10.4 Input Impedance of Open and Short Circuited Lines

Consider a transmission line of length l. Let voltage V_s and current I_s be applied at the source.

Also, let V_R be the voltage at termination, I_R be the current at termination and γ be the propagation constant.

The wave equations on the line are given by Eq. (9.31) and Eq. (9.32), reproduced below.

$$V = V_s \cosh \gamma x - I_s Z_0 \sinh \gamma x, \tag{10.5}$$

$$I = I_s \cosh \gamma x - \frac{V_s}{Z_0} \sinh \gamma x. \tag{10.6}$$

At $x = l$,

$$V_R = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l, \tag{10.7}$$

$$I_R = I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l. \tag{10.8}$$

Open-circuited line When the load is open, the termination current becomes zero.

That is, $I_R = 0$.

From Eq. (10.8),

$$I_S \cosh \gamma l - \frac{V_S}{Z_0} \sinh \gamma l = 0.$$

or $$\frac{V_S}{I_S} = Z_0 \coth \gamma l.$$

Let Z_{oc} be the input impedance of the open line.

Then, $$Z_{oc} = \frac{V_S}{I_S} = Z_0 \coth \gamma l. \tag{10.9}$$

Short-circuited line When the load is shorted, the termination voltage becomes zero.

That is, $V_R = 0$.

From Eq. (10.7),

$$V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l = 0.$$

or $$\frac{V_s}{I_s} = Z_0 \tanh \gamma l.$$

Let Z_{sc} be the input impedance of the short circuited line.

Then, $$Z_{sc} = \frac{V_s}{I_s} = Z_0 \tanh \gamma l. \tag{10.10}$$

Z_0 and γ in terms of Z_{sc} and Z_{oc}:

From Eq. (10.9) and Eq. (10.10),

$$Z_{oc}\, Z_{sc} = Z_0^2\ (\coth \gamma l)(\tanh \gamma l\ ).$$

$$Z_0 = \sqrt{Z_{oc}\, Z_{sc}} \tag{10.11}$$

and $$\frac{Z_{sc}}{Z_{oc}} = \frac{Z_0 \tanh \gamma l}{Z_0 \coth \gamma l} = \tanh^2 \gamma l$$

$$\tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

or $$\gamma = \frac{1}{l} \tanh^{-1}\left(\sqrt{\frac{Z_{sc}}{Z_{oc}}}\right). \tag{10.12}$$

Thus by measuring the short-circuit and open-circuit impedances at the termination of a transmission line, we can calculate the characteristic impedance and propagation constant of the line.

Again, from Z_0 and γ, we can compute the primary constants R, G, C and L.

10.17 Smith Chart

Phillip H. Smith in the year 1939 developed a polar chart for calculating transmission line characteristics. This chart is called Smith chart. It consists of two sets of orthogonal circles which represent the values of normalised impedance. One set of circles represents the resistive component R, called R circles, and the other set of circles represent the reactive component X, called X circles.

10.18 Properties of the Smith Chart

Normalisation of impedance The Smith chart represents the normalised values of R and X circles. If $Z_R = R_R + jX_R$ is the load impedance and $Z_0 = R_0$ is the characteristic impedance of a lossless line, then the R circles give $\frac{R_R}{Z_0}$ values and the X circles give $\frac{X_R}{Z_0}$ values.

Thus, to obtain actual values, the values from the chart should be multiplied by Z_0.

Load impedance plot The intersection points of R and X circles give normalised load impedance (z_R) values. If X is positive, the point is above the $K_R = 0$ axis and if X is negative, the point is below $K_R = 0$ axis.

VSWR plot The VSWR values can be obtained by drawing S-circles on the chart, as shown in Fig. 10.21. The circles with a centre at the origin (0, 0) with radius $|z_R|$ are called S circles.

The radius of the S circle $|z_R|$ gives VSWR value $S = \left|\frac{Z_R}{Z_0}\right|$.

If M is the intersection point of an S circle with the horizontal axis AB, the normalised resistance at M is equal to the value of VSWR. Therefore, $S = \text{OM}$.

Reflection coefficient values Draw the line OP and extend it to the outer circle, It cuts the outer circle at P'. The angles are indicated on the outer circle. The angle (θ) of the line OP gives the angle of the reflection coefficient. The value (K_R, K_X) at the point P gives the magnitude of K. $|K|$ can be obtained from the K scale provided in the chart. The length of OP on the K scale gives the magnitude of K. Also $|K|$ can be calculated directly as $|K| = \dfrac{\mathrm{OP}}{\mathrm{OP}'}$.

