EE LAB REPORT-6

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CONTENTS

Sallen-Key Second-Order Low-Pass and High-Pass Filters

I. Introduction

An Operational Amplifier (Op-Amp(LM358)) is a high-gain electronic voltage amplifier with differential inputs and a single-ended output. It is widely used in analog circuits for signal amplification, filtering, mathematical operations, and waveform shaping. By configuring external components like resistors and capacitors, op-amps can perform functions such as integration, differentiation, and summation.

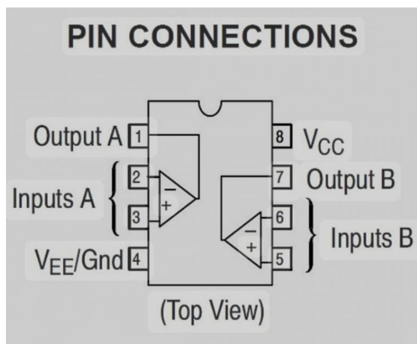


Fig. 1: LM358 connections

The LM358 is a low-power, dual operational amplifier (op-amp) designed for general-purpose applications. It operates with a wide voltage range (3V to 32V) and has low power consumption, making it ideal for battery-powered circuits. The LM358 consists of two independent op-amps in a single package, providing high gain and internal frequency compensation. It is commonly used in signal conditioning, filtering, and amplification circuits.

A. ***Sallen-Key:***

The Sallen-Key topology is a popular active filter design that allows the implementation of second-order low-pass and high-pass filters using operational amplifiers (op-amps), resistors, and capacitors. These filters are widely used in signal processing applications to either pass or attenuate specific frequency ranges.

B. *Quality Factor (Q) in Sallen-Key Second-Order Filters*

The **quality factor** (Q) is a key parameter in second-order **Sallen-Key filters** (both low-pass and high-pass) that determines the selectivity and damping of the filter response. It is given by:

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(R_1 + R_2) C_2}$$

where:

- R_1, R_2 are the resistances in the circuit,
- C_1, C_2 are the capacitances,
- Q influences the filter's damping and peak response.

This formula applies **identically** to both **low-pass (LPF)** and **high-pass (HPF)** Sallen-Key filters because the overall circuit topology remains the same, with only resistors and capacitors swapping positions.

C. Effect of Q on Filter Response

The value of Q determines the nature of the filter's frequency response:

D. Butterworth Response ($Q \approx 0.707$)

- **Maximally flat** response in the passband.
- No ripples, smooth transition to the stopband.
- Standard choice for general filtering applications.

E. Bessel Response ($Q \approx 0.577$)

- **Linear phase response**, preserving the waveform shape.
- Slower roll-off compared to Butterworth.
- Used in audio and pulse signal processing.

F. Chebyshev Response ($Q > 0.707$)

- **Sharper cutoff** but introduces ripples in the passband.
- Higher selectivity but at the cost of increased ringing.
- Used when a steeper roll-off is needed, like in communication systems.

G. High Q ($Q > 1$, underdamped)

- Leads to **resonance or peaking** near the cutoff frequency.
- In extreme cases, it can cause instability or oscillations.

H. Low Q ($Q < 0.5$, overdamped)

- Results in a **slow, gradual roll-off** with no peaking.
- Less selective filtering, useful in some smoothing applications.

I. Cutoff Frequency Formula

The **cutoff frequency** (f_c) for a Sallen-Key second-order filter is given by:

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

J. Conclusion

- Choosing Q depends on the application—**Butterworth** is ideal for general filtering, **Bessel** for phase accuracy, and **Chebyshev** for aggressive filtering.
- The **cutoff frequency** is calculated separately using f_c, which determines the transition between the passband and stopband.

This provides insight into designing Sallen-Key filters with precise frequency and damping characteristics. We will be using V_{CC} as 12V and -12V for this experiment

Fig. 2: V_{CC}

II. Sallen-Key Low-Pass Filter (LPF)

A low-pass filter allows low-frequency signals to pass while attenuating high-frequency signals. The Sallen-Key second-order low-pass filter is designed using two resistors (R_1, R_2) and two capacitors (C_1, C_2) connected in a feedback loop with an operational amplifier.

A. Cirucit Diagram

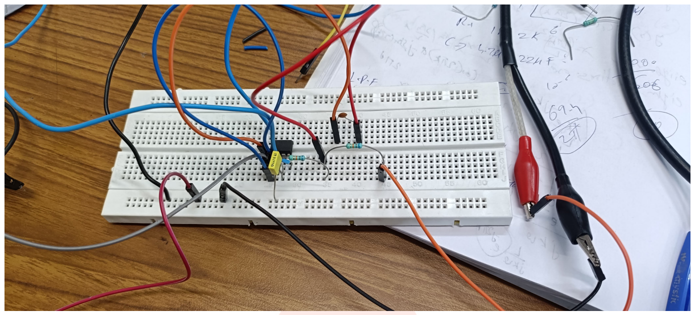
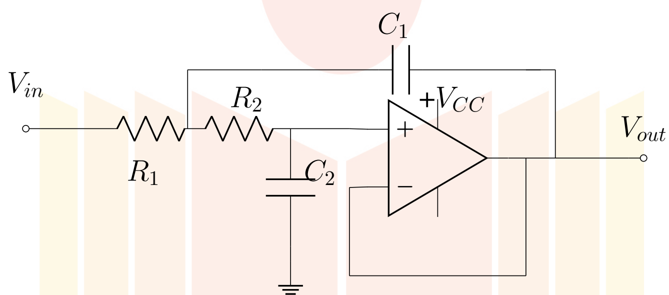


Fig. 3: Ciruit for LPF Sallen-key



B. Derivation of Transfer Function

Using Kirchhoff's Current Law (KCL) at the intermediate node:

$$\frac{V_{\text{in}} - V_x}{R_1} = C_1 \frac{dV_x}{dt} + \frac{V_x - V_{\text{out}}}{R_2} \tag{1}$$

where V_x is the voltage at the junction of R_1, R_2, and C_1.

