Chapter 22

Definite Integrals

1. $\int_0^{\pi} [\cot x] dx$, where [.] denotes the greatest integer function, is equal to **[AIEEE-2009]**

(1) 1 (2) –1

(3) $-\frac{\pi}{2}$ (4) $\frac{\pi}{2}$

2. Let $p(x)$ be a function defined on R such that $\lim\limits_{x\to\infty}\frac{f(3x)}{f(x)}=1$ $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)dx$ equals **[AIEEE-2010]**

(1) $\sqrt{41}$ (2) 21

(3) 41 (4) 42

3. Let [.] denote the greatest integer function, then the value of $\int_0^{1.5} x[x^2]\, dx$ is **[AIEEE-2011]**

(1) $\frac{3}{4}$ (2) $\frac{5}{4}$

(3) 0 (4) $\frac{3}{2}$

4. If $g(x) = \int_0^x \cos 4t\, dt$, then $g(x + \pi)$ equals **[AIEEE-2012]**

(1) $g(x) + g(\pi)$ (2) $g(x) - g(\pi)$

(3) $g(x)\cdot g(\pi)$ (4) $\frac{g(x)}{g(\pi)}$

5. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is **[JEE (Main)-2013]**

(1) 2500 (2) 3000

(3) 3500 (4) 45000

6. **Statement - I :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$

Statement - II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx.$

[JEE (Main)-2013]

(1) Statement - I is true; Statement - II is true; Statement - II is a **correct** explanation for Statement - I.

(2) Statement - I is true; Statement - II is true; Statement - II is **not** a correct explanation for Statement - I.

(3) Statement - I is true; Statement - II is false.

(4) Statement - I is false; Statement - II is true.

7. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\, dt, x \in R$, which are parallel to the line $y = 2x$, are equal to **[JEE (Main)-2013]**

(1) ± 1 (2) ± 2

(3) ± 3 (4) ± 4

8. The integral

$$\int_0^{\pi} \sqrt{1+4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}\, dx \text{ equals}$$

[JEE (Main)-2014]

(1) $4\sqrt{3} - 4$ (2) $4\sqrt{3} - 4 - \frac{\pi}{3}$

(3) $\pi - 4$ (4) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

9. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$ is equal to **[JEE (Main)-2015]**

(1) 2 (2) 4

(3) 1 (4) 6

10. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to

[JEE (Main)-2017]

(1) 2 (2) 4

(3) –1 (4) –2

11. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is **[JEE (Main)-2018]**

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{2}$

(3) 4π (4) $\frac{\pi}{4}$

12. The value of $\int_0^{\pi} |\cos x|^3 dx$ is **[JEE (Main)-2019]**

(1) 0 (2) $\frac{2}{3}$

(3) $-\frac{4}{3}$ (4) $\frac{4}{3}$

13. If $\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{2k\sec\theta}} d\theta = 1-\frac{1}{\sqrt{2}}, (k>0)$, then the value of k is **[JEE (Main)-2019]**

(1) 4 (2) 2

(3) 1 (4) $\frac{1}{2}$

14. Let $I=\int_a^b (x^4-2x^2)dx$. If I is minimum then the ordered pair (a, b) is **[JEE (Main)-2019]**

(1) $(-\sqrt{2},0)$ (2) $(0,\sqrt{2})$

(3) $(\sqrt{2},-\sqrt{2})$ (4) $(-\sqrt{2},\sqrt{2})$

15. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x]+[\sin x]+4}$, where [$t$] denotes the greatest integer less than or equal to t, is

[JEE (Main)-2019]

(1) $\frac{1}{12}(7\pi-5)$ (2) $\frac{3}{10}(4\pi-3)$

(3) $\frac{3}{20}(4\pi-3)$ (4) $\frac{1}{12}(7\pi+5)$

16. If $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$, then $f'(½)$ is **[JEE (Main)-2019]**

(1) $\frac{6}{25}$ (2) $\frac{24}{25}$

(3) $\frac{4}{5}$ (4) $\frac{18}{25}$

17. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$ (where [x] denotes the greatest integer less than or equal to x) is **[JEE (Main)-2019]**

(1) sin 4 (2) 4 – sin 4

(3) 0 (4) 4

18. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$ equals

[JEE (Main)-2019]

(1) $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(2) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

(3) $\frac{\pi}{40}$

(4) $\frac{1}{5}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

19. Let f and g be continuous functions on [0, a] such that $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$, then $\int_0^a f(x)g(x)\,dx$ is equal to **[JEE (Main)-2019]**

(1) $\int_0^a f(x)dx$ (2) $4\int_0^a f(x)dx$

(3) $-3\int_0^a f(x)dx$ (4) $2\int_0^a f(x)dx$

20. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^{x}\right\} \log_e x\,dx$ is equal to **[JEE (Main)-2019]**

(1) $\frac{3}{2} - e - \frac{1}{2e^2}$ (2) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$

(3) $\frac{1}{2} - e - \frac{1}{e^2}$ (4) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$

21. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \cdots + \frac{1}{5n}\right)$ is equal to **[JEE (Main)-2019]**

(1) $\pi/4$ (2) $\tan^{-1}(3)$

(3) $\tan^{-1}(2)$ (4) $\pi/2$

22. If $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ and $g(x) = \log_e x$, $(x > 0)$ then the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x))dx$ is : **[JEE (Main)-2019]**

(1) $\log_e 1$ (2) $\log_e 3$

(3) $\log_e 2$ (4) $\log_e e$

23. Let $f(x) = \int_0^x g(t)dt$, where g is a non-zero even function. If $f(x + 5) = g(x)$, then $\int_0^x f(t)dt$, equals: **[JEE (Main)-2019]**

(1) $\int_{x+5}^{5} g(t)dt$ (2) $2\int_{5}^{x+5} g(t)dt$

(3) $\int_{5}^{x+5} g(t)dt$ (4) $5\int_{x+5}^{5} g(t)dt$

24. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x}dx$ is **[JEE (Main)-2019]**

(1) $\frac{\pi - 2}{4}$ (2) $\frac{\pi - 2}{8}$

(3) $\frac{\pi - 1}{4}$ (4) $\frac{\pi - 1}{2}$

25. The value of the integral $\int_0^1 x\cot^{-1}(1 - x^2 + x^4)\,dx$ is **[JEE (Main)-2019]**

(1) $\frac{\pi}{4} - \log_e 2$ (2) $\frac{\pi}{2} - \log_e 2$

(3) $\frac{\pi}{2} - \frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4} - \frac{1}{2}\log_e 2$

26. If $f : R \to R$ is a differentiable function and $f(2) = 6$, then $\lim\limits_{x\to 2}\int_6^{f(x)} \frac{2t\,dt}{(x-2)}$ is **[JEE (Main)-2019]**

(1) 0 (2) $2f'(2)$

(3) $12f'(2)$ (4) $24f'(2)$

27. The value of $\int_0^{2\pi} [\sin 2x(1 + \cos 3x)]dx$, where $[t]$ denotes the greatest integer function, is **[JEE (Main)-2019]**

(1) π (2) $-\pi$

(3) -2π (4) 2π

28. $\lim\limits_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + \ldots\ldots + \frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{4}{3}(2)^{4/3}$ (2) $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$

(3) $\frac{4}{3}(2)^{3/4}$ (4) $\frac{3}{4}(2)^{4/3} - \frac{4}{3}$

29. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3} x\,\mathrm{cosec}^{4/3} x\,dx$ is equal to **[JEE (Main)-2019]**

(1) $3^{7/6} - 3^{5/6}$ (2) $3^{5/3} - 3^{1/3}$

(3) $3^{5/6} - 3^{2/3}$ (4) $3^{4/3} - 3^{1/3}$

30. Let $f : R \to R$ be a continuously differentiable function such that $f(2) = 6$ and $f'(2) = \frac{1}{48}$. If $\int_6^{f(x)} 4t^3 dt = (x-2)g(x)$, then $\lim\limits_{x\to 2} g(x)$ is equal to **[JEE (Main)-2019]**

(1) 18 (2) 36

(3) 24 (4) 12

31. If $\int_0^{\frac{\pi}{2}} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then $m \cdot n$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{2}$ (2) 1

(3) –1 (4) $-\frac{1}{2}$

32. A value of α such that

$$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e\left(\frac{9}{8}\right) \text{ is}$$

[JEE (Main)-2019]

(1) $\frac{1}{2}$ (2) –2

(3) $-\frac{1}{2}$ (4) 2

33. The value of α for which $4\alpha \int_{-1}^{2} e^{-\alpha|x|} dx = 5$, is **[JEE (Main)-2020]**

(1) $\log_e\left(\frac{4}{3}\right)$ (2) $\log_e\left(\frac{3}{2}\right)$

(3) $\log_e 2$ (4) $\log_e \sqrt{2}$

34. If $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then **[JEE (Main)-2020]**

(1) $\frac{1}{9} < I^2 < \frac{1}{8}$ (2) $\frac{1}{8} < I^2 < \frac{1}{4}$

(3) $\frac{1}{6} < I^2 < \frac{1}{2}$ (4) $\frac{1}{16} < I^2 < \frac{1}{9}$

35. $\lim\limits_{x \to 0} \frac{\int_0^x t \sin(10t) dt}{x}$ is equal to **[JEE (Main)-2020]**

(1) 0 (2) $\frac{1}{10}$

(3) $-\frac{1}{5}$ (4) $-\frac{1}{10}$

36. The value of $\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ is equal to **[JEE (Main)-2020]**

(1) π^2 (2) 2π

(3) $2\pi^2$ (4) 4π

37. If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$; then $\int_0^1 f(x) dx$ is equal to **[JEE (Main)-2020]**

(1) $2\left\{3f(1) + 2f\left(\frac{1}{2}\right)\right\}$

(2) $\frac{1}{2}\left\{f(1) + 3f\left(\frac{1}{2}\right)\right\}$

(3) $\frac{1}{6}\left\{f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right\}$

(4) $\frac{1}{3}\left\{f(0) + f\left(\frac{1}{2}\right)\right\}$

38. $\int_{-\pi}^{\pi} \left|\pi - |x|\right| dx$ is equal to **[JEE (Main)-2020]**

(1) π^2 (2) $2\pi^2$

(3) $\sqrt{2}\pi^2$ (4) $\frac{\pi^2}{2}$

39. If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$ is $\frac{k}{6}$ then k is equal to **[JEE (Main)-2020]**

(1) $2\sqrt{3} + \pi$ (2) $3\sqrt{2} - \pi$

(3) $3\sqrt{2} + \pi$ (4) $2\sqrt{3} - \pi$

40. Let $f(x) = |x - 2|$ and $g(x) = f(f(x))$, $x \in [0, 4]$. Then $\int_0^3 (g(x) - f(x))\, dx$ is equal to

[JEE (Main)-2020]

(1) 0 (2) $\frac{3}{2}$

(3) $\frac{1}{2}$ (4) 1

41. The integral

$$\int_{\pi/6}^{\pi/3} \tan^3 x \cdot \sin^2 3x (2\sec^2 x \cdot \sin^2 3x + 3\tan x \cdot \sin 6x) dx$$

is equal to **[JEE (Main)-2020]**

(1) $\frac{7}{18}$ (2) $\frac{9}{2}$

(3) $-\frac{1}{18}$ (4) $-\frac{1}{9}$

42. The value of $\int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$ is

[JEE (Main)-2020]

(1) $\frac{\pi}{4}$ (2) π

(3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{2}$

43. If $I_1 = \int_0^1 (1-x^{50})^{100} dx$ and $I_2 = \int_0^1 (1-x^{50})^{101} dx$ such that $I_2 = \alpha I_1$ then α equals to

[JEE (Main)-2020]

(1) $\frac{5051}{5050}$ (2) $\frac{5050}{5051}$

(3) $\frac{5050}{5049}$ (4) $\frac{5049}{5050}$

44. The integral $\int_1^2 e^x . x^x (2+\log_e x) dx$ equals

[JEE (Main)-2020]

(1) e(2e – 1) (2) e(4e – 1)

(3) $4e^2 - 1$ (4) e(4e + 1)

45. The integral $\int_0^2 || x-1| - x| \, dx$ is equal to ______.

[JEE (Main)-2020]

46. Let $[t]$ denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x - [3x]| \, dx$ is ______.

[JEE (Main)-2020]

47. Let $\{x\}$ and $[x]$ denote the fractional part of x and the greatest integer $\leq x$ respectively of a real number x. If $\int_0^n \{x\} \, dx$, $\int_0^n [x] \, dx$ and $10(n^2 - n)$, ($n \in N$, $n > 1$) are three consecutive terms of a G.P., then n is equal to ________.

[JEE (Main)-2020]

48. $\lim_{x\to 0} \frac{\int_0^{x^2} \left(\sin\sqrt{t}\right) dt}{x^3}$ is equal to :

(1) 0 (2) $\frac{1}{15}$

(3) $\frac{2}{3}$ (4) $\frac{3}{2}$

[JEE (Main)-2021]

49. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$, is equal to **[JEE (Main)-2020]**

(1) $\frac{\pi}{3} + \frac{1}{6}$ (2) $\frac{\pi}{3}$

(3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{9}$

50. If $\int_{-a}^{a} (|x| + |x-2|) dx = 22, (a > 2)$ and $[x]$ denotes the greatest integer $\leq$ x, then $\int_{a}^{-a} (x + [x]) dx$ is equal to __________. **[JEE (Main)-2021]**

51. Let f(x) be a differentiable function defined on [0, 2] such that f′(x) = f′(2 – x) for all $x \in (0, 2)$, f(0) = 1 and f(2) = e^2. Then the value of $\int_0^2 f(x) dx$ is :

[JEE (Main)-2021]

(1) $2(1 + e^2)$ (2) $1 + e^2$

(3) $2(1 - e^2)$ (4) $1 - e^2$

52. The value of the integral, $\int_1^3 [x^2 - 2x - 2] dx$, where [x] denotes the greatest integer less than or equal to x, is: **[JEE (Main)-2021]**

(1) –5 (2) $-\sqrt{2} - \sqrt{3} + 1$

(3) $-\sqrt{2} - \sqrt{3} - 1$ (4) –4

53. If a curve y = f(x) passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x) dx = \frac{62}{5}$? **[JEE (Main)-2021]**

(1) $\frac{31}{5}$ (2) $\frac{62}{5}$

(3) 10 (4) 5

54. The value of $\int_{-1}^{1} x^2 e^{[x^3]} dx$, where [t] denotes the greatest integer $\le t$, is : **[JEE (Main)-2021]**

(1) $\frac{e+1}{3}$ (2) $\frac{1}{3e}$

(3) $\frac{e+1}{3e}$ (4) $\frac{e-1}{3e}$

55. If $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x dx$, then : **[JEE (Main)-2021]**

(1) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

(2) $I_2+I_4, I_3+I_5, I_4+I_6$ are in A.P.

(3) $I_2+I_4, (I_3+I_5)^2, I_4+I_6$ are in G.P.

(4) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

56. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\ldots+\frac{n}{(2n-1)^2}\right]$ is equal to : **[JEE (Main)-2021]**

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$

(3) 1 (4) $\frac{1}{4}$

57. The value of $\int_{-2}^{2}\left|3x^2-3x-6\right|dx$ is __________. **[JEE (Main)-2021]**

58. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+3^x} dx$ is : **[JEE (Main)-2021]**

(1) $\frac{\pi}{4}$ (2) 2π

(3) $\frac{\pi}{2}$ (4) 4π

59. The value of $\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]} dx$, where [x] is the greatest integer $\le$ x, is : **[JEE (Main)-2021]**

(1) 100(e – 1) (2) 100(1 – e)

(3) 100e (4) 100(1 + e)

60. The value of the integral $\int_0^{\pi}|\sin 2x|dx$ is __________. **[JEE (Main)-2021]**

61. For x > 0, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to : **[JEE (Main)-2021]**

(1) 1 (2) $\frac{1}{2}$

(3) 0 (4) –1

62. If $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1} dx$, for m, n $\ge$ 1, and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx = \alpha I_{m,n}$, $\alpha \in R$, then α equals _________. **[JEE (Main)-2021]**

63. Let $f:(0,2)\to R$ be defined as $f(x)=\log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right)$. Then, $\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+\ldots+f(1)\right)$ is equal to ______. **[JEE (Main)-2021]**

64. If the normal to the curve $y(x)=\int_0^x (2t^2-15t+10)dt$ at a point (a, b) is parallel to the line x + 3y = –5, a > 1, then the value of |a + 6b| is equal to ______. **[JEE (Main)-2021]**

65. Let f : R $\to$ R be a continuous function such that f(x) + f(x + 1) = 2, for $x \in R$. If $I_1 = \int_0^8 f(x)dx$ and $I_2 = \int_{-1}^{3} f(x)dx$, then the value of $I_1 + 2I_2$ is equal to _____. **[JEE (Main)-2021]**

66. Let P(x) = x^2 + bx + c be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and P(x) leaves remainder 5 when it is divided by (x – 2). Then the value of 9(b + c) is equal to : **[JEE (Main)-2021]**

(1) 11 (2) 9

(3) 15 (4) 7

67. Consider the integral

$$I=\int_0^{10}\frac{[x]e^{[x]}}{e^{x-1}}dx,$$

where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to :

[JEE (Main)-2021]

(1) 9(e – 1) (2) 45(e – 1)

(3) 9(e + 1) (4) 45(e + 1)

68. Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha\in R$ such that

[JEE (Main)-2021]

$$g(\alpha)=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin^{\alpha}x}{\cos^{\alpha}x+\sin^{\alpha}x}dx$$

(1) $g(\alpha)$ is a strictly increasing function

(2) $g(\alpha)$ has an inflection point at $\alpha=-\frac{1}{2}$

(3) $g(\alpha)$ is a strictly decreasing function

(4) $g(\alpha)$ is an even function

69. If [·] represent the greatest integer function, then the value of $\left|\int_0^{\sqrt{\frac{\pi}{2}}}\left[[x^2]-\cos x\right]dx\right|$ is ______.

[JEE (Main)-2021]

70. If the integral $\int_0^{10}\frac{[\sin 2\pi x]}{e^{x-[x]}}dx=\alpha e^{-1}+\beta e^{-\frac{1}{2}}+\gamma$, where α, β, γ are integers and [x] denotes the greatest integer less than or equal to x, then the value of $\alpha+\beta+\gamma$ is equal to :

[JEE (Main)-2021]

(1) 25 (2) 10

(3) 0 (4) 20

71. Let $f:R\to R$ be defined as $f(x) = e^{-x}\sin x$. If $F:[0,1]\to R$ is a differentiable function such that $F(x)=\int_0^x f(t)dt$, then the value of

$\int_0^1(F'(x)+f(x))e^x dx$ lies in the interval

[JEE (Main)-2021]

(1) $\left[\frac{335}{360},\frac{336}{360}\right]$ (2) $\left[\frac{327}{360},\frac{329}{360}\right]$

(3) $\left[\frac{330}{360},\frac{331}{360}\right]$ (4) $\left[\frac{331}{360},\frac{334}{360}\right]$

72. Let $I_n=\int_1^e x^{19}(\log|x|)^n\,dx$, where $n\in N$. If $(20)I_{10}=\alpha I_9+\beta I_8$, for natural numbers α and β, then $\alpha-\beta$ equals to ______.

[JEE (Main)-2021]

73. Let f(x) and g(x) be two functions satisfying $f(x^2)+g(4-x)=4x^3$ and $g(4-x)+g(x)=0$, then the value of $\int_{-4}^{4}f(x^2)dx$ is ______. **[JEE (Main)-2021]**

74. Let $g(x)=\int_0^x f(t)dt$, where f is continuous function in [0, 3] such that $\frac{1}{3}\le f(t)\le 1$ for all $t\in[0,1]$ and $0\le f(t)\le\frac{1}{2}$ for all $t\in[1,3]$. The largest possible interval in which g(3) lies is :

[JEE (Main)-2021]

(1) [1, 3] (2) $\left[\frac{1}{3},2\right]$

(3) $\left[-1,-\frac{1}{2}\right]$ (4) $\left[-\frac{3}{2},-1\right]$

75. Let P(x) be a real polynomial of degree 3 which vanishes at x = –3. Let P(x) have local minima at x=1, local maxima at x = –1 and $\int_{-1}^{1}P(x)dx=18$, then the sum of all the coefficients of the polynomial P(x) is equal to ______.

