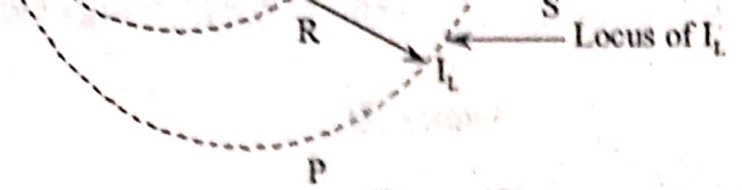


Figure (2)

Q35. Mention the applications of a differentiator and integrator circuit.

Ans:

Applications of Differentiator

Differentiator (or) High pass RC circuit finds is applications in,

(i) Wave shaping circuits (such as conversion of pulses into spikes).

(ii) FM demodulator.

Applications of Integrator

Integrator circuits (or) low pass RC circuits are widely used in,

(i) Analog computers

(ii) Analog to digital computers

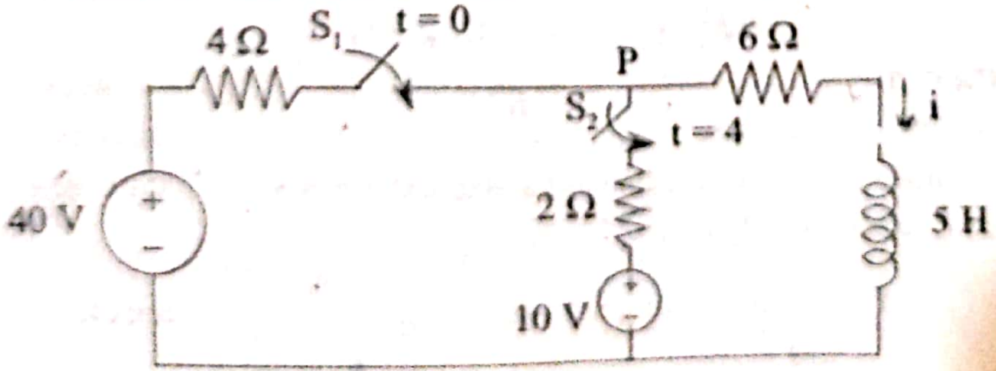
(iii) Wave shaping circuits (such as conversion of square waves into triangular waves)

(iv) Filter

(v) Ramp generators

(vi) Servo control system.

Q36. At t = 0, switch 1 in figure is closed, and switch 2 is closed 4s later. Find i(t) for t > 0. Calculate i for t = 2s and t = 5s.



Figure

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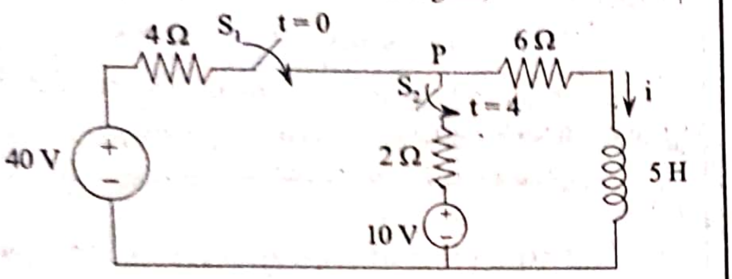
Ans:

The given circuit is shown in figure,



Figure

To determine,

$i(t)$ for $t > 0$ and

$i(t)$ for $t = 2$s and $t = 5$s

In order to find $i(t)$ considering three time intervals i.e., $t \leq 0$, $0 \leq t \leq 4$ and $t \geq 4$ separately. The switches S_1 and S_2 are opened for $t < 0$ and current through the inductor is zero i.e., $i = 0$, because inductor doesn't allow the sudden changes in current.

Thus,

$$i(0^-) = i(0) = i(0^+) = 0$$

For $0 \leq t \leq 4$, switch S_1 is closed and S_2 is still open.

Then,

$$i(\infty) = \frac{40}{4+6} = \frac{40}{10}$$

$$\therefore \quad i(\infty) = 4 \text{ A}$$

Now,

Equivalent resistance, $R = 4 + 6 = 10\ \Omega$

Time constant, $\tau = \frac{L}{R}$

$$\tau = \frac{5}{10} = \frac{1}{2} s = 0.5s$$

Thus,

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$= 4 + [0 - 4]e^{-t/0.5}$$

$$= 4 - 4e^{-2t}$$

$$\therefore \quad i(t) = 4(1 - e^{-2t}) \text{ A}, \quad 0 \leq t \leq 4$$

For $t \leq 4$, switch S_2 is closed. Since the inductor doesn't allow the sudden changes in current, the initial current at $t = 4$ is,

$$i(4) = i(4^-) = 4(1 - e^{-2(4)})$$

$$= 4(1 - e^{-8})$$

$$= 4(1 - 3.35 \times 10^{-4})$$

$$= 4(0.99)$$

$$i(4) = 3.96 \simeq 4$$

$$\therefore \quad i(4) = 4 \text{ A}$$

To find $i(\infty)$, assume the voltage 'V' at node P. Now, applying KCL at node P, we get,

$$\frac{40 - V}{4} + \frac{10 - V}{2} = \frac{V}{6}$$

$$V\left(\frac{1}{6} + \frac{1}{4} + \frac{1}{2}\right) = \frac{40}{4} + \frac{10}{2}$$

