Chapter 12

Conic Sections (Parabola, Ellipse and Hyperbola)

1. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is **[AIEEE-2009]**

   (1) $x^2 + 12y^2 = 16$ (2) $4x^2 + 48y^2 = 48$

   (3) $4x^2 + 64y^2 = 48$ (4) $x^2 + 16y^2 = 16$

2. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of p is **[AIEEE-2010]**

   (1) $x = 1$ (2) $2x + 1 = 0$

   (3) $x = -1$ (4) $2x - 1 = 0$

3. The equation of the hyperbola whose foci are (–2, 0) and (2, 0) and eccentricity is 2 is given by: **[AIEEE-2011]**

   (1) $-x^2 + 3y^2 = 3$ (2) $-3x^2 + y^2 = 3$

   (3) $x^2 - 3y^2 = 3$ (4) $3x^2 - y^2 = 3$

4. **Statement 1 :** An equation of a common tangent to the hyperbola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$. **[AIEEE-2012]**

   Statement 2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

   (1) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

   (2) Statement 1 is true, Statement 2 is true, Statement 2 is **not** a correct explanation for Statement 1

   (3) Statement 1 is true, Statement 2 is false

   (4) Statement 1 is false, Statement 2 is true

5. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the original and its axes are the coordinate axes, then the equation of the ellipse is **[AIEEE-2012]**

   (1) $x^2 + 4y^2 = 8$ (2) $4x^2 + y^2 = 8$

   (3) $x^2 + 4y^2 = 16$ (4) $4x^2 + y^2 = 4$

6. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}\,x$

   Statement-I : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

   Statement-II : If the line, $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$. **[JEE (Main)-2013]**

   (1) Statement-I is true; statement-II is true; statement-II is a **correct** explanation for statement-I

   (2) Statement-I is true; statement-II is true; statement-II is **not** a correct explanation for statement-I

   (3) Statement-I is true; statement-II is false

   (4) Statement-I is false; statement-II, is true

7. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is **[JEE (Main)-2014]**

   (1) $(x^2 + y^2)^2 = 6x^2 + 2y^2$

   (2) $(x^2 + y^2)^2 = 6x^2 - 2y^2$

   (3) $(x^2 - y^2)^2 = 6x^2 + 2y^2$

   (4) $(x^2 - y^2)^2 = 6x^2 - 2y^2$

8. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is **[JEE (Main)-2014]**

   (1) $\frac{1}{8}$ (2) $\frac{2}{3}$

   (3) $\frac{1}{2}$ (4) $\frac{3}{2}$

9. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is

[JEE (Main)-2015]

(1) $\frac{27}{4}$ (2) 18

(3) $\frac{27}{2}$ (4) 27

10. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is

[JEE (Main)-2015]

(1) $x^2 = y$ (2) $y^2 = x$

(3) $y^2 = 2x$ (4) $x^2 = 2y$

11. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is

[JEE (Main)-2016]

(1) $x^2 + y^2 - x + 4y - 12 = 0$

(2) $x^2+y^2-\frac{x}{4}+2y-24=0$

(3) $x^2 + y^2 - 4x + 9y + 18 = 0$

(4) $x^2 + y^2 - 4x + 8y + 12 = 0$

12. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is

[JEE (Main)-2016]

(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\sqrt{3}$ (4) $\frac{4}{3}$

13. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is

[JEE (Main)-2017]

(1) $4x - 2y = 1$ (2) $4x + 2y = 7$

(3) $x + 2y = 4$ (4) $2y - x = 2$

14. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at (±2, 0). Then the tangent to this hyperbola at P also passes through the point

[JEE (Main)-2017]

(1) $(2\sqrt{2}, 3\sqrt{3})$

(2) $(\sqrt{3}, \sqrt{2})$

(3) $(-\sqrt{2}, -\sqrt{3})$

(4) $(3\sqrt{2}, 2\sqrt{3})$

15. Two sets A and B are as under :

$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$

$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \le 36\}$, then **[JEE (Main)-2018]**

(1) $B \subset A$

(2) $A \subset B$

(3) $A \cap B = \phi$ (an empty set)

(4) Neither $A \subset B$ nor $B \subset A$

16. Tangent and normal are drawn at $P(16, 16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is

[JEE (Main)-2018]

(1) $\frac{1}{2}$ (2) 2

(3) 3 (4) $\frac{4}{3}$

17. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point $T(0, 3)$ then the area (in sq. units) of ΔPTQ is **[JEE (Main)-2018]**

(1) $45\sqrt{5}$ (2) $54\sqrt{3}$

(3) $60\sqrt{3}$ (4) $36\sqrt{5}$

18. Let $0<\theta<\frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta}=1$ is greater than 2, then the length of its latus rectum lies in the interval

[JEE (Main)-2019]

(1) (2, 3] (2) (3/2, 2]

(3) (1, 3/2] (4) $(3, \infty)$

19. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

[JEE (Main)-2019]

(1) $(4, -4)$ (2) $(5, 2\sqrt{6})$

(3) $(6, 4\sqrt{2})$ (4) $(8, 6)$

20. Let $A(4, -4)$ and $B(9, 6)$ be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is **[JEE (Main)-2019]**

(1) 32 (2) $31\frac{3}{4}$

(3) $31\frac{1}{4}$ (4) $30\frac{1}{2}$

21. A hyperbola has its centre at the origin, passes through the point (4, 2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is **[JEE (Main)-2019]**

(1) $\frac{3}{2}$ (2) $\sqrt{3}$

(3) $\frac{2}{\sqrt{3}}$ (4) 2

22. If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a, b, c)?

[JEE (Main)-2019]

(1) $\left(\frac{1}{2}, 2, 0\right)$ (2) $\left(\frac{1}{2}, 2, 3\right)$

(3) (1, 1, 0) (4) (1, 1, 3)

23. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line $x - y = 2$ is

[JEE (Main)-2019]

(1) $x - y + 7 = 0$ (2) $x - y + 1 = 0$

(3) $x - y - 3 = 0$ (4) $x - y + 9 = 0$

24. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is

[JEE (Main)-2019]

(1) $3\sqrt{2}$ (2) $6\sqrt{3}$

(3) $2\sqrt{11}$ (4) $8\sqrt{2}$

25. Let $S = \left\{(x, y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$, where $r \neq \pm 1$. Then S represents

[JEE (Main)-2019]

(1) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when $r > 1$.

(2) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$.

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, when $0 < r < 1$.

(4) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.

26. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is

[JEE (Main)-2019]

(1) $4x + 2y + 1 = 0$

(2) $x + 2y + 4 = 0$

(3) $x - 2y + 4 = 0$

(4) $x + y + 1 = 0$

27. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve **[JEE (Main)-2019]**

(1) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$ (2) $\frac{x^2}{2} + \frac{y^2}{4} = 1$

(3) $\frac{x^2}{4} + \frac{y^2}{2} = 1$ (4) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

28. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is **[JEE (Main)-2019]**

(1) $5\sqrt{5}$

(2) $(10)^{2/3}$

(3) $5(2^{1/3})$

(4) 5

29. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is **[JEE (Main)-2019]**

(1) 2 (2) $\frac{13}{8}$

(3) $\frac{13}{6}$ (4) $\frac{13}{12}$

30. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it? **[JEE (Main)-2019]**

(1) $\left(4\sqrt{3}, 2\sqrt{3}\right)$ (2) $\left(4\sqrt{3}, 2\sqrt{2}\right)$

(3) $\left(4\sqrt{2}, 2\sqrt{2}\right)$ (4) $\left(4\sqrt{2}, 2\sqrt{3}\right)$

31. Let $P(4, -4)$ and $Q(9, 6)$ be two points on the parabola, $y^2 = 4x$ and let X be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum. Then this maximum area (in sq. units) is **[JEE (Main)-2019]**

(1) $\frac{75}{2}$ (2) $\frac{125}{4}$

(3) $\frac{625}{4}$ (4) $\frac{125}{2}$

32. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12 - x^2$ such that the rectangle lies inside the parabola, is **[JEE (Main)-2019]**

(1) 32 (2) 36

(3) $20\sqrt{2}$ (4) $18\sqrt{3}$

33. If the vertices of a hyperbola be at $(-2, 0)$ and $(2, 0)$ and one of its foci be at $(-3, 0)$, then which one of the following points does not lie on this hyperbola? **[JEE (Main)-2019]**

(1) $\left(4, \sqrt{15}\right)$ (2) $\left(6, 5\sqrt{2}\right)$

(3) $\left(2\sqrt{6}, 5\right)$ (4) $\left(-6, 2\sqrt{10}\right)$

34. Let S and S' be the foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area $(\Delta S'BS)$ = 8 sq. units, then the length of a latus rectum of the ellipse is **[JEE (Main)-2019]**

(1) $4\sqrt{2}$ (2) 4

(3) $2\sqrt{2}$ (4) 2

35. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points $(1, 2)$ and (a, b) are perpendicular to each other, then a^2 is equal to **[JEE (Main)-2019]**

(1) $\frac{64}{17}$ (2) $\frac{2}{17}$

(3) $\frac{4}{17}$ (4) $\frac{128}{17}$

36. Let $O(0, 0)$ and $A(0, 1)$ be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is **[JEE (Main)-2019]**

(1) $8x^2 - 9y^2 + 9y = 18$

(2) $9x^2 + 8y^2 - 8y = 16$

(3) $9x^2 - 8y^2 + 8y = 16$

(4) $8x^2 + 9y^2 - 9y = 18$

37. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $\left(0, 5\sqrt{3}\right)$, then the length of its latus rectum is **[JEE (Main)-2019]**

(1) 5 (2) 6

(3) 8 (4) 10

38. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point **[JEE (Main)-2019]**

(1) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (2) $\left(-\frac{1}{3}, \frac{4}{3}\right)$

(3) $\left(\frac{1}{4}, \frac{3}{4}\right)$ (4) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

39. If the eccentricity of the standard hyperbola passing through the point $(4, 6)$ is 2, then the equation of the tangent to the hyperbola at $(4, 6)$ is **[JEE (Main)-2019]**

(1) $2x - 3y + 10 = 0$

(2) $x - 2y + 8 = 0$

(3) $3x - 2y = 0$

(4) $2x - y - 2 = 0$

40. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is **[JEE (Main)-2019]**

(1) 24 (2) 20

(3) 22 (4) 25

41. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is **[JEE (Main)-2019]**

(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{3}{\sqrt{5}}$

(3) $\frac{\sqrt{15}}{2}$ (4) $\frac{\sqrt{5}}{2}$

42. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is **[JEE (Main)-2019]**

(1) $4\pi\left(3+\sqrt{2}\right)$ (2) $8\pi\left(2-\sqrt{2}\right)$

(3) $8\pi\left(3-2\sqrt{2}\right)$ (4) $4\pi\left(2-\sqrt{2}\right)$

43. If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to **[JEE (Main)-2019]**

(1) $\sqrt{2}-1$ (2) $\sqrt{2}+1$

(3) $2\sqrt{2}+1$ (4) $2\sqrt{2}-1$

44. If a directrix of a hyperbola centred at the origin and passing through the point $\left(4, -2\sqrt{3}\right)$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then : **[JEE (Main)-2019]**

