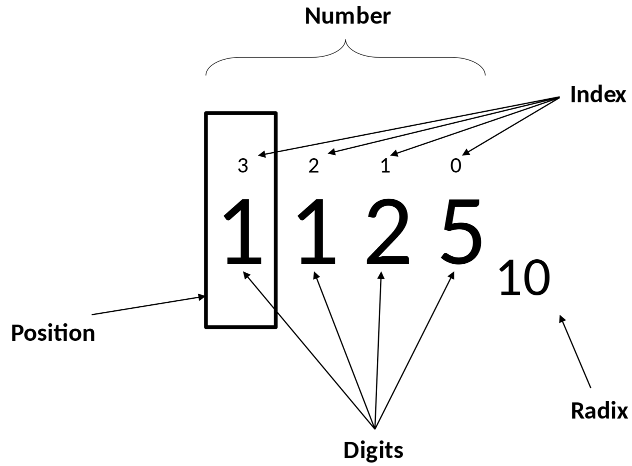
Number systems

- A way to represent numbers
  - numbers are expressed in a certain **base**
- Why study number systems in **CS**?
  - to understand data representation
- Examples of number systems
  - binary
  - decimal
  - octal
  - hexadecimal

Number Systems

System	Base	Digits
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F

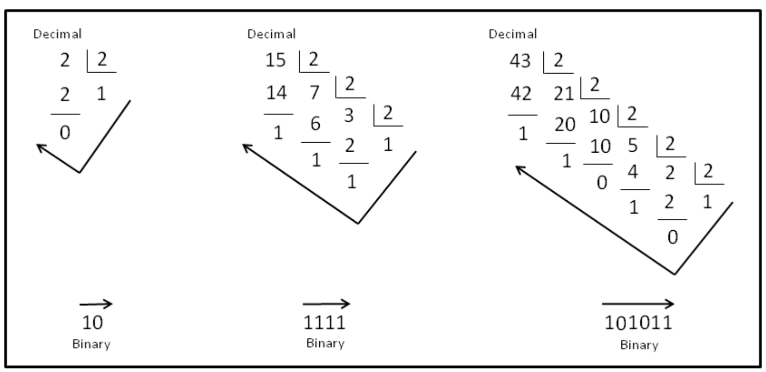
Positional notation



https://en.wikipedia.org/wiki/Positional_notation

Conversions to decimal

Conversions from decimal



https://en.wikiversity.org/wiki/Numeral_systems

Examples

Binary to hexadecimal

Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Bin	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Dec	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Oct	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17

Humans think in **base 10.** Computers think in **base 2.**
Humans use **base 16** to easily manipulate data in **base 2.**

Color codes

	Name	Hex (RGB)
	White	#FFFFFF
	Silver	#C0C0C0
	Gray	#808080
	Black	#000000
	Red	#FF0000
	Maroon	#800000
	Yellow	#FFFF00
	Olive	#808000
	Lime	#00FF00
	Green	#008000
	Aqua	#00FFFF
	Teal	#008080
	Blue	#0000FF
	Navy	#000080
	Fuchsia	#FF00FF
	Purple	#800080

Integer literals in C/C++

```
int d = 42;
int o = 052;
int x = 0x2a;
int X = 0X2A;
int b = 0b101010; // C++14
```

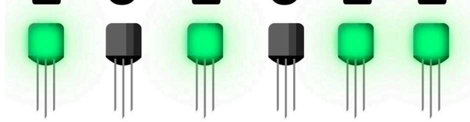
- **decimal-literal** is a non-zero decimal digit (1, 2, 3, 4, 5, 6, 7, 8, 9), followed by zero or more decimal digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
- **octal-literal** is the digit zero (0) followed by zero or more octal digits (0, 1, 2, 3, 4, 5, 6, 7)
- **hex-literal** is the character sequence `0x` or the character sequence `0X` followed by one or more hexadecimal digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, A, b, B, c, C, d, D, e, E, f, F)
- **binary-literal** is the character sequence `0b` or the character sequence `0B` followed by one or more binary digits (0, 1)

Bits and Bytes

Bits and computers

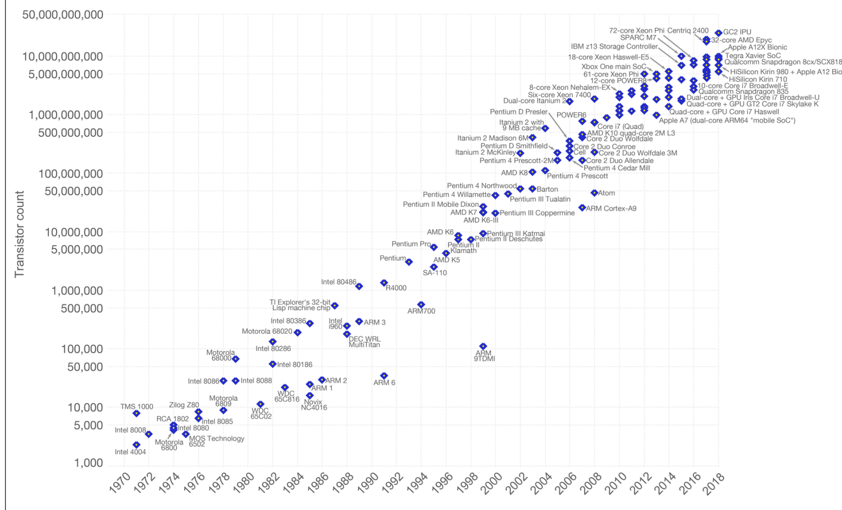
- A bit can only have two values (states)
  - easy to embed into physical devices
- Transistor
  - processors have billions of transistors
  - transistors can be switched on and off

1 0 1 0 1 1



Moore's Law – The number of transistors on integrated circuit chips (1971-2018)

Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important as other aspects of technological progress – such as processing speed or the price of electronic products – are linked to Moore's law.