$$V\left(\frac{2+3+6}{12}\right) = 10 + 5$$

$$V\left(\frac{11}{12}\right) = 15$$

$$\therefore \quad V = \frac{180}{11} = 16.363 \text{ V}$$

Thus,

$$i(\infty) = \frac{V}{6} = \frac{16.363}{6}$$

$$i(\infty) = 2.727 \text{ A}$$

At the inductor terminal, the equivalent resistance is,

$$R = 4 \parallel 2 + 6$$

$$= \frac{4 \times 2}{4+2} + 6$$

$$= \frac{8}{6} + 6 = \frac{8+36}{6} = \frac{44}{6}$$

$$R = \frac{22}{3}\ \Omega = 7.333\ \Omega$$

And,

$$\tau = \frac{L}{R} = \frac{5}{7.333} = 0.6818s$$

Hence,

$$i(t) = i(\infty) + [i(4) - i(\infty)]e^{-(t-4)/\tau}, \quad t \geq 4$$

The term $(t - 4)$ in the exponential is due to time delay. Thus, substituting $i(\infty)$, $i(4)$ and τ values in the above equation, we get,

$$i(t) = 2.727 + [4 - 2.727]e^{-(t-4)/0.6818}$$

$$= 2.727 + [1.273]e^{-(t-4)(1.466)}$$

$$\therefore \quad i(t) = 2.727 + 1.273e^{-(t-4)(1.466)}, \quad t \geq 4$$

Hence,

$$i(t) = \begin{cases} 0, & t \leq 0 \\ 4(1 - e^{-2t}), & 0 \leq t \leq 4 \\ 2.727 + 1.273e^{-1.466(t-4)}, & t \geq 4 \end{cases}$$

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Now,

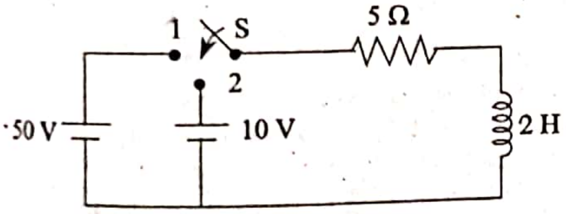
At $t = 2$s,

$$i(2) = 4\,(1 - e^{-2(2)})$$
$$= 4\,(1 - e^{-4})$$
$$= 4\,(1 - 0.0183)$$
$$= 4\,(0.98)$$
$$\therefore \quad i(2) = 3.92 \text{ A}$$

At $t = 5$s,

$$i(5) = 2.727 + 1.273e^{-1.466(5-4)}$$
$$= 2.727 + 1.273e^{-1.466}$$
$$= 2.727 + 1.273(0.2308)$$
$$= 2.727 + 0.293$$
$$\therefore i(5) = 3.020 \text{ A}$$

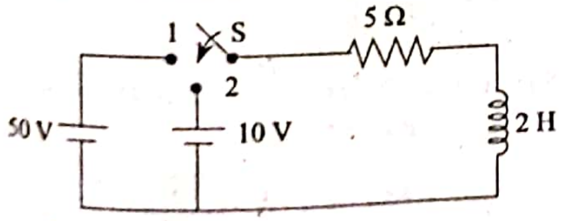
Q37. In the circuit as shown in figure, the switch S is in position 1 for a long time and brought the position 2 at time t = 0. Determine the circuit current.



Figure

Ans:

The given circuit is shown in figure,



Figure

Let, current i be flowing through the circuit when the switches are close.

In figure, switch 'S' is at position-1 for steady state condition.

At, $t = 0$.

$\therefore$ Curren through the inductor is given as,

$$i = \frac{V_1}{R}$$
$$= \frac{50}{5} = 10 \text{ A}$$

The inductor will not allow to change the current through it.

$$\therefore \quad i(0^-) = i(0^+) = 10 \text{ A}$$

It is required to obtain the current response after the switch is moved to position 2.

$\therefore$ The current in the circuit is,

$$i(t) = c_1 + c_2 e^{-\frac{R}{L}t} \qquad \text{... (1)}$$

We know that,

At $t = 0$, $i = 10$ A

Substituting the values in equation (1),

$$\Rightarrow \quad 10 = c_1 + c_2\, e_0$$
$$\Rightarrow \quad c_1 + c_2 = 10 \qquad \text{... (2)}$$

Now, switch is moved to position 2 i.e., at $t = \infty$

$$i = \frac{V_2}{R}$$
$$= \frac{10}{5} = 2 \text{ A}$$

Substituting the values in equation (1), we have,

$$2 = c_1 + c_2(0)$$
$$c_1 = 2$$

Substituting c_1 in equation (2), we get,

$$c_1 + c_2 = 10$$
$$c_2 = 10 - 2$$
$$c_2 = 8$$

$\therefore$ The current in the circuit is,

$$i(t) = 2 + 8\,e^{-\frac{5}{2}t}$$
$$= 2\,(1 + 4e^{-25t})$$

Q38. A symmetrical square wave whose peak-to-peak amplitude is 2 V and whose average value is zero as applied to an RC integrating circuit. The time constant is equal to half period of the square wave. Find the peak-to-peak value of the output amplitude.

Ans: Given that,

Peak-to-peak amplitude of symmetrical square wave, $V = 2$ Volts

Time constant is equal to half the time period of the square wave.

$$\Rightarrow \quad RC = \frac{T}{2}$$

Peak-to-peak Value of the Output Amplitude: Since, it is given that a symmetrical square wave with zero average value is applied to an RC integrating circuit (i.e., low-pass circuit), the output waveform will also be symmetrical. The positive and negative peaks in the output waveform will be equal in magnitude but opposite in direction. The output waveform is shown in figure.