Applying KCL at the output node:

$$C_2 \frac{dV_{\text{out}}}{dt} = \frac{V_x - V_{\text{out}}}{R_2} \tag{2}$$

Taking the Laplace transform:

$$\frac{V_{\text{in}} - V_x}{R_1} = sC_1 V_x + \frac{V_x - V_{\text{out}}}{R_2} \tag{3}$$

$$sC_2 V_{\text{out}} = \frac{V_x - V_{\text{out}}}{R_2} \tag{4}$$

Solving these equations step by step:

Express V_x in terms of V_{in} and V_{out}.

Substitute into the second equation.

Solve for $H(s) = \frac{V_{\text{out}}}{V_{\text{in}}}$.

After simplification, the transfer function is:

$$H(s) = \frac{A}{s^2 + \frac{\omega_c}{Q}s + \omega_c^2} \tag{5}$$

where:

$$\omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \tag{6}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(R_1 + R_2)C_2} \tag{7}$$

Where C_2 is the capacitor whose one end is connected to ground.

For a unity-gain Sallen-Key low-pass filter, $A = 1$.

We have a unity-gain Sallen-Key low-pass filter so our transfer function equation is

$$H(s) = \frac{1}{s^2 + \frac{\omega_c}{Q}s + \omega_c^2} \tag{8}$$

The values of R and C for this LPF are

$$R_1 = 68k\Omega$$
$$R_2 = 68k\Omega$$
$$C_1 = 10nF$$
$$C_2 = 4.7nF$$

Theoritical cutoff frequency will be

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 R_3 R_4}}$$

$$f_c = \frac{1}{2\pi\sqrt{68 \times 10^3 \times 68 \times 10^3 \times 10 \times 10^{-9} \times 4.7 \times 10^{-9}}}$$

$$f_c = 341.40HZ$$

At cutoff frequency we should be getting magnitude of -3Db
Calculation of Quality factor::

$$\sqrt{68k \times 68k \times 10n \times 4.7n} = \sqrt{2.176} \approx 1.475$$

$$(68k + 68k) \times 4.7n = 136k \times 4.7n = 639.2 \times 10^{-6}$$

$$Q = \frac{1.475}{639.2 \times 10^{-6}} \approx 0.72$$

Final Answer::