[JEE (Main)-2021]

76. Let *a* be a positive real number such that

$$\int_0^a e^{x-[x]}dx=10e-9$$

where [*x*] is the greatest integer less than or equal to *x*. Then *a* is equal to : **[JEE (Main)-2021]**

(1) $10+\log_e 3$ (2) $10-\log_e(1+e)$

(3) $10+\log_e(1+e)$ (4) $10+\log_e 2$

77. The value of the integral $\int_{-1}^{1}\log_e\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$ is equal to **[JEE (Main)-2021]**

(1) $2\log_e 2+\frac{\pi}{4}-1$ (2) $\log_e 2+\frac{\pi}{2}-1$

(3) $\frac{1}{2}\log_e 2+\frac{\pi}{4}-\frac{3}{2}$ (4) $2\log_e 2+\frac{\pi}{2}-\frac{1}{2}$

78. Let $g(t)=\int_{-\pi/2}^{\pi/2}\cos\left(\frac{\pi}{4}t+f(x)\right)dx$,

where $f(x)=\log_e\left(x+\sqrt{x^2+1}\right)$, $x\in\mathbf{R}$. Then which one of the following is correct?

[JEE (Main)-2021]

(1) $g(1)+g(0)=0$ (2) $g(1)=\sqrt{2}\,g(0)$

(3) $\sqrt{2}\,g(1)=g(0)$ (4) $g(1)=g(0)$

79. If $[x]$ denotes the greatest integer less than or equal to x, then the value of the integral $\int_{-\pi/2}^{\pi/2}[[x]-\sin x]dx$ is equal to :

[JEE (Main)-2021]

(1) $-\pi$ (2) 0

(3) π (4) 1

80. If the real part of the complex number $(1-\cos\theta+2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta\in(0,\pi)$, then the value of the integral $\int_0^{\theta}\sin x\,dx$ is equal to :

[JEE (Main)-2021]

(1) 1 (2) 2

(3) 0 (4) –1

81. If $\int_0^{100\pi}\frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}}dx=\frac{\alpha\pi^3}{1+4\pi^2},\alpha\in\mathbf{R}$,

where $[x]$ is the greatest integer less than or equal to x, then the value of α is : **[JEE (Main)-2021]**

(1) $50(e-1)$ (2) $100(1-e)$

(3) $150(e^{-1}-1)$ (4) $200(1-e^{-1})$

82. The value of the definite integral $\int_{\pi/24}^{5\pi/24}\frac{dx}{1+\sqrt[3]{\tan 2x}}$ is

[JEE (Main)-2021]

(1) $\frac{\pi}{18}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{12}$

83. The value of the integral $\int_{-1}^{1}\log\left(x+\sqrt{x^2+1}\right)dx$ is

[JEE (Main)-2021]

(1) 0 (2) –1

(3) 2 (4) 1

84. The value of $\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\frac{(2j-1)+8n}{(2j-1)+4n}$ is equal to :

[JEE (Main)-2021]

(1) $1+2\log_e\left(\frac{3}{2}\right)$ (2) $2-\log_e\left(\frac{2}{3}\right)$

(3) $3+2\log_e\left(\frac{2}{3}\right)$ (4) $5+\log_e\left(\frac{3}{2}\right)$

85. The value of the definite integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{dx}{\left(1+e^{x\cos x}\right)\left(\sin^4 x+\cos^4 x\right)}$$

is equal to : **[JEE (Main)-2021]**

(1) $-\frac{\pi}{4}$ (2) $\frac{\pi}{2\sqrt{2}}$

(3) $-\frac{\pi}{2}$ (4) $\frac{\pi}{\sqrt{2}}$

86. Let the domain of the function

$f(x)=\log_4(\log_5(\log_3(18x-x^2-77)))$ be (a, b).

Then the value of the integral

$\int_a^b\frac{\sin^3 x}{\left(\sin^3 x+\sin^3(a+b-x)\right)}dx$ is equal to ________.

[JEE (Main)-2021]

87. If $\int_0^{\pi}(\sin^3 x)\,e^{-\sin^2 x}\,dx=\alpha-\frac{\beta}{e}\int_0^1\sqrt{t}\,e^t dt$, then $\alpha+\beta$ is equal to ________. **[JEE (Main)-2021]**

88. The value of $\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$ is

[JEE (Main)-2021]

(1) $\frac{1}{2}\tan^{-1}(2)$ (2) $\frac{1}{2}\tan^{-1}(4)$

(3) $\tan^{-1}(4)$ (4) $\frac{1}{4}\tan^{-1}(4)$

89. The value of $\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left(\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right)^{1/2}dx$ is

[JEE (Main)-2021]

(1) $2\log_e 16$ (2) $\log_e 16$

(3) $4\log_e\left(3+2\sqrt{2}\right)$ (4) $\log_e 4$

90. If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta$, where $\alpha, \beta \in \mathbf{R}$, $5\alpha + 6\beta = 0$, and $[x]$ denotes the greatest integer less than or equal to x; then the value of $(\alpha + \beta)^2$ is equal to : **[JEE (Main)-2021]**

(1) 25 (2) 16

(3) 36 (4) 100

91. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right) dx$ is

[JEE (Main)-2021]

(1) $\frac{3\pi}{2}$ (2) $\frac{\pi}{2}$

(3) $\frac{5\pi}{4}$ (4) $\frac{3\pi}{4}$

92. $\int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e\left(x^2 - 44x + 484\right)} dx$ is equal to

[JEE (Main)-2021]

(1) 6 (2) 10

(3) 5 (4) 8

93. If $U_n = \left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2 \dots \left(1+\frac{n^2}{n^2}\right)^n$, then $\lim_{n\to\infty} (U_n)^{\frac{-4}{n^2}}$ is equal to **[JEE (Main)-2021]**

(1) $\frac{4}{e^2}$ (2) $\frac{4}{e}$

(3) $\frac{16}{e^2}$ (4) $\frac{e^2}{16}$

94. The value of the integral $\int_0^1 \frac{\sqrt{x}dx}{(1+x)(1+3x)(3+x)}$ is

[JEE (Main)-2021]

(1) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$ (2) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

(3) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$ (4) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$

95. Let f be a non-negative function in [0, 1] and twice differentiable in (0, 1).

If $\int_0^x \sqrt{1-(f'(t))^2}\, dt = \int_0^x f(t)dt$, $0 \le x \le 1$ and $f(0) = 0$, then $\lim_{x\to\infty} \frac{1}{x^2}\int_0^x f(t)dt$

[JEE (Main)-2021]

(1) Equals 1 (2) Does not exist

(3) Equals 0 (4) Equals $\frac{1}{2}$

96. Let $[t]$ denote the greatest integer $\le t$. Then the value of $8\cdot\int_{-\frac{1}{2}}^{1}([2x]+|x|)dx$ is _____.

[JEE (Main)-2021]

97. If $[x]$ is the greatest integer $\le x$, then $\pi^2\int_0^2 \left(\sin\frac{\pi x}{2}\right)(x-[x])^{[x]}dx$ is equal to

[JEE (Main)-2021]

(1) $4(\pi + 1)$ (2) $2(\pi - 1)$

(3) $4(\pi - 1)$ (4) $2(\pi + 1)$

98. The function $f(x)$, that satisfies the condition $f(x) = x + \int_0^{\pi/2} \sin x \cdot \cos y\, f(y)dy$, is :

[JEE (Main)-2021]

(1) $x + (\pi + 2) \sin x$ (2) $x+\frac{2}{3}(\pi-2)\sin x$

(3) $x+\frac{\pi}{2}\sin x$ (4) $x + (\pi - 2) \sin x$

99. Let $J_{n,\, m} = \int_0^{1/2} \frac{x^n}{x^m - 1}dx$, $\forall\, n > m$ and $n, m \in \mathbb{N}$.

Consider a matrix $A = \left[a_{ij}\right]_{3\times3}$ where

$a_{ij} = \begin{cases} J_{6+i,\, 3} - J_{i+3,\, 3}, & i \le j \\ 0, & i > j \end{cases}$. Then $\left|\text{adj } A^{-1}\right|$ is

[JEE (Main)-2021]

(1) $(105)^2 \times 2^{38}$ (2) $(15)^2 \times 2^{42}$

(3) $(15)^2 \times 2^{34}$ (4) $(105)^2 \times 2^{36}$

100. Let $f(x) = \int_0^x e^t f(t)dt + e^x$ be a differentiable function for all $x \in R$. Then f(x) equals : **[JEE (Main)-2021]**

(1) $e^{(e^x - 1)}$ (2) $2e^{e^x} - 1$

(3) $2e^{(e^x - 1)} - 1$ (4) $e^{e^x} - 1$

101. Let $f : [0, \infty) \to [0, \infty)$ be defined as

$$f(x) = \int_0^x [y]\, dy$$

where $[x]$ is the greatest integer less than or equal to x. Which of the following is true? **[JEE (Main)-2021]**

(1) f is continuous at every point in $[0, \infty)$ and differentiable except at the integer points

(2) f is both continuous and differentiable except at the integer points in $[0, \infty)$

(3) f is continuous everywhere except at the integer points in $[0, \infty)$

(4) f is differentiable at every point in $[0, \infty)$

102. Let $f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta) f(t)dt$. Then the value of $\left| \int_0^{\pi/2} f(\theta)d\theta \right|$ is ________. **[JEE (Main)-2022]**

103. The value of $\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x} + e^{-\cos x})}dx$ is equal to : **[JEE (Main)-2022]**

(1) $\frac{\pi^2}{4}$ (2) $\frac{\pi^2}{2}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

104. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x}dx, n \in \mathbb{N}$, then **[JEE (Main)-2022]**

(1) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference –2

(2) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A. P. with common difference 2

(3) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

(4) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference –2

105. The value of $b > 3$ for which $12\int_3^b \frac{1}{(x^2-1)(x^2-4)}dx = \log_e\left(\frac{49}{40}\right)$, is equal to **[JEE (Main)-2022]**

106. The integral $\frac{24}{\pi}\int_0^{\sqrt{2}} \frac{(2-x^2)dx}{(2+x^2)\sqrt{4+x^4}}$ is equal to ______. **[JEE (Main)-2022]**

107. The value of the integral $\int_{-2}^{2} \frac{|x^3 + x|}{(e^{x|x|} + 1)}dx$ is equal to: **[JEE (Main)-2022]**

(1) $5e^2$ (2) $3e^{-2}$

(3) 4 (4) 6

108. Let f be a differentiable function in $\left(0, \frac{\pi}{2}\right)$. If $\int_{\cos x}^{1} t^2 f(t)dt = \sin^3 x + \cos x$, then $\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to **[JEE (Main)-2022]**

(1) $6 - 9\sqrt{2}$ (2) $6 - \frac{9}{\sqrt{2}}$

(3) $\frac{9}{2} - 6\sqrt{2}$ (4) $\frac{9}{\sqrt{2}} - 6$

109. The integral $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}}dx$, where $[\cdot]$ denotes the greatest integer function, is equal to **[JEE (Main)-2022]**

(1) $1 + 6\log_e\left(\frac{6}{7}\right)$ (2) $1 - 6\log_e\left(\frac{6}{7}\right)$

(3) $\log_e\left(\frac{7}{6}\right)$ (4) $1 - 7\log_e\left(\frac{6}{7}\right)$

110. Let $[t]$ denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 [-8x^2 + 6x - 1]dx$ is equal to **[JEE (Main)-2022]**

(1) –1 (2) $\frac{-5}{4}$

(3) $\frac{\sqrt{17} - 13}{8}$ (4) $\frac{\sqrt{17} - 16}{8}$

111. Let $f : R \to R$ be a differentiable function such that $f\left(\frac{\pi}{4}\right) = \sqrt{2}$, $f\left(\frac{\pi}{2}\right) = 0$ and $f'\left(\frac{\pi}{2}\right) = 1$ and let

$g(x) = \int_x^{\frac{\pi}{4}} \left(f'(t)\sec t + \tan t \sec t\, f(t)\right) dt$ for $x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then $\lim\limits_{x \to \left(\frac{\pi}{2}\right)^-} g(x)$ is equal to

[JEE (Main)-2022]

(1) 2 (2) 3

(3) 4 (4) –3

112. Let $f : R \to R$ be a continuous function satisfying $f(x) + f(x + k) = n$, for all $x \in R$ where $k > 0$ and n is a positive integer. If $I_1 = \int_0^{4nk} f(x)dx$ and $I_2 = \int_{-k}^{3k} f(x)dx$, then

[JEE (Main)-2022]

(1) $I_1 + 2I_2 = 4nk$ (2) $I_1 + 2I_2 = 2nk$

(3) $I_1 + nI_2 = 4n^2k$ (4) $I_1 + nI_2 = 6n^2k$

113. Let f : R $\Rightarrow$ R be a function defined by :

$$f(x) = \begin{cases} \max\limits_{t \le x}\{t^3 - 3t\} & ; \quad x \le 2 \\ x^2 + 2x - 6 & ; \quad 2 < x < 3 \\ [x-3] + 9 & ; \quad 3 \le x \le 5 \\ 2x + 1 & ; \quad x > 5 \end{cases}$$

where $[t]$ is the greatest integer less than or equal to t. Let m be the number of points where f is not differentiable and $I = \int_{-2}^{2} f(x)dx$. Then the ordered pair (m, I) is equal to :

[JEE (Main)-2022]

(1) $\left(3, \frac{27}{4}\right)$ (2) $\left(3, \frac{23}{4}\right)$

(3) $\left(4, \frac{27}{4}\right)$ (4) $\left(4, \frac{23}{4}\right)$

114. $\int_0^5 \cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right) dx$, where $[t]$ denotes greatest integer less than or equal to t, is equal to

[JEE (Main)-2022]

(1) –3 (2) –2

(3) 2 (4) 0

115. Let f be a real valued continuous function on [0, 1] and $f(x) = x + \int_0^1 (x-t) f(t)dt$. Then, which of the following points (x, y) lies on the curve $y = f(x)$?

[JEE (Main)-2022]

(1) (2, 4) (2) (1, 2)

(3) (4, 17) (4) (6, 8)

116. If $\int_0^2 \left(\sqrt{2x} - \sqrt{2x - x^2}\right) dx = \int_0^1 \left(1 - \sqrt{1 - y^2} - \frac{y^2}{2}\right) dy + \int_1^2 \left(2 - \frac{y^2}{2}\right) dy + I$ then I equal is

[JEE (Main)-2022]

(1) $\int_0^1 \left(1 + \sqrt{1 - y^2}\right) dy$

(2) $\int_0^1 \left(\frac{y^2}{2} - \sqrt{1 - y^2} + 1\right) dy$

(3) $\int_0^1 \left(1 - \sqrt{1 - y^2}\right) dy$

(4) $\int_0^1 \left(\frac{y^2}{2} + \sqrt{1 - y^2} + 1\right) dy$

117. For any real number x, let $[x]$ denote the largest integer less than equal to x. Let f be a real valued function defined on the interval [–10, 10] by

$$f(x) = \begin{cases} x - [x], \text{ if } [x] \text{ is odd} \\ 1 + [x] - x, \text{ if } [x] \text{ is even}. \end{cases}$$

Then the value of $\frac{\pi^2}{10} \int_{-10}^{10} f(x) \cos \pi x \, dx$ is

[JEE (Main)-2022]

(1) 4 (2) 2

(3) 1 (4) 0

118. If $n(2n+1)\int_0^1 \left(1-x^n\right)^{2n} dx = 1177\int_0^1 (1-x^n)^{2n+1} dx$, then $n \in \boldsymbol{N}$ is equal to ______.

[JEE (Main)-2022]

119. Let $\mathbb{R} \to \mathbb{R}$ be function defined as

$f(x) = a\sin\left(\frac{\pi[x]}{2}\right) + [2-x], a \in \mathbb{R},$ where [*t*] is the greatest integer less than or equal to *t*. If $\lim_{x\to -1} f(x)$ exists, then the value of $\int_0^4 f(x)dx$ is equal to

[JEE (Main)-2022]

(1) –1 (2) –2

(3) 1 (4) 2

120. The minimum value of the twice differentiable function $f(x) = \int_0^x e^{x-t} f'(t)\mathrm{d}t - (x^2 - x + 1)e^x$, $x \in \mathbb{R}$, is

[JEE (Main)-2022]

(1) $-\frac{2}{\sqrt{e}}$ (2) $-2\sqrt{e}$

(3) $-\sqrt{e}$ (4) $\frac{2}{\sqrt{e}}$

121. If $\int_0^{\sqrt{3}} \frac{15x^3}{\sqrt{1+x^2+\sqrt{\left(1+x^2\right)^3}}} dx = \alpha\sqrt{2} + \beta\sqrt{3}$, where α, β are integers, then α + β is equal to

[JEE (Main)-2022]

122. Let $I_n(x) = \int_0^x \frac{1}{\left(t^2+5\right)^n} dt$, $n = 1, 2, 3, \ldots$ Then

[JEE (Main)-2022]

(1) $50I_6 - 9I_5 = xI_5'$ (2) $50I_6 - 11I_5 = xI_5'$

(3) $50I_6 - 9I_5 = I_5'$ (4) $50I_6 - 11I_5 = I_5'$

123. The value of the integral $\int_0^{\frac{\pi}{2}} 60\frac{\sin(6x)}{\sin x}dx$ is equal to ______.

[JEE (Main)-2022]

124. The integral $\int_0^{\frac{\pi}{2}} \frac{1}{3+2\sin x+\cos x}dx$ is equal to

[JEE (Main)-2022]

(1) $\tan^{-1}(2)$

(2) $\tan^{-1}(2) - \frac{\pi}{4}$

(3) $\frac{1}{2}\tan^{-1}(2) - \frac{\pi}{8}$

(4) $\frac{1}{2}$

125. If [*t*] denotes the greatest integer $\le t$, then the value of $\int_0^1 \left[2x - |3x^2 - 5x + 2| + 1\right] dx$ is

[JEE (Main)-2022]

(1) $\frac{\sqrt{37}+\sqrt{13}-4}{6}$ (2) $\frac{\sqrt{37}-\sqrt{13}-4}{6}$

(3) $\frac{-\sqrt{37}-\sqrt{13}+4}{6}$ (4) $\frac{-\sqrt{37}+\sqrt{13}+4}{6}$

126. $\int_0^{20\pi} \left(|\sin x| + |\cos x|\right)^2 dx$ is equal to

[JEE (Main)-2022]

(1) $10(\pi+4)$ (2) $10(\pi+2)$

(3) $20(\pi-2)$ (4) $20(\pi+2)$

127. Let *f* be a twice differentiable function on R. If $f'(0) = 4$ and $f(x) + \int_0^x (x-t) f'(t) dt$ $= \left(e^{2x} + e^{-2x}\right)\cos 2x + \frac{2}{a}x$, then $(2a + 1)^5 a^2$ is equal to ______________.

