



ENGINEERING ECONOMICS AND FINANCIAL MANAGEMENT

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Synopsis:

When engineering is considered, it is usually the technical aspect that is associated with engineering. However, engineering also consists of the economical and financial aspect, as a construction project take basis in economy and financing through a client.

This report take basis in three main chapters. Chapter 2 gives and understanding of the overall macroeconomics and microeconomics, which can influence the building industry. Important elements within these have been emphasised, as these two areas are very broad. A brief overview of the phases in the building industry is also covered.

Through chapter 3 it is possible to assess how the economic planning for a construction project is carried out. This consist of how cost for a project is estimated, how time affect money, and how different decisions are made with basis in income or probability.

Chapter 4 covers elements within accounting through assets, liability, and the double entry book keeping. These overall make is possible to determine revenue and expenses, where the profit for the specified period can be deduced. Further, depreciation with three different methods is outlined.

In the discussion it is suggested to implement value engineering, as it can affect the project positively.

With basis in these three main areas, it is possible to get a general understanding of how the economical perspective for engineers is perceived and performed.

| Preface

This project is completed during the 10th semester by Chan D. Nguyen in the *Project Management in the Building Industry* master programme. The project is within the topic *Engineering Economics and Financial Management*, and contains a basic insight into the economical as well as financial aspect of the building industry.

The project takes basis in a course offered on University of New South Wales located in Australia on semester 2 in 2017. Knowledge and arguments are therefore with relation to what have been presented through lectures, course literature, and lecturer Steven Davis.

Acknowledgements

During the process of the project work I have received guidance from the supervisor Lene Faber Ussing, whom I would like to address my gratitude. She has showed commitment and support for the topic, which I really have appreciated.

Participants

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Signature:

Resumé

Når ingeniørarbejde betragtes, er det almindeligvis det tekniske aspekt, der er forbundet med ingeniørarbejde. Dog dækker ingeniørarbejde ligeledes det økonomiske og finansielle aspekt, da konstruktionsprojekter tager udgangspunkt i økonomi og finansiering gennem en klient.

Denne rapport tager basis i tre hovedkapitler. Kapitel 2 giver en forståelse af den overordnede makro- og mikroøkonomi, som kan påvirke byggebranchen. Vigtige elementer inden for disse er blevet fremhævet, da disse to områder er meget brede. En let gennemgang af faserne i byggebranchen er ligeledes dækket.

Gennem kapitel 3 er det muligt at evaluere, hvordan den økonomiske planlægning for et konstruktionsprojekt er udført. Dette omhandler, hvordan omkostninger for et projekt estimeres, hvordan tiden påvirker penge, samt hvordan forskellige beslutninger er taget med udgangspunkt i indkomst og sandsynlighed.

Kapitel 4 dækker elementer inden for regnskabsførelse gennem aktiver, passiver samt det dobbelte bogholderi. Dette giver overordnet mulighed for at bestemme omsætning og omkostninger, hvor profit for en bestemt periode kan beregnes. Yderligere er afskrivning i form af tre metoder beskrevet.

I diskussionen er det anbefalet at implementere value engineering, da det kan påvirke projektet positivt.

Ud fra de tre hovedområder er det muligt at få en general forståelse for hvordan det økonomiske perspektiv for ingeniører er opfattet samt udført.

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1 | Introduction

Engineering economics is all about determining if engineering projects are worth carrying out. This in other words entails the question, will the benefits outweigh the costs? One of the biggest issues for engineering projects is the costs are usually incurred almost immediately, while the benefits might take a long time to occur, which can take decades. Engineering projects are therefore long term investments, which must be assessed in great detail, before they are initiated in physical form. However, long term investments hold different difficulties, which must be taking into account.

The long term investments is complicated by the fact people have a high tendency of valuing objects in the future with less worth than the value of the same objects at present time. Future value is therefore perceived with less worth than the current value. The cost and benefits consequently need to be adjusted to account for this. Another issue is the uncertainty of the future. There are many unknown factors that are difficult to predict, as the exact weather forecast, possible production errors, delays, budget overrun, etc. The expected outcome of the investment may thereby entail a smaller amount of benefits or may become obsolete. This can however also go the other way around, where the benefits are bigger than expected and produce an increased amount of benefits.

An understanding of accounting for better manage of the financial aspect in projects and basics economics are therefore required, to be able to understand the different elements that can influence an engineering project. Furthermore, it gives an insight to the wider picture and how the different elements fit together within engineering economics and financial management.

2 | Macroeconomics and microeconomics

In this chapter the overall macroeconomics is outlined. Important aspects have been emphasised, which can influence the economic perspective of different companies in the building industry. Microeconomics is also covered, where the focus is on individual companies to how they their decisions affect utilisation and distribution of resources. Relevant areas are also described within Microeconomics. The different construction phases as well as the financial planning in these are also accounted for.

2.1 Macroeconomics

Before diving into specific economical aspects in the building industry such as economy planning, accounting, etc., which in general is focused on individual companies, the overall picture of economy must be understood through macroeconomics. Macroeconomics is a system of the overall performance of the national and global economies. This covers a broad field of areas, where some key areas within macroeconomics will be accounted for. These areas are therefore external factors, which can influence the building industry to for instance how projects are financed.

2.1.1 Output and GDP

One very important aspect of the overall national economy is output and GDP. Output is the amount of economic goods an economy produces, meaning a higher output provide more goods and services for the people to enjoy. Following the higher output more labour is required resulting to a lower unemployment, which will create an economic growth. Economic growth is the main focus for many countries, as it not only will increase stability in one country but can also affect the global perspective. Economic growth is caused by five main factors, which is the overall focus:

- Improvement in the labour force
- Improvement in the capital stock
- Increases in natural resources (e.g. mineral, oil)
- Improvements in efficiency
- Improvements in technology

Growth is part of a business cycle, which can have three scenarios. Periods with high growth is referred as expansions, low growth is referred as downturns, and periods of negative growth is referred as recessions. In relation to output, when the economy is expanding the businesses will in general produce at full output, where a need for new factories and capital goods may be necessary. When the growth is slowed down the businesses are no longer producing at full output, where people can lose jobs and have less income. Focusing on stabilising and smoothing the growth is therefore critical.

Output is measured in Gross Domestic Product (GDP), which is the total value of all the final goods produced for sale to the market value in an economy for a period (quarterly or yearly). The market value is determined through an aggregate demand and aggregate supply. Many aspects must be considered for the GDP, for instance are raw materials not considered final goods, as they would be double counted when produced into a product. Due to GDP being a monetary measure, it holds many deviation, which does not show the correct picture of a society. This can be the distribution of income, unaccounted value

of work like household, etc. The output is often broken down to the formula given in Eq. 2.1.

$$Y = C + G + I + X - M \quad (2.1)$$

Y	Output
C	Consumption by consumers
G	Consumption by government
I	Investment by business
X	Export to other countries
M	Import from other countries

By using Eq. 2.1 it is possible to understand fiscal policy, which gives the understanding of how tax can influence the output.

2.1.2 Fiscal policy and monetary policy

Fiscal policy is the use of fluctuation of government spending and taxation to influence the overall economy. If the government collects more taxes than it spends it is in surplus, and if it spends more than collected tax it is in deficit. By using fiscal policy it is possible to influence the growth of the country. By this Eq. 2.1 is assessed. If the government increases the taxes, the consumption of consumers, C , will decrease, and if the government spends the same amount the consumption of the government, G , will increase. By this the output, Y , will not change. However, if the government increases their spending to for instance big infrastructural projects without increasing tax, the output will increase, which will create growth. For this growth there is a potential multiplier effect. As the output is increased due increased government spending, more people will have jobs, meaning the consumption by consumers, C , will increase as well, and even further increase the output. Increasing spending by government is more used by traditionally democratic governments than reducing the taxes. However, if the desired outcome is not fulfilled it will lead to an increasing debt.

