Question-7-7.2-8

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Question

The centre of circle is $\begin{pmatrix} 2a \\ a-7 \end{pmatrix}$. Find the values of a if the circle passes through the point $\vec{A}\begin{pmatrix} 11 \\ -9 \end{pmatrix}$ and has diameter $10\sqrt{2}$ units.

Inputs

Description	Given value
Centre	$\begin{pmatrix} 2a \\ a-7 \end{pmatrix}$
Diameter	$10\sqrt{2}$
point $\vec{A}$	$\begin{pmatrix} 11 \\ -9 \end{pmatrix}$

Formulas

Conic	Expression
Circle	$\left\|\vec{x}\right\|^2 + 2\vec{u}^\top \vec{x} + f = 0$

Solution

The radius of circle is $\frac{diameter}{2}$

$$\implies radius = 5\sqrt{2}$$

The equation of a circle is given by

$$\|\vec{x}\|^2 + 2\vec{u}^\top \vec{x} + f = 0 \tag{1}$$

for

$$\vec{u} = -\vec{c}, f = \|\vec{c}\|^2 - r^2 \tag{2}$$

Where $\vec{c}$ is centre and r is the radius of the circle

Solution

Now,

$$\vec{u} = -\begin{pmatrix} 2a \\ a-7 \end{pmatrix}, f = 5a^2 - 14a - 1 \tag{3}$$

On substituting $x = \begin{pmatrix} 11 \\ -9 \end{pmatrix}$ in (1) We get,

$$\begin{aligned} 202 - 26a - 126 + 5a^2 - 14a - 1 &= 0 \\ 5a^2 - 40a + 75 &= 0 \\ a^2 - 8a + 15 &= 0 \\ a = 3, a &= 5 \end{aligned}$$

C Code

```
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <math.h>
#include <unistd.h>
#include "libs/matfun.h"
#include "libs/geofun.h"

void point_gen(FILE *fptr, double **A, double **B, int no_rows, int
    no_cols, int num_points) {
    for (int i = 0; i < num_points; i++) {
        double t = (double)i / (num_points - 1);
        double **output = Matadd(A, Matscale(Matsub(B, A, no_rows,
            no_cols), no_rows, no_cols, t), no_rows, no_cols);
        fprintf(fptr, "%lf,%lf\n", output[0][0], output[1][0]);
        freeMat(output, no_rows);}}
```

C Code

```
double** centre_gen(double x1, double x2, double y1, double y2,
    double **ptA, double diameter){
    double m = y1/x1;
    double c = y2-m*x2;
    double radius = diameter/2;
    double A=(1+m*m);
    double B=(2*m*(c-ptA[1][0])-2*ptA[0][0]);
    double C=(ptA[0][0]*ptA[0][0])+((ptA[1][0]-c)*(ptA[1][0]-c)) - (
        radius*radius);
    double** points = createMat(2,2);
    points[0][0] = (-B+sqrt(B*B-4*A*C))/(2*A);
    points[1][0] = m*points[0][0]+c;
    points[0][1] = (-B-sqrt(B*B-4*A*C))/(2*A);
    points[1][1] = m*points[0][1]+c;
    return points;
}
```

C Code

```
void circle_point_gen(FILE *fptr, double radius, double **center, int
    num_points) {
    double **output;
    for (int i = 0; i <= num_points; i++) {
        double angle = (2 * M_PI * i) / num_points;
        output = createMat(2, 1);
        output[0][0] = center[0][0] + radius * cos(angle);
        output[1][0] = center[1][0] + radius * sin(angle);
        fprintf(fptr, "%lf,%lf\n", output[0][0], output[1][0]);
        freeMat(output, 2);
    }
}
```

C Code

```
int main() {
    double a = 1.0; //for graphing
    double** A = createMat(2,1);
    A[0][0] = 11;
    A[1][0] = -9;
    double diameter = 10*sqrt(2);
    double** centers = centre_gen(2,0,1,-7,A,diameter);
    double** center1 = createMat(2,1);
    double** center2 = createMat(2,1);
    center1[0][0] = centers[0][0];
    center1[1][0] = centers[1][0];
    center2[0][0] = centers[0][1];
    center2[1][0] = centers[1][1];
    printMat(center1,2,1);
    printMat(center2,2,1);
    printf("%lf",diameter/2);
```

C Code

```
FILE *fptr = fopen("points.dat", "w");
    if (fptr == NULL) {
        printf("Error opening file!\n");
        return 1;
    }

    circle_point_gen(fptr, diameter/2, center1, 100);
    circle_point_gen(fptr, diameter/2, center2, 100);

    fclose(fptr);
    return 0;
}
```

