EE LAB REPORT-2

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CONTENTS

I. EXPERIMENT

A. Introduction

An RC circuit is a fundamental building block in electronics, consisting of a resistor (R) and a capacitor (C). When subjected to a time-varying input, such as a square wave, the circuit exhibits distinct charging and discharging behaviors, governed by its time constant $\tau = RC$. The response of the circuit depends on the relationship between the time constant (τ) and the Time period of the input square wave (T).

B. Aim

To analyze the voltage response across a capacitor in a series RC circuit subjected to a square wave input, and to study the effects of varying the input frequency on the charging and discharging behavior of the capacitor. We will be studying three cases here

1) $RC \ll T$
2) RC=T
3) $RC \gg T$

C. Apparatus Used

- Cathode Ray Oscilloscope (CRO).
- Breadboard.
- Function Generator
- BNC Cables.
- Connecting wires.
- Capacitor($100\mu F$)
- Resistor($2K\Omega$)

In our experiment, the value of RC is **0.2 sec**

D. Procedure/Experiment

1) Connect the resistor and capacitor in series on a breadboard
2) Use a function generator to apply a +5V square wave input across the RC circuit
3) Use the oscilloscope to display both the input and output waveforms
   - Connect Channel 1 across the capacitor to observe the output.
   - Connect Channel 2 to the input (square wave).
4) To observe the three cases on a CRO, We will change the frequency of square wave keeping RC constant such that it adjusts to all our cases and observe the input and output waveforms, measuring the peak voltages and time intervals using the oscilloscope markers

The circuit will be as following

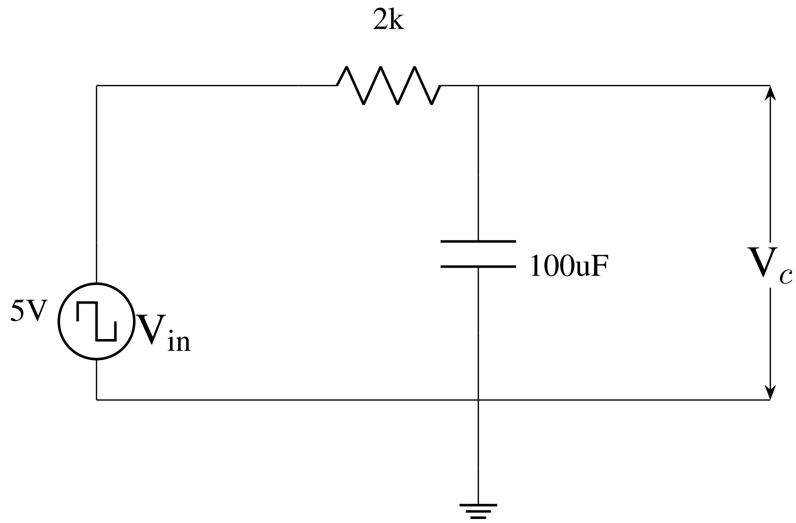

Fig. 1: RC Circuit with Voltage Output V_c

Where V_{in} in the input signal and V_c is the voltage across capacitor

*E. **RC = T***

1) Here RC=T=0.2s
2) When the square wave goes high (from 0 V to 5 V), the capacitor starts charging. However, it doesn't reach the full 5 V within the half-period $\frac{T}{2}$ because reaching approximately full charge takes about 5τ.
3) When the square wave goes low (from 5 V to 0 V), the capacitor begins discharging. Again, it doesn't fully discharge to 0 V before the next transition occurs.
4) The peak and trough values of V_c depend on the exact ratio of RC to T, and since RC=T, the output reaches a fraction of the input amplitude.

Following parameters are used for square wave

Parameter	**Value**
High Voltage (V_{high})	5 V
Low Voltage (V_{low})	0 V
Time Period (T)	$0.2s$
Duty Cycle (D)	50%

