Probability and Statistics, Spring 2018

Homework 4

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4.2.2 (a)

$$\begin{aligned}\int_{-5}^{7} 2c(v+5)dv &= 1\\ (\frac{v^2}{2}+5v)\Big|_{-5}^{7} &= 1/2c\\ c &= 1/144.\end{aligned}$$

(b)

$$\int_{4}^{7} 1/144 \cdot 2(v+5)dv = \frac{63}{144} = \frac{7}{16}.$$

(c)

$$\int_{-3}^{0} 1/144 \cdot 2(v+5)dv = \frac{7}{48}.$$

(d)

$$\begin{aligned}\int_{-5}^{a} 1/144 \cdot 2(v+5)dv &= 1/3\\ a^2+10a-23 &= 0\\ a &= -5+4\sqrt{3}.\end{aligned}$$

4.3.3

$$\frac{dF_U(u)}{du} = f_U(u) = \begin{cases} 0 & u < -5,\\ 1/8 & -5 \le u < -3,\\ 0 & -3 \le u < 3,\\ 3/8 & 3 \le u < 5,\\ 0 & u \ge 5.\end{cases}$$

4.4.5 (a)

$$f_Y(y) = \begin{cases} 1/2 & -1 \le y \le 1,\\ 0 & \text{otherwise.}\end{cases}$$

$$\mathrm{E}[Y] = \int_{-1}^{1} y/2dy = 0.$$

(b)

$$\begin{aligned}\mathrm{E}[Y^2] &= \int_{-1}^{1} \frac{y^2}{2}dy = 1/3,\\ \mathrm{Var}[Y] &= \mathrm{E}[Y^2] - \mathrm{E}[Y]^2 = 1/3 - 0 = 1/3.\end{aligned}$$

4.5.10 (a)

$$f_X(x) = \begin{cases} 1/10 & -5 \le x \le 5, \\ 0 & \text{otherwise.} \end{cases}$$

(b)

$$\int_{-5}^{x} \frac{1}{10} dy = \frac{x}{10} + \frac{1}{2}.$$

$$F_X(x) = \begin{cases} 0 & x < -5 \\ \frac{x}{10} + \frac{1}{2} & -5 \le x \le 5 \\ 1 & x > 5. \end{cases}$$

(c)

$$\mathrm{E}[X] = 0.$$

(d)

$$\mathrm{E}[X^5] = \int_{-5}^{5} \frac{1}{10} x^5 dx = 0.$$

(e)

$$\mathrm{E}[e^x] = \int_{-5}^{5} e^x \frac{1}{10} dx = \frac{1}{10}(e^5 - e^{-5}).$$

4.6.4 (a)

$$\mathrm{P}[Y \le 10] = 0.933 = \Phi(1.5) = \Phi(z)$$
$$\Rightarrow z = 1.5 = \frac{x - \mu}{\sigma} = \frac{10 - \mu}{10} \Rightarrow \mu = -5.$$

(b)

$$\mathrm{P}[Y \le 0] = 0.067 = 1 - 0.933 = 1 - \Phi(1.5) = 1 - \Phi(z) = \Phi(-z)$$
$$\Rightarrow z = -1.5 = \frac{x - \mu}{\sigma} = \frac{0 - \mu}{10} \Rightarrow \mu = 15.$$

(c)

$$\mathrm{P}[Y \le 10] = 0.977 \approx \Phi(1.99) = 0.9767 = \Phi(z)$$
$$\Rightarrow z = 1.99 = \frac{x - \mu}{\sigma} = \frac{10 - \mu}{\sigma} \Rightarrow \mu = 10 - 1.99\sigma.$$

(d)

$$\mathrm{P}[Y > 5] = 1 - F_Y(5) = \frac{1}{2} \Rightarrow \mu = 5.$$

4.7.6

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x} & x \ge 0, \\ 0 & \text{otherwise.} \end{cases}$$

$$f_X(x) = \frac{dF_X(x)}{dx} = \begin{cases} \lambda e^{-\lambda x} & x \ge 0, \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathrm{E}[X] = 3 = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{3}.$$

(a) Because $w = 60x$, $x = w/60$.

$$F_X(x) = \begin{cases} 1 - e^{-x/3} & x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

$$F_W(x) = \begin{cases} 1 - e^{-w/180} & w \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(b)

$$f_W(w) = \frac{dF_W(w)}{dw} = \begin{cases} 0.2 + 0.3 = 0.5 & w = 0 \\ \frac{1}{180} e^{-w/180} & w > 0, \\ 0 & \text{otherwise.} \end{cases}$$

(c)

$$\begin{aligned} \mathrm{E}[W] &= \int_0^\infty \frac{1}{180} e^{-w/180} w dw & \left(\text{let } \frac{-w}{180}\Big|_0^\infty = t\Big|_0^{-\infty}\right) \\ &= \int_0^{-\infty} (-t) e^t (-180 dt) \\ &= 180. \end{aligned}$$

$$\begin{aligned} \mathrm{Var}[W] &= \mathrm{E}[W^2] - (\mathrm{E}[W])^2 \\ &= \int_0^\infty f(w) w^2 dw - (\mathrm{E}[W])^2 \\ &= \int_0^\infty \frac{1}{180} e^{-w/180} w^2 dw - 180^2 \\ &= 32400. \end{aligned}$$