Python Code for Plotting

```
import numpy as np
import matplotlib.pyplot as plt

# Load points from the data file
points = np.loadtxt("points.dat", delimiter=',')
x_circle1 = points[:101, 0]
y_circle1 = points[:101, 1]
x_circle2 = points[101:, 0]
y_circle2 = points[101:, 1]

# Circle 1 center and radius
center1 = ((x_circle1[0]+x_circle1[50])/2, (y_circle1[0]+y_circle1[50])/2)
radius1 = np.sqrt((x_circle1[0] - center1[0])**2 + (y_circle1[0] - center1
    [1])**2)
circle_eq1 = f'(x-{center1[0]:.2f})^2 + (y-{center1[1]:.2f})^2 = {radius1
    **2:.2f}'
```

Python Code for Plotting

```
center2 = ((x_circle2[0]+x_circle2[50])/2, (y_circle2[0]+y_circle2[50])/2)
radius2 = np.sqrt((x_circle2[0] - center2[0])**2 + (y_circle2[0] - center2
    [1])**2)
circle_eq2 = f'(x-{center2[0]:.2f})^2 + (y-{center2[1]:.2f})^2 = {radius2
    **2:.2f}'

# Create subplots
fig, axes = plt.subplots(1, 2, figsize=(12, 6))
```

Python Code for Plotting

```
# Plot the first circle in the first subplot
axes[0].plot(x_circle1, y_circle1, label='Circle-1', color='orange')
axes[0].fill(x_circle1, y_circle1, 'lightblue', alpha=0.5)
axes[0].scatter(*center1, color='red', label='Center-1')
axes[0].text(center1[0], center1[1], f'({center1[0]:.2f},-{center1[1]:.2f})',
    fontsize=10, ha='right')
axes[0].text(center1[0], center1[1] + radius1, circle_eq1, fontsize=12, ha='
    center', color='black')
axes[0].set_title("Circle-1")
axes[0].set_xlabel("x")
axes[0].set_ylabel("y")
axes[0].axis('equal')
axes[0].grid(True)
axes[0].legend(loc="upper-right")
```