TABLE I: Parameters of a Square Wave

The plot looks like as shown

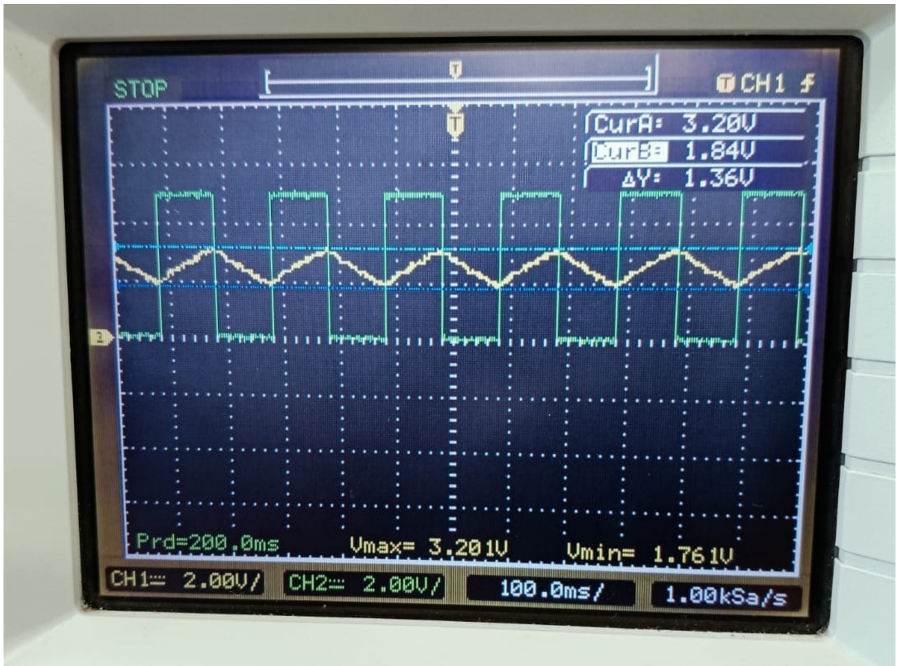

Fig. 2: Steady State response for RC=T

The graph represents the steady state of voltage across capacitor. From the CRO readings we can observe that

$$V_{max} = 3.201V$$
$$V_{min} = 1.84V$$

Mathematical process for calculating steady state voltages:
During the charging phase, the voltage across the capacitor is given by:

$$V_{crest} = V_{\text{source}}\left(1 - e^{-\frac{T/2}{\tau}}\right) + V_{trough} \cdot e^{-\frac{T/2}{\tau}},$$

During the discharging phase, the capacitor discharges from the before crest. We will get the voltage of subsequent trough as:

$$V_{trough} = V_{\text{initial}} e^{-\frac{T/2}{\tau}}. \tag{1}$$

To calculate steady state voltages We will loop through these equations continously to calculate crests and troughs.By hand calculations in steady state we get peak voltage as(derivation given at end of report)

$$V_{max} = \frac{V_{source}}{1 + e^{-\frac{t}{\tau}}}$$
$$V_{min} = \frac{V_{source}}{1 + e^{-\frac{t}{\tau}}} \cdot e^{-}\frac{t}{\tau}$$

The values given by calculation are

$$V_{max} = 3.1122V$$
$$V_{min} = 1.8877V$$

Which are very close to the values observed on CRO

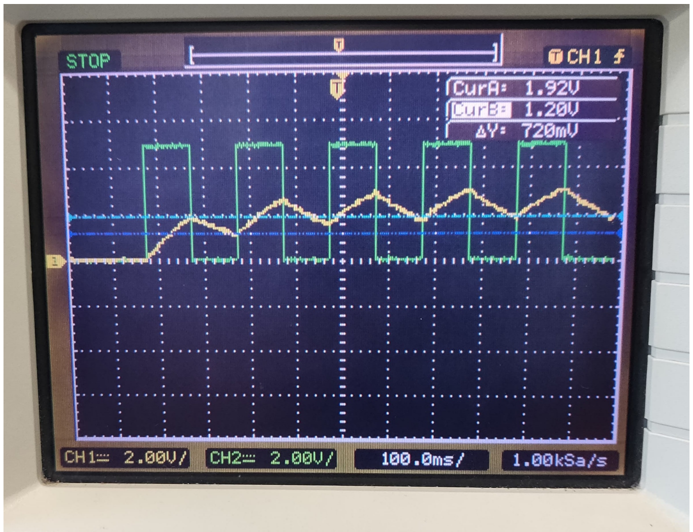

Fig. 3: Transient response for RC=T

This shows the transient response for 5 cycles.By cursors we can see the values of first crest and trough. We will try to compute the values on graph using mathematical calculations.

