Now, comparing equation (14) with equation (1), we get,

$$h_{11} = \frac{B}{D}$$

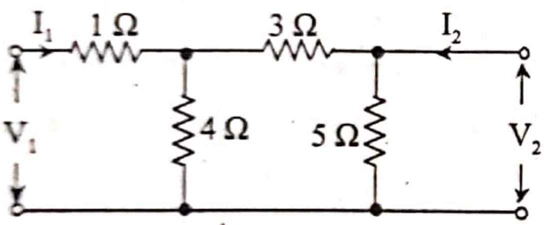
$$h_{12} = \frac{AD - BC}{D}$$

Similarly, comparing equation (13) with equation (2), we get,

$$h_{21} = \frac{-1}{D}$$

$$h_{22} = \frac{C}{D}$$

Q50. Determine the Z-parameter of the network shown in figure.



Figure

Ans:

Given circuit shown in figure (1).

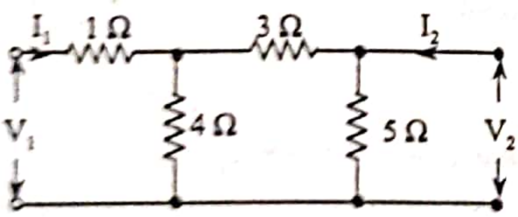


Figure (1)

Now, converting the delta connection of 3 Ω, 4 Ω and 5 Ω into star connection as shown in figure (2).

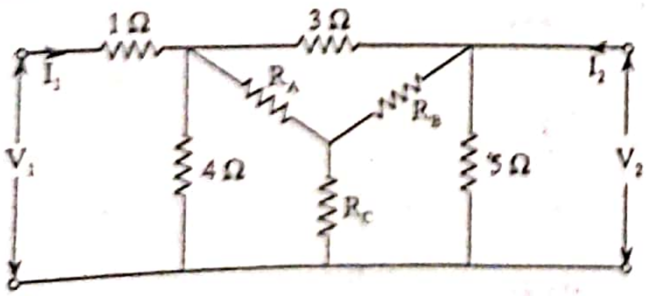


Figure (2)

$$R_A = \frac{3 \times 4}{3+4+5} = 1\ \Omega$$

$$R_B = \frac{3 \times 5}{3+4+5} = 1.25\ \Omega$$

$$R_C = \frac{4 \times 5}{3+4+5} = 1.67\ \Omega$$

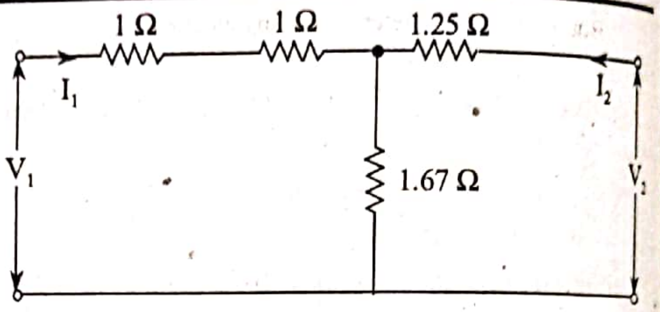


Figure (3)

Two resistors are of 1 Ω each connected in series. Therefore, the circuit in figure (3) gets modified as,

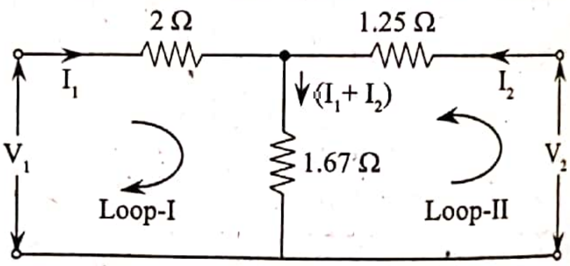


Figure (4)

Now, applying KVL in loop-I, we get,

$$V_1 = 2I_1 + 1.67\,(I_1 + I_2)$$

$$\Rightarrow\ V_1 = 3.67\,I_1 + 1.67\,I_2 \qquad \text{... (1)}$$

Similarly, applying KVL in loop-II, we get,

$$V_2 = 1.67\,(I_1 + I_2) + 1.25\,I_2$$

$$\Rightarrow\ V_2 = 1.67\,I_1 + 2.92\,I_2 \qquad \text{... (2)}$$

Open-circuiting terminal-2 i.e., making, $I_2 = 0$.

From equations (1) and (2), we have,

$$V_1 = 3.67\,I_1$$

$$\Rightarrow\ \frac{V_1}{I_1} = 3.67\ \Omega \text{ and}$$

$$V_2 = 1.67\,I_1$$

$$\Rightarrow\ \frac{V_2}{I_1} = 1.67\ \Omega$$

Now, open-circuiting terminal-1 i.e., making, $I_1 = 0$

From equations (1) and (2), we have,

$$V_1 = 1.67\,I_2$$

$$\Rightarrow\ \frac{V_1}{I_2} = 1.67\ \Omega \text{ and}$$

$$V_2 = 2.92\,I_2$$

$$\Rightarrow\ \frac{V_2}{I_2} = 2.92\ \Omega$$

But, from Z-parameter equations, we have,

$$Z_{11} = \frac{V_1}{I_1} \Rightarrow Z_{11} = 3.67\ \Omega$$

$$Z_{12} = \frac{V_1}{I_2} \Rightarrow Z_{12} = 1.67\ \Omega$$

$$Z_{21} = \frac{V_2}{I_1} \Rightarrow Z_{21} = 1.67\ \Omega \text{ and}$$

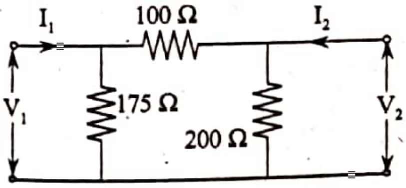
$$Z_{22} = \frac{V_2}{I_2} \Rightarrow Z_{22} = 2.92\ \Omega$$

The Z-parameters can also be written in matrix form as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3.67 & 1.67 \\ 1.67 & 2.92 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

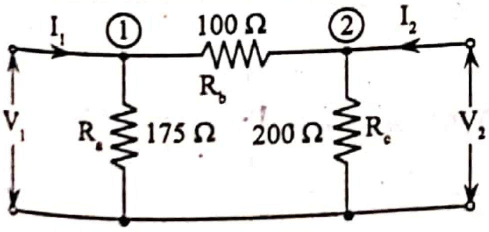
Q51. Find the Y-parameters of the network shown in figure.



Figure

Ans:

The given network shown in figure below,



Figure

Let, the resistances of 175 Ω, 100 Ω and 200 Ω be R_a, R_b and R_c respectively.

