MATHEMATICS

MATHEMATICS (050) (E)

QUESTION PAPER - 1

STD. 12th

Time : 3 Hours

JULY-2018

Total Mark : 100

PART - A : 50 Marsk • Part - B : 50 Marks

Time : 1 Hour

PART - A

Maximum Marks : 50

Instructions :

1. There are 50 objective type (M.C.Q.) question in Part-A and all questions are compulsory.
2. The questions are serially numbered from 1 to 50 and each carries 1 marks.
3. Read each question carefully, select proper alternative and answere in the O.M.R. Sheet.
4. The OMR sheet is given for answereing the questions. The answer of each question is represented by (A) ○, (B) ○, (C) ○, (D) ○. Darken the cricle ● of the correct answere with ball-pen.
5. Rough work is to be done in the space provided for this purpose in the Test Booklet only.
6. Set No. of Question paper on the upper-most right side of the Question paper is to be written in the column provided in the OMR sheet.
7. A candidate may use a simple calculator and log table if required.
8. Notation used in this question paper have proper meaning.

1. If $x = at^2, y = 2at$ then $y_2 =$ .... $(t \neq 0)$

(A) $-\frac{1}{2at^3}$ (B) $\frac{1}{2at^2}$ (C) $-\frac{1}{t^2}$ (D) $\frac{1}{2at}$

2. $\int \frac{x \cdot (x\sin x + \cos x)^{-2}}{\sec x} dx =$ .... $+ c$

(A) $\frac{1}{\sin x + x \cdot \cos x}$ (B) $\frac{-1}{\sin x + x \cdot \cos x}$ (C) $\frac{x}{x \cdot \sin x + \cos x}$ (D) $\frac{-1}{x \cdot \sin x + \cos x}$

3. $f'(x) = 3\sin x - 4\sin^3 x$ and $f(0) = \frac{1}{3}$ then $c =$ .... (where c is integrating constant)

(A) $-\frac{3}{2}$ (B) $\frac{2}{3}$ (C) 0 (D) $-\frac{2}{3}$

4. $\int \frac{x^4 + 5^{x-1} \cdot \log_e^5}{x^5 + 5^x} \cdot dx =$ .... $+ c$

(A) $\frac{-1}{\log 5} \cdot \log|x^5 + 5^x|$ (B) $-\frac{1}{5}\log|x^5 + 5^x|$

(C) $\frac{1}{\log 5} \cdot \log|x^5 + 5^x|$ (D) $\frac{1}{5}\log|x^5 + 5^x|$

5. $\int \frac{1}{x(x^{100}+1)} \cdot dx =$ .... $+ c; (x > 0)$.

(A) $\frac{-1}{100}\log\left|\frac{x^{100}+1}{x^{100}}\right|$ (B) $\frac{1}{100}\log\left|\frac{x^{100}}{x^{100}+1}\right|$ (C) $\frac{1}{100}\log\left|\frac{x^{100}+1}{x^{100}}\right|$ (D) $\frac{-1}{100}\log\left|\frac{x^{100}}{x^{100}+1}\right|$

6. If P(A) = 0.25, P (B) = 0.55 and P $(A \cup B)$ = 0.65 then P (B'/A) = ...

(A) 0.0004 (B) 0.04 (C) 0.4 (D) 0.004

7. If A_1 and A_2 are two independent events such that $P(A_1 \cup A_2) = 0.5$ and $P(A_1) = 0.2$ then $P(A_2) =$ ....

(A) $\frac{3}{8}$ (B) $\frac{3}{5}$ (C) $\frac{3}{4}$ (D) $\frac{3}{7}$

8. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively then P (X = 3) = ...

(A) $\frac{19}{32}$ (B) $\frac{13}{32}$ (C) $\frac{17}{32}$ (D) $\frac{7}{32}$

9. If $P(x) = c.\binom{4}{x}$; where $x = 0, 1, 2, 3, 4$ then $c =$ ....

(A) $\frac{1}{21}$ (B) $\frac{1}{16}$ (C) $\frac{1}{19}$ (D) $\frac{1}{22}$

10. Two unbiased coins are tossed. If one coin shows head, the probability that the other also shows head is ....

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$

11. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (10, 15), (15, 25), (0, 30). Let $z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of z occurs at both the points (15, 25) and (0, 30) is ....

(A) p : q = 2 : 1 (B) p : q = 1 : 1 (C) p : q = 2 : 3 (D) p : q = 1 : 3

12. For an LP problem the objective function $z = 3x + 2y$ the coordinates of the corner points of the bounded feasible region are A (3, 3), B (20, 3), C (20, 10), D(18, 12) and E (12, 12) the minimum value of z is ....

(A) 05 (B) 15 (C) 10 (D) 49

13. There is 2% error in measuring the period of a simple pendulum. The percentage error in length is .... (Where $T = 2\pi \cdot \sqrt{\frac{l}{g}}$)

(A) 6% (B) 8% (C) 2% (D) 4%

14. The rate of change of length (R) of diagonal of square with respect to its area (A) is .... (where A = 20 unit²)

(A) $\frac{1}{4\sqrt{5}}$ unit (B) $\frac{1}{\sqrt{10}}$ unit (C) $\frac{1}{5\sqrt{2}}$ unit (D) $\frac{1}{2\sqrt{10}}$ unit

15. $y = a \cdot e^x$, $y = b \cdot e^{-x}$ intersect at right angles if .... ($a \neq 0, b \neq 0.$)

(A) $a + b = 0$ (B) $a = b$ (C) $a = -\frac{1}{b}$ (D) $a = \frac{1}{b}$

16. $\int (\sqrt{e})^x \cdot \sin\left(\frac{x}{3}\right) \cdot dx$ = ... + c

(A) $\frac{(\sqrt{e})^x}{\sqrt{13}} \cos\left(2x - \tan^{-1}\frac{2}{3}\right)$ (B) $\frac{(\sqrt{e})^x}{\sqrt{13}} \cos\left(\frac{x}{2} - \tan^{-1}\frac{2}{3}\right)$

(C) $\frac{(\sqrt{e})^x}{\sqrt{13}} \sin\left(\frac{x}{2} - \cot^{-1}\frac{3}{2}\right)$ (D) $\frac{e^x}{\sqrt{13}} \sin\left(\frac{x}{2} - \tan^{-1}\frac{2}{3}\right)$

17. $\int \frac{1}{(x-2020)(x-2018)} \cdot dx = \text{..............} \ (x > 2020).$

(A) $\log|2x-4038|+c$ (B) $\frac{1}{2}\log\left|\frac{x-2018}{x-2020}\right|+c$

(C) $\log|(x-2020)(x-2018)|+c$ (D) $\frac{1}{2}\log\left|\frac{x-2020}{x-2018}\right|+c$

18. $\int\sqrt{\frac{x}{1-x^3}} \cdot dx = \text{..............} + c; \ (0 < x < 1).$

(A) $\frac{1}{2}\sin^{-1}\left(x^{\frac{3}{2}}\right)$ (B) $\frac{3}{2}\sin^{-1}\left(x^{\frac{3}{2}}\right)$ (C) $\frac{1}{3}\sin^{-1}\left(x^{\frac{2}{3}}\right)$ (D) $\frac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right)$

19. $\int_{-1}^{0} x \cdot |x| \cdot dx = \text{..............}$

(A) 0 (B) -1 (C) $-\frac{1}{3}$ (D) $\frac{1}{3}$

20. $\int_{n}^{n+1} f(x), dx = 2n-1; (n \geq 1)$, તો $\int_{1}^{11} f(x)\, dx = \text{..............}$

(A) 113 (B) 111 (C) 123 (D) 100

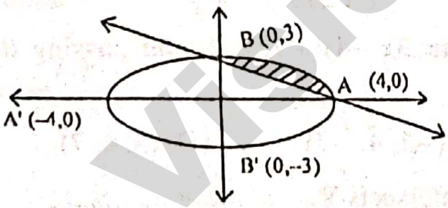
21. $\int_{0}^{4036} \frac{2^x}{2^x + 1^{4036-x}} dx = \text{..............}$

(A) 2018 (B) 4036 (C) 2017 (D) -2015

22. Area of the region bounded by the lines $x = 2y + 8 = 0, y = -3, y = -1$ and y-axis is .... units.

(A) 16 (B) 4 (C) 8 (D) 6

23. As shown in the figure of an ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. The area of the shaded region is ....



