MATHEMATICS (050) (E)

MATHEMATICS

QUESTION PAPER – 2

STD. 12th

Time : 3 Hours

MARCH-2018

Total Mark : 100

PART – A : 50 Marsk ● Part – B : 50 Marks

Time : 1 Hour **PART – A** Maximum Marks : 50

Instructions : According to Question Paper - 1.

1. If the perpendicular distance from the point $(-1, 2, -2)$ to the plane $3x-4y+2z+k=0$ is $\sqrt{29}$, then $k=$ ...., $(k<0)$

(A) 44 (B) 44 and -14
(C) -44 (D) -14

2. If the image of the point A $(1, 2, -3)$ relative to the plane π is B $(-3, 6, 4)$, then equation of plane π is ....

(A) $8x+8y+14z-47=0$ (B) $8x-8y-14z+47=0$
(C) $8x-8y-14z-47=0$ (D) $8x+8y-14z+47=0$

3. The equation of the line perpendicular to the plane $2x+3y+5z+1=0$ and passing through the point $(1, 2, -3)$ is ....

(A) $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z+3}{5}$ (B) $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z+3}{5}$
(C) $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{5}$ (D) $\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z+3}{5}$

4. The number of binary operation on $\{1, 2, 3, 4\}$ is ....

(A) 4^2 (B) 4^8 (C) 4^3 (D) 4^{16}

5. If $a*b=a^3+b^3$ on z, then $(1*2)*0$ = ...

(A) 0 (B) 729 (C) 81 (D) 27

6. Let $f: N\times N \to N-\{1\}$, be defined as $f(m, n)=m+n$, then function f is ....

(A) one-one and onto (B) many - on and not onto
(C) one - one and not onto (D) many - on and onto

7. If the relation is defined on $R-\{0\}$ by $(x, y)\in S \Leftrightarrow xy>0$, then S is ....

(A) an equivalence relation (B) symmetric only
(C) reflexive only (D) transitive only

8. If, $f: R\to R, f(x)=\begin{cases} 1 & x>0 \\ 0 & x=0 \\ -1 & x<0 \end{cases}$ and $g: R\to R, g(x)=[x]$, then $fog(\pi)$ = ...

(A) π (B) 0 (C) 1 (D) -1

9. $\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)=$ ....

(A) 6 (B) 15 (C) 13 (D) 25

10. $\sin\left(\cos^{-1}\left(-\frac{1}{7}\right)+\sin^{-1}\left(-\frac{1}{7}\right)\right)=$ ....

(A) 1 (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{7}$

11. If $3\cos^{-1}x+\sin^{-1}x=\pi$ then $x=$ ....

(A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{\sqrt{2}}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

12. $\cot^{-1}\left(\frac{1}{2}\right)+\cot^{-1}\left(\frac{1}{3}\right)=$ ....

(A) $\frac{\pi}{4}$ (B) $\frac{5\pi}{4}$ (C) $-\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$

13. The number of solutions of simultaneous equations $4x+3y=6xy$, $8x+6y=9xy$ are ....

(A) 0 (B) 2 (C) 1 (D) Infinite

14. If the matrix $\begin{bmatrix} a & 2 & -3 \\ b & 0 & 4 \\ c & -4 & 0 \end{bmatrix}$ is skew - symmetric then, $a+b+c$ = ...

(A) 1 (B) 0 (C) -1 (D) 5

15. For 3×3 matrices A and B, If $|B|=1$ and $A=2B$ then, $|A|=$ ....

(A) 1 (B) 4 (C) 2 (D) 8

16. $f(x)=\begin{cases} \dfrac{\tan 4x\times\cos 3x}{x} & x\neq0 \\ k & x=0 \end{cases}$ If f is continuous at $x=0$ then, $k=$ ....

(A) 0 (B) 4 (C) $\frac{4}{3}$ (D) $\frac{3}{4}$

17. $\frac{d}{dx}(\sqrt{x\sin x})=$ ...., $0<x<\pi$

(A) $\frac{1}{2\sqrt{x\sin x}}$ (B) $\frac{x\cos x}{2\sqrt{x\sin x}}$ (C) $\frac{x\cos x+\sin x}{2\sqrt{x\sin x}}$ (D) $\frac{x\sin x+\cos x}{\sqrt{x\sin x}}$

18. $\left[\frac{d}{dx}\operatorname{cosec}^{-1}x\right]_{x=-2}=$ ....

(A) $-\frac{1}{2\sqrt{3}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$

19. If $x=at^2, y=2at$ then, $\frac{dy}{dx}=$ .... ; $t\neq0$

(A) at (B) $\frac{t}{2}$ (C) $\frac{2}{t}$ (D) $\frac{1}{t}$

20. For the function $y = \tan^{-1} x$, $(1+x^2)y_2 =$ ....

(A) $-2xy_1$ (B) xy_1 (C) $2xy_1$ (D) $-xy_1$

21. $\int \frac{1}{\sqrt{e^{2x}-1}}dx =$ .... $+ c$

(A) $2\sec^{-1}(e^x)$ (B) $\operatorname{cosec}^{-1}(e^x)$ (C) $2\operatorname{cosec}^{-1}(e^x)$ (D) $\sec^{-1}(e^x)$

22. If $\int f(x)dx = \frac{(\log x)^5}{5} + c$, then $f(x) =$ ....

(A) $\frac{(\log x)^6}{6}$ (B) $\frac{\log x^4}{x}$ (C) $\frac{(\log x)^4}{x}$ (D) $\frac{\log x}{4}$

23. If the probability of selecting a defective bolt from 400 bolts is 0.1, then the mean for the distribution = ...

(A) 0.09 (B) 40 (C) 36 (D) 360

24. The mean and the standard deviation of a random variable x are given by 5 and 3 respectively. The standard deviation of $2 - 3x$ is ....

