manipal PROlearn

Session 2

Time value of Money

Course Plan

Date & Time	Session No.	Name of the Unit	Topic	Assessment
30-05-2018 09.00AM -05:00 PM	1	An Introduction to Financial Management & Planning	Forms of Business Orgninsation	
			Financial Decissions in a Firm	
			Finance Functions	
			The fundamental principle of Finance	
			Financial System - Functions	
			Growth & Trends in the Indian Financial System	
			Financial Staments & Ratio Analysis	
31-05-2018 09.00AM -05:00 PM	2	Time Value of Money	Power of Compounding	Quiz-1
			Power of Discounting	
			Annuity	
			Perpetuity	
07-06-2018 09:00AM -05:00 PM	3	Valuation of Bonds & Stocks	Valuation of Bonds	Assignment_1 announcement
			Valuation of Stocks	
08-06-2018 09.00AM -05:00 PM	4	Leverage	Operating Leverage	
			Financial Leverage	
			Combined Leverage	
14-06-2018 09.00AM -05:00 PM	5	Cost of Capital	Concept of Average Cost of Capital	Quiz-2; Assignment_2 announcement
			Cost of Optimum Capital Structure	
			Cost of different sources of finance	
18-06-2018 09.00AM -05:00 PM	6	Capital Structture	Factors affecting Cost of Capital	
			Theories of Capital Structure	
19-06-2018 09.00AM -05:00 PM	7	Capital Budgeting	Importance of Capital Budgeting	
			Capital Budgeting Process	
20-06-2018 09.00AM -05:00 PM	8		Techniques of Capital Budgeting	Assignment_3 announcement
			Importance of Capital Rationing and its Process	
			Types and sources of risks in Capital	
25-06-2018 09.00AM -05:00 PM	9		Techniques used in Capital Budgeting Decision	
			CAPM Model	
26-06-2018 09.00AM -05:00 PM	10	Working Capital Manaement	Itroduction to Working Capital	
			Operating Cycle; Determinants of Working Capital	
			Estimation of Working Capital; Cash Management	
27-06-2018 09.00AM -05:00 PM	11		Inventory Management	Quiz-3
			Receivables Management	

Learning Objectives:

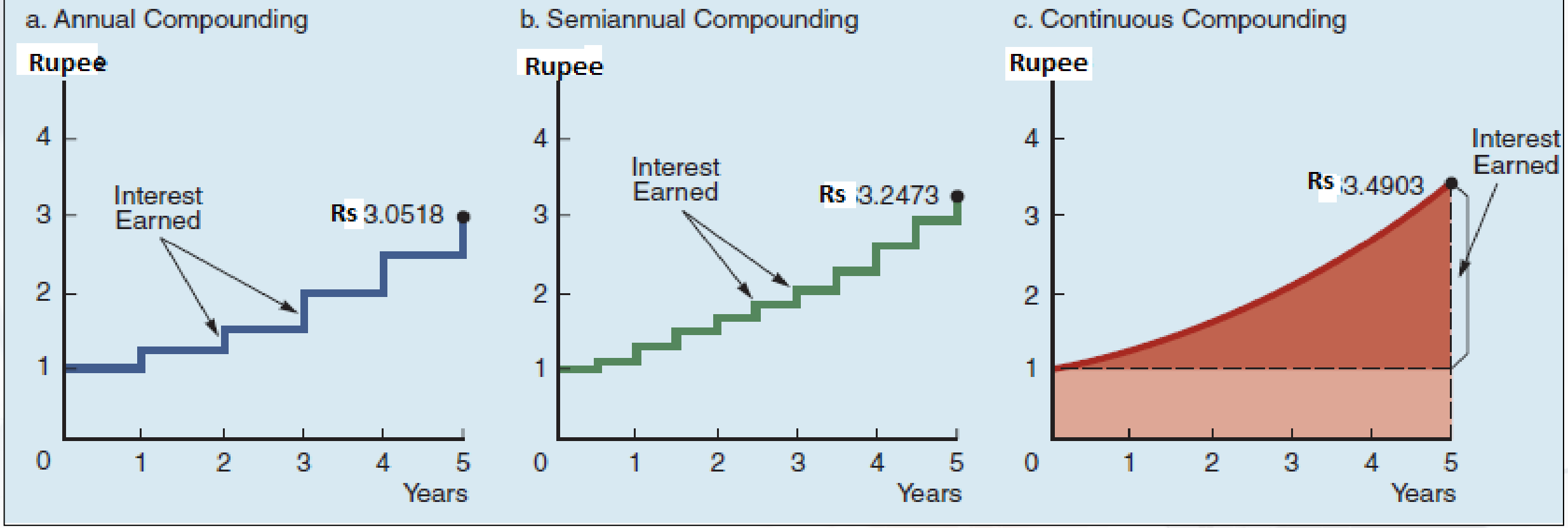
- Calculate the future and present value of single amount
- Calculate the future and present value of an annuity
- Preparation of amortization table.
- Understanding the impact of effective, nominal, real rate of interest.
- Calculate the present value of perpetuity
- Applying the concepts of time value of money in real-time.

