ASANSOL ENGINEERING COLLEGE

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- Sub code - OECIT 701 A
- Sec - B

"Operation Research"

OEC-IT 701 A

"SEC-B"

1. A firm plans to purchase at 200 quintals of ~~sco~~ scrap Containing high quality metal x and low quality metal y. It decidees that the scrap to be purchased must contain at least 100 quintals of x metal and not more than 35 quintals of y metal. The firm can purchase the scrap from two supliers (A and B) in unlimited ~~quantials~~ quantities. The purchase of x and y that in scraps supplied by B and 75% and 20% respectively. The price of A's scrap is Rs. 200 per quintal and that of B's is 400 per quintal. Formulate this problem as LP model and solve it to determine the quantities that the firm should buy from the two suppliers so as to minimize total purchase cost.

Answer

We have,

Metal	Supplier A	Supplier B
~~A~~, x	25 %	75 %
y	10 %	20%

Let us consider the following decision Variable,
The following LP model of given problem can be express as,

$$\text{Minimize } z = 200x_1 + 400x_2$$

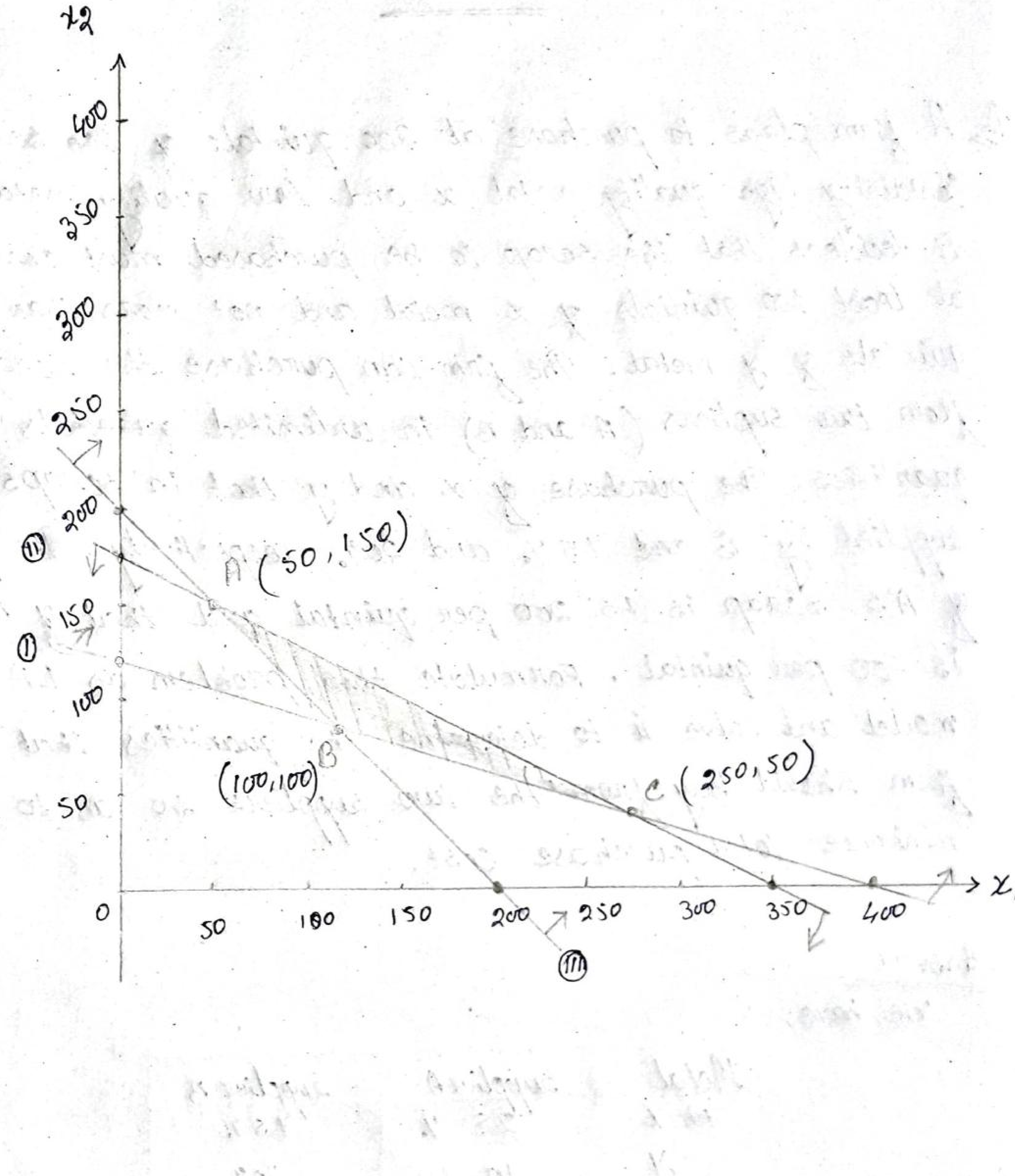
Subject to constrains,

$$x_1 + x_2 \geq 200$$

$$\frac{x_1}{4} + \frac{3x_2}{4} \geq 100$$

$$\frac{x_1}{10} + \frac{x_2}{5} \leq 35$$

Graph :



x_2
400
350
300
250
200
150
100
50
0
50
100
150
200
250
300
350
400
x_1
(I)
(II)
(III)
A (50, 150)
B
(100,100)
C (250,50)

Now, we can write the equations like,

$\therefore \quad \frac{x_1}{4} + \frac{3x_2}{4} \geq 100 \Rightarrow x_1 + 3x_2 \geq 400$

$\therefore \quad \frac{x_1}{100} + \frac{x_2}{5} \leq 35 \Rightarrow x_1 + 2x_2 \leq 350$

$x_1 + x_2 \geq 200$

1s eq

$x_1 + 3x_2 = 400$ —— (I)

when $x_1 = 0$,

$x_2 = \frac{400}{3} = 133.33$

when, $x_2 = 0$

$x_1 = 400$

x_1	0	400
x_2	133.33	0

2nd eq

$x_1 + 2x_2 = 350$ —— (II)

When, $x_1 = 0$

