Probability and Statistics, Spring 2018

Homework 3

DUE DATE: April 24, 2018

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3.2.6 To find the PMF of Y, we need to know the set of Y first.

It's easy to see that $Y = \{0, 1, 2\}$.

$$\mathrm{P}_Y(y) = \begin{cases} 1-p & y = 0 \text{ (first goes out)}, \\ p(1-p) & y = 1 \text{ (first goes in, then second goes out)}, \\ p^2 & y = 2 \text{ (first goes in, then second goes in)}, \\ 0 & \text{otherwise.} \end{cases}$$

3.3.1 (a)

$$\mathrm{P}_Y(y) = \begin{cases} 1/11 & y = 5, 6, 7, \ldots, 15, \\ 0 & \text{otherwise.} \end{cases}$$

(b)

$$\mathrm{P}[Y < 10] = \sum_{i=5}^{9} \mathrm{P}_Y(i) = 5/11.$$

(c)

$$\mathrm{P}[Y > 12] = \sum_{i=13}^{1} 5\mathrm{P}_Y(i) = 3/11.$$

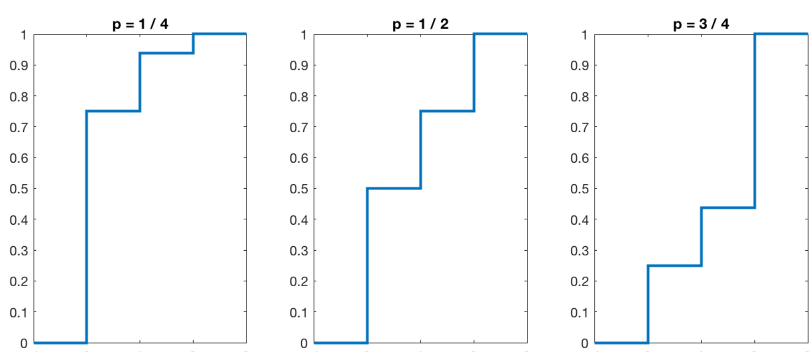
(d)

$$\mathrm{P}[8 \le Y \le 12] = \sum_{i=8}^{1} 2\mathrm{P}_Y(i) = 5/11.$$

3.4.6 The CDF are

$$\mathrm{F}_Y(y) = \begin{cases} 0 & y < 0, \\ 1-p & 0 \le y < 1, \\ 1-p^2 & 1 \le y < 2, \\ 1 & y \ge 2. \end{cases}$$

The plottings are



3.5.10 (a)

$$\mathrm{P}_R(r) = \begin{cases} 1-(\frac{1}{2})^{20} & r = 0, \\ (\frac{1}{2})^{20} & r = 20,000,000, \\ 0 & \text{otherwise.} \end{cases}$$

(b)

$$\mathrm{P}_L(l) = \begin{cases} (\frac{1}{2})^{20}(1-(\frac{1}{2})^{20})^{l-1} & l = 1, 2, \ldots, \\ 0 & \text{otherwise.} \end{cases}$$

(c) The expected reward a customer earned

$$E[R] = 20,000,000 \cdot (\frac{1}{2})^{20} \approx 19.07 < 20.$$

The casino company should offer this game.

3.6.2 (a) From 3.4.2, we have

$$\mathrm{P}_X(x) = \begin{cases} 0.2 & x = -1, \\ 0.5 & x = 0, \\ 0.3 & x = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Therefore

$$\mathrm{P}_V(v) = \mathrm{P}_{|X|}(v) = \begin{cases} 0.5 & v = 0, \\ 0.5 & v = 1, \\ 0 & \text{otherwise.} \end{cases}$$

(b)

$$\mathrm{F}_V(v) = \begin{cases} 0 & v < 0, \\ 0.5 & 0 \le v < 1, \\ 1 & 1 \le v. \end{cases}$$

(c)

$$\mathrm{E}[V] = 0 \cdot 0.5 + 1 \cdot 0.5 = 0.5.$$