# EE LAB REPORT-1

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# I. TASK-A

## A. Introduction

Lissajous figures are intricate patterns formed when two perpendicular harmonic oscillations with different frequencies and phases are combined. These patterns are commonly observed on an oscilloscope by applying sinusoidal signals to its X and Y inputs. The resulting shapes, such as straight lines, ellipses, circles, and complex loops, depend on the ratio of the frequencies and the phase difference between the signals. Lissajous figures have applications in signal analysis, system identification, and the study of harmonic motion.

## B. Aim

(a) Obtaining at least six Lissajous figures on a CRO by combining two signals with different phase and frequency combinations and mathematically proving the structure of each figure

## C. Apparatus Used

- Cathode Ray Oscilloscope (CRO).
- BNC Cables.
- Function Generator.
- Oscilloscope Probes

## D. Procedure/Experiment

1) Connect the X and Y inputs of the oscilloscope to the two channels of the function generator using BNC Cables.
2) Configure the function generators. Set both output same signals initially. Adjust the frequencies and amplitudes as required. Introduce phase differences and frequency changes and amplitude offsets as needed for specific observations.
3) Ensure proper grounding and check all connections.
4) Observe the patterns obtained on oscilloscope.

1) **When both signals are same**
Consider a sinusodial signal

$$y = A\sin(\omega t + \phi)$$

Since we are plotting one against each other the other signal will be

$$x = A\sin(\omega t)$$

In this case $\phi$ will be 0 for simplicity. The parameters of both signals are provided in TABLE 1

| Parameter | Channel 1 | Channel 2 |
|---|---|---|
| Frequency | 200.000 Hz | 200.000 Hz |
| Phase | 0° | 0° |
| High | 1 V | 1 V |
| Low | 0 mV | 0 mV |

TABLE I: Function Generator Settings

Following is obtained on oscilloscope when both signals are plotted in X-Y mode

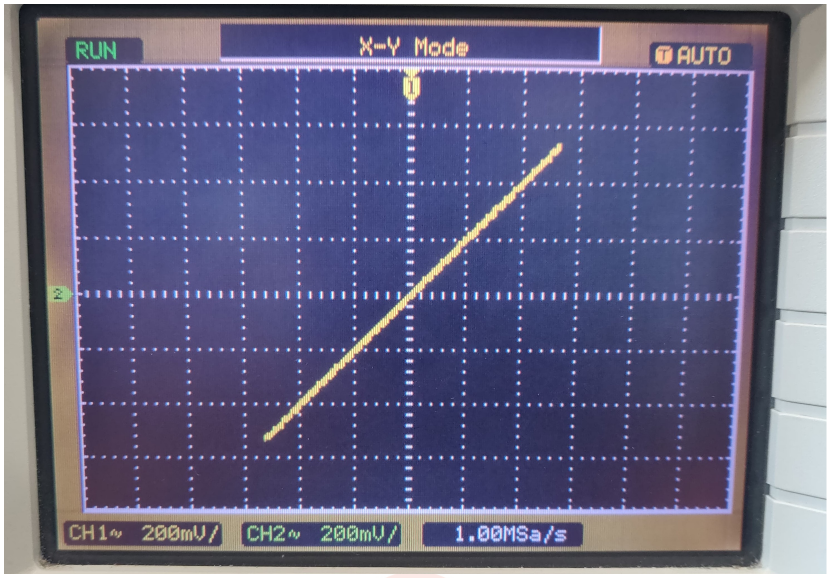


Fig. 1

Practically there should be continuos straight lines side by side like those but the CRO only shows the graph for only one period. It means the graph shown on CRO will repeat itself continuosly.

**Mathematical Proof:**
Since the two waves are equal

$$x = y$$

The graph on CRO is clearly $x = y$

2) **When there is phase difference between two signals:**
Let us introduce a phase difference of $\phi$=45°between two signals and see what happens

The parameters of both signals are provided in TABLE 2

| **Parameter** | **Channel 1** | **Channel 2** |
|---|---|---|
| Frequency | 200.000 Hz | 200.000 Hz |
| Phase | 45° | 0° |
| High | 1.000V | 1.000V |
| Low | 0 V | 0 V |

TABLE II: Function Generator Settings

For these parameters, We will get the following figure on CRO


Fig. 2

Let us try mathematically why we got that figure on CRO.
**Mathematical proof:**
Consider two sinusoidal signals applied to the X and Y channels of the CRO. The signals have the same frequency and amplitude but a phase difference $\phi$:

$$x(t) = A\sin(\omega t)$$
$$y(t) = A\sin(\omega t + \phi)$$

Here:
- $A$ is the amplitude of both signals.
- $\omega$ is the angular frequency ($\omega = 2\pi f$).
- $\phi$ is the phase difference between the two signals.

