Probability and Statistics, Spring 2018

Homework 2

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2.1.3

$$\begin{aligned}\mathrm{P}[\text{overtime}] &= \mathrm{P}[\text{1st good}]\mathrm{P}[\text{2nd miss}] + \mathrm{P}[\text{1st miss}]\mathrm{P}[\text{2nd good}] \\ &= \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \\ &= \frac{1}{4}\end{aligned}$$

2.1.9 (a)

$$\begin{aligned}\mathrm{P}[D \mid A] &= \frac{\mathrm{P}[D]\mathrm{P}[A \mid D]}{\mathrm{P}[A]} \\ &= \frac{10^{-4} \cdot 0.99}{10^{-4} \cdot 0.99 + (1 - 10^{-4}) \cdot 0.1} \\ &\approx 0.000989.\end{aligned}$$

(b)

$$\begin{aligned}\mathrm{P}[H \mid D] &= \mathrm{P}[A \mid D]\mathrm{P}[T^+ \mid A \cap D] \\ &= 0.99 \cdot 0.999 \\ &= 0.98901.\end{aligned}$$

(c)

$$\begin{aligned}\mathrm{P}[H \mid D^C] &= \mathrm{P}[A \mid D^C]\mathrm{P}[T^+ \mid A \cap D^C] \\ &= 0.1 \cdot 0.001 \\ &= 0.0001.\end{aligned}$$

(d)

$$\begin{aligned}\mathrm{P}[H] &= \mathrm{P}[D]\mathrm{P}[A \mid D]\mathrm{P}[T^+ \mid A \cap D] + \mathrm{P}[D^C]\mathrm{P}[A \mid D^C]\mathrm{P}[T^+ \mid A \cap D^C] \\ &= 10^{-4} \cdot 0.99 \cdot 0.999 + (1 - 10^{-4}) \cdot 0.1 \cdot 0.001 \\ &= 0.000198891.\end{aligned}$$

(e)

$$\begin{aligned}\mathrm{P}[D^C \mid H] &= \frac{\mathrm{P}[D^C]\mathrm{P}[A \mid D^C]\mathrm{P}[T^+ \mid A \cap D^C]}{\mathrm{P}[H]} \\ &\approx 0.50273.\end{aligned}$$

2.2.10 (a) $\mathrm{P}[A] = C_{19}^{40}(\frac{19}{40})^{19} \cdot C_{19}^{21}(\frac{19}{40})^{19} \cdot C_{2}^{2}(\frac{2}{40})^{2}$.

(b) $\mathrm{P}[G_{19}] = C_{19}^{40}(\frac{19}{40})^{19} \cdot C_{21}^{21}(\frac{21}{40})^{21}$.

(c)

$$\begin{aligned}\mathrm{P}[\text{You win}] &= \mathrm{P}[\text{You bet red}]\mathrm{P}[\text{red}] + \mathrm{P}[\text{You bet green}]\mathrm{P}[\text{green}] \\ &= \frac{1}{2}\cdot\frac{19}{40} + \frac{1}{2}\cdot\frac{19}{40} \\ &= \frac{19}{40}.\end{aligned}$$

2.3.1 (a) $\mathrm{P}[00111] = 0.8^2 \cdot 0.2^3 = 0.00512$.

(b) $\mathrm{P}[\text{exactly three ones}] = C_3^5(0.2)^3 \cdot C_2^2(0.8)^2 = 0.0512$.

2.3.4 Let A denotes away.

$$\begin{aligned}\mathrm{P}[H] &= \mathrm{P}[H_1]\mathrm{P}[H_2] + \mathrm{P}[H_1]\mathrm{P}[A_2]\mathrm{P}[H_3] + \mathrm{P}[A_1]\mathrm{P}[H_2]\mathrm{P}[H_3] \\ &= p^2 + p(1-p)p + (1-p)p^2 \\ &= p^2(1 + (1-p) + (1-p)) \\ &= p^2(3-2p) \\ &= 3p^2 - 2p^3.\end{aligned}$$

If $p \geq 1/2$, then $3p^2 - 2p^3 \geq p \Rightarrow \mathrm{P}[H] \geq p$.