Applying KCL at node (1), we get,

$$\Rightarrow \quad -I_1 + \frac{V_1}{R_a} + \frac{V_1 - V_2}{R_b} = 0$$

$$\Rightarrow \quad I_1 = \left(\frac{1}{R_a} + \frac{1}{R_b}\right)V_1 + \left(-\frac{1}{R_b}\right)V_2 \quad \dots (1)$$

Applying KCL at node (2), we get,

$$\Rightarrow \quad -I_2 + \frac{V_2}{R_c} + \frac{V_2 - V_1}{R_b} = 0$$

$$\Rightarrow \quad I_2 = \left(\frac{-1}{R_b}\right)V_1 + \left(\frac{1}{R_b} + \frac{1}{R_c}\right)V_2 \quad \dots (2)$$

In a two port network, the input and the output currents, I_1 and I_2 can be expressed in terms of input and output voltages, V_1 and V_2 respectively as,

$$[I] = [Y]\,[V]$$

$$\Rightarrow \quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow \quad I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{and} \quad \dots (3)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \dots (4)$$

On comparing equations (1) and (3), we get,

$$Y_{11} = \frac{1}{R_a} + \frac{1}{R_b} = \frac{1}{175} + \frac{1}{100}$$

$$= 15.714 \times 10^{-3}\ \mho \text{ and}$$

$$Y_{12} = \frac{-1}{R_b} = \frac{-1}{100} = -10 \times 10^{-3}\ \mho$$

Similarly, on comparing equations (2) and (4), we get,

$$Y_{21} = \frac{-1}{R_b} = \frac{-1}{100} = -10 \times 10^{-3}\ \mho$$

$$\text{And } Y_{22} = \frac{1}{R_b} + \frac{1}{R_c} = \frac{1}{100} + \frac{1}{200}$$

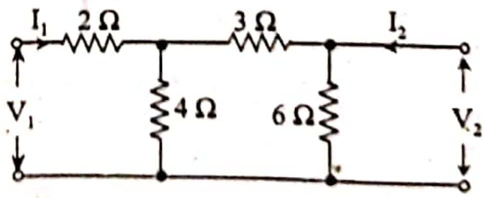
$$= 15 \times 10^{-3}\ \mho$$

∴ Admittance matrix,

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$= \begin{bmatrix} (15.714 \times 10^{-3}) & (-10 \times 10^{-3}) \\ (-10 \times 10^{-3}) & (15 \times 10^{-3}) \end{bmatrix}$$

Q52. Determine the ABCD parameters of the network shown in figure.



Figure

Ans:

The given network shown in figure (1).

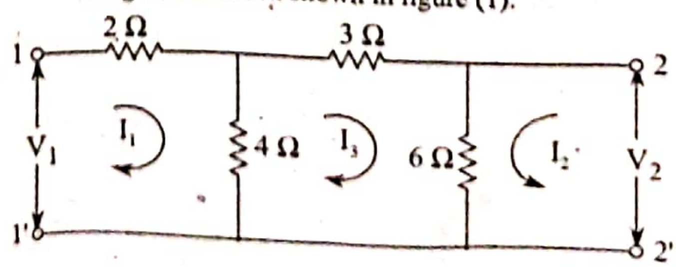


Figure (1)

The characteristic equation of $ABCD$ parameters are given by,

$$V_1 = AV_2 + B(-I_2) \quad ...(1)$$

$$I_1 = CV_2 + D(-I_2) \quad ...(2)$$

In matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Now, open-circuiting the terminals 2-2', which makes current $I_2 = 0$. The circuit then changes to figure (2),

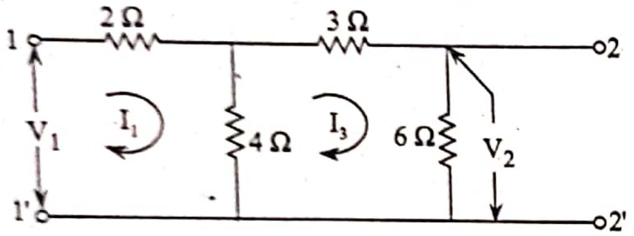


Figure (2)

Applying KVL in loop-1, we get,

$$V_1 = 2I_1 + 4(I_1 - I_3)$$

$$\Rightarrow \quad V_1 = 6I_1 - 4I_3 \quad ...(3)$$

Similarly, applying KVL in loop-2, we get,

$$0 = 3I_3 + 6I_3 + 4(I_3 - I_1)$$

$$\Rightarrow \quad 4I_1 = 13I_3 \Rightarrow I_3 = \frac{4}{13} I_1 \quad ...(4)$$

Substituting equation (4) in equation (3), we get,

$$V_1 = 6I_1 - 4\left(\frac{4}{13}\right)I_1$$

$$\Rightarrow \quad V_1 = 6I_1 - \frac{16}{13}I_1$$

$$\Rightarrow \quad V_1 = \frac{62}{13}I_1 \quad ...(5)$$

And, $V_2 = 6I_3$

$$\Rightarrow \quad V_2 = 6 \times \frac{4}{13}I_1$$

$$\Rightarrow \quad V_2 = \frac{24}{13}I_1 \quad ...(6)$$

We know that,

$$A = \frac{V_1}{V_2} = \frac{62}{13}I_1 \times \frac{13}{24I_1}$$

[$\because$ From equations (5) and (6)]

$$\Rightarrow \quad A = \frac{31}{12}$$

And, $C = \dfrac{I_1}{V_2} = \dfrac{13}{24}$ ℧ [$\because$ From equation (6)]

Now, short-circuiting the terminals 2-2', which makes voltage $V_2 = 0$. The circuit, gets modified to figure (3).

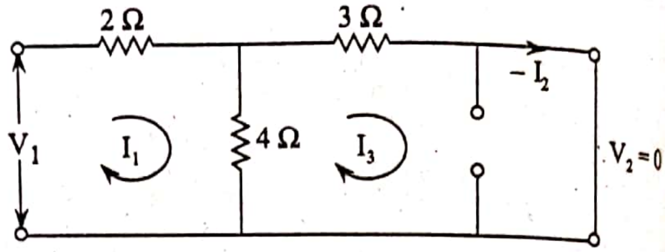


Figure (3)