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Here, $V_1 = -V_2$

We know that in an RC integrating circuit, when a symmetrical square wave is applied, V_1 is given by,

$$V_1 = \frac{V}{2}\left[\frac{1-e^{-T/2RC}}{1+e^{-T/2RC}}\right]$$

Substituting the given values, we get,

$$\Rightarrow \quad V_1 = \frac{2}{2}\left[\frac{1-e^{\frac{-T}{2}\cdot\frac{2}{T}}}{1+e^{\frac{-T}{2}\cdot\frac{2}{T}}}\right]$$

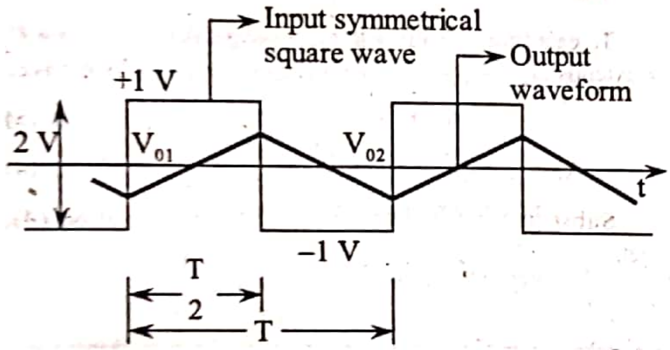
$$\Rightarrow \quad V_1 = \left[\frac{1-e^{-1}}{1+e^{-1}}\right]$$

$$\Rightarrow \quad V_1 = \frac{0.632}{1.368}$$

$$\Rightarrow \quad V_1 = 0.4621$$

$$\therefore \quad V_2 = -V_1$$

$$= -0.4621 \text{ Volts}$$



Figure

So, peak-to-peak value of the output amplitude varies from −0.4621 V to 0.4621 V.

Peak-to-peak amplitude = 0.4621 − (−0.4621)

= 0.9242 V

$$\boxed{\therefore \text{Peak-to-peak amplitude} = 0.924 \text{ V}}$$

Q39. An RC differentiator circuit is driven from a 500 Hz symmetrical square wave of 10 V peak-to peak. Calculate the output voltage levels under steady state if RC = 1 msec.

Ans: Given that,

For an RC differentiator circuit,

Peak-to-peak amplitude of input square wave,

$V = 10$ volts

Time constant, $\tau = RC = 1$ msec

Frequency of symmetrical square wave, $f = 500$ Hz

The square wave applied at the input of differentiator is shown in figure (1).

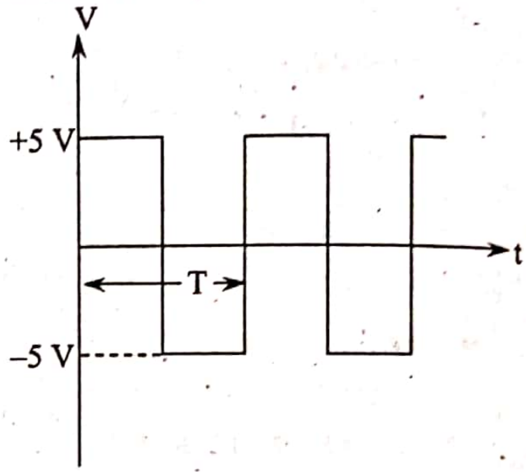


Figure (1)

The output voltage levels of high pass filter are V_1, V_1', V_2 and V_2'.

The steady state maximum output voltage is, V_1' and is given as,

$$V_1' = V_1 e^{-T/RC} \qquad \text{... (1)}$$

The steady state minimum output voltage is, V_2' and is given as,

$$V_2' = V_2 e^{-T/RC} \qquad \text{... (2)}$$

To find out the values of V_1' and V_2', first calculate the value of V_1 and V_2. By using the equations, $V_2 = -V_1$, $V_1 = V_2' = V$ and $V_2' = -V_1'$, we can write,

$$V_1 = \frac{V}{1+e^{-T/RC}}$$

$$= \frac{10}{1+e^{-0.002/1\times10^{-3}}} = 8.8$$

$$\boxed{\therefore V_1 = 8.8 \text{ volts}}$$

Substitute the value of V_1 in equation (1), we get,

$$V_1' = V_1 e^{-T/RC}$$

$$= 8.8(e^{-2\times10^{-3}/1\times10^{-3}})$$

$$= 1.19$$

$$\boxed{\therefore V_1' = 1.19 \text{ volts}}$$

Similarly, we have,

$$V_1' - V_2 = V$$

$$\therefore \quad V_2 = \frac{-Ve^{-T/RC}}{1+e^{-T/RC}}$$

$$= \frac{-1.35}{1+0.135}$$

$$= -1.189$$

$$\boxed{\therefore V_2 = -1.189}$$

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Substitute the value of 'V_2' in equation (2), we get,

$$V_2' = V_2 e^{-T/RC}$$

$$= (-1.189)(e^{-2\times10^{-3}/1\times10^{-3}})$$

$$= (-1.189)(0.135)$$

$$= -0.16$$

$$\boxed{\therefore V_2' = -0.16 \text{ volts}}$$

$\therefore$ Therefore, the output voltage levels under steady state are,

$V_1 = 8.8$ volts

$V_1' = 1.19$ volts

$V_2 = -1.189$ volts

$V_2' = -0.16$ volts.

