INTRODUCTION TO COMPUTATIONAL LOGIC
HOMEWORK 3
DUE DATE: NOVEMBER 8, 2017

Let S be a binary predicate symbol, P and Q unary predicate symbols.

(1) Find a natural deduction proof to show

$$\exists x \exists y (S(x,y) \vee S(y,x)) \vdash \exists x \exists y S(x,y).$$

(2) Find a natural deduction proof to show

$$\forall x \forall y \forall z (S(x,y) \wedge S(y,z) \implies S(x,z)), \forall x \neg S(x,x) \vdash \forall x \forall y (S(x,y) \implies \neg S(y,x)).$$

(3) Find a natural deduction proof to show

$$\exists x \exists y (S(x,y) \vee S(y,x)), \neg \exists x S(x,x) \vdash \exists x \exists y \neg (x = y).$$

(4) Show that there is no natural deduction proof for

$$\forall x (P(x) \vee Q(x)) \vdash \forall x P(x) \vee \forall x Q(x).$$

(5) Semantically show

$$\forall x \neg \phi \models \neg \exists x \phi.$$