


# Chapter 4

## Beyond Classical Search



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Acknowledgements: This presentation is created by Jane hsu based on the lecture slides from *The Artificial Intelligence: A Modern Approach* by Russell & Norvig, a PowerPoint version by Min-Yen Kan, as well as various materials from the web.

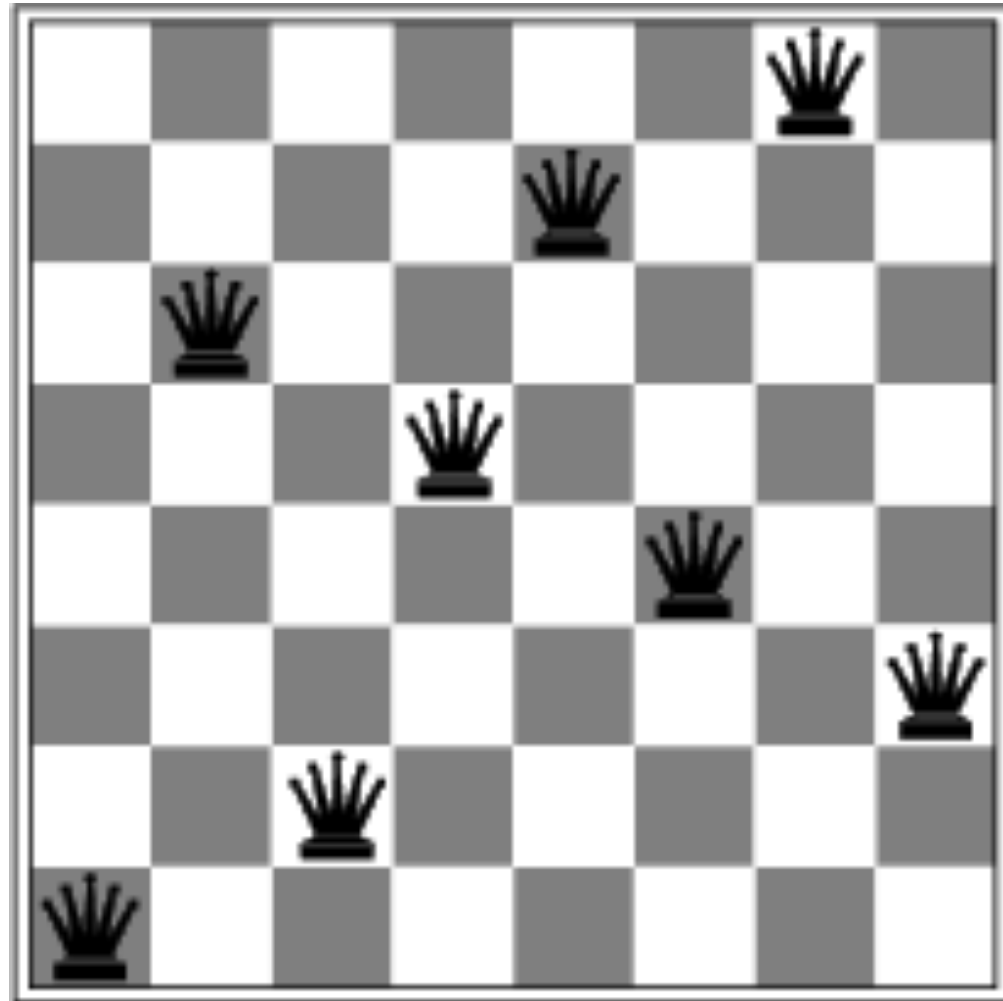
# Outline

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- Local Search Algorithms
  - Hill climbing
  - Simulated annealing
  - Genetic algorithms
- Local search in continuous spaces
- Searching with nondeterministic actions
- Searching with partial observations
- Online Search

# Example: 8-Queens Problem

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# Standard Search Problem

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- State space: the set of all states reachable from the initial state
- State is a “black box” – any data structure that supports
  - successor function
  - heuristic function, and
  - goal test

