Chapter 26

Three Dimensional Geometry

1. Let the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals **[AIEEE-2009]**

   (1) (–6, 7) (2) (5, –15)

   (3) (–5, 5) (4) (6, –17)

2. A line *AB* in three-dimensional space makes angles 45° and 120° with the positive *x*-axis and the positive *y*-axis respectively. If *AB* makes an acute angle θ with the positive *z*-axis, then θ equals **[AIEEE-2010]**

   (1) 30° (2) 45°

   (3) 60° (4) 75°

3. **Statement-1 :** The point *A*(3, 1, 6) is the mirror image of the point *B*(1, 3, 4) in the plane $x - y + z = 5$.

   Statement-2 : The plane $x - y + z = 5$ bisects the line segment joining *A*(3, 1, 6) and *B*(1, 3, 4). **[AIEEE-2010]**

   (1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

   (2) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

   (3) Statement-1 is true, Statement-2 is false

   (4) Statement-1 is false, Statement-2 is true

4. There are 10 points in a plane, out of these 6 are collinear. If *N* is the number of triangles formed by joining these points, then **[AIEEE-2011]**

   (1) $140 < N \le 190$ (2) $N > 190$

   (3) $N \le 100$ (4) $100 < N \le 140$

5. The length of the perpendicular drawn from the point (3, –1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is **[AIEEE-2011]**

   (1) $\sqrt{53}$ (2) $\sqrt{66}$

   (3) $\sqrt{29}$ (4) $\sqrt{33}$

6. The distance of the point (1, – 5, 9) from the plane $x - y + z = 5$ measured along a straight line $x = y = z$ is **[AIEEE-2011]**

   (1) $3\sqrt{10}$ (2) $3\sqrt{5}$

   (3) $10\sqrt{3}$ (4) $5\sqrt{3}$

7. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is **[AIEEE-2012]**

   (1) $x - 2y + 2z + 1 = 0$

   (2) $x - 2y + 2z - 1 = 0$

   (3) $x - 2y + 2z + 5 = 0$

   (4) $x - 2y + 2z - 3 = 0$

8. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then *k* is equal to **[AIEEE-2012]**

   (1) $\frac{2}{9}$ (2) $\frac{9}{2}$

   (3) 0 (4) –1

9. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is **[JEE (Main)-2013]**

   (1) $\frac{3}{2}$ (2) $\frac{5}{2}$

   (3) $\frac{7}{2}$ (4) $\frac{9}{2}$

10. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then *k* can have **[JEE (Main)-2013]**

   (1) Any value

   (2) Exactly one value

   (3) Exactly two values

   (4) Exactly three values

11. The image of the line

$\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line **[JEE (Main)-2014]**

(1) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

(2) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

(3) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

(4) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

12. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x - y + z = 16$, is **[JEE (Main)-2015]**

(1) $2\sqrt{14}$ (2) 8

(3) $3\sqrt{21}$ (4) 13

13. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is **[JEE (Main)-2015]**

(1) $2x+6y+12z=13$

(2) $x+3y+6z=-7$

(3) $x+3y+6z=7$

(4) $2x+6y+12z=-13$

14. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $lx+my-z=9$, then l^2+m^2 is equal to **[JEE (Main)-2016]**

(1) 18 (2) 5

(3) 2 (4) 26

15. The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is **[JEE (Main)-2016]**

(1) $10\sqrt{3}$ (2) $\frac{10}{\sqrt{3}}$

(3) $\frac{20}{3}$ (4) $3\sqrt{10}$

16. The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is **[JEE (Main)-2017]**

(1) $\frac{10}{\sqrt{83}}$ (2) $\frac{5}{\sqrt{83}}$

(3) $\frac{10}{\sqrt{74}}$ (4) $\frac{20}{\sqrt{74}}$

17. If the image of the point $P(1, -2, 3)$ in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to the line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to **[JEE (Main)-2017]**

(1) $2\sqrt{42}$ (2) $\sqrt{42}$

(3) $6\sqrt{5}$ (4) $3\sqrt{5}$

18. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane containing the lines L_1 and L_2, is **[JEE (Main)-2018]**

(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$

(3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

19. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x + y + z = 7$ is: **[JEE (Main)-2018]**

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) $\sqrt{\frac{2}{3}}$

20. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point **[JEE (Main)-2019]**

(1) (3, 2, 1) (2) (3, 3, –1)

(3) (–3, 0, –1) (4) (–3, 1, 1)

21. The equation of the line passing through (–4, 3, 1), parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is **[JEE (Main)-2019]**

(1) $\frac{x-4}{2} = \frac{y+3}{2} = \frac{z+1}{4}$

(2) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

(3) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(4) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

22. If the lines $x = ay + b$, $z = cy + d$ and $x = a'z + b'$, $y = c'z + d'$ are perpendicular, then **[JEE (Main)-2019]**

(1) $ab' + bc' + 1 = 0$ (2) $cc' + a + a' = 0$

(3) $aa' + c + c' = 0$ (4) $bb' + cc' + 1 = 0$

23. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is **[JEE (Main)-2019]**

(1) $x - 2y + z = 0$ (2) $x + 2y - 2z = 0$

(3) $5x + 2y - 4z = 0$ (4) $3x + 2y - 3z = 0$

24. The plane passing through the point (4, –1, 2) and parallel to the lines $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ also passes through the point **[JEE (Main)-2019]**

(1) (–1, –1, –1) (2) (–1, –1, 1)

(3) (1, 1, 1) (4) (1, 1, –1)

25. Let A be a point on the line $\vec{r} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$ and $B(3, 2, 6)$ be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane $x - 4y + 3z = 1$ is **[JEE (Main)-2019]**

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$

(3) $\frac{1}{8}$ (4) $-\frac{1}{4}$

26. The plane which bisects the line segment joining the points (–3, –3, 4) and (3, 7, 6) at right angles, passes through which one of the following points? **[JEE (Main)-2019]**

(1) (–2, 3, 5) (2) (4, 1, –2)

(3) (2, 1, 3) (4) (4, –1, 7)

27. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $x + y + z = 2$? **[JEE (Main)-2019]**

(1) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

(2) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

(3) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

(4) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

28. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points? **[JEE (Main)-2019]**

(1) (0, –2, 2) (2) (2, 2, 0)

(3) (–2, 2, 2) (4) (2, 0, –2)

29. The direction ratios of normal to the plane through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$ are **[JEE (Main)-2019]**

(1) $2\sqrt{3}, 1, -1$ (2) $2, \sqrt{2}, -\sqrt{2}$

(3) 2, –1, 1 (4) 2, 1, –1

30. Two lines $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1}$ and $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates **[JEE (Main)-2019]**

(1) (–2, 4, 7) (2) (2, 4, 7)

(3) (2, –4, –7) (4) (2, –4, 7)

31. If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to **[JEE (Main)-2019]**

(1) 5 (2) 12

(3) 17 (4) 7

32. The perpendicular distance from the origin to the plane containing the two lines,

$\frac{x+2}{3} = \frac{y-2}{5} = \frac{z+5}{7}$ and $\frac{x-1}{1} = \frac{y-4}{4} = \frac{z+4}{7}$, is

[JEE (Main)-2019]

(1) $11\sqrt{6}$ (2) $6\sqrt{11}$

(3) 11 (4) $\frac{11}{\sqrt{6}}$

33. A tetrahedron has vertices $P(1, 2, 1)$, $Q(2, 1, 3)$, $R(-1, 1, 2)$ and $O(0, 0, 0)$. The angle between the faces OPQ and PQR is **[JEE (Main)-2019]**

(1) $\cos^{-1}\left(\frac{19}{35}\right)$ (2) $\cos^{-1}\left(\frac{9}{35}\right)$

(3) $\cos^{-1}\left(\frac{17}{31}\right)$ (4) $\cos^{-1}\left(\frac{7}{31}\right)$

34. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point $(-1, -1, 1)$. Then S is equal to

[JEE (Main)-2019]

(1) $\{1, -1\}$ (2) $\{\sqrt{3}\}$

(3) $\{\sqrt{3}, -\sqrt{3}\}$ (4) $\{3, -3\}$

35. If an angle between the line, $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ and the plane, $x - 2y - kz = 3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is **[JEE (Main)-2019]**

(1) $\sqrt{\frac{3}{5}}$ (2) $\sqrt{\frac{5}{3}}$

(3) $-\frac{5}{3}$ (4) $-\frac{3}{5}$

36. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point (1, 1, 0) is : **[JEE (Main)-2019]**

(1) $2x - z = 2$ (2) $x - 3y - 2z = -2$

(3) $x - y - z = 0$ (4) $x + 3y + z = 4$

37. The length of the perpendicular from the point $(2, -1, 4)$ on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is

[JEE (Main)-2019]

(1) Greater than 3 but less than 4

(2) Greater than 2 but less than 3

(3) Greater than 4

(4) Less than 2

38. If a point $R(4, y, z)$ lies on the line segment joining the points $P(2, -3, 4)$ and $Q(8, 0, 10)$, then the distance of R from the origin is **[JEE (Main)-2019]**

(1) $\sqrt{53}$ (2) $2\sqrt{21}$

(3) 6 (4) $2\sqrt{14}$

39. If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is **[JEE (Main)-2019]**

(1) $2\sqrt{5}$ (2) $\frac{9}{2}$

(3) $\frac{7}{2}$ (4) $\frac{\sqrt{5}}{2}$

40. A plane passing through the points $(0, -1, 0)$ and $(0, 0, 1)$ and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$, also passes through the point

[JEE (Main)-2019]

(1) $(\sqrt{2}, 1, 4)$ (2) $(\sqrt{2}, -1, 4)$

(3) $(-\sqrt{2}, -1, -4)$ (4) $(-\sqrt{2}, 1, -4)$

41. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that $BC = 5$ units. Then the area (in sq. units) of this triangle, given that the point $A(1, -1, 2)$, is **[JEE (Main)-2019]**

(1) $5\sqrt{17}$ (2) $\sqrt{34}$

(3) 6 (4) $2\sqrt{34}$

42. Let P be the plane, which contains the line of intersection of the planes, $x + y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$ and it is perpendicular to the xy-plane. Then the distance of the point $(0, 0, 256)$ from P is equal to **[JEE (Main)-2019]**

(1) $63\sqrt{5}$ (2) $\frac{17}{\sqrt{5}}$

(3) $205\sqrt{5}$ (4) $\frac{11}{\sqrt{5}}$

43. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to **[JEE (Main)-2019]**

(1) -1 (2) -2

(3) 1 (4) 2

44. Let $A(3, 0 -1)$, $B(2, 10, 6)$ and $C(1, 2, 1)$ be the vertices of a triangle and M be the mid point of AC. If G divides BM in the ratio, $2 : 1$ then $\cos(\angle GOA)$ (O being the origin) is equal to

[JEE (Main)-2019]

(1) $\frac{1}{6\sqrt{10}}$ (2) $\frac{1}{\sqrt{30}}$

(3) $\frac{1}{2\sqrt{15}}$ (4) $\frac{1}{\sqrt{15}}$

45. If $Q(0, -1, -3)$ is the image of the point P in the plane $3x - y + 4z = 2$ and R is the point $(3, -1, -2)$, then the area (in sq. units) of ΔPQR is :

[JEE (Main)-2019]

(1) $\frac{\sqrt{65}}{2}$ (2) $\frac{\sqrt{91}}{4}$

(3) $2\sqrt{13}$ (4) $\frac{\sqrt{91}}{2}$

46. A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are **[JEE (Main)-2019]**

(1) $(1, 0, 2)$ (2) $(2, 0, 1)$

(3) $(4, 0, -1)$ (4) $(-1, 0, 4)$

47. If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to

[JEE (Main)-2019]

(1) 13 (2) 15

(3) 5 (4) 9

48. If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the plane $2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to **[JEE (Main)-2019]**

(1) $2\sqrt{14}$ (2) 14

(3) $2\sqrt{7}$ (4) $\sqrt{14}$

49. The length of the perpendicular drawn from the point $(2, 1, 4)$ to the plane containing the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ is **[JEE (Main)-2019]**

(1) $\frac{1}{3}$ (2) 3

(3) $\frac{1}{\sqrt{3}}$ (4) $\sqrt{3}$

50. Let P be a plane through the points $(2, 1, 0)$, $(4, 1, 1)$ and $(5, 0, 1)$ and R be any point $(2, 1, 6)$. Then the image of R in the plane P is

[JEE (Main)-2020]

(1) $(6, 5, 2)$ (2) $(4, 3, 2)$

(3) $(3, 4, -2)$ (4) $(6, 5, -2)$

51. The shortest distance between the lines

$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{2}$ is

[JEE (Main)-2020]

(1) $2\sqrt{30}$ (2) $\frac{7}{2}\sqrt{30}$

(3) $3\sqrt{30}$ (4) 3

52. The mirror image of the point $(1, 2, 3)$ in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane? **[JEE (Main)-2020]**

(1) $(-1, -1, -1)$ (2) $(1, 1, 1)$

(3) $(-1, -1, 1)$ (4) $(1, -1, 1)$

53. The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, $2x = 3y$, $z = 1$ also passes through the point **[JEE (Main)-2020]**

(1) (0, 6, –2) (2) (–2, 0, 1)

(3) (0, –6, 2) (4) (2, 0 –1)

54. A plane passing through the point (3, 1, 1) contains two lines whose direction ratios are 1, –2, 2 and 2, 3, –1 respectively. If this plane also passes through the point (α, –3, 5), then α is equal to **[JEE (Main)-2020]**

(1) 5 (2) 10

(3) –10 (4) –5

55. The foot of the perpendicular drawn from the point (4, 2, 3) to the line joining the points (1, –2, 3) and (1, 1, 0) lies on the plane

[JEE (Main)-2020]

(1) $x - 2y + z = 1$ (2) $x + 2y - z = 1$

(3) $x - y - 2z = 1$ (4) $2x + y - z = 1$

56. The lines

$\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$ and

$\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$ **[JEE (Main)-2020]**

(1) do not intersect for any values of l and m

(2) intersect for all values of l and m

(3) intersect when $l = 2$ and $m = \frac{1}{2}$

(4) intersect when $l = 1$ and $m = 2$

57. The plane which bisects the line joining, the points (4, –2, 3) and (2, 4, –1) at right angles also passes through the point **[JEE (Main)-2020]**

(1) (4, 0, 1) (2) (0, –1, 1)

(3) (0, 1, –1) (4) (4, 0, –1)

58. The distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is **[JEE (Main)-2020]**

(1) 7 (2) $\frac{7}{5}$

(3) 1 (4) $\frac{1}{7}$

59. If (a, b, c) is the image of the point (1, 2, –3) in the line, $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then $a + b + c$ is equal to **[JEE (Main)-2020]**

(1) 2 (2) 1

(3) 3 (4) –1

60. If for some $\alpha \in R$, the lines

$L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and

$L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar,

then the line L_2 passes through the point

[JEE (Main)-2020]

(1) (10, 2, 2) (2) (2, –10, –2)

(3) (10, –2, –2) (4) (–2, 10, 2)

61. The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and $x + y + z + 1 = 0$, $2x - y + z + 3 = 0$ is **[JEE (Main)-2020]**

(1) $\frac{1}{\sqrt{2}}$ (2) 1

(3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{2}$

62. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1, 1, 2). Then the equation of the line through this centroid and perpendicular to the plane P is

[JEE (Main)-2020]

(1) $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

(2) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$

(3) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$

(4) $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$

63. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through (α, 7, 1) is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to ____________.

[JEE (Main)-2020]

64. The projection of the line segment joining the points (1, –1, 3) and (2, –4, 11) on the line joining the points (–1, 2, 3) and (3, –2, 10) is ________. **[JEE (Main)-2020]**

65. If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines

$$\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3} \text{ and}$$

$$\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}(\lambda \in R)$$

is equal to $\frac{k}{\sqrt{633}}$, then k is equal to

[JEE (Main)-2020]

66. Let a plane P contain two lines

$\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}),\ \lambda \in R$ and

$\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}),\ \mu \in R$

If $Q(\alpha, \beta, \gamma)$ is the foot of the perpendicular drawn from the point $M(1, 0, 1)$ to P, then $3(\alpha + \beta + \gamma)$ equals ________. **[JEE (Main)-2020]**

67. If the equation of a plane P, passing through the intersection of the planes $x + 4y - z + 7 = 0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for some $a, b \in R$, then the distance of the point (3, 2, –1) from the plane P is ________. **[JEE (Main)-2020]**

68. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is:

[JEE (Main)-2021]

(1) $3x - 10y - 2z + 11 = 0$

(2) $6x - 5y - 2z - 2 = 0$

(3) $6x - 5y + 2z + 10 = 0$

(4) $11x + y + 17z + 38 = 0$

69. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane $x + y + z = 17$ is: **[JEE (Main)-2021]**

(1) 38 (2) $19\sqrt{2}$

(3) $2\sqrt{19}$ (4) $\sqrt{38}$

70. The vector equation of the plane passing through the intersection of the planes

$\vec{r}\cdot(\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r}\cdot(\hat{i} - 2\hat{j}) = -2$, and the point (1, 0, 2) is : **[JEE (Main)-2021]**

(1) $\vec{r}\cdot(\hat{i} + 7\hat{j} + 3\hat{k}) = \frac{7}{3}$ (2) $\vec{r}\cdot(3\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

(3) $\vec{r}\cdot(\hat{i} - 7\hat{j} + 3\hat{k}) = \frac{7}{3}$ (4) $\vec{r}\cdot(\hat{i} + 7\hat{j} + 3\hat{k}) = 7$

71. Let a, b $\in$ R. If the mirror image of the point P(a, 6, 9) with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is (20, b, –a –9), then |a + b| is equal to :

[JEE (Main)-2021]

(1) 86 (2) 88

(3) 90 (4) 84

72. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is ________. **[JEE (Main)-2021]**

73. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is : **[JEE (Main)-2021]**

(1) $\frac{3}{4}$ (2) $\frac{5}{8}$

(3) $\frac{1}{2}$ (4) $\frac{3}{8}$

74. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is : **[JEE (Main)-2021]**

(1) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$ (2) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

(3) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$ (4) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

75. A line ‘l’ passing through origin is perpendicular to the lines

$l_1 : \vec{r} = (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}$

$l_2 : \vec{r} = (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}$

If the co-ordinates of the point in the first octant on ‘l_2” at a distance of $\sqrt{17}$ from the point of intersection of ‘l’ and ‘l_1’ are (a, b, c) then 18(a + b + c) is equal to ________.

[JEE (Main)-2021]

76. If (1, 5, 35), (7, 5, 5), (1, λ, 7) and (2λ, 1, 2) are coplanar, then the sum of all possible values of λ is **[JEE (Main)-2021]**

(1) $\frac{44}{5}$ (2) $-\frac{44}{5}$

(3) $\frac{39}{5}$ (4) $-\frac{39}{5}$

77. Let (λ, 2, 1) be a point on the plane which passes through the point (4, –2, 2). If the plane is perpendicular to the line joining the points (–2, –21, 29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to ___________.

[JEE (Main)-2021]

78. If the mirror image of the point (1, 3, 5) with respect to the plane 4x – 5y + 2z = 8 is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals :

(1) 43 (2) 47

(3) 41 (4) 39

[JEE (Main)-2021]

79. Let L be a line obtained from the intersection of two planes x + 2y + z = 6 and y + 2z = 4. If point P(α, β, γ) is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha + \beta + \gamma)$ equals : **[JEE (Main)-2021]**

(1) 68 (2) 102

(3) 142 (4) 136

80. If for a > 0, the feet of perpendiculars from the points A(a, –2a, 3) and B(0, 4, 5) on the plane lx + my + nz = 0 are points C(0, –a, –1) and D respectively, then the length of line segment CD is equal to : **[JEE (Main)-2021]**

(1) $\sqrt{41}$ (2) $\sqrt{66}$

(3) $\sqrt{55}$ (4) $\sqrt{31}$

81. Let P be a plane lx + my + nz = 0 containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(–3, –6, 1) and B(2, 4, –3) in ratio k : 1 then the value of k is equal to: **[JEE (Main)-2021]**

(1) 2 (2) 4

(3) 1.5 (4) 3

82. Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, –1, 2) and (–2, 1, –2), respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

[JEE (Main)-2021]

(1) $\sqrt{482}$ (2) $\sqrt{227}$

(3) $\sqrt{5}$ (4) $\sqrt{171}$

83. If (x, y, z) be an arbitrary point lying on a plane P which passes through the point (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of the expression

$$3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(x-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

is equal to : **[JEE (Main)-2021]**

(1) 39 (2) 3

(3) –45 (4) 0

84. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}$, $l \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to :

[JEE (Main)-2021]

(1) $\frac{1}{2}$ (2) $\sqrt{\frac{2}{3}}$

(3) $\sqrt{\frac{1}{\sqrt{6}}}$ (4) $\sqrt{\frac{1}{\sqrt{3}}}$

85. If the distance of the point (1, –2, 3) from the plane x + 2y – 3z + 10 = 0 measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of |m| is equal to _________. **[JEE (Main)-2021]**

86. The equation of the plane which contains the y-axis and passes through the point (1, 2, 3) is:

[JEE (Main)-2021]

(1) 3x + z = 6 (2) x + 3x = 0

(3) 3x – z = 0 (4) x + 3z = 10

87. If the equation of the plane passing through the line of intersection of the planes 2x – 7y + 4z – 3 = 0, 3x – 5y + 4z + 11 = 0 and the point (–2, 1, 3) is ax + by + cz – 7 = 0, then the value of 2a + b + c – 7 is _____. **[JEE (Main)-2021]**

88. If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$ and containing the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha + \beta + \gamma$ is equal to : **[JEE (Main)-2021]**

(1) 20 (2) 18

(3) 19 (4) 21

89. Let P be an arbitrary point having sum of the squares of the distances from the planes $x + y + z = 0$, $lx - nz = 0$ and $x - 2y + z = 0$, equal to 9. If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of $l - n$ is equal to_____.

