Chapter 24

Differential Equations

1. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1 and c_2 are arbitrary constants, is **[AIEEE-2009]**
   (1) $y'' = y'y$
   (2) $yy'' = y'$
   (3) $yy'' = (y')^2$
   (4) $y' = y^2$

2. Solution of the differential equation
   $\cos x dy = y(\sin x - y)dx,\ 0 < x < \frac{\pi}{2}$ is **[AIEEE-2010]**
   (1) $\sec x = (\tan x + c)y$
   (2) $y\sec x = \tan x + c$
   (3) $y\tan x = \sec x + c$
   (4) $\tan x = (\sec x + c)y$

3. Consider the differential equation
   $y^2 dx + \left(x - \frac{1}{y}\right)dy = 0$. If $y(1) = 1$, then x is given by **[AIEEE-2011]**
   (1) $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$
   (2) $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$
   (3) $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$
   (4) $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

4. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is **[AIEEE-2012]**
   (1) ln 9
   (2) $\frac{1}{2}$ ln 18
   (3) ln 18
   (4) 2 ln 18

5. Let $y(x)$ be the solution of the differential equation
   $(x\log x)\frac{dy}{dx} + y = 2x\log x, (x \ge 1), y(0) = 1$
   Then $y(e)$ is equal to **[JEE (Main)-2015]**
   (1) e
   (2) 0
   (3) 2
   (4) $2e$

6. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)\, dx = x\, dy$, then $f\left(-\frac{1}{2}\right)$ is equal to **[JEE (Main)-2016]**
   (1) $-\frac{4}{5}$
   (2) $\frac{2}{5}$
   (3) $\frac{4}{5}$
   (4) $-\frac{2}{5}$

7. If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to **[JEE (Main)-2017]**
   (1) $-\frac{2}{3}$
   (2) $-\frac{1}{3}$
   (3) $\frac{4}{3}$
   (4) $\frac{1}{3}$

8. let $y = y(x)$ be the solution of the differential equation $\sin x\frac{dy}{dx} + y\cos x = 4x$, $x \in (0, \pi)$. If $y = \left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to **[JEE (Main)-2018]**
   (1) $\frac{4}{9\sqrt{3}}\pi^2$
   (2) $\frac{-8}{9\sqrt{3}}\pi^2$
   (3) $-\frac{8}{9}\pi^2$
   (4) $-\frac{4}{9}\pi^2$

9. If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx}+2y = x^2$ satisfying $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{13}{16}$ (2) $\frac{7}{64}$

(3) $\frac{1}{4}$ (4) $\frac{49}{16}$

10. Let $f : [0, 1] \to R$ be such that $f(xy) = f(x).f(y)$, for all $x, y \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to **[JEE (Main)-2019]**

(1) 4 (2) 3

(3) 2 (4) 5

11. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y = \frac{1}{\cos^2 x}, x \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$ and $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals

[JEE (Main)-2019]

(1) $\frac{1}{3}+e^6$ (2) $\frac{1}{3}+e^3$

(3) $\frac{1}{3}$ (4) $-\frac{4}{3}$

12. The curve amongst the family of curves represented by the differential equation, $(x^2 - y^2)dx + 2xydy = 0$ which passes through (1, 1) is

[JEE (Main)-2019]

(1) A hyperbola with transverse axis along the x-axis.

(2) A circle with centre on the y-axis.

(3) An ellipse with major axis along the y-axis.

(4) A circle with centre on the x-axis.

13. If $y(x)$ is the solution of the differential equation $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y = e^{-2x}, x > 0,$ where $y(1) = \frac{1}{2}e^{-2}$, then **[JEE (Main)-2019]**

(1) $y(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$

(2) $y(\log_e 2) = \frac{\log_e 2}{4}$

(3) $y(\log_e 2) = \log_e 4$

(4) $y(x)$ is decreasing in (0, 1)

14. The solution of the differential equation, $\frac{dy}{dx} = (x-y)^2$, when $y(1) = 1$, is

[JEE (Main)-2019]

(1) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

(2) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x+y-2$

(3) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

(4) $\log_e\left|\frac{2-x}{2-y}\right| = x-y$

15. Let $y = y(x)$ be the solution of the differential equation, $x\frac{dy}{dx}+y = x\log_e x, (x > 1)$. If $2y(2) = \log_e 4 - 1$, then $y(e)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{e^2}{4}$ (2) $-\frac{e}{2}$

(3) $-\frac{e^2}{2}$ (4) $\frac{e}{4}$

16. Let $y = y(x)$ be the solution of the differential equation, $(x^2+1)^2\frac{dy}{dx}+2x(x^2+1)y = 1$ such that $y(0) = 0$. If $\sqrt{a}\,y(1) = \frac{\pi}{32}$, then the value of '$a$' is

[JEE (Main)-2019]

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) 1 (4) $\frac{1}{16}$

17. Given that the slope of the tangent to a curve $y = y(x)$ at any point (x,y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is

[JEE (Main)-2019]

(1) $x \log_e |y| = -2(x - 1)$

(2) $x \log_e |y| = x - 1$

(3) $x \log_e |y| = 2(x - 1)$

(4) $x^2 \log_e |y| = -2(x - 1)$

18. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 (x \neq 0)$ with $y(1) = 1$, is

[JEE (Main)-2019]

(1) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$ (2) $y = \frac{x^3}{5} + \frac{1}{5x^2}$

(3) $y = \frac{x^2}{4} + \frac{3}{4x^2}$ (4) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$

19. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to

[JEE (Main)-2019]

(1) $\frac{1}{e} - 2$ (2) $\frac{1}{2} - e$

(3) $e - 2$ (4) $2 + \frac{1}{e}$

20. Let $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 1$.Then **[JEE (Main)-2019]**

(1) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

(2) $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$

(3) $y'\left(\frac{\pi}{4}\right) + y'\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

(4) $y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

21. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $x = 0$ is equal to **[JEE (Main)-2019]**

(1) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ (2) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

(3) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$ (4) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

22. Consider the differential equation, $y^2 dx + \left(x - \frac{1}{y}\right)dy = 0$. If value of y is 1 when $x = 1$, then the value of x for which $y = 2$, is

[JEE (Main)-2019]

(1) $\frac{5}{2} + \frac{1}{\sqrt{e}}$ (2) $\frac{3}{2} - \sqrt{e}$

(3) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (4) $\frac{1}{2} + \frac{1}{\sqrt{e}}$

23. The general solution of the differential equation $(y^2 - x^3)\, dx - xydy = 0$ $(x \neq 0)$ is
(where c is a constant of integration)

[JEE (Main)-2019]

(1) $y^2 + 2x^3 + cx^2 = 0$ (2) $y^2 - 2x^2 + cx^3 = 0$

(3) $y^2 - 2x^3 + cx^2 = 0$ (4) $y^2 + 2x^2 + cx^3 = 0$

24. Let $x^k + y^k = a^k$, $(a, k > 0)$ and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is **[JEE (Main)-2020]**

(1) $\frac{3}{2}$ (2) $\frac{1}{3}$

(3) $\frac{4}{3}$ (4) $\frac{2}{3}$

25. If $y = y(x)$ is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that $y(0) = 0$, then $y(1)$ is equal to **[JEE (Main)-2020]**

(1) $2e$

(2) $\log_e 2$

(3) $2 + \log_e 2$

(4) $1 + \log_e 2$

26. Let $y = y(x)$ be the solution curve of the differential equation, $\left(y^2 - x\right)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x-axis at a point whose abscissa is **[JEE (Main)-2020]**

(1) $2 - e$

(2) $2 + e$

(3) $-e$

(4) 2

27. The differential equation of the family of curves, $x^2 = 4b(y + b),\ b \in R$, is **[JEE (Main)-2020]**

(1) $x(y')^2 = x - 2yy'$

(2) $x(y')^2 = 2yy' - x$

(3) $x(y')^2 = x + 2yy'$

(4) $xy'' = y'$

28. If $\frac{dy}{dx} = \frac{xy}{x^2 + y^2};\ y(1) = 1;$ then a value of x satisfying $y(x) = e$ is **[JEE (Main)-2020]**

(1) $\frac{e}{\sqrt{2}}$ (2) $\sqrt{3}e$

(3) $\sqrt{2}e$ (4) $\frac{1}{2}\sqrt{3}e$

29. Let y = y(x) be the solution of the differential equation,

$$\frac{2 + \sin x}{y + 1} \cdot \frac{dy}{dx} = -\cos x,\ y > 0,\ y(0) = 1.$$ If

$y(\pi) = a$ and $\frac{dy}{dx}$ at $x = \pi$ is b, then the ordered pair (a, b) is equal to **[JEE (Main)-2020]**

(1) (2, 1)

(2) (1, –1)

(3) $\left(2, \frac{3}{2}\right)$

(4) (1, 1)

30. If a curve $y = f(x)$, passing through the point (1, 2), is the solution of the differential equation, $2x^2dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to

[JEE (Main)-2020]

(1) $\frac{1}{1 + \log_e 2}$ (2) $\frac{-1}{1 + \log_e 2}$

(3) $1 + \log_e 2$ (4) $\frac{1}{1 - \log_e 2}$

31. The solution curve of the differential equation, $\left(1 + e^{-x}\right)\left(1 + y^2\right)\frac{dy}{dx} = y^2$, which passes through the point (0, 1), is **[JEE (Main)-2020]**

(1) $y^2 + 1 = y\left(\log_e\left(\frac{1 + e^{-x}}{2}\right) + 2\right)$

(2) $y^2 + 1 = y\left(\log_e\left(\frac{1 + e^{x}}{2}\right) + 2\right)$

(3) $y^2 = 1 + y\log_e\left(\frac{1 + e^{-x}}{2}\right)$

(4) $y^2 = 1 + y\log_e\left(\frac{1 + e^{x}}{2}\right)$

32. If $x^3dy + xy\,dx = x^2dy + 2y\,dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{\sqrt{e}}{2}$ (2) $\frac{1}{2} + \sqrt{e}$

(3) $\frac{3}{2} + \sqrt{e}$ (4) $\frac{3}{2}\sqrt{e}$

33. Let $y = y(x)$ be the solution of the differential equation,

$xy' - y = x^2(x\cos x + \sin x)$, $x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to **[JEE (Main)-2020]**

(1) $1 + \frac{\pi}{2}$ (2) $1 + \frac{\pi}{2} + \frac{x^2}{4}$

(3) $2 + \frac{\pi}{2}$ (4) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$

34. The solution of the differential equation $\frac{dy}{dx} - \frac{y + 3x}{\log_e(y + 3x)} + 3 = 0$ is

(where C is a constant of integration.)

[JEE (Main)-2020]

(1) $x-\frac{1}{2}\left(\log_e(y+3x)\right)^2=C$

(2) $y+3x-\frac{1}{2}\left(\log_e x\right)^2=C$

(3) $x-2\log_e(y+3x)=C$

(4) $x-\log_e(y+3x)=C$

35. If $y = y(x)$ is the solution of the differential equation $\frac{5+e^x}{2+y}\cdot\frac{dy}{dx}+e^x=0$ satisfying $y(0) = 1$, then a value of $y(\log_e 13)$ is **[JEE (Main)-2020]**

(1) –1 (2) 2

(3) 0 (4) 1

36. Let $y = y(x)$ be the solution of the differential equation $\cos x\frac{dy}{dx}+2y\sin x=\sin 2x,\ x\in\left(0,\frac{\pi}{2}\right)$. If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to

[JEE (Main)-2020]

(1) $\frac{1}{\sqrt{2}}-1$ (2) $\sqrt{2}-2$

(3) $2-\sqrt{2}$ (4) $2+\sqrt{2}$

37. The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$ is

(where C is a constant of integration)

[JEE (Main)-2020]

(1) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

(2) $\sqrt{1+y^2}+\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

(3) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right)+C$

(4) $\sqrt{1+y^2}-\sqrt{1+x^2}=\frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$

38. If $y=\left(\frac{2}{\pi}x-1\right)$ cosec x is the solution of the differential equation, $\frac{dy}{dx}+p(x)y=\frac{2}{\pi}\operatorname{cosec} x$, $0<x<\frac{\pi}{2}$, then the function $p(x)$ is equal to

[JEE (Main)-2020]

(1) tan x (2) cot x

(3) cosec x (4) sec x

39. If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation,

$(x + 1)dy = ((x + 1)^2 + y - 3)dx$, $y(2) = 0$,

then $y(3)$ is equal to _______. **[JEE (Main)-2020]**

40. The population $P = P(t)$ at time 't' of a certain species follows the differential equation $\frac{dP}{dt}=0.5P-450$. If $P(0) = 850$, then the time at which population becomes zero is :

[JEE (Main)-2021]

(1) $\log_e 18$ (2) $\frac{1}{2}\log_e 18$

(3) $\log_e 9$ (4) $2\log_e 18$

41. Let f be a twice differentiable function defined on R such that $f(0) = 1$, $f'(0) = 2$ and $f'(x) \neq 0$ for all $x \in$ R. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix}=0$, for all $x \in$ R, then the value of f(1) lies in the interval : **[JEE (Main)-2021]**

(1) (0, 3) (2) (9, 12)

(3) (3, 6) (4) (6, 9)

42. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point : **[JEE (Main)-2021]**

(1) (5, 5) (2) (4, 5)

(3) (4, 4) (4) (5, 4)

43. If the curve, $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)\,dx + xdy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to __________. **[JEE (Main)-2021]**

44. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time $t = 0$. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to :

[JEE (Main)-2021]

(1) 16 (2) 4
(3) 8 (4) 2

45. If $y = y(x)$ is the solution of the equation $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x,\ y(0) = 0;$ then $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to ________. **[JEE (Main)-2021]**

46. Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2 + y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y, for which the point (3, y) lies on the curve, is : **[JEE (Main)-2021]**

(1) $\frac{18}{35}$ (2) $-\frac{4}{3}$

(3) $-\frac{18}{11}$ (4) $-\frac{18}{19}$

47. If $y = y(x)$ is the solution of the differential equation, $\frac{dy}{dx} + 2y\tan x = \sin x,\ y\left(\frac{\pi}{3}\right) = 0,$ then the maximum value of the function y(x) over **R** is equal to : **[JEE (Main)-2021]**

(1) $-\frac{15}{4}$ (2) $\frac{1}{8}$

(3) $\frac{1}{2}$ (4) 8

48. Let the curve $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} = 2(x+1)$. If the numerical value of area bounded by the curve $y = y(x)$ and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of y(1) is equal to ____. **[JEE (Main)-2021]**

49. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx} = y^2 - x^2, x > 0$. Let the curve C_2 be the solution of $\frac{2xy}{x^2 - y^2} = \frac{dy}{dx}$. If both the curves pass through (1, 1), then the area enclosed by the curves C_1 and C_2 is equal to : **[JEE (Main)-2021]**

(1) $\pi - 1$ (2) $\pi + 1$

(3) $\frac{\pi}{4} + 1$ (4) $\frac{\pi}{2} - 1$

50. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + (\tan x)y = \sin x,\ 0 \le x \le \frac{\pi}{3},$ with $y(0) = 0,$ then $y\left(\frac{\pi}{4}\right)$ equal to : **[JEE (Main)-2021]**

(1) $\log_e 2$ (2) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

(3) $\frac{1}{2}\log_e 2$ (4) $\frac{1}{4}\log_e 2$

51. Which of the following is true for y(x) that satisfies the differential equation $\frac{dy}{dx} = xy - 1 + x - y;\ y(0) - 0:$ **[JEE (Main)-2021]**

(1) $y(1) = e^{\frac{1}{2}} - e^{-\frac{1}{2}}$ (2) y(1) = 1

(3) $y(1) = e^{\frac{1}{2}} - 1$ (4) $y(1) = e^{-\frac{1}{2}} - 1$

52. Let $y = y(x)$ be the solution of the differential equation $\cos x(3\sin x + \cos x + 3)dy = (1 + y\sin x(3\sin x + \cos x + 3))dx, 0 \le x \le \frac{\pi}{2}$, $y(0) = 0$. Then, $y\left(\frac{\pi}{3}\right)$ is equal to **[JEE (Main)-2021]**

(1) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$ (2) $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

(3) $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$ (4) $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$

53. If the curve $y = y(x)$ is the solution of the differential equation

$$2(x^2 + x^{5/4})\,dy - y(x + x^{1/4})\,dx = 2x^{9/4}\,dx,\ x > 0$$

which passes through the point $\left(1, 1 - \frac{4}{3}\log_e 2\right)$, then the value of y(16) is equal to : **[JEE (Main)-2021]**

(1) $\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$ (2) $\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

(3) $4\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$ (4) $4\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$

54. The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is **[JEE (Main)-2021]**

(1) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

(2) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

(3) $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

(4) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

55. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = (y+1)\left((y+1)e^{x^2/2} - x\right)$, $0 < x < 2.1$, with $y(2) = 0$. Then the value of $\frac{dy}{dx}$ at $x = 1$ is equal to : **[JEE (Main)-2021]**

(1) $\frac{e^{5/2}}{(1+e^2)^2}$ (2) $\frac{-e^{3/2}}{(e^2+1)^2}$

(3) $\frac{5e^{1/2}}{(e^2+1)^2}$ (4) $-\frac{2e^2}{(1+e^2)^2}$

56. Let $y = y(x)$ be the solution of the differential equation $xdy - ydx = \sqrt{(x^2 - y^2)}dx,\ x \geq 1$, with $y(1) = 0$. If the area bounded by the line $x = 1$, $x = e^{\pi}$, $y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + \beta$, then the value of $10(\alpha + \beta)$ is equal to _________.

[JEE (Main)-2021]

57. Let $y = y(x)$ be the solution of the differential equation

$$x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx,$$

$$-1 \leq x \leq 1,\ y\left(\frac{1}{2}\right) = \frac{\pi}{6}.$$

Then the area of the region bounded by the curves $x = 0$, $x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is **[JEE (Main)-2021]**

(1) $\frac{1}{6}(\pi - 1)$ (2) $\frac{1}{12}(\pi - 3)$

(3) $\frac{1}{8}(\pi - 1)$ (4) $\frac{1}{4}(\pi - 2)$

58. Let $y = y(x)$ be the solution of the differential equation $e^x\sqrt{1-y^2}\ dx + \left(\frac{y}{x}\right)dy = 0,\ y(1) = -1.$

Then the value of $(y(3))^2$ is equal to

[JEE (Main)-2021]

(1) $1 + 4e^6$ (2) $1 - 4e^6$

(3) $1 - 4e^3$ (4) $1 + 4e^3$

59. Let $y = y(x)$ satisfies the equation $\frac{dy}{dx} - |A| = 0$, for all $x > 0$, where $A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$. If $y(\pi) = \pi + 2$, then the value of $y\left(\frac{\pi}{2}\right)$ is

[JEE (Main)-2021]

(1) $\frac{\pi}{2} - \frac{4}{\pi}$ (2) $\frac{\pi}{2} + \frac{4}{\pi}$

(3) $\frac{3\pi}{2} - \frac{1}{\pi}$ (4) $\frac{\pi}{2} - \frac{1}{\pi}$

60. Let a curve $y = y(x)$ be given by the solution of the differential equation

$$\cos\left(\frac{1}{2}\cos^{-1}\left(e^{-x}\right)\right)dx = \sqrt{e^{2x} - 1}\ dy$$

If it intersects y-axis at $y = -1$, and the intersection point of the curve with x-axis is $(\alpha, 0)$, then e^{α} is equal to ______. **[JEE (Main)-2021]**

61. Let $y = y(x)$ be the solution of the differential equation $\operatorname{cosec}^2 x\ dy + 2dx = (1 + y\cos 2x)\operatorname{cosec}^2 x\ dx$, with $y\left(\frac{\pi}{4}\right) = 0$. Then, the value of $(y(0) + 1)^2$ is equal to

[JEE (Main)-2021]

(1) e (2) $e^{1/2}$

(3) e^{-1} (4) $e^{-1/2}$

62. Let $y = y(x)$ be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right)dx = (x+2)dy$, $y(1) = 1$. If the domain of $y = y(x)$ is an open interval (α, β), then $|\alpha + \beta|$ is equal to _______. **[JEE (Main)-2021]**

63. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 1 + xe^{y-x}, -\sqrt{2} < x < \sqrt{2}, y(0) = 0$

then, the minimum value of $y(x), x \in \left(-\sqrt{2}, \sqrt{2}\right)$ is equal to **[JEE (Main)-2021]**

(1) $\left(2-\sqrt{3}\right) - \log_e 2$

(2) $\left(1-\sqrt{3}\right) - \log_e\left(\sqrt{3}-1\right)$

(3) $\left(1+\sqrt{3}\right) - \log_e\left(\sqrt{3}-1\right)$

(4) $\left(2+\sqrt{3}\right) + \log_e 2$

64. Let $y = y(x)$ be solution of the following differential equation $e^y \frac{dy}{dx} - 2e^y \sin x + \sin x \cos^2 x = 0$,

$y\left(\frac{\pi}{2}\right) = 0$. If $y(0) = \log_e(\alpha + \beta e^{-2})$, then $4(\alpha + \beta)$ is equal to _______. **[JEE (Main)-2021]**

65. Let $y = y(x)$ be the solution of the differential equation $xdy = (y + x^3 \cos x)dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to **[JEE (Main)-2021]**

(1) $\frac{\pi^2}{2} + \frac{\pi}{4}$ (2) $\frac{\pi^2}{2} - \frac{\pi}{4}$

(3) $\frac{\pi^2}{4} - \frac{\pi}{2}$ (4) $\frac{\pi^2}{4} + \frac{\pi}{2}$

66. Let a curve $y = f(x)$ pass through the point $(2, (\log_e 2)^2)$ and have slope $\frac{2y}{x \log_e x}$ for all positive real value of x. Then the value of $f(e)$ is equal to _________. **[JEE (Main)-2021]**

67. Let $y = y(x)$ be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, with $y(0) = 0$. If

$y\left(-\frac{2}{3}\log_e 2\right) = \alpha \log_e 2$, then the value of α is equal to **[JEE (Main)-2021]**

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$

(3) $\frac{1}{4}$ (4) 2

68. Let $F : [3, 5] \to \mathbf{R}$ be a twice differentiable function on (3, 5) such that $F(x) = e^{-x}\int_3^x (3t^2 + 2t + 4F'(t))dt$.