Mathematical proof:

For the given case RC=T, where the time period $T = 0.2\,\mathrm{s}$, the half-period is:

$$T/2 = 0.1\,\mathrm{s}.$$

During the charging phase, the voltage across the capacitor is given by:

$$V_C(T/2) = V_{\text{max}}\left(1 - e^{-\frac{T/2}{\tau}}\right),$$

where $\tau = RC = 0.2\,\mathrm{s}$. Substituting the values:

$$V_C(T/2) = 5\left(1 - e^{-\frac{0.1}{0.2}}\right) = 5\left(1 - e^{-0.5}\right).$$

Using $e^{-0.5} \approx 0.60653$:

$$V_C(T/2) \approx 5 \cdot (1 - 0.60653) = 5 \cdot 0.39347 = 1.9673\,\mathrm{V}.$$

During the discharging phase, the capacitor discharges from $V_{\text{initial}} = 1.9675\,\mathrm{V}$. The voltage is given by:

$$V_C(T) = V_{\text{initial}} e^{-\frac{T/2}{\tau}}.$$

Substituting the values:

$$V_C(T) = 1.9673 \cdot e^{-\frac{0.1}{0.2}} = 1.9673 \cdot e^{-0.5}.$$

Using $e^{-0.5} \approx 0.60653$:

$$V_C(T) \approx 1.9673 \cdot 0.60653 \approx 1.1932\,\text{V}.$$

The capacitor charges up to $1.9673\,\text{V}$ during the charging phase and discharges to $1.1932\,\text{V}$ during the discharging phase.
By using equations 3 and 4 we can get the values of next crests and troughs. By hand calculations we get values of first crest and first trough as

$$V_{crest_1} = 1.973V$$
$$V_{trough_1} = 1.1932V$$

From Fig 3 We can see that value of 1st crest is nearly equal to 2V

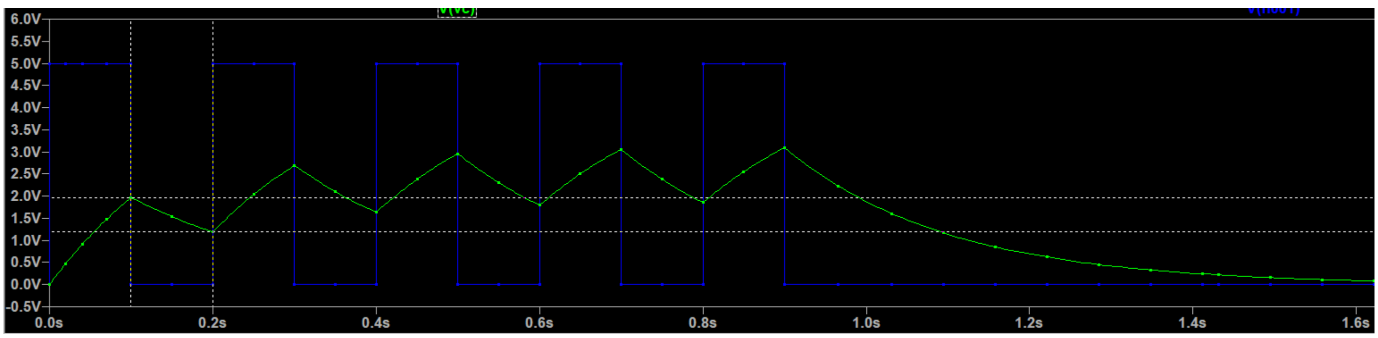

Fig. 4: Simulation of RC=T transient response(5 cycles)

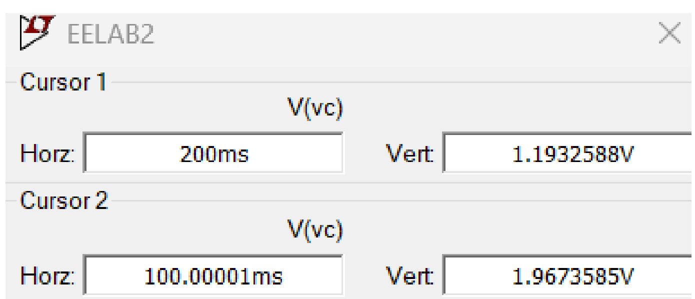

Fig. 5: Values of first trough and crest

From Fig:4 we can see the simulation of transient response of voltage across capacitor along with the square wave. Using cursor we get the last peak of the transient response as 3.088V which is very close to the value on oscilloscope.
By using cursors from Fig:5 We have

$$V_{crest_1} = 1.973585V$$
$$V_{trough_1} = 1.1932588V$$

Which are almost equal to the values from mathematical calculation.

