Chapter 19

Continuity and Differentiability

1. Let $f(x) = x\,|x|$ and $g(x) = \sin x$.

Statement-1 : gof is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement-2 : gof is twice differentiable at $x = 0$. **[AIEEE-2009]**

(1) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

2. Let $f : (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$ **[AIEEE-2010]**

(1) 4 (2) –4

(3) 0 (4) –2

3. Let $f : \boldsymbol{R} \to \boldsymbol{R}$ be a continuous function defined by

$$f(x) = \frac{1}{e^x + 2e^{-x}}$$

Statement-1 : $f(c) = \frac{1}{3}$, for some $c \in \boldsymbol{R}$.

Statement-2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in \boldsymbol{R}$.

[AIEEE-2010]

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

4. Define $F(x)$ as the product of two real functions

$f_1(x) = x, x \in \mathbb{R}$, and $f_2(x) = \begin{cases} \sin\frac{1}{x}, & if\ x \ne 0 \\ 0, & if\ x = 0 \end{cases}$

as follows:

$$F(x) = \begin{cases} f_1(x).f_2(x) & \text{if } x \ne 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Statement - 1 : $F(x)$ is continuous on $\mathbb{R}$.

Statement - 2 : $\boldsymbol{f_1(x)}$ and $\boldsymbol{f_2(x)}$ are continuous on R. **[AIEEE-2011]**

(1) Statement-1 is true, Statement-2 is false

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation of Statement-1

5. If function $f(x)$ is differentiable at $x = a$, then

$\lim\limits_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$ is **[AIEEE-2011]**

(1) $2a\, f(a) - a^2 f'(a)$

(2) $2a\, f(a) + a^2 f'(a)$

(3) $-a^2 f'(a)$

(4) $a\, f(a) - a^2 f'(a)$

6. If $f : R \to R$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is **[AIEEE-2012]**

(1) Discontinuous only at $x = 0$

(2) Discontinuous only at non-zero integral values of x

(3) Continuous only at $x = 0$

(4) Continuous for every real x

7. **Consider the function,**

$f(x)=|x-2|+|x-5|, x \in R$

Statement-1 : $f'(4)=0$

Statement-2 : f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$.

[AIEEE-2012]

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

8. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$ **[JEE (Main)-2014]**

(1) $f'(c) = g'(c)$ (2) $f'(c) = 2g'(c)$

(3) $2f'(c) = g'(c)$ (4) $2f'(c) = 3g'(c)$

9. If the function.

$$g(x)=\begin{cases} k\sqrt{x+1} & , \quad 0 \le x \le 3 \\ mx+2 & , \quad 3 < x \le 5 \end{cases}$$

is differentiable, the value of $k + m$ is **[JEE (Main)-2015]**

(1) 2 (2) $\frac{16}{5}$

(3) $\frac{10}{3}$ (4) 4

10. For $x \in R, f(x)=|\log 2-\sin x|$ and

$g(x)=f(f(x))$, then **[JEE (Main)-2016]**

(1) $g'(0)=\cos(\log 2)$

(2) $g'(0)=-\cos(\log 2)$

(3) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(4) g is not differentiable at $x = 0$

11. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals **[JEE (Main)-2017]**

(1) $\frac{3x\sqrt{x}}{1-9x^3}$ (2) $\frac{3x}{1-9x^3}$

(3) $\frac{3}{1+9x^3}$ (4) $\frac{9}{1+9x^3}$

12. Let $S=\{t \in R : f(x)=|x-\pi| \cdot (e^{|x|}-1)\sin|x|$ is not differentiable at $t\}$. Then the set S is equal to

[JEE (Main)-2018]

(1) ϕ (an empty set) (2) $\{0\}$

(3) $\{\pi\}$ (4) $\{0, \pi\}$

13. Let $f : R \to R$ be a function defined as

$$f(x)=\begin{cases} 5, & \text{if } x \le 1 \\ a+bx, & \text{if } 1 < x < 3 \\ b+5x, & \text{if } 3 \le x < 5 \\ 30, & \text{if } x \ge 5 \end{cases}$$

Then, f is **[JEE (Main)-2019]**

(1) Continuous if $a = -5$ and $b = 10$

(2) Continuous if $a = 5$ and $b = 5$

(3) Continuous if $a = 0$ and $b = 5$

(4) Not continuous for any values of a and b

14. Let f be a differentiable function from R to R such that $|f(x)-f(y)| \le 2|x-y|^{\frac{3}{2}}$, for all $x, y \in$ R. If $f(0) = 1$ then $\int_0^1 f^2(x)dx$ equal to **[JEE (Main)-2019]**

(1) 1 (2) 0

(3) $\frac{1}{2}$ (4) 2

15. If $x = 3\tan t$ and $y = 3\sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$, is **[JEE (Main)-2019]**

(1) $\frac{1}{6\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$

(3) $\frac{3}{2\sqrt{2}}$ (4) $\frac{1}{6}$

16. Let $f(x)=\begin{cases}\max\{|x|,x^2\}, & |x|\le 2\\ 8-2|x|, & 2<|x|\le 4\end{cases}$

Let S be the set of points in the interval $(-4, 4)$ at which f is not differentiable. Then S

[JEE (Main)-2019]

(1) Equals $\{-2, -1, 0, 1, 2\}$

(2) Equals $\{-2, 2\}$

(3) Is an empty set

(4) Equals $\{-2, -1, 1, 2\}$

17. Let $f:(-1, 1)\to R$ be a function defined by $f(x)=\max\left\{-|x|,-\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly **[JEE (Main)-2019]**

(1) Three elements (2) Two elements

(3) One element (4) Five elements

18. Let f be a differentiable function such that $f'(x)=7-\frac{3}{4}\frac{f(x)}{x}$, $(x > 0)$ and $f(1) \neq 4$. Then $\lim\limits_{x\to 0^+} xf\left(\frac{1}{x}\right)$ **[JEE (Main)-2019]**

(1) Exist and equals 4

(2) Does not exist

(3) Exists and equals $\frac{4}{7}$

(4) Exists and equals 0

19. Let $f(x)=\begin{cases}-1, & -2\le x<0\\ x^2-1, & 0\le x\le 2\end{cases}$ and

$g(x) = |f(x)|+f(|x|)$. Then, in the interval $(-2, 2)$, g is **[JEE (Main)-2019]**

(1) Not differentiable at two points

(2) Not differentiable at one point

(3) Not continuous

(4) Differentiable at all points

20. If $x \log_e (\log_e x) - x^2 + y^2 = 4$ $(y > 0)$, then $\frac{dy}{dx}$ at $x = e$ is equal to **[JEE (Main)-2019]**

(1) $\frac{(2e-1)}{2\sqrt{4+e^2}}$ (2) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

(3) $\frac{(1+2e)}{\sqrt{4+e^2}}$ (4) $\frac{e}{\sqrt{4+e^2}}$

21. Let K be the set of all real values of x where the function $f(x) = \sin|x| - |x| + 2(x - \pi)$ is not differentiable. Then the set K is equal to

[JEE (Main)-2019]

(1) $\{\pi\}$

(2) ϕ (an empty set)

(3) $\{0\}$

(4) $\{0, \pi\}$

22. For $x > 1$, if $(2x)^{2y} = 4e^{2x - 2y}$, then $(1+\log_e 2x)^2\frac{dy}{dx}$ is equal to **[JEE (Main)-2019]**

(1) $\log_e 2x$

(2) $x\log_e 2x$

(3) $\frac{x\log_e 2x+\log_e 2}{x}$

(4) $\frac{x\log_e 2x-\log_e 2}{x}$

23. Let S be the set of all points in $(-\pi, \pi)$ at which the function, $f(x) = \min\{\sin x, \cos x\}$ is not differentiable. Then S is a subset of which of the following?

[JEE (Main)-2019]

(1) $\left\{-\frac{3\pi}{4},-\frac{\pi}{4},\frac{3\pi}{4},\frac{\pi}{4}\right\}$

(2) $\left\{-\frac{3\pi}{4},-\frac{\pi}{2},\frac{\pi}{2},\frac{3\pi}{4}\right\}$

(3) $\left\{-\frac{\pi}{2},-\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{2}\right\}$

(4) $\left\{-\frac{\pi}{4},0\,\frac{\pi}{4}\right\}$

24. Let f be a differentiable function such that $f(1) = 2$ and $f'(x) = f(x)$ for all $x \in R$. If $h(x) = f(f(x))$, then $h'(1)$ is equal to **[JEE (Main)-2019]**

(1) $2e$ (2) $2e^2$

(3) $4e$ (4) $4e^2$

25. If $2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^2, x\in\left(0,\frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to **[JEE (Main)-2019]**

(1) $2x-\frac{\pi}{3}$ (2) $x-\frac{\pi}{6}$

(3) $\frac{\pi}{3}-x$ (4) $\frac{\pi}{6}-x$

26. If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by

$$f(x) = \begin{cases} \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \neq \dfrac{\pi}{4} \\ k, & x = \dfrac{\pi}{4} \end{cases}$$

is continuous, then k is equal to

[JEE (Main)-2019]

(1) 1 (2) $\frac{1}{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) 2

27. If the function $f(x) = \begin{cases} a|\pi - x| + 1, & x \le 5 \\ b|x - \pi| + 3, & x > 5 \end{cases}$ is continuous at $x = 5$, then the value of $a - b$ is

[JEE (Main)-2019]

(1) $\frac{2}{\pi - 5}$ (2) $\frac{-2}{\pi + 5}$

(3) $\frac{2}{\pi + 5}$ (4) $\frac{2}{5 - \pi}$

28. If $f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$

is continuous at $x = 0$, then the ordered pair (p, q) is equal to : **[JEE (Main)-2019]**

(1) $\left(\frac{5}{2}, \frac{1}{2}\right)$ (2) $\left(-\frac{3}{2}, \frac{1}{2}\right)$

(3) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (4) $\left(-\frac{1}{2}, \frac{3}{2}\right)$

29. Let $f : R \to R$ be differentiable at $c \in R$ and $f(c) = 0$. If $g(x) = |f(x)|$, then at $x = c$, g is :

[JEE (Main)-2019]

(1) Not differentiable if $f'(c) = 0$

(2) Differentiable if $f'(c) = 0$

(3) Not differentiable

(4) Differentiable if $f'(c) \neq 0$

30. The derivative of $\tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ is

[JEE (Main)-2019]

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) 2 (4) 1

31. Let the function, $f : [-7, 0] \to R$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x) \le 2$, for all $x \in (-7, 0)$, then for all such functions f, $f(-1) + f(0)$ lies in the interval

[JEE (Main)-2020]

(1) $[-3, 11]$ (2) $(-\infty, 20]$

(3) $(-\infty, 11]$ (4) $[-6, 20]$

32. If $y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1 + \tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}, \alpha \in \left(\frac{3\pi}{4}, \pi\right)$, then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is **[JEE (Main)-2020]**

(1) 4 (2) $\frac{4}{3}$

(3) $-\frac{1}{4}$ (4) -4

33. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0,1]$ is **[JEE (Main)-2020]**

(1) $\frac{4 - \sqrt{7}}{3}$ (2) $\frac{4 - \sqrt{5}}{3}$

(3) $\frac{2}{3}$ (4) $\frac{\sqrt{7} - 2}{3}$

34. If c is a point at which Rolle's theorem holds for the function, $f(x) = \log_e\left(\frac{x^2 + \alpha}{7x}\right)$ in the interval $[3, 4]$, where $\alpha \in R$, then $f''(c)$ is equal to

[JEE (Main)-2020]

(1) $-\frac{1}{24}$ (2) $\frac{1}{12}$

(3) $\frac{\sqrt{3}}{7}$ (4) $-\frac{1}{12}$

35. Let f be any function continuous on $[a, b]$ and twice differentiable on (a, b). If for all $x \in (a, b)$, $f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$, $\dfrac{f(c)-f(a)}{f(b)-f(c)}$ is greater than

[JEE (Main)-2020]

(1) 1 (2) $\dfrac{b+a}{b-a}$

(3) $\dfrac{c-a}{b-c}$ (4) $\dfrac{b-c}{c-a}$

36. If $f(x)=\begin{cases}\dfrac{\sin(a+2)x+\sin x}{x} & ; \; x<0\\ b & ; \; x=0\\ \dfrac{(x+3x^2)^{1/3}-x^{1/3}}{x^{4/3}} & ; \; x>0\end{cases}$

is continuous at $x = 0$, then $a + 2b$ is equal to

[JEE (Main)-2020]

(1) –1 (2) 1

(3) 0 (4) –2

37. Let f and g be differentiable functions on R such that fog is the identity function. If for some a, $b \in R$, $g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to **[JEE (Main)-2020]**

(1) 1 (2) 5

(3) $\dfrac{1}{5}$ (4) $\dfrac{2}{5}$

38. If $x = 2\sin\theta - \sin2\theta$ and $y = 2\cos\theta - \cos2\theta$, $\theta \in [0, 2\pi]$, then $\dfrac{d^2y}{dx^2}$ at $\theta = \pi$ is

[JEE (Main)-2020]

(1) $\dfrac{3}{4}$ (2) $\dfrac{3}{8}$

(3) $-\dfrac{3}{8}$ (4) $-\dfrac{3}{4}$

39. Let $[t]$ denote the greatest integer $\le t$ and $\lim\limits_{x\to 0} x\left[\dfrac{4}{x}\right] = A$. Then the function, $f(x) = [x^2]\sin(\pi x)$ is discontinuous, when x is equal to

[JEE (Main)-2020]

(1) $\sqrt{A+21}$ (2) $\sqrt{A}$

(3) $\sqrt{A+1}$ (4) $\sqrt{A+5}$

40. If a function $f(x)$ defined by **[JEE (Main)-2020]**

$$f(x)=\begin{cases}ae^x+be^{-x}, & -1 \le x<1\\ cx^2 & , \; 1 \le x \le 3\\ ax^2+2cx & , \; 3<x \le 4\end{cases}$$

be continuous for some a, b, $c \in R$ and $f'(0) + f'(2) = e$, then the value of a is

[JEE (Main)-2020]

(1) $\dfrac{e}{e^2+3e+13}$ (2) $\dfrac{e}{e^2-3e+13}$

(3) $\dfrac{e}{e^2-3e-13}$ (4) $\dfrac{1}{e^2-3e+13}$

41. If $(a+\sqrt{2}b\cos x)(a-\sqrt{2}b\cos y)=a^2-b^2$, where $a > b > 0$, then $\dfrac{dx}{dy}$ at $\left(\dfrac{\pi}{4},\dfrac{\pi}{4}\right)$ is

[JEE (Main)-2020]

(1) $\dfrac{2a+b}{2a-b}$ (2) $\dfrac{a-b}{a+b}$

(3) $\dfrac{a+b}{a-b}$ (4) $\dfrac{a-2b}{a+2b}$

42. The function $f(x)=\begin{cases}\dfrac{\pi}{4}+\tan^{-1}x, |x|\le 1\\ \dfrac{1}{2}(|x|-1), |x|>1\end{cases}$ is

[JEE (Main)-2020]

(1) both continuous and differentiable on $R-\{-1\}$

(2) both continuous and differentiable on $R-\{1\}$

(3) continuous on $R - \{-1\}$ and differentiable on $R - \{-1, 1\}$

(4) continuous on $R - \{1\}$ and differentiable on $R - \{-1, 1\}$.

43. If the function

$f(x)=\begin{cases}k_1(x-\pi)^2-1, & x\le\pi\\ k_2\cos x, & x>\pi\end{cases}$ is twice differentiable, then the ordered pair (k_1, k_2) is equal to **[JEE (Main)-2020]**

(1) (1, 0) (2) $\left(\dfrac{1}{2}, 1\right)$

(3) (1, 1) (4) $\left(\dfrac{1}{2}, -1\right)$

44. Let $f: R \to R$ be defined as

$$f(x)=\begin{cases} x^5\sin\left(\frac{1}{x}\right)+5x^2, & x<0 \\ 0, & x=0 \\ x^5\cos\left(\frac{1}{x}\right)+\lambda x^2, & x>0 \end{cases}$$

The value of λ for which $f'(0)$ exists, is______.

[JEE (Main)-2020]

45. Let $f: R \to R$ be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in R, where f is not differentiable. Then

[JEE (Main)-2020]

(1) ϕ(an empty set)

(2) $\{0, 1\}$

(3) $\{1\}$

(4) $\{0\}$

46. If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by

$$f(x)=\begin{cases} \frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x\neq 0 \\ k & \text{, when } x=0 \end{cases}$$

is continuous, then k is equal to _________.