The output waveform is shown in figure (2).

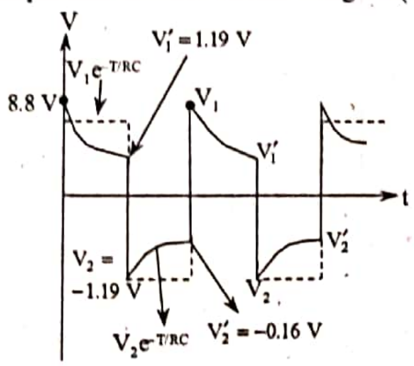

Figure (2)

Q40. A square wave whose peak-to-peak value is 1 V extends ± 0.5 V with reference to ground. The half period is 0.1 sec. This voltage impressed upon an RC differentiating circuit whose time constant is 0.2 sec. Determine the maximum and minimum values of output voltage in the steady state.

Ans: Given that,

Peak to peak amplitude of input is $V = 1$ V and extends ± 0.5 V with respect to ground.

The positive duration is $T_1 = 0.1$ sec

The negative duration is $T_2 = 0.2$ sec

The time constant RC, $\tau = 0.2$ sec

Figure (a) illustrates the input waveform applied to the high-pass RC circuit.

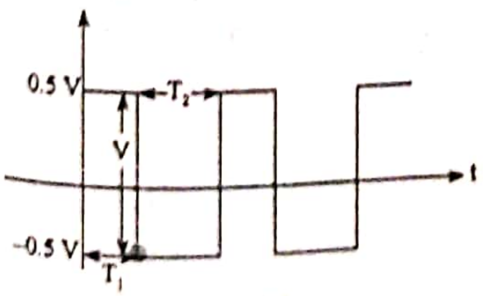

Figure (a)

From the data, the time constant RC is comparable with T_1 and T_2.

Since the shape of the output of a high-pass RC circuit for square wave input (any input) depends on time constant, the output waveform is as illustrated in figure (b).

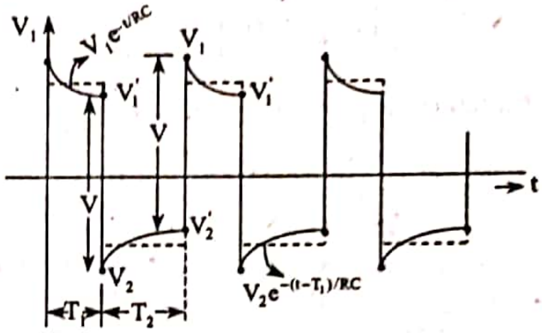

Figure (b): Output Waveform of RC High-pass Circuit for Square Wave Input

From figure (b),

$$V_1' = V_1 e^{-T_1/RC} \quad \text{... (1)}$$

$$V_2' = V_2 e^{-T_2/RC} \quad \text{... (2)}$$

The steady state maximum value of output is V_1' and

The steady state minimum value of output is V_2'

$\therefore$ To calculate V_1' and V_2' we have to calculate V_1 and V_2 as V extends equally as ± 0.5 with respect to ground we have.

$$V_1 = -V_2 \quad \text{... (3)}$$

$$\text{Also, } V_1 - V_2' = V \quad \text{... (4)}$$

Substituting equations (2) and (3) in equation (4), we get,

$$V_1 + V_1 e^{-T_2/RC} = V$$

$$\Rightarrow \quad V_1 = \frac{V}{1+e^{-T_2/RC}}$$

$$\Rightarrow \quad V_1 = \frac{1}{1+e^{-0.2/0.2}} = 0.7310$$

$$\Rightarrow \quad V_1' = V_1 e^{-T_1/RC}$$

$$\Rightarrow \quad V_1' = 0.7310 e^{-0.1/0.2}$$

$$\Rightarrow \quad V_1' = 0.443$$

Similarly we have,

$$V_1' - V_2' = V$$

$$\Rightarrow \quad \frac{V_1}{e^{-T_1/RC}} - V_2 = V$$

$$\Rightarrow \quad \frac{-V_2}{e^{-T_1/RC}} - V_2 = V$$

$$\therefore \quad V_2 = \frac{-Ve^{-T_1/RC}}{1+e^{-T_1/RC}} \quad \text{... (5)}$$

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We have, $V_2' = V_2 e^{-T_2/RC}$

Substituting equation (5) in V_2', we get,

$$V_2' = \frac{(-Ve^{-T_1/RC})(e^{-T_2/RC})}{(1+e^{-T_1/RC})}$$

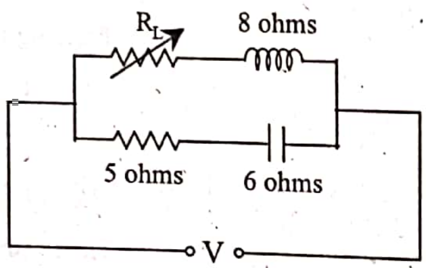
$$\Rightarrow \quad V_2' = \frac{-Ve^{-(T_1+T_2)/RC}}{(1+e^{-T_1/RC})}$$

$$\Rightarrow \quad V_2' = \frac{-1e^{-(0.3)/0.2}}{1+e^{-0.1/0.2}} = \frac{-0.223}{1.606}$$

$$\therefore \quad V_2' = -0.1388$$

$\therefore$ The steady state maximum and minimum values of the output of high-pass RC circuit for square wave input are $V_1' =$ 0.443 V and $V_2' = -0.1388$ respectively.