From figure (3),

$$I_3 = -I_2$$

Applying KVL in loop-1, we get,

$$V_1 = 2I_1 + 4(I_1 - I_3)$$

$$\Rightarrow \quad V_1 = 6I_1 + 4I_2 \quad [\because I_3 = -I_2] \quad ...(7)$$

Similarly, applying KVL in loop-2, we get,

$$3(I_3) + 4(I_3 - I_1) = 0$$

$$7I_3 = 4I_1$$

$$7(-I_2) = 4I_1 \quad [\because I_3 = -I_2]$$

$$\frac{-I_1}{I_2} = \frac{7}{4} = D$$

$$D = \frac{7}{4}$$

From equation (7), we have,

$$V_1 = 6I_1 + 4I_2$$

$$\Rightarrow \quad V_1 = 6\left(\frac{-7}{4}I_2\right) + 4I_2 \quad \left[\because I_1 = \frac{-7}{4}I_2\right]$$

$$\Rightarrow \quad V_1 = \frac{-42}{4}I_2 + 4I_2$$

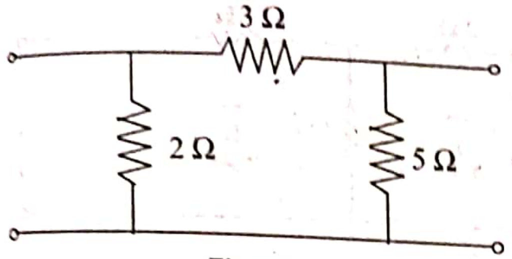
$$\Rightarrow \quad V_1 = \frac{-13}{2}I_2 \Rightarrow \frac{V_1}{I_2} = \frac{-13}{2}$$

$$B = \frac{-V_1}{I_2} = \frac{13}{2}\ \Omega$$

$\therefore$ The $ABCD$ parameters are,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 31/12 & 13/2 \\ 13/34 & 7/4 \end{bmatrix}$$

Q53. Determine the h parameters for the circuit shown in figure below.



Figure

Ans: **Model Paper-I, Q6(b)**

The given circuit is shown in figure (1)

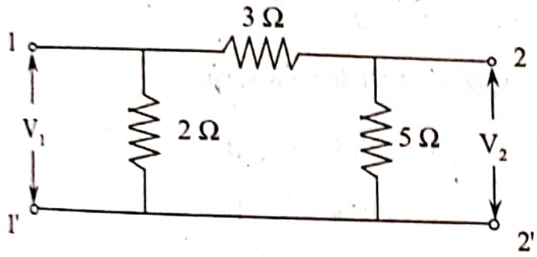


Figure (1)

The hybrid or h -parameters are expressed in terms of V_1 and I_2 as,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{... (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{... (2)}$$

The individual h-parameters are given as, by short circuiting the port 2 – 2', V_2 becomes zero.

$$\therefore \; h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

By open circuiting the port 1–1', I_1 becomes zero.

$$\therefore \; h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Now, short circuiting the port 2 – 2' for determining the parameters h_{11} and h_{21}. The voltage V_2 becomes zero as shown in figure (2).

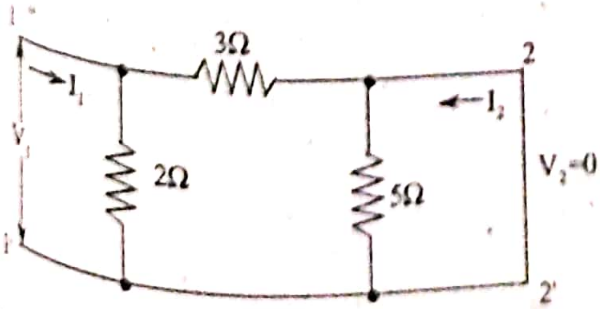


Figure (2)

The modified circuit can be drawn as shown in figure (3).

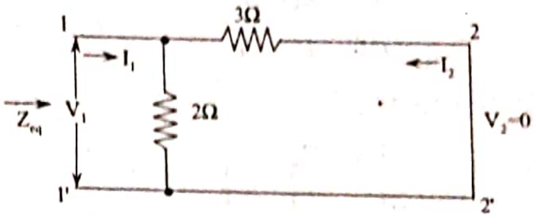


Figure (3)

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$V_1 = I_1 Z_{eq}$$

From figure (3), the equivalent impedance is given as,

$$Z_{eq} = \frac{2\times 3}{2+3} = \frac{6}{5}\,\Omega$$

$$\therefore \; V_1 = I_1\left(\frac{6}{5}\right)$$

$$\frac{V_1}{I_1} = \frac{6}{5}\,\Omega$$

$$\therefore \quad h_{11} = \frac{V_1}{I_1} = \frac{6}{5}\,\Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$-I_2 = I_1 \times \left(\frac{2}{2+3}\right)$$

$$= I_1\left(\frac{2}{5}\right)$$

$$\therefore \; \frac{I_2}{I_1} = \frac{-2}{3}$$

$$\therefore \; h_{21} = \frac{-2}{3}$$

Now, making the port 1–1' open for determining the parameters h_{12} and h_{22}. The current I_1 becomes zero as shown in figure (4).

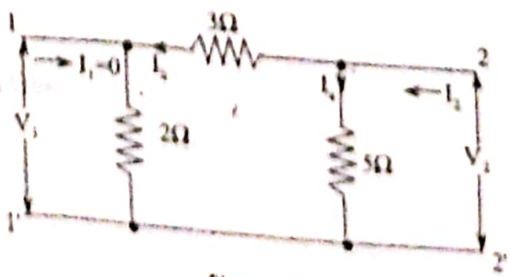


Figure (4)

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$V_1 = I_b\,(2) \qquad \text{... (3)}$$

$$V_2 = I_2\,Z_{eq} \qquad \text{... (4)}$$

$$Z_{eq} = (2+3) \parallel 5$$

$$= \frac{5\times5}{5+5} = \frac{25}{10} = \frac{5}{2}$$

$$\therefore\; V_2 = I_2\left(\frac{5}{2}\right) \qquad \text{... (5)}$$

$$I_a = I_2\left(\frac{5}{5+5}\right)$$

$$I_a = I_2\left(\frac{1}{2}\right) \qquad \text{... (6)}$$

$$I_b = I_2 - I_a$$

$$\Rightarrow I_b = I_2 - I_2\left(\frac{1}{2}\right)$$

$$\Rightarrow I_b = I_2\left(1-\frac{1}{2}\right)$$

$$\therefore\; I_b = I_2\left(\frac{1}{2}\right) \qquad \text{... (7)}$$

Substituting equation (7) in equation (3), we get,

$$\therefore\; V_1 = \frac{I_2}{2}\times 2 = I_2 \qquad \text{... (8)}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$= \frac{I_2}{I_2\left(\frac{5}{2}\right)}$$

$$= \frac{2}{5}$$

$$h_{12} = \frac{2}{5}$$

From equation (5), we get,

$$\frac{I_2}{V_2} = \frac{2}{5}$$

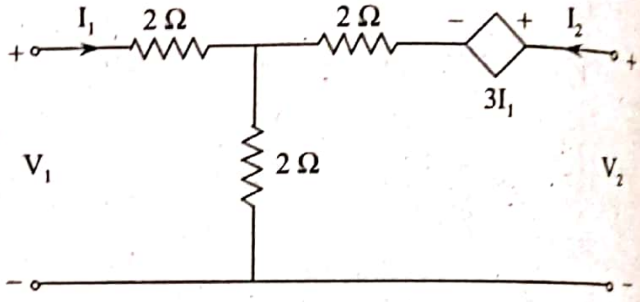
$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{2}{5}$$

$$\therefore\; h_{22} = \frac{2}{5}$$

$\therefore$ The h-parameters are expressed as,

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{2}{5} \\ \frac{-2}{5} & \frac{2}{5} \end{bmatrix}$$

Q54. Determine the g-parameters of given network as shown in figure.