[JEE (Main)-2020]

47. Suppose a differentiable function $f(x)$ satisfies the identity $f(x + y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y. If $\lim\limits_{x\to 0}\frac{f(x)}{x}=1$, then $f'(3)$ is equal to _______. **[JEE (Main)-2020]**

48. Let $f(x)=x\cdot\left[\frac{x}{2}\right]$, for $-10 < x < 10$, where $[t]$ denotes the greatest integer function. Then the number of points of discontinuity of f is equal to ______. **[JEE (Main)-2020]**

49. If $f: R \to R$ is a function defined by $f(x) = [x - 1]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[\cdot]$ denotes the greatest integer function, then f is : **[JEE (Main)-2021]**

(1) discontinuous only at x = 1

(2) continuous for every real x

(3) discontinuous at all integral values of x except at x = 1

(4) continuous only at x = 1

50. If Rolle's theorem holds for the function $f(x)=x^3-ax^2+bx-4, x\in[1, 2]$ with $f'\left(\frac{4}{3}\right)=0$, then ordered pair (a, b) is equal to :

[JEE (Main)-2021]

(1) $(5,-8)$ (2) $(5, 8)$

(3) $(-5,-8)$ (4) $(-5, 8)$

51. The number of points, at which the function $f(x)=|2x+1|-3|x+2|+|x^2+x-2|, x\in R$ is not differentiable is _________. **[JEE (Main)-2021]**

52. A function f is defined on [–3, 3] as

$$f(x)=\begin{cases} \min\{|x|, 2-x^2\} & , -2\le x\le 2 \\ [|x|] & , 2<|x|\le 3 \end{cases}$$

where [x] denotes the greatest integer $\le$ x. The number of points, where f is not differentiable in (–3, 3) is __________. **[JEE (Main)-2021]**

53. Let f be any function defined on R and let it satisfy the condition : **[JEE (Main)-2021]**

$$|f(x)-f(y)|\le|(x-y)^2|, \forall(x,y)\in R$$

If $f(0)=1$, then :

(1) $f(x)$ can take any value in R

(2) $f(x)<0, \forall x\in R$

(3) $f(x)=0, \forall x\in R$

(4) $f(x)>0, \forall x\in R$

54. Let $f: R \to R$ be defined as

$$f(x)=\begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x<-1 \\ |ax^2+x+b|, & \text{if } -1\le x\le 1 \\ \sin(\pi x), & \text{if } x>1 \end{cases}$$

If f(x) is continuous on R, then a + b equals :

[JEE (Main)-2021]

(1) –1 (2) –3

(3) 3 (4) 1

55. Let a be an integer such that all the real roots of the polynomial $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval (a, a + 1). Then, | a | is equal to _________. **[JEE (Main)-2021]**

56. Let the function $f: \mathbf{R} \to \mathbf{R}$ and $g: \mathbf{R} \to \mathbf{R}$ be defined as:

$$f(x) = \begin{cases} x+2, & x<0 \\ x^2, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x^3, & x<1 \\ 3x-2, & x \geq 1 \end{cases}$$

Then, the number of points in **R** where (fog) (x) is NOT differentiable is equal to :

[JEE (Main)-2021]

(1) 3 (2) 0

(3) 2 (4) 1

57. If $\lim\limits_{x\to 0} \dfrac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$, then a + b + c is equal to ______. **[JEE (Main)-2021]**

58. Let $\alpha \in R$ be such that the function

$$f(x) = \begin{cases} \dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$$ is

continuous at x = 0, where {x} = x – [x], [x] is the greatest integer less than or equal to x.

[JEE (Main)-2021]

(1) No such α exists (2) $\alpha = \dfrac{\pi}{\sqrt{2}}$

(3) $\alpha = 0$ (4) $\alpha = \dfrac{\pi}{4}$

59. Let $f: S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x+1) = xf(x)$. If $g: S \to R$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to :

[JEE (Main)-2021]

(1) $\dfrac{205}{144}$ (2) 1

(3) $\dfrac{187}{144}$ (4) $\dfrac{197}{144}$

60. Let $f: R \to R$ and $g: R \to R$ be defined as

$$f(x) = \begin{cases} x+a, & x<0 \\ |x-1|, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x+1, & x<0 \\ (x-1)^2+b, & x \geq 0' \end{cases}$$

where a, b are non-negative real numbers. If (go$f$) (x) is continuous for all $x \in R$, then a + b is equal to________. **[JEE (Main)-2021]**

61. The value of $\lim\limits_{x\to 0^+} \dfrac{\cos^{-1}(x-[x]^2)\cdot \sin^{-1}(x-[x]^2)}{x-x^3}$, where [x] denote the greatest integer $\leq x$ is

[JEE (Main)-2021]

(1) $\dfrac{\pi}{4}$ (2) 0

(3) $\dfrac{\pi}{2}$ (4) π

62. If $f(x) = \sin\left(\cos^{-1}\left(\dfrac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\dfrac{b}{a}\log_e 2$ when x = 1, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is ________. **[JEE (Main)-2021]**

63. If the function $f(x) = \dfrac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \dfrac{1}{k}$, then k is ______. **[JEE (Main)-2021]**

64. If $\lim\limits_{x\to 0} \dfrac{\sin^{-1}x - \tan^{-1}x}{3x^3}$ is equal to L, then the value of (6L + 1) is : **[JEE (Main)-2021]**

(1) 6 (2) 2

(3) $\dfrac{1}{2}$ (4) $\dfrac{1}{6}$

65. If $f(x) = \begin{cases} \dfrac{1}{|x|} & ; \ |x| \geq 1 \\ ax^2 + b & ; \ |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively : **[JEE (Main)-2021]**

(1) $-\dfrac{1}{2}, \dfrac{3}{2}$ (2) $\dfrac{5}{2}, -\dfrac{3}{2}$

(3) $\dfrac{1}{2}, -\dfrac{3}{2}$ (4) $\dfrac{1}{2}, \dfrac{1}{2}$

66. Let $f: R \to R$ be a function defined as

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin 2x}{2x}, & \text{if } x<0 \\ b, & \text{if } x=0 \\ \dfrac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}, & \text{if } x>0 \end{cases}$$

If f is continuous at x = 0, then the value of a + b is equal to : **[JEE (Main)-2021]**

(1) $-\dfrac{5}{2}$ (2) $-\dfrac{3}{2}$

(3) –3 (4) –2

67. Let f : R → R satisfy the equation f(x + y) = f(x)·f(y) for all x, y∈R and f(x) ≠ 0 for any x∈R. If the function f is differentiable at x = 0 and f′(0) = 3, then $\lim_{h\to 0}\frac{1}{h}(f(h)-1)$ is equal to ________ .

[JEE (Main)-2021]

68. Let a function $f: \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x)=\begin{cases}\sin x - e^x & \text{if } x \le 0\\ a+[-x] & \text{if } 0 < x < 1\\ 2x-b & \text{if } x \ge 1\end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous on **R**, then (a + b) is equal to :

[JEE (Main)-2021]

(1) 5 (2) 3

(3) 4 (4) 2

69. If the value of $\lim_{x\to 0}\left(2-\cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to ________.

[JEE (Main)-2021]

70. If $f: \mathbf{R} \to \mathbf{R}$ is given by f(x) = x + 1, then the value of $\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+...+f\left(\frac{5(n-1)}{n}\right)\right]$, is

[JEE (Main)-2021]

(1) $\frac{7}{2}$ (2) $\frac{3}{2}$

(3) $\frac{5}{2}$ (4) $\frac{1}{2}$

71. If

$$\lim_{x\to 0}\frac{\alpha x e^x - \beta\log_e(1+x)+\gamma x^2 e^{-x}}{x\sin^2 x}=10,\ \alpha,\ \beta,\ \gamma\ \in \mathbf{R},$$

then the value of α + β + γ is________.

________.

[JEE (Main)-2021]

72. Let $f: \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x)=\begin{cases}\dfrac{x^3}{(1-\cos 2x)^2}\log_e\left(\dfrac{1+2xe^{-2x}}{\left(1-xe^{-x}\right)^2}\right), & x\ne 0\\ \alpha & , x=0\end{cases}$$

If f is continuous at x = 0, then α is equal to

[JEE (Main)-2021]

(1) 0 (2) 1

(3) 2 (4) 3

73. Let $f:\mathbf{R}\to\mathbf{R}$ be a function defined as

$$f(x)=\begin{cases}3\left(1-\dfrac{|x|}{2}\right) & \text{if } |x|\le 2\\ 0 & \text{if } |x|>2\end{cases}$$

Let $g:\mathbf{R}\to\mathbf{R}$ be given by g(x) = f(x + 2) – f(x – 2). If n and m denote the number of points in **R** where g is not continuous and not differentiable, respectively, then n + m is equal to ________.

[JEE (Main)-2021]

74. If $f(x)=\begin{cases}\int_0^x (5+|1-t|)dt, & x>2\\ 5x+1, & x\le 2\end{cases}$, then

(1) f(x) is everywhere differentiable

(2) f(x) is not continuous at x = 2

(3) f(x) is not differentiable at x = 1

(4) f(x) is continuous but not differentiable at x = 2

[JEE (Main)-2021]

75. Consider the function $f(x)=\begin{cases}\dfrac{P(x)}{\sin(x-2)}, & x\ne 2\\ =7 & , x=2\end{cases}$

where P(x) is a polynomial such that P″(x) is always a constant and P(3) = 9. If f(x) is continuous at x = 2, then P(5) is equal to______.

[JEE (Main)-2021]

76. Let $f: \mathbf{R} \to \mathbf{R}$ be a function such that f(2) = 4 and f′(2) = 1. Then, the value of $\lim_{x\to 2}\frac{x^2 f(2)-4f(x)}{x-2}$ is equal to

[JEE (Main)-2021]

(1) 16 (2) 8

(3) 4 (4) 12

77. Let $f:\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\to\mathbf{R}$ be defined as

$$f(x)=\begin{cases}(1+|\sin x|)^{\frac{3a}{|\sin x|}} & , -\frac{\pi}{4}<x<0\\ b & , x=0\\ e^{\cot 4x/\cot 2x} & , 0<x<\frac{\pi}{4}\end{cases}$$

If f is continuous at x = 0, then the value of $6a + b^2$ is equal to

[JEE (Main)-2021]

(1) 1 + e (2) 1 – e

(3) e (4) e – 1

78. Let $f : [0, 3] \to \mathbf{R}$ be defined by

$f(x) = \min\{x - [x], 1 + [x] - x\}$

where $[x]$ is the greatest integer less than or equal to x.

Let P denote the set containing all $x \in [0, 3]$ where f is discontinuous, and Q denote the set containing all $x \in (0, 3)$ where f is not differentiable. Then the sum of number of elements in P and Q is equal to _______.

[JEE (Main)-2021]

79. Let $f : (a, b) \to R$ be twice differentiable function such that $f(x) = \int_a^x g(t)dt$ for a differentiable function $g(x)$. If $f(x) = 0$ has exactly five distinct roots in (a, b), then $g(x)g'(x) = 0$ has at least

[JEE (Main)-2021]

(1) Twelve roots in (a, b) (2) Three roots in (a, b)

(3) Five roots in (a, b) (4) Seven roots in (a, b)

80. Let $f : [0, \infty) \to [0, 3]$ be a function defined by

$$f(x) = \begin{cases} \max\{\sin t : 0 \le t \le x\}, & 0 \le x \le \pi \\ 2 + \cos x, & x > \pi \end{cases}$$

Then which of the following is true?

[JEE (Main)-2021]

(1) f is continuous everywhere but not differentiable exactly at two points in $(0, \infty)$

(2) f is continuous everywhere but not differentiable exactly at one point in $(0, \infty)$

(3) f is differentiable everywhere in $(0, \infty)$

(4) f is not continuous exactly at two points in $(0, \infty)$

81. Let $f(x) = \cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\dfrac{1-x}{x}}\right)\right), 0 < x < 1.$

Then **[JEE (Main)-2021]**

(1) $(1-x)^2 f'(x) - 2(f(x))^2 = 0$

(2) $(1-x)^2 f'(x) + 2(f(x))^2 = 0$

(3) $(1+x)^2 f'(x) - 2(f(x))^2 = 0$

(4) $(1+x)^2 f'(x) + 2(f(x))^2 = 0$

82. If $y = y(x)$ is an implicit function of x such that $\log_e(x + y) = 4xy$, then $\dfrac{d^2y}{dx^2}$ at $x = 0$ is equal to

[JEE (Main)-2021]

83. Let a, b $\in \mathbf{R}$, b $\ne$ 0. Define a function

$$f(x) = \begin{cases} a\sin\dfrac{\pi}{2}(x-1), & \text{for } x \le 0 \\ \dfrac{\tan 2x - \sin 2x}{bx^3}, & \text{for } x > 0. \end{cases}$$

If f is continuous at $x = 0$, then 10 – ab is equal to _______. **[JEE (Main)-2021]**

84. Let [t] denote the greatest integer less than or equal to t. Let $f(x) = x - [x]$, $g(x) = 1 - x + [x]$, and $h(x) = \min\{f(x), g(x)\}$, $x \in [-2, 2]$. **[JEE (Main)-2021]**

(1) Not continuous at exactly four points in [–2, 2]

(2) Continuous in [–2, 2] but not differentiable at more than four points in (–2, 2)

(3) Not continuous at exactly three points in [–2, 2]

(4) Continuous in [–2, 2] but not differentiable at exactly three points in (–2, 2)

85. The function $f(x) = |x^2 - 2x - 3| \cdot e^{|9x^2 - 12x + 4|}$ is not differentiable at exactly : **[JEE (Main)-2021]**

(1) One point (2) Four points

(3) Two points (4) Three points

86. If the function $f(x) = \begin{cases} \dfrac{1}{x}\log_e\left(\dfrac{1+\dfrac{x}{a}}{1-\dfrac{x}{b}}\right), & x < 0 \\ k, & x = 0 \\ \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1}, & x > 0 \end{cases}$

is continuous at x = 0, then $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{4}{k}$ is equal to :

[JEE (Main)-2021]

(1) 5 (2) 4

(3) –4 (4) –5

87. Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). If $f(0) = 0$, $f(1) = 1$ and $f(2) = 2$, then **[JEE (Main)-2021]**

(1) $f''(x) > 0$ for all $x \in (0, 2)$

(2) $f'(x) = 0$ for some $x \in [0, 2]$

(3) $f''(x) = 0$ for some $x \in (0, 2)$

(4) $f''(x) = 0$ for all $x \in (0, 2)$

88. If $\alpha = \lim\limits_{x \to \pi/4} \dfrac{\tan^3 x - \tan x}{\cos\left(x + \dfrac{\pi}{4}\right)}$ and $\beta = \lim\limits_{x \to 0} (\cos x)^{\cot x}$ are the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is

[JEE (Main)-2021]

(1) $(-1, -3)$ (2) $(-1, 3)$

(3) $(1, 3)$ (4) $(1, -3)$

89. Let $f : \mathbf{R} \to \mathbf{R}$ be a continuous function. Then

$$\lim_{x \to \frac{\pi}{4}} \frac{\frac{\pi}{4}\int_{2}^{\sec^2 x} f(x)dx}{x^2 - \frac{\pi^2}{16}}$$ is equal to

[JEE (Main)-2021]

(1) $f(2)$ (2) $4f(2)$

(3) $2f(2)$ (4) $2f\left(\sqrt{2}\right)$

90. Let [t] denote the greatest integer $\le$ t. The number of points where the function

$f(x) = [x]\,|x^2 - 1| + \sin\left(\dfrac{\pi}{[x]+3}\right) - [x+1],\ x \in (-2, 2)$

is not continuous is ______. **[JEE (Main)-2021]**

91. Let $f(x) = x^6 + 2x^4 + x^3 + 2x + 3$, $x \in \mathbf{R}$. Then the natural number n for which $\lim\limits_{x \to 1} \dfrac{x^n f(1) - f(x)}{x - 1} = 44$ is _______. **[JEE (Main)-2021]**

92. Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in R$. Then $f(2)$ equals

[JEE (Main)-2021]

(1) 8 (2) –4

(3) –2 (4) 30

93. Let $f : [-1, 3] \to R$ be defined as

$$f(x) = \begin{cases} |x| + [x], & -1 \le x < 1 \\ x + |x|, & 1 \le x < 2 \\ x + [x], & 2 \le x \le 3, \end{cases}$$

where $[t]$ denotes the greatest integer less than or equal to t. Then, f is discontinuous at :

[JEE (Main)-2021]

(1) Only one point (2) Only two points

(3) Only three points (4) Four or more points

94. If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is **[JEE (Main)-2021]**

(1) 33 (2) 12

(3) 9 (4) 15

95. Let $f(x) = 15 - |x - 10|$; $x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is **[JEE (Main)-2021]**

(1) (10, 15) (2) {5, 10, 15, 20}

(3) {10} (4) {5, 10, 15}

96. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \ge 0)$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then

[JEE (Main)-2021]

(1) $a\alpha^2 - b\alpha - a = 1$ (2) $a\alpha^2 + b\alpha + a = 0$

(3) $a\alpha^2 - b\alpha - a = 0$ (4) $a\alpha^2 + b\alpha - a = -2\alpha^2$

97. Let S be the set of points where the function, $f(x) = |2 - |x - 3||,\ x \in R,$ is not differentiable. Then $\sum\limits_{x \in S} f(f(x))$ is equal to ________.

[JEE (Main)-2021]

98. For all twice differentiable functions $f : R \to R$, with $f(0) = f(1) = f'(0) = 0$, **[JEE (Main)-2021]**

(1) $f''(0) = 0$

(2) $f''(x) = 0$, for some $x \in (0, 1)$

(3) $f''(x) = 0$, at every point $x \in (0, 1)$

(4) $f''(x) \ne 0$, at every point $x \in (0, 1)$

99. Let a function $g : [0, 4] \to \mathbf{R}$ be defined as

$$g(x) = \begin{cases} \max\limits_{0 \le t \le x} \{t^3 - 6t^2 + 9t - 3\}, & 0 \le x \le 3 \\ 4 - x & , 3 < x \le 4 \end{cases}$$,then

the number of points in the interval (0, 4) where $g(x)$ is NOT differentiable, is______.