# Local Search Algorithms



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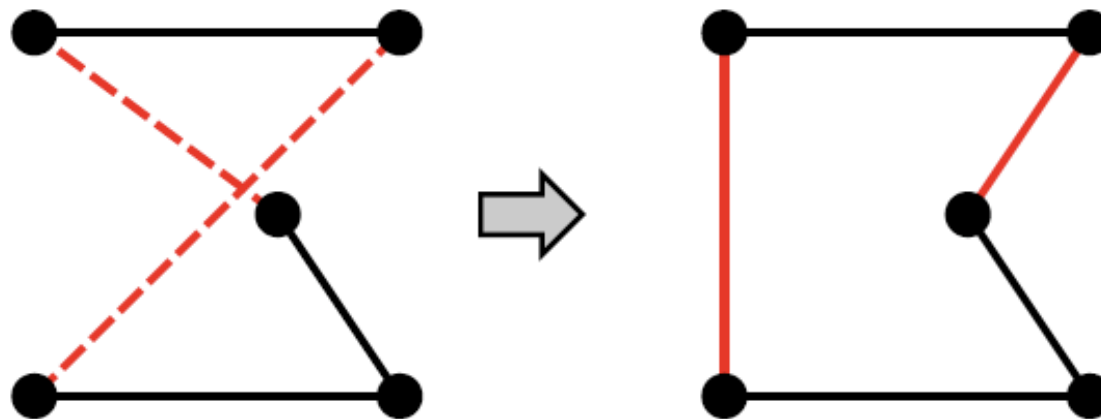
# Iterative Improvement Algorithms

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- In many optimization problems, *path* is irrelevant; the *goal state* itself is the solution
- State space = set of "complete" configurations
  - Find *optimal* configuration, e.g., TSP
  - Find configuration satisfying constraints, e.g., n-queens
- In such cases, we can use *iterative improvement*, also called *local search* algorithms
  - keep a single "current" state, try to improve it
  - Constant space
  - Online or offline search

# Traveling Salesperson Problem

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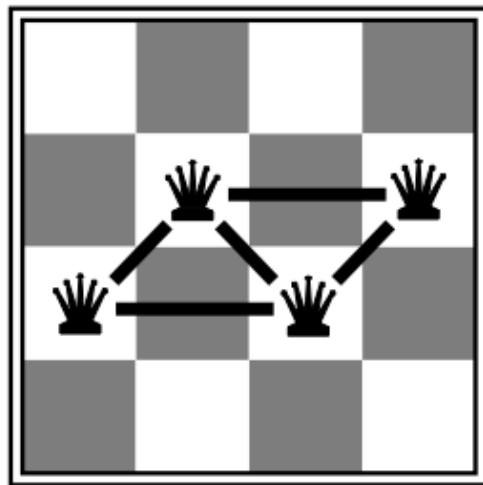


- Start with any complete tour, perform pair-wise exchanges.
- Variants of this approach get within 1% of optimal very quickly with thousands of cities.

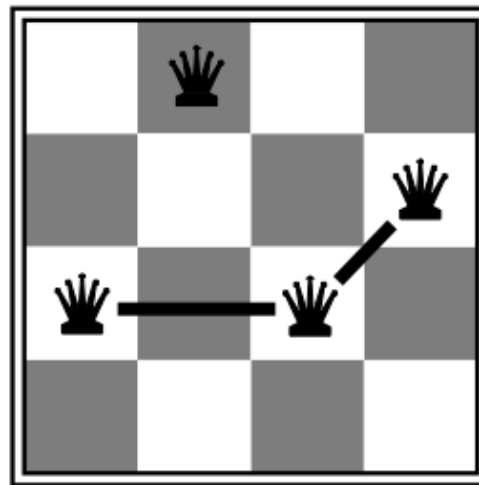
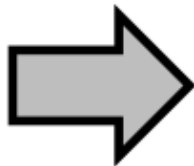
## Example: $n$ -queens

Put  $n$  queens on an  $n \times n$  board with no two queens on the same row, column, or diagonal