Figure

Ans:

The circuit is given as,

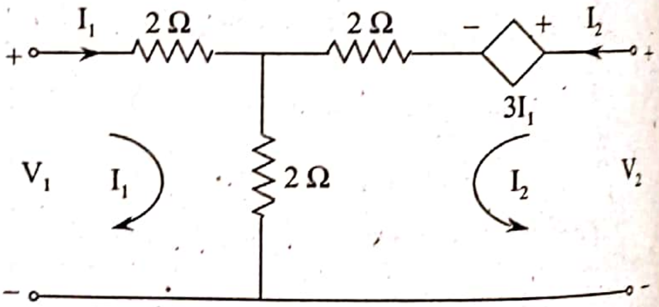


Figure (1)

To find g-parameters of the given network

Applying KVL to loop-1, we get,

$$V_1 = 2I_1 + 2(I_1 + I_2)$$

$$V_1 = 2I_1 + 2I_1 + 2I_2$$

$$V_1 = 4I_1 + 2I_2 \qquad \text{... (1)}$$

Applying KVL to loop-2, we get,

$$V_2 = 3I_1 + 2I_2 + 2(I_2 + I_1)$$

$$V_2 = 3I_1 + 2I_2 + 2I_2 + 2I_1$$

$$V_2 = 5I_1 + 4I_2 \qquad \text{... (2)}$$

To find inverse hybrid parameters lets open circuit the output terminals i.e., $I_2 = 0$.

$\therefore$ From equation (1)

$$V_1 = 4I_1 + 0$$

$$\frac{I_1}{V_1} = \frac{1}{4} \qquad \text{... (3)}$$

$$\therefore\; g_{11} = \frac{I_1}{V_1} = 4$$

And from equation (2),

$$V_2 = 5I_1 + 0$$

$$V_2 = 5I_1$$

$$V_2 = 5\left(\frac{V_1}{4}\right) \quad \left(\because I_1 = \frac{V_1}{4} \text{ From equation (3)}\right)$$

$$\frac{V_2}{V_1} = \frac{5}{4}$$

$$\therefore g_{21} = \frac{V_2}{V_1} = \frac{5}{4}$$

Now, let short-circuit the input port i.e., $V_1 = 0$.

Then from equation (1),

$$0 = 4I_1 + 2I_2$$

$$4I_1 = -2I_2$$

$$\frac{I_1}{I_2} = \frac{-2}{4}$$

$$\frac{I_1}{I_2} = \frac{-1}{2} \quad \text{... (4)}$$

$$\therefore g_{12} = \frac{I_1}{I_2} = \frac{-1}{2}$$

From equation (2)

$$V_2 = 5\left(\frac{-I_2}{2}\right) + 4I_2$$

$$\left(\because I_1 = \frac{-I_2}{2} \text{ From equation (4)}\right)$$

$$V_2 = \frac{-5I_2 + 8I_2}{2}$$

$$2V_2 = 3I_2$$

$$\frac{V_2}{I_2} = \frac{3}{2}$$

$$\therefore g_{22} = \frac{V_2}{I_2} = \frac{3}{2}$$

Result

$$g_{11} = \frac{1}{4}$$

$$g_{12} = \frac{-1}{2}$$

$$g_{21} = \frac{5}{4}$$

$$g_{22} = \frac{3}{2}$$

The inverse hybrid equations are,

$$I_1 = \frac{1}{4}V_1 - \frac{1}{2}I_2$$

$$V_2 = \frac{5}{4}V_1 + \frac{3}{2}I_2$$

3.2 NETWORK FUNCTION, DRIVING POINT AND TRANSFER FUNCTIONS – USING TRANSFORMED (S) VARIABLES, POLES AND ZERO'S

Q55. Explain the concept of transformed network.

Ans:

The representation of the transformed network is nothing but the representation of the three passive elements (R, L and C) by their respective transform impedances or transform admittances.

Resistance

The relation between the voltage and current of a resistor in time domain is given by,

$$v_R(t) = Ri_R(t) \quad \text{(or)}$$

$$i_R(t) = \frac{v_R(t)}{R} = Gv_R(t)$$

The corresponding transform equations are,

$$V_R(s) = RI_R(s) \quad \text{(or)}$$

$$I_R(s) = GV_R(s)$$

Transform impedance of the resistor,

$$Z_R(s) = \frac{V_R(s)}{I_R(s)} = R$$

Transform admittance of the resistor,

$$Y_R(s) = \frac{1}{Z_R(s)} = \frac{I_R(s)}{V_R(s)} = G$$

The representation of the resistor with its corresponding transform network is shown in figure (1).

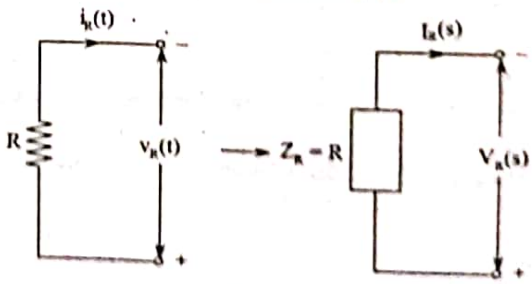


Figure (1)

Inductance

The relation between voltage and current of an inductor in time domain is given by,

$$v_L(t) = L\frac{di_L(t)}{dt} \quad \text{... (1)}$$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)dt \quad \text{... (2)}$$

$$\left.\frac{\ldots}{\ldots(s-s_a)(s-s_b)(s-s_c)\ldots(s-s_m)}\right](s-s_p)\Big|_{s=s_p} \quad \ldots (7)$$

Where, s_p, s_m, s_n are the complex numbers.

$$\Rightarrow K_p = H\left[\frac{(s_p-s_1)(s_p-s_2)(s_p-s_3)\ldots(s_p-s_n)}{(s_p-s_a)(s_p-s_b)(s_p-s_c)\ldots(s_p-s_m)}\right] \quad \ldots (8)$$

The numerator and denominator is in the from of (s_p-s_x) and (s_p-s_y) which are complex numbers the term $(s_p-s_1)(s_p-s_2)$ $(s_p-s_3)\ldots(s_p-s)$ can be expressed as,

$$(s_p-s_x) = N_{px}\, e^{j\alpha_{px}} \quad \ldots (9)$$

Where,

$x = 1, 2, 3, \ldots n$

N_{px} is the magnitude

α_{px} is the phase angle

Similarly, denominator terms $(s-s_a)(s-s_b)\ldots(s-s_m)$ of equation (8) can be expressed as,

$$(s_p-s_y) = D_{py}\, e^{j\beta p_y}$$

Where,

$y = a, b, c, \ldots m$

D_{py} is the magnitude

β_{py} is the phase angle.

