Optimization-Advanced

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Get Python code for the figure from

https://github.com/SurabhiSeetha/Fwciith2022/tree/main/Assignment%201/codes/src

Get LaTex code from

https://github.com/SurabhiSeetha/Fwciith2022/tree/main/avr%20gcc

1 Question

Q(24), Class - 12, CBSE Paper, 2013

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

2 Solution

Let **h** be the height of the Cylinder

Let **r** be the radius of the Cylinder

Let **R** be the radius of the Sphere

Since the Cylinder is inscribed in the sphere we get,

$$h = 2\sqrt{R^2 - r^2} \tag{2.0.1}$$

We know that the Volume of the Cylinder is,

$$V = \pi r^2 h \tag{2.0.2}$$

By substituting Eq. (2.0.1) in (2.0.2) we get,

$$V = 2\pi r^2 \sqrt{R^2 - r^2} \tag{2.0.3}$$

Given to prove that $h = \frac{2R}{\sqrt{3}}$ for $\mathbf{V_{max}}$. And to find the $\mathbf{V_{max}}$.

Hence, differentiating Eq. (2.0.3) w.r.t **r** we get,

$$\frac{dV}{dr} = \frac{d}{dr}[2\pi r^2 \sqrt{R^2 - r^2}]$$

By simplification we get,

$$\frac{dV}{dr} = \frac{4\pi r R^2 - 6\pi r^3}{\sqrt{R^2 - r^2}} \tag{2.0.4}$$

Now equating the Eq. (2.0.4) to zero and solving for **r** we get,

$$r^2 = \frac{2R^2}{3} \tag{2.0.5}$$

Differentiating Eq. (2.0.4) once again w.r.t **r** and substituting (2.0.5) it is observed that,

$$\frac{d^2V}{dr^2} < 0 \tag{2.0.6}$$

Hence it is a point of Maxima.

Substituting Eq. (2.0.5) in Eq. (2.0.1),

$$h = 2\sqrt{R^2 - \frac{2R^2}{3}}$$

$$\therefore \mathbf{h} = \frac{2R}{\sqrt{3}} \tag{2.0.7}$$

Therefore, the Height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

Hence Proved.

Now, to find V_{max}, Substitute Eq. (2.0.7) in Eq. (2.0.2). We get,

$$\mathbf{V_{max}} = \frac{4\pi R^3}{3\sqrt{3}}$$

3 Verification Using Geomtric Programming

Disciplined geometric programming, DGP is a subset of log-log-convex program (LLCP). An LLCP is defined as,

$$
\begin{aligned}
&\text{minimize } f_0(x) \\
&\text{subject to } f_i(x) \leq \tilde{f}_i, \text{ i = 1,2,...,m.} \\
&\qquad\qquad g_i(x) = \tilde{g}_i, \text{ i = 1,2,...,p.}
\end{aligned}
\tag{3.0.1}
$$

where the functions f_i are log-log convex, $\tilde{f}_i$ are log-log concave, and the functions g_i and $\tilde{g}_i$ are log-log affine. An optimization problem with constraints of the above form in which the goal is to maximize or minimize a log-log concave function is also an LLCP. These LLCPs generalize geometric programming.

The given problem can be formulated as a DGP as,

$$V = \max_{r,h} \ \pi r^2 h \tag{3.0.2}$$

$$s.t \quad h = 2\sqrt{R^2 - r^2} \tag{3.0.3}$$

By assuming R = 4 as input, and solving the above DGP Equations using Cvxpy we get,

$$V_{max} = 309.55 \tag{3.0.4}$$

$$h = 4.61 \tag{3.0.5}$$

$$r = 3.26 \tag{3.0.6}$$

Hence it is proved that $h = \frac{2R}{\sqrt{3}}$ for V_{max}.