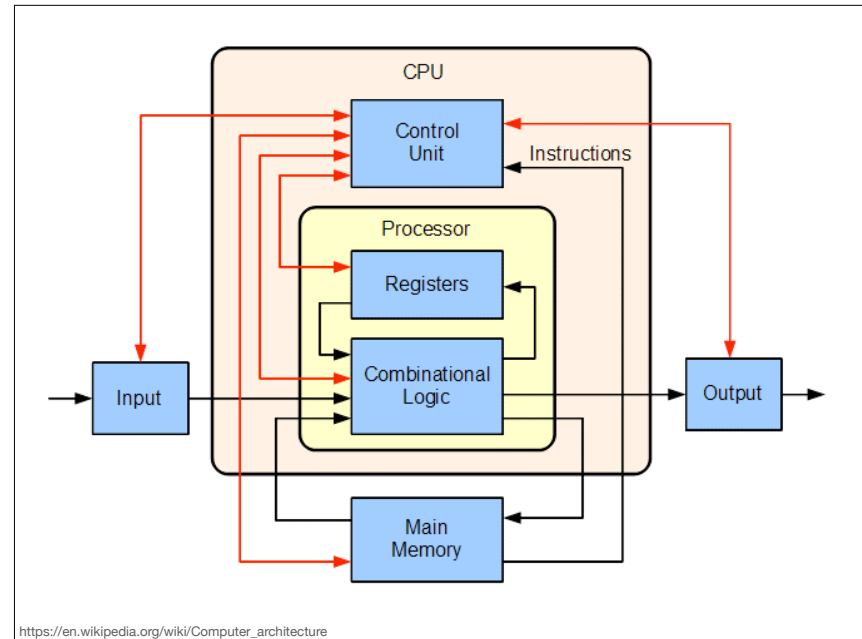


# CSC 411

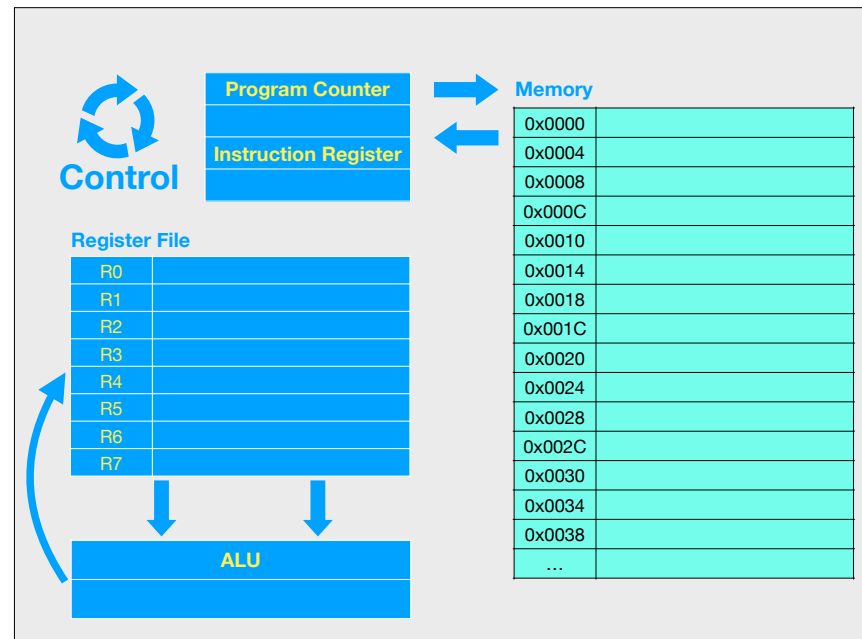
Computer Organization (Spring 2022)

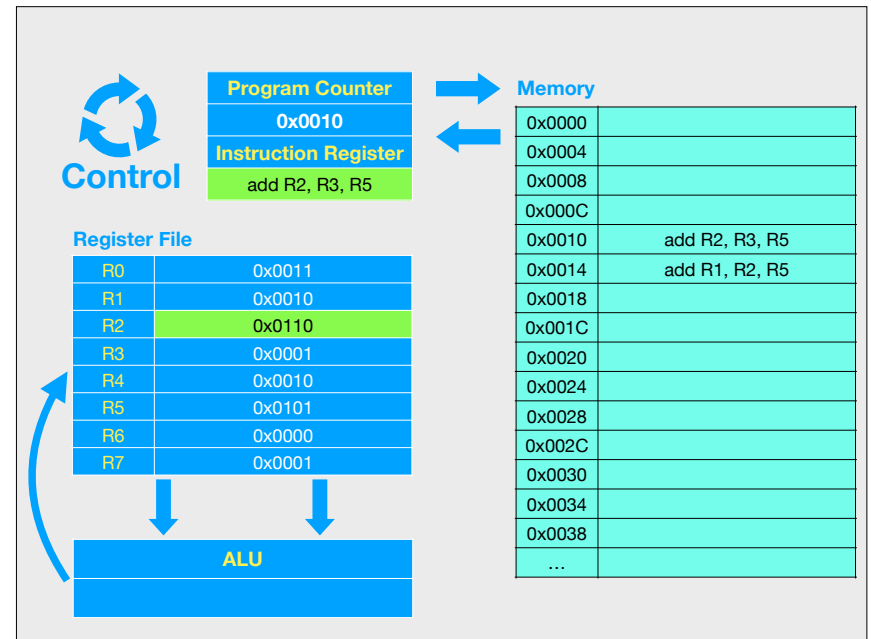
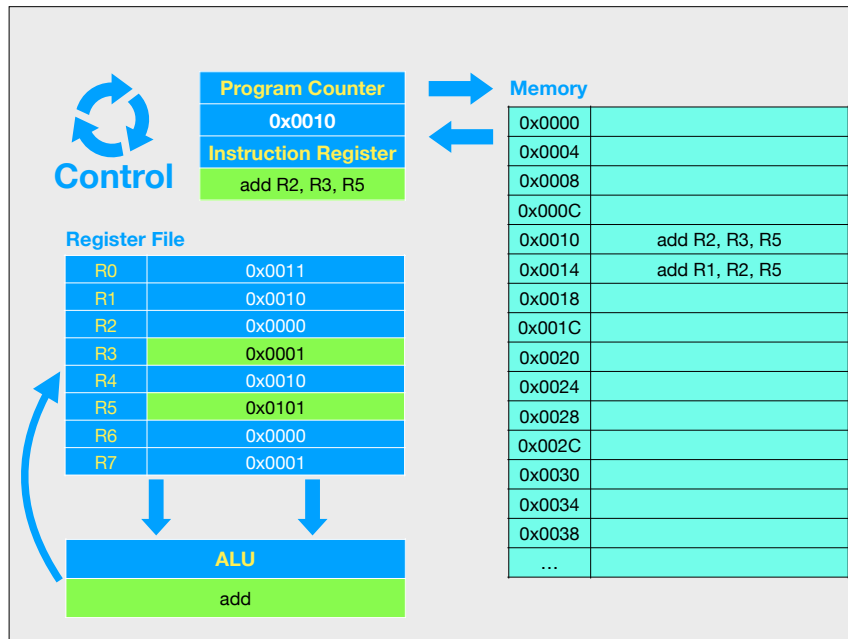
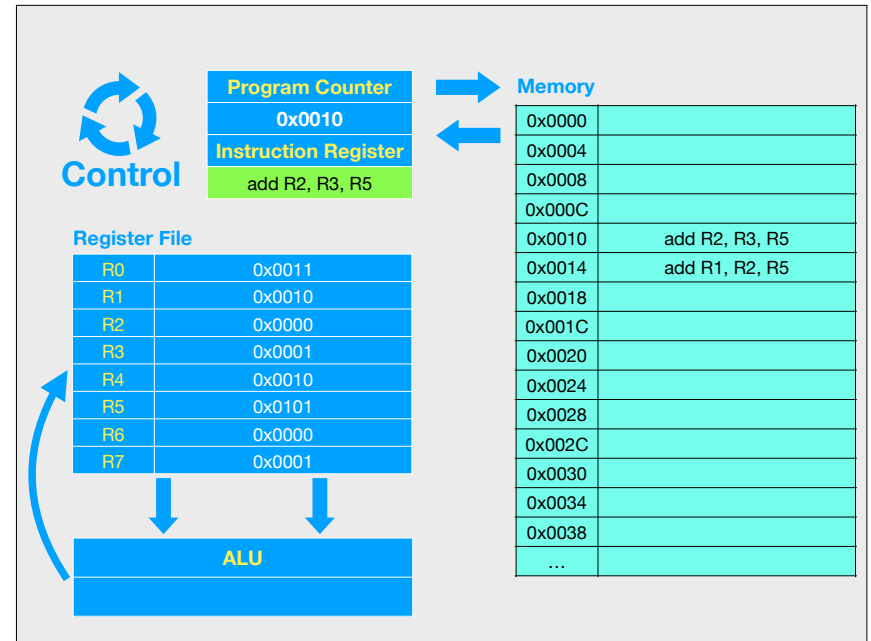
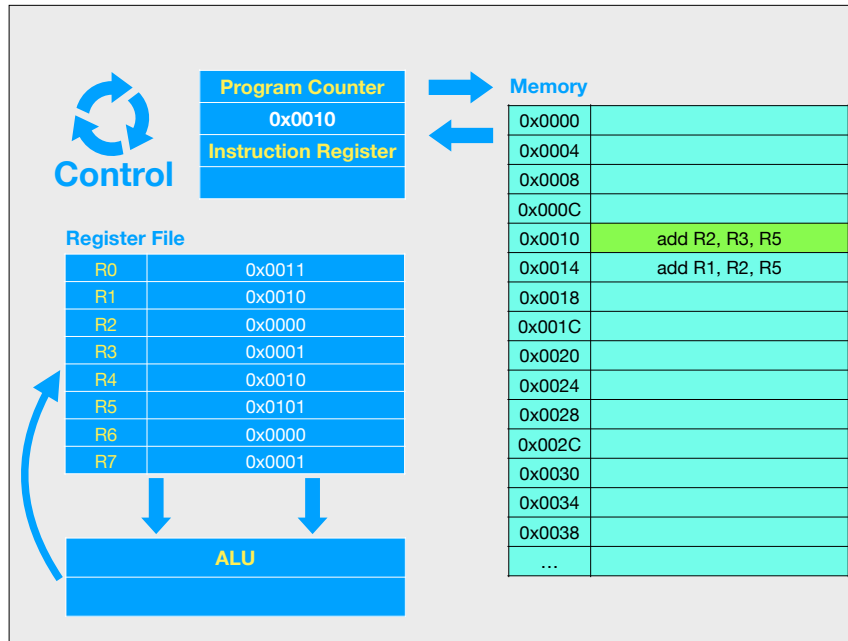
Lecture 6: Executing Instructions and Integer Representation