Time Value of Money

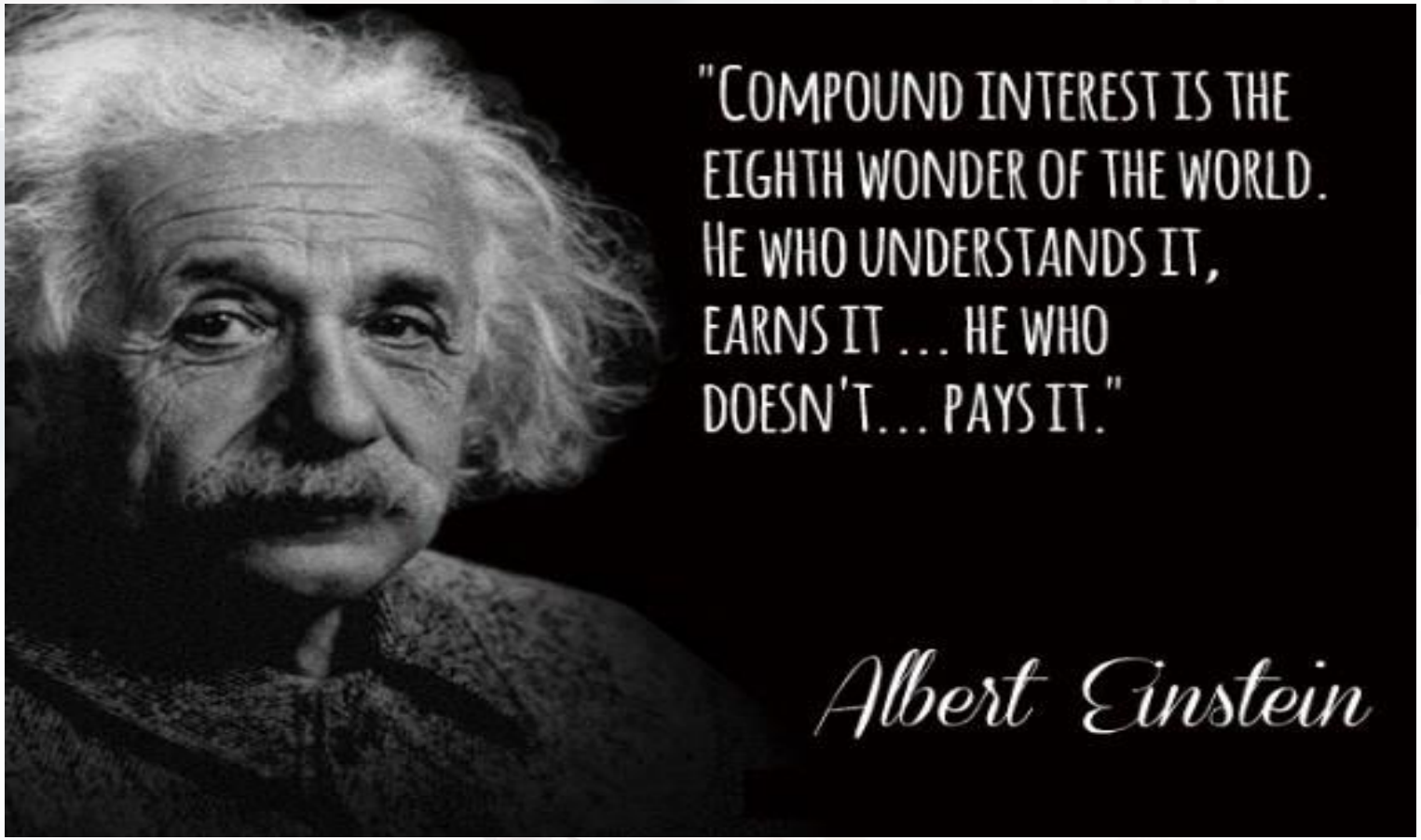
Sources: https://www.andertoons.com/money/cartoon/5105/time-money-order-more-clocks

WHY TIME VALUE

The more frequent the compounding period, the larger the final compounded amount as interest is earned on interest more often. Future value of Rs1 with r of 25%.



Power of Compounding



Sources: http://alpashah.in/2016/07/21/power-of-compounding-wealthstart-commanding-your-wealth/

A rupee in hand today is worth more than a rupee to be received in the future because, if you had it now, you could invest it, earn interest, and end up with more than a rupee in the future. The process of going to future values (FVs) from present values (PVs) is called compounding.

To illustrate, refer to our 3-year time line and assume that you plan to deposit Rs100 in a bank that pays a guaranteed 5% interest each year. How much would you have at the end of Year 3?

Summary of Future Value Calculations

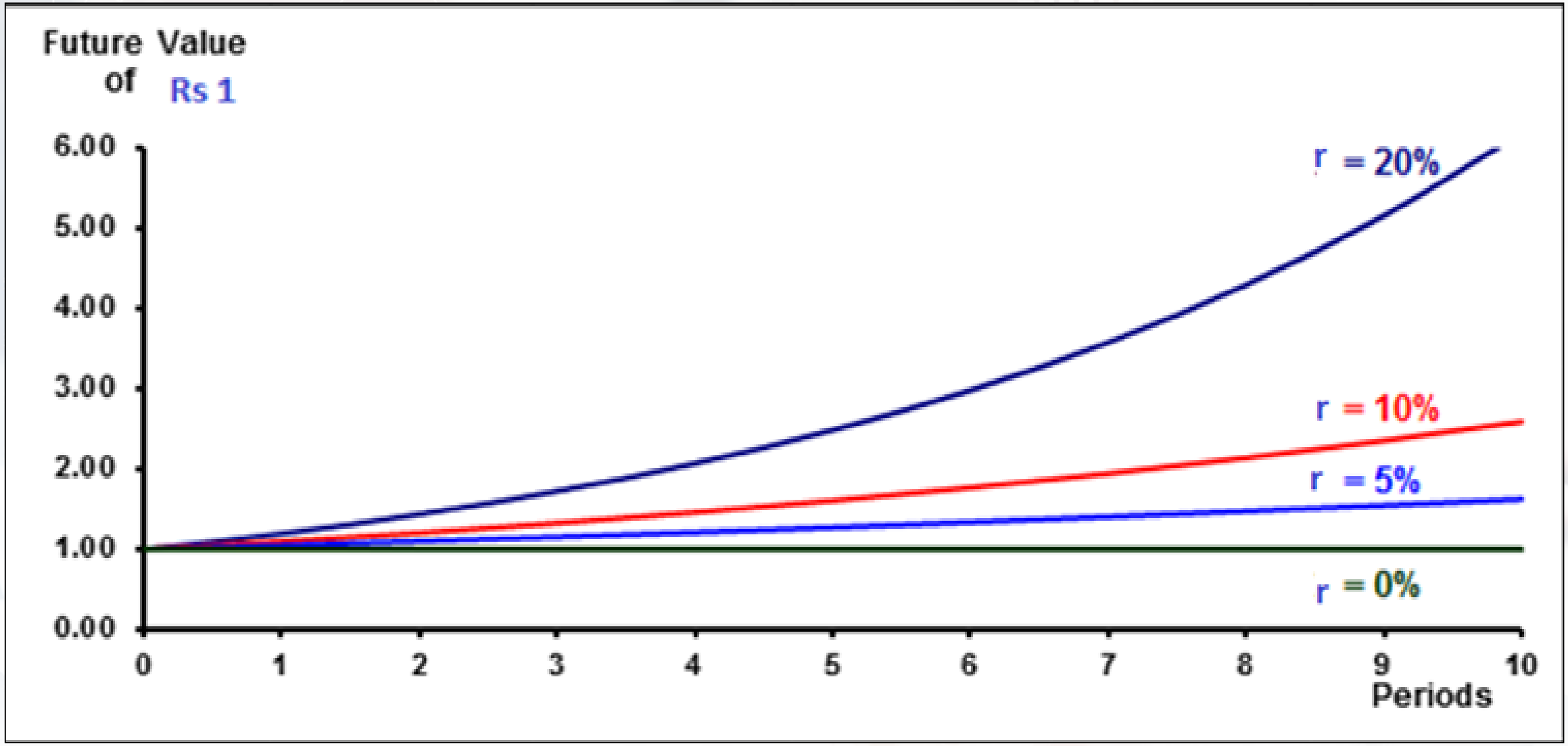
Investment = CF_0 = PV = -Rs100.00
Interest rate = r = 5.00%
No. of periods = N = 3