$$Q \approx 0.72$$

It is nearer to 0.77 so we will get clean bode plot

C. Results

Fig. 4

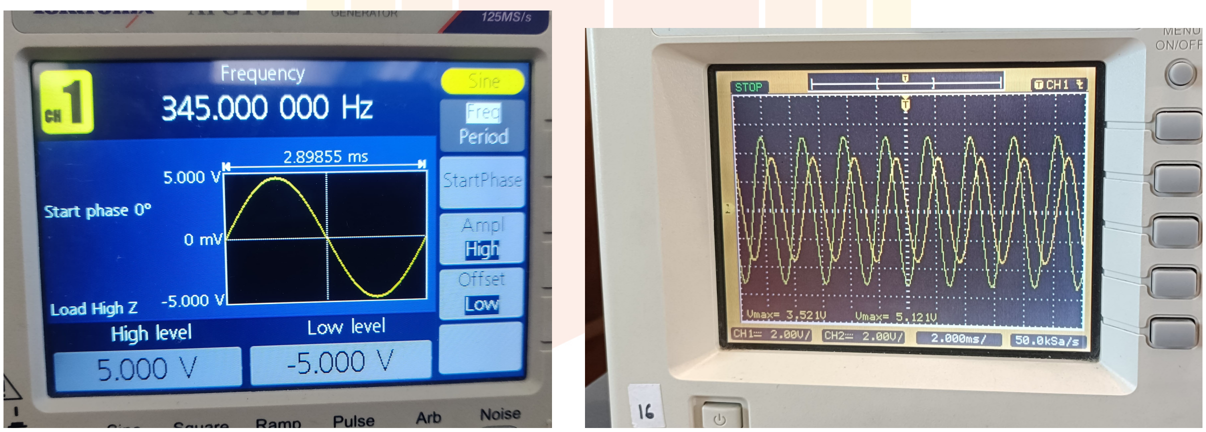


Fig. 5

Fig. 6

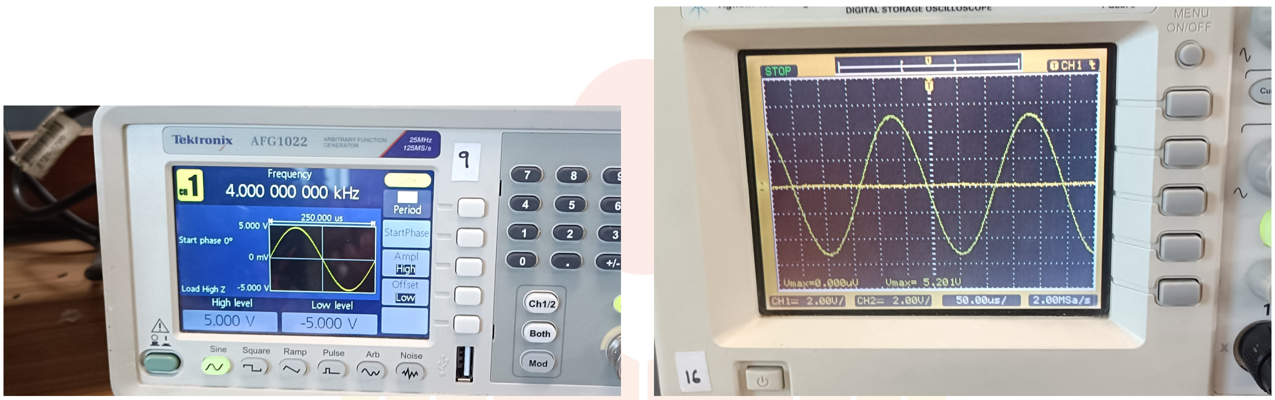


Fig. 7

Frequency (Hz)	Magnitude (dB)
0.1	-0.07
345	-3.05
1000	-19.02
4000	-33.98

TABLE I: Results of LPF

D. Bode plot

Bode plots are plotted using Results obtained

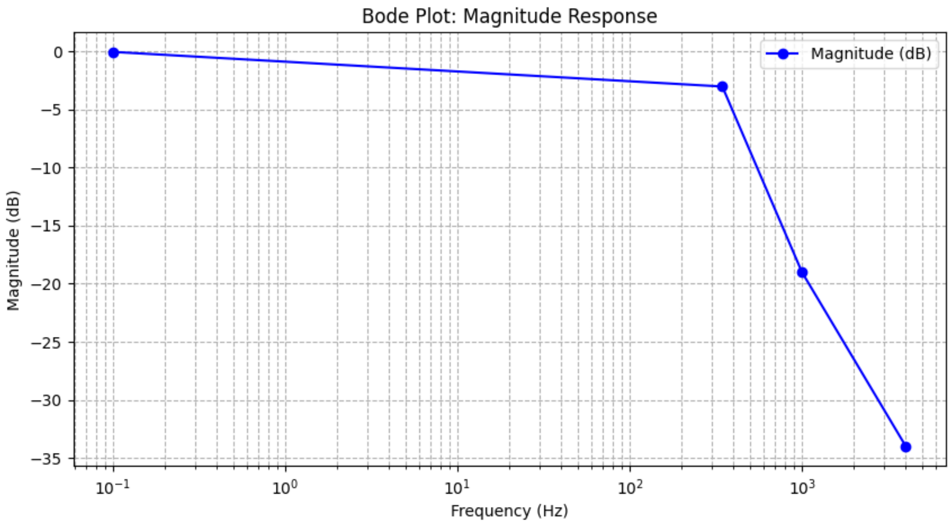


Fig. 9: Python generated plot from values obtained

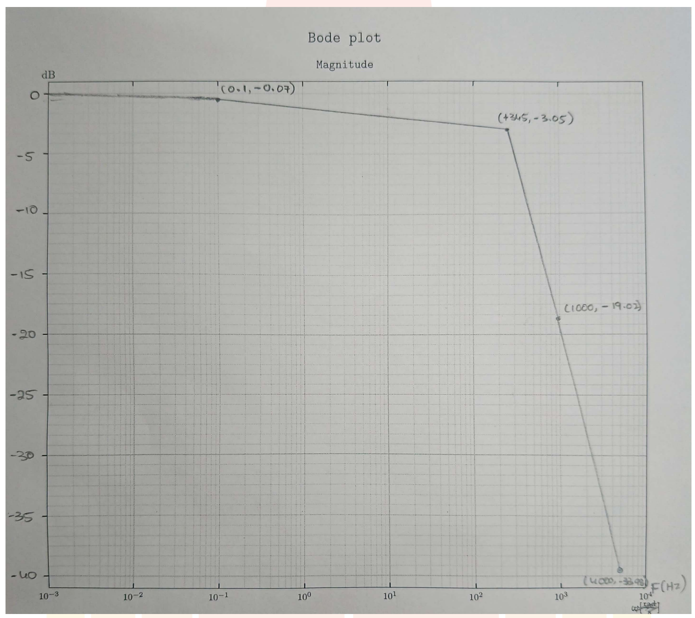


Fig. 8: Hand-plotted LPF bode plot(magnitude vs frequency)

III. Sallen-Key High-Pass Filter (HPF)

A high-pass filter allows high-frequency signals to pass while attenuating low-frequency signals. The Sallen-Key second-order high-pass filter uses a similar topology but with capacitors and resistors swapped in position.

A. Circuit Diagram

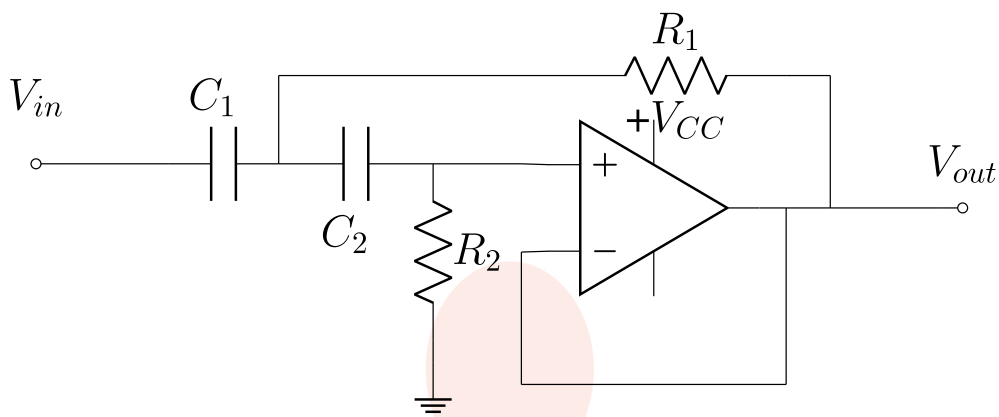


Fig. 10: Circuit Diagram for sallen-key HPF

B. Derivation of Transfer Function

Using KCL at the intermediate node:

$$C_1 \frac{dV_{\text{in}}}{dt} = \frac{V_x}{R_1} + C_1 \frac{dV_x}{dt} \tag{9}$$

Applying KCL at the output node:

$$\frac{V_x}{R_2} + C_2 \frac{dV_{\text{out}}}{dt} = 0 \tag{10}$$

Taking the Laplace transform:

$$sC_1 V_{\text{in}} = \frac{V_x}{R_1} + sC_1 V_x \tag{11}$$

$$\frac{V_x}{R_2} + sC_2 V_{\text{out}} = 0 \tag{12}$$

Solving these equations step by step, the transfer function is:

$$H(s) = \frac{As^2}{s^2 + \frac{\omega_c}{Q}s + \omega_c^2} \tag{13}$$

where ω_c and Q are defined as in the low-pass filter case.

$$\omega_c = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \tag{14}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(R_1 + R_2) C_2} \tag{15}$$

For a unity-gain Sallen-Key low-pass filter, $A = 1$.