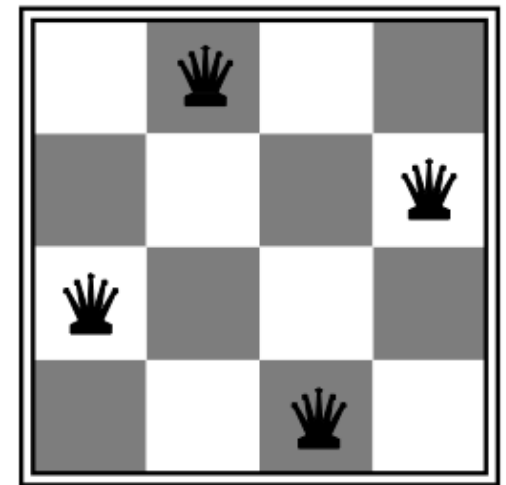
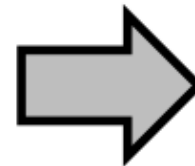
Move a queen to reduce number of conflicts



$h = 5$



$h = 2$



$h = 0$

Almost always solves  $n$ -queens problems almost instantaneously for very large  $n$ , e.g.,  $n = 1\text{million}$



# Hill-Climbing (Gradient Ascent/Descent)

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“Like climbing Everest in thick fog with amnesia”

**function** HILL-CLIMBING(*problem*) **returns** a state that is a local maximum

*current*  $\leftarrow$  MAKE-NODE(*problem*.INITIAL-STATE)

**loop do**

*neighbor*  $\leftarrow$  a highest-valued successor of *current*

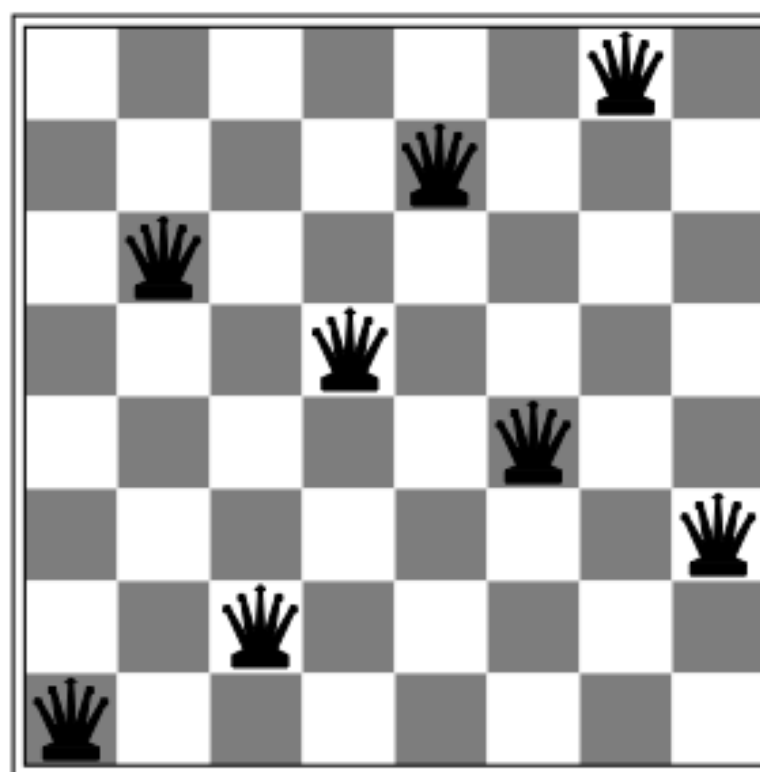
**if** *neighbor*.VALUE  $\leq$  *current*.VALUE **then return** *current*.STATE

*current*  $\leftarrow$  *neighbor*

# Local Maxima

18	12	14	13	13	12	14	14
14	16	13	15	12	14	12	16
14	12	18	13	15	12	14	14
15	14	14	♚	13	16	13	16
♚	14	17	15	♚	14	16	16
17	♚	16	18	15	♚	15	♚
18	14	♚	15	15	14	♚	16
14	14	13	17	12	14	12	18