Step-by-Step Approach: Rs 100 → Rs105.00 → Rs 110.25 → Rs115.76

Formula Approach: $FV_N = PV(1+r)^N$ FV_N = Rs100(1.05)^3 = Rs115.76

$$FV_1 = PV + Int = PV + PV\ X\ \ r = PV(1+r)$$

$$FV_2 = FV_1(1+r) = PV(1+r)(1+r) = PV(1+r)^2$$


Future Value of Rs 1
6.00
5.00
4.00
3.00
2.00
1.00
0.00
r = 20%
r = 10%
r = 5%
r = 0%
0
1
2
3
4
5
6
7
8
9
10
Periods

DOUBLING PERIOD

Thumb Rule : Rule of 72

$$\text{Doubling period} = \frac{72}{\text{Interest rate}}$$

Interest rate : 15 percent

$$\text{Doubling period} = \frac{72}{15} = 4.8 \text{ years}$$

A more accurate thumb rule : Rule of 69

$$\text{Doubling period} = 0.35 + \frac{69}{\text{Interest rate}}$$

Interest rate : 15 percent

$$\text{Doubling period} = 0.35 + \frac{69}{15} = 4.95 \text{ years}$$

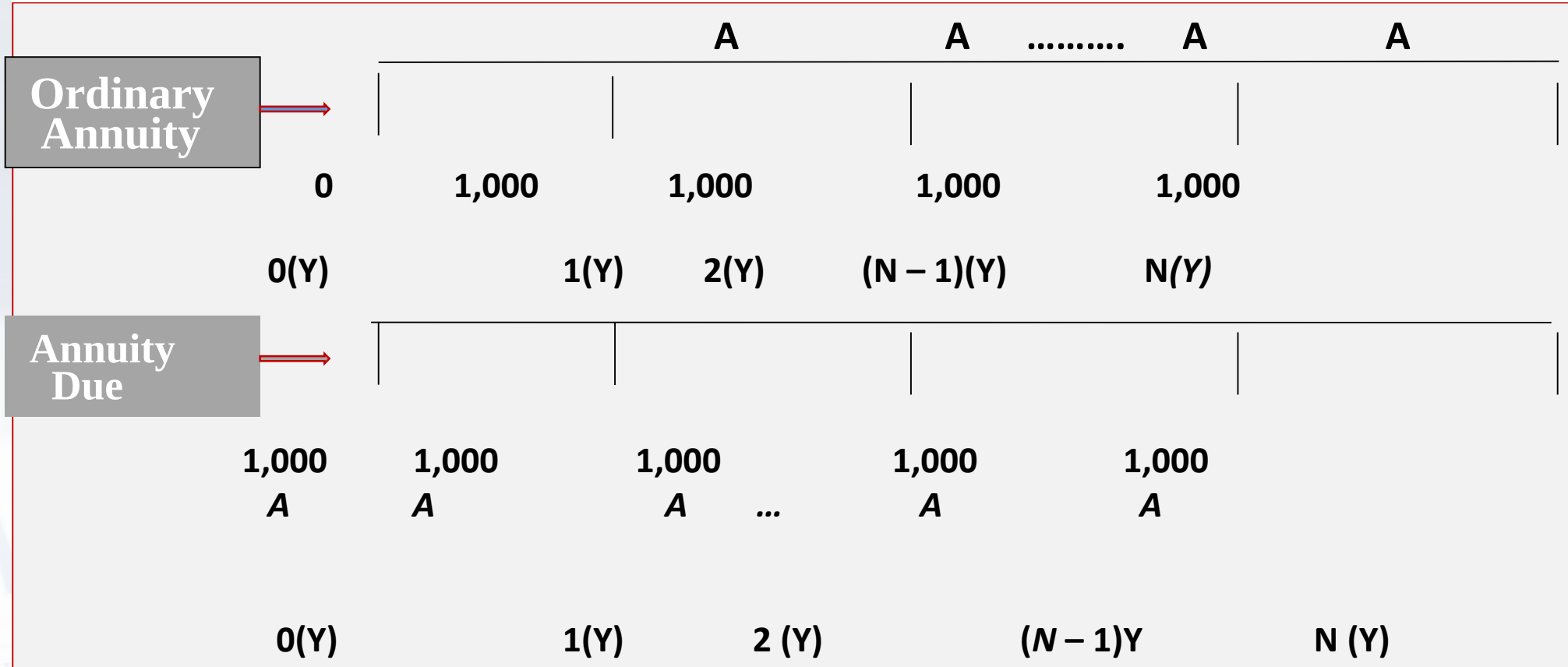
PV = F V*n* [1/ (1 + *r*)^n]= F V*n* X $PVIF_{r,n}$, $PVIF_{r,n}$= [1/ (1 + *r*)^n]

Like F V*n*= *PV* X $PVIF_{r,n}$, $PVIF_{r,n}$= (1 + *r*)^n

Year	*6%*	*8%*	*10%*	*12%*	*14%*
1	0.943	0.926	0.909	0.893	0.877
2	0.890	0.857	0.826	0.797	0.770
4	0.792	0.735	0.683	0.636	0.592
6	0.705	0.630	0.565	0.507	0.456
8	0.626	0.540	0.467	0.404	0.351
10	0.558	0.463	0.386	0.322	0.270
12	0.497	0.397	0.319	0.257	0.208

ANNUITY

A stream of uniform periodic cash flows
Ordinary: Cash flow starts at the end of 1st year, Annuity Due: Cash flow Starts from day 1.



Thus , Annuity due value = Ordinary annuity value (1 + *r*)

This applies to both present and future values

FUTURE VALUE OF AN ORDINARY ANNUITY

$$Future value\ of\ the\ annuity: (3 Years)$$

$$Rs100(1.05)^2 + Rs100(1.05) + Rs100 = Rs315.25$$

$$FVA_3 = A(1+r)^{3-1} + A(1+r)^{3-2} + A(1+r)^{3-3}$$

$$FVA_N = A(1+r)^{N-1} + A(1+r)^{N-2} \ldots\ldots A(1+r) + A \ldots\ldots\ldots (1)$$

$$FVA_N(1+r) = A(1+r)^N + A(1+r)^{N-1} \ldots A(1+r)^2 + A(1+r) \ldots\ldots (2)$$

$$(2) - (1), FVA_N r = A(1+r)^N - A = A\{(1+r)^N - 1\}$$

$$FVA_N = A\{(1+r)^N - 1\} / r,$$

$$FVIFA_{r,N}(Future\ Value\ \ Intersest\ Factor\ \ Annuity) = \frac{(1+r)^N - 1}{r}$$