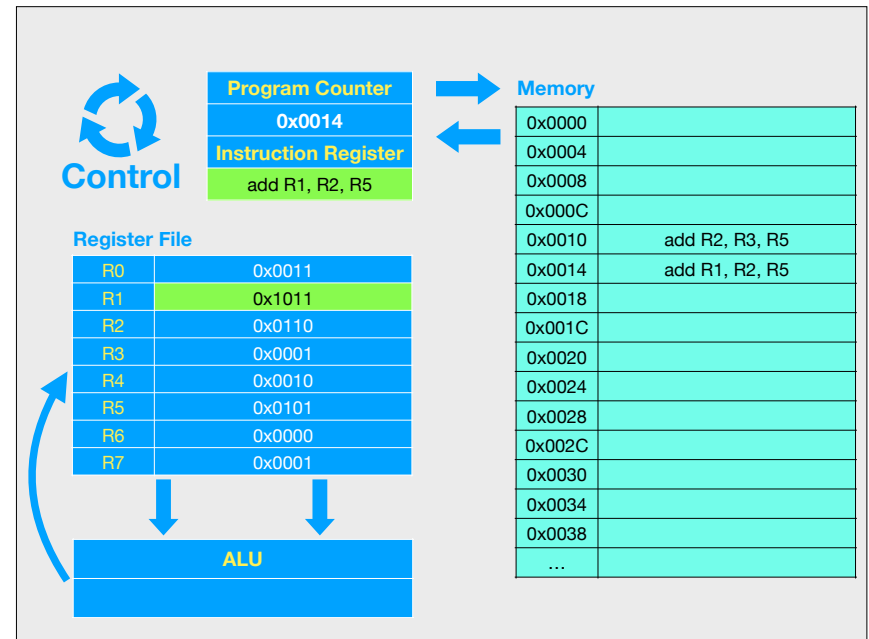
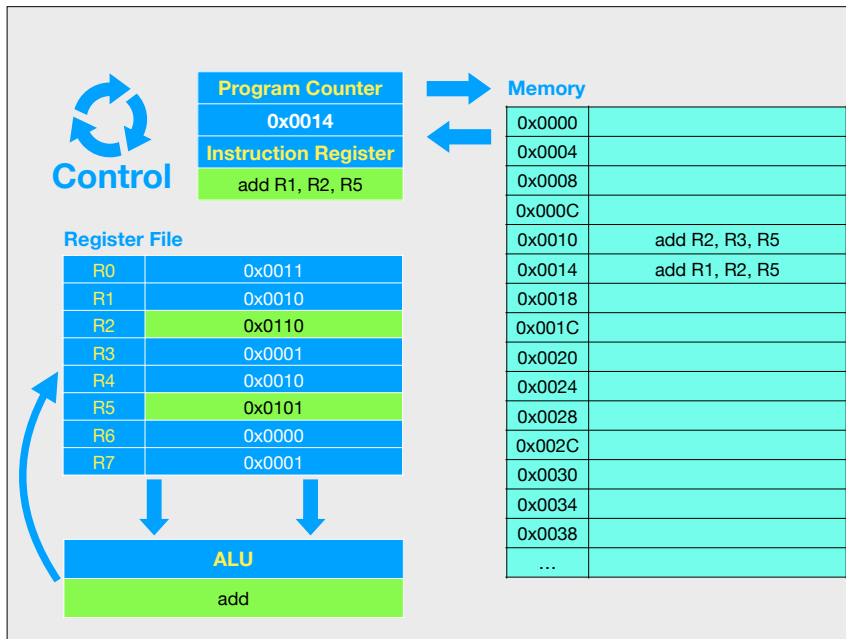
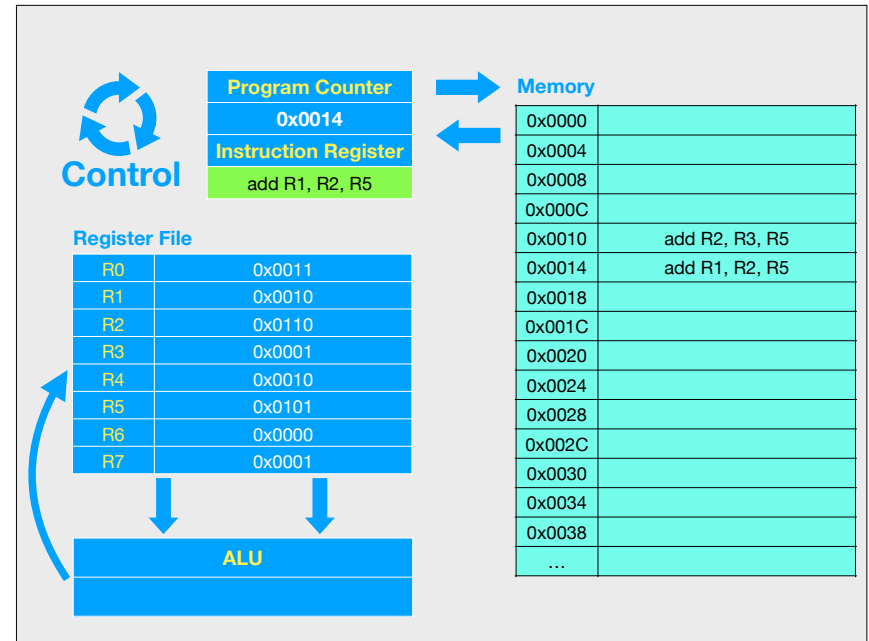
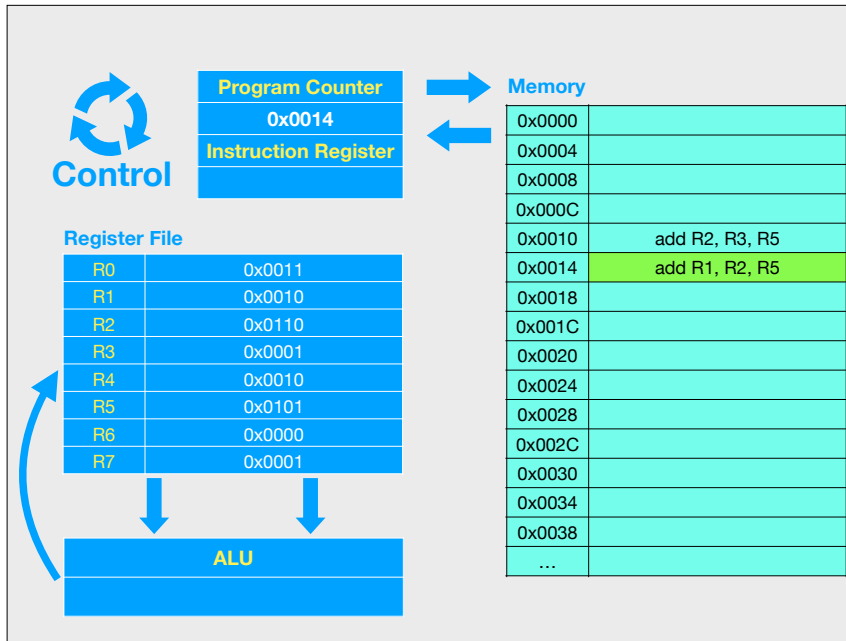
Prof. Marco Alvarez, University of Rhode Island