F. *RC*$\gg T$

1) For this case we will take period of square wave as 0.02s to match the required condition
2) When $RC >> T$, the capacitor charges and discharges very slowly compared to the rate at which the input square wave changes.
3) When the input square wave goes high (from 0 V to 5 V), the capacitor begins charging. However, the charging is so slow that the capacitor voltage (V_c) barely increases during the high phase of the square wave.
4) Similarly, when the square wave goes low (from 5 V to 0 V), the capacitor discharges just as slowly. As a result, the voltage across the capacitor decreases only slightly during the low phase.
5) The capacitor voltage (V_c) changes very gradually and never catches up to the fast transitions of the square wave.
6) Instead of sharp transitions, V_c exhibits a slow, almost linear rise and fall over several periods of the input wave.
7) Following parameters are used for square wave

Parameter	**Value**
High Voltage (V_{high})	5 V
Low Voltage (V_{low})	0 V
Time Period (T)	$0.02s$
Duty Cycle (D)	50%

TABLE II: Parameters of a Square Wave

8) The graph is as shown

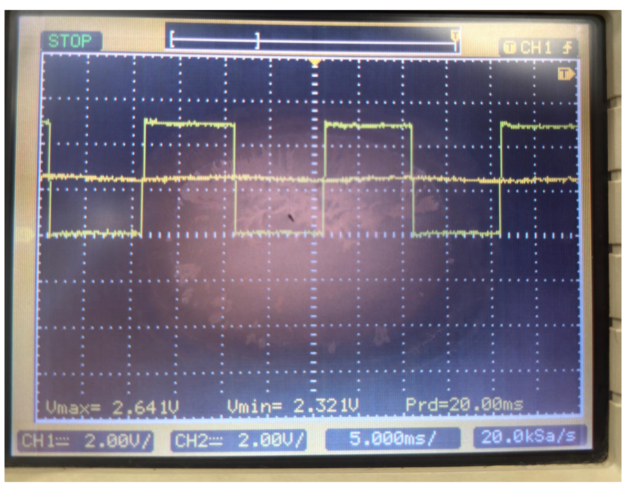

Fig. 6: Steady state response for RC>>T

9) We can see that the steady state voltages are

$$V_{max} = 2.641V$$
$$V_{min} = 2.321V$$

and period of wave is 0.02s

10) **Mathematical process for calculating steadys state voltages:**
During the charging phase, the voltage across the capacitor is given by:

$$V_{crest} = V_{\text{source}} \left(1 - e^{-\frac{T/2}{\tau}}\right) + V_{trough} \cdot e^{-\frac{T/2}{\tau}},$$

During the discharging phase, the capacitor discharges from the before crest. We will get the voltage of subsequent trough as

$$V_{trough} = V_{\text{initial}} e^{-\frac{T/2}{\tau}}. \tag{2}$$

Using the equations

$$V_{max} = \frac{V_{source}}{1 + e^{-\frac{t}{\tau}}}$$
$$V_{min} = \frac{V_{source}}{1 + e^{-\frac{t}{\tau}}} \cdot e^{-\frac{t}{\tau}}$$

To calculate steady state voltages We will loop through these equations continously to calculate crests and troughs. We will stop when we get the values of continuos crests and troughs same which represents steady state.
Using the equations

$$V_{max} = \frac{V_{source}}{1 + e^{-\frac{t}{\tau}}}$$
$$V_{min} = \frac{V_{source}}{1 + e^{-\frac{t}{\tau}}} \cdot e^{-\frac{t}{\tau}}$$

The values obtained through calculation are

$$V_{max} = 2.5624V$$
$$V_{min} = 2.4375V$$

We can observe that the values from CRO and given by the python code are approximately equal.

