Exam for Vega-Prep

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While implementing the functions below, you need to account for the singularities of the type 0/0.

Cost-of-carry from forward and maturity

Input:

$S(t_0)$: the spot price.

t_0 : the initial time given as year fraction.

Output:

$q = (q(F(t), t))_{t>t_0}$: the continuously compounded surface of *cost-of-carry* rates.

We recall that the relation between the forward price and the cost-of-carry rate:

$$F(t) = S(t_0) \exp(q(F(t), t)(t - t_0)), \quad t > t_0.$$

Forward curve for exchange rate

Input:

$S(t_0)$: the spot exchange rate (the number of units of domestic currency per one unit of foreign currency).

$D^d = (D^d(t))_{t \geq t_0}$: the discount curve in domestic currency.

$D^f = (D^f(t))_{t \geq t_0}$: the discount curve in foreign currency.

Output:

$F = (F(t))_{t \geq t_0}$: the forward FX rates. Here t is the maturity of the contract and t_0 is the issue time.

The Svensson yield curve

Input:

$(c_i)_{i=0,1,2,3}$: the constant coefficients of the Svensson model of interest rates.

$(\lambda_i)_{i=1,2}$: the strictly positive mean-reversion rates, $\lambda_1 \neq \lambda_2$.

t_0 : the initial time given as year fraction.

Output: the Svensson yield curve $\gamma = (\gamma(t))_{t \geq t_0}$. It has the form:

$$\begin{aligned}\gamma(t) = c_0 + c_1 \frac{1 - e^{-\lambda_1(t-t_0)}}{\lambda_1(t-t_0)} + c_2 \left(\frac{1 - e^{-\lambda_1(t-t_0)}}{\lambda_1(t-t_0)} - e^{-\lambda_1(t-t_0)} \right) \\ + c_3 \left(\frac{1 - e^{-\lambda_2(t-t_0)}}{\lambda_2(t-t_0)} - e^{-\lambda_2(t-t_0)} \right), \quad t \geq t_0.\end{aligned}$$

Implied volatility curve for the Black model

Input:

$\lambda \geq 0$: the mean-reversion rate.

$\sigma > 0$: the short-term volatility.

t_0 : the initial time given as year fraction.

Output: the stationary implied volatility curve for the Black model. It has the form:

$$\Sigma(t) = \sigma\sqrt{\frac{1-\exp(-2\lambda(t-t_0))}{2\lambda(t-t_0)}}, \quad t \geq t_0.$$

Forward LIBORs

Input:

δt : the time interval for LIBOR.

$(B(t_0,t))_{t\geq t_0}$: the discount curve; t_0 is the initial time.

Output:

$(L^f(t,t+\delta t) = L^f(t,t+\delta t;t_0))_{t\geq t_0}$: the forward LIBORs for period δt computed at t_0.

It costs nothing to enter the forward rate agreement (FRA) with maturity t and period δt as either a borrower or a lender. Forward LIBOR $L^f(t,t+\delta t)$ is set at issue time t_0. In the contract,

1. At maturity t, the borrower receives notional N.
2. At time $t+\delta t$, the borrower pays the notional plus the interest computed at the forward LIBOR:

$$N(1+L^f(t,t+\delta t)\delta t).$$

Forward curve obtained by linear interpolation of cost-of-carry rates

Input:

S_0 : the current spot price.

$(t_i)_{i=1,\dots,M}$: the maturities of forward contracts, $t_0 < t_1$.

$(F_i)_{i=1,\dots,M}$: the market forward prices.

t_0 : the initial time given as year fraction.

Output: the forward curve

$$F(t) = S_0 \exp(q(t)(t - t_0)), \quad t \in [t_0, t_M],$$

where *cost-of-carry* curve $q = (q(t))_{t\in[t_0,t_M]}$ is obtained by the linear interpolation of market cost-of-carry rates $(q_i)_{i=1,\ldots,M}$. We recall that

$$F_i = S_0 \exp(q_i(t_i - t_0)), \quad i = 1, \ldots, M.$$

We assume that on $[t_0, t_1]$ the cost-of-carry rate is constant:

$$q(t) = q_1, \quad t \in [t_0, t_1].$$