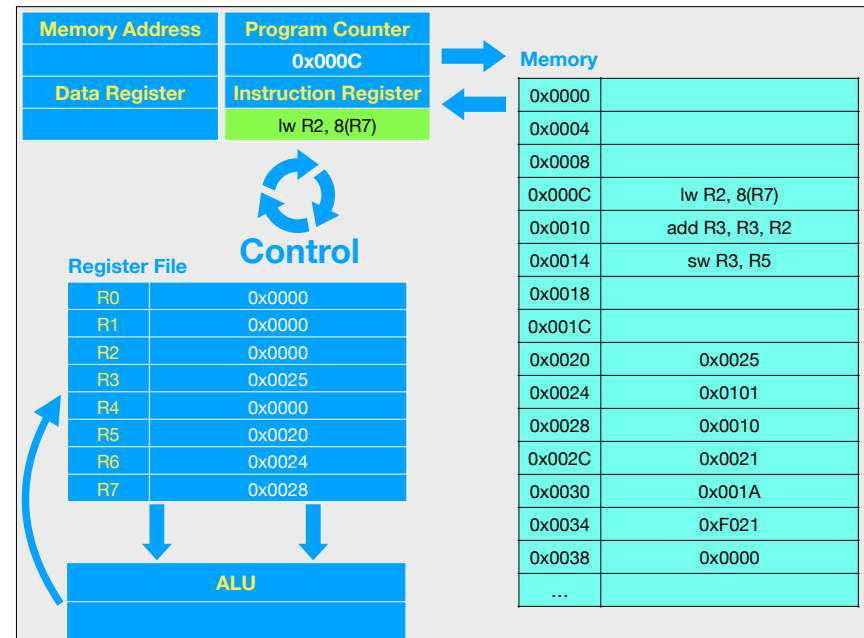
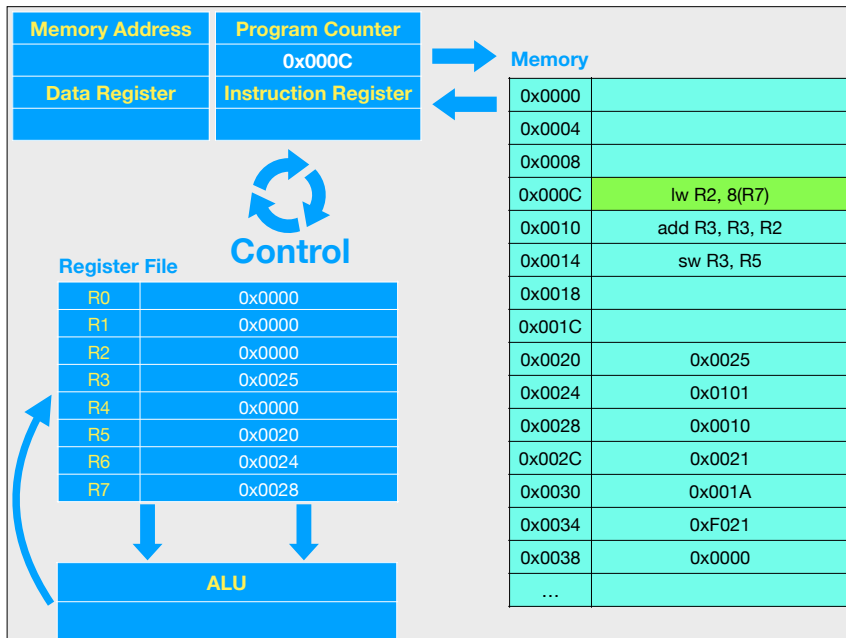
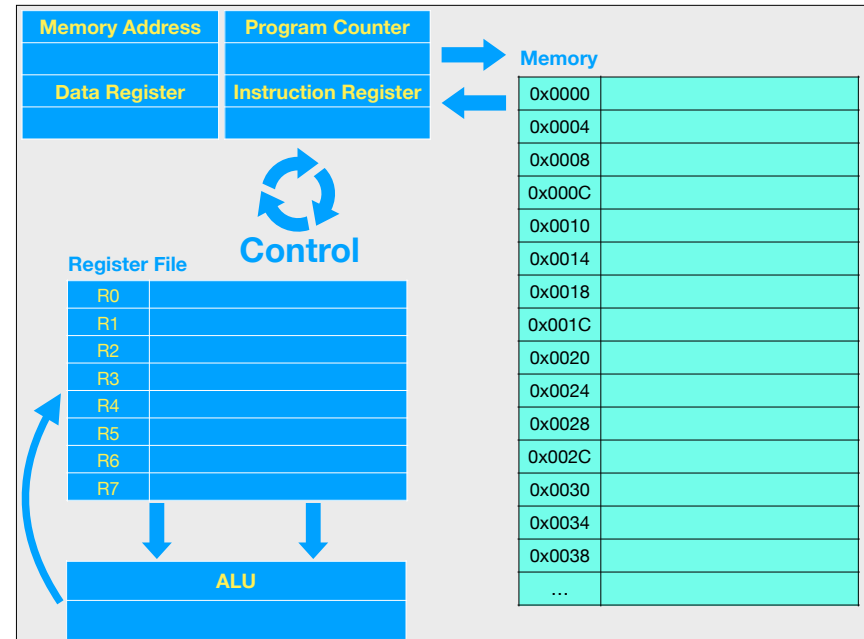
## Executing instructions

