QUEEN MARY UNIVERSITY OF LONDON

SCHOOL OF ENGINEERING AND MATERIAL SCIENCE

DEN6335: Spacecraft Design: Manoeuvring and Orbital Mechanics – 2022/23

Coursework 1: Interplanetary Orbit Transfers

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Table 1: Provided Data

Spacecraft orbit	Comet properties				
Initial orbit radius (km)	Semi-major axis a, (Au)	Eccentricity e, (-)	Inclination i, (°)	Argument of Perihelion ω, (°)	Comet mass (kg)
7552.838048	2.989170458	0.685673899	11.32197626	198.8097995	2.14E+15

Date: 6th April 2023

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Part A:

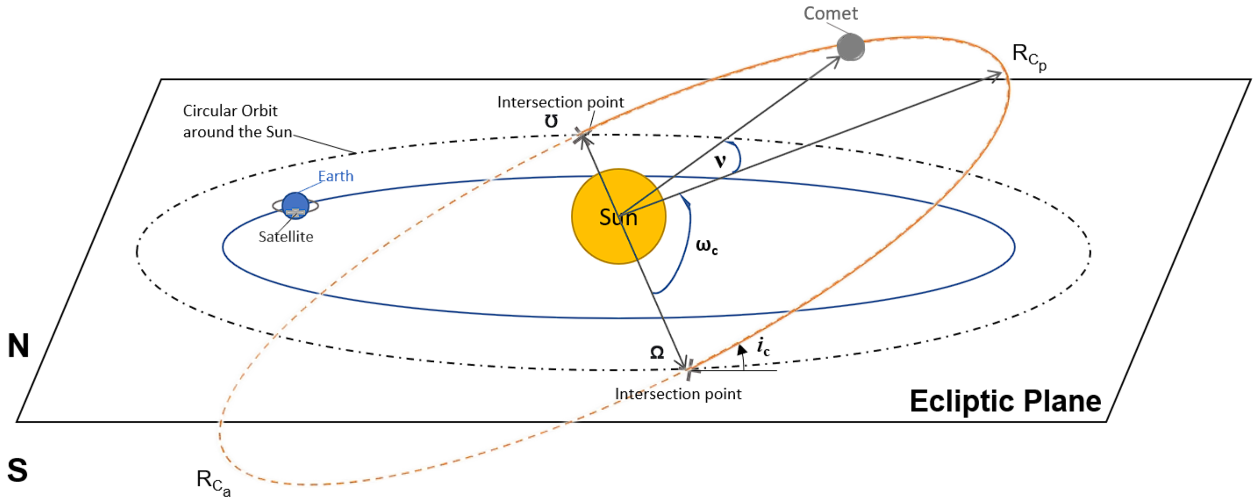

Figure 1: The comet's orbit intersects with the ecliptic plane.

Table 1: Parameters

Symbol	Quantity	Unit
θ or v	True anomaly	Degrees / Radians
ω	Angle of perihelion	Degrees / Radians
i_c	Inclination angle	Degrees / Radians
a	Semi-major axis	km / Au

Semi-major axis a = 2.989170458 Au × 1.4959787 × 10^8 = 4.4717353358372 × 10^8 km

When a comet's orbit intersects ecliptic plane at θ = -ω, that intersection point on the ecliptic plane is called "ascending node". (Comet Orbit direction S ⟹N)

➔ At ascending node, the sun-comet distance, (θ = -198.8097995° = -3.4698855 radian)

$$\boldsymbol{r_{asc}} = \frac{a(1-e^2)}{1+e\cos\theta} = \frac{4.47\times10^8(1-(0.6857)^2)}{1+(0.6857\cos(-3.4698855))} = \boldsymbol{6.7513542\times10^8\ km} = 4.5130015\ Au$$

When a comet's orbit intersects ecliptic plane at θ = (180°- ω), that intersection point on the ecliptic plane is called "descending node". (Comet Orbit direction N ⟹S)

➔ At descending node, the sun-comet distance, (θ =180° -198.8097995° = -0.32829 radian)

$$\boldsymbol{r_{dsc}} = \frac{a(1-e^2)}{1+e\cos\theta} = \frac{4.47\times10^8(1-(0.6857)^2)}{1+(0.6857\cos(-0.32829))} = \boldsymbol{1.4367955\ \times10^8\ km} = 0.960438\ Au$$

Part B:

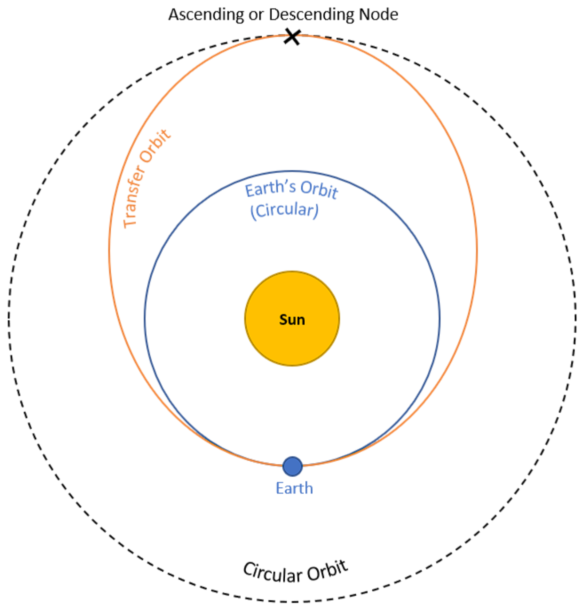

Figure 2: Hohmann orbit transfer

The patch conic method can be used to evaluate ΔV with simple assumptions be made such as the all the celestial body (Sun, Earth, comet) are taken as the point masses. The other assumption is the satellite only experiences the comet gravity throughout the journey.

To evaluate transfer orbit radius at ascending and descending node can be followed below method,

$$\boldsymbol{a_{tr-asc}} = \frac{r_{asc} + r_E}{2} = \frac{6.7513542 + 1.4959787}{2} \times 10^8 = \mathbf{4.12366645 \times 10^8}\ \boldsymbol{km}$$

$$\boldsymbol{a_{tr-dsc}} = \frac{r_{dsc} + r_E}{2} = \frac{1.4367955 + 1.4959787}{2} \times 10^8 = \mathbf{1.4663871 \times 10^8}\ \boldsymbol{km}$$

Using visa-viva energy equation to evaluate the transfer velocity at the both nodes can be deduced by,

$$\varepsilon_{asc} = -\frac{\mu_{sun}}{2a_{tr-asc}} = -\frac{1.327474512 \times 10^{11}}{2(4.12366645 \times 10^8)} = -160.958036\ (km/s)^2$$

$$\varepsilon_{dsc} = -\frac{\mu_{sun}}{2a_{tr-dsc}} = -\frac{1.327474512 \times 10^{11}}{2(1.4663871 \times 10^8)} = -452.63440738\ (km/s)^2$$

$$V_{asc}^2 = 2\left(\frac{\mu_{sun}}{r_E} + \varepsilon\right)$$

$$\boldsymbol{V_{asc}} = \sqrt{2\left(\frac{\mu_{sun}}{r_E} + \varepsilon_{asc}\right)} = \sqrt{2\left(\frac{1.327474512 \times 10^{11}}{1.4959787 \times 10^8} - 160.958036\right)} = \mathbf{38.115715137}\ \boldsymbol{km/s}$$

$$\boldsymbol{V_{dsc}} = \sqrt{2\left(\frac{\mu_{sun}}{r_E} + \varepsilon_{dsc}\right)} = \sqrt{2\left(\frac{1.327474512 \times 10^{11}}{1.4959787 \times 10^{8}} - 452.63440738\right)} = \mathbf{29.486522330}\ \boldsymbol{km/s}$$

The earth's velocity is accounted for by following equation with an assumption of earth's eccentricity zero (circular orbit).

$$\boldsymbol{V_E} = \sqrt{2\left(\frac{\mu_{sun}}{r_E} + \varepsilon_{earth}\right)} = \sqrt{2\left(\frac{\mu_{sun}}{r_E} - \frac{\mu_{sun}}{2r_E}\right)} = \sqrt{\frac{\mu_{sun}}{r_E}} = \sqrt{\frac{1.327474512 \times 10^{11}}{1.4959787 \times 10^{8}}} = \mathbf{29.788620}\ \boldsymbol{km/s}$$

Likewise, the satellite's circular velocity within its parking orbit can be calculated but μ_{earth} is required to find as it is needed to put in V_S equation.