Location of voltage maximum and minimum: There are two intersection points of the S circle with the horizontal axis AB. The point at the left side of the centre represents voltage minima $(V_{\min})$ and the point at the right side of the centre represents voltage maxima $(V_{\max})$. Thus the points M and L respectively gives the positions voltage maximum and voltage minimum.

The location of the first $V_{\max}$ can be obtained from the wavelength scale on the outer circle. The arc AP' gives the distance of $V_{\min}$ from the load. Similarly, the arc $P'AB$ gives the distance of the first $V_{\max}$ from the load.

Open and short-circuited line: At point B on the right side end of the horizontal axis, both R and X are infinite which represents an open circuit termination of the line. Similarly, at point A on the left side end of the horizontal axis, both R and X are zero which represent short circuit termination of the line.

Movement along the periphery of the chart: On the outer circle or periphery of the chart, moving in the clockwise direction corresponds to travelling from the load towards the generator. Similarly, moving in the anti-clockwise direction corresponds to travelling from the generator towards the load. The full rotation around the chart gives a distance of $\lambda/2$. If the line length is greater than $\lambda/2$, rotate around the circle n times to reach the line length.

Matched load: The circle $R = 1$ represents $z_R = \dfrac{\Re e[Z_R]}{Z_0} = 1$. It passes through the point $(1, 0)$. The resistive part of the load impedance is equal to the characteristic impedance of the line. This circle represents impedance only when the reactive component varies on the line. A stub can be used at this location to nullify the reactive component.

Therefore, the centre point of the chart is known as the matched load point.

Applications of a $\lambda/4$ line transformer

(1) To match the impedance between a transmission line and an antenna.
For example, if a line with characteristic impedance Z_0 is connected to a mismatched load Z_R, then for proper matching, a quarter wave transformer is inserted in between the line and load.
The characteristic impedance of the quarter wave transformer should be

$$Z_0 = \sqrt{Z_0 Z_R}. \tag{10.82}$$

(2) To step up or step down the characteristic impedance Z_0 of a transmission line.
If the load of a line is not resistive, the impedance of the line at node points is either SZ_0 or $\frac{Z_0}{S}$, irrespective of the load impedance, where S is the VSWR.

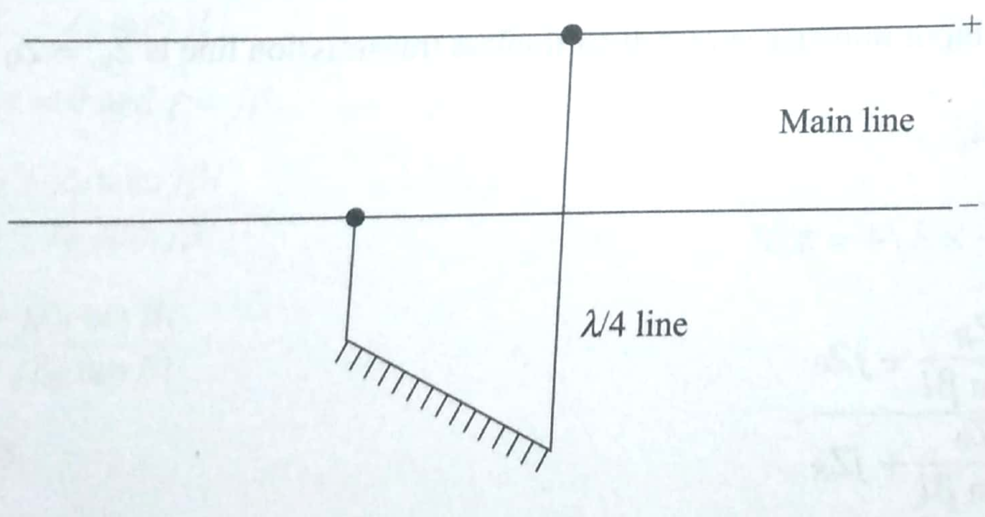
$\therefore$ The characteristic impedance of the $\lambda/4$ line is

$$Z_0' = \sqrt{Z_0 Z_0 S} = Z_0\sqrt{S}$$

or
$$Z_0' = \sqrt{Z_0 \frac{Z_0}{S}} = \frac{Z_0}{\sqrt{S}}. \tag{10.83}$$

Since $S > 1$, the impedance may be either step up or step down.