[JEE (Main)-2021]

90. Let the plane $ax + by + cz + d = 0$ bisect the line joining the points (4, –3, 1) and (2, 3, –5) at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is ______.

[JEE (Main)-2021]

91. The equation of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which are at unit distance from the point (1, 2, 3) is $ax + by + cz + d = 0$. If $(b - d) = K(c - a)$, then the positive value of K is _______.

[JEE (Main)-2021]

92. Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})-b=0$ be (–3, 5, 2). Then, the value of $|a + b|$ is equal to ________. **[JEE (Main)-2021]**

93. Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-3}{-3}=\frac{z+5}{7}$. If the point (1, –1, α) lies on the plane P, then the value of $|5\alpha|$ is equal to ______. **[JEE (Main)-2021]**

94. If the shortest distance between the lines $\vec{r_1}=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right)$, $\lambda\in\mathbf{R}$, $\alpha>0$ and $\vec{r_2}=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$, $\mu\in\mathbf{R}$ is 9, then α is equal to _______. **[JEE (Main)-2021]**

95. The lines $x = ay - 1 = z - 2$ and $x = 3y - 2 = bz - 2$, $(ab \neq 0)$ are coplanar, if

[JEE (Main)-2021]

(1) a = 2, b = 2
(2) a = 2, b = 3
(3) a = 1, $b \in \mathbf{R} - \{0\}$
(4) b = 1, $a \in \mathbf{R} - \{0\}$

96. Consider the line L given by the equation $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$. Let Q be the mirror image of the point (2, 3, –1) with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P? **[JEE (Main)-2021]**

(1) (–1, 1, 2)
(2) (1, 2, 2)
(3) (1, 1, 1)
(4) (1, 1, 2)

97. Let L be the line of intersection of planes $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=2$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})=2$. If $P(\alpha, \beta, \gamma)$ is the foot of perpendicular on L from the point (1, 2, 0), then the value of $35(\alpha + \beta + \gamma)$ is equal to **[JEE (Main)-2021]**

(1) 143
(2) 101
(3) 134
(4) 119

98. If the shortest distance between the straight lines $3(x - 1) = 6(y - 2) = 2(z - 1)$ and $4(x - 2) = 2(y - \lambda) = (z - 3)$, $\lambda \in R$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to **[JEE (Main)-2021]**

(1) 2
(2) 5
(3) 3
(4) –1

99. If the lines $\frac{x-k}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x+1}{3}=\frac{y+2}{2}=\frac{z+3}{1}$ are co-planar, then the value of k is ______.

[JEE (Main)-2021]

100. Let the plane passing through the point (–1, 0, –2) and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$ be $ax + by + cz + 8 = 0$. Then the value of $a + b + c$ is equal to : **[JEE (Main)-2021]**

(1) 3
(2) 5
(3) 8
(4) 4

101. Let a plane P pass through the point (3, 7, –7) and contain the line, $\frac{x-2}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 is equal to _______, **[JEE (Main)-2021]**

102. For real numbers α and $\beta \neq 0$, if the point of intersection of the straight lines

$$\frac{x-\alpha}{1}=\frac{y-1}{2}=\frac{z-1}{3} \text{ and } \frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3},$$

lies on the plane $x + 2y - z = 8$, then $\alpha - \beta$ is equal to **[JEE (Main)-2021]**

(1) 9
(2) 5
(3) 3
(4) 7

103. The distance of the point P(3, 4, 4) from the point of intersection of the line joining the points Q(3, –4, –5) and R(2, –3, 1) and the plane $2x + y + z = 7$, is equal to _______.

[JEE (Main)-2021]

104. A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$, and is perpendicular to the plane $-2x + y + z + 8 = 0$. Then which of the following points lies on P? **[JEE (Main)-2021]**

(1) (1, 0, 1)
(2) (2, –1, 1)
(3) (0, 1, 1)
(4) (–1, 1, 2)

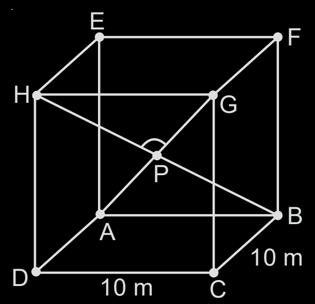
105. Let the line L be the projection of the line

$$\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$$

in the plane $x - 2y - z = 3$. If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to ________. **[JEE (Main)-2021]**

106. A hall has a square floor of dimension 10 m × 10 m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is

[JEE (Main)-2021]



(1) $2\sqrt{10}$ (2) $5\sqrt{2}$

(3) $5\sqrt{3}$ (4) 5

107. Let P be the plane passing through the point (1, 2, 3) and the line of intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+4\hat{k})=16$ and $\vec{r}\cdot(-\hat{i}+\hat{j}+\hat{k})=6$.

Then which of the following points does NOT lie on P? **[JEE (Main)-2021]**

(1) (–8, 8, 6) (2) (4, 2, 2)

(3) (3, 3, 2) (4) (6, – 6, 2)

108. Let Q be the foot of the perpendicular from the point P(7, –2, 13) on the plane containing the lines

$$\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8} \text{ and } \frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}.$$

Then $(PQ)^2$, is equal to ________.

[JEE (Main)-2021]

109. Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes $x - y - z - 1 = 0$ and $2x + y - 3z + 4 = 0$, is :

[JEE (Main)-2021]

(1) $4x - y - 5z + 2 = 0$

(2) $3x - 4z + 3 = 0$

(3) $-x + 2y + 2z - 3 = 0$

(4) $3x - y - 5z + 2 = 0$

110. The distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to a line, whose direction ratios are 2, 3, –6 is

[JEE (Main)-2021]

(1) 2 (2) 5

(3) 3 (4) 1

111. The angle between the straight lines, whose direction cosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$, is

[JEE (Main)-2021]

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$

(3) $\pi-\cos^{-1}\left(\frac{4}{9}\right)$ (4) $\cos^{-1}\left(\frac{8}{9}\right)$

112. The equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to the x-axis is

[JEE (Main)-2021]

(1) $\vec{r}\cdot(\hat{i}+3\hat{k})+6=0$ (2) $\vec{r}\cdot(\hat{j}-3\hat{k})+6=0$

(3) $\vec{r}\cdot(\hat{j}-3\hat{k})-6=0$ (4) $\vec{r}\cdot(\hat{i}-3\hat{k})+6=0$

113. Let S be the mirror image of the point $Q(1, 3, 4)$ with respect to the plane $2x - y + z + 3 = 0$ and let $R(3, 5, \gamma)$ be a point of this plane. Then the square of the length of the line segment SR is ______.

[JEE (Main)-2021]

114. Let the equation of the plane, that passes through the point (1, 4, –3) and contains the line of intersection of the planes $3x - 2y + 4z - 7 = 0$ and $x + 5y - 2z + 9 = 0$, be $\alpha x + \beta y + \gamma z + 3 = 0$, then $\alpha + \beta + \gamma$ is equal to :

(1) –23 (2) –15

(3) 23 (4) 15

[JEE (Main)-2021]

115. The square of the distance of the point of intersection of the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ and the plane $2x - y + z = 6$ from the point (–1, –1, 2) is ______. **[JEE (Main)-2021]**

116. The distance of the point (–1, 2, –2) from the line of intersection of the planes $2x + 3y + 2z = 0$ and $x - 2y + z = 0$ is **[JEE (Main)-2021]**

(1) $\frac{5}{2}$ (2) $\frac{\sqrt{34}}{2}$

(3) $\frac{\sqrt{42}}{2}$ (4) $\frac{1}{\sqrt{2}}$

117. Suppose the line $\frac{x-2}{\alpha}=\frac{y-2}{-5}=\frac{z+2}{2}$ lies on the plane $x + 3y - 2z + \beta = 0$. Then $(\alpha + \beta)$ is equal to _______. **[JEE (Main)-2021]**

118. Let the acute angle bisector of the two planes $x - 2y - 2z + 1$ and $2x - 3y - 6z + 1 = 0$ be the plane P. Then which of the following points lies on P? **[JEE (Main)-2021]**

(1) $\left(-2, 0, -\frac{1}{2}\right)$

(2) $(0, 2, -4)$

(3) $(4, 0, -2)$

(4) $\left(3, 1, -\frac{1}{2}\right)$

119. The distance of line $3y - 2z - 1 = 0 = 3x - z + 4$ from the point $(2, -1, 6)$ is **[JEE (Main)-2021]**

(1) $2\sqrt{6}$

(2) $\sqrt{26}$

(3) $4\sqrt{2}$

(4) $2\sqrt{5}$

120. Let P be a plane passing through the points (1, 0, 1), (1, –2, 1) and (0, 1, –2). Let a vector $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\vec{a}\cdot\left(\hat{i}+\hat{j}+2\hat{k}\right)=2$, then $(\alpha - \beta + \gamma)^2$ equals _______. **[JEE (Main)-2021]**

121. Let a line having direction ratios 1, –4, 2 intersect the lines $\frac{x-7}{3}=\frac{y-1}{-1}=\frac{z+2}{1}$ and $\frac{x}{2}=\frac{y-7}{3}=\frac{z}{1}$ at the points A and B. Then $(AB)^2$ is equal to _________. **[JEE (Main)-2022]**

122. If the shortest distance between the lines $\vec{r}=\left(-\hat{i}+3\hat{k}\right)+\lambda\left(\hat{i}-a\hat{j}\right)$ and $\vec{r}=\left(-\hat{j}+2\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$ is $\sqrt{\frac{2}{3}}$, then the integral value of a is equal to ____________. **[JEE (Main)-2022]**

123. If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible values of λ is :

(1) 16

(2) 6

(3) 12

(4) 15

[JEE (Main)-2022]

124. Let the points on the plane P be equidistant from the points (–4, 2, 1) and (2, –2, 3). Then the acute angle between the plane P and the plane $2x + y + 3z = 1$ is

(1) $\frac{\pi}{6}$

(2) $\frac{\pi}{4}$

(3) $\frac{\pi}{3}$

(4) $\frac{5\pi}{12}$

[JEE (Main)-2022]

125. Let Q be the mirror image of the point $P(1, 0, 1)$ with respect to the plane S: $x + y + z = 5$. If a line L passing through (1, –1, –1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to :

(1) 2

(2) 5

(3) 7

(4) 11

[JEE (Main)-2022]

126. Let the lines

$$L_1 : \vec{r}=\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right), \lambda \in \mathrm{R}$$

$$L_2 : \vec{r}=\left(\hat{i}+3\hat{j}+\hat{k}\right)+\mu\left(\hat{i}+\hat{j}+5\hat{k}\right); \mu \in \mathrm{R},$$

intersect at the point S. If a plane $ax + by - z + d = 0$ passes through S and is parallel to both the lines L_1 and L_2, then the value of $a + b + d$ is equal to _______. **[JEE (Main)-2022]**

127. Let P be the plane passing through the intersection of the planes

$\vec{r}\cdot\left(\hat{i}+3\hat{j}-\hat{k}\right)=5$ and $\vec{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)=3$, and the point (2, 1, –2). Let the position vectors of the points X and Y be $\hat{i}-2\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+2\hat{k}$ respectively. Then the points

(1) X and $X + Y$ are on the same side of P

(2) Y and $Y - X$ are on the opposite sides of P

(3) X and Y are on the opposite sides of P

(4) $X + Y$ and $X - Y$ are on the same side of P

[JEE (Main)-2022]

128. Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively and l_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to __________ .

[JEE (Main)-2022]

129. If the two lines $l_1 : \frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $l_2 : \frac{x-1}{1} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$ are perpendicular, then an angle between the lines l_2 and $l_3 : \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$ is :

(1) $\cos^{-1}\left(\frac{29}{4}\right)$ (2) $\sec^{-1}\left(\frac{29}{4}\right)$

(3) $\cos^{-1}\left(\frac{2}{29}\right)$ (4) $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

[JEE (Main)-2022]

130. Let the plane $2x + 3y + z + 20 = 0$ be rotated through a right angle about its line of intersection with the plane $x - 3y + 5z = 8$. If the mirror image of the point $\left(2, -\frac{1}{2}, 2\right)$ in the rotated plane is $B(a, b, c)$, then :

(1) $\frac{a}{8} = \frac{b}{5} = \frac{c}{-4}$ (2) $\frac{a}{4} = \frac{b}{5} = \frac{c}{-2}$

(3) $\frac{a}{8} = \frac{b}{-5} = \frac{c}{4}$ (4) $\frac{a}{4} = \frac{b}{5} = \frac{c}{2}$

[JEE (Main)-2022]

131. If the plane $2x + y - 5z = 0$ is rotated about its line of intersection with the plane $3x - y + 4z - 7 = 0$ by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point: **[JEE (Main)-2022]**

(1) $(2, -2, 0)$ (2) $(-2, 2, 0)$

(3) $(1, 0, 2)$ (4) $(-1, 0, -2)$

132. If the lines $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{j} - \hat{k})$ and $\vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{j} - 3\hat{k})$ are co-planar, then the distance of the plane containing these two lines from the point $(\alpha, 0, 0)$ is :

(1) $\frac{2}{9}$ (2) $\frac{2}{11}$

(3) $\frac{4}{11}$ (4) 2

[JEE (Main)-2022]

133. Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be $(a - 6, \beta, \gamma)$. If $a + b + c = 5$, then $7\beta - 9\gamma$ is equal to _____.

[JEE (Main)-2022]

134. Let the foot of the perpendicular from the point $(1, 2, 4)$ on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P, Then the distance of P from the plane $3x + 4y + 12z + 23 = 0$ is **[JEE (Main)-2022]**

(1) 5 (2) $\frac{50}{13}$

(3) 4 (4) $\frac{63}{13}$

135. The shortest distance between the lines $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$ and $\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$, is

(1) $\frac{18}{\sqrt{5}}$ (2) $\frac{22}{3\sqrt{5}}$

(3) $\frac{46}{3\sqrt{5}}$ (4) $6\sqrt{3}$

[JEE (Main)-2022]

136. If two distinct points Q, R lie on the line of intersection of the planes $-x + 2y - z = 0$ and $3x - 5y + 2z = 0$ and $PQ = PR = \sqrt{18}$ where the point P is $(1, -2, 3)$, then the area of the triangle PQR is equal to

(1) $\frac{2}{3}\sqrt{38}$ (2) $\frac{4}{3}\sqrt{38}$

(3) $\frac{8}{3}\sqrt{38}$ (4) $\sqrt{\frac{152}{3}}$

[JEE (Main)-2022]

137. The acute angle between the planes P_1 and P_2, when P_1 and P_2 are the planes passing through the intersection of the planes $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the points (2, 1, 3) and (0, 1, 2), respectively, is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{12}$

[JEE (Main)-2022]

138. Let the plane $P : \vec{r} \cdot \vec{a} = d$ contain the line of intersection of two planes $\vec{r} \cdot \left(\hat{i} + 3\hat{j} - \hat{k}\right) = 6$ and $\vec{r} \cdot \left(-6\hat{i} + 5\hat{j} - \hat{k}\right) = 7$. If the plane P passes through the point $\left(2, 3, \frac{1}{2}\right)$, then the value of $\frac{|13\,\vec{a}\,|^2}{d^2}$ is equal to **[JEE (Main)-2022]**

(1) 90 (2) 93

(3) 95 (4) 97

139. Let the plane $ax + by + cz = d$ pass through (2, 3, –5) and is perpendicular to the planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$.

If a, b, c, d are integers $d > 0$ and $gcd\,(|a|, |b|, |c|, d) = 1$, then the value of $a + 7b + c + 20d$ is equal to :

(1) 18 (2) 20

(3) 24 (4) 22

[JEE (Main)-2022]

140. Let the image of the point $P(1, 2, 3)$ in the line $L : \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be Q. Let $R\,(\alpha, \beta, \gamma)$ be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22(\alpha + \beta + \gamma)$ is equal to ________. **[JEE (Main)-2022]**

141. If the mirror image of the point (2, 4, 7) in the plane $3x - y + 4z = 2$ is (a, b, c), then $2a + b + 2c$ is equal to : **[JEE (Main)-2022]**

(1) 54 (2) 50

(3) –6 (4) –42

142. Let d be the distance between the foot of perpendiculars of the point $P(1, 2, -1)$ and $Q(2, -1, 3)$ on the plane $-x + y + z = 1$. Then d^2 is equal to _____.

[JEE (Main)-2022]

143. Let $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z+3}{-1}$ lie on the plane $px - qy + z = 5$, for some $p, q \in \mathbb{R}$. The shortest distance of the plane from the origin is : **[JEE (Main)-2022]**

(1) $\sqrt{\frac{3}{109}}$ (2) $\sqrt{\frac{5}{142}}$

(3) $\frac{5}{\sqrt{71}}$ (4) $\frac{1}{\sqrt{142}}$

144. Let Q be the mirror image of the point $P(1, 2, 1)$ with respect to the plane $x + 2y + 2z = 16$. Let T be a plane passing through the point Q and contains the line $\vec{r} = -\hat{k} + \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right), \lambda \in \mathbb{R}$. Then, which of the following points lies on T? **[JEE (Main)-2022]**

(1) (2, 1, 0) (2) (1, 2, 1)

(3) (1, 2, 2) (4) (1, 3, 2)

145. The length of the perpendicular from the point (1, –2, 5) on the line passing through (1, 2, 4) and parallel to the line $x + y - z = 0 = x - 2y + 3z - 5$ is

(1) $\sqrt{\frac{21}{2}}$ (2) $\sqrt{\frac{9}{2}}$

(3) $\sqrt{\frac{73}{2}}$ (4) 1

[JEE (Main)-2022]

146. Let Q and R be two points on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point P(4, 2, 7). Then the square of the area of the triangle PQR is ________.

[JEE (Main)-2022]

147. If the plane P passes through the intersection of two mutually perpendicular planes $2x + ky - 5z = 1$ and $3kx - ky + z = 5$, $k < 3$ and intercepts a unit length on positive x-axis, then the intercept made by the plane P on the y-axis is

(1) $\frac{1}{11}$ (2) $\frac{5}{11}$

(3) 6 (4) 7

[JEE (Main)-2022]

148. If the length of the perpendicular drawn from the point $P(a, 4, 2)$, $a > 0$ on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ is $2\sqrt{6}$ units and $Q(\alpha_1, \alpha_2, \alpha_3)$ is the image of the point P in this line, then $a + \sum_{i=1}^{3} \alpha_i$ is equal to :

(1) 7 (2) 8

(3) 12 (4) 14

[JEE (Main)-2022]

149. If the line of intersection of the planes $ax + by = 3$ and $ax + by + cz = 0$, $a > 0$ makes an angle $30°$ with the plane $y - z + 2 = 0$, then the direction cosines of the line are : **[JEE (Main)-2022]**

(1) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (2) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0$

(3) $\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 0$ (4) $\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0$

150. The foot of the perpendicular from a point on the circle $x^2 + y^2 = 1$, $z = 0$ to the plane $2x + 3y + z = 6$ lies on which one of the following curves?

(1) $(6x + 5y - 12)^2 + 4(3x + 7y - 8)^2 = 1, z = 6 - 2x - 3y$

(2) $(5x + 6y - 12)^2 + 4(3x + 5y - 9)^2 = 1, z = 6 - 2x - 3y$

(3) $(6x + 5y - 14)^2 + 9(3x + 5y - 7)^2 = 1, z = 6 - 2x - 3y$

(4) $(5x + 6y - 14)^2 + 9(3x + 7y - 8)^2 = 1, z = 6 - 2x - 3y$

[JEE (Main)-2022]

151. Let $P(-2, -1, 1)$ and $Q\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ be the vertices of the rhombus $PRQS$. If the direction ratios of the diagonal RS are $\alpha, -1, \beta$, where both α and β are integers of minimum absolute values, then $\alpha^2 + \beta^2$ is equal to ____________. **[JEE (Main)-2022]**

152. Let the lines $\frac{x-1}{\lambda} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x+26}{-2} = \frac{y+18}{3} = \frac{z+28}{\lambda}$ be coplanar and P be the plane containing these two lines. Then which of the following points does **NOT** lie on P?