$$\boldsymbol{\mu_{earth}} = GM = (6.67408 \times 10^{-20}\ Nkm^2/kg^2) \times (5.97 \times 10^{24}\ kg) = \mathbf{398442.576}\ \boldsymbol{km^3/s^2}$$

$$\boldsymbol{V_S} = \sqrt{\frac{\mu_{earth}}{r_S}} = \sqrt{\frac{398442.576}{7552.838048}} = \mathbf{7.2631963}\ \boldsymbol{km/s}$$

When the spacecraft leave the Earth's SOI and follows hyperbolic trajectory, the Hyperbolic excess velocities at ascending node and descending node are,

$$\boldsymbol{V_{\infty-asc}} = V_{asc} - V_E = 38.115715137 - 29.788620 = \mathbf{8.3270951371}\ \boldsymbol{km/s}$$

$$\boldsymbol{V_{\infty-dsc}} = V_{dsc} - V_E = 29.486522330 - 29.788620 = \mathbf{-0.302097669}\ \boldsymbol{km/s}$$

The negative $\boldsymbol{V_{\infty-dsc}}$ is not possible thus it is assumed as **0 km/s** and proceed with following calculations.

The satellite's hyperbolic departure velocity is calculated as shown below,

$$\varepsilon_{\infty-asc} = \frac{V_{\infty-asc}^2}{2} = \frac{(8.3270951371)^2}{2} = 34.670256711\ (km/s)^2$$

$$\boldsymbol{V_{h-asc}} = \sqrt{2\left(\frac{\mu_{earth}}{r_{satellite}} + \varepsilon_{\infty-asc}\right)} = \sqrt{2\left(\frac{398442.576}{7552.838048} + 34.670256711\right)} = \mathbf{13.2230312}\ \boldsymbol{km/s}$$

$$\varepsilon_{\infty-dsc} = \frac{V_{\infty-dsc}^2}{2} = \frac{(0)^2}{2} = 0\ (km/s)^2$$

$$\boldsymbol{V_{h-dsc}} = \sqrt{2\left(\frac{\mu_{earth}}{r_{satellite}} + \varepsilon_{\infty-dsc}\right)} = \sqrt{2\left(\frac{398442.576}{7552.838048} + 0\right)} = \mathbf{10.271710750}\ \boldsymbol{km/s}$$

The ΔV_1 of satellite to escape from its parking orbit about earth and intersect the comet's orbit at the two location (ascending and descending node) is deduced by,

$$\boldsymbol{\Delta V_{1-asc}} = V_{h-asc} - V_S = 13.2230312 - 7.2631963 = \mathbf{5.95983494}\ \boldsymbol{km/s}$$

$$\boldsymbol{\Delta V_{1-dsc}} = V_{h-dsc} - V_S = 10.271710750 - 7.2631963 = \mathbf{3.0085144}\ \boldsymbol{km/s}$$

Part C:

The tangential velocity for satellite at both nodes using conservation of momentum (v·r) equation is presented by below equation,

$$\boldsymbol{V_{asc}^{T}} = \frac{V_{asc} \times r_E}{r_{asc}} = \frac{38.115715137 \times 1.4959787 \times 10^8}{6.7513542 \times 10^8} = \mathbf{8.4457571460\ \ km/s}$$

$$\boldsymbol{V_{dsc}^{T}} = \frac{V_{dsc} \times r_E}{r_{dsc}} = \frac{29.486522330 \times 1.4959787 \times 10^8}{1.4367955\ \times 10^8} = \mathbf{30.701104884\ \ km/s}$$

The circumferential velocity at comet's orbit and ecliptic plane intersection point can be evaluated by following equation,

$$\boldsymbol{V_{\theta,asc}} = \frac{\sqrt{\mu_{sun} \times a \times (1-e^2)}}{r_{asc}} = \frac{\sqrt{1.327474512 \times 4.47173533 \times 10^{19} \times (1-(0.6857)^2)}}{6.7513542 \times 10^8}$$
$$= \mathbf{8.3068669\ \ km/s}$$

$$\boldsymbol{V_{\theta,dsc}} = \frac{\sqrt{\mu_{sun} \times a \times (1-e^2)}}{r_{dsc}} = \frac{\sqrt{1.327474512 \times 4.47173533 \times 10^{19} \times (1-(0.6857)^2)}}{1.4367955\ \times 10^8}$$
$$= \mathbf{39.033113\ \ km/s}$$