If $F'(4) = \frac{\alpha e^{\beta} - 224}{(e^{\beta} - 4)^2}$, then $\alpha + \beta$ is equal to _______. **[JEE (Main)-2021]**

69. If $y = y(x)$, $y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation

$\sec y \frac{dy}{dx} - \sin(x+y) - \sin(x-y) = 0$, with $y(0) = 0$,

then $5y'\left(\frac{\pi}{2}\right)$ is equal to _______. **[JEE (Main)-2021]**

70. Let $y = y(x)$ be the solution of the differential equation $(x - x^3)dy = (y + yx^2 - 3x^4)dx$, $x > 2$. If $y(3) = 3$, then $y(4)$ is equal to **[JEE (Main)-2021]**

(1) 12 (2) 8

(3) 4 (4) 16

71. Let $y = y(x)$ be the solution of the differential equation $dy = e^{\alpha x + y}dx$; $\alpha \in N$. If $y(\log_e 2) = \log_e 2$ and $y(0) = \log_e\left(\frac{1}{2}\right)$, then the value of α is equal to _______. **[JEE (Main)-2021]**

72. Let $y = y(x)$ be a solution curve of the differential equation $(y + 1)\tan^2 x\, dx + \tan x\, dy + y dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $\lim_{x \to 0+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is **[JEE (Main)-2021]**

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{4} + 1$

(3) $\frac{\pi}{4} - 1$ (4) $-\frac{\pi}{4}$

73. Let $y(x)$ be the solution of the differential equation $2x^2 dy + (e^y - 2x)dx = 0$, $x > 0$. If $y(e) = 1$, then $y(1)$ is equal to **[JEE (Main)-2021]**

(1) 2 (2) 0

(3) $\log_e(2e)$ (4) $\log_e 2$

74. Let us consider a curve, $y = f(x)$ passing through the point (–2, 2) and slope of the tangent to the curve at any point $(x, f(x))$ is given by $f(x) + xf'(x) = x^2$. Then **[JEE (Main)-2021]**

(1) $x^3 + xf(x) + 12 = 0$ (2) $x^2 + 2xf(x) + 4 = 0$

(3) $x^2 + 2xf(x) - 12 = 0$ (4) $x^3 - 3xf(x) - 4 = 0$

75. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2\sin x - 5)x - 2\cos x$ such that $y(0) = 7$. Then $y(\pi)$ is equal to **[JEE (Main)-2021]**

(1) $7e^{\pi^2} + 5$ (2) $2e^{\pi^2} + 5$

(3) $e^{\pi^2} + 5$ (4) $3e^{\pi^2} + 5$

76. If $y^{1/4} + y^{-1/4} = 2x$, and

$$(x^2 - 1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0,$$

then $|\alpha - \beta|$ is equal to ______. **[JEE (Main)-2021]**

77. If the solution curve of the differential equation $(2x - 10y^3)dy + ydx = 0$, passes through the points (0, 1) and (2, β), then β is a root of the equation **[JEE (Main)-2021]**

(1) $2y^5 - y^2 - 2 = 0$ (2) $y^5 - y^2 - 1 = 0$

(3) $y^5 - 2y - 2 = 0$ (4) $2y^5 - 2y - 1 = 0$

78. A differential equation representing the family of parabolas with axis parallel to y-axis and whose length of latus rectum is the distance of the point (2, –3) from the line $3x + 4y = 5$, is given by **[JEE (Main)-2021]**

(1) $11\frac{d^2y}{dx^2} = 10$ (2) $11\frac{d^2x}{dy^2} = 10$

(3) $10\frac{d^2x}{dy^2} = 11$ (4) $10\frac{d^2y}{dx^2} = 11$

79. If $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}, y(0) = 1$, then $y(1)$ is equal to : **[JEE (Main)-2021]**

(1) $\log_2(2e)$ (2) $\log_2(1 + e^2)$

(3) $\log_2(1 + e)$ (4) $\log_2(2 + e)$

80. If $x\phi(x) = \int_5^x (3t^2 - 2\phi'(t))dt$, $x > -2$, and $\phi(0) = 4$, then $\phi(2)$ is ______. **[JEE (Main)-2021]**

81. If $\frac{dy}{dx} = \frac{2^x y + 2^y \cdot 2^x}{2^x + 2^{x+y}\log_e 2}, y(0) = 0$, then for $y = 1$, the value of x lies in the interval **[JEE (Main)-2021]**

(1) $\left(\frac{1}{2}, 1\right]$ (2) $\left(0, \frac{1}{2}\right]$

(3) (1, 2) (4) (2, 3)

82. If $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, $x > 0$, $\phi > 0$, and $y(1) = -1$, then $\phi\left(\frac{y^2}{4}\right)$ is equal to **[JEE (Main)-2021]**

(1) $4\phi(2)$ (2) $4\phi(1)$

(3) $2\phi(1)$ (4) $\phi(1)$

83. If $y = y(x)$ is the solution curve of the differential equation $x^2dy + \left(y - \frac{1}{x}\right)dx = 0; x > 0$, and $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to : **[JEE (Main)-2021]**

(1) $3 - e$ (2) $\frac{3}{2} - \frac{1}{\sqrt{e}}$

(3) $3 + \frac{1}{\sqrt{e}}$ (4) $3 + e$

84. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is ________. **[JEE (Main)-2021]**

85. If $x = x(y)$ is the solution of the differential equation $y\frac{dx}{dy} = 2x + y^3(y+1)e^y, x(1) = 0;$ then $x(e)$ is equal to : **[JEE (Main)-2022]**

(1) $e^3(e^e - 1)$ (2) $e^e(e^3 - 1)$

(3) $e^2(e^e + 1)$ (4) $e^e(e^2 - 1)$

86. Let $y = y(x)$ be the solution of the differential equation $(x + 1)y' - y = e^{3x}(x + 1)^2$, with $y(0) = \frac{1}{3}$. Then, the point $x = -\frac{4}{3}$ for the curve $y = y(x)$ is : **[JEE (Main)-2022]**

(1) not a critical point

(2) a point of local minima

(3) a point of local maxima

(4) a point of inflection

87. If the solution curve $y = y(x)$ of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$, which passes through the point (1,1) and intersects the line $y=\sqrt{3}x$ at the point $\left(\alpha, \sqrt{3}\alpha\right)$, then value of $\log_e\left(\sqrt{3}\alpha\right)$ is equal to : **[JEE (Main)-2022]**

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{12}$ (4) $\frac{\pi}{6}$

88. If $y = y(x)$ is the solution of the differential equation $2x^2\frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(e) = \frac{e}{3}$, then $y(1)$ is equal to **[JEE (Main)-2022]**

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) 3

89. Let the solution curve $y = y(x)$ of the differential equation $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$ pass through the origin. Then $y(2)$ is equal to ________.

[JEE (Main)-2022]

90. Let $S = (0, 2\pi) - \left\{\frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\right\}$. Let $y = y(x)$, $x \in S$, be the solution curve of the differential equation $\frac{dy}{dx} = \frac{1}{1+\sin 2x}$, $y\left(\frac{\pi}{4}\right) = \frac{1}{2}$. If the sum of abscissas of all the points of intersection of the curve $y = y(x)$ with the curve $y = \sqrt{2}\sin x$ is $\frac{k\pi}{12}$, then k is equal to _________. [JEE (Main)-2022]

91. If the solution of the differential equation

$\frac{dy}{dx} + e^x\left(x^2 - 2\right)y = \left(x^2 - 2x\right)\left(x^2 - 2\right)e^{2x}$ satisfies $y(0) = 0$, then the value of $y(2)$ is _______.

[JEE (Main)-2022]

(1) –1 (2) 1

(3) 0 (4) e

92. Let $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$, where a, b, c are constants, represent a circle passing through the point (2, 5). Then the shortest distance of the point (11, 6) from this circle is [JEE (Main)-2022]

(1) 10 (2) 8

(3) 7 (4) 5

93. If $\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0$, $x, y > 0$, $y(1) = 1$, then $y(2)$ is equal to : **[JEE (Main)-2022]**

(1) $2 + \log_2 3$ (2) $2 + \log_3 2$

(3) $2 - \log_3 2$ (4) $2 - \log_2 3$

94. If the solution curve of the differential equation $\left(\left(\tan^{-1} y\right) - x\right)dy = \left(1 + y^2\right)dx$ passes through the point (1, 0), then the abscissa of the point on the curve whose ordinate is tan(1), is[JEE (Main)-2022]

(1) $2e$ (2) $\frac{2}{e}$

(3) 2 (4) $\frac{1}{e}$

95. Let $y = y(x)$ be the solution of the differential equation $\left(1 - x^2\right)dy = \left(xy + \left(x^3 + 2\right)\sqrt{1 - x^2}\right)dx$,

$-1 < x < 1$, and $y(0) = 0$. If $\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1 - x^2}\, y(x)\,dx = k$, then k^{-1} is equal to _______. **[JEE (Main)-2022]**

96. Let the solution curve $y = y(x)$ of the differential equation

$$\left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right]x\frac{dy}{dx} = x + \left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right]y$$

pass through the points (1, 0) and $(2\alpha, \alpha)$, $\alpha > 0$. Then α is equal to **[JEE (Main)-2022]**

(1) $\frac{1}{2}\exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$ (2) $\frac{1}{2}\exp\left(\frac{\pi}{3} + e - 1\right)$

(3) $\exp\left(\frac{\pi}{6} + \sqrt{e} + 1\right)$ (4) $2\exp\left(\frac{\pi}{3} + \sqrt{e} - 1\right)$

97. Let $y = y(x)$ be the solution of the differential equation $x\left(1-x^2\right)\frac{dy}{dx}+\left(3x^2y-y-4x^3\right)=0,\ x>1$, with $y(2) = -2$. Then $y(3)$ is equal to **[JEE (Main)-2022]**

(1) –18
(2) –12
(3) –6
(4) –3

98. Let $x = x(y)$ be the solution of the differential equation

$$2ye^{\frac{x}{y^2}}dx+\left(y^2-4xe^{\frac{x}{y^2}}\right)dy=0$$ such that $x(1) = 0$. Then, $x(e)$ is equal to **[JEE (Main)-2022]**

(1) $e\log_e(2)$
(2) $-e\log_e(2)$
(3) $e^2\log_e(2)$
(4) $-e^2\log_e(2)$

99. Let the solution curve of the differential equation $x\frac{dy}{dx}-y=\sqrt{y^2+16x^2}$, $y(1) = 3$ be $y = y(x)$. Then $y(2)$ is equal to : [JEE (Main)-2022]

(1) 15
(2) 11
(3) 13
(4) 17

100. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx}+\frac{\sqrt{2}y}{2\cos^4x-\cos 2x}=xe^{\tan^{-1}\left(\sqrt{2}\cot 2x\right)}$, $0<x<\frac{\pi}{2}$ with $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{32}$. If $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{18}e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to ______.

[JEE (Main)-2022]

101. If $y = y(x)$ is the solution of the differential equation $\left(1+e^{2x}\right)\frac{dy}{dx}+2\left(1+y^2\right)e^x=0$ and $y(0)=0$, then $6\left(y'(0)+\left(y\left(\log_e\sqrt{3}\right)\right)^2\right)$ is equal to

[JEE (Main)-2022]

(1) 2
(2) –2
(3) –4
(4) –1

102. Let $y = y(x)$, $x > 1$, be the solution of the differential equation $(x-1)\frac{dy}{dx}+2xy=\frac{1}{x-1}$, with $y(2)=\frac{1+e^4}{2e^4}$. If $y(3)=\frac{e^{\alpha}+1}{\beta e^{\alpha}}$, then the value of $\alpha + \beta$ is equal to ____. **[JEE (Main)-2022]**

103. The slope of the tangent to a curve $C : y = y(x)$ at any point (x, y) on it is $\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$. If C passes through the points $\left(0,\frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha,\frac{1}{2}e^{2\alpha}\right)$, then e^{α} is equal to

(1) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$
(2) $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$
(3) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$
(4) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

104. The general solution of the differential equation $(x - y^2)dx + y(5x + y^2)dy = 0$ is : **[JEE (Main)-2022]**

(1) $\left(y^2+x\right)^4=C\left|\left(y^2+2x\right)^3\right|$
(2) $\left(y^2+2x\right)^4=C\left|\left(y^2+x\right)^3\right|$
(3) $\left|\left(y^2+x\right)^3\right|=C\left(2y^2+x\right)^4$
(4) $\left|\left(y^2+2x\right)^3\right|=C\left(2y^2+x\right)^4$

105. If $\frac{dy}{dx}+2y\tan x=\sin x,\ 0<x<\frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right)=0$, then the maximum value of $y(x)$ is:

[JEE (Main)-2022]

(1) $\frac{1}{8}$
(2) $\frac{3}{4}$
(3) $\frac{1}{4}$
(4) $\frac{3}{8}$

106. Let a curve $y = y(x)$ pass through the point $(3, 3)$ and the area of the region under this curve, above the x-axis and between the abscissae 3 and $x(> 3)$ be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $\left(\alpha, 6\sqrt{10}\right)$ in the first quadrant, then α is equal to ________. **[JEE (Main)-2022]**

107. Let $y = y_1(x)$ and $y = y_2(x)$ be two distinct solution of the differential equation $\frac{dy}{dx} = x + y$, with $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then, the number of points of intersection of $y = y_1(x)$ and $y = y_2(x)$ is [JEE (Main)-2022]

(1) 0
(2) 1
(3) 2
(4) 3

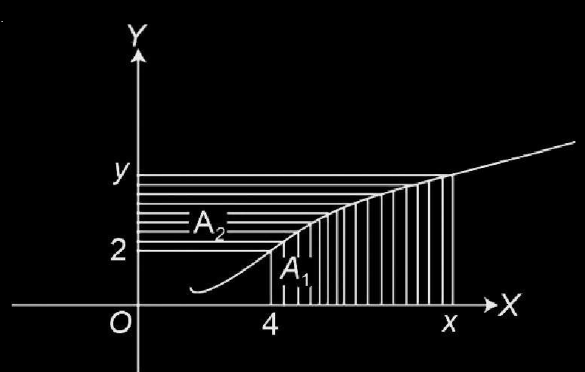
108. Let $y = y(x)$ be the solution curve of the differential equation [JEE (Main)-2022]

$$\sin\left(2x^2\right)\log_e\left(\tan x^2\right)dy + \left(4xy - 4\sqrt{2}x\sin\left(x^2 - \frac{\pi}{4}\right)\right)dx = 0,$$

$0 < x < \sqrt{\frac{\pi}{2}}$, which passes through the point $\left(\sqrt{\frac{\pi}{6}}, 1\right)$. Then $\left|y\left(\sqrt{\frac{\pi}{3}}\right)\right|$ is equal to ________.

[JEE (Main)-2022]

109. Consider a curve $y = y(x)$ in the first quadrant as shown in the figure. Let the area A_1 is twice the area A_2. Then the normal to the curve perpendicular to the line $2x - 12y = 15$ does **NOT** pass through the point.



(1) (6, 21)
(2) (8, 9)
(3) (10, –4)
(4) (12, –15)

110. Let the solution curve of the differential equation $xdy = \left(\sqrt{x^2 + y^2} + y\right)dx,\ x > 0,$ intersect the line $x = 1$ at $y = 0$ and the line $x = 2$ at $y = \alpha$. Then the value of α is [JEE (Main)-2022]

(1) $\frac{1}{2}$
(2) $\frac{3}{2}$
(3) $-\frac{3}{2}$
(4) $\frac{5}{2}$

111. If $y = y(x)$, $x \in \left(0, \frac{\pi}{2}\right)$ be the solution curve of the differential equation $(\sin^2 2x)\frac{dy}{dx} + (8\sin^2 2x + 2\sin 4x)y = 2e^{-4x}(2\sin 2x + \cos 2x)$, with $y\left(\frac{\pi}{4}\right) = e^{-\pi}$, then $y\left(\frac{\pi}{6}\right)$ is equal to [JEE (Main)-2022]

(1) $\frac{2}{\sqrt{3}}e^{-2\pi/3}$
(2) $\frac{2}{\sqrt{3}}e^{2\pi/3}$
(3) $\frac{1}{\sqrt{3}}e^{-2\pi/3}$
(4) $\frac{1}{\sqrt{3}}e^{2\pi/3}$

112. The differential equation of the family of circles passing through the points (0, 2) and (0, –2) is

[JEE (Main)-2022]

(1) $2xy\frac{dy}{dx} + (x^2 - y^2 + 4) = 0$
(2) $2xy\frac{dy}{dx} + (x^2 + y^2 - 4) = 0$
(3) $2xy\frac{dy}{dx} + (y^2 - x^2 + 4) = 0$
(4) $2xy\frac{dy}{dx} - (x^2 - y^2 + 4) = 0$

113. If the solution curve of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passes through the points (2, 1) and (k + 1, 2), k > 0, then [JEE (Main)-2022]

(1) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$

(2) $\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$

(3) $2\tan^{-1}\left(\frac{1}{k+1}\right)=\log_e\left(k^2+2k+2\right)$

(4) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(\frac{k^2+1}{k^2}\right)$

114. Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y=\frac{(x+3)}{x+1}$, $x>-1$, which passes through the point (0, 1). Then $y(1)$ is equal to [JEE (Main)-2022]

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$

(3) $\frac{5}{2}$ (4) $\frac{7}{2}$

115. Let a smooth curve $y = f(x)$ be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the points (1, 2) and (8, 1), then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to

[JEE (Main)-2022]

(1) $2\log_e 2$ (2) 4

(3) 1 (4) $4\log_e 2$

116. Suppose $y = y(x)$ be the solution curve to the differential equation $\frac{dy}{dx}-y=2-e^{-x}$ such that $\lim\limits_{x\to\infty} y(x)$ is finite. If a and b are respectively the x- and y- intercepts of the tangent to the curve at $x = 0$, then the value of $a - 4b$ is equal to _____.