(3) It can provide a mechanical support to the transmission line in addition to the impedance. For example, the line connected between the transmission line and the ground acts as an insulator at the point of contact as shown in Fig. 10.11.



10.11 *Quarter wave transformer as insulator*

10.8 Standing Wave Ratio

When the lossless line is not terminated with characteristic impedance, the combination of incident and reflected waves gives rise to standing waves as shown in Fig. 10.7.

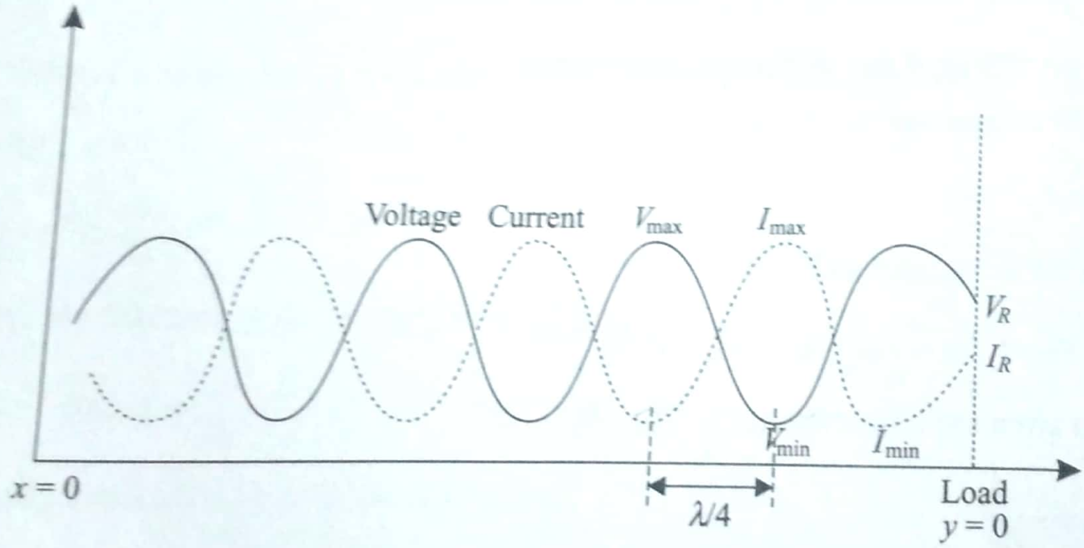


Fig. 10.7 *Standing wave pattern along a lossless high frequency transmission line* ($Z_R \neq Z_o$)

Let V_{max} be the maximum voltage, V_{min} be the minimum voltage, I_{max} be the maximum current and I_{min} be the minimum current.

The distance between two maximum or minimum points is $\lambda/2$. The maximum values occur when the incident and reflected waves are added.

i.e., $$|V_{\max}| = |V_r| + |V_i| \tag{10.48}$$

and $$|I_{\max}| = |I_r| + |I_i|. \tag{10.49}$$

The minimum values occur when the incident and reflected waves are subtracted.

i.e., $$|V_{\min}| = |V_i| - |V_r| \tag{10.50}$$

and $$|I_{\min}| = |I_i| - |I_r|. \tag{10.51}$$

Voltage standing wave ratio

The ratio of the maximum magnitude of the voltage to the minimum magnitude of the voltage is called voltage standing wave ratio. It is abbreviated as VSWR and denoted by S.

Thus $$\text{VSWR} = S = \frac{|V_{\max}|}{|V_{\min}|}. \tag{10.52}$$

Current standing wave ratio

The ratio of the maximum magnitude of the current to the minimum magnitude of the current is called current standing wave ratio.

It is abbreviated as ISWR.

$$\therefore \quad \text{ISWR} = \frac{|I_{\max}|}{|I_{\min}|}. \tag{10.53}$$

VSWR in terms of reflection coefficient (K):

We know that

$$S = \frac{|V_{\max}|}{|V_{\min}|}.$$

$$S = \frac{|V_r| + |V_i|}{|V_r| - |V_i|} = \frac{1 + \left|\dfrac{V_r}{V_i}\right|}{1 - \left|\dfrac{V_r}{V_i}\right|}.$$

But we know that the reflection coefficient

$$K = \frac{V_r}{V_i} \quad \text{or} \quad |K| = \left|\frac{V_r}{V_i}\right|. \tag{10.54}$$

$$\therefore \quad S = \frac{1 + |K|}{1 - |K|} \tag{10.55}$$

or $$|K| = \frac{S-1}{S+1}.$$

VSWR is a real quantity. It is always greater than 1.