(1) $(0, -2, -2)$ (2) $(-5, 0, -1)$

(3) $(3, -1, 0)$ (4) $(0, 4, 5)$

[JEE (Main)-2022]

153. A plane P is parallel to two lines whose direction rations are $-2, 1, -3$ and $-1, 2, -2$ and it contains the point $(2, 2, -2)$. Let P intersect the co-ordinate axes at the points A, B, C making the intercepts α, β, γ. If V is the volume of the tetrahedron $OABC$, where O is the origin and $p = \alpha + \beta + \gamma$, then the ordered pair (V, p) is equal to :

(1) $(48, -13)$ (2) $(24, -13)$

(3) $(48, 11)$ (4) $(24, -5)$

[JEE (Main)-2022]

154. Let Q be the foot of perpendicular drawn from the point $P(1, 2, 3)$ to the plane $x + 2y + z = 14$. If R is a point on the plane such that $\angle PRQ = 60°$, then the area of ΔPQR is equal to :

(1) $\frac{\sqrt{3}}{2}$ (2) $\sqrt{3}$

(3) $2\sqrt{3}$ (4) 3

155. If $(2, 3, 9)$, $(5, 2, 1)$, $(1, \lambda, 8)$ and $(\lambda, 2, 3)$ are coplanar, then the product of all possible values of λ is :

(1) $\frac{21}{2}$ (2) $\frac{59}{8}$

(3) $\frac{57}{8}$ (4) $\frac{95}{8}$

[JEE (Main)-2022]

156. Let a line with direction ratios a, $-4a$, -7 be perpendicular to the lines with direction ratios 3, -1, $2b$ and b, a, -2. If the point of intersection of the line $\frac{x+1}{a^2+b^2}=\frac{y-2}{a^2-b^2}=\frac{z}{1}$ and the plane $x-y+z=0$ is (α, β, γ), then $\alpha+\beta+\gamma$ is equal to ______.
[JEE (Main)-2022]

157. The largest value of a, for which the perpendicular distance of the plane containing the lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+a\,\hat{j}-\hat{k})$ and $\vec{r}=(\hat{i}+\hat{j})+\mu(-\hat{i}+\hat{j}-a\hat{k})$ from the point (2, 1, 4) is, $\sqrt{3}$ is ____________.
[JEE (Main)-2022]

158. A vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}, \hat{i}+\hat{k}$. The obtuse angle between $\vec{a}$ and the vector $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$ is
[JEE (Main)-2022]

(1) $\frac{3\pi}{4}$
(2) $\frac{2\pi}{3}$
(3) $\frac{4\pi}{5}$
(4) $\frac{5\pi}{6}$

159. Let the line $\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z-3}{-4}$ intersect the plane containing the lines $\frac{x-4}{1}=\frac{y+1}{-2}=\frac{z}{1}$ and $4ax-y+5z-7a=0=2x-5y-z-3$, $a \in \mathbb{R}$ at the point $P(\alpha, \beta, \gamma)$. Then the value of $\alpha+\beta+\gamma$ equals _____.
[JEE (Main)-2022]

160. A plane E is perpendicular to the two planes $2x-2y+z=0$ and $x-y+2z=4$, and passes through the point $P(1, -1, 1)$. If the distance of the plane E from the point $Q(a, a, 2)$ is $3\sqrt{2}$, then $(PQ)^2$ is equal to
[JEE (Main)-2022]

(1) 9
(2) 12
(3) 21
(4) 33

161. If the foot of the perpendicular from the point $A(-1, 4, 3)$ on the plane $P: 2x+my+nz=4$, is $\left(-2, \frac{7}{2}, \frac{3}{2}\right)$, then the distance of the point A from the plane P, measured parallel to a line with direction ratios 3, -1, -4, is equal to **[JEE (Main)-2022]**

(1) 1
(2) $\sqrt{26}$
(3) $2\sqrt{2}$
(4) $\sqrt{14}$

162. The plane passing through the line $L: lx-y+3(1-l)z=1$, $x+2y-z=2$ and perpendicular to the plane $3x+2y+z=6$ is $3x-8y+7z=4$. If θ is the acute angle between the line L and the y-axis, then $415\cos^2\theta$ is equal to ________.
[JEE (Main)-2022]

163. The shortest distance between the lines $\frac{x+7}{-6}=\frac{y-6}{7}=z$ and $\frac{7-x}{2}=y-2=z-6$ is
[JEE (Main)-2022]

(1) $2\sqrt{29}$
(2) 1
(3) $\sqrt{\frac{37}{29}}$
(4) $\frac{\sqrt{29}}{2}$

❑ ❑ ❑

Chapter 26

Three Dimensional Geometry

1. Answer (1)

The point (2, 1, –2) is on the plane $x + 3y - \alpha z + \beta = 0$

Hence $\quad 2 + 3 + 2\alpha + \beta = 0$

$2\alpha + \beta = -5 \quad \ldots \text{(i)}$

Also $\quad 1(3) + 3(-5) + -\alpha(2) = 0$

$3 - 15 - 2\alpha = 0$

$2\alpha = -12$

$\alpha = -6$

Put $\alpha = -6$ in (i)

$\beta = 12 - 5 = 7$

$\therefore \quad (\alpha, \beta) \equiv (-6, 7)$

2. Answer (3)

$\cos^2 45° + \cos^2 120° + \cos^2\theta = 1$

$$\frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$$

$$\therefore \cos^2\theta = \frac{1}{4}$$

$$\cos\theta = \pm\frac{1}{2} \Rightarrow \theta = 60^\circ \text{ or } 120^\circ$$

3. Answer (1)

The image of the point (3, 1, 6) w.r.t. the plane $x - y + z = 5$ is

$$\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-6}{1} = \frac{-2(3-1+6-5)}{1+1+1}$$

$$\Rightarrow \frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-6}{1} = -2$$

$\Rightarrow \quad x = 3 - 2 = 1$

$y = 1 + 2 = 3$

$z = 6 - 2 = 4$

which shows that statement-1 is true.

We observe that the line segment joining the points $A(3, 1, 6)$ and $B(1, 3, 4)$ has direction ratios 2, –2, 2 which one proportional to 1, –1, 1 the direction ratios of the normal to the plane. Hence statement-2 is true.

4. Answer (3)

Maximum number of triangle

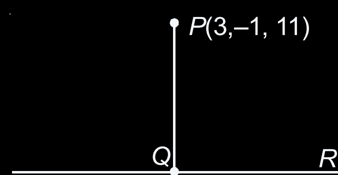
$= {}^{10}C_3 - {}^{6}C_3$

$$= \frac{10\times9\times8}{0} - \frac{6\times5\times4}{3\times2}$$

$= 100$

5. Answer (1)

Let co-ordinates of Q be



$$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = t$$

$x = 2t$

$y = 2 + 3t$

$z = 3 + 4t$

Direction ratios of PQ are $(2t - 3, 3 + 3t, 4t - 8)$

Direction ratios of . Q.R. are (2, 3, 4)

$PQ \perp QR$

$\therefore \quad 2(2t - 3) + 3(3 + 3t) + 4(4t - 8) = 0$

$29t - 29t = 0 \quad \Rightarrow \quad t = 1$

Co-ordinates of Q are

$x = 2, y = 5, z = 7$

The length of the perpendicular PQ

$$= \sqrt{(3-2)^2 + (-1-5)^2 + (11-7)^2}$$

$$= \sqrt{1^2 + 6^2 + 4^2}$$

$$= \sqrt{53}$$

6. Answer (3)

Any point on a line parallel to the given line $x = y = z$ and passing through $(1, -5, 9)$ is

$(\lambda + 1, \lambda - 5, \lambda + 9)$

It lies on given plane

$\therefore (\lambda + 1) - (\lambda - 5) + (\lambda + 9) = 5$

$\lambda + 15 = 5$

$\lambda = -10$

Point is $(-9, -15, -1)$

Required distance = $\sqrt{10^2 + 10^2 + 10^2} = 10\sqrt{3}$

7. Answer (4)

8. Answer (2)

9. Answer (3)

Distance between the planes $= \left|\dfrac{\frac{5}{2} + 8}{3}\right| = \dfrac{7}{2}$ units

10. Answer (3)

Given lines are coplanar if $\begin{vmatrix} 1 & 1 & -k \\ k & 2 & 1 \\ 1 & -1 & -1 \end{vmatrix} = 0$

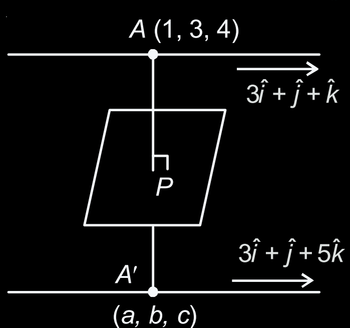
$\Rightarrow 1(-2 + 1) - 1(-k - 1) - k(-k - 2) = 0$

$-1 + k + 1 + k^2 + 2k = 0$

$\Rightarrow k = 0$ or -3

$\therefore$ Exactly two values of k.

11. Answer (3)



$$\frac{a-1}{2} = \frac{b-3}{-1} = \frac{c-4}{1} = \lambda$$

$\Rightarrow a = 2\lambda + 1$

$b = 3 - \lambda$

$c = 4 + \lambda$

$$P \equiv \left(\lambda + 1,\ 3 - \frac{\lambda}{2},\ 4 + \frac{\lambda}{2}\right)$$

$$2(\lambda + 1) - \left(3 - \frac{\lambda}{2}\right) + \left(4 + \frac{\lambda}{2}\right) + 3 = 0$$

$$2\lambda + 2 - 3 + \frac{\lambda}{2} + 4 + \frac{\lambda}{2} + 3 = 0$$

$3\lambda + 6 = 0 \Rightarrow \lambda = -2$

$a = -3,\ b = 5,\ c = 2$

So the equation of the required line is

$$\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$$

12. Answer (4)

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda$$

$P(3\lambda + 2,\ 4\lambda - 1,\ 12\lambda + 2)$

Lies on plane $x - y + z = 16$

Then,

$3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 = 16$

$11\lambda + 5 = 16$

$\lambda = 1 \qquad P(5, 3, 14)$

Distance = $\sqrt{16 + 9 + 144} = \sqrt{169} = 13$

13. Answer (3)

Required plane is

$(2x - 5y + z - 3) + \lambda(x + y + 4z - 5) = 0$

It is parallel to $x + 3y + 6z = 1$

$$\therefore \quad \frac{2+\lambda}{1} = \frac{-5+\lambda}{3} = \frac{1+4\lambda}{6}$$

Solving $\lambda = \dfrac{-11}{2}$

$\therefore$ Required plane is

$$(2x - 5y + z - 3) - \frac{11}{2}(x + y + 4z - 5) = 0$$

$\therefore \quad x + 3y + 6z - 7 = 0$

14. Answer (3)

Line is perpendicular to normal of plane

$\Rightarrow \left(2\hat{i}-\hat{j}+3\hat{k}\right)\bullet\left(l\hat{i}+m\hat{j}-\hat{k}\right)=0$

$2l - m - 3 = 0$ ...(i)

(3, –2, –4) lies on the plane

$3l - 2m + 4 = 9$

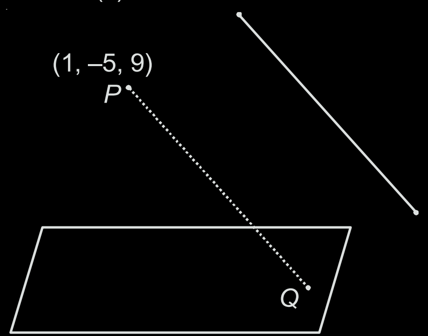
$3l - 2m = 5$ ...(ii)

Solving (i) and (ii)

$l = 1,\ m = -1$

$l^2 + m^2 = 2$

15. Answer (1)



$L:\quad x = y = z$

Equation of line PQ:

An x point Q on the line PQ is $(\lambda + 1, \lambda - 5. \lambda + 9)$

$\because$ Point Q lies on the plane : $x - y + z = 5$

$(\lambda + 1) - (\lambda - 5) + \lambda + 9 = 5$

$\lambda + 10 = 0$

$\lambda = -10$

Point Q is $(-9, -15, -1)$

$PQ = \sqrt{(1+9)^2+(-5+15)^2+(9+1)^2} = 10\sqrt{3}$

16. Answer (1)

Let the plane be

$a(x-1)+b(y+1)+c(z+1)=0$

It is perpendicular to the given lines

$a - 2b + 3c = 0$

$2a - b - c = 0$

Solving, $a : b : c = 5 : 7 : 3$

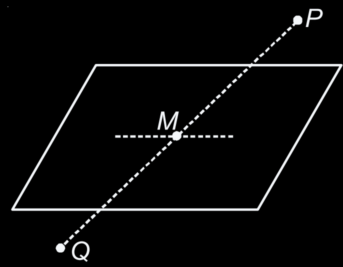
$\therefore$ The plane is $5x + 7y + 3z + 5 = 0$

Distance of (1, 3, –7) from this plane $= \dfrac{10}{\sqrt{83}}$

17. Answer (1)

Equation of PQ, $\dfrac{x-1}{1}=\dfrac{y+2}{4}=\dfrac{z-3}{5}$

Let M be $(\lambda+1, 4\lambda-2, 5\lambda+3)$



As it lies on $2x + 3y - 4z + 22 = 0$

$\lambda = 1$

For Q, $\lambda = 2$

Distance $PQ = 2\sqrt{1^2+4^2+5^2} = 2\sqrt{42}$

18. Answer (2)

L_1 is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}+\hat{j}$

L_2 is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 3\hat{i}-5\hat{j}-7\hat{k}$

Also, L_2 passes through $\left(\dfrac{5}{7}, \dfrac{8}{7}, 0\right)$

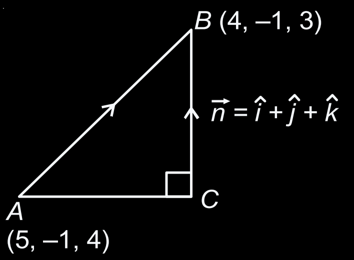
So, required plane is $\begin{vmatrix} x-\dfrac{5}{7} & y-\dfrac{8}{7} & z \\ 1 & 1 & 0 \\ 3 & -5 & -7 \end{vmatrix} = 0$

$\Rightarrow 7x - 7y + 8z + 3 = 0$

Now, perpendicular distance $= \dfrac{3}{\sqrt{162}}$

$= \dfrac{1}{3\sqrt{2}}$

19. Answer (4)



Normal to the plane $x + y + z = 7$ is $\vec{n} = \hat{i} + \hat{j} + \hat{k}$

$\overrightarrow{AB} = -\hat{i} - \hat{k} \Rightarrow \; |\overrightarrow{AB}| \; = AB = \sqrt{2}$

BC = Length of projection of $\overrightarrow{AB}$ on $\vec{n}$ $= |\overrightarrow{AB} \cdot \hat{n}|$

$$= \left|\left(-\hat{i} - \hat{k}\right) \cdot \frac{\left(\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$$

Length of projection of the line segment on the plane is AC

$$AC^2 = AB^2 - BC^2 = 2 - \frac{4}{3} = \frac{2}{3}$$

$$AC^2 = \sqrt{\frac{2}{3}}$$

20. Answer (1)

Equation of plane through intersection of planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$(2x + 3y - z + 4) + \lambda(x + y + z - 1) = 0$

$(2 + \lambda)x + (3 + \lambda)y + (-1 + \lambda)z + (4 - \lambda) = 0$ ...(1)

$\because$ This plane is parallel to y-axis.

$\Rightarrow$ $0 \times (2 + \lambda) + 1 \times (3 + \lambda) + 0 \times (-1 + \lambda) = 0$

$\Rightarrow$ $\lambda = -3$

$\therefore$ Equation of required plane

$-x - 4z + 7 = 0$

$x + 4z - 7 = 0$

$\therefore$ (3, 2, 1) lies on the plane.

21. Answer (3)

Let any point on the intersecting line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1} = \lambda$$

$\Rightarrow (-3\lambda - 1,\; 2\lambda + 3,\; -\lambda + 2)$

which also lie on a line which passes through (–4, 3, 1)

So D.R. of line

$= \langle -3\lambda - 1 + 4, 2\lambda + 3 - 3, -\lambda + 2 - 1 \rangle$

$= \langle -3\lambda + 3, 2\lambda, -\lambda + 1 \rangle$

and this line is parallel to the plane

$x + 2y - z - 5 = 0$

so perpendicular vector to the line is $\hat{i} + 2\hat{j} - \hat{k}$

Now $(-3\lambda + 3)(1) + (2\lambda)(2) + (-\lambda + 1)(-1) = 0$

$\boxed{\lambda = -1}$

Now D.R. of line = $\langle 3, -1, 1 \rangle$

Now equation of line is

$$\frac{(x+4)}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

22. Answer (3)

First line is : $x = ay + b$, $z = cy + d$

$$\Rightarrow \quad \frac{x-b}{a} = y = \frac{z-d}{c}$$

and another line is: $x = a'z + b'$, $y = c'z + d'$

$$\Rightarrow \quad \frac{x-b'}{a'} = \frac{y-d'}{c'} = z$$

$\because$ both lines are perpendicular to each other

$\therefore$ $aa' + c' + c = 0$

23. Answer (1)

Let the direction ratios of the plane containing lines

$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is $\langle a, b, c \rangle$

$\therefore$ $3a + 4b + 2c = 0$

$4a + 2b + 3c = 0$

$$\therefore \quad \frac{a}{12-4} = \frac{b}{8-9} = \frac{c}{6-16}$$

$$\frac{a}{8} = \frac{b}{-1} = \frac{c}{-10}$$

$\therefore$ Direction ratio of plane = $\langle -8, 1, 10 \rangle$.

The direction ratio of required plane is $\langle l, m, n \rangle$

Then $-8l + m + 10n = 0$ ...(3)

and $2l + 3m + 4n = 0$ ...(4)

From (3) and (4),

$$\frac{l}{-26} = \frac{m}{52} = \frac{n}{-26}$$

$\therefore$ D.R.s are $\langle 1, -2, 1 \rangle$

$\therefore$ Equation of plane : $x - 2y + z = 0$

24. Answer (3)

Equation of required plane is

$$\begin{vmatrix} x-4 & y+1 & z-2 \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$(x-4)(-3-4)-(y+1)(9-2)+(z-2)(6+1)=0$$

$$-7(x-4)-7(y+1)+7(z-2)=0$$

$$x-4+y+1-z+2=0$$

$$x+y-z-1=0$$

$\therefore$ Point (1, 1, 1) lies on the plane

25. Answer (1)

$\because$ A be a point on given line.

$\therefore$ Position vector of

$$A=\overline{OA}=\vec{r}=(1-3\mu)\hat{i}+(\mu-1)\hat{j}+(2+5\mu)\hat{k}$$

position vector of $B=\overline{OB}=3\hat{i}+2\hat{j}+6\hat{k}$

$\therefore$ $\overline{AB}=\overline{OB}-\overline{OA}$

$= (3\mu+2)\hat{i}+(3-\mu)\hat{j}+(4-5\mu)\hat{k}$

equation of plane is: $x - 4y + 3z = 1$

$\because$ $\overline{AB}$ is parallel to plane.

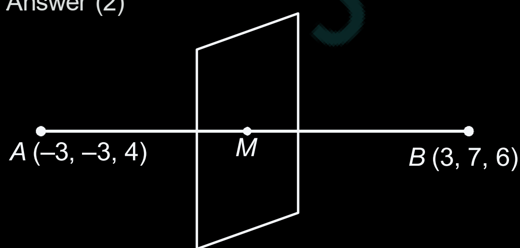
$\therefore$ $1(3\mu+2)-4(3-\mu)+3(4-5\mu)=0$

$$3\mu+2-12+4\mu+12-15\mu=0$$

$$2-8\mu=0$$

$$\mu=\frac{1}{4}$$

26. Answer (2)



M (0, 2, 5)

D.R's of normal to the plane is $\vec{n}=6\hat{i}+10\hat{j}+2\hat{k}$

$\Rightarrow$ equation of the plane is

$(x - 0)6 + (y-2)10 + (z - 5)2 = 0$

$3x + 5y - 10 + z - 5 = 0$

$3x + 5y + z = 15$ ...(i)

plane (i) passes through (4, 1, –2)

option (2) is correct.

27. Answer (1)

Any point on the line $\frac{x-4}{2}=\frac{y-5}{2}=\frac{z-3}{1}$ is $P(2\lambda + 4, 2\lambda + 5, \lambda + 3)$ lies on the plane $x + y + z = 2$

$\Rightarrow$ $2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$

$\Rightarrow$ $5\lambda = -10 \Rightarrow \lambda = -2$

$\Rightarrow$ Point of intersection is (0, 1, 1)

which lies on the line $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$

28. Answer (4)

Let normal to the required plane is $\vec{n}$

$\Rightarrow$ $\vec{n}$ is perpendicular to both vector $2\hat{i}-\hat{j}+3\hat{k}$ and $2\hat{i}+3\hat{j}-\hat{k}$.

$$\Rightarrow \vec{n}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 2 & 3 & -1 \end{vmatrix}=-8\hat{i}+8\hat{j}+8\hat{k}$$

$\Rightarrow$ equation of the required plane is

$(x - 3)(-1) + (y + 2) \times 1 + (z - 1) \times 1 = 0$

$x - 3 - y - 2 - z + 1 = 0$

$x - y - z = 4$ passes through (2, 0, –2)

$\Rightarrow$ Option (4) is correct

29. Answer (2)

Let the d.r's of the normal be $\langle a, b, c\rangle$

Equation of the plane is

$$a(x-0)+b(y+1)+c(2-0)=0$$

It passes through (0, 0, 1)

$\therefore$ $b + c = 0$

Also $\frac{0\cdot a+b-c}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{2}}=\frac{1}{\sqrt{2}}$

$$\Rightarrow b\cdot c=\sqrt{a^2+b^2+c^2}$$

And $b + c = 0$

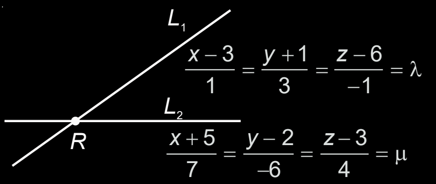
Solving we get $b = \pm\frac{1}{\sqrt{2}}a$.

$\therefore$ The d.r's are $\sqrt{2}, 1, -1$

Or $2, \sqrt{2}, -\sqrt{2}$

30. Answer (3)

Let the coordinate of A with respect to line



$L_1 = (\lambda + 3, 3\lambda - 1, -\lambda + 6)$

and coordinate of A w.r.t.

line $L_2 = (7\mu - 5, -6\mu + 2, 4\mu + 3)$.