(A) - 7 (B) 81 (C) 34 (D) 9

25. If A and B are independent events, $P(A) = 0.1$ and $P(B) = 0.9$ then, $P(A \cup B) =$ ....

(A) 0.91 (B) 0.09 (C) 0.99 (D) 0.90

26. If $P(A) = 0.40$, $P(B) = 0.35$ and $P(A \cup B) = 0.55$, then $P(A / B) =$ ....

(A) $\frac{1}{5}$ (B) $\frac{8}{11}$ (C) $\frac{4}{7}$ (D) $\frac{3}{4}$

27. The feasible solution of an LP problem, is ....

(A) must satisfies all of the problems constraints simultaneously.
(B) must be a corner point of the fesible region.
(C) need not satisfy all of the constraints, only some of them.
(D) must optimize the value of the objective function.

28. The corner point of the feasible region and A (0, 0), B (16, 0), C (8, 16) and D (0, 24). The minimum value of the objective function $z = 300x + 190y$ is ....

(A) 5440 (B) 4800 (C) 4560 (D) 0

29. Find the approximate error in the volume of a cube with edge x cm, when the edge is increased by 2%

(A) 4% (B) 2% (C) 6% (D) 8%

30. The slope of the tangent and normal to $y = x^2 - 3x + 5$ at (2, 3) are .... and .... respectively.

(A) 1, 1 (B) 1, −1 (C) −1, 1 (D) 2, −2

31. $\int (ex)^x (2 + \log x)\, dx =$ .... $+ c$; $x \in R^+ - \{1\}$

(A) x^x (B) $(ex)^x$ (C) e^x (D) $(1 + \log x)(ex)^x$

32. $\int (e^x + e^{-x})(e^{2x} - e^{-2x})\, dx =$ .... $+ c$

(A) $\frac{(e^x + e^{-x})^3}{3}$ (B) $\frac{e^{3x} + e^{-3x}}{3}$ (C) $\frac{(e^x - e^{-x})^3}{3}$ (D) $\frac{e^{3x} - e^{-3x}}{3}$

33. $\int\left(3\cdot x^2\cdot\tan^{-1}x+\frac{x^3}{1+x^2}\right)dx=$ .... $+c$

(A) $x^3\tan^{-1}x$ (B) $\frac{x^3}{3}\tan^{-1}x$ (C) $x^2\tan^{-1}x$ (D) $\frac{x^2}{2}\tan^{-1}x$

34. $\int\sqrt{5-x^2}dx=$ .... $+c$; $x^2<5$

(A) $\frac{x}{2}\sqrt{\sqrt{5}-x^2}+\frac{\sqrt{5}}{2}\sin^{-1}\frac{x}{\sqrt{5}}$ (B) $\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}$

(C) $\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{5}$ (D) $\frac{x}{2}\sqrt{5-x^2}+\frac{\sqrt{5}}{2}\sin^{-1}\frac{x}{\sqrt{5}}$

35. If $\int_{n}^{n+1}f(x)dx=\binom{n}{n-1}$, $n\in N$, then $\int_{1}^{11}f(x)dx=$ ....

(A) 1023 (B) 1024 (C) 10 (D) 55

36. The area of the region bounded by the curve $y=9-x^2$, X -axis and the lines $x=0$ and $x=3$ is ....

(A) 9 (B) 27 (C) 18 (D) 36

37. The area of the region bounded by the curves $y=\sin\pi x$ and X-axis is ...., where $x\in[0,2]$.

(A) π (B) $\frac{4}{\pi}$ (C) 2π (D) $\frac{\pi}{4}$

38. Area bounded by the ellipse $4x^2+9y^2=1$ is ....

(A) $\frac{\pi}{36}$ (B) 6π (C) 36π (D) $\frac{\pi}{6}$

39. The area bounded by the parabota $y^2=4x$ and its latus rectum is ....

(A) $\frac{4}{3}$ (B) $\frac{16}{3}$ (C) $\frac{8}{3}$ (D) $\frac{32}{3}$

40. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)=\int ydx$ are .... respectively.

(A) 2 and 3 (B) 2 and 2 (C) 3 and 1 (D) 3 and 2

41. An integrarting factor of the differential equation $x\frac{dy}{dx}-y=x^3,(x>0)$ is ....

(A) x (B) $-\frac{1}{x}$ (C) $-x$ (D) $\frac{1}{x}$

42. The number of arbitary constants in the particular solution of a differential equation of third order is ....

(A) 0 (B) 2 (C) 1 (D) 3

43. The differential equation of $y=cx^3$, c arbitary constant is ....

(A) $x\cdot\frac{dy}{dx}=3y$ (B) $x^3\cdot\frac{dy}{dx}=3y$ (C) $3x\frac{dy}{dx}=y$ (D) $x\cdot\frac{dy}{dx}=3y^2$

44. If the angle between unit vector $\bar{a}$ and $\bar{b}$ of R^3 is θ, then $\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{b}\cdot\bar{a}\\ \bar{a}\cdot\bar{b} & \bar{b}\cdot\bar{b}\end{vmatrix}+|\bar{a}\times\bar{b}|^2=$ ....