Using re-arranged form of visa-viva equation, the velocity of comet at both nodes can be found by below method,

$$\boldsymbol{V_{comet-asc}} = \sqrt{2\left(\frac{\mu_{sun}}{r_{asc}} - \frac{\mu_{sun}}{2a}\right)} = \sqrt{2\left(\frac{1.327474512 \times 10^{11}}{6.7513542 \times 10^8} - \frac{1.327474512 \times 10^{11}}{2 \times 4.47173533 \times 10^8}\right)}$$
$$= \mathbf{9.8177374978\ \ km/s}$$

$$\boldsymbol{V_{comet-dsc}} = \sqrt{2\left(\frac{\mu_{sun}}{r_{dsc}} - \frac{\mu_{sun}}{2a}\right)} = \sqrt{2\left(\frac{1.327474512 \times 10^{11}}{1.4367955\ \times 10^8} - \frac{1.327474512 \times 10^{11}}{2 \times 4.47173533 \times 10^8}\right)}$$
$$= \mathbf{39.382327740\ \ km/s}$$

The Euler angle of rotation required to calculate for the x-axis, can be presented by,

$$V_{S-A/D}^{T} = X(i) \times V_{asc/dsc}^{T}$$

$$X(i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{bmatrix}, \quad V_{asc/dsc}^{T} = \begin{bmatrix} 0 \\ v \\ 0 \end{bmatrix}, \qquad i = 11.32197626°$$

$$V_{S-A}^{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ V_{asc}^{T} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(11.32197626°) & -\sin(11.32197626°) \\ 0 & \sin(11.32197626°) & \cos(11.32197626°) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 8.4458 \\ 0 \end{bmatrix}$$

$$\boldsymbol{V_{S-A}^{T}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{8.28139789} \\ \mathbf{1.65809007} \end{bmatrix}$$

$$\left|\boldsymbol{V_{S-A}^T}\right| = \sqrt{(8.28139789)^2 + (1.65809007)^2} = \mathbf{8.4457571460}\ \boldsymbol{km/s}$$

$$V_{S-D}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ V_{dsc}^T \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(11.32197626°) & -\sin(11.32197626°) \\ 0 & \sin(11.32197626°) & \cos(11.32197626°) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 30.701105 \\ 0 \end{bmatrix}$$

$$\boldsymbol{V_{S-D}^T} = \begin{bmatrix} \mathbf{0} \\ \mathbf{30.1036439} \\ \mathbf{6.02731006} \end{bmatrix}$$

$$\left|\boldsymbol{V_{S-D}^T}\right| = \sqrt{(30.1036439)^2 + (6.02731006)^2} = \mathbf{30.7011049}\ \boldsymbol{km/s}$$

To find the angle at which the satellite fly alongside the comet in the same orbital plane of comet the below process needs to be considered for both nodes,

$$V_{asc}^c = [V_{comet-asc} \quad V_{\theta,asc} \quad 0] = [9.8177374978 \quad 8.3068669 \quad 0]$$

$$|\boldsymbol{V_{asc}^c}| = \sqrt{(9.8177374978)^2 + (8.3068669)^2} = \mathbf{12.860482388}\ \boldsymbol{km/s}$$

$$V_{dsc}^c = [V_{comet-dsc} \quad V_{\theta,dsc} \quad 0] = [39.382327740 \quad 39.033113 \quad 0]$$

$$|\boldsymbol{V_{dsc}^c}| = \sqrt{(39.382327740)^2 + (39.033113)^2} = \mathbf{55.448639737}\ \boldsymbol{km/s}$$

The angle is now to be found as,

$$\phi_{asc} = \cos^{-1}\left(\frac{V_{asc}^c \cdot V_{S-A}^T}{|V_{asc}^c|\ |V_{S-A}^T|}\right) = \cos^{-1}\left(\frac{[9.8177374978 \quad 8.3068669 \quad 0]\begin{bmatrix} 0 \\ 8.28139789 \\ 1.65809007 \end{bmatrix}}{12.860482388 \times 8.4457571460}\right)$$

$$\boldsymbol{\phi_{asc}} = \cos^{-1}\left(\frac{68.792470018}{108.616511035}\right) = \mathbf{50.7021476°}$$

$$\phi_{dsc} = \cos^{-1}\left(\frac{V_{dsc}^c \cdot V_{S-D}^T}{|V_{dsc}^c|\ |V_{S-D}^T|}\right) = \cos^{-1}\left(\frac{[39.382327740 \quad 39.033113 \quad 0] \cdot \begin{bmatrix} 0 \\ 30.1036439 \\ 6.02731006 \end{bmatrix}}{55.448639737 \times 30.7011049}\right)$$

$$\boldsymbol{\phi_{dsc}} = \cos^{-1}\left(\frac{1175.0389340}{1702.334505}\right) = \mathbf{46.34997997°}$$