Fig. 7: Transient response for RC>>T

The cursors in the figure shows values of first crest and torugh
Let’s Calculate the first crest and trough in mathematical way

For the given case, where the time period $T = 0.02\,\text{s}$, the half-period is:

$$T/2 = 0.01\,\text{s}.$$

During the charging phase, the voltage across the capacitor is given by:

$$V_C(T/2) = V_{\text{max}}\left(1 - e^{-\frac{T/2}{\tau}}\right),$$

where $\tau = RC = 0.2\,\text{s}$. Substituting the values:

$$V_C(T/2) = 5\left(1 - e^{-\frac{0.01}{0.2}}\right) = 5\left(1 - e^{-0.05}\right).$$

Using $e^{-0.05} \approx 0.9512$:

$$V_C(T/2) \approx 5 \cdot (1 - 0.9512) = 5 \cdot 0.0488 = 0.244\,\text{V}.$$

During the discharging phase, the capacitor discharges from $V_{\text{initial}} = 0.244\,\text{V}$. The voltage is given by:

$$V_C(T) = V_{\text{initial}} e^{-\frac{T/2}{\tau}}.$$

Substituting the values:

$$V_C(T) = 0.244 \cdot e^{-\frac{0.01}{0.2}} = 0.244 \cdot e^{-0.05}.$$

Using $e^{-0.05} \approx 0.9512$:

$$V_C(T) \approx 0.244 \cdot 0.9512 \approx 0.232\,\text{V}.$$

The capacitor never fully charges or discharges due to the very short time period of the square wave. By hand calculations we get values of first crest and trough as

$$V_{crest_1} = 0.244V$$
$$V_{trough_1} = 0.232V$$

Let us simulate the graph and verify the values we got

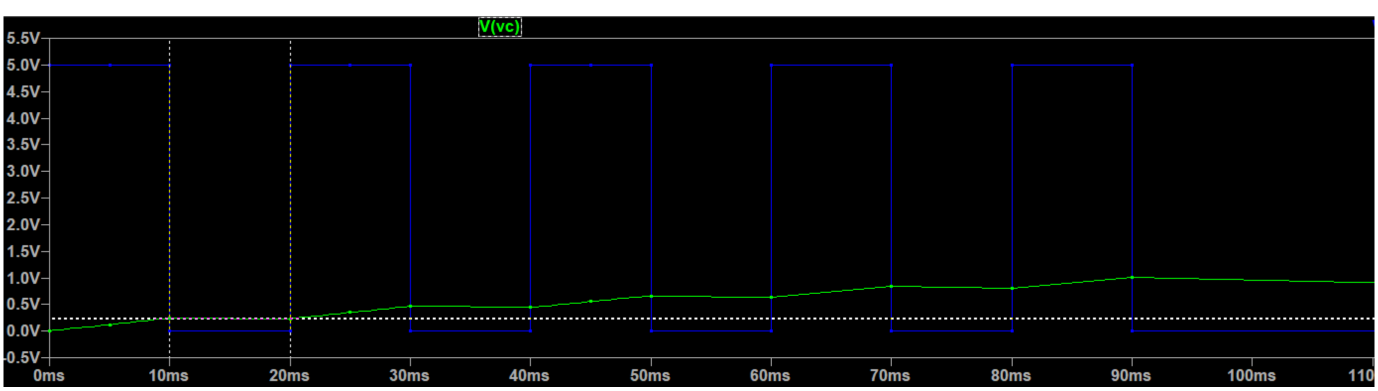

Fig. 8: Simulation of transient signal for RC¿¿T(5 cycles)

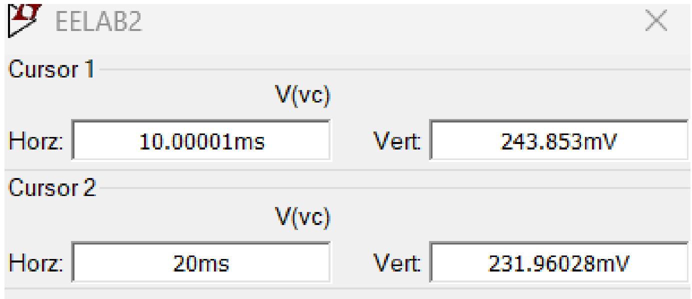

Fig. 9: Values of first crest and trough

From Fig:8 we can see the simulation of transient response of voltage across capacitor along with the square wave. Using cursor we get the last peak of the transient response as 1.008V which is very close to the value on oscilloscope.
By using cursors from Fig:9 We have

$$V_{crest_1} = 243.853mV$$
$$V_{trough_1} = 231.96028mV$$

Which are almost equal to the values from mathematical calculation.