(A) $\sin^2\theta$ (B) $1-\cos2\theta$ (C) $1+\cos2\theta$ (D) $\cos^2\theta$

45. The vector of magnitude $3\sqrt{21}$ in the direction of vector (4, 1, −2) is ....

(A) (−12, −3, 6) (B) $\frac{1}{\sqrt{21}}(12,3,-6)$ (C) (12, 3, −6) (D) $\frac{1}{\sqrt{21}}(4,1,-2)$

46. For the vector $\bar{x}=(1,2,1), \bar{y}=(2,-3,-1)$ $\text{Comp}_{\bar{y}}\bar{x}$ = ...

(A) $\frac{5}{\sqrt{14}}$ (B) $-\frac{5}{14}$ (C) $\frac{5}{14}$ (D) $-\frac{5}{\sqrt{14}}$

47. For which values of 'a' the different vectors $\bar{x}=(2a,3a,0)$ and $\bar{y}=(0,0,4a)$ are orthogonal vectors.

(A) $a\in R$ (B) $a\in N\cup\{0\}$ (C) $a\in R-\{0\}$ (D) $a=0$

48. The area of parallelogram whose diagonals are $\bar{j}+\bar{k}$ and $\bar{i}+\bar{k}$ is ....

(A) $\frac{\sqrt{3}}{2}$ (B) 3 (C) $\frac{3}{2}$ (D) $\sqrt{3}$

49. If the foot of the perpendicular from the origin to a plane is (1, 2, −3), then the equation of the plane is ....

(A) $x+2y-3z=0$ (B) $\frac{x}{1}+\frac{y}{2}-\frac{z}{3}=1$ (C) $x+2y-3z=-6$ (D) $x+2y-3z=14$

50. The measure of the angle between the line $\frac{x-1}{2}=\frac{y-3}{2}=\frac{z+1}{1}$ and the plane $\bar{r}\cdot(-2,2,-1)=$ is ....

(A) $\sin^{-1}\left(\frac{1}{9}\right)$ (B) $\sin^{-1}\left(\frac{2\sqrt{5}}{9}\right)$ (C) $\cos^{-1}\left(\frac{4\sqrt{5}}{9}\right)$ (D) $\sin^{-1}\left(\frac{\sqrt{5}}{9}\right)$

MARCH-2018 (050) (E)

Time : 2 Hour **PART – B** **Maximum Marks : 5**

Instructions : According to Question Paper - 1.

SECTION : A

● **Question Nos. 1 to 8 do as directed. Each question carries 2 marks.** **[16]**

1. If $a*b=\frac{ab}{10}$ on Q^+, then find the identify for $*$.

2. Find the solution set of $\begin{vmatrix} 2+x & 2-x & 2-x \\ 2-x & 2+x & 2-x \\ 2-x & 2-x & 2+x \end{vmatrix}=0$

3. For $x=\sqrt{a^{\tan^{-1}t}}, y=\sqrt{a^{\cot^{-1}(t)}}, t\in R$ find $\frac{dy}{dx}$.

4. Maximize $z=3x+4y$ subject to $x+y\le 4, x\ge 0, y\ge 0$.

5. Obtain the approximate value of $\sqrt[3]{28}$.

6. Repretent $\int_0^1 e^x\,dx$ as the limit of sum. **OR** 6. Repretent $\int_2^3 (3x+8)\,dx$ as the limit of sum

7. Find the area of the region bounded by the curves $y=x^2+2$, $y=x, x=2$ and $x=0$.

OR

7. Find the area of region bounded by curves $x=\sqrt{y-1}$, X -axis and the lines $x=1$ and $x=$

8. If a vector $\bar{x}$ makes angle with measure $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ with X-axis and Y-axis respectively, then fine the measure of the angle made by $\bar{x}$ with Z-axis.

SECTION : B

- **Question Nos. 9 to 14 do as directed. Each question carries 3 marks. [18]**

9. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, then prove that $x^2+y^2+z^2+2xyz=1$.

10. Prove : $\begin{vmatrix} x^2 & y^2 & z^2 \\ (x+1)^2 & (y+1)^2 & (z+1)^2 \\ (x-1)^2 & (y-1)^2 & (z-1)^2 \end{vmatrix} = -4(x-y)(y-z)(x-z)$

11. An urn contains 5-red and 6-blue balls. Two balls are drawn at random with replacement. Fine the probability of getting.
(1) both red balls. (2) both blue balls.
(3) one - red and one - blue ball.

OR

11. In a shooting competition, the probability of a man hitting a target is $\frac{3}{5}$. If he fires 4 times, what is the probability of hitting target.
1. at least twice
2. at most twice.

12. Obtain : $\int \frac{dx}{\cos x+\sin 2x}$

13. A certain radioactive meterial has a one-fourth of its life in 4000 years. Find the time required for a given amount to become one-tenth of its original mass.

14. Express $2\bar{i}+2\bar{j}+\bar{k}$ as a sum of two vectors out of which one vector is perpendicular to $2\bar{i}-4\bar{j}+\bar{k}$ and another is parallel to $2\bar{i}-4\bar{j}+\bar{k}$,

SECTION : C

- **Question Nos. 15 to 18 do as directed. Each question carries 4 marks. [16]**

15. Obtain : $\int\sqrt{\tan x}\,dx$, $\left(0<x<\frac{\pi}{2}\right)$

16. Useing matrix method to solve the following system of linear equations :
$x+y+z=0$, $2x-y+z=-3$, $x+2y-z=8$

17. Determine the Intervals in which f is increasing and decreasing
$f:R\to R, f(x)=3x^4-4x^3-12x^2+5$

OR

17. Find the maximum and minimum value of $f(x)=\sin^4 x+\cos^4 x$, $x\in\left[0,\frac{\pi}{2}\right]$

18. Obtain values : $\int_0^{\frac{\pi}{2}}\frac{\sin x}{1+\cos x+\sin x}dx$.

QUESTION PAPER - 2 - SOLUTION (050) (E) (MARCH - 2018)