(1) $4e^4 + 8e^2 - 35 = 0$

(2) $4e^4 - 24e^2 + 35 = 0$

(3) $4e^4 - 12e^2 - 27 = 0$

(4) $4e^4 - 24e^2 + 27 = 0$

45. If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $\left(3, \frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is **[JEE (Main)-2019]**

(1) 5 (2) $8\sqrt{3}$

(3) $12\sqrt{2}$ (4) 9

46. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is **[JEE (Main)-2019]**

(1) $\left(\frac{5}{3}, 0\right)$ (2) $\left(-\frac{5}{3}, 0\right)$

(3) (–5, 0) (4) (5, 0)

47. If the line $ax + y = c$, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then $|c|$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$

(3) 2 (4) $\sqrt{2}$

48. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point $P(2, 2)$ meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is **[JEE (Main)-2019]**

(1) $\frac{16}{3}$ (2) $\frac{14}{3}$

(3) $\frac{34}{15}$ (4) $\frac{68}{15}$

49. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through $Q(4, 4)$ then PQ is equal to **[JEE (Main)-2019]**

(1) $\frac{\sqrt{61}}{2}$ (2) $\frac{5\sqrt{5}}{2}$

(3) $\frac{\sqrt{157}}{2}$ (4) $\frac{\sqrt{221}}{2}$

50. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio **[JEE (Main)-2019]**

(1) 13 : 11 (2) 14 : 13

(3) 5 : 4 (4) 2 : 1

51. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points? **[JEE (Main)-2019]**

(1) $\left(\sqrt{2}, 2\right)$ (2) $\left(2, 2\sqrt{2}\right)$

(3) $\left(1, 2\sqrt{2}\right)$ (4) $\left(2, \sqrt{2}\right)$

52. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$, is **[JEE (Main)-2019]**

(1) $x + y + 4 = 0$

(2) $2x - y + 2 = 0$

(3) $x - 2y + 16 = 0$

(4) $x - y + 4 = 0$

53. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to

[JEE (Main)-2020]

(1) –64 (2) 128

(3) –32 (4) –128

54. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is

[JEE (Main)-2020]

(1) $\frac{3}{\sqrt{2}}$ (2) $\sqrt{3}$

(3) $3\sqrt{2}$ (4) $2\sqrt{3}$

55. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in R$, then the distance between the foci of the ellipse is

[JEE (Main)-2020]

(1) $2\sqrt{5}$ (2) $2\sqrt{7}$

(3) 4 (4) $2\sqrt{2}$

56. The locus of a point which divides the line segment joining the point (0, –1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1 : 2, is

[JEE (Main)-2020]

(1) $9x^2 - 12y = 8$

(2) $4x^2 - 3y = 2$

(3) $x^2 - 3y = 2$

(4) $9x^2 - 3y = 2$

57. Let the line $y = mx$ and the ellipse $2x^2 + y^2 = 1$ intersect at a point P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}}, 0\right)$ and $(0, \beta)$, then β is equal to

[JEE (Main)-2020]

(1) $\frac{\sqrt{2}}{3}$ (2) $\frac{2}{3}$

(3) $\frac{2}{\sqrt{3}}$ (4) $\frac{2\sqrt{2}}{3}$

58. The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2, 2) is

[JEE (Main)-2020]

(1) $2\sqrt{2}$ (2) $\sqrt{2}$

(3) $4\sqrt{2}$ (4) 2

59. If a hyperbola passes through the point $P(10, 16)$ and it has vertices at $(\pm 6, 0)$, then the equation of the normal to it at P is

[JEE (Main)-2020]

(1) $x + 2y = 42$ (2) $2x + 5y = 100$

(3) $x + 3y = 58$ (4) $3x + 4y = 94$

60. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to

[JEE (Main)-2020]

(1) 14 (2) 15

(3) 17 (4) 16

61. The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$; then its eccentricity is **[JEE (Main)-2020]**

(1) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (2) $\frac{1}{2}\sqrt{\frac{5}{3}}$

(3) $\sqrt{\frac{5}{6}}$ (4) $\frac{1}{2}\sqrt{\frac{11}{3}}$

62. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2}, -2\right)$, then the equation of the tangent to it at B is **[JEE (Main)-2020]**

(1) $x - 2y + 8 = 0$ (2) $x + 2y + 8 = 0$

(3) $2x - y - 24 = 0$ (4) $2x + y - 24 = 0$

63. A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to

[JEE (Main)-2020]

(1) 8 (2) 6

(3) 10 (4) 5

64. For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2\sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is

[JEE (Main)-2020]

(1) $2\sqrt{6}$ (2) $\frac{2\sqrt{5}}{3}$

(3) $\frac{4\sqrt{5}}{3}$ (4) $\sqrt{30}$

65. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is

[JEE (Main)-2020]

(1) $64\sqrt{3}$ (2) $256\sqrt{3}$

(3) $128\sqrt{3}$ (4) $192\sqrt{3}$

66. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then **[JEE (Main)-2020]**

(1) $MQ = \frac{1}{4}$ (2) $PN = 3$

(3) $PN = 4$ (4) $MQ = \frac{1}{3}$

67. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse of $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points?

[JEE (Main)-2020]

(1) $\left(-\sqrt{\frac{3}{2}}, 1\right)$ (2) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$

(3) $\left(\frac{1}{\sqrt{2}}, 0\right)$ (4) $\left(1, -\frac{1}{\sqrt{2}}\right)$

68. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ $(b < 5)$ and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively satisfying $e_1e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to **[JEE (Main)-2020]**

(1) (8, 10) (2) $\left(\frac{24}{5}, 10\right)$

(3) $\left(\frac{20}{3}, 12\right)$ (4) (8, 12)

69. Let $P(3, 3)$ be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to

[JEE (Main)-2020]

(1) (9, 3) (2) $\left(\frac{9}{2}, 3\right)$

(3) $\left(\frac{3}{2}, 2\right)$ (4) $\left(\frac{9}{2}, 2\right)$

70. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to

[JEE (Main)-2020]

(1) 135 (2) 116

(3) 126 (4) 145

71. Let $x = 4$ be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If $P(1, \beta)$, $\beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is **[JEE (Main)-2020]**

(1) $7x - 4y = 1$ (2) $4x - 2y = 1$

(3) $4x - 3y = 2$ (4) $8x - 2y = 5$

72. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to **[JEE (Main)-2020]**

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

73. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point $Q(0, -4)$, then PQ^2 is equal to **[JEE (Main)-2020]**

(1) 29 (2) 48

(3) 21 (4) 36

74. If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true ? **[JEE (Main)-2020]**

(1) $5m = 4$ (2) $8m + 5 = 0$

(3) $c^2 = 369$ (4) $4c^2 = 369$

75. Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line

[JEE (Main)-2020]

(1) $2x + 1 = 0$ (2) $x + 3 = 0$

(3) $x + 2y = 0$ (4) $x + 2 = 0$

76. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4} + \frac{y^2}{2} = 1$ from any of its foci?

[JEE (Main)-2020]

(1) (1, 2) (2) $\left(-2, \sqrt{3}\right)$

(3) $\left(-1, \sqrt{3}\right)$ (4) $\left(-1, \sqrt{2}\right)$

77. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies

[JEE (Main)-2020]

(1) $e^2 + 2e - 1 = 0$

(2) $e^2 + e - 1 = 0$

(3) $e^4 + 2e^2 - 1 = 0$

(4) $e^4 + e^2 - 1 = 0$

78. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y-axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, then $|m|$ is equal to ___________.

[JEE (Main)-2020]

79. Let a line $y = mx$ $(m > 0)$ intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q. If area (ΔOPQ) = 4 sq. units, then m is equal to_________. **[JEE (Main)-2020]**

80. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is ________.

[JEE (Main)-2020]

81. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is **[JEE (Main)-2021]**

(1) $x = a$ (2) $x = \frac{a}{2}$

(3) $x = 0$ (4) $x = -\frac{a}{2}$

82. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are : **[JEE (Main)-2021]**

(1) (–2, 8) (2) (1, 5)

(3) (3, 13) (4) (2, 8)

83. If the curves, $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$ intersect each other at an angle of 90º, then which of the following relations is TRUE?

[JEE (Main)-2021]

(1) $a - c = b + d$ (2) $a + b = c + d$

(3) $a - b = c - d$ (4) $ab = \frac{c + d}{a + b}$

84. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?

[JEE (Main)-2021]

(1) (0, 3) (2) (–6, 0)

(3) (4, 5) (4) (5, 4)

85. The locus of the point of intersection of the lines $\left(\sqrt{3}\right)kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4\left(\sqrt{3}\right)k = 0$ is a conic, whose eccentricity is ____________.

[JEE (Main)-2021]

86. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is : **[JEE (Main)-2021]**

(1) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (2) $x^2 - y^2 = 9$

(3) $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (4) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

87. If the curve $x^2+2y^2=2$ intersects the line $x+y=1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is : **[JEE (Main)-2021]**

(1) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)$ (2) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{3}\right)$

(3) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{3}\right)$ (4) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$

88. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is : **[JEE (Main)-2021]**

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$

(3) 0 (4) $\frac{1}{2\sqrt{2}}$

89. A line is a common tangent to the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then 2(a + c) is equal to _________. **[JEE (Main)-2021]**

90. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to : **[JEE (Main)-2021]**

(1) $\frac{1}{3}$ (2) 1

(3) $\frac{1}{4}$ (4) $\frac{1}{2}$

91. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is _________. **[JEE (Main)-2021]**

92. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point (a, 0) $a \neq 0$, then 'a' must be greater than **[JEE (Main)-2021]**

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$

(3) 1 (4) –1

93. The locus of the mid-points of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9}-\frac{y^2}{16}=1$ is **[JEE (Main)-2021]**

(1) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

(2) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

(3) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

(4) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

94. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line y = x. Then the equation of tangent to C at P(2, 1) is : **[JEE (Main)-2021]**

(1) $x + 2y = 4$ (2) $2x + y = 5$

(3) $x - y = 1$ (4) $x + 3y = 5$

95. If the points of intersections of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to : **[JEE (Main)-2021]**

(1) 5 (2) 6

(3) 12 (4) 10

96. The line $2x - y + 1 = 0$ is a tangent to the circle at the point (2, 5) and the centre of the circle lies on $x - 2y = 4$. Then, the radius of the circle is **[JEE (Main)-2021]**

(1) $3\sqrt{5}$ (2) $5\sqrt{3}$

(3) $4\sqrt{5}$ (4) $5\sqrt{4}$

97. Let L be a tangent line to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{b}=1$, then the value of b is equal to : **[JEE (Main)-2021]**