G. *RC* ≪ *T*

1) Since the time constant oof circuit is 0.2s we will take the time period of wave as 2s to set the given condition
2) The time constant RC is much smaller than the signal period T, so the capacitor has enough time to fully(approximately) charge and discharge during each cycle of the input signal.
3) When the square wave goes high (from 0 V to 5 V), the capacitor charges almost instantly to match the input signal.
4) When the square wave goes low (from 5 V to 0 V), the capacitor discharges almost instantly.
5) However, V_c lags slightly behind the input square wave due to the finite charging/discharging time of the capacitor.
6) On an oscilloscope, the capacitor voltage waveform looks almost identical to the square wave input. Following are parameters for square wave

Parameter	**Value**
High Voltage (V_{high})	5 V
Low Voltage (V_{low})	0 V
Time Period (T)	$2s$
Duty Cycle (D)	50%

TABLE III: Parameters of a Square Wave

7) The plot looks like as shown

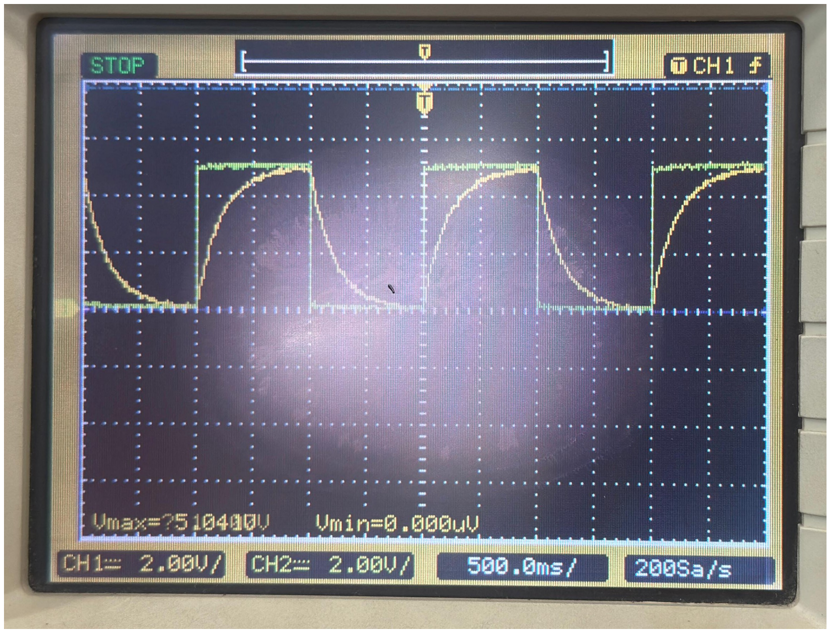

Fig. 10: Steady state when RC≪ T

From the graph we can see the period of signal as 4x(500ms)=2s which matches the time period of square wave and peak voltage of 5.041V which is not so accurate(because of some defect in oscilloscope) about an 0.08 error
Using the equations

$$V_{max} = \frac{V_{source}}{1 + e^{-\frac{t}{\tau}}}$$

$$V_{min} = \frac{V_{source}}{1 + e^{-\frac{t}{\tau}}} \cdot e^{-\frac{t}{\tau}}$$

By calculations we get

$$V_{max} = 4.9665V$$

$$V_{min} = 33.431mV$$

Let us look at the transient response of the output signal which will be same as steady state because the capacitor is being almost fully charged during high and almost fully discharging when input signal is low. This is because the time constant of circuit is much lesser than the period of the wave steady state is reached early

