Comparative Study of Approaches Used To Solve Mathematical Word Problems

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ABSTRACT :
This paper presents a survey on the applications of text categorization for design and implementation of Automatic Mathematical Word Problem Classifier and Solver. We put forth the discussion on combined use of state-of-the-art NLP (Information Extraction), ML (SVM) techniques to implement a classifier which is capable of classifying text describing MWP in classes like, Joint-Separate, Part-Part-Whole and Comparison Problems etc. MWP solvers are some of the most complicated and sophisticated automatic solvers as there is no standard definitive method to analyze the context and semantics of the text describing the problem. We present a brief report on the novel methods used in AMWPS (Automatic Math Word Problem Solvers) and various techniques to identify the features (keywords, average number of sentences etc.) for categorization.

INDEX TERM: MWP, NLP, Text Categorization, Neural Networks, Information Extraction etc.

1. INTRODUCTION:

Understanding Natural Language is the most trivial task for humans. As we know humans are good at reasoning and hence, one of the first reasoning tasks given to kids is the quantitative reasoning. Solving MWP involves the use of both of the above abilities (Understanding Natural Language and Quantitative Reasoning). When it comes to Arithmetic calculations (Counting) computers are outperforming humans since their early days.

In Mathematics, word problem is the term used to describe any mathematical exercise on which significant background information is presented as text rather than in mathematical notations. This information is critical to solving the problems as it presents relevant data which is useful in identifying problem type and to identify the components of the problem.

Historical prototype systems such as WORDPRO (Fletcher et al,1985), SOLUTION (Dellarosa, 1985) are representations of cognitive models of the human's process of mathematical word problem-solving.

We present a comparative study on the current research in AMWPS, describing the overall approach incorporated by studies(section 3). Which shows that there two main categories of these studies, first is solution with the help of text classification and second is the solution with the help of custom data-structure to bridge the gap between mathematical notation and text e.g. Shi et al, 2015 which uses DOL to construct tree representation.

2. RELATED WORK:

Previous work to solve mathematical word problems falls into two categories: symbolic approaches and statistical learning methods. In symbolic approaches, math problem sentences are transformed to the certain structure by pattern matching or verb categorization. Equations are then derived from the structure. Statistical learning methods are employed using probabilistic model.

Research by Kushman et al., 2014[1], proposes a system which automatically learns to solve algebra word problems. In this paper, a problem is mapped to equations in two steps: First, a template is selected to define a structure of equation system. Next, a template is instantiated with numbers and nouns from the text. It uses data from algebra.com, a crowd-sourced tutoring website. It can correctly answer 70% data in the data set.

Another study(Hosseini et al. 2014)[2], designs a system named ARIS which is based on state transition. It analyzes each sentence in the problem to find relevant variables and their values. Verb categorization is important during state transition. There are seven categories of verbs. This is 81.2% accurate in classifying verbs categories and could solve 77.7% of problems of standard primary school. Data set consists of only addition and subtraction problems. No verb is shared between training and test data sets.

A system called ALGES proposed in Rik Koncel-Kedziorski et al., 2015[3], solves multi-sentence algebraic word problems as that of generating and scoring equation trees. Integer Linear Programming(ILP) is used to generate equation trees. Scoring is done via learning local and global discrimination models. Training data set for this consists of problems and their equations. No manual annotation is done on training data. It can cover all mathematical operations (+, -, *, / , =).

The state-of-the-art system proposed by Roy and Roth, 2015[8], solves arithmetic word problems without using predefined templates or additional manual annotation. There is a notion of expression tree which is helpful in representing and evaluating the target expression. This approach outperforms existing system achieving state of art performance. The system is evaluated on three datasets. Each dataset has different category of problems. The dataset consists of problems of all arithmetic types(+,-,*,/).

The paper(Suleyman Cetintas et al., 2009)[4] describes text categorization of mathematical word problems into multiplicative compare and equal group problems. It allows selective stopword removal and stemming. POS tagging is used to distinguish between discriminative and non-discriminative words. It uses SVM classifier with a selective preprocessing technique which outperforms default SVM classifier. This approach is totally based on unigram model of words.

The method discussed in (Amnueypornsakul et al., 2014)[5] follows a systematic approach to solve mathematical word problems. There are three categories of problems which are handled by this: join and separate problems, part part whole problems and compare problems. Various stages to solve problems are: Firstly, the problem is classified, which is followed by sentence function identification and sign prediction. Next,an equation is generated using information extraction followed by solving it.