Using the trigonometric identity:

$$\sin(\omega t + \phi) = \sin(\omega t)\cos(\phi) + \cos(\omega t)\sin(\phi)$$

Substitute $y(t)$:

$$y(t) = A\sin(\omega t)\cos(\phi) + A\cos(\omega t)\sin(\phi)$$

Divide both $x(t)$ and $y(t)$ by $A$ to normalize the signals:

$$\frac{x(t)}{A} = \sin(\omega t), \quad \frac{y(t)}{A} = \sin(\omega t)\cos(\phi) + \cos(\omega t)\sin(\phi)$$

Let $X = \frac{x}{A}$ and $Y = \frac{y}{A}$, so:

$$Y = X\cos(\phi) + \sqrt{1 - X^2}\sin(\phi)$$

Square both sides and simplify using $\sin^2(\omega t) + \cos^2(\omega t) = 1$:

$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{A}\right)^2 - 2\frac{x}{A}\frac{y}{A}\cos(\phi) = \sin^2(\phi)$$

Rewriting this:

$$x^2 + y^2 - 2xy\cos(\phi) = A^2\sin^2(\phi) \tag{1}$$

This is the general equation of an ellipse. Let us see what will happen if $\phi$=45°and Amplitude A=0.5V. The general equation is modified as

$$x^2 + y^2 - \sqrt{2}xy = \frac{1}{8}$$

The mathematical graph of the equation will be

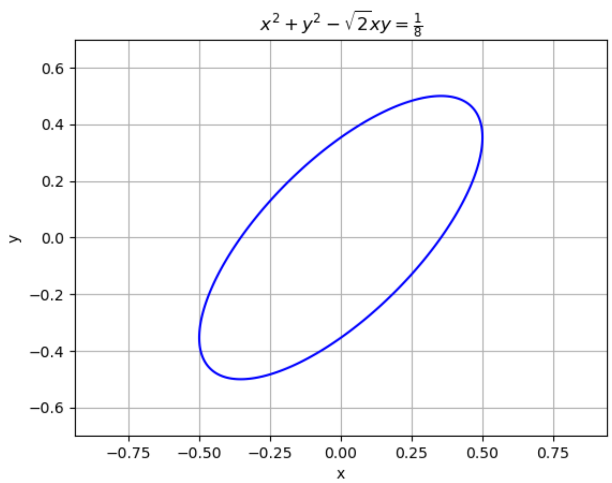


Fig. 3

3) **If $\phi$ between two signals is 90°:**
Let us see what will happen if we introduce a phase difference of 90°between the signals keeping the amplitude and frequencies same. Following parameters are used in respective channels

| **Parameter** | **Channel 1** | **Channel 2** |
|---|---|---|
| Frequency | 200.000 Hz | 200.000 Hz |
| Phase | 90° | 0° |
| High | 1.000V | 1.000V |
| Low | 0 V | 0 V |