[JEE (Main)-2022]

117. If $y = y(x)$ is the solution of the differential equation $x\frac{dy}{dx}+2y=xe^x, y(1)=0$ then the local maximum value of the function $z(x)=x^2y(x)-e^x, x\in R$ is

(1) $1-e$ (2) 0

(3) $\frac{1}{2}$ (4) $\frac{4}{e}-e$

[JEE (Main)-2022]

118. Let the slope of the tangent to a curve $y = f(x)$ at (x, y) be given by 2 tan$x$(cos$x$ – y). If the curve passes through the point $\left(\frac{\pi}{4},0\right)$, then the value of $\int_0^{\pi/2} y\,dx$ is equal to : **[JEE (Main)-2022]**

(1) $(2-\sqrt{2})+\frac{\pi}{\sqrt{2}}$ (2) $2-\frac{\pi}{\sqrt{2}}$

(3) $(2+\sqrt{2})+\frac{\pi}{\sqrt{2}}$ (4) $2+\frac{\pi}{\sqrt{2}}$

119. Let the solution curve $y = f(x)$ of the differential equation $\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^4+2x}{\sqrt{1-x^2}}, x\in(-1, 1)$ pass through the origin. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is

[JEE (Main)-2022]

(1) $\frac{\pi}{3}-\frac{1}{4}$ (2) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$

(3) $\frac{\pi}{6}-\frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{6}-\frac{\sqrt{3}}{2}$

120. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx}=\frac{4y^3+2yx^2}{3xy^2+x^3}$, $y(1) = 1$.

If for some $n \in N$, $y(2) \in [n - 1, n)$, then n is equal to

Chapter 24

Differential Equations

1. Answer (3)

Put $e^{c_2} = k$

Then $y = c_1.k^x$

$\Rightarrow \log_e y = \log_e c_1 + x \log_e k$

$$\Rightarrow \frac{1}{y}y' = \log_e k$$

$$\Rightarrow \frac{1}{y}y'' - \frac{1}{y^2}(y')^2 = 0$$

$\Rightarrow yy'' = (y')^2$

2. Answer (1)

The given differential equation can be put in the form

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$$

$$\Rightarrow \frac{dz}{dx} + (\tan x)z = +\sec x,\ z = \frac{1}{y}$$

which is linear in z

$$\text{I.F} = e^{\int \tan x\, dx} = e^{\ln \sec x} = \sec x$$

The solution is

$$z.\sec x = \int \sec^2 x dx = \tan x + c$$

where c is a constant of integration

$$\Rightarrow \sec x = y(\tan x + c)$$

3. Answer (1)

The given diff. equation reduces to $\dfrac{dy}{du} = \dfrac{y^3}{1 - xy}$

$$\Rightarrow \frac{dx}{dy} = \frac{1 - xy}{y^3} = \frac{1}{y^3} - \frac{x}{y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$$

$$\therefore \text{ I.F.} = e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$$

The required solution is

$$x.e^{-\frac{1}{y}} = \int e^{-\frac{1}{y}} \cdot \frac{1}{y^3}dy$$

Put $-\dfrac{1}{y} = t$

$$= -\int e^t.t\, dt \qquad \Rightarrow \frac{1}{y^2}dy = dt$$

$$= -(te^t - e^t) + c$$

$$= (e^t - te^t) + c$$

$$= e^{-\frac{1}{y}} + \frac{1}{y}e^{-\frac{1}{y}} + c$$

$$\Rightarrow x = 1 + \frac{1}{y} + ce^{\frac{1}{y}}$$

Put $x = 1,\ y = 1$

$1 = 1 + 1 + ce$

$\Rightarrow ce = -1$

$$\Rightarrow c = -\frac{1}{e}$$

$$\Rightarrow x = 1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$$

4. Answer (4)

$$\int_{850}^{0} \frac{dp}{\frac{1}{2}p - 450} = \int_0^t dt$$

$$2\ln\left|\frac{1}{2}p - 450\right|_{850}^{0} = t$$

$\Rightarrow t = 2 \ln 18$

5. Answer (3)

$x \log x \dfrac{dy}{dx} + y = 2x \log x$ *If* $x = 1$ then $y = 0$

$$\frac{dy}{dx} + \frac{y}{x\log x} = 2$$

$$\text{I.F.} = e^{\int \frac{1}{x\log x}dx} = e^{\log\log x} = \log x$$

Solution is $y \cdot \log x = \int 2\log x\, dx + c$

$y\log x = 2(x\log x - x) + c$

$x = 1$, $y = 0$

Then, $c = 2$, $y(e) = 2$

6. Answer (3)

$ydx - xdy = -y^2xdx$

$$\Rightarrow \frac{ydx - xdy}{y^2} = -xdx$$

$$\Rightarrow d\left(\frac{x}{y}\right) = -xdx$$

On integrating both sides

$$\frac{x}{y} = \frac{-x^2}{2} + c$$

it passes through (1, –1)

$$\Rightarrow -1 = \frac{1}{2} + c \Rightarrow c = \frac{-1}{2}$$

So, $\dfrac{x}{y} = \dfrac{-x^2}{2} - \dfrac{1}{2}$

$\Rightarrow y = \dfrac{-2x}{x^2+1}$ *i.e.*, $f\left(-\dfrac{1}{2}\right) = \dfrac{4}{5}$

7. Answer (4)

$$(2+\sin x)\frac{dy}{dx} + (y+1)\cos x = 0$$

$$y(0) = 1,\ y\left(\frac{\pi}{2}\right) = ?$$

$$\frac{1}{y+1}dy + \frac{\cos x}{2+\sin x}dx = 0$$

$$\ln|y+1| + \ln(2+\sin x) = \ln C$$

$$(y+1)(2+\sin x) = C$$

Put $x = 0$, $y = 1$

$(1+1)\cdot 2 = C \Rightarrow C = 4$

Now, $(y+1)(2+\sin x) = 4$

For, $x = \dfrac{\pi}{2}$

$$(y+1)(2+1) = 4$$

$$y + 1 = \frac{4}{3}$$

$$y = \frac{4}{3} - 1 = \frac{1}{3}$$

8. Answer (3)

$$\sin x\frac{dy}{dx} + y\cos x = 4x,\ x \in (0, \pi)$$

$$\frac{dy}{dx} + y\cot x = \frac{4x}{\sin x}$$

$\therefore$ $\text{I.F.} = e^{\int \cot x\, dx} = \sin x$

$\therefore$ Solution is given by

$$y\sin x = \int \frac{4x}{\sin x}\cdot \sin x\, dx$$

$y \cdot \sin x = 2x^2 + c$

when $x = \dfrac{\pi}{2}$, $y = 0$ $\Rightarrow$ $c = -\dfrac{\pi^2}{2}$

$\therefore$ Equation is $y\sin x = 2x^2 - \dfrac{\pi^2}{2}$

when $x = \dfrac{\pi}{6}$ then $y \cdot \dfrac{1}{2} = 2\cdot\dfrac{\pi^2}{36} - \dfrac{\pi^2}{2}$

$\therefore$ $y = -\dfrac{8\pi^2}{9}$

9. Answer (4)

$\because$ $x\dfrac{dy}{dx} + 2y = x^2$

$$\frac{dy}{dx} + \frac{2}{x}y = x$$

$\text{I.F.} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2.$

Solution of differential equation is:

$$y \cdot x^2 = \int x \cdot x^2 dx$$

$$y\cdot x^2=\frac{x^4}{4}+c$$

$\because\quad y(1)=1$

$\therefore\quad c=\frac{3}{4}$

$$y\cdot x^2=\frac{x^2}{4}+\frac{3}{4}$$

$$\therefore\quad y=\frac{x^2}{4}+\frac{3}{4x^2}$$

$$\therefore\quad y\left(\frac{1}{2}\right)=\frac{1}{16}+3=\frac{49}{16}$$

10. Answer (2)

$f(xy) = f(x).f(y)$ ...(1)

Put $x = y = 0$ in (1) to get $f(0) = 1$

Put $x = y = 1$ in (1) to get $f(1) = 0$ or $f(1) = 1$

$f(1) = 0$ is rejected else $y = 1$ in (1) gives $f(x) = 0$ imply $f(0) = 0$.

Hence, $f(0) = 1$ and $f(1) = 1$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}f(x)\left(\frac{f\left(1+\frac{h}{x}\right)-f(1)}{h}\right)$$

$$=\frac{f(x)}{x}f'(1)$$

$$\Rightarrow\quad \frac{f'(x)}{f(x)}=\frac{k}{x}\Rightarrow \ln f(x) = k\ln x + c$$

$f(1) = 1 \quad\Rightarrow \ln 1 = k\ln 1 + c \Rightarrow c = 0$

$\Rightarrow\quad \ln f(x) = k\ln x \quad\Rightarrow\quad f(x) = x^k$ but $f(0) = 1$

$\Rightarrow\quad k = 0$

$\therefore\quad \boxed{f(x)=1}$

$$\frac{dy}{dx}=f(x)=1\ \Rightarrow\ y=x+c,\ y(0)=1\ \Rightarrow c=1$$

$\Rightarrow\quad y = x + 1$

$$\therefore\quad y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)=\frac{1}{4}+1+\frac{3}{4}+1=3$$

11. Answer (1)

$$\frac{dy}{dx}=\sec^2 x(1-3y)$$

$$\Rightarrow\int\frac{dy}{(1-3y)}=\int\sec^2 x\,dx$$

$$\Rightarrow-\frac{1}{3}\ln|1-3y|=\tan x+C$$

$$y\left(\frac{\pi}{4}\right)=\frac{4}{3}$$

$$\Rightarrow-\frac{1}{3}\ln|1-4|=\tan\frac{\pi}{4}+C$$

$$\Rightarrow-\frac{1}{3}\ln 3=C+1\Rightarrow C=-1-\frac{1}{3}\ln 3$$

$$x=\frac{\pi}{4}$$

$$\Rightarrow-\frac{1}{3}\ln|1-3y|=\tan\left(-\frac{\pi}{4}\right)+C=-1+C$$

$$=-1-1-\frac{1}{3}\ln 3$$

$\Rightarrow \ln|1-3y| = 6 + \ln 3$

$$\Rightarrow\ln\left|\frac{1}{3}-y\right|=6\ \Rightarrow\ \left|\frac{1}{3}-y\right|=e^6\Rightarrow y=\frac{1}{3}\pm e^6$$

12. Answer (4)

$(x^2 - y^2)dx + 2xy\,dy = 0$

$y^2dx - 2xydy = x^2dx$

$2xydy - y^2dx = -x^2dx$

$$\frac{xd(y^2)-y^2d(x)}{x^2}=-dx$$

$$d\left(\frac{y^2}{x}\right)=-dx$$

$$\int d\left(\frac{y^2}{x}\right)=-\int dx$$

$\frac{y^2}{x}=-x+C$ passes through (1, 1) $\Rightarrow C = 2$

$y^2 = -x^2 + 2x$

$\Rightarrow\quad y^2 = -(x-1)^2 + 1$

$(x-1)^2 + y^2 = 1$, circle with centre (1, 0)

centre lies on x-axis

13. Answer (1)

$$\frac{dy}{dx}+\left(2+\frac{1}{x}\right)y=e^{-2x},\ x>0$$

$$\text{IF} = e^{\int\left(2+\frac{1}{x}\right)dx}\quad e^{2x+\ln x}$$

Complete solution is given by

$$y(x)\cdot e^{2x+\ln x}=\int e^{2x+\ln x}\cdot e^{-2x}dx+c$$

$$=\int x\,dx+c$$

$$y(x)\cdot e^{2x}\cdot x=\frac{x^2}{2}+c$$

$$y(1)=\frac{1}{2}e^{-2}\text{ gives }\frac{1}{2}e^{-2}\cdot e^{2}\cdot 1=\frac{1}{2}+c\quad\Rightarrow\quad c=0$$

$$\therefore\quad y(x)=\frac{x^2}{2}\cdot\frac{e^{-2x}}{x}$$

$$y(x)=\frac{x}{2}\cdot e^{-2x}$$

$$y'(x)=\frac{e^{-2x}}{2}(1-2x)<0\ \forall\ x\in\left(\frac{1}{2},1\right)$$

Hence, y(x) is decreasing in $\left(\frac{1}{2},1\right)$

14. Answer (3)

$$\frac{dy}{dx}=(x-y)^2\qquad\qquad ...(i)$$

Let $x - y = t$

$$1-\frac{dy}{dx}=\frac{dt}{dx}$$

$$\Rightarrow\ \frac{dy}{dx}=1-\frac{dt}{dx}$$

From equation (i)

$$\left(1-\frac{dt}{dx}\right)=(t)^2\ \Rightarrow\ 1-t^2=\frac{dt}{dx}\Rightarrow\int dx=\int\frac{dt}{1-t^2}$$

$$\Rightarrow\ -x=\frac{1}{2\times 1}\ln\left|\frac{t-1}{t+1}\right|+c$$

$$\Rightarrow\ -x=\frac{1}{2}\ln\left|\frac{x-y-1}{x-y+1}\right|+c\quad\because\ \text{given } y(1)=1$$

$$-1=\frac{1}{2}\ln\left|\frac{1-1-1}{1-1+1}\right|+c\qquad\Rightarrow\ \boxed{c=-1}$$

$$\text{So, } 2(x-1)=\ln\left|\frac{1-x+y}{1-y+x}\right|$$

15. Answer (4)

$$\frac{dy}{dx}+\frac{y}{x}=\log_e x$$

$$\text{IF} = e^{\int\frac{1}{x}dx}=x$$

Solution is $\quad yx=\int x\ln x\,dx$

$$\Rightarrow\quad xy=\ln x\cdot\frac{x^2}{2}-\int\frac{1}{x}\cdot\frac{x^2}{2}dx$$

$$\Rightarrow\quad xy=\frac{x^2}{2}\cdot\ln x-\frac{x^2}{4}+c$$

At $x = 2$,

$$2y = 2\ln 2-1+c$$

$$\ln 4-1=\ln 4-1+c$$

i.e. $\quad c = 0$

$$\Rightarrow\quad xy=\frac{x^2}{2}\ln x-\frac{x^2}{4}$$

$$\Rightarrow\quad y=\frac{x}{2}\ln x-\frac{x}{4}$$

$$\Rightarrow\quad y(e)=\frac{e}{2}-\frac{e}{4}=\frac{e}{4}$$

16. Answer (4)

$$(1+x^2)^2\frac{dy}{dx}+2x\left(1+x^2\right)y=1$$

$$\Rightarrow\ \frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{1}{(1+x^2)^2}$$

It is a linear differential equation

$$\text{I·F·}=e^{\int\frac{2x}{1+x^2}dx}=e^{\ln\left(1+x^2\right)}=1+x^2$$

$$\Rightarrow\ y\cdot\left(1+x^2\right)=\int\frac{dx}{1+x^2}+c$$

$\Rightarrow\ y\,(1+x^2) = \tan^{-1}x + c$

If $x = 0$ then $y = 0$

So, $0 = 0 + c$

$\Rightarrow\ c = 0$

$\Rightarrow\ y(1+x^2) = \tan^{-1}x$

put $x = 1$

$$2y=\frac{\pi}{4}$$

$$\Rightarrow\ 2\left(\frac{\pi}{32\sqrt{a}}\right)=\frac{\pi}{4}$$

$\Rightarrow \quad \sqrt{a} = \dfrac{1}{4}$

$\Rightarrow \quad a = \dfrac{1}{16}$

17. Answer (3)

$$\frac{dy}{dx} = \frac{2y}{x^2} \quad \Rightarrow \quad \int \frac{dy}{y} = 2\int \frac{dx}{x^2}$$

$$\Rightarrow \quad \ln|y| = -\frac{2}{x} + C \qquad \text{...(i)}$$

(i) passes through (1, 1)

$\Rightarrow \quad C = 2$

$$\Rightarrow \quad \ln|y| = -\frac{2}{x} + 2$$

$$x\ln|y| = -2 + 2x$$

$$x\ln|y| = -2\,(1-x) = 2(x-1)$$

18. Answer (3)

$$\frac{dy}{dx} + \frac{2}{x}y = x \qquad y(1) = 1 \text{ (given)}$$

$$\text{I·F} = e^{\int \frac{2}{x}dx} = x^2$$

$$y \times x^2 = \int x^3\ dx$$

$$\Rightarrow \quad yx^2 = \frac{x^4}{4} + c$$

$\because$ at $x = 1$; $y = 1$

$$\Rightarrow \quad c = \frac{3}{4}$$

$$\therefore \quad y = \frac{x^2}{4} + \frac{3}{4x^2}$$

19. Answer (3)

$\dfrac{dy}{dx} + y\sec^2 x = \sec^2 x\tan x \ \rightarrow$ This is linear differential equation

$$\text{IF} = e^{\int \sec^2 x dx} = e^{\tan x}$$

Now solution is

$$y \cdot e^{\tan x} = \int e^{\tan x}\sec^2 x \tan x dx$$

$\because$ Let $\tan x = t$

$\sec^2 x dx = dt$

$$ye^{\tan x} = \int \underset{II}{e^t}\,\underset{I}{t}\,dt$$

$$ye^{\tan x} = te^t - e^t + c$$

$$ye^{\tan x} = (\tan x - 1)e^{\tan x} + c$$

$$y = (\tan x - 1) + c \cdot e^{-\tan x}$$

Given $y(0) = 0$

$\Rightarrow \quad 0 = -1 + c$

$\Rightarrow \quad c = 1$

$$y\left(-\frac{\pi}{4}\right) = -1 - 1 + e = -2 + e$$

20. Answer (4)

$$\frac{dy}{dx} + y\tan x = 2x + x^2\tan x$$

$P = \tan x,\ Q = 2x + x^2\tan x$

$$\text{I.F.} = e^{\int \tan x\, dx} = e^{\ln|\sec x|} = |\sec x|$$

$$y(\sec x) = \int (2x + x^2\tan x)\sec x dx$$

$$= \int x^2\tan x\sec x\, dx + \int 2x\sec x\, dx$$

$$= x^2\sec x - \int 2x\sec x\, dx + \int 2x\sec x\, dx$$

$$= x^2\sec x + c$$

As $y(0) = 1$, $c = 1$

$\therefore \quad y = x^2 + \cos x$

At $x = \dfrac{\pi}{4}$, $\quad y\left(\dfrac{\pi}{4}\right) = \dfrac{\pi^2}{16} + \dfrac{1}{\sqrt{2}}$

$$y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$$

$$y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = 0$$

$$\frac{dy}{dx} = 2x - \sin x$$

$$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}},\ y'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} + \frac{1}{\sqrt{2}}$$

$$y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$$

21. Answer (3)

$$e^y + xy = e \qquad \text{...(i)}$$

Put $x = 0$ in (i)

$\Rightarrow \quad e^y = e \ \Rightarrow y = 1$

Differentiate (i) w.r.t. x

$$e^y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0 \qquad \text{...(ii)}$$

Put $y = 1$ in (ii)

$$e\frac{dy}{dx} + 0 + 1 = 0 \ \Rightarrow \frac{dy}{dx} = -\frac{1}{e}$$

Differentiate (ii) w.r. to x

$$e^y\frac{d^2y}{dx^2}+\frac{dy}{dx}.e^y\frac{dy}{dx}+x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=0 \quad ...(iii)$$

Put $y = 1$, $x = 0$, $\frac{dy}{dx}=-\frac{1}{e}$

$$e\frac{d^2y}{dx^2}+\frac{1}{e}-\frac{2}{e}=0\Rightarrow\frac{d^2y}{dx^2}=\frac{1}{e^2}$$

$$\Rightarrow \left(\frac{dy}{dx},\frac{d^2y}{dx^2}\right)\equiv\left(-\frac{1}{e},\frac{1}{e^2}\right)$$

22. Answer (3)

$$y^2dx+\left(x-\frac{1}{y}\right)dy=0$$

$$\frac{dx}{dy}+\left(\frac{1}{y^2}\right)x=\frac{1}{y^3}$$

I.F. $=e^{\int\frac{1}{y^2}dy}=e^{-\frac{1}{y}}$

its solution is

$$x.e^{-\frac{1}{y}}=\int e^{-\frac{1}{y}}\frac{1}{y^3}dy+c$$

put $-\frac{1}{y}=t\Rightarrow\frac{1}{y^2}dy=dt$

$$\Rightarrow \quad x.e^{-\frac{1}{y}}=-\int te^t dt+c=-te^t+e^t+c$$

$x.e^{-\frac{1}{y}}=e^{-\frac{1}{y}}\left(\frac{1}{y}+1\right)+c$ passes through (1, 1)