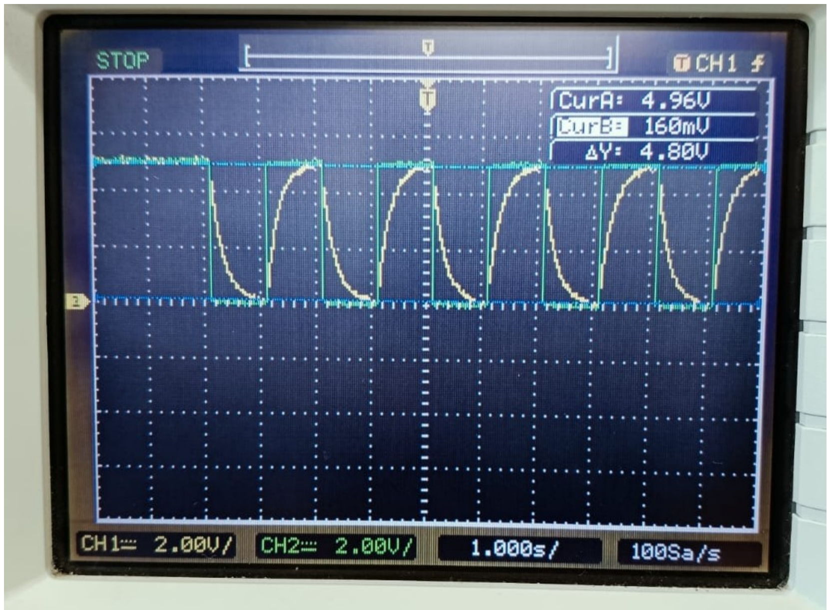

Fig. 11: Transient response when RC$\ll$ T

Let us try mathematical calculations of capacitor voltages.
Mathematical proof:

1) In square wave since it is on high voltage for half time and low for the other half, the capacitor charges for half time and discharges half time
2) For the given case, where the time period $T = 2\,\text{s}$, the half-period is:

$$T/2 = 1\,\text{s}.$$

During the charging phase, the voltage across the capacitor is given by:

$$V_{crest} = V_{\text{source}} \left(1 - e^{-\frac{T/2}{\tau}}\right) + V_{trough} \cdot e^{-\frac{T/2}{\tau}}, \tag{3}$$

where $\tau = RC = 0.2\,\text{s}$, V_{crest}= value of next crest(at the end of charging phase crest will be obtained) and V_{trough} represents the value of the preceding trough i.e the starting voltage of charging phase. Substituting the values:

$$V_{crest} = 5\left(1 - e^{-\frac{1}{0.2}}\right) = 5\left(1 - e^{-5}\right).$$

Using $e^{-5} \approx 0.00673$:

$$V_{crest} \approx 5 \cdot (1 - 0.00673) = 5 \cdot 0.99327 = 4.9663\,\text{V}.$$

During the discharging phase, the capacitor discharges from $V_{\text{initial}} = 4.9663\,\text{V}$. We will get the voltage of subsequent trough as:

$$V_{trough} = V_{\text{initial}} e^{-\frac{T/2}{\tau}}. \tag{4}$$

Substituting the values:

$$V_{trough} = 4.9663 \cdot e^{-\frac{1}{0.2}} = 4.9663 \cdot e^{-5}.$$

Using $e^{-5} \approx 0.00673$:

$$V_{trough} \approx 4.9663 \cdot 0.00673 \approx 0.0334231\,\text{V} = 33.4231 mV$$

From equation 3 and 4 we can loop those equations until we get a steady state. By hand calculations it is difficult so, we will be using a python code for the calculation of steady state crest and trough. The values given by code are At steady state