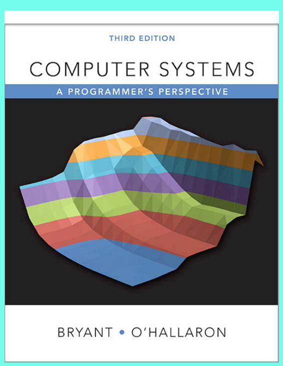
Data source: Wikipedia (https://en.wikipedia.org/wiki/Transistor_count)
The data visualization is available at OurWorldinData.org. There you find more visualizations and research on this topic.

Disclaimer

The following slides are from:

Computer Systems (Bryant and O'Hallaron)

A Programmer's Perspective



Everything is bits

- **Each bit is 0 or 1**
- **By encoding/interpreting sets of bits in various ways**
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- **Why bits? Electronic Implementation**
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires

An Amazing & Successful Abstraction.

(which we won't dig into in 213)

Encoding Byte Values

- **Byte = 8 bits**
  - Binary 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Hexadecimal 00_{16} to FF_{16}
    - Base 16 number representation
    - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
    - Write $FA1D37B_{16}$ in C as
      - 0xFA1D37B
      - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

$$15213: \underbrace{0011}_{3}\ \underbrace{1011}_{B}\ \underbrace{0110}_{6}\ \underbrace{1101}_{D}$$

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1
`short`	2	2	2
`int`	4	4	4
`long`	4	8	8
`float`	4	4	4
`double`	8	8	8
pointer	4	8	8

Boolean Algebra

- **Developed by George Boole in 19th Century**
  - Algebraic representation of logic
    - Encode “True” as 1 and “False” as 0

And

- **A&B = 1 when both A=1 and B=1**

&	0	1
0	0	0
1	0	1

Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

Not

- **~A = 1 when A=0**

~	
0	1
1	0

Exclusive-Or (Xor)

- **A^B = 1 when either A=1 or B=1, but not both**

^	0	1
0	0	1
1	1	0

General Boolean Algebras

- **Operate on Bit Vectors**
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

- **All of the Properties of Boolean Algebra Apply**

Example: Representing & Manipulating Sets

- **Representation**
  - Width w bit vector represents subsets of {0, …, w–1}
  - $a_j = 1$ if $j \in A$
    - 01101001 { 0, 3, 5, 6 }
    - *76543210*
    - 01010101 { 0, 2, 4, 6 }
    - *76543210*
- **Operations**
  - & Intersection 01000001 { 0, 6 }
  - | Union 01111101 { 0, 2, 3, 4, 5, 6 }
  - ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
  - ~ Complement 10101010 { 1, 3, 5, 7 }

Bit-Level Operations in C

- **Operations &, |, ~, ^ Available in C**
  - Apply to any "integral" data type
    - long, int, short, char, unsigned
  - View arguments as bit vectors
  - Arguments applied bit-wise
- **Examples (Char data type)**
  - ~0x41 →
  - ~0x00 →
  - 0x69 & 0x55 →
  - 0x69 | 0x55 →

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
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- **Operations &, |, ~, ^ Available in C**
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  - View arguments as bit vectors
  - Arguments applied bit-wise
- **Examples (Char data type)**
  - ~0x41 → 0xBE
    - ~0100 0001_2 → 1011 1110_2
  - ~0x00 → 0xFF
    - ~0000 0000_2 → 1111 1111_2
  - 0x69 & 0x55 → 0x41
    - 0110 1001_2 & 0101 0101_2 → 0100 0001_2
  - 0x69 | 0x55 → 0x7D
    - 0110 1001_2 | 0101 0101_2 → 0111 1101_2

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Contrast: Logic Operations in C

- **Contrast to Bit-Level Operators**
  - **Logic Operations: &&, ||, !**
    - View 0 as "False"
    - Anything nonzero as "True"
    - Always return 0 or 1
    - Early termination
- **Examples (char data type)**
  - !0x41 → 0x00
  - !0x00 → 0x01
  - !!0x41→ 0x01
  - 0x69 && 0x55 → 0x01
  - 0x69 || 0x55 → 0x01
  - p && *p (avoids null pointer access)

Watch out for && vs. & (and || vs. |)… Super common C programming pitfall!

Shift Operations

- **Left Shift: `x << y`**
  - Shift bit-vector **x** left **y** positions
    - – Throw away extra bits on left
    - Fill with 0's on right
- **Right Shift: `x >> y`**
  - Shift bit-vector **x** right **y** positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- **Undefined Behavior**
  - Shift amount < 0 or ≥ word size

Argument x	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument x	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000