(a)

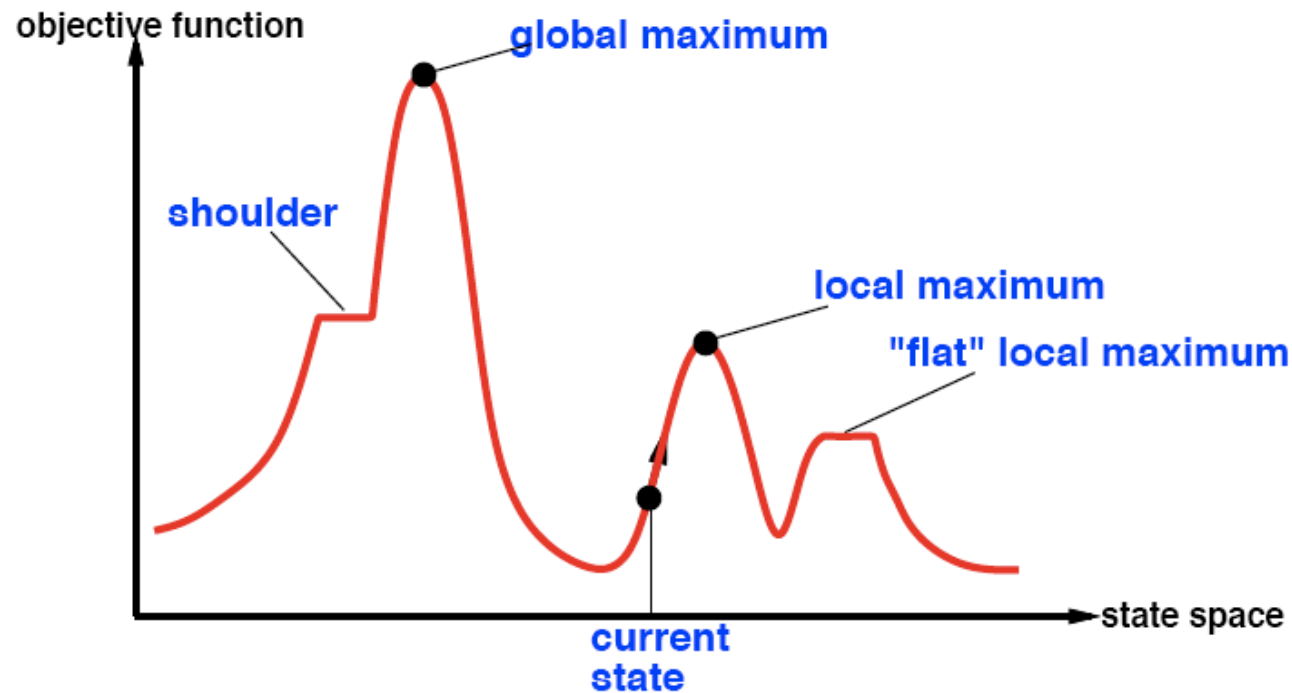


(b)

**Figure 4.3** FILES: figures/8queens-successors.eps figures/8queens-local-minimum.eps. (a) An 8-queens state with heuristic cost estimate  $h = 17$ , showing the value of  $h$  for each possible successor obtained by moving a queen within its column. The best moves are marked. (b) A local minimum in the 8-queens state space; the state has  $h = 1$  but every successor has a higher cost.

