

11.16.3.3.2

EE24BTECH11033 - Kolluru Suraj

Question:

A die is rolled, Find the probability that a number greater than 3 will appear

Solution:

Textual solution:

Probability of a given event 'A'(A: Outcome is greater than 3),

$$P(A) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2} \quad (0.1)$$

Computational solution:

COMPUTATION OF PROBABILITIES FOR ROLLING A DIE

To compute the probability of obtaining specific outcomes when rolling a six-sided die, we rely on two key concepts: the **Probability Mass Function (PMF)** and the **Cumulative Distribution Function (CDF)**.

Definitions

Probability Mass Function (PMF): The PMF represents the probability of each individual outcome in the sample space S . For a six-sided die:

$$S = \{1, 2, 3, 4, 5, 6\},$$

the PMF is given as:

$$P(X = x) = \begin{cases} \frac{1}{6}, & x \in S, \\ 0, & x \notin S. \end{cases}$$

Cumulative Distribution Function (CDF): The CDF represents the cumulative probability of outcomes up to a given value x , defined as:

$$F(x) = P(X \leq x) = \sum_{k=1}^x P(X = k).$$

For a six-sided die:

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{x}{6}, & x \in \{1, 2, 3, 4, 5, 6\}, \\ 1, & x > 6. \end{cases}$$

Simulation Process

We simulate the rolling of a die using the following steps:

- 1) A six-sided die produces outcomes in the set:

$$S = \{1, 2, 3, 4, 5, 6\}.$$

- 2) For each simulated roll, a random integer X is generated such that $X \in S$, using a random number generator function:

$$X = (\text{rand}() \bmod 6) + 1.$$

- 3) The number of occurrences of each outcome is tracked over N trials, where N is the total number of simulations.
- 4) Both the PMF and CDF are computed:
 - ****PMF****: The frequency of each outcome is divided by the total trials to compute the probability of each face.
 - ****CDF****: The cumulative probabilities are calculated as the running total of the PMF values.

Calculation of Probabilities

Probability of Each Outcome (PMF): The probability of rolling each face i ($i \in \{1, 2, 3, 4, 5, 6\}$) is computed as:

$$P(i) = \frac{\text{Number of rolls resulting in } i}{N}.$$

Cumulative Probability (CDF): The cumulative probability up to face i is:

$$F(i) = \sum_{k=1}^i P(k).$$

Probability of Rolling $X > 3$: The probability of rolling a number greater than 3 is:

$$P(X > 3) = \frac{\text{Number of rolls resulting in } X > 3}{N}.$$

For a standard six-sided die, $P(X > 3) = P(4) + P(5) + P(6) = \frac{3}{6} = \frac{1}{2}$.

Output Representation

The computed probabilities are represented in two forms:

- ****PMF****: The probabilities of rolling each face $\{1, 2, 3, 4, 5, 6\}$, as well as the probability of $X > 3$.
- ****CDF****: The cumulative probabilities up to each face, $\{1, 2, 3, 4, 5, 6\}$, showing the cumulative likelihood of outcomes.

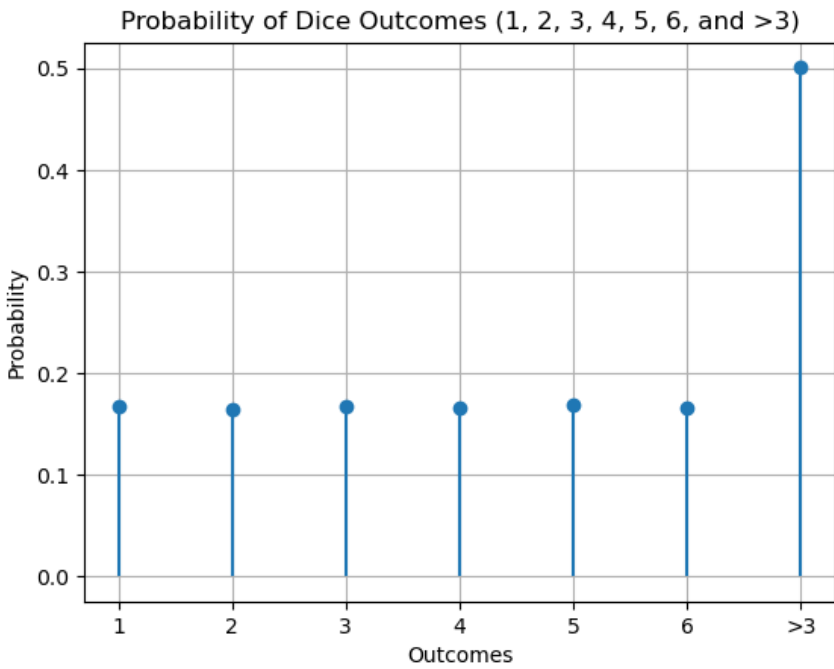


Fig. 4.1: Solution of the system of linear equations