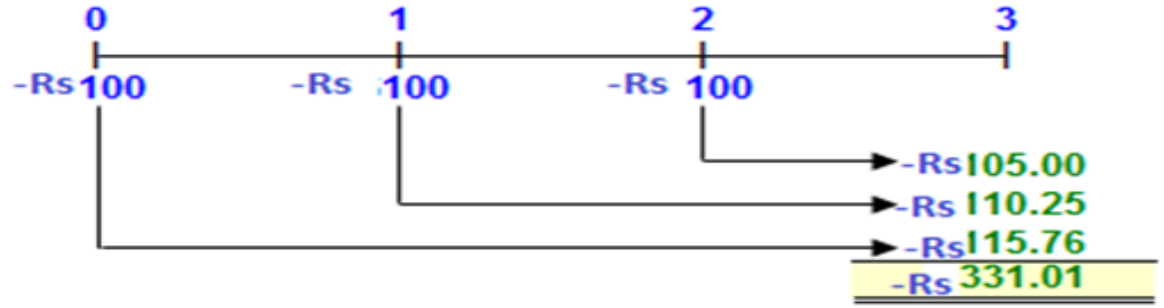
FUTURE VALUE OF AN ANNUITY DUE

Since each payment occurs one period earlier with an annuity due, the payments will all earn interest for one additional period.

$$FVA_{Due} = FVA_{Ord}(1+r)$$

Future Value of an Annuity Due

Payment amount = PMT = Rs 100.00
Interest rate = r = 5.00%
Number of periods = N = 3

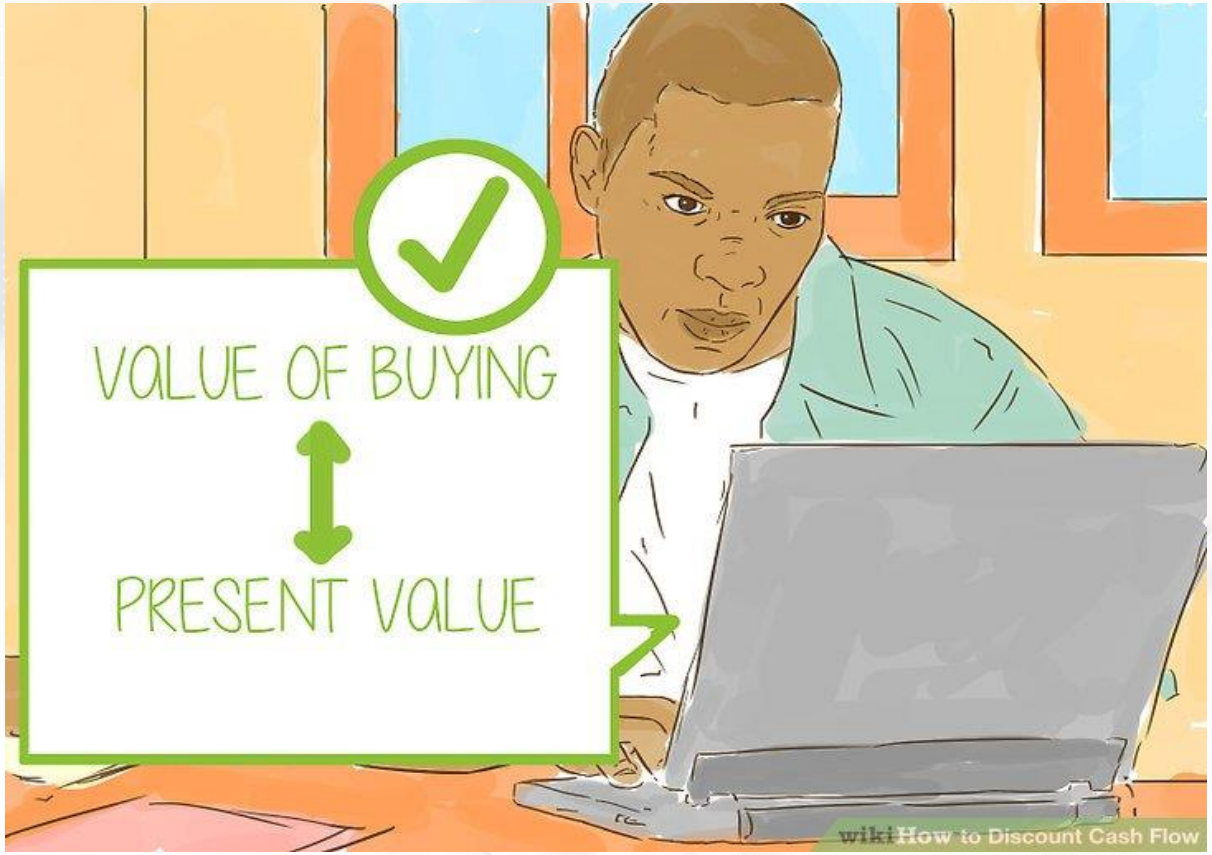


Step-By-Step Approach.
Multiply each payment by $(1+r)^{N-t}$ and sum these FVs to find FVA_N:

Formula Approach:

$$FVA_{N(due)} = PMT \times \left(\frac{(1+r)^N}{r} - \frac{1}{r} \right) x(1+r) = \text{Rs } 331.01$$

