Q17. The propagation constant of a lossy transmission line is (1 + j2) m⁻¹ and its characteristic impedance is 20 Ω at ω = 1M rad/s. Find L, C, R and G for the line.

(or)

For a lossy transmission line, the propagation constant is 1 + j2 m⁻¹, characteristic impedance is 20 Ω at ω = 1 Mrad/s. Determine L, C, R and G for the line.

Ans:

Given that,

For a lossy transmission line,

Propagation constant, $\gamma = 1 + j2\ \text{m}^{-1}$

Characteristic impedance, $Z_o = 20\ \Omega$

Angular frequency, $\omega = 1$ M rad/s $= 1 \times 10^6$ rad/s

Inductance, L = ?

Capacitance, C =?

Resistance, R = ?

Conductance, G = ?

The characteristic impedance of a transmission line is given by the expression,

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$20 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Squaring on both sides, we get,

$$400 = \frac{R + j\omega L}{G + j\omega C}$$

$$\Rightarrow \quad R + j\omega L = 400\,(G + j\omega C)$$

Since,

$$\Rightarrow \quad \gamma^2 = (R + j\omega L)(G + j\omega C)$$

$$\Rightarrow \quad (1 + j2)^2 = (R + j\omega L)\,(G + j\omega C)^2$$

$$\Rightarrow \quad (1 + j2)^2 = 400\,(G + j\omega C)^2 \qquad [\because \text{From equation (1)}]$$

$$\Rightarrow 1 + 4j - 2 = 400\,G^2 + j\,800\,G\omega C - 400\,\omega^2 C^2$$

$$\Rightarrow \quad -1 + 4j = 400\,G^2 - 400\,\omega^2 C^2 + j800\,G\omega C$$

Comparing the real and imaginary parts on both sides, we get,

$$400\,(G^2 - \omega^2 C^2) = -1$$

And $800\,G\omega C = 4$

Solving equations (2) and (3), we get,

$$\boxed{G = 0.05\ \text{s/m}}$$

$$\boxed{C = 0.1\ \mu\text{F/m}}$$

From characteristic impedance,

$$Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\Rightarrow \quad 20 = \sqrt{\frac{R+j\omega L}{0.05+j\times10^{-6}\times0.1\times10^{-6}}}$$

$$\Rightarrow \quad 400 = \frac{R+j\omega L}{0.05+0.1j}$$

$$\Rightarrow \quad 400\,(0.05+0.1j) = R+j\omega L$$

$$\Rightarrow \quad 20+40j = R+j\omega L$$

Comparing the real and imaginary parts on both sides, we get,

$$\boxed{R = 20\Omega/\text{m}}$$

$$L = \frac{40}{\omega} = \frac{40}{10^6} = 40\times10^{-6}\ \text{H/m}$$

$$\boxed{\therefore L = 40\,\mu\text{H/m}}$$

Q18. Given R = 10.4 Ω/mt
L = 0.00367 H/mt
G = 0.8 × 10⁻⁴ mhos/mt
C = 0.00835 μF/mt.
Calculate Z_0 and γ at 1.0 kHz.

Ans: Model Paper-III, Q8(b)

Given that,

Resistance, $R = 10.4\ \Omega$/mt

Inductance, $L = 0.00367$ H/mt

Capacitance, $C = 0.00835\ \mu\text{F} = 0.00835\times10^{-6}$ F/mt

Conductance, $G = 0.8\times10^{-4}$ mhos/mt

Operating frequency, $f = 1.0$ kHz $= 1\times10^3$ Hz

Characteristic impedance, $Z_0 = ?$

Propagation constant, $\gamma = ?$

Characteristic Impedance (Z_o)

The expression for characteristic impedance of the line is given by,

$$Z_o = \sqrt{\frac{Z}{Y}} \quad \ldots (1)$$

Where, 'Z' is series impedance, i.e.,

$$Z = R+j\omega L$$

And 'Y' is shunt admittance, i.e.,

$$Y = G+j\omega C$$

Series Impedance, $Z = R+j\omega L$

$$= 10.4 + j2\pi f L$$
$$= 10.4 + j2\times3.14\times(1.0\times10^3)\,(0.00367)$$
$$= 10.4 + j23.04$$
$$\therefore Z = 25.27\ \angle 65.70\ \Omega$$

Shunt Admittance (Y),

$$Y = G+j\omega C.$$
$$= 0.8\times10^{-4} + j2\pi f C$$
$$= 0.8\times10^{-4} + j\,2\times3.14\times(1.0\times10^3)\,(0.00835\times10^{-6})$$
$$= 0.8\times10^{-4} + j\,6.28\times10^{-3}\times0.00835$$
$$= 0.8\times10^{-4} + j\,0.0524\times10^{-3}$$
$$\therefore Y = 9.563\times10^{-5}\ \angle 33.22\ \mho$$

