# Assignment 3

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Download all python codes from

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https://github.com/Suraj11050/Assignments-AI1103/blob/main/Assignment%203/Assignment3.py
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Download Latex-tikz codes from

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https://github.com/Suraj11050/Assignments-AI1103/blob/main/Assignment%203/Assignment3.tex
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## 1 GATE 2009 (MA) PROBLEM 16

Let F, G and H be pair wise independent events such that $\Pr(F) = \Pr(G) = \Pr(H) = \frac{1}{3}$ and $\Pr(F \cap G \cap H) = \frac{1}{4}$ Then the probability that at least one event among F, G and H occurs is

(A) $\frac{11}{12}$ (B) $\frac{7}{12}$ (C) $\frac{5}{12}$ (D) $\frac{3}{4}$

## 2 SOLUTION

Let f,g,h be three random variables taking values 0 or 1 (Bernoulli random variable) Which represent the occurrence of event F, G, H respectively such that

$$\Pr(f = 0) = \frac{2}{3} \quad \Pr(f = 1) = \frac{1}{3}$$
$$\Pr(g = 0) = \frac{2}{3} \quad \Pr(g = 1) = \frac{1}{3}$$
$$\Pr(h = 0) = \frac{2}{3} \quad \Pr(h = 1) = \frac{1}{3}$$

If two Random variables $X_1$ and $X_2$ are independent then

$$\Pr(X_1 \wedge X_2) = \Pr(X_1) \times \Pr(X_2) \tag{2.0.1}$$

Using equation (2.0.1) we get the following results

$$\Pr(f = 1 \wedge g = 1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$
$$\Pr(g = 1 \wedge h = 1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$
$$\Pr(f = 1 \wedge g = 1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

At least one event among F, G, H should occur is $\Pr(F \cup G \cup H)$ from Principal of inclusion and exclusion it is calculated using random variable as

$$\begin{aligned}\Pr(f = 1 \vee g = 1 \vee h = 1) = \\ (\Pr(f = 1) + \Pr(g = 1) + \Pr(h = 1)) \\ - \Pr(f = 1 \wedge g = 1) - \Pr(g = 1 \wedge h = 1) \\ - \Pr(h = 1 \wedge f = 1) + \Pr(f = 1 \wedge g = 1 \wedge h = 1)\end{aligned}$$

$$\Pr(f = 1 \vee g = 1 \vee h = 1) = 3\left(\frac{1}{3}\right) - 3\left(\frac{1}{9}\right) + \frac{1}{4}$$
$$\therefore \Pr(f = 1 \vee g = 1 \vee h = 1) = \frac{11}{12}$$

Hence Probability that at least one event among F, G, H occurs is $\Pr(F \cup G \cup H) = \frac{11}{12}$ and correct answer is **Option (A)**

But we know that

$$(F \cap G \cap H) \subseteq (F \cap G)$$
$$\therefore \Pr(F \cap G \cap H) \leq \Pr(F \cap G)$$

In the given question

$$\Pr(F \cap G \cap H) = \frac{1}{4}$$
$$\Pr(F \cap G) = \frac{1}{9}$$
$$\Pr(F \cap G \cap H) > \Pr(F \cap G)$$

Which is not possible

Some of the probabilities turnout to be negative like

$$\Pr(F \cap G \cap H^{c}) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36}$$
$$\Pr(F \cap G^{c} \cap H) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36}$$
$$\Pr(F^{c} \cap G \cap H) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36}$$

Similar case with $\Pr(G \cap H)$ and $\Pr(H \cap F)$
Therefore **Question is incorrect**