Power of Discounting



Source: https://www.wikihow.com/Discount-Cash-Flow

Summary of Present Value Calculations

Future payment = CF_N = FV = Rs 115.76

Interest rate = r = 5.00%

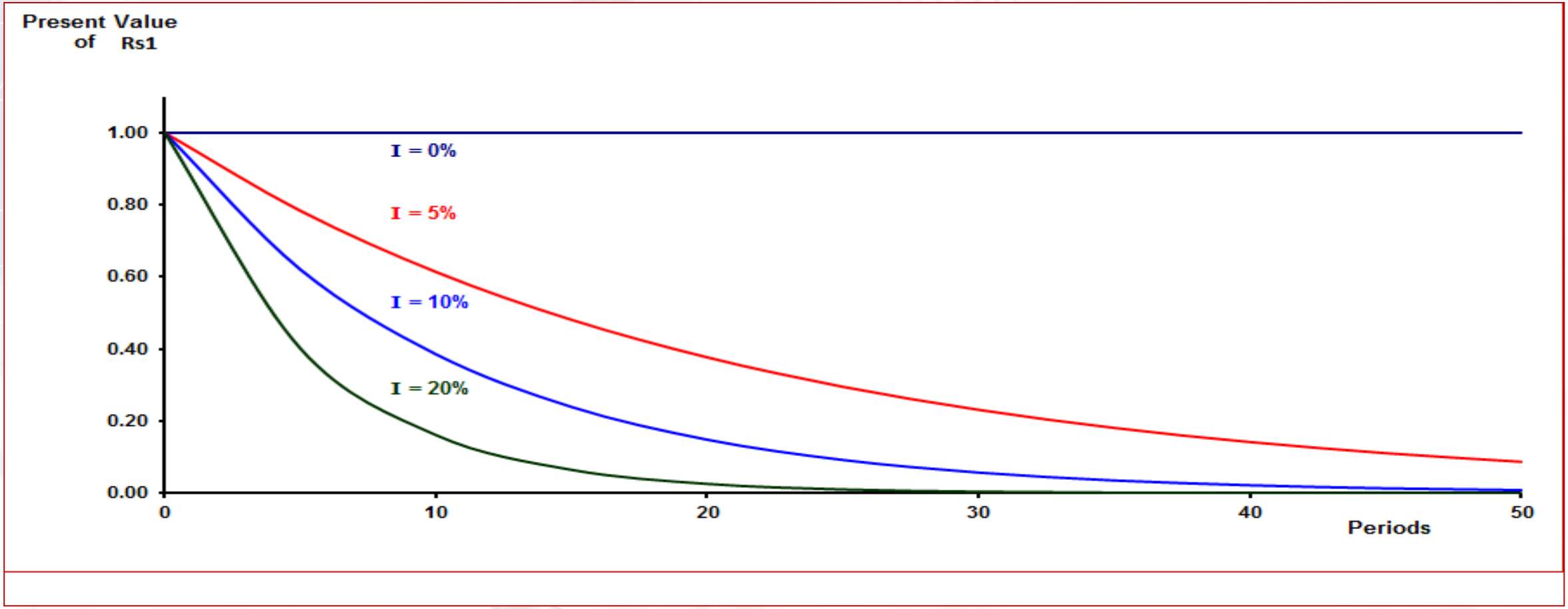
No. of periods = N = 3

Periods: 0 1 2 3

Cash Flow Time Line: PV = ? ← ← ← Rs 115.76

Step-by-Step Approach: Rs 100.00 ← Rs 105.00 ← Rs 110.25 ← Rs 115.76

Formula Approach: $PV = FV_N / (1 + r)^N$ $PV = Rs\ 115.76/(1.05)^3$ = Rs 100.00


Present Value
of Rs1
1.00
0.80
0.60
0.40
0.20
0.00
I = 0%
I = 5%
I = 10%
I = 20%
0
10
20
30
40
50
Periods

PRESENT VALUE OF AN ANNUITY :ORDINARY (Section 2.9)

Present value of an ordinary annuity is the sum of the PVs of the individual cash flows. Methods for solving the present value of an ordinary annuity are under.

$$\text{Present value of the annuity : (3 Years)}$$

$$Rs100(1.05)^{-1} + Rs100(1.05)^{-2} + Rs100(1.05)^{-3} = Rs272.32$$

$$PVA = A(1+r)^{-1} + A(1+r)^{-2} + A(1+r)^{-3} \ldots\ldots A(1+r)^{1-N} + .A(1+r)^{-N} \ldots\ldots\ldots (1)$$

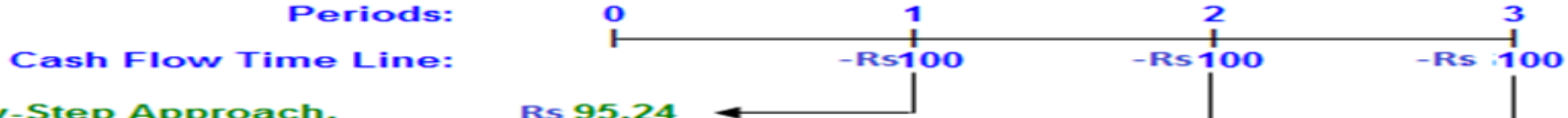
$$PVA(1+r) = A + A(1+r)^{-1} + A(1+r)^{-2} \ldots . A(1+r)^{1-N} \ldots\ldots (2)$$

$$(2)-(1), PVA\ X\ r = A - A(1+r)^{-N} = A\{(1-(1+r)^{-N}\}$$

$$PVA = A\{(1-1/(1+r)^{N}\}/r$$

Present Value of an Ordinary Annuity

Payment amount = PMT = Rs 100.00 (A)
Interest rate = r = 5.00%
Number of periods = N = 3



Step-By-Step Approach.
Divide each payment by $(1+r)^t$ and sum these PVs to find PVA_N:

Rs 95.24
Rs 90.70
Rs 86.38
Rs 272.32

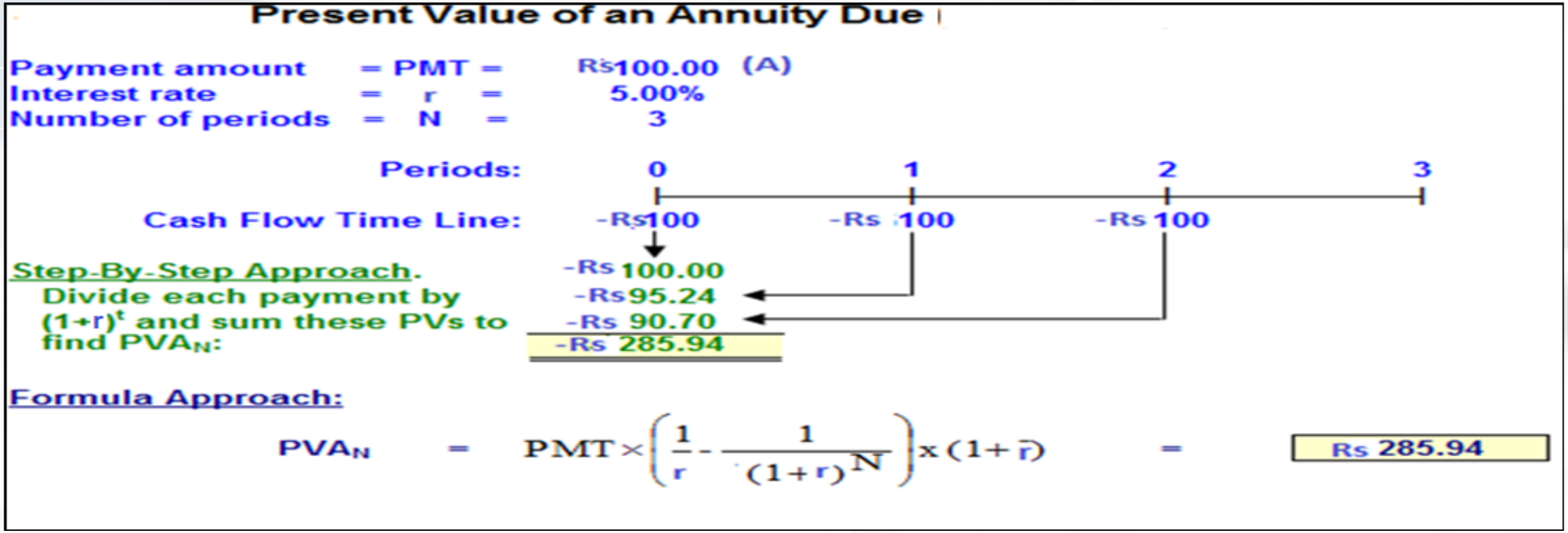
Formula Approach:

$$PVA_N = PMT \times \left(\frac{1}{r} - \frac{1}{r(1+r)^N} \right) = \text{Rs } 272.32$$

