# 11.16.3.3.2

EE24BTECH11033 - Kolluru Suraj

**Question**:
A die is rolled, Find the probability that a number greater than 3 will appear
**Solution:**
**Textual solution:**
Probability of a given event 'A'(A: Outcome is greater than 3),

$$P(A) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2} \tag{0.1}$$

**Computational solution:**

## Computation of Probabilities for Rolling a Die

To compute the probability of obtaining specific outcomes when rolling a six-sided die, we rely on two key concepts: the **Probability Mass Function (PMF)** and the **Cumulative Distribution Function (CDF)**.

### *Definitions*

*Probability Mass Function (PMF):* The PMF represents the probability of each individual outcome in the sample space $S$. For a six-sided die:

$$S = \{1, 2, 3, 4, 5, 6\},$$

the PMF is given as:

$$P(X = x) = \begin{cases} \frac{1}{6}, & x \in S, \\ 0, & x \notin S. \end{cases} \tag{0.2}$$

*Cumulative Distribution Function (CDF):* The CDF represents the cumulative probability of outcomes up to a given value $x$, defined as:

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{x}{6}, & x \in \{1, 2, 3, 4, 5, 6\}, \\ 1, & x > 6. \end{cases} \tag{0.3}$$

The table below shows the CDF for a six-sided die:

| Outcome (x) | $F_X(x)$ |
|---|---|
| 1 | 0.1667 |
| 2 | 0.3332 |
| 3 | 0.4998 |
| 4 | 0.6666 |
| 5 | 0.8335 |
| 6 | 1.0000 |

The CDF starts with the PMF of $x = 1$ and accumulates to 1.0 at $x = 6$, confirming the correctness of the cumulative probabilities.

**Getting required Probability using CDF:**

The probability $P(X > 3)$ can be derived using the CDF:

$$P(X > 3) = 1 - P(X \leq 3) = 1 - F(3). \tag{0.4}$$

From the simplified CDF:

$$F(3) = \frac{3}{6} = 0.5. \tag{0.5}$$

Thus:

$$P(X > 3) = 1 - 0.5 = 0.5. \tag{0.6}$$

*Simulation Process*

We simulate the rolling of a die using the following steps:

1) A six-sided die produces outcomes in the set:

$$S = \{1, 2, 3, 4, 5, 6\}.$$

2) For each simulated roll, a random integer $X$ is generated such that $X \in S$, using a random number generator function:

$$X = (\text{rand}() \mod 6) + 1. \tag{2.1}$$

3) The number of occurrences of each outcome is tracked over $N$ trials, where $N$ is the total number of simulations.
4) Both the PMF and CDF are computed:
   - **PMF**: The frequency of each outcome is divided by the total trials to compute the probability of each face.
   - **CDF**: The cumulative probabilities are calculated as the running total of the PMF values.

*Calculation of Probabilities*

*Probability of Each Outcome (PMF):* The probability of rolling each face $i$ ($i \in \{1, 2, 3, 4, 5, 6\}$) is computed as:

$$P(i) = \frac{\text{Number of rolls resulting in } i}{N}. \tag{4.1}$$

*Cumulative Probability (CDF):* The cumulative probability up to face $i$ is:

$$F(i) = \sum_{k=1}^{i} P(k). \tag{4.2}$$

*Probability of Rolling $X > 3$:* The probability of rolling a number greater than 3 is:

$$P(X > 3) = \frac{\text{Number of rolls resulting in } X > 3}{N}. \tag{4.3}$$

For a standard six-sided die, $P(X > 3) = P(4) + P(5) + P(6) = \frac{3}{6} = \frac{1}{2}$.

*Output Representation*

The computed probabilities are represented in two forms:

- **PMF**: The probabilities of rolling each face $\{1, 2, 3, 4, 5, 6\}$, as well as the probability of $X > 3$.
- **CDF**: The cumulative probabilities up to each face, $\{1, 2, 3, 4, 5, 6\}$, showing the cumulative likelihood of outcomes.

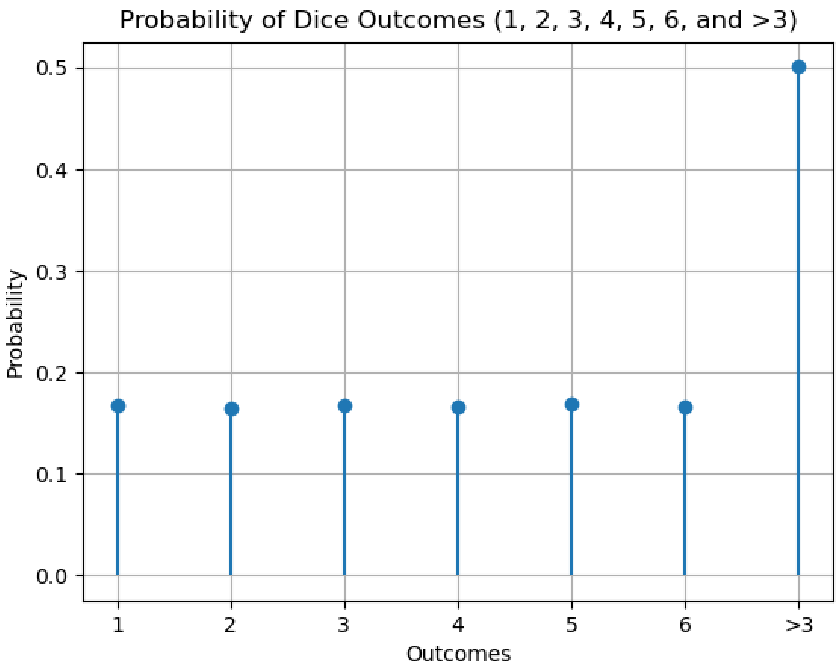


Fig. 4.1: Solution of the system of linear equations