# Problems in Hill-Climbing Search

- Consider the state space landscape



Random-restart hill climbing overcomes local maxima—trivially complete

Random sideways moves 😊 escape from shoulders 😞 loop on flat maxima

# Simulated Annealing

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- Idea: escape local maxima by allowing some "bad" moves but **gradually decrease** their size & frequency

**function** SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state

**inputs:** *problem*, a problem

*schedule*, a mapping from time to “temperature”

*current*  $\leftarrow$  MAKE-NODE(*problem*.INITIAL-STATE)

**for**  $t = 1$  **to**  $\infty$  **do**

$T \leftarrow$  *schedule*( $t$ )

**if**  $T = 0$  **then return** *current*

*next*  $\leftarrow$  a randomly selected successor of *current*

$\Delta E \leftarrow$  *next*.VALUE – *current*.VALUE

**if**  $\Delta E > 0$  **then** *current*  $\leftarrow$  *next*

**else** *current*  $\leftarrow$  *next* only with probability  $e^{\Delta E/T}$

# Properties of Simulated Annealing

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At fixed “temperature”  $T$ , state occupation probability reaches Boltzman distribution

$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$

$T$  decreased slowly enough  $\implies$  always reach best state  $x^*$   
because  $e^{\frac{E(x^*)}{kT}} / e^{\frac{E(x)}{kT}} = e^{\frac{E(x^*) - E(x)}{kT}} \gg 1$  for small  $T$

Is this necessarily an interesting guarantee??

Devised by Metropolis et al., 1953, for physical process modelling

Widely used in VLSI layout, airline scheduling, etc.

# Local Beam Search

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- Keep track of (top)  $k$  states rather than just one
  - Start with  $k$  randomly generated states
  - At each iteration, all the successors of all  $k$  states are generated
  - If any one is a goal state, stop; else select the  $k$  best successors from the complete list and repeat.
- Question: is it  $k$  searches run in parallel?
- Problem: all  $k$  states end up on same local hill
- Solution idea: choose  $k$  successors randomly, biased towards good ones.

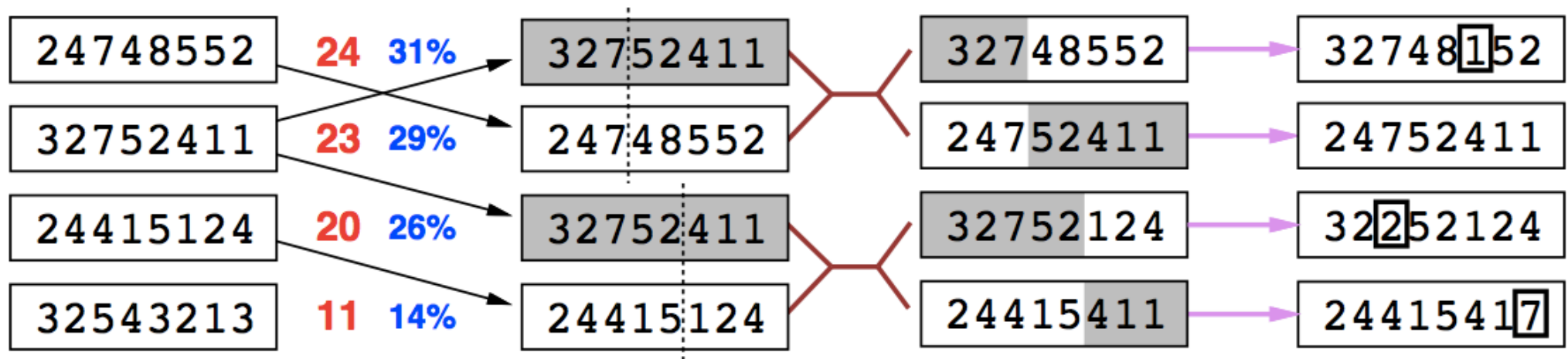
# Genetic Algorithms

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- **Population:** Start with  $k$  randomly generated individuals (i.e. states)
  - **Individual:** each is represented as a string over a finite alphabet (often a string of 0s and 1s)
  - **Fitness function:** evaluation of the “goodness” of a given state.
- A successor is generated by combining two parents from the current population.
- Produce the next generation of states by selection, crossover, and mutation