The study in (Arindam Mitra et al. 2016)[6], uses formulas to solve arithmetic word problems. The system developed identifies the variables and their attributes, and maps this information to higher level representation. This representation is then used to recognize the presence of formula along with associated variables. Finally, an equation is generated from formal description of the formula. It is able to produce 86.07% accuracy. AddSub dataset is used which consists of 395 addition-subtraction problems.

The paper (Shuming Shi et al., 2015)[7] presents a system called SigmaDolphin which automatically solves math word problems by semantic parsing and reasoning. DOL language is used as a structured semantic representation of NL text. A semantic parser is used to transform math problem text into DOL trees. A reasoning module is used to derive math expressions from DOL trees and calculate final answers. Dataset used contains 1878 math problems.

3. METHODOLOGIES :

3.1 Template Based Approach: (Kushman et al., 2014)[1]

System follows two-step process to map word problems to an equation and to ultimately find the solution.

Step 1 : Template selection so that it defines overall structure of the equation system.

Step 2 : Template is instantiated with numbers and nouns.

It uses two kinds of slot to fill instantiate the template,

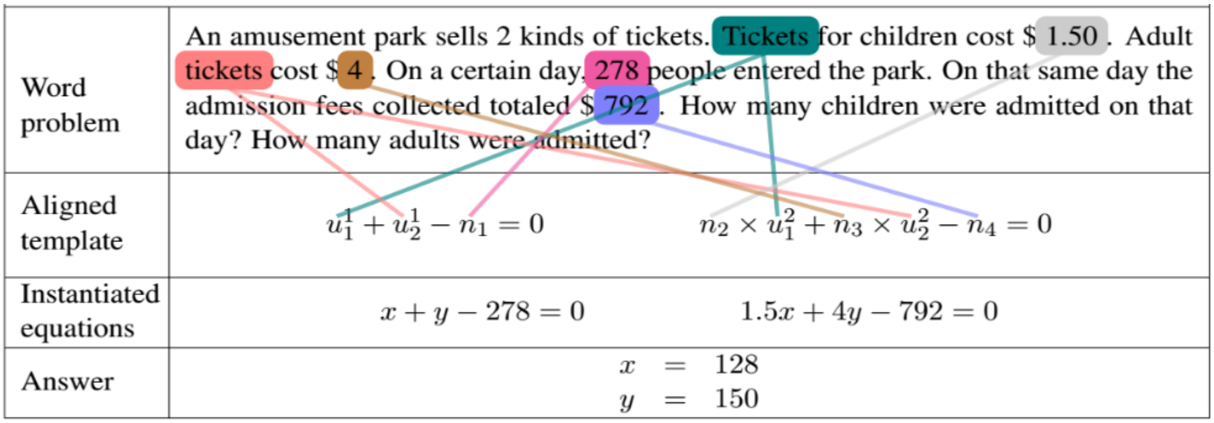

Figure 1: Derivation for problem which shows an alignment where two instances of same slot are align to same word

***u* slot** *: unknowns from the problem,*
***n* slot** *: numerical entity.*

Figure 1. shows the derivation of a simple problem statement and describes the use of slots

3.2 Approach of verb categorization: (Hosseini et al., 2014)[2]

ARIS revolves around three features of the problem statement; It is dependent on Stanford's coreNLP toolkit to ground these features (Quantities, Entities, Containers)

Quantities: numerical weight of any object.

Entities: Object whose quantities are observed to be changed as problem progresses.(Noun)
Containers: Each entity is associated with one or more container which is responsible for holding the full of a partial quantity of entity.

Category	**Example**
Observation	There **were** 26 bales of hay in the barn.
Positive	*Joan* **went** to 4 football games this year.
Negative	*John* **lost** 3 of the violet balloons.
Positive Transfer	*Mike's* dad **borrowed** 7 nickels form mike.
Negative Transfer	*Jason* **placed** 131 erasers in the drawer.
Construct	*Karen* **added** ¼ of a cup of walnut to a *batch* of trail mix.
Destroy	The *rabbit* **ate** 4 of *Dan's* potatoes.