Reflection coefficient

Reflection coefficient is defined as *the ratio of reflected voltage to the incident voltage or reflected current to the incident current.*

It is a vector quantity and designated by letter K.

$$\therefore \quad K = \frac{V_r}{V_i} \tag{10.33}$$

$$\text{or} \quad K = \frac{-I_r}{I_i}, \tag{10.34}$$

where V_r and V_i are the incident and reflected voltages respectively and I_r and an I_i are the incident and reflected currents respectively. The negative sign indicates that I_r is in an opposite direction to I_i.

10.15 Single Stub Matching

In this method, to achieve impedance matching, an open or short circuited short length transmission line is connected in parallel to the main line at a certain distance from the load. Since the stub is connected in parallel, it is easy to use admittance instead of impedance for analysis.

When the load admittance, $Y_R = 1/Z_R$ is connected to the line and if it is not equal to the characteristic admittance ($Y_0 = 1/Z_0 \neq Y_R$), a mismatch occurs. Standing waves exist on the line. When we move from the load towards the source, the admittance on the line varies from the max/min value to the min/max value depending on the line length. The admittance variation repeats after every $\lambda/2$ length. At some point on the line, the real part of the admittance is equal to the characteristic admittance, i.e., $\Re e[Y] = \Re e[Y_0]$.

If a short or open circuited stub having the same characteristic admittance (Y_0) is added in shunt at this point and the length of the stub is chosen such that the stub susceptance is matched to the line susceptance, then the admittance of the line at this point is matched to the admittance of the load and stub combination.

To avoid losses, the stub should be connected as near the load as possible.

A short circuited stub is generally preferred to an open circuited stub because of the following reasons.

1) It provides strong construction and support to the main line.
2) The short circuited stub can be easily established with a large metal plate.
3) Radiation loss is very less compared to an open circuited stub.

(10.111)

Disadvantages of single matching

1 Since the location and length of the single stub matching depend on frequency, if the frequency of the wave changes, the location and length of the stub should be changed. However, it is very difficult to change the stub once it is fixed.

2 In practical cases, the location of the stub has to be moved along the line for final adjustment. This tuning is possible only on open wire lines. However, it is very difficult to place a stub on coaxial cables.

10.16 Double Stub Matching

To overcome the disadvantages of single stub matching, two stubs can be used at different locations. This is called double stub matching.

Consider a double stub matching system consisting of two short circuited stubs connected in parallel to the line near the load as shown in Fig. 10.17.

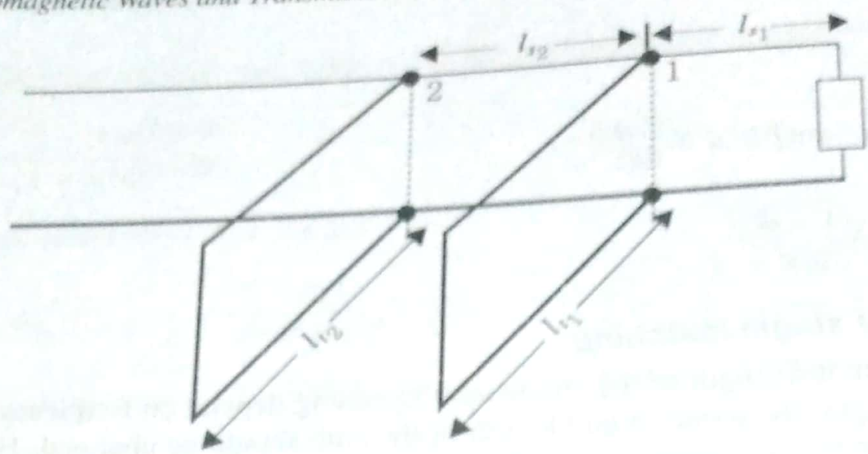


Fig. 10.17 *Double stub matching*

Let l_{s_1} = location of stub A from load, l_{s_2} = location of stub B from load,
l_{t_1} = length of stub A, l_{t_2} = length of stub B, l_s = separation between stubs and
Y_R = load admittance.

The characteristic admittance of the stubs should be equal to the line characteristic admittance Y_0.

In this system, the locations of the stubs have to be chosen arbitrarily. But in practice, it is very difficult to design stub matching with arbitrary locations. Due to some problems encountered in the design, the locations of stubs should be restricted.

