CSC 212
Data Structures & Algorithms

Fall 2022 | Jonathan Schrader

Sorts

Housekeeping

If programming and or the c++ is a *significant* issue…

- Last Day To Withdraw Without Affecting Transcript
  - September 29
- 50% complete or less on Assignment 1, should truly weigh this option
- Those who did not submit, should outright withdraw…

Internships & Jobs

Google

https://careers.google.com/how-we-hire/interview

Amazon

https://www.amazon.jobs/en/landing_pages/software-development-topics

Meta

https://www.metacareers.com/life/

Worst-case, Average-case, Best-case

Analyzing Code

```
unsigned int argmin (const std::vector<int> &values) {
  unsigned int length = values.size();
  assert(length > 0);
  unsigned int idx = 0;
  int current = values[0];
  for (unsigned int i = 1; i < length; i++) {
    if (values[i] < current) {
      current = values[i];
      idx = i;
    }
  }
  return idx;
}
```

$$T(n) = ?$$

based on number of comparisons

Analyzing Code

```
bool argk (const std::vector<int> &values, int k, unsigned int &idx) {
  unsigned int length = values.size();
  for (unsigned int i = 0; i < length; i++) {
    if (values[i] == k) {
      idx = i;
      return true;
    }
  }
  return false;
}
```

$$T(n) = ?$$

based on number of comparisons

Different types of analysis

😕 Worst-case

- *maximum* time of algorithm on *any* input

🤔 Average-case

- *expected* time of algorithm over *all* inputs

🦄 Best-case

- *minimum* time of algorithm on *some* (optimal) input

Different types of analysis, con't

While *asymptotic analysis* describes $T(n)$ => ∞…

- *asymptotic notation: O, Ω, Θ*

Case analysis looks into the different input types

- best-case, worst-case, average-case

Both analysis types are orthogonal to each other

Worst-case, Average-case, Best-case

Examples

Factorial of a number (iterative algorithm)

Sequential search (return first occurrence)

Sequential search (return last occurrence)

Basic Sorting Algorithms

Sorting

Given n elements that can be compared according to a *total order* relation

- we want to rearrange them in non-increasing/non-decreasing order
- for example (non-decreasing):
  - `input`: sequence of items
    - $A = [k_0, k_1, \ldots, k_{n-1}]$
  - `output`: permutation of A
    - $B | B[0] <= B[1] <= \ldots B[n-1]$

Central problem in computer science

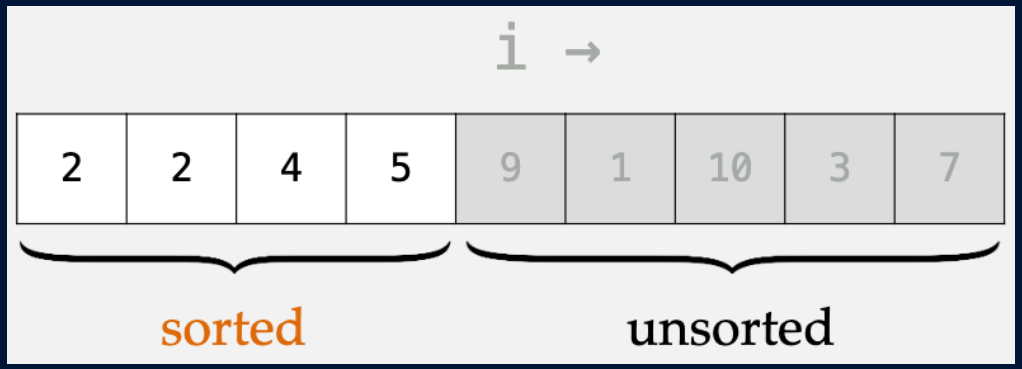
Insertion Sort

Array is divided into sorted and unsorted parts

- algorithm scans array from left to right

Invariants

- elements in sorted are in ascending order
- elements in unsorted have not been seen



Insertion Sort

```c
void insertionsort (int *A, unsigned int n) {
  int temp;
  unsigned int i,j;
  //grows the left part (sorted)
  for (i = 0; i < n; i++) {
    //inserts A[j] in sorted part
    for (j = i; j > 0; j--) {
      if(A[j] < A[j-1]) {
        temp = A[j];
        A[j] = A[j-1];
        A[j-1] = temp;
      } else
        break;
    }
  }
}
```

Number of comparisons? | Number of exchanges?

