Question-7-7.2-8

EE24BTECH11033 - KOLLURU SURAJ

Question:

The centre of circle is $\begin{pmatrix} 2a \\ a-7 \end{pmatrix}$. Find the values of a if the circle passes through the point $\mathbf{A}\begin{pmatrix} 11 \\ -9 \end{pmatrix}$and has diameter $10\sqrt{2}$ units

Solution:

Description	Given value
Centre	$\begin{pmatrix} 2a \\ a-7 \end{pmatrix}$
Diameter	$10\sqrt{2}$
point $\mathbf{A}$	$\begin{pmatrix} 11 \\ -9 \end{pmatrix}$

TABLE 0: variables used

The radius of circle is $\frac{diameter}{2}$

$$\implies radius = 5\sqrt{2} \tag{0.1}$$

The equation of a circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{0.2}$$

for

$$\mathbf{u} = -\mathbf{c}, f = \|\mathbf{c}\|^2 - r^2 \tag{0.3}$$

Now,

$$\mathbf{u} = -\begin{pmatrix} 2a \\ a-7 \end{pmatrix}, f = 5a^2 - 14a - 1 \tag{0.4}$$

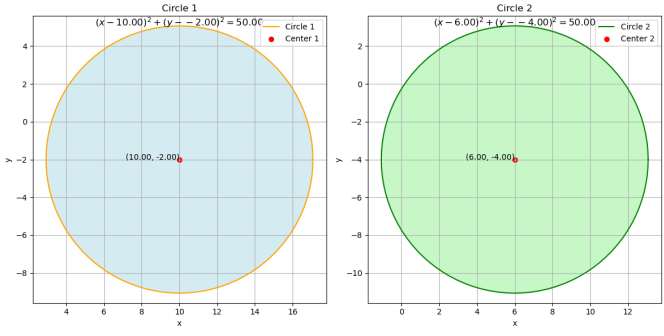
On substituting x=$\begin{pmatrix} 11 \\ -9 \end{pmatrix}$

$$202 - 26a - 126 + 5a^2 - 14a - 1 = 0 \tag{0.5}$$

$$5a^2 - 40a + 75 = 0 \tag{0.6}$$

$$a^2 - 8a + 15 = 0 \tag{0.7}$$

$$a = 3, a = 5 \tag{0.8}$$


Circle 1
$(x-10.00)^2+(y--2.00)^2=50.00$
Circle 1
Center 1
(10.00, -2.00)
x
y
Circle 2
$(x-6.00)^2+(y--4.00)^2=50.00$
Circle 2
Center 2
(6.00, -4.00)
x
y