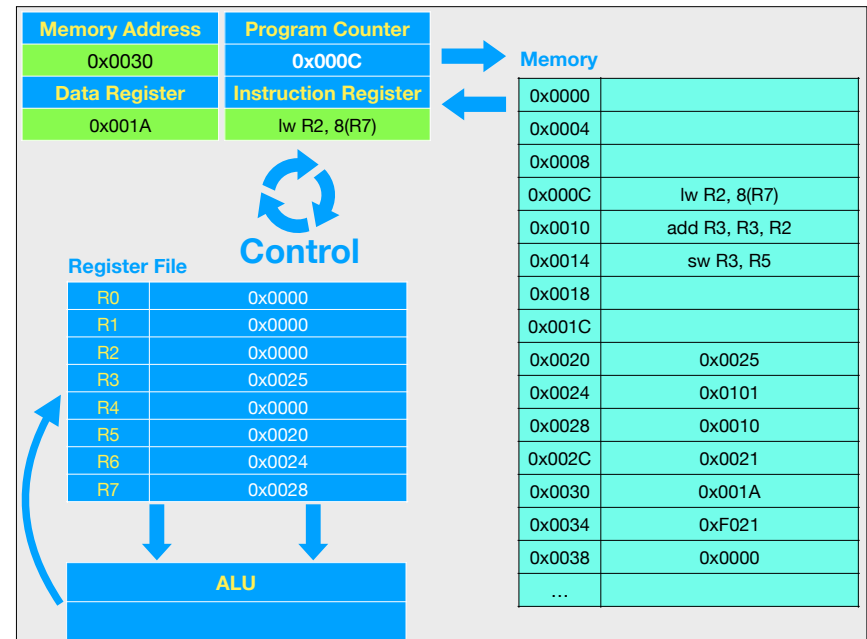
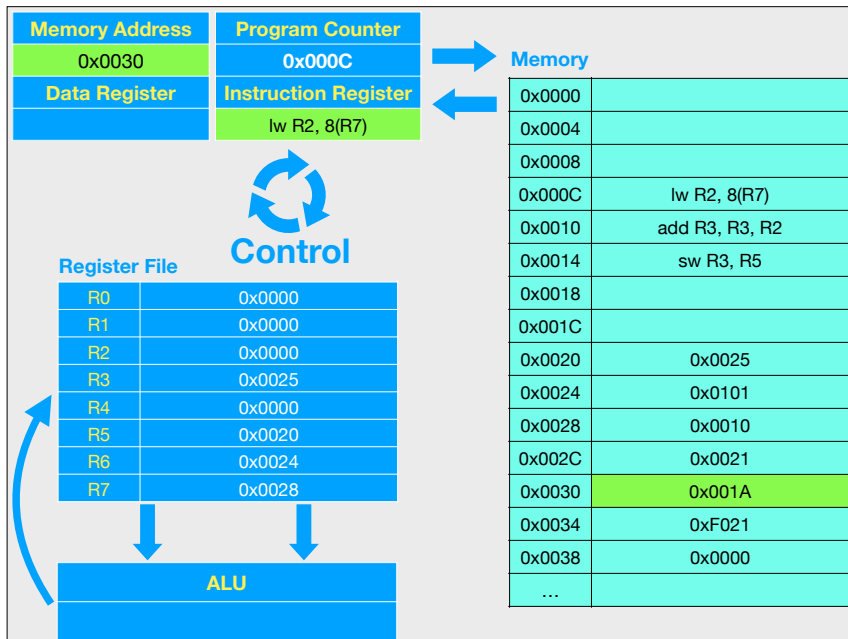
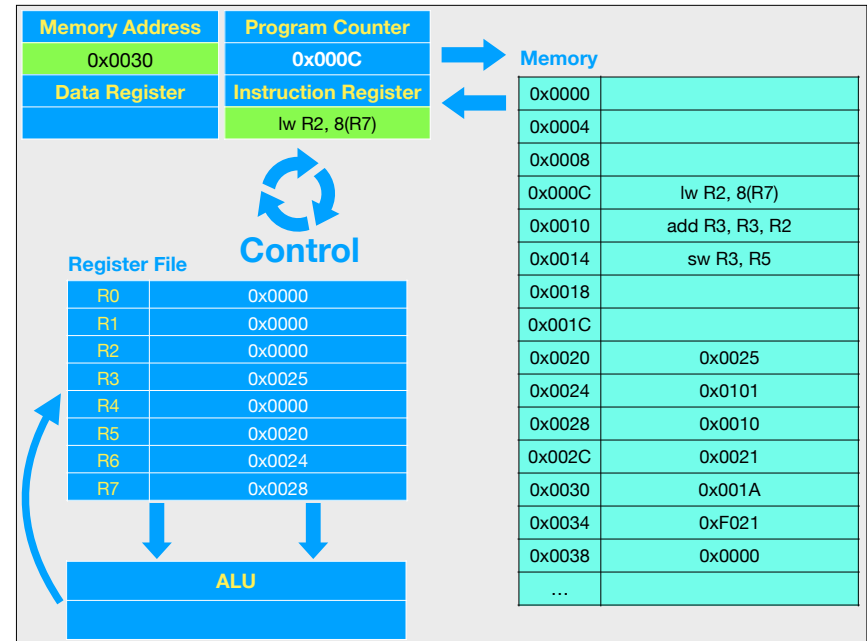
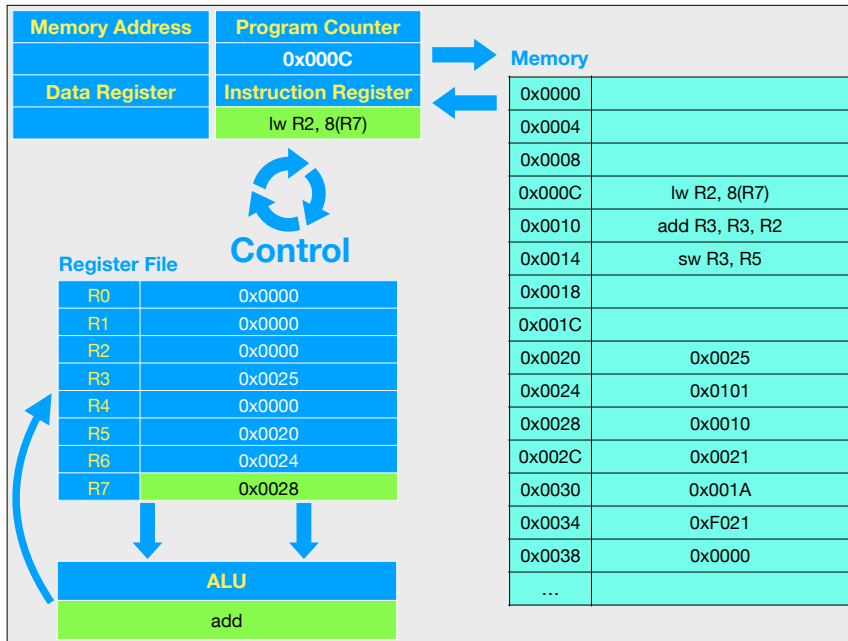


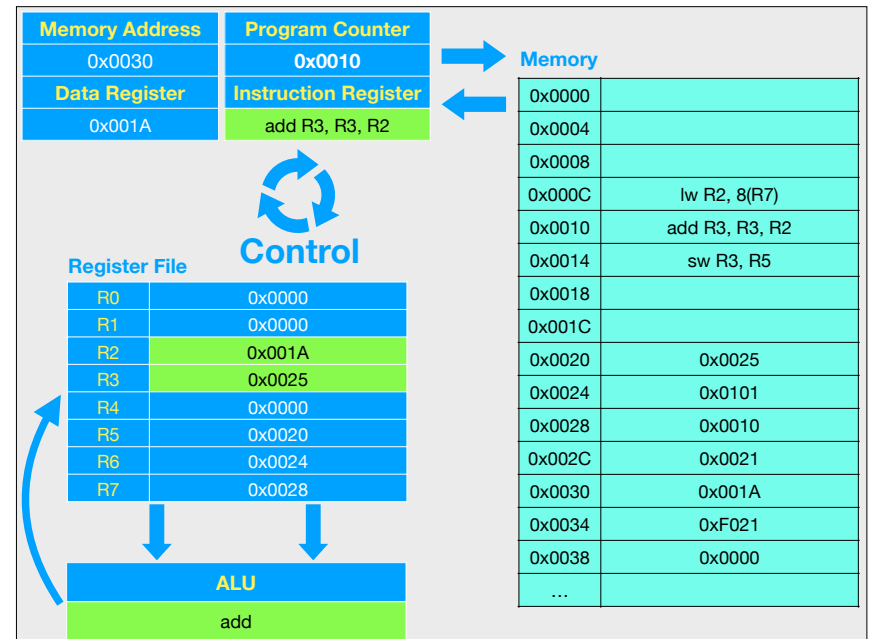
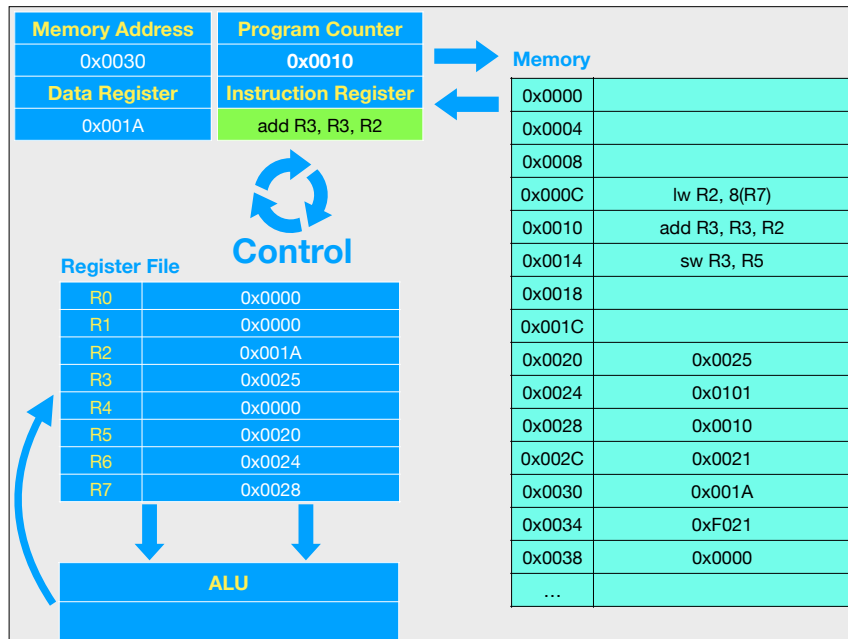
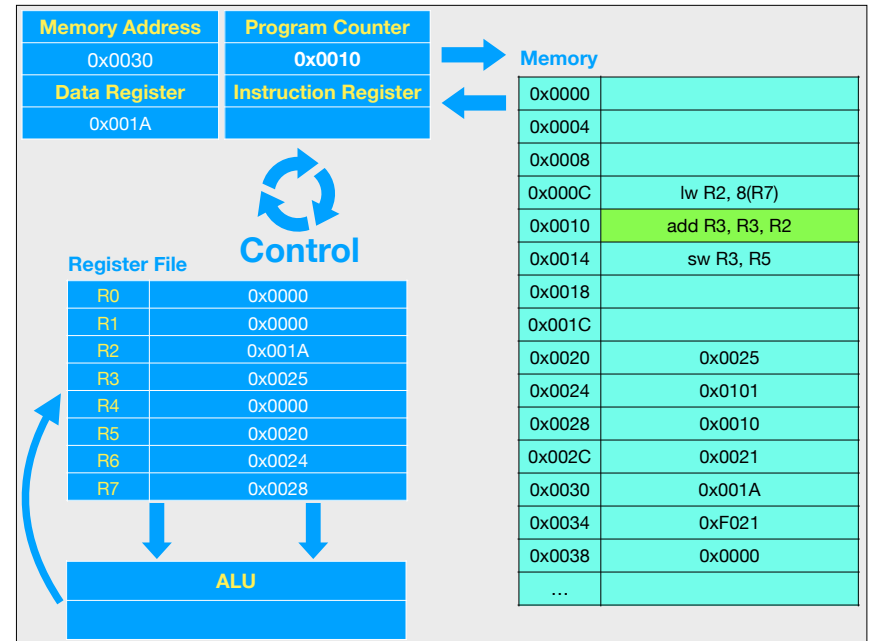
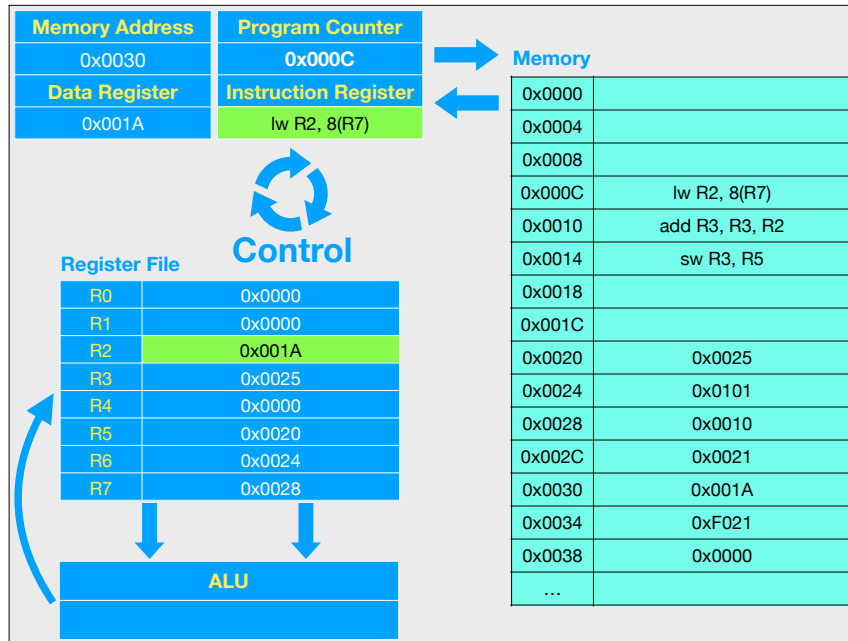