[JEE (Main)-2021]

100. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\dfrac{1}{2}\right) = -\dfrac{1}{4}$. Then $\dfrac{dy}{dx}$ at $x = \dfrac{1}{2}$, is equal to

[JEE (Main)-2021]

(1) $-\dfrac{\sqrt{5}}{2}$ (2) $\dfrac{2}{\sqrt{5}}$

(3) $\dfrac{\sqrt{5}}{2}$ (4) $-\dfrac{\sqrt{5}}{4}$

101. If $y^2 + \log_e(\cos^2 x) = y,\ x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$, then

[JEE (Main)-2021]

(1) $|y'(0)| + |y''(0)| = 3$

(2) $|y'(0)| + |y''(0)| = 1$

(3) $y''(0) = 0$

(4) $|y''(0)| = 2$

102. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} \dfrac{\lambda\left|x^2 - 5x + 6\right|}{\mu(5x - x^2 - 6)}, & x < 2 \\ e^{\frac{\tan(x-2)}{x-(x)}} & , x > 2 \\ \mu & , x = 2 \end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous at $x = 2$, then $\lambda + \mu$ is equal to **[JEE (Main)-2021]**

(1) 1 (2) e(e – 2)

(3) e(–e + 1) (4) 2e – 1

103. The number of points where the function

$$f(x) = \begin{cases} |2x^2 - 3x - 7| & \text{if } \quad x \le -1 \\ \left[4x^2 - 1\right] & \text{if } \quad -1 < x < 1 \\ |x+1| + |x-2| & \text{if } \quad x \ge 1, \end{cases}$$

[t] denotes the greatest integer $\le t$, is discontinuous is ______________.

[JEE (Main)-2022]

104. Let $f(x) = \begin{cases} \dfrac{\sin(x - [x])}{x - [x]}, & x \in (-2, -1) \\ \max\{2x,\ 3[|x|]\}, & |x| < 1 \\ 1 & , \quad \text{otherwise} \end{cases}$

Where [t] denotes greatest integer $\le t$. If m is the number of points where f is not continuous and n is the number of points where f is not differentiable, then the ordered pair (m, n) is

[JEE (Main)-2022]

(1) (3, 3) (2) (2, 4)

(3) (2, 3) (4) (3, 4)

105. Let $f(x)$ be a polynomial function such that $f(x) + f'(x) + f''(x) = x^5 + 64$. Then, the value of $\lim\limits_{x \to 1} \dfrac{f(x)}{x - 1}$ is equal to : **[JEE (Main)-2022]**

(1) –15 (2) –60

(3) 60 (4) 15

106. Let $f(x) = [2x^2 + 1]$ and $g(x) = \begin{cases} 2x - 3, & x < 0 \\ 2x + 3, & x \ge 0 \end{cases}$,

where [t] is the greatest integer $\le t$. Then, in the open interval (–1, 1), the number of points where *fog* is discontinuous is equal to ________.

[JEE (Main)-2022]

107. $f,\ g : \mathbf{R} \to \mathbf{R}$ be two real valued functions defined as $f(x) = \begin{cases} -|x + 3|, & x < 0 \\ e^x \quad , & x \ge 0 \end{cases}$ and

$g(x) = \begin{cases} x^2 + k_1 x\ , & x < 0 \\ 4x + k_2\ \ , & x \ge 0 \end{cases}$, where k_1 and k_2 are real constants. If (gof) is differentiable at $x = 0$, then $(gof)(-4) + (gof)(4)$ is equal to :

[JEE (Main)-2022]

(1) $4(e^4 + 1)$ (2) $2(2e^4 + 1)$

(3) $4e^4$ (4) $2(2e^4 - 1)$

108. Let $f(x) = \min\{1, 1 + x \sin x\}$, $0 \le x \le 2\pi$. If m is the number of points, where f is not differentiable and n is the number of points, where f is not continuous, then the ordered pair (m, n) is equal to **[JEE (Main)-2022]**

(1) (2, 0) (2) (1, 0)

(3) (1, 1) (4) (2, 1)

109. If $\cos^{-1}\left(\dfrac{y}{2}\right) = \log_e\left(\dfrac{x}{5}\right)^5, |y| < 2$, then :

[JEE (Main)-2022]

(1) $x^2y'' + xy' - 25y = 0$

(2) $x^2y'' - xy' - 25y = 0$

(3) $x^2y'' - xy' + 25y = 0$

(4) $x^2y'' + xy' + 25y = 0$

110. If $y(x) = (x^x)^x$, $x > 0$, then $\dfrac{d^2x}{dy^2} + 20$ at $x = 1$ is equal to __________. **[JEE (Main)-2022]**

111. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{bmatrix} \left[e^x\right], & x<0 \\ ae^x + [x-1], & 0 \le x < 1 \\ b + [\sin(\pi x)], & 1 \le x < 2 \\ \left[e^{-x}\right] - c, & x \ge 2 \end{bmatrix}$$

Where $a, b, c \in \mathbb{R}$ and $[t]$ denotes greatest integer less than or equal to t. Then, which of the following statements is true? **[JEE (Main)-2022]**

(1) There exists a, b, c $\in \mathbb{R}$ such that f is continuous on $\mathbb{R}$.

(2) If f is discontinuous at exactly one point, then $a + b + c = 1$

(3) If f is discontinuous at exactly one point, then $a + b + c \neq 1$

(4) f is discontinuous at atleast two points, for any values of a, b and c

112. Let $f, g : R \to R$ be functions defined by

$$f(x) = \begin{cases} [x] & , \quad x < 0 \\ |1-x| , & x \ge 0 \end{cases} \text{ and } g(x) = \begin{cases} e^x - x & , \quad x < 0 \\ (x-1)^2 - 1 , & x \ge 0 \end{cases}$$

Where $[x]$ denote the greatest integer less than or equal to x. Then, the function fog is discontinuous at exactly : **[JEE (Main)-2022]**

(1) one point (2) two points

(3) three points (4) four points

113. Let $f(x) = \begin{cases} \left|4x^2 - 8x + 5\right|, & \text{if } 8x^2 - 6x + 1 \ge 0 \\ \left[4x^2 - 8x + 5\right], & \text{if } 8x^2 - 6x + 1 < 0 \end{cases}$

where $[\alpha]$ denotes the greatest integer less than or equal to α. Then the number of points in **R** where f is not differentiable is **[JEE (Main)-2022]**

114. Let f : ***R*** → ***R*** be a continuous function such that $f(3x) - f(x) = x$. If $f(8) = 7$, then $f(14)$ is equal to **[JEE (Main)-2022]**

(1) 4 (2) 10

(3) 11 (4) 16

115. If the function

$$f(x) = \begin{cases} \dfrac{\log_e\left(1 - x + x^2\right) + \log_e\left(1 + x + x^2\right)}{\sec x - \cos x}, & x \in \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right) - \{0\} \\ k \qquad , & x = 0 \end{cases}$$

is continuous at $x = 0$, then k is equal to **[JEE (Main)-2022]**

(1) 1 (2) –1

(3) e (4) 0

116. If

$$f(x) = \begin{cases} x + a & , \ x \le 0 \\ |x-4| , & x > 0 \end{cases} \text{ and } g(x) = \begin{cases} x+1 & , \ x < 0 \\ (x-4)^2 + b , & x \ge 0 \end{cases}$$

are continuous on ***R***, then (gof) (2) + (fog) (–2) is equal to **[JEE (Main)-2022]**

(1) –10 (2) 10

(3) 8 (4) –8

117. Let a function $f : \mathbb{R} \to \mathbb{R}$ be defined as :

$$f(x) = \begin{cases} \displaystyle\int_0^x (5 - |t-3|)\,dt, & x > 4 \\ x^2 + bx, & x \le 4 \end{cases}$$

where $b \in \mathbb{R}$. If f is continuous at $x = 4$ then which of the following statements is **NOT** true? **[JEE (Main)-2022]**

(1) f is not differentiable at $x = 4$

(2) $f'(3) + f'(5) = \dfrac{35}{4}$

(3) f is increasing in $\left(-\infty, \dfrac{1}{8}\right) \cup (8, \infty)$

(4) f has a local minima at $x = \dfrac{1}{8}$

118. If for $p \neq q \neq 0$, the function $f(x) = \dfrac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx}-9}$ is continuous at $x = 0$, then

[JEE (Main)-2022]

(1) $7pq\, f(0) - 1 = 0$ (2) $63q\, f(0) - p^2 = 0$

(3) $21q\, f(0) - p^2 = 0$ (4) $7pq\, f(0) - 9 = 0$

119. For the curve $C : (x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$, the value of $3y' - y^3y''$, at the point (α, α), $\alpha > 0$, on C is equal to ________.

[JEE (Main)-2022]

120. The function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \lim_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1 + x^{2n+1} - x^{2n}} \text{ is}$$

continuous for all x in

(1) $\mathbb{R} - \{-1\}$ (2) $\mathbb{R} - \{-1, 1\}$

(3) $\mathbb{R} - \{1\}$ (4) $\mathbb{R} - \{0\}$

121. Let $x(t) = 2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t) = 2\sqrt{2}\sin t\ \sqrt{\sin 2t}$, $t \in \left(0, \dfrac{\pi}{2}\right)$. Then

$$\frac{1+\left(\dfrac{dy}{dx}\right)^2}{\dfrac{d^2y}{dx^2}} \text{ at } t = \frac{\pi}{4} \text{ is equal to}$$

[JEE (Main)-2022]

(1) $\dfrac{-2\sqrt{2}}{3}$ (2) $\dfrac{2}{3}$

(3) $\dfrac{1}{3}$ (4) $\dfrac{-2}{3}$

122. Let the function

$$f(x) = \begin{cases} \dfrac{\log_e(1+5x) - \log_e(1+\alpha x)}{x}; & \text{if } x \in 0 \\ 10 & ; \text{if } x = 0 \end{cases}$$

be continuous at $x = 0$. Then α is equal to

[JEE (Main)-2022]

(1) 10 (2) –10

(3) 5 (4) –5

123. If $[t]$ denotes the greatest integer $\leq t$, then the number of points, at which the function $f(x) = 4|2x+3| + 9\left[x + \dfrac{1}{2}\right] - 12[x+20]$ is not differentiable in the open interval (–20, 20), is ________.

[JEE (Main)-2022]

124. The number of points, where the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = |x-1|\cos|x-2|\sin|x-1| + (x-3)|x^2 - 5x + 4|$, is **NOT** differentiable, is

[JEE (Main)-2022]

(1) 1 (2) 2

(3) 3 (4) 4

125. The value of $\log_e 2\dfrac{d}{dx}(\log_{\cos x}\operatorname{cosec} x)$ at $x = \dfrac{\pi}{4}$ is

[JEE (Main)-2022]

(1) $-2\sqrt{2}$ (2) $2\sqrt{2}$

(3) –4 (4) 4

126. If $y = \tan^{-1}\left(\sec x^3 - \tan x^3\right)$, $\dfrac{\pi}{2} < x^3 < \dfrac{3\pi}{2}$, then

[JEE (Main)-2022]

(1) $xy'' + 2y' = 0$ (3) $x^2y'' - 6y + \dfrac{3\pi}{2} = 0$

(3) $x^2y'' - 6y + 3\pi = 0$ (4) $xy'' - 4y' = 0$

❑ ❑ ❑

Chapter 19

Continuity and Differentiability

1. Answer (2)

$f(x) = x\,|x|$ and $g(x) = \sin x$

$$(gof)(x) = \begin{cases} -\sin x^2 & x < 0 \\ 0 & x = 0 \\ \sin x^2 & x > 0 \end{cases}$$

For first derivative

$$\text{LHD} = \lim_{x\to 0} \frac{-\sin x^2}{x} = \lim_{x\to 0^-} \frac{-x\sin x^2}{x^2} = 0$$

$= 0$

$$\text{RHD} = \lim_{x\to 0^+} \frac{\sin x^2}{x} \times \frac{x}{x} = 0$$

$\therefore$ *gof* is differentiable at $x = 0$.

$$(gof)'(x) = \begin{cases} -2x\cos x^2 & x < 0 \\ 0 & x = 0 \\ 2x\cos x^2 & x > 0 \end{cases}$$

For second derivative,

$$\text{LHD} = \lim_{x\to 0^-} \frac{-2x\cos x^2}{x} = -2$$

$$\text{RHD} = \lim_{x\to 0^+} \frac{2x\cos x^2}{x} = 2$$

$\therefore$ (gof) is not twice differentiable at $x = 2$.

2. Answer (2)

We have,

$$f:(-1,1) \longrightarrow R$$

$$f(0) = -1 \qquad f'(0) = 1$$

$g(x) = [f(2f(x) + 2)]^2$

$g'(x) = 2[f(2f(x) + 2)] \times f'(2f(x) + 2) \times 2f'(x)$

$\Rightarrow$ $g'(0) = 2[f(2f(0) + 2)] \times f'(2f(0) + 2) \times 2f'(0)$

$= 2[f(0)] \times f'(0) \times 2f'(0)$

$= 2 \times -1 \times 1 \times 2 \times 1 = -4$

3. Answer (1)

$$f(0) = \frac{1}{3}$$

$\therefore$ Statement-1 is true.

$$f(x) = \frac{1}{\frac{e^x}{2} + \frac{e^x}{2} + e^{-x} + e^{-x}}$$

By AM – GM

$$\frac{e^x}{2} + \frac{e^x}{2} + e^{-x} + e^{-x} \geq 4\sqrt[4]{\frac{1}{4}} = 4^{3/4}$$

$$\therefore \quad 0 < f(x) \leq \frac{1}{4^{3/4}} = \frac{1}{2\sqrt{2}}$$

Equality holds if $e^x = 2e^{-x} \Rightarrow e^{2x} = 2$.

Since $\frac{1}{3} \leq \frac{1}{2\sqrt{2}}$ by intermediate value theorem

$f(c) = \frac{1}{3}$ same $c \in R$.

4. Answer (1)

$$F(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$$

Statement-1

$$\lim_{x\to 0} F(x) = \lim_{x\to 0} x\sin\left(\frac{1}{x}\right) = 0$$

Also, $F(0) = 0$

$\Rightarrow$ $\lim_{x\to 0} F(x) = F(0)$

$\Rightarrow$ $F(x)$ is countinuous at $x = 0$

$\Rightarrow$ $F(x)$ is countinuous $\forall\, x \in R$

Statement-2

$f_1(x) = x$

$\Rightarrow$ It is continuous on R

$$f_2(x)=\begin{cases}\sin\left(\dfrac{1}{x}\right) & , \ x\neq 0\\ 0 & , \ x=0\end{cases}$$

$\lim\limits_{x\to 0}\sin\dfrac{1}{x}$ does not exist

$\Rightarrow$ It is not countinuous at $x = 0$

$\Rightarrow$ $f_2(x)$ is discontinuous on R

Thus statement-2 is false.

5. Answer (1)

$$\lim_{x\to a}\frac{x^2f(a)-a^2f(x)}{x-a}\left(\frac{0}{0}\right)$$

Applying L' hospital rule

$$\lim_{x\to a}\frac{2xf(a)-a^2f'(x)}{1}$$

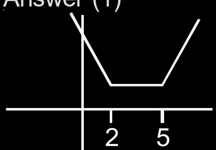
$= 2a\ f(a) - a_2\ f\ '(a)$

6. Answer (3)

At $x = 0$

LHL = 0 = RHL = $f(0)$

7. Answer (1)



By graph clearly both (1) & (2) are correct

8. Answer (2)

Using, mean value theorem

$$f'(c)=\frac{f(1)-f(0)}{1-0}=4$$

$$g'(c)=\frac{g(1)-g(0)}{1-0}=2$$

so, $\boxed{f'(c)=2g'(c)}$

9. Answer (1)

$$g(x)=\begin{cases}k\sqrt{x+1} & , \ 0\le x\le 3\\ mx+2 & , \ 3<x\le 5\end{cases}$$

R.H.D.

$$\lim_{h\to 0}\frac{g(3+h)-g(3)}{h}$$

$$=\lim_{h\to 0}\frac{m(3+h)+2-2k}{h}$$

$$=\lim_{h\to 0}\frac{(3m-2k)+mh+2}{h}=m$$

and $3m - 2k + 2 = 0$

L.H.D.

$$\lim_{h\to 0}\frac{k\sqrt{(3-h)+1}-2k}{-h}$$

$$\lim_{h\to 0}\frac{-k[\sqrt{4-h}-2]}{h}$$

$$\lim_{h\to 0}-k\times\frac{4-h-4}{h(\sqrt{4-h}+2)}=\frac{k}{4}$$

From above,

$\dfrac{k}{4}=m$ and $3m - 2k + 2 = 0$

$m=\dfrac{2}{5}$ and $k=\dfrac{8}{5}$

$$k+m=\frac{8}{5}+\frac{2}{5}=\frac{10}{5}=2$$

Alternative Answer

$$g(x)=\begin{cases}k\sqrt{x+1} & , \ 0\le x\le 3\\ mx+2 & , \ 3<x\le 5\end{cases}$$

g is constant at $x = 3$

$k\sqrt{4}=3m+2$

$2k = 3m + 2$ …(i)

Also $\left(\dfrac{k}{2\sqrt{x+1}}\right)_{x=3}=m$

$\dfrac{k}{4}=m$

$k = 4\ m$ …(ii)

$8\ m = 3\ m + 2$

$m=\dfrac{2}{5}, k=\dfrac{8}{5}$

$$m+k=\frac{2}{5}+\frac{8}{5}=2$$

10. Answer (1)

$g(x)=f(f(x))=|\log 2-\sin|\log 2-\sin x||$

$g(x) = f(f(x)) = \log 2 - \sin(\log 2 - \sin x)$

$g'(x) = \cos(\log 2 - \sin x)x - \cos x$

$g'(0) = \cos(\log 2)$

11. Answer (4)

$f(x) = 2\tan^{-1}(3x\sqrt{x})$ For $x \in \left(0,\ \frac{1}{4}\right)$

$f'(x) = \dfrac{9\sqrt{x}}{1+9x^3}$

$g(x) = \dfrac{9}{1+9x^3}$

12. Answer (1)

$f(x) = |x-\pi|(e^{|x|}-1)\sin|x|$

$x = \pi$, 0 are repeated roots and also continuous.