[JEE (Main)-2022]

128. If $f(\alpha)=\int_{1}^{\alpha}\frac{\log_{10}t}{1+t}dt$, $\alpha > 0$, then $f(e^3)+f(e^{-3})$ is equal to : **[JEE (Main)-2022]**

(1) 9 (2) $\frac{9}{2}$

(3) $\frac{9}{\log_e(10)}$ (4) $\frac{9}{2\log_e(10)}$

129. Let $[t]$ denote the greatest integer less than or equal to t. Then the value of the integral $\int_{-3}^{101}\left(\left[\sin(\pi x)\right]+e^{\left[\cos(2\pi x)\right]}\right)dx$ is equal to

[JEE (Main)-2022]

(1) $\frac{52(1-e)}{e}$ (2) $\frac{52}{e}$

(3) $\frac{52(2+e)}{e}$ (4) $\frac{104}{e}$

130. $\lim\limits_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+.....+\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$ is equal to

(1) $\frac{1}{2}$ (2) 1

(3) 2 (4) –2

[JEE (Main)-2022]

131. $\lim\limits_{n\to\infty}\left(\frac{n^2}{(n^2+1)(n+1)}+\frac{n^2}{(n^2+4)(n+2)}+\frac{n^2}{(n^2+9)(n+3)}+\ldots+\frac{n^2}{(n^2+n^2)(n+n)}\right)$

is equal to **[JEE (Main)-2022]**

(1) $\frac{\pi}{8}+\frac{1}{4}\log_e 2$ (2) $\frac{\pi}{4}+\frac{1}{8}\log_e 2$

(3) $\frac{\pi}{4}-\frac{1}{8}\log_e 2$ (4) $\frac{\pi}{8}+\frac{1}{8}\log_e\sqrt{2}$

132. Let $a_n=\int_{-1}^{n}\left(1+\frac{x}{2}+\frac{x^2}{3}+....+\frac{x^{n-1}}{n}\right)dx$ for every $n\in\mathbb{N}$. Then the sum of all the elements of the set $\{n\in\mathbb{N} : a_n\in(2, 30)\}$ is ________________ .

[JEE (Main)-2022]

133. Let $f(x)=\max\{|x+1|, |x+2|, ..., |x+5|\}$. Then $\int_{-6}^{0}f(x)dx$ is equal to ______.

[JEE (Main)-2022]

134. The value of the integral

$\frac{48}{\pi^4}\int_{0}^{\pi}\left(\frac{3\pi x^2}{2}-x^3\right)\frac{\sin x}{1+\cos^2 x}dx$ is equal to ________.

[JEE (Main)-2022]

135. If m and n respectively are the number of local maximum and local minimum points of the function $f(x)=\int_{0}^{x^2}\frac{t^2-5t+4}{2+e^t}dt$, then the ordered pair (m, n) is equal to

[JEE (Main)-2022]

(1) (3, 2) (2) (2, 3)

(3) (2, 2) (4) (3, 4)

136. If $\lim\limits_{n\to\infty}\frac{(n+1)^{k-1}}{n^{k+1}}[(nk+1)+(nk+2)+\ldots+(nk+n)]$

$=33\cdot\lim\limits_{n\to\infty}\frac{1}{n^{k+1}}\cdot\left[1^k+2^k+3^k+\ldots+n^k\right]$, then the integral value of k is equal to _______.

[JEE (Main)-2022]

137. If $a=\lim\limits_{n\to\infty}\sum_{k=1}^{n}\frac{2n}{n^2+k^2}$ and $f(x)=\sqrt{\frac{1-\cos x}{1+\cos x}}, x\in(0, 1)$, then

[JEE (Main)-2022]

(1) $2\sqrt{2}f\left(\frac{a}{2}\right)=f'\left(\frac{a}{2}\right)$ (2) $f\left(\frac{a}{2}\right)f'\left(\frac{a}{2}\right)=\sqrt{2}$

(3) $\sqrt{2}f\left(\frac{a}{2}\right)=f'\left(\frac{a}{2}\right)$ (4) $f\left(\frac{a}{2}\right)=\sqrt{2}f'\left(\frac{a}{2}\right)$

138. Let $f(x) = 2+|x|-|x-1|+|x+1|, x \in R$. Consider

$$(S1): f'\left(-\frac{3}{2}\right)+f'\left(-\frac{1}{2}\right)+f'\left(\frac{1}{2}\right)+f'\left(\frac{3}{2}\right)=2$$

$$(S2): \int_{-2}^{2} f(x)dx = 12$$

Then,

[JEE (Main)-2022]

(1) Both (S1) and (S2) are correct

(2) Both (S1) and (S2) are wrong

(3) Only (S1) is correct

(4) Only (S2) is correct

139. $\int_0^2 \left(\left|2x^2-3x\right|+\left[x-\frac{1}{2}\right]\right)dx$, where $[t]$ is the greatest integer function, is equal to

[JEE (Main)-2022]

(1) $\frac{7}{6}$ (2) $\frac{19}{12}$

(3) $\frac{31}{12}$ (4) $\frac{3}{2}$

140. Let $f(x) = \min \{[x-1], [x-2], ..., [x-10]\}$ where $[t]$ denotes the greatest integer $\le t$. Then $\int_0^{10} f(x)dx + \int_0^{10} (f(x))^2 dx + \int_0^{10} |f(x)|dx$ is equal to ________.

[JEE (Main)-2021]

141. Let f be a differential function satisfying

$$f(x) = \frac{2}{\sqrt{3}}\int_0^{\sqrt{3}} f\left(\frac{\lambda^2 x}{3}\right)d\lambda, x>0$$ and $f(1)=\sqrt{3}$. If $y = f(x)$ passes through the point $(\alpha, 6)$, then α is equal to _____

[JEE (Main)-2022]

142. The value of the integral

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{\left(1+e^x\right)\left(\sin^6 x+\cos^6 x\right)}$$ is equal to

[JEE (Main)-2022]

(1) 2π (2) 0

(3) π (4) $\frac{\pi}{2}$

143. Let $\underset{0\le x\le 2}{\text{Max}}\left\{\frac{9-x^2}{5-x}\right\}=\alpha$ and $\underset{0\le x\le 2}{\text{Min}}\left\{\frac{9-x^2}{5-x}\right\}=\beta$.

If $\int_{\beta-\frac{8}{3}}^{2\alpha-1} \text{Max}\left\{\frac{9-x^2}{5-x}, x\right\}dx = \alpha_1+\alpha_2\log_e\left(\frac{8}{15}\right)$ then $\alpha_1+\alpha_2$ is equal to __________.

[JEE (Main)-2022]

❑ ❑ ❑

Definite Integrals

1. Answer (3)

$$I = \int_0^{\pi} [\cot x] dx$$

$$I = \int_0^{\pi} [\cot(\pi - x)] dx$$

$$2I = \int_0^{\pi} ([\cot x] + [-\cot x]) dx$$

$$2I = \int_0^{\pi} (-1) dx = -\pi$$

$$I = -\frac{\pi}{2}$$

2. Answer (2)

We have,

$p'(x) = p'(1 - x)$, $\forall\ x \in [0, 1]$, $p(0) = 1$, $p(1) = 41$

$p(x) = -p(1 - x) + C$

$\Rightarrow\ 1 = -41 + C$

$\Rightarrow\ C = 42$

$\Rightarrow\ p(x) + p(1 - x) = 42$

$$I = \int_0^1 p(x) dx = \int_0^1 p(1-x) dx$$

$$\Rightarrow\ 2I = \int_0^1 (p(x) + p(1-x)) dx = \int_0^1 42.dx = 42$$

$\Rightarrow\ I = 21$

3. Answer (1)

$$\int_0^{1.5} x[x^2] dx$$

$$= \int_0^1 x[x^2]\, dx + \int_1^{\sqrt{2}} x[x^2]\, dx + \int_{\sqrt{2}}^{1.5} x[x^2]\, dx$$

$$= \int_0^1 x.0 dx + \int_1^{\sqrt{2}} x dx + 2\int_{\sqrt{2}}^{1.5} x dx$$

$$= 0 + \left.\frac{x^2}{2}\right|_1^{\sqrt{2}} + \left. x^2 \right|_{\sqrt{2}}^{1.5}$$

$$= \frac{1}{2} + \frac{1}{4}$$

$$= \frac{3}{4}$$

4. Answer (1)

5. Answer (3)

$$\int_{2000}^{P} dp = \int_0^{25} \left(100 - 12\sqrt{x}\right) dx$$

$$\Rightarrow\ P - 2000 = 2500 - 12 \times \frac{2}{3} \times 125 = 1500$$

$$\Rightarrow\ P = 3500$$

6. Answer (4)

Statement (1)

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}}$$

$$\Rightarrow\ I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\cot x}}$$

$$\Rightarrow\ 2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$$

$$\Rightarrow\ 2I = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}$$

Statement (1) is false, Statement (2) is true.

$$y=\int_0^{x}|t|\,dt$$

$$\frac{dy}{dx}=|x|=2$$

$x = \pm 2$

Case-1, $x = 2$

$$y=\int_0^{2}t\,dt=2$$

Equation of tangent is $y - 2 = 2(x - 2)$

$$\Rightarrow \frac{y}{-2}+\frac{x}{1}=1$$

x-intercept = 1

When $x = -2$

$$y=\int_0^{-2}-t\,dt=\left[-\frac{t^2}{2}\right]_0^{-2}=-2$$

$y + 2 = 2(x + y)$

$\Rightarrow$ $y = 2x + 2$

Hence, here x-intercept is -1

$\therefore$ x-intercepts = ± 1

8. Answer (2)

$$\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$$

$$=\int_0^{\pi}\left|2\sin\frac{x}{2}-1\right|dx \qquad \left[\begin{array}{l}\sin\frac{x}{2}=\frac{1}{2}\\ \Rightarrow \frac{x}{2}=\frac{\pi}{6}\to x=\frac{\pi}{3}\\ \frac{x}{2}=\frac{5\pi}{6}\to x=\frac{5\pi}{3}\end{array}\right]$$

$$=\int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx+\int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx$$

$$=\left[x+4\cos\frac{x}{2}\right]_0^{\pi/3}+\left[-4\cos\frac{x}{2}-x\right]_{\pi/3}^{\pi}$$

$$=\frac{\pi}{3}+4\frac{\sqrt{3}}{2}-4+\left(0-\pi+4\frac{\sqrt{3}}{2}+\frac{\pi}{3}\right)$$

$$=4\sqrt{3}-4-\frac{\pi}{3}$$

$$I=\int_2^{4}\frac{\log x^2\,dx}{\log x^2+\log(36-12x+x^2)}$$

$$I=\int_2^{4}\frac{\log(6-x)^2\,dx}{\log x^2+\log(6-x)^2}$$

$$2I=\int_2^{4}1\,dx$$

$2I = 2$

$I = 1$

10. Answer (1)

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{dx}{2\cos^2\frac{x}{2}}dx=\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\sec^2\frac{x}{2}dx$$

$$=\frac{1}{2}\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$=\tan\frac{3\pi}{8}-\tan\frac{\pi}{8}$$

$$\left[\tan\frac{\pi}{8}=\sqrt{\frac{1-\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}}=\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}=\frac{\sqrt{2}-1}{1}\right.$$

$$\left.\tan\frac{3\pi}{8}=\sqrt{\frac{1-\cos\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}}}=\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}=\sqrt{2}+1\right]$$

$$=(\sqrt{2}+1)-(\sqrt{2}-1)$$

$$=2$$

11. Answer (4)

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin^2 x\,dx}{1+2^x} \qquad \text{... (i)}$$

$$\text{Also, } I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{2^x\sin^2 x\,dx}{1+2^x} \qquad \text{... (ii)}$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$$

$$2I = 2\int_0^{\frac{\pi}{2}} \sin^2 x dx \quad \Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \quad \text{... (iii)}$$

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x dx \qquad \text{... (iv)}$$

Adding (iii) & (iv)

$$2I = \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

12. Answer (4)

$$I = \int_0^{\pi} |\cos x|^3 \, dx$$

$$= 2\int_0^{\pi/2} \cos^3 x \, dx$$

$$= \frac{2}{4}\int_0^{\pi/2} (3\cos x + \cos 3x) \, dx$$

$$= \frac{1}{2}\left(3\sin x + \frac{\sin 3x}{3}\right)_0^{\pi/2} \quad = \frac{1}{2}\left(3 - \frac{1}{3}\right) \quad = \frac{4}{3}$$

13. Answer (2)

$$I = \int_0^{\pi/3} \frac{\tan\theta}{\sqrt{2k\sec\theta}} d\theta$$

$$= \frac{1}{\sqrt{2k}} \int_0^{\pi/3} \frac{\sin\theta}{\sqrt{\cos\theta}} d\theta$$

Let $\cos\theta = t^2$

$\therefore \quad \sin\theta \, d\theta = -2t \, dt$

$$= \frac{1}{\sqrt{2k}} \int_1^{\sqrt{\frac{1}{2}}} \frac{-2t \, dt}{t}$$

$$\sqrt{k} \quad \frac{1}{\sqrt{2}}$$

$$= \sqrt{\frac{2}{k}}\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{2}-1}{\sqrt{k}}$$

$$= 1 - \frac{1}{\sqrt{2}} \qquad \text{(Given)}$$

$\therefore \quad k = 2$

14. Answer (4)

$$I = \int_a^b \left(x^4 - 2x^2\right) dx$$

$$\Rightarrow \frac{dI}{dx} = x^4 - 2x^2 = 0 \quad \Rightarrow x = 0, \pm\sqrt{2}$$

Also, $I = \left[\frac{x^5}{5} - \frac{2x^3}{3}\right]_a^b = x^3\left(\frac{x^2}{5} - \frac{2}{3}\right)$

$|I|$ is maximum when $b = -\sqrt{2}$ and $a = \sqrt{2}$

$\therefore \quad I$ is minimum when $(a, b) = \left(-\sqrt{2}, \sqrt{2}\right)$

15. Answer (3)

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x] + [\sin x] + 4}$$

$$= \int_{\frac{-\pi}{2}}^{-1} \frac{dx}{-2-1+4} + \int_{-1}^{0} \frac{dx}{-1-1+4} + \int_0^1 \frac{dx}{0+0+4} + \int_1^{\frac{\pi}{2}} \frac{dx}{1+0+4}$$

$$= \left(-1 + \frac{\pi}{2}\right) + \frac{1}{2}(0+1) + \frac{1}{4}(1-0) + \frac{1}{5}\left(\frac{\pi}{2} - 1\right)$$

$$= \frac{3\pi}{5} - \frac{9}{20} = \frac{3}{20}(4\pi - 3)$$

Option (3) is correct.

$$\int_0^{x} f(t)dt = x^2 + \int_x^{1} t^2 f(t)dt$$

$\Rightarrow$ $f(x) = 2x - x^2 f(x)$

$\Rightarrow$ $f(x) = \dfrac{2x}{1+x^2}$

$\Rightarrow$ $f'(x) = \dfrac{2(1-x^2)}{(1+x^2)^2}$

$$f'(1/2) = \frac{2\left(1-\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2} = \frac{3}{2}\times\frac{16}{25} = \frac{24}{25}$$

17. Answer (3)

Let $f(x) = \dfrac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}$

Now $f(-x) = \dfrac{\sin^2(-x)}{\left[\frac{-x}{\pi}\right]+\frac{1}{2}}$ $\quad \because [-x] = -1-[x]$

$$f(-x) = \frac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}} = \frac{\sin^2 x}{-\frac{1}{2}-\left[\frac{x}{\pi}\right]} = -f(x)$$

So $f(x)$ is odd function

So $\displaystyle\int_{-2}^{2} f(x)\,dx = 0$

18. Answer (1)

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{dx}{\sin 2x\left(\tan^5 x + \cot^5 x\right)}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan^5 x\cdot\sec^2 x}{2\frac{\sin x}{\cos x}\left(\left(\tan^5 x\right)^2+1\right)}$$

$$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan^4 x\cdot\sec^2 x}{\left(\tan^5 x\right)^2+1}\,dx.$$

Let $\tan^5 x = t$.

$5\tan^4 x\cdot\sec^2 x\,dx = dt$.

$$= \frac{1}{10}\int_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1} \frac{dt}{t^2+1}$$

$$= \frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$$

$g(x) + g(a-x) = 4$

$$I = \int_0^a f(x)g(x)dx$$

$$= \int_0^a f(a-x)\cdot g(a-x)dx$$

$$I = \int_0^a f(x)[4-g(x)]dx$$

$$I = \int_0^a 4f(x)dx - \int_0^a f(x)\cdot g(x)dx$$

$$I = \int_0^a 4f(x)dx - I$$

$$2I = \int_0^a 4f(x)dx$$

$$\boxed{I = 2\int_0^a f(x)dx}$$

20. Answer (1)

$$I = \int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^x\right\}\log_e x dx$$

Let $\left(\frac{x}{e}\right)^x = t$

$\Rightarrow$ $x\ln\left(\frac{x}{e}\right) = \ln t$

$\Rightarrow$ $x(\ln x - 1) = \ln t$

On differentiating both sides w.r.t x we get.

$$\ln x\cdot dx = \frac{dt}{t}$$

$$I = \int_{\frac{1}{e}}^{1}\left(t^2 - \frac{1}{t}\right)\cdot\frac{dt}{t}$$

$$= \int_{\frac{1}{e}}^{1}\left(t - \frac{1}{t^2}\right)dt$$

$$= \left(\frac{t^2}{2} + \frac{1}{t}\right)_{\frac{1}{e}}^{1}$$

$$= \left(\frac{1}{2}+1\right) - \left(\frac{1}{2e^2} + e\right)$$

$$= \frac{3}{2} - e - \frac{1}{2e^2}$$

$$I = \lim_{n\to\infty}\sum_{r=1} \frac{n}{n^2+r^2}$$

$$= \int_0^2 \frac{dx}{1+x^2} \qquad \frac{r}{n} \to x,\ \frac{1}{r} \to dx$$

$$= \left[\tan^{-1} x\right]_0^2$$

$$= \tan^{-1} 2$$

22. Answer (1)

$$g(f(x)) = \ln\left(\frac{2 - x\cos x}{2 + x\cos x}\right)$$

$$\text{Let } I = \int_{-\pi/4}^{\pi/4} \ln\left(\frac{2 - x\cos x}{2 + x\cos x}\right)dx \qquad \text{...(i)}$$

$$\left(\text{Using property } \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$$

$$I = \int_{-\pi/4}^{\pi/4} \ln\left(\frac{2 + x\cos x}{2 - x\cos x}\right)dx \qquad \text{...(ii)}$$

Adding (i) and (ii),

$$2I = \int_{-\pi/2}^{\pi/2} \ln(1)dx = 0$$

$\Rightarrow$ $I = 0 = \ln 1$

23. Answer (1)

$$f(x) = \int_0^x g(t)dt, \qquad \ldots\text{(i)}$$

$g(-x) = g(x)$, …(ii)

$f(x + 5) = g(x)$ …(iii)

From (i),

$f'(x) = g(x)$

$$\text{Let } I = \int_0^x f(t)dt,$$

Put $t = \lambda - 5$

$\because$ $f(x + 5) = g(x)$

$\Rightarrow$ $f(-x + 5) = g(-x) = g(x)$ …(iv)

$$I = \int_5^{x+5} f(\lambda - 5)d\lambda,$$

$$I = \int_5^{x+5} -f(5-\lambda)d\lambda$$

($\because$ $f(0) = 0$, $g(x)$ is even $\Rightarrow$ $f(x)$ is odd)

$$I = -\int_5^{x+5} g(\lambda)d\lambda = \int_{x+5}^{5} g(t)dt \qquad \text{(from (iv))}$$

