# 12.8.1.2

EE24BTECH11033 - KOLLURU SURAJ

**Question:** Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

**Solution:**
If the area is calulated directly using integration it would be

$$A = \int_2^4 3\sqrt{x}dx \tag{0.1}$$

$$A = \left[3 \cdot \frac{2}{3}x^{3/2}\right]_2^4 \tag{0.2}$$

$$A = 10.343146 \tag{0.3}$$

Let's try to verify this computationally using Trapezoidal rule
**Some theory about Trapezoidal rule:**
The idea behind the trapezoidal rule is to approximate the region under the curve of the function as a series of trapezoids, rather than using rectangles. The area of each trapezoid is then computed and summed to estimate the total area under the curve (the integral). First we need to set up the integral
Area under curve is given by

$$A = \int_2^4 3\sqrt{x}dx \tag{0.4}$$

We will approximate this using trapezoidal rule. Divide the interval [2,4] into 100 subintervals with each of width $h = \frac{4-2}{100} = \frac{1}{50}$, Let:

$$x_0 = 2, x_1 = x_0 + h, x_2 = x_1 + h, ....x_n = 4 \tag{0.5}$$

Let $A(x_n)$ be the area enclosed by the curve $y(x)$ from $x = x_0$ to $x = x_n$, $(x_0, x_1, \dots x_n)$ be equidistant points with step-size $h$.

$$A(x_n + h) = A(x_n) + \frac{1}{2}h(y(x_n + h) + y(x_n)) \tag{0.6}$$

where $\frac{1}{2}h(y(x_n + h) + y(x_n))$ is area of difference trapezium We can repeat this till we get the required area.
Let $A(x_n) = A_n$ and $y(x_n) = y_n$

$$A_{n+1} = A_n + \frac{1}{2}h(y_{n+1} + y_n) \tag{0.7}$$

We can write $y_{n+1}$ in terms of $y_n$ using first principle of derivative. $y_{n+1} = y_n + hy'_n$

$$A_{n+1} = A_n + \frac{1}{2}h\left(\left(y_n + hy'_n\right) + y_n\right) \tag{0.8}$$

$$A_{n+1} = A_n + \frac{1}{2}h\left(2y_n + hy'_n\right) \tag{0.9}$$

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \tag{0.10}$$

$$\tag{0.11}$$

from 0.5

$$x_{n+1} = x_n + h \tag{0.12}$$

In the given question, $y_n^2 = 9x_n$ and $y'_n = \frac{3}{2\sqrt{x}}$
General Difference Equation is given by, from 0.10

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \tag{0.13}$$

$$= A_n + h\left(3\sqrt{x_n}\right) + \frac{1}{2}h^2\left(\frac{3}{2\sqrt{x_n}}\right) \tag{0.14}$$

$$x_{n+1} = x_n + h \tag{0.15}$$

Using a C code we get the final area as

$$A \approx 10.34317 \tag{0.16}$$

which is closer to the theoritical solution hence this is correct

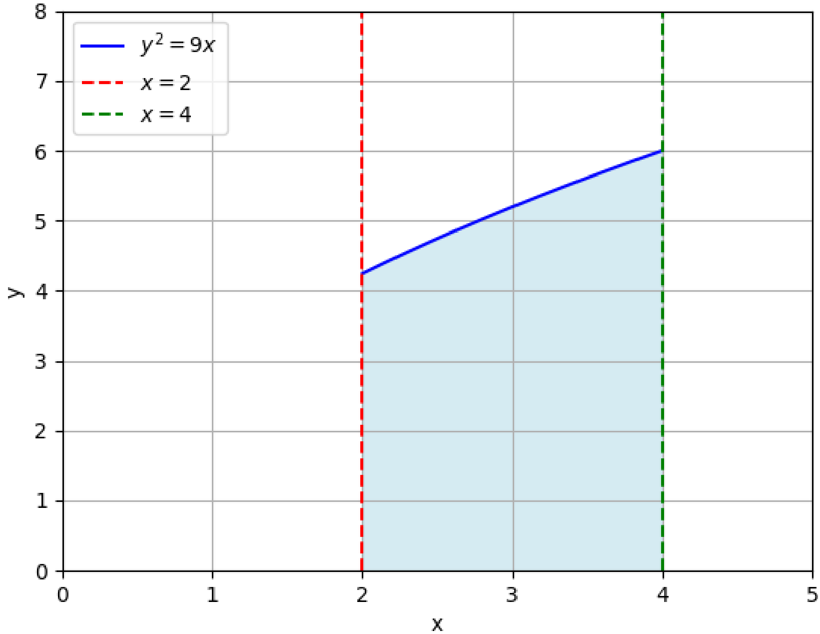


Fig. 0.1