We have a unity-gain Sallen-Key low-pass filter so our transfer function equation is

$$H(s) = \frac{1}{s^2 + \frac{\omega_c}{Q} s + \omega_c^2} \tag{16}$$

The values of R and C for this HPF are

$$R_1 = 68k\Omega$$
$$R_2 = 68k\Omega$$
$$C_1 = 10\mu F$$
$$C_2 = 1\mu F$$

The critical frequency f_c is given by:

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

Substituting the values:

$$f_c = \frac{1}{2\pi\sqrt{(68 \times 10^3)(68 \times 10^3)(10 \times 10^{-6})(1 \times 10^{-6})}}$$

$$f_c = \frac{1}{2\pi \times 0.000215}$$

$$f_c \approx 0.740 \text{ Hz}$$

C. Results

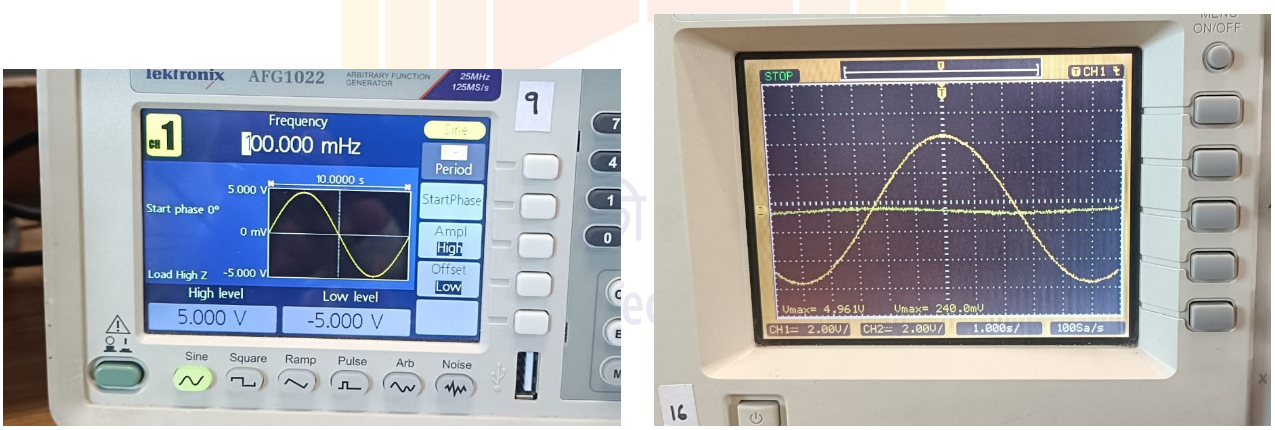


Fig. 11

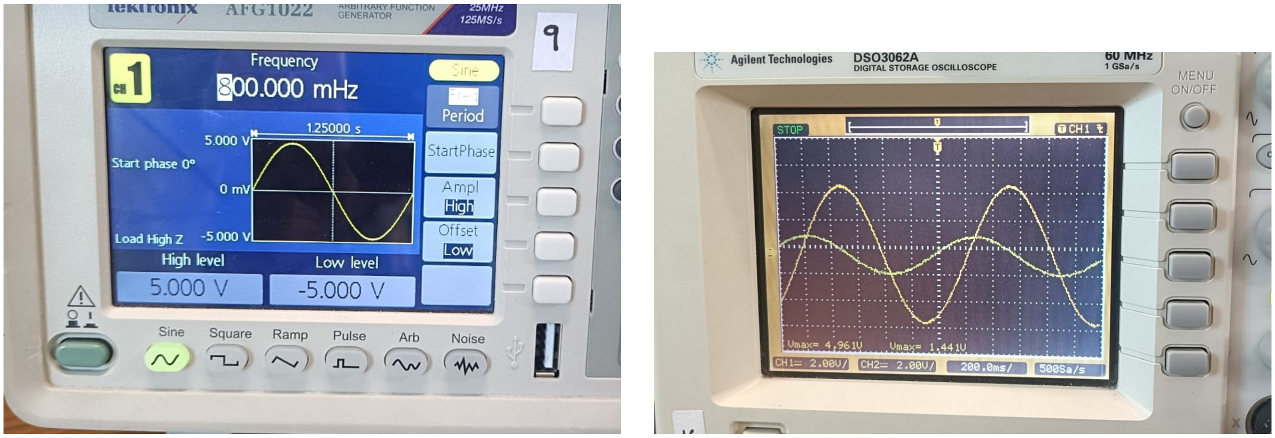


Fig. 12

Fig. 13

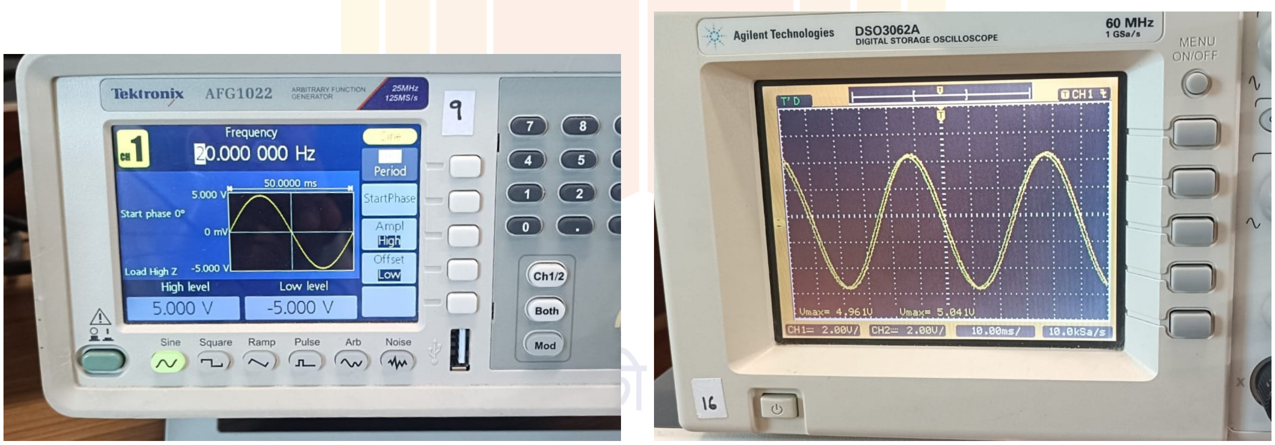


Fig. 14

Fig. 15

Frequency (Hz)	Magnitude (dB)
0.1	-26.375
0.8	-10.81
2.6	-3.24
20	-0.07
100	-0.02