Substituting the values of 'Z' and 'Y' in equation (1), we get,

$$Z_o = \sqrt{\frac{25.27\ \angle 65.70}{9.563\times10^{-5}\ \angle 33.22}}$$
$$= 514.05\ \angle\frac{65.70-33.22}{2}$$
$$= 514.05\ \angle 16.24\ \Omega$$
$$= 514.05\,(\cos(16.24) + j\sin(16.24))$$
$$= 514.05\,(0.96 + j\,0.27)$$

$$\boxed{\therefore Z_o = 493.4 + j138.7\ \Omega}$$

Propagation Constant

The expression for propagation constant is given by,

$$\gamma = \sqrt{ZY} \quad \ldots (2)$$

Substituting 'Z' and 'Y' values in equation (2), we get,

$$\gamma = \sqrt{(25.27\angle 65.70°)(9.563\times10^{-5}\ \angle 33.22)}$$
$$= 0.049\ \angle\frac{65.70+33.22}{2}$$
$$= 0.049\ \angle 49.46$$
$$= 0.049\,(\cos 49.46 + j\sin 49.46)$$
$$= 0.049\,(0.649 + j\ 0.759)$$

$$\boxed{\therefore \gamma = 0.031 + j0.037}$$

Q19. A telephone line has R = 30Ω/km, L = 0.1H/km, C = 20μF/m and G = 0. At f = 10K Hz, find the secondary constants and phase velocity.

Ans:

Given that

For a telephone line,

Resistance, $R = 30\ \Omega$/km

Inductance, $L = 0.1$ H/km

Capacitance, $C = 20\ \mu$F/m $= 20$ mF/km

Conductance, $G = 0$

Operating frequency, $f = 10$ kHz

1. Secondary constants = ?
2. Phase velocity, $V_p = ?$

1. **Secondary Constants**

Secondary constants of a transmission line are characteristic impedance, Z_o and propagation constant γ.

(i) **Characteristic Impedance Z_o**

The expression for characteristic impedance of the line is given by,

$$Z_o = \sqrt{\frac{Z}{Y}} \quad \text{... (1)}$$

Where, 'Z' is series impedance, i.e.,

$$Z = R + j\omega L$$

and 'Y' is shunt admittance, i.e.,

$$Y = G + j\omega C$$

Series Impedance, $Z = R + j\omega L$

$$= 30\ \Omega + j2\pi fL$$

$$= 30 + j2\pi(10\times10^3)\,(0.1)$$

$$= 30 + j\,6.283 \times 10^3$$

$$\therefore Z = 6283.07 \angle 89.72\ \Omega$$

Shunt Admittance, $Y = G + j\omega C$

$$= G + j2\pi fC$$

$$= 0 + j2\pi\,(10 \times 10^3)\,(20 \times 10^{-3})$$

$$= 0 + j\,1.256 \times 10^3$$

$$\therefore Y = 1256 \angle 90^\circ\ \mho$$

Substituting the values of 'Z' and 'Y' in equation (1), we get,

$$Z_o = \sqrt{\frac{6283.07 \angle 89.72^\circ}{1256 \angle 90^\circ}}$$

$$= 2.236 \angle \frac{89.72^\circ - 90^\circ}{2}$$

$$= 2.236 \angle -0.14^\circ\ \Omega$$

$$= 2.236 - j\,0.0055\ \Omega$$

$$\boxed{\therefore Z_o = 2.236 + j(-0.0055)}\ \Omega$$

(ii) ***Propagation Constant***

The expression for propagation constant is given by,

$$\boxed{\gamma = \sqrt{ZY}}$$

Substituting 'Z' and 'Y' values in the above expression, we get,

$$\gamma = \sqrt{(6283.07 \angle 89.72^\circ)(1256 \angle 90^\circ)}$$

$$= 2809 \angle \frac{89.72^\circ + 90^\circ}{2}$$

$$= 2809.1 \angle 89.86^\circ$$

$$= 6.863 + j\,2809.09$$

$$\boxed{\therefore \gamma = \alpha + j\beta = 6.863 + j2809.09}$$

From the above expression,

Attenuation constant, $\alpha = 6.744$ NP/m

Phase constant, $\beta = 2809.62$ radians/m

2. **Phase Velocity (V_p)**

The expression for phase velocity, v_p is given by,

$$V_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta}$$

$$\Rightarrow \quad V_p = \frac{2\pi(10\times10^3)}{2809.09}$$

$$\Rightarrow \quad V_p = 22.367 \text{ m/sec.}$$

$$\boxed{\therefore \text{ Phase velocity}(v_p) = 22.367 \text{ m/sec.}}$$

Q20. A distortionless transmission line has the following parameter $Z_o = 60\ \Omega$, $\alpha = 20$ Np/m and $V_p = 21 \times 10^7$ m. Find R, L, G and C.