$x_2 = 350/2 = 175$

When, $x_2 = 0$

$x_1 = 350$

x_1	0	350
x_2	175	0

3rd eq

$x_1 + x_2 = 200$ —— (III)

$x_1 = 0, \ x_2 = 200$

$x_2 = 0, \ x_1 = 200$

x_1	0	200
x_2	200	0

To get point A,

solve eq (II) & (III),

$$\begin{array}{r} \cancel{x_1} + 2x_2 = 350 \\ \cancel{x_1} + x_2 = 200 \\ \hline x_2 = 150 \end{array}$$

$\therefore \quad x_1 + 150 = 200$

$x_1 = 50$

$\therefore$ point A (50, 150)

Get B, solve (i), (iii)

$$x_1 + 3x_2 = 400$$
$$x_1 + x_2 = 200$$
$$- \quad - \quad -$$
$$2x_2 = 200$$
$$x_2 = 100$$

$\therefore x_1 = 100$

point B (100, 100).

Get C; solve (i), (ii)

$$x_1 + 3x_2 = 400$$
$$x_1 + 2x_2 = 350$$
$$- \quad -$$
$$x_2 = 50$$

$\therefore x_1 = 250$

point C (250, 50)

Objective Function value,

$$Z = 200x_1 + 400x_2$$

point A (50, 150) $\therefore Z = 200(50) + 400(150)$
$= 70,000$

point B (100, 100) $\therefore Z = 200(100) + 400(100)$
$= 60,000$

point C (250, 50) $\therefore Z = 200(250) + 400(50)$
$= 70,000$

So, $x_1 = 100$, $x_2 = 100$

and, Min $Z = 60,000$ [Solved]

$\therefore$ 100 quintals of scrap each from suplier A and B. in order to minimize the total cost of purchase.

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2 Solve the following linear programming problem by simplex method.

Maximize $Z = 3x + 2y$, subject to

constraints, $x + y <= 4$

and $x - y <= 2$

and $x, y >= 0$

Answer.

By Introducing slack variables s_1, s_2 convert the problem in standard form.

Max, $Z = 3x_1 + 2y_2 + 0s_1 + 0s_2$

Subject to, $x_1 + y_2 + s_1 = 4$

$x_1 - y_2 + s_2 = 2$

Initial simplex table,

		c_j	3	2	0	0	
C_B	Basic	x_B	x_1	y_1	s_1	s_2	min $\frac{x_B}{x_1}$
0	s_1	4	1	1	1	0	$4/1 = 4$
0	s_2	2	1	-1	0	1	$2/1 = 2$ (min)
	Z_j	0	0	0	0	0	
	$Z_j - C_j$		-3 ↑	-2	0	0	

[where $Z_j = C_B . a_j$]

Leaving variables in the basic variable s_2, This row is called the key row.