24. Answer (3)

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x\, dx}{\sin x + \cos x}$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{\cos^3 x\, dx}{\sin x + \cos x}$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} \left(1 - \frac{1}{2}\sin(2x)\right)dx$$

$$\Rightarrow \quad I = \frac{1}{2}\left[x + \frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{2}}$$

$$I = \frac{1}{2}\left(\frac{\pi - 1}{2}\right) = \frac{\pi - 1}{4}$$

25. Answer (4)

$$\int_0^1 x\cot^{-1}\left(1 - x^2 + x^4\right)dx = \int_0^1 x\tan^{-1}\left(\frac{1}{1 + x^4 - x^2}\right)$$

$$\Rightarrow \quad \int_0^1 x\tan^{-1}\left(\frac{x^2 - \left(x^2 - 1\right)}{1 + x^2\left(x^2 - 1\right)}\right)dx$$

$$\Rightarrow \quad \int_0^1 x\tan^{-1} x^2 dx - \int_0^1 x\tan^{-1}\left(x^2 - 1\right)dx$$

$$\Rightarrow \quad \frac{1}{2}\int_0^1 1\tan^{-1} t\,dt - \frac{1}{2}\int_{-1}^0 1\tan^{-1} k\,dk$$

$$\Rightarrow \quad \frac{1}{2}\left(t\tan^{-1}t\Big|_0^1 - \int_0^1 \frac{t}{1+t^2}dt\right) - \frac{1}{2}\left(k\tan^{-1}k\Big|_{-1}^0 - \int_{-1}^0 \frac{k}{1+k^2}dk\right)$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{\pi}{4} - \left(\frac{1}{2}\ln\left(1+t^2\right)\right)\Big|_0^1\right) - \frac{1}{2}\left(0 - \frac{\pi}{4} - \left(\frac{1}{2}\ln\left(1+k^2\right)\right)\Big|_{-1}^0\right)$$

$$\Rightarrow \quad \left(\frac{\pi}{8} - \frac{1}{4}\ln 2\right) - \left(\frac{-\pi}{8} - \frac{1}{4}10 - \ln 2\right)$$

$$\Rightarrow \quad \frac{\pi}{4} - \frac{1}{2}\ln 2$$

26. Answer (3)

Using L' Hospital rule and Leibnitz theorem,

$$\lim_{x\to 2}\frac{\int_6^{f(x)} 2t\,dt}{(x-2)}$$

$$\lim_{x\to 2}\frac{2f(x)f'(x) - 0}{1}$$

$2f(2)f'(2) = 12f'(2)$

27. Answer (2)

$$I = \int_0^{2\pi}\left[\sin 2x\left(1+\cos 3x\right)\right]dx \qquad \text{...(i)}$$

$$\because \quad \int_0^a f(x) = \int_0^a f(a-x)\,dx$$

$$\therefore \quad I = \int_0^{2\pi}\left[-\sin 2x\left(1+\cos 3x\right)\right]dx \qquad \text{...(ii)}$$

By (i) + (ii),

$$2I = \int_0^{2\pi}(-1)dx$$

$$2I = -(x)_0^{2\pi}$$

$\Rightarrow \quad I = -\pi$

$$\lim_{n\to\infty}\frac{(n+1)^{\frac{1}{3}} + (n+2)^{\frac{1}{3}} + \ldots + (n+n)^{\frac{1}{3}}}{n(n)^{\frac{1}{3}}}1$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{(n+r)^{\frac{1}{3}}}{n.n^{\frac{1}{3}}} \qquad \frac{r}{n}\to x \text{ and } \frac{1}{n}\to dx$$

$$= \int_0^1 (1+x)^{\frac{1}{3}}dx$$

$$= \left[\frac{3}{4}(1+x)^{\frac{4}{3}}\right]_0^1$$

$$= \frac{3}{4}(2)^{\frac{4}{3}} - \frac{3}{4}$$

29. Answer (1)

$$I = \int_{\pi/6}^{\pi/3}\sec^{2/3}x.\mathrm{cosec}^{4/3}x\,dx$$

$$= \int_{\pi/6}^{\pi/3}\frac{1.dx}{\cos^{2/3}x.\sin^{4/3}x}$$

$$= \int_{\pi/6}^{\pi/3}\frac{1 \quad dx}{\cos^2 x.\tan^{4/3}x} = \int_{\pi/6}^{\pi/3}\frac{\sec^2 x dx}{\tan^{4/3}x}$$

Let $\tan x = t$

$$I = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} t^{-4/3}dt = \frac{3\left[t^{-1/3}\right]_{1/\sqrt{3}}^{\sqrt{3}}}{-1}$$

$$= -3\left[3^{-1/6} - \frac{1}{3^{-1/6}}\right]$$

$$= -3(3^{-1/6} - 3^{-1/6})$$

$$= 3(3^{1/6} - 3^{-1/6})$$

$$= 3^{7/6} - 3^{5/6}$$

$$\int_{6} 4t^3 dt = (x-2)g(x)$$

$4(f(x))^3.f'(x) = g'(x)(x-2) + g(x)$

put $x = 2$,

$$\frac{4(6)^3.1}{48} = g(2)$$

$$\lim_{x\to 2} g(x) = 18$$

31. Answer (3)

$$\int_0^{\pi/2} \frac{\cot x\, dx}{\cot x + \operatorname{cosec} x} = \int_0^{\pi/2} \frac{\cos x\, dx}{1+\cos x}$$

$$= \int_0^{\pi/2} \left(1 - \frac{1}{1+\cos x}\right) dx$$

$$= [x]_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2\cos^2 \frac{x}{2}} dx$$

$$= \frac{\pi}{2} - \frac{1}{2}\int_0^{\pi/2} \sec^2 \frac{x}{2} dx$$

$$= \frac{\pi}{2} - \left[\tan\frac{x}{2}\right]_0^{\pi/2}$$

$$= \frac{\pi}{2} - [1] = \left(\frac{\pi}{2} - 1\right)$$

$$m = \frac{1}{2}, n = -2$$

$\Rightarrow$ $mn = -1$

32. Answer (2)

$$\int_\alpha^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \int_\alpha^{\alpha+1} \left[\frac{1}{x+\alpha} - \frac{1}{x+\alpha+1}\right] dx$$

$$= \ln\left(\frac{x+\alpha}{x+\alpha+1}\right)\Bigg|_\alpha^{\alpha+1}$$

$$= \ln\left(\frac{2\alpha+1}{2\alpha+2}\cdot\frac{2\alpha+1}{2\alpha}\right) = \ln\frac{9}{8}$$

So, $\dfrac{(2\alpha+1)^2}{\alpha(\alpha+1)} = \dfrac{9}{2}$

$\Rightarrow$ $8\alpha^2 + 8\alpha + 2 = 9\alpha^2 + 9\alpha$

$\Rightarrow$ $\alpha^2 + \alpha - 2 = 0$

$\Rightarrow$ $\alpha = 1, -2$

$$\because \quad 4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5$$

$$\Rightarrow \quad 4\alpha\left\{\int_{-1}^{0} e^{\alpha x}dx + \int_0^2 e^{-\alpha x}dx\right\} = 5$$

$$4\alpha\left\{\left(\frac{e^{\alpha x}}{\alpha}\right)_{-1}^{0} + \left(\frac{e^{-\alpha x}}{-\alpha}\right)_0^2\right\} = 5$$

$4(1 - e^{-\alpha} - e^{-2\alpha} + 1) = 5$

$4(2 - e^{-\alpha} - e^{-2\alpha}) = 5$

$4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$

$(2e^{-\alpha} + 3)(2e^{-\alpha} - 1) = 0$

$$\therefore \quad e^{-\alpha} = \frac{1}{2} \Rightarrow \alpha = \ln 2$$

34. Answer (1)

Let $f(x) = 2x^3 - 9x^2 + 12x + 4$

$\Rightarrow$ $f'(x) = 6(x^2 - 3x + 2)$

$\therefore$ $f(x)$ decreases in $(1, 2)$, $f(1) = 9$

$f(2) = 8$

$$\Rightarrow \quad \frac{1}{3} < I < \frac{1}{\sqrt{8}}$$

$$\Rightarrow \quad \boxed{\frac{1}{9} < I^2 < \frac{1}{8}}$$

35. Answer (1)

$$\lim_{x\to 0} \int_0^x \frac{t\sin(10t)dt}{x}$$

by L′ hospital rule

$$\lim_{x\to 0} \frac{x\sin(10x)}{1} = \boxed{0}$$

36. Answer (1)

Let $I = \displaystyle\int_0^{2\pi} \frac{x\cdot\sin^8 x}{\sin^8 x + \cos^8 x}dx$ ...(i)

$I = \displaystyle\int_0^{2\pi} \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x}dx$ ...(ii)

(i) + (ii)

$$2I = 2\pi\int_0^{2\pi} \frac{\sin^8 x}{\sin^8 x + \cos^8 x}dx$$

Again $I = 4\pi\int_0^{\pi/2} \dfrac{\cos^8 x}{\sin^8 x + \cos^8 x}dx$ ...(iv)

(iii) + (iv)

$2I = 4\pi\int_0^{\pi/2} dx = 2\pi^2 \Rightarrow I = \pi^2$

37. Answer (3)

$$\int_0^1 (a + bx + cx^2)dx = \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]_0^1$$

$$= \frac{1}{6}(6a + 3b + 2c)$$

$\because$ f(0) = a ....(i)

f(1) = a + b + c ...(ii)

$f\left(\dfrac{1}{2}\right) = a + \dfrac{b}{2} + \dfrac{c}{4}$

$\Rightarrow 4f\left(\dfrac{1}{2}\right) = 4a + 2b + c$ ....(iii)

(i) + (ii) + (iii)

$6a + 3b + 2c = f(0) + f(1) + 4f\left(\dfrac{1}{2}\right)$

Hence, $\int_0^1 f(x)dx = \dfrac{1}{6}\left(f(0) + f(1) + 4f\left(\dfrac{1}{2}\right)\right)$

38. Answer (1)

$I = \int_{-\pi}^{\pi} \big|\pi - |x|\big| dx$

$= 2\int_0^{\pi} \big|\pi - |x|\big| dx$

$= 2\int_0^{\pi} (\pi - x)dx$

$= 2\left[\pi x - \dfrac{x^2}{2}\right]_0^{\pi}$

$= 2\left(\pi^2 - \dfrac{\pi^2}{2}\right) = \pi^2$

39. Answer (4)

$\int_0^{1/2} \dfrac{x^2}{(1-x^2)^{3/2}}dx$ let x = sinθ, dx = cosθdθ

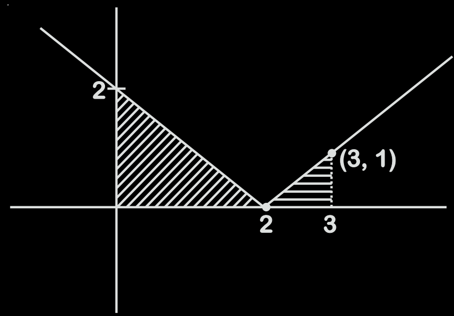
$= \int_0^{\pi/6} \dfrac{\sin^2\theta \cdot \cos\theta d\theta}{\cos^3\theta} = \int_0^{\pi/6} \tan^2\theta\, d\theta$

$= \int_0^{\pi/6} (\sec^2\theta - 1)d\theta = [\tan\theta - \theta]_0^{\pi/6} = \dfrac{1}{\sqrt{3}} - \dfrac{\pi}{6}$

$\because \dfrac{k}{6} = \dfrac{1}{\sqrt{3}} - \dfrac{\pi}{6}$

$\Rightarrow k = 2\sqrt{3} - \pi$

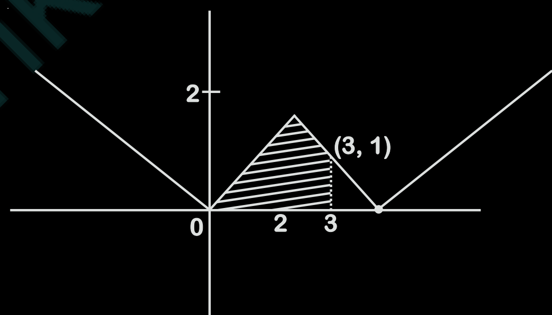
So $\int_0^3 f(x)dx = \dfrac{1}{2}(2\times 2 + 1\times 1) = \dfrac{5}{2}$



$\because g(x) = \big||x-2| - 2\big|$

So $\int_0^3 g(x)dx = \dfrac{1}{2}(2\times 2) + \dfrac{1}{2}\times 1\times(1+2)$

$= \dfrac{7}{2}$



Now $\int_0^3 (g(x) - f(x))dx = \dfrac{7}{2} - \dfrac{5}{2} = 1$

41. Answer (3)

$\tan^3 x\sin^2 3x\ (2\sec^2 x\ \sin^2 3x + 3\tan x\sin 6x)$

$= \dfrac{d}{dx}\dfrac{(\tan^4 x\sin^4 3x)}{2}$

$\therefore \displaystyle\int_{\pi/6}^{\pi/3} \tan^3 x\sin^2 3x\left(2\sec^2 x\sin^2 3x + 3\tan x\sin 6x\right)dx$

$= \left.\dfrac{\tan^4 x\sin^4 3x}{2}\right|_{\pi/6}^{\pi/3}$

$= \dfrac{9.0}{2} - \dfrac{\frac{1}{9}.1}{2} = \dfrac{-1}{18}$

$$I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$$

$$= \int_0^{\pi/2} \left(\frac{1}{1+e^{\sin x}} + \frac{1}{1+e^{-\sin x}}\right) dx$$

$$= \int_0^{\pi/2} \frac{1+e^{\sin x}}{1+e^{\sin x}} dx$$

$$= \frac{\pi}{2}$$

43. Answer (2)

$$I_1 = \int_0^1 (1-x^{50})^{100} dx,\ I_2 = \int_0^1 (1-x^{50})^{101} dx$$

Here, $\alpha = \dfrac{I_2}{I_1}$

Now, $I_2 = \int_0^1 (1-x^{50})^{101} dx = \int_0^1 (1-x^{50})(1-x^{50})^{100} dx$

$$I_2 = \int_0^1 (1-x^{50})^{100} - \int_0^1 \underset{I}{x}\cdot \underbrace{x^{49}(1-x^{50})^{100}}_{II} dx$$

$$I_2 = I_1 + \left[\frac{x}{5050}(1-x)^{101}\right]_0^1 - \int_0^1 \frac{(1-x^{50})^{101}}{5050} dx$$

$$I_2 = I_1 + 0 - \frac{I_2}{5050}$$

$$\frac{5051}{5050} I_2 = I_1$$

$$\Rightarrow \quad \frac{I_2}{I_1} = \frac{5050}{5051} = \alpha$$

44. Answer (2)

$$I = \int_1^2 e^x x^x (2 + \log_e x) dx$$

$$I = \int_1^2 e^x x^x \left[1 + (1 + \log_e x)\right] dx$$

$$= \int_1^2 e^x \left[x^x + x^x (1 + \log_e x)\right] dx$$

$$= \left[e^x x^x\right]_1^2 \quad \left\{\int e^x (f(x) + f'(x)) dx \ = e^x f(x) + c\right\}$$

$= e^2 \times 4 - e \times 1$

$= 4e^2 - e$

$= e(4e - 1)$

$$I = \int_0^1 |1 - x - x| dx + \int_1^2 |x - 1 - x| dx$$

$$= \int_0^1 |1 - 2x| dx + \int_1^2 dx$$

$$= \int_0^{\frac{1}{2}} (1 - 2x) dx + \int_{\frac{1}{2}}^1 (2x - 1) dx + \int_1^2 dx$$

$$= \left[x - x^2\right]_0^{\frac{1}{2}} + \left[x^2 - x\right]_{\frac{1}{2}}^1 + (2 - 1)$$

$$= \left(\frac{1}{2} - \frac{1}{4}\right) + (-)\left[\frac{1}{4} - \frac{1}{2}\right] + 1$$

$$= \frac{3}{2}$$

46. Answer (1)

$$\int_1^2 |2x - [3x]|\ dx$$

$$= \int_1^2 |\{3x\} - x|\ dx$$

$$= \int_1^2 (x - \{3x\})\ dx$$

$$= \int_1^2 x dx - \int_1^2 \{3x\}\ dx$$

$$= \frac{(4-1)}{2} - 3\int_0^{\frac{1}{3}} 3x dx$$

$$= \frac{3}{2} - \frac{1}{2} = 1$$

47. Answer (21)

$$\int_0^n \{x\} dx = n\int_0^1 x.dx = \frac{n}{2}$$

$$\int_0^n [x] dx = \int_0^n (x - \{x\}) dx = \frac{n^2}{2} - \frac{n}{2}$$

Given that $\dfrac{n}{2}, \dfrac{n^2 - n}{2}, 10(n^2 - n)$ are in GP

$$\therefore \quad \left(\frac{n^2 - n}{2}\right)^2 = \frac{n}{2} \times 10(n^2 - n)$$

$\Rightarrow \quad n^2 = 21n$

$\therefore \quad n = 21$

$$\lim_{x\to 0}\frac{\int_0^{x}\left(\sin\sqrt{t}\right)dt}{x^3}=\frac{0}{0}\text{ (form)}$$

$\Rightarrow$ By D, L Hospital rule

$$\lim_{x\to 0}\frac{2x\sin x}{3x^2}=\frac{2}{3}\lim_{x\to 0}\left(\frac{\sin x}{x}\right)$$

$$=\frac{2}{3}\times 1=\frac{2}{3}$$

49. Answer (2)

$$\because\quad 2\cot^2\theta-\frac{5}{\sin\theta}+4=0$$

$$2+2\operatorname{cosec}^2\theta-5\operatorname{cosec}\theta=0$$

$$2\operatorname{cosec}^2\theta-4\operatorname{cosec}\theta-\operatorname{cosec}\theta+2=0$$

$$\therefore\quad (2\operatorname{cosec}\theta-1)(\operatorname{cosec}\theta-2)=0$$

$$\therefore\quad \operatorname{cosec}\theta=\frac{1}{2}\text{ or } 2.$$

$$\therefore\quad \sin\theta=2\text{ or }\frac{1}{2}.\qquad \because\ \theta\in(0,\ 2\pi)$$

$$\therefore\quad \theta_1=\frac{\pi}{6}\ \text{ and }\ \theta_2=\frac{5\pi}{6},\qquad \because\ \theta_1<\theta_2$$

$$\therefore\quad I=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\cos^2 3\theta\,d\theta=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\frac{1+\cos 6\theta}{2}d\theta$$

$$=\frac{1}{2}\left[\theta+\frac{\sin 6\theta}{6}\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}=\frac{\pi}{3}$$

50. Answer (3)

$$\int_{-a}^{a}(|x|+|x-2|)dx=\frac{1}{2}a^2+\frac{1}{2}a^2+\frac{1}{2}(a-2)^2+\frac{1}{2}(a+2)^2$$

$\Rightarrow$ $22 = 2a^2 + 4 \Rightarrow a = 3$

Now,

$$\int_{3}^{-3}(x+[x]dx=-\int_{-3}^{3}[x]dx=-\int_{-3}^{-2}[x]dx=-\int_{-3}^{-2}(-3)dx$$

$\Rightarrow$ $f'(x) - f'(2 - x) = 0$

Integrating both sides, we get

$f(x) + f(2 - x) = c$ ...(i)

Put $x = 0$, we get

$c = f(0) + f(2) = 1 + e^2$

Integrating 0 to 2 equation (i) both sides, we get

$$\int_0^2 f(x)dx+\int_0^2 f(2-x)dx=(1+e^2)\times 8\Big|_0^2$$

Also $\displaystyle\int_0^2 f(x)dx=\int_0^2 f(2-x)dx$

Hence $\displaystyle 2\int_0^2 f(x)dx=2(1+e^2)$

$$\Rightarrow\quad \int_0^2 f(x)dx=1+e^2$$

52. Answer (3)

$$I=\int_1^3[x^2-2x-2]dx$$

$$=\int_1^3\left[\left(x^2-2x+1\right)-3\right]dx=\int_1^3\left[(x-1)^2\right]dx-\int_1^3 3dx$$