(A) 12π

(B) $3(\pi-2)$

(C) $4(\pi-2)$

(D) $12(\pi-2)$

24. Area of the region bounded by the parabola $y^2 = 8x$ and line $x + y = 0$ is ....

(A) $\frac{39}{2}$ (B) $\frac{32}{3}$ (C) $\frac{35}{2}$ (D) $\frac{37}{2}$

25. The order and degree of the differential equation $\sqrt{y_2} = \sqrt[3]{y_1}$ are .... respectively.

(A) 3 and 3 (B) 3 and 2 (C) 2 and 2 (D) 2 and 3

26. The general solution of the differential equation $y \cdot \frac{dy}{dx} = x \cdot \left(\frac{dy}{dx}\right)^2 + 1$ is ....

(A) $y = c - \frac{1}{cx}$; (c is an arbitary constant) (B) $y = cx - \frac{1}{c}$; (c is an arbitary constant)

(C) $y = cx + \frac{1}{c}$; (c is an arbitary constant) (D) $y = c + \frac{1}{cx}$; (c is an arbitary constant)

27. The particular solution of the differential equation $\sec^2 x \cdot \tan y \cdot dx + \sec^2 y \cdot \tan x \cdot dy = 0$; $y\left(\frac{\pi}{4}\right) = \frac{-\pi}{4}$ is ....

(A) $\tan x \cdot \tan y = \frac{\pi}{4}$ (B) $\tan x \cdot \tan y = c$ (C) $\tan x \cdot \tan y = -2$ (D) $\tan x \cdot \tan y = 1$

28. Unit vectors orthogonal to both (1, 2, 3) and (2, −1, 4) are ....

(A) $\pm\left(\frac{-11}{5\sqrt{6}}, \frac{-2}{5\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$ (B) $\pm\left(\frac{11}{5\sqrt{6}}, \frac{-2}{5\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

(C) $\pm\left(\frac{-11}{5\sqrt{6}}, \frac{2}{5\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ (D) $\pm\left(\frac{11}{5\sqrt{6}}, \frac{2}{5\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

29. If $\bar{a} = (-3, 1, 0)$ and $\bar{b} = (1, -1, -1)$ then $Comp_{\bar{a}}\bar{b}$ = ...

(A) $\frac{-\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{4}$ (C) $\frac{-4}{\sqrt{10}}$ (D) $\frac{4}{\sqrt{10}}$

30. The area of the parallelogram whose adjacent sides are $2\hat{i} + 3\hat{j}$ and $3\hat{i} + 4\hat{k}$ is ....

(A) 23 (B) 19 (C) 21 (D) 17

31. A (3, 1), B (2, 3) and C (5, 1) then $m\angle B$ = ...

(A) $\frac{\pi}{2}$ (B) $\cos^{-1}\left(\frac{7}{\sqrt{65}}\right)$ (C) $\sin^{-1}\left(\frac{5}{\sqrt{34}}\right)$ (D) $\cos^{-1}\left(\frac{3}{\sqrt{34}}\right)$

32. The measure of the angle between the line $\frac{x-2}{2} = \frac{2-y}{3} = \frac{z-1}{2}$ and plane $2x + y - 3z + 4 = 0$ is ....

(A) $\frac{\pi}{2}$ (B) $\cos^{-1}\left(\sqrt{\frac{213}{238}}\right)$ (C) $\sin^{-1}\left(\sqrt{\frac{213}{238}}\right)$ (D) $\sin^{-1}\left(\frac{7}{\sqrt{238}}\right)$

33. Direction of the line perpendicular to the plane $3x - 4y + 7z = 2$ and passing through (−1, 2, 4) is ....

(A) (−1, 2, 4) (B) (4, −6, 3) (C) (−3, 4, −7) (D) (3, 4, 7)

34. Line passing through (2, −3, 1) and (3, −4, −5) intersects YZ - plane at ....

(A) (−1, 0, 13) (B) (0, 1, −13) (C) (−1, 0, 19) (D) (0, −1, 13)

35. Lines L: $\frac{x+2}{3} = \frac{y-2}{-1}, z + 1 = 0$ and M: $\{(4 + 2k, 0, -1 + 3k) / k \in R\}$ then $L \cap M$ = ...

(A) (4, 0, −1) (B) (4, −1, 0) (C) (−1, 4, 0) (D) (0, 4, −1)

36. If set A = {x / x is a measure of an angle of scalen triangle}, then the number of equivalence relations containing (measure of minimum angle, measure of maximum angle) is ....

(A) 8 (B) 2 (C) 3 (D) 1

37. The number of binary operations on set $\{3k / 1 \le k \le n; k, n \in N\}$ is ....

(A) n^{2n} (B) n^{n^2} (C) n^3 (D) 2^n

38. જો $f: R \to R, f(x) = x^3, g: R \to R, g(x) = 3^x$ then $\{x / fog(x) = gof(x)\}$ = ...

(A) R (B) { } (C) $\{0, \sqrt{3}, -\sqrt{3}\}$ (D) $\{0, \sqrt{3}\}$

39. Range of $[\cos^{-1} x]$ is .... (where [ ] = greatest integer part)

(A) $\{0, 1, 2, 3\}$ (B) $[0, 3]$ (C) $\{1, 2, 3\}$ (D) $[0, \pi]$

40. $\sec^{-1}\left(\frac{5}{x}\right)+\sin^{-1}\left(\frac{4}{5}\right)=\frac{\pi}{2};\ x \neq 0$ then $x =$ ....

(A) 4 (B) 3 (C) 5 (D) 1

41. If $2.\cos(2\tan^{-1} x)=1$ then $x =$ ....

(A) $\sqrt{3}$ (B) $1-\sqrt{3}$ (C) $1-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{3}}$

42. If $0<x<1$ and if $\tan^{-1}(1-x)$, $\tan^{-1} x$ and $\tan^{-1}(1+x)$ are in arithmetic progression then x^3 = ...

(A) x^2 (B) x^2-1 (C) $1-x^2$ (D) $1+x^2$

43. If area of a triangle whose vertices are (8, 2), (*k*, 4) and (6, 7) is 13 units then the possible integer value of k is ....

(A) 3 (B) 1 (C) 2 (D) 0

44. If $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & b^2c & 0 \end{vmatrix} = m(abc)^k$, then $m+k =$ ....

(A) 5 (B) 3 (C) 0 (D) 2

45. $\begin{vmatrix} \sin\left(\frac{2\pi}{9}\right) & \cos\left(\frac{2\pi}{9}\right) \\ \sin\left(\frac{5\pi}{18}\right) & \cos\left(\frac{5\pi}{18}\right) \end{vmatrix} =$ ....