PRESENT VALUE OF AN ANNUITY DUE

The difference between the present value of an ordinary annuity and an annuity due is that payments are received earlier in an annuity due.

$$PVA_{Due} = PVA_{Ord}(1+r)$$



$$PVA_{Due} = Rs\,272.32(1.05) = Rs\,285.94$$

PRESENT VALUE OF AN UNEVEN SERIES

$$PV_n = \frac{A1}{(1+r)} + \frac{A2}{(1+r)^2} \quad \ldots\ldots \quad + \frac{An}{(1+r)^n}$$

$$= \sum_{t=1}^{n} \frac{At}{(1+r)^t} \qquad (r = 12\%, \quad t = 8 \text{ years})$$

Year	Cash Flow Rs.	$PVIF_{12\%,n}$	Present Value of Individual Cash Flow
1	1,000 (A1)	0.893	893
2	2,000 (A2)	0.797	1,594
3	2,000 (A3)	0.712	1,424
4	3,000 (A4)	0.636	1,908
5	3,000 (A5)	0.567	1,701
6	4,000 (A6)	0.507	2,028
7	4,000 (A7)	0.452	1,808
8	5,000 (A8)	0.404	2,020
Present Value of the Cash Flow Stream			13,376

PRESENT VALUE OF A PERPETUITY

A perpetuity is an annuity if infinite duration. For example, the British Government has issued bonds called "Consols" which pay yearly interest forever.

The present value of a perpetuity may be expressed as follows

$$P_{\infty} = A \; X \; PVIFA_{r,\infty}$$

Where $PVIFA_{r,\infty}$ **is the present value of interest factor of annuity,** P_{∞} **is the present value of a perpetuity and A is the constant annual payment**

$$PVIFA_{r,\infty} = \sum_{t=1}^{\infty} \frac{A}{(1+r)^t} = \frac{1}{r}$$

CONCEPT

$$PVIFA_{r,\infty} = A/(1+r) + A/(1+r)^2 .... A/(1+r)^{\alpha} \quad (Eq1)$$

$$PVIFA_{r,\infty}(1+r) = A + A/(1+r) ..... A/(1+r)^{\alpha-1} \quad (EQ2 : Multiply \; Eq1 by \; (1+r)$$

$$PVIFA_{r,\infty} r = A - A/(1+r)^{\infty} \quad (Eq2 - Eq1)$$

$$PVIFA_{r,\infty} = \frac{A}{r} \; , As \; A/(1+r)^{\infty} \; vanishes$$

EFFECTIVE VERSUS NOMINAL RATE(P: T161)

The general relationship between the effective interest rate and the stated annual interest rate is as follows:

$$r = (1+k/m)^m - 1$$

r = effective rate of interest,

k = stated annual rate of interest (Say, 12%)

m = frequency of compounding per year

If a rate is quoted as 10% compounded semiannually, means the investment actually pays 5% every six months. If you invest Rs100 at 10% pa, you will have Rs110/ at the end of the year. But if you invest 5% every six months the, value will be the summation of two six month's periods i.e 100(1.05)(1.05)=Rs110.25 i.e. Rs 0.25 more .

Effective Interest Rate with Annual compounding ⟹ $\left[1+\frac{0.12}{1}\right]^1 - 1 = 0.12(12\%)$

Effective Interest Rate with HLY compounding ⟹ $\left[1+\frac{0.12}{2}\right]^2 - 1 = 0.1236(12.36\%)$

Effective Interest Rate with Qtly compounding ⟹ $\left[1+\frac{0.12}{4}\right]^4 - 1 = 0.1255(12.55\%)$

Effective Rate %

Nominal Rate %	*Annual Compounding*	*Semi-annual Compounding*	*Quarterly Compounding*	*Monthly Compounding*
8	8.00	8.16	8.24	8.30
12	12.00	12.36	12.55	12.68

GROWING ANNUITY

The formula for the present value of a growing annuity(PVGA) is derived as follows:

$$PGVA = \frac{A(1+g)}{(1+r)} + \frac{A(1+g)^2}{(1+r)^2} \cdots\cdots \frac{A(1+g)^n}{(1+r)^n} \quad \ldots\ldots(1)$$

Multiplying both sides of (1) *by* $\frac{(1+g)}{(1+r)}$ *gives,*

$$PGVA \ X \ \frac{(1+g)}{(1+r)} = \frac{A(1+g)^2}{(1+r)^2} + \frac{A(1+g)^3}{(1+r)^3} \cdots\cdots \frac{A(1+g)^{n+1}}{(1+r)^{n+1}} \quad \ldots\ldots(2)$$

$$PGVA\left(1 - \frac{(1+g)}{(1+r)}\right) = \frac{A(1+g)}{(1+r)} - \frac{A(1+g)^{n+1}}{(1+r)^{n+1}} \qquad \textbf{(1-2)}$$

$$PVGA\frac{(r-g)}{(1+r)} = \frac{A(1+g)}{(1+r)}\left(1 - \frac{(1+g)^n}{(1+r)^n}\right)$$

$$PGVA = A(1+g)\left[\frac{1 - \frac{(1+g)^n}{(1+r)^n}}{r-g}\right]$$

PRESENT VALUE OF AN ANNUITY

Present value of an annuity = $A\left(\frac{1 - \frac{1}{(1+r)^n}}{r}\right)$

Value of $PVIFA_{r,n}$ for Various Combinations of r and n

Yr	6 %	8 %	10 %	12 %	14 %
1	0.943	0.926	0.909	0.893	0.877
2	1.833	1.783	1.737	1.690	1.647
4	3.465	3.312	3.170	3.037	2.914
6	4.917	4.623	4.355	4.111	3.889
8	~~7~~ 6.210	5.747	5.335	4.968	4.639
10	~~8~~ 7.360	6.710	6.145	5.650	5.216
12	8.384	7.536	6.814	6.194	5.660