This can in some extent be connected to monetary policy, as monetary policy is the control of money supply to influence the economy. If the money supply is increased people will have more money to spend, which will increase the aggregate demand and therefore increase the output and the general price level. As described earlier, increasing the output will reduce unemployment. However, instead of the government controlling it, central banks make the actions. These actions can be modifying the interest rate, inflation rate, changing the bank reserves, etc., which therefore focus on price stability with basis in monetary supply.

Before being able to use for instance fiscal policy it must be recognised by the government first. This is called lags, which can be inside lag and outside lag. Inside lag is the time it takes for a government to identify a change to the economy, and then respond to it through for instance fiscal policy, while outside lag is the time between the response and an actual effect to the economy. In relation to fiscal policy the inside lag tends to be long, as the government has to do internal debates and discussions about the changes. However, if it is initiated the outside lag tends to be short, because changing tax rates and governmental spending have an immediate effect. For monetary policy the inside lag tends to be short, because the central banks can make actions fast, while outside lags tends to be longer as for instance interest rates affect investment decisions. Fiscal policy and monetary policy therefore are a matter of balance to be able to keep a stable economy.

2.2 Microeconomics

For the microeconomics the individual companies will be assessed to how, they can affect prices on the market and why they can compete better compared to other competitors. Microeconomics shows how and why different goods have different value, even though the goods as final product is almost the same. Goods and services is based on four aspects.

- **Land:** Refers to any kind of natural resource.
- **Labour:** The physical and mental efforts of people, but it depends on the quality of labour in form of human capital. Human capital is the knowledge and skills the labour possess.
- **Capital:** Buildings, equipments, plant, etc. to make goods and services.
- **Entrepreneurship:** The human resource that organises the other three aspects.

These four aspects is the basis of how the goods and services are created. One aspect itself can change the quality of the final product, which will have more value for a consumer. Different companies can by this choose what they want to invest into to be able to obtain a competitive advantage over other rivals on the market. However, this only concerns similar goods and services. If they choose to make a product not similar to others, the consumer may tend to buy that product or not at all. The different choices therefore hold different possibilities. In general, companies are only able to focus on some aspects due to scarcity of resources. A traditional example is a production between guns and butter. If a company puts more effort into making guns, there will be put less effort into making butter. The same thought concerns mass production versus not mass production. If a company only produce a small number of guns and bigger amount of butter, better quality resource is used for the small amount which in this case are the guns. Overall, many economic theories is based on competition. It is therefore in this case assumed that the amount produced is small compared to the whole industry. The maximum output of a firm's production must also be taking into account, as they are restricted by the numbers and capabilities of the firm. Therefore they must focus on other aspects to get consumers. Relevant for this is the use of blue ocean strategy.

2.2.1 Blue Ocean Strategy

Blue ocean strategy overall describes how different markets depends on the market concentration. If the market concentration is high, it has many competitors fighting for the same consumers. A market like this is referred as a red ocean. They then "win" these consumers by for instance marketing or investing more into the four aspects. By this a consumer may choose goods if for instance the natural resources are better quality. However, it is suggested through the strategy to not invest into market with high concentration. The companies must identify blue oceans, which are markets with little or almost no competition. In these markets it is not needed to fight for the consumers, which will generate a lot of profit. Blue ocean strategy is just one of many strategies that can be used for competitive advantage.

In continuation of blue ocean strategy, the blue oceans tend to have monopoly. This means a single firm can influence the price of its goods by varying the quantity produced. Producing less intentionally can therefore increase profit. These firms have high market power, as they have the ability to increase product price without losing their consumers. Monopoly is caused by for instance a new market, patents, copyright, barriers to entry if compared to Porter's five forces, branding, etc. In case of red oceans monopoly would not exist. For instance an increase in price of a product a company produces without good justification for the consumer, the consumer would just go to other competitors.

2.2.2 FIFO vs LIFO

When the different products are produced the producer can choose two methods for inventory management.

FIFO is first in first out. It implies that the goods that were produced first are used first. Thus the cost of goods sold uses the earlier prices, and the value of current inventory uses the more recent prices.

LIFO is last in first out. It implies that the goods that were produced latest are used first. Thus the value of current inventory uses the earlier prices and the cost of goods sold uses the more recent prices.

FIFO is advantageous to the company when costs are decreasing, as the first product sold are the most expensive. This means the cost of goods is increased, why you report fewer profit and therefore pay less tax. LIFO is advantageous when costs are increasing, where the same apply. Some jurisdictions will not allow the use of LIFO. Where it is allowed businesses must choose one method and stick to it for all stocks and all years.

2.2.3 Price discrimination

Besides focusing on different markets other factors can influence the price of goods. It has already been described briefly, but will be clarified even further. Alternatively creating products with different basis such as quality can influence the price. Some elements will be outlined:

- Reliability, e.g. thermostat accuracy
- Capacity, e.g. memory of computer
- Pleasantness, e.g. high service level (waiter)
- Convenience, e.g. coupons
- Quantity, e.g. bulk discount
- Channel, e.g. internet vs. physical shop
- Geography, e.g. supermarket locations
- Add-ons, e.g. internet access at hotels
- Delivery, e.g. hard cover vs. paperback books

These are just some elements that can affect the price. Many more exist, which the different companies use at their disposal to get more consumers. These elements can also be transferred to the building industry, where choosing for instance a sub-contractor with more reliability would be relevant.

2.3 Phases in the building industry

The macroeconomics has a great effect on the building industry, as it is external factors influencing potential investments into construction projects both private and governmental. The microeconomics give an insight to how, competition in not only the building industry is formed. To be able to understand the specific economical aspects in the building industry the lifecycle of phases a construction project pass through is described. In the construction industry a project is defined by objectives, starting and finish points clarified by the developer. These phases will be described with the focus of an overall financial management and are illustrated in Figure 2.1.

Initiation and concept phase

Generating relevant information in this phase is essential, as it will determine if the project is into interest or not. Based on the information a cost benefit analysis can be carried out to be able to evaluate the project

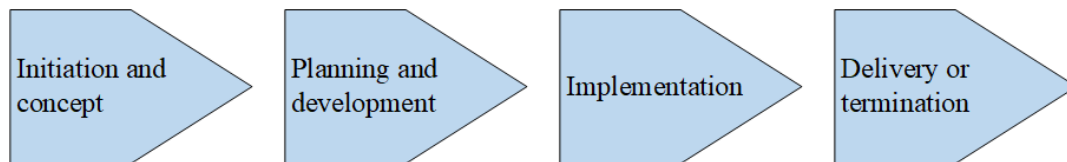


Fig. 2.1: Phases for a construction project.

feasibility. This is also the phase where it is easiest and most inexpensive to make changes to the project, if for instance a reduced cost or increased value is desired.

Planning and development phase

In this phase a detailed plan and design are performed for the project. The financial planning include defining budget for different elements in the project, and refining them continuously when more information is available. Different options will also be assessed, which will have a major role in selection.

Implementation phase

Within the implementation phase the actual work is performed, which is the major work. The financial aspect will be continuous evaluation of information regarding money spent in comparison with budgets, as well as forecasts remaining costs. Depending on this information it is possible to identify, if anything needs to be revised such as the planned project work in order to keep the project within budget.

Delivery or termination phase

This is the final stage of the construction project, where the phase overall concerns the handover of products to the client. An error and defect assessment by both the client and contractors are carried out. If both parties accept, the project will be handed over. The last financial management will be finalising the project review of the cost outcomes, where improvements, etc. are evaluated for future projects.

With basis in the different phases, it is possible to understand the deeper level for economics. This is outlined in Chap. 3, where a more profound assessment of the financial aspect is described.