(A) $\sin\left(\frac{\pi}{18}\right)$ (B) $\tan\left(\frac{\pi}{4}\right)$ (C) $\cot\left(\frac{3\pi}{4}\right)$ (D) $-\sin\left(\frac{\pi}{18}\right)$

46. If $A=[x\ y\ z]$; $B=\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$; $C=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and (AB) C is $m \times n$ matric then ....

(A) $m+n=5$ (B) $m<n$ (C) $m=n$ (D) $m>n$

47. If $\begin{bmatrix} a_1+a_2 & 4 \\ 3 & a_3+a_4 \end{bmatrix} - \begin{bmatrix} 3a_2 & 3a_1 \\ 3a_4 & 3a_3 \end{bmatrix} = \begin{bmatrix} -6 & -a_1 \\ -2a_4 & 1 \end{bmatrix}$ then $\sum_{i=1}^{4} a_i =$ ....

(A) 16 (B) 10 (C) 12 (D) 8

48. $A=\begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$ then $A^{-1} =$ ....

(A) $\begin{bmatrix} \frac{4}{23} & \frac{3}{23} \\ \frac{5}{23} & \frac{2}{23} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{4}{23} & \frac{-3}{23} \\ \frac{-5}{23} & \frac{2}{23} \end{bmatrix}$ (C) $\begin{bmatrix} \frac{-4}{23} & \frac{-3}{23} \\ \frac{-5}{23} & \frac{-2}{23} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{4}{23} & \frac{3}{23} \\ \frac{-5}{23} & \frac{2}{23} \end{bmatrix}$

49. $f(x)=\begin{cases}\dfrac{\sin 5x \cdot \tan kx}{x^2}; & x \neq 0,\\ 1 & ; x=0\end{cases}$ is continuous at $x=0$ then $k=$ .... ; $(k \neq 0)$

(A) $\frac{5}{3}$ (B) $\frac{1}{5}$ (C) $\frac{1}{15}$ (D) $\frac{1}{3}$

50. $\left\{\frac{d}{dx}(Sec\, x^{0})\right\}_{(x=30^0)}=$ ....

(A) $\frac{\pi}{90}$ (B) $\frac{\pi}{240}$ (C) $\frac{\pi}{270}$ (D) $\frac{\pi}{180}$

JULY-2018 (050) (E)

Time : 2 Hour **PART – B** **Maximum Marks : 50**

Instructions :

1. Write in a clear legible handwriting.
2. There are three section in part – B of the question paper and total 1 to 18 question are there.
3. All the questions are compulsory. Internal option are given.
4. The number at right side represent the marks of the question.
5. Start new section on new page.
6. Maintain sequence.
7. Use of simple calculator and log table is allowed, if required.

SECTION : A

- **Question Nos. 1 to 8 do as directed. Each question carries 2 marks.** **[16]**

1. For the function $f: R-\left\{-\frac{2}{3}\right\} \rightarrow R,\ f(x)=\frac{4x+3}{6x+4}$ If $fof(a)=1$ then find a.

2. Define binary operation $*$ on R by $m * n = m + n - (mn)^2$. Then find possible inverses of 1 $(m, n \in R)$

3. Evaluate : $\int \frac{dx}{\cos x - \cos 3x}$ **OR** 3. Evaluate : $\int \frac{x^2-1}{x^4+6x^3+1}dx$

4. Find the number of times a fair coin must be tossed so that the probability of getting at least one tail is at least 0.95.

5. Maximize $z=x+y$ subject to $x+y \leq 1, -3x+y \geq 3$ and $x \geq 0, y \geq 0$, if possible.

6. Where does the normal to $x^2-xy+y^2=3$ at $(-1, 1)$ intersect the curve again ?

OR

6. The position of a particle is given by $S=f(t)=t^3-6t^2+9t$, S is in meters, t is in seconds. Then find the distance travelled in first 5 seconds.

7. Solve the differential equation : $x \cdot \frac{dy}{dx} - y + x \cdot \sin\left(\frac{y}{x}\right)=0$

8. If the length of the subnormal of a curve is constant and such curve passes through the origin then find its equation.

SECTION : B

- **Question Nos. 9 to 14 do as directed. Each question carries 3 marks.** **[18]**

9. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$, then prove that

$$x\cdot\sqrt{1-x^2}+y\cdot\sqrt{1-y^2}+z\cdot\sqrt{1-z^2}=2xyz$$

10. Prove that : $\begin{vmatrix}(y+z)^2 & xy & zx\\ xy & (z+x)^2 & yz\\ zx & yz & (x+y)^2\end{vmatrix}=2xyz\cdot(x+y+z)^3$

11. If the following system of equations has unique solution then find the solution set using matrix.

$$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10$$

$$\frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13 \quad (xyz\neq 0)$$

12. A cylinder is inscribed in a sphere of radius R. Prove that its volume is maximum if its height is $\frac{2R}{\sqrt{3}}$.

OR

12. Determine maximum and minimum values of $f(x)=x-2\cos x;\ x\in[-\pi,\pi]$

13. Find the area of the region enclosed by two parabolas $y^2=4ax$ and $x^2=4by$. $(a>0,b>0)$

OR

13. Find the area of the region : $\{(x,y)/0\le y\le x^2, 0\le y\le x+2, 0\le x\le 4\}$

14. $\bar{a}$ is a unit vector and $\bar{b}=(3,0,-4)$. The measure of the angle between them is $\frac{3\pi}{8}$. If the diagonals of the parallelogram are $(3\bar{a}+\bar{b})$ and $(\bar{a}+3\bar{b})$, then obtain the area of the parallelogram.

SECTION : C

- **Question Nos. 15 to 18 do as directed. Each question carries 4 marks.** **[16]**

15. If $x=(a+bx)\cdot e^{\frac{y}{x}}$, prove $xy_2=\left(y_1-\frac{y}{x}\right)^2$, $(a>0,b>0,x\in R^+)$ **OR**

15. Find value of C when the mean value theorem is applied to $f(x)=2\sin x+\sin 2x$; $[0,\pi]$

16. Evaluate : $\int(x+2)\sqrt{\frac{x+3}{x-3}}\cdot dx;\ (x>3)$

17. Prove that : $\int_0^{\frac{\pi}{2}}\log\sin x\cdot dx=\frac{-\pi}{2}\log 2$ **OR** 17. Evaluate : $\int_{-\frac{1}{2}}^{\frac{1}{2}}\left\{\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right\}^{\frac{1}{2}}\cdot dx$

18. Find the equation of the plane passing through the intersecting of the planes $2x+3y+z-1=0$ and $x+y-z-7=0$ and also passing through the point (1, 2, 3). Also obtain the equation of the line of these planes.