Hence, 'f' is differentiable at all x.

13. Answer (4)

If $f(x)$ is continuous at $x = 1$, then

$f(1^-) = f(1) = f(1^+)$

$\Rightarrow\ 5 = a + b$ ...(1)

If $f(x)$ is continuous at $x = 3$, then

$f(3^-) = f(3) = f(3^+)$

$\Rightarrow\ a + 3b = b + 15$ ...(2)

If $f(x)$ is continuous at $x = 5$, then

$f(5^-) = f(5) = f(5^+)$

$\Rightarrow\ b + 25 = 30$ ...(3)

From (3) $b = 5 \Rightarrow$ from (1), $a = 0$

but $a = 0$, $b = 5$ do not satisfy equation (2)

$\Rightarrow f(x)$ is not continuous for any values of a and b

14. Answer (1)

$\because\ f : R \to R$

and $|f(x) - f(y)| \le 2 \cdot |x - y|^{3/2}$

$\Rightarrow\ \left|\dfrac{f(x)-f(y)}{x-y}\right| \le 2\sqrt{x-y}$

$\Rightarrow\ \lim\limits_{x\to y}\left|\dfrac{f(x)-f(y)}{x-y}\right| \le \lim\limits_{x\to y} 2\sqrt{x-y}$

$\Rightarrow\ |f'(x)| = 0$

$\therefore\ f(x)$ is a constant function.

$\because\ f(0) = 1 \quad \Rightarrow\ f(x) = 1$

$\therefore\ \displaystyle\int_0^1 f^2(x)dx = \int_0^1 1dx = [x]_0^1 = 1$

15. Answer (1)

$\because\ x = 3\tan t \Rightarrow \dfrac{dx}{dt} = 3\sec^2 t$

and $y = 3\sec t \Rightarrow \dfrac{dy}{dt} = 3\sec t \cdot \tan t$

$\therefore\ \dfrac{dy}{dx} = \dfrac{\tan t}{\sec t} = \sin t$

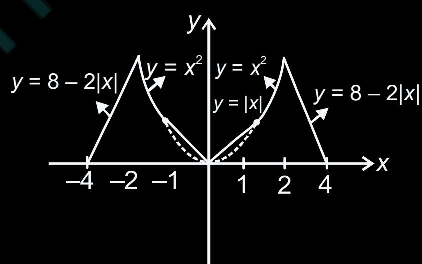
$\therefore\ \dfrac{d^2y}{dx^2} = \dfrac{d}{dx}(\sin t)\cdot\dfrac{dt}{dx}$

$= \cos t \cdot \dfrac{1}{3\sec^2 t}$

$= \dfrac{1}{3}\cos^3 t$

$\therefore\ \dfrac{d^2y}{dx^2}\left(\text{at } t = \dfrac{\pi}{4}\right) = \dfrac{1}{3}\cdot\left(\dfrac{1}{\sqrt{2}}\right)^3 = \dfrac{1}{6\sqrt{2}}$

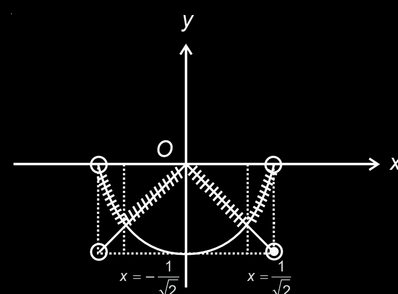
16. Answer (1)



Clearly, $S = \{-2, -1, 0, 1, 2\}$

17. Answer (1)

$f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$



$f(x)$ is not differentiable at $x \in \left\{-\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right\}$

$\Rightarrow K = \left\{-\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right\}$

18. Answer (1)

Let $y = f(x)$

$$\frac{dy}{dx} + \left(\frac{3}{4x}\right)y = 7$$

$$\text{I.F} = e^{\int \frac{3}{4x}dx} = e^{\frac{3}{4}\ln x} = x^{\left(\frac{3}{4}\right)}$$

Solution of differential equation

$$y \cdot x^{\frac{3}{4}} = \int 7 \cdot x^{\frac{3}{4}} dx + C$$

$$y \cdot x^{\frac{3}{4}} = 7 \cdot \frac{x^{\frac{7}{4}}}{\left(\frac{7}{4}\right)} + C = 4x^{\frac{7}{4}} + C$$

$$f(x) = 4x + Cx^{-\frac{3}{4}}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{4}{x} + Cx^{\frac{3}{4}}$$

$$\Rightarrow \lim_{x \to 0^+} x \cdot f\left(\frac{1}{x}\right) = \lim_{x \to 0^+}\left(4 + Cx^{\frac{7}{4}}\right) = 4$$

Option (1) is correct.

19. Answer (2)

$$f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$$

$$f(|x|) = \begin{cases} -1, & -2 \le |x| < 0 \\ |x|^2 - 1, & 0 \le |x| \le 2 \end{cases}$$

$f(|x|) = x^2 - 1, -2 \le x \le 2$

$$\Rightarrow g(x) = \begin{cases} x^2, & -2 \le x < 0 \\ (x^2 - 1) + |x^2 - 1|, & 0 \le x \le 2 \end{cases}$$

$$= \begin{cases} x^2, & -2 \le x < 0 \\ 0, & 0 \le x < 1 \\ 2(x^2 - 1), & 1 \le x \le 2 \end{cases}$$

$g'(0^-) = 0,\ g'(0^+) = 0,\ g'(1^-) = 0,\ g'(1^+) = 4$

$\Rightarrow$ $g(x)$ is non-differentiable at $x = 1$

$\Rightarrow$ Option (2) is correct.

20. Answer (1)

$x \log_e(\log_e x) - x^2 + y^2 = 4$

Differentiate both sides w.r.t. x, we get

$$\log_e(\log_e x) + x \cdot \frac{1}{x \cdot \log_e x} - 2x + 2y\frac{dy}{dx} = 0$$

$$\log_e(\log_e x) + \frac{1}{\log_e x} - 2x + 2y\frac{dy}{dx} = 0 \quad \text{...(1)}$$

When $x = e$, $y = \sqrt{4 + e^2}$.

When $x = e$ in equation (1)

$$0 + 1 - 2e + 2\sqrt{4 + e^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2e - 1}{2\sqrt{4 + e^2}}.$$

21. Answer (2)

$f(x) = \sin|x| - |x| + 2(x - \pi)\cos|x|$

For $x > 0$

$f(x) = \sin x - x + 2(x - \pi)\cos x$

$f'(x) = \cos x - 1 + 2(1 - 0)\cos x - 2\sin(x - \pi)$

$f'(x) = 3\cos x - 2(x - \pi)\sin x - 1$

By observing this, it is differentiable for $x > 0$

Now for $x < 0$

$f(x) = -\sin x + x + 2(x - \pi)\cos x$

$f'(x) = -\cos x + 1 - 2(x - \pi)\sin x + 2\cos x$

$f'(x) = \cos x + 1 - 2(x - \pi)\sin x$

By observing this, it is differentiable for all $x < 0$

Now check for $x = 0$

$f'(0^+)$ R.H.D. = 3 – 1 = 2

$f'(0^-)$ L.H.D. = 1 + 1 = 2

L.H.D. = R.H.D.

It is differentiable for $x = 0$,

So it is differentiable everywhere

22. Answer (4)

$(2x)^{2y} = 4e^{2x-2y}$

Taking log on both sides

$2y\ln(2x) = \ln 4 + (2x - 2y)$ ...(i)

Differentiate w.r.t x

$$2y\frac{1}{2x}2 + 2\ln(2x)\frac{dy}{dx} = 0 + 2 - 2\frac{dy}{dx}$$

$$2\frac{dy}{dx}(1 + \ln(2x)) = 2 - \frac{2y}{x} = \frac{2x - 2y}{x} \quad \text{...(ii)}$$

From (i) and (ii),

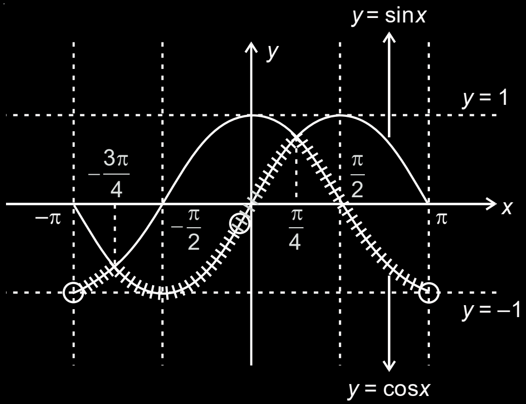
$$\frac{dy}{dx}(1 + \ln 2x) = 1 - \frac{1}{x}\left(\frac{\ln 2 + x}{1 + \ln 2x}\right)$$

$$(1 + \ln 2x)^2\frac{dy}{dx} = 1 + \ln(2x) - \left(\frac{x + \ln 2}{x}\right)$$

$$= \frac{x\ln(2x) - \ln 2}{x}$$

23. Answer (1)

$f(x) = \min\{\sin x, \cos x\}$



$\Rightarrow$ $f(x)$ is not differentiable at $x = -\frac{3\pi}{4}, \frac{\pi}{4}$

$\Rightarrow \quad S = \left\{-\frac{3\pi}{4}, \frac{\pi}{4}\right\}$

24. Answer (3)

$f'(x) = f(x)$

$$\frac{f'(x)}{f(x)} = 1$$

$$\Rightarrow \quad \frac{f'(x)\,dx}{f(x)} = dx$$

$\Rightarrow \quad \ln|f(x)| = x + c$

$f(x) = \pm\, e^{x+c}$

$\because \quad f(1) = 2$

$\Rightarrow \quad f(x) = e^{x+c} = e^c e^x$

$\Rightarrow \quad 2 = e^{1+c} = e \cdot e^c$

$$\Rightarrow \quad f(x) = \frac{2}{e} e^x$$

$$\Rightarrow \quad f'(x) = \frac{2}{e} e^x$$

$h(x) = f(f(x))$

$h'(x) = f'(f(x)) \cdot f'(x)$

$$h'(1) = f'(2) \cdot f'(1) = \frac{2}{e} e^2 \cdot \frac{2}{e} \cdot e = 4e$$

$\Rightarrow$ Option (3) is correct.

25. Answer (2)

$$2y = \left[\cot^{-1}\left(\frac{\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x}{\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x}\right)\right]^2$$

$$\Rightarrow \quad 2y = \left[\cot^{-1}\left(\frac{\cos\left(\frac{\pi}{6} - x\right)}{\sin\left(\frac{\pi}{6} - x\right)}\right)\right]^2$$

$$\Rightarrow \quad 2y = \left[\cot^{-1}\left(\cot\left(\frac{\pi}{6} - x\right)\right)\right]^2$$

$$\because \quad \frac{\pi}{6} - x \in \left(-\frac{\pi}{3}, \frac{\pi}{6}\right)$$

$$\Rightarrow \quad 2y = \begin{cases} \left(\frac{7\pi}{6} - x\right)^2 & \text{if } \frac{\pi}{6} - x \in \left(-\frac{\pi}{3}, 0\right) \\ \left(\frac{\pi}{6} - x\right)^2 & \text{if } \frac{\pi}{6} - x \in \left(0, \frac{\pi}{6}\right) \end{cases}$$

$$\Rightarrow \quad \frac{dy}{dx} = \begin{cases} x - \frac{7\pi}{6} & \text{if } x \in \left(\frac{\pi}{6}, \frac{\pi}{2}\right) \\ x - \frac{\pi}{6} & \text{if } x \in \left(0, \frac{\pi}{6}\right) \end{cases}$$

Note: Only one given option is correct.

26. Answer (2)

$$\lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k \qquad \therefore \text{ By } L \text{ hospital rule}$$

$$\Rightarrow \quad \lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\sin x}{\operatorname{cosec}^2 x} = k \qquad \Rightarrow \quad k = \frac{1}{2}$$

27. Answer (4)

L.H.L $\lim_{x \to 5} b|\pi - 5| + 3 = (5 - \pi)b + 3$

$f(5)$ = R.H.L. $\lim_{x \to 5} a|5 - \pi| + 1 = a(5 - \pi) + 1$

For continuity LHL = RHL

$(5 - \pi)b + 3 = (5 - \pi)a + 1$

$\Rightarrow \quad 2 = (a - b)(5 - \pi)$

$$\Rightarrow \quad a - b = \frac{2}{5 - \pi}$$

28. Answer (2)

$$f(x)=\begin{cases}\dfrac{\sin(p+1)x+\sin x}{x} & x<0\\ q & x=0\\ \dfrac{\sqrt{x^2+x}-\sqrt{x}}{x^{\frac{3}{2}}} & x>0\end{cases}$$

is continuous at $x = 0$

So, $f(0^-) = f(0) = f(0^+)$ ...(1)

$$f(0^-)=\lim_{h\to 0} f(0-h)$$

$$=\lim_{h\to 0}\frac{\sin(p+1)(-h)+\sin(-h)}{-h}$$

$$=\lim_{h\to 0}\left[\frac{-\sin(p+1)h}{-h}+\frac{\sin h}{h}\right]$$

$$=\lim_{h\to 0}\frac{\sin(p+1)h}{h(p+1)}\times(p+1)+\lim_{h\to 0}\frac{\sin h}{h}$$

$= (p + 1) + 1 = p + 2$ ...(2)

$$\text{Now, } f(0^+)=\lim_{h\to 0} f(0+h)=\lim_{h\to 0}\frac{\sqrt{h^2+h}-\sqrt{h}}{h^{3/2}}$$

$$=\lim_{h\to 0}\frac{(h)^{\frac{1}{2}}\left[\sqrt{h+1}-1\right]}{h\left(h^{\frac{1}{2}}\right)}$$

$$=\lim_{h\to 0}\frac{\sqrt{h+1}-1}{h}\times\frac{\sqrt{h+1}+1}{\sqrt{h+1}+1}$$

$$=\lim_{h\to 0}\frac{h+1-1}{h\left(\sqrt{h+1}+1\right)}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{h+1}+1}=\frac{1}{1+1}=\frac{1}{2} \quad ...(3)$$

Now, from equation (1)

$f(0^-) = f(0) = f(0^+)$

$$p+2=q=\frac{1}{2}$$

$$\text{So, } q=\frac{1}{2} \text{ and } p=\frac{1}{2}-2=\frac{-3}{2}$$

$$(p,q)\equiv\left(-\frac{3}{2},\frac{1}{2}\right)$$

29. Answer (2)

$$\because\quad g'(c)=\lim_{x\to c}\frac{g(x)-g(c)}{x-c}$$

$$\Rightarrow\quad g'(c)=\lim_{x\to c}\frac{|f(x)|-|f(c)|}{x-c}$$

$\because\quad f(c) = 0$

$$\Rightarrow\quad g'(c)=\lim_{x\to c}\frac{|f(x)|}{x-c}$$

$$\Rightarrow\quad g'(c)=\lim_{x\to c}\frac{f(x)}{x-c} \text{ if } f(x)>0$$

$$\text{and } g'(c)=\lim_{x\to c}\frac{-f(x)}{x-c} \text{ if } f(x)<0$$

$\Rightarrow\quad g'(c) = f'(c) = -f'(c)$

$\Rightarrow\quad 2f'(c) = 0$

$\Rightarrow\quad f'(c) = 0$

30. Answer (3)

$$f(x)=\tan^{-1}\left(\frac{\tan x-1}{\tan x+1}\right)=-\tan^{-1}\left(\tan\left(\frac{\pi}{4}-x\right)\right)$$

$$\because\quad \frac{\pi}{4}-x\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$$

$$\text{So, } f(x)=-\left(\frac{\pi}{4}-x\right)=x-\frac{\pi}{4}$$

$$\text{Let } y=\frac{x}{2}$$

$$\frac{d}{dy}f(x)=\frac{\frac{d}{dx}f(x)}{\frac{d}{dx}y}=\frac{1}{1/2}=2$$

31. Answer (2)

Apply LMVT in [–7, –1]

$$\frac{f(-1)-f(-7)}{6}\le 2$$

$$\Rightarrow\ f(-1)+3\le 12$$

$$f(-1)\le 9$$

Now apply LMVT in [–7, 0]

$$\frac{f(0)-f(-7)}{7}\le 2$$

$$f(0)\le 11$$

Hence $f(-1) + f(0) \le 20$

32. Answer (1)

Converting $\tan\alpha$ and $\cot\alpha$ in $\sin\alpha$ and $\cos\alpha$:

$$y=\sqrt{2\cot\alpha+\operatorname{cosec}^2\alpha}=\sqrt{2\cot\alpha+1+\cot^2\alpha}$$

$$=\left|\cot\alpha+1\right|$$

As $\alpha\in\left(\frac{3\pi}{4},\pi\right)$

$y=-1-\cot\alpha$

$$\frac{dy}{d\alpha}=\operatorname{cosec}^2\alpha$$

At $\alpha=\frac{5\pi}{6},\ \frac{dy}{d\alpha}=4$

33. Answer (1)

$$\because\quad f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$\therefore\quad 3c^2-8c+8=\frac{f(1)-f(0)}{1-0}$$