# Genetic Algorithms

= stochastic local beam search + generate successors from **pairs** of states



**Fitness**   **Selection**   **Pairs**   **Cross-Over**   **Mutation**

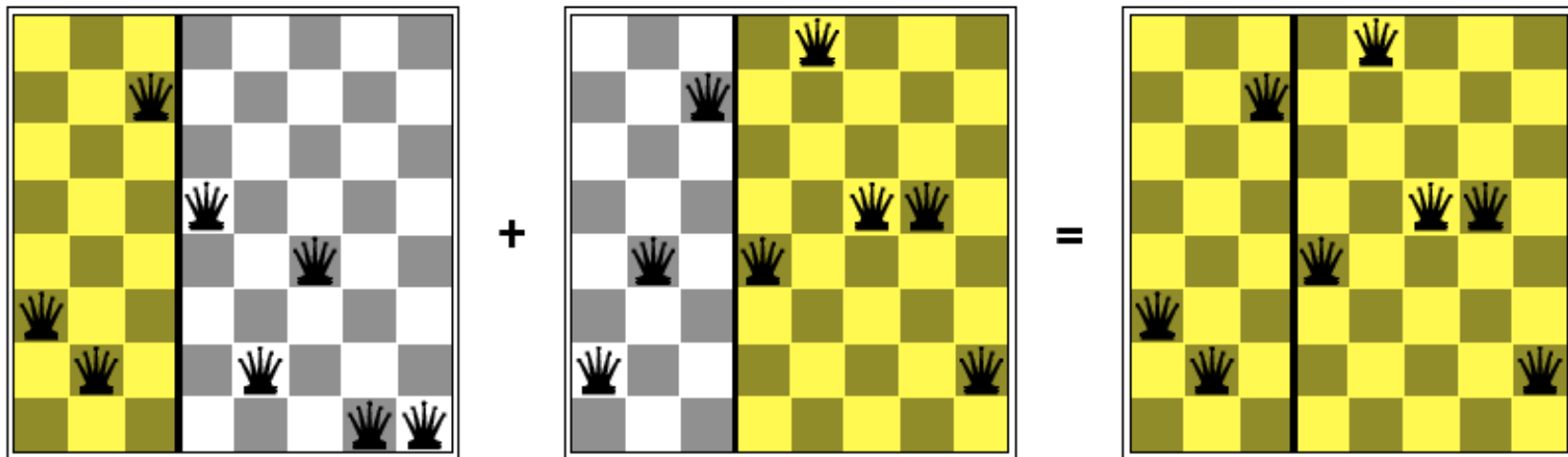
- Fitness function: number of **non-attacking** pairs of queens (min = 0, max =  $8 \times 7/2 = 28$ )
- $24/(24+23+20+11) = 31\%$
- $23/(24+23+20+11) = 29\%$  etc



# Crossover Operators

- States are encoded as strings

Crossover helps **iff substrings are meaningful components**



**function** GENETIC-ALGORITHM(*population*, FITNESS-FN) **returns** an individual

**inputs:** *population*, a set of individuals

FITNESS-FN, a function that measures the fitness of an individual

**repeat**

*new\_population*  $\leftarrow$  empty set

**for**  $i = 1$  **to** SIZE(*population*) **do**

$x \leftarrow$  RANDOM-SELECTION(*population*, FITNESS-FN)

$y \leftarrow$  RANDOM-SELECTION(*population*, FITNESS-FN)

*child*  $\leftarrow$  REPRODUCE( $x, y$ )

**if** (small random probability) **then** *child*  $\leftarrow$  MUTATE(*child*)

add *child* to *new\_population*

*population*  $\leftarrow$  *new\_population*

**until** some individual is fit enough, or enough time has elapsed

**return** the best individual in *population*, according to FITNESS-FN

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**function** REPRODUCE( $x, y$ ) **returns** an individual