QUESTION PAPER – 1 – SOLUTION (050) (E) (JULY - 2018)

PART – A

1. (A) $-\frac{1}{2at^3}$

→ **Hint :**

$x = at^2$ and $y = 2at, (t \neq 0)$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$

(Derivative of parametric function)

$\therefore \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{1}{t}\right) \cdot \frac{dt}{dx}$

$= -\frac{1}{t^2}\left(\frac{1}{2at}\right) = \frac{-1}{2at^3}$

2. (D) $\frac{-1}{x \cdot \sin x + \cos x}$

→ **Hint :**

$I = \int \frac{x(x\sin x + \cos x)^2}{\sec x} dx$

$= \int \frac{x\cos x}{(x\sin x + \cos x)^2} dx$

Take $x\sin x + \cos x = t$

Now differential w.r. to t

$\therefore (x\cos x + \sin x - \sin x)\, dx = dt$

$\therefore x\cos x\, dx = dt$

$\therefore I = \int \frac{dt}{t^2} = -\frac{1}{t} + c$

$\therefore I = -\frac{1}{x\sin x + \cos x} + c$

3. (B) $\frac{2}{3}$

→ **Hint :**

$f'(x) = 3\sin x - 4\sin^3 x$

Now integrate both sides term wise

$\therefore f(x) = 3\int \sin x - 4\int \sin^3 x\, dx$

$= -3\cos x - 4\int \sin x(1 - \cos^2 x) dx$

Take $\cos x = t$

Now differentiate w.r. to t

$\therefore -\sin x\, dx = dt$

$= -3\cos x + 4\int (1 - t^2)\, dt$

$= -3\cos x + 4\left(t - \frac{t^3}{3}\right) + c$

$f(x) = -3\cos x + 4\left(\cos x - \frac{\cos^3 x}{3}\right) + c$

$f(0) = -3\cos 0 + 4\left(\cos 0 - \frac{\cos^3 0}{3}\right) + c$

$\therefore \frac{1}{3} = -3 + 4\left(1 - \frac{1}{3}\right) + c$

$\frac{1}{3} = -3 + \left(4\left(\frac{2}{3}\right)\right) + c$

$= \frac{-9+8}{3} + c$

$\therefore \frac{1}{3} = -\frac{1}{3} + c$

$\therefore c = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

4. (D) $\frac{1}{5}\log|x^5 + 5^x|$

→ **Hint :**

Take $x^5 + 5^x = t$

Now differentiate w.r. to t

$\therefore (5x^4 + 5^x \cdot \log_e{}^5)\, dx = dt$

$\therefore 5(x^4 + 5^{x-1}\log_e{}^5)\, dx = dt$

$\therefore \left(x^4 + 5^{x-1}\log 5\right) dx = \frac{1}{5} dt$

$\therefore I = \int \frac{x^4 + 5^{x-1} \cdot \log_e{}^5}{x^5 + 5^x} dx$

$= \frac{1}{5}\int \frac{1}{t} dx$

$= \frac{1}{5}\log|t| + c$

$\therefore I = \frac{1}{5}\log|x^5 + 5^x| + c$

5. (B) $\frac{1}{100}\log\left|\frac{x^{100}}{x^{100}+1}\right|$

→ **Hint :**

$$I=\int\frac{1}{x(x^{100}+1)}$$

Take $x^{100}+1=t$

$\therefore\quad x^{100}=t-1$ ...... (i)

Now differentiate w.r. to t

$\therefore\quad 100x^{99}dx=dt$

$$dx=\frac{1}{100x^{99}}dt$$

$$\therefore\ I=\frac{1}{100}\int\frac{1}{x^{99}\cdot x(t)}dt$$

$$=\frac{1}{100}\int\frac{1}{x^{100}\cdot t}dt$$

$$=\frac{1}{100}\int\frac{1}{t(t-1)}dt\quad(\because\text{ using result (i)})$$

$$=\frac{1}{100}\int\frac{1-t+t}{t(t-1)}dt$$

$$=\frac{1}{100}\int\frac{t-(t-1)}{t(t-1)}dt$$

$$=\frac{1}{100}\int\left(\frac{1}{t-1}-\frac{1}{t}\right)dt$$

$$=\frac{1}{100}\log\left|\frac{t-1}{t}\right|+c$$

$$\therefore\ I=\frac{1}{100}\int\frac{x^{100}}{x^{100}+1}+c$$

Note : Take $n=100$ in integral

$$\int\frac{1}{x(x^n+1)}dx=\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+c$$

we get correct answer.

6. (C) 0.4

→ **Hint :**

Here $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore\quad 0.65=0.25+0.55-P(A\cap B)$

$\therefore\quad P(A\cap B)=0.80-0.65=0.15$

Now $P(B'|A)=1-P(B|A)$

$$\therefore\quad P(B'|A)=1-\frac{P(B\cap A)}{P(A)}$$

$$=1-\frac{0.15}{0.25}$$

$$=1-\frac{3}{5}=\frac{2}{5}=\frac{4}{10}=0.4$$

7. (A) $\frac{3}{8}$

→ **Hint :**

$P(A_1\cup A_2)=P(A_1)+P(A_2)-P(A_1\cap A_2)$

$\therefore\quad 0.5=0.2+A(A_2)-P(A_1)\cdot P(A_2)$

$\therefore\quad 0.5-0.2=P(A_2)-[(1-P(A_1)]$

$\therefore\quad 0.3=P(A_2)[1-0.2]$

$\therefore\quad 0.3=P(A_2)(0.8)$

$\therefore\quad \frac{3}{8}=P(A_2)$

8. (D) $\frac{7}{32}$

→ **Hint :**

Mean $=np$

$\therefore\ np=4$

$n\left(\frac{1}{2}\right)=4$

$\therefore\ n=8$

Variance $=npq$

$npq=2$

$\therefore\ 4q=2$

$\therefore\ q=\frac{1}{2}$

$\therefore\ p=1-q$

$=1-\frac{1}{2}=\frac{1}{2}$ મળે.

Now $p(x)=\binom{n}{x}p^x\cdot q^{n-x}$

$$\therefore\ P(3)=\binom{8}{3}\cdot\left(\frac{1}{2}\right)^3\cdot\left(\frac{1}{2}\right)^{8-3}$$

$$=\frac{{}_8P_3}{3!}\cdot\left(\frac{1}{2}\right)^3\cdot\left(\frac{1}{2}\right)^5$$

$$=\frac{8\cdot7\cdot6}{1\cdot2\cdot3}\left(\frac{1}{8}\right)\left(\frac{1}{32}\right)$$

$$=56\cdot\frac{1}{8}\cdot\frac{1}{32}=\frac{7}{32}$$

9. (C) $\frac{1}{19}$

→ **Hint :**

Here

$P(0)+P(1)+P(2)+P(3)+P(4)=1$

$\therefore\ C\left\{\binom{4}{0}+\binom{4}{1}+\binom{4}{2}+\binom{4}{3}+\binom{4}{4}\right\}=1$

$\because \binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots\ldots+\binom{n}{n}=2^n$

$\therefore\ C\{2^4\}=1$

$\therefore\ 16\cdot C=1$

$\therefore\ C=\frac{1}{16}$

10. (B) $\frac{1}{2}$

→ **Hint :**

S = Sample space of tossing two coins together

= {HH, HT, TT, TH}

Event A = Head is obtained on first coin

= {HH, HT}

$\therefore P(A)=\frac{2}{4}=\frac{1}{2}$

Event B = Head is obtained on second coin

= {HH, TH}

$\therefore P(B)=\frac{2}{4}=\frac{1}{2}$

And $(A\cap B)=\{HH\}$

$\therefore P(A\cap B)=\frac{1}{4}$

Event B/A = Head is obtained on second when first coin has head

$$\therefore P(B\,|\,A)=\frac{P(B\cap A)}{P(A)}=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$$

11. (D) p : q = 1 : 3

→ **Hint :**

$z=px+qy$

$\therefore$ z has maximum value for (15, 25) and (0, 30)

$\therefore\ 15p+25q=0+30q$

$\therefore\ 15p=5q$

$\therefore\ 3p=q$

$\therefore\ \frac{p}{q}=\frac{1}{3}$

$\therefore\ p:q=1:3$

12. (B) 15

→ **Hint :**

Out of the given corner points. A has small co-ordinates (3, 3)

$\therefore$ Maximum value of $z=3x+2y$ exists at A (3, 3)

$\therefore\ z=3(3)+2(3)$

$\therefore\ z=9+6$

$\therefore\ z=15$

Minimum value of z is 15.