$\therefore\ \lambda - 7\mu = -8,\ 3\lambda + 6\mu = 3,\ \lambda + 4\mu = 3$

from above equations : $\lambda = -1$, $\mu = 1$

$\therefore$ Coordinate of point of intersection $R = (2, -4, 7)$.

Image of R w.r.t. xy plane = $(2, -4, -7)$.

31. Answer (4)

Let the normal to the required plane is $\vec{n}$

$$\Rightarrow\ \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -4 & 4 \\ 2 & -5 & 0 \end{vmatrix} = 20\hat{i} + 8\hat{j} - 12\hat{j}$$

$\Rightarrow$ Equation of the plane is

$(x-3) \times 20 + (y-4) \times 8 + (z-2) \times (-12) = 0$

$5x - 15 + 2y - 8 - 3z + 6 = 0$

$5x + 2y - 3z - 17 = 0$ passes through $(2, \alpha, \beta)$

$\Rightarrow\ 10 + 2\alpha - 3\beta - 17 = 0 \ \Rightarrow\ 2\alpha - 3\beta = 7$

So, option (4) is correct

32. Answer (4)

Equation of plane containing both lines.

$$\text{D.R. of plane} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 7\hat{i} - 14\hat{j} + 7\hat{k}$$

Now, equation of plane

$\Rightarrow\ 7(x-1) - 14(y-4) + 7(z+4) = 0$

$x - 1 - 2y + 8 + z + 4 = 0$

$x - 2y + z + 11 = 0$

Now, distance from (0, 0, 0) to the plane

$$= \frac{11}{\sqrt{1+4+1}} = \frac{11}{\sqrt{6}}$$

33. Answer (1)

Let $\vec{x}_1$ and $\vec{x}_2$ be the vectors perpendicular to the plane OPQ and PQR respectively.

$$\vec{x}_1 = \overrightarrow{OP} \times \overrightarrow{OQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5\hat{i} - \hat{j} - 3\hat{k}$$

$$\vec{x}_2 = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = \hat{i} - 5\hat{j} - 3\hat{k}$$

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{5+5+9}{25+1+9} = \frac{19}{35}$$

$$\theta = \cos^{-1}\left(\frac{19}{35}\right)$$

34. Answer (3)

$P(-\lambda^2, 1, 1)$, $Q(1, -\lambda^2, 1)$, $R(1, 1, -\lambda^2)$, $S(-1, -1, 1)$ lie on same plane

$$\therefore \begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & 1-\lambda^2 & 0 \\ 2 & 2 & -\lambda^2 - 1 \end{vmatrix} = 0$$

$\Rightarrow\ (\lambda^2 + 1)((1 - \lambda^2)^2 - 4) = 0$

$\Rightarrow\ (3 - \lambda^2)(\lambda^2 + 1) = 0$

$\lambda^2 = 3$

$\lambda = \pm\sqrt{3}$

$S = \{-\sqrt{3}, \sqrt{3}\}$

35. Answer (2)

Let angle between line and plane is θ

$$\sin\theta = \left|\frac{\vec{b} \cdot \vec{n}}{|\vec{b}| \cdot |\vec{n}|}\right|$$

$$= \left|\frac{(2\hat{i} + \hat{j} - 2\hat{k}) \cdot (\hat{i} - 2\hat{j} - K\hat{k})}{\sqrt{9} \cdot \sqrt{1 + 4 + K^2}}\right|$$

$$= \left|\frac{2 - 2 + 2K}{3\sqrt{5 + K^2}}\right|$$

$$= \frac{2|K|}{3\sqrt{5 + K^2}}$$

But $\cos\theta = \frac{2\sqrt{2}}{3} \quad \Rightarrow\ \sin\theta = \frac{1}{3}$

$$\frac{2|K|}{3\sqrt{5} + K^2} = \frac{1}{3}$$

$4K^2 = 5 + K^2$

$3K^2 = 5$

$$K = \pm\sqrt{\frac{5}{3}}$$

36. Answer (3)

Let the equation of required plane be

$(2x - y - 4) + \lambda(y + 2z - 4) = 0$

$\because$ This plane passes through (1, 1, 0), then

$(2 - 1 - 4) + \lambda(1 + 0 - 4) = 0 \Rightarrow \lambda = -1$

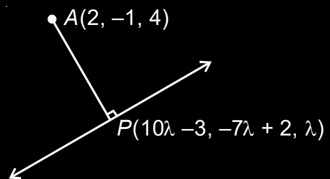
Equation of required plane will be

$(2x - y - 4) - (y + 2z - 4) = 0$

$\Rightarrow 2x - 2y - 2z = 0$

$\Rightarrow x - y - z = 0$

37. Answer (1)



Let P be the foot of perpendicular from point $A(2, -1, 4)$ on the given line. So P can be assumed as $P(10\lambda - 3, -7\lambda + 2, \lambda)$

DR's of $AP \propto$ to $10\lambda - 5, -7\lambda + 3, \lambda - 4$

$\because$ AP and given line are perpendicular, so

$10(10\lambda - 5) - 7(-7\lambda + 3) + 1(\lambda - 4) = 0$

$$\Rightarrow \lambda = \frac{1}{2}$$

$$AP = \sqrt{(10\lambda - 5)^2 + (-7\lambda + 3)^2 + (\lambda - 4)^2}$$

$$= \sqrt{0 + \frac{1}{4} + \frac{49}{4}}$$

$$= \sqrt{12.5}\,;\ \sqrt{12.5} \in (3, 4)$$

38. Answer (4)

P, Q, R are collinear.

$$\Rightarrow \overrightarrow{PR} = \lambda\overrightarrow{PQ}$$

$$2\hat{i} + (y+3)\hat{j} + (z-4)\hat{k} = \lambda[6\hat{i} + 3\hat{j} + 6\hat{k}]$$

$\Rightarrow 6\lambda = 2,\ y + 3 = 3\lambda,\ z - 4 = 6\lambda$

$$\Rightarrow \lambda = \frac{1}{3},\ y = -2,\ z = 6$$

$\Rightarrow$ point $R(4, -2, 6)$

$$\Rightarrow OR = \sqrt{(4)^2 + (-2)^2 + (6)^2}$$

$$= \sqrt{16 + 4 + 36} = \sqrt{56} = 2\sqrt{14}$$

39. Answer (2)

Let point on line is $p(2r + 1, 3r - 1, 4r + 2)$

It lies on the plane $x + 2y + 3z = 15$

$$\therefore\ 2r + 1 + 6r - 2 + 12r + 6 = 15 \Rightarrow r = \frac{1}{2}$$

$$\therefore\ P = \left(2, \frac{1}{2}, 4\right)$$

$$\therefore\ OP = \sqrt{4 + \frac{1}{4} + 16} = \sqrt{\frac{81}{4}} = \frac{9}{2}$$

40. Answer (1)

Let the required plane be $\frac{x}{a} + \frac{y}{-1} + \frac{z}{1} = 1$ given plane is $y - z + 5 = 0$

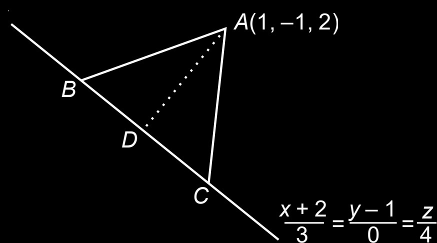
$$\therefore\ \cos\frac{\pi}{4} = \frac{-1-1}{\sqrt{\frac{1}{a^2} + 1 + 1}\ \sqrt{2}} \Rightarrow a^2 = \frac{1}{2}$$

$$\Rightarrow \boxed{\frac{1}{a} = \pm\sqrt{2}}$$

$$\Rightarrow \boxed{\pm\sqrt{2}x - y + z = 1}$$

$$\therefore\ \left(\sqrt{2}, 1, 4\right) \text{ satisfies } -\sqrt{2}\,x - y + z = 1$$

41. Answer (2)



Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

Given $BC = 5$ so we need perpendicular distance of A from line BC.

Let a point D on $BC = (3\lambda - 2,\ 1,\ 4\lambda)$

$$\overrightarrow{AD} = (3\lambda - 3)\hat{i} + 2\hat{j} + (4\lambda - 2)\hat{k}$$

Also $\overrightarrow{AD}$ and $\overrightarrow{BC}$ should be perpendicular

$$\overrightarrow{AD} \cdot \overrightarrow{BC} = 0$$

$$(3\lambda - 3)3 + 2(0) + (4\lambda - 2)4 = 0$$

$$9\lambda - 9 + 16\lambda - 8 = 0 \Rightarrow \lambda = \frac{17}{25}$$

Hence, $D = \left(\frac{1}{25}, 1, \frac{68}{25}\right)$

$$|\overrightarrow{AD}| = \sqrt{\left(\frac{1}{25}-1\right)^2 + (2)^2 + \left(\frac{68}{25}-2\right)^2}$$

$$= \sqrt{\left(\frac{-24}{25}\right)^2 + 4 + \left(\frac{18}{25}\right)^2}$$

$$= \sqrt{\frac{(24)^2 + 4(25)^2 + (18)^2}{25^2}}$$

$$= \sqrt{\frac{576 + 2500 + 324}{25^2}}$$

$$= \sqrt{\frac{3400}{25^2}}$$

$$= \frac{\sqrt{34}\cdot 10}{25} = \frac{2\sqrt{34}}{5}$$

Area of triangle $= \frac{1}{2} \times \left|\overline{BC}\right| \times \left|\overline{AD}\right|$

$$= \frac{1}{2} \times 5 \times \frac{2\sqrt{34}}{5}$$

$$= \sqrt{34}$$

42. Answer (4)

Let the plane be

$P \equiv (2x + 3y + z + 5) + \lambda(x + y + z - 6) = 0$

As the above plane is perpendicular to xy plane

$\Rightarrow \quad ((2+\lambda)\hat{i} + (3+\lambda)\hat{j} + (1+\lambda)\hat{k}) \cdot \hat{k} = 0$

$\Rightarrow \quad \lambda = -1$

$P \equiv x + 2y + 11 = 0$

Distance from (0, 0, 256)

$$\left|\frac{0+0+11}{\sqrt{5}}\right| = \frac{11}{\sqrt{5}}$$

43. Answer (1)

$\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1} = p \qquad P(\beta, 0, \beta)$

any point on line $A = (p, 1, -p-1)$

Now, DR of $AP \equiv < p - \beta, 1 - 0, -p - 1 - \beta >$

Which is perpendicular to line so

$(p - \beta).\ 1 + 0.1 - 1(-p - 1 - \beta) = 0$

$\Rightarrow \quad p - \beta + p + 1 + \beta = 0$

$p = \frac{-1}{2}$

Point $A\left(\frac{-1}{2}, 1 - \frac{1}{2}\right)$

Now, distance $AP = \sqrt{\frac{3}{2}}$

$\Rightarrow \quad AP^2 = \frac{3}{2}$

$\Rightarrow \quad \left(\beta + \frac{1}{2}\right)^2 + 1 + \left(\beta + \frac{1}{2}\right)^2 = \frac{3}{2}$

$2\left(\beta + \frac{1}{2}\right)^2 = \frac{1}{2}$

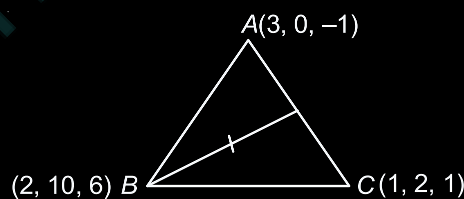
$\Rightarrow \quad \left(\beta + \frac{1}{2}\right)^2 = \frac{1}{4}$

$\Rightarrow \quad \beta = 0, \quad -1, \ (\beta \neq 0)$

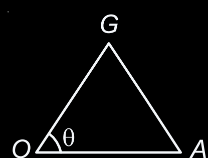
$\therefore \quad \boxed{\beta = -1}$

44. Answer (4)

G is the centroid of ΔABC



G (2, 4, 2)



$OG = \sqrt{4+16+4}, \qquad OA = \sqrt{9+1}$

$AG = \sqrt{1+16+9}$

$$\cos\theta = \frac{24+10-26}{2\sqrt{24}\sqrt{10}}$$

$$= \frac{8}{2\sqrt{8\times 3\times 2\times 5}}$$

$$= \frac{4}{4\sqrt{15}} = \frac{1}{\sqrt{15}}$$

45. Answer (4)

Image of Q in plane

$$\frac{(x-0)}{3}=\frac{(y+1)}{-1}=\frac{z+3}{+4}=\frac{-2(1-12-2)}{9+1+16}=1$$

$x = 3,\ y = -2,\ z = 1$

$P(3, -2, 1),\ \ Q(0, -1, -3),\ \ R(3, -1, -2)$

Now area of ΔPQR is

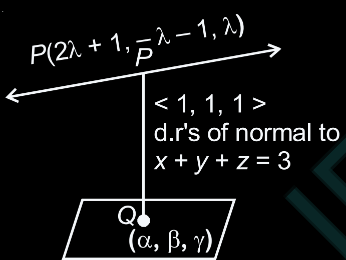
$$\frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{QR}| = \frac{1}{2}\left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 4\\ 3 & 0 & 1\end{vmatrix}\right|$$

$$=\frac{1}{2}\left|\left\{\hat{i}(-1)-\hat{j}(3-12)+\hat{k}(3)\right\}\right|$$

$$=\frac{1}{2}\sqrt{(1+81+9)}$$

$$=\frac{\sqrt{91}}{2}$$

46. Answer (2)



Let Q be (α, β, γ)

$\alpha + \beta + \gamma = 3$ ...(i)

$\alpha - \beta + \gamma = 3$ ...(ii)

$\therefore\ \ \alpha + \gamma = 3$ and $\beta = 0$

Equating DR's of PQ :

$$\frac{\alpha-2\lambda-1}{1}=\frac{\lambda+1}{1}=\frac{\gamma-\lambda}{1}$$

$\Rightarrow\ \alpha = 3\lambda + 2,\ \gamma = 2\lambda + 1$

Substituting in equation (i), we get

$\Rightarrow\ 5\lambda + 3 = 3$

$\lambda = 0$

Point is $Q(2, 0, 1)$

47. Answer (1)

$P_1 : 2x - y + 2z + 3 = 0$

$P_2 : 2x - y + 2z + \dfrac{\lambda}{2} = 0$

$P_3 : 2x - y + 2z + \mu = 0$

Given, $\dfrac{1}{3}=\dfrac{\left|3-\dfrac{\lambda}{2}\right|}{\sqrt{9}} \Rightarrow \left|3-\dfrac{\lambda}{2}\right|=1$

$\lambda_{max} = 8$

Also, $\dfrac{2}{3}=\dfrac{|\mu-3|}{\sqrt{9}} \Rightarrow \mu_{max}=5$

$(\lambda + \mu)_{max} = 13$

48. Answer (1)

Let $P(3\lambda + 2, 2\lambda - 1, -\lambda + 1)$ and

$Q(3\mu + 2, 2\mu - 1, -\mu + 1)$

As P lies on $2x + 3y - z + 13 = 0$

$6\lambda + 4 + 6\lambda - 3 + \lambda - 1 + 13 = 0$

$\Rightarrow\ 13\lambda = -13$

$\Rightarrow\ \lambda = -1$

$\therefore\ \ P(-1, -3, 2)$

Q lies on $3x + y + 4z = 16$

$9\mu + 6 + 2\mu - 1 - 4\mu + 4 = 16$

$\Rightarrow\ 7\mu = 7$

$\Rightarrow\ \mu = 1$

Q is $(5, 1, 0)$

$PQ=\sqrt{36+16+4}=\sqrt{56}=2\sqrt{14}$

49. Answer (4)

Equation of plane containing two given lines;

$$\begin{vmatrix}x-1 & y-1 & z\\ 1 & 2 & -1\\ -1 & 1 & -2\end{vmatrix}=0$$

$\Rightarrow\ x - y - z = 0$

The length of perpendicular from (2, 1, 4) to this plane = $\left|\dfrac{2-1-4}{\sqrt{1^2+1^2+1^2}}\right|=\sqrt{3}$

50. Answer (4)

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 4-2 & 1-1 & 1-0 \\ 5-2 & 0-1 & 1-0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

Plane is x + y – 2z = 3

Image of (2, 1, 6) is given by

$$\text{Thus, } \frac{x_1-2}{1} = \frac{y_1-1}{1} = \frac{z_1-6}{-2} = \frac{-2(2+1-12-3)}{6} = 4$$

∴ Image of R = (6, 5, – 2)

Point is (6, 5, –2)

51. Answer (3)

$$\text{Shortest distance} = \left| \frac{[\vec{a}_2 - \vec{a}_1 \ \ \vec{b}_1 \ \ \vec{b}_2]}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ -3 & 2 & 4 \end{vmatrix} = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{36+225+9} = \sqrt{270}$$

and $\left[\vec{a}_2 - \vec{a}_1 \ \ \vec{b}_1 \ \ \vec{b}_2\right]$ = – 36 – 225 – 9 = –270

⇒ Shortest distance = $\sqrt{270} = 3\sqrt{30}$

52. Answer (4)

Let A(1, 2, 3), $B\left(\frac{-7}{3}, \frac{-4}{3}, \frac{-1}{3}\right)$

Mid-point of $AB \equiv \left(\frac{-2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

Let equation of plane is

$$a\left(x+\frac{2}{3}\right) + b\left(y-\frac{1}{3}\right) + c\left(z-\frac{4}{3}\right) = 0 \qquad \text{...(i)}$$

dr's of $AB \equiv \frac{10}{3}, \frac{10}{3}, \frac{10}{3}$

∴ Equation of plane is

$$x+\frac{2}{3}+y-\frac{1}{3}+z-\frac{4}{3} = 0$$

$$\Rightarrow \boxed{x+y+z=1}$$

∴ (1, –1, +1) lies on the plane

53. Answer (2)

Let the plane be Ax + By + Cz + 1 = 0

Satisfying the conditions we get

A + 2B + C + 1 = 0

...(1) (passes through (1, 2, 1))

2A + B + 2C + 1 = 0

... (2) (passes through (2, 1, 2))

$\frac{A}{2} + \frac{B}{3} = 0$ ...(3) (parallel to 2x = 3y, z = 1)

Solving we get $A = \frac{2}{9}, B = -\frac{1}{3}$ & $C = -\frac{5}{9}$

Hence plane is 2x – 3y – 5z + 9 = 0

∴ Passes through (–2, 0, 1)

54. Answer (1)

The equation of plane passing through (3, 1, 1) is-

a(x – 3) + b (y – 1) + c(z – 1) = 0 ... (i)

∵ This plane contains the lines having drs (1, –2, 2) & (2, 3, –1)

so,

$$a - 2b + 2c = 0$$
$$2a + 3b - c = 0$$

$$\frac{a}{\begin{matrix}-2 & 2 \\ 3 & -1\end{matrix}} = \frac{b}{\begin{matrix}1 & 2 \\ 2 & -1\end{matrix}} = \frac{c}{\begin{matrix}1 & -2 \\ 2 & 3\end{matrix}}$$

$$\Rightarrow \frac{a}{2-6} = \frac{b}{-(-1-4)} = \frac{c}{3+4}$$

$$\Rightarrow \frac{a}{-4} = \frac{b}{5} = \frac{c}{7}$$

So, equation of plane is

–4(x – 3) + 5(y – 1) + 7(z – 1) = 0

–4x + 12 + 5y – 5 + 7z – 7 = 0

–4x + 5y + 7z = 0

This also passes through (α, –3, 5)

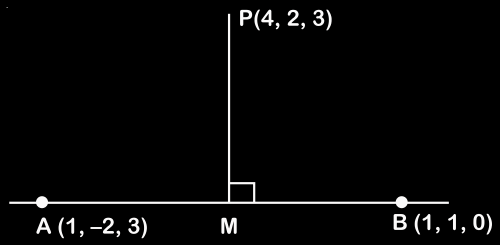
So, –4α – 15 + 35 = 0

⇒ –4α = –20

⇒ α = 5

55. Answer (4)

Equation of line through points (1, –2, 3) and (1, 1, 0) is



$$\frac{x-1}{0}=\frac{y-1}{-3}=\frac{z-0}{3-0}\ (=\lambda \text{ say})$$

A point on above line M = $(1, -\lambda + 1, \lambda)$

D.R^s of PM = $<-3, -\lambda - 1, \lambda - 3>$

$\because$ PM $\perp$ AB

$\therefore$ $(-3).0 + (-1-\lambda)(-1) + (\lambda - 3).1 = 0$

$\therefore$ $\lambda = 1$

$\therefore$ foot of perpendicular = (1, 0, 1)

This point lies on plane 2x + y – z = 1

56. Answer (1)

$$L_1 \equiv \vec{r} = \left(\hat{i}-\hat{j}\right)+l\left(2\hat{i}+\hat{k}\right)$$

$$L_2 \equiv \vec{r} = \left(2\hat{i}-\hat{j}\right)+m\left(\hat{i}+\hat{j}-\hat{k}\right)$$

Equating coeff. of $\hat{i}, \hat{j}$ and $\hat{k}$ of L_1 and L_2

2l + 1 = m + 2 ... (1)

–1 = –1 + m ...(2)

l = –m ...(3)

$\Rightarrow$ m = l = 0 Which gives absurd result hence lines are skew (do not intersect) for any value of l and m.

57. Answer (4)

Direction ratios of normal to plane are < 2, –6, 4 >

Also plane passes through (3, 1, 1)

$\therefore$ Equation of plane 2(x – 3) – 6(y – 1) + 4(z – 1) = 0

x – 3y + 2z = 2

Clearly it passes through (4, 0, –1)

58. Answer (3)

Equation of line through point (1, –2, 3) and parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=(\lambda \text{ Say})$$

A point on whole line = $(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$.

