Introduction to Computational Logic
Homework 1

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(1) Show $q \Rightarrow r \vdash (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$ is valid.

1	$q \Rightarrow r$	premise
2	$p \Rightarrow q$	assumption
3	p	assumption
4	q	$\Rightarrow$ e $3, 2$
5	r	$\Rightarrow$ e $4, 1$
6	$p \Rightarrow r$	$\Rightarrow$ i $3 - 5$
7	$(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$	$\Rightarrow$ i $2 - 6$

(2) Show $\vdash \neg p \Rightarrow (p \Rightarrow (p \Rightarrow q))$ is valid.

1	$\neg p$	assumption
2	p	assumption
3	p	assumption
4	$\bot$	$\neg$e $3, 1$
5	q	$\bot$e 4
6	$p \Rightarrow q$	$\Rightarrow$ i $3 - 5$
7	$p \Rightarrow (p \Rightarrow q)$	$\Rightarrow$ i $2 - 6$
8	$\neg p \Rightarrow (p \Rightarrow (p \Rightarrow q))$	$\Rightarrow$ i $1 - 7$

(3) Show $p \Rightarrow q \vdash \neg p \vee q$ is valid.

1	$p \Rightarrow q$	premise
2	$\neg(\neg p \vee q)$	assumption
3	p	assumption
4	q	$\Rightarrow$ e $3, 1$
5	$\neg p \vee q$	$\vee$i$_2$ 4
6	$\bot$	$\neg$e $5, 2$
7	$\neg p$	$\neg$i $3 - 6$
8	$\neg p \vee q$	$\vee$i$_1$ 7
9	$\bot$	$\neg$e $8, 2$
10	$\neg\neg(\neg p \vee q)$	$\neg$i $2 - 9$
11	$\neg p \vee q$	$\neg\neg$e 10

(4) Show $(s \Rightarrow p) \lor (t \Rightarrow q) \vdash (s \Rightarrow q) \lor (t \Rightarrow p)$ is valid.

1	$(s \Rightarrow p) \lor (t \Rightarrow q)$	premise
2	$s \Rightarrow p$	assumption
3	$p \lor \neg p$	LEM
4	p	assumption
5	$t \Rightarrow p$	$\Rightarrow$ i 4
6	$(s \Rightarrow q) \lor (t \Rightarrow p)$	$\lor$i$_2$ 5
7	$\neg p$	assumption
8	s	assumption
9	p	$\Rightarrow$ e 8, 2
10	$\bot$	$\neg$e 9, 7
11	q	$\bot$e 10
12	$s \Rightarrow q$	$\Rightarrow$ i 8 − 11
13	$(s \Rightarrow q) \lor (t \Rightarrow p)$	$\lor$i$_1$ 12
14	$(s \Rightarrow q) \lor (t \Rightarrow p)$	$\lor$e 3, 4 − 6, 7 − 13
15	$t \Rightarrow q$	assumption
16	$q \lor \neg q$	LEM
17	q	assumption
18	$s \Rightarrow q$	$\Rightarrow$ i 17
19	$(s \Rightarrow q) \lor (t \Rightarrow p)$	$\lor$i$_1$ 18
20	$\neg q$	assumption
21	t	assumption
22	q	$\Rightarrow$ e 21, 15
23	$\bot$	$\neg$e 22, 20
24	p	$\bot$e 23
25	$t \Rightarrow p$	$\Rightarrow$ i 21 − 24
26	$(s \Rightarrow q) \lor (t \Rightarrow p)$	$\lor$i$_2$ 25
27	$(s \Rightarrow q) \lor (t \Rightarrow p)$	$\lor$e 16, 17 − 19, 20 − 26
28	$(s \Rightarrow q) \lor (t \Rightarrow p)$	$\lor$e 1, 2 − 14, 15 − 27

(5) Show $(p \land q) \Rightarrow r, r \Rightarrow s, q \land \neg s \vdash \neg p$ is valid.

1	$p \land q \Rightarrow r$	premise
2	$r \Rightarrow s$	premise
3	$q \land \neg s$	premise
4	q	$\land$e$_1$ 3
5	$\neg s$	$\land$e$_2$ 3
6	$\neg r$	MT 2, 5
7	$\neg(p \land q)$	MT 1, 6
8	p	assumption
9	$p \land q$	$\land$i 8, 4
10	$\bot$	$\neg$e 9, 7
11	$\neg p$	$\neg$i 8 − 10