TABLE II: Results of HPF

D. Bode plot

Bode plots are plotted using Results obtained

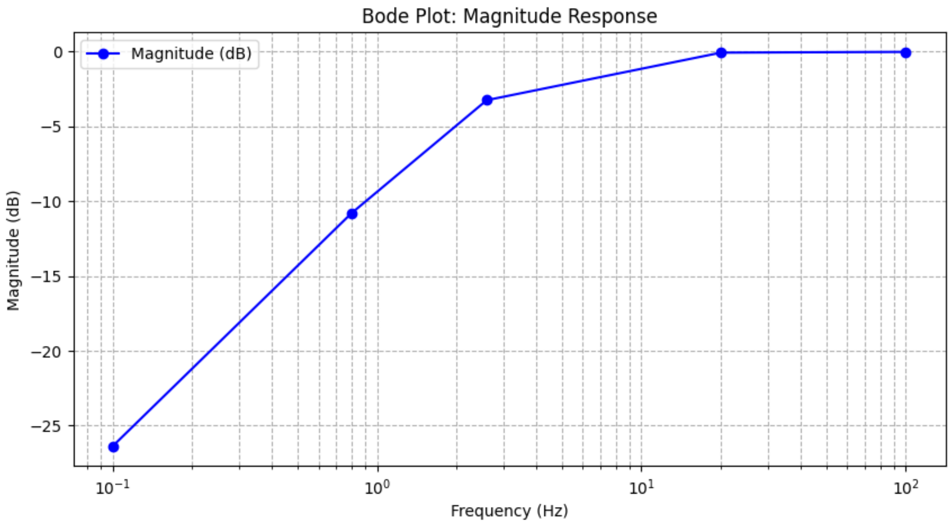


Fig. 17: Python generated plot from values obtained

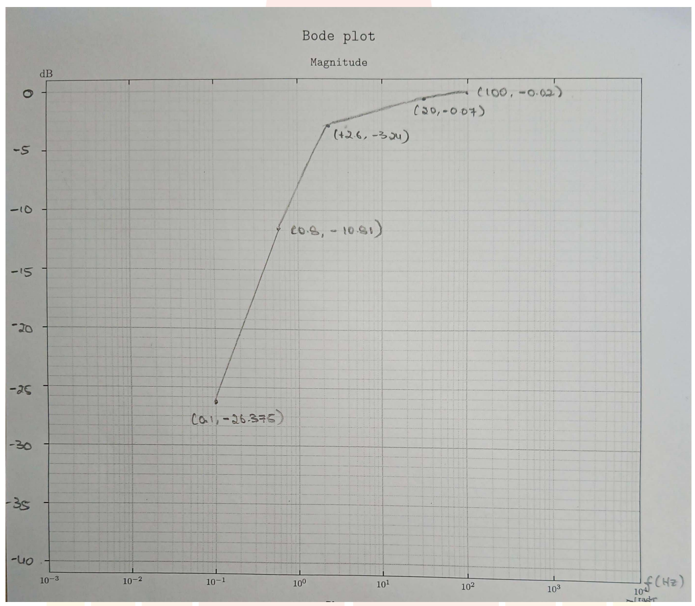


Fig. 16: Hand-plotted HPF bode plot(magnitude vs frequency)

IV. Band-Pass Filter Using High-Pass and Low-Pass Filters

A. Introduction to Band-Pass Filters

A **Band-Pass Filter (BPF)** is an electronic circuit that allows frequencies within a specific range to pass while attenuating frequencies outside this range. It is essentially a combination of a **low-pass filter (LPF)** and a **high-pass filter (HPF)**, where the LPF eliminates high-frequency components, and the HPF removes low-frequency components, allowing only the desired frequency range to pass through. Band-pass filters are widely used in communication systems, audio processing, signal processing, and various electronic applications.

B. Concept of Band-Pass Filter Using High-Pass and Low-Pass Filters

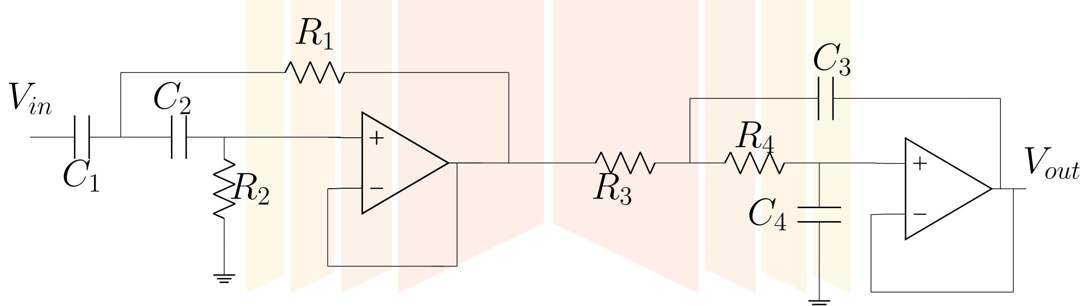
A **band-pass filter (BPF)** can be constructed by cascading a high-pass filter and a low-pass filter in series. The high-pass filter removes low-frequency components, while the low-pass filter suppresses high-frequency components. As a result, only frequencies between the cutoff frequencies ω_L and ω_H are passed, forming a band of frequencies.

The output of the HPF should be connected to the input of the LPF to achieve the desired band-pass response.