Therefore, equation (8) can be written in magnitude and phase angle as,

$$K_p = H\left[\frac{N_{px}e^{j\alpha_{px}}}{D_{py}e^{j\beta_{py}}}\right] \qquad [\text{Where, } x = 1, 2, 3, \ldots n,\ y = a, b, c, \ldots m]$$

$$= H\left[\frac{N_{p1}e^{j\alpha_{p1}}.N_{p2}e^{j\alpha_{p2}}.N_{p3}e^{j\alpha_{p3}}\ldots N_{pn}e^{j\alpha_{pn}}}{D_{pa}e^{j\beta_{pa}}.D_{pb}e^{j\beta_{pb}}.D_{pc}e^{j\beta_{pc}}\ldots D_{pm}e^{j\beta_{pm}}}\right]$$

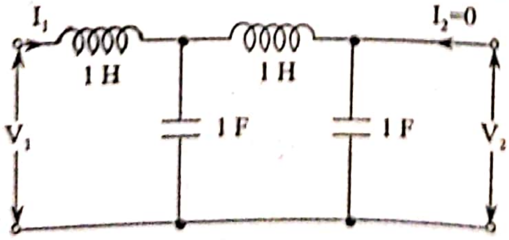
$$= H\left[\frac{N_{p1}N_{p2}N_{p3}\ldots N_{pn}}{D_{pa}D_{pb}D_{pc}\ldots D_{pm}}\times\frac{e^{j\alpha p_1}e^{j\alpha p_2}e^{j\alpha p_3}\ldots e^{j\alpha p_n}}{e^{j\beta p_a}e^{j\beta p_b}e^{j\beta p_c}\ldots e^{j\beta p_n}}\right]$$

$$= H\left[\frac{N_{p1}N_{p2}N_{p3}\ldots N_{pn}}{D_{pa}D_{pb}D_{pc}\ldots D_{pm}}\times\frac{e^{j[\alpha_{p1}+\alpha_{p2}+\alpha_{p3}+\ldots+\alpha_{pn}]}}{e^{j[\beta_{pa}+\beta_{pb}+\beta_{pc}+\ldots+\beta_{pn}]}}\right]$$

$$K_p = H\left[\frac{N_{p1}N_{p2}N_{p3}\ldots N_{pn}}{D_{pa}D_{pb}D_{pc}\ldots D_{pm}}\right]\times e^{j[\alpha_{p1}+\alpha_{p2}+\alpha_{p3}\ldots+\alpha_{pn}]-[\beta_{pa}+\beta_{pb}+\beta_{pc}+\ldots+\beta_{pm}]}$$

Similarly, the other coefficients $K_1, K_2, \ldots K_m$ can be evaluated.

Q63. What is the driving point and transfer impedance of the network shown figure below?



Figure

Ans:

The given circuit is as shown in figure (1).

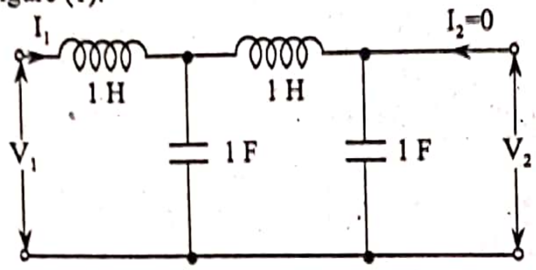


Figure (1)

The equivalent *s*-domain form is shown in figure (2).

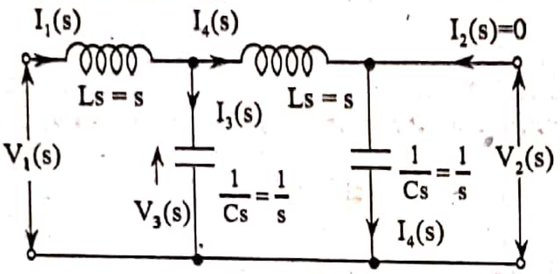


Figure (2)

From figure (2),

$$I_4(s) = \frac{V_2(s)}{1/Cs} = sV_2(s) \qquad [\because C = 1\text{ F}]$$

Also,

$$V_3(s) = V_2(s) + I_4(s).Ls$$
$$= V_2(s) + s\,V_2(s).Ls$$
$$= V_2(s)\,[1+Ls^2]$$
$$V_3(s) = V_2(s)\,[1+s^2] \qquad [\because L = 1\text{ H}]$$

Now,

$$I_1(s) = I_3(s) + I_4(s)$$
$$= \frac{V_3(s)}{1/Cs} + I_4(s)$$

Substituting $V_3(s)$ and $I_4(s)$ in $I_1(s)$, we get,

$$I_1(s) = Cs\,V_2(s)\,[1+s^2] + s\,V_2(s)$$
$$= s\,V_2(s)\,[1+s^2] + V_2(s)\,s \qquad [\because C = 1\text{ F}]$$
$$= V_2(s)\,[s(1+s^2)+s]$$
$$= V_2(s)\,(s+s^3+s)$$
$$I_1(s) = V_2(s)\,(s^3+2s)$$

Now,

$$V_1(s) = V_3(s) + I_1(s)\,Ls$$

Substituting $V_3(s)$ and $I_1(s)$ in $V_1(s)$, we get,

$$V_1(s) = V_2(s)[1+s^2] + V_2(s)\,[s^3+2s]s \qquad [\because L = 1\text{ H}]$$
$$V_1(s) = V_2(s)\,[1+s^2+s^4+2s^2]$$
$$V_1(s) = V_2(s)\,[s^4+3s^2+1]$$

$\therefore$ Driving point impedance $= \dfrac{V_1(s)}{I_1(s)}$

Substituting $V_1(s)$ and $I_1(s)$ values in above equation, we get,

$$\frac{V_1(s)}{I_1(s)} = \frac{V_2(s)[s^4+3s^2+1]}{V_2(s)[s^3+2s]}$$

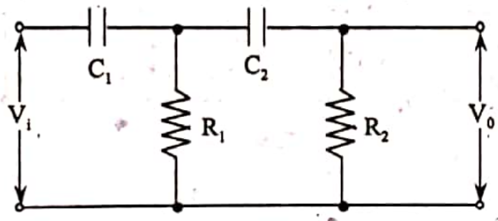
$\therefore$ Driving point impedance, $\dfrac{V_1(s)}{I_1(s)} = \dfrac{s^4+3s^2+1}{s^3+2s}$

Similarly,

Transfer impedance, $\dfrac{V_2(s)}{I_1(s)} = \dfrac{V_2(s)}{V_2(s)[s^3+2s]}$

$\therefore$ Transfer impedance $= \dfrac{1}{s^3+2s}$

Q64. Find the expression for voltage transformation ratio for the network shown figure below.