TABLE III: Function Generator Settings

Following graph is observed on CRO

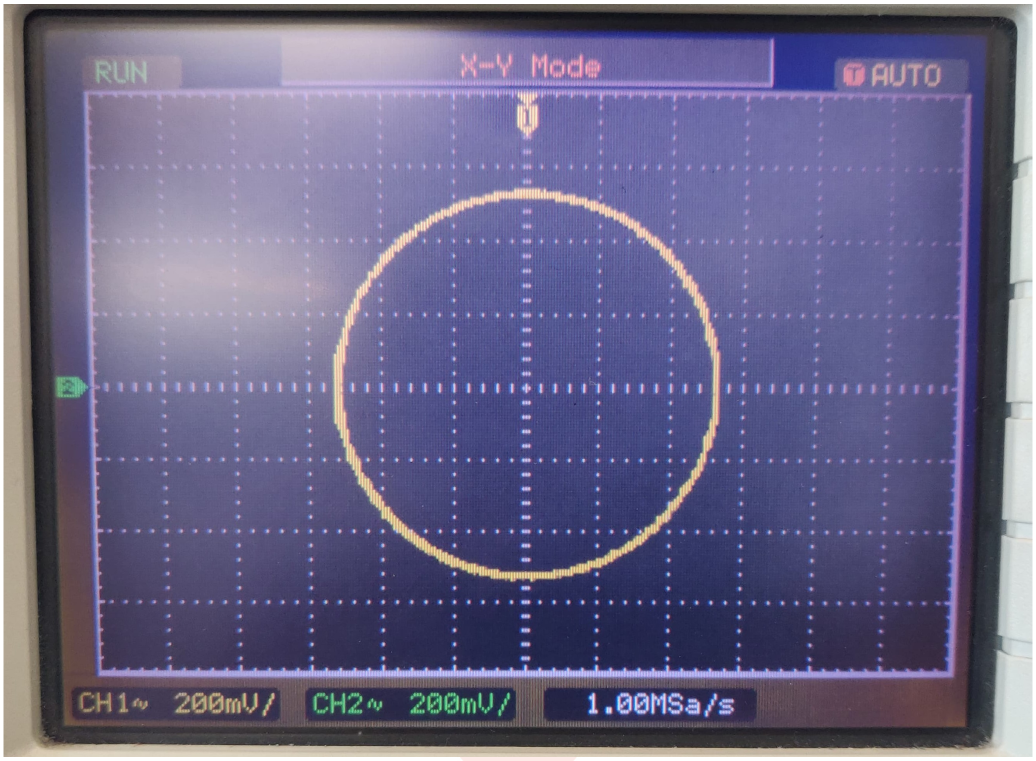


Fig. 4

Rewriting the general equation of Ellipse from 1 with $\phi = 90°$ and A=0.5V. We get

$$x^2 + y^2 = \frac{1}{4}$$

The mathematical graph of the equation will be

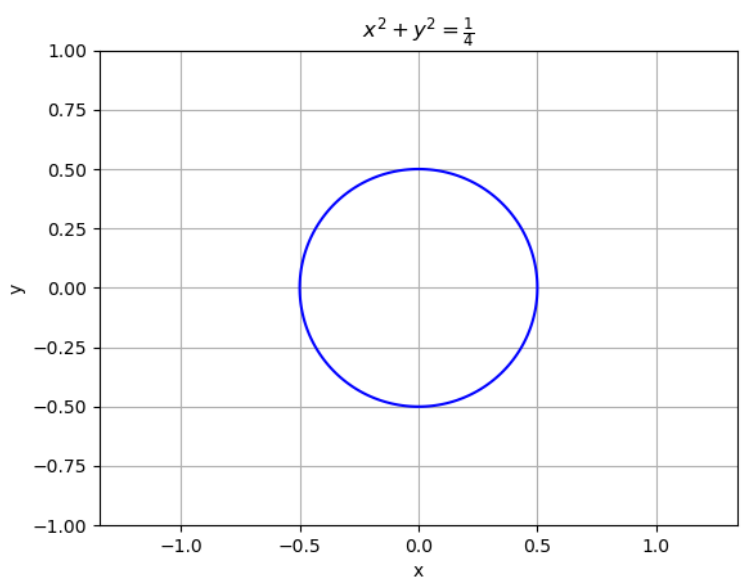


Fig. 5

4) **When frequencies are in ratio of 1:2**

Now we will keep the frequencies of two signals in the ratio 1:2 keeping the amplitudes and phase difference the same. Following parameters are used for signals:

| **Parameter** | **Channel 1** | **Channel 2** |
|---|---|---|
| Frequency | 200.000 Hz | 400.000 Hz |
| Phase | 0° | 0° |
| High | 1 V | 1 V |
| Low | 0 V | 0 V |

TABLE IV: Function Generator Settings

Following graph is observed on CRO:

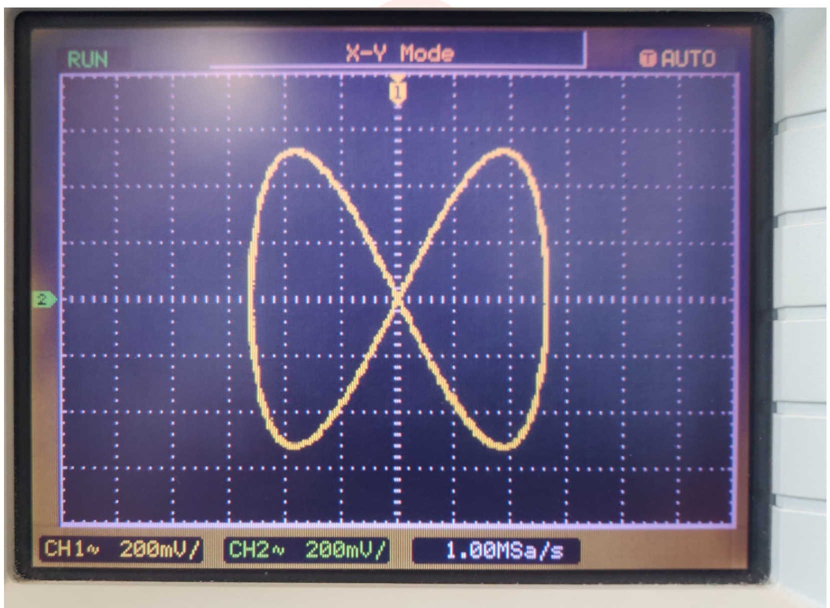


Fig. 6

We will look into the mathematical proof of the figure.

**Mathematical Proof:**

Let the equations for the two waves be:

$$x(t) = A\sin(\omega t)$$
$$y(t) = A\sin(2\omega t)$$

where:

- $A$ is the amplitude of the wave.
- $\omega$ is the angular frequency of the wave.
- $t$ is the time.

The frequency ratio of $1:2$ means that the second wave completes twice as many oscillations in the same period as the first wave.

**Elimination of Time Variable**

To obtain the Lissajous figure, we eliminate the time variable $t$ by solving for $t$ from the equation for $x(t)$:

$$x(t) = A\sin(\omega t) \implies \sin(\omega t) = \frac{x}{A}$$

Substitute this into the equation for $y(t)$:

$$y(t) = A\sin(2\omega t)$$

Using the identity for $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$, we have:

$$y(t) = A \cdot 2\sin(\omega t)\cos(\omega t)$$

Now, substituting $\sin(\omega t) = \frac{x}{A}$ into this:

$$y(t) = 2A \cdot \frac{x}{A}\cos(\omega t)$$

Simplifying:

$$y(t) = 2x\cos(\omega t)$$

Next, we need an expression for $\cos(\omega t)$. From $x(t) = A\sin(\omega t)$ and using $\sin^2\theta + \cos^2\theta = 1$ we can write:

$$\cos(\omega t) = \pm\sqrt{1 - \left(\frac{x}{A}\right)^2}$$

Thus, the equation for $y(t)$ becomes:

$$y = \pm 2x\sqrt{1 - \left(\frac{x}{A}\right)^2}$$

Here $A = \frac{1}{2}$,

$$y = \pm 2x\sqrt{1 - 4x^2}$$

The mathematical graph of equation will be This represents the characteristic shape of the Lissajous

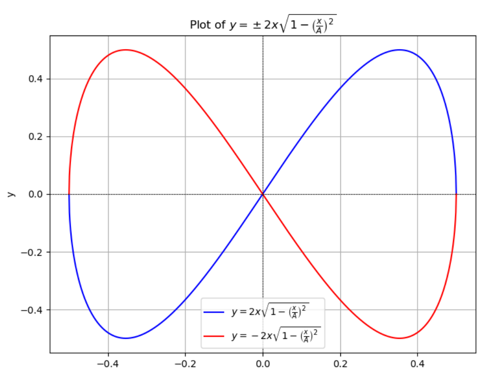


Fig. 7



figure formed on the cathode ray oscilloscope (CRO) when the two waves are in-phase and have a 1:2 frequency ratio.