13. (D) 4%

→ **Hint :**

Suppose periodic time of simple pendulum is T and its length is l

ΔT = Error in Periodic time

= 2%

$=\frac{2T}{100}$

Now $T=2\pi\left(\sqrt{\frac{l}{g}}\right)$

$\therefore\ T^2=4\pi^2\cdot\frac{l}{g}$

$\therefore\ l=\frac{gT^2}{4\pi^2}$

$\therefore\ \frac{dl}{dT}=\frac{2gT}{4\pi^2}$

$\therefore$ Error in length =

$\frac{dl}{dT}\cdot\Delta T=\frac{2gT}{4\pi^2}\times\frac{2T}{100}$

$=\frac{gT^2}{\pi^2(100)}$

$=\frac{g}{\pi^2}\times\frac{4\pi^2 l}{g}\times\frac{1}{100}$

SA - I
SA - II
MCQ's

$= \frac{4l}{100} = 4\%$

Error in length = 4%

14. (D) $\frac{1}{2\sqrt{10}}$ unit

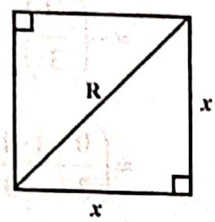
→ **Hint :**

Suppose length of side of square is x

$\therefore \quad x^2 + x^2 = R^2$

$\therefore \quad 2x^2 = R^2$

$\therefore \quad x^2 = \frac{R^2}{2}$



Now area of square $A = x^2$

$\therefore \quad A = \frac{R^2}{2}$ ..... (i)

$\therefore \quad 20 = \frac{R^2}{2}$

$\therefore \quad R^2 = 40$

$\therefore \quad R = 2\sqrt{10}$

$2A^2 = R^2$ (From result (i))

$\therefore \quad 4A\frac{dA}{dR} = 2R$

$\therefore \quad \frac{dA}{dR} = \frac{R}{2A} = \frac{2\sqrt{10}}{2(20)}$

$= \frac{\sqrt{10}}{10 \times 2} = \frac{1}{2\sqrt{10}}$

$\therefore$ Rate of change in diagonal R w.r. to area A $= \frac{1}{2\sqrt{10}}$ unit

15. (D) $a = \frac{1}{b}$

→ **Hint :**

m_1 = Slope of curve $y = ae^x$

$\therefore \quad m_1 = \frac{dy}{dx} = ae^x$

$\therefore \quad m_1 = ae^x$

m_2 = Slope of curve $y = be^{-x}$

$\therefore \quad m_2 = \frac{dy}{dx} = -be^{-x}$

$\therefore \quad m_2 = -be^{-x}$

Now given curves are orthogonal so product of their slopes is -1.

$\therefore \quad m_1 m_2 = -1$

$(ae^x)(-be^{-x}) = -1$

$\therefore \quad ab = 1$

$\therefore \quad a = \frac{1}{b}$

16.

→ **Hint :**

$$I = \int \left(\sqrt{e}\right)^x \sin\left(\frac{x}{3}\right) dx$$

$$= \int e^{\left(\frac{x}{2}\right)} \sin\left(\frac{x}{3}\right) dx$$

Now compare given integral with

$$I = \int e^{ax} \sin(bx + k)\, dx$$

$$= \frac{e^{ax}}{\sqrt{a2}} \sin\left(bx + k - \tan^{-1}\left(\frac{b}{a}\right)\right) + c$$

$\therefore$ Here, we have $a = \frac{1}{2}, b = \frac{1}{3}$ and $k = 0$

$\therefore \quad a^2 + b^2 = \frac{1}{4} + \frac{1}{9} = \frac{9+4}{36} = \frac{13}{36}$

$\therefore \quad \sqrt{a^2 + b^2} = \frac{\sqrt{13}}{6}$

$$\therefore \quad I = \frac{6 \cdot e^{\frac{x}{2}}}{\sqrt{13}} \sin\left(\frac{1}{3} \cdot x - \tan^{-1}\left(\frac{\frac{1}{3}}{\frac{1}{2}}\right)\right) + c$$

$$= \frac{6 \cdot \left(\sqrt{e}\right)^x}{\sqrt{13}} \sin\left(\frac{x}{3} - \tan^{-1}\left(\frac{2}{3}\right)\right) + c$$

Note : There is an error in given options.

17. (D) $\frac{1}{2}\log\left|\frac{x - 2020}{x - 2018}\right| + c$

→ **Hint :**

$$I = \int \frac{1}{(x - 2000) \cdot (x - 2018)} dx$$

$$=\frac{1}{2}\int\frac{2}{(x-2000)(x-2018)}dx$$

$$=\frac{1}{2}\int\frac{(x-2018)-(x-2000)}{(x-2000)(x-2018)}dx$$

$$=\frac{1}{2}\left\{\int\left(\frac{1}{x-2000}-\frac{1}{x-2018}\right)dx\right\}$$

$$=\frac{1}{2}\{\log|x-2000|-\log|x-2018|\}\,dx$$

$$=\frac{1}{2}\{\log|x-2000|-\log|x-2018|\}\,dx$$

18. (D) $\frac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right)$

→ **Hint :**

$$I=\int\sqrt{\frac{x}{1-x^3}}\,dx,\ 0<x<1$$

$$=\int\frac{\sqrt{x}}{\sqrt{1-x^3}}\,dx$$

Take $x^{\frac{3}{2}}=t$

$$\therefore\ \frac{3}{2}\left(x^{\frac{1}{2}}\right)dx=dt$$

$$\therefore\ \sqrt{x}\,dx=\frac{2}{3}dt$$

and $x^{\frac{3}{2}}=t\Rightarrow x^3=t^2$

$$I=\frac{2}{3}\int\frac{dt}{\sqrt{1-t^2}}=\frac{2}{3}\sin^{-1}(t)+c$$

$$\therefore I=\frac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right)$$

19. (C) $-\frac{1}{3}$

→ **Hint :**

$$I=\int_{-1}^{0}x\cdot|x|\,dx$$

Here $x\in(-1,0)$

$\therefore$ x is negative.