PART – A

1. (D) -14

→ **Hint :**

Distance of plane from the point

$$p=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

$$\therefore \sqrt{29}=\frac{|3(-1)+(-4)2+2(-2)+k|}{\sqrt{9+16+4}}$$

$$\therefore \quad =\frac{|-3-8-4+k|}{\sqrt{29}}$$

$\therefore$ $29 = |-15+k|$

$\therefore$ $29 = -(-15+k)$ ($\because$ Here $k<0$)

$\therefore$ $29 = 15+k$

$\therefore$ $k = 15-29$

$= -14$

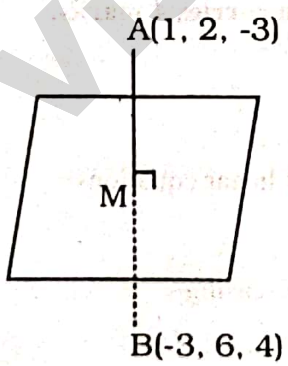
2. (B) $8x-8y-14z+47=0$

→ **Hint :**

M (m) = mid point of $\overline{AB}$

$$=\left(\frac{-3+1}{2},\frac{6+2}{2},\frac{4-3}{2}\right)$$

$$=\left(-1,4,\frac{1}{2}\right)$$



Here, plane passes from point M (m). Equation which satisfy co-ordinates of M (m) is answer.

Here, option (B) : $8x-8y-14z+47=0$ satisfy the co-ordinates of point M (m).

$\therefore$ Required plane is

$8x-8y-14z+47=0$

3. (A) $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z+3}{5}$

→ **Hint :**

Line is perpendicuar to plane.

$\therefore$ Direction of line l=normal to the plane.

$l=(2,3,5)$

Line passes from the point

A$(\bar{a})$ = (1, 2, -3)

$\therefore$ using equation

$$\frac{x+x_1}{l_1}=\frac{y-y_1}{l_2}=\frac{z-z_1}{l_3}$$

we get required line as follows

$$\therefore \frac{x-1}{2}=\frac{y-2}{3}=\frac{z+3}{5}$$

$$\therefore \frac{x+1}{2}=\frac{2-y}{-3}=\frac{z+3}{5}$$

4. (D) 4^{16}

→ **Hint :**

A = {1, 2, 3, 4}

$\therefore$ $n(A)=4$

$\therefore$ $n(A\times A)=4\times 4=16$

$\therefore$ Total number of binary operations on set A= 4^{16}

5. (B) 729

→ **Hint :**

$a*b=a^3+b^3$

$\therefore$ $1*2=1^3+2^3=1+8=9$

$\therefore$ $9\times 0=9^3+0^3=729+0=729$

$\therefore$ $(1\times 2)*0=729$

6. (D) many - one and not onto

→ **Hint :**

$f: N\times N\to N-\{1\}$, $f(m,n)=m+n$

Now, $f(2,4)=2+4=6$

$f(4,2)=4+2=6$

$\therefore$ $(2,4)\neq(4,2)\Rightarrow f(2,4)=f(4,2)$

$\therefore$ f is not one one

Now, $m+n\in N, m\geq 1, n\geq 1$

$\therefore$ $m+n\geq 2$

$f(m, n) \geq 2$

$Rf \geq 2$

$\therefore$ Range of $f = [2, \infty)$

$= N - \{1\}$

$\therefore$ Function f is onto function.

7. (A) An equivalence relation

→ **Hint :**

$(x, y) \in S \Leftrightarrow x \cdot y > 0$

Now, $(x, x) \in S \Leftrightarrow x \cdot x > 0$ થાય.

$\therefore$ Relation S is reflective.

$(x, y) \in S \Rightarrow x \cdot y > 0$

$\Rightarrow y \cdot x > 0$

$\Rightarrow (y \cdot x) \in S$

$\therefore$ Relation S is symmetric also.

Now, $S(x, y) \Leftrightarrow x \cdot y > 0$

$S(y, z) \Rightarrow y \cdot z > 0$

$S(x, z) \Rightarrow x \cdot y \cdot y \cdot z > 0$

$\Rightarrow x \cdot y^2 \cdot z > 0$

$\Rightarrow x \cdot z > 0 \quad [\because y^2 > 0]$

Hence, $(x, z) \in S$

$\therefore$ Relation S is transistive.

$\therefore$ S is equivelence relation.

8. (C) 1

→ **Hint :**

Here, $g(\pi) = g([\pi])$

$= g([3 \cdot 14])$

$= 3$

$\therefore fog(\pi) = f(g[\pi])$

$= f(3 \cdot 14)$

$= f(3)$

$\therefore fog(\pi) = 1 \quad (\because f(x) = 1, x > 0)$

9. (B) 15

→ **Hint :**

$\sec^2(\tan^{-1} 2) + \operatorname{cosec}^2(\cot^{-1} 3)$

Take, $\tan^{-1} 2 = \alpha$ and $\cot^{-1} 3 = \beta$

$\therefore 2 = \tan\alpha$ and $3 = \cot\beta$

$= \sec^2(\alpha) + \operatorname{cosec}^2(\beta)$

$= (1 + \tan^2\alpha) + (1 + \cot^2\beta)$

$= 1 + 4 + 1 + 9$

$= 5 + 10$

$= 15$

10. (A) 1

→ **Hint :**

$$\sin\left(\cos^{-1}\left(-\frac{1}{7}\right) + \sin^{-1}\left(-\frac{1}{7}\right)\right)$$

$$= \sin\left(\pi - \cos^{-1}\left(\frac{1}{7}\right) - \sin^{-1}\left(\frac{1}{7}\right)\right)$$