Now when $x\in[1,\ 3]$

we see that $(x-1)^2\in[0, 4]$

$$\text{So, } I=\int_0^1 0dx+\int_1^{\sqrt{2}}\left[(x-1)^2\right]dx+\int_{\sqrt{2}}^{\sqrt{3}}\left[(x-1)^2\right]dx+\int_{\sqrt{3}}^{2}\left[(x-1)^2\right]dx-\int_1^3 3dx$$

$$=0+\int_1^{\sqrt{2}}1dx+\int_{\sqrt{2}}^{\sqrt{3}}2dx+\int_{\sqrt{3}}^{2}3dx-6$$

$$=0+[x]_1^{\sqrt{2}}+2[x]_{\sqrt{2}}^{\sqrt{3}}+3[x]_{\sqrt{3}}^{2}$$

$$=\sqrt{2}-1+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}-6$$

$$=-\sqrt{3}-\sqrt{2}-1$$

$$=-\sqrt{2}-\sqrt{3}-1$$

$$x\frac{dy}{dx}+y=bx$$

$$\Rightarrow \frac{dy}{dx}+\frac{y}{x}=bx^3$$

$$\text{I.F} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

$$\Rightarrow yx=\int bx^4dx$$

$$\Rightarrow xy=\frac{bx^5}{5}+c$$

$$\downarrow (1,\ 2)$$

$$\Rightarrow 2=\frac{b}{5}+c \Rightarrow c=2-\frac{b}{5}$$

$$\therefore y=\frac{bx^4}{5}+\frac{1}{x}\left(2-\frac{b}{5}\right)$$

$$\int_1^2 f(x)dx=\frac{bx^5}{25}\bigg|_1^2+\left(2-\frac{b}{5}\right)\ln\ x\big|_1^2$$

$$\frac{31b}{25}+\left(2-\frac{b}{5}\right)\ln 2=\frac{62}{5}$$

$$\Rightarrow \left(2-\frac{b}{5}\right)\ln 2=\left(2-\frac{b}{5}\right)\frac{31}{5}$$

$$\Rightarrow 2-\frac{b}{5}=0 \Rightarrow b = 10$$

54. Answer (3)

$$\int_{-1}^{1} x^2e^{[x^3]}dx=\int_{-1}^{0} x^2e^{[x^3]}dx+\int_{0}^{1} x^2e^{[x^3]}dx$$

$$=\int_{-1}^{0} x^2\cdot e^{-1}dx+\int_{0}^{1} x^2\cdot e^{0}dx$$

$$=\frac{1}{e}\int_{-1}^{0} x^2dx+\int_{0}^{1} x^2dx$$

$$=\frac{1}{e}\frac{x^3}{3}\bigg|_{-1}^{0}+\frac{x^3}{3}\bigg|_{0}^{1}$$

$$=\frac{1}{3e}+\frac{1}{3}=\frac{e+1}{3e}$$

55. Answer (4)

$$I_n=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{n-2} x\left(\cot^2 x\right)dx$$

$$=\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{n-2} x \operatorname{cosec}^2 x\ dx-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{n-2} x dx$$

$$=-\frac{\cot^{n-1}x}{n-1}\Bigg]_{\frac{\pi}{4}}^{\frac{\pi}{2}}-I_{n-2}=\frac{1}{n-1}-I_{n-2}$$

$$I_n+I_{n-2}=\frac{1}{n-1} \Rightarrow \frac{1}{I_{n-2}+I_n}=n-1$$

= a linear expression in n.

$\therefore$ Sequence $\frac{1}{I_{n-2}+I_n}$ is an A.P.

56. Answer (1)

$$\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{\left(1+\frac{r}{n}\right)^2}$$

$$\int_0^1 \frac{dx}{(1+x)^2}=-\frac{1}{1+x}\bigg|_0^1=-\frac{1}{2}+1=\frac{1}{2}$$

57. Answer (19)

$\because$ $3x^2 - 3x - 6 = 3(x^2 - x - 2)$

$= 3(x - 2)(x + 1)$

$$\therefore \int_{-2}^{2}\left|3x^2-3x-6\right|dx$$

$$=\int_{-2}^{-1}(3x^2-3x-6)dx+\int_{-1}^{2}(6+3x-3x^2)dx$$

$$=3\left\{\int_{-2}^{-1}(x^2-x-2)dx+\int_{-1}^{2}(2+x-x^2)dx\right\}$$

$$=3\left\{\left(\frac{x^3}{3}-\frac{x^2}{2}-2x\right)_{-2}^{-1}+\left(2x+\frac{x^2}{2}-\frac{x^3}{3}\right)_{-1}^{2}\right\}$$

$$=3\left\{\left(-\frac{1}{3}-\frac{1}{2}+2\right)-\left(-\frac{8}{3}-2+4\right)+\left(4+2-\frac{8}{3}\right)-\left(-2+\frac{1}{2}+\frac{1}{3}\right)\right\}$$

$$= 19$$

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a}(f(x)+f(a-x))dx$$

$$\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^x}dx = \int_{0}^{\pi/2}\frac{\cos^2 x}{1+3^x}+\frac{\cos^2(-x)}{1+3^{-x}}dx$$

$$= \int_{0}^{\pi/2}\cos^2 x\left(\frac{1}{1+3^x}+\frac{3^x}{1+3^x}\right)dx = \int_{0}^{\pi/2}\cos^2 x\,dx$$

$$= \frac{1}{2}\int_{0}^{\pi/2}(1+\cos 2x)dx = \frac{\pi}{4}$$

59. Answer (1)

$$\int_{n-1}^{n} e^{x-[x]}\,dx = \int_{0}^{1} e^{x}\,dx = (e-1)$$

$$\therefore \quad \sum_{n=1}^{100}(e-1) = 100(e-1)$$

60. Answer (02)

$$\int_{0}^{\pi}|\sin 2x|\,dx$$

$$= \int_{0}^{\frac{\pi}{2}}\sin 2x dx + \int_{\frac{\pi}{2}}^{\pi}-\sin 2x dx$$

$$= \left[-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{2}} + \left[\frac{\cos 2x}{2}\right]_{\frac{\pi}{2}}^{\pi}$$

$$= \frac{1}{2}+\frac{1}{2}+\left(\frac{1}{2}+\frac{1}{2}\right)$$

$= 2$

61. Answer (2)

$$f(x) = \int_1^x \frac{\ln t}{1+t}dt$$

then $f\left(\frac{1}{x}\right) = \int_1^{1/x}\frac{\ln t}{1+t}dt$

Let $t = \frac{1}{u} \Rightarrow dt = -\frac{1}{u^2}du$

$$\Rightarrow \quad f\left(\frac{1}{x}\right) = \int_1^x \frac{\ln\frac{1}{u}}{1+\frac{1}{u}}\left(-\frac{1}{u^2}\right)dx$$

$$\therefore \quad f(x)+f\left(\frac{1}{x}\right) = \int_1^x \ln t\left(\frac{1}{1+t}+\frac{1}{t(1+t)}\right)dt$$

$$= \int_1^x \ln t\left(\frac{1}{1+t}+\frac{1}{t}-\frac{1}{t+1}\right)dt$$

$$= \int_1^x \frac{\ln t}{t}dt = \frac{1}{2}(\ln x)^2$$

$$\therefore \quad f(e)+f\left(\frac{1}{e}\right) = \frac{1}{2}(\ln e)^2 = \frac{1}{2}$$

62. Answer (1)

$\because \quad I_{m,n} = \beta_{m,n}$

$= \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$ let $x = \tan^2\theta$

$$= \int_0^{\pi/4}\frac{\tan^{2m-2}\theta+\tan^{2n-2}\theta}{\sec^{2(m+n)}\theta}.2\tan\theta\sec^2\theta\, d\theta$$

$$= 2\int_0^{\pi/4}\frac{\tan^{2m-1}\theta+\tan^{2n-1}\theta}{\sec^{2(m+n-1)}\theta}d\theta$$

$$= 2\int_0^{\pi/4}\left[\sin^{2m-1}\theta.\cos^{2n-1}\theta+\sin^{2n-1}\theta.\cos^{2m-1}\theta\right]d\theta$$

$$= 2\int_0^{\pi/2}\sin^{2m-1}\theta\cdot\cos^{2n-1}\theta d\theta$$

$= \beta_{m,\,n}$

Clearly $\alpha = 1$

63. Answer (01)

$$\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+....+f\left(\frac{n}{n}\right)\right)$$

$$\Rightarrow \quad \lim_{n\to\infty}\frac{2}{n}\sum_{r=1}^{n}\log_2\left(1+\tan\frac{\pi r}{4n}\right)$$

$$\Rightarrow \quad I = 2\int_0^1 \log_2\left(1+\tan\frac{\pi x}{4}\right)dx$$

Using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ we get,

$$I = 2\int_0^1 \log_2\left(1+\tan\frac{\pi}{4}(1-x)\right)dx$$

$$\Rightarrow \quad 2I = 2\int_0^1 \log_2\left(\left(1+\tan\frac{\pi x}{4}\right)\left(1+\tan\frac{\pi(1-x)}{4}\right)\right)dx$$

$$\Rightarrow \quad 2I = 2\int_0^1 \log_2^2 dx$$

$$\Rightarrow \quad I = \int_0^1 dx = 1$$

$$y(x)=\int_0^x (2t^2-15t+10)dx$$

by Leibnitz's rule

$y'(x)=2x^2-15x+10$ (given $m_N = 3$)

$\Rightarrow\ 2x^2 - 15x + 10 = 3 \Rightarrow x=\frac{1}{2}$ or 7 (but a > 1)

$\therefore\ a = 7$

$$b = y(7) = \int_0^7 (2t^2-15t+10)dt$$

$$=\left(\frac{2t^3}{3}-\frac{15t^2}{2}+10t\right)\Bigg|_0^7=\frac{686}{3}-\frac{735}{2}+70$$

$$=\frac{1372-2205+420}{6}$$

$|a + 6b| = |7 - 413| = 406$

65. Answer (16)

$$f(x)+f(x+1)=2$$

$$\Rightarrow\ \frac{f(x+1)+f(x+2)=2}{f(x)-f(x+2)=0}\ \Rightarrow f(x) \text{ has fundamental period} = 2$$

$\therefore\ f(x) = 2 - f(x + 1)$

$$\Rightarrow\ \int_0^2 f(x)dx=\int_0^2 2dx-\int_0^2 f(x+1)dx$$

Now, $I=\int_0^2 f(x+1)dx$

Put x + 1 = t

dx = dt

$$\Rightarrow\ I=\int_1^3 f(t)dt$$

$$\Rightarrow\ I=\int_0^2 f(x)dx=\int_0^2 f(x+1)dx=\int_0^2 dx=2=I$$

by periodicity $I_1 = 4I$ and $I_2 = 2I$

$\Rightarrow\ I_1 + 2I_2 = 8I = 16$

$$\int_0 x^2+bx+c=1$$

$$\left.\frac{x^3}{3}+\frac{bx^2}{2}+cx\right]_0^1=1$$

$$\frac{1}{3}+\frac{b}{2}+c=1$$

$3b + 6c = 4$ ...(i)

$P(2) = 5 \Rightarrow 4 + 2b + c = 5$

$2b + c = 1$ ...(ii)

(i) & (ii) $\Rightarrow b=\frac{2}{9}, c=\frac{5}{9}$

$9(b + c) = 7$

67. Answer (2)

$$I=\sum_{k=0}^{9}\int_k^{k+1} ke^{[x]}.e^{1-x}dx=\sum_{k=0}^{9}\int_k^{k+1} ke^{1-\{x\}}dx$$

$$=\sum_{k=0}^{9}k\int_0^1 e^{1-x}dx$$

[$\because$ {x} is periodic function with period 1]

$$=\sum_{k=0}^{9}k\left(-e^{(1-x)}\right)_0^1=\sum_{k=0}^{9}k(e-1)$$

$$=(e-1)\sum_{k=0}^{9}k=45(e-1)$$

68. Answer (4)

$$g(\alpha)=\int_{\pi/6}^{\pi/3}\frac{\sin^\alpha x}{\cos^\alpha x+\sin^\alpha x}dx.$$

$$g(\alpha)=\int_{\pi/6}^{\pi/3}\frac{\sin^\alpha\left(\frac{\pi}{2}-x\right)}{\cos^\alpha\left(\frac{\pi}{2}-x\right)x+\sin^\alpha\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_{\pi/6}^{\pi/3}\frac{\cos^\alpha x}{\sin^\alpha x+\cos^\alpha x}dx$$

$$2.g(\alpha)=\int_{\pi/6}^{\pi/3}\frac{\sin^\alpha x+\cos^\alpha x}{\sin^\alpha x+\cos^\alpha x}dx=\int_{\pi/6}^{\pi/3}dx=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}.$$

$g(\alpha)=\frac{\pi}{12}$ i.e. a constant function hence an even function.

$$\left|\int_0^{\sqrt{\frac{\pi}{2}}}\left[[x^2]-\cos x\right]dx\right| = \left|\int_0^1(-1)dx + \int_1^{\sqrt{\frac{\pi}{2}}} 0\cdot dx\right|$$

$= 1$

70. Answer (3)

$$\int_0^{10}\frac{[\sin 2\pi x]}{e^{\{x\}}}dx = 10\int_0^1\frac{[\sin 2\pi x]}{e^x}dx$$

$$= -10\int_{1/2}^1\frac{dx}{e^x} = 10e^{-x}\int_{1/2}^1 = 10 = (e^{-1} - e^{-1/2})$$

$\Rightarrow \ \alpha + \beta + \gamma = 0$

71. Answer (3)

$$I = \int_0^1 e^x f'(x)dx + \int_0^1 e^x \cdot f(x)dx$$

$$2\int_0^1 e^x f(x)dx = 2\int_0^1 \sin x\ dx$$

$$2\int_0^1\left(x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{720} + ...\right)dx$$

$$\Rightarrow \quad 2\left[\frac{1}{2} - \frac{1}{4.6}\right] < I < 2\left[\frac{1}{2} - \frac{1}{4.6} + \frac{1}{6.120}\right]$$

$$\Rightarrow \quad \frac{11}{12} < I < \frac{331}{360}$$

72. Answer (1)

$$I_n = \int_1^e x^{19}.(\ln x)^n\, dx$$

$$\Rightarrow I_n = \left.\frac{(\ln x)^n \cdot x^{20}}{20}\right|_1^e - \int_1^e n(\ln x)^{n-1}\frac{x^{19}}{20}dx$$

$\Rightarrow 20I_n = e^{20} - nI_{n-1}$

So, $20I_{10} = e^{20} - 10I_9$

and $20I_9 = e^{20} - 9I_8$

$\overline{20I_{10} = 10I_9 + 9I_8}$

$$I = \int_{-4}^{4} f(x)^2 dx = 2\cdot\int_0^4 f(x)^2 dx$$

$\because \quad f(x^2) = 4x^3 - g(4 - x)$

$$I = 2\cdot\int_0^4\left(4x^3 - g(4-x)\right)dx = 2\cdot 4\frac{x^4}{4}\int_0^4 -2\cdot\int_0^4 g(4.x)dx$$

$I = 512 - 2 \cdot I_1$

$$I_1 = \int_0^4 g(4-x)dx =$$

$$= \int_0^4 g\left(4-(0+4-x)\right)dx = \int_0^4 g(x)dx$$

$$I_1 = -\int_0^4 g(4-x)dx$$

$\Rightarrow \ I_1 = 0$

Hence $I = 512$

74. Answer (2)

$$g(3) = \int_0^3 f(t)dt$$

$$g(3) = \int_0^1 f(t)dt + \int_1^2 f(t)dt \qquad \ldots(i)$$

$$\int_0^1 \frac{1}{3}dt \le \int_0^1 f(t)dt \le \int_0^1 1dt$$

$$\frac{1}{3} \le \int_0^1 f(t)dt \le 1 \qquad \ldots(ii)$$

$$\int_1^3 0\,dt \le \int_1^3 f(t)dt \le \int_1^3 \frac{1}{2}dt$$

$$0 \le \int_1^3 f(t)dt \le \frac{1}{2}\times 2 = 1 \qquad \ldots(iii)$$

(i), (ii), (iii) $\quad\Rightarrow\quad \frac{1}{3} \le g(3) \le 2$

$x = \pm 1$

let $P'(x) = k(x-1)(x+1) = kx^2 - k$

$\Rightarrow P(x) = \frac{k}{3}.x^3 - kx + \lambda$

$\int_{-1}^{1} p(x)dx = 18 \Rightarrow \int_{-1}^{1}\left(\frac{k}{3}.x^3 - kx + \lambda\right)dx = 2\lambda = 18$

$\Rightarrow \lambda = 9$

Also P(–3) = 0

$\Rightarrow \frac{k}{3}(-3)^3 - k(-3) + 9 = 0 \Rightarrow k = \frac{3}{2}$

$P(x) = \frac{1}{2}x^3 - \frac{3}{2}x + 9 \Rightarrow$ sum of all the coefficients = 8

76. Answer (4)

$\because \int_0^a e^{x-[x]}dx = 10e - 9$

Here $e^{x-[x]}$ is periodic function of period 1

$\therefore \int_0^{[a]+\{a\}} e^{\{x\}}dx = 10e - 9$

$\Rightarrow [a]\int_0^1 e^x dx + \int_0^{\{a\}} e^x dx = 10e - 9$

$\Rightarrow [a](e-1) + (e^{\{a\}} - 1) = 10e - 9$

$\Rightarrow [a]e - e^{\{a\}} - [a] - 1 = 10e - 9$

$\therefore$ Possible value of a = $10 + \log_e 2$

77. Answer (2)

Let x = cos2θ

dx = –2sin2θ

$$I = \int_0^{\frac{\pi}{2}} 2\sin 2\theta \ln\left(\sqrt{2}(\sin\theta + \cos\theta)\right)$$

$$= 2\left[\frac{-\cos 2\theta}{2} \times \ln\left(\sqrt{2}(\sin\theta + \cos\theta)\right)\Big|_0^{\pi/2} - \int_0^{\frac{\pi}{2}}\left(\frac{-\cos 2\theta}{2}\right) \times \left(\frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}\right)d\theta\right]$$

$$= 2\left[\ln\sqrt{2} + \frac{1}{2}\int_0^{\frac{\pi}{2}}(\cos\theta - \sin\theta)^2 d\theta\right]$$

$$= \ln 2 + \int_0^{\frac{\pi}{2}}(1 - \sin 2\theta)d\theta = \ln 2 + \frac{\pi}{2} - 1$$

$\therefore f(x) = \ln\left(x + \sqrt{x^2+1}\right)$

$\therefore$

$f(x) + f(-x) = \ln\left(\sqrt{x^2+1} + x\right) + \ln\left(\sqrt{x^2+1} - x\right)$

$\therefore f(x) + f(-x) = 0$ ...(i)

$\because g(t) = \int_{-\pi/2}^{\pi/2}\cos\left(\frac{\pi}{4}t + f(x)\right)dx.$

$$= \int_0^{\pi/2}\left\{\cos\left(\frac{\pi}{4}t + f(x)\right) + \cos\left(\frac{\pi}{4}t + f(-x)\right)\right\}dx.$$

$$= \int_0^{\pi/2}\left\{\cos\left(\frac{\pi t}{4} + f(x)\right) + \cos\left(\frac{\pi t}{4} - f(x)\right)\right\}dx.$$

$$g(t) = 2\int_0^{\pi/2}\cos\frac{\pi t}{4}\cdot\cos(f(x))\,dx.$$

$\therefore g(1) = \sqrt{2}\int_0^{\pi/2}\cos(f(x))\,dx$

and $g(0) = 2\int_0^{\pi/2}\cos(f(x))\,dx.$

$\therefore \sqrt{2}\,g(1) = g(0)$

79. Answer (1)

$$I = \int_{-\pi/2}^{\pi/2}[[x] - \sin x]dx$$

$$= \int_{-\pi/2}^{\pi/2}([x] + [-\sin x])dx$$

$$= \int_0^{\pi/2}([x] + [-\sin x] + [-x] + [\sin x])dx$$

$$= \int_0^{\pi/2}(-2)dx$$

$= -\pi$

80. Answer (1)

$$Z = \frac{1}{1 - \cos\theta + 2i\sin\theta} = \frac{(1-\cos\theta) - 2i\sin\theta}{(1-\cos\theta)^2 + 4\sin^2\theta}$$