3 | Economic planning

In this chapter the preliminary work for initiating a construction project is accounted for. This include how cost is estimated, the time value of money that changes over time, and the external factors affecting the financial management. How decision making with basis in income or probability is also covered.

3.1 Cost estimation

When an organisation or government chooses to initiate a construction project, they usually expect to have more benefits than actual cost, or else there would be no purpose of carrying out the project. To be able to evaluate this it is necessary to estimate what benefits will accrue from it as well as the actual costs. However, these estimates depend on the extent and detail of data at disposal for the organisation. The more and detailed the data is, the more accurate the estimates will be. Often in the earliest stages of a construction project the estimates are not very accurate as the information at disposal is limited. Some of the first cost estimations are done in the two first phases illustrated in Figure 2.1, which is the procurement phase. Developing information consist of costs for the organisation and developing good information for the estimates can be beneficial, and even approximate estimates may be useful for the decision-making to for instance whether extra initiatives, which may be more costly, can be carried out. In contradictory, developing information based on limited data can give a false future prediction of the actual scenario, why a fine balance must be taking into account. Naturally, as the project develops through the procurement phases of a construction project more information is also developed, where estimates become more accurate. The different phases describes the procurement phase as illustrated in Figure 3.1, where the approximate standard deviation development also are shown.

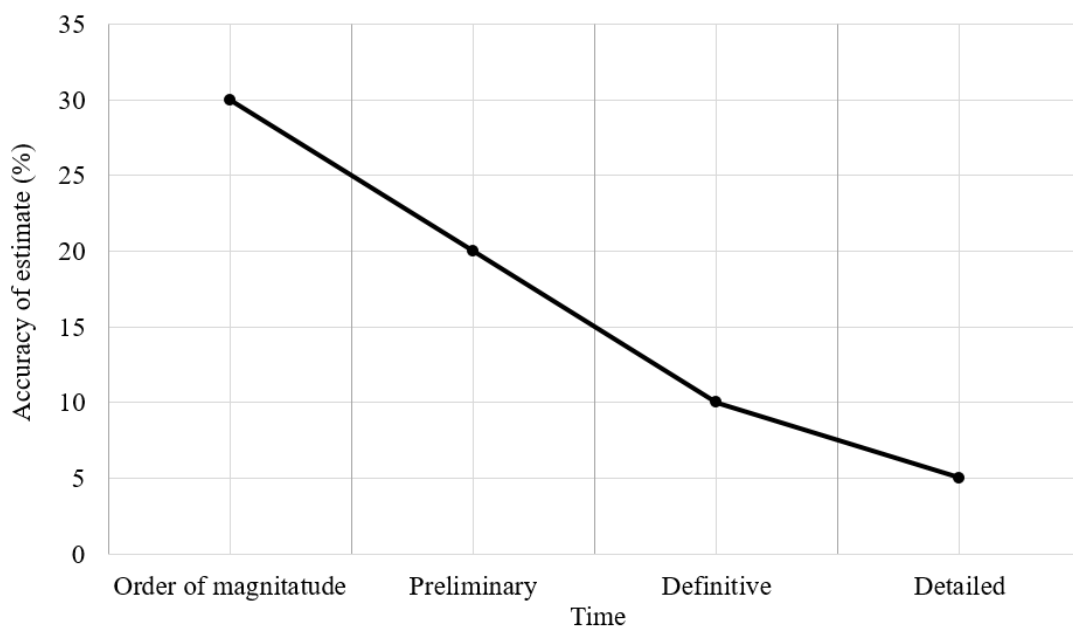


Fig. 3.1: Standard deviation development through procurement phases.

The different phases of the procurement phase will be accounted for.

Order of magnitude magnitudes estimates

The first initiatives when a project is commenced to establish the broad viability of the overall project, which in other words is the first budget for the fiscal planning. Fiscal planning can be understood as a financial business plan in accordance to a fiscal financial year to for instance the annual tax return. This is not calculated in the traditional calendar year that starts on the first of January. Estimates in the stage is usually based on previous experience with similar projects.

Preliminary estimates

This phase is performed when some information on the project has been developed to be able to more accurately determine the extent of the overall financial commitment. It is also done to evaluate alternatives.

definitive estimates

This is the process of the detailed design phase, to ensure that the cost of the structure or facility developing from the design process is within budget. If there is a risk of a budget overrun, the probability of that risk must be determined as well as the extent.

Detailed estimates

The last phase of the process of preparing a tender for the construction project, where a creation of a detailed work plan is finalised.

3.1.1 Work Breakdown Structure (WBS)

The procurement phases described overall is the phases different companies go through when forming a tender. However, all companies have their own systems and phases they go through when forming a tender. The described one is therefore just an example. Making a tender is easier said than done, where many different methods can be used as a system. A widely used system in the building industry is Work Breakdown Structure (WBS) formed in the mid 1960's originally for the U.S. Department of Defence.

WBS is a hierarchical decomposition of a construction project, where any significant project can be broken down into several sub-projects. A fundamental comprehension is the 100 % rule. If one element is divided into smaller sub-projects, those sub-projects in total define the divided element 100 %, which applies to all levels. Therefore, those sub-projects can be divided even further if needed. [Fried et al., 2011] The breakdown of sub-projects is generally divided into individual activities or tasks. The lowest level must consist of a delegation of time, cost, and resources to the individual activities, tasks, so the responsibilities can be assigned. The lowest level must also be manageable and measurable. If this is not possible the elements must be broken down further. The levels can be infinite, however the levels normally range from 2 to 5. To be able to determine the number of levels, different aspects must be taking into consideration: [Haugan, 2002]

- Size and complexity
- Structure of the organisations involved
- Phase of the project
- Degree of uncertainty and risks
- Time available for planning

An illustration of the overall basis of WBS with 4 levels is illustrated in Figure 3.2.

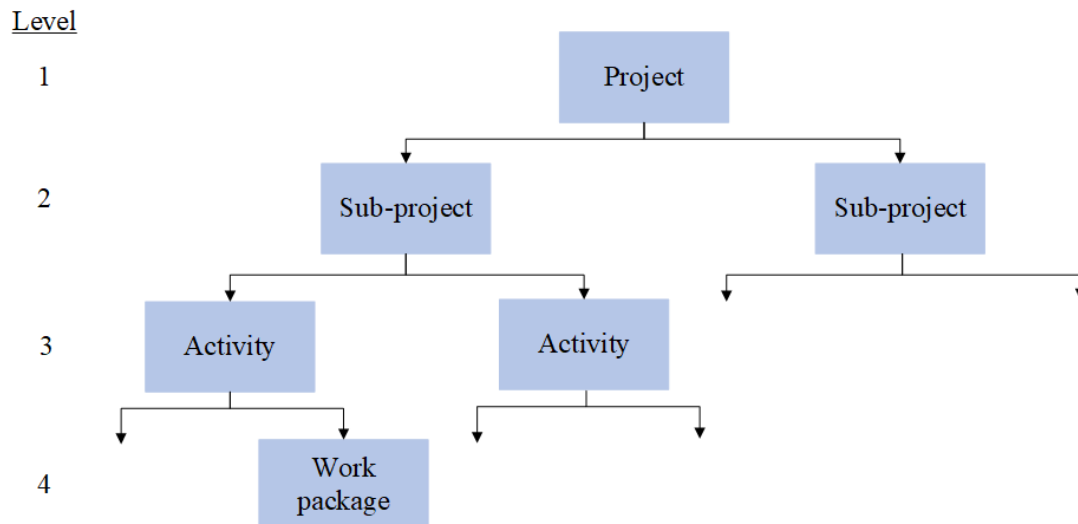


Fig. 3.2: The basis of WBS.