$\Rightarrow$ 1 = 2 + $ce \Rightarrow$ $c=-\frac{1}{e}$

$\Rightarrow$ $x=\left(1+\frac{1}{y}\right)-\frac{1}{e}e^{\frac{1}{y}}$ passes through (k, 2)

$$\Rightarrow \quad k=\frac{3}{2}-\frac{1}{\sqrt{e}}$$

23. Answer (1)

$y^2dx - xydy = x^3dx$

$$\Rightarrow \quad \frac{(ydx-xdy)y}{x^2}=xdx$$

$$\Rightarrow \quad -yd\left(\frac{y}{x}\right)=xdx$$

$$\Rightarrow \quad -\frac{y}{x}\cdot d\left(\frac{y}{x}\right)=dx$$

$$\Rightarrow \quad -\frac{1}{2}\left(\frac{y}{x}\right)^2=x+c_1$$

$\Rightarrow$ $2x^3 + cx^2 + y^2 = 0$

24. Answer (4)

Given $x^k + y^k = a^k$

Differentiating, $k\cdot x^{k-1}+k\cdot y^{k-1}\frac{dy}{dx}=0$

$$\therefore \quad \frac{dy}{dx}=-\left(\frac{x}{y}\right)^{k-1}$$

$$\left(\frac{dy}{dx}\right)+\left(\frac{y}{x}\right)^{1-k}=0$$

$$\therefore \quad 1-k=\frac{1}{3}$$

$$\Rightarrow \quad k=\frac{2}{3}$$

25. Answer (4)

$e^y = t$

$$e^y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\Rightarrow \quad \frac{dt}{dx}-t=e^x$$

I.F. = $e^{\int -1\,dx}=e^{-x}$

$\Rightarrow$ $t.e^{-x} = x + c$

$e^{y-x} = x + c$

$c = 1$

$\Rightarrow$ $f(1) = 1 + \ln 2$

26. Answer (1)

$\because\ \left(y^2-x\right)\dfrac{dy}{dx}=1$

$\therefore$ $y^2dy - xdy = dx$

$\Rightarrow$ $e^y \cdot y^2dy = e^ydx + xe^ydy$

$\Rightarrow$ $e^y \cdot y^2dy = d(e^y \cdot x)$

On integrating both sides we get

$$\int e^y \cdot y^2 dy = \int d\left(e^y \cdot x\right)$$

$$y^2 \cdot e^y - \int 2y \cdot e^y dy = e^y \cdot x$$

$$y^2 \cdot e^y - 2\left\{y \cdot e^y - \int e^y dy\right\} = e^y \cdot x$$

$\therefore$ $y^2 \cdot e^y - 2ye^y + 2e^y = e^y \cdot x + c$

$\therefore$ $y(0) = 1$

$\Rightarrow$ $c = e$

$\therefore$ $y^2 - 2y + 2 = x + e \cdot e^{-y}$

$\therefore$ when $y = 0$ then $x = 2 - e$

27. Answer (3)

$x^2 = 4b(y + b)$ …(i)

$$\Rightarrow\ 2x = 4b\left(\frac{dy}{dx}\right) \Rightarrow x = 2b\frac{dy}{dx}$$

$$\Rightarrow\ b = \frac{x}{2\left(\frac{dy}{dx}\right)} \quad \text{…(ii)}$$

Put b from (ii) in (i)

$$\Rightarrow\ x^2 = \frac{4\times x}{2\times\frac{dy}{dx}}\left(y+\frac{x}{2\left(\frac{dy}{dx}\right)}\right)$$

$$\Rightarrow x\frac{dy}{dx} = \frac{\left(2y\frac{dy}{dx}+x\right)\times 2}{2\left(\frac{dy}{dx}\right)} \Rightarrow x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx}+x$$

28. Answer (2)

$\dfrac{dy}{dx}=\dfrac{xy}{x^2+y^2}$, it is a homogenous differential equation.

Put $y = Vx \Rightarrow \dfrac{dy}{dx} = V + x\dfrac{dV}{dx}$

$$\Rightarrow\ V + x\frac{dV}{dx} = \frac{V}{1+V^2} \Rightarrow \int\frac{1+V^2}{V^3}dV = -\int\frac{1}{x}dx$$

$$\Rightarrow\ -\frac{1}{2v^2}+\ln V = -\ln x + c$$

$$\Rightarrow\ -\frac{x^2}{2y^2} = -\ln y + c$$

$y(1) = 1 \Rightarrow c = -\dfrac{1}{2}$

$\therefore$ Solution is given by $x^2 = y^2(1+2\ln y)$

$\Rightarrow$ $x^2 = 3e^2$

$\Rightarrow$ $x = \pm\sqrt{3}e$

29. Answer (4)

$$\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x,\ y>0$$

$$\int\frac{dy}{y+1} = \int\frac{(-\cos x)dx}{2+\sin x}$$

$$\ln|y+1| = -\ln|2+\sin x| + \ln c$$

$$\ln|(y+1)(2+\sin x)| = \ln c$$

$\because\ y(0)=1\ \Rightarrow \ln 4 = \ln c \Rightarrow c = 4$

$\therefore\ (y+1)(2+\sin x) = 4$

$\therefore\ y = \dfrac{2-\sin x}{2+\sin x}$

$\therefore\ a = y(\pi) = 1$

and $\dfrac{dy}{dx} = \dfrac{(2+\sin x)(-\cos x)-(2-\sin x).\cos x}{(2+\sin x)^2}$

$$\left.\frac{dy}{dx}\right|_{x=\pi} = 1 = b$$

30. Answer (1)

$$\frac{dy}{dx} = \frac{2xy+y^2}{2x^2}$$

Put $y = vx$

$$v + x\frac{dv}{dx} = v + \frac{v^2}{2}$$

$$\Rightarrow\ 2\frac{dv}{v^2} = \frac{dx}{x}$$

$\Rightarrow \quad \frac{-2}{v} = \ln x + c$

$\Rightarrow \quad \frac{-2x}{y} = \ln x + c$

$\downarrow(1, 2)$

$\Rightarrow$ c = –1

$\Rightarrow \quad \frac{-2x}{y} = \ln x - 1$

Hence, for $x = \frac{1}{2} \Rightarrow y = \frac{1}{1+\ln 2}$

31. Answer (4)

$$\int\left(\frac{y^2+1}{y^2}\right)dy = \int\frac{e^x dx}{e^x+1}$$

$$\Rightarrow \quad y - \frac{1}{y} = \ln\left|e^x+1\right| + c$$

$\downarrow(0, 1)$

c = –ln2

$$\Rightarrow \quad y^2 - 1 = y\ln\left(\frac{e^x+1}{2}\right)$$

$$\Rightarrow \quad y^2 = 1 + y\ln\left(\frac{e^x+1}{2}\right)$$

32. Answer (4)

$(x^3 - x^2)dy = y(2 - x)dx$

$$\Rightarrow \quad \int\frac{dy}{y} = \int\frac{2-x}{x^2(x-1)}dx \qquad \ldots(1)$$

Making partial fractions for RHS

$$\frac{2-x}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

$\therefore$ A = –1, B = –2, C = 1

$$\Rightarrow \quad \int\frac{dy}{y} = \int\left(\frac{-1}{x} - \frac{2}{x^2} + \frac{1}{x-1}\right)dx$$

$$\Rightarrow \quad \ln y = -\ln x + \frac{2}{x} + \ln(x-1) + c$$

as y(2) = e $\Rightarrow$ c = ln2

$$\Rightarrow \quad y(4) = \frac{3}{2}\sqrt{e}$$

33. Answer (3)

$$\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x)$$

$$\text{I.F} = e^{-\int\frac{1}{x}dx} = \frac{1}{x}$$

$$\Rightarrow \quad \int d\left(\frac{y}{x}\right) = \int(x\cos x + \sin x)dx$$

$$\Rightarrow \quad \frac{y}{x} = x\sin x + c$$

Also y(π) = π $\Rightarrow$ c = 1

$$y = x^2\sin x + x \ \Rightarrow \ y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$$

$y' = 2x\sin x + x^2\cos x + 1$

$$y'' = 2\sin x - x^2\sin x \ \Rightarrow \ y''\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4}$$

$$\therefore \quad y\left(\frac{\pi}{2}\right) + y''\left(\frac{\pi}{2}\right) = 2 + \frac{\pi}{2}$$

34. Answer (1)

Let y + 3x = t

$$\Rightarrow \quad \frac{dy}{dx} + 3 = \frac{dt}{dx}$$

Then $\frac{dt}{dx} = \frac{t}{\ln t}$

$$\Rightarrow \quad \frac{\ln t}{t}dt = dx$$

$$\Rightarrow \quad \frac{(\ln t)^2}{2} = x - C$$

$$\Rightarrow \quad x - \frac{1}{2}(\ln(y+3x))^2 = C$$

35. Answer (1)

$$\frac{5+e^x}{2+y} \cdot \frac{dy}{dx} + e^x = 0$$

$$\int\frac{dy}{2+y} = -\int\frac{e^x}{5+e^x}dx$$

$\ln|2 + y| + \ln|5 + e^x| = \ln C$

$\because$ y(0) = 1

$\Rightarrow$ ln C = ln 18

$\therefore$ $|(2 + y) \cdot (5 + e^x)| = 18$

When x = ln 13 then $|(2 + y)\cdot 18| = 18$

2 + y = ±1

$\therefore$ y = –1, –3

$\therefore$ y(ln 13) = –1

36. Answer (2)

$$\frac{dy}{dx}+2y\tan x=2\sin x$$

$$\text{I.F.}=e^{\int 2\tan x\,dx}=\sec^2 x$$

$$\Rightarrow\quad y.\sec^2 x=\int 2\sin x.\sec^2 x dx+c$$

$\Rightarrow$ ysec^{2}x = 2secx + c

When $x=\frac{\pi}{3}$, $y=0$; then $c=-4$

$$\Rightarrow\quad y\sec^2 x=2\sec x-4\Rightarrow y=\frac{2\sec x-4}{\sec^2 x}$$

$$\Rightarrow\quad y\left(\frac{\pi}{4}\right)=\sqrt{2}-2$$

37. Answer (1)

$$\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$$

$$\sqrt{1+x^2}\cdot\sqrt{1+y^2}=-xy\frac{dy}{dx}$$

$$\int\frac{\sqrt{1+x^2}}{x}dx=-\int\frac{y}{\sqrt{1+y^2}}dy$$

Let I = $\int\frac{\sqrt{1+x^1}}{x}dx$ Let $x=\tan\theta\Rightarrow dx=\sec^2\theta d\theta$

$$=\int\frac{\sec^3\theta d\theta}{\tan\theta}=\int\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cdot\cos^2\theta}d\theta$$

$$=\int(\tan\theta\cdot\sec\theta+\text{cosec}\theta)d\theta$$

$$=\sec\theta+\ln|\text{cosec}\theta-\cot\theta|+C$$

$$=\sqrt{1+x^2}+\ln\left|\frac{\sqrt{1+x^2}-1}{x}\right|+C$$

$$\therefore\quad\sqrt{1+x^2}+\ln\left|\frac{\sqrt{1+x^2}-1}{x}\right|=-\sqrt{1+y^2}+C$$

$$\therefore\quad\sqrt{1+y^2}+\sqrt{1+x^2}=\ln\left|\frac{x}{\sqrt{1+x^2}-1}\right|+$$

$$C=\frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right)+C$$

38. Answer (2)

$$\because y=\left(\frac{2}{\pi}x-1\right)\text{cosec}\,x$$

$$\frac{dy}{dx}=\frac{2}{\pi}\text{cosec}\,x-\left(\frac{2}{\pi}x-1\right)\text{cosec}\,x\cdot\cot x$$

$$=\text{cosec}\,x\left[\frac{2}{\pi}-\left(\frac{2}{\pi}x-1\right)\cot x\right]$$

$$\therefore\frac{dy}{dx}-\frac{2}{\pi}\text{cosec}\,x=-yp(x)$$

$$\Rightarrow\quad yp(x)=\left(\frac{2}{\pi}x-1\right)\cot x\cdot\text{cosec}\,\tilde{x}$$

$\Rightarrow$ p(x) = cotx

39. Answer (3.00)

$$\because\quad\frac{(1+x)dy-(y-3)dx}{(1+x)^2}=dx$$

$$\Rightarrow\quad d\left(\frac{y-3}{1+x}\right)=dx$$

$$\Rightarrow\quad\frac{y-3}{1+x}=x+c$$

When x = 2, y = 0 $\Rightarrow$ c = –3

When x = 3; $\frac{y-3}{4}=0$ $\Rightarrow$ y = 3

40. Answer (4)

$$\because\quad\frac{dP}{dt}=\frac{1}{2}(P-900)$$

$$\Rightarrow\quad\frac{dP}{P-900}=\frac{1}{2}dt\Rightarrow\ln|P-900|=\frac{1}{2}t+c$$

When t = 0, P = 850 $\Rightarrow$ c = ln50

When P = 0, t = 2(ln900 – ln50) = 2ln18

41. Answer (4)

$$\because\quad\begin{vmatrix}f(x) & f'(x)\\ f'(x) & f''(x)\end{vmatrix}=0,$$

$$\Rightarrow\quad\frac{f(x)\cdot f''(x)-(f'(x))^2}{(f(x))^2}=0\quad\because f(x)\neq 0.$$

$$\therefore\quad\frac{d}{dx}\left(\frac{f'(x)}{f(x)}\right)=0.$$

$$\Rightarrow\quad\frac{f'(x)}{f(x)}=c,\qquad\because f(0)=1\text{ and }f'(0)=2$$

$\therefore$ c = 2

$\therefore$ f′(x) = 2f(x)

$$\Rightarrow\quad\frac{f'(x)}{f(x)}=2$$

$\Rightarrow$ ln|f(x)| = 2x + d.

$\Rightarrow$ |f(x)| = e^{2x}. $\quad\because$ f(0) = 1.

$\therefore$ f(1) = e^2

$\therefore$ f(1) $\in$ (6, 9)

42. Answer (1)

$$\frac{dy}{dx}=\frac{x^2-4x+y+8}{x-2}=\frac{(x-2)^2+(y+4)}{(x-2)}$$

Put x – 2 = t

$\Rightarrow$ dx = dt

$$\Rightarrow \frac{dy}{dt}=\frac{t^2+y+4}{t}$$

$$\Rightarrow \frac{dy}{dt}-\frac{y}{t}=t+\frac{4}{t}$$

$$\text{I.F} = e^{-\int\frac{1}{t}dt}=\frac{1}{t}$$

$$\Rightarrow \frac{y}{t}=t-\frac{4}{t}+C$$

$y = (x-2)^2 - 4 + C(x-2)$

$\downarrow$ (0, 0)

C = 0

$y = (x-2)^2 - 4$ also passes through (5, 5)

43. Answer (01)

$\because$ $(2xy^2 - y)dx + xdy = 0$

$$\Rightarrow 2xdx = \frac{ydx-xdy}{y^2}$$

$$\Rightarrow 2xdx = d\left(\frac{x}{y}\right)$$

On integrating both sides we get

$$x^2=\frac{x}{y}+c \qquad ...(1)$$

The point of intersection of lines 2x – 3y = 1 and 3x + 2y = 8 is (2, 1)

$\because$ Curve (1) passes through (2, 1) then c = 2

$$\therefore\ y(x)=\frac{x}{x^2-2}$$

$$\therefore\ y(1)=\frac{1}{1-2}=-1$$

$$\therefore\ |y(1)|=1$$

44. Answer (2)

At t = 0 $\quad B_0 = 1000$

$$\frac{dB}{dt}\propto B$$

$$\Rightarrow \int_{B_0}^{1\cdot 2B_0}\frac{dB}{B}=\int_0^2 kt \qquad \text{[Given]}$$

$$\ln\left(\frac{1\cdot 2B_0}{B_0}\right)=2k$$

$$\Rightarrow k=\frac{1}{2}\ln(1\cdot 2)$$

To find time when B = 2000

$$\Rightarrow \int_{B_0}^{2B_0}\frac{dB}{B}=\frac{1}{2}\ln(1\cdot 2)\int_0^t dt$$

$$\ln 2=\frac{1}{2}\ln(1\cdot 2)t$$

$$\Rightarrow t=\frac{\ln 4}{\ln\left(\frac{6}{5}\right)}\text{hrs.}$$

$\therefore$ R = ln = 4

$$\text{Thus}\left(\frac{K}{\ln}\right)^2=2^2=4$$

45. Answer (1)

$$e^{\sin y}\cdot\cos x\frac{dy}{dx}+e^{\sin y}\cdot\cos x=\cos x$$

Let $e^{\sin y} = Y$

$$\Rightarrow \frac{dY}{dx}+Y\cos x=\cos x$$

$$\Rightarrow I\cdot F=e^{\sin x}$$

$$\Rightarrow Y\cdot e^{\sin x}=\int e^{\sin x}\cdot\cos dx+c$$

$$\Rightarrow e^{\sin y}\cdot e^{\sin x}=e^{\sin x}+c$$

When x = 0, y = 0 then c = 0

$$\Rightarrow e^{\sin x+\sin y}=e^{\sin x}\Rightarrow \sin y=0$$

$\Rightarrow$ y = 0

$\Rightarrow$ y(x) = 0

$$\text{hence, } 1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)=1$$

46. Answer (4)

$$\frac{dy}{dx}=y^2+\frac{y}{x}$$

$$\frac{dy}{dx}-\frac{y}{x}=y^2$$

$$\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{x}\times\frac{1}{y}=1$$

$$\text{Let }\frac{1}{y}=z$$

$$\frac{-1}{y^2}\frac{dy}{dx}=\frac{dz}{dx}$$

$$\frac{-dz}{dx}-\frac{1}{x}z=1$$

$$\frac{dz}{dx}+\frac{1}{x}z=-1$$

$$\text{IF} = e^{\int\frac{1}{x}dx}=e^{\ell n x}=x$$

$$z\cdot x=\int -1\cdot x\,dx$$

$$z\cdot x=\frac{-x^2}{2}+c$$

$$\frac{x}{y}=\frac{-x^2}{2}+c \qquad \ldots(i)$$

Putting $x = -2$ in $x + 2y = 4$, we get $y = 3$

Put $(-2, 3)$ in (i)

$$\Rightarrow \quad c=\frac{4}{3}$$

$$(i) \Rightarrow \frac{x}{y}=\frac{-x^2}{2}+\frac{4}{3} \qquad \ldots(ii)$$

Put $x = 3$ in (ii)

$$\frac{3}{y}=\frac{-9}{2}+\frac{4}{3}$$

$$y=\frac{-18}{19}$$

47. Answer (2)

$$\frac{dy}{dx}+2\tan x\cdot y=\sin x$$

$$\text{I. F.} = e^{\int 2\tan x\,dx}=\sec^2 x$$

$$\Rightarrow \quad y\cdot\sec^2 x = \int \sin x\cdot\sec^2 x\,dx$$

$$\Rightarrow \quad y\sec^2 x = \sec x + C$$

$$\because \quad y\left(\frac{\pi}{3}\right)=0$$

$$\Rightarrow \quad C = -2$$

$$\Rightarrow \quad y = \cos x - 2\cos^2 x = \frac{1}{8}-\left(\sqrt{2}\cos x-\frac{1}{2\sqrt{2}}\right)^2$$

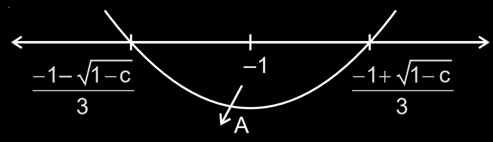
Maximum value of f(x) is $\frac{1}{8}$.

