

1.6.12

EE24BTECH11033 - Kolluru Suraj

Question:

Point $(-4, 2)$ lies on the line segment joining the points $\mathbf{A}(-4, 6)$ and $\mathbf{B}(-4, -6)$.

Solution:

point	Coordinates
A	$(-4, 6)$
B	$(-4, -6)$
C	$(-4, 2)$

TABLE 0: variables used

Here $\mathbf{A} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -4 \\ -6 \end{pmatrix}$, and $\mathbf{C} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

Points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are defined to be collinear if

$$\text{rank}(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A}) = 1 \quad (0.1)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 0 \\ -12 \end{pmatrix} \quad (0.2)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \quad (0.3)$$

From equations 0.1 and using 0.2 and 0.3, the collinearity matrix can be expressed as

$$\begin{pmatrix} 0 & 0 \\ -12 & -4 \end{pmatrix} \quad (0.4)$$

which is a rank 1 matrix. The number of nonzero rows in the row reduced matrix (also known as echelon form) is defined as the rank.

Since the rank of collinearity matrix is 1 the given three points lie on the same line.

To find the ratio which **C** divides **A,B**. Using section formula,

$$\begin{pmatrix} -4 \\ 2 \end{pmatrix} = \frac{\begin{pmatrix} -4 \\ 6 \end{pmatrix} + k \begin{pmatrix} -4 \\ -6 \end{pmatrix}}{1 + k} \quad (0.5)$$

$$\Rightarrow 2k \begin{pmatrix} 0 \\ 4 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad (0.6)$$

$$\text{or, } k = \frac{1}{2}. \quad (0.7)$$

Since the ratio is positive and the 3 points are collinear we can conclude that point **C** lies on the line segment joining **A,B**

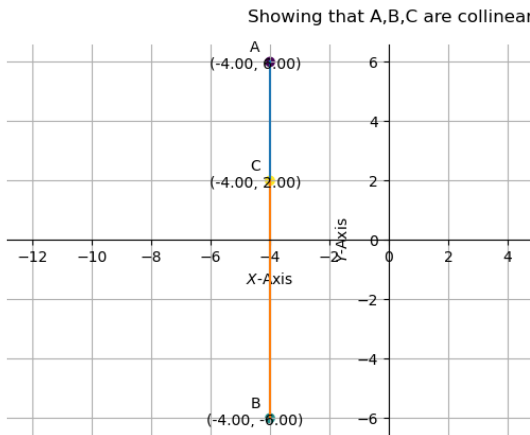


Fig. 0.1: Line connecting ABC