$\therefore \text{Re}(Z) = \frac{1-\cos\theta}{2 - 2\cos\theta + 3\sin^2\theta} = \frac{1}{5}$

$\therefore$ 5 – 5 cosθ = 2 – 2 cosθ + 3sin²θ

3cosθ(1 – cosθ) = 0

$\therefore \theta = \frac{\pi}{2}$, when θ ∈ (0, π)

$\therefore \int_0^{\theta}\sin x\,dx = \int_0^{\pi/2}\sin x\,dx$

= 1

$$I = \int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}}dx$$

$\because$ Integrand is periodic with period 1

$$\therefore \quad I = 100\int_0^{\pi}\frac{\sin^2 x}{e^{\left\{\frac{x}{\pi}\right\}}}dx$$

Let $\frac{x}{\pi} = t \Rightarrow dx = \pi dt$

$$= 100\pi\int_0^1 \frac{\sin^2(\pi t)dt}{e^t}$$

$$= 50\pi\int_0^1 e^{-t}(1-\cos 2\pi t)dt$$

$$= 50\pi\int_0^1 e^{-t}dt - 50\pi\int_0^1 e^{-t}\cos(2\pi t)dt$$

$$= -50\pi\left[e^{-t}\right]_0^1$$

$$-50\pi\left[\frac{e^{-t}}{1+4\pi^2}(-\cos 2\pi t + 2\pi\sin 2\pi t)\right]_0^1$$

$$= -50\pi\,(e^{-1}-1) - \frac{50\pi}{1+4\pi^2}(e^{-1}(-1+0)-(-1+0))$$

$$= -50\pi\,(e^{-1}-1) - \frac{50\pi}{1+4\pi^2}\left(1-e^{-1}\right)$$

$$= 50\pi\,(1-e^{-1}) - \frac{50\pi\left(1-e^{-1}\right)}{1+4\pi^2}$$

$$= \frac{200\pi^3\left(1-e^{-1}\right)}{1+4\pi^2} = \frac{\alpha\pi^3}{1+4\pi^3} \quad \text{(Given)}$$

$\therefore \quad \alpha = 200\,(1-e^{-1})$

82. Answer (4)

$$I = \int_{\pi/24}^{5\pi/24}\frac{dx}{1+(\tan 2x)^{1/3}}$$

applying $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$$I = \int_{\pi/24}^{5\pi/24}\frac{dx}{1+\left(\tan\left(\frac{\pi}{2}-2x\right)\right)^{1/3}}$$

$$\int_{\pi/24} \frac{\quad}{1+(\tan 2x)^{2/3}} \qquad \int_{\pi/24}\frac{\quad}{1+(\tan 2x)^{1/3}}$$

$$= \int_{\pi/24}^{5\pi/24} dx$$

$$2I = \frac{5\pi}{24} - \frac{\pi}{24} \Rightarrow I = \frac{\pi}{12}$$

83. Answer (1)

Let $I = \int_{-1}^{1}\log\left(x+\sqrt{x^2+1}\right)dx$

As $f(x)$ is odd, $\int_{-a}^{a} f(x) = 0$

$\therefore \quad I = 0$

84. Answer (1)

$$\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\frac{\frac{2j-1}{n}+8}{\frac{2j-1}{n}+4}$$

$$\Rightarrow \quad \frac{1}{2}\int_0^2\left(1+\frac{4}{x+8}\right)dx$$

$$\Rightarrow \quad \frac{1}{2}\left(2+4\ln\frac{3}{2}\right) = 1+2\ln\left(\frac{3}{2}\right)$$

85. Answer (2)

$$I = \int_{-\pi/4}^{\pi/4}\frac{dx}{\left(1+e^{x\cos x}\right)\left(\sin^4 x+\cos^4 x\right)}$$

applying $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$I = \int_{-\pi/4}^{\pi/4}\frac{dx}{\left(1+e^{-x\cos x}\right)\left(\sin^4 x+\cos^4 x\right)}$$

$$2I = \int_{-\pi/4}^{\pi/4}\frac{dx}{\sin^4 x+\cos^4 x} = \int_{-\pi/4}^{\pi/4}\frac{\sec^2 x\left(1+\tan^2 x\right)dx}{\tan^4 x+1}$$

Put $\tan x = t$ we get

Put $t-\frac{1}{t}=R$

$$I=\int_{-\infty}^{0}\frac{dk}{k^2+2}$$

$$\Rightarrow\quad I=\frac{1}{\sqrt{2}}\tan^{-1}\frac{k}{\sqrt{2}}\Big|_{-\infty}^{0}=\frac{1}{\sqrt{2}}\left(0+\frac{\pi}{2}\right)$$

$$=\frac{\pi}{2\sqrt{2}}$$

86. Answer (1)

$\because\quad 18x-x^2-77>3\Rightarrow x^2-18x+80<0$

$\Rightarrow x\in(8,\,10)$

a = 8 and b = 10

Now, $I=\int_a^b\frac{\sin^3 x}{\sin^3 x+\sin^3(a+b-x)}dx$

So, $I=\int_a^b\frac{\sin^3(a+b-x)}{\sin^3(a+b-x)+\sin^3 x}dx$

hence $2I=\int_a^b dx=b-a$

$\Rightarrow\quad I$ = 1

87. Answer (5)Ans`wer (5)

$\int_0^{\pi}\left(\sin^3 x\right)\cdot e^{\sin^{-2x}}\,dx=\frac{1}{e}\int_0^{\pi}\sin^2 x\cdot e^{\cos^2 x}\cdot\sin x dx$

Let cos$x$ = t, sind$x$ = –dt

$$=\frac{1}{e}\int_1^{-1}\left(t^2-1\right)e^{t^2}dt=\frac{2}{e}\int_0^1\left(1-t^2\right)e^{t^2}dt$$

Let $t^2=z$, $dt=\frac{dz}{2\sqrt{z}}$

$$=\frac{1}{e}\int_0^1\left(\frac{1}{\sqrt{z}}-\sqrt{z}\right)e^z dz$$

$$=\frac{1}{e}\left[e^z\cdot 2\sqrt{z}\Big|_0^1-\int_0^1 2e^z\cdot\sqrt{z}dz-\int_0^1\sqrt{z}e^z dz\right]$$

$$=\frac{1}{e}\left[2e-3\int_0^1 e^t\cdot\sqrt{t}\,dt\right]$$

Clearly α = 2 and β = 3

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}\frac{1}{1+4\left(\frac{r}{n}\right)^2}$$

$$=\int_0^2\frac{dx}{1+4x^2}=\frac{1}{2}\tan^{-1}(2x)\Big|_0^2=\frac{1}{2}\tan^{-1}4$$

89. Answer (2)

$$I=\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left(\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right)^{1/2}dx$$

$$=\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left(\left(\frac{x+1}{x-1}-\frac{x-1}{x+1}\right)^2\right)^{1/2}dx$$

$$=\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left|\frac{4x}{x^2-1}\right|dx=2\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\frac{2|x|}{1-x^2}dx$$

$$=2\left\{\int_{-1/\sqrt{2}}^{0}\frac{-2x}{1-x^2}dx+\int_0^{1/\sqrt{2}}\frac{2x}{1-x^2}dx\right\}$$

$$=2\left\{\ln\left(1-x^2\right)\Big|_{-1/\sqrt{2}}^{0}+\left(-\ln\left(1-x^2\right)\right)\Big|_0^{1/\sqrt{2}}\right.$$

$$=2\left\{\left(0-\ln\frac{1}{2}\right)-\left(\ln\frac{1}{2}-0\right)\right\}=4\ln 2=\log_e 16$$

90. Answer (1)

$$I=\int_0^5\frac{x+[x]}{e^{x-[x]}}dx$$

$$I=\int_0^1\frac{x+0}{e^{x-0}}dx+\int_1^2\frac{x+1}{e^{x-1}}dx+\ldots+\int_4^5\frac{x+4}{e^{x-4}}dx$$

$$\therefore\quad I=\sum_{k=0}^{4}\int_k^{k+1}\frac{x+k}{e^{x-k}}\,dx$$

$$=\sum_{k=0}^{4}e^k\int_k^{k+1}(x+k)e^{-x}\,dx$$

$$=\sum_{k=0}^{4}e^k\left|-(x+k)e^{-x}-e^{-x}\right)\Big|_k^{k+1}$$

$$\sum_{k=0}$$

$$= \sum_{k=0}^{4} \left((-2k-1)e^{-1} - e^{-1} + (2k+1)\right)$$

$\Rightarrow$ $-25e^{-1} - 5e^{-1} + 25 = 30e^{-1} + 25$

$\Rightarrow$ $\alpha = -30$ and $\beta = 25$

$\Rightarrow$ $(\alpha + \beta)^2 = 25$

91. Answer (4)

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right) dx = \int_{0}^{\frac{\pi}{2}} \left(\frac{1+\sin^2 x}{1+\pi^{\sin x}} + \frac{1+\sin^2 x}{1+\pi^{-\sin x}}\right) dx$$

$$= \int_{0}^{\frac{\pi}{2}} \left(\frac{1+\sin^2 x}{1+\pi^{\sin x}} + \frac{\pi^{\sin x}\left(1+\sin^2 x\right)}{1+\pi^{\sin x}}\right) dx$$

$$= \int_{0}^{\frac{\pi}{2}} \frac{\left(1+\sin^2 x\right)\left(1+\pi^{\sin x}\right)}{\left(1+\pi^{\sin x}\right)} dx$$

$$= \int_{0}^{\frac{\pi}{2}} \left(1+\sin^2 x\right) dx = \int_{0}^{\frac{\pi}{2}} 1 + \frac{1-\cos 2x}{2} dx$$

$$= \int_{0}^{\frac{\pi}{2}} \left(\frac{3}{2} - \frac{1}{2}\cos 2x\right) dx = \left.\frac{3}{2}x - \frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{3}{2}\left(\frac{\pi}{2}\right) - \frac{1}{4}(0)$$

$$= \frac{3\pi}{4}$$

$$I = \int_{6} \frac{2\ln x}{2\ln x + 2\ln(22-x)} dx$$

$$I = \int_{6}^{16} \frac{\ln x}{\ln x + \ln(22-x)} dx \qquad ...(1)$$

$$I = \int_{6}^{16} \frac{\ln(22-x)}{\ln(22-x) + \ln x} dx \qquad ...(2)$$

Adding (1) and (2) we get

$$2I = \int_{6}^{16} dx = 10$$

$\Rightarrow$ $I = 5$

93. Answer (4)

Let $L = \lim_{n\to\infty} \left(U_n\right)^{-\frac{4}{n^2}}$

taking log of both sides we get:

$$\log L = \lim_{n\to\infty} -\frac{4}{n^2}\left\{\ln\left(1+\frac{1}{n^2}\right) + 2\ln\left(1+\frac{2^2}{n^2}\right) + 3\cdot\ln\left(1+\frac{3^2}{n^2}\right) + ... + n\cdot\ln\left(1+\frac{n^2}{n^2}\right)\right\}$$

$$\log L = -4\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \frac{r}{n}\ln\left(1+\left(\frac{r}{n}\right)^2\right)$$

$$= -4\int_0^1 x\ln\left(1+x^2\right)dx$$

$$= -2\int_0^1 2x\ln\left(1+x^2\right)dx \qquad \begin{cases}\text{Let } 1+x^2 = t \\ \therefore 2xdx = dt\end{cases}$$

$$= -2\int_1^2 \ln t\, dt$$

$$= -2(2\ln 2 - 1) = \ln\left(\frac{e^2}{16}\right)$$

$$\therefore \quad L = \frac{e^2}{16}$$

94. Answer (2)

Put $x = t^2$

$$\int_0^1 \frac{2t^2}{(1+t^2)(1+3t^2)(3+t^2)} dt$$

$$\frac{2t^2}{(1+t^2)(1+3t^2)(3+t^2)} = \frac{A}{1+t^2} + \frac{B}{1+3t^2} + \frac{C}{3+t^2}$$

$$\Rightarrow \quad \frac{1}{2}\int_0^1 \frac{dt}{1+t^2} - \frac{3}{8}\int_0^1 \frac{dt}{1+3t^2} - \frac{3}{8}\int_0^1 \frac{dt}{3+t^2}$$

$$= \frac{\pi}{8} - \frac{3}{8}\left(\frac{1}{\sqrt{3}}\cdot\frac{\pi}{3}\right) - \frac{3}{8}\left(\frac{1}{\sqrt{3}}\frac{\pi}{6}\right)$$

$$= \frac{\pi}{8} - \frac{\sqrt{3}}{16}\pi = \frac{(2-\sqrt{3})\pi}{16}$$

95. Answer (4)

$$\int_0^x \sqrt{1-(f'(t)^2}\,dt = \int_0^x f(t)dt,\ x \in [0, 1]$$

On differentiating both sides we get

$$\sqrt{1-(f'(x))^2} = f(x)$$

$$1-(f(x))^2 = (f'(x))^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \sqrt{1-y^2}$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int dx \Rightarrow \sin^{-1} y = x + c$$

$\because \quad f(0) = 0 \Rightarrow c = 0$

$\therefore \quad y = f(x) = \sin x$

$$\therefore \quad \lim_{x\to 0} \frac{\int_0^x f(t)dt}{x^2} = \lim_{x\to 0} \frac{f(x)}{2x} = \lim_{x\to 0} \frac{\sin x}{2x}$$

$$= \frac{1}{2}$$

96. Answer (5)

$$\int_{-\frac{1}{2}}^{1} ([2x] + |x|)dx = \int_{-\frac{1}{2}}^{1} [2x]dx + \int_{-\frac{1}{2}}^{0} -x dx + \int_0^1 x dx$$

$$= \int_{-\frac{1}{2}}^{0} -1 dx + \int_0^{\frac{1}{2}} 0\, dx + \int_{\frac{1}{2}}^{1} 1 dx + \int_{-\frac{1}{2}}^{0} -x\, dx + \int_0^1 x\, dx$$

$$= \left(-\frac{1}{2} + 0 + \frac{1}{2}\right) + \frac{-x^2}{2}\Bigg|_{-\frac{1}{2}}^{0} + \frac{x^2}{2}\Bigg|_0^1$$

$$= 0 - \left(\frac{-1}{8}\right) + \frac{1}{2} = \frac{5}{8}$$

$$\pi \int_0 \left(\sin\frac{\pi x}{2}\right)(x-[x]) \cdot dx$$

$$= \pi^2\left[\int_0^1 \sin\frac{\pi x}{2}dx + \int_1^2 (x-1)\sin\frac{\pi x}{2}dx\right]$$

$$= \pi^2\left[-\frac{2}{\pi}\cos\frac{\pi x}{2}\Bigg|_0^1 - (x-1)\frac{2}{\pi}\cos\frac{\pi x}{2}\Bigg|_1^2 + \int_1^2 \frac{2}{\pi}\cos\frac{\pi x}{2}dx\right]$$

$$= \pi^2\left[\frac{2}{\pi} + \frac{2}{\pi} + \frac{4}{\pi^2}\sin\frac{\pi x}{2}\Bigg|_1^2\right]$$

$$= \pi^2\left[\frac{4}{\pi} - \frac{4}{\pi^2}\right] = 4(\pi - 1)$$

98. Answer (4)

$$f(x) = x + \int_0^{\pi/2} \sin x \cdot \cos y\ f(y)dy$$

Let $\int_0^{\pi/2} \cos y\ f(y)dy = k$

then $f(x) = x + k\sin x$

So, $k = \int_0^{\pi/2} \cos y(y + k\sin y)dy = [y\sin y + \cos y]_0^{\pi/2} - \frac{k}{4}\cos 2y\Big|_0^{\pi/2}$

$$\Rightarrow \quad k = \left(\frac{\pi}{2} - 1\right) + \frac{k}{2}$$

$\Rightarrow \quad k = \pi - 2$

So $f(x) = x + (\pi - 2)\sin x$

99. Answer (1)

$$J_{6+i,\,3} - J_{i+3,\,3} = \int_0^{1/2} \frac{x^{6+i}}{x^3-1}dx - \int_0^{1/2} \frac{x^{i+3}}{x^3-1}dx$$

$$= \int_0^{1/2} \frac{x^{i+3}\left(x^3-1\right)}{\left(x^3-1\right)}dx$$

$$= \left[\frac{x^{i+4}}{i+4}\right]_0^{1/2} = \frac{\left(\frac{1}{2}\right)^{i+4}}{i+4}$$

$\because \quad \det(A) = a_{11}\cdot a_{22}\cdot a_{33}$

$$= \frac{\left(\frac{1}{2}\right)^5}{5}\cdot\frac{\left(\frac{1}{2}\right)^6}{6}\cdot\frac{\left(\frac{1}{2}\right)^7}{7} = \frac{1}{105\times 2^{19}}$$

Now, $\left|\text{adj } A^{-1}\right| = \frac{1}{(\det A)^2} = (105)^2 \times 2^{38}$

$f'(x) = e^x + (y) + e^x$

$$\int \frac{dy}{y+1} = \int e^x dx$$

$\Rightarrow$ $\ln (y + 1) = e^x + c$

$\downarrow (0,\ 1)$

$$c = \ln\left(\frac{2}{e}\right)$$

$$y + 1 = e^{ex} \cdot \frac{2}{e} \Rightarrow y = \left(2 \cdot e^{e^x - 1}\right) - 1$$

101. Answer (1)

Let $[x] = n$

$$f(x) = \int_0^x [y]dy = \int_0^1 [y]dy + \int_1^2 [y]dy + \ldots\ldots \int_{n-1}^n [y]dy + \int_n^x [y]dy$$

$$= \frac{n(n-1)}{2} + n(x-n)$$

$$f(x) = [x]x - \frac{[x]([x]+1)}{2}$$

$f(x)$ is continuous at $x = k$, $(k \in I)$

$$\because \quad f(k^-) = f(k^+) = f(k) = \frac{k^2 - k}{2} = \frac{k(k-1)}{2}$$

LHD = $f'(k^-) = k - 1$

RHD = $f'(k^+) = k$

Not differentiable at $x = k$ where $k \in I$

102. Answer (1)

$$f(\theta) = \sin\theta\left(1 + \int_{-\pi/2}^{\pi/2} f(t)\, dt\right) + \cos\theta\left(\int_{-\pi/2}^{\pi/2} tf(t)\, dt\right)$$

Clearly $f(\theta) = a\sin\theta + b\cos\theta$

Where $a = 1 + \int_{-\pi/2}^{\pi/2} (a\sin t + b\cos t)\, dt \Rightarrow a = 1 + 2b$ …(1)

and $b = \int_{-\pi/2}^{\pi/2} (at\sin t + bt\cos t)dt \Rightarrow b = 2a$ …(2)

from (1) and (2) we get

$$a = -\frac{1}{3} \text{ and } b = -\frac{2}{3}$$

So, $f(\theta) = -\frac{1}{3}(\sin\theta + 2\cos\theta)$

$$\Rightarrow \left|\int_0^{\pi/2} f(\theta)\, d\theta\right| = \frac{1}{3}(1 + 2 \times 1) = 1$$

$$\int_0 \frac{e \quad \sin x}{\left(1+\cos^2 x\right)\left(e^{\cos x} + e^{-\cos x}\right)} dx$$

Let $\cos x = t$

$\sin x\, dx = dt$

$$= \int_1^{-1} \frac{-e^t dt}{\left(1+t^2\right)\left(e^t + e^{-t}\right)}$$

$$I = \int_{-1}^{1} \frac{e^t}{\left(1+t^2\right)\left(e^t + e^{-t}\right)} dt \quad \ldots\text{(i)}$$

$$I = \int_{-1}^{1} \frac{e^{-t}}{\left(1+t^2\right)\left(e^{-t} + e^t\right)} dt \quad \ldots\text{(ii)}$$

Adding (i) and (ii)

$$2I = \int_{-1}^{1} \frac{dt}{1+t^2}$$

$$2I = \tan^{-t}\Big]_{-1}^{1}$$

$$2I = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right)$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

104. Answer (4)

$$b_n - b_{n-1} = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx - \cos^2 (n-1)x}{\sin x} dx$$

$$= \int_0^{\pi/2} \frac{-\sin(2n-1)x \cdot \sin x}{\sin x} dx$$

$$= \frac{\cos(2n-1)x}{2n-1}\Bigg|_0^{\pi/2} = -\frac{1}{2n-1}$$

So, $b_3 - b_2$, $b_4 - b_3$, $b_5 - b_4$ are in H.P.

