1.6.12

EE24BTECH11033 - Kolluru Suraj

Question:
Point $(-4, 2)$ lies on the line segment joining the points $\mathbf{A}(-4, 6)$ and $\mathbf{B}(-4, -6)$.
Solution:
Here $\mathbf{A} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -4 \\ -6 \end{pmatrix}$, and $\mathbf{C}$ be $\mathbf{C} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

Points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are defined to be collinear if

$$\text{rank}\begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix} = 1 \tag{0.1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 0 \\ -12 \end{pmatrix} \tag{0.2}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{0.3}$$

From equations 0.1 and using 0.2 and 0.3, the collinearity matrix can be expressed as

$$\begin{pmatrix} 0 & 0 \\ -12 & -4 \end{pmatrix} \tag{0.4}$$

which is a rank 1 matrix. The number of nonzero rows in the row reduced matrix (also known as echelon form)is defined as the rank.
Since the rank of collinearity matrix is 1 the given three points lie on the same line.

To find the ratio which $\mathbf{C}$ divides $\mathbf{A}, \mathbf{B}$. Using section formula,

$$\begin{pmatrix} -4 \\ 2 \end{pmatrix} = \frac{\begin{pmatrix} -4 \\ 6 \end{pmatrix} + k\begin{pmatrix} -4 \\ -6 \end{pmatrix}}{1 + k} \tag{0.5}$$

$$\implies 2k\begin{pmatrix} 0 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{0.6}$$

$$\text{or, } k = \frac{1}{2}. \tag{0.7}$$

Since the ratio is positive and the 3 points are collinear we can conclude that point $\mathbf{C}$ lies on the line segment joining $\mathbf{A}, \mathbf{B}$

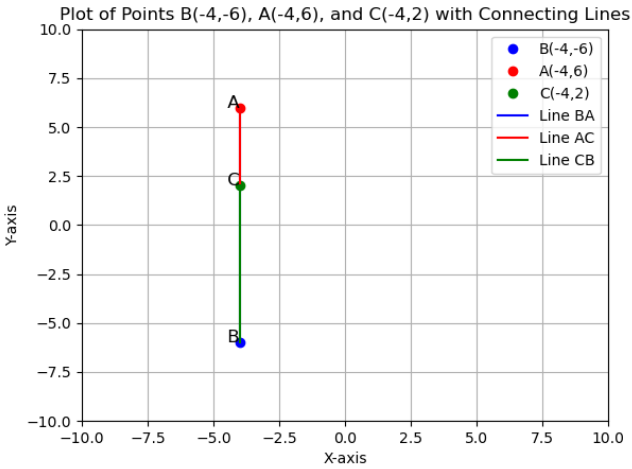

Fig. 0.1: Line connecting ABC