6.5.1.4

EE24BTECH11033 - KOLLURU SURAJ

Question:
Find the Maximum and Minimum values of the function if exists

$$y(x) = x^3 + 1$$

Solution :
Threotical Solution :

Finding the derivative of the function

$$\frac{dy}{dx} = 3x^2 \tag{0.1}$$

We can observe that for any value of x the value of $\frac{dy}{dx}$ is always ≥ 0.It is zero at $x = 0$. Which says that the function is increasing that means there is no absolute maximum or minimum
Computational solution :

Minimum value of the function can be done by **Gradient decent** method:
Update the position iteratively:

$$x_{n+1} = x_n - \eta \cdot \frac{dy}{dx} \tag{0.2}$$

Taking $x_0 = 1$ as starting point
Here, $\eta = 0.01$, η is learning rate.

$$x_{n+1} = x_n - \eta \cdot 3x^2 \tag{0.3}$$

The gradeint descent method stops when $\frac{dy}{dx} = 0$. Because of that the minimum value of the function as computed from gradient descent will be $y \approx 1, x \approx 0$

Maximum value of the function can be done by **Gradient ascent** method:
Update the position iteratively:

$$x_{n+1} = x_n + \eta \cdot \frac{dy}{dx} \tag{0.4}$$

Taking $x_0 = 1$ as starting point
Here, $\eta = 0.01$, η is learning rate.

$$x_{n+1} = x_n + \eta \cdot 3x^2 \tag{0.5}$$

This causes x_n to increase indefinitely, because this method stops at x=0 but since we started at x=1 and value of x keeps increasing.

The function increases without bound as $x \to \infty$, so **gradient ascent** will not converge to a maximum. The iteration will continue indefinitely. so, **No Maximum exists**

Computational results :
-Absolute Minimum

$$x \approx 0, \;\; y(x) \approx 1 \tag{0.6}$$

-No Absolute Maximum

If we take starting point as x=-1(anything such that $x < 0$) we will get minimum as -∞ and maximum as $x \approx 0, \;\; y(x) \approx 1$. Because at x=0 $\frac{dy}{dx} = 0$.