$\therefore$ we have $|x|=-x$

$$I=\int_{-1}^{0}x\cdot(-x)\,dx$$

$$=-\int_{-1}^{0}x^2\,dx$$

$$=-\left(\frac{x^3}{3}\right)_{-1}^{0}$$

$$=\left(\frac{0}{3}-\frac{(-1)^3}{3}\right)$$

$$=-\left(-\frac{(-1)}{3}\right)$$

$$=-\frac{1}{3}$$

$$\therefore\ I=\frac{-1}{3}$$

20. (D) 100

→ **Hint :**

Given that $\int_{n}^{n+1}f(x)\,dx=2n-1$

Now $\int_{1}^{11}f(x)\,dx$

$$=\int_{1}^{2}f(x)\,dx+\int_{2}^{3}f(x)+\int_{3}^{4}f(x)\,dx+\ldots+\int_{10}^{11}f(x)\,dx$$

(10 term)

$=1+3+5+7+\ .....+19$ (10 term)

$=(10)^2$ $(\because 1+3+5+7+\ldots+n=n^2, n\in N)$

$=100$

21. (A) 2018

→ **Hint :**

$$I=\int_{0}^{4036}\frac{2^x}{2^x+2^{4036-x}}dx \quad \text{.... (i)}$$

$$I=\int_{0}^{4036}\frac{2^{4036-x}}{2^{4036-x}+2^{4036-(4036-x)}}dx$$

$$\left(\because \int_{0}^{a}f(x)\,dx=\int_{0}^{a}f(a-x)\,dx\right)$$

$$\therefore\ I=\int_{0}^{4036}\frac{2^{4036-x}}{2^{4036-x}+2^x}dx \quad \text{.... (ii)}$$

Take result (i) + result (ii)

$$2I=\int_{0}^{4036}\left(\frac{2^x}{2^x-2^{4036-x}}+\frac{2^{4036-x}}{2^{4036-x}+2^x}\right)dx$$

$$= \int_0^{4036} 1\,dx$$

$$= [x]_0^{4036}$$

$\therefore 2I = 4036$

$I = 2018$

22. (C) 8

→ **Hint :**

$x + 2y + 8 = 0$

$\therefore\ x = -2y - 8$

$\therefore$ Required area $A = \left|\int_{-3}^{-1} x\,dy\right|$

$$= \left|\int_{-3}^{-1} (-2y-8)\,dy\right|$$

$$= \left|-y^2 - 8y\right|_{-3}^{-1}$$

$$= \left|\left(-(-1)^2 - 8(-1)\right) - \left(-(-3)^2 - 8(-3)\right)\right|$$

$$= |(-1+8) - (-9+24)|$$

$$= |(-1+8) - (-9+24)|$$

$$= |7 - 15|$$

$$= |-8|$$

$\therefore A = 8$

23. (B) $3(\pi - 2)$

→ **Hint :**

As per Note given in text book in illustration (5) page No. 141 the area of the region between ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and line $\frac{x}{a} + \frac{y}{b} = 1$ is $\frac{(\pi-2)\,a \cdot b}{4}$

Here $a = 4$ and $b = 3$

$\therefore$ Required area $= \frac{(\pi-2)\,(4)\cdot(3)}{4}$

$= 3(\pi - 2)$

24. (B) $\frac{32}{3}$

→ **Hint :**

First we find point of intersection of line $x + y = 0$ and curve $y^2 = 8x$

$x + y = 0 \Rightarrow y = -x$

$\therefore\ y^2 = 8x$

$\therefore\ x^2 = 8x$

$\therefore\ x^2 - 8x = 0$

$\therefore\ x(x-8) = 0$

$x = 0$ and $x = 8$

$\therefore$ If $x = 0$ then $y = 0$

and if $x = 8$ then $y = -8$

$\therefore$ Required area $A = |I_1| - |I_2|$

Where $I_1 = \int_0^8 \sqrt{8x}\,dx$

$$= 2\sqrt{2}\int_0^8 x^{\frac{1}{2}}\,dx$$

$$= 2\sqrt{2}\left[\left(x^{\frac{3}{2}}\right)\frac{2}{3}\right]_0^8$$

$$= \frac{4\sqrt{2}}{3}\left(x^{\frac{3}{2}}\right)_0^8$$

$$= \frac{4\sqrt{2}}{3}(8)^{\frac{3}{2}}$$

$$= \frac{4\sqrt{2}}{3}\left(8 \cdot 8^{\frac{1}{2}}\right)$$

$$= \frac{32}{3}\sqrt{2}\cdot(2\sqrt{2})$$

$$= \frac{64 \times 2}{3} = \frac{128}{3}$$

Also $I_2 = -\int_0^8 x\,dx$

$$= \left(-\frac{x^2}{2}\right)_0^8$$

$$= \frac{-(8)^2}{2} = \frac{-64}{2} = -32$$

$\therefore$ Required area $= |I_1| - |I_2|$

$$= \frac{128}{3} - |-32|$$

$$= \frac{128}{3} - 32$$
$$= \frac{128-96}{3}$$
$$= \frac{32}{3} \text{ Sq. Unit}$$

25.

→ **Hint :**

$\sqrt{y_2} = \sqrt[3]{y_1}$

$\therefore \quad y_2^{\frac{1}{2}} = (y_1)^{\frac{3}{2}}$

Now square on both sides

$\therefore \quad y_2 = y_1^3$

$\therefore \quad \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^3$

Given differential equation has second order derivative.

∴ Its order is 2 and its highest power is 1.

∴ Its degree is 1

Note : There is an error in question.

26. (C) $y = cx + \frac{1}{c}$;

(Where c is arbitrary Constant)

→ **Hint :**

Out of given option we differentiate option (C)

$y = cx + \frac{1}{c}$;

$\therefore \quad \frac{dy}{dx} = c$

Substitute this value in given Equation

$\therefore$ L.H.S. $y\frac{dy}{dx}$

$= \left(cx + \frac{1}{c}\right) \cdot c$

$= c^2x + 1$

R.H.S. $x\left(\frac{dy}{dx}\right)^2 + 1$

$= x \cdot c^2 + 1$

∴ L.H.S. = R.H.S.

∴ General solution of the differential

Equation is $y = cx + \frac{1}{c}$

27. (D) $\tan x \cdot \tan y = 1$

→ **Hint :**

$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dx = 0$

Devide both sides by $\tan x \tan y$

$\therefore \quad \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$

$\therefore \quad \int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy + \log c$

$\therefore \quad \int \frac{f'(x)}{f(x)} dx = -\int \frac{f'(y)}{f(y)} dy + \log c$

Where $f(x) = \tan x$, $f(y) = \tan y$

$\therefore \quad \log|f(x)| = \log|f(y)| + \log c$

$\therefore \quad \log \tan x = -\log \tan y + \log c$

$\therefore \quad \log \tan (x) + \log \tan y = \log c$

$\therefore \quad \tan x \cdot \tan y = c$

Given that $y\left(\frac{\pi}{4}\right) = \frac{-\pi}{4}$

$\therefore$ Take $x = \frac{\pi}{4}$ and $y = \frac{-\pi}{4}$

$\therefore \quad \tan\left(\frac{\pi}{4}\right) \cdot \tan\left(\frac{-\pi}{4}\right) = c$

$\therefore \quad 1(-1) = c$

$\therefore \quad c = -1$

$\therefore \quad -c = 1$

$\therefore \quad c' = 1$

$\therefore \quad c = 1$ Where take constant $c' = c$

∴ Perticular solution is $\tan x \cdot \tan y = 1$

28. (D) $\pm\left(\frac{11}{5\sqrt{6}}, \frac{2}{5\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

→ **Hint :**

$\bar{x} = (1, 2, 3)$

$\bar{y} = (2, -1, 4)$

$$\therefore \quad \bar{x} \times \bar{y} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{vmatrix}$$

$= i(8+3) - j(4-6) + k(-1-4)$

$= 11i + 2j - 5k$

$= (11, 2, -5)$

$\therefore \ |\bar{x} \times \bar{y}| = \sqrt{121+4+25}$

$= \sqrt{150}$

$= 5\sqrt{6}$

$\therefore$ Unit Vector to $\bar{x}$ and $\bar{y}$

$= \pm \frac{(\bar{x} \times \bar{y})}{|(\bar{x} \times \bar{y})|}$

$= \pm \frac{(11, 2, -5)}{|5\sqrt{6}|}$

$= \pm \left(\frac{11}{5\sqrt{6}}, \frac{2}{5\sqrt{6}}, \frac{-1}{\sqrt{6}} \right)$