Analysis — Insertion Sort(comparisons)

Running time depends on the input

Worst-case?

- input reverse sorted

Best-case?

- input already sorted

Average-case?

- expect every element to move O(n/2) times

Partially sorted arrays

An *inversion* is a pair of keys that are out of order

1	3	4	5	2	6	10	15	7

"array is *partially sorted* if the number of pairs that are out-of-order is $O(n)$"

For partially-sorted arrays, insertion sort runs in *linear time*.

$$\Theta(n)$$

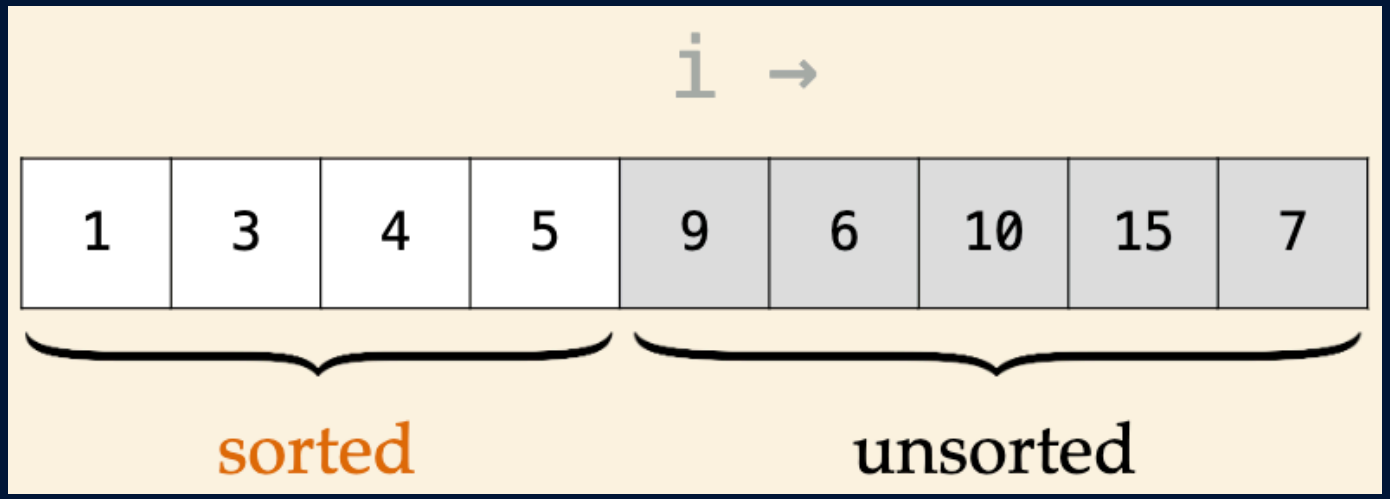
Selection Sort

Array is divided into *sorted* and *unsorted* parts

- algorithm scans array from left to right

Invariants

- elements in *sorted* are *fixed* and in ascending order
- no element in *unsorted* is smaller than any element in *sorted*



```c
void selectionsort (int *A, unsigned int n) {
  int temp;
  unsigned int i, j, min;
  // grows the left part (sorted)
  for (i = 0; i < n; i++) {
    min = i;
    // find min in unsorted part
    for (j = i + 1; j < n; j++) {
      if (A[j] < A[min]) {
        min = j;
      }
    }
    // swap A[i] and A[min]
    temp = A[i];
    A[i] = A[min];
    A[min] = temp;
  }
}
```

Number of comparisons? | Number of exchanges?

Summary

	Best-Case	Average-Case	Worst-Case
Selection Sort	$\theta(n^2)$	$\theta(n^2)$	$\theta(n^2)$
Insertion Sort	$\theta(n)$	$\theta(n^2)$	$\theta(n^2)$