Cost Breakdown Structure (CBS)

The WBS can also be translated into Cost Breakdown Structure (CBS), which overall is the same as WBS. The difference is that the activities are broken down into smaller cost groups with different cost code. Each cost code can for instance refer to materials, labour, equipment, etc.

3.1.2 Factors to consider in estimating

When a system has been set for the estimation such as WBS, there are different factors that must be taken into account when carrying out the estimation. These will be accounted for

Hazards and local factors

This overall concerns a risk management study of how to identify risks, analyse and evaluate risks, and what action that must be taken to deal with the risk. Risks could for instance be availability of labour and materials, weather, fire, flood, etc.

Productivity factors

For this a plan for how to sustain an acceptable productivity with focus on facility management must be carried out. This therefore concerns the following, plant and equipment:

- Age
- Location of project
- Skill of operator, e.g. truck driver
- Maintenance program before and/or under breakdown

Labour:

- Environment on and around project, e.g. temperature
- Job situation, e.g. work in high altitude
- Working hours
- Learning curves

The mentioned learning curves within labour is important to understand, why this will be accounted for in more detail.

Learning curves

Learning curves is basically about, the more an activity is performed and repeated, the more efficient the workers become, which result the work being done in progressively less time and lower cost [Peck, 1984]. The learning curves are defined through Eq. 3.1 and Eq. 3.2.

$$\bar{T}_n = T_1 \cdot k^{\log_2 n} \quad (3.1)$$

$$T_n = T_1 \cdot [nk^{\log_2 n} - (n-1)k^{\log_2(n-1)}] \quad (3.2)$$

k	Improvement factor
\bar{T}_n	Average man hours, unit after n cycles
T_n	Man hours, unit for the n^{th} cycle

The smaller the factor k is, the faster the workers learn. On average it is around 0.8, but can range from 0.65 to 0.95. This means, if the task is more complex, the factor k will naturally be bigger. By using the learning curves it is possible to calculate the overall future time consumption of a repetitive activity.

Cost elements

There are many different cost elements such as direct costs, indirect costs, and overheads/administration costs. Another aspect is a contingency, which is connected to risk management. By creating a contingency plan a safety net is developed. Contingency is the allowance for errors in the estimate in the form of for instance calculation errors, delivery damage, etc. In the client's perspective this is the amount of money that must be allocated to the project in case of errors, scope change etc., while it for the contractor is a kind of protection for the profit. If the project costs more than expected the profit would diminish if there was no contingency or if the contingency is already used up.

contract type

Many varieties of a contract to calculate payments exist. Overall there are three main methods: Lump sum, cost reimbursable, and unit rate contract. For a lump sum contract the total cost for the contract is fixed. All the risk is therefore on the contractor. For the cost reimbursable contract, also known as cost plus, the risk is on the client. Here the client pays all the contractor bills, where the contractors can charge a fee to for instance overheads costs. The unit rates contract is a shared risk between the client and the contractor. A specified rate for measurable such as work performed is defined.

3.2 The time value of money

Besides being able to do cost estimation with basis in different factors, a very critical aspect must be considered. Huge sums of money is invested into construction projects for the design, constructing, as well as the operation. Therefore an engineer is not only concerned with the technical perspective, but must also evaluate the economical perspective. When a construction project is initiated, long periods of time can pass before the actual building is constructed and fully operational. Therefore, when planning and evaluating decisions the time value of money must be taking into account. Money at current state is not the same in the future. Investing into the future can hold many uncertainties and risks. Another area is money can also earn interest in the time invested into a project, whereas a future return can be less than just using interests obtained through banks.

The economic estimations described in Sec. 3.1 is therefore more complex than stated. The economic evaluations cannot be performed by simply using the net benefits and costs, as it can change over time. The money by this has to be "translated"(discounted) to comparable amounts. For this different discounting formulae is available, which overall have two parameters; the duration of the project, and the discount rate.

Discount rate

The discount rate define how the money is worth more at the current state than in the future, which is determined by for instance the possibilities of good use of money, or the effects of inflation. It however hold deviation, as it is based on assumptions. The discount rate can therefore vary from project to project, and company to company. In general the rate must be bigger than the interest rate, or else the investor would get less value than the interest they pay for loaning the money, which by this have no point. The formulae for discounting will be accounted for.

3.2.1 Discounting and compounding

Discounting and compounding are two fundamental concepts for economic evaluation. Compounding describes how an investment changes at a specified rate compounded periodically for an interval of time. Discounting is the relationship between the present value of the investment and its future value also with basis in the rate and interval of time. These two concepts is defined through Eq. 3.3, which is described as *compound interest (single payment)*.

$$F = P(1 + i)^n \quad (3.3)$$

F	Future value
P	Present value
i	Rate
n	Number of interest periods

For instance the present value of \$1000 in 5 years with a rate on 5 % would be the following:

$$P = \frac{F}{(1 + i)^n} = \frac{1000}{(1 + 0.05)^5} = \$783.53 \quad (3.4)$$

The formula outlined in Eq. 3.3 is based on a single payment, which is the overall basis of the future value perspective. Other commonly used discounting formulae describe other situations. These will also be mentioned briefly:

- **Compound interest (single payment):** Relation between present and future value
- **Compound interest (uniform series):** Future value of a series of uniform payments over a period of n years
- **Sinking fund (uniform series):** Inverse of the compound interest uniform series, where uniform payment is determined
- **Uniform series present value:** Uniform payment with relation between present and future value
- **Capital recovery factor:** Inverse of the uniform series present value, where uniform annual payment is determined
- **Uniform gradient series:** An constant increasing or decreasing uniform payment
- **Geometric gradient series:** An increasing or decreasing uniform payment with constant percentage rate

3.2.2 Determining discount rate

All the mentioned situations are all depending on the rate i , where the choice of the discount rate is very critical for the economic evaluation. Small changes can affect the economic viability of a project, why it is must be evaluated with care and a critical mind. Overall high discount rates are unfavourable to long term investments, as the long term benefits will "disappear". When the discount rate must be determined it is different between the private sector and the public sector.

Private sector projects

When investing in the private sector the general level of interest is important to assess, as it is the cost of acquiring investment capital. This also concerns the creditworthiness of the company, which overall is the possibility a borrower may default their debt obligations. Factors such as repayment history and credit score must be judged. Riskiness of the project is also a part of the assessment of the discount rate. A method used is setting the discount rate equal to the minimum acceptable rate of return of the company's capital, which can be the general interest rate.

Public sector projects

In public sector projects the influence of the factors in the private sector is applicable. However, there are more aspects to consider, as the public sector has added complications by for instance public interests, non-economic benefits, etc. The public sector also tend to invest into very long term construction projects with huge sums of money, which can be tunnels, bridges, etc. These huge infrastructural projects are usually funded through tax receipts and are a very long term investment, where having a discount rate on zero would make sense. Tax receipts is also more "safe" capital compared to the interest rates.

3.3 Project appraisal

With basis in Sec. 3.1 it is possible to understand the overall picture of how cost estimations are made. Imagine there are different projects with their individual income and costs, how are these projects chosen by an investor? Three widely used methods for economic evaluations will be accounted for.

3.3.1 Benefit cost ratio

A method used is the benefit cost ratio, see Eq. 3.5, where a ratio above 1.0 is desirable due a more benefits.

$$\text{Benefit Cost Ratio} = \frac{\text{Discounted value of net benefits}}{\text{Discounted value of capital cost}} \quad (3.5)$$

To get an understanding of how it works an example has been outlined. The full calculations can be seen in App. A.

A developer is considering two projects.

Project A has a capital cost of \$760,000 and a life of 12 years. Income for the project will be \$95,000 per year. At the end of 12 years the project can be sold for \$300,000. The project's discount rate is 16 %. Project B has a capital cost of \$1,400,000 and a life of 20 years. Income for the project will be \$200,000 per year. At the end of 20 years the project can be sold for \$200,000. The project's discount rate is 9 %.