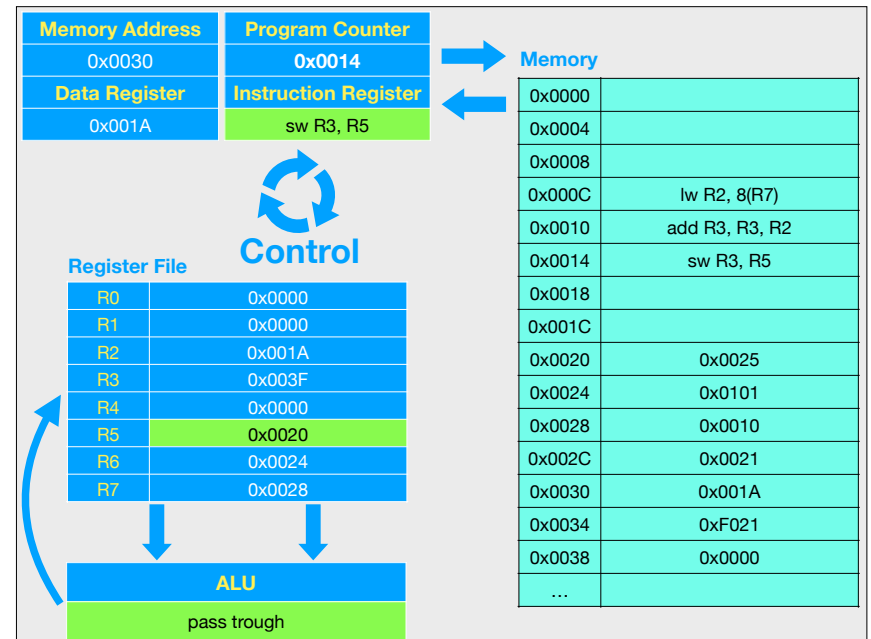
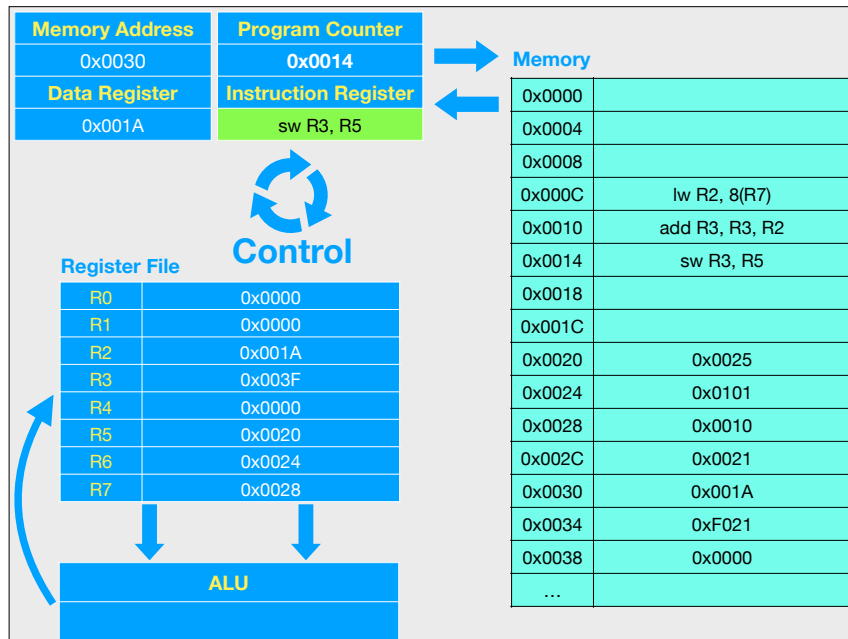
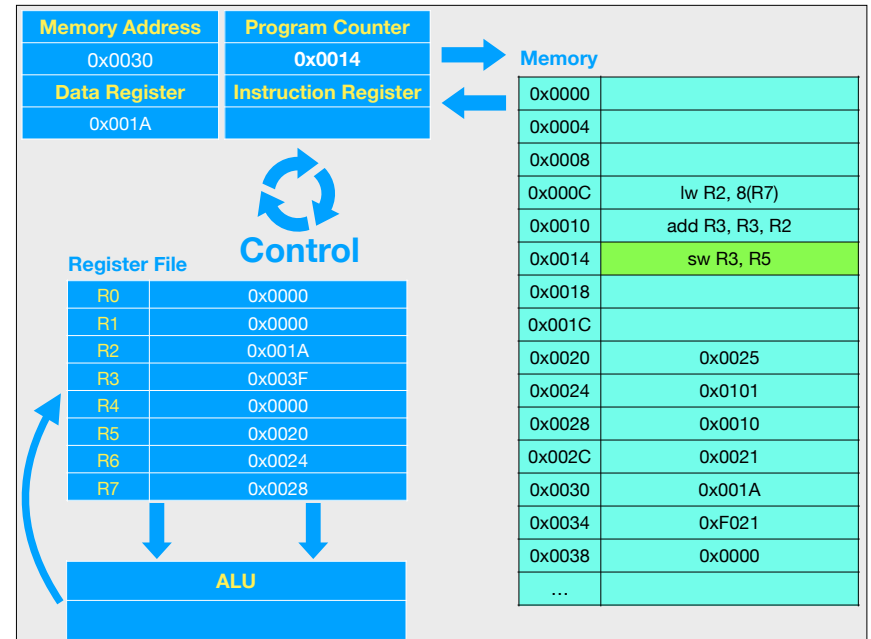
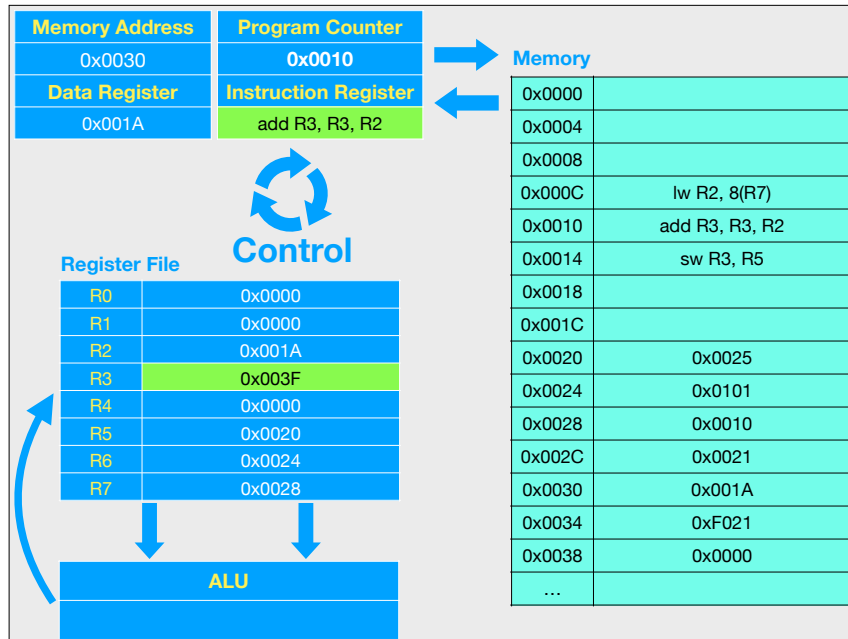


