Probability and Statistics, Spring 2018

Homework 1

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1.1.3 (a) $S = \{aaa, aaf, afa, aff, faa, faf, ffa, fff\}$

(b) $Z_F = \{aaf, aff, faf, fff\}$
$X_A = \{aaa, aaf, afa, aff\}$

(c) Since $Z_F \cap X_A = \{aaf, aff\} \neq \emptyset$, Z_F and X_A are not mutually exclusive.

(d) Since $Z_F \cup X_A = \{aaa, aaf, afa, aff, faf, fff\} \neq S$, Z_F and X_A are not collectively exhaustive.

(e) $C = \{aaa, aaf, afa, faa\}$
$D = \{aff, faf, ffa, fff\}$

(f) Since $C \cap D = \emptyset$, C and D are mutually exclusive.

(g) Since $C \cup D = S$, C and D are collectively exhaustive.

1.2.10 (a) If A and B are mutually exclusive, $\mathrm{P}[A \cup B] = \mathrm{P}[A] + \mathrm{P}[B] \geq \mathrm{P}[A]$.
If A and B are not mutually exclusive, $\mathrm{P}[A \cup B] = \mathrm{P}[A] + \mathrm{P}[B] - \mathrm{P}[A \cap B] \geq P[A]$.

(b) If A and B are mutually exclusive, $\mathrm{P}[A \cup B] = \mathrm{P}[A] + \mathrm{P}[B] \geq \mathrm{P}[B]$.
If A and B are not mutually exclusive, $\mathrm{P}[A \cup B] = \mathrm{P}[A] + \mathrm{P}[B] - \mathrm{P}[A \cap B] \geq P[B]$.

(c) If A and B are mutually exclusive, $\mathrm{P}[A \cap B] = \emptyset \leq \mathrm{P}[A]$.
If A and B are not mutually exclusive, $\mathrm{P}[A \cap B] = \mathrm{P}[A] - \mathrm{P}[B] \leq P[A]$.

(d) If A and B are mutually exclusive, $\mathrm{P}[A \cap B] = \emptyset \leq \mathrm{P}[B]$.
If A and B are not mutually exclusive, $\mathrm{P}[A \cap B] = \mathrm{P}[B] - \mathrm{P}[A] \leq P[B]$.

1.3.4 Let A represents Apricots and B represents Bananas.
$S = \{AA, AB, BA, BB\}$, $\mathrm{P}[BB] = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$.

1.4.3 (a) Let

	H_0	H_1	H_2
F	p_0	p_1	p_2
V	q_0	q_2	q_2

With $p_0 + p_1 + p_2 = \frac{5}{12}$, $q_0 + q_1 + q_2 = \frac{7}{12}$, $p_i + q_i = \frac{1}{3}$ for $i = 0, 1$ and 2. We can get following possible solutions:

	H_0	H_1	H_2
F	0	1/6	1/4
V	1/3	1/6	1/12

	H_0	H_1	H_2
F	1/6	0	1/4
V	1/6	1/3	1/12

	H_0	H_1	H_2
F	1/6	1/4	0
V	1/6	1/12	1/3

(b) In the beginning, the table looks like:

	H_0	H_1	H_2
F	1/4		
V		1/6	

Since $\mathrm{P}[H_0] = \mathrm{P}[H_1] = \mathrm{P}[H_2] = 1/3$, $\mathrm{P}[VH_0] = 1/3 - 1/4 = 1/12$ and $\mathrm{P}[FH_1] = 1/3 - 1/6 = 1/6$.

	H_0	H_1	H_2
F	1/4	1/6	
V	1/12	1/6	

Since $\mathrm{P}[F] = 5/12$ and $\mathrm{P}[V] = 7/12$, $\mathrm{P}[FH_2] = 5/12 - 1/4 - 1/6 = 0$ and $\mathrm{P}[VH_2] = 7/12 - 1/12 - 1/6 = 1/3$.

	H_0	H_1	H_2
F	1/4	1/6	0
V	1/12	1/6	1/3

1.5.9 Let $A = \{1, 2\}$, $B = \{1, 3\}$ and $C = \{2, 3\}$, then A, B, and C are pairwise independent but are not independent. Since

$$\begin{aligned}
\mathrm{P}[A \cap B] = \mathrm{P}[\{1\}] = \frac{1}{4} = \mathrm{P}[A]\mathrm{P}[B] = \frac{1}{2} \cdot \frac{1}{2} \\
\mathrm{P}[B \cap C] = \mathrm{P}[\{3\}] = \frac{1}{4} = \mathrm{P}[B]\mathrm{P}[C] = \frac{1}{2} \cdot \frac{1}{2} \\
\mathrm{P}[C \cap A] = \mathrm{P}[\{2\}] = \frac{1}{4} = \mathrm{P}[C]\mathrm{P}[A] = \frac{1}{2} \cdot \frac{1}{2} \\
\mathrm{P}[A \cap B \cap C] = \mathrm{P}[\phi] = 0 \neq \mathrm{P}[A]P[B]\mathrm{P}[C] = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}.
\end{aligned}$$