CSC 544 Homework 1

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Problem 1

Let $Z = \{01^k0 \mid k \geq 0\}$.

Part (a)

Give a GNFA that recognizes Z.
TODO

Part (b)

Give a regular expression that generates Z.
TODO

Problem 2

Part (a)

Let $L = \{\text{abra}(\text{cad})^k\text{abra} \mid k \geq 1\}$. Prove or disprove: L is regular.

Proof. We will show that L is regular by construction. Let $A = \{\text{abra}\}$ and $C = \{\text{cad}\}$. Since A and C have finite cardinality, they are regular. Define $L' = A \circ C \circ C^* \circ A$. Since A and C are regular, and the class of regular languages is closed under $\circ$ and $*$, L' is regular.

We will argue that $L' = L$. Note that

$$\begin{aligned}
L' &= A \circ C \circ C^* \circ A \\
&= \{\text{abra}\} \circ \{\text{cad}\} \circ \{(\text{cad})^k \mid k \geq 0\} \circ \{\text{abra}\} \\
&= \{\text{abra}\} \circ \{(\text{cad})^k \mid k \geq 1\} \circ \{\text{abra}\} \\
&= \{\text{abra}(\text{cad})^k\text{abra} \mid k \geq 1\} \\
&= L
\end{aligned}$$

so L is regular as desired. //

Part (b)

Let $L = \{(\text{abra})^r(\text{cad})^k(\text{abra})^r \mid k \geq 1, r \geq 0\}$. Prove or disprove: L is regular.

Proof. TODO //

Problem 3

Let $\Sigma = \{0, 1\}$.

Part (a)

Let $L = \{xyx \mid x, y \in \Sigma^*\}$. Prove that L is regular.

Proof. We will show that L is regular by construction. The main idea is that L contains all strings (e.g. take $x = \epsilon$). Since Σ has finite cardinality, Σ is regular. Define $L' = \Sigma^*$. Since Σ is regular, and the class of regular languages is closed under $*$, L' is regular.

We will argue that $L' = L$. Clearly $L \subseteq L' = \Sigma^*$. Fix some $s \in L'$. Let $x = \epsilon$ and $y = s$. Then $x \in \Sigma^*$ and $y \in L' = \Sigma^*$. By definition, $xyx \in L$, so $xyx = \epsilon s \epsilon = s \in L$. Therefore, $L' \subseteq L$. Hence, $L' = L$, so L is regular as desired. //

Part (b)

Let $L = \{xyxy \mid x, y \in \Sigma^*\}$. Prove that L is not regular.

Proof. We will show that L is not regular via the pumping lemma. Assume that L is regular, and let p be the pumping length. Take $x = 0^p1^p$ and $y = 0^p1^p$. Then $x, y \in \Sigma^*$, so $s = xyxy \in L$.

We will show that s cannot be pumped. For any choice of u, v, w such that $uvw = s$ and $1 \leq |uv| \leq p$, $v = 0^k$ for some $1 \leq k \leq p$ since the first p symbols in s are all zero. By the pumping lemma, $s' = uv^0w = 0^{p-k}1^p0^p1^p \in L$. Suppose there exists some choice of x', y' such that $s' = x'y'x'y'$. Then, the first half of s' is $x'y' = 0^{p-k}1^p1^{k/2}$ and the second half of s' is $x'y' = 1^{p-k/2}0^p$. However, the first half is not the same as the second half, so $x'y' \neq x'y'$ which is impossible. Thus, there is no choice of $x'y'$ such that $s' = x'y'x'y'$, so $s' \notin L$, which is impossible. Hence L is not regular as desired. //

Problem 4

Part (a)

Prove or disprove: the class of regular languages is closed under union.

Proof. Let L_1 and L_2 be regular languages. We will show that $L = L_1 \cup L_2$ is regular by construction.

Since L_1 is regular, there exists some DFA $M_1 = (Q_1, \Sigma_1, \delta_1, q_0^1, F_1)$ that recognizes L_1, and since L_2 is regular, there exists some DFA $M_2 = (Q_2, \Sigma_2, \delta_2, q_0^2, F_2)$ that recognizes L_2. Define DFA

$$M = ((Q_1 \cup q_{\text{reject}}) \times (Q_2 \cup q_{\text{reject}}), \Sigma_1 \cup \Sigma_2, \delta, (q_0^1, q_0^2), (F_1 \times Q_2) \cup (Q_1 \times F_2))$$

where

$$\delta(s_k, (q_i^1, q_j^2)) = \begin{cases} (\delta_1(q_i^1), \delta_2(q_j^2)) & \text{if } s_k \in \Sigma_1 \wedge s_k \in \Sigma_2 \wedge q_i^1 \in Q_1 \wedge q_j^2 \in Q_2 \\ (\delta_1(q_i^1), q_{\text{reject}}) & \text{if } s_k \in \Sigma_1 \wedge s_k \notin \Sigma_2 \wedge q_i^1 \in Q_1 \\ (q_{\text{reject}}, \delta_2(q_j^2)) & \text{if } s_k \notin \Sigma_1 \wedge s_k \in \Sigma_2 \wedge q_j^2 \in Q_2 \\ (q_{\text{reject}}, q_{\text{reject}}) & \text{otherwise} \end{cases}$$

We will show that M recognizes L.

Fix some $s \in L$. Then $s = s_1 s_2 \ldots s_m$. TODO. //

Part (b)

Prove or disprove: the class of regular languages is closed under concatenation.

Proof. TODO //

Part (c)

Prove or disprove: the class of regular languages is closed under star.

Proof. TODO //