JEE ADVANCED

Suraj Kolluru
EE24BTECH110033

1 Subjective Problems

1) Let 'd' be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent drawn at a point $\mathbf{P}$ on the ellipse. If $\mathbf{F_1}$ and $\mathbf{F_2}$ are the two *foci* of the ellipse, then show that $(PF_1-PF_2)^2=4a^2\left(1-\frac{b^2}{d^2}\right)$. $(1995-5marks)$
2) Points $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ lie on a parabola $y^2=4ax$. The tangents to the parabola at $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ taken in pairs, intersect at points $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$. Determine the ratios of the areas of triangles ABC and PQR. $(1996-3marks)$
3) From a point $\mathbf{A}$ common tangents are drawn to the circle $x^2+y^2=\frac{a^2}{2}$ and the parabola $y^2=4ax$. Find the area of the quadrilateral formed by the common tangents, the chord of contact of the circle, and the chord of contact of the parabola. $(1996-2marks)$
4) A tangent to the ellipse $x^2+4y^2=4$ meets the ellipse $x^2+2y^2=6$ at $\mathbf{P}$ and $\mathbf{Q}$. Prove that the tangents at $\mathbf{P}$ and $\mathbf{Q}$ of the ellipse $x^2+2y^2=6$ are at right angles. $(1997-5marks)$
5) The angle between a pair of tangents drawn from a point $\mathbf{P}$ to the parabola $y^2=4ax$ is $45°$. Show that the locus of the point $\mathbf{P}$ is a hyperbola. $(1998-8marks)$
6) Consider the family of circles $x^2+y^2=r^2$, $2<r<5$. If in the first quadrant, the common tangent to a circle of this family and the ellipse $4x^2+25y^2=100$ meets the coordinate axes at $\mathbf{A}$ and $\mathbf{B}$, then find the equation of the locus of the midpoint of AB. $(1999-10marks)$
7) Find the coordinates of all the points $\mathbf{P}$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1, for which the area of the triangle PON is maximum, where $\mathbf{O}$ denotes the origin and $\mathbf{N}$, the foot of the perpendicular from $\mathbf{O}$ to the tangent at $\mathbf{P}$. $(1999-10marks)$
8) Let ABC be an equilateral triangle inscribed in the circle $x^2+y^2=a^2$. Suppose perpendiculars from A, B, C to the major axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1, $(a>b)$ meet the ellipse respectively at P, Q, R such that P, Q, R lie on the same side of the major axis as A, B, C respectively. Prove that the normals to the ellipse drawn at the points $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{R}$ are concurrent. $(2000-7marks)$
9) Let C_1 and C_2 be respectively, the parabolas $x^2=y-1$ and $y^2=x-1$. Let $\mathbf{P}$ be any point on C_1 and $\mathbf{Q}$ be any point on C_2. Let P_1 and Q_1 be the reflections of $\mathbf{P}$ and $\mathbf{Q}$ respectively with respect to the line $y=x$. Prove that P_1 lies on C_2, Q_1 lies on C_1, and $PQ\geq\min(PP_1,QQ_1)$. Hence or otherwise determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that $P_0Q_0\leq PQ$ for all pairs of points $(\mathbf{P},\mathbf{Q})$ with $\mathbf{P}$ on C_1 and $\mathbf{Q}$ on C_2. $(2000-10marks)$
10) Let $\mathbf{P}$ be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $0<b<a$. Let the line parallel to the y-axis passing through $\mathbf{P}$ meet the circle $x^2+y^2=a^2$ at the point $\mathbf{Q}$ such that $\mathbf{P}$ and $\mathbf{Q}$ are on the same side of the x-axis. For two positive real numbers r and s,

find the locus of the point **R** on PQ such that $PR = r$ as **P** varies over the ellipse. $(2001 - 4marks)$

11) Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the center of the ellipse to the point of contact meet on the corresponding directrix. $(2002 - 5marks)$
12) Normals are drawn from the point **P** with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If the locus of **P** with $m_1 m_2 = \alpha$ is a part of the parabola itself, then find α. $(2003 - 4marks)$
13) A tangent is drawn to the parabola $y^2 - 2y - 4x + 5 = 0$ at a point P which cuts the directrix at the point **Q**. A point **R** is such that it divides QP externally in the ratio 1:2. Find the locus of the point **R**. $(2004 - 4marks)$
14) Tangents are drawn from any point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ to the circle $x^2 + y^2 = 9$. Find the locus of the midpoint of the chord of contact. $(2005 - 4marks)$
15) Find the equation of the common tangent in the 1st quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Also, find the length of the intercept of the tangent between the coordinate axes. $(2005 - 4marks)$