Table 1. shows the verb categories employed for operation selection

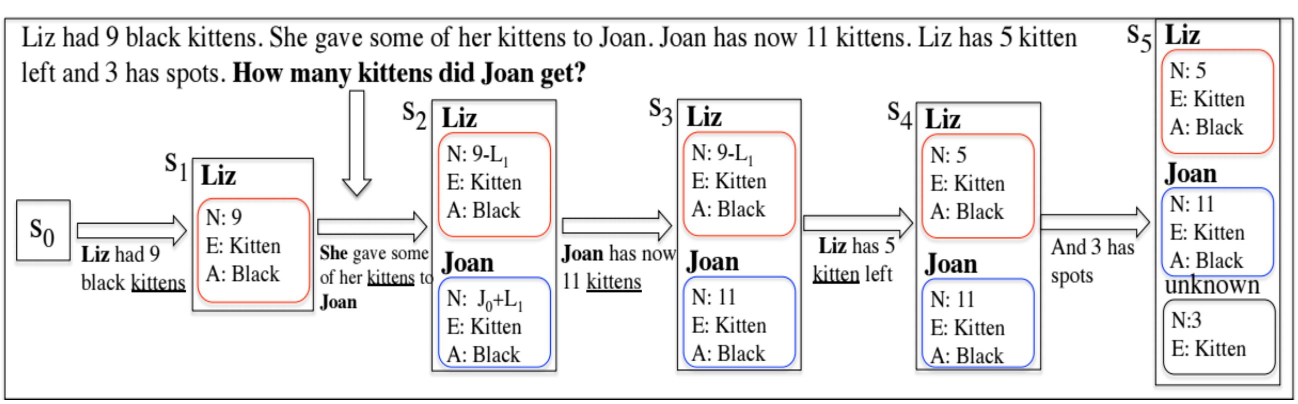

Figure 2. Sketches different steps involved in method of **(Hosseini et al., 2014)[2])**

3.3 Use of DOL (dolphin language): (Shi et al., 2015)[7]

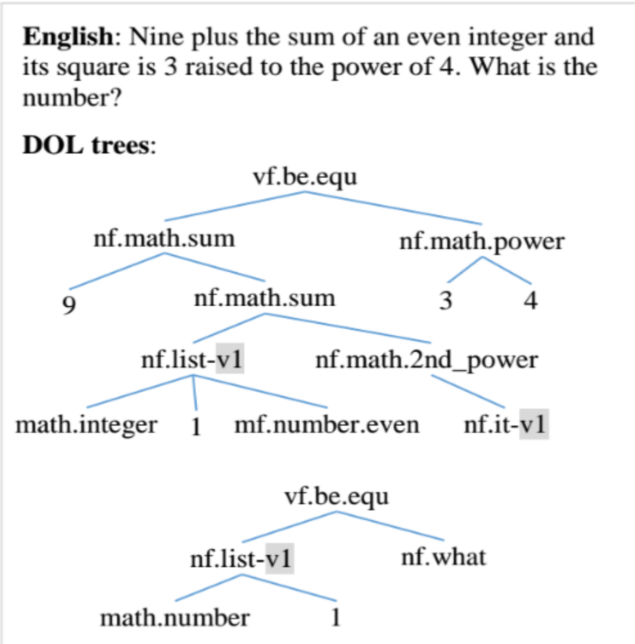

Figure 3. DOL example

3.4 Use of Equation trees and ILP : ALGES (Koncel-Kedziorski et al.,2015)[3]

ALGES uses customized data structure Q-set which is used for developing the syntactically valid trees which carry the essence of the equation required to solve the problem presented to it. ALGES parses all the sentences and ground all the entities to respective ordered Q-sets and generates a list of type consistent candidate training equation that yield the correct result. System follows local and global discrimination with which it identifies,

Local features = <s1, s2>, where s1, s2 ∈ Q-set
Global features = <w, t> , where w = word problem; t = tree representing possible output equation.

ALGES finds all the Q-sets and then rearranges them, rearrangement rules are described in Figure 4. Further down the process, it uses Integer Linear Programming which generates the semantically valid representation. These trees are subjected to *global* discrimination and if *t* gives correct result it is stored otherwise discarded.

These trees are parsed in a bottom-up fashion to generate equation from them. Then if the equation gives the valid answer it is included in a *positive* set. Otherwise labeled as *negative*. All positive entries yield same equation with a different arrangement to find the answer.