5) **When frequencies are in ratio 1:3**
Now we will keep the frequencies of two signals in the ratio 1:3 keeping the amplitudes and phase difference same. Following parameters are used for signals

| Parameter | Channel 1 | Channel 2 |
|---|---|---|
| Frequency | 200.000 Hz | 600.000 Hz |
| Phase | 0° | 0° |
| High | 1.00V | 1.00V |
| Low | 0 V | 0 V |

TABLE V: Function Generator Settings

Following graph is observed on CRO

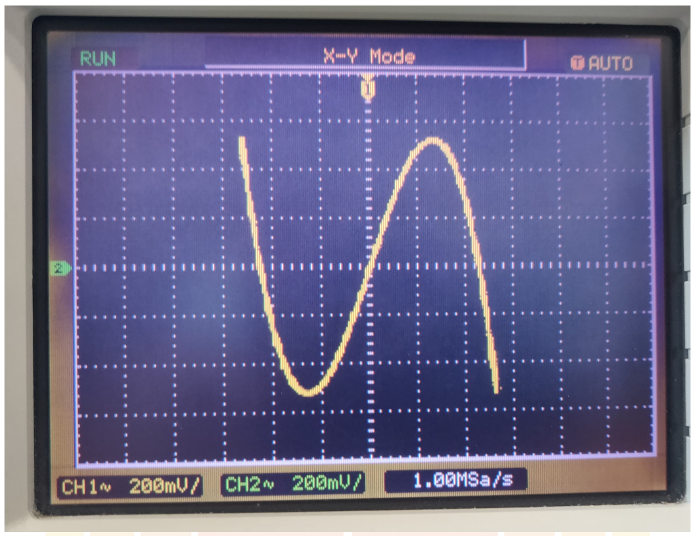


Fig. 8

Here is the mathematical explanation for the graph on CRO
**Mathematical Explanation of Lissajous Pattern for Frequency Ratio 1:3**
The two signals are given as:

$$x(t) = A\sin(2\pi f_x t),$$
$$y(t) = A\sin(2\pi f_y t),$$

where:

- $A$ is the amplitude of both signals (same amplitude),
- $f_x$ and $f_y$ are the frequencies of the signals, and
- $f_x : f_y = 1 : 3$, so $f_y = 3f_x$.

Substituting the frequency ratio:

$$x(t) = A\sin(2\pi f t), \quad \text{where } f_x = f,$$
$$y(t) = A\sin(6\pi f t), \quad \text{where } f_y = 3f.$$

**Eliminating Time $t$**
To find the relationship between $x(t)$ and $y(t)$, we eliminate $t$:

$$\sin(2\pi f t) = \frac{x(t)}{A}.$$

Now, substitute this into the second signal:

$$y(t) = A\sin(6\pi f t).$$

Using the trigonometric identity $\sin(3\theta) = 3\sin(\theta) - 4\sin^3(\theta)$ with $\theta = 2\pi f t$:

$$y(t) = A\left[3\sin(2\pi f t) - 4\sin^3(2\pi f t)\right].$$

Substituting $\sin(2\pi f t) = \frac{x}{A}$:

$$y = A\left[3\frac{x}{A} - 4\left(\frac{x}{A}\right)^3\right].$$

Simplify:

$$y = 3x - \frac{4x^3}{A^2}.$$

Here $A = \frac{1}{2}$

$$y = 3x - 16x^3.$$

**Final Lissajous Pattern Equation:**
The Lissajous curve for the frequency ratio $1:3$ is:

$$y = 3x - 16x^3.$$

The mathematical graph of equation will be

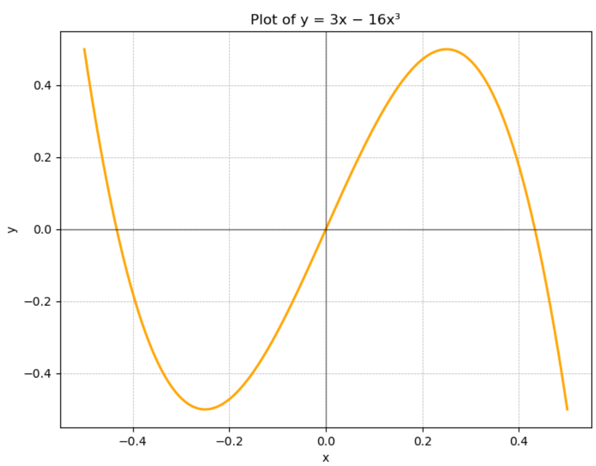


Fig. 9

**Observations:**

- The shape of the pattern is a cubic curve that repeats periodically.