$$V_{crest} = 4.966536V$$
$$V_{trough} = 0.033464V$$

The values are very close to ones which are obtained on CRO

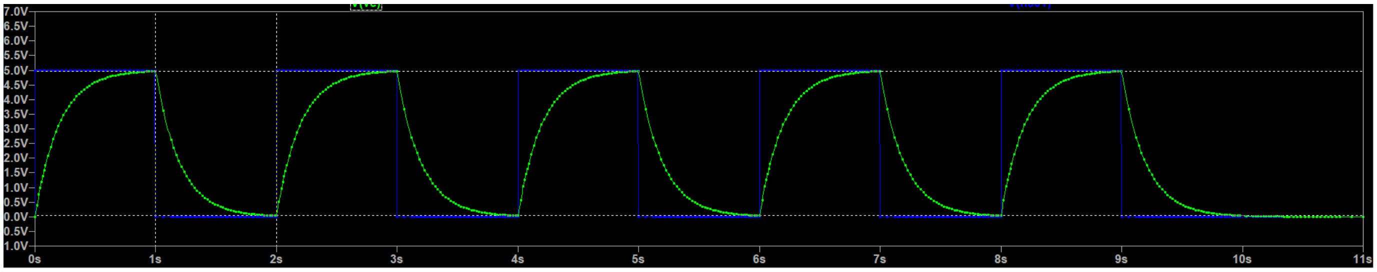

Fig. 12: Simulation of signal

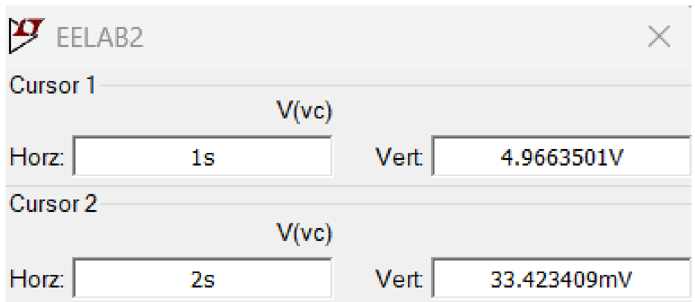

Fig. 13: Caption

This is the simluation of the transient response of 5 cycles in LTspice(software for simlating responses) Here

- Blue signal represents square wave(only 5 cycles)
- Green signal represents voltage across capacitor

This is the simluation of the transient response of 5 cycles in LTspice(software for simlating responses). By using cursors in LTspice we get

- Value of first crest as V_c= 4.96635V
- Value of first trough as V_c= 33.4234mV

We can observe that in this condition steady state is obtained very fast.
So the maximum value of V_c is 4.96657V. minimum value is 33.4248mV
We can observe that the values obtained from mathematical calculations are approximately same as from simulation

H. Applications of RC circuits

1) RC circuits are used in filtering to remove noise, smooth signals, or create specific frequency responses.
2) RC circuits are useful for shaping signals in oscillators, waveform generators, and pulse circuits.
3) They play a crucial role in analog circuits where signal conditioning is needed.

HAND CALCULATIONS TO CALCULATE STEADY STATE VOLTAGE

A series RC circuit is driven by a square wave input switching between **+5V (high)** and **0V (low)**. We derive the voltage across the capacitor, $V_C(t)$, in both charging and discharging phases.

Charging Phase ($V_{in} = 5V$)

When the input switches high at $t = 0$, the capacitor starts charging from 0V towards 5V. The governing differential equation is:

$$RC\frac{dV_C}{dt} + V_C = V_{\text{in}} \tag{5}$$

Substituting $V_{\text{in}} = 5V$:

$$RC\frac{dV_C}{dt} + V_C = 5 \tag{6}$$

Solving this first-order equation, the general solution is:

$$V_C(t) = V_{\text{final}} + (V_{\text{initial}} - V_{\text{final}})e^{-t/\tau} \tag{7}$$

where $\tau = RC$ is the time constant.

Since the capacitor starts at $V_C(0) = 0$ and charges toward 5V:

$$V_C(t) = 5(1 - e^{-t/\tau}) \tag{8}$$

Discharging Phase ($V_{in} = 0V$)

When the input switches low at $t = T/2$, the capacitor discharges toward 0V. The governing equation is:

$$RC\frac{dV_C}{dt} + V_C = 0 \tag{9}$$

Solving for $V_C(t)$:

$$V_C(t) = V_{\text{initial}}e^{-t/\tau} \tag{10}$$

At the moment of switching ($t = T/2$), the capacitor voltage is:

$$V_C(T/2) = 5(1 - e^{-T/(2\tau)}) \tag{11}$$

So the discharging equation for $t > T/2$ is:

$$V_C(t) = 5(1 - e^{-T/(2\tau)})e^{-(t-T/2)/\tau} \tag{12}$$

Steady-State Behavior

In a periodic steady-state response, after multiple cycles, the capacitor voltage oscillates between:

To find $V_{\max}$, we analyze the charging equation at $t = T/2$:

$$V_{\max} = 5(1 - e^{-T/(2\tau)}) \tag{13}$$

To find $V_{\min}$, we substitute $V_{\max}$ into the discharging equation at $t = T/2$:

$$V_{\min} = V_{\max} e^{-T/(2\tau)} \tag{14}$$

which simplifies to:

$$V_{\max} = \frac{V_{source}}{1 + e^{-T/(2\tau)}} \tag{15}$$

$$V_{\min} = \frac{V_{Source}(e^{-T/(2\tau)})}{1 + e^{-T/(2\tau)}} \tag{16}$$

where $V_{\text{source}} = 5V$.