This point lies on plane x – y + 2 = 5

$\therefore$ $2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$

$\therefore$ $\lambda = \frac{1}{7}$

$\therefore$ Point on plane

$$=\left(\frac{2}{7}+1, \frac{3}{7}-2, \frac{-6}{7}+3\right)=\left(\frac{9}{7}, \frac{11}{7}, \frac{15}{7}\right)$$

$\therefore$ Required distance

$$=\sqrt{\left(\frac{9}{7}-1\right)^2+\left(-\frac{11}{7}+2\right)^2+\left(\frac{15}{7}-3\right)^2}$$

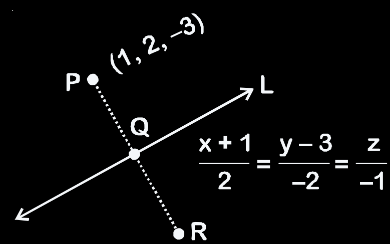
= 1

59. Answer (1)

Equation of line : $\frac{x+1}{2}=\frac{y-3}{-2}=\frac{z}{-1}(=\lambda \text{ say})$

a point on line L is

= $Q(2\lambda - 1, -2\lambda + 3, -\lambda)$



D·R^S of PQ = $< 2\lambda - 2, -2\lambda + 1, -\lambda + 3 >$

$\because$ PQ is perpendicular to line L

$\therefore$ $2(2\lambda - 2) - 2(-2\lambda + 1) - 1(-\lambda + 3) = 0$

$4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$

$9\lambda - 9 = 0$

$\Rightarrow$ $\lambda = 1$

$\therefore$ Coordinate of foot of $\perp$ = Q = (1, 1, –1)

$\therefore$ Coordinate of image R = (1, 0, 1) = (a, b, c)

$\therefore$ a + b + c = 2

60. Answer (2)

The lines are $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$

and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar

$$\therefore \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

$1(-1-5+\alpha) - 3(2-\alpha) + 2(10-2\alpha+\alpha) = 0$

$\therefore \quad \alpha = -4$

$\therefore$ Equation of $L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

$\therefore$ Point $(2, -10, -2)$ lies on line L_2

61. Answer (3)

First we will find the equation of line $x + y + z + 1 = 0 = 2x - y + z + 3$ in symmetrical form.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} = 2\hat{i} + \hat{j} - 3\hat{k}$$

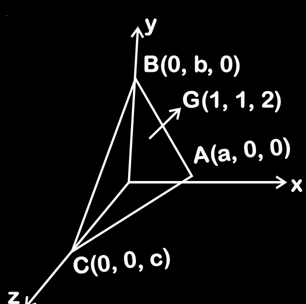
$$L_1 : \frac{x+2}{2} = \frac{y}{1} = \frac{z-1}{-3} \equiv \bar{r} = \left(-2\hat{i} + \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - 3\hat{k}\right)$$

And

Here $\bar{b}_1 \times \bar{b}_2 = -2\left[\hat{i} + \hat{j} + \hat{k}\right]$

Shortest distance $= \frac{1}{\sqrt{3}}$

62. Answer (1)



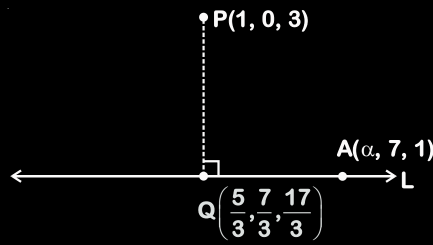
$\therefore$ a = 3, b = 3 and c = 6 as G is centroid

$\overrightarrow{AB} = 3\hat{j} - 3\hat{i}$ and $\overrightarrow{AC} = 6\hat{k} - 3\hat{i}$

$\overrightarrow{AB} \times \overrightarrow{AC} = 9(2\hat{i} + 2\hat{j} + \hat{k})$

Required line $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$

63. Answer (4)



Direction Ratio of PQ are

$$< \frac{5}{3} - 1, \frac{7}{3} - 0, \frac{17}{3} - 3 >$$

$$= < \frac{2}{3}, \frac{7}{3}, \frac{8}{3} > = < 2, 7, 8 >$$

Direction ratio of line L are

$$< \alpha - \frac{5}{3}, 7 - \frac{7}{3}, 1 - \frac{17}{3} >$$

$= <3\alpha - 5, 14, -14>$

$\therefore$ PQ is perpendicular to line L.

$\therefore \quad 2(3\alpha - 5) + 7.14 + (-14).8 = 0$

$\therefore \quad \alpha = 4$

64. Answer (8.00)

$\overline{AB} = \hat{i} - 3\hat{j} + 8\hat{k}$

and $\overline{CD} = 4\hat{i} - 4\hat{j} + 7\hat{k}$

Projection of $\overline{AB}$ on $\overline{CD}$ $= \left|\frac{\overline{AB} \cdot \overline{CD}}{\left|\overline{CD}\right|}\right|$

$$= \frac{4+12+56}{\sqrt{16+16+49}} = \frac{72}{9} = 8 \text{ units}$$

65. Answer (03)

$\because$ Point $(-1, 3, -1)$ lies on the plane containing given two lines. So distance between two plane = length of perpendicular from $(-1, 3, -1)$ to the given plane.

$$= \left|\frac{-23-30+2+48}{\sqrt{23^2+10^2+2^2}}\right|$$

$$= \frac{3}{\sqrt{633}}$$

Clearly k = 3

66. Answer (5)

Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} - \hat{k}$

$\vec{a} \times \vec{b} = -\hat{i} + \hat{j} + \hat{k}$

Equation of plane is –x + y + z + d = 0

It passes through (1, 0, 0)

∴ Plane is x – y – z – 1 = 0

Foot of perpendicular

$$\frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{-1} = \frac{-(1-1-1)}{1+1+1}$$

$$\therefore \quad (x, y, z) = \left(\frac{4}{3}, \frac{-1}{3}, \frac{2}{3}\right)$$

$$3(\alpha + \beta + \gamma) = 3\left(\frac{4}{3} - \frac{1}{3} + \frac{2}{3}\right) = 5$$

67. Answer (3)

Equation of plane P is

(x + 4y – z + 7) + λ(3x + y + 5z – 8) = 0

⇒ x(1 + 3λ) + y(4 + λ) + z(–1 + 5λ) + (7 – 8λ) = 0

$$\frac{1+3\lambda}{a} = \frac{4+\lambda}{b} = \frac{5\lambda-1}{6} = \frac{7-8\lambda}{-15}$$

From last two : λ = –1

$$\frac{-2}{a} = \frac{3}{b} = -1$$

∴ a = 2, b = –3

Plane is 2x – 3y + 6z – 15 = 0

$$\text{Distance} = \frac{|6-6-6-15|}{7} = \frac{21}{7} = 3$$

68. Answer (4)

The given planes are 3x + y – 2z = 5 ...(1)

2x – 5y – z = 7 ...(2)

Since the required plane passes through (1, 2, –3)

So equation of this plane is

a(x – 1) + b(y – 2) + c(z + 3) = 0 ...(3)

Now this plane (3) is ⊥ to the planes (1) & (2)

So 3a + b – 2c = 0

& 2a – 5b – c = 0

$$\Rightarrow \quad \frac{a}{-11} = \frac{b}{-1} = \frac{c}{-17}$$

So equation of plane is 11(x – 1) + (y – 2) + 17(2 + 3) = 0

⇒ 11x + y + 17z + 38 = 0

69. Answer (4)

Let a point P(λ) on the line

$$\Rightarrow \quad \frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda$$

∴ P(λ + 3, 2λ + 4, 2λ + 5)

as P also satisfies the given plane

λ + 3 + 2λ + 4 + 2λ + 5 = 17

⇒ 5λ = 5 ⇒ λ = 1

∴ P ≡ (4, 6, 7)

Distance from (1, 1, 9) is

$$\sqrt{(4-1)^2 + (6-1)^2 + (7-9)^2}$$

$$= \sqrt{9+25+4} = \sqrt{38}$$

70. Answer (4)

$P_1 + \lambda P_2 = 0$

$$\Rightarrow \left(\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)-1\right) + \lambda\left(\vec{r}\cdot\left(\hat{i}-2\hat{j}\right)+2\right) = 0 \quad ...(1)$$

$$\downarrow \left(\hat{i}+2\hat{k}\right)$$

$$\Rightarrow \quad (1 + 2 - 1) + \lambda\,(1 + 2) = 0 \Rightarrow \lambda = -\frac{2}{3} \quad ...(2)$$

by (1) and (2)

$$\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}-\frac{2}{3}\left(\hat{i}-2\hat{j}\right)\right)-1-\frac{4}{3}=0$$

$$\Rightarrow \quad \vec{r}\cdot\left(\frac{\hat{i}}{3}+\frac{7}{3}\hat{j}+\hat{k}\right)=\frac{7}{3}$$

$$\Rightarrow \quad \vec{r}\cdot\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$$

71. Answer (2)

∵ Mirror image of P(a, 6, 9) is Q(20, b, –a –9). so

Mid-point of PQ $\left(\text{i.e,}\left(\frac{a}{2}+10, \frac{b}{2}+3, -\frac{a}{2}\right)\right)$ lies on the given line

$$\frac{\frac{a}{2}+7}{7} = \frac{\frac{b}{2}+1}{5} = \frac{-\frac{a}{2}-1}{-9} \Rightarrow a = -56 \text{ and } b = -32$$

a + b = – 88

|a + b| = 88

72. Answer (01)

$$L_1 : \frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$$

$$L_2 : \frac{x}{1} = \frac{y-2\lambda}{1} = \frac{z-\lambda}{1}$$

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix} = \hat{i} - \frac{3}{2}\hat{j} + \frac{1}{2}\hat{k}$$

$$\overline{a_1} - \overline{a_2} = \lambda\hat{i} + \left(\frac{1}{2} + 2\lambda\right)\hat{j} - \lambda\hat{k}$$

$$d = \left|\frac{(\overline{a_1} - \overline{a_2}) \cdot (\overline{b_1} \times \overline{b_2})}{(\overline{b_1} \times \overline{b_2})}\right| = \left|\frac{\frac{5\lambda}{2} + \frac{3}{4}}{\sqrt{\frac{7}{2}}}\right| = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\Rightarrow \ = \left|\frac{5}{2}\lambda + \frac{3}{4}\right| = \frac{7}{4} \Rightarrow \lambda = \frac{2}{5} \text{ or } -1$$

$$\Rightarrow |\lambda| = 1$$

73. Answer (2)

$l + m - n = 0 \Rightarrow l = n - m$ ...(i)

$l^2 + m^2 - n^2 = 0$ ...(ii)

Substitute l from (i) into (ii)

$\Rightarrow (n - m)^2 + m^2 - n^2 = 0$

$2m(m - n) = 0$

$m = 0$ or $m = n$

Case-I

$m = 0 \Rightarrow l = n$

$$l^2 + m^2 + n^2 = 1 \Rightarrow l^2 = \frac{1}{2} \Rightarrow l_1, l_2 = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$$

$$l = n \Rightarrow n_1, n_2 = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$$

DCs $\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ or $\left(\frac{-1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right)$ are DCs of same line $\rightarrow l_1$

Case-II

$m = n \Rightarrow l = 0 \Rightarrow l_1, l_2 = 0$

$$l^2 + m^2 + n^2 = 1 \Rightarrow m^2 = \frac{1}{2} \Rightarrow m_1, m_2 = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$$

$$m = n \Rightarrow n_1, n_2 = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$$

DCs $\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(0, \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ are DCs of l_2

$$\cos\alpha = l_1l_2 + m_1m_2 + n_1n_2 = 0 + 0 \pm \frac{1}{2} = \pm\frac{1}{2}$$

$$\cos^2\alpha = \frac{1}{4}, \sin^2\alpha = \frac{3}{4} \Rightarrow \sin^4\alpha + \cos^4\alpha = \frac{5}{8}$$

74. Answer (3)

Let equation of line $\frac{x}{a} = \frac{y-1}{b} = \frac{z-2}{c}$

for being perpendicular to $\frac{x}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ we get $2a + 3b - 2c = 0$

Hence satisfying this equation $a : b : c = -3 : 4 : 3$

Hence required line is $\frac{x-1}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

75. Answer (44)

$l_1 : \frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2}$ and

$l_2 : \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1}$

$$\overline{b}_1 \times \overline{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i} + 3\hat{j} - 2\hat{k}$$

So, $l : \frac{x}{-2} = \frac{y}{3} = \frac{z}{-2}$

Point of intersection of l and l_1 can be considered as

$P(-2\lambda, 3\lambda, -2\lambda)$ and $\frac{-2\lambda-3}{1} = \frac{3\lambda+1}{2} = \frac{-2\lambda-4}{2}$

$\Rightarrow P(2, -3, 2)$

Let a point Q on l_2 as $Q(2\mu + 3, 2\mu + 3, \mu + 2)$

$\because PQ = \sqrt{17}$

$\Rightarrow (2\mu+1)^2 + (2\mu+6)^2 + \mu^2 = 17$

$\Rightarrow \mu = -\frac{10}{9}$ or -2

As Q lies in 1st octant, then $Q\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$,

Hence $18(a + b + c) = 44$

76. Answer (1)

Four points (1, 5, 35), (7, 5, 5), (1, λ, 7) and (2λ, 1, 2) are coplanar then

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$$\begin{vmatrix} 6 & 0 & 0 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & 10\lambda-38 \end{vmatrix} (R_3 \to (C_3 + 5 + C_1) = 0$$

$$6((\lambda-5)(10\lambda-38) - 112) = 0$$

$\therefore\ 10\lambda^2 - 88\lambda + 78 = 0$

$\Rightarrow 5\lambda^2 - 44\lambda + 39 = 0$

$\therefore$ Sum of all possible values of $\lambda = \frac{44}{5}$

77. Answer (8)

Vector perpendicular to the plane is

$\bar{n} = \hat{i} + 5\hat{j} - 6\hat{k}.$

Given $A(\lambda, 2, 1)$ and $B(4, -2, 2)$

$\because$ $\overline{AB} \perp \bar{n}$, so

$(\lambda - 4) + 5 \times 4 - 6(-1) = 0$

$\Rightarrow \lambda - 4 + 20 + 6 = 0$

$\Rightarrow \lambda = -22$

$\Rightarrow \dfrac{\lambda}{11} = -2$

hence $\left(\dfrac{\lambda}{11}\right)^2 - 4\left(\dfrac{\lambda}{11}\right) - 4 = 8$

78. Answer (2)

$$\frac{\alpha-1}{4} = \frac{\beta-3}{-5} = \frac{\gamma-5}{2} = -2\,\frac{(4\times1-5\times3+2\times5-8)}{16+25+4}$$

$$\frac{\alpha-1}{4} = \frac{\beta-3}{-5} = \frac{\gamma-5}{2} = \frac{2}{5}$$

$$\alpha = \frac{8}{5}+1,\ \beta = \frac{-10}{5}+3,\ \gamma = \frac{4}{5}+5$$

$5|\alpha+\beta+\gamma| = |5\alpha+5\beta+5\gamma| = 47$

79. Answer (2)

Direction of line L = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3\hat{i} - 2\hat{j} + \hat{k}$

d.r's = < 3, –2, 1>

A point on line (–2, 4, 0)

Line = $\dfrac{x+2}{3} = \dfrac{y-4}{-2} = \dfrac{z}{1}$

Foot of perpendicular from (3, 2, 1) be $(3\lambda - 2, -2\lambda + 4, \lambda)$

$(3\lambda - 5) \cdot 3 + (-2\lambda + 2)(-2) + (\lambda - 1)1 = 0$

$9\lambda - 15 + 4\lambda - 4 + \lambda - 1 = 0$

$14\lambda - 20 = 0 \Rightarrow \lambda = \dfrac{10}{7}$

$(\alpha, \beta, \gamma) = \left(\dfrac{16}{7}, \dfrac{8}{7}, \dfrac{10}{7}\right)$

$\therefore 21(\alpha + \beta + \gamma) = (16 + 8 + 10)3 = 102$

80. Answer (2)

DR's of AC $\propto a, -a, 4$

So equation of the plane will be $ax - ay + 4z = 0$.

$\because$ Point C lies on this plane, so $a^2 = 4 \Rightarrow a = 2$

Equation of plane : $x - y + 2z = 0$.

Projection of B(0, 4, 5) on this plane is D(–1, 5, 3).

$CD = \sqrt{66}$

81. Answer (1)

$L : \dfrac{x-1}{-1} = \dfrac{y+4}{2} = \dfrac{z+2}{3}$

$\because$ Plane P passes through origin and contains line L, then equation of plane P is

$$\begin{vmatrix} x & y & z \\ -1 & 2 & 3 \\ 1 & -4 & -2 \end{vmatrix} = 0$$

$\Rightarrow 8x + y + 2z = 0$

$\because$ Point $\left(\dfrac{2k-3}{k+1}, \dfrac{4k-6}{k+1}, \dfrac{-3k+1}{k+1}\right)$ lies on plane P, then $8(2k - 3) + (4k - 6) + 2(-3k + 1) = 0$

$\Rightarrow k = 2$

82. Answer (4)

Equation of PR : $\dfrac{x-3}{4} = \dfrac{y+1}{-1} = \dfrac{z-2}{2}$

Equation of QS : $\dfrac{x-1}{-2} = \dfrac{y-2}{1} = \dfrac{z+4}{-2}$

Their point of intersection of PR and QS is T(11, –3, 6)

$$\overline{PQ} \times \overline{QS} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix} = 2\hat{j} + \hat{k}$$

Clearly $\overline{TA} = \pm(2\hat{j} + \hat{k})$

Position vector of $A = 11\hat{i} - 3\hat{j} + 6\hat{k} \pm (2\hat{j} + \hat{k})$

$11\hat{i} - \hat{j} + 7\hat{k}$ or $11\hat{i} - 5\hat{j} + 5\hat{k}$

Modulus of P. V. of $A = \sqrt{171}$

83. Answer (2)

Equation of plane $x + y + z = 42$ ...(i)

Given expression is

$$E = 3 + \frac{(x-11)^3 + (y-19)^3 + (z-12)^3}{(x-11)^2(y-19)^2(z-12)^2} - \frac{42}{14(x-11)(y-19)(z-12)} \text{ (using (i))}$$

Now $(x - 11) + (y - 19) + (z - 12)$

$= x + y + z - 42 = 0$ (using (i))

$\therefore$ $(x - 11)^3 + (y - 19)^3 + (z - 12)^3$

$= 3(x - 11)(y - 19)(z - 12)$ ...(ii)

$$E = 3 + \frac{3(x-11)(y-19)(z-12)}{((x-11)(y-19)(z-12))^2} - \frac{3}{(x-11)(y-19)(z-12)}$$

84. Answer (3)

Let A(4, 3, 8), B(3, 5, 7)

DRs of AB(1, –2, 1)

AB $\perp$ L_1 $\Rightarrow$ l – 6 + 4 = 0 $\Rightarrow$ l = 2

Equation of L_1

$$\frac{x-3}{2}=\frac{y-5}{3}=\frac{z-7}{4}$$

$$L_2 = \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$$

$$SD=\frac{\left\|\begin{matrix}1 & 1 & 2\\ 2 & 3 & 4\\ 3 & 4 & 5\end{matrix}\right\|}{\left\|\begin{matrix}i & j & k\\ 2 & 3 & 4\\ 3 & 4 & 5\end{matrix}\right\|}=\frac{|-1|}{\left|-\hat{i}+2\hat{j}-\hat{k}\right|}=\frac{1}{\sqrt{6}}$$

85. Answer (2)

Line through (1, –2, 3) is

$$L_1:\frac{x-1}{3}=\frac{y+2}{-m}=\frac{z-3}{1}=r$$

Foot of $\perp$ Q(3r + 1, –mr –2, r + 3)

Q lies on x + 2y –3z + 10 = 0

3r + 1 –2mr – 4 –3r –9 + 10 = 0

$\Rightarrow$ mr = –1

$$PQ=\sqrt{\frac{7}{2}}\Rightarrow 10r^2+m^2r=\frac{7}{2}$$

$$r^2=\frac{1}{4}$$

$m^2 = 4$

|m| = 2

86. Answer (3)

Any plane containing y-axis is of the form

x + λz = 0

It passes through (1, 2, 3)

$$1 + 3\lambda = 0,\ \Rightarrow \lambda=\frac{-1}{3}$$

Required plane is

3x – z = 0

87. Answer (4)

Let $p_1 \equiv 2x-7y+4z-3=0$

and $p_2 = 3x-5y+4z+11=0$

Any plane through line of intersection of p_1 and p_2 is

(2x – 7y + 4z – 3) + λ(3x – 5y + 4z + 11) = 0

If passes through (–2, 1, 3)

$$-2+12\lambda=0 \Rightarrow \lambda=\frac{1}{6}$$

Required plane is

15x – 47y + 28z – 7 = 0

a = 15, b = –47, c = 28

2a + b + c – 7 = 4

88. Answer (3)

Let foot of perpendicular from P(2, 3, 1) on the line.