In this configuration:

- The **low-pass filter (LPF)** consists of resistors R_3, R_4 and capacitors C_3, C_4.
- The **high-pass filter (HPF)** consists of resistors R_1, R_2 and capacitors C_1, C_2.

C. Circuit Diagram



D. Transfer Function of a Band-Pass Filter

The transfer function of a second-order **Sallen-Key Band-Pass Filter** formed using a separate HPF and LPF is given by:

$$H(s) = H_{LPF}(s) \cdot H_{HPF}(s) \tag{17}$$

where:

$$H_{LPF}(s) = \frac{1}{1 + s\left(\frac{R_3+R_4}{R_3R_4C_4}\right) + s^2\left(\frac{1}{R_3R_4C_3C_4}\right)} \tag{18}$$

$$H_{HPF}(s) = \frac{s^2}{s^2 + s\left(\frac{R_1+R_2}{R_1R_2C_2}\right) + \frac{1}{R_1R_2C_1C_2}} \tag{19}$$

Center Frequency and Quality Factor:
The **center frequency** ω_C (resonant frequency) of the band-pass filter is given by:

$$\boxed{\omega_C = \sqrt{\omega_L \cdot \omega_H}} \tag{20}$$

At the resonance frequency f_C, the output gain is maximum, and the phase shift is typically $0°$ (for Butterworth filters). The circuit exhibits maximum energy exchange between reactive components, with the quality factor Q determining the bandwidth, where higher Q leads to a narrower passband. The group delay is minimal or stable near f_C, ensuring minimal signal distortion.
where ω_L and ω_H are the cutoff frequencies of the HPF and LPF, respectively.

Characteristics of Band-Pass Filters:

- **Frequency Selection:** Band-pass filters allow frequencies within a certain range to pass while attenuating both low and high frequencies outside the passband.
- **Center Frequency (f_0):** The frequency at which the gain is maximum.
- **Bandwidth (BW):** The range of frequencies between the lower cutoff frequency f_L and the upper cutoff frequency f_H, given by:

$$BW = f_H - f_L \tag{21}$$

- **Quality Factor (Q):** The measure of how selective the filter is, given by:

$$Q = \frac{f_C}{f_H - f_L} \tag{22}$$

  - A higher Q results in a **narrow** band-pass filter, allowing only a small range of frequencies.
  - A lower Q value results in a **wider bandwidth**.