Q41. Draw the locus diagram and obtain the value of R_L in the circuit shown in below figure which results in resonance for the circuit.



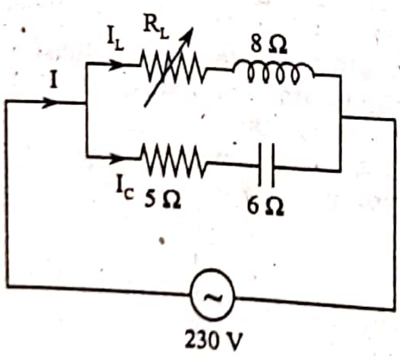
Figure

Ans:

Model Paper-II, Q5(a)

Note: In the given problem the value of the source voltage is not specified. Hence assume it to be 230 V (r.m.s).

The given circuit is shown in figure.



Figure

Now,

Impedance of branch-1,

$$Z_C = R_C - jX_C$$
$$= 5 - j6$$
$$= 7.81\angle -50.19$$

Current through branch-1,

$$I_C = \frac{V}{Z_C}$$
$$= \frac{230\angle 0°}{7.81\angle -50.19°}$$
$$= 29.45\angle 50.19° \text{ A}$$

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The branch current I_c is of fixed magnitude equal to 29.45 A and leads the applied voltage by 50.19°.

Now,

Impedance of branch-2,

$$Z_L = R_L + jX_L$$
$$= R_L + j8$$

When, $R = 0$

$$Z_L = j8$$
$$= 8\angle 90°$$

∴ Current through branch-2

$$I_L = \frac{V}{Z_L}$$
$$= \frac{230\angle 0°}{8\angle 90°}$$
$$= 28.75\angle 90°$$

∴ $I_L = 28.75\angle 90°$ A

The current I_L lags behind the applied voltage by 90°.

As R_L increases, Z_L increase and hence I_L decreases.

The locus of the current I_L is a semi-circle lying below the voltage phasor, since I_L lags behind the voltage.

Also the diameter of the semicircle is = 28.75 A

Steps to Draw Locus Diagram

1. Take the applied voltage $V = 230$ V the reference phasor on x-axis.
2. Take a scale of 1 cm = 4 A on the y-axis.
3. Draw $I_L = OA = 28.75$ A = 7.2 cm on the negative y-axis since I_L lags V by 90°.
4. With midpoint B of OA draw the semicircle.
5. Now, draw $I_C = OC = 29.45$ A = 7.36 cm at an angle $\phi = 50.19$ with respect to the applied voltage.
6. Draw CD parallel to OA and magnitude equal to OA.
7. With centre E, the midpoint of CD draw the semicircle.
8. Consider a point 'F' on the semicircle arbitrarily such that $CF = OG = I_L$.
9. Join CF, OF and FG
10. Measure OF from the locus diagram.

    Thus $OF = I = 8.5$ cm $= 8.5 \times 4 = 34$ A
11. Now from triangle OFC,

$$FC = \sqrt{OF^2 - OC^2}$$
$$= \sqrt{34^2 - 29.45^2}$$
$$= \sqrt{288.7}$$
$$= 16.99$$
$$\simeq 17 \text{ A}$$

12. $FC = I_L = 17$ A

But, $I_L = \frac{V}{Z_L}$

$$\Rightarrow \quad I_L = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$$\Rightarrow \quad 17 = \frac{230}{\sqrt{R^2 + 8^2}}$$

$$\Rightarrow \quad \sqrt{R^2 + 8^2} = \frac{230}{17}$$

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$$\Rightarrow \quad R^2 + 8^2 = \left[\frac{230}{17}\right]^2$$