(1) 11 (2) 16

(3) 14 (4) 20

98. A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is _______. **[JEE (Main)-2021]**

99. Let a tangent be drawn to the ellipse $\frac{x^2}{27}+y^2=1$ at $\left(3\sqrt{3}\cos\theta,\sin\theta\right)$ where $\theta\in\left(0,\frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to : **[JEE (Main)-2021]**

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{8}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

100. Consider a hyperbola H : $x^2 - 2y^2 = 4$. Let the tangent at a point $P\left(4,\sqrt{6}\right)$ meet the x-axis at Q and latus rectum at $R(x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to. **[JEE (Main)-2021]**

(1) $\sqrt{6}-1$ (2) $4\sqrt{6}$

(3) $4\sqrt{6}-1$ (4) $\frac{7}{\sqrt{6}}-2$

101. Let the tangent to the parabola S : $y^2 = 2x$ at the point P(2, 2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to

[JEE (Main)-2021]

(1) $\frac{25}{2}$ (2) $\frac{15}{2}$

(3) $\frac{35}{2}$ (4) 25

102. Let $y = mx + c$, $m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m+c)$ is equal to _______.

[JEE (Main)-2021]

103. Let P be a variable point on the parabola $y = 4x^2 + 1$. Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line $y = x$ is :

[JEE (Main)-2021]

(1) $(3x - y)^2 + 2(x - 3y) + 2 = 0$

(2) $2(3x - y)^2 + (x - 3y) + 2 = 0$

(3) $2(x - 3y)^2 + (3x - y) + 2 = 0$

(4) $(3x - y)^2 + (x - 3y) + 2 = 0$

104. Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is

[JEE (Main)-2021]

(1) $\frac{-1+\sqrt{6}}{2}$ (2) $\frac{-1+\sqrt{8}}{2}$

(3) $\frac{-1+\sqrt{3}}{2}$ (4) $\frac{-1+\sqrt{5}}{2}$

105. Let a line L : $2x + y = k$, $k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, the α is equal to :

[JEE (Main)-2021]

(1) –24 (2) 24

(3) 12 (4) –12

106. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is

[JEE (Main)-2021]

(1) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

(2) $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

(3) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

(4) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

107. Let a parabola P be such that its vertex and focus lie on the positive x-axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from O(0, 0) to the parabola P which meet P at S and R, then the area (in sq. units) of ΔSOR is equal to **[JEE (Main)-2021]**

(1) 16 (2) 32

(3) $16\sqrt{2}$ (4) $8\sqrt{2}$

108. Let the foot of perpendicular from a point P(1, 2, –1) to the straight line L : $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane $x + y + 2z = 0$ which meets L at point Q. If α is the acute angle between the lines PN and PQ, then $\cos\alpha$ is equal to_______. **[JEE (Main)-2021]**

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{2\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{\sqrt{5}}$

109. If a tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point **[JEE (Main)-2021]**

(1) (–1, 1) (2) $(\sqrt{3}, 0)$

(3) (1, 1) (4) $(\sqrt{2}, 0)$

110. The equation of a circle is $\mathrm{Re}(z^2) + 2(\mathrm{Im}(z^2)) + 2\mathrm{Re}(z) = 0$, where $z = x + iy$. A line which passes through the centre of the given circle and the vertex of the parabola, $x^2 - 6x - y + 13 = 0$, has y-intercept equal to _________. **[JEE (Main)-2021]**

111. A ray of light through (2, 1) is reflected at a point P on the y-axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be **[JEE (Main)-2021]**

(1) $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$

(2) $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$

(3) $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$

(4) $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$

112. Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at (3, –4), one focus at (4, –4) and one vertex at (5, –4). If $mx - y = 4$, $m > 0$ is a tangent to the ellipse E, then the value of $5m^2$ is equal to _______.

[JEE (Main)-2021]

113. If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point $(-30, 0)$ and is tangent to the parabola $y^2 = 30x$, then the length of this chord is **[JEE (Main)-2021]**

(1) $3\sqrt{5}$ (2) $5\sqrt{3}$

(3) 7 (4) 5

114. On the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$, let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x + 2y = 0$. Let S and S′ be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS′, then the value of $(5 - e^2)\cdot A$ is **[JEE (Main)-2021]**

(1) 12 (2) 14

(3) 6 (4) 24

115. The point $P\left(-2\sqrt{6}, \sqrt{3}\right)$ lies on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the points Q and R respectively, then QR is equal to

[JEE (Main)-2021]

(1) $3\sqrt{6}$ (2) 6

(3) $6\sqrt{3}$ (4) $4\sqrt{3}$

116. The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is **[JEE (Main)-2021]**

(1) $y^2(x - 2) = x^3$ (2) $y^3(x - 2) = x^2$

(3) $x^3(x - 2) = y^2$ (4) $x^2(x - 2) = y^3$

117. A tangent and a normal are drawn at the point $P(2, -4)$ on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If $Q(a, b)$ is a point such that $AQBP$ is a square, then $2a + b$ is equal to

[JEE (Main)-2021]

(1) –18 (2) –12

(3) –16 (4) –20

118. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2}+\frac{y^2}{4a^2}=1$ and the co-ordinate axis is kab, then k is equal to _____.

[JEE (Main)-2021]

119. If two tangents drawn from a point P to the parabola $y^2 = 16(x - 3)$ are at right angles, then the locus of point P is **[JEE (Main)-2021]**

(1) $x + 2 = 0$ (2) $x + 4 = 0$

(3) $x + 1 = 0$ (4) $x + 3 = 0$

120. Let z_1 and z_2 be two complex numbers such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the question $|z - 3| = \text{Re}(z)$. Then the imaginary part of $z_1 + z_2$ is equal to ________ . **[JEE (Main)-2021]**

121. The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves? **[JEE (Main)-2021]**

(1) $25x^2 + 12y^2 = 3600$

(2) $144x^2 + 25y^2 = 3600$

(3) $x^2 + y^2 = 169$

(4) $x^2 + y^2 = 60$

122. The length of the latus rectum of a parabola, whose vertex and focus are on the positive x-axis at a distance R and S $(>R)$ respectively from origin, is

[JEE (Main)-2021]

(1) $2(S - R)$ (2) $4(S - R)$

(3) $2(S + R)$ (4) $4(S + R)$

123. The locus of mid-points of the line segments joining $(-3, -5)$ and the points on the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is

[JEE (Main)-2021]

(1) $36x^2 + 16y^2 + 90x + 56y + 145 = 0$

(2) $9x^2 + 4y^2 + 18x + 8y + 145 = 0$

(3) $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

(4) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

124. A tangent line L is drawn at the point $(2, -4)$ on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then 'a' is equal to ______.

[JEE (Main)-2021]

125. Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9}+\frac{y^2}{1}=1$ and the circle $x^2 + y^2 = 3$ at their point of intersection in the first quadrant. Then $\tan\theta$ is equal to : **[JEE (Main)-2021]**

(1) $\frac{2}{\sqrt{3}}$ (2) 2

(3) $\frac{5}{2\sqrt{3}}$ (4) $\frac{4}{\sqrt{3}}$

126. Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to

[JEE (Main)-2021]

(1) $\frac{15}{2}$ (2) $\frac{125}{16}$

(3) $\frac{75}{8}$ (4) $\frac{25}{2}$

127. Let an ellipse E: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a^2 > b^2$, passes through $\left(\sqrt{\frac{3}{2}},1\right)$, and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus F(α, 0), α > 0, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to **[JEE (Main)-2021]**

(1) 3 (2) $\frac{16}{3}$

(3) $\frac{8}{3}$ (4) $\frac{4}{3}$

128. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is

[JEE (Main)-2021]

(1) $\sqrt{3}y = x+3$ (2) $2\sqrt{3}y = 12x+1$

(3) $\sqrt{3}y = 3x+1$ (4) $2\sqrt{3}y = -x-12$

129. Let the latus ractum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is

[JEE (Main)-2021]

(1) 8

(2) 12

(3) $8\sqrt{5}$

(4) $4\sqrt{5}$

130. The centre of the circle passing through the point (0, 1) and touching the parabola $y = x^2$ at the point (2, 4) is **[JEE (Main)-2021]**

(1) $\left(\frac{-53}{10}, \frac{16}{5}\right)$ (2) $\left(\frac{6}{5}, \frac{53}{10}\right)$

(3) $\left(\frac{-16}{5}, \frac{53}{10}\right)$ (4) $\left(\frac{3}{10}, \frac{16}{5}\right)$

131. If the co-ordinates of two points A and B are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ respectively and P is any point on the conic, $9x^2 + 16y^2 = 144$, then $PA + PB$ is equal to **[JEE (Main)-2021]**

(1) 9

(2) 16

(3) 6

(4) 8

132. Let $x^2+y^2+Ax+By+C=0$ be a circle passing through (0, 6) and touching the parabola $y = x^2$ at (2, 4). Then $A + C$ is equal to ________.

[JEE (Main)-2022]

(1) 16 (2) $\frac{88}{5}$

(3) 72 (4) –8

133. Let $\lambda x - 2y = \mu$ be a tangent to the hyperbola $a^2x^2 - y^2 = b^2$. Then $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is equal to :

[JEE (Main)-2022]

(1) –2 (2) –4

(3) 2 (4) 4

134. If two tangents drawn from a point (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ to the parabola $y^2 = 4x$ are such that the slope of one tangent is four times the other, then the value of $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$ equals ____________.

