I. Subjective Problems

1) Let 'd' be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1 to the tangent drawn at a point P on the ellipse.If $F_1 and F_2$ are the two foci of the ellipse,then show that $(PF_1-PF_2)^2 = 4a^2(1-\frac{b^2}{d^2})$. (1995- 5 marks)

2) Points A,B and C lie on a parabola $y^2 = 4ax$.The tangents to the parabola at A,B and C taken in pairs , intersect at points P,Q and R.Determine the ratios of the areas of triangles ABC and PQR (1996- 3 marks)

3) From a point A common tangents are drawn to circle $x^2+y^2 = \frac{a^2}{2}$ and parabola $y^2 = 4ax$.FInd the area of the quardilateral formed by the common tangents,the chord of contact of circle and the chord of contact of parabola. (1996- 2 marks)

4) A tangent to the ellipse $x^2+4y^2 = 4$ meets the ellipse $x^2+2y^2 = 6$ at P and Q.Prove that the tangents at P and Q of the ellipse $x^2+2y^2 = 6$ are at right angles. (1997- 5 marks)

5.The angle between a pair of tangents drawn from a point Pto the parabola $y^2 = 4ax$ is 45 .Show that the locus of point P is hyperbola. (1998- 8 marks)

5) Consider the family of Circles $x^2+y^2 = r^2$,$2 < r < 5$.If in the first quadrant,the common tangent to a circle of this family and the ellipse $4x^2+25y^2 = 100$ meets the co-ordinate axes at A and B,then find the equation of the locus of the mid-point of AB. (1999- 10 marks)

6) Find the co-ordinates of all the points P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1, for which the area of the triangle PON is maximum ,where O denotes origin and N,the foot of the perpendicular from O to the tangent P. (1999- 10 marks)

7) Let ABC be equilateral triangle inscribed in the circle $x^2+y^2 = a^2$.Supose perpendiculars from A,B,C to the major axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1,$(a > b)$ meets the ellipse respectively,at P,Q,R.so that P,Q,R lie on the same side of major axis as A,B,C respectively .Prove that the normals to the ellipse drawn at the points P,Q and R are concurrent. (2000- 7 marks)

8) Let C_1 and C_2 be respectively ,the parabolas $x^2 = y-1$ and $y^2 = x-1$.Let P be any point on C_1 and q be any point on C_2.Let P_1 and Q_1 be the reflections of P and Q respectively with respect to the line y=x.Prove that P_1 lies on C_2,Q_1 lies on C_1 and PQ $\geq \min(PP_1, QQ_1)$.Hence or otherwise determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that $P_0Q_0 \leq$PQ for all pairs od points (P,Q) with P on C_1 and Q on C_2. (2000- 10 marks)

9) Let P be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1 ,$0 < b < a$.Let the line parallel to y-axis passing through P meet the circle $x^2+y^2 = a^2$ at the point Q such that P and Q are on the same side of x-axis.For two positive real numbers r and s,find the locus of the point R on PQ such that PR : RQ=r:s as P varies over the ellipse. (2001- 4 marks)

10) Prove that,in an ellipse,the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix. (2002- 5 marks)

11) Normals are drawn from the point P with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$.If locus of P with $m_1m_2 = \alpha$ is a part of parabola itslef then find α (2003- 4 marks)

12) Tangent is drawn to parabola $y^2-2y-4x+5 = 0$ at a point P which cuts the directrix at the

point Q.A point R is such that it divides QP externally in the ratio 1/2:1.Find the locus of point R (2004 - 4 marks)

13) Tangentes are drawn from any point on hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ to the circle $x^2+y^2=9$.Find the locus of mid-point of the chord of contact. (2005 - 4 marks)

14) Find the equation of the common tangent in 1^{st} quadrant to the circle $x^2+y^2=16$ and the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$.Also find the length of the intercept of the tangent between the coordinate axes. (2005 - 4 marks)