48. Answer (02)

$$\frac{dy}{dx}=2x+2 \Rightarrow y=x^2+2x+c \qquad \ldots(1)$$

Represents parabola with vertex at $x = -1$

Given area $\frac{4\sqrt{8}}{3}=A$



$\therefore$ Shifting origin to $(-1, 0)$ won't change the area

Hence equation (1) becomes $y = x^2 - 1 + c$

$$\therefore \quad A=2\sum_{0}^{\sqrt{1-c}}(x^2+(c-1)dx=\frac{4\sqrt{8}}{3}$$

$$\Rightarrow \quad \frac{x^3}{3}+(c-1)x\Big|_0^{\sqrt{1-c}}=\frac{2\sqrt{8}}{3}$$

$$\Rightarrow \quad \sqrt{1-c}\,(1-c+3(-3)=2\sqrt{p}$$

$$\Rightarrow \quad \sqrt{1-c}\,(c-1)=\sqrt{8}\Rightarrow c=-1$$

Hence $y = x^2 + 2x - 1 \Rightarrow y(1) = 2$

49. Answer (4)

$$2\frac{dy}{dx}=\frac{y^2-x^2}{xy}=\frac{y}{x}-\frac{1}{\left(\frac{y}{x}\right)}$$

Let $\frac{y}{x}=u$

$$2\left(u+x\frac{du}{dx}\right)=u-\frac{1}{u}$$

$$\frac{2u}{u^2-1}du=\frac{-dx}{x}$$

$$\ln\left(u^2+1\right)+\ln_x=\ln C$$

$$x^2 + y^2 = Cx$$

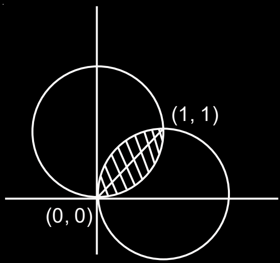
Curve passes through (1, 1) $\Rightarrow$ C = 2

$$x^2 + y^2 = 2x \qquad \ldots(i)$$

Similarly second curve can be obtained by interchanging x and y

$$x^2 + y^2 = 2y \qquad \ldots(ii)$$

Required region is $= 2\left(\frac{\pi}{4}-\frac{1}{2}\right)=\frac{\pi}{2}-1$



50. Answer (2)

$$IF = e^{\int \tan x\,dx} = e^{\ln(\sec x)} = \sec x$$

$$y \sec x = \int (\sin x)\sec x\ dx = \ln(\sec x) + C$$

$$y\ (0) = 0 \Rightarrow C = 0$$

$$\therefore\quad y = \cos x \ln|\sec x|$$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\ln\left(\sqrt{2}\right) = \frac{1}{2\sqrt{2}}\ln 2$$

51. Answer (4)

$$\frac{dy}{dx} = xy - 1 + x - y$$

$$\Rightarrow\quad \frac{dy}{dx} = (x-1)(y+1)$$

$$\Rightarrow\quad \frac{dy}{y+1} = (x-1)dx$$

$$\Rightarrow\quad \ln(y+1) = \frac{x^2}{2} - x + c$$

$y(0) = 0 \quad \Rightarrow c = 0$

Hence $y(x) = e^{\left(\frac{x^2}{2}-x\right)} - 1$

$$y(1) = e^{\frac{-1}{2}} - 1$$

52. Answer (1)

$(3\sin x + \cos x + 3)\ (\cos x\,dy - y \sin x\,dx) = dx$

$$\Rightarrow\quad d(\cos x \cdot y) = \frac{dx}{3\sin x + \cos x + 3}$$

$$\Rightarrow\quad d(y\cdot \cos x) = \frac{\sec^2 \frac{x}{2}}{4 + 6\tan\frac{x}{2} + 2\tan^2\frac{x}{2}}dx$$

$$\Rightarrow\quad y\cos x = \int \frac{dt}{t^2+3t+2} \quad \text{where } t = \tan\frac{x}{2}$$

$$y\cos x = \ln\left|\frac{\tan\frac{x}{2}+1}{\tan\frac{x}{2}+2}\right| + C$$

$\because\ y(0) = 0 \Rightarrow C = \ln 2$

$$\text{Then } y\left(\frac{\pi}{3}\right) = 2\left[\ln\left(\frac{\sqrt{3}+1}{2\sqrt{3}+1}\right) + \ln 2\right]$$

$$= 2\ln\left(\frac{2\sqrt{3}+2}{2\sqrt{3}+1}\right) = 2\ln\left(\frac{10+2\sqrt{3}}{11}\right)$$

53. Answer (4)

$$\frac{dy}{dx} - \frac{y}{2x} = \frac{x}{1+x^{3/4}}$$

$$\text{I.F.} = e^{-\int \frac{1}{2x}dx} = \frac{1}{\sqrt{x}}$$

$$\Rightarrow\quad y\cdot\frac{1}{\sqrt{x}} = \int \frac{\sqrt{x}\,dx}{1+x^{3/4}} + C$$

$$\Rightarrow\quad \frac{y}{\sqrt{x}} = \int \frac{4t^5}{1+t^3}dt + C \quad \text{where } t^4 = x$$

$$\Rightarrow\quad \frac{y}{\sqrt{x}} = \frac{4}{3}\left[1 + x^{3/4} - \ln(1+x^{3/4})\right] + C$$

Put x = 1, y = $1 - \frac{4}{3}\ln 2;\ 1 - \frac{4}{3}\ln 2 = \frac{4}{3}[2 - \ln 2] + C$

$$\Rightarrow\quad C = -\frac{5}{3}$$

Put x = 16, $\frac{y}{4} = \frac{4}{3}[1 + 8 - \ln 9] - \frac{5}{3}$

$$\Rightarrow\quad y = 4\left[\frac{31}{3} - \frac{8}{3}\ln 3\right]$$

54. Answer (4)

$y^2 = 4ax + 4a^2$ ...(i)

Differentiate both sides we get

$2yy' = 4a \quad \Rightarrow a = \frac{yy'}{2}$ ...(ii)

By (i) and (ii) we get

$$y^2 = \frac{4\cdot yy'}{2}\cdot x + \frac{4y^2(y')^2}{4}$$

$$\Rightarrow\quad y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$$

55. Answer (2)

$$\frac{dy}{dx} = (y+1)^2 e^{x^2/2} - x(y+1) \quad ...(A)$$

$$\frac{dy}{dx} + x(y+1) = (y+1)^2 e^{x^2/2}$$

$$\frac{1}{(y+1)^2}\frac{dy}{dx} + x \times \frac{1}{y+1} = e^{x^2/2} \quad ...(i)$$

Let $\frac{1}{y+1} = z \Rightarrow \frac{-1}{(y+1)^2}\frac{dy}{dx} = \frac{dz}{dx}$

$$-\frac{dz}{dx} + xz = e^{x^2/2}$$

$$\frac{dz}{dx} - x \cdot z = -e^{x^2/2} \quad ...(ii)$$

$$\text{IF} = e^{\int -x dx} = e^{\frac{-x^2}{2}}$$

$$z.e^{\frac{-x^2}{2}} = \int -e^{\frac{x^2}{2}} \times e^{\frac{-x^2}{2}} dx = \int -1 dx = -x + C$$

$$\frac{1}{(y+1)e^{x^2/2}} = -x + C \quad ...(iii)$$

$$y(2) = 0 \Rightarrow \frac{1}{1 \times e^2} = -2 + C$$

$$C = 2 + \frac{1}{e^2}$$

$$(iii) \Rightarrow \frac{1}{(y+1)e^{x^2/2}} = -x + 2 + \frac{1}{e^2}$$

at x = 1,

$$\frac{1}{(y+1)e^{\frac{1}{2}}} = 1 + \frac{1}{e^2} = \frac{e^2+1}{e^2}$$

$$(y+1)e^{\frac{1}{2}} = \frac{e^2}{e^2+1}$$

$$y + 1 = \frac{e^{3/2}}{e^2+1}$$

$$(A) \Rightarrow \frac{dy}{dx} = \frac{e^{3/2}}{e^2+1}\left(\frac{e^{3/2}}{e^2+1}e^{1/2} - 1\right)$$

$$= \frac{e^{3/2}}{e^2+1}\left(\frac{e^2}{e^2+1} - 1\right)$$

$$= \frac{-e^{3/2}}{\left(e^2+1\right)^2}$$

56. Answer (4)

$$x dy - y dx = \sqrt{x^2 - y^2}\, dx$$

$$\Rightarrow \frac{x dy - y dx}{x^2} = \frac{1}{x}\sqrt{1 - \left(\frac{y}{x}\right)^2}\, dx$$

$$\Rightarrow \frac{d\left(\frac{y}{x}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \frac{dx}{x}$$

$$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ln x + C$$

$\Rightarrow$ y(1) = 0 $\Rightarrow$ C = 0

Here y = x·sin(lnx)

A = The required area bounded is

$$= \int_1^{e^{\pi}} x \cdot \sin(\ln x)\, dx$$

Put $x = e^t$

$\Rightarrow$ $dx = e^t\, dt$

$$A = \int_0^{\pi} e^t \cdot \sin t \cdot e^t dt$$

$$A = \int_0^{\pi} e^{2t} \cdot \sin t\, dt$$

$$= \frac{e^{2t}}{1^2 + 2^2} \cdot (2\sin t - \cos t)\Big|_0^{\pi}$$

$$= \frac{1}{5}\left(e^{2\pi}(0+1) - 1\,(0-1)\right)$$

$$= \frac{1}{5} \cdot e^{2\pi} + \frac{1}{5} = \alpha \cdot e^{2\pi} + \beta$$

$$\alpha = -\frac{1}{5} \cdot \beta = \frac{1}{5}$$

$10(\alpha + \beta) = 4$

57. Answer (3)

$$\tan\left(\frac{y}{x}\right)\frac{dy}{dx} = \left(\frac{y}{x}\right)\tan\left(\frac{y}{x}\right) - 1$$

Put $y = ux$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$\Rightarrow \quad u + x\frac{du}{dx} = \frac{u\tan u}{\tan u} - \cot u$$

$$\Rightarrow \quad x\frac{du}{dx} = -\cot u$$

$$\Rightarrow \quad \frac{dx}{x} + \tan u\, du = 0$$

$$\Rightarrow \quad \ln x + \ln\left|\sec\left|\frac{y}{x}\right|\right| = c$$

Given $y\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$$\ln\left(\frac{1}{2}\right) + \ln\left|\sec\frac{\pi}{3}\right| = c$$

$\Rightarrow \quad c = 0$

$\therefore \quad y = x\cos^{-1}x$

$$\text{Area} = \int_0^{1/\sqrt{2}} x\cos^{-1}x\,dx$$

Put $x = \cos\theta$

$$\therefore \quad \int_{\pi/2}^{\pi/4} \cos\theta\cdot\theta\cdot(-\sin\theta)\,d\theta = A$$

$$\therefore \quad A = \int_{\pi/4}^{\pi/2}\left(\frac{\theta}{2}\right)\sin 2\theta\, d\theta$$

$$A = -\frac{\theta}{2}\frac{\cos 2\theta}{2}\Bigg|_{\pi/4}^{\pi/2} + \int_{\pi/4}^{\pi/2}\frac{1}{2}\frac{\cos 2\theta}{2}\,d\theta$$

$$= \left(\frac{-\pi}{4}\left(\frac{-1}{2}\right)\right) - \left(\frac{-\pi}{8}(0)\right) + \frac{1}{4}\frac{\sin 2\theta}{2}\Bigg|_{\pi/4}^{\pi/2}$$

$$= \frac{\pi}{8} - 0 - \frac{1}{8} = \frac{\pi - 1}{8}$$

58. Answer (2)

$$e^x\sqrt{1-y^2}\,dx = -\frac{y}{x}dy$$

$$\Rightarrow \quad \int xe^x dx = \int\frac{-y}{\sqrt{1-y^2}}dy$$

$$\Rightarrow \quad xe^x - e^x = \sqrt{1-y^2} + c$$

$y(1) = -1$

$\Rightarrow \quad 0 = 0 + c \quad \Rightarrow \quad c = 0$

$$\therefore \quad xe^x - e^x = \sqrt{1-y^2}$$

for $y(3)$ put $x = 3$

$$3e^3 - e^3 = \sqrt{1-y^2}$$

$4e^6 = 1 - y^2$

$\Rightarrow \quad (y(3))^2 = 1 - 4e^6$

59. Answer (2)

$$|A| = \frac{-y}{x} - \sin x(-2) + 1(2)$$

$$= 2 + 2\sin x - \frac{y}{x}$$

$$\frac{dy}{dx} + \frac{y}{x} = 2 + 2\sin x$$

I.f $= e^{\ln x} = x$

$$\int d(xy) = \int 2x(1+\sin x)\,dx$$

$\Rightarrow \quad xy = x^2 - 2x\cos x + \int 2\cos x\,dx$

$\Rightarrow \quad xy = x^2 - 2x\cos x + 2\sin x + c$

$\therefore \quad y(\pi) = \pi + 2$

$\Rightarrow \quad \pi(\pi + 2) = \pi^2 - 2\pi(-1) + 0 + c$

$\Rightarrow \quad c = 0$

For $y\left(\pi/2\right)$

$$\frac{\pi}{2}y = \frac{\pi^2}{4} - \frac{2\pi}{2}(0) + 2$$

$$\Rightarrow \quad y\left(\pi/2\right) = \frac{\pi}{2} + \frac{4}{\pi}$$

60. Answer (2)

$$\int dy = \int\frac{\cos\frac{1}{2}\cos^{-1}(e^{-x})}{\sqrt{e^{2x}-1}}dx$$

Let $\frac{1}{2}\cos^{-1}(e^{-x}) = \theta$

$e^{-x} = \cos 2\theta$

$x = \ln\sec 2\theta$

$dx = 2\tan 2\theta\, d\theta$

$y = \int 2\cos\theta\, d\theta = 2\sin\theta + C = \sqrt{2}\sqrt{1-\cos 2\theta} + C$

$= \sqrt{2}\sqrt{1-e^{-x}} + C$

$y(0) = -1 \Rightarrow C = -1$

$y = \sqrt{2(1-e^{-x})} - 1$

$y = 0 \Rightarrow e^{\alpha} = 2$

61. Answer (3)

$\operatorname{cosec}^2 x\, dy = (\operatorname{cosec}^2 x - 2)\, dx + (\cos 2x \operatorname{cosec}^2 x) y\, dx$

$\frac{dy}{dx} = \left(1 - 2\sin^2 x\right) + \cos 2x \cdot y$

$\frac{dy}{dx} - \cos 2x \cdot y = \cos 2x$

If $e^{-\int \cos 2x\, dx} = e^{\frac{-\sin 2x}{2}}$

$y \cdot e^{\frac{-\sin 2x}{2}} = \int \cos 2x \cdot e^{\frac{-\sin 2x}{2}} dx$

$\Rightarrow \quad y \cdot e^{\frac{-\sin 2x}{2}} = -e^{\frac{-\sin 2x}{2}} + c \Rightarrow y = -1 + ce^{\frac{\sin 2x}{2}}$

$y\left(\frac{\pi}{4}\right) = 0 \Rightarrow c = e^{\frac{-1}{2}}$

$\Rightarrow \quad y = -1 + e^{\frac{-1}{2}(1-\sin 2x)}$

$\Rightarrow \quad y(0) = -1 + e^{\frac{-1}{2}}$

$\Rightarrow \quad (y(0) + 1)^2 = e^{-1}$

62. Answer (4)

Let $y + 1 = Y$ and $x + 2 = X$

$dy = dY \qquad dx = dX$

$\left(Xe^{\frac{X}{X}} + Y\right) dX = X dY$

$\Rightarrow \quad \frac{X dY - Y dX}{X^2} = \frac{e^{\frac{Y}{X}}}{X} dX$

$\Rightarrow \quad e^{-\frac{Y}{X}} d\left(\frac{Y}{X}\right) = \frac{dX}{X}$

$\Rightarrow \quad -e^{-\frac{Y}{X}} = \ln|X| + c$

$\Rightarrow \quad -e^{-\left(\frac{y+1}{x+2}\right)} = \ln|x+2| + c$

$\because$ (1, 1) satisfy this equation

So, $c = -e^{-\frac{2}{3}} - \ln 3$

Now $y = -1 - (x+2) \ln\left(\ln\left(\left|\frac{3}{x+2}\right|\right) + e^{-\frac{2}{3}}\right)$

Domain :

$\ln\left|\frac{3}{x+2}\right| > e^{-e^{-\frac{2}{3}}}$

$\Rightarrow \quad \frac{3}{|x+2|} > \bar{e}^{e^{-\frac{2}{3}}}$

$\Rightarrow \quad |x+2| < 3e^{e^{-\frac{2}{3}}}$

$\Rightarrow \quad -3e^{e^{-\frac{2}{3}}} - 2 < x < 3e^{e^{-\frac{2}{3}}} - 2$

So $\alpha + \beta = -4$

$\Rightarrow \quad |\alpha + \beta| = 4$

63. Answer (2)

$\frac{dy}{dx} = 1 + xe^{y-x}$

$\Rightarrow \quad e^{-y}\frac{dy}{dx} = e^{-y} + xe^{-x}$

Let $e^{-y} = t \Rightarrow -e^{-y}\frac{dy}{dx} = \frac{dt}{dx}$.

$-\frac{dt}{dx} = t + xe^{-x}$

$\frac{dt}{dx} + t = -xe^{-x}$

$\therefore$ Integrating function $= e^{\int 1 dx} = e^x$

$\therefore$ Solution is : $t \cdot e^x = \int -xe^{-x} \cdot e^x dx$

$t \cdot e^x = -\frac{x^2}{2} + c$

$e^{x-y} = -\frac{x^2}{2} + c$

$\because$ $y(0) = 0 \Rightarrow c = 1$

$\therefore$ $e^{x-y} = 1 - \frac{x^2}{2} \Rightarrow y(x) = x - \ln\left(1 - \frac{x^2}{2}\right)$

Now $g'(x) = 1 - \frac{1}{1-\frac{x^2}{2}}(0-x)$

$$= 1 + \frac{x}{1-\frac{x^2}{2}} = \frac{2-x^2+2x}{2-x^2}$$

$$= \frac{(x-1)^2-3}{x^2-2}$$

$\therefore$ Critical points are $-\sqrt{2}, \sqrt{2}, 1-\sqrt{3}, 1+\sqrt{3}$

$+ \quad - \quad + \quad - \quad +$

$-\sqrt{2} \quad 1-\sqrt{3} \quad \sqrt{2} \quad 1+\sqrt{3}$

$\because$ $x \in (-\sqrt{2}, \sqrt{2})$ then $x = 1-\sqrt{3}$ is point of local minima

$\therefore$ Minimum value of $y(x) = y(1-\sqrt{3})$

$$= (1-\sqrt{3}) - \log_e\left(1-\frac{(1-\sqrt{3})^2}{2}\right)$$

$$= (1-\sqrt{3}) - \log_e(\sqrt{3}-1)$$

64. Answer (4)

Let $e^y = y$, $\frac{dy}{dx} - 2y\sin x = -\sin x \cdot \cos^2 x$

I.F. $= e^{2\cos x}$

$\Rightarrow$ $y \cdot e^{2\cos x} = -\int e^{2\cos x}(\sin x \cdot \cos^2 x)dx + C$

$\Rightarrow$ $e^{2\cos x} \cdot e^y = \frac{1}{4}e^{2\cos x}(2\cos^2 x - 2\cos x + 1) + C$

$\because$ $y\left(\frac{\pi}{2}\right) = 0 \quad \Rightarrow C = \frac{3}{4}$

Now, $y(0) = \ln\left(\frac{1}{4} + \frac{3}{4}e^{-2}\right)$

$\Rightarrow$ $\alpha = \frac{1}{4}$ and $\beta = \frac{3}{4}$

65. Answer (4)

$$\frac{x\,dy - y\,dx}{x^2} = x\cos x\,dx$$

$$\int d\left(\frac{4}{x}\right) = \int x\cos x\,dx$$

$\Rightarrow$ $\frac{y}{x} = x\sin x + \cos x + c$

$\downarrow y(\pi) = 0 \Rightarrow c = 1$

$\Rightarrow$ $y = x^2\sin x + x\cos x + x$

$$y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$$

66. Answer (1)

$\because$ $\frac{dy}{dx} = \frac{2y}{x\ln x} \Rightarrow \frac{dy}{y} = \frac{2dx}{x\ln x}$