$\Rightarrow \dfrac{1}{b_3 - b_2}, \dfrac{1}{b_4 - b_3}, \dfrac{1}{b_5 - b_4}$ are in A. P. with common difference –2.

$$I=\int\frac{1}{(x^2-1)(x^2-4)}dx=\frac{1}{3}\int\left(\frac{1}{x^2-4}-\frac{1}{x^2-1}\right)dx$$

$$=\frac{1}{3}\left(\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|-\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\right)+C$$

$$12I=\ln\left|\frac{x-2}{x+2}\right|-2\ln\left|\frac{x-1}{x+1}\right|+C$$

$$12\int_3^b\frac{dx}{(x^2-4)(x^2-1)}$$

$$=\ln\left(\frac{b-2}{b+2}\right)-2\ln\left(\frac{b-1}{b+1}\right)-\left(\ln\left(\frac{1}{5}\right)-2\ln\left(\frac{1}{2}\right)\right)$$

$$=\ln\left(\left(\frac{b-2}{b+2}\right)\cdot\frac{(b+1)^2}{(b-1)^2}\right)-\left(\ln\frac{4}{5}\right)$$

So, $\frac{49}{40}=\frac{(b-2)}{(b+2)}\frac{(b+1)^2}{(b-1)^2}\cdot\frac{5}{4}$

$\Rightarrow\ b=6$

106. Answer (3)

$$I=\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{2-x^2}{\left(2+x^2\right)\sqrt{4+x^4}}dx$$

Let $x=\sqrt{2}t\ \Rightarrow dx=\sqrt{2}dt$

$$I=\frac{24}{\pi}\int_0^1\frac{\left(2-2t^2\right)\cdot\sqrt{2}dt}{\left(2+2t^2\right)\sqrt{4+4t^4}}$$

$$=\frac{12\sqrt{2}}{\pi}\int_0^1\frac{\left(\frac{1}{t^2}-1\right)dt}{\left(t+\frac{1}{t}\right)\sqrt{\left(t+\frac{1}{t}\right)^2-2}}$$

Let $t+\frac{1}{t}=u$

$$\Rightarrow\left(1-\frac{1}{t^2}\right)dt=du$$

$$\pi \qquad u\sqrt{u^2-2}$$

$$=\frac{12\sqrt{2}}{\pi}\int_2^{\infty}\frac{du}{u^2\sqrt{1-\left(\frac{\sqrt{2}}{u}\right)^2}}$$

$$=\frac{12\sqrt{2}}{\pi}\int_{\frac{1}{\sqrt{2}}}^{0}\frac{-\frac{1}{\sqrt{2}}dp}{\sqrt{1-p^2}}$$

$$=\frac{12}{\pi}\left[\sin^{-1}p\right]_0^{\frac{1}{\sqrt{2}}}$$

$$=\frac{12}{\pi}\cdot\frac{\pi}{4}$$

$=3$

107. Answer (4)

$$I=\int_{-2}^{2}\frac{|x^3+x|}{e^{x|x|}+1}dx\quad\ldots(i)$$

$$I=\int_{-2}^{2}\frac{|x^3+x|}{e^{-x|x|}+1}dx\quad\ldots(ii)$$

$$2I=\int_{-2}^{2}|x^3+x|dx$$

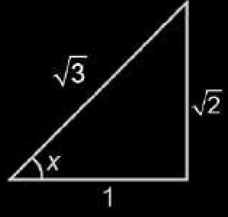
$$2I=2\int_0^2(x^3+x)dx$$

$$I=\int_0^2(x^3+x)dx$$

$$=\left.\frac{x^4}{4}+\frac{x^2}{2}\right]_0^2$$

$$=\left(\frac{16}{4}+\frac{4}{2}\right)-0$$

$=4+2=6$



$$\int_{\cos x}^{1} t^2 f(t)\,dt = \sin^3 x + \cos x$$

$$\Rightarrow \sin x \cos^2 x\, f(\cos x) = 3\sin^2 x \cos x - \sin x$$

$$\Rightarrow f(\cos x) = 3\tan x - \sec^2 x$$

$$\Rightarrow f'(\cos x)\cdot(-\sin x) = 3\sec^2 x - 2\sec^2 x \tan x$$

Put, $\cos x = \frac{1}{\sqrt{3},}$

$$\therefore\ f'\left(\frac{1}{\sqrt{3}}\right)\left(-\frac{\sqrt{2}}{\sqrt{3}}\right) = 9 - 6\sqrt{2}$$

$$\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right) = 6 - \frac{9}{\sqrt{2}}$$

109. Answer (1)

$$\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}}dx \quad \text{let } \frac{1}{x} = t$$

$$\frac{-1}{x^2}dx = dt$$

$$= \int_\infty^1 \frac{1}{-t^2 7^{[t]}}dt = \int_1^\infty \frac{1}{t^2 7^{[t]}}dt$$

$$= \int_1^2 \frac{1}{7t^2}dt + \int_2^3 \frac{1}{7^2 t^2}dt + \ldots.$$

$$= \frac{1}{7}\left[-\frac{1}{t}\right]_1^2 + \frac{1}{7^2}\left[\frac{-1}{t}\right]_2^3 + \frac{1}{7^3}\left[-\frac{1}{t}\right]_2^3 + \ldots.$$

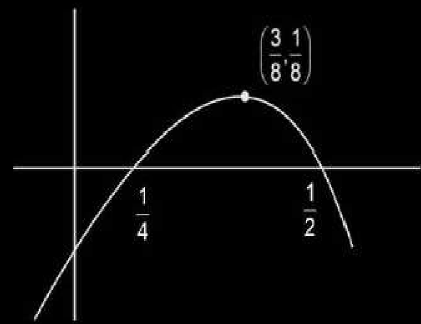
$$= \sum_{n=1}^{\infty} \frac{1}{7^n}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= \sum_{n=1} \frac{(\ldots)}{n} - 7\sum_{n=1} \frac{(\ldots)}{n+1}$$

$$= -\log\left(1 - \frac{1}{7}\right) + 7\log\left(1 - \frac{1}{7}\right) + 1$$

$$= 1 + 6\log\frac{6}{7}$$

110. Answer (3)



$$\int_0^1 \left[-8x^2 + 6x - 1\right]dx$$

$$= \int_0^{\frac{1}{4}} (-1)dx + \int_{\frac{1}{4}}^{\frac{3}{4}} 0dx + \int_{\frac{1}{2}}^{\frac{3}{4}} -1dx + \int_{\frac{3}{4}}^{\frac{3+\sqrt{17}}{8}} -2dx + \int_{\frac{3+\sqrt{17}}{8}}^{1} -3dx$$

$$= -\frac{1}{4} - \frac{1}{4} - 2\left(\frac{3+\sqrt{17}}{8} - \frac{3}{4}\right) - 3\left(1 - \frac{3+\sqrt{17}}{8}\right)$$

$$= \frac{\sqrt{17} - 13}{8}$$

111. Answer (2)

Given : $f\left(\frac{\pi}{4}\right) = \sqrt{2},\ f\left(\frac{\pi}{2}\right) = 0$ and $f'\left(\frac{\pi}{2}\right) = 1$

$$g(x) = \int_x^{\frac{\pi}{4}} \left(f'(t)\sec t + \tan t \sec t\, f(t)\right)dt$$

$$= \left[\sec t + f(t)\right]_x^{\frac{\pi}{4}} = 2 - \sec x\, f(x)$$

Now, $\lim_{x\to\frac{\pi}{2}}$

$$= \lim_{h\to 0} 2-(\operatorname{cosec} h)\, f\left(\frac{\pi}{2}-h\right)$$

$$= \lim_{h\to 0}\left[2-\frac{f\left(\frac{\pi}{2}-h\right)}{\sin h}\right]$$

$$= \lim_{h\to 0}\left[2+\frac{f'\left(\frac{\pi}{2}-h\right)}{\cos h}\right]$$

$= 3$

112. Answer (3)

$f: R \to R$ and $f(x)+f(x+k)=n \quad \forall x \in R$

$x \to x + k$

$f(x+k)+f(x+2k)=n$

$\therefore\ f(x+2k)=f(x)$

So, period of $f(x)$ is $2k$

Now, $I_1 = \int_0^{4nk} f(x)dx = 2n\int_0^{2k} f(x)dx$

$$= 2n\left[\int_0^k f(x)dx + \int_k^{2k} f(x)dx\right]$$

$x = t + k \Rightarrow dx = dt$ (in second integral)

$$= 2n\left[\int_0^k f(x)dx + \int_0^k f(t+k)dt\right]$$

$$= 2n^2k$$

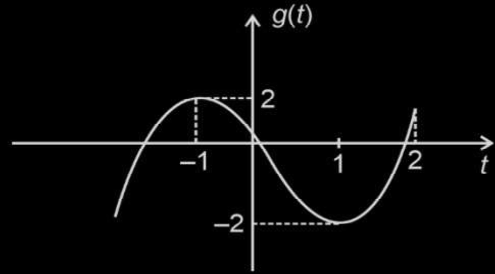
Now, $I_2 = \int_{-k}^{3k} f(x)dx = 2\int_0^{2k} f(x)dx$

$I_2 = 2(nk)$

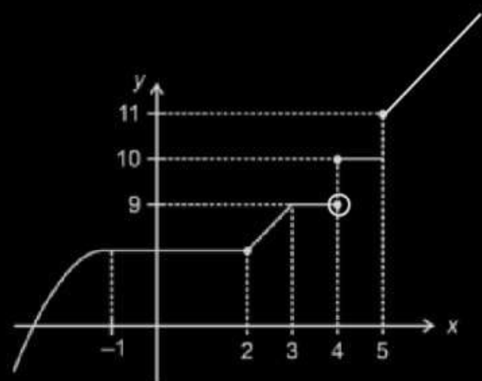
$\therefore\ I_1 + nI_2 = 4n^2k$

$\max_{t\le x}\{t^3-3t\}$; $x \le 2$

$g(t) = t^3 - 3t \Rightarrow g'(t) = 3t^2 - 3 = 3(t-1)(t+1)$



$$f(x)=\begin{cases} x^3-3x & x<-1 \\ 2 & -1\le x\le 2 \\ x^2+2x-6 & 2<x<3 \\ 9 & 3\le x<4 \\ 10 & 4\le x<5 \\ 11 & x=5 \\ 2x+1 & x>5 \end{cases}$$



Points of non-differentiability = {2, 3, 4, 5}

$\Rightarrow\ m = 4$

$$I = \int_{-2}^{2} f(x)dx = \int_{-2}^{-1}(x^3-3x)dx + \int_{-1}^{2} 2dx$$

$$= \left[\frac{x^4}{4}-\frac{3x^2}{2}\right]_{-2}^{-1} + 2(2+1) = \left(\frac{1}{4}-\frac{3}{2}\right)-(4-6)+6$$

$$= \frac{27}{4}$$

$$\int_0^5 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx$$

$$=\int_0^2 \cos(\pi x)dx+\int_2^4 \cos(\pi(x-1))dx+\int_4^5 \cos(\pi(x-2))dx$$

$$\left.\frac{\sin \pi x}{\pi}\right|_0^2+\left.\frac{\sin(\pi(x-1))}{\pi}\right|_2^4+\left.\frac{\sin(\pi(x-2))}{\pi}\right|_4^5$$

= 0 + 0 + 0 = 0

115. Answer (4)

$$f(x)=x\int_0^1 (x-t)f(t)dt$$

$$f(x)=x+x\int_0^1 f(t)dt-\int_0^1 tf(t)dt$$

$$f(x)=x\left(1+\int_0^1 f(t)dt\right)-\int_0^1 tf(t)dt$$

Let, $1+\int_0^1 f(t)dt=a$ and $\int_0^1 tf(t)dt=1$

$$f(x)=ax-b$$

Now, $a=1+\int_0^1(at-b)dt=1+\frac{a}{2}-b \Rightarrow \frac{a}{2}+b=1$

$$b=\int_0^1 t(at-b)dt=\frac{a}{3}-\frac{b}{2} \quad \Rightarrow \frac{3b}{2}=\frac{a}{3} \Rightarrow \boxed{b=\frac{2a}{9}}$$

$$\frac{a}{2}+\frac{2a}{9}=1$$

$$\Rightarrow \boxed{a=\frac{18}{13}}\ \boxed{b=\frac{4}{13}}$$

$$f(x)=\frac{18x-4}{13}$$

(6, 8) lies on $f(x)$ i.e. option (4)

$$\int_0^2 \sqrt{2x}dx-\int_0^2 \sqrt{1-(x-1)^2}dx=\int_0^2\left(1-\frac{y^2}{2}\right)dy-\int_0^1\sqrt{1-y^2}dy$$

$$+1+I$$

$$\Rightarrow \frac{8}{3}-2\int_0^1\sqrt{1-y^2}dy=\frac{2}{3}+1-\int_0^1\sqrt{1-y^2}dy+I$$

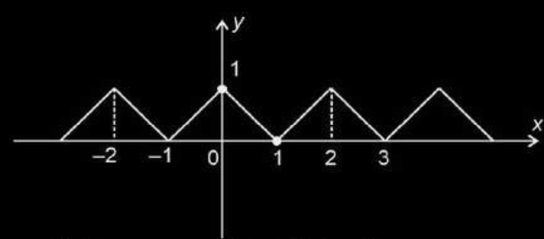
$$\Rightarrow I=1-\int_0^1\sqrt{1-y^2}dy$$

$$\Rightarrow I=\int_0^1\left(1-\sqrt{1-y^2}\right)dy$$

117. Answer (1)

$$f(x)=\begin{cases} x-[x], & \text{if } [x] \text{ is odd} \\ 1+[x]-x, & \text{if } [x] \text{ is even}\end{cases}$$

Graph of $f(x)$



$f(x)$ is an even and periodic function

So, $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx=\frac{\pi^2}{10}.20\int_0^1 f(x)\cos \pi x\, dx$

$$=2\pi^2\int_0^1(1-x)\cos \pi x\, dx$$

$$=2\pi^2\left\{\left.(1-x)\frac{\sin \pi x}{\pi}\right|_0^1-\left.\frac{\cos \pi x}{\pi^2}\right|_0^1\right\}=4$$

$$\int_0^1 (1-x^n)^{2n+1}dx = \int_0^1 1\cdot(1-x^n)^{2n+1}dx$$

$$=\left[(1-x^n)^{2n+1}\cdot x\right]_0^1 - \int_0^1 x\cdot(2n+1)(1-x^n)^{2n}\cdot -nx^{n-1}dx$$

$$= n(2n+1)\int_0^1\left(1-(1-x^n)\right)(1-x^n)^{2n}dx$$

$$= n(2n+1)\int_0^1(1-x^n)^{2n}dx - n(2n+1)\int_0^1(1-x^n)^{2n+1}dx$$

$$(1+n(2n+1))\int_0^1(1-x^n)^{2n+1}dx = n(2n+1)\int_0^1(1-x^n)^{2n}dx$$

$$(2n^2+n+1)\int_0^1(1-x^n)^{2n+1}dx = 1177\int_0^1(1-x^n)^{2n+1}dx$$

$\therefore\ 2n^2 + n + 1 = 1177$

$2n^2 + n - 1176 = 0$

$\therefore\ n = 24$ or $-\dfrac{49}{2}$

$\therefore\ n = 24$

119. Answer (2)

$$f(x) = a\sin\left(\frac{\pi[x]}{2}\right) + [2-x]\ a\in R$$

Now,

$\because\ \lim\limits_{x\to-1} f(x)$ exist

$$\therefore\ \lim_{x\to-1^-} f(x) = \lim_{x\to-1^+} f(x)$$

$$\Rightarrow a\sin\left(\frac{-2\pi}{2}\right)+3 = a\sin\left(\frac{-\pi}{2}\right)+2$$

$\Rightarrow\ -a = 1 \Rightarrow \boxed{a=-1}$

Now, $\displaystyle\int_0^4 f(x)dx = \int_0^4\left(-\sin\left(\frac{\pi[x]}{2}\right)+[2-x]\right)dx$

$$= \int_0^1 1dx + \int_1^2 -1dx + \int_2^3 -1dx + \int_3^4 (1-2)dx$$

$= 1 - 1 - 1 - 1 = -2$

$$f(x) = \int_0^x e^{x-t}f'(t)dt - (x^2-x+1)e^x$$

$$f(x) = e^x\int_0^x e^{-t}f'(t)dt - (x^2-x+1)e^x$$

$$e^{-x}f(x) = \int_0^x e^{-t}f'(t)dt - (x^2-x+1)$$

Differentiate on both side

$$e^{-x}f'(x) + \left(-f(x)e^{-x}\right) = e^{-x}f'(x) - 2x + 1$$

$$f(x) = e^x(2x-1)$$

$$f'(x) = e^x(2) + e^x(2x-1)$$

$$= e^x(2x+1)$$

$$x = -\frac{1}{2}$$

$$f''(x) = e^x(2) + (2x+1)e^x$$

$$= e^x(2x+3)$$

For $x = -\dfrac{1}{2}$ $f''(x) > 0$

$\Rightarrow$ Maxima

$\therefore$ Max. $= e^{-\frac{1}{2}}(-1-1)$

$\therefore\ -\dfrac{2}{\sqrt{e}}$

121. Answer (10)

Put $x = \tan\theta \quad \Rightarrow dx = \sec^2\theta\, d\theta$

$$\Rightarrow I = \int_0^{\frac{\pi}{3}} \frac{15\tan^3\theta\cdot\sec^2\theta\, d\theta}{\sqrt{1+\tan^2\theta+\sqrt{\sec^6\theta}}}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{3}} \frac{15\tan^2\theta\sec^2\theta\, d\theta}{\sec\theta\sqrt{1+\sec\theta}}$$

$$\Rightarrow I = \int_0 \frac{\left(\quad\right)}{\left(\sqrt{1+\sec\theta}\right)}$$

Now put 1 + secθ = t^2

$\Rightarrow$ secθ tanθ $d\theta = 2tdt$

$$\Rightarrow I = \int_{\sqrt{2}}^{\sqrt{3}} \frac{15\left(\left(t^2-1\right)^2-1\right)2t\ dt}{t}$$

$$\Rightarrow I = 30\int_{\sqrt{2}}^{\sqrt{3}}\left(t^4-2t^2+1-1\right)dt$$

$$\Rightarrow I = 30\int_{\sqrt{2}}^{\sqrt{3}}\left(t^4-2t^2\right)dt$$

$$\Rightarrow I = 30\left(\frac{t^5}{5}-\frac{2t^3}{3}\right)\Bigg|_{\sqrt{2}}^{\sqrt{3}}$$

$$= 30\left[\left(\frac{9}{5}\sqrt{3}-2\sqrt{3}\right)-\left(\frac{4\sqrt{2}}{5}-\frac{4\sqrt{2}}{3}\right)\right]$$

$$= \left(54\sqrt{3}-60\sqrt{3}\right)-\left(24\sqrt{2}-40\sqrt{2}\right)$$

$$= 16\sqrt{2}-6\sqrt{3}$$

$\therefore$ $\alpha = 16$ and $\beta = -6$

$\alpha + \beta = 10$.

