10.3.2.3.2

EE24BTECH11033 - Kolluru Suraj

Question:
On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}, and \frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent

$$2x - 3y = 8, 4x - 6y = 9 \tag{0.1}$$

Theoritical Solution: To determine whether the given pair of linear equations is consistent or inconsistent, we compare the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$, and $\frac{c_1}{c_2}$, where:

$$a_1x + b_1y = c_1 \quad \text{and} \quad a_2x + b_2y = c_2 \tag{0.2}$$

From the equations:

$$2x - 3y = 8 \quad \text{and} \quad 4x - 6y = 9, \tag{0.3}$$

we identify:

$$a_1 = 2,\ b_1 = -3,\ c_1 = 8,\ a_2 = 4,\ b_2 = -6,\ c_2 = 9. \tag{0.4}$$

Now calculate the ratios:

$$\frac{a_1}{a_2} = 2, \quad \frac{b_1}{b_2} = 2, \quad \frac{c_1}{c_2} = \frac{9}{8}. \tag{0.5}$$

Since:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}, \tag{0.6}$$

the given pair of equations is **inconsistent** and the lines represented by the equations are **not intersecting**. Therefore, the system of equations has no solution.

Computational Solution:
Let us assume the given system of equations are consistent and we will try solving using LU decomposition

Given the system of linear equations:

$$2x - 3y = 8, \tag{0.7}$$

$$4x - 6y = 9. \tag{0.8}$$

We rewrite the equations as:

$$x_1 = x, \tag{0.9}$$

$$x_2 = y, \tag{0.10}$$

giving the system:

$$2x_1 - 3x_2 = 8, \tag{0.11}$$

$$4x_1 - 6x_2 = 9. \tag{0.12}$$

Step 1: Convert to Matrix Form

We write the system as:

$$A\mathbf{x} = \mathbf{b}, \tag{0.13}$$

where:

$$A = \begin{bmatrix} 2 & -3 \\ 4 & -6 \end{bmatrix}, \tag{0.14}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{0.15}$$

$$\mathbf{b} = \begin{bmatrix} 8 \\ 9 \end{bmatrix}. \tag{0.16}$$

Step 2: LU factorization using update equaitons

Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:
Step-by-Step Procedure:
1. Initialization: - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.

2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.

3. Result: - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix $\mathbf{U}$ by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix $\mathbf{L}$, where each entry in the column is determined by the values in the rows above it.
Using a code we get L,U as

$$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} U = \begin{bmatrix} 2 & -3 \\ 0 & 0 \end{bmatrix} \tag{0.17}$$

Step 3: Solve $L\mathbf{y} = \mathbf{b}$ (Forward Substitution)

We solve:

$$L\mathbf{y} = \mathbf{b} \quad \text{or} \quad \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \end{bmatrix}. \tag{0.18}$$

From the first row:

$$y_1 = 8. \tag{0.19}$$

From the second row:

$$2y_1 + y_2 = 9 \quad \implies \quad 2 \cdot 8 + y_2 = 9 \quad \implies \quad y_2 = -9. \tag{0.20}$$

Thus:

$$\mathbf{y} = \begin{bmatrix} 8 \\ -7 \end{bmatrix}. \tag{0.21}$$

Step 4: Solve $U\mathbf{x} = \mathbf{y}$ (Backward Substitution)

We solve:

$$U\mathbf{x} = \mathbf{y} \quad \text{or} \quad \begin{bmatrix} 2 & -3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \end{bmatrix}. \tag{0.22}$$

From the second row, We get

$$0 = -7 \tag{0.23}$$

which is inappropriate. That means our assumption is wrong

Final Solution

Since we are getting inappropriate equations using LU method that means the given system of equations have no solution.
Hence they are inconsistent

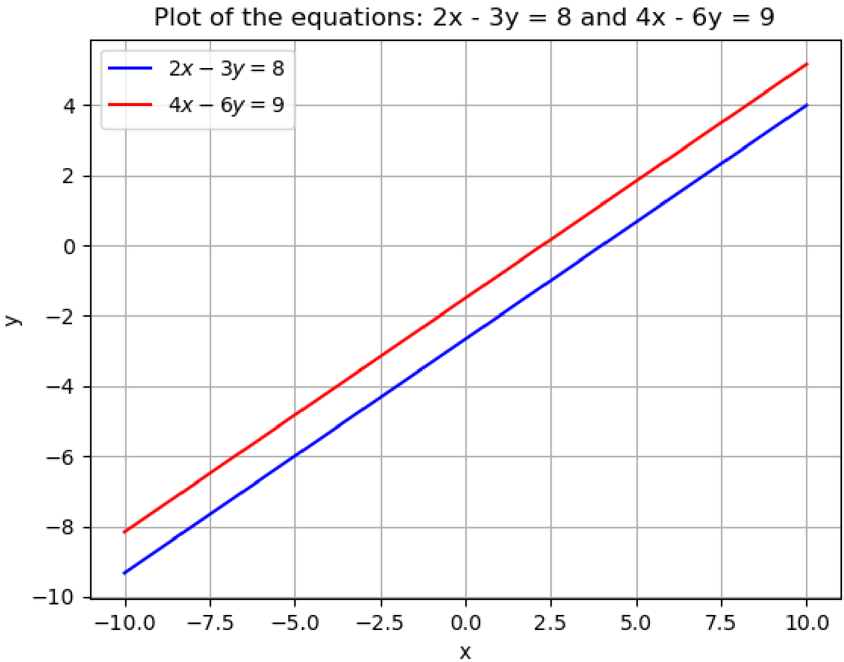
Plot of the equations: 2x - 3y = 8 and 4x - 6y = 9
$2x - 3y = 8$
$4x - 6y = 9$
4
2
0
−2
−4
−6
−8
−10
y
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x

Fig. 0.1