$\Rightarrow$ $\ln y = 2\ln(\ln x) + C$

$\because$ $(2, (\ln 2)^2$ lies on the curve, so

$2\ln(\ln 2) = 2\ln(\ln 2) + C \Rightarrow C = 0$

Now, at $x = e$

$\ln y = 0 \Rightarrow y = 1 = f(e)$

67. Answer (2)

$\because$ $\ln\left(\frac{dy}{dx}\right) = 3x + 4y$

$\Rightarrow$ $\frac{dy}{dx} = e^{3x+4y}$

$\Rightarrow$ $e^{-4y}dy = e^{3x}dx$

$\Rightarrow$ $\int e^{-4y}dy = \int e^{3x}dx$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$$

$\because$ $y(0) = 0$

$\Rightarrow$ $C = -\frac{7}{12}$

$\therefore$ $e^{-4y} = \frac{7}{3} + \frac{e^{3x}}{3}$

$$e^{4y} = \frac{3}{7-4e^{3x}}$$

$$y = \frac{1}{4}\ln\left(\frac{3}{7-4e^{3x}}\right)$$

$\therefore$ $y = \left(-\frac{2}{3}\ln 2\right) = \frac{1}{4}\ln\left(\frac{3}{6}\right) = -\frac{1}{4}\ln 2$

$\therefore$ $\alpha = -\frac{1}{4}$

68. Answer (16)

$$e^x \cdot F(x) = \int_3^x (3t^2 + 2t + 4F'(t))dt, F(3) = 0$$

Differentiating w.r.t. x

$e^xF(x) + e^xF'(x) = 3x^2 + 2x + 4F'(x)$

$$\Rightarrow \quad F'(x) + \left(\frac{e^x}{e^x - 4}\right)F(x) = \frac{3x^2 + 2x}{e^x - 4} \qquad \ldots(1)$$

I.F. = $e^x - 4$

$$F(x).(e^x - 4) = \int (3x^2 + 2x)dx + c$$

$$\Rightarrow \quad F(x) = \frac{x^3 + x^2 + c}{e^x - 4} \qquad (\because\ F(3) = 0 \Rightarrow c = -36)$$

$$\Rightarrow \quad F(x) = \frac{x^3 + x^2 - 36}{e^x - 4} \Rightarrow F(4) = \frac{44}{e^4 - 4}$$

From (1)

$$F'(4) + \left(\frac{e^4}{e^4 - 4}\right)F(4) = \frac{56}{e^4 - 4}$$

$$\Rightarrow \quad F'(4) = \frac{56}{e^4 - 4} - \frac{44e^4}{(e^4 - 4)^2}$$

$$\Rightarrow \quad F'(4) = \frac{12e^4 - 224}{(e^4 - 4)^2}$$

Clearly $\alpha = 12$, $\beta = 4$

69. Answer (2)

$$\sec y \frac{dy}{dx} = 2\sin x \cdot \cos y$$

$\Rightarrow \quad \sec^2 y dy = 2\sin x dx$

$\Rightarrow \quad \tan y = -2\cos x + c$

put $x = 0$ and $y = 0$

$\Rightarrow \quad c = 2$

put $x = \frac{\pi}{2}$ then $y = \tan^{-1}2$

Now, $\frac{dy}{dx} = 2\sin x \cdot \cos^2 y$

put $x = \frac{\pi}{2}$, $y = \tan^{-1}2$

$$\Rightarrow \quad y'\left(\frac{\pi}{2}\right) = 2\left(\frac{1}{5}\right)$$

70. Answer (1)

$(x - x^3)dy = y(1 + x^2)dx - 3x^4dx$

$$\therefore \quad \frac{dy}{dx} + y\frac{1 + x^2}{x(x^2 - 1)} = \frac{3x^3}{x^2 - 1}$$

$$\therefore \quad \text{I.F.} = e^{\int \frac{1+x^2}{x(x^2-1)}dx} = e^{\int\left(\frac{1}{x-1} + \frac{1}{x+1} - \frac{1}{x}\right)dx}$$

$$= e^{\ln\left(\frac{x^2-1}{x}\right)} = \frac{x^2 - 1}{x}$$

$$\therefore \quad \text{Solution is } y \cdot \left(\frac{x^2 - 1}{x}\right) = \int \frac{3x^3}{x^2 - 1} \cdot \frac{x^2 - 1}{x}dx$$

$$y\left(\frac{x^2 - 1}{x}\right) = x^3 + c$$

$\because \quad y(3) = 3$ then $c = -1$

$$\therefore \quad y(x) = \frac{(x^3 - 19) \cdot x}{x^2 - 1}$$

$$\therefore \quad y(4) = \frac{45 \times 4}{15} = 12$$

71. Answer (2)

$e^{-y}\,dy = e^{\alpha x}\,dx$

$$\Rightarrow \quad -e^{-y} = \frac{1}{\alpha}e^{\alpha x} + c$$

Put $x = y = \ln 2$ and $x = 0$, $y = -\ln 2$

$$-\frac{1}{2} = \frac{2^\alpha}{\alpha} + c \qquad -2 = \frac{1}{\alpha} + c$$

$$\Rightarrow \quad \alpha = 2 \text{ and } c = -\frac{5}{2}$$

72. Answer (1)

$$(y + 1)\tan^2 x\,dx + \tan x\,dy + y\,dx = 0$$

$$(y + 1)(\sec^2 x - 1)dx + \tan x\,d(y + 1) + y\,dx = 0$$

$$(y + 1)\sec^2 x\,dx + \tan x\,d(y + 1) - (y + 1)dx + ydx = 0$$

$$(y + 1)d(\tan x) + \tan x\,d(y + 1) = dx$$

$$\int d((y + 1)\tan x) = \int dx$$

$$(y + 1)\tan x = x + C$$

$$y = \frac{x + C}{\tan x} - 1$$

$$\lim_{x \to 0^+} xy = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{x}{\tan x}(C+x) - x = 1$$

$\Rightarrow$ 1(C + 0) – 0 = 1

$\Rightarrow$ C = 1

$$y = \frac{x+1}{\tan x} - 1$$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$$

73. Answer (4)

$$\frac{dy}{dx} = \frac{2x - e^y}{2x^2}$$

$$\Rightarrow -e^{-y}\frac{dy}{dx} + \frac{e^{-y}}{x} = \frac{1}{2x^2} \qquad \text{Let } e^{-y} = \gamma$$

$$\Rightarrow \frac{d\gamma}{dx} + \frac{\gamma}{x} = \frac{1}{2x^2}$$

$$\text{I}\cdot\text{f}\cdot = e^{\int \frac{1}{x}dx} = x$$

$$\Rightarrow \gamma \cdot x = \int \frac{1}{2x}dx + C$$

$$\Rightarrow xe^{-y} = \frac{1}{2}\ln x + C$$

$$\because \; y(e) = 1 \Rightarrow e \cdot e^{-1} = \frac{1}{2} + C \Rightarrow C = \frac{1}{2}$$

Now put $x = 1$, $e^{-y} = \frac{1}{2} \Rightarrow y = \ln 2$

74. Answer (4)

Let $f(x) = y$

$$\Rightarrow y + x\frac{dy}{dx} = x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 - y}{x}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x \qquad \text{(linear D.E)}$$

$$\text{I.F} = e^{\int \frac{1}{x}dx} = x$$

$$\Rightarrow \int d(xy) = \frac{x^3}{3} + C$$

$\downarrow (-2, 2)$

$$-4 = \frac{-8}{3} + C$$

$$\Rightarrow C = \frac{-4}{3}$$

$\Rightarrow$ $3xy = x^3 - 4$

$\Rightarrow$ $x^3 - 3xf(x) - 4 = 0$

75. Answer (2)

$$\frac{dy}{dx} - 2xy = 4x\sin x - 2\cos x - 10x$$

$$\text{I.F.} = e^{\int -2x dx} = e^{-x^2}$$

$$y \cdot e^{-x^2} = \int e^{-x^2}(4x\sin x - 2\cos x - 10x)dx + C$$

$$\Rightarrow ye^{-x^2} = \int e^{-x^2}\cdot(-2x)(-2\sin x)dx - \int 2\cos x \cdot e^{-x^2}dx + 5\int\left(-2xe^{-x^2}\right)dx + C$$

$$\Rightarrow ye^{-x^2} = -2\sin x \cdot e^{-x^2} + 5e^{-x^2} + C$$

Put $x = 0$, $7 = 5 + C \Rightarrow C = 2$

Put $x = \pi$ $\quad y = 5 + 2e^{\pi^2}$

76. Answer (17)

$$y^{1/4} + y^{-1/4} = 2x \Rightarrow \left(y^{1/4} - y^{-1/4}\right)^2$$

$$= \left(y^{1/4} + y^{-1/4}\right)^2 - 4 = 4(x^2 - 1)$$

$$\frac{1}{4y}\cdot\left(y^{1/4} - y^{-1/4}\right)\cdot\frac{dy}{dx} = 2 \Rightarrow$$

$$\frac{1}{8}\cdot\left(y^{1/4} - y^{-1/4}\right)\cdot\frac{dy}{dx} = y \qquad \text{...(i)}$$

$$\left(y^{1/4} - y^{-1/4}\right)\frac{dy}{dx} = 8y$$

$$\frac{1}{4y\cdot}\left(y^{1/4} + y^{-1/4}\right)\cdot\left(\frac{dy}{dx}\right)^2 + \left(y^{1/4} - y^{-1/4}\right)\cdot\frac{d^2y}{dx^2} = 8.\frac{dy}{dx}$$

$$\left(y^{1/4} + y^{-1/4}\right)\cdot\left(\frac{dy}{dx}\right)^2 + \left(y^{1/4} - y^{-1/4}\right)\frac{d^2y}{dx^2}(4y)$$

$$= 4\cdot(8y)\frac{dy}{dx}$$

$$2x\cdot\left(\frac{dy}{dx}\right)^2 + \left(y^{1/4} - y^{-1/4}\right)\cdot\frac{d^2y}{dx^2}$$

$$\cdot\left(\frac{y^{1/4}-y^{-1/4}}{2}\right)\frac{dy}{dx}=4\cdot(8y)\frac{dy}{dx}$$

$$2x\cdot\frac{dy}{dx}+\frac{4(x^2-1)}{2}\cdot\frac{d^2y}{dx^2}=32.y$$

$$(x^2-1)\cdot\frac{d^2y}{dx^2}+x\cdot\frac{dy}{dx}-16y=0 \Rightarrow \alpha = 1,\ \beta = -16$$

$$|\alpha-\beta|=17$$

77. Answer (2)

$2xdy - 10y^3\,dy + ydx = 0$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{10y^3-2x}$$

$$\Rightarrow \frac{dx}{dy}=\frac{10y^3-2x}{y}$$

$$\Rightarrow \frac{dx}{dy}+\frac{2x}{y}=10y^2 \qquad \text{(Linear D.E.)}$$

$$\text{I.F.} = e^{\int\frac{2}{y}dy}=y^2$$

$$\Rightarrow \int d(xy^2)=\frac{10y^5}{5}$$

$$\Rightarrow xy^2 = 2y^5+C$$

$$\downarrow (0, 1)$$

$$C=-2$$

$$\Rightarrow 2y^5-xy^2-2=0$$

(Put $x = 2$ gives equation whose root is β)

i.e. $y^5 - y^2 - 1 = 0$

78. Answer (1)

$\because$ Length of latus rectum $= \dfrac{11}{5}$

Let equation of parabola : $(x-a)^2=\dfrac{11}{5}(y-b)$

$$\Rightarrow 2(x-a)=\frac{11}{5}\frac{dy}{dx}$$

$$\Rightarrow 2=\frac{11}{5}\frac{d^2y}{dx^2}$$

$$\Rightarrow 11\frac{d^2y}{dx^2}=10$$

79. Answer (3)

$$\frac{dy}{dx}=2^x\left(\frac{2^y-1}{2^y}\right)$$

$$\Rightarrow \frac{2^y\ln 2\,dy}{2^y-1}=2^x\ln 2\,dx$$

$\Rightarrow \ln(2^y - 1) = 2^x + c$

put $x = 0$ $\qquad$ $c = -1$

put $x = 1$ $\qquad$ $\ln(2^y - 1) = 1$

$\Rightarrow 2^y - 1 = e$

$\Rightarrow y = \log_2(1 + e)$

80. Answer (4)

$$x\phi(x)=\int_5^x\left(3t^2-2\phi'(t)\right)dx$$

Differentiating both side w.r.t. x.

$$\phi(x)+x\cdot\phi'(x)=3x^2-2\phi'(x)$$

Let $\phi(x) = y$

$$(x+2)\cdot\frac{dy}{dx}+y=3x^2$$

$$\frac{dy}{dx}+\frac{y}{x+2}=\frac{3x^2}{x+2}$$

$$\text{I.F} = e^{\int\frac{1}{x+2}dx}=(x+2)$$

Solution of differential equation

$$\Rightarrow y\cdot(x+2)=\int\left(\frac{3x^2}{x+2}\right)(x+2)\,dx+c$$

$y(x + 2) = x^3 + c$

at $x = 0$, $y = 4$

$c = 8$

$y(x + 2) = x^3 + 8$

at $x = 2$

$y = 4$

81. Answer (3)

$$\frac{dy}{dx} = \frac{2^x\left(y+2^y\right)}{2^x\left(1+2^y \ln 2\right)}$$

$$\Rightarrow \int \frac{\left(1+2^y \ln 2\right)dy}{\left(y+2^y\right)} = \int dx$$

For LHS put $y + 2^y = t$

$\Rightarrow$ $(1 + 2^y \ln 2)\, dy = dt$

$\Rightarrow$ $\ln (y + 2^y) = x + c$

$\downarrow (0, 0)$

$c = 0$

$\therefore$ $\ln(y + 2^y) = x$

If $y = 1$

$\Rightarrow$ $x = \ln 3$

$x \in (1, 2)$

82. Answer (2)

Let $\dfrac{y^2}{x^2} = u$

$\Rightarrow$ $y^2 = ux^2$

$2yy' = u'x^2 + 2ux$ ...(i)

Given

$$yy' = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$$

$$\Rightarrow \quad yy' = x\left[u + \frac{\phi(u)}{\phi'(u)}\right]$$

$$\Rightarrow \quad \frac{1}{2}\left[u'x^2 + 2ux\right] = x\left[u + \frac{\phi(u)}{\phi'(u)}\right]$$

$$\frac{1}{2}u'x^2 = x\frac{\phi(u)}{\phi'(u)}$$

$$\Rightarrow \quad xu' = \frac{2\phi(u)}{\phi'(u)}$$

$$x\frac{du}{dx} = \frac{2\phi(u)}{\phi'(u)}$$

$$\int \frac{\phi'(u)\,du}{\phi(u)} = \int \frac{2dx}{x}$$

$\ln \phi(u) = 2 \ln x + \ln c$

$\phi(u) = cx^2$

$$\phi\left(\frac{y^2}{x^2}\right) = cx^2 \qquad \text{...(ii)}$$

$x = 1 \Rightarrow y = -1$

$\therefore$ $\phi(1) = c$

$$\Rightarrow \quad \phi\left(\frac{y^2}{x^2}\right) = \phi(1)\,x^2$$

Put $x = 2$

$$\phi\left(\frac{y^2}{4}\right) = 4\phi(1)$$

83. Answer (1)

$$x^2dy = \left(\frac{1}{x} - y\right)dx$$

$$x^2\frac{dy}{dx} = \frac{1}{x} - y$$

$$x^2\frac{dy}{dx} + y = \frac{1}{x}$$

$$\frac{dy}{dx} + \frac{1}{x^2}\cdot y = \frac{1}{x^3} \qquad \text{...(i)}$$

$$\text{IF} = e^{\int \frac{1}{x^2}dx} = e^{\frac{-1}{x}}$$

$$y \cdot e^{\frac{-1}{x}} = \int e^{\frac{-1}{x}} \times \frac{1}{x^3}dx$$

Let $\dfrac{-1}{x} = t$, $\dfrac{1}{x^2}dx = dt$

$$y \cdot e^{\frac{-1}{x}} = -\int e^t t\,dt = -e^t(t-1) + c$$

$$ye^{\frac{-1}{x}} = -e^{\frac{-1}{x}}\left(\frac{-1}{x} - 1\right) + c$$

$$y = \frac{1}{x} + 1 + ce^{\frac{1}{x}}$$

$$y(1) = 1 \Rightarrow 1 = 2 + ce \Rightarrow c = \frac{-1}{e}$$

$$y\left(\frac{1}{2}\right) = 2 + 1 + \left(\frac{-1}{e}\right)\cdot e^2$$

$$= 3 - e$$

84. Answer (02)

$$y^2 = a\left(x + \frac{\sqrt{a}}{2}\right) \quad \ldots(1)$$

$$2y.\frac{dy}{dx} = a \quad \ldots(2)$$

From (1) and (2)

$$y^2 = 2y\frac{dy}{dx}\left(x + \frac{1}{2}\sqrt{2y\frac{dy}{dx}}\right)$$

$$y - 2x\frac{dy}{dx} = y\frac{dy}{dx}.\sqrt{2y\frac{dy}{dx}}$$

$$\Rightarrow \left(y - 2x\frac{dy}{dx}\right)^2 = 2y^3\left(\frac{dy}{dx}\right)^3$$

$\Rightarrow$ Order 1 and degree 3.

85. Answer (1)

$$\frac{dx}{dy} - \frac{2x}{y} = y^2(y+1)e^y$$

$$\text{If } = e^{\int -\frac{2}{y}dy} = e^{-2\ln y} = \frac{1}{y^2}$$

Solution is given by

$$x.\frac{1}{y^2} = \int y^2(y+1)e^y.\frac{1}{y^2}dy$$

$$\Rightarrow \frac{x}{y^2} = \int (y+1)e^y dy$$

$$\Rightarrow \frac{x}{y^2} = ye^y + c$$

$\Rightarrow$ $x = y^2\,(ye^y + c)$

at, $y = 1$, $x = 0$

$\Rightarrow$ $0 = 1(1.e^1 + c) \Rightarrow c = -e$

at $y = e$,

$x = e^2(e.e^e - e)$

86. Answer (2)

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$$

$$\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$$

$$\text{If } e^{-\int \frac{1}{x+1}x} = e^{-\log(x+1)} = \frac{1}{x+1}$$

$$\therefore \quad y\left(\frac{1}{x+1}\right) = \int \frac{e^{3x}(x+1)}{x+1}dx$$

$$\frac{y}{x+1} = \int e^{3x}dx$$

$$\boxed{\frac{y}{x+1} = \frac{e^{3x}}{3} + c}$$

$$\because \; y(0) = \frac{1}{3}$$

$$\frac{1}{3} = \frac{1}{3} + c$$

$\therefore$ $c = 0$

$$\text{So} : \boxed{y = \frac{e^{3x}}{3}(x+1)}$$

$$y' = e^{3x}(x+1) + \frac{e^{3x}}{3} = e^{3x}\left(x + \frac{4}{3}\right)$$

$$y'' = 3e^{3x}\left(x + \frac{4}{3}\right) + e^{3x} = e^{3x}(3x+5)$$

$$y' = 0 \text{ at } x = \frac{-4}{3} \;\&\; y'' = e^{-4}(1) > 0 \text{ at } x = \frac{-4}{3}$$

$\Rightarrow$ $x = \frac{-4}{3}$ is point of local minima

87. Answer (3)

$$\frac{dy}{dx} = \frac{y^2}{xy - x^2 - y^2}$$

Put $y = vx$ we get

$$v + x\frac{dv}{dx} = \frac{v^2}{v - 1 - v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v^2 - v^2 + v + v^3}{v - 1 - v^2}$$

$$\Rightarrow \int \frac{v - 1 - v^2}{v(1+v^2)}dv = \int \frac{dx}{x}$$

$$\tan^{-1}\left(\frac{y}{x}\right) - \ln\left(\frac{y}{x}\right) = \ln x + c$$

As it passes through (1, 1)

$$c = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) - \ln\left(\frac{y}{x}\right) = \ln x + \frac{\pi}{4}$$

Put $y = \sqrt{3}x$ we get

$$\Rightarrow \frac{\pi}{3} - \ln\sqrt{3} = \ln x + \frac{\pi}{4}$$

$$\Rightarrow \ln x = \frac{\pi}{12} - \ln\sqrt{3} = \ln\alpha$$

$$\therefore \ln\left(\sqrt{3}\alpha\right) = \ln\sqrt{3} + \ln\alpha$$

$$= \ln\sqrt{3} + \frac{\pi}{12} - \ln\sqrt{3} = \frac{\pi}{12}$$

88. Answer (2)

$$2x^2\frac{dy}{dx} - 2xy + 3y^2 = 0$$

$$\Rightarrow 2x(xdy - ydx) + 3y^2dx = 0$$

$$\Rightarrow 2\left(\frac{xdy - ydx}{y^2}\right) + 3\frac{dx}{x} = 0$$

$$\Rightarrow -\frac{2x}{y} + 3\ln x = C$$

$$\because y(e) = \frac{e}{3} \Rightarrow -6 + 3 = C \Rightarrow C = -3$$

Now, at $x = 1,\ -\frac{2}{y} + 0 = -3$

$$y = \frac{2}{3}$$

89. Answer (12)

$(4 + x^2)\,dy - 2x(x^2 + 3y + 4)dx = 0$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{6x}{x^2+4}\right)y + 2x$$

$$\Rightarrow \frac{dy}{dx} - \left(\frac{6x}{x^2+4}\right)y = 2x$$

I.F. $= e^{-3\ln\left(x^2+4\right)} = \frac{1}{\left(x^2+4\right)^3}$

So $\frac{y}{\left(x^2+4\right)^3} = \int \frac{2x}{\left(x^2+4\right)^3}dx + c$

$$\Rightarrow y = -\frac{1}{2}\left(x^2+4\right) + c\left(x^2+4\right)^3$$

When $x = 0$, $y = 0$ gives $c = \frac{1}{32}$,

So, for $x = 2$, $y = 12$

90. Answer (42)

$$\frac{dy}{dx} = \frac{1}{1+\sin 2x}$$

$$\Rightarrow dy = \frac{\sec^2 x\,dx}{(1+\tan x)^2}$$

$$\Rightarrow y = -\frac{1}{1+\tan x} + c$$

When $x = \frac{\pi}{4},\ y = \frac{1}{2}$ gives $c = 1$

So $y = \frac{\tan x}{1+\tan x} \Rightarrow y = \frac{\sin x}{\sin x + \cos x}$

Now, $y = \sqrt{2}\sin x \Rightarrow \sin x = 0$

or $\sin x + \cos x = \frac{1}{\sqrt{2}}$

$\sin x = 0$ gives $x = \pi$ only.

and $\sin x + \cos x = \frac{1}{\sqrt{2}} \Rightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{1}{2}$

So $x + \frac{\pi}{4} = \frac{5\pi}{6}$ or $\frac{13\pi}{6} \Rightarrow x = \frac{7\pi}{12}$ or $\frac{23\pi}{12}$

Sum of all solutions $= \pi + \frac{7\pi}{12} + \frac{23\pi}{12} = \frac{42\pi}{12}$

Hence $k = 42$.