$3c^2-8c+8=16-11$

$3c^2-8c+3=0$

$$\therefore\quad c=\frac{8\pm2\sqrt{7}}{6}$$

$$\therefore\quad c=\frac{4-\sqrt{7}}{3}\ \text{ as } c\in(0,1)$$

34. Answer (2)

For application of Rolle's theorem

$f(3)=f(4)$

$$\frac{9+\alpha}{21}=\frac{16+\alpha}{28}\Rightarrow 36+4\alpha=48+3\alpha\Rightarrow\alpha=12$$

also $f'(c)=0$

$$\Rightarrow\quad f'(x)=\frac{1}{x^2+\alpha}.2x-\frac{1}{x}$$

$$\Rightarrow\quad \frac{2c}{c^2+12}=\frac{1}{c}\Rightarrow 2c^2=c^2+12\Rightarrow c^2=12$$

$$f''(x)=\frac{\left(x^2+12\right)2-2x(2x)}{\left(x^2+12\right)^2}+\frac{1}{x^2}$$

$$f''(c)=\frac{(24)(2)-4(12)}{(12+12)^2}+\frac{1}{12}=\frac{1}{12}$$

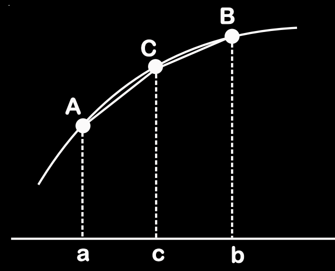
35. Answer (3)

$\because\quad f'(x)>0$ and $f''(x)<0$

So graph of function f(x) is increasing and concave up

$\because\quad a<c<b$ so $f(a)<f(c)<f(b)$

Also slope of AC > slope of BC



because $f'(x)$ is decreasing function

$$\Rightarrow\quad \frac{f(c)-f(a)}{c-a}>\frac{f(b)-f(c)}{b-c}$$

$$\Rightarrow\quad \frac{f(c)-f(a)}{f(b)-f(c)}>\frac{c-a}{b-c}$$

36. Answer (3)

$$\lim_{x\to0^-}\frac{\sin(a+2)x+\sin x}{x}=b=\lim_{x\to0^+}\frac{\left(x+3x^2\right)^{\frac{1}{3}}-x^{\frac{1}{3}}}{x^{\frac{4}{3}}}$$

$$\Rightarrow\quad (a+2)+1=b=\lim_{x\to0^+}\frac{(1+3x)^{\frac{1}{3}}-1}{x}$$

$$\Rightarrow\quad a+3=b=\lim_{x\to0^+}\left(\frac{(1+3x)^{\frac{1}{3}}-1}{(1+3x)-1}\right)\cdot3$$

$$\Rightarrow\quad a+3=b=\frac{1}{3}\cdot3=1$$

$\Rightarrow\quad a=-2,\ b=1$

So, $a+2b=0$

37. Answer (3)

$\because\quad f(g(x)=x$

Differentiating w.r.t. x

$f'(g(x).g'(x)=1$

Put $x=a$

$f'(g(a).g'(a)=1$

$\Rightarrow\quad f'(b).5=1$

$$\Rightarrow\quad f'(b)=\frac{1}{5}$$

38. Answer (2)

$x = 2\sin\theta - \sin 2\theta,\ y = 2\cos\theta - \cos 2\theta$

$\frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta,\ \frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\sin 2\theta - \sin\theta}{\cos\theta - \cos 2\theta} = \frac{\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}{\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}$$

$\Rightarrow \frac{dy}{dx} = \cot\frac{3\theta}{2}$, again diff w.r.t x

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\cot\frac{3\theta}{2}\right)$$

$$= \frac{-3}{2}\text{cosec}^2\frac{3\theta}{2}\cdot\frac{d\theta}{dx}$$

$$= \frac{-3}{2}\text{cosec}^2\frac{3\theta}{2}\cdot\frac{1}{2(\cos\theta - \cos 2\theta)}$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=\pi} = \frac{-3}{2}\cdot 1\cdot\frac{1}{2(-1-1)} = \frac{3}{8}$$

39. Answer (3)

$$\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$$

$$\Rightarrow \lim_{x\to 0} x\left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right) = A$$

$\Rightarrow 4 - \lim_{x\to 0} x\left\{\frac{4}{x}\right\} = A \quad \Rightarrow$ A = 4

Now, at $x = \sqrt{A+1}$ i.e. $x = \sqrt{5}$, f(x) = $[x^2]$ $\sin\pi x$ is discontinuous

Whereas at x = 5, 2 and 3, f(x) is continuous.

40. Answer (2)

$\because$ f(x) is continuous function

$\therefore f(1^-) = f(1^+)$

$\Rightarrow ae + \frac{b}{e} = c$ ...(i)

and $f(3^-) = f(3^+) \Rightarrow 9c = 9a + 6c$

$\Rightarrow c = 3a$ ...(ii)

For $f'(0) + f'(2) = e$

a – b + 4c = e

a – 3ae + ae^2 + 12a = e

$13a - 3ae + ae^2 = e$

$$\therefore a = \frac{e}{e^2 - 3e + 13}$$

41. Answer (3)

$$(a + \sqrt{2}b\cos x)(a - \sqrt{2}b\cos y) = a^2 - b^2$$

Differentiating both sides

$$(-\sqrt{2}b\sin x)(a - \sqrt{2}\cos y) + (a + \sqrt{2}b\cos x)(\sqrt{2}b\sin y)y' = 0$$

at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$

–b(a – b) + (a + b)by′ = 0

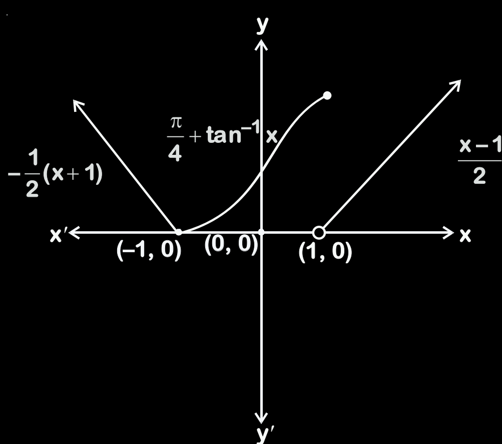
$$\frac{dy}{dx} = \frac{a-b}{a+b}$$

$$\Rightarrow \frac{dx}{dy} = \frac{a+b}{a-b}$$

42. Answer (4)

$$f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1}x, & |x| \le 1 \\ \frac{1}{2}(|x| - 1), & |x| > 1 \end{cases}$$

$$\therefore \quad f(x) = \begin{cases} \frac{-x-1}{2}, & x < -1 \\ \frac{\pi}{4} + \tan^{-1}x, & -1 \le x \le 1 \\ \frac{1}{2}(x-1), & x > 1 \end{cases}$$



f(x) is discontinuous at x = 1.

f(x) is non differentiable at x = –1, 1.

43. Answer (2)

$$f(x)=\begin{cases}k_1(x-\pi)^2-1 & ; x\le \pi\\ k_2\cos x & ; x>\pi\end{cases}$$

$$f'(x)=\begin{cases}2k_1(x-\pi) & ; x\le \pi\\ -k_2\sin x & ; x>\pi\end{cases}$$

and $f''(x)=\begin{cases}2k_1 & ; x\le \pi\\ -k_2\cos x & ; x>\pi\end{cases}$

$\because$ f(x) is twice differentiable at x = π, then

(i) $\lim\limits_{x\to\pi^+} f(x)=\lim\limits_{x\to\pi^-} f(x) \Rightarrow -1=-k_2 \Rightarrow k_2=1$

(ii) $\lim\limits_{x\to\pi^+} f''(x)=\lim\limits_{x\to\pi^-} f''(x) \Rightarrow k_2=2k_1 \Rightarrow k_1=\dfrac{1}{2}$

44. Answer (5)

If $g(x)=x^5\sin\left(\dfrac{1}{x}\right)$ and $h(x)=x^5\cos\left(\dfrac{1}{x}\right)$

then g″(0) = 0 and h″(0) = 0

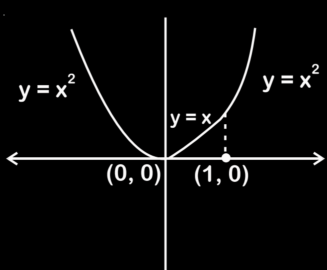
So, $f''(0^+) = g''(0^+) + 10 = 10$

and $f''(0^-) = h''(0^-) + 2\lambda = f''(0^+)$

$\Rightarrow$ $2\lambda = 10$

$\lambda = 5$

45. Answer (2)



$$f(x)=\begin{cases}x^2, & x<0\\ x, & 0\le x<1\\ x^2, & x\ge 1\end{cases}$$

$\therefore$ f(x) is not differentiable at x = 0, 1

46. Answer (5)

$$K=\lim_{x\to 0^+}\frac{1}{x}\ln\left(\frac{1+3x}{1-2x}\right)$$

$$=\lim_{x\to 0^+}\left(\frac{1-2x}{1+3x}\right)\left(\frac{(1-2x)3-(1+3x)(-2)}{(1-2x)^2}\right)$$

= 3 + 2 = 5

47. Answer (10)

$f(x + y) = f(x) + f(y) + xy^2 + x^2y$

Differentiate w.r.t. x ;

$f'(x + y) = f'(x) + y^2 + 2xy$

Put y = – x

$f'(0) = f'(x) + x^2 - 2x^2$ ...(i)

As $\lim\limits_{x\to 0}\dfrac{f(x)}{x}=1 \Rightarrow$ f(0) = 0

$\therefore$ f′(0) = 1 ...(ii)

From (i), (ii);

$f'(x) = (x^2 + 1)$

f′(3) = 10

48. Answer (8)

$f(x)=x\left[\dfrac{x}{2}\right]$ may be discontinuous where $\dfrac{x}{2}$ is an integer.

So possible points of discontinuity are;

x = ± 2, ± 4, ± 6, ± 8 and 0

but at x = 0

$\lim\limits_{x\to 0^+} f(x)=0=f(0)=\lim\limits_{x\to 0^-} f(x)$

So f(x) will be discontinuous at x = ± 2, ± 4, ± 6 and ± 8

49. Answer (2)

$$f(x)=[x-1]\ \cos\left(\frac{2x-1}{2}\right)\pi$$

at x = 1

$$\lim_{x\to 1^-}[x-1]\ \cos\left(\frac{2x-1}{2}\right)\pi=0$$

$$\lim_{x\to 1^+}[x-1]\ \cos\left(\frac{2x-1}{2}\right)\pi=0$$

f(1) = 0

at any general integer x = k

$$\lim_{x\to K^-}[x-1]\ \cos\left(\frac{2k-1}{2}\right)\pi=0$$

$$\lim_{x\to K^+}[x-1]\cos\left(\frac{2k-1}{2}\right)\pi = 0$$

f(k) = 0

$\therefore$ f(x) is continuous $\forall\ x \in R$

50. Answer (2)

$f(x) = x^3 - ax^2 + bx - 4$

f(1) = f(2)

$\Rightarrow$ 3a – b = 7 ...(i)

$f'(x) = 3x^2 - 2ax + b$

$$f'\left(\frac{4}{3}\right) = 0$$

$\Rightarrow$ 8a – 3b = 16 ...(ii)

(i) and (ii)

$\Rightarrow$ a = 5, b = 8

51. Answer (2)

$f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$

$= |2x + 1| - 3|x + 2| + |(x + 2)(x - 1)|$

$$\therefore\ f(x) = \begin{cases} x^2+2x+3 & x<-2 \\ -x^2-6x-5 & -2\le x<-\frac{1}{2} \\ -x^2-2x-3 & \frac{-1}{2}\le x<1 \\ x^2-7 & 1\le x \end{cases}$$

at x = –2 f(x) is continuous,

LHD = –2 & RHD = –2 Hence differentiable

at $x=\frac{-1}{2}$ f(x) is continuous,

LHD = –5 & RHD = –1 Hence non-differentiable

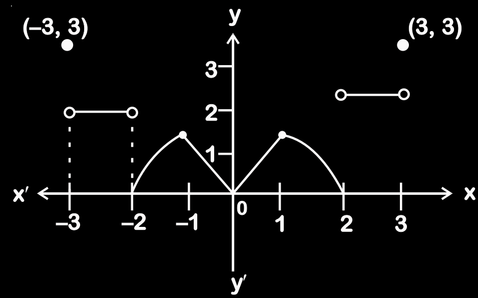
at x = 1 f(x) is continuous,

LHD = –4 & RHD = 2 Hence non-differentiable

$\therefore$ f(x) is non differentiable at $x=\frac{-1}{2}$ and 1

52. Answer (05)

$$f(x) = \begin{cases} \min\{|x|, 2-x^2\}, & -2\le x\le 2 \\ [|x|], & 2<|x|\le 3 \end{cases}$$



$$\text{Now, } f(x) = \begin{cases} 3 & , x=-3 \\ 2 & , -3<x<-2 \\ 2-x^2 & , -2\le x<-1 \\ -x & , -1\le x<0 \\ x & , 0\le x<1 \\ 2-x^2 & , 1\le x\le 2 \\ 2 & , 2<x<3 \\ 3 & , x=3 \end{cases}$$

$\therefore$ The points in (–3, 3) where function is not differentiable is x = –2, –1, 0, 1 and 2.

$\therefore$ Total number of non differentiable points = 5

53. Answer (4)

$$|f(x)-f(y)| \le |(x-y)^2|$$

$$\Rightarrow \left|\frac{f(x)-f(y)}{x-y}\right| \le |x-y|$$

$$\Rightarrow \left|\lim_{x\to y}\frac{f(x)-f(y)}{x-y}\right| \le \left|\lim_{x\to y}(x-y)\right|$$

$\Rightarrow$ $|\mathbf{f'(x)}| \le \mathbf{0}$

$\Rightarrow$ f′(x) = 0

$\Rightarrow$ f(x) is constant function.

$\because$ f(0) = 1 then f(x) = 1

54. Answer (1)

$f(-1^-) = 2$

$f(-1^+) = |a+b-1|$

$|a+b-1| = 2$ ...(i)

$f(1^-) = |a+b+1|$

$f(1^+) = 0$

$|a+b+1| = 0 \Rightarrow a+b+1=0$

$\Rightarrow$ a + b = –1 ...(ii)

55. Answer (2)

Let $f(x) = 2x^5 + 5x^4 + 10(x^3 + x^2 + x + 1)$

$\because\ f(-1) = 3$

and $f(-2) = -34$

hence roots of f(x) lies in (–2, –1)

Clearly, $|a| = 2$

56. Answer (4)

$\because$ g(x) is always differentiable and f(x) is non-differentiable at x = 0

Clearly f(g(x)) is non-differentiable when g(x) = 0 (i.e. x = 0)

57. Answer (04)

Put x = 0 we get $N_r = a - b + c = 0$ ...(i)

(for indeterminancy to be present)

$$\therefore \quad \lim_{x\to 0}\frac{ae^{x}-b\cos x+ce^{-x}}{x^2}=2 \quad \left(\lim_{x\to 0}\frac{\sin x}{x}=1\right)$$

using L′ Hospital rule we get

$$\lim_{x\to 0}\frac{ae^{x}+b\sin x-cex^{-x}}{2x}=2$$

Put x = 0 $\Rightarrow$ a – c = 0 ...(ii)

Again by L′ hospital rule we get

$$\lim_{x\to 0}\frac{ae^{x}+b\cos x+ce^{-x}}{2}=2$$

$\Rightarrow$ a + b + c = 4 ...(iii)

58. Answer (1)

When $x \to 0^-$, {x} = 1 – h where $h \to 0$

$$\text{LHL}=\lim_{h\to 0}\frac{\cos^{-1}\left(1-(1-h)^2\right)\sin^{-1}(h)}{(1-h)\left(1-(1-h)^2\right)}$$

$$=\lim_{h\to 0}\frac{\cos^{-1}\left(1-(1-h)^2\right)\sin^{-1}h}{(1-h)\ h\ (2-h)}=\frac{\frac{\pi}{2}\times 1}{1\times 2}=\frac{\pi}{4}$$

When $x \to 0^+$, {x} = h where $h \to 0$

$$\text{RHL}=\lim_{h\to 0}\frac{\cos^{-1}\left(1-h^2\right)\sin^{-1}(1-h)}{h-h^3}$$

$$=\lim_{h\to 0}\frac{\sin^{-1}\sqrt{1-(1-h)^2}\ \sin^{-1}(1-h)}{h\left(1-h^2\right)}$$

$$=\lim_{h\to 0}\frac{\sin^{-1}\left(h\sqrt{2-h^2}\right)\sin^{-1}(1-h)}{h\left(1-h^2\right)}$$

$$=\lim_{h\to 0}\frac{\sin^{-1}\left(h\sqrt{2-h^2}\right)}{h\sqrt{2-h^2}}\times\frac{h\sqrt{2-h^2}}{h\left(1-h^2\right)}\times\sin^{-1}(1-h)$$

$$=1\times\frac{\sqrt{2}}{1}\times\frac{\pi}{2}=\frac{\pi}{\sqrt{2}}$$

LHL $\neq$ RHL

59. Answer (1)

$f(x + 1) = x\, f(x)$

$\ln\left(f(x+1)\right)=\ln x+\ln f(x)$

g(x + 1) = ln x + g(x)

g(x + 1) – g(x) = ln x ...(i)