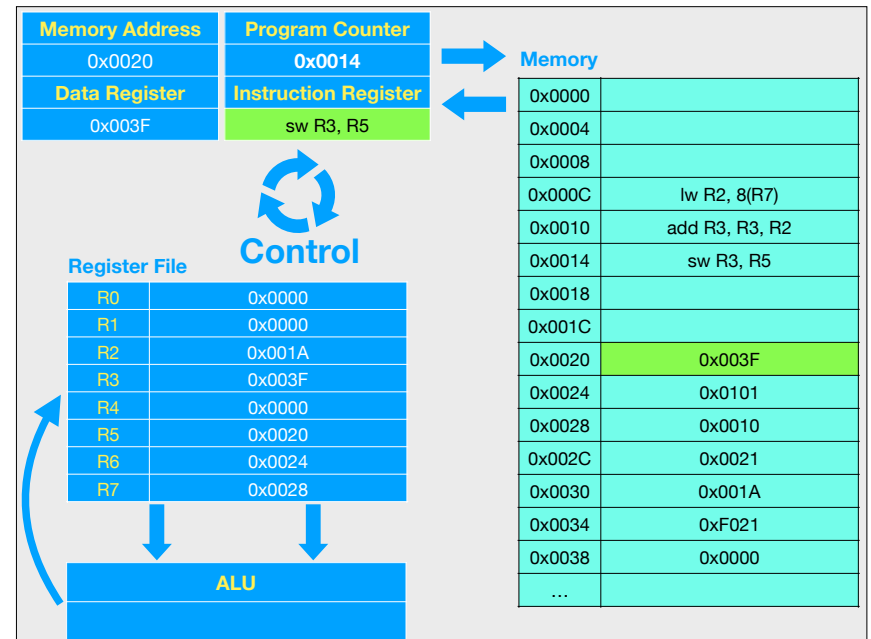
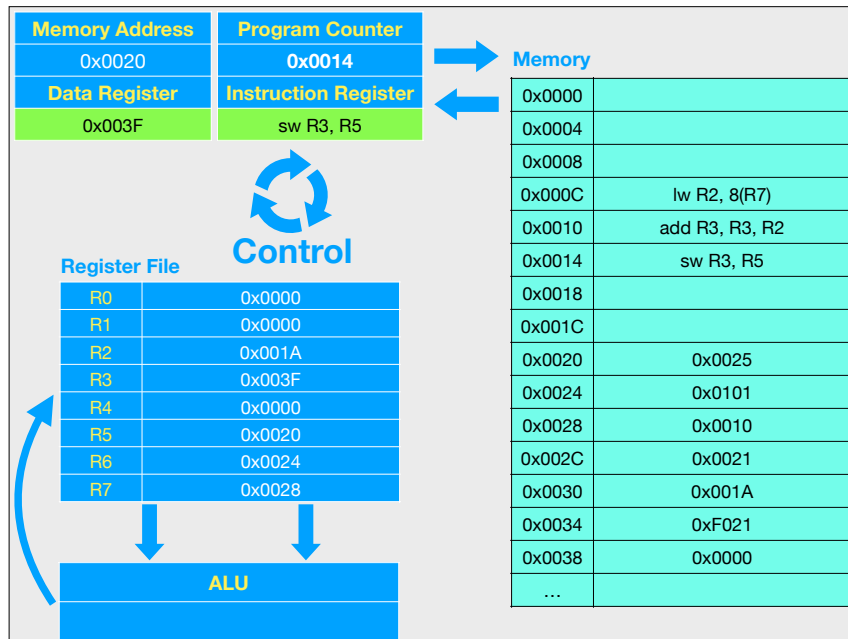
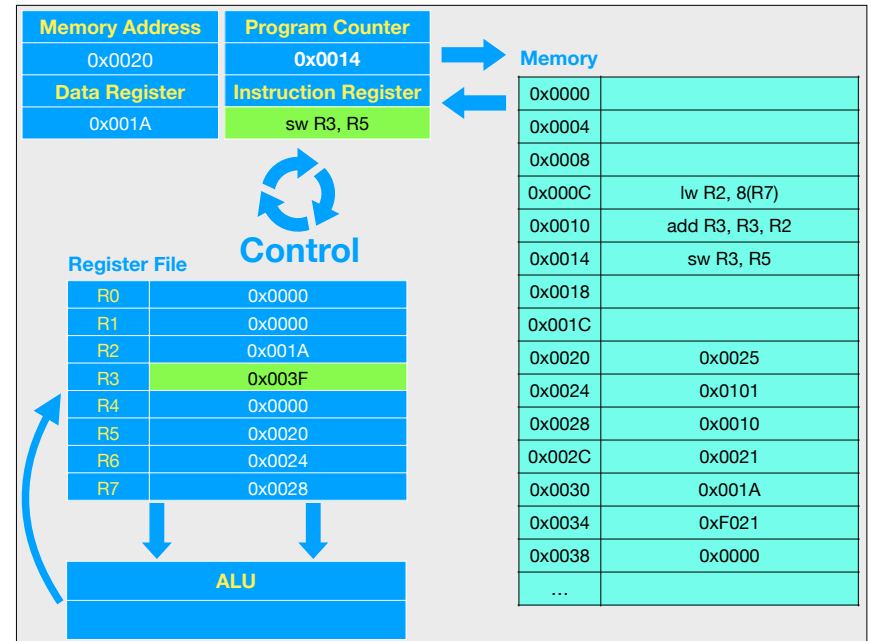
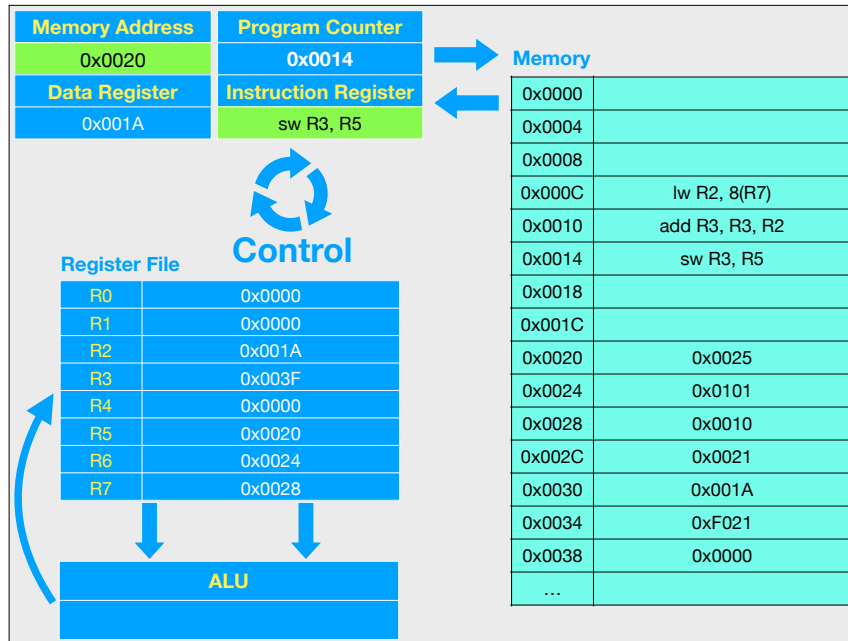
# Executing memory instructions