[JEE (Main)-2022]

135. A particle is moving in the xy-plane along a curve C passing through the point (3, 3). The tangent to the curve C at the point P meets the x-axis at Q. If the y-axis bisects the segment PQ, then C is a parabola with **[JEE (Main)-2022]**

(1) Length of latus rectum 3

(2) Length of latus rectum 6

(3) Focus$\left(\frac{4}{3},0\right)$

(4) Focus$\left(0,\frac{3}{4}\right)$

136. Let the maximum area of the triangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{4}=1,\ a>2,$ having one of its vertices at one end of the major axis of the ellipse and one of its sides parallel to the y-axis, be $6\sqrt{3}$. Then the eccentricity of the ellipse is

[JEE (Main)-2022]

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{4}$

137. Let the hyperbola $H:\frac{x^2}{a^2}-y^2=1$ and the ellipse $E:3x^2+4y^2=12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If e_H and e_E are the eccentricities of H and E respectively, then the value of $12\left(e_H^2+e_E^2\right)$ is equal to ____. [JEE (Main)-2022]

138. Let P_1 be a parabola with vertex (3, 2) and focus (4, 4) and P_2 be its mirror image with respect to the line $x + 2y = 6$. Then the directrix of P_2 is $x + 2y =$ ________. [JEE (Main)-2022]

139. If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1m_2|$ is equal to :

[JEE (Main)-2022]

(1) $3+4\sqrt{2}$ (2) $-5+6\sqrt{2}$

(3) $-4+3\sqrt{2}$ (4) $7+6\sqrt{2}$

140. Let $x=2t,\ y=\frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that $SA \perp BA$, where A is any point on the conic. If k is the ordinate of the centroid of the ΔSAB, then $\lim\limits_{t\to 1} k$ is equal to [JEE (Main)-2022]

(1) $\frac{17}{18}$ (2) $\frac{19}{18}$

(3) $\frac{11}{18}$ (4) $\frac{13}{18}$

141. If the line $y = 4 + kx$, $k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is : [JEE (Main)-2022]

(1) $\frac{3}{2}$ (2) $\frac{26}{9}$

(3) $\frac{5}{2}$ (4) $\frac{23}{6}$

142. The line $y = x + 1$ meets the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to :

[JEE (Main)-2022]

(1) 20 (2) 12

(3) 11 (4) 8

143. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x+\beta y=\lambda$, then $\lambda-\beta$ is equal to ____. [JEE (Main)-2022]

144. Let the normal at the point P on the parabola $y^2 = 6x$ pass through the point (5, –8). If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is :

[JEE (Main)-2022]

(1) –3 (2) $-\frac{9}{4}$

(3) $-\frac{5}{2}$ (4) –2

145. Let the common tangents to the curves $4(x^2 + y^2) = 9$ and $y^2 = 4x$ intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and l respectively denote the eccentricity and the length of the latus rectum of this ellipse, then $\frac{l}{e^2}$ is equal to ____. [JEE (Main)-2022]

146. If m is the slope of a common tangent to the curves $\frac{x^2}{16}+\frac{y^2}{9}=1$ and $x^2 + y^2 = 12$, then $12m^2$ is equal to:

[JEE (Main)-2022]

(1) 6 (2) 9

(3) 10 (4) 12

147. The locus of the mid-point of the line segment joining the point (4, 3) and the points on the ellipse $x^2 + 2y^2 = 4$ is an ellipse with eccentricity:

[JEE (Main)-2022]

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

148. The normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{9}=1$ at the point $\left(8, 3\sqrt{3}\right)$ on it passes through the point:

[JEE (Main)-2022]

(1) $\left(15, -2\sqrt{3}\right)$ (2) $\left(9, 2\sqrt{3}\right)$

(3) $\left(-1, 9\sqrt{3}\right)$ (4) $\left(-1, 6\sqrt{3}\right)$

149. Let a line L_1 be tangent to the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ and let L_2 be the line passing through the origin and perpendicular to L_1. If the locus of the point of intersection of L_1 and L_2 is $(x^2+y^2)^2=\alpha x^2+\beta y^2$, then $\alpha+\beta$ is equal to____. [JEE (Main)-2022]

150. Let the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b,$ be $\frac{1}{4}$. If this ellipse passes through the point $\left(-4\sqrt{\frac{2}{5}}, 3\right)$, then $a^2 + b^2$ is equal to :

[JEE (Main)-2022]

(1) 29 (2) 31

(3) 32 (4) 34

150. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y=\left(x-\frac{1}{4}\right)^2+\alpha$, where $\alpha > 0$. Then $(4\alpha - 8)^2$ is equal to ___________.

[JEE (Main)-2022]

151. If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x + y - 29 = 0$, is $x^2 + ay^2 + bxy + cx + dy + k = 0$, then $a + b + c + d + k$ is equal to [JEE (Main)-2022]

(1) 575 (2) –575

(3) 576 (4) –576

152. Let the eccentricity of the hyperbola $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\sqrt{\frac{5}{2}}$ and length of its latus rectum be $6\sqrt{2}$. If $y = 2x + c$ is a tangent to the hyperbola H. then the value of c^2 is equal to [JEE (Main)-2022]

(1) 18 (2) 20

(3) 24 (4) 32

153. Let l be a line which is normal to the curve $y = 2x^2 + x + 2$ at a point P on the curve. If the point $Q(6, 4)$ lies on the line l and O is origin, then the area of the triangle OPQ is equal to ___________.

[JEE (Main)-2022]

154. Let a > 0, b > 0. Let e and l respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Let e' and l' respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2=\frac{11}{14}l$ and $(e')^2=\frac{11}{8}l'$, then the value of $77a + 44b$ is equal to : [JEE (Main)-2022]

(1) 100 (2) 110

(3) 120 (4) 130

155. If vertex of a parabola is $(2, -1)$ and the equation of its directrix is $4x - 3y = 21$, then the length of its latus rectum is : [JEE (Main)-2022]

(1) 2 (2) 8

(3) 12 (4) 16

156. Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point $(3, 0)$. Let the line segment PQ be also a focal chord of the ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is equal to [JEE (Main)-2022]

(1) $1+\sqrt{2}$ (2) $3+2\sqrt{2}$

(3) $1+2\sqrt{3}$ (4) $4+5\sqrt{3}$

157. Let H: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, a > 0, b > 0,$ be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4\left(2\sqrt{2} + \sqrt{14}\right)$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then the value of $a^2 + b^2$ is equal to ____________. [JEE (Main)-2022]

158. Let $P : y^2 = 4ax$, $a > 0$ be a parabola with focus S. Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line $y = 3x + 5$ touch the parabola P at A and B. Then the value of a for which A, B and S are collinear is [JEE (Main)-2022]

(1) 8 only (2) 2 only

(3) $\frac{1}{4}$ only (4) any $a > 0$

159. Let the equation of two diameters of a circle $x^2 + y^2 - 2x + 2fy + 1 = 0$ be $2px - y = 1$ and $2x + py = 4p$. Then the slope $m \in (0, \infty)$ of the tangent to the hyperbola $3x^2 - y^2 = 3$ passing through the centre of the circle is equal to _______.

[JEE (Main)-2022]

160. The sum of diameters of the circles that touch (i) the parabola $75x^2 = 64(5y - 3)$ at the point $\left(\frac{8}{5}, \frac{6}{5}\right)$ and (ii) the y-axis, is equal to _______.

[JEE (Main)-2022]

161. Let the tangent drawn to the parabola $y^2 = 24x$ at the point (α, β) is perpendicular to the line $2x + 2y = 5$. Then the normal to the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at the point $(\alpha + 4, \beta + 4)$ does **NOT** pass through the point [JEE (Main)-2022]

(1) (25, 10) (2) (20, 12)

(3) (30, 8) (4) (15, 13)

162. Let P and Q be any points on the curves $(x - 1)^2 + (y + 1)^2 = 1$ and $y = x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval [JEE (Main)-2022]

(1) $\left(0, \frac{1}{4}\right)$ (2) $\left(\frac{1}{2}, \frac{3}{4}\right)$

(3) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (4) $\left(\frac{3}{4}, 1\right)$

163. Let $P(a, b)$ be a point on the parabola $y^2 = 8x$ such that the tangent at P passes through the centre of the circle $x^2 + y^2 - 10x - 14y + 65 = 0$. Let A be the product of all possible values of a and B be the product of all possible values of b. Then the value of $A + B$ is equal to [JEE (Main)-2022]

(1) 0 (2) 25

(3) 40 (4) 65

164. An ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H: \frac{x^2}{49} - \frac{y^2}{64} = -1$. Let the major and minor axes of the ellipse E coincide with the transverse and conjugate axes of the hyperbola H, respectively. Let the product of the eccentricities of E and H be $\frac{1}{2}$. If the length of the latus rectum of the ellipse E, then the value of $113l$ is equal to _____.

[JEE (Main)-2022]

165. If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4, and l is the length of its major axis, then $\lambda + l$ is equal to ________.

[JEE (Main)-2022]

166. If the length of the latus rectum of a parabola, whose focus is (a, a) and the tangent at its vertex is $x + y = a$, is 16, then $|a|$ is equal to : [JEE (Main)-2022]

(1) $2\sqrt{2}$ (2) $2\sqrt{3}$

(3) $4\sqrt{2}$ (4) 4

167. A common tangent T to the curves $C_1 : \dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ and $C_2 : \dfrac{x^2}{42} - \dfrac{y^2}{143} = 1$ does not pass through the fourth quadrant. If T touches C_1 at (x_1, y_1) and C_2 at (x_2, y_2), then $|2x_1 + x_2|$ is equal to ______.

[JEE (Main)-2022]

168. If the tangents drawn at the points P and Q on the parabola $y^2 = 2x - 3$ intersect at the point $R(0, 1)$, then the orthocentre of the triangle PQR is :

[JEE (Main)-2022]

(1) (0, 1) (2) (2, –1)

(3) (6, 3) (4) (2, 1)

169. For the hyperbola H: $x^2 - y^2 = 1$ and the ellipse $\text{E} : \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, a > b > 0,$ let the

(1) eccentricity of E be reciprocal of the eccentricity of H, and

(2) the line $y = \sqrt{\dfrac{5}{2}}x + K$ be a common tangent of E and H.

Then $4(a^2 + b^2)$ is equal to _______.

[JEE (Main)-2022]

170. Let the hyperbola $H : \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ pass through the point $(2\sqrt{2}, -2\sqrt{2})$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola?

[JEE (Main)-2022]

(1) $(2\sqrt{3}, 3\sqrt{2})$ (2) $(3\sqrt{3}, -6\sqrt{2})$

(3) $(\sqrt{3}, -\sqrt{6})$ (4) $(3\sqrt{6}, 6\sqrt{2})$

171. Let the tangents at the points P and Q on the ellipse $\dfrac{x^2}{2} + \dfrac{y^2}{4} = 1$ meet at the point $R\left(\sqrt{2}, 2\sqrt{2} - 2\right)$. If S is the focus of the ellipse on its negative major axis, then $SP^2 + SQ^2$ is equal to ________.

[JEE (Main)-2022]

172. Two tangent lines l_1 and l_2 are drawn from the point (2, 0) to the parabola $2y^2 = -x$. If the lines l_1 and l_2 are also tangent to the circle $(x - 5)^2 + y^2 = r$, then $17r$ is equal to _________. [JEE (Main)-2022]

173. Let a line L pass through the point intersection of the lines $bx + 10y - 8 = 0$ and $2x - 3y = 0$, $b \in \boldsymbol{R} - \left\{\dfrac{4}{3}\right\}$. If the line L also passes through the point (1, 1) and touches the circle $17(x^2 + y^2) = 16$, then the eccentricity of the ellipse $\dfrac{x^2}{5} + \dfrac{y^2}{5} = 1$ is

[JEE (Main)-2022]

(1) $\dfrac{2}{\sqrt{5}}$ (2) $\sqrt{\dfrac{3}{5}}$

(3) $\dfrac{1}{\sqrt{5}}$ (4) $\sqrt{\dfrac{2}{5}}$

174. Let $S = \{(x, y) \in \mathbb{N} \times \mathbb{N} : 9(x-3)^2 + 16(y-4)^2 \le 144\}$

and $T = \{(x, y) \in \mathbb{R} \times \mathbb{R} : (x-7)^2 + (y-4)^2 \le 36\}$.