$g'(x + 1) - g'(x) = \frac{1}{x}$

$$g''(x+1)-g''(x)=\frac{-1}{x^2}$$

$$g''(2)-g'(1)=\frac{-1}{1} \quad \text{...(ii)}$$

$$g''(3)-g''(2)=\frac{-1}{4} \quad \text{...(iii)}$$

$$g''(4)-g''(3)=\frac{-1}{9} \quad \text{...(iv)}$$

$$g''(5)-g''(4)=\frac{-1}{16} \quad \text{...(v)}$$

Adding (ii), (iii), (iv) & (v)

$$g''(5)-g''(1)=-\left(\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}\right)=\frac{-205}{144}$$

$$\left|g''(5)-g''(1)\right|=\frac{205}{144}$$

60. Answer (1)

f(x) should be continuous at x = 0

$\Rightarrow$ a = 1

g(x) should be continuous at x = 0

$\Rightarrow$ 1 = 1 + b $\Rightarrow$ b = 0

a + b = 1

61. Answer (3)

$$\lim_{x\to 0^+}\frac{\cos^{-1}\left(x-[x]^2\right)\cdot\sin^{-1}\left(x-[x]^2\right)}{x-x^3}$$

$$=\lim_{x\to 0^+}\frac{\cos^{-1}x}{1-x^2}\cdot\frac{\sin^{-1}x}{x}$$

$$=\cos^{-1}0=\frac{\pi}{2}$$

62. Answer (481)

$$f(x)=\sin\cos^{-1}\left(\frac{1-(2^x)^2}{1+(2^x)^2}\right)$$

$$=\sin(2\tan^{-1}2^x)$$

$$f'(x)=\cos(2\tan^{-1}2^x).2.\frac{1}{1+(2^x)^2}\times 2^x.\log_e 2$$

$$f(1)=\cos(2\tan^{-1}2)\frac{2}{1+4}\times 2\times\log_e 2$$

$$\Rightarrow\quad f(1)=\cos\cos^{-1}\left(\frac{1-2^2}{1+2^2}\right).\frac{4}{5}\log_e 2$$

$$=-\frac{12}{25}\log_e 2$$

$\Rightarrow$ a = 25, b = 12

$|a^2-b^2| = |625-144| = 481$

63. Answer (6)

$$\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$$

$$=\lim_{x\to 0}\frac{2\sin\left(\frac{x+\sin x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$$

$$=\lim_{x\to 0}\frac{2\sin\left(\frac{x+\sin x}{2}\right)}{\left(\frac{x+\sin x}{2}\right)}\times\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\times\frac{x^2-\sin^2 x}{4x^4}$$

$$=\lim_{x\to 0}2\times 1\times 1\times\left(\frac{x+\sin x}{x}\right)\frac{(x-\sin x)}{x^3}\times\frac{1}{4}$$

$$=2\times 2\times\frac{1}{6}\times\frac{1}{4}=\frac{1}{6}$$

For continuity at x = 0, $f(0)=\frac{1}{6}=\frac{1}{k}\Rightarrow k=6$

64. Answer (2)

$$\lim_{x\to 0}\frac{\sin^{-1}x-\tan^{-1}x}{3x^3}$$

$$\lim_{x\to 0}\frac{\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)-\tan^{-1}x}{3x^3}$$

$$\Rightarrow\lim_{x\to 0}\frac{\tan^{-1}\left(\frac{x-x\sqrt{1-x^2}}{\sqrt{1-x^2}+x^2}\right)}{3x^3}$$

$$\Rightarrow\lim_{x\to 0}\frac{x\left(1-\sqrt{1-x^2}\right)}{\left(x^2+\sqrt{1-x^2}\right)\cdot 3x^3}$$

$$\Rightarrow\lim_{x\to 0}\frac{\left(1-\sqrt{1-x^2}\right)}{3x^2}\frac{\left(1+\sqrt{1-x^2}\right)}{\left(1+\sqrt{1-x^2}\right)}$$

$$\Rightarrow\lim_{x\to 0}\frac{x^2}{3x^2\left(1+\sqrt{1-x^2}\right)}=\frac{1}{6}=L$$

$\therefore$ 6L + 1 = 2

65. Answer (1)

f(x) must be continuous at x = ± 1

$\Rightarrow$ 1 = a + b ...(i)

For differentiable at x = 1

$$f(x)=\begin{matrix}1/x & x\ge 1\\ ax^2+b & x<1\end{matrix}$$

$$\text{LHD}=\lim_{h\to 0}\frac{a(1-h)^2+b-1}{-h}=\frac{-2ah+h^2}{-h}=2a$$

(For existence a + b = 1)

$$\text{RHD} = \lim_{h\to 0} = \frac{\frac{1}{1+h}-1}{h} = -1$$

$$\Rightarrow \quad a = \frac{-1}{2}, b = \frac{3}{2}$$

(Similar can be done for x = –1)

66. Answer (1)

$\therefore$ f(x) is continuous at x = 0

$$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$$\Rightarrow \lim_{h\to 0} \frac{\sin(a+1)(-h) - \sin 2h}{-2h} = \lim_{h\to 0} \frac{\sqrt{h+bh^3} - \sqrt{h}}{bh^{5/2}} = b$$

$$\Rightarrow \frac{a+1}{2} + 1 = \lim_{h\to 0} \frac{bh^2}{bh^2\left(\sqrt{4+bh^2}+1\right)} = b$$

$$\Rightarrow \frac{a+3}{2} = \frac{1}{2} = b$$

$$\Rightarrow a = -3,\ b = \frac{1}{2}$$

$$\therefore\ a + b = -3 + \frac{1}{2} = -\frac{5}{2}.$$

67. Answer (3)

$\because f(x+y) = f(x)f(y) \qquad \forall\, x, y \in R$

x = y = 0 $\Rightarrow$ f(0) = $(f(0))^2 \Rightarrow$ f(0) =0 or f(0) = 1

f(x) $\neq$ for any $x \in R \Rightarrow$ f(0) =1

Given $f'(0) = 3$

$$\Rightarrow \lim_{h\to 0} \frac{f(0+h)+f(0)}{h} = 3$$

$$\Rightarrow \lim_{h\to 0} \frac{f(h)-1}{h} = 3$$

68. Answer (2)

$$\because f(x) = \begin{cases} \sin x - e^x, & x \le 0 \\ a + [-x], & 0 < x < 1 \\ 2x - b, & x \ge 1 \end{cases}$$

$\because$ f(x) is continuous everywhere

$\therefore$ f(x) is continuous at x = 0

$\therefore$ $f(0^-) = f(0) = f(0^+)$

$\therefore$ –1 = –1 = a – 1 $\Rightarrow$ a = 0

f(x) is continuous at x = 1

$\therefore$ $f(1^-) = f(1) = f(1^+)$

a – 1 = 2 – b = 2 – b

$\therefore$ b = 3

$\therefore$ a + b = 3

69. Answer (3)

$$L = e^{\underset{x\to 0}{Lt}(1-\cos x\sqrt{\cos 2x})\times\frac{x+2}{x^2}}$$

$$= e^{\underset{x\to 0}{Lt}\left(\frac{1-\cos^2 x(\cos 2x)}{1+\cos x\sqrt{\cos 2x}}\right)\times\frac{x+2}{x^2}}$$

$$= e^{\underset{x\to 0}{Lt}\frac{1-(1-\sin^2 x)(1-2\sin^2 x)}{x^2}\times\frac{x+2}{1+\cos x\sqrt{\cos 2x}}}$$

$$= e^{\underset{x\to 0}{Lt}\frac{1-(1-3\sin^2 x+2\sin^4 x)}{x^2}\times\frac{2}{1+1}}$$

$= e^3$

$\Rightarrow$ a = 3

70. Answer (1)

$$f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + ... + f\left(\frac{5(n-1)}{n}\right)$$

$$\Rightarrow \ 1 + 1 + \frac{5}{n} + 1 + \frac{10}{n} + ... + 1 + \frac{5(n-1)}{n}$$

$$\Rightarrow \ n + \frac{5}{n}\,\frac{(n-1)n}{2} = \frac{2n+5n-5}{2} = \frac{7n-5}{2}$$

$$\lim_{n\to\infty} \frac{1}{n}\left(\frac{7n-5}{2}\right) = \frac{7}{2}$$

71. Answer (3)

$$\lim_{x\to 0} \frac{\alpha x\left(1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + ...\right) - \beta\left(x - \frac{x^2}{2} + \frac{x^3}{3} + ...\right) + \gamma x^2\left(1 - \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + ...\right)}{x^3\left(\frac{\sin x}{x}\right)^2} = 10$$

$$\Rightarrow \ \lim_{x\to 0} \frac{x(\alpha-\beta) + x^2\left(\alpha + \frac{\beta}{2} + \gamma\right) + x^3\left(\frac{\alpha}{2} - \frac{\beta}{3} - \gamma\right) + ...}{x^3} = 10$$

$$\Rightarrow \alpha - \beta = 0,\ \alpha + \frac{\beta}{2} + \gamma = 0,\ \frac{\alpha}{2} - \frac{\beta}{3} - \gamma = 10$$

$\Rightarrow \alpha$ = 6, β = 6, γ = – 9

72. Answer (2)

$\therefore$ f(x) is continuous at x = 0

$$\therefore \quad \alpha = \lim_{x\to 0}\frac{x^3}{(1-\cos 2x)^2}\log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right)$$

$$\alpha = \lim_{x\to 0}\frac{x^4}{4\sin^4 x}\cdot\frac{1}{x}\log_e\left(\frac{e^{2x}+2x}{x^2-2xe^x+e^{2x}}\right)$$

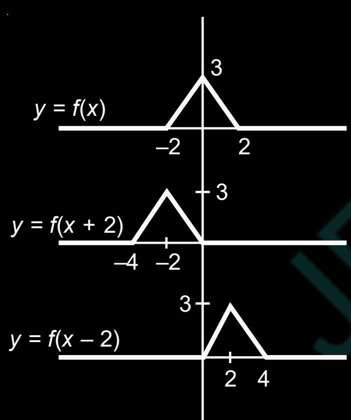
$$=\frac{1}{4}\lim_{x\to 0}\left\{\frac{\ln(e^{2x}+2x)}{x}-\frac{\ln(x^2-2xe^x+e^{2x})}{x}\right\}$$

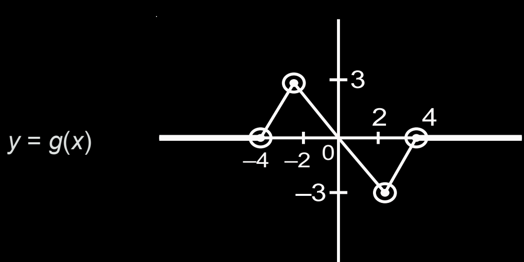
$$=\frac{1}{4}\lim_{x\to 0}\left\{\frac{2e^{2x}+2}{e^{2x}+2x}-\frac{2x-2e^x(x+1)+2e^{2x}}{x^2-2xe^x+e^{2x}}\right\}$$

$$=\frac{1}{4}(4-0)$$

$= 1$

73. Answer (4)





Clearly $n = 0$ and $m = 4$.

74. Answer (4)

$$\lim_{x\to 2^-} f(x) = \int_0^2 (5+|1-t|)\,dt$$

$$=\int_0^1 (5+1-t)\,dt+\int_1^2(5+t-1)\,dt$$

$$= 6t-\frac{t^2}{2}\Big|_0^1+4t+\frac{t^2}{2}\Big|_1^2$$

$$= 6-\frac{1}{2}+\left(8+2-4-\frac{1}{2}\right)$$

$$= 16-4-1=11=\lim_{x\to 2^+} f(x)=f(2)$$

at $x = 2$ (checking differentiability)

$$\text{LHD}=\lim_{h\to 0^-}\frac{f(2-h)-f(2)}{-h}=\lim_{h\to 0}\frac{5(2-h)+1-11}{-h}=5$$

$$\text{RHD}=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{\int_0^{2+h}(5+|1-t|)dt-11}{h}$$

$$=\lim_{h\to 0} 5+|1-(2+h)|=6$$

$\therefore$ $f(x)$ is continuous and non-differentiable

75. Answer (39)

P(x) = (x – 2) (ax + b)

$\lim_{x\to 2} P(x)=7 \quad \Rightarrow 2a + b = 7$

P(3) = 9 $\qquad \Rightarrow 3a + b = 9$

a = 2, b = 3

P(x) = (x – 2)(2x + 3)

P(5) = 39

76. Answer (4)

$$\lim_{x\to 2}\frac{x^2f(2)-4f(x)}{x-2}\left[\frac{0}{0}\right]$$

$$=\lim_{x\to 2}2x\cdot f(2)-4f'(x)$$

$= 4f(2) - 4\cdot f'(2)$

$= 4 \times 4 - 4 \times 1$

$= 12$

77. Answer (1)

LHL = $f(0)$ = RHL

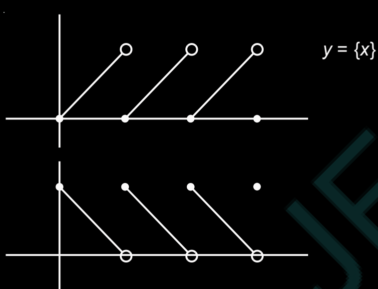
$$e^{3a} = b = e^{\lim_{x\to 0}\frac{\tan 2x}{2x}\times\frac{4x}{\tan 4x}\times\frac{1}{2}}$$

$$e^{3a} = b = e^{\frac{1}{2}}$$

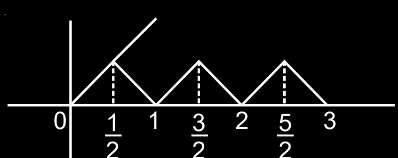
$6a + b^2 = 1 + e$

78. Answer (5)

$f(x) = \min\{\{x\}, 1 - \{x\}\}$



$y = f(x)$



Cont. everywhere & non diff. at $x=\frac{1}{2},1,\frac{3}{2},2,\frac{5}{2}$

79. Answer (4)

$f'(x) = g(x)$

As $f(x)$ has 5 roots $f'(x) = 0$, 4 times for $x \in (a, b)$

$\therefore$ $g(x)$ has 4 roots in $x \in (a, b)$

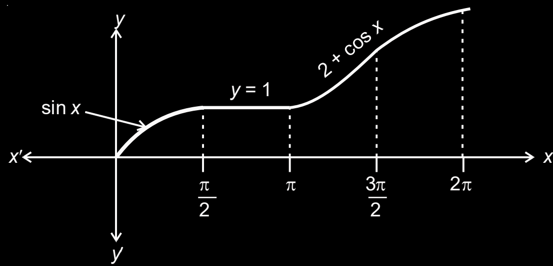
$\therefore$ $g'(x)$ has 3 roots in $x \in (a, b)$

$\therefore$ $g(x)\,g'(x)$ has 7 roots in $x \in (a, b)$

80. Answer (3)

$f : [0, \infty) \to [0, 3]$

and $f(x)=\begin{cases}\max\{\sin t : 0\le t\le x\}, & 0\le x\le\pi\\ 2+\cos x, & x>\pi\end{cases}$



Clearly $f(x)$ is continuous everywhere

and $f(x)$ is differentiable at $x=\frac{\pi}{2}$ and $x = \pi$

$\therefore$ $f(x)$ is differentiable everywhere

81. Answer (2)

$$f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$$

Let $\frac{\sqrt{1-x}}{\sqrt{x}}=\cot\theta$

$\Rightarrow$ $f(x)=\cos\left(2\tan^{-1}\sin\theta\right)=\cos\left(2\tan^{-1}\left(\sqrt{x}\right)\right)$

let $\sqrt{x}=\tan\theta$

$\Rightarrow$ $f(x) = \cos 2\theta = 2\cos^2\theta - 1 = \frac{2}{1+x}-1=\frac{1-x}{1+x}$

$\Rightarrow$ $f'(x)=-\frac{2}{(1+x)^2}$

$\Rightarrow$ $(1-x)^2f'(x)=-2\left(\frac{1-x}{1+x}\right)^2$

$\Rightarrow$ $(1 - x)^2f'(x) + 2f^2(x) = 0$

82. Answer (40)

$\log_e(x + y) = 4xy$ at $x = 0$, $y = 1$

$$\frac{1}{x+y}\left[1+\frac{dy}{dx}\right]=4\left[y+x\cdot\frac{dy}{dx}\right]$$

$$\frac{1}{x+y}-4y=\left(4x-\frac{1}{x+y}\right)\frac{dy}{dx}$$

$$1-4xy-4y^2=(4x^2+4xy-1)\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{1-4xy-4y^2}{4x^2+4xy-1},\qquad \text{at } x=0,\ \frac{dy}{dx}=\frac{1-4}{-1}=3$$

$$\frac{d^2y}{dx^2}=\frac{\left(4x^2+4xy-1\right)\left[-\left(4y+4x\frac{dy}{dx}\right)-8y\cdot\frac{dy}{dx}\right]-\left(1-4xy-4y^2\right)\left(8x+4y+4x\cdot\frac{dy}{dx}\right)}{\left(4x^2+4xy-1\right)^2}$$

at $x = 0$, $y = 1$, $\frac{dy}{dx}=3$

$$\frac{d^2y}{dx^2}=\frac{(-1)[-4-8\times 3]-(1-4)(4)}{1}=28+12=40$$

83. Answer (14)

L.H.L. at $x = 0$ for $f(x)$

$$\underset{x\to 0^-}{\text{lt}}\ a\sin\frac{\pi}{2}(x-1)=a\sin\left(-\frac{\pi}{2}\right)=-a$$

RHL at $x = 0$ for $f(x)$

$$\underset{x\to 0^+}{\text{lt}}\ \frac{\tan 2x-\sin 2x}{bx^3}=\underset{x\to 0^+}{\text{lt}}\ \frac{\tan 2x}{x}\frac{(1-\cos 2x)}{x^2}\cdot\frac{1}{b}$$

$$=\frac{1}{b}\underset{x\to 0^+}{\text{lt}}\ \frac{\tan 2x}{x}\cdot\frac{1-\cos 2x}{4x^2}\cdot 4$$

$$=\frac{1}{b}\cdot 2\cdot\frac{1}{2}\cdot 4=\frac{4}{b}$$

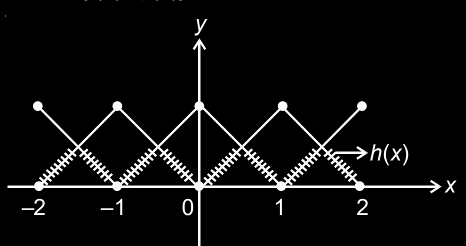
For continuity $-a=\frac{4}{b}\Rightarrow ab=-4$

So, $10 - ab = 14$

84. Answer (2)

$f(x) = \{x\}$ and $g(x) = 1 - \{x\}$

$h(x) = \min\{f(x), g(x)\}$



$\therefore$ $h(x)$ is continuous everywhere and non-differentiable at 7 points

85. Answer (3)

$f(x) = |(x + 1)(x - 3)|\cdot e^{(3x-2)^2}$

Clearly $f(x)$ is not differentiable at $x = -1$ and 3.