$L:\frac{x+1}{2}=\frac{y-3}{1}=\frac{z+2}{-1}$ be Q(2λ –1, λ + 3, –λ –2)

$\therefore$ PQ is perpendicular to L, then

$$2(2\lambda-3)+\lambda-(-\lambda-3)=0 \quad\Rightarrow\quad \lambda=\frac{1}{2}$$

$$Q\left(0,\frac{7}{2},\ \frac{-5}{2}\right)$$

So image of P in L is R(–2, 4 –6)

Equation of required plane,

$$\begin{vmatrix}x+2 & y-4 & z+6\\ 3 & -2 & 1\\ 4 & -3 & 5\end{vmatrix}=0$$

$\Rightarrow$ 7x + 11y + z = 24

89. Answer (0)

$$\because\quad \left(\frac{x+y+z}{\sqrt{3}}\right)^2+\left(\frac{lx-nz}{\sqrt{l^2+n^2}}\right)^2+\left(\frac{x-2y+z}{\sqrt{6}}\right)^2=9$$

$\Rightarrow$

$$x^2\left(\frac{1}{2}+\frac{l^2}{l^2+n^2}\right)+y^2(1)+z^2\left(\frac{1}{2}+\frac{n^2}{l^2+n^2}\right)+2xz\left(\frac{1}{3}-\frac{ln}{l^2+n^2}+\frac{1}{6}\right)=9$$

Clearly $\frac{l^2}{l^2+n^2}=\frac{1}{2}\Rightarrow l=\pm n$ and $\frac{ln}{l^2+n^2}=\frac{1}{2}$

then l = n

90. Answer (28)

Slope of normal to the plane <2, –6, 6>

Hence plane is π = 2x – 6y + 6z + k = 0

(where k = λd)

Also mid point of (4, –3, 1) and 2, 3, 5)

i.e., (3, 0, –2)

satisfies π

$\Rightarrow$ 6 – 0 – 12 + k = 0

$\Rightarrow$ k = 6

Hence, π = 2x – 6y + 6z + 6 = 0

Minimum $(a^2 + b^2 + c^2 + d^2) = (1^2 + 3^2 + 3^2 + 3^2)$ = 28

91. Answer (04)

Required plane is of form

$x - 2y + 2z + d = 0$

Also it is at unit distance from (1, 2, 3)

$\Rightarrow \left|\frac{1-4+6+d}{3}\right| = 1$

$\Rightarrow |d + 3| = 3 \Rightarrow d = 0$ or -6

$\therefore$ Plane is $x - 2y + 2z = 0$ or $x - 2y + 2z - 6 = 0$

$(-2 - 0) = K(2 - 1)$ or $(-2 + 6) = K(2 - 1)$

$K = -2$ or $K = 4$

92. Answer (1)

Equation of plane : $2x - y + z = b$

Midpoint of (1, 3, a) & (–3, 5, 2) lies on this plane

$\Rightarrow 2\left(\frac{1-3}{2}\right)-\left(\frac{3+5}{2}\right)+\left(\frac{2+a}{2}\right)=b$

$\Rightarrow a - 2b = 14$ ...(i)

Also vector joining (1, 3, a) & (–3, 5, 2) is parallel to $2\hat{i}-\hat{j}+\hat{k}$

i.e., $4\hat{i}-2\hat{j}+(a-2)\hat{k}=\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$

$\Rightarrow \lambda = 2 \Rightarrow a - 2 = 2 \Rightarrow a = 4$

From (i), $b = -5$

Hence $|a + b| = 1$

93. Answer (38)

The plane is parallel to vectors

$\overline{n_1} = 3\hat{i}+4\hat{j}+2\hat{k}$ & $\overline{n_2} = 4\hat{i}-3\hat{j}+7\hat{k}$

Vector normal to plane = $\bar{n} = \overline{n_1}\times\overline{n_2}$

$\Rightarrow \bar{n} = 34\hat{i}-13\hat{j}-25\hat{k}$

Point (1, –6, –5) lies on the plane

Hence equation of plane is

$\left((x-1)\hat{i}+(y+6)\hat{j}+(z+5)\hat{k}\right).\left(34\hat{i}-13\hat{j}-25\hat{k}\right)=0$

Passes through $(1, -1, \alpha)$

$\Rightarrow \left(0\hat{i}+5\hat{j}+(\alpha+5)\hat{k}\right).\left(34\hat{i}-13\hat{j}-25\hat{k}\right)=0$

$\Rightarrow 0-65-25\alpha-125=0$

$\Rightarrow 5\alpha = 38$

$|5\alpha| = 38$

94. Answer (6)

$\bar{a}_1-\bar{a}_2=(\alpha+4)\hat{i}+2\hat{j}+3\hat{k}$

$$\bar{b}_1\times\bar{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 3 & -2 & -2\end{vmatrix}=8\hat{i}+8\hat{j}+4\hat{k}=4(2\hat{i}+2\hat{j}+\hat{k})$$

Shortest distance

$$=\left|\frac{(\bar{a}_1-\bar{a}_2).(\bar{b}_1\times\bar{b}_2)}{|\bar{b}_1\times\bar{b}_2|}\right|=\left|\frac{2(\alpha+4)+4+3}{3}\right|=9$$

$\Rightarrow (2\alpha + 15) = 27$

$\Rightarrow \alpha = 6$

95. Answer (4)

Lines are $x = ay - 1 = z - 2$

$$\therefore \frac{x}{1}=\frac{y-1/a}{1/a}=\frac{z-2}{1} \quad \text{... (i)}$$

and $x = 3y - 2 = bz - 2$

$$\therefore \frac{x}{1}=\frac{y-2/3}{1/3}=\frac{z-2/b}{1/b} \quad \text{...(ii)}$$

$\therefore$ lines are co-planar

$$\therefore \begin{vmatrix}0 & -\frac{1}{a}+\frac{2}{3} & -2+\frac{2}{b}\\ 1 & \frac{1}{a} & 1\\ 1 & \frac{1}{3} & \frac{1}{b}\end{vmatrix}=0$$

$$\therefore \begin{vmatrix}0 & \frac{2}{3}-\frac{1}{a} & \frac{2}{b}-2\\ 0 & \frac{1}{a}-\frac{1}{3} & 1-\frac{1}{b}\\ 1 & \frac{1}{3} & \frac{1}{b}\end{vmatrix}=0$$

$$\therefore \frac{1}{a}-\frac{1}{ab}=0$$

$\Rightarrow b = 1$ and $a \in \mathbf{R} - \{0\}$

96. Answer (2)

L is normal to P and plane P will pass through (2, 3, –1)

Equation of P is $2x + y + z = \lambda = 6$ which is satisfied by (1, 2, 2)

97. Answer (4)

$P_1 \equiv x - y + 2z = 2$ & $P_2 \equiv 2x + y - z = 2$

Line of intersection

$$L \equiv \frac{x-\frac{4}{3}}{-1} = \frac{y-\frac{2}{3}}{5} = \frac{z-0}{3} = \lambda$$

general point on $L \equiv \left(-\lambda + \frac{4}{3}, 5\lambda + \frac{2}{3}, 3\lambda\right)$

for it being foot of perpendicular from (1, 2, 0)

$$\left(-\lambda + \frac{1}{3}\right)(-1) + \left(5\lambda - \frac{4}{3}\right)5 + (3\lambda)3 = 0$$

$$\lambda - \frac{1}{3} + 25\lambda - \frac{20}{3} + 9\lambda = 0$$

$$\Rightarrow \quad 35\lambda = 7 \Rightarrow \lambda = \frac{7}{35}$$

$35(\alpha + \beta + \gamma) = 35(7\lambda + 2) \Rightarrow 70 + 49 = 119$

98. Answer (3)

The given equation lines are

$3(x - 1) = 6(y - 2) = 2(z - 1)$

$$\Rightarrow \quad \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{3}$$

$$\therefore \quad \bar{y} = (\hat{i} + 2\hat{j} + \hat{k}) + t(2\hat{i} + \hat{j} + 3\hat{k}) \quad \text{...(i)}$$

and $4(x - 2) = 2(y - \lambda) = (z - 3)$

$$\Rightarrow \quad \frac{x-2}{1} = \frac{y-\lambda}{2} = \frac{z-3}{4}$$

$$\therefore \quad \bar{y} = (2\hat{i} + \lambda\hat{j} + 3\hat{k}) + s(\hat{i} + 2\hat{j} + 4\hat{k}) \text{ ...(ii)}$$

$$\therefore \quad \vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}, \vec{a}_2 = 2\hat{i} + \lambda\hat{j} + 3\hat{k}$$

$$\therefore \quad \vec{a}_1 - \vec{a}_2 = -\hat{i} + (2-\lambda)\hat{j} + 2\hat{k}$$

$$\bar{b}_1 \times \bar{b}_2 = (2\hat{i} + \hat{j} + 3\hat{k}) \times (\hat{i} + 2\hat{j} + 4\hat{k})$$

$$= -2\hat{i} - 5\hat{j} + 3\hat{k}$$

$$\therefore \quad \text{S.D.} = \left|\frac{(\bar{a}_1 - \bar{a}_2) \cdot \bar{b}_1 \times \bar{b}_2}{|\bar{b}_1 \times \bar{b}_2|}\right| = \frac{1}{\sqrt{38}}$$

$\therefore \quad |5\lambda - 14| = 1$

$\therefore \quad \lambda = 3$

99. Answer (1)

$$\because \quad \begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ k+1 & 4 & 6 \end{vmatrix} = 0$$

$\Rightarrow \quad -4(k + 1) - 4(-8) - 4(6) = 0$

$\Rightarrow \quad k = 1$

100. Answer (4)

Let plane $\equiv A(x + 1) + B(y) + C(z + 2) = 0$

$$\therefore \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix} = A\hat{i} + B\hat{j} + C\hat{k}$$

$$\Rightarrow \quad 2\hat{i} - \hat{j}(-1) + \hat{k}(-3) = A\hat{i} + B\hat{j} + C\hat{k}$$

$\therefore \quad A = -2, B = 1, C = -3$

$\therefore \quad$ required plane is

$-2x - 2 + y - 3z - 6 = 0$

$\Rightarrow \quad 2x - y + 3z + 8 = 0$

$\therefore \quad a + b + c = 4$

101. Answer (3)

Equation of plane through point (3, 7, –7) and containing line $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ is

$$\begin{vmatrix} x-2 & y-3 & z+2 \\ 3-2 & 7-3 & -7+2 \\ -3 & 2 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-3 & z+2 \\ 1 & 4 & -5 \\ -3 & 2 & 1 \end{vmatrix} = 0$$

$x - y + z + 3 = 0$

$$\therefore \quad \text{Distance from origin} = d = \left|\frac{3}{\sqrt{1^2 + 1^2 + 1}}\right|$$

$\therefore \quad d^2 = 3$

102. Answer (4)

Let point on line L_1 be $(\lambda + \alpha, 2\lambda + 1, 3\lambda + 1)$ and a point on line L_2 be $(\mu\beta + 4, 3\mu + 6, 3\mu + 7)$

$\therefore \quad \lambda + \alpha = \mu\beta + 4,\ 2\lambda + 1 = 3\mu + 6$ & $3\lambda + 1 = 3\mu + 7$

$\lambda = 1$ and $\mu = 1$

$\Rightarrow \quad 1 + \alpha = -\beta + 4 \Rightarrow \alpha + \beta = 3$

$\therefore \quad$ Point of intersection $(1 + \alpha, 3, 4)$

$1 + \alpha + 6 - 4 = 8 \quad \Rightarrow \quad \alpha = 5, \beta = -2$

$\alpha - \beta = 7$

103. Answer (7)

$$QR : \frac{x-2}{-1} = \frac{y+3}{1} = \frac{z-1}{6}$$

Let point of intersection be S($-\lambda + 2, \lambda - 3, 6\lambda + 1$)

$2(-\lambda + 2) + \lambda - 3 + 6\lambda + 1 = 7 \Rightarrow \lambda = 1$

So S(1, –2, 7)

$$PS = \sqrt{2^2 + 6^2 + 3^2} = 7$$

104. Answer (3)

Required plane is a plane passing through the line of intersection of planes

$P_1 \equiv x + 2y + 3z + 1 = 0$

and $P_2 \equiv x - y - z - 6 = 0$

Its equation: $P_1 + \lambda P_2 = 0$

$\Rightarrow (x + 2y + 3z + 1) + \lambda(x - y - z - 6) = 0$

$\Rightarrow (1 + \lambda)x + (2 - \lambda)y + (3 - \lambda)z + 1 - 6\lambda = 0$

$\because$ Perpendicular to $-2x + y + z + 8 = 0$

$\therefore -2(1 + \lambda) + (2 - \lambda) + (3 - \lambda) = 0$

$$\Rightarrow \lambda = \frac{3}{4}$$

$\Rightarrow$ Required plane is $7x + 5y + 9z = 14$

Checking the option shows that (0, 1, 1) satisfies it.

105. Answer (26)

$$L : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$$

$P_1 : x - 2y - z = 3$

Equation of a plane P_2 which contains L and perpendicular to P_1,

$$\begin{vmatrix} x-1 & y-3 & z-4 \\ 1 & -2 & -1 \\ 2 & 1 & 2 \end{vmatrix} = 0$$

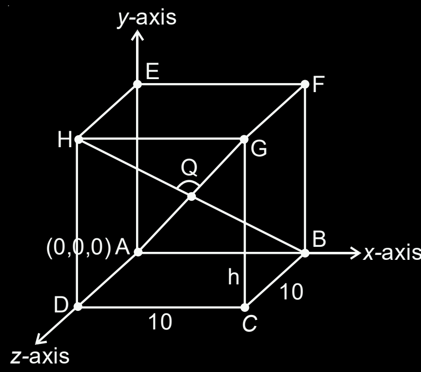
$\Rightarrow 3x + 4y - 5z + 5 = 0$

Distances of point (0, 0, 6) from P_1 and P_2 are

$$d_1 = \frac{25}{\sqrt{50}} = \sqrt{\frac{25}{2}} \text{ and } d_2 = \frac{9}{\sqrt{6}} = \sqrt{\frac{27}{2}}$$

Now $d^2 = d_1^2 + d_2^2 = 26$

106. Answer (2)



Let height be h.

A ≡ (0, 0, 0)

G ≡ (10, h, 10)

B ≡ (10, 0, 0)

H ≡ (0, h, 10)

DRs of AG ≡ (10, h, 10)

DRs of BH ≡ (10, –h, –10)

$$\cos\theta = \left|\frac{10 \times 10 + h(-h) + 10(-10)}{\sqrt{10^2 + h^2 + 10^2} \times \sqrt{10^2 + h^2 + 10^2}}\right|$$

$$= \frac{h^2}{200 + h^2} = \frac{1}{5}$$

$$\Rightarrow h = 5\sqrt{2}$$

107. Answer (2)

Equation of plane through point of intersection of planes

$\vec{r} \cdot (\hat{i} + \hat{j} + 4\hat{k}) = 16$ and $\vec{r} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 6$ is

$(x + y + 4z - 16) + \lambda(-x + y + z - 6) = 0$ ...(i)

This plane passes through point (1, 2, 3) then

$-1 - 2\lambda = 0$

$$\therefore \lambda = -\frac{1}{2}$$

$\therefore$ Equation of plane is :

$2x + 2y + 8z - 32 + x - y - z + 6 = 0$

$\therefore 3x + y + 7z - 26 = 0$

Clearly (4, 2, 2) does not lies on the plane.

108. Answer (96)

Normal vector for plane $\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix}$

$= 9\hat{i} - 18\hat{j} + 9\hat{k} = 9(\hat{i} - 2\hat{j} + \hat{k})$

$\Rightarrow$ Normal is parallel to $\hat{i} - 2\hat{j} + \hat{k}$

Plane passes through (1, 2, 3) as it is a point on L_2 so equation of plane

$1(x - 1) - 2(y - 2) + 1(z - 3) = 0$

$x - 2y + z = 0$

$$PQ = \frac{7 - 2(-2) + 13}{\sqrt{6}} \Rightarrow PQ^2 = 96$$

109. Answer (1)

Let the equation of required plane be,

$(x - y - z - 1) + \lambda\,(2x + y - 3z + 4) = 0$

$$\because \left|\frac{4\lambda - 1}{(2\lambda+1)^2 + (\lambda-1)^2 + (3\lambda+1)^2}\right| = \sqrt{\frac{2}{21}}$$

$\Rightarrow$ $21\,(16\lambda^2 - 8\lambda + 1) = 2(14\lambda^2 + 8\lambda + 3)$

$\Rightarrow$ $308\lambda^2 - 184\lambda + 15 = 0$

$\Rightarrow$ $(2\lambda - 1)\,(154\lambda - 15) = 0$

$\Rightarrow$ $\lambda = \frac{1}{2}$ and $\frac{15}{154}$

Put $\lambda = \frac{1}{2}$ we get $4x - y - 5z + 2 = 0$

110. Answer (4)

Equation of line through point (1, –2, 3) and parallel to line with direction ratios 2, 3, –6 is

$$L : \frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = (\lambda \text{ say})$$

a point on line L is $P(2\lambda + 1,\ 3\lambda - 2,\ -6\lambda + 3)$

$\because$ P lies on plane $x - y + z = 5$ we get

$2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$

$\therefore$ $\lambda = \frac{1}{7}$

$\therefore$ Coordinate of $P = \left(\frac{9}{7},\ -\frac{11}{7},\ \frac{15}{7}\right)$

$\therefore$ Required distance

$$= \sqrt{\left(\frac{9}{7} - 1\right)^2 + \left(-\frac{11}{7} + 2\right)^2 + \left(\frac{15}{7} - 3\right)^2}$$

= 1

111. Answer (2)

$\because$ $2l + 2m - n = 0$ …(i)

and $mn + nl + lm = 0$ …(ii)

From equation (i) and (ii)

$(m + l)\,(2l + 2m) + lm = 0$

$2l^2 + 5lm + 2m^2 = 0$

$2l^2 + 4lm + lm + 2m^2 = 0$

$2l(l + 2m) + m(l + 2m) = 0$

$\therefore$ $(2l + m)\,(l + 2m) = 0$

$\therefore$ D.Rs of lines are < 1, –2, –2 > and < 2, –1, 2 >

Here $l_1l_2 + m_1m_2 + n_1n_2 = 0$

$\therefore$ Lines are perpendicular to each other.

112. Answer (2)

Equation of plane passing through line of intersection of planes $\vec{r}\cdot(\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r}\cdot(2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ is

$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$

$\Rightarrow$ $(1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + (4\lambda - 1) = 0$ …(i)

$\because$ This plane is parallel to x-axis

$\therefore$ $1\cdot(1 + 2\lambda) + 0\cdot(1 + 3\lambda) + 0\cdot(1 - \lambda) = 0$

$\therefore$ $\lambda = -\frac{1}{2}$

$\therefore$ Required equation of plane is

$$-\frac{1}{2}y + \frac{3}{z}z - 3 = 0$$

$\therefore$ $y - 3z + 6 = 0$

$\therefore$ $\vec{r}\cdot(\hat{j} - 3\hat{k}) + 6 = 0$

113. Answer (72)

Let S be (x_1, y_1, z_1)

$$\frac{x_1 - 1}{2} = \frac{y_1 - 3}{-1} = \frac{z_1 - 4}{1} = -2\cdot\frac{2\times1 - 3 + 4}{2^2 + (-1)^2 + 1^2}$$

$\Rightarrow$ $x_1 = -3,\ y_1 = 5,\ z_1 = 2$

Hence S is (–3, 5, 2)

as $R(3, 5, y)$ lies on $2x - y + z + 3 = 0$

$\Rightarrow$ $6 - 5 + \gamma + 3 = 0 \Rightarrow \gamma = -4$

Hence R is (3, 5, –4)

$(SR)^2 = 36 + 0 + 36 = 72$

114. Answer (1)

Let the equation of required plane is :

$(3x - 2y + 4z - 7) + \lambda(x + 5y - 2z + 9) = 0$

$\therefore$ $(3 + \lambda)x + (-2 + 5\lambda)y + (4 - 2\lambda)z + (9\lambda - 7) = 0$

$\because$ This plane passing through point (1, 4, –3), we get

$3 + \lambda - 8 + 20\lambda - 12 + 6\lambda + 9\lambda - 7 = 0$

$$36\lambda - 24 = 0 \Rightarrow \lambda = \frac{2}{3}$$

$$\therefore \quad \frac{11}{3}x + \frac{4}{3}y + \frac{8}{3}z - 1 = 0$$

$\therefore$ $-11x - 4y - 8z + 3 = 0$

On comparing with $\alpha x + \beta y + \gamma z + 3 = 0$, we get

$\alpha + \beta + \gamma = -23$

115. Answer (61)

Any point on line $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z+1}{6}$

is $P(2\lambda + 1, 3\lambda + 2, 6\lambda - 1)$

If P lies on the plane $2x - y + z = 6$

$\Rightarrow$ $2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1) = 6$

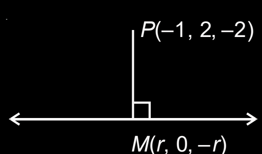
$\Rightarrow$ $\lambda = 1$

Hence point of intersection is $P(3, 5, 5)$

Distance of P from (–1, –1, 2)

$$= \left(\sqrt{16 + 36 + 9}\right)^2 = 61$$

116. Answer (2)



DRs of line of intersection (LOI).

$$\begin{vmatrix} i & j & k \\ 2 & 3 & 2 \\ 1 & -2 & 1 \end{vmatrix} = 7\vec{i} - 0\vec{j} - 7\vec{k}$$

DRs $\equiv$ (1, 0, –1)

Equation of line of intersection

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{-1} = r$$

$M(r, 0, -r)$

Direction ratios of PM $(r + 1, -2, -r + 2)$

Apply PM perpendicualr line $1(r + 1) + 0(-2) - 1(-r + 2) = 0$

$2r - 1 = 0$

$$r = \frac{1}{2}$$

$$M\left(\frac{1}{2}, 0, \frac{-1}{2}\right)$$

$$\text{PM} = \sqrt{\frac{9}{4} + 4 + \frac{9}{4}} = \frac{\sqrt{34}}{2}$$

117. Answer (7)

Line $\dfrac{x-2}{\alpha} = \dfrac{y-2}{-5} = \dfrac{z+2}{2}$ lies in $x + 3y - 2z + \beta = 0$

Angle b/w line and plane = 0

$\Rightarrow$ $\sin 0° = 0$

$(1)\cdot(\alpha) + (-5)(3) + (2)(-2) = 0$

$\alpha = 19$

Point (2, 2, –2) will be on plane

$(1)(2) + 3(2) - 2(-2) + \beta = 0$

$\beta = -12$

$\alpha + \beta = 19 - 12 = 7$

118. Answer (1)

$P_1 \equiv x - 2y - 2z + 1 = 0\ ;\ P_2 \equiv 2x - 3y - 6z + 1 = 0$

Pair of bisectors be

$$\frac{x - 2y - 2z + 1}{3} = \pm\frac{2x - 3y - 6z + 1}{7}$$

As $a_1a_2 + b_1b_2 + c_1c_2 = 1(2) + (-2)(-3) + (-2)(-6) > 0$

Ogive sign gives acute angle bisector

i.e., $7(x - 2y - 2z + 1) = -3(2x - 3y - 6z + 1)$

$\Rightarrow$ $13x - 23y - 32z + 10 = 0$

Clearly (–2, 0, –1/2) satisfy above plane.

