

JEE ADVANCED

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1 SUBJECTIVE PROBLEMS

- 1) Let 'd' be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the tangent drawn at a point **P** on the ellipse. If **F**₁ and **F**₂ are the two foci of the ellipse, then show that $(PF_1 - PF_2)^2 = 4a^2(1 - \frac{b^2}{a^2})$. (1995 – 5marks)
- 2) Points **A**, **B** and **C** lie on a parabola $y^2 = 4ax$. The tangents to the parabola at **A**, **B** and **C** taken in pairs, intersect at points **P**, **Q** and **R**. Determine the ratios of the areas of triangles **ABC** and **PQR**. (1996 – 3marks)
- 3) From a point **A** common tangents are drawn to circle $x^2 + y^2 = \frac{a^2}{2}$ and parabola $y^2 = 4ax$. Find the area of the quadrilateral formed by the common tangents, the chord of contact of circle and the chord of contact of parabola. (1996 – 2marks)
- 4) A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at **P** and **Q**. Prove that the tangents at **P** and **Q** of the ellipse $x^2 + 2y^2 = 6$ are at right angles. (1997 – 5marks)
- 5) The angle between a pair of tangents drawn from a point **P** to the parabola $y^2 = 4ax$ is 45° . Show that the locus of point **P** is hyperbola. (1998 – 8marks)
- 6) Consider the family of Circles $x^2 + y^2 = r^2$, $2 < r < 5$. If in the first quadrant, the common tangent to a circle of this family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at **A** and **B**, then find the equation of the locus of the mid-point of **AB**. (1999 – 10marks)
- 7) Find the co-ordinates of all the points **P** on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, for which the area of the triangle **PON** is maximum, where **O** denotes origin and **N**, the foot of the perpendicular from **O** to the tangent **P**. (1999 – 10marks)
- 8) Let **ABC** be equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose perpendiculars from **A**, **B**, **C** to the major axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, ($a > b$) meets the ellipse respectively, at **P**, **Q**, **R**. so that **P**, **Q**, **R** lie on the same side of major axis as **A**, **B**, **C** respectively. Prove that the normals to the ellipse drawn at the points **P**, **Q** and **R** are concurrent. (2000 – 7marks)
- 9) Let **C**₁ and **C**₂ be respectively, the parabolas $x^2 = y - 1$ and $y^2 = x - 1$. Let **P** be any point on **C**₁ and **q** be any point on **C**₂. Let **P**₁ and **Q**₁ be the reflections of **P** and **Q** respectively with respect to the line $y = x$. Prove that **P**₁ lies on **C**₂, **Q**₁ lies on **C**₁ and $PQ \geq \min(PP_1, QQ_1)$. Hence or otherwise determine points **P**₀ and **Q**₀ on the parabolas **C**₁ and **C**₂ respectively such that $P_0Q_0 \leq PQ$ for all pairs of points (**P**, **Q**) with **P** on **C**₁ and **Q** on **C**₂. (2000 – 10marks)
- 10) Let **P** be a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $0 < b < a$. Let the line parallel to y-axis passing through **P** meet the circle $x^2 + y^2 = a^2$ at the point **Q** such that **P** and **Q** are on the same side of x-axis. For two positive real numbers **r** and **s**, find the locus of the point **R** on **PQ** such that $PR : RQ = r : s$ as **P** varies over the ellipse. (2001 – 4marks)

- 11) Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix.
(2002 – 5marks)
- 12) Normals are drawn from the point **P** with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If locus of **P** with $m_1 m_2 = \alpha$ is a part of parabola itself then find α . (2003 – 4marks)
- 13) Tangent is drawn to parabola $y^2 - 2y - 4x + 5 = 0$ at a point **P** which cuts the directrix at the point **Q**. A point **R** is such that it divides QP externally in the ratio 1:2. Find the locus of point **R**.
(2004 – 4marks)
- 14) Tangents are drawn from any point on hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ to the circle $x^2 + y^2 = 9$. Find the locus of mid-point of the chord of contact.
(2005 – 4marks)
- 15) Find the equation of the common tangent in 1st quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Also find the length of the intercept of the tangent between the coordinate axes.
(2005 – 4marks)