$(\because \cos^{-1}(-x) = \pi - \cos^{-1} x$ and $\sin^{-1}(-x) = -\sin^{-1} x)$

$$= \sin\left[\left(\pi - \left(\sin^{-1}\frac{1}{7} + \cos^{-1}\left(\frac{1}{7}\right)\right)\right)\right]$$

$$= \sin\left(\pi - \frac{\pi}{2}\right) \quad \left(\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right)$$

$$= \sin\left(\frac{\pi}{2}\right)$$

$= 1$

11. (C) $\frac{1}{\sqrt{2}}$

→ **Hint :**

$3\cos^{-1} x + \sin^{-1} x = \pi$

$\therefore 2\cos^{-1} x + \cos^{-1} x + \sin^{-1} x = \pi$

$\therefore 2\cos^{-1} x + \frac{\pi}{2} = \pi$

$\therefore 2\cos^{-1} x = \pi - \frac{\pi}{2}$

$\therefore 2\cos^{-1} x = \frac{\pi}{2}$

$\therefore \cos^{-1} x = \frac{\pi}{4}$

$\therefore x = \cos\left(\frac{\pi}{4}\right)$

$\therefore x = \frac{1}{\sqrt{2}}$

12. (C) $-\frac{\pi}{4}$

→ **Hint :**

$\cot^{-1}\left(\frac{1}{2}\right)+\cot^{-1}\left(\frac{1}{3}\right)$

$= \tan^{-1}2+\tan^{-1}3$

$= \tan^{-1}\left(\frac{2+3}{1-6}\right)$

$= \tan^{-1}\left(\frac{5}{-5}\right)$

$= \tan^{-1}(-1)$

$= -\tan^{-1}(1)$

$(\because \tan^{-1}(-x) = -\tan^{-1}x)$

$= -\tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$= -\frac{\pi}{4}$

13. (D) Infinite

→ **Hint :**

$4x+3y=6y$

$8x+6y=9xy$

Devide equation by xy

$\frac{4}{y}+\frac{3}{x}=6, \frac{8}{y}+\frac{6}{x}=9$

Take, $\frac{1}{x}=a$ and $\frac{1}{y}=b$

$\therefore$ We get $3a+4b=b$ and $6a+8b=a$

$\therefore \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 6 & 8 \end{vmatrix}$

$= 24-24$

$= 0$

$\therefore$ Equation has infinite solution.

14. (A) 1

→ **Hint :**

Given matrics is skew symmetric.

$\therefore a=0, b=-2$ and $c=3$

$\therefore a+b+c = 0+(-2)+3$

$= -2+3$

$= 1$

15. (D) 8

→ **Hint :**

$A = 2B$

$|A| = |2B|$

$= |2|\,|B|$

$= 2^3\,|B|$

$= 8 \times 1 = 8$

16. (B) 4

→ **Hint :**

$f(x)$ is continuous function.

$\therefore \lim_{x\to 0} f(x) = f(0)$

$\therefore \lim_{x\to 0} \frac{\tan(4x)\cdot\cos(3x)}{x} = k$

$\therefore \lim_{x\to 0} 4\left(\frac{\tan(4x)}{4x}\right) \lim_{x\to 0}\cos(3x) = k$

$\therefore 4(1)\cdot\cos 0 = k$

$\therefore k = 4$

17. (C) $\frac{x\cos x+\sin x}{2\sqrt{x\sin x}}$

→ **Hint :**

$\frac{d}{dx}(\sqrt{x\sin x})$

$= \frac{1}{2\sqrt{x\sin x}}\frac{d}{dx}(x\sin x)$

$= \frac{1}{2\sqrt{x\cdot\sin x}}\left(x\frac{d}{dx}\cos x+\sin x\frac{dx}{dx}\right)$

$= \frac{x\cos x+\sin x}{2\sqrt{x\sin x}}$

18. (A) $-\frac{1}{2\sqrt{3}}$

→ **Hint :**

$\frac{d}{dx}\left(\operatorname{cosec}^{-1}x\right)_{x=-2} = \frac{-1}{(x)\sqrt{x^2-1}}$

$= \frac{-1}{|-2|\sqrt{(-2)^2-1}}$

$= -\frac{1}{2\sqrt{3}}$

19. (D) $\frac{1}{t}$

→ **Hint :**

$y = 2at$ and $x = at^2$

$\therefore \frac{dy}{dt} = 2a$ | $\therefore \frac{dx}{dt} = 2at$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ (Derivative of the parameteric function)

$= \frac{2a}{2at}$

$\therefore \frac{dy}{dx} = \frac{1}{t}$

20. (A) $-2xy_1$

→ **Hint :**

$y = \tan^{-1} x$

Now, differentiate w.r. to x

$\therefore y_1 = \frac{1}{1+x^2}$

$\therefore (1+x^2)y_1 = 1$

Again differentiate w.r. to x

$\therefore (1+x^2)y_2 + (2x)y_1 = 0$

$\therefore (1+x^2)y_2 = -2xy_1$

21. (D) $\sec^{-1}(e^x)$

→ **Hint :**

$I = \int \frac{1}{\sqrt{e^{2x}-1}} dx$

Take $e^x = t$

$\therefore e^x \, dx = dt$

$\therefore dx = \frac{1}{e^x} dt = \frac{1}{t} dt$

$I = \int \frac{1}{t\sqrt{t^2-1}} dt$

$= \sec^{-1}(t) + c$

$I = \sec^{-1}(e^x)$

22. (C) $\frac{(\log x)^4}{x}$

→ **Hint :**

$\int f(x)dx = \frac{(\log x)^5}{5} + c$

Differentiate both the side w.r. to x

$\therefore f(x) = \frac{1}{5}\frac{d}{dx}(\log x)^5$

$= \frac{1}{5}(5(\log x)^4)\frac{d}{dx}(\log x)$

$\therefore f(x) = \frac{(\log x)^4}{x}$

23. (B) 40

→ **Hint :**

Here, $n = 400$, $p = 0.1$ છે.