Ans:

Given that,

For a distortionless transmission line,

Characteristic impedance, $Z_o = 60\ \Omega$

Attenuation constant, $\alpha = 20$ Np/m

Phase velocity, $V_p = 21 \times 10^7$ m/s

Resistance, $R = ?$

Inductance, $L = ?$

Conductance, $G = ?$

Capacitance, $C = ?$

Resistance (R)

The condition for a distortionless line is,

$$\frac{R}{L} = \frac{G}{C}$$

For a distortionless line, the attenuation constant is given by,

$$\alpha = \sqrt{RG}$$

$$= \sqrt{R\frac{CR}{L}} \quad \left(\because G = \frac{CR}{L}\right)$$

$$= \sqrt{\frac{R^2C}{L}} = R\sqrt{\frac{C}{L}}$$

$$\Rightarrow \quad \alpha = \frac{R}{Z_o} \quad \left[\because Z_o = \sqrt{\frac{L}{C}}\right]$$

$$\Rightarrow \quad R = \alpha Z_o$$

On substituting the corresponding values in equation (2), we get,

$$R = (20 \times 10^{-3})(60)$$

$$= 1.2\ \Omega/\text{m}$$

$$\boxed{\therefore R = 1.2\ \Omega/\text{m}}$$

Inductance (L)

The expression for characteristic impedance is,

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{L}{(1/LV_p^2)}} \quad \left(\begin{array}{l} \because V_p = \dfrac{1}{\sqrt{LC}} \\ \Rightarrow C = \dfrac{1}{LV_p^2} \end{array}\right)$$

$$\Rightarrow \quad Z_0 = \sqrt{L^2V_p^2}$$

$$\Rightarrow \quad Z_0 = LV_p$$

$$\Rightarrow \quad L = \frac{Z_0}{V_p} \quad \ldots (3)$$

On substituting the corresponding values in equation (3), we get,

$$L = \frac{60}{21 \times 10^7}$$

$$= 2.8571 \times 10^{-7}\,\text{H}$$

$$\boxed{\therefore L = 285.71\ \text{nH}}$$

Capacitance (C)

The expression for characteristic impedance is,

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{1}{V_p^2C^2}} \quad \left(\begin{array}{l} \because V_p = \dfrac{1}{\sqrt{LC}} \\ \Rightarrow L = \dfrac{1}{V_p^2C} \end{array}\right)$$

$$\Rightarrow \quad Z_0 = \frac{1}{V_pC}$$

$$\Rightarrow \quad C = \frac{1}{Z_0V_p} \quad \ldots (4)$$

On substituting the corresponding values in equation (4), we get,

$$C = \frac{1}{60 \times 21 \times 10^7}$$

$$= 7.936 \times 10^{-11}\,\text{F/m}$$

$$\boxed{\therefore C = 79.36\ \text{pF/m}}$$

Conductane (G)

From equation (1),

$$G = \frac{CR}{L}$$

$$\Rightarrow \quad G = \frac{\alpha^2}{R} \quad \ldots (5)$$

On substituting the corresponding values in equation (5), we get,

$$G = \frac{(20 \times 10^{-3})^2}{1.20}$$

$$= 3.333 \times 10^{-4}\,\text{mhos/m}$$

$$\boxed{\therefore G = 333\ \mu\mho/\text{m}}$$

4.2 INFINITE LINE CONCEPTS, LOSS LESS/LOW LOSS CHARACTERIZATION, TYPES OF DISTORTION – CONDITION FOR DISTORTIONLESS LINE, MINIMUM ATTENUATION

Q21. Explain the concept of infinite line.

Ans: *Model Paper-I, Q9(a)*

A transmission line whose length is infinite is known as Infinite line. An infinite length transmission line is as shown in figure (1).

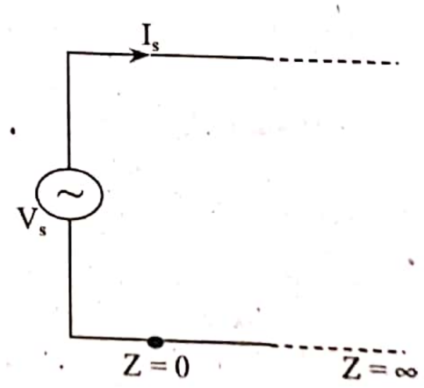

Figure (1) : Infinite Line

Basically, an infinite line is formed by cascading a number of symmetrical sections as shown in figure (2).

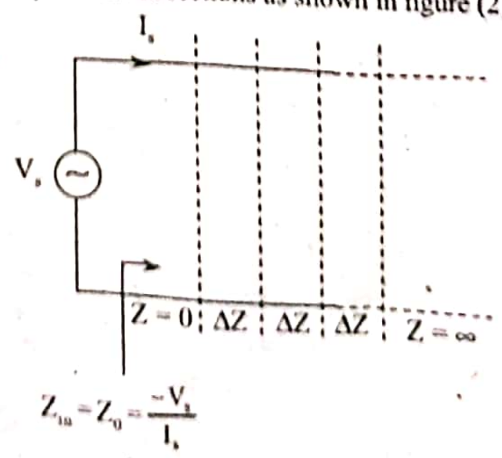

Figure (2)