EQUATED MONTHLY INSTALMENT

EMI CONCEPT:

Home Loan = 10,00,000, Interest = 1% p.m,

Repayment period = 180 months

$$10,00,000 = A \times \frac{[1-1/(1.01)^{180}]}{0.01}$$

or

$$10000 = A\,X \left[\left[1-\left(\frac{1}{1.01}\right)^{180}\right]\right]$$

0.8318

$$PVIFA_{r,n} \longrightarrow \left(\frac{1-\frac{1}{(1+r)^n}}{r}\right)$$

A = Rs.12,022

SHORTER COMPOUNDING PERIOD

Present Value ⟶ $$PV = FV_n\left[\frac{1}{1+r/m}\right]^{mn}$$

Future value ⟶ $$FV_n = PV(1+\frac{r}{m})^{mn}$$

Where r = nominal annual interest rate

m = number of times compounding is done in a year

n = number of years over which compounding is done

<u>Example</u> : Rs.5000, 12 percent, 4 times a year, 6 years

$FV = 5000(1+ 0.12/4)^{4x6} = 5000\ (1.03)^{24}$ = 5000 X 2.029

= Rs.10,145

NEED TO REMEMBER

(1) $FV_n = PV \ X \ FVIF_{r,n}$

(2) $FVA_n = A \, X \ FVIFA_{r,n}$

(3) $PV = FV_n \ X \ PVIF_{r,n}$

(4) $PVA_n = A \ X \ PVIFA_{r,n}$

A

LOAN AMORTISATION SCHEDULE

Loan : Rs10,00,000, where r = 15%, n = 5 years

10,00,000 = A x PVIFA (n =5, r =15%)

= A x 3.3522

Therefore, A = Rs 2,98,312

The term ,amortization, is used in two senses
(i) Repayment of loan over a period of time
(ii) Write-off of an expenditure (Issue cost of shares) over a period of time.

Year	*Beginning Amount* (1)	*Annual Instalment* (2)	*Interest*	*Principal Repayment* (3)	*Remaining Balance* (2)-(3) = (4)	(1)-(4) = (5)
1	1,000,000	298,312	150,000	148,312	851,688	
2	851,688	298,312	127,753	170,559	681,129	
3	681,129	298,312	102,169	196,143	484,986	
4	484,986	298,312	727,482	225,564	259,422	
5	259,422	298,312	38,913	259,399	23*	

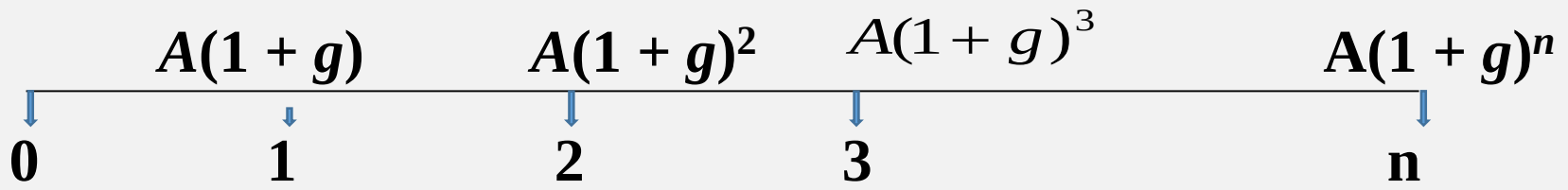
a **Interest is calculated by multiplying the beginning loan balance by the interest rate.**

b. **Principal repayment is equal to annual instalment minus interest.**

* **Due to rounding off error a small balance is shown**

PRESENT VALUE OF A GROWING ANNUITY

A cash flow that grows at a constant rate for a specified period of time is a growing annuity. The time line of a growing annuity is shown below:



The present value of a growing annuity can be determined using the following equation:

(Arrived: Next Slide)

$$\text{PV of a Growing Annuity} = A(1+g)\left[\frac{1-\frac{(1+g)^n}{(1+r)^n}}{r-g}\right]$$

The above formula can be used when the growth rate is less than the discount rate ($g < r$) as well as when the growth rate is more than the discount rate ($g > r$). However, it does not work when the growth rate is equal to the discount rate ($g = r$) – in this case, the present value is simply tends to Zero.

Problem 1

Suppose you have decided to deposit Rs.30,000 per year in your Public Provident Fund A/C for 30 years. What will be the accumulated amount in your Public Provident Fund Account at the end of 30 years if the interest rate is 11 percent ?

The accumulated sum will be :

Rs.30,000 ($FVIFA_{11\%,30yrs}$)(Future Value of Interest Factor of Annuity)

= Rs.30,000 $\left(\dfrac{(1.11)^{30} - 1}{0.11} \right)$ → $\dfrac{(1+r)^{N} - 1}{r}$

= Rs.30,000 [199.02] [by log calculation: 198.73]

= Rs.5,970,600

Problem 2

You want to buy a house after 5 years when it is expected to cost Rs.20Lac. How much should you save annually if your savings earn a compound return of 12 percent pa?

Future value interest factor annuity(FVIFA)for a 5 year annuity, given an interest rate of 12 percent pa, is :

$$FVIFA_{n=5,\ r=12\%} = \frac{(1+0.12)^5 - 1}{0.12} = 6.383$$

The annual savings should be :

$$\text{Rs.}\ \frac{20,00,000}{6.383} = \text{Rs.}313,332/$$

Problem 3

Sudharshan has estimated that he is going to need enough in his retirement fund to withdraw 75,000 per year beginning on his 66th birthday and for 19 additional years thereafter. How much will Sudharshan need in his retirement account at age 65 if his fund is expected to earn an annual return of 9.0%?

Ans: 684640.93

(Hint: Concept : Present value annuity)

Problem 4

Shwetha, who has just celebrated her 28th birthday, will retire on her 62nd birthday, and she has just set up a retirement plan to pay her income starting on her retirement day, and to continue paying for 19 more years. Shwetha's goal is to receive $130,000 for each of these twenty years. In creating her retirement account, Shwetha has committed to set aside equal investments at the end of each year, for the next 33 years starting on her 29th birthday. If the annual interest rate is 7%, how big should Shwetha's equal investments be?

Ans: 11580

(Hint: Concept : Present & future value annuity)

Problem 5

Raghu and Ravindra are purchasing their first house. The house costs $380,000. They have put a 15% down payment (that is, an amount that banks should require you to pay out-of-pocket), but will therefore finance the rest. They are considering a fixed rate 15-year mortgage at a 6.75% APR with monthly payments. How much will the Ravindra‘s first monthly payment be?