$$\Rightarrow \quad R^2 + 8^2 = 183.044$$

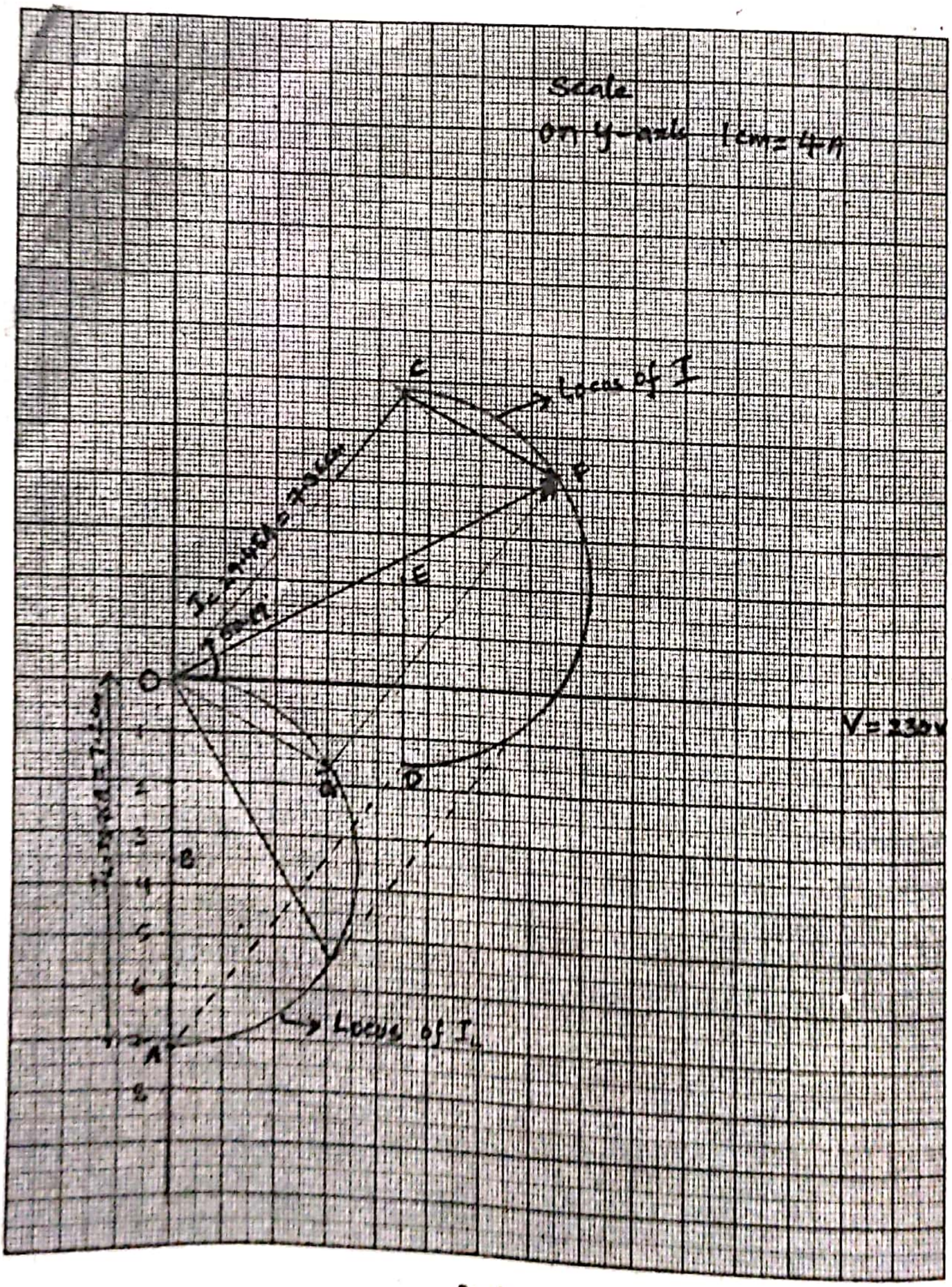
$$\Rightarrow \quad R^2 = 183.044 - 64$$

$$\Rightarrow \quad R^2 = 119.045$$

$$\Rightarrow \quad R = \sqrt{119.045}$$

$$\Rightarrow \quad R = 10.91\ \Omega$$

$$\boxed{\therefore R = 10.91\Omega}$$



Graph

For Capacitor

The maximum energy stored in capacitor $= \frac{1}{2}CV^2$

Energy released per cycle $= P \times T$

$$= \frac{V^2}{2 \times R} \times \frac{1}{f}$$

$$= \frac{V^2}{2Rf}$$

$$\therefore \quad \text{Quality factor, } Q = \frac{2\pi \times \frac{1}{2}CV^2}{\frac{V^2}{2Rf}}$$

$$= \frac{2\pi CV^2}{2} \times \frac{2Rf}{V^2}$$

$$= \omega CR$$

$$\boxed{\therefore Q = \omega CR}$$

Q48. A parallel circuit has two branches, branch 1 has a resistance of 5 Ω connected in series with an inductance 10 mH. A capacitor is connected in branch 2, the parallel circuit is connected across a 230 V, 50 Hz supply. If the circuit is to be in resonance, find the value of the capacitance. Find the current drawn from the supply and also find the currents in branches (1) and (2).

Ans:

Given that,

Resistance of branch 1, $R = 5\ \Omega$

Inductance of branch 1, $L = 10$ mH $= 0.01$

Supply voltage, $V = 230$ V

Frequency, $f = 50$ Hz

According to given data, the circuit obtained is as shown in figure (a).

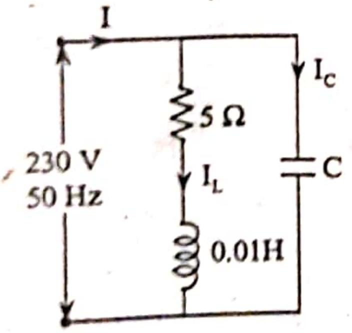


Figure (a)

Let, Y_1, Y_2 be the admittances of the two parallel branches.

