JEE ADVANCED

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1 Subjective Problems

1) Let 'd' be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2}$=1 to the tangent drawn at a point **P** on the ellipse.If $\mathbf{F_1}$ and $\mathbf{F_2}$ are the two $foci$ of the ellipse,then show that $(PF_1 - PF_2)^2 = 4a^2(1 - \frac{b^2}{d^2})$. $(1995 - 5marks)$
2) Points **A**,**B** and **C** lie on a parabola $y^2 = 4ax$.The tangents to the parabola at A, B and C taken in pairs,intersect at points **P**,**Q** and **R**.Determine the ratios of the areas of triangles ABC and PQR. $(1996 - 3marks)$
3) From a point **A** common tangents are drawn to circle $x^2 + y^2 = \frac{a^2}{2}$ and parabola $y^2 = 4ax$.Find the area of the quadrilateral formed by the common tangents,the chord of contact of circle and the chord of contact of parabola. $(1996 - 2marks)$
4) A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at **P** and **Q**.Prove that the tangents at **P** and **Q** of the ellipse $x^2 + 2y^2 = 6$ are at right angles. $(1997 - 5marks)$
5) The angle between a pair of tangents drawn from a point **P** to the parabola $y^2 = 4ax$ is 45°.Show that the locus of point **P** is hyperbola. $(1998 - 8marks)$
6) Consider the family of Circles $x^2 + y^2 = r^2$,$2 < r < 5$.If in the first quadrant,the common tangent to a circle of this family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at **A** and **B**,then find the equation of the locus of the mid-point of AB. $(1999 - 10marks)$
7) Find the co-ordinates of all the points **P** on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2}$=1, for which the area of the triangle PON is maximum ,where **O** denotes origin and **N**,the foot of the perpendicular from **O** to the tangent P. $(1999 - 10marks)$
8) Let ABC be equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$.Suppose perpendiculars from A, B, C to the major axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2}$=1,$(a > b)$ meets the ellipse respectively,at P, Q, R.so that P, Q, R lie on the same side of major axis as A, B, C respectively .Prove that the normals to the ellipse drawn at the points $\mathbf{P}, \mathbf{Q}$ and **R** are concurrent. $(2000 - 7marks)$
9) Let C_1 and C_2 be respectively ,the parabolas $x^2 = y - 1$ and $y^2 = x - 1$.Let **P** be any point on C_1 and **q** be any point on C_2.Let P_1 and Q_1 be the reflections of **P** and **Q** respectively with respect to the line $y = x$.Prove that P_1 lies on C_2,Q_1 lies on C_1 and $PQ \geq \min(PP_1, QQ_1)$.Hence or otherwise determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that $P_0Q_0 \leq$PQ for all pairs of points $(\mathbf{P}, \mathbf{Q})$ with **P** on C_1 and **Q** on C_2. $(2000 - 10marks)$
10) Let **P** be a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2}$=1 ,$0 < b < a$.Let the line parallel to y-axis passing through P meet the circle $x^2 + y^2 = a^2$ at the point **Q** such that **P** and **Q** are

on the same side of x-axis.For two positive real numbers r and s,find the locus of the point **R** on PQ such that $PR : RQ = r : s$ as **P** varies over the ellipse. (2001 – 4*marks*)

11) Prove that,in an ellipse,the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix.
(2002 – 5*marks*)

12) Normals are drawn from the point **P** with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$.If locus of **P** with $m_1 m_2 = \alpha$ is a part of parabola itself then find α. (2003 – 4*marks*)

13) Tangent is drawn to parabola $y^2 - 2y - 4x + 5 = 0$ at a point P which cuts the directrix at the point **Q**.A point **R** is such that it divides QP externally in the ratio 1:2.Find the locus of point **R**. (2004 – 4*marks*)

14) Tangents are drawn from any point on hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ to the circle $x^2 + y^2 = 9$.Find the locus of mid-point of the chord of contact. (2005 – 4*marks*)

15) Find the equation of the common tangent in 1^{st} quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$.Also find the length of the intercept of the tangent between the coordinate axes. (2005 – 4*marks*)