9.6.18

EE24BTECH11033 - KOLLURU SURAJ

Question: $x\frac{dy}{dx} - y = 2x^2$

Solution:
Divide the given equation with x

$$\frac{dy}{dx} - \frac{y}{x} = 2x \tag{0.1}$$

Which is a linear differential equation of the type $\frac{dy}{dx} + Py = Q$
where

$$P = -\frac{1}{x} \tag{0.2}$$

$$Q = 2x \tag{0.3}$$

I.F. $= e^{\int -\frac{1}{x}\,dx} = e^{-\log x} = e^{\log \frac{1}{x}} = \frac{1}{x}$ [as $e^{\log f(x)} = f(x)$]

Hence, the Integrating factor is $\frac{1}{x}$
Therefore, solution of the given equation is given by

$$\frac{y}{x} = \int 2dx \tag{0.4}$$

$$\frac{y}{x} = 2x + c \tag{0.5}$$

$$y = 2x^2 + cx \tag{0.6}$$

Take initial condition as x= 1, y= 1. We get c= -1

$$y = 2x^2 - x \tag{0.7}$$

Now let us this computationally from the definition of $\frac{dy}{dx}$

$$Y_{n+1} = Y_n + \frac{dy}{dx} \cdot h \tag{0.8}$$

From the differential equation 0.1

$$\frac{dy}{dx} = \frac{y}{x} + 2x \tag{0.9}$$

$$y_{n+1} = y_n + \left(\frac{y_n}{x_n} + 2x_n\right) \cdot h \tag{0.10}$$

BY taking x_0=1 and y_0=1 and h=0.01 by iterating through the loop a 100 times and finding $y_2, y_3, y_4, \cdots$ and plotting the graph. we can verify the function we got by solving the differential equation mathematically

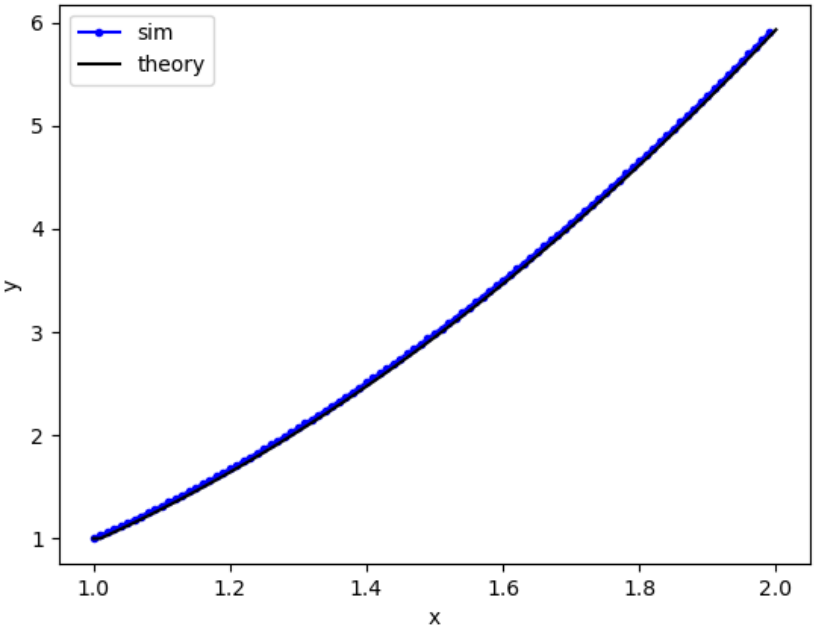

Fig. 0.1