119. Answer (1)

Direction of given line

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -2 \\ 3 & 0 & -1 \end{vmatrix} = \hat{i}(-3) - \hat{j}(6) + \hat{k}(-9)$$

$$= -3\hat{i} - 6\hat{j} - 9\hat{k}$$

Let $z = 0 \Rightarrow y = \frac{1}{3}$ and $x = -\frac{4}{3}$

$\therefore$ Line in Cartesian form is

$$\frac{x+\frac{4}{3}}{-3} = \frac{y-\frac{1}{3}}{-6} = \frac{z}{-9}$$

Let point of shortest distance be $P(\lambda)$ i.e.

$P\left(-\lambda-\frac{4}{3}, -2\lambda+\frac{1}{3}, -3\lambda\right)$ and $Q(2, -1, 6)$

For shortest distance $\overrightarrow{PQ}\cdot(\hat{i}+2\hat{j}+3\hat{k}) = 0$

$$\left(\left(\frac{10}{3}+\lambda\right)\hat{i} + \left(2\lambda-\frac{4}{3}\right)\hat{j} + (6+3\lambda)\hat{k}\right)\cdot(\hat{i}+2\hat{j}+3\hat{k}) = 0$$

$\Rightarrow \lambda = -\frac{4}{3}$

$\therefore P \equiv (0, 3, 4)$

$\therefore |PQ| = 2\sqrt{6}$

120. Answer (81)

Let $\overline{n}$ be the normal vector of the given plane.

$$\overline{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 0 \\ 1 & -1 & 3 \end{vmatrix} = 6\hat{i} - 2\hat{k}$$

$\because$ $\overline{a}$ is perpendicular to $\overline{n}$ and $\hat{i}+2\hat{j}+3\hat{k}$

So, $\overline{a} = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ 1 & 2 & 3 \end{vmatrix} = \lambda(2\hat{i} - 10\hat{j} + 6\hat{k})$

$\because$ $\overline{a}.(i+j+2k) = 2 \Rightarrow \lambda(2-10+12) = 2 \Rightarrow \lambda = \frac{1}{2}$

hence $\overline{a} = \hat{i} - 5\hat{j} + 3\hat{k}$

So $(\alpha - \beta + \gamma)^2 = (1 + 5 + 3)^2 = 81$

121. Answer (84)

Let $A(3\lambda + 7, -\lambda + 1, \lambda - 2)$ and $B(2\mu, 3\mu + 7, \mu)$

So, DR's of AB α $3\lambda - 2\mu + 7, -(\lambda + 3\mu + 6), \lambda - \mu - 2$

Clearly $\frac{3\lambda-2\mu+7}{1} = \frac{\lambda+3\mu+6}{4} = \frac{\lambda-\mu-2}{2}$

$\Rightarrow 5\lambda - 3\mu = -16$ …(i)

And $\lambda - 5\mu = 10$ …(ii)

From (i) and (ii) we get $\lambda = -5, \mu = -3$

So, A is $(-8, 6, -7)$ and B is $(-6, -2, -3)$

$$AB = \sqrt{4+64+16} \Rightarrow (AB)^2 = 84$$

122. Answer (2)

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix} = -a\hat{i} - \hat{j} + (a-1)\hat{k}$$

$$\vec{a}_1 - \vec{a}_2 = -\hat{i} + \hat{j} + \hat{k}$$

Shortest distance $= \left|\frac{(\vec{a}_1-\vec{a}_2)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$$\Rightarrow \sqrt{\frac{2}{3}} = \frac{2(a-1)}{\sqrt{a^2+1+(a-1)^2}}$$

$$\Rightarrow 6(a^2-2a+1) = 2a^2-2a+2$$

$\Rightarrow (a-2)(2a-1) = 0 \Rightarrow a = 2$ because $a \in z$.

123. Answer (1)

Let $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{p} = 2\hat{i} + 3\hat{j} + \lambda\hat{k},\ \vec{q} = \hat{i} + 4\hat{j} + 5\hat{k}$

$\therefore \vec{p}\times\vec{q} = (15-4\lambda)\hat{i} - (10-\lambda)\hat{j} + 5\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore$ Shortest distance

$$= \left|\frac{(15-4\lambda)-2(10-\lambda)+10}{\sqrt{(15-4\lambda)^2+(10-\lambda)^2+25}}\right| = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad 3(5-2\lambda)^2 = (15-4\lambda)^2+(10-\lambda)^2+25$

$\Rightarrow \quad 5\lambda^2 - 80\lambda + 275 = 0$

$\therefore$ Sum of values of $\lambda = \frac{80}{5} = 16$

124. Answer (3)

Let $P(x, y, z)$ be any point on plane P_1

Then $(x+4)^2+(y-2)^2+(z-1)^2$
$= (x-2)^2+(y+2)^2+(z-3)^2$

$\Rightarrow \quad 12x-8y+4z+4=0$

$\Rightarrow \quad 3x - 2y + z + 1 = 0$

And $P_2 : 2x + y + 3z = 1$

$\therefore$ angle between P_1 and P_2

$$\cos\theta\left|\frac{6-2+3}{14}\right| \Rightarrow \theta = \frac{\pi}{3}$$

125. Answer (2)

As L is parallel to PQ d.r.s of S is <1, 1, 1>

$\therefore \quad L \equiv \frac{x-1}{1} = \frac{y+1}{1} = \frac{z+1}{1}$

Point of intersection of L and S be λ

$\Rightarrow$ $(\lambda + 1) + (\lambda - 1) + (\lambda - 1) = S$

$\Rightarrow$ $\lambda = 2$

$\therefore$ $R \equiv (3, 1, 1)$

Let $Q(\alpha, \beta, \gamma)$

$$\Rightarrow \quad \frac{\alpha-1}{1} = \frac{\beta}{1} = \frac{\gamma-1}{1} = \frac{-2(-3)}{3}$$

$\Rightarrow$ $\alpha = 3, \beta = 2, \gamma = 3$

$\Rightarrow$ $Q \equiv (3, 2, 3)$

$(QR)^2 = 0^2 + (1)^2 + (2)^2 = 5$

126. Answer (5)

As plane is parallel to both the lines we have d.r's of normal to the plane as < 7, – 2, –1>

$$\left(\text{from} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix} = 7\hat{i} - \hat{j}(2) + \hat{k}(-1)\right)$$

Also point of intersection of lines is $2\hat{i}+4\hat{j}+6\hat{k}$

$\therefore$ Equation of plane is

$7(x - 2) - 2(y - 4) - 1(z - 6) = 0$

$\Rightarrow$ $7x - 2y - z = 0$

$a + b + d = 7 - 2 + 0 = 5$

127. Answer (3)

Let the equation of required plane

$\pi : (x+3y-z-5)+\lambda(2x-y+z-3)=0$

$\because$ $(2, 1, -2)$ lies on it so, $2 + \lambda(-2) = 0$

$\Rightarrow$ $\lambda = 1$

Hence, $\pi : 3x+2y-8=0$

$\therefore$ $\pi_x = -9,\ \pi_y = 5,\ \pi_{x+y} = 4$

$\pi_{x-y} = -22$ and $\pi_{y-x} = 6$

Clearly X and Y are on opposite sides of plane π

128. Answer (51)

$$\frac{x-\frac{1}{8}}{\frac{1}{8}} = \frac{y}{-\frac{1}{4\sqrt{2}}} = \frac{z}{0} \qquad \text{\_\_\_\_} L_1$$

or $\frac{x-\frac{1}{8}}{1} = \frac{y}{-\sqrt{2}} = \frac{z}{0}$ …(i)

Equation of L_2

$\frac{x+\frac{1}{8}}{-6\sqrt{3}} = \frac{y}{0} = \frac{z}{8}$ …(ii)

$$d = \left|\frac{(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})}{|\vec{b}\times\vec{d}|}\right|$$

$$= \frac{\left(\frac{1}{4}\hat{i}\right)\cdot\left(4\sqrt{2}\,\hat{i}+4\hat{j}+3\sqrt{6}\,\hat{k}\right)}{\sqrt{\left(4\sqrt{2}\right)^2+4^2+\left(3\sqrt{6}\right)^2}}$$

$$= \frac{\sqrt{2}}{\sqrt{32+16+54}} = \frac{1}{\sqrt{51}}$$

$d^{-2} = 51$

129. Answer (2)

$\because$ L_1 and L_2 are perpendicular, so

$$3\times1+(-2)\left(\frac{\alpha}{2}\right)+0\times2=0$$

$\Rightarrow$ $\alpha = 3$

Now angle between l_2 and l_3,

$$\cos\theta=\frac{1(-3)+\frac{\alpha}{2}(-2)+2(4)}{\sqrt{1+\frac{\alpha^2}{4}+4}\sqrt{9+4+16}}$$

$$\Rightarrow \cos\theta=\frac{2}{\frac{29}{2}}\Rightarrow\theta=\cos^{-1}\left(\frac{4}{29}\right)=\sec^{-1}\left(\frac{29}{4}\right)$$

130. Answer (1)

Consider the equation of plane,

$P : (2x + 3y + z + 20) + \lambda(x - 3y + 5z - 8) = 0$

$P : (2 + \lambda)x + (3 - 3\lambda)y + (1 + 5\lambda)z + (20 - 8\lambda) = 0$

$\because$ Plane P is perpendicular to $2x + 3y + z + 20 = 0$

So, $4 + 2\lambda + 9 - 9\lambda + 1 + 5\lambda = 0$

$\Rightarrow$ $\lambda = 7$

$P : 9x - 18y + 36z - 36 = 0$

Or $P : x - 2y + 4z = 4$

If image of $\left(2,-\frac{1}{2},2\right)$ in plane P is (a, b, c) then

$$\frac{a-2}{1}=\frac{b+\frac{1}{2}}{-2}=\frac{c-2}{4}$$

and $\left(\frac{a+2}{2}\right)-2\left(\frac{b-\frac{1}{2}}{2}\right)+4\left(\frac{c+2}{2}\right)=4$

Clearly $a=\frac{4}{3}, b=\frac{5}{6}$ and $c=-\frac{2}{3}$

So, $a : b : c = 8 : 5 : -4$

131. Answer (3)

$P_1 : 2x + y - 52 = 0$, $P_2 : 3x - y + 4z - 7 = 0$

Family of planes P_1 and P_2

$P : P_1 + \lambda P_2$

$\therefore$ $P : (2+3\lambda)x+(1-\lambda)y+(-5+4\lambda)z-7\lambda=0$

$\because$ $P \perp P_1$ $\therefore 4+6\lambda+1-\lambda+25-20\lambda=0$

$\boxed{\lambda=2}$

$\therefore$ $P : 8x - y + 32 - 14 = 0$

It passes through the point (1, 0, 2)

132. Answer (2)

$\because$ Both lines are coplanar, so

$$\begin{vmatrix}\alpha-1 & 0 & -1\\ 0 & 3 & -1\\ 2 & 0 & -3\end{vmatrix}=0$$

$\Rightarrow$ $\alpha=\frac{5}{3}$

Equation of plane containing both lines

$$\begin{vmatrix}x-1 & y+1 & z-1\\ 0 & 3 & -1\\ 2 & 0 & -3\end{vmatrix}=0$$

$\Rightarrow$ $9x + 2y + 6z = 13$

So, distance of $\left(\frac{5}{3},0,0\right)$ from this plane

$$=\frac{2}{\sqrt{81+4+36}}=\frac{2}{11}$$

133. Answer (137)

$$\frac{x-a}{3}=\frac{y-b}{-4}=\frac{z-c}{12}=\frac{-2(3a-4b+12c+19)}{3^2+(-4)^2+12^2}$$

$$\frac{x-a}{3}=\frac{y-b}{-4}=\frac{z-c}{12}=\frac{-6a+8b-24c-38}{169}$$

$$(x, y, z)\equiv(a-6,\ \beta,\ \gamma)$$

$$\frac{(a-6)-a}{3}=\frac{\beta-b}{-4}=\frac{\gamma-c}{12}=\frac{-6a+8b-24c-38}{169}$$

$$\frac{\beta-b}{-4}=-2\Rightarrow\boxed{\beta=8+b}$$

$$\frac{\gamma-c}{12}=-2\Rightarrow\boxed{\gamma=-24+c}$$

$$\frac{-6a+8b-24c-38}{169}=-2$$

$\Rightarrow \quad 3a - 4b + 12c = 150 \quad ...(1)$

$a + b + c = 5$

$3a + 3b + 3c = 15 \quad ....(2)$

Applying (1) – (2)

$-7b + 9c = 135$

$7b - 9c = -135$

$7\beta - 9\gamma = 7(8 + b) - 9(-24 + c)$

$= 56 + 216 + 7b - 9c.$

$= 56 + 216 - 135 = 137$

134. Answer (1)

$$L: \frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = t$$

Let $P = (4t - 2, 2t + 1, 3t - 1)$

$\because$ P is the foot of perpendicular of (1, 2, 4)

$\therefore$ $4(4t - 3) + 2(2t - 1) + 3(3t - 5) = 0$

$\Rightarrow$ $29t = 29 \Rightarrow \boxed{t = 1}$

$P = (2, 3, 2)$

Now, distance of P from the plane

$3x + 4y + 12z + 23 = 0$, is

$$\left|\frac{6 + 12 + 24 + 23}{\sqrt{9 + 16 + 144}}\right| = \frac{65}{13} = 5$$

135. Answer (1)

$$L_1: \frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$$

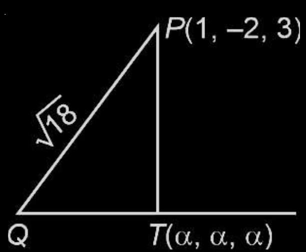
$$L_2: \frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$$

Now, $\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix} = 10\hat{i} - 8\hat{j} - 4\hat{k}$

and $\vec{a}_2 - \vec{a}_1 = 6\hat{i} - 4\hat{j} - 4\hat{k}$

$$\therefore \text{ S.D} = \left|\frac{60 + 32 + 16}{\sqrt{100 + 64 + 16}}\right| = \frac{108}{\sqrt{180}} = \frac{18}{\sqrt{5}}$$

136. Answer (2)



Line L is $x = y = z$

$\overrightarrow{PQ}.(\hat{i} + \hat{j} + \hat{k}) = 0$

$\Rightarrow$ $(\alpha - 3) + \alpha + 2 + \alpha - 1 = 0$

$\Rightarrow$ $\alpha = \frac{2}{3}$ so, $T = \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$PT = \sqrt{\frac{38}{3}}$

$\Rightarrow$ $QT = \frac{4}{\sqrt{3}}$

So, Area $= \left(\frac{1}{2} \times \frac{4}{\sqrt{3}} \times \frac{\sqrt{38}}{\sqrt{3}}\right).2$

$= \frac{4\sqrt{38}}{3}$ sq units

137. Answer (1)

Family of Plane's equation can be given by

$(5 + 8\lambda)x + (8 - 7\lambda)y + (13 + \lambda) z - (29 + 20\lambda) = 0$

P_1 passes through (2, 1, 3)

$\Rightarrow$ $(10 + 16\lambda) + (8 - 7\lambda) + (39 + 3\lambda) - (29 + 20\lambda) = 0$

$\Rightarrow$ $-8\lambda + 28 = 0 \Rightarrow \lambda = \frac{7}{2}$

d.r, s of normal to P_1

$\left\langle 33, \frac{-33}{2}, \frac{33}{2} \right\rangle$ or $\left\langle 1, -\frac{1}{2}, \frac{1}{2} \right\rangle$

P_2 passes through (0, 1, 2)

$\Rightarrow$ $8 - 7\lambda + 26 + 2\lambda - (29 + 20\lambda) = 0$

$\Rightarrow$ $5 - 25\lambda = 0$

$\Rightarrow$ $\lambda = \frac{1}{5}$

d.r, s of normal to P_2

$\left\langle \frac{33}{5}, \frac{33}{5}, \frac{66}{5} \right\rangle$ or $\langle 1, 1, 2 \rangle$

Angle between normals

$$= \frac{\left(\hat{i} - \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}\right) \cdot \left(\hat{i} + \hat{j} + 2\hat{k}\right)}{\frac{\sqrt{3}}{2} \qquad \sqrt{6}}$$

$$\cos\theta = \frac{1 - \frac{1}{2} + 1}{3} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

138. Answer (2)

$P_1 : x + 3y - z = 6$

$P_2 : -6x + 5y - z = 7$

Family of planes passing through line of intersection of P_1 and P_2 is given by $x(1 - 6\lambda) + y(3 + 5\lambda) + z(-1 - \lambda) - (6 + 7\lambda) = 0$

It passes through $\left(2, 3, \frac{1}{2}\right)$

So, $2(1-6\lambda)+3(3+5\lambda)+\frac{1}{2}(-1-\lambda)-(6+7\lambda)=0$

$\Rightarrow \quad 2-12\lambda+9+15\lambda-\frac{1}{2}-\frac{\lambda}{2}-6-7\lambda=0$

$\Rightarrow \quad \frac{9}{2}-\frac{9\lambda}{2}=0 \Rightarrow \lambda=1$

Required plane is

$-5x + 8y - 2z - 13 = 0$

Or $\vec{r}\cdot(-5\hat{i}+8\hat{j}-2\hat{k})=13$

$$\frac{|13\vec{a}|^2}{|d|^2}=\frac{13^2}{(13)^2}\cdot|\vec{a}|^2=93$$

139. Answer (4)

Equation of plane through point (2, 3, –5) and perpendicular to planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$ is

$$\begin{vmatrix} x-2 & y-3 & z+5 \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix}=0$$

$\therefore$ Equation of plane is $(x - 2)(-7 + 25) - (y - 3)(-14 + 15) + (z + 5)\cdot 7 = 0$

$\therefore \quad 18x - y + 7z + 2 = 0$

$\Rightarrow \quad 18x - y + 7z = -2$

$\therefore \quad -18x + y - 7z = 2$

On comparing with $ax + by + cz = d$ where $d > 0$ is

$a = -18,\ b = 1,\ c = -7,\ d = 2$

$\therefore \quad a + 7b + c + 20d = 22$

140. Answer (125)

The point dividing PQ in the ratio 1 : 3 will be mid-point of P & foot of perpendicular from P on the line.