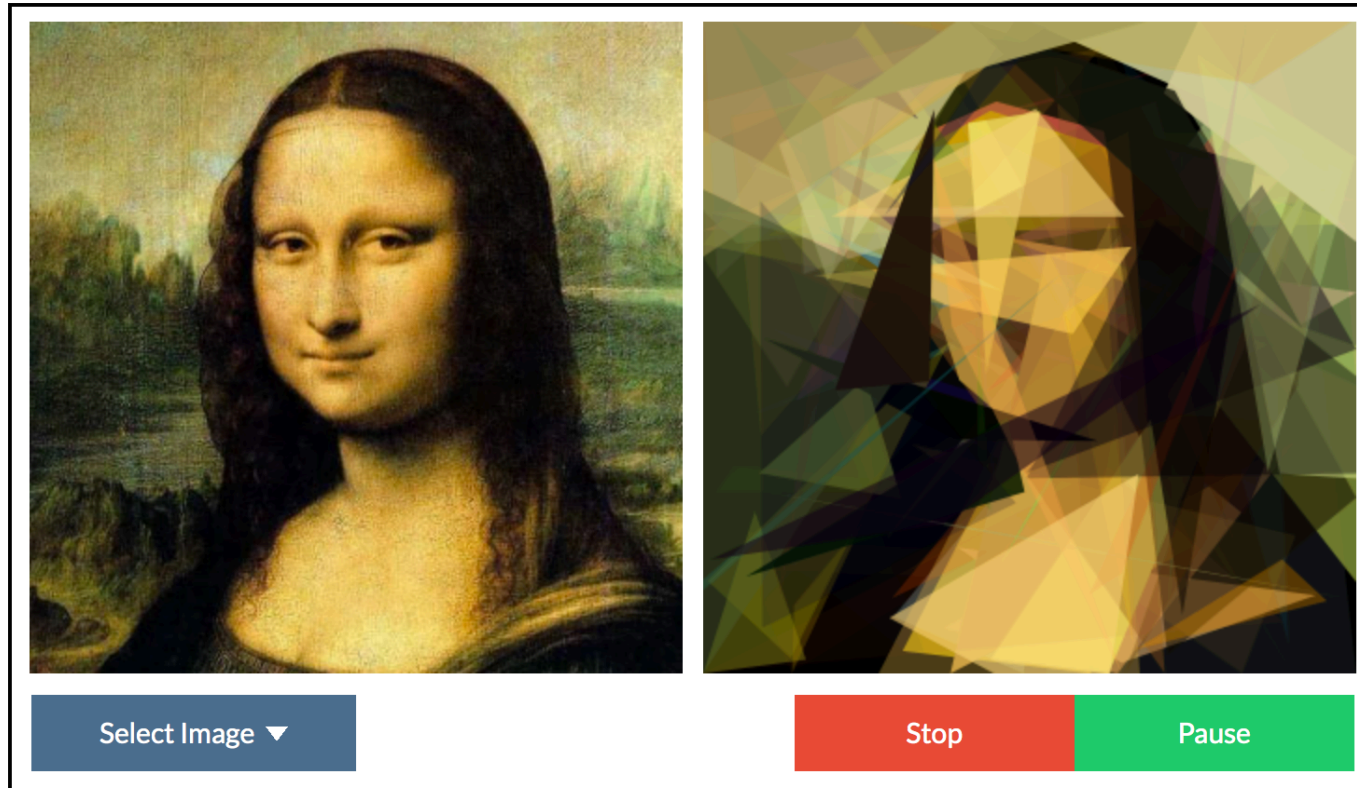
**inputs:**  $x, y$ , parent individuals

$n \leftarrow$  LENGTH( $x$ );  $c \leftarrow$  random number from 1 to  $n$

**return** APPEND(SUBSTRING( $x, 1, c$ ), SUBSTRING( $y, c + 1, n$ ))

# Grow Your Own Picture

Genetic Algorithms & Generative Art



## Evolving the Mona Lisa

Welcome! Click start to see genetics in action, as a randomly generated collection of shapes evolve to resemble a given picture.

### HOW IT WORKS

This page uses a genetic algorithm to model a population of individuals, each

⚙️ Configuration

🔬 Analytics

### GENETICS

Population Size



50

<http://chriscummins.cc/s/genetics/>

# Evolutionary Computation