Figure

Ans:

Given circuit is shown in figure (1).

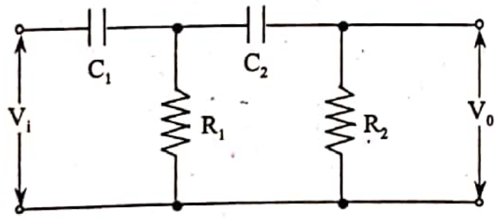


Figure (1)

Transforming the given circuit into s-domain, the circuit is modified as shown in figure (2).

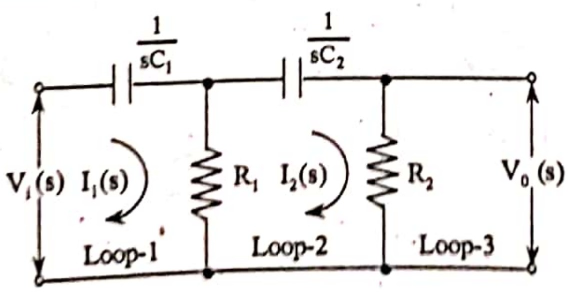


Figure (2)

Applying KVL in loop-1, we get,

$$V_i(s) = I_1(s)R_1 + \frac{1}{sC_1}I_1(s) - R_1 I_2(s)$$

$$V_i(s) = I_1(s)\left[R_1 + \frac{1}{sC_1}\right] - R_1 I_2(s) \quad \text{... (1)}$$

Applying KVL in loop-2, we get,

$$-R_1 I_1(s) + \frac{1}{sC_2} I_2(s) + R_2 I_2(s) + R_1 I_2(s) = 0$$

$$-R_1 I_1(s) + \left[R_1 + R_2 + \frac{1}{sC_2}\right] I_2(s) = 0 \quad \text{...(2)}$$

Applying KVL in loop-3, we get,

$$R_2 I_2(s) = V_0(s) \quad \text{...(3)}$$

Solving for $I_2(s)$,

$$I_2(s) = \frac{\Delta}{\Delta_1}$$

From equation (1) and (2),

$$\Delta = \begin{vmatrix} R_1 + \frac{1}{sC_1} & V_i(s) \\ -R_1 & 0 \end{vmatrix}$$

$$= 0 + V_i(s)\, R_1 = V_i(s)\, R_1$$

$$\Delta_1 = \begin{vmatrix} R_1 + \frac{1}{sC_1} & -R_1 \\ -R_1 & R_1 + R_2 + \frac{1}{sC_2} \end{vmatrix}$$

$$\Delta_1 = \left(R_1 + \frac{1}{sC_1}\right)\left(R_1 + R_2 + \frac{1}{sC_2}\right) - R_1^2$$

$$\Delta_1 = \frac{(R_1C_1s+1)(R_1C_2s + R_2C_2s + 1)}{(sC_1)(sC_2)} - R_1^2$$

$$\therefore \; I_2(s) = \frac{V_i(s)R_1}{\frac{(R_1C_1s+1)(R_1C_2s+R_2C_2s+1)}{(sC_1)(sC_2)} - R_1^2}$$

$$= \frac{R_1V_i(s)C_1sC_2s}{R_1^2C_1C_2s^2 + R_1R_2C_1C_2s^2 + R_1C_2s + 1 - R_1^2C_1C_2s^2 + R_1C_1s + R_2C_2s}$$

$$I_2(s) = \frac{R_1V_i(s)C_1sC_2s}{C_2s(R_1+R_2) + 1 + R_1C_1s(R_2C_2s+1)} \quad \text{...(4)}$$

Substituting equation (4) in equation (3), we get

$$V_o(s) = R_2 \cdot \frac{R_1V_i(s)C_1sC_2s}{(R_1+R_2)C_2s + 1 + R_1C_1s(R_2C_2s+1)}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{R_1R_2C_1sC_2s}{(R_1+R_2)C_2s + 1 + R_1C_1s(R_2C_2s+1)}$$

$\therefore$ Voltage transformation ratio is,

$$\frac{V_0(s)}{V_i(s)} = \frac{(R_1C_1)(R_2C_2)s^2}{(R_1+R_2)C_2s + 1 + R_1C_1s(R_2C_2s+1)}$$

$$G_{12}\left(\frac{V_2(s)}{V_1(s)}\right) = \frac{L}{C\left(R_1R_2 + R_1sL + \frac{R_1}{sC} + \frac{R_2}{sC} + \frac{L}{C}\right)}$$

$$= \frac{L}{R_1R_1C + R_1sLC + \frac{R_1}{s} + \frac{R_2}{s} + L}$$

$$G_{12}\left(\frac{V_2(s)}{V_1(s)}\right) = \frac{sL}{s^2R_1LC - s(R_1R_2C + L) + R_1 + R_2}$$

Q66. For the given network function, draw the pole zero diagram and hence obtain the time domain response i(t).

$I(s) = 5s / (s+1)(s^2 + 4s + 8)$

Ans: **Model Paper-III, Q7(b)**

Given that,

$$I(s) = \frac{5s}{(s+1)(s^2+4s+8)} \quad \text{...(1)}$$

Now, finding the roots for $s^2 + 4s + 8$

i.e., $$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1)(8)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16-32}}{2}$$

$$s = -2 \pm j2$$

Therefore, equation (1) now reduces to,

$$I(s) = \frac{5s}{(s+1)((s+2)-j2)((s+2)+j2)} \quad \text{...(2)}$$

The number of zeroes = 1, i.e., . The number of poles = 3, i.e., $s = -1, -2 + j2, -2 - j2$.

The pole-zero diagram is shown figure.

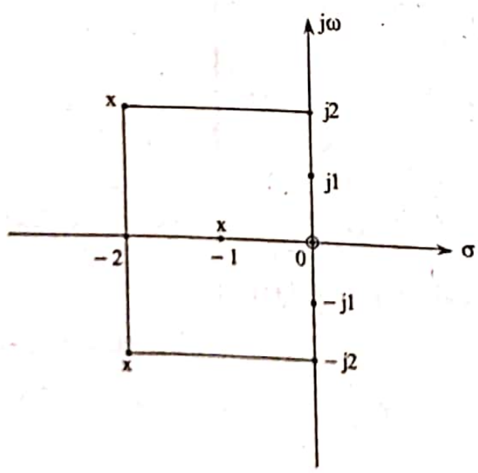


Figure: Pole-zero Diagram

The zeroes are marked as "0" and poles are marked as "×".

