10.4.3.1.2

EE24BTECH11033 - Kolluru Suraj

Question:
Find the roots of the equation $2x^2 + x - 4 = 0$
Solution:
Theoritical solution:

Applying quadratic formula gives solution as

$$x_1 = \frac{-1 - \sqrt{33}}{4} \tag{0.1}$$

$$x_2 = \frac{-1 + \sqrt{33}}{4} \tag{0.2}$$

Computational solution:
We can solve the above equation using fixed point iterations. First we separate x, from the above equation and make an update equation of the below sort.

$$x = g(x) = \frac{4 - 2x^2}{2} \tag{0.3}$$

Applying the above update equation on our equation, we get

$$x_{n+1} = \frac{4 - 2x_n^2}{2} \tag{0.4}$$

Now we start with an initial guess $x_0 = 10$
But we realize that the updated values always approach infinity for any initial value.
So we will use Newton-Rapshon method
Newton-Raphson Method:
Start with an initial guess x_0, and then run the following logical loop,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.5}$$

where,

$$f(x) = 2x^2 + x + -4 \tag{0.6}$$

$$f'(x) = 4x + 1 \tag{0.7}$$

The update equation will be

$$x_{n+1} = x_n - \frac{2{x_n}^2 + x_n - 4}{4x_n + 1} \tag{0.8}$$

$$\tag{0.9}$$

The problem with this method is if the roots are complex but the coeffcients are real, x_n either converges to an extrema or grows continuously without any bound. To get the complex solutions, however , we can just take the initial guess point to be a random complex number.
The output of a program written to find roots is shown below:

$$r_1 = -1.6861 \tag{0.10}$$

$$r_2 = 1.1859 \tag{0.11}$$

CODING LOGIC FOR FINDING EIGENVALUES :-

The quadratic equation

$$2x^2 + x - 4 = 0 \tag{0.12}$$

is rewritten in matrix form:

$$\text{Matrix} = \begin{pmatrix} 0 & -\frac{c}{a} \\ 1 & -\frac{b}{a} \end{pmatrix} \tag{0.13}$$

$$a = 2, \quad b = 1, \quad c = -4. \tag{0.14}$$

Substituting the values of a, b and c, the matrix becomes:
Let

$$\text{A} = \begin{pmatrix} 0 & 2 \\ 1 & -\frac{1}{2} \end{pmatrix} \tag{0.15}$$

QR-Algorithm

The QR method is an iterative algorithm used to compute the eigenvalues of a square matrix A. The algorithm works as follows:

1) Initialization
   Let $A_0 = A$, where A is the given matrix.
2) QR Decomposition
   For each iteration $k = 0, 1, 2, \ldots$:
   a) Compute the QR decomposition of A_k, such that:

$$A_k = Q_k R_k \tag{2.1}$$

      where:
      i) Q_k is an orthogonal matrix ($Q_k^\top Q_k = I$).
      ii) R_k is an upper triangular matrix.

      The decomposition ensures $A_k = Q_k R_k$.
   b) Form the next matrix A_{k+1} as:

$$A_{k+1} = R_k Q_k \tag{2.2}$$

3) Convergence
   Repeat Step 2 until A_k converges to an upper triangular matrix T. The diagonal

entries of T are the eigenvalues of A.

4) The eigenvalues of matrix will be the roots of the equation.
The roots we get by using this method are

$$x_1 = 1.18614066 \tag{4.1}$$

$$x_2 = -1.68614066 \tag{4.2}$$

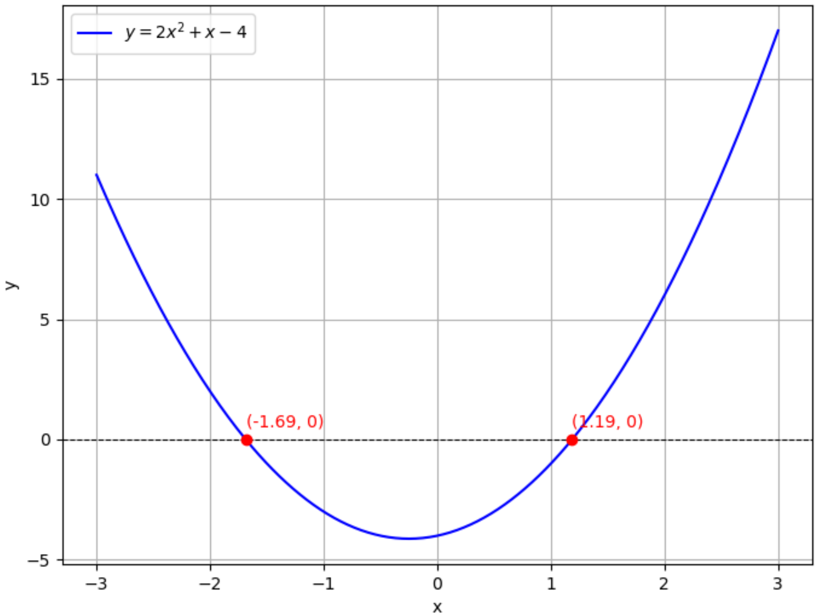

Fig. 4.1