New element = Old element - $\left[\frac{\text{product of elements in key row and key column}}{\text{key element}}\right]$

First Iteration,

		C_j	3	2	0	0	
C_B	Basic	x_B	x_1	y_1	s_1	s_2	Min $\frac{x_B}{y_1}$
0	s_1	2	0	2	1	-1	$2/2 = 1$
3	x_1	2	1	-1	0	1	$2/-1 = -2$
	Z_j	6	3	-3	0	3	
	$Z_j - C_j$		0	-5	0	3	

↑

[① $1 - \frac{1 \times 1}{1} = 0$ ⓘⓘⓘ $1 - \frac{1 \times 0}{1} = 1$

ⓘ $1 - \frac{1(-1)}{1} = 2$ ⓘⓥ $0 - \frac{1 \times 1}{1} = -1$

∴ $4 - \frac{2 \times 1}{1} = 2$]

Second Iteration,

		C_j	3	2	0	0
C_B	Basic	x_B	x_1	y_1	s_1	s_2
2	y_1	1	0	1	$1/2$	$-1/2$
3	x_1	3	1	0	$1/2$	$-1/2$
	Z_j	11	3	2	$5/2$	$1/2$
	$Z_j - C_j$		0	0	$5/2$	$1/2$

[① $1 - \frac{(0 \times -1)}{2} = 1$ ⓘⓘⓘ $\cancel{1/2}\ 0 - \left(\frac{1 \times -1}{2}\right) = 1/2$ $2 - \frac{(2 \times -1)}{2} = 3$]

ⓘ $-1 - \frac{(2 \times -1)}{2} = 0$ ⓘⓥ $1 - \left(\frac{-1 \times -1}{2}\right) = -1/2$

So, all $Z_j - C_j \geq 0$, the solution is optimum.

The optimum solution is,

Max $Z = 11$

$x = 3$

and $y = 2$ [solved]

3. Solve the following linear programming problem by graphical method,

Maximum $Z = 3x + 2y$, subject to

Contraints : $x + y \leq 4$

and $x - y \leq 2$

and $x, y \geq 0$

Answer :