91. Answer (3)

$\because \ \dfrac{dy}{dx}+e^x\left(x^2-2\right)y=\left(x^2-2x\right)\left(x^2-2\right)e^{2x}$

Here, I.F. $=e^{\int e^x\left(x^2-2\right)dx}$

$=e^{\left(x^2-2x\right)e^x}$

$\therefore$ Solution of the differential equation is

$$y\cdot e^{\left(x^2-2x\right)e^x}=\int\left(x^2-2x\right)\left(x^2-2\right)e^{2x}\cdot e^{\left(x^2-2x\right)e^x}dx$$

$$=\int\left(x^2-2x\right)e^x\cdot\left(x^2-2\right)e^x\cdot e^{\left(x^2-2x\right)e^x}dx$$

Let $\left(x^2-2x\right)e^x=t$

$\therefore \ \left(x^2-2\right)e^x dx=dt$

$$y\cdot e^{\left(x^2-2x\right)e^x}=\int t\cdot e^t dt$$

$$y\cdot e^{\left(x^2-2x\right)e^x}=\left(x^2-2x-1\right)e^{\left(x^2-2x\right)e^x}+c$$

$\therefore$ $y(0) = 0$

$\therefore$ $c = 1$

$\therefore \ y=\left(x^2-2x-1\right)+e^{\left(2x-x^2\right)e^x}$

$\therefore$ $y(2) = -1 + 1$

$= 0$

92. Answer (2)

$\dfrac{dy}{dx}=\dfrac{ax-by+a}{bx+cy+a}$

$= bx\,dy + cy\,dy + a\,dy = ax\,dx - by\,dx + a\,dx$

$= cy\,dy + a\,dy - ax\,dx - a\,dx + b(x\,dy + y\,dx) = 0$

$= c\int y\,dy+a\int x\,dx-a\int dx+b\int d(xy)=0$

$=\dfrac{cy^2}{2}+ay-\dfrac{ax^2}{2}-ax+bxy=k$

$= ax^2 - cy^2 + 2ax - 2ay - 2bxy = k$

Above equation is circle

$\Rightarrow$ $a = -c$ and $b = 0$

$ax^2 + ay^2 + 2ax - 2ay = k$

$\Rightarrow$ $x^2 + y^2 + 2x - 2y = \lambda$ $\quad\left[\lambda=\dfrac{k}{a}\right]$

Passes through (2, 5)

$4 + 25 + 4 - 10 = \lambda \Rightarrow \lambda = 23$

Circle $\equiv x^2 + y^2 + 2x - 2y - 23 = 0$

Centre $(-1, 1)$ $r=\sqrt{(-1)^2+1^2+23}=5$

Shortest distance of (11, 6) $=\sqrt{12^2+5^2}-5$

$= 13 - 5$

$= 8$

93. Answer (4)

$\dfrac{dy}{dx}+\dfrac{2^{x-y}(2^y-1)}{2^x-1}=0$, $x, y > 0$, $y(1) = 1$

$\dfrac{dy}{dx}=-\dfrac{2^x(2^y-1)}{2^y(2^x-1)}$

$\int\dfrac{2^y}{2^y-1}dy=-\int\dfrac{2^x}{2^x-1}dx$

$=\dfrac{\log_e(2^y-1)}{\log_e 2}=-\dfrac{\log_e(2^x-1)}{\log_e 2}+\dfrac{\log_e c}{\log_e 2}$

$= |(2^y - 1)(2^x - 1)| = c$

$\because$ $y(1) = 1$

$\therefore$ $c = 1$

$= |(2^y - 1)(2^x - 1)| = 1$

For $x = 2$

$|(2^y - 1)3| = 1$

$2^y-1=\dfrac{1}{3}\Rightarrow 2y=\dfrac{4}{3}$

Taking log to base 2.

$\therefore$ $\boxed{y=2-\log_2 3}$

94. Answer (2)

$\left(\left(\tan^{-1}y\right)-x\right)dy=\left(1+y^2\right)dx$

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$$

I.F. = $e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$

$\therefore$ Solution

$$x\cdot e^{\tan^{-1}y}=\int\frac{e^{\tan^{-1}y}\tan^{-1}y}{1+y^2}dy$$

Let $e^{\tan^{-1}y}=t$

$$\frac{e^{\tan^{-1}y}}{1+y^2}=dt$$

$$=xe^{\tan^{-1}y}=\int \ln t\,dt=t\ln t-t+c$$

$$\therefore\ =xe^{\tan^{-1}y}=e^{\tan^{-1}y}\tan^{-1}y-e^{\tan^{-1}}y+c \quad \ldots\text{(i)}$$

$\because$ It passes through (1, 0) $\Rightarrow c = 2$

Now put $y = \tan 1$, then

$ex = e - e + 2$

$$\Rightarrow\ x=\frac{2}{e}$$

95. Answer (320)

$$\left(1-x^2\right)dy=\left(xy+\left(x^3+2\right)\sqrt{1-x^2}\right)dx$$

$$\therefore\ \frac{dy}{dx}-\frac{x}{1-x^2}y=\frac{x^3+3}{\sqrt{1-x^2}}$$

$\therefore$ I.F. = $e^{\int-\frac{x}{1-x^2}dx}=\sqrt{1-x^2}$

Solution is

$$y\cdot\sqrt{1-x^2}=\int\left(x^3+3\right)dx$$

$$y\cdot\sqrt{1-x^2}=\frac{x^4}{4}+3x+c$$

$$\because\ y(0)=0\Rightarrow c=0$$

$$\therefore\ y(x)=\frac{x^4+12x}{4\sqrt{1-x^2}}$$

$$\therefore\ \int_{-\frac{1}{2}}^{\frac{1}{2}}\sqrt{1-x^2}\ y(x)\,dx=\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\frac{x^4+12x}{4}\right)dx$$

$$=\int_0^{\frac{1}{2}}\frac{x^4}{2}dx$$

$$\therefore\ k=\frac{1}{320}$$

$$\therefore\ k^{-1}=320$$

96. Answer (1)

$$\left(\frac{1}{\sqrt{1-\frac{y^2}{x^2}}}+e^{\frac{y}{x}}\right)\frac{dy}{dx}=1+\left(\frac{1}{\sqrt{1-\frac{y^2}{x^2}}}+e^{\frac{y}{x}}\right)\frac{y}{x}$$

Putting $y = tx$

$$\left(\frac{1}{\sqrt{1-t^2}}+e^t\right)\left(t+x\frac{dt}{dx}\right)=1+\left(\frac{1}{\sqrt{1-t^2}}+e^t\right)t$$

$$\Rightarrow\ x\left(\frac{1}{\sqrt{1-t^2}}+e^t\right)\frac{dt}{dx}=1$$

$\Rightarrow\ \sin^{-1}t + e^t = \ln x + C$

$$\Rightarrow\ \sin^{-1}\left(\frac{y}{x}\right)+e^{y/x}=\ln x+C$$

at $x = 1$, $y = 0$

So, $0 + e^0 = 0 + C \Rightarrow C = 1$

at $(2\alpha, \alpha)$

$$\sin^{-1}\left(\frac{y}{x}\right)+e^{y/x}=\ln x+1$$

$$\Rightarrow\ \frac{\pi}{6}+e^{\frac{1}{2}}-1=\ln(2\alpha)$$

$$\Rightarrow\ \alpha=\frac{1}{2}e^{\left(\frac{\pi}{6}+e^{\frac{1}{2}}-1\right)}$$

97. Answer (1)

$$\frac{dy}{dx}+\frac{y(3x^2-1)}{x(1-x^2)}=\frac{4x^3}{x(1-x^2)}$$

$$IF=e^{\int\frac{3x^2-1}{x-x^3}dx}=e^{-\ln|x^3-x|}=e^{-\ln(x^3-x)}$$

$$=\frac{1}{x^3-x}$$

Solution of D.E. can be given by

$$y.\frac{1}{x^3-x}=\int\frac{4x^3}{x(1-x^2)}.\frac{1}{x(x^2-1)}dx$$

$$\Rightarrow\quad \frac{y}{x^3-x}=\int\frac{-4x}{(x^2-1)^2}dx$$

$$\Rightarrow\quad \frac{y}{x^3-x}=\frac{2}{(x^2-1)}+c$$

at $x = 2$, $y = -2$

$$\frac{-2}{6}=\frac{2}{3}+c \;\Rightarrow\; c=-1$$

$$\text{at } x=3\Rightarrow\frac{y}{24}=\frac{2}{8}-1\Rightarrow y=-18$$

98. Answer (4)

Given differential equation

$$2ye^{\frac{x}{y^2}}dx+\left(y^2-4xe^{\frac{x}{y^2}}\right)dy=0,\; x(1)=0$$

$$\Rightarrow\quad e^{\frac{x}{y^2}}\left[2ydx-4xdy\right]=-y^2dy$$

$$\Rightarrow\quad e^{\frac{x}{y^2}}\left[\frac{2y^2dx-4xydy}{y^4}\right]=\frac{-1}{y}dy$$

$$\Rightarrow\quad 2e^{\frac{x}{y^2}}d\left(\frac{x}{y^2}\right)=-\frac{1}{y}dy$$

$$\Rightarrow\quad 2e^{\frac{x}{y^2}}=-\ln y+c \qquad \ldots(i)$$

Now, using $x(1) = 0$, $c = 2$

So, for $x(e)$, Put $y = e$ in (i)

$$2e^{\frac{x}{e^2}}=-1+2$$

$$\Rightarrow\quad \frac{x}{e^2}=\ln\left(\frac{1}{2}\right)\;\Rightarrow x(e)=-e^2\ln 2$$

99. Answer (A)

$$x\frac{dy}{dx}-y=\sqrt{y^2+16x^2}$$

$y = 4x\tan\theta$

$$\frac{dy}{dx}=4\tan\theta+4x\sec^2\theta\frac{d\theta}{dx}$$

$$4x\tan\theta+4x^2\sec^2\theta\frac{d\theta}{dx}-4x\tan\theta=4x\sec\theta$$

$$\int\sec\theta\, d\theta=\int\frac{dx}{x}$$

$\log|\sec\theta+\tan\theta| = \log|x| + C$

$y(1) = 3 \;\Rightarrow 3 = 4\tan\theta$

$$=\tan\theta=\frac{3}{4}\;\Rightarrow\sec\theta=\frac{5}{4}$$

$$\ln\left|\frac{8}{4}\right|=\ln|1|+C$$

$\Rightarrow\quad C = \ln 2$

$\therefore\quad |\sec\theta + \tan\theta| = 2|x|$

To find $y(2)$ put $x = 2$

$$\Rightarrow\quad \tan\theta=\frac{y}{8}$$

$(\sec\theta + \tan\theta)^2 = 16$

$$\begin{array}{l}\sec\theta+\tan\theta=\pm 4\\ \sec\theta-\tan\theta=\pm\frac{1}{4}\\ \hline 2\tan\theta=\frac{15}{4}=2\times\frac{y}{8}\end{array}$$

$$\Rightarrow\quad \boxed{y=15}$$

100. Answer (2)

$$\frac{dy}{dx}+\frac{2\sqrt{2}y}{1+\cos^2 2x}=xe^{\tan^{-1}\left(\sqrt{2}\cot 2x\right)}$$

$$\text{I.F.}=e^{\int\frac{2\sqrt{2}dx}{1+\cos^2 2x}}=e^{\sqrt{2}\int\frac{2\sec^2 2x}{2+\tan^2 2x}dx}$$

$$=e^{\tan^{-1}\left(\frac{\tan 2x}{\sqrt{2}}\right)}$$

$$\Rightarrow\quad y\cdot e^{\tan^{-1}\left(\frac{\tan 2x}{\sqrt{2}}\right)}=\int xe^{\tan^{-1}\left(\sqrt{2}\cot 2x\right)}\cdot e^{\tan^{-1}\left(\frac{\tan 2x}{\sqrt{2}}\right)}dx+c$$

$$\Rightarrow\quad y\cdot e^{\tan^{-1}\left(\frac{\tan 2x}{\sqrt{2}}\right)}=e^{\frac{\pi}{2}}\cdot\frac{x^2}{2}+c$$

When $x=\frac{\pi}{4}, y=\frac{\pi^2}{32}$ gives $c=0$

When $x=\frac{\pi}{3}, y=\frac{\pi^2}{18}\,e^{-\tan^{-1}\alpha}$

So $\frac{\pi^2}{18}e^{-\tan^{-1}\alpha}\cdot e^{-\tan^{-1}\left(-\sqrt{\frac{3}{2}}\right)}=e^{\pi/2}\frac{\pi^2}{18}$

$$\Rightarrow\quad -\tan^{-1}\alpha+\tan^{-1}\left(\sqrt{\frac{3}{2}}\right)=\frac{\pi}{2}$$

$$\Rightarrow\quad \tan^{-1}(-\alpha)=\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

$$\Rightarrow\quad \alpha=-\sqrt{\frac{2}{3}}\ \Rightarrow\ 3\alpha^2=2$$

101. Answer (3)

$$\left(1+e^{2x}\right)\frac{dy}{dx}+2\left(1+y^2\right)e^x=0$$

$$\int\frac{dy}{1+y^2}=-\int\frac{2e^x}{1+e^{2x}}dx \qquad e^x=t$$

$$e^x dx=dt$$

$$\tan^{-1}y=-2\int\frac{dt}{1+t^2}$$

$$\tan^{-1}y=2\tan^{-1}\left(e^x\right)+c$$

$$y(0)\Rightarrow c=\frac{\pi}{2}$$

$$\tan^{-1}y=-2\tan^{-1}\left(e^x\right)+\frac{\pi}{2}$$

$$y=\cot\left(2\tan^{-1}e^x\right)$$

$$\frac{dy}{dx}=-\operatorname{cosec}^2\left(2\tan^{-1}e^x\right)\left(\frac{2e^x}{1+e^{2x}}\right)$$

$$y'(0)=\left.\frac{dy}{dx}\right|_{x=0}=\frac{-2}{2}=-1$$

$$y=\cot\left(2\tan^{-1}e^x\right)$$

$$y\left(\ln\sqrt{3}\right)=\cot\left(2\tan^{-1}e^{\log_e\sqrt{3}}\right)$$

$$=\cot\left(2\tan^{-1}\sqrt{3}\right)=\cot\left(\frac{2\pi}{3}\right)=-\cot\frac{\pi}{3}=-\frac{1}{\sqrt{3}}$$

$$6\left(y'(0)+\left(y\left(\ln\sqrt{3}\right)\right)^2\right)=6\left(-1+\left(-\frac{1}{\sqrt{3}}\right)^2\right)$$

$$=6\left(-1+\frac{1}{3}\right)=-4$$

102. Answer (14)

$$\frac{dy}{dx}+y\left(\frac{2x}{x-1}\right)=\frac{1}{(x-1)^2}$$

$$\text{I.F}=e^{\int\frac{2x}{x-1}dx}$$

$$=e^{2\int\left(\frac{x-1}{x-1}+\frac{1}{x-1}\right)dx}$$

$$=e^{2x+2\ln(x-1)}$$

$$=e^{2x}(x-1)^2$$

$$\Rightarrow \int d\left(y.e^{2x}(x-1)^2\right)=\int e^{2x}dx$$

$$\Rightarrow y.e^{2x}(x-1)^2=\frac{e^{2x}}{2}+c$$

$$\downarrow y(2)=\frac{1+e^4}{2e^4}$$

$$\frac{1+e^4}{2e^4}\cdot e^4=\frac{e^4}{2}+c$$

$$\Rightarrow c=\frac{e^4}{2}\left(\frac{1+e^4-e^4}{e^4}\right)=\frac{1}{2}$$

$$\Rightarrow y\cdot e^{2x}(x-1)^2=\frac{e^{2x}+1}{2}$$

$$\downarrow y(3)=\frac{e^{\alpha}+1}{\beta e^{\alpha}}$$

$$\Rightarrow \frac{e^{\alpha}+1}{\beta e^{\alpha}}\cdot e^6\cdot 4=\frac{e^6+1}{2}$$

$\Rightarrow$ $\alpha = 6$ and $\beta = 8 \Rightarrow \alpha+\beta = 14$

103. Answer (2)

$$\frac{dy}{dx}=\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}=e^{2x}-\frac{6e^{-x}}{2+9e^{-2x}}$$

$$\int dy=\int e^{2x}dx-3\int\underbrace{\frac{e^{-x}}{1+\left(\frac{3e^{-x}}{\sqrt{2}}\right)^2}}_{\text{put } e^{-x}=t}dx$$

$$=\frac{e^{2x}}{2}+3\int\frac{dt}{1+\left(\frac{3t}{\sqrt{2}}\right)^2}$$

$$=\frac{e^{2x}}{2}+\sqrt{2}\tan^{-1}\frac{3t}{\sqrt{2}}+C$$

$$y=\frac{e^{2x}}{2}+\sqrt{2}\tan^{-1}\left(\frac{3e^{-x}}{\sqrt{2}}\right)+C$$

It is given that the curve passes through

$$\left(0,\frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$$

$$\frac{1}{2}+\frac{\pi}{2\sqrt{2}}=\frac{1}{2}+\sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)+C$$

$$\Rightarrow C=\frac{\pi}{2\sqrt{2}}-\sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$$

Now if $\left(\alpha,\frac{1}{2}e^{2\alpha}\right)$ satisfies the curve, then

$$\frac{1}{2}e^{2\alpha}=\frac{e^{2\alpha}}{2}+\sqrt{2}\tan^{-1}\left(\frac{3e^{-\alpha}}{\sqrt{2}}\right)+\frac{\pi}{2\sqrt{2}}-\sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$$

$$\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)-\tan^{-1}\left(\frac{3e^{-\alpha}}{\sqrt{2}}\right)=\frac{\pi}{2\sqrt{2}}\times\frac{1}{\sqrt{2}}=\frac{\pi}{4}$$

$$\frac{\frac{3}{\sqrt{2}}-\frac{3e^{-\alpha}}{\sqrt{2}}}{1+\frac{9}{2}e^{-\alpha}}=1$$

$$\frac{3}{\sqrt{2}}e^{\alpha}-\frac{3}{\sqrt{2}}=e^{\alpha}+\frac{9}{2}$$

$$e^{\alpha}=\frac{\frac{9}{2}+\frac{3}{\sqrt{2}}}{\frac{3}{\sqrt{2}}-1}=\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$$

104. Answer (1)

$$\left(x-y^2\right)dx+y\left(5x+y^2\right)dy=0$$

$$y\frac{dy}{dx}=\frac{y^2-x}{5x+y^2}$$

Let $y^2 = t$

$$\frac{1}{2}\cdot\frac{dt}{dx}=\frac{t-x}{5x+t}$$

Now substitute, $t = vx$

$$\frac{dt}{dx}=v+x\frac{dv}{dx}$$

$$\frac{1}{2}\left\{v+x\frac{dv}{dx}\right\}=\frac{v-1}{5+v}$$

$$x\frac{dv}{dx}=\frac{2v-2}{5+v}-v=\frac{-3v-v^2-2}{5+v}$$

$$\int\frac{5+v}{v^2+3v+2}\,dv=\int-\frac{dx}{x}$$

$$\int\frac{4}{v+1}\,dv-\int\frac{3}{v+2}dv=-\int\frac{dx}{x}$$

$$4\ln|v+1|-3\ln|v+2|=-\ln x+\ln C$$

$$\left|\frac{(v+1)^4}{(v+2)^3}\right|=\frac{C}{x}$$

$$\left|\frac{\left(\frac{y^2}{x}+1\right)^4}{\left(\frac{y^2}{x}+2\right)^3}\right|=\frac{C}{x}$$

$$\left|\left(y^2+x\right)^4\right|=C\left|\left(y^2+2x\right)^3\right|$$

105. Answer (1)

$$\frac{dy}{dx}+2y\tan x=\sin x$$

which is a first order linear differential equation.