Then $n(S \cap T)$ is equal to ______.

[JEE (Main)-2022]

175. If the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ meets the line $\dfrac{x}{7} + \dfrac{y}{2\sqrt{6}} = 1$ on the x-axis and the line $\dfrac{x}{7} - \dfrac{y}{2\sqrt{6}} = 1$ on the y-axis, then the eccentricity of the ellipse is

[JEE (Main)-2022]

(1) $\dfrac{5}{7}$ (2) $\dfrac{2\sqrt{6}}{7}$

(3) $\dfrac{3}{7}$ (4) $\dfrac{2\sqrt{5}}{7}$

176. If the line $x - 1 = 0$ is a directrix of the hyperbola $kx^2 - y^2 = 6$, then the hyperbola passes through the point

[JEE (Main)-2022]

(1) $(-2\sqrt{5}, 6)$ (2) $(-\sqrt{5}, 3)$

(3) $(\sqrt{5}, -2)$ (4) $(2\sqrt{5}, 3\sqrt{6})$

177. Let the foci of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{7} = 1$ and the hyperbola $\dfrac{x^2}{144} - \dfrac{y^2}{\alpha} = \dfrac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is :

[JEE (Main)-2022]

(1) $\dfrac{32}{9}$ (2) $\dfrac{18}{5}$

(3) $\dfrac{27}{4}$ (4) $\dfrac{27}{10}$

178. Let the focal chord of the parabola $P : y^2 = 4x$ along the line $L : y = mx + c$, $m > 0$ meet the parabola at the points M and N. Let the line L be a tangent to the hyperbola $H : x^2 - y^2 = 4$. If O is the vertex of P and F is the focus of H on the positive x-axis, then the area of the quadrilateral $OMFN$ is

[JEE (Main)-2022]

(1) $2\sqrt{6}$ (2) $2\sqrt{14}$

(3) $4\sqrt{6}$ (4) $4\sqrt{14}$

179. The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is **[JEE (Main)-2022]**

(1) $y = 4(x - 2)$

(2) $y = 4(x - 1)$

(3) $y = 4(x + 1)$

(4) $y = 4(x + 2)$

180. The tangents at the points $A(1, 3)$ and $B(1, -1)$ on the parabola $y^2 - 2x - 2y = 1$ meet at the point P. Then the area (in unit^2) of the triangle PAB is :

[JEE (Main)-2022]

(1) 4 (2) 6

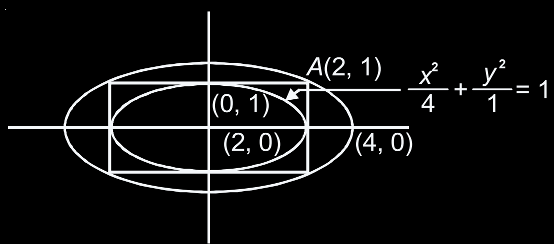
(3) 7 (4) 8

181. Let the hyperbola $H : \dfrac{x^2}{a^2} - y^2 = 1$ and the ellipse $E : 3x^2 + 4y^2 = 12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If e_H and e_E are the eccentricities of H and E respectively, then the value of $12(e_H^2 + e_E^2)$ is equal to ____. **[JEE (Main)-2022]**

Chapter 12

Conic Sections (Parabola, Ellipse and Hyperbola)

1. Answer (1)



$$\frac{x^2}{16}+\frac{y^2}{b^2}=1$$

Let the equation of the required ellipse is

But the ellipse passes through (2, 1)

$$\Rightarrow \frac{1}{4}+\frac{1}{b^2}=1$$

$$\Rightarrow \frac{1}{b^2}=\frac{3}{4}$$

$$\Rightarrow b^2=\frac{4}{3}$$

Hence equation is

$$\frac{x^2}{16}+\frac{y^2\times 3}{4}=1$$

$\Rightarrow$ $x^2 + 12y^2 = 16$

2. Answer (3)

Locus of P from which two perpendicular tangents are drawn to the parabola is the directrix of the parabola

Hence locus is, $x = -1$

3. Answer (4)

Co-ordinates of focus are $(\pm 2, 0)$

$(\pm a\,e, 0)$

$a\,e = 2$

$e = 2$

$a = 1$

For hyperbola $b^2 = a^2(e^2 - 1)$

$b^2 = 4 - 1$

$= 3$

Standard equation of hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\Rightarrow \frac{x^2}{1}-\frac{y^2}{3}=1$$

$\Rightarrow$ $3x^2 - y^2 = 3$

4. Answer (1)
5. Answer (4)
6. Answer (2)

$$x^2+y^2=\frac{5}{2}$$

$$y^2=4\sqrt{5}x$$

Equation of tangent to parabola is

$$y=mx+\frac{\sqrt{5}}{m} \quad \text{...(i)}$$

For circle,

$$y=mx\pm\frac{\sqrt{5}}{\sqrt{2}}\sqrt{1+m^2} \quad \text{...(ii)}$$

(i) and (ii) are identical,

$$\frac{5}{m^2}=\frac{5}{2}(1+m^2)$$

$2 = m^4 + m^2$

$\Rightarrow$ $m^4 + m^2 - 2 = 0$

$\Rightarrow$ $m = \pm 1$

which satisfy given equation

Statement (1) is true and statement (2) is true.

7. Answer (1)

Here ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a^2 = 6$, $b^2 = 2$

Now, equation of any variable tangent is

$$y = mx \pm \sqrt{a^2m^2+b^2} \quad ...(i)$$

where m is slope of the tangent

So, equation of perpendicular line drawn from centre to tangent is

$$y = \frac{-x}{m} \quad ...(ii)$$

Eliminating m, we get

$(x^2+y^2)^2 = a^2x^2+b^2y^2$

$\Rightarrow \boxed{(x^2+y^2)^2 = 6x^2+2y^2}$

8. Answer (3)

$y^2 = 4x$ …(1)

$x^2 = -32y$ …(2)

m be slope of common tangent

Equation of tangent (1)

$$y = mx + \frac{1}{m} \quad \text{…(i)}$$

Equation of tangent (2)

$$y = mx + 8m^2 \quad \text{…(ii)}$$

(i) and (ii) are identical

$$\frac{1}{m} = 8m^2$$

$\Rightarrow m^3 = \frac{1}{8}$

$\boxed{m=\frac{1}{2}}$

Alternative method :

Let tangent to $y^2 = 4x$ be

$$y = mx + \frac{1}{m}$$

as this is also tangent to $x^2 = -32y$

Solving $x^2 + 32mx + \frac{32}{m} = 0$

Since roots are equal

$\therefore$ $D = 0$

$\Rightarrow (32)^2 - 4\times\frac{32}{m} = 0$

$\Rightarrow m^3 = \frac{4}{32}$

$\Rightarrow m = \frac{1}{2}$

9. Answer (4)

Ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$

i.e., $a^2 = 9$, $b^2 = 5$

So, $e = \frac{2}{3}$

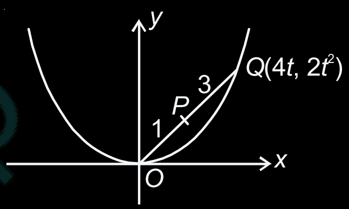
As, required area $= \frac{2a^2}{e} = \frac{2\times 9}{(2/3)} = 27$

10. Answer (4)

$x^2 = 8y$

Let Q be $(4t, 2t^2)$

$\therefore$ $P = \left(t, \frac{t^2}{2}\right)$



Let P be (h, k)

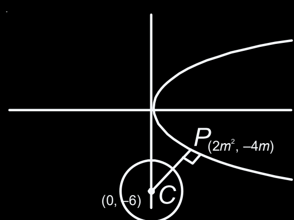
$\therefore$ $h = t$, $k = \frac{t^2}{2}$

$\therefore$ $2k = h^2$

$\therefore$ Locus of (h, k) is $x^2 = 2y$.

11. Answer (4)

Let the normal of parabola be



$y = mx - 4m - 2m^3$

$(0, -6)$ lies on it

$\therefore$ $-6 = -4m - 2m^3$

$\Rightarrow$ $m^3 + 2m - 3 = 0$

$(m-1)(m^2+m+3) = 0$

$m = 1$

$\therefore$ Point P: $(2m^2, -4m)$

$= (2, -4)$

$\therefore$ Equation of circle is

$(x-2)^2 + (y+4)^2 = (4+4)$

$\Rightarrow$ $x^2 + y^2 - 4x + 8y + 12 = 0$

12. Answer (2)

Given $\frac{2b^2}{a} = 8,\ 2b = ae$

$\frac{b}{a} = \frac{e}{2}$

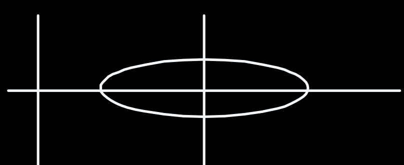
We know that $b^2 = a^2(e^2 - 1)$

$\frac{b^2}{a^2} = e^2 - 1$

$\frac{e^2}{4} = e^2 - 1,\ e^2 = \frac{4}{3}$

$e = \frac{2}{\sqrt{3}}$

13. Answer (1)



$e = \frac{1}{2}$

$\frac{-a}{e} = -4$

$-a = -4 \times e$

$\boxed{a = 2}$

Now, $b^2 = a^2(1 - e^2) = 3$

Equation to ellipse

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

Equation of normal is

$$\frac{x-1}{\frac{1}{4}} = \frac{y - \frac{3}{2}}{\frac{3}{2\times 3}} \Rightarrow 4x - 2y - 1 = 0$$

14. Answer (1)

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$a^2 + b^2 = 4$

and $\frac{2}{a^2} - \frac{3}{b^2} = 1$

$\frac{2}{4-b^2} - \frac{3}{b^2} = 1$

$\Rightarrow\ b^2 = 3$

$\therefore\ a^2 = 1$

$\therefore\ x^2 - \frac{y^2}{3} = 1$

$\therefore$ Tangent at $P(\sqrt{2}, \sqrt{3})$ is $\sqrt{2}x - \frac{y}{\sqrt{3}} = 1$

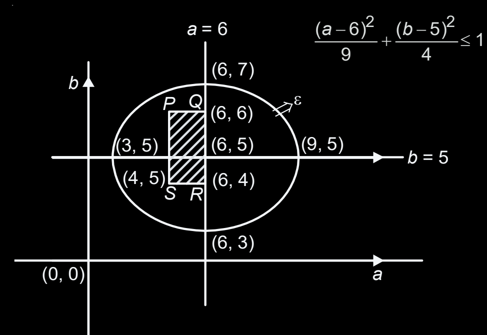
Clearly it passes through $(2\sqrt{2}, 3\sqrt{3})$

15. Answer (2)

As, $|a - 5| < 1$ and $|b - 5| < 1$

$\Rightarrow\ 4 < a, b < 6$ and $\frac{(a-6)^2}{9} + \frac{(b-5)^2}{4} \le 1$

Taking axes as *a*-axis and *b*-axis



The set *A* represents square *PQRS* inside set *B* representing ellipse and hence $A \subset B$.

