9.4.4

EE24BTECH11033 - KOLLURU SURAJ

Question: $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

Solution:
Divide the given equation with $\tan x \tan y$

$$\frac{\sec^2 x \, dx}{\tan x} + \frac{\sec^2 y \, dy}{\tan y} = 0 \tag{0.1}$$

Substitute $\tan x$ as u and $\tan y$ as v

$$\frac{du}{u} + \frac{dv}{v} = 0 \tag{0.2}$$

Integrate

$$\int \frac{du}{u} + \int \frac{dv}{v} = \int 0 \tag{0.3}$$

$$\ln u + \ln v = a \tag{0.4}$$

$$\ln uv = a \tag{0.5}$$

$$\tan x \tan y = e^a \tag{0.6}$$

e^a can be written as another constant c

$$\tan x \tan y = c \tag{0.7}$$

Here no initial condition is given so let us take $X_0 = \pi/4, Y_0 = \pi/4$ which gives $c = 1$

$$\tan x \tan y = 1 \tag{0.8}$$

$$\tan y = \cot x \tag{0.9}$$

$$y = \tan^{-1}(\cot x) \tag{0.10}$$

$$y = \frac{\pi}{2} - x \tag{0.11}$$

Now let us this computationally from the definition of $\frac{dy}{dx}$

$$Y_{n+1} = Y_n + \frac{dy}{dx}.h \tag{0.12}$$

From the differential equation

$$\frac{dy}{dx} = \frac{y_n - x_n}{y_n + x_n}.h \tag{0.13}$$

$$y_{n+1} = y_n + \left(\frac{y_n - x_n}{y_n + x_n}\right).h \tag{0.14}$$

BY taking x_0=0 and y_0=1 and h=0.01 by iterating through the loop a 100 times and finding $y_2, y_3, y_4, \cdots$ and plotting the graph. we can verify the function we got by solving the differential equation mathematically

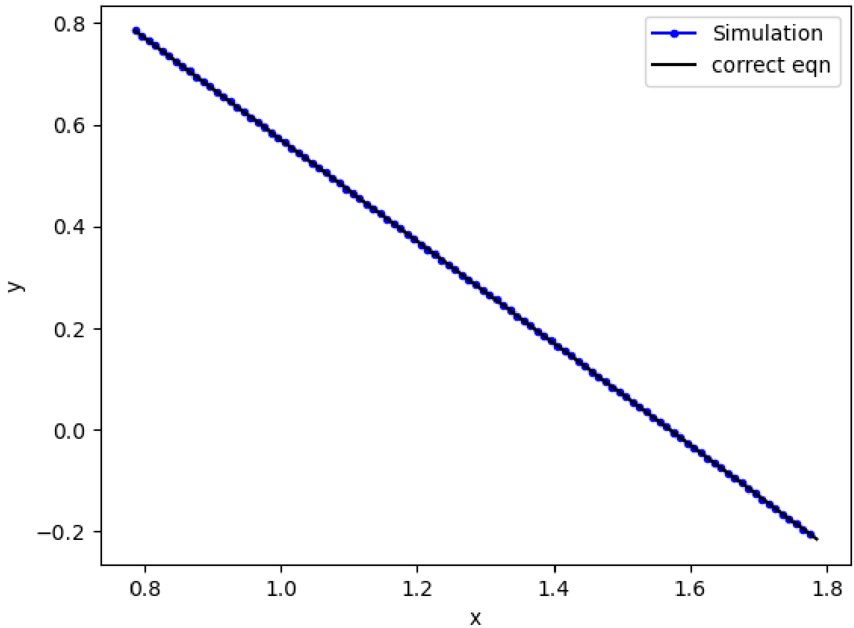

Fig. 0.1