E. Results

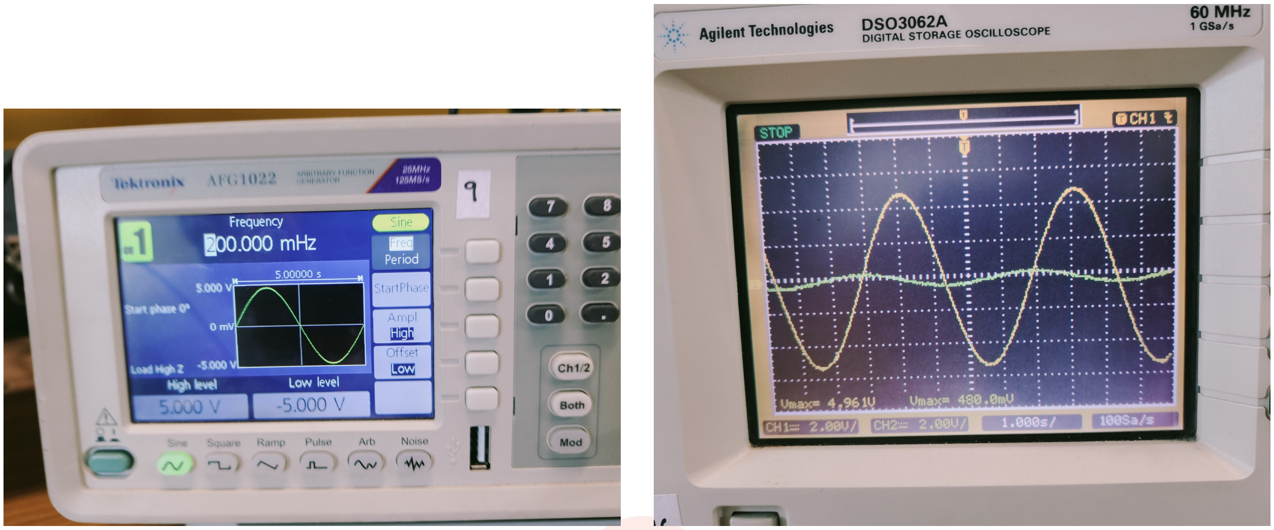


Fig. 18

Fig. 19

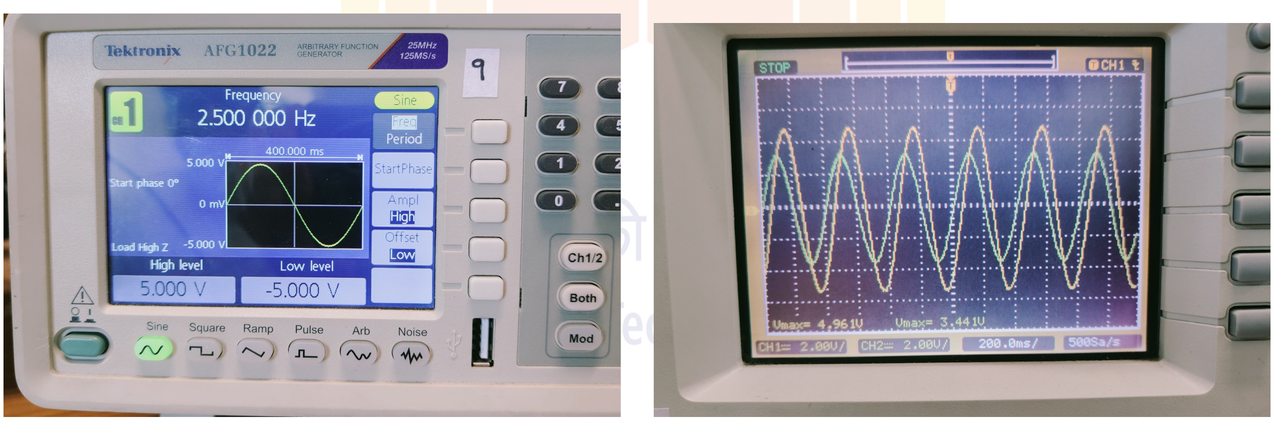


Fig. 20

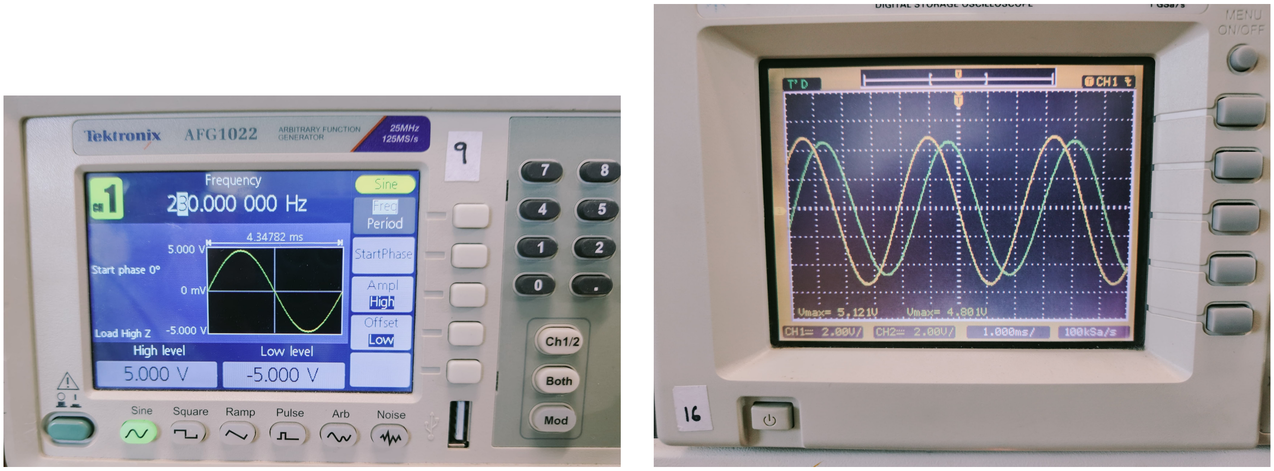


Fig. 21

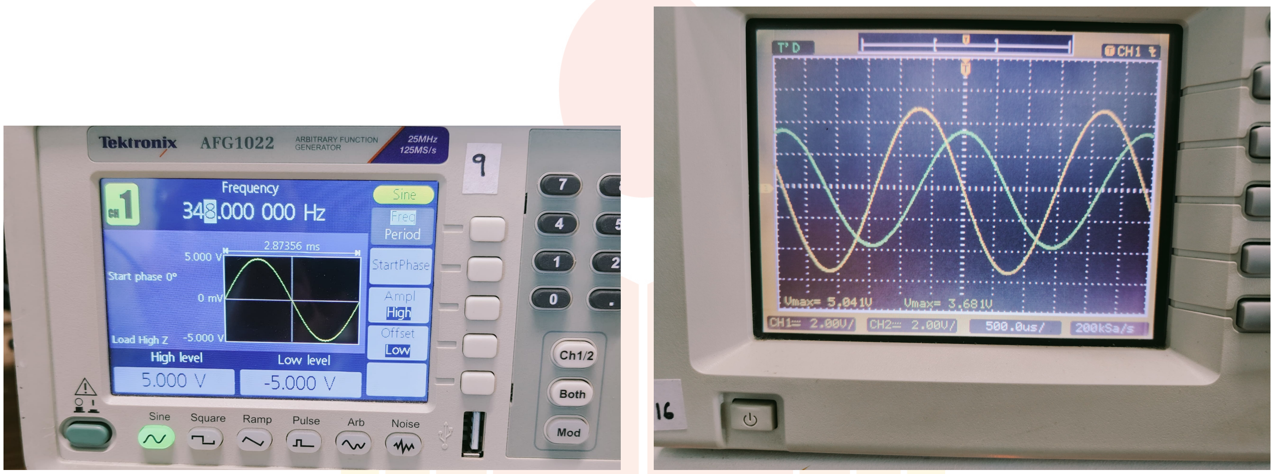


Fig. 22

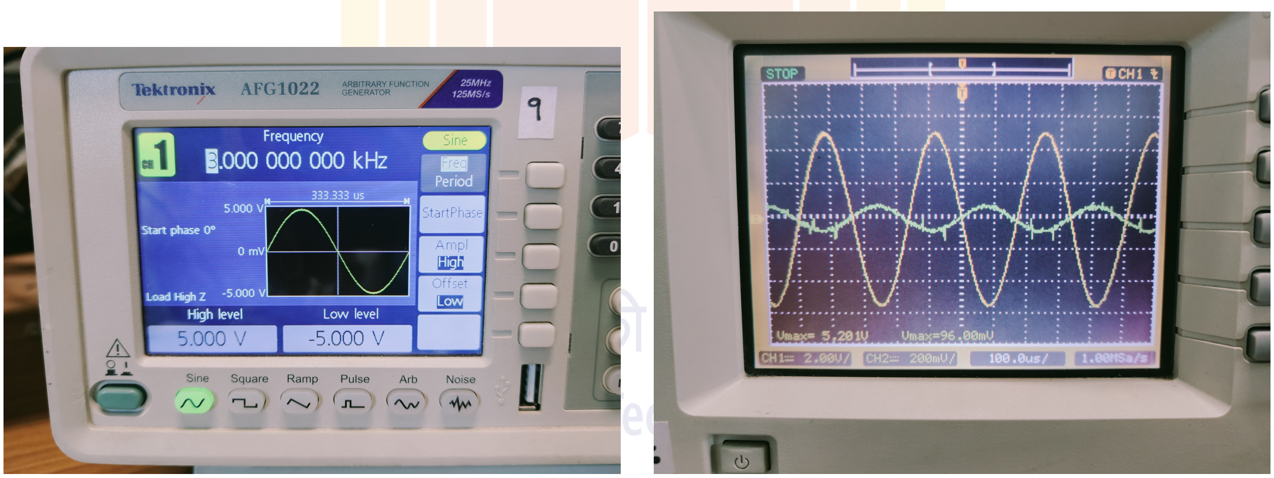


Fig. 23

Here is case where output is maximum i.e resonant frequency

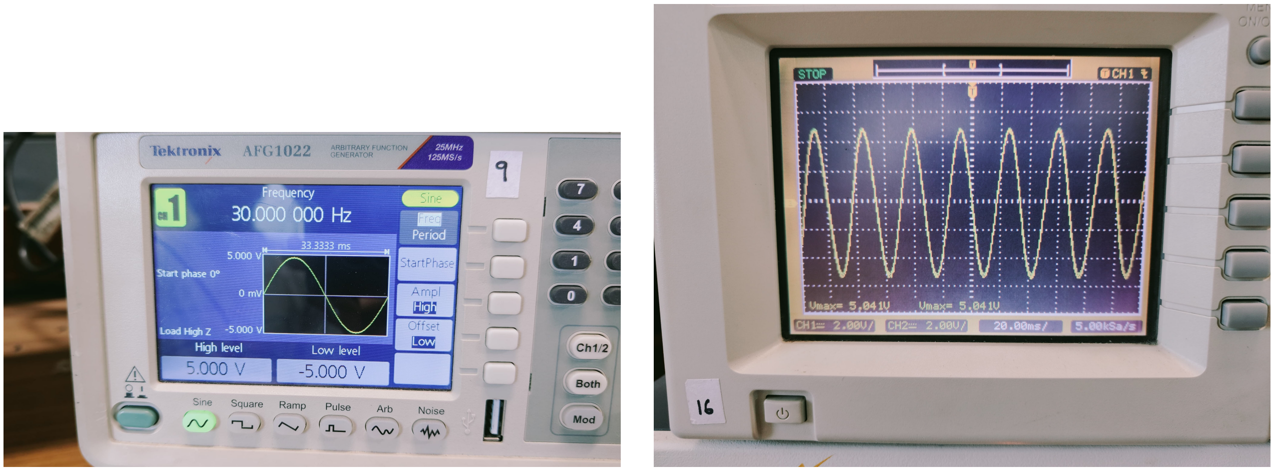


Fig. 24