Taking partial fraction of equation (2), we get,

$$\frac{5s}{(s+1)((s+2)-j2)((s+2)+j2)} = \frac{A_1}{s+1} + \frac{A_2}{(s+2)-j2} + \frac{A_3}{(s+2)+j2} \quad \text{...(}$$

The constants A_1, A_2 and A_3 are calculated as,

$$A_1 = [(s+1)\times I(s)]_{s=-1}$$

$$= \left[(s+1)\times\frac{5s}{(s+1)(s^2+4s+8)}\right]_{s=-1}$$

$$= \left.\frac{5s}{s^2+4s+8}\right|_{s=-1}$$

$$= \frac{5(-1)}{(-1)^2+4(-1)+8}$$

$$\boxed{A_1 = -1}$$

$$A_2 = [((s+2)-j2)\times I(s)]_{s=-2+j2}$$

$$= \left[(s+2)-j2)\times\frac{5s}{(s+1)((s+2)-j2)((s+2)+j2)}\right]_{s=-2+j2}$$

$$= \left.\frac{5s}{(s+1)((s+2)+j2)}\right|_{s=-2+j2}$$

$$= \frac{5(-2+j2)}{(-2+j2+1)(-2+j2+2+j2)}$$

$$= \frac{-10+j10}{(-1+j2)(j4)}$$

$$\boxed{A_2 = 0.5-j1.5}$$

$$A_3 = [((s+2)+j2)\times I(s)]_{s=-2-j2}$$

$$= [(s+2)+j2]\times\left.\frac{5s}{(s+1)((s+2)-j2)(s+2)+j2)}\right|_{s=-2-j2}$$

$$= \left.\frac{5s}{(s+1)((s+2)-j2)}\right|_{s=-2-j2}$$

$$= \frac{5(-2-j2)}{(-2-j2+1)(-2-j2+2-j2)} = \frac{-10-j10}{(-1-j2)(-j4)}$$

$$\boxed{A_3 = 0.5+j1.5}$$

Substituting A_1, A_2 and A_3 values in equation (3), we get,

$$I(s) = \frac{-1}{s+1} + \frac{0.5-j1.5}{(s+2)-j2} + \frac{0.5+j1.5}{(s+2)+j2}$$

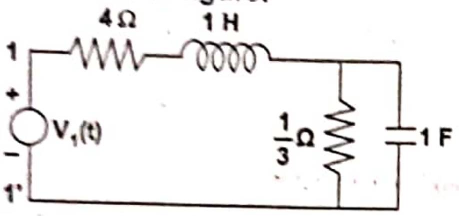
Transforming the above equation into time domain response by taking inverse transform.

$$i(t) = [-1\,e^{-t} + (0.5-j1.5)\,e^{(-2+j2)t} + (0.5+j1.5)\,e^{(-2-j2)t}]\,\text{A}$$

(or)

$$i(t) = [-e^{-t} + 1.58\,e^{j(71.56^\circ)}\,e^{-(2-j2)t} + 1.58\,e^{-j(71.56^\circ)}\,e^{-(2+j2)t}]\,\text{A}$$

Q67. Sketch the pole zero plots of Z_{11} of the network shown in below figure.



Figure

Ans:

The given network is as shown in figure (1).

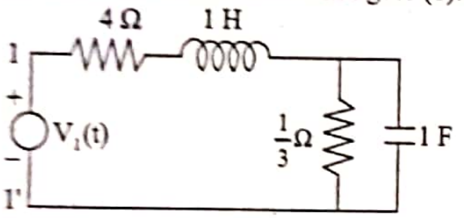


Figure (1)

The transform network in s-domain of figure (1) is as shown in figure (2).

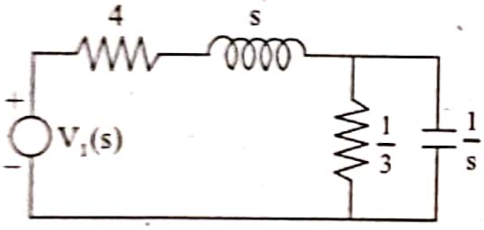


Figure (2)

Z_{11} is the driving point impedance which is equal to the ratio of V_1 and I_1.

Now, 4 and s are in series and $\frac{1}{3}$ and $\frac{1}{s}$ are in parallel.

The circuit now reduces as shown in figure (3).

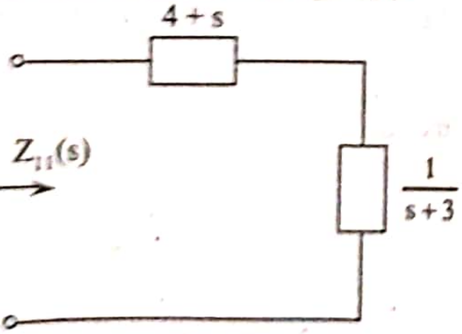


Figure (3)

$$\Rightarrow \frac{1}{3} \| \frac{1}{s}$$

$$\Rightarrow \frac{\frac{1}{3} \times \frac{1}{s}}{\frac{1}{3} + \frac{1}{s}}$$

$$\Rightarrow \frac{\frac{1}{3s}}{\frac{s+3}{3s}} = \frac{1}{s+3}$$

Now, the impedance $(4 + s)$ and $\frac{1}{s+3}$ are in series. The equivalent impedance of the network will be,

$$Z_{11}(s) = 4 + s + \frac{1}{s+3}$$

$$= \frac{4(s+3) + s(s+3) + 1}{s+3}$$

$$= \frac{4s + 12 + s^2 + 3s + 1}{s+3}$$

$$Z_{11}(s) = \frac{s^2 + 7s + 13}{s+3}$$

Now, finding the roots for $s^2 + 7s + 13$

i.e., $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(13)}}{2(1)}$

$$= \frac{-7 \pm \sqrt{-3}}{2}$$

$$= -3.5 \pm j0.86$$

$$\therefore \quad Z_{11}(s) = \frac{(s + 3.5 - j0.86)(s + 3.5 + j0.86)}{s+3}$$

The roots of numerator represents the zeros of the system and are $-3.5 + j0.86$ and $-3.5 - j0.86$. The roots of denominator represents poles of the given system and is -3. Now, the pole zero plot of $Z_{11}(s)$ is shown in figure (4).