29. (C) $\frac{-4}{\sqrt{10}}$

→ **Hint :**

$\bar{a} = (-3, 1, 0)$

$\bar{b} = (1, -1, -1)$

$\text{Com}_{\bar{a}} \bar{b} = \frac{\bar{b} \cdot \bar{a}}{|\bar{a}|}$

$= \frac{(1, -1, -1) \cdot (-3, 1, 0)}{\sqrt{9+1}}$

$= \frac{-3-1+0}{\sqrt{10}}$

$= \frac{-4}{\sqrt{10}}$

30. (D) 17

→ **Hint :**

Take $\bar{a} = 2\hat{i} + 3j$

$\therefore \ \bar{a} = (2, 3, 0)$

and take $\bar{b} = 2\hat{i} + 4\hat{k}$

$= (3, 0, 4)$

$\therefore$ Required area $= |\bar{a} \times \bar{b}|$

$$\bar{a} \times \bar{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 3 & 0 & 4 \end{vmatrix}$$

$= i(12) - j(8) + k(-9)$

$= (12, -8, -9)$

$\therefore \ |\bar{a} \times \bar{b}| = \sqrt{144+64+81}$

$= \sqrt{289}$

$= 17$ unit

31.

→ **Hint :**

$\bar{a} = \vec{BC}$

$= C - B$

$= (5-2, 1-3)$

$= (3, -2)$

And $\bar{c} = \vec{AB} = B - A = (2, 3) - (3, 1)$

$= (2-3, 3-1)$

$= (-1, 4)\ (-1, 2)$

Now $B = \bar{a} \wedge \bar{c}$

$\therefore \ \cos B = \frac{\bar{a} \cdot \bar{c}}{|\bar{a}| \cdot |\bar{c}|}$

$= \frac{(3, -2) \cdot (-1, 2)}{\sqrt{9+4} \cdot \sqrt{1+4}}$

$= \frac{-3-4}{\sqrt{13} \cdot \sqrt{5}}$

$= \frac{-7}{\sqrt{65}}$

$B = \cos^{-1}\left(\frac{-7}{\sqrt{65}} \right)$

Note : There is an error in option B.

Option (B) will be either $\cos^{-1}\left(\frac{-7}{\sqrt{65}} \right)$

Or $\pi - \cos^{-1}\left(\frac{-7}{\sqrt{65}} \right)$.

32. (B) $\cos^{-1}\left(\sqrt{\frac{213}{238}} \right)$

→ **Hint :**

From the Equation of line direction will be $\bar{l} = (2, -3, 2)$

And from Equation of plane Normal will be $\bar{n} = (2, 1, -3)$

$\therefore\ \bar{l}\cdot\bar{n}=(2,-3,2)\cdot(2,1,-3)$

$=2(2)+(-3)(1)+2(-3)$

$=4-3-6$

$=-5$

Now $|\bar{l}|=\sqrt{4+9+4}=\sqrt{17}$

$|\bar{n}|=\sqrt{4+1+9}=\sqrt{14}$

Angle between plane and line is

$$\sin\theta=\frac{|\bar{l}\cdot\bar{n}|}{|\bar{l}|\cdot|\bar{n}|}=\frac{|-5|}{\sqrt{17}\cdot\sqrt{14}}$$

$$\therefore\ \sin\theta=\frac{5}{\sqrt{17\cdot14}}=\frac{5}{\sqrt{238}}$$

$\therefore\ \cos^2\theta=1-\sin^2\theta$

$$=1-\frac{25}{238}$$

$$=\frac{238-25}{238}=\frac{213}{238}$$

$$\therefore\ \cos\theta=\sqrt{\frac{213}{238}}$$

$\therefore$ Angle between plane and line will be

$$\theta=\cos^{-1}\left(\sqrt{\frac{213}{238}}\right)$$

33. (C) $(-3, 4, -7)$

→ **Hint :**

Line is always perpendicular to plane.

$\therefore$ Normal $\bar{n}=(3,-4,7)$ will be direction of line.

$\therefore$ Direction of line $\bar{l}=(3,-4,7)$

OR $\bar{l}=(-3,4,-7)$

34. (D) $(0, -1, 13)$

→ **Hint :**

We get Equation of line passes from $A(\bar{a})=(2,-3,1)$ and $B(\bar{b})=(3,-4,-5)$ using $\bar{r}=\bar{a}+k(\bar{b}-\bar{a})$

$\therefore\ \bar{r}=(2,-3,1)+k(3-2,-4+3,-5-1)$

$=(2,-3,1)+k(1,-1,-6)$

For YZ Plane take $(0, y, z)$

$\therefore\ (0,y,z)=(2,-3,1)+k(1,-1,-6)$

$=(2+k,-3-k,1-6k)$

$\therefore\ 2+k=0$

$\therefore\ k=-2$

$\therefore\ y=-3-k=-3+2=-1$

and $z=1-6k=1-6(-2)=1+12=13$

$\therefore\ (0,-1,13)$ is the point of YZ Plane

$\therefore$ Line intersect Plane at $(0,-1,13)$

35. (A) $(4, 0, -1)$

→ **Hint :**

Vector Equation of line L is follows

$\bar{l}=(-2,2,-1)+k(3,-1,0)$ .... (i), $k\in R$

And vector Equation of line M will be

$\bar{r}=(4,0,-1)+\lambda(2,0,3)$ .... (ii)

where $\lambda\in R$

Suppose line intersect at (x, y, z)

$\therefore\ (x,y,z)=(-2,2,-1)+k(3,-1,0)$

$(x,y,z)=(-2+3k,2-k,-1+0)$ ....(A)

And

$(x,y,z)=(4,0,-1)+(2\lambda,0,-1+3\lambda)$

$\therefore\ (x,y,z)=(4+2\lambda,0,-1+3\lambda)$ ....(B)

From result (A) and (B)

we gets $2-k=0$

$\therefore\ k=2$

Substitute in result (i)