The cosine rule is applied to deduce ΔV_2,

$$\boldsymbol{\Delta V_{2-asc}} = \sqrt{(12.8605^2) + (8.44576^2) - 2(12.8605)(8.4457)\cos(50.702)} = \mathbf{9.9568007}\ \boldsymbol{km/s}$$

$$\boldsymbol{\Delta V_{2-dsc}} = \sqrt{(55.4486^2) + (30.701^2) - 2(55.4486)(30.7011)\cos(46.350)} = \mathbf{40.82929858}\ \boldsymbol{km/s}$$

Total ΔV for the mission is,

$$\boldsymbol{\Delta V_{asc}} = \Delta V_{1-asc} + \Delta V_{2-asc} = 5.95983494 + 9.9568007 = \mathbf{15.9166357}\ \boldsymbol{km/s}$$

$$\boldsymbol{\Delta V_{dsc}} = \Delta V_{1-dsc} + \Delta V_{2-dsc} = 3.008514 + 40.82929858 = \mathbf{43.8378126}\ \boldsymbol{km/s}$$

Part D:

The sphere of influence of the comet from ascending and descending nodes are calculated by the following equation,

$$R_c \approx R_d \left(\frac{\mu_{comet}}{\mu_{sun}}\right)^{\frac{2}{5}} = R_d \left(\frac{M_{comet}}{M_{sun}}\right)^{\frac{2}{5}}$$

$$\boldsymbol{R_{c-asc}} \approx R_{d-asc} \left(\frac{M_{comet}}{M_{sun}}\right)^{\frac{2}{5}} = 6.7513542 \times 10^8 \left(\frac{2.14 \times 10^{15}}{1.99 \times 10^{30}}\right)^{\frac{2}{5}} = \boldsymbol{695.04859855\ km}$$

$$\boldsymbol{R_{c-dsc}} \approx R_{d-dsc} \left(\frac{M_{comet}}{M_{sun}}\right)^{\frac{2}{5}} = 1.4367955 \times 10^8 \left(\frac{2.14 \times 10^{15}}{1.99 \times 10^{30}}\right)^{\frac{2}{5}} = \boldsymbol{147.91739095\ km}$$

Part E:

The transfer time of this mission will be deduced in the days from both ascending node and descending node,

$$t_{asc} = \pi \sqrt{\frac{a_{tr-asc}^3}{\mu_{sun}}} \times \frac{1}{24 \times 3600} = \pi \sqrt{\frac{(4.12366645 \times 10^8)^3}{1.327474512 \times 10^{11}}} \times \frac{1}{24 \times 3600}$$

$$\boldsymbol{t_{asc} = 835.6960\ days}$$

$$t_{dsc} = \pi \sqrt{\frac{a_{tr-dsc}^3}{\mu_{sun}}} \times \frac{1}{24 \times 3600} = \pi \sqrt{\frac{(1.4663871 \times 10^8)^3}{1.327474512 \times 10^{11}}} \times \frac{1}{24 \times 3600}$$

$$\boldsymbol{t_{dsc} = 177.2132\ days}$$

The above results show that the descending node would be prove better than the ascending node since it only takes roughly 177 days to set the satellite in the comet's orbit which is approximately 659 days less than the ascending node used to launch satellite.

Also, from the Part C, the total ΔV_{dsc} is 43.8378126 km/s compared to ΔV_{asc} = 15.9166357.

$$\boldsymbol{\Delta V_{asc}} < \boldsymbol{\Delta V_{dsc}} \Rightarrow 15.9166357 < 43.8378126$$

The velocity difference at descending node is nearly three times higher than the ascending node. Due to the high velocity difference between ascending node and descending node, the descending node able to set the satellite in the comet's orbit in less days compared to ascending node release. However, the mission would be performed better at the cost of the time if the ascending node would be used since it allows to reduce the amount of the propellant which results into reduction of overall weight of the spacecraft by fulling up less fuel and reducing costs as well.

References:

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