Introduction to Computational Logic
Homework 6

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(1) Show $\phi\mathbf{U}\psi \equiv \psi\mathbf{R}(\phi\vee\psi)\wedge\mathbf{F}\psi$ using semantic equivalences.

Since $\phi\mathbf{U}\psi \equiv \phi\mathbf{W}\psi\wedge F\psi$, it remains to prove that $\phi\mathbf{W}\psi \equiv \psi\mathbf{R}(\phi\vee\psi)$ and it's in the problem (2).

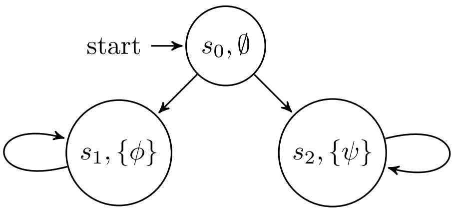
(2) Show $\phi\mathbf{W}\psi \equiv \psi\mathbf{R}(\phi\vee\psi)$ from definitions.

- $\phi\mathbf{W}\psi \rightarrow \psi\mathbf{R}(\phi\vee\psi)$. Suppose $\pi\vDash\phi\mathbf{W}\psi$. Then there are two conditions:
  (a) Either there is some $i\geq 0$ such that $\pi^i\vDash\psi$ and for all $0\leq j<i$ we have $\pi^j\vDash\phi$;
  (b) or for all $k\geq 0$ we have $\pi^k\vDash\phi$.

  In case (a), there is a minimal such i, say i_0, and so $\pi^{i_0}\vDash\psi$ and, for all $0\leq j<i_0$, $\pi^j\nvDash\psi$ and $\pi^j\vDash\phi$, hence $\pi\vDash\psi\mathbf{R}(\phi\vee\psi)$.
  In case (b), $\pi^k\vDash\phi$ for all $k\geq 0$, hence $\pi\vDash\psi\mathbf{R}(\phi\vee\psi)$.
  Therefore, we have proved that in both case (a) and case (b), $\pi\vDash\psi\mathbf{R}(\phi\vee\psi)$.
- $\phi\mathbf{W}\psi \leftarrow \psi\mathbf{R}(\phi\vee\psi)$. Suppose $\pi\vDash\psi\mathbf{R}(\phi\vee\psi)$. Then there are two conditions:
  (a) Either there is some $i\geq 0$ such that $\pi^i\vDash\psi$ and for all $0\leq j\leq i$ we have $\pi^i\vDash\phi\vee\psi$;
  (b) or for all $k\geq 0$ we have $\pi^k\vDash\phi\vee\psi$

  In case (a), there is a minimal such i, say i_0, and so $\pi^{i_0}\vDash\psi$ and, for all $0\leq j<i_0$, $\pi^j\nvDash\psi$ and $\pi^j\vDash\phi$; in particular, $\pi^0\vDash\phi$ i.e. $\pi\vDash\phi$, hence $\pi\vDash\phi\mathbf{W}\psi$.
  In case (b), either $\pi^k\vDash\phi$ for all $k\geq 0$, and hence $\pi\vDash\phi\mathbf{W}\psi$; or there is $h\geq 0$ such that $\pi^h\nvDash\phi$ and $\pi^h\vDash\psi$, hence there is a minimal such h, say h_0, and so $\pi^i\vDash\phi$ for all $0\leq i<h_0$, thus $\pi\vDash\phi\mathbf{W}\psi$.
  Therefore, we have proved that in both case (a) and case (b), $\pi\vDash\phi\mathbf{W}\psi$.

(3) Give a model $\mathcal{M}=(S,\rightarrow,L)$ and $s\in S$ such that $\mathcal{M},s\vDash\mathbf{AF}(\phi\vee\psi)$ but $\mathcal{M},s\nvDash\mathbf{AF}\phi\vee\mathbf{AF}\psi$.
Let $\mathcal{M}=(S,\rightarrow,L)$ with $S=\{s_0,s_1,s_2\}, \rightarrow=\{(s_0,s_1),(s_0,s_2),(s_1,s_1),(s_2,s_2)\}$, and $L(s_0)=\emptyset, L(s_1)=\{\phi\}$, and $L(s_2)=\{\psi\}$ in below:



For each path that starts in state s_0 we have that $\mathbf{AF}(\phi\vee\psi)$ holds. This follows directly from the fact that each path visits either state s_1 or state s_2 eventually, and $s_1\vDash(\phi\vee\psi)$ and $s_2\vDash(\phi\vee\psi)$.

However, state s_0 does not satisfy $\mathbf{AF}\phi\vee\mathbf{AF}\psi$. For instance, path $s_0(s_1)^\omega\vDash\mathbf{F}\phi$ but $s_0(s_1)^\omega\nvDash\mathbf{F}\psi$. Thus, $\mathcal{M},s_0\nvDash\mathbf{AF}\psi$. By a similar reasoning applied to path $s_0(s_2)^\omega$ it follows $\mathcal{M},s_0\nvDash\mathbf{AF}\phi$. Thus, $\mathcal{M},s_0\nvDash\mathbf{AF}\phi\vee\mathbf{AF}\psi$.

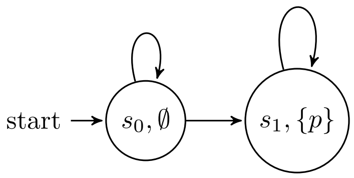
(4) Express the following statement in CTL^*:

"the event p is never true between the event q and r on a path."

$[\mathbf{AG}(q\rightarrow\neg\mathbf{EF}(p\wedge\mathbf{EF}r))]\wedge[\mathbf{AG}(r\rightarrow\neg\mathbf{EF}(p\wedge\mathbf{EF}q))]$.

(5) Show $\mathbf{AGF}p$ and $\mathbf{AGEF}p$ specify different properties.

Give a model $\mathcal{M} = (S, \rightarrow, L)$ with $S = \{s_0, s_1\}, \rightarrow = \{(s_0, s_0), (s_0, s_1), (s_1, s_1)\}$, and $L(s_0) = \emptyset$, and $L(s_1) = \{p\}$ in below:



The formula $\mathbf{AGF}p$ is true in model $\mathcal{M}$ at state s_0 if from every state on every path from $s_0(\mathbf{AG})$, p is eventually true($\mathbf{F}p$) on that same path, and $s_0^{\omega_0}$ doesn't hold.

The formula $\mathbf{AGEF}p$ is true in model $\mathcal{M}$ at state s_0 if from every state on every path from $s_0(\mathbf{AG})$, there exists a path on which p is eventually true($\mathbf{EF}p$), and $s_0(s_1)^{\omega_1}$ holds.

Thus $\mathcal{M}, s_0 \nvDash \mathbf{AGF}p$ but $\mathcal{M}, s_0 \vDash \mathbf{AGEF}p$.