So in this situation a developer has two options to choose from. The present value of the income per year as well as the income from selling the project after a period of years must be determined. By using the different methods outlined in Sec. 3.2.1, which in this case is the *uniform series present value* for the income per year, and the *compound interest (single payment)* for the selling price. The capital cost is already given for both, why it is possible to calculate the benefit cost ratio. Project A is 0.72, while project B is 1.33. Therefore in this case project B would be the best to choose, as there is more income for the developer.

3.3.2 Net Present Value (NPV)

The NPV is simply the difference between discounted benefits, B , and costs, C for a construction project, see Eq. 3.6, where all income and spendings in the future must be discounted to the present time. If Eq. 3.6 is positive the project is feasible. The different methods outlined in Sec. 3.2.1 can be used for this.

$$NPV = B - C \quad (3.6)$$

An extended version of this method is the annual equivalent amount, where it is possible to compare different projects with different lives. Here the NPV is transferred into an annual amount.

3.3.3 Internal Rate of Return (IRR)

Another method used is by comparing the possible discount rate, r , with the market interest rate. If this is higher, the investment for the project is feasible, or else it would make more sense to just get income from interest rates. IRR is used to estimate the profitability of potential investments, where IRR is defined as a specific discount rate, which makes the NPV of all cash flows zero. In this situation the present value of returns equals the present value of costs, why Eq. 3.7 is valid.

$$\sum \frac{B_t}{(1+r)^t} = \sum \frac{C_t}{(1+r)^t} \quad (3.7)$$

A similar example between two projects has also been made for this, where the full calculations can be seen in App. B.

Project A has a capital cost of \$2,818,440.272582 and a life of 12 years. Income for the project will be \$455,000 per year. There is no salvage value. Project B has a capital cost of \$899,060.598323 and a life of 14 years. Income for the project will be \$195,000 per year. There is no salvage value.

Yet again there are two options. By using Eq. 3.7 and isolating the discount rate, r , project A has an IRR on 12 % and project B has an IIR on 20 %. Project B is therefore the most beneficial.

With basis in these three methods it is possible to choose a project most beneficial for the investor.

3.3.4 Risk and uncertainty

In many cases, choosing a project is not as easy as the stated examples for benefit cost ratio, and IRR. In all cases there are some risks and uncertainties, which must be taking into account. For this some criteria for decision making exist. Figure 3.3 is used to illustrate how a decision maker has to take different potential scenarios into account with different profit depending on the level of demand, which is unknown.

	States of Nature		
	Low Demand for land	Moderate Demand for land	High Demand for land
Develop small subdivision	\$3,500,000	\$3,000,000	\$2,700,000
Develop medium subdivision	\$1,000,000	\$12,500,000	\$12,400,000
Develop large subdivision	\$-500,000	\$-250,000	\$25,000,000

Fig. 3.3: Project choice.

Maximin

Maximising the the minimum. The decision maker will select the action with the largest minimum of different scenarios. The minimum for each column would be evaluated: \$2700 (low demand), \$1000 (moderate demand), \$-250 (high demand). This is a pessimistic criteria, as the *developing small subdivision* would be the choice based on maximin.

Maximax

Maximising the maximum. Here the decision maker will choose the scenario, which can give the highest profit possible, even though it holds risk. By this, develop large subdivision would be chosen.

Minimax

Minimising the maximum. This criterion is based on what you can lose and therefore "regret". Therefore, the decision maker will select the minimum of the maximum the decision maker can loose. Through Figure 3.3, Figure 3.4 is generated, why medium subdivision would be chosen.

	Low Demand	Moderate Demand	High Demand
small subdivision	0	9,500,000	22,300,000
medium subdivision	2,500,000	0	12,600,000
large subdivision	4,000,000	12,750,000	0

Fig. 3.4: Project choice based in minimax.

3.3.5 Decision making based on probability

There exist many other different scenarios, which have their own use of system. One scenario concerns probability.

The different criteria to for instance maximin, etc. is based on potential profit, however in some cases the scenarios can be depending on probability. By this the probability can be calculated as weighted average payoff. This is called Expected Monetary Value (EMV). The EMVs can then be presented into a decision tree to get an overview of the most feasible choice. This can be for an as simple thing as choosing to bring an umbrella. The illustration can be seen in Figure 3.5, where choosing not to bring an umbrella would be the best choice.

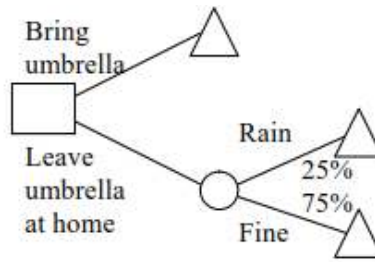


Fig. 3.5: Decision tree.

Monte Carlo Simulation

In some cases the probability is much more advanced and cannot be easily predicted. For instance, knowing certain probabilities for specific scenarios can be very complex. However, using Monte Carlo Simulation based on random variables it is possible to understand the impact of risks and uncertainties in prediction and forecasting models. However, Monte Carlo simulations is a broad class of computational algorithms, why it will only be touched briefly in this report. When different scenarios contain significant uncertainty in the process of making a forecast or estimation, using Monte Carlo Simulation instead of using an average number can be better. Within finance many random variables exist, where the use of Monte Carlo Simulation can estimate the probability of cost overruns and the probability to how an asset price will fluctuate. Overall, Monte Carlo Simulation is based on using different building models of possible results, and apply it to a factor holding uncertainty. By the building model it calculates the result over and over with a different set of random values from the probability functions. After the many thousands of recalculations for completion a probability distribution is formed with basis in input data. Therefore, many distributions with different methods exist within the Monte Carlo Simulation.

A simple one is the normal distribution, where the mean, variance, and standard deviation are key numbers to the algorithm. This means values near the mean are more likely to occur. Within the normal distribution a method named Box-Muller method can be used. Box-Muller method involves a polar transformation and calculates two standard normal random numbers, Z_1 and Z_2 , from two standard uniform random numbers, U_1 and U_2 , see Eq. 3.8 and Eq. 3.9.

$$Z_1 = \sqrt{-2\ln(U_1)} \cos(2\pi U_2) \quad (3.8)$$

$$Z_2 = \sqrt{-2\ln(U_1)} \sin(2\pi U_2) \quad (3.9)$$

By using these transformation for a normal distribution the probability of certain prices can be predicted. In a case where a project manager wants to predict a cost of an activity, however the cost hold

uncertainties, by using a mean value and a variance it is possible to predict for instance 10 different prices. Here the standard normal random numbers, Z_1 or Z_2 , multiplied with variance gives the deviation to the mean. By this it is possible to estimate the probability of how often the cost will be higher than for instance the project manager's budget.

This is mere one distribution for Monte Carlo Simulation, where many others exist like exponentially distributed, etc. A Monte Carlo simulation example with different methods is illustrated in App. C. A set of random numbers, also used for the calculations, can be seen at Figure 3.6.

