9.4.4

EE24BTECH11033 - KOLLURU SURAJ

Question: $x\frac{dy}{dx} - y = 2x^2$

Solution:
Divide the given equation with x

$$\frac{dy}{dx} - \frac{y}{x} = 2x \tag{0.1}$$

Which is a linear differential equation of the type $\frac{dy}{dx} + Py = Q$
where

$$P = -\frac{1}{x} \tag{0.2}$$
$$Q = 2x \tag{0.3}$$

$$\text{I.F.} = e^{\int -\frac{1}{x}\,dx} = e^{-\log x} = e^{\log \frac{1}{x}} = \frac{1}{x} \quad [\text{as } e^{\log f(x)} = f(x)]$$

Hence, the Integrating factor is $\frac{1}{x}$
Therefore, solution of the given equation is given by

$$\frac{y}{x} = \int 2dx \tag{0.4}$$
$$\frac{y}{x} = 2x + c \tag{0.5}$$
$$y = 2x^2 + cx \tag{0.6}$$

Take initial condition as x= 1, y= 1. We get c= -1

$$y = 2x^2 - x \tag{0.7}$$

Numerical Approach:
1. I used a for loop for finding the y values as the loop proceeds with iterative formula given below. I took some initial value of x and as loop proceeds I assigned it the value as $x + h$. where h is the step size, representing the rate of change.
2. Assigned the values of y for different x-values using a for loop.
3. Here the initial value of x is 1 and y is 1

Using the Method of Finite Differences
The Method of Finite Differences is a numerical technique used to approximate solutions to differential equations.

We know that:

$$\lim_{h\to 0}\frac{y(x+h)-y(x)}{h} = \frac{dy}{dx} \tag{0.8}$$

For the given differential equation 0.1,

$$\frac{dy}{dx} = \frac{y}{x} + 2x \tag{0.9}$$

$$\frac{y_{n+1}-y_n}{h} \approx \frac{y_n}{x_n} + 2x_n \tag{0.10}$$

$$y_{n+1} = y_n + \left(\frac{y_n}{x_n} + 2x_n\right)\cdot h \tag{0.11}$$

The iterative formula for updating x-values is:

$$x_n = x_{n-1} + h \tag{0.12}$$

Using Matplotlib, I plotted the computed points and the graph of the exact solution to verify that they approximately match

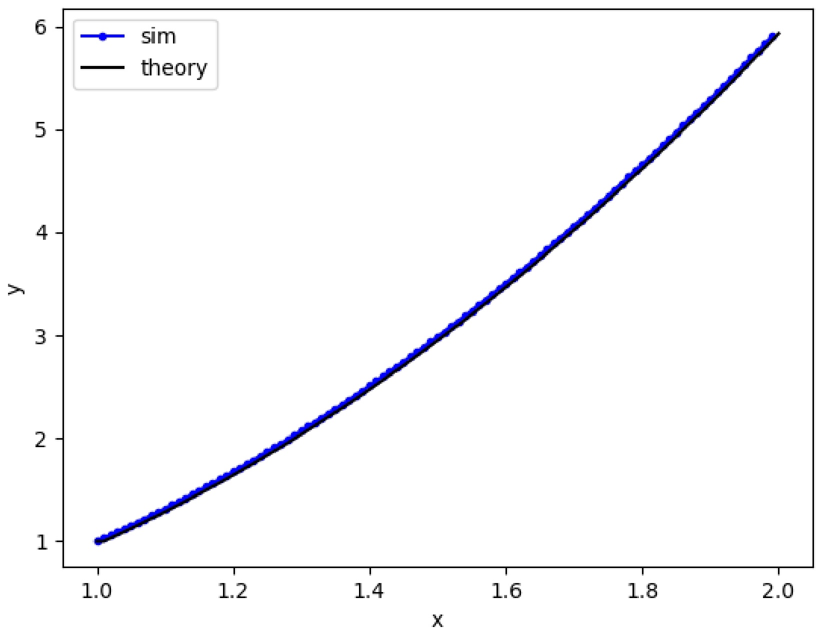

Fig. 0.1