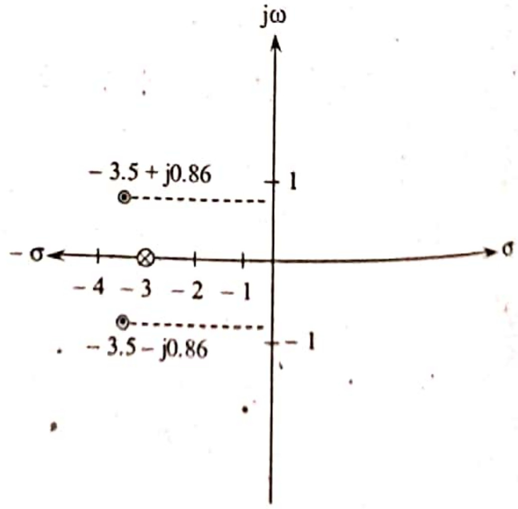


Figure (4)

Q68. Draw the pole-zero plots for a system with following network function.

$$Z(s) = \frac{(s^3 + 2s^2 + 3s + 2)}{s^4 + 6s^3 + 8s^2}$$

Ans:

Given that,

Network function, $Z(s) = \frac{(s^3 + 2s^2 + 3s + 2)}{(s^4 + 6s^3 + 8s^2)}$

$$\text{Let, } Z(s) = \frac{C(s)}{R(s)}$$

$$= \frac{s^3+2s^2+3s+2}{s^4+6s^3+8s^2}$$

Now to plot the poles and zeros, we need to factorize the numerator and denominator.

$\therefore \quad c(s) = s^3 + 2s^2 + 3s + 2$

$$\begin{array}{c|cccc} s=-1 & 1 & 2 & 3 & 2 \\ & 0 & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

$\therefore \quad s^3 + 2s^2 + 3s + 2 = (s+1)(s^2+s+2)$ ... (1)

Now, finding the roots for $(s^2 + s + 2)$

$$\text{i.e., } \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-1\pm\sqrt{1^2-4(1)(2)}}{2(1)}$$

$$= \frac{-1\pm\sqrt{-7}}{2} = \frac{-1\pm\sqrt{7}i}{2}$$

Therefore equation (1), now reduces to,

$$s^3+2s^2+3s+2 = (s+1)\left[s-\left(\frac{-1+\sqrt{7}i}{2}\right)\right]\left[s-\left(\frac{-1-\sqrt{7}i}{2}\right)\right]$$

$$= (s+1)\left(s+\frac{1-\sqrt{7}i}{2}\right)\left(s+\frac{1+\sqrt{7}i}{2}\right)$$

$$\therefore \quad c(s) = (s+1)\left(s+\frac{1-\sqrt{7}i}{2}\right)\left(s+\frac{1+\sqrt{7}i}{2}\right) = (s+1)(s+0.5+j1.32)(s+0.5-j1.32) \quad \text{... (2)}$$

$\therefore$ These roots of the numerator i.e., $-1, \frac{1+\sqrt{7}i}{2}, \frac{-1-\sqrt{7}i}{2}$ represents the zero's of the given system.

Now, $R(s) = s^4 + 6s^3 + 8s^2$

$= s^2(s^2 + 6s + 8)$

$= s^2(s^2 + 2s + 4s + 8)$

$= s^2[s(s+2) + 4(s+2)]$

$= s^2[(s+4)(s+2)]$

$\therefore \quad R(s) = s^2(s+4)(s+2)$

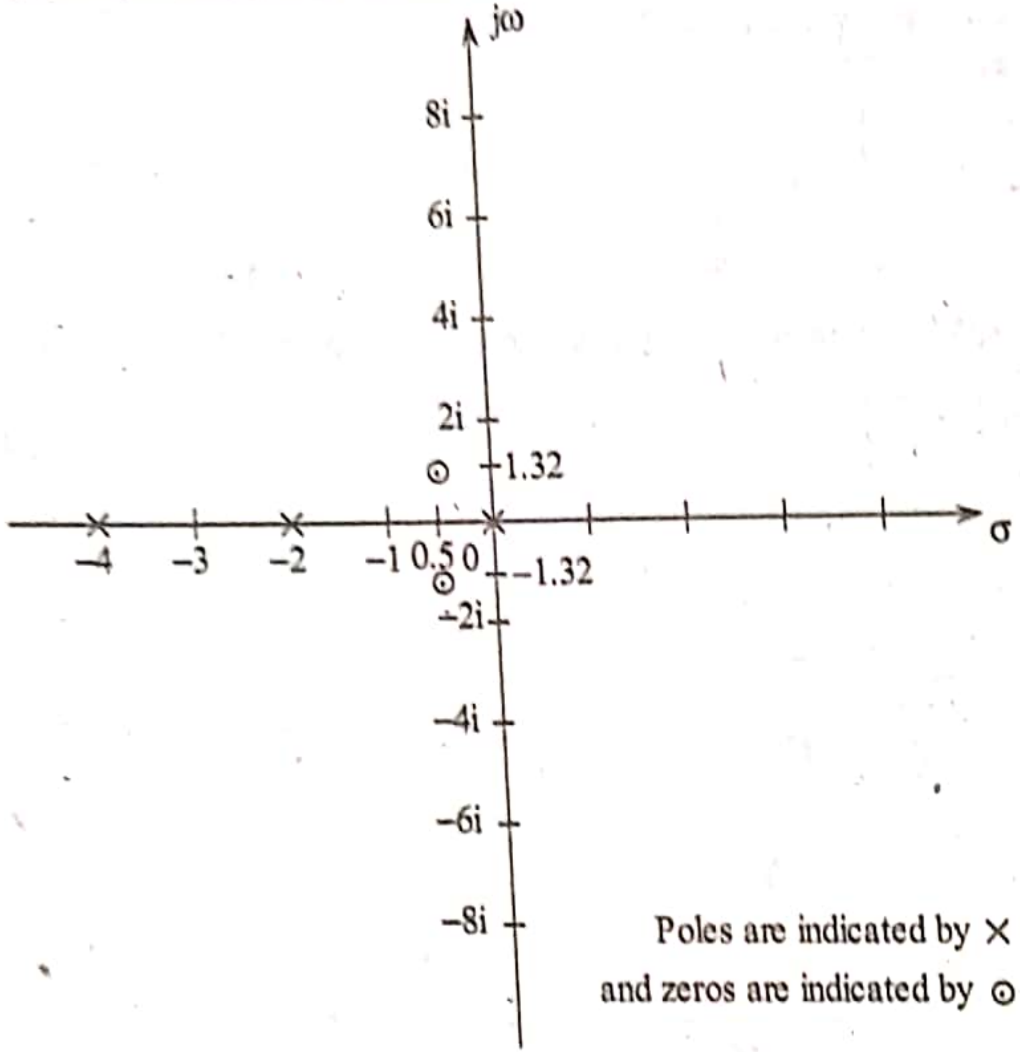
Therefore, the roots of denominator represents the poles of the given system and are 0, 0, –2, –4.

$\therefore$ For the given network function.

Poles = 0, 0, –2, –4

And zeros = –1, $-0.5 + j1.32, -0.5 - j1.32$

Now, the pole-zero plot for the system with the given transfer function is shown in figure.



Figure

3.3 STANDARD T, π, L SECTIONS, CHARACTERISTIC IMPEDANCE, IMAGE TRANSFER CONSTANTS, DESIGN OF ATTENUATORS, IMPEDANCE MATCHING NETWORK