86. Answer (4)

$$\lim_{x\to 0^-}f(x)=\lim_{x\to 0^+}f(x)=f(0)$$

$$\Rightarrow\ \lim_{x\to 0^-}\frac{\ln\left(1+\frac{x}{a}\right)-\ln\left(1-\frac{x}{b}\right)}{x}=\lim_{x\to 0^+}\frac{-2\sin^2 x}{\sqrt{x^2+1}-1}=k$$

$$\Rightarrow\ \frac{1}{a}+\frac{1}{b}=\lim_{x\to 0^+}\left(\frac{-2\sin^2 x}{x^2}\right)\left(\sqrt{x^2+1}+1\right)=k$$

$$\Rightarrow\ \frac{1}{a}+\frac{1}{b}=-4=k$$

So, $\frac{1}{a}+\frac{1}{b}=\frac{4}{k}=-4-1=-5$

87. Answer (3)

There exists a $C_1\in(0,1)$

Such that $f'(C_1)=\frac{f(1)-f(0)}{1-0}=1$

and there exists a $C_2\in(1, 2)$

Such that $f'(C_2)=\frac{f(2)-f(1)}{2-1}=1$

Hence there exists a $C\in(C_1, C_2)$ such that

$$f''(C)=\frac{f'(C_1)-f'(C_2)}{C_1-C_2}=0$$

88. Answer (3)

$$\alpha=\lim_{x\to\frac{\pi}{4}}\frac{\tan x(-\cos 2x)}{\cos^2 x\cdot\cos\left(x+\frac{\pi}{4}\right)}=-2\lim_{x\to\frac{\pi}{4}}\frac{\cos 2x}{\cos\left(x+\frac{\pi}{4}\right)}=-4$$

$$\beta=\lim_{x\to 0}(\cos x)^{\cot x}=e^{\lim_{x\to 0}(\cos x-1)\cot x}=e^0=1$$

Quadratic equation having roots $\alpha_1\beta$ is

$x^2 + 3x - 4 = 0$

Clearly $a = 1$, $b = 3$

89. Answer (3)

$$\lim_{x\to\frac{\pi}{4}}\frac{\frac{\pi}{4}\int_2^{\sec^2 x}f(x)dx}{x^2-\frac{\pi^2}{16}}\ \left[\frac{0}{0}\right]$$

$$=\frac{\pi}{4}\lim_{x\to\frac{\pi}{4}}\frac{2\sec^2 x\cdot\tan x\cdot f\left(\sec^2 x\right)}{2x}$$

$$=\frac{\pi}{4}\cdot\frac{2\cdot 1\cdot f(2)}{\frac{\pi}{4}}$$

$= 2f(2)$

90. Answer (2)

$$f(x)=\begin{cases}-2|x^2-1|+1 & x\in(-2,-1)\\ -|x^2-1|+1 & x\in[-1,0)\\ \sin\dfrac{\pi}{3}+1 & x\in[0,1)\\ |x^2-1|+\dfrac{1}{\sqrt{2}}-2 & x\in[1,2)\end{cases}$$

$\therefore$ at $x = -1$ $\lim\limits_{x\to-1^-} f(x)=1$ and $\lim\limits_{x\to-1^+} f(x)=1$

Hence continuous at $x = -1$

Similarly check at $x = 0$

$\lim\limits_{x\to 0^-} f(x)=-1$ and $\lim\limits_{x\to 0^+} f(x)=1+\dfrac{\sqrt{3}}{2}$

$\Rightarrow$ discontinuous

and at $x = 1$

$\lim\limits_{x\to 1^-} f(x)=1+\dfrac{\sqrt{3}}{2}$ and $\lim\limits_{x\to 1^+} f(x)=\dfrac{1}{\sqrt{2}}-2$

$\Rightarrow$ discontinuous

Hence 2 points of discontinuity.

91. Answer (7)

$$\underset{x\to1}{\text{lt}}\ \frac{x^n f(1)-f(x)}{x-1}=44$$

By L.H. Rule

$$\underset{x\to1}{\text{lt}}\left(nx^{n-1}f(1)-f'(x)\right)=44$$

$n\cdot f(1) - f'(1) = 44$

$n(9) - 19 = 44$

$n = 7$

92. Answer (3)

Let $f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b \quad \Rightarrow f'(1) = 3 + 2a + b$

$f''(x) = 6x + 2a \quad \Rightarrow f''(2) = 12 + 2a$

$f'''(x) = 6 \quad \Rightarrow f'''(3) = 6$

$\because f(x) = x^3 + f'(1)x^2 + f''(2)\,x + f'''(3)$

$\Rightarrow f'(1) = a \Rightarrow 3 + 2a + b = a \Rightarrow a + b = -3 \quad$ ...(1)

$\Rightarrow f''(2) = b \Rightarrow 12 + 2a = b \Rightarrow 2a - b = -12 \quad$ ...(2)

From (1) and (2)

$3a = -15 \Rightarrow a = -5 \Rightarrow b = 2$

$\Rightarrow f(x) = x^3 - 5x^2 + 2x + 6$

$\Rightarrow f(2) = 8 - 20 + 4 + 6 = -2$

93. Answer (3)

$$f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3\end{cases}$$

$$=\begin{cases}-x-1, & -1\le x<0\\ x, & 0\le x<1\\ 2x, & 1\le x<2\\ x+2, & 2\le x<3\\ 6, & x=3\end{cases}$$

$\Rightarrow f(-1) = 0,\ f(-1^+) = 0$

$f(0^-) = -1,\ f(0) = 0,\ f(0^+) = 0$

$f(1^-) = 1,\ f(1) = 2,\ f(1^+) = 2$

$f(2^-) = 4,\ f(2) = 4,\ f(2^+) = 4$

$f(3^-) = 5,\ f(3) = 6$

$f(x)$ is discontinuous at $x = \{0, 1, 3\}$

94. Answer (1)

Let $g(x) = f(f(f(x))) + (f(x))^2$

On differentiating both sides w.r.t. x, we get

$g'(x) = f'(f(f(x)))\, f'(f(x))\, f'(x) + 2f(x)\, f'(x)$

$g'(1) = f'(f(f(1)))\, f'(f(1))\, f'(1) + 2f(1)\, f'(1)$

$= f'(f(1))\, f'(1)\, f'(1) + 2f(1)\, f'(1)$

$= 3 \times 3 \times 3 + 2 \times 1 \times 3 = 27 + 6 = 33$

95. Answer (4)

Given $f(x) = 15 - |(10 - x)|$

$\Rightarrow f(f(x)) = 15 - ||10 - x| - 5|$

$\therefore$ Non-differentiable at points where

$10 - x = 0$ and $|10 - x| = 5$

$\Rightarrow x = 10$ and $x - 10 = \pm 5$

$\Rightarrow x = 10$ and $x = 15, 5$

96. Answer (1)

$f(x) = \ln(\sin x),\ g(x) = \sin^{-1}(e^{-x})$

$f(g(x)) = \ln(\sin(\sin^{-1} e^{-x})) = -x$

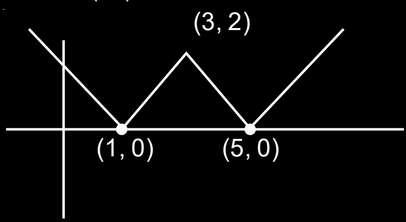
$\Rightarrow -\alpha = b$

$f'(g(\alpha)) = a$

i.e., $a = -1$

$\therefore a\alpha^2 - b\alpha + 1 = -\alpha^2 + \alpha^2 + 1 = -a$

97. Answer (03)



f(x) is non-differentiable at x = 1, 3, 5

S is {1, 3, 5}

At x = 1, f(x) = 0

At x = 3, f(x) = 2

At x = 5, f(x) = 0

$\therefore$ f(0) + f(2) + f(0) = 1 + 1 + 1 = 3

98. Answer (2)

$f : R \to R$, with f(0) = f(1) = 0

and $f'(0) = 0$

$\because$ f(x) is differentiable and continuous

and f(0) = f(1) = 0

So by Rolle's theorem

For $c \in (0, 1)$, $f'(c) = 0$

Now again

$\because$ $f'(c) = 0$, $f'(0) = 0$

So by Rolle's theorem

$f''(x) = 0$ for some $x \in (0, 1)$

99. Answer (1)

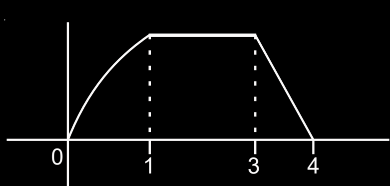
$f(t) = t^3 - 6t^2 + 9t - 3$

$f'(t) = 3(t - 1)(t - 3)$

Local max at $x = 1$, $f(1) = 1$

$$g(x) = \begin{cases} f(x) = x^3 - 6x^2 + 9x - 3, & t \in [0, 1] \\ 1 & t \in (1, 3] \\ 4 - x & 3 < x \le 4 \end{cases}$$

$y = g(x)$ =



Not diff. at $x = 3$

100. Answer (1)

$\because \quad y\sqrt{1-x^2} = k - x\sqrt{1-y^2} \qquad$ ...(i)

On differentiating both side of eq. (i) w.r.t. x we get,

$$\frac{dy}{dx}\sqrt{1-x^2} - y\frac{2x}{2\sqrt{1-x^2}} = 0 - \sqrt{1-y^2} + \frac{x \cdot y}{\sqrt{1-y^2}}\frac{dy}{dx}$$

Put $x = \frac{1}{2}$ and $y = -\frac{1}{4}$ we get

$$\frac{dy}{dx}\cdot\frac{\sqrt{3}}{2} - \left(-\frac{1}{4}\right)\cdot\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{\sqrt{15}}{4} + \frac{-\frac{1}{8}}{\frac{\sqrt{15}}{4}}\cdot\frac{dy}{dx}$$

$$\therefore \quad \frac{dy}{dx} = -\frac{\sqrt{5}}{2}$$

101. Answer (4)

$y^2 + 2\ln(\cos x) = y \qquad$ ...(1)

$\Rightarrow 2yy' - 2\tan x = y' \qquad$ ...(2)

From (1) $\quad$ y (0) = 0 or 1

$\therefore$ $y'(0) = 0$

Again differentiating (2) we get

$2(y')^2 + 2yy'' - 2\sec^2 x = y''$

gives $|y''(0)| = 2$

102. Answer (3)

For continuity $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) = f(2)$

$$\therefore \quad \lim_{x\to 2^-}\frac{\lambda|(x-2)(x-3)|}{\mu(x-2)(3-x)} = \lim_{x\to 2^+} e^{\frac{\tan(x-2)}{x-2}} = \mu$$

$$\Rightarrow \quad \frac{\lambda(-1)}{\mu(1)} = e = \mu$$

$\Rightarrow \quad \mu = e$ and $\lambda \equiv e^2$

$\lambda + \mu = e - e^2 = e(1 - e)$

103. Answer (7)

$\because$ $f(-1) = 2$ and $f(1) = 3$

For $x \in (-1, 1)$, $(4x^2 - 1) \in [-1, 3)$

hence $f(x)$ will be discontinuous at $x = 1$ and also

whenever $4x^2 - 1 = 0$, 1 or 2

$$\Rightarrow \quad x = \pm\frac{1}{2}, \ \pm\frac{1}{\sqrt{2}} \text{ and } \pm\frac{\sqrt{3}}{2}$$

So there are total 7 points of discontinuity.

104. Answer (3)

$$f(x)=\begin{cases}\dfrac{\sin(x-[x])}{x[x]} & , \quad x\in(-2,-1)\\ \max\{2x, 3[|x|]\} & , \quad |x|<1\\ 1 & , \quad \text{otherwise}\end{cases}$$

$$f(x)=\begin{cases}\dfrac{\sin(x+2)}{x+2} & , \quad x\in(-2,-1)\\ 0 & , \quad x\in(-1, 0]\\ 2x & , \quad x\in(0, 1)\\ 1 & , \quad \text{otherwise}\end{cases}$$

It clearly shows that $f(x)$ is discontinuous

At $x = -1$, 1 also non differentiable

and at $x = 0$, L.H.D = $\lim\limits_{h\to 0}\dfrac{f(0-h)-f(0)}{-h}=0$

R.H.D = $\lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=2$

$\therefore$ $f(x)$ is not differentiable at $x = 0$

$\therefore$ $m = 2$, $n = 3$

105. Answer (1)

$$\underset{x\to 1}{\text{limit}}\frac{f(x)}{x-1}$$

$f(x) + f'(x) + f''(x) = x^5 + 64$

Let $f(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$

$f'(x) = 5x^4 + 4ax^3 + 3bx^2 + 2cx + d$

$f''(x) = 20x^3 + 12ax^2 + 6bx + 2c$

$x^5 + (a + 5)x^4 + (b + 4a + 20)\, x^3 + (c + 3b + 12a)\, x^2 + (d + 2c + 6b)\, x + e + d + 2c = x^5 + 64$

$\Rightarrow$ $a + 5 = 0$

$b + 4a + 20 = 0$

$c + 3b + 12a = 0$

$d + 2c + 6b = 0$

$e + d + 2c = 64$

$\therefore$ $a = -5$, $b = 0$, $c = 60$, $d = -120$, $e = 64$

$\therefore$ $f(x) = x^5 - 5x^4 + 60x^2 - 120x + 64$

Now,

$$\underset{x\to 1}{\text{limit}}\frac{x^5-5x^4+60x^2-120x+64}{x-1}\text{ is }\left(\frac{0}{0}\text{ form}\right)$$

By L' Hospital rule

$$\underset{x\to 1}{\text{limit}}\frac{5x^4-20x^3+120x-120}{1}$$

$= -15$

106. Answer (62)

$$f(g(x)) = \begin{cases}[2(2x-3)^2]+1, & x<0\\ [2(2x+3)^2]+1, & x\ge 0\end{cases}$$

The possible points where $fog(x)$ may be discontinuous are

$2(2x-3)^2 \in I$ & $x \in (-1, 0)$

$2(2x+3)^2 \in I$ & $x \in [0, 1)$

$x \in (-1, 0)$

$2x - 3 \in (-5, -3)$

$2(2x-3)^2 \in (18, 50)$

So, no. of points = 31

$x \in [0, 1)$

$2x + 3 \in [3, 5)$

$2(2x+3)^2 \in [18, 50)$

It is discontinuous at all points except $x = 0$ of no. points = 31

107. Answer (4)

$\because$ gof is differentiable at $x = 0$

So R.H.D = L.H.D

$$\frac{d}{dx}\left(4e^x+k_2\right)=\frac{d}{dx}\left(\left(-|x+3|\right)^2-k_1|x+3|\right)$$