Now, mean $= np$

$= 400 \times \frac{1}{100}$

$= 40$

24. (D) 9

→ **Hint :**

Standard deviation of,

$2-3x = |-3| \cdot$ Given standard deviation

$= |-3| \cdot 3$

$= 3 \times 3$

$= 9$

($\because$ Standard deviation of $a_i x + b = |a| S$

$i = 1, 2, 3, ...n$)

25. (A) 0.91

→ **Hint :**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.1 + 0.9 - P(A) \cdot P(B)$

$= 1 - (0.1)(0.9)$

$= 1 - 0.09$

$= 0.91$

26. (C) $\frac{4}{7}$

→ **Hint :**

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.40 + 0.35 - 0.55$

$= 0.2$

Now, $P[A|B] = \frac{P(A \cap B)}{P(B)}$

$$= \frac{2}{10} \times \frac{100}{35}$$

$$= \frac{4}{7}$$

27. (A) must satisfies all of the problems constraints simultaneously.

→ **Hint :**

L.P. problem must satisfy all problems constraints simultaneously.

28. (D) 0

→ **Hint :**

Corner point	Objective functiona $z = 300x + 190y$
(0, 0)	$z = 300(0) + 190\,(0) = 0$
(16, 0)	$z = 300(16) + 190\,(0) = 4800$
(8, 16)	$z = 300(8) + 190\,(16)$ $= 5400 + 3040 = 8440$
(0, 24)	$z = 300(0) + 190\,(24)$ $= 0 + 4560 = 4560$ minim.

$\therefore$ Minimum value $= 0$

29. (C) 6%

→ **Hint :**

Volume of cube $V = x^3$

$$\therefore \frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

Side increases by 2%

$$\therefore \frac{dx}{dt} = 2\% \text{ of } x = \frac{2x}{100}$$

$$\therefore \frac{dV}{dt} = 3x^2\left(\frac{2x}{100}\right)$$

$$= \frac{6}{100}x^3$$

$$= \frac{6V}{100}$$

$\therefore$ Approximate error in volume = 6%

30. (B) 1, −1

→ **Hint :**

Slope of tangent $\frac{dy}{dx} = 2x - 3$

$$\therefore \left(\frac{dy}{dx}\right)_{(2,3)} = 2(2) - 3$$

$$\therefore m = 4 - 3 = 1$$

slope of normal $= -\frac{1}{m}$

$= -\frac{1}{1}$

$= -1$

$\therefore$ slope of tangent $= 1$,

slope of normal $= -1$

31. (D) $(1 + \log x)\,(ex)^x$

→ **Hint :**

$$I = \int (ex)^x (2 + \log x)\,dx$$

$$= \int e^x \cdot x^x [1 + \log x + 1]\,dx$$

$$= \int e^x \left(x^x(1 + \log x) + x^x\right) dx$$

Take $f(x) = x^x$

$\therefore$ $f'(x) = x^x\,(1 + \log x)$

$$\therefore I = \int e^x (f(x) + f'(x))\,dx$$

$$= e^x f(x) + c$$

$$= e^x \cdot x^x (1 + \log x)$$

$$\therefore I = (ex)^x (1 + \log x)$$

32. (A) $\frac{(e^x + e^{-x})^3}{3}$

→ **Hint :**

$$I = \int \left(e^x + e^{-x}\right)\left(e^{2x} - e^{-2x}\right)dx$$

$$= \int \left(e^x + e^{-x}\right)\left(\left(e^x\right)^2 - \left(e^{-x}\right)^2\right)dx$$

$$= \int \left(e^x + e^{-x}\right)\left(e^x + e^{-x}\right)\left(e^x - e^{-x}\right)dx$$

→ $$I = \int \left(e^x + e^{-x}\right)^2 \cdot \left(e^x - e^{-x}\right)dx$$

Take $e^x + e^{-x} = t$

Now, differentiate to t

$$\therefore (e^x - e^{-x})\,dx = dt$$

$$\therefore I = \int t^2 dt$$

$$= \frac{t^3}{3} + c$$

$$\therefore \quad I = \frac{(e^x + e^{-x})^3}{3} + c$$

33. (A) $x^3 \tan^{-1} x$

→ **Hint :**

$$I = \int \left(3x^2 \tan^{-1} x + \frac{x^3}{1+x^2}\right) dx$$

$$= \int 3x^2 \tan^{-1} x \, dx + \int \frac{x^3}{1+x^2} dx$$

$$= I_1 + I_2$$

Where, $I_1 = 3\int x^2 \tan^{-1} x \, dx$ and rule of integration by parts,

$$I_2 = \int \frac{x^3}{1+x^2} dx$$

$$I_1 = 3\int x^2 \tan^{-1} x \, dx$$

Take $u = \tan^{-1} x$, and $v = x^2$

$$\therefore \quad I_1 = 3\left\{u \int v \, dx - \int (u') \, v \, dx \int dx\right\}$$

$$= 3\left\{\tan^{-1} x \left(\frac{x^3}{3}\right) - \int \frac{1}{1+x^2} \cdot \frac{x^3}{3} dx\right\}$$

$$\therefore \quad I = x^3 \tan^{-1} x - \int \frac{x^3}{1+x^2} dx + \int \frac{x^3}{1+x^2} dx$$

$$\therefore \quad I = x^3 \tan^{-1} x$$

34. (B) $\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$

→ **Hint :**

$$I = \int \sqrt{5-x^2} \, dx$$

$$= \int \sqrt{(\sqrt{5})^2 - x^2} \, dx$$

$$I = \frac{x\sqrt{5-x^2}}{2} + \frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$$