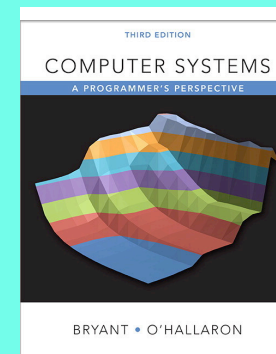


# Representing integers

## Disclaimer

The following slides are from:

Computer Systems (Bryant and O'Hallaron)  
A Programmer's Perspective



## Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings
- Summary

## Encoding Integers

### Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

### Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

```
short int x = 15213;  
short int y = -15213;
```

Sign Bit

- C does not mandate using two's complement
  - But, most machines do, and we will assume so
- C short 2 bytes long

|          | Decimal | Hex   | Binary            |
|----------|---------|-------|-------------------|
| <b>x</b> | 15213   | 3B 6D | 00111011 01101101 |
| <b>y</b> | -15213  | C4 93 | 11000100 10010011 |

- Sign Bit
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative

## Two-complement: Simple Example

$$\begin{array}{ccccccc} & -16 & 8 & 4 & 2 & 1 & \\ 10 = & 0 & 1 & 0 & 1 & 0 & \end{array} \quad 8+2 = 10$$

$$\begin{array}{ccccccc} & -16 & 8 & 4 & 2 & 1 & \\ -10 = & 1 & 0 & 1 & 1 & 0 & \end{array} \quad -16+4+2 = -10$$

## Two-complement Encoding Example (Cont.)

**x =** 15213: 00111011 01101101  
**y =** -15213: 11000100 10010011

| Weight     | 15213        |      | -15213        |        |
|------------|--------------|------|---------------|--------|
| 1          | 1            | 1    | 1             | 1      |
| 2          | 0            | 0    | 1             | 2      |
| 4          | 1            | 4    | 0             | 0      |
| 8          | 1            | 8    | 0             | 0      |
| 16         | 0            | 0    | 1             | 16     |
| 32         | 1            | 32   | 0             | 0      |
| 64         | 1            | 64   | 0             | 0      |
| 128        | 0            | 0    | 1             | 128    |
| 256        | 1            | 256  | 0             | 0      |
| 512        | 1            | 512  | 0             | 0      |
| 1024       | 0            | 0    | 1             | 1024   |
| 2048       | 1            | 2048 | 0             | 0      |
| 4096       | 1            | 4096 | 0             | 0      |
| 8192       | 1            | 8192 | 0             | 0      |
| 16384      | 0            | 0    | 1             | 16384  |
| -32768     | 0            | 0    | 1             | -32768 |
| <b>Sum</b> | <b>15213</b> |      | <b>-15213</b> |        |

## Numeric Ranges

### ■ Unsigned Values

- $UMin = 0$   
000...0
- $UMax = 2^w - 1$   
111...1

### ■ Two's Complement Values

- $TMin = -2^{w-1}$   
100...0
- $TMax = 2^{w-1} - 1$   
011...1
- Minus 1  
111...1

Values for  $W = 16$

|             | Decimal       | Hex          | Binary            |
|-------------|---------------|--------------|-------------------|
| <b>UMax</b> | <b>65535</b>  | <b>FF FF</b> | 11111111 11111111 |
| <b>TMax</b> | <b>32767</b>  | <b>7F FF</b> | 01111111 11111111 |
| <b>TMin</b> | <b>-32768</b> | <b>80 00</b> | 10000000 00000000 |
| <b>-1</b>   | <b>-1</b>     | <b>FF FF</b> | 11111111 11111111 |
| <b>0</b>    | <b>0</b>      | <b>00 00</b> | 00000000 00000000 |

## Values for Different Word Sizes

|             | W    |         |                |                            |
|-------------|------|---------|----------------|----------------------------|
|             | 8    | 16      | 32             | 64                         |
| <b>UMax</b> | 255  | 65,535  | 4,294,967,295  | 18,446,744,073,709,551,615 |
| <b>TMax</b> | 127  | 32,767  | 2,147,483,647  | 9,223,372,036,854,775,807  |
| <b>TMin</b> | -128 | -32,768 | -2,147,483,648 | -9,223,372,036,854,775,808 |

### ■ Observations

- $|TMin| = TMax + 1$ 
  - Asymmetric range
- $UMax = 2 * TMax + 1$
- Question:  $\text{abs}(TMin)$ ?

### ■ C Programming

- `#include <limits.h>`
- Declares constants, e.g.,
  - `ULONG_MAX`
  - `LONG_MAX`
  - `LONG_MIN`
- Values platform specific

## Unsigned & Signed Numeric Values

| X    | B2U(X) | B2T(X) |
|------|--------|--------|
| 0000 | 0      | 0      |
| 0001 | 1      | 1      |
| 0010 | 2      | 2      |
| 0011 | 3      | 3      |
| 0100 | 4      | 4      |
| 0101 | 5      | 5      |
| 0110 | 6      | 6      |
| 0111 | 7      | 7      |
| 1000 | 8      | -8     |
| 1001 | 9      | -7     |
| 1010 | 10     | -6     |
| 1011 | 11     | -5     |
| 1100 | 12     | -4     |
| 1101 | 13     | -3     |
| 1110 | 14     | -2     |
| 1111 | 15     | -1     |

### ■ Equivalence

- Same encodings for nonnegative values

### ■ Uniqueness

- Every bit pattern represents unique integer value
- Each representable integer has unique bit encoding

### ■ ⇒ Can Invert Mappings

- $U2B(x) = B2U^{-1}(x)$ 
  - Bit pattern for unsigned integer
- $T2B(x) = B2T^{-1}(x)$ 
  - Bit pattern for two's comp integer

## Today: Bits, Bytes, and Integers

### ■ Representing information as bits

### ■ Bit-level manipulations

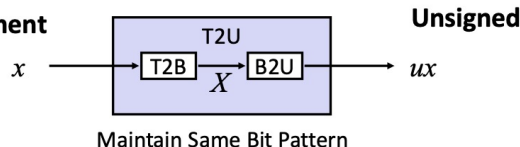
### ■ Integers

- Representation: unsigned and signed
- Conversion, casting
- Expanding, truncating
- Addition, negation, multiplication, shifting
- Summary

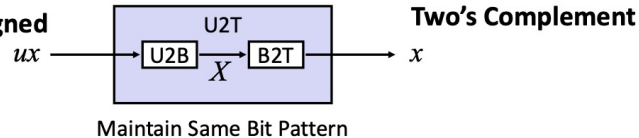
### ■ Representations in memory, pointers, strings

## Mapping Between Signed & Unsigned

Two's Complement



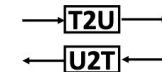
Unsigned



- Mappings between unsigned and two's complement numbers:  
**Keep bit representations and reinterpret**

## Mapping Signed ↔ Unsigned

| Bits | Signed | Unsigned |
|------|--------|----------|
| 0000 | 0      | 0        |
| 0001 | 1      | 1        |
| 0010 | 2      | 2        |
| 0011 | 3      | 3        |
| 0100 | 4      | 4        |
| 0101 | 5      | 5        |
| 0110 | 6      | 6        |
| 0111 | 7      | 7        |
| 1000 | -8     | 8        |
| 1001 | -7     | 9        |
| 1010 | -6     | 10       |
| 1011 | -5     | 11       |
| 1100 | -4     | 12       |
| 1101 | -3     | 13       |
| 1110 | -2     | 14       |
| 1111 | -1     | 15       |



## Mapping Signed ↔ Unsigned

| Bits | Signed | Unsigned |
|------|--------|----------|
| 0000 | 0      | 0        |
| 0001 | 1      | 1        |
| 0010 | 2      | 2        |
| 0011 | 3      | 3        |
| 0100 | 4      | 4        |
| 0101 | 5      | 5        |
| 0110 | 6      | 6        |
| 0111 | 7      | 7        |
| 1000 | -8     | 8        |
| 1001 | -7     | 9        |
| 1010 | -6     | 10       |
| 1011 | -5     | 11       |
| 1100 | -4     | 12       |
| 1101 | -3     | 13       |
| 1110 | -2     | 14       |
| 1111 | -1     | 15       |

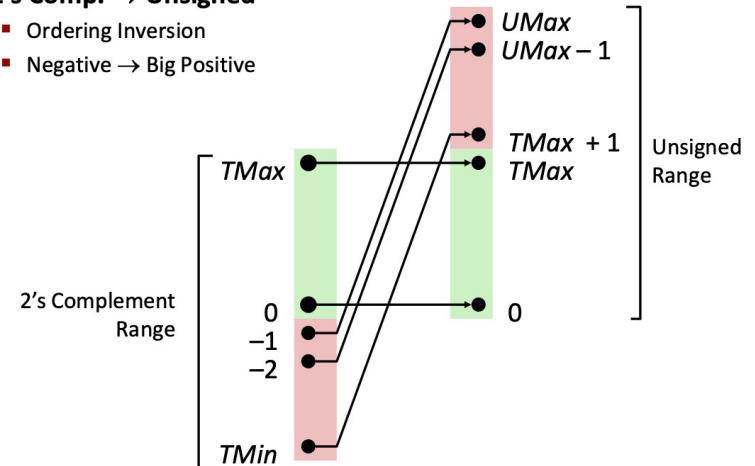
↔ =

↔ +/- 16

## Conversion Visualized

### 2's Comp. → Unsigned

- Ordering Inversion
- Negative → Big Positive



## Signed vs. Unsigned in C

### Constants

- By default are considered to be signed
- Unsigned if have "U" as suffix, or if too big to be signed  
`0U, 2147483648`
- Watch out! A leading minus sign is not part of the constant!  
`-2147483648 == 2147483648` on 32-bit machines (why?)