16. Answer (2)

$y^2 = 16x$

Tangent at $P(16, 16)$ is $2y = x + 16$ ... (1)

Normal at $P(16, 16)$ is $y = -2x + 48$ ... (2)

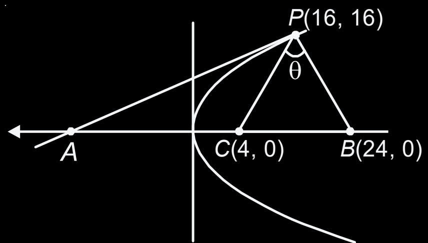
i.e., *A* is $(-16, 0)$; *B* is $(24, 0)$

Now, Centre of circle is $(4, 0)$

Now, $m_{PC} = \frac{4}{3}$

$m_{PB} = -2$

i.e., $\tan\theta = \left|\frac{\frac{4}{3}+2}{1-\frac{8}{3}}\right| = 2$



17. Answer (1)

Clearly PQ is a chord of contact,

i.e., equation of PQ is $T \equiv 0$

$\Rightarrow \; y = -12$

Solving with the curve, $4x^2 - y^2 = 36$

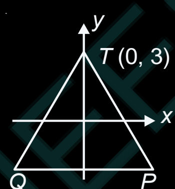
$\Rightarrow \; x = \pm 3\sqrt{5},\, y = -12$

i.e., $P(3\sqrt{5}, -12);\, Q(-3\sqrt{5}, -12);\, T(0, 3)$

Area of ΔPQT is

$\Delta = \frac{1}{2} \times 6\sqrt{5} \times 15$

$= 45\sqrt{5}$



18. Answer (4)

$a^2 = \cos^2\theta,\, b^2 = \sin^2\theta$

$e > 2 \Rightarrow 1 + b^2/a^2 > 4 \Rightarrow 1 + \tan^2\theta > 4$

$\Rightarrow \sec^2\theta > 4 = \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Latus rectum,

$$LR = \frac{2b^2}{a} = \frac{2\sin^2\theta}{\cos\theta} = 2(\sec\theta - \cos\theta)$$

$$\frac{d(LR)}{d\theta} = 2(\sec\theta\tan\theta + \sin\theta) > 0 \;\forall\theta\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

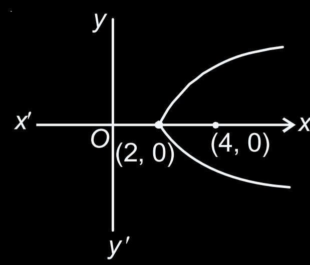
$\therefore$ min (LR) $= 2\left(\sec\frac{\pi}{3} - \cos\frac{\pi}{3}\right) = 2\left(2 - \frac{1}{2}\right) = 3$

max (LR) tends to infinity as $\theta \to \frac{\pi}{2}$

19. Answer (4)

Vertex and focus of given parabola is (2, 0) and (4, 0) respectively

Equation of parabola is

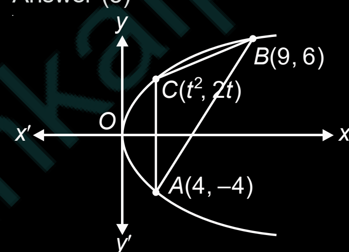


$(y - 0)^2 = 4 \times 2(x - 2)$

$y^2 = 8x - 16$

Clearly (8, 6) does not lies on given parabola.

20. Answer (3)



Let the coordinates of C is $(t^2, 2t)$.

$\therefore$ Area of ΔACB

$$= \frac{1}{2}\left\|\begin{matrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{matrix}\right\|$$

$$= \frac{1}{2}|t^2(6+4) - 2t(9-4) + 1(-36-24)|$$

$$= \frac{1}{2}|10t^2 - 10t - 60|$$

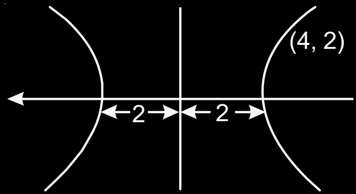
$= 5|t^2 - t - 6|$

$= 5\left|\left(t - \frac{1}{2}\right)^2 - \frac{25}{4}\right|$ [Here, $t \in (0, 3)$]

For maximum area, $t = \frac{1}{2}$

$\therefore$ Maximum area $= \frac{125}{4} = 31\frac{1}{4}$ sq. units

21. Answer (3)



Let equation of hyperbola

$$\frac{x^2}{2^2} - \frac{y^2}{b^2} = 1$$

$\because$ (4, 2) lies on hyperbola

$$\therefore \quad \frac{16}{4} - \frac{4}{b^2} = 1$$

$$\therefore \quad b^2 = \frac{4}{3}$$

$$\therefore \quad \text{Eccentricity} = \sqrt{1 + \frac{\frac{4}{3}}{4}} = \sqrt{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}}$$

22. Answer (4)

Note: x-axis is a common normal. Hence all the options are correct for $m = 0$.

Normal to $y^2 = 8ax$ is

$y = mx - 4am - 2am^3$ ...(1)

Normal to $y^2 = 4b(x - c)$ with slope m is

$y = m(x - c) - 2bm - bm^3$ ...(2)

(1) and (2) are identical

$\Rightarrow \quad 4am + 2am^3 = cm + 2bm + bm^3$

$\Rightarrow \quad 4a + 2am^2 = c + 2b + bm^2$ or $m = 0$

$\Rightarrow \quad 4a - c - 2b = (b - 2a)\, m^2$

or (X-axis is common normal always)

$$\Rightarrow \quad m^2 = \frac{4a - 2b - c}{b - 2a} = -2 - \frac{c}{b - 2a}$$

$$m^2 \geq 0 \;\Rightarrow\; -2 - \frac{c}{b - 2a} \geq 0 \;\Rightarrow\; \frac{c}{b - 2a} + 2 \leq 0$$

only (1, 1, 3) satisfies $\dfrac{c}{b - 2a} + 2 \leq 0$

23. Answer (2)

Slope, $m = 1$

$$\frac{x^2}{5} - \frac{y^2}{4} = 1$$

$a^2 = 5, \quad b^2 = 4$

$$y = mx \pm \sqrt{a^2 m^2 - b^2}$$

$$= x \pm \sqrt{5 - 4}$$

$\Rightarrow y = x \pm 1$

24. Answer (2)

For intersection point $A(x_1, y_1)$ and $B(x_2, y_2)$

$$x - \sqrt{2}\frac{x^2}{4} + 4\sqrt{2} = 0$$

$$\sqrt{2}x^2 - 4x - 16\sqrt{2} = 0$$

$$x_1 + x_2 = 2\sqrt{2}, x_1 x_2 = -16, (x_1 - x_2)^2 = 8 + 64 = 72$$

$$\because \quad x_1 - \sqrt{2}y_1 + 4\sqrt{2} = 0$$

$$x_2 - \sqrt{2}y_2 + 4\sqrt{2} = 0$$

$$(x_2 - x_1) = \sqrt{2}(y_2 - y_1) \Rightarrow (x_2 - x_1)^2 = 2(y_2 - y_1)^2$$

$$\Rightarrow \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + \frac{(x_2 - x_1)^2}{2}}$$

$$= |x_2 - x_1| \cdot \frac{\sqrt{3}}{\sqrt{2}} = 6\sqrt{2} \times \frac{\sqrt{3}}{\sqrt{2}} = 6\sqrt{3}$$

$$= 6\sqrt{2} \times \frac{\sqrt{3}}{\sqrt{2}}$$

$$= 6\sqrt{3}$$

25. Answer (2)

When $r > 1$

$$\frac{x^2}{r - 1} + \frac{y^2}{r + 1} = 1 \qquad \text{(Ellipse)}$$

$$(r - 1) = (r + 1)(1 - e^2) \quad \Rightarrow \quad 1 - e^2 = \frac{(r - 1)}{(r + 1)}$$

$$\Rightarrow \quad e^2 = 1 - \frac{(r - 1)}{(r + 1)} = \frac{2}{(r + 1)} \;\Rightarrow\; e = \sqrt{\frac{2}{(r + 1)}}$$

when $0 < r < 1$

$$\frac{x^2}{1 - r} - \frac{y^2}{1 + r} = -1 \quad \text{(Hyperbola)}$$

$$(1 - r) = (1 + r)(e^2 - 1) \Rightarrow e^2 = 1 + \frac{(r - 1)}{(r + 1)} = \frac{2r}{(r + 1)}$$

$$\Rightarrow \quad e = \sqrt{\frac{2r}{r + 1}}$$

Option (2) is correct.

26. Answer (2)

Equation of a tangent to parabola $y^2 = 4x$ is :

$$y = mx + \frac{1}{m}$$

This line is a tangent to $xy = 2$

$$\therefore \quad x\left(mx + \frac{1}{m}\right) = 2$$

$$mx^2 + \frac{1}{m}x - 2 = 0$$

$$\therefore \quad D = \left(\frac{1}{m}\right)^2 - 4 \cdot m \cdot (-2) = 0$$

$$\frac{1}{m^2} + 8m = 0$$

$$1 + 8m^3 = 0$$

$$m^3 = -\frac{1}{8}$$

$$m = -\frac{1}{2}$$

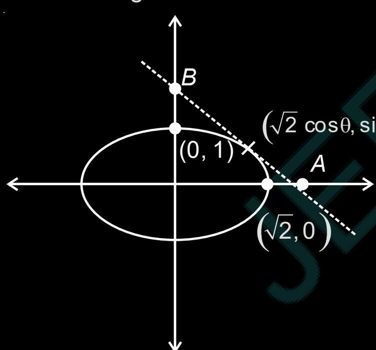
$\therefore$ Equation of common tangent: $y = -\frac{1}{2}x - 2$

$$2y = -x - 4$$

$\therefore \quad x + 2y + 4 = 0$

27. Answer (4)

Equation of tangent is



$$\frac{\sqrt{2}\cos\theta\, x}{2} + y\sin\theta = 1$$

$$A\left(\frac{\sqrt{2}}{\cos\theta}, 0\right) \text{ and } B\left(0, \frac{1}{\sin\theta}\right)$$

Let mid point be (h, k)

$$\Rightarrow \quad h = \frac{1}{\sqrt{2}\cos\theta}, k = \frac{1}{2\sin\theta}$$

