

# JEE ADVANCED

1

Suraj Kolluru  
EE24BTECH110033

## 1 SUBJECTIVE PROBLEMS

- 1) Let ' $d$ ' be the perpendicular distance from the centre of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  to the tangent drawn at a point  $\mathbf{P}$  on the ellipse. If  $\mathbf{F}_1$  and  $\mathbf{F}_2$  are the two foci of the ellipse, then show that  $(PF_1 - PF_2)^2 = 4a^2 \left(1 - \frac{b^2}{d^2}\right)$ . (1995 – 5marks)
- 2) Points  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  lie on a parabola  $y^2 = 4ax$ . The tangents to the parabola at  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  taken in pairs, intersect at points  $\mathbf{P}$ ,  $\mathbf{Q}$  and  $\mathbf{R}$ . Determine the ratios of the areas of triangles  $ABC$  and  $PQR$ . (1996 – 3marks)
- 3) From a point  $\mathbf{A}$  common tangents are drawn to the circle  $x^2 + y^2 = \frac{a^2}{2}$  and the parabola  $y^2 = 4ax$ . Find the area of the quadrilateral formed by the common tangents, the chord of contact of the circle, and the chord of contact of the parabola. (1996 – 2marks)
- 4) A tangent to the ellipse  $x^2 + 4y^2 = 4$  meets the ellipse  $x^2 + 2y^2 = 6$  at  $\mathbf{P}$  and  $\mathbf{Q}$ . Prove that the tangents at  $\mathbf{P}$  and  $\mathbf{Q}$  of the ellipse  $x^2 + 2y^2 = 6$  are at right angles. (1997 – 5marks)
- 5) The angle between a pair of tangents drawn from a point  $\mathbf{P}$  to the parabola  $y^2 = 4ax$  is  $45^\circ$ . Show that the locus of the point  $\mathbf{P}$  is a hyperbola. (1998 – 8marks)
- 6) Consider the family of circles  $x^2 + y^2 = r^2$ ,  $2 < r < 5$ . If in the first quadrant, the common tangent to a circle of this family and the ellipse  $4x^2 + 25y^2 = 100$  meets the coordinate axes at  $\mathbf{A}$  and  $\mathbf{B}$ , then find the equation of the locus of the midpoint of  $AB$ . (1999 – 10marks)
- 7) Find the coordinates of all the points  $\mathbf{P}$  on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , for which the area of the triangle  $PON$  is maximum, where  $\mathbf{O}$  denotes the origin and  $\mathbf{N}$ , the foot of the perpendicular from  $\mathbf{O}$  to the tangent at  $\mathbf{P}$ . (1999 – 10marks)
- 8) Let  $ABC$  be an equilateral triangle inscribed in the circle  $x^2 + y^2 = a^2$ . Suppose perpendiculars from  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$  to the major axis of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , ( $a > b$ ) meet the ellipse respectively at  $\mathbf{P}$ ,  $\mathbf{Q}$ ,  $\mathbf{R}$  such that  $\mathbf{P}$ ,  $\mathbf{Q}$ ,  $\mathbf{R}$  lie on the same side of the major axis as  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$  respectively. Prove that the normals to the ellipse drawn at the points  $\mathbf{P}$ ,  $\mathbf{Q}$ , and  $\mathbf{R}$  are concurrent. (2000 – 7marks)
- 9) Let  $C_1$  and  $C_2$  be respectively, the parabolas  $x^2 = y - 1$  and  $y^2 = x - 1$ . Let  $\mathbf{P}$  be any point on  $C_1$  and  $\mathbf{Q}$  be any point on  $C_2$ . Let  $P_1$  and  $Q_1$  be the reflections of  $\mathbf{P}$  and  $\mathbf{Q}$  respectively with respect to the line  $y = x$ . Prove that  $P_1$  lies on  $C_2$ ,  $Q_1$  lies on  $C_1$ , and  $PQ \geq \min(PP_1, QQ_1)$ . Hence or otherwise determine points  $P_0$  and  $Q_0$  on the parabolas  $C_1$  and  $C_2$  respectively such that  $P_0Q_0 \leq PQ$  for all pairs of points  $(\mathbf{P}, \mathbf{Q})$  with  $\mathbf{P}$  on  $C_1$  and  $\mathbf{Q}$  on  $C_2$ . (2000 – 10marks)
- 10) Let  $\mathbf{P}$  be a point on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $0 < b < a$ . Let the line parallel to the  $y$ -axis passing through  $\mathbf{P}$  meet the circle  $x^2 + y^2 = a^2$  at the point  $\mathbf{Q}$  such that  $\mathbf{P}$  and  $\mathbf{Q}$  are on the same side of the  $x$ -axis. For two positive real numbers  $r$  and  $s$ ,

find the locus of the point **R** on  $PQ$  such that  $PR = r$  as **P** varies over the ellipse.  
(2001 – 4marks)

- 11) Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the center of the ellipse to the point of contact meet on the corresponding directrix. (2002 – 5marks)
- 12) Normals are drawn from the point **P** with slopes  $m_1, m_2, m_3$  to the parabola  $y^2 = 4x$ . If the locus of **P** with  $m_1 m_2 = \alpha$  is a part of the parabola itself, then find  $\alpha$ . (2003 – 4marks)
- 13) A tangent is drawn to the parabola  $y^2 - 2y - 4x + 5 = 0$  at a point  $P$  which cuts the directrix at the point **Q**. A point **R** is such that it divides  $QP$  externally in the ratio 1:2. Find the locus of the point **R**. (2004 – 4marks)
- 14) Tangents are drawn from any point on the hyperbola  $\frac{x^2}{9} - \frac{y^2}{4} = 1$  to the circle  $x^2 + y^2 = 9$ . Find the locus of the midpoint of the chord of contact. (2005 – 4marks)
- 15) Find the equation of the common tangent in the 1st quadrant to the circle  $x^2 + y^2 = 16$  and the ellipse  $\frac{x^2}{25} + \frac{y^2}{4} = 1$ . Also, find the length of the intercept of the tangent between the coordinate axes. (2005 – 4marks)