94737	08225	35614	24826	88319	05595	58701	57365	74759
87259	85982	13296	89326	74863	99986	68558	06391	50248
63856	14016	18527	11634	96908	52146	53496	51730	03500
66612	54714	46783	61934	30258	61674	07471	67566	31635
30712	58582	05704	23172	86689	94834	99057	55832	21012
69607	24145	43886	86477	05317	30445	33456	34029	09603
37792	27282	94107	41967	21425	04743	42822	28111	09757
01488	56680	73847	64930	11108	44834	45390	86043	23973
66248	97697	38244	50918	55441	51217	54786	04940	50807
51453	03462	61157	65366	61130	26204	15016	85665	97714
92168	82530	19271	86999	96499	12765	20926	25282	39119
36463	07331	54590	00546	03337	41583	46439	40173	46455
47097	78780	04210	87084	44484	75377	57753	41415	09890
80400	45972	44111	99708	45935	03694	81421	60170	58457
94554	13863	88239	91624	00022	40471	78462	96265	55360
31567	53597	08490	73544	72573	30961	12282	97033	13676
07821	24759	47266	21747	72496	77755	50391	59554	31177
09056	10709	69314	11449	40531	02917	95878	74587	60906
19922	37025	80731	26179	16039	01518	82697	73227	13160
29923	02570	80164	36108	73689	26342	35712	49137	13482
29602	29464	99219	20308	82109	03898	82072	85199	13103
94135	94661	87724	88187	62191	70607	63099	40494	49069
87926	34092	34334	55064	43152	01610	03126	47312	59578
85039	19212	59160	83537	54414	19856	90527	21756	64783
66070	38480	74636	45095	86576	79337	39578	40851	53503
78166	82521	79261	12570	10930	47564	77869	16480	43972
94672	07912	26153	10531	12715	63142	88937	94466	31388
56406	70023	27734	22254	27685	67518	63966	33203	70803
67726	57805	94264	77009	08682	18784	47554	59869	66320
07516	45979	76735	46509	17696	67177	92600	55572	17245
43070	22671	00152	81326	89428	16368	57659	79424	57604
36917	60370	80812	87225	02850	47118	23790	55043	75117
03919	82922	02312	31106	44335	05573	17470	25900	91080
46724	22558	64303	78804	05762	70650	56117	06707	90035
16108	61281	86823	20286	14025	24909	38391	12183	89393
74541	75808	89669	87680	72758	60851	55292	95663	88326
82919	31285	01850	72550	42986	57518	01159	01786	98145
31388	26809	77258	99360	92362	21979	41319	75739	98082
17190	75522	15687	07161	99745	48767	03121	20046	28013
00466	88068	68631	98745	97810	35886	14497	90230	69264

Fig. 3.6: A set of random digits generated for Monte Carlo Simulation.

3.4 Continuous evaluation

By using all the discussed areas it is possible to make cost estimations and therefore make an economic planning for not only the procurement phase. If the project is then won, the the project goes to the implementation phase. In the process of the project it is a good idea to make midway evaluations between planned cost and actual cost. An example is illustrated in Figure 3.7, which is a good basis for cost management and evaluation.

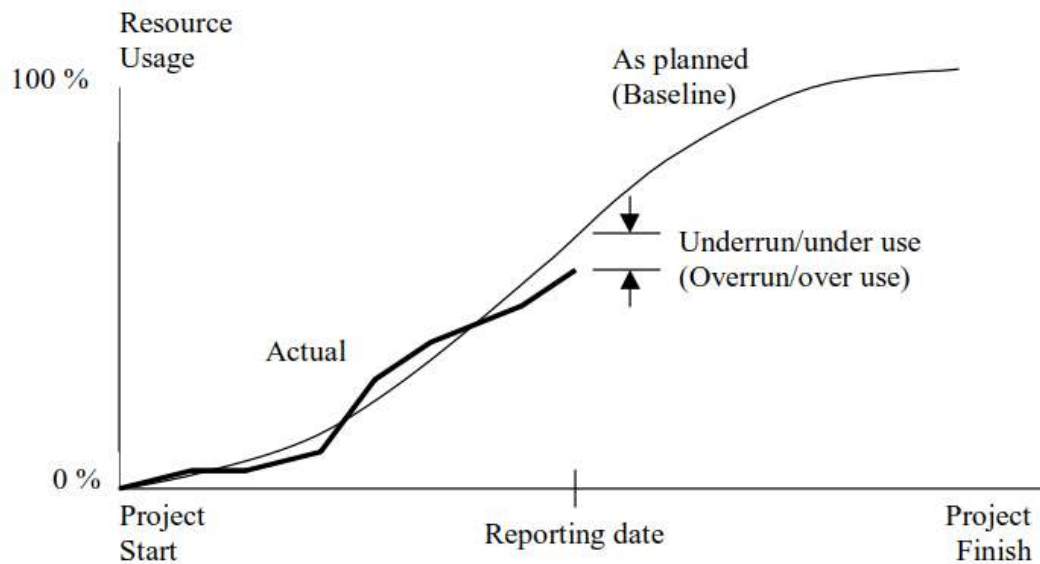


Fig. 3.7: Evaluation of implementation phase.

4 | Accounting

In this chapter basic accounting will be covered to get an understanding to how accounting is managed. Within accounting depreciation is also touched.

Using accounting it is possible to evaluate how well a company is performing economically, and if it is necessary to make certain actions to be able get a stronger and more stable economy. It is therefore for internal investment decisions, and performance evaluation. However, it is also used for external purposes such as attracting people outside the company for more capital: Equity (shareholders), debt, (loans), credit (supplier), costumers, and employees. Key areas for accounting will be described.

4.1 Balance sheet

The balance sheet is a statement of the financial position of a business on a specific date, which contains assets and liabilities. The balance is defined through Eq. 4.1. The difference between them is the owners' equity in the company.

$$\text{Assets} = \text{Liabilities} + \text{Owners' equity} \quad (4.1)$$

The purpose of a balance sheet is to give an insight into the value of assets and liabilities.

Assets

There are many different kind of assets for a company. Overall assets are what the company owns. Assets is acquired through transactions to for instance buying physical things as tools (asset is what is bought) or selling (asset is the money received or promised). More specifically, assets is cash, inventory (machinery, buildings, land), intangible assets (patents, copyright, goodwill). The assets is categorised into two groups: fixed assets and current assets.

Liabilities

Liabilities are what the company owes, which must be paid through cash, goods, or services. This is for instance obligations for salary, interest, tax, bonds, mortgages, etc. Liabilities are based on two groups: Capital, and borrowed capital.

4.1.1 Double entry book keeping

This is a method used to track all transactions, which makes it possible to prevent employees stealing from the business. The principle behind the system is that every transaction is recorded two times, one time in debits (left) and one time in credits (right). The system is a partial system of the overall system, which means there are many double entry book keeping. Increases in assets are debits, decreases are credits. Increases in liabilities or owners equity are credits, decreases are debits.

4.1.2 Revenue vs. expenses

With basis in assets and liabilities it is possible to identify revenue and expenses. Revenue is the money the company earns from conducting business. Revenue is recognised when a service has been provided or a title for goods has been transferred, the customer has been billed, and there is a reasonable certainty of cash collection. Expenses are the cost for obtaining the revenue, which for instance is wages, rent,

utilities, etc. Revenue minus the expenses are then the profit the company made in its business, see Eq. 4.2.

$$\text{Profit} = \text{Revenue} - \text{Expenses} \quad (4.2)$$

A positive profit is desired, or else it requires actions to change it.

4.2 Depreciation

An important aspect to consider is depreciation, as it is the decline of value of an asset over time. Most assets depreciate, such as material, or tools. If depreciation is not used the assets will be valued more than actual value, which will create a misrepresenting. Ideally the depreciation would equal the exact value when sold, however depreciation holds uncertainties until an actual buyer is found. Methods used for depreciation is therefore approximates for the book value of an asset. As depreciation lowers value of assets it is considered an expense for taxation. Higher depreciation for assets therefore decreases the tax paid. Thus, companies try to depreciate assets fast for taxation purposes, as this gives them a tax deduction sooner rather than later. Money now is worth more than money later.

There are different methods used for depreciation, which can determine the depreciation charged each year. This is also used for the book value. Some widely used will be accounted for. Full examples of how the different methods are used are illustrated in App. D

Straight line depreciation

This is the most simple method, as the value decreases by equal annual amounts.

Diminishing value method

Instead of using an equal annual amount for depreciation a fixed percentage of the remaining value can be used.