Frequency (Hz)	Magnitude (dB)
0.2	-20.35
2.5	-3.24
30	0.00
230	-0.53
1000	-15.09
3000	-34.33

TABLE III: Results of Bandpass filter

F. Bode plot

Bode plots are plotted using Results obtained

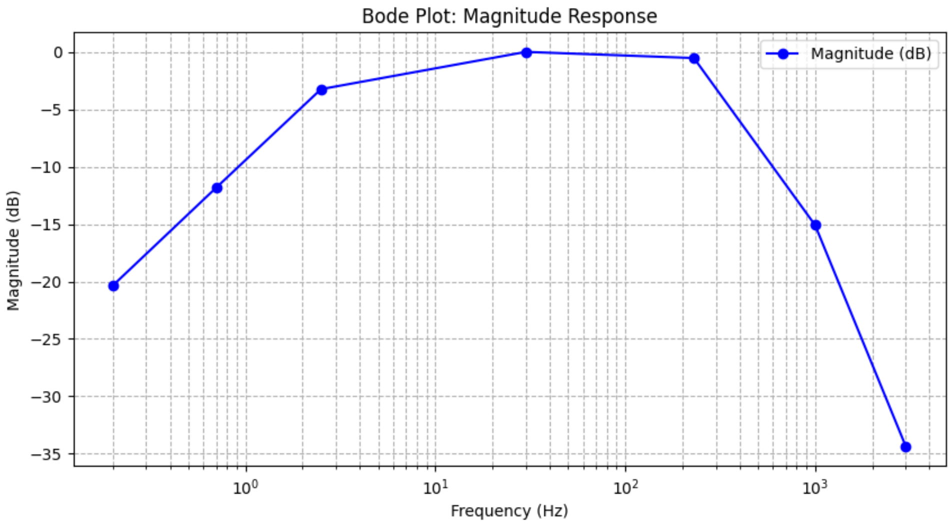


Fig. 26: Python generated plot from values obtained

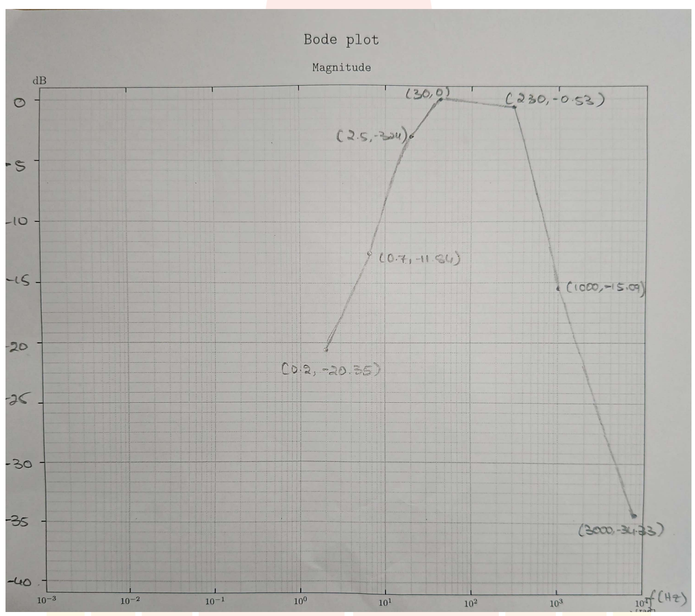


Fig. 25: Hand-plotted BPF bode plot(magnitude vs frequency)