6) **When frequencies are in ratio 1:4**
Now we will keep the frequencies of two signals in the ratio 1:4 keeping the amplitudes and phase difference same. Following parameters are used for signals
Following graph is observed on CRO
Here is the mathematical explanation for the graph on CRO
**Mathematical Explanation of Lissajous Pattern for Frequency Ratio 1:4**

| Parameter | Channel 1 | Channel 2 |
|---|---|---|
| Frequency | 200.000 Hz | 800.000 Hz |
| Phase | 0° | 0° |
| High | 1.00V | 1.00V |
| Low | 0 V | 0 V |

TABLE VI: Function Generator Settings

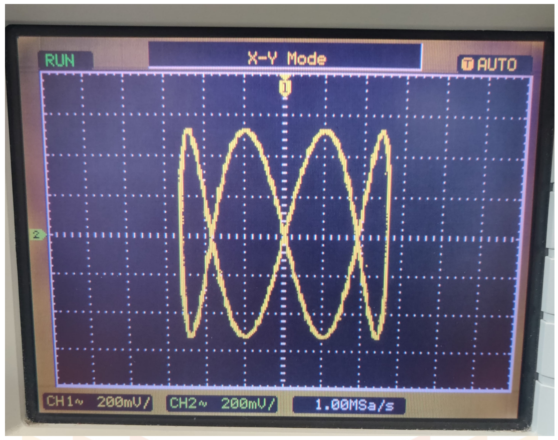


Fig. 10

The two signals are given as:

$$x(t) = A\sin(2\pi f_x t),$$
$$y(t) = A\sin(2\pi f_y t),$$

where:
- $A$ is the amplitude of both signals (same amplitude),
- $f_x$ and $f_y$ are the frequencies of the signals, and
- $f_x : f_y = 1 : 4$, so $f_y = 4f_x$.

Substituting the frequency ratio:

$$x(t) = A\sin(2\pi f t), \quad \text{where } f_x = f,$$
$$y(t) = A\sin(8\pi f t), \quad \text{where } f_y = 4f.$$

**Eliminating Time $t$**

To find the relationship between $x(t)$ and $y(t)$, we eliminate $t$:

$$\sin(2\pi f t) = \frac{x(t)}{A}.$$

Using the trigonometric identity $\sin^2(\theta) + \cos^2(\theta) = 1$:

$$\cos(2\pi f t) = \pm\sqrt{1 - \sin^2(2\pi f t)} = \pm\sqrt{1 - \left(\frac{x}{A}\right)^2}.$$

Now, substitute this into the second signal:

$$y(t) = A\sin(8\pi f t).$$

Using the trigonometric identity $\sin(4\theta) = 8\sin(\theta)\cos^3(\theta) - 4\sin(\theta)\cos(\theta)$ with $\theta = 2\pi ft$:

$$y(t) = A\left[8\sin(2\pi ft)\cos^3(2\pi ft) - 4\sin(2\pi ft)\cos(2\pi ft)\right].$$

Substituting $\sin(2\pi ft) = \frac{x}{A}$ and $\cos(2\pi ft) = \pm\sqrt{1 - \left(\frac{x}{A}\right)^2}$:

$$y = \pm A\left[8\frac{x}{A}\left(\sqrt{1 - \left(\frac{x}{A}\right)^2}\right)^3 - 4\frac{x}{A}\sqrt{1 - \left(\frac{x}{A}\right)^2}\right].$$

Here $A = \frac{1}{2}$

$$y = \pm\left(8x\left(1 - 4x^2\right)^{3/2} - 4x\sqrt{1 - 4x^2}\right).$$

**Final Lissajous Pattern Equation**

The Lissajous curve for the frequency ratio $1:4$ is:

$$y = \pm\left(8x\left(1 - 4x^2\right)^{3/2} - 4x\sqrt{1 - 4x^2}\right).$$

The mathematical graph of equation will be

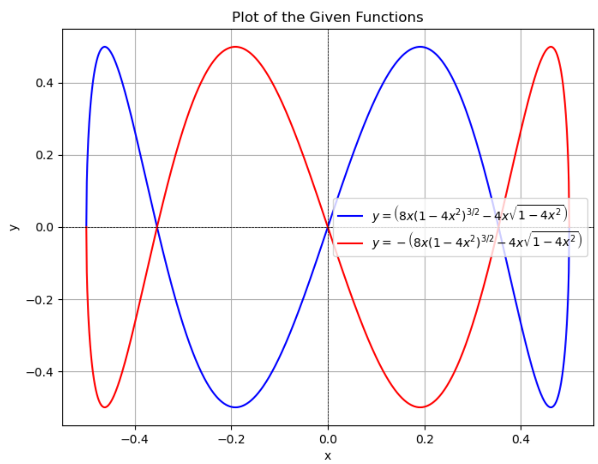


Fig. 11

**Observations**

- The shape of the pattern is a more complex periodic curve.