Ans: 2859

(Hint: Concept : Present value annuity)

Problem 6

Two years ago Kavya purchased a $12,000 car; she paid $2,000 down and borrowed the rest. She took a fixed rate 48-month instalment loan at a stated rate of 8.0% per year. Interest rates have fallen during the last two years and she can refinance her car by borrowing the amount she still owes on the car at a new fixed rate of 4% per year for 2 years. Should Kavya refinance her loan? How much will she save per month for the remainder of the loan life if she decides to refinance?

Ans: 11475

(Hint: Concept : Present value annuity)

Problem 7

Shashank wants to buy a new telescope. He estimates that it will take him one year to save the money and that the telescope will cost $200. At an interest rate of 7%, how much does Shashank need to set aside today to purchase the telescope in one year?

Ans: 186

(Hint: Concept : Present value)

Problem 8

Harish has $1,400 that he invests in a safe financial instrument expected to return 4% annually. Aravind has $700 and invests in a more risky venture that is expected to return 8% annually. Who has more after 27 years? And how much does he have in FV terms?

Ans: 1554.93

(Hint: Concept : Future value)

Problem 9

Haripriya's grandparents opened a savings account for her and placed $900 in the account. The account pays 5.0% interest. Haripriya wants to be a singer and she has her heart set on a new karaoke machine. The machine costs $250. How much less will the account be worth in 10 years if she buys the karaoke machine now versus leaving the account untouched?

Ans: $244.33

(Hint: Concept : Future value)

Problem 10

Shreya is in the market for a new car. She has narrowed her search down to 2 models. Model A costs $29,000 and Model B costs $17,000. With both cars she plans to pay cash and own them for 4 years before trading in for a new car. Her research indicates that the trade in value for Model A after 4 years is 51% of the initial purchase price, while the trade in value for Model B is 44%. Shreya has no emotional attachment to either model and wants to make a strictly financial decision. The interest rate is 5%. For simplicity assume that operating and maintenance costs for the models are identical every year. Which model is the better decision and how much "cheaper" is it than the alternative?

Ans: Model B $5986.04

(Hint: Present value and future value)

Problem 11

College tuition has been rising at a rate of 8% per year. Currently the average tuition of a state college is $13,600 per year. Faraz's son Anikhan will begin college in 11 years. Faraz's portfolio is making 3% annually. How much does Faraz need to have set aside today/now to pay for 4 years of college for Anikhan? (Note: Tuition will continue to change annually and Faraz's portfolio balance will continue to accrue interest while Anikhan is in school. Also, tuition is due at the beginning of each year.)

1st year fee	₹ 22,908.19	-31710.29036
2nd year fee	₹ 24,020.24	-34247.11359
3rd year fee	₹ 25,186.27	-36986.88268
4th year fee	₹ 26,408.90	-39945.83329
total	₹ 98,523.59	

Ans: 98523.59

Problem 12

Fund Limited has an obligation to redeem Rs.5000Lakhs bonds 6 years hence. How much should the company deposit annually in a sinking fund account wherein it earns 14 percent interest to accumulate Rs.5000Lakhs in 6 years time ?

The future value interest factor for a 5 year annuity, given an interest rate of 14 percent is :

$$FVIFA_{n=6,\ r=14\%} = \frac{(1+0.14)^6 - 1}{0.14} = 8.5$$

The annual sinking fund deposit should be :

$$\frac{\text{Rs.5000lakhs}}{8.5} = \text{Rs.588.23 lakhs}$$

A fund to which a firm makes periodic contributions to facilitate retirement of debt.

Problem 13

Suppose you have the right to harvest a teak plantation for the next 20 years over which you expect to get 100,000 cubic feet of teak per year. The current price per cubic feet of teak is Rs 500, but it is expected to increase at a rate of 15 percent per year. The discount rate is 8 percent. The present value of the teak that you can harvest from the teak forest can be determined as follows

$$\text{PV of teak} = \text{Rs } 500 \times 100{,}000\ (1.08)\left[\frac{1 - \frac{1.08^{20}}{1.15^{20}}}{0.15 - 0.08}\right] = \text{Rs}55{,}20{,}31{,}200$$

If g=8%,r=15%

If r=8%,g=15%

$$\text{PV of teak} = \text{Rs } 500 \times 100{,}000\ (1.15)\left[\frac{1 - \frac{1.15^{20}}{1.08^{20}}}{.08 - .15}\right] = \text{Rs } 206{,}29{,}35{,}715/(\text{CHECK})$$

If r=g,

PV tends to zero

Problem 14

You want to take up a trip to the moon which costs Rs.10,00,000 & the cost is expected to remain unchanged in nominal terms. You can save annually Rs.50,000 to fulfill your desire. How long will you have to wait if your savings earn an interest of 12 percent ?

The future value of an annuity of Rs.50,000 that earns 12 percent is equated to Rs.10,00,000.

$$50{,}000 \times FVIFA_{n=?,12\%} = 1{,}000{,}000$$

$$50{,}000 \times \left[\frac{1.12^n - 1}{0.12}\right] = 10{,}00{,}000$$

$$1.12^n - 1 = \frac{10{,}00{,}000}{50{,}000} \times 0.12 = 2.4$$

$$1.12^n = 2.4 + 1 = 3.4$$

$$n \log 1.12 = \log 3.4$$

$$n \times 0.0492 = 0.5315$$

$$n = \frac{0.5315}{0.0492} = 10.8 \text{ years, You will have to wait for about 11 years.}$$

SUMMING UP

- **Money has time value. A rupee today is more valuable than a rupee a year hence.**
- **The general formula for the future value of a single amount is :**

$$\text{Future value} = \text{Present value } (1+r)^n$$

- **The value of the compounding factor, $(1+r)^n$, depends on the interest rate (r) and the life of the investment (n).**
- **According to the rule of 72, the doubling period is obtained by dividing 72 by the interest rate.**
- **The general formula for the future value of a single cash amount when compounding is done more frequently than annually is:**

$$\text{Future value} = \text{Present value } [1+r/m]^{m*n}$$

SUMMING UP: Contd

- An annuity is a series of periodic cash flows (payments and receipts) of equal amounts. The future value of an annuity is:

$$\text{Future value of an annuity} = \text{Constant periodic flow(A)} \left[(1+r)^n - 1\right)/r]$$

- The process of discounting, used for calculating the present value, is simply the inverse of compounding. The present value of a single amount is:

$$\text{Present value} = \text{Future value} \times 1/(1+r)^n$$

- The present value of an annuity is:

$$\text{Present value of an annuity} = \text{Constant periodic flow(A)} \left[1 - 1/ (1+r)^n\right] /r$$

- A perpetuity is an annuity of infinite duration. In general terms:

$$\text{Present value of a perpetuity} = \text{Constant periodic flow(A)} [1/r]$$

THANK YOU

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