Since the admittance repeats at every $\lambda/2$, the total distance $l_{s_1} + l_{s_2}$ can never be more than or equal to $\lambda/2$. In practice, l_{s_1} is taken in between $0.1\ \lambda$ and $0.15\ \lambda$. Sometimes, $l_{s_1} = 0$ (at load) can be chosen. The space between two stubs is generally taken as $\lambda/8$, $\lambda/4$ or $3\lambda/8$ distance. The total distance of the double stab matching should be kept as small as possible.

Therefore, in the design of a double stub matching, it is better to keep the locations of the stubs fixed. Impedance matching is done by finding the lengths of the stubs. When the signal frequency changes, the stub lengths can be adjusted to achieve impedance matching.

10.19 Applications of the Smith Chart

The following are various applications of a Smith chart.

Smith chart as an admittance diagram: Generally, Smith chart is used as an impedance diagram. The impedance Z can be obtained from the intersection of R and X circles. We can also draw the Smith chart for admittance.

Normalised admittance is given by $Y = G + jB$, where $G = \frac{1}{R}$ normalised conductance, and $B = \frac{1}{X}$ normalised susceptance.

$\therefore$ At termination, $y_R = \frac{1}{z_R} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2}$.

So, $G - jB = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2}$,

also, $y_R = \frac{1}{z_R} = \frac{1 - K}{1 + K}$.

$$G - jB = \frac{1 - K}{1 + K}.$$

The G and B circles on the K plane can be drawn similar to R and X circles.

A Smith chart with G and B circles is called an admittance diagram. The admittance diagram is the mirror image of the impedance diagram; all measurements will be taken in the reverse direction.

Converting impedance into admittance: We know that for a lossless quarter wave transformer, if Z_R is the characteristic impedance and Z_0 is the termination impedance, then the input impedance is

$$Z_{\text{in}} = \frac{Z_0^2}{Z_R}.$$

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$$\frac{Z_{in}}{Z_0} \times \frac{Z_R}{Z_0} = 1 \text{ or } \quad z_{in} \times z_R = 1.$$

$$y_R = z_{in}.$$

Thus, in order to find the admittance on the chart, we first locate the impedance point and rotate it to a distance $\lambda/4$ (quarter wave) towards the generator. The $\lambda/4$ distance is an opposite point on the chart. Hence, the point opposite to the impedance point on the circle gives the admittance point.

Determination of an input impedance: Consider a transmission line of length l terminated with a load impedance Z_R. Denote the normalised load impedance z_R as point P on the Smith chart as shown in Fig. 10.22. With centre O and radius OP, draw the S circle. Extend the OP line to the outer circle which cuts at point P′. Rotate towards the generator (clockwise) up to a length l/λ. Denote this point as N'. Draw the line ON′ which cuts the S circle at point N. This point N represents the normalised input impedance z_{in}.

The angle NOP gives the electric length βl of the line.

Determination of the load impedance: Consider a transmission line of length l. Given the VSWR and the location of the first V_{min} point from the load. To locate the load impedance, first draw the S circle with centre O and radius VSWR. Locate point A on the outer circle at the left side end of the horizontal axis as the position of V_{min}.

Move towards the load (clockwise direction) to a given length l/λ on the wavelength scale and locate the point P. Draw the line OP which cuts the S circle at point P. The location of the point P gives the normalised load impedance.

Input impedance and admittance of an SC (short-circuited) line We know that the input impedance of an SC line is purely reactive and $R = 0$. The short circuit termination represents the position of V_{min}, i.e., a point A on the outer circle at left side end of the horizontal axis (i.e., $R = 0$ on the R circle). From point A, move towards the generator (anti-clockwise direction) to a given length l/λ on the wavelength scale and locate the point P. The location of point P gives normalised input impedance of the SC line. The opposite point Q gives the normalised input admittance.

Input impedance and admittance of an OC (open-circuited) line: We know that the input impedance of an OC line is purely reactive and $R = \infty$. The open circuit termination represents the position of V_{max}, i.e., a point B on the outer circle at a right side end of the horizontal axis (i.e., $R = \infty$ on R-circle). From point B, move towards the generator to a given length l/λ on the wavelength scale and locate the point P. The location of the point P gives normalised input impedance of the OC line. The opposite point Q gives the normalised input admittance.

Determination of locations and lengths of stubs by Smith chart: Design of impedance matching can be easily done by using Smith charts. The locations and lengths of single and double stub matching can be obtained by locating the admittances on the chart. Since the stubs are connected in parallel, it is much easier to combine admittance in parallel than impedances. Also, short-circuited stubs are preferred over open-circuited stubs to avoid radiation losses.