($\because$ Using formula of $\int \sqrt{a^2 - x^2}$)

35. (D) 55

→ **Hint :**

Here, $\int_n^{n+1} f(x) dx = \binom{n}{n-1}$

$$= \binom{n}{n-(n-1)} \left(\because \binom{n}{r} = \binom{n}{n-r}\right)$$

$$= \binom{n}{1}$$

$$= n \qquad \text{.... (i)}$$

$$\therefore \quad \int_n^{n+1} f(x) dx = n$$

Now, $\int_1^{11} f(x) dx$

$$= \int_1^2 f(x) dx + \int_2^3 f(x) dx + ... + \int_{10}^{11} f(x) dx$$

$$= 1 + 2 + 3 + 4 + ... + 10$$

(From result (i))

$$= \frac{10(10+1)}{2} \quad \left(\because \Sigma n = \frac{n(n+1)}{2}\right)$$

$$= 5\,(11)$$

$$= 55$$

36. (C) 18

→ **Hint :**

Required area $A = \int_0^3 (9 - x^2) dx$

$$= \left(9x - \frac{x^3}{3}\right)_0^3$$

$$= 9(3) - \frac{27}{3}$$

$$= 27 - 9$$

$$= 18 \text{ Sq. unit}$$

37. (B) $\frac{4}{\pi}$

→ **Hint :**

$\sin(\pi x)$ has period 2

$$\therefore \quad \text{Required area} = 2\left|\int_0^1 \sin(\pi x) dx\right|$$

$$= \left|\frac{2}{\pi}(\cos \pi x)\right|_0^1$$

$$= \left|\frac{2}{\pi}(\cos \pi - \cos 0)\right|$$

$$= \left|\frac{2}{\pi}(-1-1)\right|$$

$$= \left|\frac{2}{\pi}(-2)\right|$$

$$= \frac{4}{\pi}$$

38. (D) $\frac{\pi}{6}$

→ **Hint :**

$4x^2 + 9y^2 = 1$

$\therefore \quad \frac{x^2}{\left(\frac{1}{2}\right)^2} + \frac{y^2}{\left(\frac{1}{9}\right)^2} = 1$

Area of ellipse $= \pi\, a \cdot b$

$$= \pi\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$$

$$= \frac{\pi}{6} \text{ sq. unit}$$

39. (C) $\frac{8}{3}$

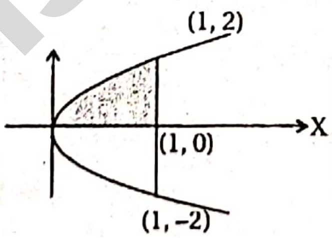
→ **Hint :**

Here, $4a = 4$

$\therefore \quad a = 1$

And $y^2 = 4x$

$y = 2\sqrt{x}$



$\therefore$ Required area $A = 2\int_0^1 y\, dx$

$$= 2\int_0^1 2\sqrt{x}\,dx$$

$$= 4\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^1$$

$$= \frac{8}{3}\left(1^{\frac{3}{2}} - 0\right)$$

$$= \frac{8}{3} \text{ sq. unit}$$

40. (A) 2 and 3

→ **Hint :**

Given differential equation conains second order derivative.

$\therefore$ Its order is 2

Its derivative has maximum power 3

$\therefore$ Its degree is 3

41. (D) $\frac{1}{x}$

→ **Hint :**

$x\frac{dy}{dx} - y = x^3, x > 0$

Now, devide by x

$\therefore \quad \frac{dy}{dx} - \frac{1}{x}(y) = x^2$

Now, compare with $\frac{dy}{dx} + P(x) \cdot y = Q(x)$

$\therefore \quad P(x) = -\frac{1}{x}$

$\therefore$ Integrating factor $= e^{\int P(x)dx}$

$$= e^{-\int \frac{1}{x}dx}$$

$$= e^{-\log x}$$

$$= e^{-\log x^{-1}}$$

$$= x^{-1}$$

$$= \frac{1}{x}$$

42. (D) 3

→ **Hint :**

Number of arbitary constants in third order differential equation are 3.

43. (A) $x \cdot \frac{dy}{dx} = 3y$

→ **Hint :**

$y = cx^3$ .... (i)

$\therefore \dfrac{dy}{dx} = c \cdot 3x^2$

$\therefore x\dfrac{dy}{dx} = 3(cx^3)$

$\therefore x\dfrac{dy}{dx} = 3y$ $\quad (\because$ from (i))

Which is required differential equation.