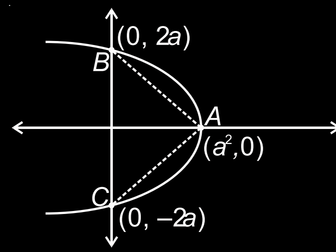
As $\cos^2\theta + \sin^2\theta = 1$

$$\therefore \quad \frac{1}{2h^2} + \frac{1}{4k^2} = 1$$

Locus is $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

28. Answer (4)

$y^2 = -4(x - a^2)$



Area $= \frac{1}{2}(4a)(a^2)$

$= 2a^3$

As $2a^2 = 250$

$\Rightarrow \quad a = 5$

29. Answer (4)

$2b = 5$

$2ae = 13$

Now, $b^2 = a^2(e^2 - 1)$

$\Rightarrow \quad a^2 = 36$

$\therefore \quad a = 6$

$$ae = \frac{13}{2} \Rightarrow e = \frac{13}{12}$$

30. Answer (2)

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Now, $\frac{2b^2}{a} = 8, \quad 2ae = b^2$ and $b^2 = a^2(1 - e^2)$

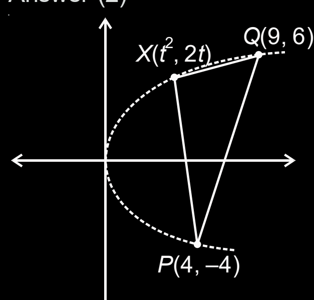
gives $a = 8$, $b^2 = 32$

$\therefore$ The equation of the ellipse is

$$\frac{x^2}{64} + \frac{y^2}{32} = 1$$

Clearly $\left(4\sqrt{3}, 2\sqrt{2}\right)$ lies on it

31. Answer (2)



$y^2 = 4x$

$$\text{Area of } \Delta PXQ = \frac{1}{2}\begin{vmatrix} t^2 & 2t & 1 \\ 4 & -4 & 1 \\ 9 & 6 & 1 \end{vmatrix}$$

$$= -5t^2 + 5t + 30$$

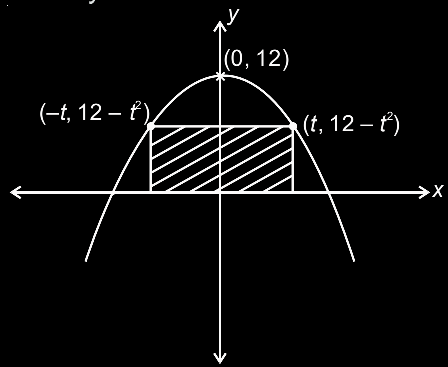
$$= -5(t^2 - t - 6)$$

$$= -5\left[\left(t - \frac{1}{2}\right)^2 - \frac{25}{4}\right]$$

$$\text{Maximum area} = 5\left(\frac{25}{4}\right) = \frac{125}{4}$$

32. Answer (1)

$x^2 = 12 - y$



Area = $(2t)(12 - t^2)$

$A = 24t - 2t^3$

$\frac{dA}{dt} = 24 - 6t^2$

$\underset{-2}{-\quad|}\quad + \quad\underset{+2}{|\quad -}$

At $t = 2$, area is maximum $= 24(2) - 2(2)^3$

$= 48 - 16 = 32$ sq. units

33. Answer (2)

$A(2, 0)$, $A'(-2, 0)$, $S(-3, 0)$

$\Rightarrow$ Centre of hyperbola is $O(0, 0)$

$AA' = 2a \Rightarrow 4 = 2a \Rightarrow a = 2$

$\therefore$ $OS = ae \Rightarrow 3 = 2e \Rightarrow e = \frac{3}{2}$

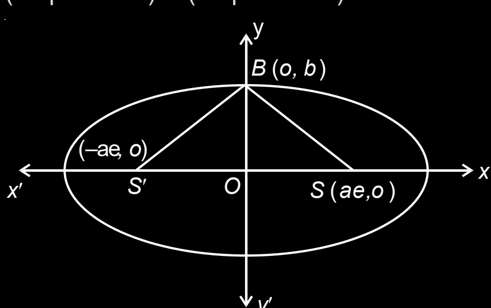
$\Rightarrow$ $b^2 = a^2(e^2 - 1) = a^2e^2 - a^2 = 9 - 4 = 5$

$\Rightarrow$ Equation of hyperbola is $\frac{x^2}{4} - \frac{y^2}{5} = 1$ …(i)

$\left(6, 5\sqrt{2}\right)$ does not lie on (i)

34. Answer (2)

(Slope of BS) × (Slope of BS′) = –1



$\frac{b}{-ae} \times \frac{b}{ae} = -1$

$b^2 = a^2e^2$ …(i)

$\because$ Area g ΔSBS′ = 8

$\Rightarrow$ $\frac{1}{2} \cdot 2ae \cdot b = 8$

$b^2 = 8$ …(ii)

$\because$ $a^2e^2 = 8$

$\because$ $e^2 = 1 - \frac{b^2}{a^2}$

$a^2e^2 = a^2 - b^2$

$8 = a^2 - 8$

$a^2 = 16$

$\therefore$ Length of latus rectum $= \frac{2b^2}{a} = \frac{2.8}{4} = 4$ units

35. Answer (2)

Equation of tangent at $A(1, 2)$;

$4x + 2y = 8 \Rightarrow 2x + y = 4$

So tangent at $B(a, b)$ can be assumed as

$x - 2y = c \Rightarrow y = \frac{1}{2}x - \frac{c}{2}$

Condition for tangency;

$$-\frac{c}{2} = \pm\sqrt{2\left(\frac{1}{2}\right)^2 + 8} = \pm\sqrt{\frac{17}{2}}$$

$\Rightarrow$ $c = \pm\sqrt{34}$

Equation of tangent; $x - 2y = \pm\sqrt{34}$ ...(i)

Equation of tangent at $P(a, b)$; $4ax + by = 8$...(ii)

Comparing both the equations;

$$\frac{4a}{1} = \frac{b}{-2} = \frac{8}{\pm\sqrt{34}}$$

$\Rightarrow$ $a = \pm\frac{2}{\sqrt{34}}$ $\Rightarrow$ $a^2 = \frac{2}{17}$

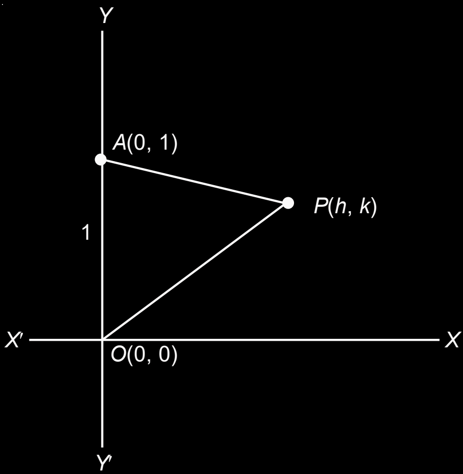
36. Answer (2)

Let point $P(h, k)$.

$\because$ $OA = 1$

So, $OP + AP = 3$

$$\sqrt{h^2 + k^2} + \sqrt{h^2 + (k-1)^2} = 3$$



$\Rightarrow\ h^2+(k-1)^2=9+h^2+k^2-6\sqrt{h^2+k^2}$

$\Rightarrow\ 6\sqrt{h^2+k^2}=2k+8$

$\Rightarrow\ 9(h^2+k^2)=k^2+16+8k$

$\Rightarrow\ 9h^2+8k^2-8k-16=0$

Locus of point P will be,

$9x^2+8y^2-8y-16=0$

37. Answer (1)

$\because$ Focus is $(0, 5\sqrt{3})$ $\Rightarrow |b|>|a|$

Let $b > a > 0$

foci $(0, \pm be)$

$a^2 = b^2 - b^2e^2$ $\Rightarrow b^2e^2 = b^2 - a^2$

$be=\sqrt{b^2-a^2}$

$\Rightarrow$ $b^2 - a^2 = 75$ …(i)

$2b - 2a = 10$

$\Rightarrow$ $b - a = 5$ …(ii)

From (i) and (ii),

$b + a = 15$ …(iii)

$\Rightarrow$ $b = 10,\ a = 5$

Length of L·R· $=\dfrac{2a^2}{b}=\dfrac{50}{10}=5$

38. Answer (1)

Intersection point of

$x^2 + y^2 = 5,\quad y^2 = 4x$

$\Rightarrow$ $x^2 + 4x - 5 = 0$

$\Rightarrow$ $x^2 + 5x - x - 5 = 0$

$\Rightarrow$ $x(x + 5) -1(x + 5) = 0$

$\therefore$ $x = 1, -5$

Intersection point in 1st quadrant be (1, 2)

equation of tangent to $y^2 = 4x$ at (1, 2) is

$y \times 2 = 2(x + 1)$

$\Rightarrow$ $y = x + 1$

$\Rightarrow$ $x - y + 1 = 0$ …(i)

$\left(\dfrac{3}{4},\dfrac{7}{4}\right)$ lies on (i)

39. Answer (4)

Let equation of hyperbola be

$\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ ...(i)

$\Rightarrow$ $b^2 = a^2(e^2 - 1)$

$\because$ $e = 2$ $\Rightarrow$ $\boxed{b^2=3a^2}$ ...(ii)

(i) passes through (4, 6),

$\Rightarrow\ \dfrac{16}{a^2}-\dfrac{36}{b^2}=1$ ...(iii)

From (ii) and (iii),

$a^2 = 4,\ b^2 = 12$

$\Rightarrow$ Equation of hyperbola is $\dfrac{x^2}{4}-\dfrac{y^2}{12}=1$

Equation of tangent to the hyperbola at (4, 6) is

$\dfrac{4x}{4}-\dfrac{6y}{12}=1$

$\Rightarrow\ x-\dfrac{y}{2}=1$

$\Rightarrow$ $2x - y = 2$

40. Answer (4)

$\because$ $y^2 = 16x$ $\Rightarrow a = 4$

One end of focal chord (1, 4) $\therefore$ $2at = 4$

$\Rightarrow t=\dfrac{1}{2}$

Length of focal chord $=a\left(t+\dfrac{1}{t}\right)^2$

$=4\times\left(2+\dfrac{1}{2}\right)^2$

$= 25$

41. Answer (1)

$mx - y + 7\sqrt{3}$ is normal to hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$

then $\frac{24}{m^2} - \frac{18}{(-1)^2} = \frac{(24+18)^2}{(7\sqrt{3})^2}$

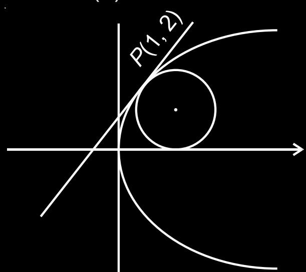
$\Rightarrow \quad \frac{24}{m^2} - 18 = \frac{42 \times 42}{7 \times 7 \times 3}$

$\Rightarrow \quad m = \frac{2}{\sqrt{5}}$

If $lx + my + n = 0$ is a normal to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

then $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

42. Answer (3)



The circle and parabola will have common tangent at $P(1, 2)$.