$\Rightarrow$ $4 = 6 - k_1 \Rightarrow k_1 = 2$

Also $g(f(0^+)) = g(f(0^-))$

$\Rightarrow$ $4 + k_2 = 9 - 3k_1 \Rightarrow k_2 = -1$

Now $g(f(-4)) + g(f(4))$

$= g(-1) + g(e^4) = (1 - k_1) + (4e^4 + k_2)$

$= 4e^4 - 2$

$= 2(2e^4 - 1)$

108. Answer (2)

$f(x) = \min\{1, 1 + x\sin x\}$, $0 \le x \le 2\pi$

$$f(x)=\begin{cases}1, & 0\le x<\pi\\ 1+x\sin x, & \pi\le x\le 2\pi\end{cases}$$

Now at $x = \pi$, $\lim\limits_{x\to\pi^-}f(x)=1=\lim\limits_{x\to\pi^+}f(x)$

$\therefore$ $f(x)$ is continuous in $[0, 2\pi]$

Now, at $x = \pi$

$$\text{L.H.D} = \lim_{h\to 0}\frac{f(\pi-h)-f(\pi)}{-h}=0$$

$$\text{R.H.D.} = \lim_{h\to 0}\frac{f(\pi+h)-f(\pi)}{h}=1-\frac{(\pi+h)\sin h-1}{h}$$

$$= -\pi$$

$\therefore$ $f(x)$ is not differentiable at $x = \pi$

$\therefore$ $(m, n) = (1, 0)$

109. Answer (4)

$$\cos^{-1}\left(\frac{y}{2}\right)=\log_e\left(\frac{x}{5}\right)^5 \quad |y|<2$$

Differentiating on both sides

$$-\frac{1}{\sqrt{1-\left(\frac{y}{2}\right)^2}}\times\frac{y'}{2}=\frac{5}{\frac{x}{5}}\times\frac{1}{5}$$

$$\frac{-xy'}{2}=5\sqrt{1-\left(\frac{y}{2}\right)^2}$$

Square on both sides

$$\frac{x^2y'^2}{4}=25\left(\frac{4-y^2}{4}\right)$$

Diff. on both sides

$$2xy'^2+2y'y''x^2=-25\times 2yy'$$

$xy' + y''x^2 + 25y = 0$

110. Answer (16)

$$\because \quad y(x)=\left(x^x\right)^x$$

$$\therefore \quad y=x^{x^2}$$

$$\therefore \quad \frac{dy}{dx}=x^2\cdot x^{x^2-1}+x^{x^2}\ln x\cdot 2x$$

$$\therefore \quad \frac{dx}{dy}=\frac{1}{x^{x^2+1}(1+2\ln x)} \quad ..(i)$$

Now, $$\frac{d^2x}{dx}=\frac{d}{dx}\left(\left(x^{x^2+1}(1+2\ln x)\right)^{-1}\right)\cdot\frac{dx}{dy}$$

$$=\frac{-x\left(x^{x^2+1}(1+2\ln x)\right)^{-2}\cdot x^{x^2}(1+2\ln x)\left(x^2+2x^2\ln x+3\right)}{x^{x^2}(1+2\ln x)}$$

$$=\frac{-x^{x^2}(1+2\ln x)\left(x^2+3+2x^2\ln x\right)}{\left(x^{x^2}(1+2\ln x)\right)^3}$$

$$\frac{d^2x}{dy^2(\text{at } x=1)}=-4$$

$$\therefore \quad \frac{d^2x}{dy^2(\text{at } x=1)}+20=16$$

111. Answer (3)

$$f(x)=\begin{cases} 0 & x<0 \\ ae^x-1 & 0\le x<1 \\ b & x=1 \\ b-1 & 1<x<2 \\ -c & x\ge 2 \end{cases}$$

To be continuous at $x = 0$

$a - 1 = 0$

to be continuous at $x = 1$

$ae - 1 = b = b - 1 \Rightarrow$ not possible

to be continuous at $x = 2$

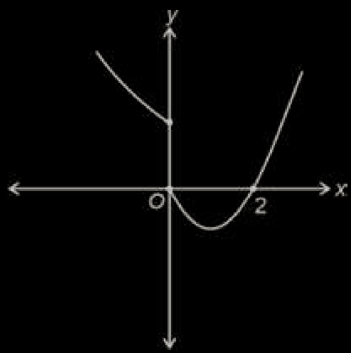
$b - 1 = -c \Rightarrow b + c = 1$

If $a = 1$ and $b + c = 1$ then $f(x)$ is discontinuous at exactly one point

112. Answer (2)

$$f(x)=\begin{cases}[x] & , \; x<0 \\ |1-x| & , \; x\ge 0\end{cases} \text{ and } g(x)=\begin{cases}e^x-x & , \; x<0 \\ (x-1)^2-1 & , \; x\ge 0\end{cases}$$

$$fog(x)=\begin{cases}[g(x)] & , \; g(x)<0 \\ |1-g(x)| & , \; g(x)\ge 0\end{cases}$$



(graph of $y = g(x)$)

$$= \begin{cases} \left|1+x-e^x\right| & , \quad x<0 \\ 1 & , \quad x=0 \\ \left[(x-1)^2-1\right] & , \quad 0<x<2 \\ \left|2-(x-1)^2\right| & , \quad x \geq 2 \end{cases}$$

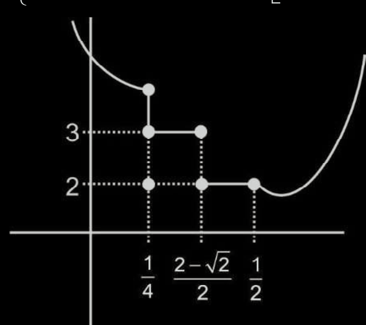
So, $x = 0, 2$ are the two points where fog is discontinuous.

113. Answer (3)

$$f(x) = \begin{cases} \left|4x^2-8x+5\right|, & \text{if } 8x^2-6x+1 \geq 0 \\ \left[4x^2-8x+5\right], & \text{if } 8x^2-6x+1<0 \end{cases}$$

$$= \begin{cases} 4x^2-8x+5, & \text{if } x \in \left[-\infty, \frac{1}{4}\right] \cup \left[\frac{1}{2}, \infty\right) \\ \left[4x^2-8x+5\right] & \text{if } x \in \left(\frac{1}{4}, \frac{1}{2}\right) \end{cases}$$

$$f(x) = \begin{cases} 4x^2-8x+5 & \text{if } x \in \left(-\infty, \frac{1}{4}\right] \cup \left[\frac{1}{2}, \infty\right) \\ 3 & x \in \left(\frac{1}{4}, \frac{2-\sqrt{2}}{2}\right) \\ 2 & x \in \left[\frac{2-\sqrt{2}}{2}, \frac{1}{2}\right) \end{cases}$$



$\therefore$ Non-diff. at $x = \frac{1}{4}, \frac{2-\sqrt{2}}{2}, \frac{1}{2}$

114. Answer (2)

$f(3x) - f(x) = x$ ...(1)

$x \to \frac{x}{3}$

$f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3}$ ...(2)

Again $x \to \frac{x}{3}$

$f\left(\frac{x}{3}\right) - f\left(\frac{x}{9}\right) = \frac{x}{3^2}$ ...(3)

Similarly

$f\left(\frac{x}{3^{n-2}}\right) - f\left(\frac{x}{3^{n-1}}\right) = \frac{x}{3^{n-1}}$..... (n)

Adding all these and applying $n \to \infty$

$$\lim_{n\to\infty}\left(f(3x) - f\left(\frac{x}{3^{n-1}}\right)\right) = x\left(1+\frac{1}{3}+\frac{1}{3^2}+.....\right)$$

$f(3x) - f(0) = \frac{3x}{2}$

Putting $x = \frac{8}{3}$

$f(8) - f(0) = 4$

$\Rightarrow f(0) = 3$

Putting $x = \frac{14}{3}$

$f(14) - 3 = 7 \Rightarrow f(14) = 10$

115. Answer (1)

$$f(x) = \begin{cases} \dfrac{\log_e\left(1-x+x^2\right)+\log_e\left(1+x+x^2\right)}{\sec x - \cos x}, & x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) - \{0\} \\ k & , x = 0 \end{cases}$$

for continuity at $x = 0$

$\lim_{x\to 0} f(x) = k$

$$\therefore \quad k = \lim_{x\to 0} \frac{\log_e\left(x^4+x^2+1\right)}{\sec x - \cos x}\left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x\to 0} \frac{\cos x \log_e\left(x^4+x^2+1\right)}{\sin^2 x}$$

$$= \lim_{x\to 0} \frac{\log_e\left(x^4+x^2+1\right)}{x^2}$$

$$= \lim_{x\to 0} \frac{\ln\left(1+x^2+x^4\right)}{x^2+x^4}\cdot\frac{x^2+x^4}{x^2}$$

$= 1$

116. Answer (4)

$$f(x)=\begin{cases} x+a & , x\le 0 \\ |x-4| & , x>0 \end{cases} \text{ and } g(x)=\begin{cases} x+1 & , x<0 \\ (x-4)^2+b, & x\ge 0 \end{cases}$$

$\because$ $f(x)$ and $g(x)$ are continuous on R

$\therefore$ $a = 4$ and $b = 1 - 16 = -15$

then $(gof)(2) + (fog)(-2)$

$= g(2) + f(-1)$

$= -11 + 3 = -8$

117. Answer (3)

$\because$ $f(x)$ is continuous at $x = 4$

$\Rightarrow$ $f(4^-) = f(4^+)$

$$\Rightarrow \quad 16+4b=\int_0^4 (5-|t-3|)\,dt$$

$$=\int_0^3 (2+t)\,dt+\int_3^4 (8-t)\,dt$$

$$=\left.2t+\frac{t^2}{2}\right)_0^3+\left.8t-\frac{t^2}{3}\right]_3^4$$

$$=6+\frac{9}{2}-0+(32-8)-\left(24-\frac{9}{2}\right)$$

$16 + 4b = 15$

$$\Rightarrow \quad b=\frac{-1}{4}$$

$$\Rightarrow \quad f(x)=\begin{cases} \int_0^x 5-|t-3|\,dt & x>4 \\ x^2-\dfrac{x}{4} & x\le 4 \end{cases}$$

$$\Rightarrow \quad f'(x)=\begin{cases} 5-|x-3| & x>4 \\ 2x-\dfrac{1}{4} & x\le 4 \end{cases}$$

$$\Rightarrow \quad f'(x)=\begin{cases} 8-x & x>4 \\ 2x-\dfrac{1}{4} & x\le 4 \end{cases}$$

$$f'(x)<0 \Rightarrow x\in\left(-\infty,\frac{1}{8}\right)\cup(8,\infty)$$

$$f'(3)+f'(5)=6-\frac{1}{4}=\frac{35}{4}$$

$$f'(x)=0 \quad\Rightarrow\quad x=\frac{1}{8} \text{ have local minima}$$

$\therefore$ (3) is only incorrect option.

118. Answer (2)

$$f(x)=\frac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx}-9}$$

for continuity at $x = 0$, $\lim_{x\to 0} f(x)=f(0)$

$$\text{Now, } \therefore \quad \lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx}-9}$$

$\Rightarrow$ $p = 3$ (To make indeterminant form)

$$\text{So, } \lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{\left(3^7+3x\right)^{\frac{1}{7}}-3}{(729+qx)^{\frac{1}{3}}-9}$$

$$=\lim_{x\to 0}\frac{3\left[\left(1+\dfrac{x}{3^6}\right)^{\frac{1}{7}}-1\right]}{9\left[\left(1+\dfrac{q}{729}x\right)^{\frac{1}{3}}-1\right]}=\frac{1}{3}\cdot\frac{\frac{1}{7}\cdot\frac{1}{3^6}}{\frac{1}{3}\cdot\frac{q}{729}}$$

$$\therefore \quad f(0)=\frac{1}{7q}$$

$\therefore$ Option (2) is correct

119. Answer (16)

$\because$ $C : (x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$ for point (α, α).

$\alpha^2 + \alpha^2 - 3 + (\alpha^2 - \alpha^2 - 1)^5 = 0$

$\therefore$ $\alpha = \sqrt{2}$ ·

On differentiating $(x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$ we get

$x + yy' + 5(x^2 - y^2 - 1)^4 (x - yy') = 0$ ...(i)

When $x = y = \sqrt{2}$ then $y' = \frac{3}{2}$.

Again on differentiating eq. (i) we get :

$1 + (y')^2 + yy'' + 20(x^2 - y^2 - 1)(2x - 2yy')$

$(x - y'y) + 5(x^2 - y^2 - 1)^4 (1 - y'^2 - yy'') = 0$

For $x = y = \sqrt{2}$ and $y' = \frac{3}{2}$ we get $y'' = -\frac{23}{4\sqrt{2}}$

$\therefore$ $3y' - y^3y'' = 3 \cdot \frac{3}{2} - \left(\sqrt{2}\right)^3 \cdot \left(-\frac{23}{4\sqrt{2}}\right)$

$= 16$

120. Answer (2)

$$f(x) = \lim_{n \to \infty} \frac{\cos(2\pi x) - x^{2n} \sin(x-1)}{1 + x^{2n+1} - x^{2n}}$$

For $|x| < 1$, $f(x) = \cos 2\pi x$, continuous function

$$|x| > 1,\ f(x) = \lim_{n \to \infty} \frac{\frac{1}{x^{2n}} \cos 2\pi x - \sin(x-1)}{\frac{1}{x^{2n}} + x - 1}$$

$= \frac{-\sin(x-1)}{x-1}$, continuous

For $|x| = 1$, $f(x) = \begin{cases} 1 & \text{if } x = 1 \\ -(1 + \sin 2) & \text{if } x = -1 \end{cases}$

Now,

$\lim_{x \to 1^+} f(x) = -1$, $\lim_{x \to 1^-} f(x) = 1$, so

discontinuous at $x = 1$

$\lim_{x \to -1^+} f(x) = 1$, $\lim_{x \to -1^-} f(x) = -\frac{\sin 2}{2}$, so

discontinuous at $x = -1$

$\therefore$ $f(x)$ is continuous for all $x \in R - \{-1, 1\}$

121. Answer (4)

$$x = 2\sqrt{2} \cos t \sqrt{\sin 2t},\ y = 2\sqrt{2} \sin t \sqrt{\sin 2t}$$

$$\therefore \quad \frac{dx}{dt} = \frac{2\sqrt{2}\cos 3t}{\sqrt{\sin 2t}},\ \frac{dy}{dt} = \frac{2\sqrt{2}\sin 3t}{\sqrt{\sin 2t}}$$

$$\therefore \quad \frac{dy}{dx} = \tan 3t,\ \left(\text{at } t = \frac{\pi}{4},\ \frac{dy}{dx} = -1\right)$$

$$\text{and } \frac{d^2y}{dx^2} = 3\sec^2 3t \cdot \frac{dt}{dx} = \frac{3\sec^2 3t \cdot \sqrt{\sin 2t}}{2\sqrt{2}\cos 3t}$$

$$\left(\text{At } t = \frac{\pi}{4},\ \frac{d^2y}{dx^2} = -3\right)$$

$$\therefore \quad \frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} = \frac{2}{-3} = \frac{-2}{3}$$

122. Answer (4)

$$\operatorname*{lt}_{x \to 0} \frac{\ln(1+5x) - \ln(1+\alpha x)}{x}$$

$= 5 - \alpha = 10$

$\Rightarrow$ $\alpha = -5$

123. Answer (79)

$$f(x) = 4|2x+3| + 9\left[x + \frac{1}{2}\right] - 12[x + 20]$$

$$= 4|2x+3| + 9\left[x + \frac{1}{2}\right] - 12[x] - 240$$

$f(x)$ is non differentiable at $x = -\frac{3}{2}$

and $f(x)$ is discontinuous at $\{-19, -18, \ldots\ldots, 18, 19\}$

as well as $\left\{-\frac{39}{2}, -\frac{37}{2}, \ldots, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \ldots, \frac{39}{2}\right\}$,

at same point they are also non differentiable

$\therefore$ Total number of points of non differentiability

$= 39 + 40$

$= 79$

124. Answer (2)

$f : R \to R.$

$f(x) = |x - 1| \cos |x - 2| \sin |x - 1| + (x - 3)\ |x^2 -$

$5x + 4|$

$= |x - 1| \cos |x - 2| \sin |x - 1| + (x - 3)\ |x - 1|\ |x - 4|$

$= |x - 1|\ [\cos |x - 2| \sin |x - 1| + (x - 3)\ |x - 4|]$

Sharp edges at $x = 1$ and $x = 4$

$\therefore$ Non-differentiable at $x = 1$ and $x = 4$

125. Answer (4)

Let $f(x) = \log_{\cos x} \text{cosec}x$

$= \dfrac{\log \text{cosec}x}{\log \cos x}$

$$f'(x) = \frac{\log\cos x \cdot \sin x \cdot \left(-\text{cosec}x \cot x - \log \text{cosec}x \cdot \frac{1}{\cos x} \cdot -\sin x\right)}{(\log\cos x)^2}$$

at $x = \dfrac{\pi}{4}$

$$f'\left(\frac{\pi}{4}\right) = \frac{-\log\left(\frac{1}{\sqrt{2}}\right) + \log\sqrt{2}}{\left(\log\frac{1}{\sqrt{2}}\right)^2} = \frac{2}{\log\sqrt{2}}$$

$\therefore \log_e 2f'(x)$ at $x = \dfrac{\pi}{4} = 4$

126. Answer (2)

Let $x^3 = \theta \Rightarrow \dfrac{\theta}{2} \in \left(\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right)$

$\therefore \quad y = \tan^{-1} (\sec\theta - \tan\theta)$

$= \tan^{-1}\left(\dfrac{1 - \sin\theta}{\cos\theta}\right)$

$\therefore \quad y = \dfrac{\pi}{4} - \dfrac{\theta}{2}.$

$y = \dfrac{\pi}{4} - \dfrac{x^3}{2}$

$\therefore \quad y' = \dfrac{-3x^2}{2}$

$y'' = -3x$

$\therefore \quad x^2 y'' - 6y + \dfrac{3\pi}{2} = 0$

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