### Casting

- Explicit casting between signed & unsigned same as U2T and T2U  

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```
- Implicit casting also occurs via assignments and procedure calls  

```
int fun(unsigned u);
tx = ux;           uy = fun(tx);
```

## Casting Surprises

### Expression Evaluation

- If there is a mix of unsigned and signed in single expression,  
*signed values implicitly cast to unsigned*
- Including comparison operations `<`, `>`, `==`, `<=`, `>=`
- Examples for  $W = 32$ :  $TMIN = -2,147,483,648$ ,  $TMAX = 2,147,483,647$

| Constant <sub>1</sub> | Constant <sub>2</sub> | Relation | Evaluation |
|-----------------------|-----------------------|----------|------------|
| 0                     | 0U                    | ==       | unsigned   |
| -1                    | 0                     | <        | signed     |
| -1                    | 0U                    | >        | unsigned   |
| 2147483647            | -2147483647-1         | >        | signed     |
| 2147483647U           | -2147483647-1         | <        | unsigned   |
| -1                    | -2                    | >        | signed     |
| (unsigned)-1          | -2                    | >        | unsigned   |
| 2147483647            | 2147483648U           | <        | unsigned   |
| 2147483647            | (int) 2147483648U     | >        | signed     |

## Summary

### Casting Signed $\leftrightarrow$ Unsigned: Basic Rules

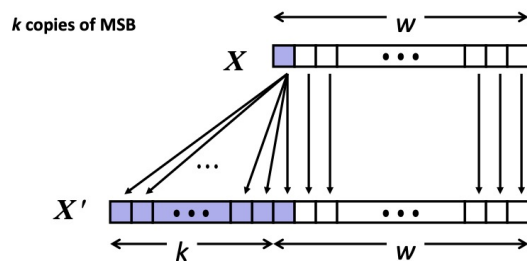
- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting  $2^w$
- Expression containing signed and unsigned int
  - `int` is cast to unsigned!!

## Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

## Sign Extension

- Task:
  - Given  $w$ -bit signed integer  $x$
  - Convert it to  $w+k$ -bit integer with same value
- Rule:
  - Make  $k$  copies of sign bit:
  - $X' = \underbrace{x_{w-1}, \dots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \dots, x_0$



## Sign Extension: Simple Example

Positive number

10 =

|  |     |   |   |   |   |
|--|-----|---|---|---|---|
|  | -16 | 8 | 4 | 2 | 1 |
|  | 0   | 1 | 0 | 1 | 0 |

10 =

|  |     |    |   |   |   |   |
|--|-----|----|---|---|---|---|
|  | -32 | 16 | 8 | 4 | 2 | 1 |
|  | 0   | 0  | 1 | 0 | 1 | 0 |

Negative number

-10 =

|  |     |   |   |   |   |
|--|-----|---|---|---|---|
|  | -16 | 8 | 4 | 2 | 1 |
|  | 1   | 0 | 1 | 1 | 0 |

-10 =

|  |     |    |   |   |   |   |
|--|-----|----|---|---|---|---|
|  | -32 | 16 | 8 | 4 | 2 | 1 |
|  | 1   | 1  | 0 | 1 | 1 | 0 |

Positive number

10 =

|  |     |   |   |   |   |
|--|-----|---|---|---|---|
|  | -16 | 8 | 4 | 2 | 1 |
|  | 0   | 1 | 0 | 1 | 0 |

10 =

|  |     |    |   |   |   |   |
|--|-----|----|---|---|---|---|
|  | -32 | 16 | 8 | 4 | 2 | 1 |
|  | 0   | 0  | 1 | 0 | 1 | 0 |

Negative number

-10 =

|  |     |   |   |   |   |
|--|-----|---|---|---|---|
|  | -16 | 8 | 4 | 2 | 1 |
|  | 1   | 0 | 1 | 1 | 0 |

-10 =

|  |     |    |   |   |   |   |
|--|-----|----|---|---|---|---|
|  | -32 | 16 | 8 | 4 | 2 | 1 |
|  | 1   | 1  | 0 | 1 | 1 | 0 |

## Larger Sign Extension Example

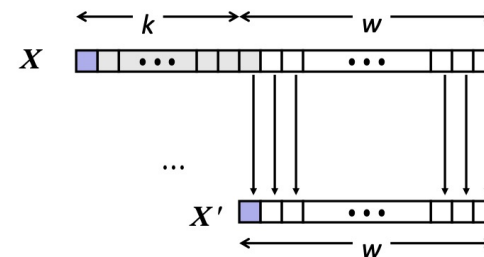
```
short int x = 15213;
int ix = (int) x;
short int y = -15213;
int iy = (int) y;
```

|    | Decimal | Hex         | Binary                              |
|----|---------|-------------|-------------------------------------|
| x  | 15213   | 3B 6D       | 00111011 01101101                   |
| ix | 15213   | 00 00 3B 6D | 00000000 00000000 00111011 01101101 |
| y  | -15213  | C4 93       | 11000100 10010011                   |
| iy | -15213  | FF FF C4 93 | 11111111 11111111 11000100 10010011 |

- Converting from smaller to larger integer data type
- C automatically performs sign extension

## Truncation

- **Task:**
  - Given  $k+w$ -bit signed or unsigned integer  $X$
  - Convert it to  $w$ -bit integer  $X'$  with same value for "small enough"  $X$
- **Rule:**
  - Drop top  $k$  bits:
  - $X' = x_{w-1}, x_{w-2}, \dots, x_0$



## Truncation: Simple Example

No sign change

|     |     |   |   |   |   |
|-----|-----|---|---|---|---|
|     | -16 | 8 | 4 | 2 | 1 |
| 2 = | 0   | 0 | 0 | 1 | 0 |

|     |    |   |   |   |
|-----|----|---|---|---|
|     | -8 | 4 | 2 | 1 |
| 2 = | 0  | 0 | 1 | 0 |

$2 \bmod 16 = 2$

|      |     |   |   |   |   |
|------|-----|---|---|---|---|
|      | -16 | 8 | 4 | 2 | 1 |
| -6 = | 1   | 1 | 0 | 1 | 0 |

|      |    |   |   |   |
|------|----|---|---|---|
|      | -8 | 4 | 2 | 1 |
| -6 = | 1  | 0 | 1 | 0 |

$-6 \bmod 16 = 26\text{U} \bmod 16 = 10\text{U} = -6$

Sign change

|      |     |   |   |   |   |
|------|-----|---|---|---|---|
|      | -16 | 8 | 4 | 2 | 1 |
| 10 = | 0   | 1 | 0 | 1 | 0 |

|      |    |   |   |   |
|------|----|---|---|---|
|      | -8 | 4 | 2 | 1 |
| -6 = | 1  | 0 | 1 | 0 |

$10 \bmod 16 = 10\text{U} \bmod 16 = 10\text{U} = -6$

|       |     |   |   |   |   |
|-------|-----|---|---|---|---|
|       | -16 | 8 | 4 | 2 | 1 |
| -10 = | 1   | 0 | 1 | 1 | 0 |

|     |    |   |   |   |
|-----|----|---|---|---|
|     | -8 | 4 | 2 | 1 |
| 6 = | 0  | 1 | 1 | 0 |

$-10 \bmod 16 = 22\text{U} \bmod 16 = 6\text{U} = 6$

## Summary: Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result
- **Truncating (e.g., unsigned to unsigned short)**
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small (in magnitude) numbers yields expected behavior