Integrating factor (I. F.) $=e^{\int 2\tan x\,dx}$

$$=e^{2\ln|\sec x|}=\sec^2 x$$

Solution of differential equation can be written as

$$y\cdot\sec^2 x=\int\sin x\cdot\sec^2 x\,dx=\int\sec x\cdot\tan x\,dx$$

$$y\sec^2 x=\sec x+C$$

$$y\left(\frac{\pi}{3}\right)=0,0=\sec\frac{\pi}{3}+C\Rightarrow\quad C=-2$$

$$y=\frac{\sec x-2}{\sec^2 x}=\cos x-2\cos^2 x$$

$$=\frac{1}{8}-2\left(\cos x-\frac{1}{4}\right)^2$$

$$y_{\max}=\frac{1}{8}$$

106. Answer (6)

$$\int_3^x f(x)\,dx=\left(\frac{f(x)}{x}\right)^3$$

$$x^3\cdot\int_3^x f(x)dx=f^3(x)$$

Differentiate w.r.t. x

$$x^3f(x)+3x^2\cdot\frac{f^3(x)}{x^3}=3f^2(x)f'(x)$$

$$\Rightarrow\quad 3y^2\frac{dy}{dx}=x^3y+\frac{3y^3}{x}$$

$$3xy\frac{dy}{dx}=x^4+3y^2$$

Let $y^2=t$

$$\frac{3}{2}\frac{dt}{dx}=x^3+\frac{3t}{x}$$

$$\frac{dt}{dx}-\frac{2t}{x}=\frac{2x^3}{3}$$

$$I.F.=\cdot e^{\int-\frac{2}{x}dx}=\frac{1}{x^2}$$

Solution of differential equation

$$t\cdot\frac{1}{x^2}=\int\frac{2}{3}x\,dx$$

$$\frac{y^2}{x^2}=\frac{x^2}{3}+C$$

$$y^2=\frac{x^4}{3}+Cx^2$$

Curve passes through (3, 3) $\Rightarrow C=-2$

$$y^2=\frac{x^4}{3}-2x^2$$

Which passes through $\left(\alpha,6\sqrt{10}\right)$

$$\frac{\alpha^4 - 6\alpha^2}{3} = 360$$

$$\alpha^4 - 6\alpha^2 - 1080 = 0$$

$\alpha = 6$

107. Answer (1)

$$\frac{dy}{dx} = x + y,$$

Let $x + y = t$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - 1 = t \Rightarrow \int \frac{dt}{t+1} = \int dx$$

$$\ln|t+1| = x + C'$$

$$|t+1| = Ce^x$$

$$|x+y+1| = Ce^x$$

For $y_1(x)$, $y_1(0) = 0 \Rightarrow C = 1$

For $y_2(x)$, $y_2(0) = 1 \Rightarrow C = 2$

$y_1(x)$ is given by $|x + y + 1| = e^x$

$y_2(x)$ is given by $|x + y + 1| = 2e^x$

At point of intersection

$e^x = 2e^x$

No solution

So, there is no point of intersection of $y_1(x)$ and $y_2(x)$.

108. Answer (1)

$$\frac{dy}{dx} + y\left(\frac{4x}{\sin(2x^2)\ln(\tan x^2)}\right) = \frac{4\sqrt{2}\,x\sin\left(x^2 - \frac{\pi}{4}\right)}{\sin(2x^2)\ln(\tan x^2)}$$

$$\text{I.F} = e^{\int \frac{4x}{\sin(2x^2)\ln(\tan x^2)}dx}$$

$$= e^{\ln|\ln(\tan x^2)|} = \ln(\tan x^2)$$

$$\therefore \int d\left(y.\ln(\tan x^2)\right) = \int \frac{4\sqrt{2}\,x\sin\left(x^2 - \frac{\pi}{4}\right)}{\sin(2x^2)}dx$$

$$\Rightarrow y\ln(\tan x^2) = \ln\left|\frac{\sec x^2 + \tan x^2}{\operatorname{cosec} x^2 - \cot x^2}\right| + C$$

$$\ln\left(\frac{1}{\sqrt{3}}\right) = \ln\left(\frac{\frac{3}{\sqrt{3}}}{2-\sqrt{3}}\right) + C$$

$$e = \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2-\sqrt{3}}\right)$$

For $y\left(\sqrt{\frac{\pi}{3}}\right)$

$$y\ln(\sqrt{3}) = \ln\left|\frac{\frac{2+\sqrt{3}}{2}}{\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}}\right| + \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2\sqrt{3}}\right)$$

$$= \ln(2+\sqrt{3}) + \ln\left(\frac{1}{\sqrt{3}}\right) + \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2-\sqrt{3}}\right)$$

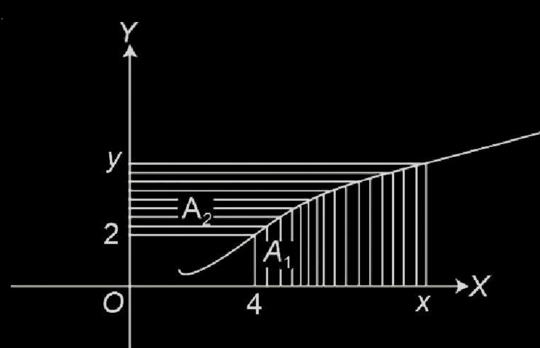
$$\Rightarrow y\ln\sqrt{3} = \ln\left(\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \frac{y}{2}\ln 3 = -\frac{1}{2}\ln 3$$

$$\Rightarrow y = 1$$

$$\therefore \left|y\left(\sqrt{\frac{\pi}{3}}\right)\right| = 1.$$

109. Answer (3)



$$A_1 + A_2 = xy - 8 \ \ \& \ \ A_1 = 2A_2$$

$$A_1 + \frac{A_1}{2} = xy - 8$$

$$A_1 = \frac{2}{3}(xy - 8)$$

$$\int_4^x f(x)dx = \frac{2}{3}(xf(x) - 8)$$

Differentiate w.r.t. x

$$f(x) = \frac{2}{3}\{xf'(x) + f(x)\}$$

$$\frac{2}{3}xf'(x) = \frac{1}{3}f(x)$$

$$2\int \frac{f'(x)}{f(x)}dx = \int \frac{dx}{x}$$

$2\ln f(x) = \ln x + \ln c$

$f^2(x) = cx$

Which passes through (4, 2)

$4 = c \times 4 \Rightarrow c = 1$

Equation of required curve

$$\boxed{y^2 = x}$$

Equation of normal having slope (–6) is

$$y = -6x - 2\left(\frac{1}{4}\right)(-6) - \frac{1}{4}(-6)^3$$

$y = -6x + 57$

Which does not pass through (10, –4)

110. Answer (2)

$$\frac{xdy - ydx}{\sqrt{x^2 + y^2}} = dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sqrt{x^2 + y^2}}{x} + \frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \sqrt{1 + \frac{y^2}{x^2}} + \frac{y}{x}$$

Let $\frac{y}{x} = v$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \sqrt{1 + v^2} + v$$

$$\Rightarrow \quad \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

OR $\ln\left(v + \sqrt{1 + v^2}\right) = \ln x + C$

at $x = 1$, $y = 0$

$\Rightarrow \quad C = 0$

$$\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = x$$

At $x = 2$,

$$\frac{y}{2} + \sqrt{1 + \frac{y^2}{4}} = 2$$

$$\Rightarrow \quad 1 + \frac{y^2}{4} = 4 + \frac{y^2}{4} - 2y$$

OR $y = \frac{3}{2}$

111. Answer (1)

$$(\sin^2 2x)\frac{dy}{dx} + (8\sin^2 2x + 2\sin 4x)y$$

$$= 2e^{-4x}(2\sin 2x + \cos 2x)$$

$$\frac{dy}{dx} + (8 + 4\cot 2x)y = 2e^{-4x}\left(\frac{2\sin 2x + \cos 2x}{\sin^2 2x}\right)$$

Integrating factor

$$(I.F.) = e^{\int (8 + 4\cot 2x)dx}$$

$$= e^{8x + 2\ln \sin 2x}$$

Solution of differential equation

$$y.e^{8x + 2\ln \sin 2x}$$

$$= \int 2e^{(4x + 2\ln \sin 2x)}\frac{(2\sin 2x + \cos 2x)}{\sin^2 2x}dx$$

$$= 2\int e^{4x}(2\sin 2x + \cos 2x)dx$$

$$y.e^{8x+2\ln\sin 2x} = e^{4x}\sin 2x + c$$

$$y\left(\frac{\pi}{4}\right) = e^{-\pi}$$

$$e^{-\pi}.e^{2\pi} = e^{\pi} + c \Rightarrow c = 0$$

$$y\left(\frac{\pi}{6}\right) = \frac{e^{\frac{2\pi}{3}}\frac{\sqrt{3}}{2}}{e^{\left(\frac{4\pi}{3}+2\ln\frac{\sqrt{3}}{2}\right)}}$$

$$= e^{\frac{-2\pi}{3}}.\frac{2}{\sqrt{3}}$$

112. Answer (1)

Family of circles passing through the points (0, 2) and (0, –2)

$x^2 + (y-2)(y+2) + \lambda x = 0,\ \lambda \in \mathbb{R}$

$x^2 + y^2 + \lambda x - 4 = 0$ ...(1)

Differentiate w.r.t x

$$2x + 2y\frac{dy}{dx} + \lambda = 0 \quad \text{...(2)}$$

Using (1) and (2), eliminate λ

$$x^2 + y^2 - \left(2x + 2y\frac{dy}{dx}\right)x - 4 = 0$$

$$2xy\frac{dy}{dx} + x^2 - y^2 + 4 = 0$$

113. Answer (1)

$$\frac{dy}{dx} = \frac{x+y-2}{x-y} = \frac{(x-1)+(y-1)}{(x-1)-(y-1)}$$

Let $x - 1 = X$, $y - 1 = Y$

$$\frac{dY}{dX} = \frac{X+Y}{X-Y}$$

Let $Y = tX \Rightarrow \dfrac{dY}{dX} = t + X\dfrac{dt}{dX}$

$$t + X\frac{dt}{dX} = \frac{1+t}{1-t}$$

$$X\frac{dt}{dX} = \frac{1+t}{1-t} - t = \frac{1+t^2}{1-t}$$

$$\int\frac{1-t}{1+t^2}dt = \int\frac{dX}{X}$$

$$\tan^{-1}t - \frac{1}{2}\ln\left(1+t^2\right) = \ln|X| + c$$

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\left(1+\left(\frac{y-1}{x-1}\right)^2\right) = \ln|x-1| + c$$

Curve passes through (2, 1)

$0 - 0 = 0 + c \Rightarrow c = 0$

If $(k + 1, 2)$ also satisfies the curve

$$\tan^{-1}\left(\frac{1}{k}\right) - \frac{1}{2}\ln\left(\frac{1+k^2}{k^2}\right) = \ln k$$

$$2\tan^{-1}\left(\frac{1}{k}\right) = \ln\left(1+k^2\right)$$

114. Answer (2)

$$\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y = \frac{(x+3)}{x+1},\ x > -1,$$

Integrating factor I.F. = $e^{\int\frac{2x^2+11x+13}{x^3+6x^2+11x+6}dx}$

Let $\dfrac{2x^2+11x+13}{(x+1)(x+2)(x+3)} = \dfrac{A}{x+1} + \dfrac{B}{x+2} + \dfrac{C}{x+3}$

$A = 2,\ B = 1,\ C = -1$

I.F. = $e^{(2\ln|x+1|+\ln|x+2|-\ln|x+3|)}$

$$= \frac{(x+1)^2(x+2)}{x+3}$$

Solution of differential equation

$$y\cdot\frac{(x+1)^2(x+2)}{x+3} = \int(x+1)(x+2)dx$$

$$y\frac{(x+1)^2(x+2)}{x+3} = \frac{x^3}{3} + \frac{3x^2}{2} + 2x + c$$

Curve passes through (0, 1)

$$1\times\frac{1\times 2}{3}=0+c \quad \Rightarrow \quad c=\frac{2}{3}$$

So, $$y(1)=\frac{\frac{1}{3}+\frac{3}{2}+2+\frac{2}{3}}{\frac{(2^2\times 3)}{4}}=\frac{3}{2}$$

115. Answer (2)

$$\frac{dy}{dx}\propto\frac{-y}{x}$$

$$\frac{dy}{dx}=\frac{-ky}{x} \Rightarrow \int\frac{dy}{y}=-K\int\frac{dx}{x}$$

$$\ln|y|=-K\ln|x|+C$$

If the above equation satisfy (1, 2) and (8, 1)

$$\ln 2=-K\times 0+C \Rightarrow C=\ln 2$$

$$\ln 1=-K\ln 8+\ln 2 \Rightarrow K=\frac{1}{3}$$

So, at $x=\frac{1}{8}$

$$\ln|y|=-\frac{1}{3}\ln\left(\frac{1}{8}\right)+\ln 2=2\ln 2$$

$$|y|=4$$

116. Answer (3)

If = e^{-x}

$$y\cdot e^{-x}=-2e^{-x}+\frac{e^{-2x}}{2}+C$$

$\Rightarrow$ $y = -2 + e^{-x} + Ce^{x}$

$\lim\limits_{x\to\infty} y(x)$ is finite so $C = 0$

$y = -2 + e^{-x}$

$$\Rightarrow \frac{dy}{dx}=-e^{-x} \Rightarrow \left.\frac{dy}{dx}\right|_{x=0}=-1$$

Equation of tangent

$y + 1 = -1\ (x - 0)$

or $y + x = -1$

So $a = -1$, $b = -1$

$\Rightarrow$ $a - 4b = 3$

117. Answer (4)

$$x\frac{dy}{dx}+2y=xe^{x},\ y(1)=0$$

$$\frac{dy}{dx}+\frac{2}{x}y=e^{x},\ \text{then } e^{\int\frac{2}{x}dx}dx=x^2$$

$$y\cdot x^2=\int x^2e^{x}dx$$

$$yx^2=x^2e^{x}-\int 2xe^{x}dx$$

$$=x^2e^{x}-2\left(xe^{x}-e^{x}\right)+c$$

$$yx^2=x^2e^{x}-2xe^{x}+2e^{x}+c$$

$$yx^2=\left(x^2-2x+2\right)e^{x}+c$$

$$0=e+c \Rightarrow c=-e$$

$$y(x)\cdot x^2-e^{x}=(x-1)^2e^{x}-e$$

$$z(x)=(x-1)^2e^{x}-e$$

For local maximum $z'(x)=0$

$$\therefore\ 2(x-1)e^{x}+(x-1)^2e^{x}=0$$

$\therefore\quad x=-1$

And local maximum value $=z(-1)$

$$=\frac{4}{e}-e$$

118. Answer (2)

$$\frac{dy}{dx}=2\tan x(\cos x-y)$$

$$\Rightarrow \frac{dy}{dx}+2\tan xy=2\sin x$$

I.F. = $e^{\int 2\tan x\,dx}=\sec^2 x$

$\therefore$ Solution of D.E. will be

$$y(x)\sec^2 x=\int 2\sin x\sec^2 x\,dx$$

$$y\sec^2 x=2\sec x+c$$

$\because$ Curve passes through $\left(\frac{\pi}{4},0\right)$

$\therefore\ c=-2\sqrt{2}$

$\therefore\ y = 2\cos x - 2\sqrt{2}\cos^2 x$

$\therefore\ \int_0^{\pi/2} y\,dx = \int_0^{\pi/2}\left(2\cos x - 2\sqrt{2}\cos^2 x\right)dx$

$= 2 - 2\sqrt{2}\cdot\frac{\pi}{4} = 2 - \frac{\pi}{\sqrt{2}}$

119. Answer (2)

$$\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}$$

which is first order linear differential equation.

Integrating factor (I.F.) = $e^{\int \frac{x}{x^2-1}dx}$

$= e^{\frac{1}{2}\ln|x^2-1|} = \sqrt{|x^2-1|}$

$= \sqrt{1-x^2} \qquad \because x \in (-1, 1)$

Solution of differential equation

$$y\sqrt{1-x^2} = \int\left(x^4+2x\right)dx = \frac{x^5}{5} + x^2 + c$$

Curve is passing through origin, $c = 0$

$$y = \frac{x^5+5x^2}{5\sqrt{1-x^2}}$$

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^5+5x^2}{5\sqrt{1-x^2}}dx = 0 + 2\int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}}dx$$

put $x = \sin\theta$

$dx = \cos\theta\, d\theta$

$$I = 2\int_0^{\frac{\pi}{3}} \frac{\sin^2\theta\cdot\cos\theta\, d\theta}{\cos\theta}$$

$$= \int_0^{\frac{\pi}{3}} (1-\cos 2\theta)\,d\theta$$

$$= \left(\theta - \frac{\sin 2\theta}{2}\right)\Bigg|_0^{\frac{\pi}{3}}$$

$$= \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

120. Answer (3)

$$\frac{dy}{dx} = \frac{y}{x}\frac{(4y^2+2x^2)}{\left(3y^2+x^2\right)}$$

Put $y = vx \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow\ v + x\frac{dv}{dx} = \frac{v(4v^2+2)}{\left(3v^2+1\right)}$

$\Rightarrow\ x\frac{dv}{dx} = v\left(\frac{(4v^2+2-3v^2-1)}{3v^2+1}\right)$

$\Rightarrow\ \int(3v^2+1)\frac{dv}{v^3+v} = \int\frac{dx}{x}$

$\Rightarrow\ \ln|v^3+v| = \ln x + c$

$\Rightarrow\ \ln\left|\left(\frac{y}{x}\right)^3 + \left(\frac{y}{x}\right)\right| = \ln x + C$

! $y(1) = 1$

$\Rightarrow\ C = \ln 2$

$\therefore$ for $y(2)$

$\ln\left(\frac{y^3}{8} + \frac{y}{2}\right) = 2\ln 2 \ \Rightarrow\ \frac{y^3}{8} + \frac{y}{2} = 4$

$\Rightarrow\ [y(2)] = 2$

$\Rightarrow\ n = 3$

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