*E. Conclusion*



In this experiment, Lissajous figures were observed on the oscilloscope by varying the phase difference and frequency ratio between two signals. The results clearly demonstrate the distinct patterns formed under different phase and frequency conditions:

- **Phase Difference:**
  - For phase differences of 0° and 180°, the figures appeared as ellipses or straight lines, indicating in-phase or out-of-phase relationships.
  - A phase difference of 45° produced a skewed elliptical pattern, while a 90° phase difference resulted in a more symmetric figure with perpendicular axes.

- **Frequency Ratio:**
  - Frequency ratios of 1:2, 1:3, and 1:4 created complex, closed-loop figures with varying numbers of lobes. Specifically, a ratio of 1:2 generated a figure with two lobes, while 1:3 and 1:4 ratios showed three and four lobes, respectively.
  - The frequency ratio directly influenced the number of lobes and the overall shape, reinforcing the relationship between the harmonic frequencies and the resulting geometric patterns.

These observations highlight the strong influence of both phase and frequency on the nature of the Lissajous figures, providing valuable insight into the behavior of coupled oscillatory systems.

# II. TASK-B

### A. Aim

To capture and analyze a one-time event using a Cathode Ray Oscilloscope (CRO).

### B. Introduction

a one-time event refers to a transient signal or a single occurrence of an event that happens only once and is not periodic or repetitive. These events are often challenging to capture and analyze because they do not repeat, requiring specialized techniques and settings on the oscilloscope.

### C. Apparatus Used

- Cathode Ray Oscilloscope (CRO).
- BNC Cables.
- Function Generator.
- Oscilloscope Probes

**Oscilloscope features for capturing one time event :-**

- **Triggering :-**
  - Edge triggering : Detects when the signal crosses a specific voltage level.
  - Pulse Width Triggering: Identifies pulses of a specified duration, useful for glitches.
  - Slope Triggering: Triggers based on the rate of signal change.
- **Sweep normal mode:-**
  - This is crucial for understanding how the system will respond to dynamic loads or vibrations at different frequencies. Operating at or near these natural frequencies can cause resonance, leading to excessive vibrations or failures.
  - Sweep normal mode helps identify the system's natural frequencies by systematically varying the excitation frequency and observing the system's response.
  - It helps in visualizing and analyzing the mode shapes associated with each natural frequency.

**Burst mode**

- Burst mode generates a fixed number of cycles of a waveform (e.g., sine, square, triangle) and stops until triggered again.
- **Applications :-**
  - Testing circuits under transient conditions.
  - Simulating signals like pulsed waveforms or one-time events.

*D. Procedure/Experiment*

- Set the oscilloscope's sweep mode to Normal.
- Choose the channel corresponding to the signal you want to capture.
- Select the appropriate trigger type
- Adjust to the voltage level where the one-time event is expected to occur.
- Steps to be followed on function generator to capture one time event
  - Activate burst mode on the function generator.
  - Set the burst count to the desired number of cycles.
  - Set the trigger mode to Manual on your setup.
- Press Single on the oscilloscope to arm the oscilloscope and prepare it to capture the signal.
- Press the trigger button on the function generator when ready to initiate the event..
- When the one-time event occurs and meets the trigger condition, the oscilloscope captures the signal.

These are the parameters taken on the function generator

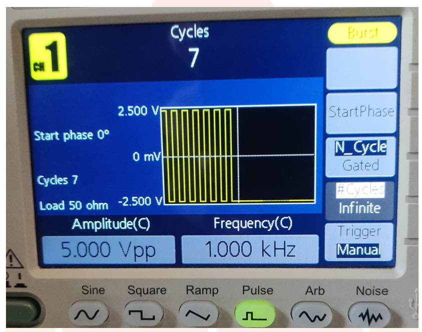


Fig. 12

The output on the oscilloscope is as shown in the below figure


Fig. 13



*E. Conclusion*

The one-time capture feature effectively allowed for the visualization of a single, non-repeating event. By configuring the trigger settings appropriately, the transient signal was captured and displayed clearly, enabling accurate analysis of the event's characteristics without repetition.