Sinking fund method

This model also gives an equal annual amounts. However, this method takes future investment into account. This method assumes a set amount must be invested each year, which makes it possible to, when the asset is no longer useful, have enough money to buy a replacement. The amount of depreciation is therefore a constant amount plus the interest, i , that would be earned that year.

$$\text{Annual amount} = \frac{(P - S) \cdot i}{(1 + i)^n - 1} \quad (4.3)$$

5 | Discussion

With an understanding of the macroeconomics and microeconomics, it is possible to get an insight into what kind of factors that can affect the building industry and the parties within. Further, the outlined phases in a construction project makes it possible to understand how cost estimation is carried out in the procurement phase. Cost estimations hold many uncertainties and has many factors which must be considered. Critical factors has been emphasised such as the time affect on money and project appraisal with focus on income an probability. To finalise the economic understand accounting has been accounted for. Taking all these areas into account, an overall picture of the economical aspect has been highlighted. To improve the building industry and the different project initiated by investors, a system has been proposed for implementation.

5.1 Value Engineering

Another system that can be used for evaluation is value engineering, which was developed accidentally during World War II. Due to a shortage of materials different companies were forced to use substitutes. However, through the use of substitutes the cost was reduced and the quality improved. Lawrence Miles was by this selected to identify these substitutions intentionally instead of accidentally. He then developed the value engineering. Value engineering is a systematic approach with the objective of optimising cost and performance for a facility or system. Value engineering focuses on dealing with errors of sub-optimisation, which are partial systems optimised for performance at the expense of the performance of the overall system. This occurs in for instance the sub-groups; architects, structural engineers, mechanical engineers, etc. Value engineering tries to substitute with less expensive alternatives without decreasing the performance, which therefore take all the sub-groups into consideration, when the optimal solutions must be found. The analysis consist of a top-down process, which means value engineering assess the overall system first, and work down trough the sub-systems. By this it is also possible to identify sub-systems not required. Value engineering is therefore an evaluation of systems to reduce cost and increase functionality for an increase of value of the product. Cost is usually educed at the expense of quality, which is a simple cost-cutting strategy. However, with the use of value engineering quality is not effected.

Value engineering is best to implement in the early stages such as the conceptual, design, and development phase, as changes can be big and inexpensive. In contrast if implemented late, it can be expensive and changes are hard to initiate. An overall picture of this is illustrated in Figure 5.1.

Value engineering consists of five phases:

- Information: Generate database and select areas for detailed assessment
- Speculative: Outline alternatives meeting requirements
- Analytical: Evaluate alternatives and optimise
- Proposal: Identify best alternative to decision making
- Final report: Define and quantify results

Following these steps it is possible to utilise value management. Different techniques can be used to support value engineering. One technique is function analysis. Through this the primary and secondary functions of the project are identified, which can give an insight into potential unnecessary secondary functions that does not create value. To support the functional analysis the verb-noun approach can be used. The function identified must only be described by two words consisting of a verb and a noun. For

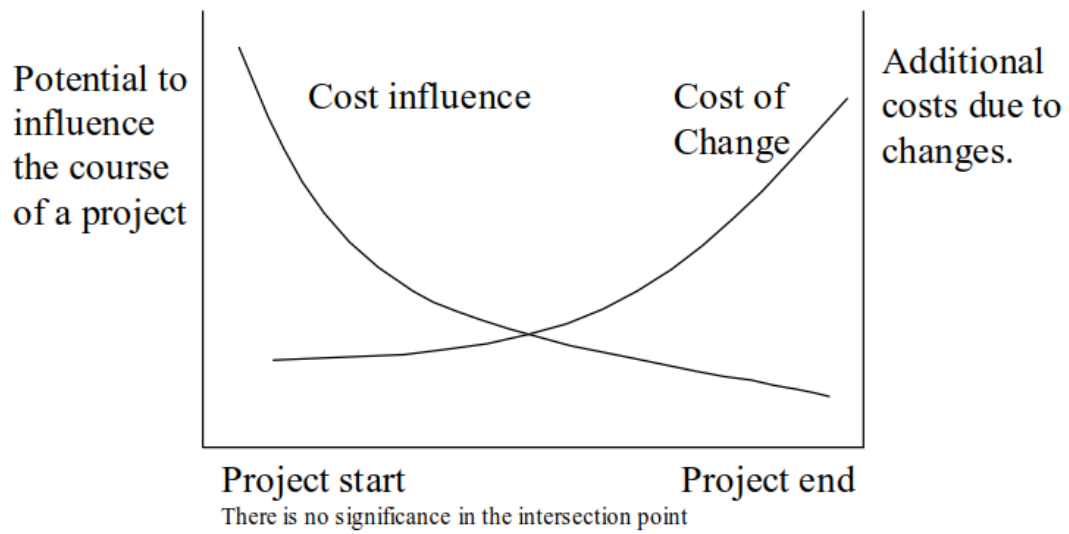


Fig. 5.1: Cost influence through development of project.

instance would a table be referred as "support weight". By this a bigger focus on the purpose of the function is clarified. Other techniques exist, however these are some that can facilitate value engineering and in the end affect the project positively.

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Part I

Appendix

A | Cost Benefit Ratio

Question 3

A developer is considering two projects.

Project A has a capital cost of \$760,000 and a life of 12 years. Income for the project will be \$95,000 per year. At the end of 12 years the project can be sold for \$300,000. The project's discount rate is 16%.

What is the Benefit Cost Ratio for Project A? (Answer to two decimal places)

Uniform series present value:

$$P = \frac{A((1+i)^n - 1)}{i(1+i)^n}$$

Present value:

$$P = \frac{F}{(1+i)^n}$$

Cost: 760,000

Benefits:

$$\frac{95000 \cdot ((1+0.16)^{12} - 1)}{0.16 \cdot (1.16)^{12}} + \frac{300000}{(1+0.16)^{12}} \rightarrow 544264.039453$$

$$\text{Benefit Cost Ratio} = \frac{\text{Discounted Values of Net Benefits}}{\text{Discounted Value of Capital Cost}}$$

$$\frac{\frac{95000 \cdot ((1+0.16)^{12} - 1)}{0.16 \cdot (1.16)^{12}} + \frac{300000}{(1+0.16)^{12}}}{760000} \rightarrow 0.716136894017$$

0.72

Project B has a capital cost of \$1,400,000 and a life of 20 years. Income for the project will be \$200,000 per year. At the end of 20 years the project can be sold for \$200,000. The project's discount rate is 9%.

What is the Benefit Cost Ratio for Project B? (Answer to two decimal places)

$$\frac{200000 \cdot \left((1+0.09)^{20} - 1 \right)}{0.09 \cdot (1+0.09)^{20}} + \frac{200000}{(1+0.09)^{20}}$$

$$\frac{1400000}{1.32956807984}$$

1.33

B | Internal Rate of Return

Question 4

A developer is considering two projects.

Project A has a capital cost of \$2,818,440.272582 and a life of 12 years. Income for the project will be \$455,000 per year. There is no salvage value.

What is the Internal Rate of Return for Project A? (Answer to the nearest percent, if you think the answer is 12% then answer 12)

$$\sum_{t=0}^n \left(\frac{B_t}{(1+r)^t} \right) = \sum_{t=0}^n \left(\frac{C_t}{(1+r)^t} \right)$$

$$\text{solve} \left(-2818440.272582 + \frac{455000}{(1+r)^1} + \frac{455000}{(1+r)^2} + \frac{455000}{(1+r)^3} + \frac{455000}{(1+r)^4} + \frac{455000}{(1+r)^5} + \frac{455000}{(1+r)^6} + \frac{455000}{(1+r)^7} + \frac{455000}{(1+r)^8} + \frac{455000}{(1+r)^9} + \frac{455000}{(1+r)^{10}} + \frac{455000}{(1+r)^{11}} + \frac{455000}{(1+r)^{12}} = 0, r \right)$$

► $r = -1.81171488034$ or $r = 0.12$ ⚠

12%

Project B has a capital cost of \$899,060.598323 and a life of 14 years. Income for the project will be \$195,000 per year. There is no salvage value.