$\therefore$ Let a point on line be λ

$\Rightarrow \quad \frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$

$\Rightarrow \quad P'(3\lambda+6, 2\lambda+1, 3\lambda+2)$

as P' is foot of perpendicular

$(3\lambda + 5)3 + (2\lambda - 1)2 + (3\lambda - 1)3 = 0$

$\Rightarrow \quad 22\lambda + 15 - 2 - 3 = 0$

$\Rightarrow \quad \lambda=\frac{-5}{11}$

$\therefore \quad P'\left(\frac{51}{11},\frac{1}{11},\frac{7}{11}\right)$

$$\text{Mid-point of } PP' \equiv \left(\frac{\frac{51}{11}+1}{2},\frac{\frac{1}{11}+2}{2},\frac{\frac{7}{11}+3}{2}\right)$$

$$\equiv\left(\frac{62}{22},\frac{23}{22},\frac{40}{22}\right)\equiv(\alpha,\beta,\gamma)$$

$\Rightarrow \quad 22(\alpha + \beta + \gamma) = 62 + 23 + 40 = 125$

141. Answer (3)

Mirror image of (2, 4, 7) in $3x - y + 4z = 2$ is (a, b, c) then

$$\frac{a-2}{3}=\frac{b-4}{-1}=\frac{c-7}{4}=\frac{-2(6-4+28-2)}{3^2+(-1)^2+4^2}$$

$$\frac{a-2}{3}=\frac{b-4}{-1}=\frac{c-7}{4}=\frac{-28}{13}$$

$a=\frac{-58}{13} \quad b=\frac{80}{13} \quad c=\frac{-21}{13}$

$$2a+b+2c=\frac{-116+80-42}{13}$$

$$=-6$$

142. Answer (26)

Foot of perpendicular from P

$$\frac{x-1}{-1}=\frac{y-2}{1}=\frac{z+1}{1}=\frac{-(-1+2-1-1)}{3}$$

$\Rightarrow \quad p'\equiv\left(\frac{2}{3},\frac{7}{3},\frac{-2}{3}\right)$

and foot of perpendicular from Q

$$\frac{x-2}{-1}=\frac{y+1}{1}=\frac{z-3}{1}=\frac{-(-2-1+3-1)}{3}$$

$\Rightarrow \quad Q'\equiv\left(\frac{5}{3},\frac{-2}{3},\frac{10}{3}\right)$

$$P'Q'=\sqrt{(1)^2+(3)^2+(4)^2}=d=\sqrt{26}$$

$\Rightarrow \quad d^2 = 26$

143. Answer (2)

$$\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{-1}=\lambda$$

$(3\lambda+2,-2\lambda-1,-\lambda-3)$ lies on plane $px-qy+z=5$

$$p(3\lambda+2)-q(-2\lambda-1)+(-\lambda-3)=5$$

$$\lambda(3p+2q-1)+(2p+q-8)=0$$

$$\left.\begin{aligned}3p+2q-1=0\\2p+q-8=0\end{aligned}\right\}\begin{aligned}p=15\\q=-22\end{aligned}$$

Equation of plane $15x+22y+z-5=0$

Shortest distance from origin $=\dfrac{|0+0+0-5|}{\sqrt{15^2+22^2+1}}$

$$=\frac{5}{\sqrt{710}}$$

$$=\sqrt{\frac{5}{142}}$$

144. Answer (2)

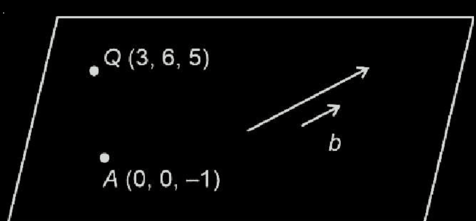
$P(1, 2, 1)$ image in plane $x+2y+2z=16$

$$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-1}{2}=\frac{-2(1+2\times2+2\times1-16)}{1^2+2^2+2^2}$$

$$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-1}{2}=2$$

$Q(3, 6, 5)$

$$\vec{r}=-\hat{k}+\lambda\left(\hat{i}+\hat{j}+2\hat{k}\right)$$



$$AQ=3\hat{i}+6\hat{j}+6\hat{k}$$

$$=3\left(\hat{i}+2\hat{j}+2\hat{k}\right)$$

$$\vec{n}=\left(\hat{i}+2\hat{j}+2\hat{k}\right)\times\left(\hat{i}+\hat{j}+2\hat{k}\right)$$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\1 & 2 & 2\\1 & 1 & 2\end{vmatrix}$$

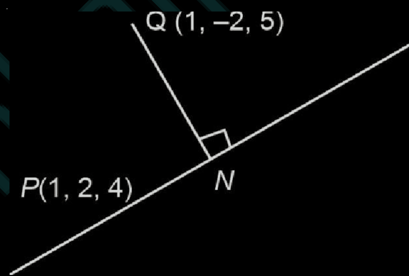
$$=2\hat{i}-0\hat{j}-\hat{k}$$

Equation of plane $\equiv 2(x-0)+0(y-0)-1(z+1)=0$

$2x-z=1$

Point lying on plane from the option is (1, 2, 1) i.e., option (2)

145. Answer (1)



The line $x+y-z=0=x-2y+3z-5$ is parallel to the vector

$$\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\1 & 1 & -1\\1 & -2 & 3\end{vmatrix}=(1, 4, -3)$$

Equation of line through $P(1, 2, 4)$ and parallel to $\vec{b}$

$$\frac{x-1}{1}=\frac{y-2}{-4}=\frac{z-4}{-3}$$

Let $N\equiv(\lambda+1, -4\lambda+2, -3\lambda+4)$

$\overrightarrow{QN}=(\lambda, -4\lambda+4, -3\lambda-1)$

$\overrightarrow{QN}$ is perpendicular to

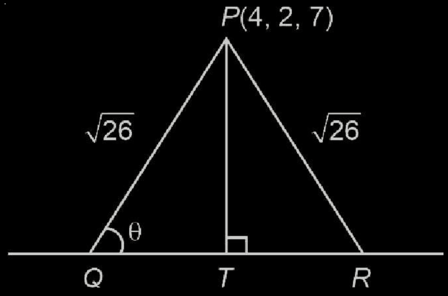
$\Rightarrow\ (\lambda, -4\lambda+4, -3\lambda-1)\cdot(1, 4, -3)=0$

$\Rightarrow\ \lambda=\dfrac{1}{2}$

Hence $\overrightarrow{QN}=\left(\dfrac{1}{2}, 2, \dfrac{-5}{2}\right)$ and $\left|\overrightarrow{QN}\right|=\sqrt{\dfrac{21}{2}}$

146. Answer (153)

$L: \frac{x+1}{2} = \frac{y+2}{3} = \frac{2-1}{2}$



Let $T(2t-1, 3t-2, 2t+1)$

$\because$ $PT \perp^r QR$

$\therefore$ $2(2t-5) + 3(3t-4) + 2(2t-6) = 0$

$17t = 34$ $\therefore$ $\boxed{t = 2}$ So $T(3, 4, 5)$

$\therefore$ $PT = \sqrt{1+4+4} = 3$

$\therefore$ $QT = \sqrt{26-9} = \sqrt{17}$

$\therefore$ Area of $\Delta PQR = \frac{1}{2} \times 2\sqrt{17} \times 3 = 3\sqrt{17}$

$\therefore$ Square of ar(ΔPQR) = 153.

147. Answer (4)

$P_1: 2x + ky - 5z = 1$

$P_2: 3kx - ky + z = 5$

$\because$ $P_1 \perp P_2 \Rightarrow 6k - k^2 + 5 = 0$

$\Rightarrow$ $k = 1, 5$

$\because$ $k < 3$

$\therefore$ $k = 1$

$P_1: 2x + y - 5z = 1$

$P_2: 3x - y + z = 5$

$P: (2x + y - 5z - 1) + \lambda(3x - y + z - 5) = 0$

Positive x-axis intercept = 1

$\Rightarrow$ $\frac{1+5\lambda}{2+3\lambda} = 1$

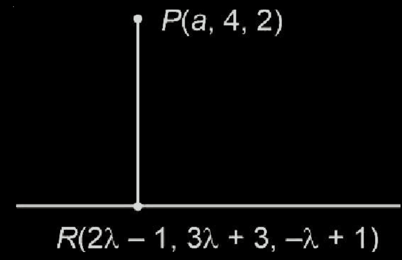
$\Rightarrow$ $\lambda = \frac{1}{2}$

$\therefore$ $P: 7x + y - 4z = 7$

y intercept = 7.

148. Answer (2)

$\because$ PR is perpendicular to given line, so



$2(2\lambda - 1 - a) + 3(3\lambda - 1) - 1(-\lambda - 1) = 0$

$\Rightarrow$ $a = 7\lambda - 2$

Now

$\because$ $PR = 2\sqrt{6}$

$\Rightarrow$ $(-5\lambda + 1)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow$ $5\lambda^2 - 2\lambda - 3 = 0 \Rightarrow \lambda = 1$ or $-\frac{3}{5}$

$\because$ $a > 0$ so $\lambda = 1$ and $a = 5$

Now $\sum_{i=1}^{3} \alpha_i = 2$(Sum of co-ordinate of R)

– (Sum of coordinates of P)

$= 2(7) - 11 = 3$

$a + \sum_{i=1}^{3} \alpha_i = 5 + 3 = 8$

149. Answer (2)

$P_1: ax + by + 0z = 3$, normal vector : $\vec{n}_1 = (a, b, 0)$

$P_2: ax + by + cz = 0$, normal vector : $\vec{n}_2 = (a, b, c)$

Vector parallel to the line of intersection = $\vec{n}_1 \times \vec{n}_2$

$\vec{n}_1 \times \vec{n}_2 = (bc, -ac, 0)$

Vector normal to $0 \cdot x + y - z + 2 = 0$ is

$\vec{n}_3 = (0, 1, -1)$

Angle between line and plane is 30°

$\Rightarrow \left|\dfrac{0-ac+0}{\sqrt{b^2c^2+c^2a^2}\sqrt{2}}\right| = \dfrac{1}{2}$

$\Rightarrow \quad a^2 = b^2$

Hence, $\vec{n}_1 \times \vec{n}_2 = (ac, -ac, 0)$

Direction ratios = $\left(\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}, 0\right)$

150. Answer (2)

Any point on $x^2 + y^2 = 1$, $z = 0$ is $p(\cos\theta, \sin\theta, 0)$

If foot of perpendicular of p on the plane $2x + 3y + z = 6$ is (h, k, l) then

$$\frac{h-\cos\theta}{2} = \frac{k-\sin\theta}{3} = \frac{l-0}{1}$$

$$= -\left(\frac{2\cos\theta + 3\sin\theta + 0 - 6}{2^2+3^2+1^2}\right) = r(\text{let})$$

$h = 2r + \cos\theta$, $k = 3r + \sin\theta$, $l = r$

Hence, $h - 2l = \cos\theta$ and $k - 3l = \sin\theta$

Hence $(h - 2l)^2 + (k - 3l)^2 = 1$

When $l = 6 - 2h - 3k$

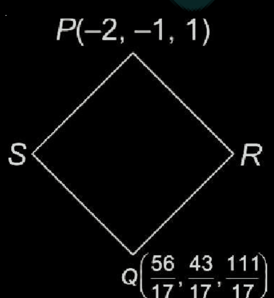
Hence required locus is

$(x - 2(6 - 2x - 3y))^2 + (y - 3(6 - 2x - 3y))^2 = 1$

$\Rightarrow \quad (5x + 6y - 12)^2 + 4(3x + 5y - 9)^2 = 1,$

$z = 6 - 2x - 3y$

151. Answer (450)



d.r's of RS = $< \alpha, -1, \beta >$

d.r's of PQ = $< \dfrac{90}{17}, \dfrac{60}{17}, \dfrac{94}{17} > = < 45, 30, 47 >$

as PQ and RS are diagonals of rhombus

$\alpha(45) + 30(-1) + 47(\beta) = 0$

$\Rightarrow \quad 45\alpha + 47\beta = 30$

i.e., $\alpha = \dfrac{30-47\beta}{45}$

for minimum integral value $\alpha = -15$ and $\beta = 15$

$\Rightarrow \quad \alpha^2 + \beta^2 = 450.$

152. Answer (4)

$L_1 : \dfrac{x-1}{\lambda} = \dfrac{y-2}{1} = \dfrac{z-3}{2},$

through a point $\vec{a}_1 \equiv (1, 2, 3)$

parallel to $\vec{b}_1 \equiv (\lambda, 1, 2)$

$L_2 : \dfrac{x+26}{-2} = \dfrac{y+18}{3} = \dfrac{z+28}{\lambda}$

through a point $\vec{a}_2 = (-26, -18, -28)$

parallel to $\vec{b}_2 = (-2, 3, 1)$

If lines are coplanar then, $(\vec{a}_2 - \vec{a}_1)\cdot \vec{b}_1 \times \vec{b}_2 = 0$

$$\Rightarrow \begin{vmatrix} 27 & 20 & 31 \\ \lambda & 1 & 2 \\ -2 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

Vector normal to the required plane $\vec{n} = \vec{b}_1 \times \vec{b}_2$

$$\Rightarrow \quad \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ -2 & 3 & 3 \end{vmatrix} = -3\hat{i} - 13\hat{j} + 11\hat{k}$$

Equation of plane

$\equiv ((x-1), (y-2), (z-3)) \cdot (-3, -13, 11) = 0$

$\Rightarrow \quad 3x + 13y - 11z + 4 = 0$

Checking the option gives (0, 4, 5) does not lie on the plane.

153. Answer (2)

Let $\vec{a}_1 = (-2, 1, -3)$ and $\vec{a}_2 = (-1, 2, -2)$

Vector normal to plane $\overline{n} = \vec{a}_1 \times \vec{a}_2$

$\overline{n} = (4, -1, -3)$

Plane through (2, 2, – 2) and normal to $\overline{n}$

$(x-2,\ y-2,\ z+2)\cdot(4,-1,-3)=0$

$\Rightarrow\ 4x-y-3z=12$

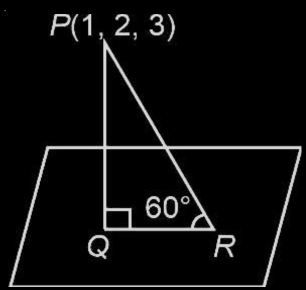
$$\frac{x}{3}+\frac{y}{-12}+\frac{z}{-4}=1$$

$$\Rightarrow\ V=\frac{1}{6}\times 3\times 12\times 4=24$$

Intercepts α, β, γ are 3, –12, –4

$P=\alpha+\beta+\gamma=-13$

154. Answer (2)



$$PQ=\left|\frac{1+4+3-14}{\sqrt{6}}\right|=\sqrt{6}$$

$$QR=\frac{PQ}{\tan 60^\circ}=\frac{\sqrt{6}}{\sqrt{3}}=\sqrt{2}$$

Area $(\Delta PQR)=\frac{1}{2}\cdot PQ\cdot QR=\sqrt{3}$

155. Answer (4)

$\because$ (2, 3, 9) (5, 2, 1), (1, λ, 8) and (λ, 2, 3) are coplanar.

$$\therefore\ \begin{vmatrix}\lambda-2 & -1 & -6\\ -1 & \lambda-3 & -1\\ 3 & -1 & -8\end{vmatrix}=0$$

$\therefore\ 8\lambda^2-67\lambda+95=0$

$\therefore$ Product of all values of $\lambda=\frac{95}{8}$

156. Answer (10)

Given $a.3+(-4a)(-1)+(-7)\,2b=0$ …(1)

and $ab-4a^2+14=0$ …(2)

$\Rightarrow\ a^2=4$ and $b^2=1$

$$\therefore\ \text{Line } L\equiv\frac{x+1}{5}=\frac{y-2}{3}=\frac{z}{1}=\lambda\ \text{(say)}$$

$\Rightarrow$ General point on line is $(5\lambda-1,\ 3\lambda+2,\ \lambda)$

for finding point of intersection with $x-y+z=0$

we get $(5\lambda-1)-(3\lambda+2)+(\lambda)=0$

$\Rightarrow\ 3\lambda-3=0\ \ \Rightarrow\lambda=1$

$\therefore$ Point at intersection (4, 5, 1)

$\therefore\ \alpha+\beta+\gamma=4+5+1=10$

157. Answer (2*)

Normal to plane $=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & a & -1\\ -1 & 1 & -a\end{vmatrix}$

$=\hat{i}(1-a^2)-\hat{j}(-a-1)+\hat{k}(1+a)$

$=(1-a)\hat{i}+\hat{j}+\hat{k}$

$\therefore$ Plane $(1-a)(x-1)+(y-1)+z=0$

Distance from (2, 1, 4) is $\sqrt{3}$ i.e.

$$\Rightarrow\ \left|\frac{(1-a)+0+4}{\sqrt{(1-a)^2+1+1}}\right|=\sqrt{3}$$

$\Rightarrow\ 25+a^2-10a=3a^2-6a+9$

$\Rightarrow\ 2a^2+4a-16=0$

$\Rightarrow\ a^2+2a-8=0$

$a=2$ or -4

$\therefore\ a_{max}=2$

158. Answer (1)

If $\vec{n}_1$ is a vector normal to the plane determined by $\hat{i}$ and $\hat{i}+\hat{j}$ then

$$\vec{n}_1=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & 0\\ 1 & 1 & 0\end{vmatrix}=\hat{k}$$

If $\vec{n}_2$ is a vector normal to the plane determined by $\hat{i}-\hat{j}$ and $\hat{i}+\hat{k}$ then

$$\bar{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = -\hat{i} - \hat{j} + \hat{k}$$

Vector $\vec{a}$ is parallel to $\vec{n}_1 \times \vec{n}_2$

i.e. $\vec{a}$ is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ -1 & -1 & 1 \end{vmatrix} = \hat{i} - \hat{j}$

Given $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$

Cosine of acute angle between

$\vec{a}$ and $\vec{b} = \left|\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}\right| = \dfrac{1}{\sqrt{2}}$

Obtuse angle between $\vec{a}$ and $\vec{b} = \dfrac{3\pi}{4}$

159. Answer (12)

Equation of plane containing the line

$4ax - y + 5z - 7a = 0 = 2x - 5y - z - 3$ can be written as

$4ax - y + 5z - 7a + \lambda(2x - 5y - z - 3) = 0$

$(4a + 2\lambda)\, x - (1 + 5\lambda)\, y + (5 - \lambda)\, z - (7a + 3\lambda) = 0$

Which is coplanar with the line

$$\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$$

$4(4a + 2\lambda) + (1 + 5\lambda) - (7a + 3\lambda) = 0$

$9a + 10\lambda + 1 = 0 \quad \ldots(1)$

$(4a + 2\lambda)1 + (1 + 5\lambda)\, 2 + 5 - \lambda = 0$

$4a + 11\lambda + 7 = 0 \quad \ldots(2)$

$a = 1, \lambda = -1$

Equation of plane is $x + 2y + 3z - 2 = 0$

Intersection with the line

$$\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4}$$

$(7t + 3) + 2\,(-t + 2) + 3\,(-4t + 3) - 2 = 0$

$-7t + 14 = 0$

$t = 2$

So, the required point is $(17, 0, -5)$

$\alpha + \beta + \gamma = 12$

160. Answer (3)

First plane, $P_1 = 2x - 2y + z = 0$, normal vector $\equiv \bar{n}_1 = (2, -2, 1)$

Second plane, $P_2 \equiv x - y + 2z = 4$, normal vector $\equiv \bar{n}_2 = (1, -1, 2)$

Plane perpendicular to P_1 and P_2 will have normal vector $\bar{n}_3$

Where $\bar{n}_3 = (\bar{n}_1 \times \bar{n}_2)$

Hence, $\bar{n}_3 = (-3, -3, 0)$

Equation of plane E through $P(1, -1, 1)$ and $\bar{n}_3$ as normal vector

$(x-1, y+1, z-1)\cdot(-3, -3, 0) = 0$

$\Rightarrow \quad x + y = 0 \equiv E$

Distance of $P\,Q(a, a, 2)$ from $E = \left|\dfrac{2a}{\sqrt{2}}\right|$

as given, $\left|\dfrac{2a}{\sqrt{2}}\right| = 3\sqrt{2} \Rightarrow a = \pm 3$

Hence, $Q \equiv (\pm 3, \pm 3, 2)$

Distance $7Q = \sqrt{21} \Rightarrow (PQ)^2 = 21$

161. Answer (2)

$\left(-2, \dfrac{7}{2}, \dfrac{3}{2}\right)$ satisfies the plane $P : 2x + my + nz = 4$

$-4 + \dfrac{7m}{2} + \dfrac{3n}{2} = 4 \Rightarrow 7m + 3n = 16 \qquad$ (i)

Line joining $A(-1, 4, 3)$ and $\left(-2, \dfrac{7}{2}, \dfrac{3}{2}\right)$ is perpendicular to $P : 2x + my + nz = 4$

$\dfrac{1}{2} = \dfrac{\frac{1}{2}}{m} = \dfrac{\frac{3}{2}}{n} \Rightarrow m = 1 \;\&\; n = 3$

Plane $P : 2x + y + 3z = 4$

Distance of P from $A(-1, 4, 3)$ parallel to the line

$\dfrac{x+1}{3} = \dfrac{y-4}{-1} = \dfrac{z-3}{-4} : L$

for point of intersection of P & L

$2(3r - 1) + (-r + 4) + 3(-4r + 3) = 4 \Rightarrow r = 1$

Point of intersection : $(2, 3, -1)$

Required distance $= \sqrt{3^2 + 1^2 + 4^2}$

$= \sqrt{26}$

162. Answer (125)

$L : lx - y + 3(1 - l)z = 1,\ x + 2y - z = 2$

and plane containing the line $p : 3x - 8y + 7z = 4$

Let $\vec{n}$ be the vector parallel to L.

then $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ l & -1 & 3(1-l) \\ 1 & 2 & -1 \end{vmatrix}$

$= (6l - 5)\hat{i} + (3 - 2l)\hat{j} + (2l + 1)\hat{k}$

$\because$ ***R*** containing L

$3(6l - 5) - 8(3 - 2l) + 7(2l + 1) = 0$

$18l + 16l + 14l - 15 - 24 + 7 = 0$

$\therefore\ l = \dfrac{32}{48} = \dfrac{2}{3}$

Let θ be the acute angle between L and y-axis

$$\therefore\ \cos\theta = \frac{\frac{5}{3}}{\sqrt{1 + \frac{25}{9} + \frac{49}{9}}} = \frac{5}{\sqrt{83}}$$

$\therefore\ 415\cos^2\theta = 125$

163. Answer (1)

$$L_1 : \frac{x+7}{-6} = \frac{y-6}{7} = \frac{z-0}{1}$$

Any point on it $\vec{a}_1(-7, 6, 0)$

and L_1 is parallel to $\vec{b}_1(-6, 7, 1)$

$$L_2 : \frac{x-7}{-2} = \frac{y-2}{1} = \frac{z-6}{1}$$

Any point on it, $\vec{a}_2(7, 2, 6)$

and L_2 is parallel to $\vec{b}_2(-2, 1, 1)$

Shortest distance between L_1 and L_2

$$= \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \left|\frac{(-14, 4, -6)\cdot(3, 2, 4)}{\sqrt{9 + 4 + 16}}\right|$$

$= 2\sqrt{29}.$

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