$\therefore$ Point of intersection is

$(x,y,z)=(-2+3(2),2-2,-1+0)$

$(4,0,-1)$

36. (A) 8

→ **Hint :**

Number of Equivalense relation $=2^3=8$

37. (B) n^{n^2}

→ **Hint :**

Given set has n elements

$\therefore$ Total number of binary operations are n^{n^2}

38. (C) $\{0,\sqrt{3},-\sqrt{3}\}$

→ **Hint :**

Here $f(x)=x^3$ and $g(x)=3^x$

$\therefore\ fog(x)=f(g(x))$

$=f(3^x)$

$=(3^x)^3=3^{3x}$

And $gof(x) = g(f(x))$

$= g(x^3)$

$= 3^{x^3}$

Also $fog(x) = gof(x)$

$\therefore\ 3^{3x} = 3^{x^3}$

$\therefore\ 3x = x^3$

$\therefore\ x^2 = 3$

$\therefore\ x = \pm\sqrt{3}$

$\therefore\ x = 0,$ and $x = \pm\sqrt{3}$

$\therefore\ x \in \{0, \sqrt{3}, -\sqrt{3}\}$

39. (D) $[0, \pi]$

→ **Hint :**

Range of $[\cos^{-1} x]$ is $[0, \pi]$

40. (A) 4

→ **Hint :**

$$\sec^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$$

$$\therefore\ \cos^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$$

Now, using complementary formula we have

$\frac{x}{5} = \frac{4}{5}\ \therefore\ x = 4$

41. (D) $\frac{1}{\sqrt{3}}$

→ **Hint :**

$2\cos(2\tan^{-1} x) = 1$

$$\therefore\ \cos(2\tan^{-1} x) = \frac{1}{2}$$

$$\therefore\ \cos(2\tan^{-1} x) = \cos\left(\frac{\pi}{3}\right)$$

$$\therefore\ 2\tan^{-1} x = \frac{\pi}{3}$$

$$\therefore\ \tan^{-1} x = \frac{\pi}{6}$$

$$\therefore\ x = \tan\left(\frac{\pi}{6}\right)$$

$$\therefore\ x = \frac{1}{\sqrt{3}}$$

42. (C) $1 - x^2$

→ **Hint :**

$\tan^{-1}(1-x), \tan^{-1} x, \tan^{-1}(1+x)$

are in Airthmatic sequence

$\therefore\ \tan^{-1}(1-x) + \tan^{-1}(1+x) = 2\tan^{-1} x$

$$\therefore\ \tan^{-1}\left(\frac{1-x+1+x}{1-(1-x)(1+x)}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\therefore\ \tan^{-1}\left(\frac{2}{1-(1-x)^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\therefore\ \frac{2}{1-1+x^2} = \frac{2x}{1-x^2}$$

$$\therefore\ \frac{2}{x^2} = \frac{2x}{1-x^2}$$

$\therefore\ 1 - x^2 = x^3$

43. (C) 2

→ **Hint :**

$$D = \begin{vmatrix} 8 & 2 & 1 \\ k & 4 & 1 \\ 6 & 7 & 1 \end{vmatrix}$$

$= 8(4-7) - 2(k-6) + 1(7k-24)$

$= 8(-3) - 2(2-6) + 1(7k-24)$

$= -24 - 2k + 12 + 7k - 24$

$= 5k - 36$

Now area of triangel is $A = \frac{1}{2}|D|$

$\therefore\ |D| = 2 \cdot A$

$\therefore\ |5k - 36| = 2(13)$

$\therefore\ |5k - 36| = 26$

$\therefore\ 5k - 36 = \pm 26$

$\therefore\ 5k - 36 = 26$ OR $5k - 36 = -26$

$\therefore\ 5k = 62$ OR $5k = 10$

$\therefore\ k = \frac{62}{5}$ OR $k = 2$

$\therefore$ Possible integer value of k is 2.

44. (A) 5

→ **Hint :**

$$D = \begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & b^2c & 0 \end{vmatrix}$$

$$D = a^2b^2c^2\begin{vmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{vmatrix}$$

$\left(\because \text{ Taking } C_1\left(\frac{1}{a^2}\right), C_2\left(\frac{1}{b^2}\right), C_3\left(\frac{1}{c^2}\right)\right)$

$= a^2b^2c^2\{-a(-bc)+a(bc)\}$

$= 2\,abc\,(a^2b^2c^2)$

$= 2\,(abc)^3$

Now compare with the given result

$\therefore$ we have $m = 2, k = 3$

$\therefore$ $m + k = 2 + 3 = 5$

45. (D) $-\sin\left(\frac{\pi}{18}\right)$

→ **Hint :**

$$D = \begin{vmatrix} \sin\left(\frac{2\pi}{9}\right) & \cos\left(\frac{2\pi}{9}\right) \\ \sin\left(\frac{5\pi}{18}\right) & \cos\left(\frac{5\pi}{18}\right) \end{vmatrix}$$

$$= \begin{vmatrix} \sin(40^0) & \cos(40^0) \\ \sin(50^0) & \cos(50^0) \end{vmatrix}$$

$= \sin(40^0)\cos(50^0) - \cos(40^0)\sin(50^0)$

$= \sin(40^0 - 50^0)$

$= \sin(-10^0)$

$= -\sin(10^0)$ $(\because \sin(-\theta) = -\sin\theta)$

$= -\sin\left(\frac{\pi}{18}\right)$

46. (C) $m = n$

→ **Hint :**

Order of matrix A = 1×3

Order of matrix B = 3×3

$\therefore$ Order of matrix $A\cdot B$ = 1×3

Now order of matrix C = 3×1

$\therefore$ Order of matrix $(A\cdot B)\cdot C$ = 1×1

$\therefore$ We have $m = 1$ and $n = 1$

$\therefore$ $m = n$

47. (B) 10

→ **Hint :**

Using definition of addition and Equality in Matrix we get following results.

$a_1 + a_2 - 3a_2 = 6$

$\therefore$ $a_1 - 2a_2 = -6$ ...... (i)

$4 - 3a_1 = -a_1$

$\therefore$ $4 = 2a_1$

$a_1 = 2$ ...... (ii)

Substitute above value in (i)

$2 - 2a_2 = -6$

$\therefore$ $8 = 2a_2$

$\therefore$ $a_2 = 4$ ...... (iii)

$3 - 3a_4 = -2a_4$

$\therefore$ $3 = 3a_4 - 2a_4$

$\therefore$ $a_4 = 3$

And

$a_3 + a_4 - 3a_3 = 1$

$\therefore$ $a_4 - 2a_3 = 1$

$\therefore$ $3 - 2a_3 = 1$

$\therefore$ $2 = 2a_3$

$\therefore$ $a_3 = 1$

$\therefore$ $\sum_{i=1}^{4} a_i = a_1 + a_2 + a_3 + a_4$

$= 2 + 4 + 1 + 3$

$= 10$

48. (D) $\begin{bmatrix} \frac{4}{23} & \frac{3}{23} \\ \frac{-5}{23} & \frac{2}{23} \end{bmatrix}$

→ **Hint :**

Here $|A| = \begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix}$

$= 8 + 15 = 23 \neq 0$

$\therefore$ A^{-1} must exists

Now $A^{-1} = \frac{1}{|A|}\, adj\, A$

$= \frac{1}{23}\begin{bmatrix} 4 & 3 \\ -5 & 2 \end{bmatrix}$

$$\therefore \quad A^{-1} = \begin{bmatrix} \frac{4}{23} & \frac{3}{23} \\ \frac{-5}{23} & \frac{2}{23} \end{bmatrix}$$

49. (B) $\frac{1}{5}$

→ **Hint :**

$f(x)$ is continus function.

$$\therefore \quad \lim_{x \to 0} f(x) = f(0)$$

$$\therefore \quad \lim_{x \to 0} \frac{\sin(5x) \cdot \tan(kx)}{x^2} = 1$$

$$\therefore \quad \lim_{x \to 0} 5\left(\frac{\sin 5x}{5x}\right) \cdot \lim_{x \to 0} k\left(\frac{\tan kx}{kx}\right) = 1$$

$$\therefore \quad 5 \cdot k = 1$$

$$\therefore \quad k = \frac{1}{5}$$

50. (C) $\frac{\pi}{270}$

→ **Hint :**

$$\frac{d}{dx}\left(\sec(x^0)\right)$$

$$x = 30^0$$

$$= \frac{d}{dx}\left(\sec \frac{\pi x}{180^0}\right)$$

$$= \sec\left(\frac{\pi x}{180^0}\right) \tan\left(\frac{\pi x}{180^0}\right) \cdot \frac{\pi}{180}$$

$$= \sec(x^0) \tan(x^0) \frac{\pi}{180}$$

$$= \sec(30^0) \tan(30^0) \frac{\pi}{180}$$

$$= \left(\frac{2}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) \frac{\pi}{180}$$

$$= \frac{2\pi}{3 \times 180} \quad = \frac{\pi}{270^0}$$