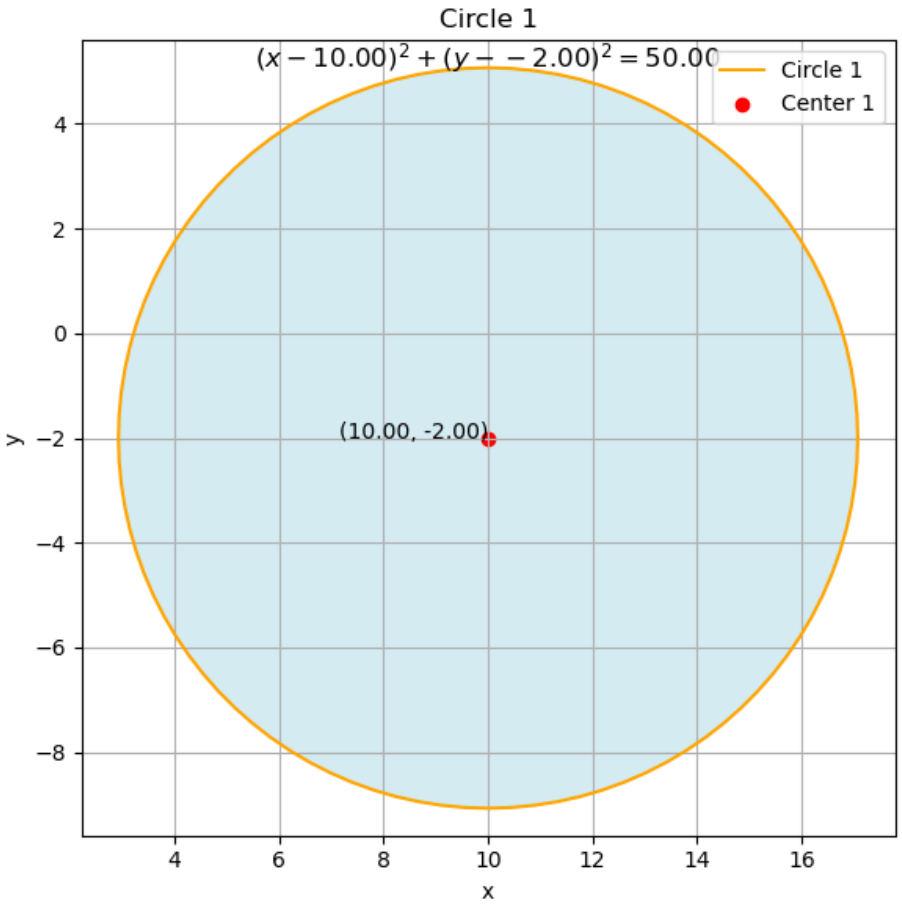
Python Code for Plotting

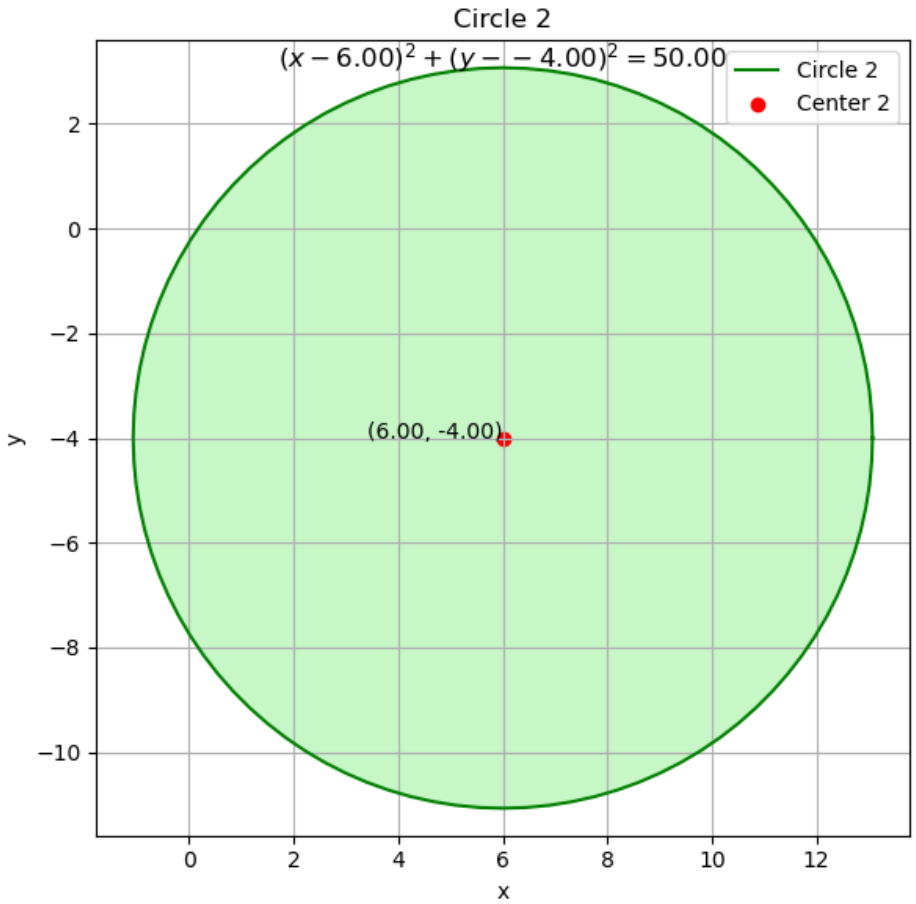
```
# Plot the second circle in the second subplot
axes[1].plot(x_circle2, y_circle2, label='Circle-2', color='green')
axes[1].fill(x_circle2, y_circle2, 'lightgreen', alpha=0.5)
axes[1].scatter(*center2, color='red', label='Center-2')
axes[1].text(center2[0], center2[1], f'({center2[0]:.2f},-{center2[1]:.2f})',
    fontsize=10, ha='right')
axes[1].text(center2[0], center2[1] + radius2, circle_eq2, fontsize=12, ha='
    center', color='black')
axes[1].set_title("Circle-2")
axes[1].set_xlabel("x")
axes[1].set_ylabel("y")
axes[1].axis('equal')
axes[1].grid(True)
axes[1].legend(loc="upper-right")
```

Python Code for Plotting

```
# Adjust layout and save the plot
plt.tight_layout()
plt.savefig('../figs/Figure_1.png')
```

Diagram


Circle 1
$(x-10.00)^2+(y--2.00)^2=50.00$
Circle 1
Center 1
(10.00, -2.00)
x
y


Circle 2
$(x-6.00)^2+(y--4.00)^2=50.00$
Circle 2
Center 2
(6.00, -4.00)
x
y