Let, X_C be the capacitive reactance of the capacitor and X_L be the inductive reactance of the inductor.

$$X_L = 2\pi fL$$
$$= 2 \times \pi \times 50 \times 0.01$$
$$= 3.14\ \Omega$$

$$Y_1 = \frac{1}{5 + j3.14}$$

$$Y_2 = \frac{1}{-jX_C}$$

The total admittance is given by,

$$Y = Y_1 + Y_2$$

$$= \frac{1}{5 + j3.14} + \frac{1}{-jX_C} \quad \text{... (1)}$$

Rationalizing the denominators of equation (1),

$$Y = \frac{1}{5 + j3.14} \times \frac{5 - j3.14}{5 - j3.14} + \frac{1}{-jX_C} \cdot \frac{jX_C}{jX_C}$$

$$= \frac{5 - j3.14}{34.86} + \frac{j}{X_C}$$

Separating the real and imaginary parts,

$$Y = \left(\frac{5}{34.86}\right) + j\left[\frac{1}{X_C} - \frac{3.14}{34.86}\right]$$

For the circuit to be at resonance, the imaginary part of admittance must be zero.

$$\text{i.e., } \frac{1}{X_C} - \frac{3.14}{34.86} = 0$$

$$\Rightarrow \quad \frac{1}{X_C} = \frac{3.14}{34.86}$$

$$\Rightarrow \quad X_C = 11.10\ \Omega$$

We know that, the capacitive reactance is given by,

$$X_C = \frac{1}{2\pi fC}$$

Capacitance,

$$C = \frac{1}{2\pi f X_C}$$

$$= \frac{1}{2\pi \times 50 \times 11.10}$$

$$= 286\ \mu\text{F}$$

Current through branch 1 is given by,

$$I_1 = \frac{V}{Z_1}$$

$$= \frac{230\angle 0^\circ}{5 + j3.14} = \frac{230\angle 0^\circ}{5.90\angle 32.12^\circ}$$

$$= 38.98\angle -32.12^\circ \text{ A}$$

Current through branch 2 is given by,

$$I_2 = \frac{V}{Z_2} = \frac{V}{X_C}$$

$$= \frac{250}{11.10}$$

$$= 22.52 \text{ A}$$

Under resonance condition, the imaginary part will be zero, hence only the real part exists i.e., only the active component. Therefore, total current taken from the supply under resonance condition is given by,

$$I = I_L \cos$$

$$= 38.98 \cos (32.12°)$$

$$= 33.01 \text{ A}$$

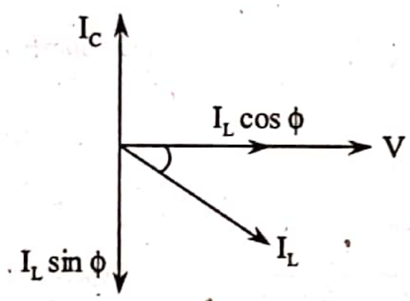


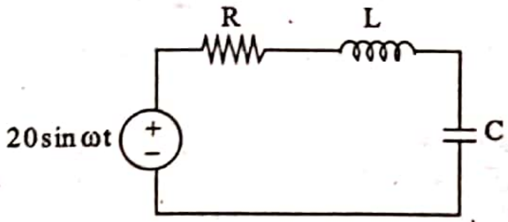
Figure (b)

Q49. In the circuit shown in below figure, R = 2 ohms, L = 1 mH and C = 0.4 μF.

(a) Find the resonant frequency and the half-power frequencies.

(b) Calculate the quality factor and bandwidth.

(c) Determine the amplitude of the currents at resonant and half-power frequencies ω_0, ω_1, and ω_2.



Figure

Ans: **Model Paper-III, Q4(b)**

Given that,

$R = 2\ \Omega$

$L = 1$ mH

$C = 0.4\ \mu$F

The given circuit diagram is shown in figure (1),

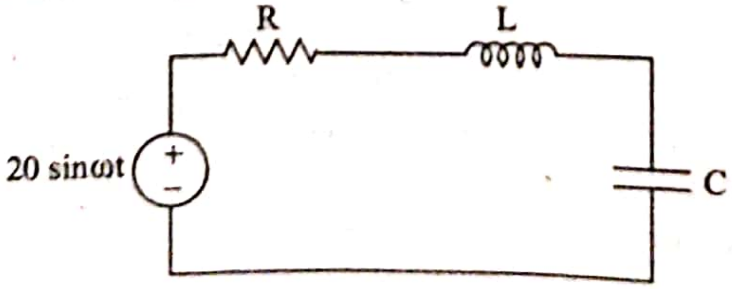


Figure (1)

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To determine,

(a) (i) Resonant frequency, $\omega_0 = ?$

(ii) Half-power frequencies, $\omega_1 = ?$ and $\omega_2 = ?$

(b) (i) Quality factor, $Q = ?$

(ii) Bandwidth, $B = ?$

(c) Amplitude of current at,

$$\omega = \omega_0, \omega_1, \omega_2 = ?$$

(a)

(i) Resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{1\times10^{-3}\times0.4\times10^{-6}}}$$

$$= \frac{1}{2\times10^{-5}}$$

$$= 50 \text{ k rad/sec}$$

(ii) Half-power frequencies

We know that,

The lower half-power frequency is given as,

$$\omega_1 = -\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}$$

$$= -\frac{2}{2\times(1\times10^{-3})}+\sqrt{\left[\frac{2}{2\times(1\times10^{-3})}\right]^2+\left[\frac{1}{(1\times10^{-3})\times(0.4\times10^{-6})}\right]}$$

$$= -1000+\sqrt{(1000)^2+25\times10^8}$$

$$= 49009.999 \text{ rad/sec}$$

$$= 49.009 \text{ k rad/sec}$$

And also,

The upper half-power frequency is given as,

$$\omega_2 = \frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}$$

$$= \frac{2}{2\times(1\times10^{-3})}+\sqrt{\left[\frac{2}{2\times(1\times10^{-3})}\right]^2+\left[\frac{1}{(1\times10^{-3})\times(0.4\times10^{-6})}\right]}$$

$$= 1000+\sqrt{(1000)^2+25\times10^8}$$

$$= 51009.999 \text{ rad/sec}$$

$$= 51.009 \text{ k rad/sec}$$

(b)