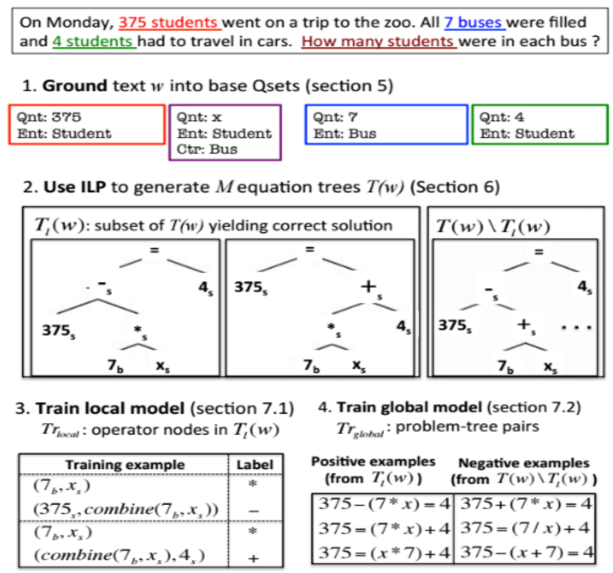

Figure 4. An overview of the process of learning for a word problem and its Q-sets (ALGES)

3.5 Approach of sentence classificatio : (Amnueypornsakul et al., 2014 - Figure 5.)[5]

Henry is walking dogs for money. There are 7 dogs to walk on Henry's street. Henry walked 4 of them. How many dogs does Henry have left to walk?

Note : The yellow highlight is the *given sentence*. The blue highlight is the *change sentence* and the pink high-light is the *result sentence* of the example problem. The remaining sentences are of the type *unknown sentence*.

Some kids are playing in a playground. 3 boys are playing on the slide. 4 girls are playing on the merry-go-round. How many kids are there in the playground?

Note : The yellow highlight is the *part sentence*. The blue highlight is the *whole sentence*. The rest of the ques-tion is the *unknown sentence*.

Equation: 3 + 4 = x

Figure 5. Use of sentence classification to solve join-separate and part part whole problem.

	A1	IXL	MA2	Avg
ARIS	83.6	75.0	74.4	77.7
KAZB	89.6	51.1	51.2	64.0
ALGES	-	-	-	77.0
Roy & Roth	-	-	-	78.0
Majority	45.5	71.4	23.7	48.9

Table 2. Comparative results of some of the studies presented in (Mitra et al, 2016)[6]

4. CONCLUSION:

In this work, we have outlined some of most effective techniques which are presented to solve MWP. This study expresses the possibility of use case and success of ML techniques in the field of NLP. We conclude with an overview of multiple efforts of scientists in the direction of building a system that can solve any mathematical problem.

5. ACKNOWLEDGMENT:

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5. REFERENCES :

[1] Nate Kushman, Yoav Artzi, Luke Zettlemoyer, and Regina Barzilay. 2014 - Learning to Automatically Solve Algebra Word Problems, *In proc. of the Annual Meeting of the Association for Computational Linguistics(ACL)*

[2] Mohammad Javad Hosseini , Hannaneh Hajishirzi, Oren Etzioni, and Nate Kushman. 2014 - Learning to Solve Arithmetic Word Problems with Verb Categorization, *In proc. of the Conference on Empirical Methods in Natural Language Processing (EMNLP), pages 523–533 and Association for Computational Linguistics (ACL)*

[3] Rik Koncel-Kedziorski, Hannaneh Hajishirzi, Ashish Sabharwal, Oren Etzioni, and Siena Dumas Ang. 2015 - Parsing Algebraic Word Problems into Equations, *In proc. of the Annual Meeting of the Association for Computational Linguistics(ACL)*

[4] Suleyman Cetintas, Luo Si, Yan Ping Xin, Dake Zhang, Joo Young Park. 2009 - Automatic Text Categorization of Mathematical Word Problems, *In proc. of Association for the Advancement of Artificial Intelligence (www.aaai.org)*

[5] Bussaba Amnueypornsakul, Suma Bhat. 2014 - Machine-Guided Solution to Mathematical Word Problems, *In Proc. of Pacific Asia Conference on Language, information and Computation*

[6] Arindam Mitra, Chitta Baral. 2016 - Learning To Use Formulas To Solve Simple Arithmetic Problems, *In proc. of the Annual Meeting of the Association for Computational Linguistics(ACL)*

[7] Shuming Shi, Yuehui Wang, Chin-Yew Lin1, Xiaojiang Liu and Yong Rui. 2015 - Automatically Solving Number Word Problems by Semantic Parsing and Reasoning, *In proc. of the Conference on Empirical Methods in Natural Language Processing*

[8] Subhro Roy and Dan Roth. 2015 - Solving general arithmetic word problems, *In proc. of EMNLP*

[9] Rik Koncel-Kedziorski, Subhro Roy , Aida Amini, Nate Kushman, and Hannaneh Hajishirzi. 2016 - MAWPS: A Math Word Problem Repository, *In proc. of NAALC*