What is the Internal Rate of Return for Project B? (Answer to the nearest percent)

$$\text{solve} \left(-899060.598434 + \frac{195000}{(1+r)^1} + \frac{195000}{(1+r)^2} + \frac{195000}{(1+r)^3} + \frac{195000}{(1+r)^4} + \frac{195000}{(1+r)^5} + \frac{195000}{(1+r)^6} + \frac{195000}{(1+r)^7} + \frac{195000}{(1+r)^8} + \frac{195000}{(1+r)^9} + \frac{195000}{(1+r)^{10}} + \frac{195000}{(1+r)^{11}} + \frac{195000}{(1+r)^{12}} + \frac{195000}{(1+r)^{13}} + \frac{195000}{(1+r)^{14}} = 0, r \right)$$

► $r = -1.85123167468$ or $r = 0.199999999969$ ⚠

20%

C | Monte Carlo Simulation

A project consists of 4 independent activities.

The cost of Activity A is exponentially distributed with a mean of \$1000.

The cost of Activity B has a 30% chance of costing \$800, 50% of \$1200 and 20% of \$1600.

The cost of activity C is normally distributed with a mean of \$1500 and std dev of \$200

The cost of activity D is normally distributed with a mean of \$2000 and std dev of \$500

1. What is the probability that the project will cost more than \$6000 assuming all activities are statistically independent?
2. What is the mean for the expected cost of the project assuming all activities are statistically independent?

Solution

The first random value in our table is 94737 so the cost for activity A in the first realisation will be $-\$1000 * \ln(0.94737) = \54.07

Activity B requires a lookup table

Activity cost	Probability	Lookup values
\$800	0.3	0 – 0.3
\$1200	0.5	0.3 – 0.8
\$1600	0.2	0.8 – 1.0

The second random variable in our table is 87259. Thus the value is between 0.8 and 1.0 giving an activity cost of \$1600.

Activities C and D need to be solved using the Box Muller method:

The third and fourth random variables are 63856 and 66612. Thus the costs for activities C and D are:

$$\begin{aligned}
 Z_1 &= \sqrt{-2 \ln(U_1)} \cos(2\pi U_2) \\
 &= \sqrt{-2 \ln(0.63856)} \cos(2\pi 0.66612) \\
 &= -0.47639
 \end{aligned}$$

$$\begin{aligned}
 Z_2 &= \sqrt{-2 \ln(U_1)} \sin(2\pi U_2) \\
 &= \sqrt{-2 \ln(0.63856)} \sin(2\pi 0.66612) \\
 &= -0.81862
 \end{aligned}$$

If we assume that the costs for activities C and D are statistically independent:

Cost of Activity C = $\$1500 + \$200 * -0.47639 = \$1404.72$

Cost of Activity D = $\$2000 + \$500 * -0.81862 = \$1590.69$

Cost of project in first realisation is $\$54.07 + \$1600 + \$1404.72 + \$1590.69 = \$4649.48$

The following table gives all of the calculations for ten realisations assuming statistical independence:

	U_1	Cost A	U_2	Cost B	U_3	U_4	Cost C	Cost D	Total
1	0.94737	54.07	0.87259	1600	0.63856	0.66612	1404.72	1590.69	4649.48
2	0.30712	1180.52	0.69607	1200	0.37792	0.01488	1777.79	2065.12	6223.43
3	0.66248	411.76	0.51453	1200	0.92168	0.36463	1446.72	2151.79	5210.27
4	0.47097	752.96	0.804	1600	0.94554	0.31567	1473.16	2153.29	5979.41
5	0.07821	2548.36	0.09056	800	0.19922	0.29923	1390.64	2855.52	7594.52
6	0.29602	1217.33	0.94135	1600	0.87926	0.85039	1559.84	1795.16	6172.33
7	0.6607	414.46	0.78166	1200	0.94672	0.56406	1439.11	1935.19	4988.75
8	0.67726	389.70	0.07516	800	0.4307	0.36917	1323.29	2475.39	4988.38
9	0.03919	3239.33	0.46724	1200	0.16108	0.74541	1488.98	1044.93	6973.24
10	0.82919	187.31	0.31388	1200	0.1719	0.00466	1875.16	2027.47	5289.94

1. The probability that the project will cost more than \$6000 can be obtained by determining the proportion of realisations where this occurs. It occurs in realisations 2, 5, 6, and 9 so the probability is 40%.
2. The mean for the expected cost for the project is the mean for all of the realisations = \$5806.97

D | Depreciation

A company purchases a piece of plant for \$220,000 with an expected life of 4 years, when it expects to be able to sell it for \$50,000. The cost of capital for this company is 9%. Determine the depreciation to be charged each year using each method in the table below. Answers should be correct to 2 decimal places. Note: the question is asking for the amount of depreciation for each year. This is the amount that should be used as an expense for that year. This means that the total sum for each column should be the same.

Straight line

$$\frac{220000 - 50000}{4} \rightarrow 42500$$

$$42500 \cdot 4 \rightarrow 170000$$

$$42500$$

Diminishing value

$$r = \left(1 - \sqrt[n]{\frac{s}{p}} \right) \cdot 100$$

$$r = \left(1 - \sqrt[4]{\frac{50000}{220000}} \right) \cdot 100 \rightarrow r = 30.9543$$

$$1: 220000 - \left(1 - \sqrt[4]{\frac{50000}{220000}} \right) \cdot 220000 \rightarrow 151901.$$

$$\left(1 - \sqrt[4]{\frac{50000}{220000}} \right) \cdot 220000 \rightarrow 68099.392168$$

$$68099.4$$

$$2: 220000 - \left(1 - \sqrt[4]{\frac{50000}{220000}} \right) \cdot 220000 - \left(1 - \sqrt[4]{\frac{50000}{220000}} \right) \cdot 151901 \rightarrow 104880.763424$$

$$\left(1 - \sqrt[4]{\frac{50000}{220000}} \right) \cdot 151901 \rightarrow 47019.8$$

47019.8

$$\begin{aligned}
 3: & 220000 - \left(1 - \sqrt[4]{\frac{50000}{220000}}\right) \cdot 220000 - \left(1 - \sqrt[4]{\frac{50000}{220000}}\right) \cdot 151901 - \left(1 - \sqrt[4]{\frac{50000}{220000}}\right) \cdot 72415.7 \\
 & - \sqrt[4]{\frac{50000}{220000}} \cdot 104880.763424 \\
 & \left(1 - \sqrt[4]{\frac{50000}{220000}}\right) \cdot 104880.763424 \rightarrow 32465.1
 \end{aligned}$$

32465.1

$$\begin{aligned}
 4: & 220000 - \left(1 - \sqrt[4]{\frac{50000}{220000}}\right) \cdot 220000 - \left(1 - \sqrt[4]{\frac{50000}{220000}}\right) \cdot 151901 - \left(1 - \sqrt[4]{\frac{50000}{220000}}\right) \cdot 49999.9 \\
 & - \sqrt[4]{\frac{50000}{220000}} \cdot 104880.763424 - \left(1 - \sqrt[4]{\frac{50000}{220000}}\right) \cdot 72415.7 \\
 & \left(1 - \sqrt[4]{\frac{50000}{220000}}\right) \cdot 72415.7 \rightarrow 22415.8
 \end{aligned}$$

22415.8

$$5/15 \cdot 80000 \rightarrow 26666.7$$

Sinking fund

$$\frac{(80000 - 6000) \cdot 0.3}{(1.1)^5 - 1} \rightarrow 36363.$$