## Introduction to Computational Logic Homework 3

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 $\forall xi \ 3-16$ 

Let S be a binary predicate symbol, P and Q unary predicate symbols.

(1) Find a natural deduction proof to show

 $\exists x \exists y (S(x,y) \lor S(y,x)) \vdash \exists x \exists y S(x,y).$ 

(2) Find a natural deduction proof to show

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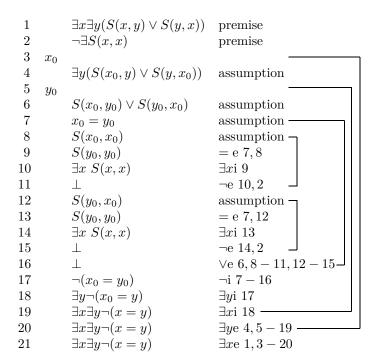
$$\forall x \forall y \forall z (S(x,y) \land S(y,z) \Rightarrow S(x,z)), \forall x \neg S(x,x) \vdash \forall x \forall y (S(x,y) \Rightarrow \neg S(y,x)).$$

$$\begin{matrix} 1 & \forall x \forall y \forall z (S(x,y) \land S(y,z) \Rightarrow S(x,z)) & \text{premise} \\ 2 & \forall x \neg S(x,x) & \text{premise} \\ 3 & a & & & & & & \\ 4 & \forall y \forall z (S(a,y) \land S(y,z) \Rightarrow S(a,z)) & \forall xe \ 1 \\ 5 & b & & & & & \\ 6 & \forall z (S(a,b) \land S(b,z) \Rightarrow S(a,z)) & \forall ye \ 4 \\ 7 & S(a,b) \land S(b,a) \Rightarrow S(a,a) & \forall ze \ 6 \\ 8 & \neg S(a,a) & \forall xe \ 2 \\ 9 & S(a,b) & \text{assumption} \\ 10 & S(b,a) & \text{assumption} \\ 11 & S(a,b) \land S(b,a) & \text{assumption} \\ 12 & S(a,a) & \Rightarrow e \ 11,7 \\ 13 & \bot & \neg e \ 12,8 \\ 14 & \neg S(b,a) & \Rightarrow i \ 9-14 \\ 15 & S(a,b) \Rightarrow \neg S(b,a) & \Rightarrow i \ 9-14 \\ 16 & \forall y S(a,y) \Rightarrow \neg S(y,a) & \forall yi \ 5-15 \end{matrix}$$

 $\forall x \forall y S(x, y) \Rightarrow \neg S(y, x)$ 

(3) Find a natural deduction proof to show

$$\exists x \exists y (S(x,y) \lor S(y,x)), \neg \exists x S(x,x) \vdash \exists x \exists y \neg (x=y).$$



(4) Show that there is no natural deduction proof for

$$\forall x (P(x) \lor Q(x)) \vdash \forall x P(x) \lor \forall x Q(x).$$

Consider the outcomes from flipping a coin: P(x): "x is heads," Q(x): "x is tails", with x belonging to the domain of all coins.

Then we can say truthfully that:

$$\forall x (P(x) \lor Q(x)).$$

But there's a problem with the RHS:

$$\forall x P(x) \lor \forall x Q(x).$$

That is, the right hand side claims that every coin tossed turns of heads, or else, every coin tossed turns of tails.

(5) Semantically show

$$\forall x \neg \phi \vDash \neg \exists x \phi.$$

Let  $\mathcal{M}$  be a model that  $\mathcal{M} \vDash \forall x \neg \phi$  (in words, an interpretation satisfying the formula).

And assume that  $\mathcal{M} \nvDash \neg \exists x \phi$ , this means that  $\mathcal{M} \vDash \exists x \phi$ , i.e., these are some  $t \in \mathcal{M}$ ,  $\phi(t/x)$  holds.

But we have:  $\mathcal{M} \vDash \forall x \neg \phi$ , i.e.,  $\forall t \in \mathcal{M}$ ,  $\neg \phi(t/x)$  holds.

There is a contradiction, thus  $\mathcal{M} \vDash \neg \exists x \phi$ .