(i) Quality factor, $Q = \frac{\omega_0 L}{R}$

$$= \frac{50\times10^3\times(1\times10^{-3})}{2}$$

$$= \frac{50\times10^3\times10^{-3}}{2} = \frac{50}{2} = 25$$

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(ii) Bandwidth, $B = \frac{R}{L} = \frac{2}{(1\times10^{-3})}$

$= 2 \times 10^3$ rad/sec

$= 2$ k rad/sec

(c) Amplitude of current

At resonating frequency, the amplitude of current is given as,

$$I = \frac{V_m}{R}$$

$$= \frac{20}{2} \quad [\because V = V_m \sin \omega t = 20 \sin \omega t]$$

$$[\therefore V_m = 20]$$

$$\boxed{I = 10 \text{ A}}$$

At half-power frequencies (i.e., $\omega = \omega_1 = \omega_2$), the amplitude of current is given as,

$$I = \frac{V_m}{\sqrt{2}R} = \frac{20}{\sqrt{2}\times2} = \frac{10}{\sqrt{2}}$$

$$\boxed{I = 7.07106 \text{ A}}$$

Result

(a) (i) Resonant frequency, $\omega_0 = 50$ k rad/sec

(ii) Half-power frequencies, $\omega_1 = 49.009$ k rad/sec

$\omega_2 = 51.009$ k rad/sec

(b) (i) Quality factor, $Q = 25$

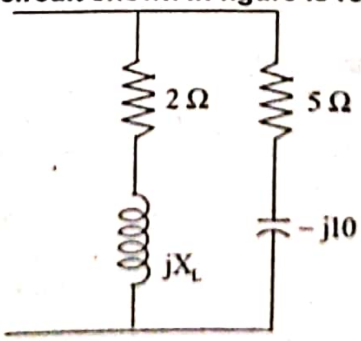
(ii) Bandwidth, $B = 2$ k rad/sec

(c) Amplitude of current,

At $\omega = \omega_0$, $I = 10$ A

At $\omega = \omega_1, \omega_2$, $I = 7.07106$ A

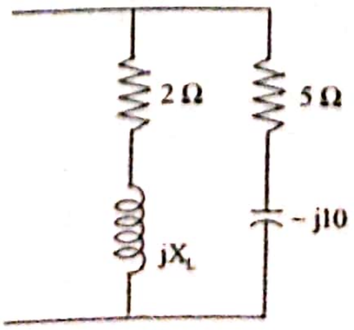
Q50. Find the value of 'L' for which the circuit shown in figure is resonant at a frequency of $\omega = 500$ rad/s.



Figure

Ans:

The given circuit is,



Figure

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To determine,

The value of inductance (L) at resonant frequency, $\omega = 500$ rad/sec. From figure, the total admittance of the circuit is given as,

$$Y_T = Y_L + Y_C$$

$$\Rightarrow \quad Y_T = \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$\Rightarrow \quad Y_T = \frac{1}{2+jX_L} + \frac{1}{5-j10}$$

$$\Rightarrow \quad Y_T = \frac{1}{2+jX_L} \times \frac{(2-jX_L)}{(2-jX_L)} + \frac{1}{5-j10} \times \frac{(5+j10)}{(5+j10)}$$

$$\Rightarrow \quad Y_T = \frac{(2-jX_L)}{2^2-(jX_L)^2} + \frac{(5+j10)}{5^2-(j10)^2}$$

$$\Rightarrow \quad Y_T = \frac{2-jX_L}{4+X_L^2} + \frac{5+j10}{25+100}$$

$$\Rightarrow \quad Y_T = \frac{2}{4+X_L^2} - j\frac{X_L}{4+X_L^2} + \frac{5}{125} + j\frac{10}{125}$$

$$Y_T = \frac{2}{4+X_L^2} - j\frac{X_L}{4+X_L^2} + \frac{1}{25} + j\frac{2}{25}$$

$$\therefore \quad Y_T = \left[\frac{2}{4+X_L^2} + \frac{1}{25}\right] + j\left[\frac{2}{25} - \frac{X_L}{4+X_L^2}\right] \quad \text{... (1)}$$

Now, under resonance condition, the imaginary term in equation (1) becomes zero.

$$\Rightarrow \quad \frac{2}{25} - \frac{X_L}{4+X_L^2} = 0$$

$$\frac{2}{25} = \frac{X_L}{4+X_L^2}$$

$$8 + 2X_L^2 = 25X_L$$

$$\Rightarrow 2X_L^2 - 25X_L + 8 = 0$$

On solving the above quadratic equation, we get,

$$X_L = 12.17\ \Omega \text{ and } X_L = 0.33\ \Omega$$

But,

$$X_L = \omega_0 L$$

$$\Rightarrow \quad L = \frac{X_L}{\omega_0}$$

Where,

$X_L = 12.17\ \Omega$, then,

$$L = \frac{12.17}{500}$$

$$= 0.024 \text{ H}$$

$$= 24.34 \text{ mH}$$

If $X_L = 0.33\ \Omega$

$$L = \frac{0.33}{500}$$

$$= 0.00066 \text{ H}$$

$$= 0.66 \text{ mH}$$

$$\therefore \quad L = 24.34 \text{ mH or } 0.66 \text{ mH}$$

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