122. Answer (1)

$$I_n(x) = \int_0^x \frac{1}{\left(t^2+5\right)^n}dt$$

$$= \int_0^x \underbrace{\frac{1}{\left(t^2+5\right)^n}}_{I} \times \underbrace{1}_{II}\,dt$$

$$= \frac{t}{\left(t^2+5\right)^n}\Bigg|_0^x - \int_0^x \frac{-2nt}{\left(t^2+5\right)^{n+1}}\times t\ dt$$

$$= \frac{x}{\left(x^2+5\right)^n} + \int_0^x 2n\left(\frac{\quad}{\left(t^2+5\right)^{n+1}}\right)dt$$

$$I_n(x) = \frac{x}{\left(x^2+5\right)^n} + 2n\ I_n(x) - 10n\ I_{n+1}(x)$$

$$10n\ I_{n+1}(x) - (2n-1)\ I_n(x) = xI_n'(x)$$

For $n = 5$

$$50I_6(x) - 9I_5(x) = xI_5'(x)$$

123. Answer (104)

$$I = \int_0^{\frac{\pi}{2}} 60.\frac{\sin 6x}{\sin x}dx$$

$$= 60.2\int_0^{\frac{\pi}{2}}(3-4\sin^2 x)(4\cos^2 x-3)\cos x dx$$

$$= 120\int_0^{\frac{\pi}{2}}(3-4\sin^2 x)(1-4\sin^2 x)\cos x dx$$

Let sin$x$ = $t \Rightarrow$ cosxdx = dt

$$= 120\int_0^1(3-4t^2)(1-4t^2)dt$$

$$= 120\int_0^1(3-16t^2+16t^4)dt$$

$$= 120\left[3t-\frac{16t^3}{3}+\frac{16t^5}{5}\right]_0^1$$

= 104

124. Answer (2)

$$I = \int_0^{\pi/2}\frac{1}{3+2\sin x+\cos x}dx$$

$$= \int_0^{\pi/2}\frac{(1+\tan^2 x/2)dx}{3(1+\tan^2 x/2)+2(2\tan x/2)+(1-\tan^2 x/2)}$$

Let, $\tan x/2 = t \;\Rightarrow\; \sec^2 x/2dx = 2dt$

$$_0\ 4+2t^2+4t$$

$$=\int_0^1 \frac{dt}{t^2+2t+2}=\int_0^1 \frac{dt}{(t+1)^2+1}$$

$$=\tan^{-1}(t+1)\Big|_0^1=\tan^{-1}2-\frac{\pi}{4}$$

125. Answer (1)

$$I=\int_0^1\left[2x-|3x^2-5x+2|+1\right]dx$$

$$I=\int_0^{2/3}\Bigg[\underbrace{-3x^2+7x-2}_{I_1}\Bigg]dx+\int_{2/3}^1\Bigg[\underbrace{3x^2-3x+2}_{I_2}\Bigg]dx+1$$

$$I_1=\int_0^{t_1}(-2)dx+\int_{t_1}^{1/3}(-1)\,dx+\int_{1/3}^{t_2}0.dx+\int_{t_2}^{2/3}dx$$

$$=-t_1-t_2+\frac{1}{3},\ \text{where}\ t_1=\frac{7-\sqrt{37}}{6},\ t_2=\frac{7-\sqrt{13}}{6}$$

$$I_2=\int_{2/3}^1 1dx=\frac{1}{3}$$

$$\therefore\ I=\frac{1}{3}-t_1-t_2+\frac{1}{3}+1=\frac{5}{3}-\left[\frac{7-\sqrt{37}}{6}+\frac{7-\sqrt{13}}{6}\right]$$

$$=\frac{\sqrt{37}+\sqrt{13}-4}{6}$$

126. Answer (4)

$$I=\int_0^{20\pi}\left(|\sin x|+|\cos x|\right)^2 dx$$

$$=20\int_0^{\pi}\left(1+|\sin 2x|\right)dx$$

$$=40\int_0^{\frac{\pi}{2}}(1+\sin 2x)\,dx$$

$$=40\left(x-\frac{\cos 2x}{2}\right)\Bigg|_0^{\frac{\pi}{2}}$$

$$=40\left(\frac{\pi}{2}+\frac{1}{2}+\frac{1}{2}\right)=20(\pi+2)$$

$$\because\ f(x)+\int_0^x (x-t)\ f'(t)dt=\left(e^{2x}+e^{-2x}\right)$$

$$\cos 2x+\frac{2x}{a}\qquad ...(i)$$

Here $f(0) = 2$ ...(ii)

On differentiating equation (i) w.r.t. x we get :

$$f'(x)+\int_0^x f'(t)dt+xf'(x)-xf'(x)=2\left(e^{2x}-e^{-2x}\right)$$

$$\cos 2x-2\left(e^{2x}+e^{-2x}\right)\sin 2x+\frac{2}{a}$$

$$\Rightarrow f'(x)+f(x)-f(0)=2(e^{2x}-e^{-2x})\cos 2x-2\,(e^{2x}+e^{-2x})$$

$$\sin 2x+\frac{2}{a}$$

Replace x by 0 we get :

$$\Rightarrow\quad 4=\frac{2}{a}\quad\Rightarrow a=\frac{1}{2}.$$

$$\therefore\quad (2a+1)^5\cdot a^2=2^5\cdot\frac{1}{2^2}=2^3=8$$

128. Answer (4)

$$f(\alpha)=\int_1^{\alpha}\frac{\log_{10}t}{1+t}\,dt\qquad ...(i)$$

$$f\left(\frac{1}{\alpha}\right)=\int_1^{\frac{1}{\alpha}}\frac{\log_{10}t}{1+t}\,dt$$

Substituting $t\rightarrow\frac{1}{p}$

$$f\left(\frac{1}{\alpha}\right)=\int_1^{\alpha}\frac{\log_{10}\left(\frac{1}{p}\right)}{1+\frac{1}{p}}\left(\frac{-1}{p^2}\right)dp$$

$$=\int_1^{\alpha}\frac{\log_{10}p}{p(p+1)}dp=\int_1^{\alpha}\left(\frac{\log_{10}t}{t}-\frac{\log_{10}t}{t+1}\right)dt\qquad ...(ii)$$

$$f(\alpha)+f\left(\frac{1}{\alpha}\right)=\int_1^{\alpha}\frac{\log_{10}t}{t}\,dt=\int_1^{\alpha}\frac{\ln t}{t}\cdot\log_{10}e\,dt$$

$$=\frac{(\ln\alpha)^2}{2\log_e 10}$$

$$\alpha=e^3\Rightarrow f\left(e^3\right)+f\left(e^{-3}\right)=\frac{9}{2\log_e 10}$$

129. Answer (2)

$$I=\int_{-3}^{101}\left(\left[\sin(\pi x)\right]+e^{[\cos(2\pi x)]}\right)dx$$

$[\sin\pi x]$ is periodic with period 2 and $e^{[\cos(2\pi x)]}$ is periodic with period 1.

So,

$$I=52\int_0^2\left([\sin\pi x]+e^{[\cos 2\pi x]}\right)dx$$

$$=52\left\{\int_1^2-1\,dx+\int_{\frac{1}{4}}^{\frac{3}{4}}e^{-1}dx+\int_{\frac{5}{4}}^{\frac{7}{4}}e^{-1}\,dx+\int_0^{\frac{1}{4}}e^0\,dx+\int_{\frac{3}{4}}^{\frac{5}{4}}e^0\,dx+\int_{\frac{7}{4}}^{2}e^0\,dx\right\}$$

$$=\frac{52}{e}$$

130. Answer (3)

$$I=\lim_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+\dots+\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$$

Let $2^n=t$ and if $n\to\infty$ then $t\to\infty$

$$I=\lim_{n\to\infty}\frac{1}{t}\left(\sum_{r=1}^{t-1}\frac{1}{\sqrt{1-\frac{r}{t}}}\right)$$

$$I=\int_0^1\frac{dx}{\sqrt{1-x}}=\int_0^1\frac{dx}{\sqrt{x}}\qquad\left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$=\left[2x^{\frac{1}{2}}\right]_0^1=2$$

$$\lim_{n\to\infty}\left(\frac{n^2}{(n^2+1)(n+1)}+\frac{n^2}{(n^2+4)(n+2)}+\dots+\frac{n^2}{(n^2+n^2)(n+n)}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{(n^2+r^2)(n+r)}$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\frac{1}{\left[1+\left(\frac{r}{n}\right)^2\right]\left[1+\left(\frac{r}{n}\right)\right]}$$

$$=\int_0^1\frac{1}{(1+x^2)(1+x)}dx$$

$$=\frac{1}{2}\int_0^1\left[\frac{1}{1+x}-\frac{(x-1)}{(1+x^2)}\right]dx$$

$$=\frac{1}{2}\left[\ln(1+x)-\frac{1}{2}\ln\left(1+x^2\right)+\tan^{-1}x\right]_0^1$$

$$=\frac{1}{2}\left[\frac{\pi}{4}+\frac{1}{2}\ln 2\right]=\frac{\pi}{8}+\frac{1}{4}\ln 2$$

132. Answer (5)

$$\because\quad a_n=\int_{-1}^{n}\left(1+\frac{x}{2}+\frac{x^2}{3}+\dots+\frac{x^{n-1}}{n}\right)dx$$

$$=\left[x+\frac{x^2}{2^2}+\frac{x^3}{3^2}+\dots+\frac{x^n}{n^2}\right]_{-1}^{n}$$

$$a_n=\frac{n+1}{1^2}+\frac{n^2-1}{2^2}+\frac{n^3+1}{3^2}+\frac{n^4-1}{4^2}+\dots+\frac{n^n+(-1)^{n+1}}{n^2}$$

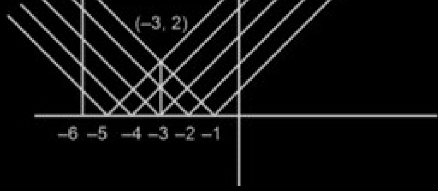
Here $a_1=2,\ a_2=\frac{2+1}{1}+\frac{2^2-1}{2}=3+\frac{3}{2}=\frac{9}{2}$

$$a_3=4+2+\frac{28}{9}=\frac{100}{9}$$

$$a_4=5+\frac{15}{4}+\frac{65}{9}+\frac{255}{16}>31.$$

$\therefore$ The required set is {2, 3}. $\because a_n\in(2,30)$

$\therefore$ Sum of elements = 5.



$$\int_{-6}^{0} f(x)dx = 2\left[\frac{1}{2}(2+5)3\right] = 21$$

134. Answer (6)

$$I = \frac{48}{\pi^4}\int_0^{\pi}\left[\left(\frac{\pi}{2}-x\right)^3 - \frac{3\pi^2}{4}\left(\frac{\pi}{2}-x\right) + \frac{\pi^3}{4}\right]\frac{\sin x\, dx}{1+\cos^2 x}$$

Using $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)dx$ we get

$$I = \frac{48}{\pi^4}\int_0^{\pi}\left[-\left(\frac{\pi}{2}-x\right)^3 + \frac{3\pi^2}{4}\left(\frac{\pi}{2}-x\right) + \frac{\pi^3}{4}\right]\frac{\sin x\, dx}{1+\cos^2 x}$$

Adding these two equations, we get

$$2I = \frac{48}{\pi^4}\int_0^{\pi}\frac{\pi^3}{2}\cdot\frac{\sin x\, dx}{1+\cos^2 x}$$

$$\Rightarrow \quad I = \frac{12}{\pi}\left[-\tan^{-1}(\cos x)\right]_0^{\pi} = \frac{12}{\pi}\cdot\frac{\pi}{2} = 6$$

135. Answer (2)

$$f(x) = \int_0^{x^2}\frac{t^2-5t+4}{2+e^t}dt$$

$$f'(x) = 2x\left(\frac{x^4-5x^2+4}{2+e^{x^2}}\right) = 0$$

$$x = 0,\ \text{or}\ \left(x^2-4\right)\left(x^2-1\right) = 0$$

$$x = 0,\ x = \pm 2,\ \pm 1$$

Now, $f'(x) = \dfrac{2x(x+1)(x-1)(x+2)(x-2)}{\left(e^{x^2}+2\right)}$

$f'(x)$ changes sign from positive to negative at $x = -1, 1$ So, number of local maximum points = 2

$f'(x)$ changes sign from negative to positive at $x = -2, 0, 2$ So, number of local minimum points = 3

$\therefore\ m = 2, n = 3$

$$\lim_{n\to\infty}\left(\frac{n+1}{n}\right)\ \frac{1}{n}\sum_{r=1}\left(k+\frac{r}{n}\right) = 33\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}\left(\frac{r}{n}\right)$$

$$\Rightarrow \int_0^1 (k+x)dx = 33\int_0^1 x^k dx$$

$$\Rightarrow \frac{2k+1}{2} = \frac{33}{k+1}$$

$$\Rightarrow k = 5$$

137. Answer (3)

$$a = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{2n}{n^2+k^2}$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{2}{1+\left(\frac{k}{n}\right)^2}$$

$$a = \int_0^1 \frac{2}{1+x^2}dx = 2\tan^{-1}x\Big|_0^1 = \frac{\pi}{2}$$

$$f(x) = \frac{1-\cos x}{\sin x} = \operatorname{cosec} x - \cot x$$

$$f'(x) = \operatorname{cosec}^2 x - \operatorname{cosec} x\cot x$$

$$\left.\begin{aligned} f\left(\frac{a}{2}\right) &= f\left(\frac{\pi}{4}\right) = \sqrt{2}-1 \\ f'\left(\frac{a}{2}\right) &= f'\left(\frac{\pi}{4}\right) = 2-\sqrt{2}\end{aligned}\right\} f'\left(\frac{a}{2}\right) = \sqrt{2}.f\left(\frac{a}{2}\right)$$

138. Answer (4)

$$f(x) = 2+|x|-|x-1|+|x+1|,\ x\in R$$

$$\therefore \quad f(x) = \begin{cases} -x & , \quad x < -1 \\ x+2 & , \quad -1\le x<0 \\ 3x+2 & , \quad 0\le x<1 \\ x+4 & , \quad x\ge 1\end{cases}$$

$\therefore$

$$f'\left(-\frac{3}{2}\right)+f'\left(-\frac{1}{2}\right)+f'\left(\frac{1}{2}\right)+f'\left(\frac{3}{2}\right) = -1+1+3+1 = 4$$

and

$$\int_{-2}^{2}f(x)dx = \int_{-2}^{-1}f(x)dx+\int_{-1}^{0}f(x)dx+\int_{0}^{1}f(x)dx+\int_{1}^{2}f(x)dx$$

$$= \frac{3}{2} + \frac{3}{2} + \frac{7}{2} + \frac{11}{2} = \frac{24}{2} = 12$$

$\therefore$ Only (S2) is correct

139. Answer (2)

$$\int_0^2 |2x^2 - 3x| dx + \int_0^2 \left[x - \frac{1}{2}\right] dx$$

$$= \int_0^{3/2} (3x - 2x^2) dx + \int_{3/2}^{2} (2x^2 - 3x) dx + \int_0^{1/2} -1 dx$$

$$+ \int_{1/2}^{3/2} 0\, dx + \int_{3/2}^{2} 1 dx$$

$$= \left(\frac{3x^2}{2} - \frac{2x^3}{3}\right)\Big|_0^{3/2} + \left(\frac{2x^3}{3} - \frac{3x^2}{2}\right)\Big|_{3/2}^{2} - \frac{1}{2} + \frac{1}{2}$$

$$= \left(\frac{27}{8} - \frac{27}{12}\right) + \left(\frac{16}{3} - 6 - \frac{27}{12} + \frac{27}{8}\right)$$

$$= \frac{19}{12}$$

140. Answer (385)

$\because$ $f(x) = \min\{[x-1], [x-2], \ldots, [x-10]\} = [x - 10]$

Also $|f(x)| = \begin{cases} -f(x), & \text{if } x \le 10 \\ f(x), & \text{if } x \ge 10 \end{cases}$

$$\therefore \int_0^{10} f(x)dx + \int_0^{10} (f(x))^2 dx + \int_0^{10} (-f(x))dx$$

$$= \int_0^{10} (f(x))^2 dx$$

$= 10^2 + 9^2 + 8^2 + \ldots\ldots + 1^2$

$$= \frac{10 \times 11 \times 21}{6}$$

$= 385$

$$\because \quad f(x) = \frac{2}{\sqrt{3}} \int_0^{\sqrt{3}} f\left(\frac{\lambda^2 x}{3}\right) d\lambda, \ x > 0$$

...(i)

On differentiating both sides w.r.t., x, we get

$$f'(x) = \frac{2}{\sqrt{3}} \int_0^{\sqrt{3}} \frac{\lambda^2}{3} f'\left(\frac{\lambda^2 x}{3}\right) d\lambda$$

$$f'(x) = \frac{1}{\sqrt{3}} \int_0^{\sqrt{3}} \lambda \cdot \frac{2\lambda}{3} f'\left(\frac{\lambda^2 x}{3}\right) d\lambda$$

$$\therefore \quad \sqrt{3} f'(x) = \left[\frac{\lambda}{x} \cdot f\left(\frac{\lambda^2 x}{3}\right)\right]_0^{\sqrt{3}} - \int_0^{\sqrt{3}} \frac{1}{x} f\left(\frac{\lambda^2 x}{3}\right) dx$$

$$\sqrt{3} x\, f'(x) = \sqrt{3} f(x) - \frac{\sqrt{3}}{2} f(x)$$

$$x f'(x) = \frac{f(x)}{2}$$

On integrating we get : $\ln y = \frac{1}{2} \ln x + \ln c$

$\because$ $f(1) = \sqrt{3}$ then $c = \sqrt{3}$

$\therefore$ $(\alpha, 6)$ lies on

$\therefore$ $y = \sqrt{3x}$

$\therefore$ $6 = \sqrt{3\alpha} \Rightarrow \alpha = 12.$

142. Answer (3)

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{(1 + e^x)(\sin^6 x + \cos^6 x)} \quad \ldots(i)$$

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{(1 + e^{-x})(\sin^6 x + \cos^6 x)} \quad \ldots(ii)$$

$$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{\sin^6 x + \cos^6 x}$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{dx}{\sin^6 x + \cos^6 x} = \int_0^{\frac{\pi}{2}} \frac{dx}{1 - \frac{3}{4}\sin^2 2x}$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \frac{4\sec^2 2x dx}{4 + \tan^2 2x} = 2\int_0^{\frac{\pi}{4}} \frac{4\sec^2 2x}{4 + \tan^2 2x} dx$$

Now, $\tan 2x = t$ when $x = 0,\ t = 0$; when, $x = \frac{\pi}{4},\ t \to \infty$

$$2\sec^2 2x\, dx = dt$$

$$= 2\frac{\pi}{2} = \pi$$

143. Answer (34)

Let $f(x) = \frac{x^2 - 9}{x - 5} \Rightarrow f'(x) = \frac{(x-1)(x-9)}{(x-5)^2}$

So, $\alpha = f(1) = 2$ and $\beta = \min(f(0), f(2)) = \frac{5}{3}$

Now, $\int_{-1}^{3} \max\left\{\frac{x^2-9}{x-5}, x\right\} dx = \int_{-1}^{9/5} \frac{x^2-9}{x-5} dx + \int_{9/5}^{3} x dx$

$$= \int_{-1}^{9/5}\left(x + 5 + \frac{16}{x-5}\right)dx + \frac{x^2}{2}\Big|_{9/5}^{3}$$

$$= \frac{28}{25} + 14 + 16\ln\left(\frac{8}{15}\right) + \frac{72}{25} = 18 + 16\ln\left(\frac{8}{15}\right)$$

Clearly $\alpha_1 = 18$ and $\alpha_2 = 16$, so $\alpha_1 + \alpha_2 = 34$.

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