Now we can write the equations like,

$$x + y = 4 \quad \text{--- (i)}$$
$$x - y = 2 \quad \text{--- (ii)}$$

1st equation

$$x + y = 4 \quad \text{--- (i)}$$

When, $x = 0$, $y = 4$

When, $y = 0$, $x = 4$

x	0	4
y	4	0

2nd equation

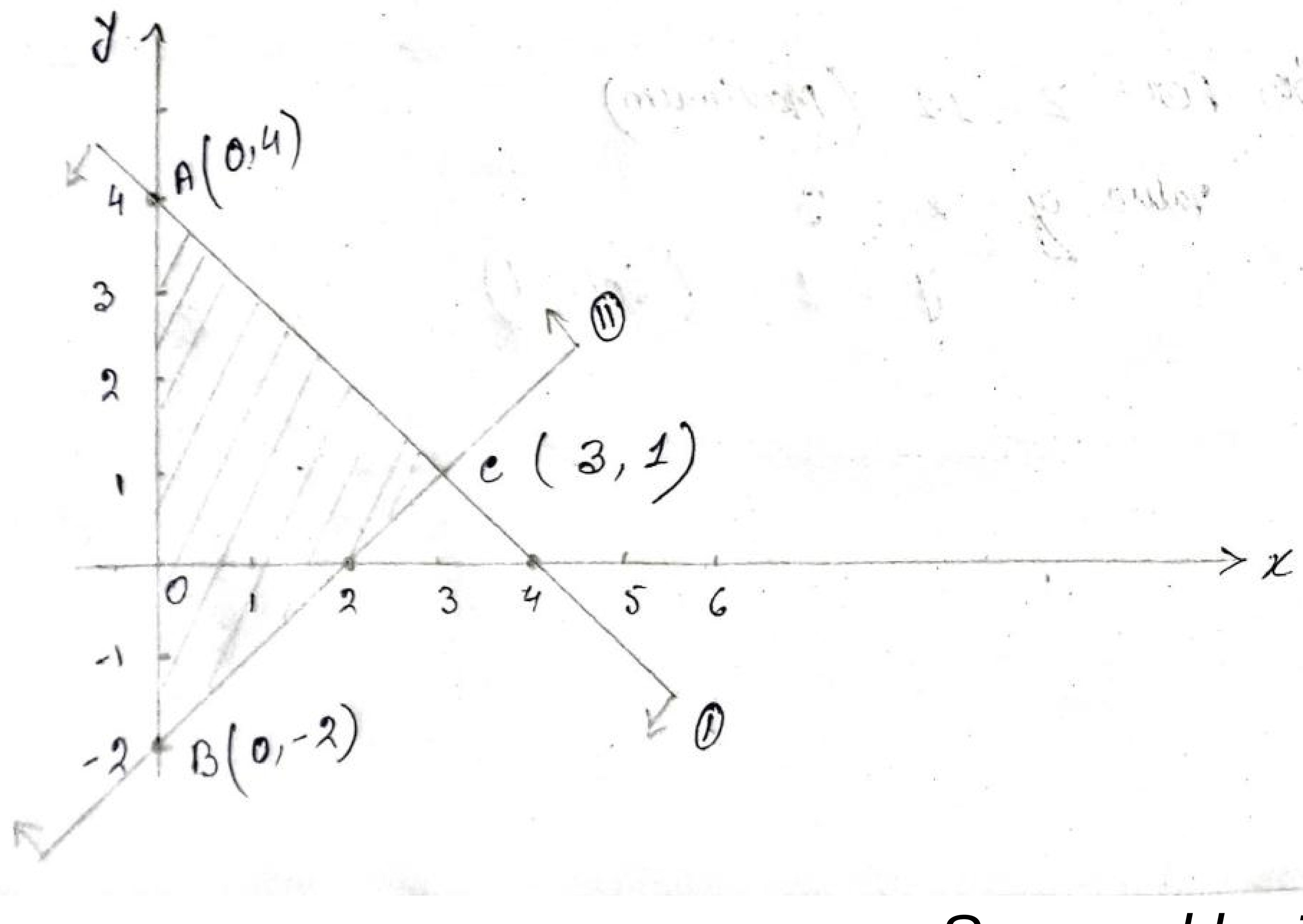
$$x - y = 2 \quad \text{--- (ii)}$$

when, $x = 0$, $y = -2$

when, $y = 0$, $x = 2$

x	0	2
y	-2	0

put it in Graph,



we get point A $(0,4)$
B $(0,-2)$

we can get point C,
by solving eq ① & ②

$\therefore \quad x + \cancel{y} = 4$
$\quad\;\; x - \cancel{y} = 2$

$2x = 6$

$x = 3$

$\therefore \quad y = 4 - 3 = 1 \qquad \therefore (3,1)$

$\therefore$ we get point C $(3,1)$

$\therefore$ Objective Function Value,

Maximum $Z = 3x + 2y$

point A $(0,4)$ $\quad \therefore \; Z = 3 \times (0) + 2(4)$
$= 8$

point B $(0,-2)$ $\quad \therefore \; Z = 3 \times (0) + 2(-2)$
$= -4$

point C $(3,1)$ $\quad \therefore \; Z = 3 \times (3) + 2 \times (1)$
$= 11$ (Maximum)

So, for $Z = 11$ (Maximum)

value of $x = 3$
$y = 1$ (Solved)

4. Compare Big-M method and Two-phase method with simplex method.

Answer

⇒ Compare Big-M method with simplex method :-

1. The Big-M method is more modernized than the simplex method. The simplex method is the method used for linear programming and is developed. While Big-M method is more advancd method of solving problems of linear programming.

2. The simplex algorithm is original and still one of the most widely used methods for solving linear maximization problems. However, to apply it, the origin (all variables equal to 0) must be a feasible point. The "Big-M" refers to a large number associated with the artificial variables, represented by "M".

3. Step-by-step explanation : Big-M method for finding the solution for a linear problem with simplex method. ~~And~~

⇒ Compare Two-phase method with simplex method :-

1. Simplex method is an iterative procedure for solving LPP in a finite number of steps while two phase method is another method to solve a given LPP involving some artificial variables and the solution is obtained in two phases.

2. In simplex method is repeated and since the number of vertices is finite, the method leads to an optimal vertex in a finite number of steps or indicates the existence of unbounded solution where as in phase Two method.

Phase I ; we construct the LPP leading to a final simplex method table containing a basic feasible solution, to the original problem, and.

Phase II, apply simplex method to the modified simplex table obtained in the end of phase I, till an optimum basic feasible solution is obtained or till there is an indication of unbounded solution.