44. (B) $1-\cos 2\theta$

→ **Hint :**

$$\begin{vmatrix} \bar{a}\cdot\bar{a} & \bar{b}\cdot\bar{a} \\ \bar{a}\cdot\bar{b} & \bar{b}\cdot\bar{b} \end{vmatrix} + |\bar{a}\times\bar{b}|^2$$

$= (\bar{a}\cdot\bar{a})(\bar{b}\cdot\bar{b}) - (\bar{a}\cdot\bar{b})(\bar{a}\cdot\bar{b}) + |\bar{a}\times\bar{b}|^2$

$= |\bar{a}|^2 \cdot |\bar{b}|^2 - (\bar{a}\cdot\bar{b})^2 + |\bar{a}|^2 \cdot |\bar{b}|^2 \cdot \sin^2\theta$

(Using Lagrange's identify)

$= 1 - (1)\cdot(1)\cos^2\theta + (1)\cdot(1)\sin^2\theta$

$(\because \bar{a}$ and $\bar{b}$ are unit vectors)

$= 1 - (1)\cdot(1)\cos^2\theta + \sin^2\theta$

$= 1 - [\cos^2\theta - \sin^2\theta]$

$= 1 - \cos(2\theta)$ $\quad (\because$ Formula of $\cos(2\theta))$

45. (C) $(12, 3, -6)$

→ **Hint :**

Take $\bar{x} = (4, 1, -2)$

$\therefore$ Unit vector in direction $\bar{x}$ having magnitude $3\sqrt{21}$

$= 3\cdot\sqrt{21}\left(\dfrac{\bar{x}}{|\bar{x}|}\right)$

$= 3\cdot\sqrt{21}\left(\dfrac{(4,1,-2)}{\sqrt{16+1+4}}\right)$

$= 3\cdot\sqrt{21}\left(\dfrac{(4,1,-2)}{\sqrt{21}}\right)$

$= 3(4, 1, -2)$

$= (12, 3, -6)$

46. (A) $\dfrac{5}{\sqrt{14}}$

→ **Hint :**

Here, $\bar{x}\cdot\bar{y} = (1, 2, 1)\cdot(2, -3, -1)$

$= 1(2) + 2(-3) + 1(-1)$

$= 2 - 6 - 1$

$= 2 - 7$

$= -5$

$\therefore |\bar{x}\cdot\bar{y}| = |-5| = 5$

and $|\bar{y}| = \sqrt{4+9+1} = \sqrt{14}$

$\therefore \text{Comp}_{\bar{y}}\bar{x} = \dfrac{|\bar{x}\cdot\bar{y}|}{|\bar{y}|} = \dfrac{5}{\sqrt{14}}$

47. (C) $a \in R - \{0\}$

→ **Hint :**

Here, $\bar{x}$ and $\bar{y}$ are perpendicular

$\therefore \bar{x}\cdot\bar{y} = 0$

$\therefore \bar{x}\cdot\bar{y} = (2a, 3a, 0)\cdot(0, 0, 4a)$

$= 2a(0) + 3a(0) + 0(4a)$

$= 0 + 0 + 0$

$\therefore \bar{x}\cdot\bar{y} = 0$

$\therefore \forall a \in R - \{0\}$ given vectors are mutually perpendicular.

NOTE : For $a = 0$ vectors $\bar{x}$ and $\bar{y}$ are equal.

48. (A) $\dfrac{\sqrt{3}}{2}$

→ **Hint :**

Take $\bar{a} = \hat{j} + \hat{k} = (0, 1, 1)$

$\bar{b} = \hat{i} + \hat{k} = (1, 0, 1)$

Area of the parallelogram $= \dfrac{1}{2}|\bar{a}\times\bar{b}|$

$$\therefore \bar{a}\times\bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$= \bar{i}(1) - \bar{j}(-1) + \bar{k}(-1)$

$= (1, 1, -1)$

$\therefore |\bar{a}\times\bar{b}| = \sqrt{1+1+1}$

$= \sqrt{3}$

$\therefore$ Area of the parallelogram $= \dfrac{\sqrt{3}}{2}$ sq. unit

49. (D) $x + 2y - 3z = 14$

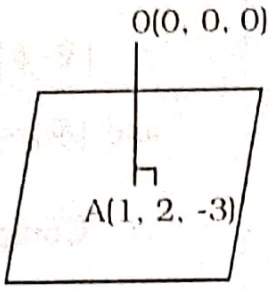
→ **Hint :**

$p = d(\text{OA})$

$= \sqrt{(0-1)^2 + (0-2)^2 + (0-3)^2}$

$= \sqrt{1+4+9}$

$= \sqrt{14}$



Now $\hat{n} = \dfrac{n}{|\hat{n}|}$

$= \dfrac{(1-0, 2-0, -3-0)}{\sqrt{14}}$

$= \dfrac{(1, 2, -3)}{\sqrt{14}}$

$\therefore$ Required plane : $\hat{r} \cdot \hat{n} = p$

$\therefore$ $(x\ y\ z) \cdot \dfrac{(1, 2, -3)}{\sqrt{14}} = \sqrt{14}$

$\therefore$ $x + 2y - 3z = 14$

50. (A) $\sin^{-1}\left(\dfrac{1}{9}\right)$

$\rightarrow$ **Hint :**

From the equation of line $\overline{l} = (2, 2, 1)$

Find from equation of plane $\overline{n} = (-2, 2, -1)$

Now, $\overline{l} \cdot \overline{n} = (2, 2, 1) \cdot (-2, 2, -1)$

$= -4 + 4 - 1$

$= -1$

$\therefore$ $|\overline{l} \cdot \overline{n}| = 1$

and $|\overline{l}| = \sqrt{4+4+1}$

$= \sqrt{9} = 3$

$|\overline{n}| = \sqrt{4+4+1}$

$= \sqrt{9} = 3$

Suppose angle between plane and line $= \alpha$

$\therefore$ $\alpha = \sin^{-1}\left(\dfrac{|\overline{l} \cdot \overline{n}|}{|\overline{l}||\overline{n}|}\right)$

$\therefore$ $\alpha = \sin^{-1}\left(\dfrac{1}{9}\right)$