$\therefore$ Equation of tangent to parabola

$\equiv y \times (2) = 4\frac{(x+1)}{2} \Rightarrow 2y = 2x + 2$

$y = x + 1$

Let equation of circle be (using family of circles)

$(x - x_1)^2 + (y - y_1)^2 + \lambda T = 0$

$\Rightarrow \quad c \equiv (x - 1)^2 + (y - 2)^2 + \lambda(x - y + 1) = 0$

Also circle touches x-axis $\Rightarrow$ y-coordinate of centre = radius

$\Rightarrow \quad c \equiv x^2 + y^2 + (\lambda - 2)x + (-\lambda - 4)y + (\lambda + 5) = 0$

$$\frac{\lambda+4}{2} = \sqrt{\left(\frac{\lambda-2}{2}\right)^2 + \left(\frac{-\lambda-4}{2}\right)^2 - (\lambda+5)}$$

$\Rightarrow \quad \frac{\lambda^2 - 4\lambda + 4}{4} = \lambda + 5 \Rightarrow \lambda^2 - 4\lambda + 4 = 4\lambda + 20$

$\Rightarrow \quad \lambda^2 - 8\lambda - 16 = 0$

$\Rightarrow \quad \lambda = \frac{8 \pm \sqrt{64+64}}{2} = 4 \pm 4\sqrt{2}$

$\lambda = 4 - 4\sqrt{2}$ (Other value forms bigger circle)

Hence, centre of circle $(2\sqrt{2} - 2, 4 - 2\sqrt{2})$

Radius $= 4 - 2\sqrt{2}$

Area $= \pi(4 - 2\sqrt{2})^2 = 8\pi(3 - 2\sqrt{2})$

43. Answer (2)

Let tangent in terms of m to parabola and ellipse i.e

$y = mx + \frac{1}{4m}$ for parabola at point $\left(\frac{1}{4m^2}, \frac{-1}{2m}\right)$ and

$y = mx \pm \sqrt{m^2 + \frac{1}{2}}$ for ellipse on comparing

$\Rightarrow \quad \frac{1}{4m} = \pm\sqrt{m^2 + \frac{1}{2}} \Rightarrow \frac{1}{16m^2} = m^2 + \frac{1}{2}$

$\Rightarrow \quad 16m^4 + 8m^2 - 1 = 0$

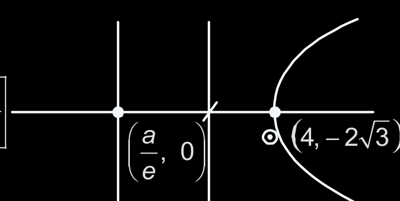
$m^2 = \frac{-8 \pm \sqrt{64+64}}{2(16)} = \frac{-8 \pm 8\sqrt{2}}{2(16)} = \frac{\sqrt{2}-1}{4}$

$\alpha = \frac{1}{4m^2} = \frac{1}{4\frac{\sqrt{2}-1}{4}} = \sqrt{2} + 1$

44. Answer (2)

$x = \frac{4}{\sqrt{5}}$

$\because \quad \boxed{\frac{a}{e} = \frac{4}{\sqrt{5}}}$



Equation of hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ it passes through $(4, -2\sqrt{3})$

$\because \quad e^2 = 1 + \frac{b^2}{a^2}$

$\Rightarrow \boxed{a^2e^2 - a^2 = b^2}$

$\Rightarrow \dfrac{16}{a^2} - \dfrac{12}{a^2e^2 - a^2} = 1$

$\Rightarrow \dfrac{4}{a^2}\left[\dfrac{4}{1} - \dfrac{3}{e^2-1}\right] = 1$

$\Rightarrow 4e^2 - 4 - 3 = (e^2 - 1)\left(\dfrac{a^2}{4}\right)$

$\Rightarrow 4(4e^3 - 7) = (e^2 - 1)\left(\dfrac{4e}{\sqrt{5}}\right)^2$

$\Rightarrow 4e^4 - 24e^2 + 35 = 0$

45. Answer (4)

Equation of tangent at $\left(3, -\dfrac{9}{2}\right)$ to $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is

$\dfrac{3x}{a^2} - \dfrac{y9}{2b^2} = 1$ which is equivalent to $x - 2y = 12$

$\dfrac{3}{a^2} = \dfrac{-9}{2b^2\cdot(-2)} = \dfrac{1}{12}$ (On comparing)

$a^2 = 3 \times 12$ and $b^2 = \dfrac{9 \times 12}{4}$

$\boxed{a = 6} \Rightarrow b = 3\sqrt{3}$

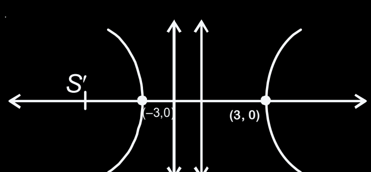
So latus rectum = $\dfrac{2b^2}{a} = \dfrac{2 \times 27}{6} = 9$

46. Answer (3)

$16x^2 - 9y^2 = 144$

i.e. $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

Focus $S'(-ae, 0)$



$x = \dfrac{-9}{5}$

$a = 3,\ b = 4$

$e^2 = 1 + \dfrac{16}{9} = \dfrac{25}{9}$

$S' \equiv \left(-3 \times \dfrac{5}{3}, 0\right) \equiv (-5, 0)$

47. Answer (4)

Tangent on $y^2 = 4\sqrt{2}x$ is $yt = x + \sqrt{2}t^2$

As it is tangent on circle also,

$$\left|\frac{\sqrt{2}t^2}{\sqrt{1+t^2}}\right| = 1$$

$2t^4 = 1 + t^2$ i.e. $t^2 = 1$

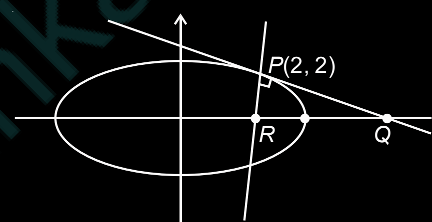
Equation is $\pm y = x + \sqrt{2}$

Hence $|c| = \sqrt{2}$

48. Answer (4)

For $\dfrac{3x^2}{32} + \dfrac{5y^2}{32} = 1$

Tangent at P is



$\dfrac{3(2)x}{32} + \dfrac{5(2)y}{32} = 1$

$\dfrac{3x}{16} + \dfrac{5y}{16} = 1$

$Q \equiv \left(\dfrac{16}{3}, 0\right)$

Normal at P is $\dfrac{32x}{3(2)} - \dfrac{32y}{5(2)} = \dfrac{32}{3} - \dfrac{32}{5}$

$R \equiv \left(\dfrac{4}{5}, 0\right)$

area of ΔPQR = $\dfrac{1}{2}$ (PQ) (PR)

$= \dfrac{1}{2}\sqrt{\dfrac{136}{3}}\cdot\sqrt{\dfrac{136}{5}} = \dfrac{68}{15}$

49. Answer (2)

Slope of tangent at point P is $\frac{1}{2}$

$3x^2 + 4y^2 = 12 \Rightarrow \frac{x^2}{2^2}+\frac{y^2}{(\sqrt{3})^2}=1$

Let point $P\left(2\cos\theta, \sqrt{3}\sin\theta\right)$

$\Rightarrow$ Equation of tangent at P is

$$\frac{x}{2}\cos\theta+\frac{y}{\sqrt{3}}\sin\theta=1$$

$$\Rightarrow \quad m_T=-\frac{\sqrt{3}}{2}\cot\theta=\frac{1}{2}$$

$$\tan\theta=-\sqrt{3} \quad \Rightarrow \quad \theta=\pi-\frac{\pi}{3} \quad \text{or} \quad \theta=2\pi-\frac{\pi}{3}$$

If $\theta=\frac{2\pi}{3}$, then $P\left(-1,\frac{3}{2}\right)$ and $PQ=\frac{5\sqrt{5}}{2}$

If $\theta=\frac{5\pi}{3}$, then tangent does not pass through $Q(4, 4)$.

50. Answer (3)

Equation of tangent to $y^2 = 12x$ is $y=mx+\frac{3}{m}$

Equation of tangent $\frac{x^2}{1}-\frac{y^2}{8}=1$ is

$$y=mx\pm\sqrt{m^2-8}$$

for common tangent,

$\frac{3}{m}=\pm\sqrt{m^2-8} \quad \Rightarrow \frac{9}{m^2}=m^2-8$

Put $m^2 = t$

$t^2 - 8t - 9 = 0 \quad \Rightarrow t^2 - 9t + t - 9 = 0$

$\Rightarrow (t + 1)(t - 9) = 0$

$\because \quad t = m^2 \geq 0 \quad \Rightarrow t = m^2 = 9$

$\Rightarrow m = \pm 3$

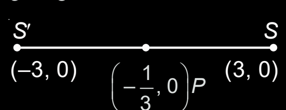
$\Rightarrow$ Equation of tangent is $y = 3x + 1$

or $\quad y = -3x - 1$

Intersection point $P\left(-\frac{1}{3}, 0\right)$

$8 = 1(e^2 - 1) \quad \Rightarrow e = 3$

foci $(\pm 3, 0)$ $\Rightarrow$



$$\frac{S'P}{SP}=\frac{3-\frac{1}{3}}{3+\frac{1}{3}}=\frac{8}{10}=\frac{4}{5}$$

51. Answer (1)

Let the equation of ellipse : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\because \quad be = 2 \quad$ and $\quad a = 2 \qquad$ (Given)

$\because \quad a^2 = b^2(1 - e^2)$

$\Rightarrow \quad 4 = b^2 - 4$

$\Rightarrow \quad b = 2\sqrt{2}$

Equation of ellipse will be $\frac{x^2}{4}+\frac{y^2}{8}=1$

Only $\left(\sqrt{2}, 2\right)$ satisfies this equation.

52. Answer (4)

$y^2 = 16x$ and $xy = -4$

Equation of tangent to the given parabola;

$$y=mx+\frac{4}{m}$$

If this is common tangent, then

$$x\left(mx+\frac{4}{m}\right)+4=0$$

$$\Rightarrow \quad mx^2+\frac{4}{m}x+4=0$$

$D = 0$

$$\frac{16}{m^2}=16m$$

$\Rightarrow \quad m^3 = 1 \qquad \Rightarrow m = 1$

Equation of common tangent is $y = x + 4$

53. Answer (4)

Tangent on $y^2 = 4x$ is $y=mx+\frac{1}{m}$

But given $y = mx + 4$

$\therefore \quad \frac{1}{m}=4$ i.e. $m=\frac{1}{4}$

Now $y=\frac{x}{4}+4$ is tangent on $x^2 = 2by$ also

$$\therefore \quad x^2=2b\left(\frac{x}{4}+4\right)$$

$$x^2-\frac{bx}{2}-8b=0$$

For tangent Discriminant = 0

$$\frac{b^2}{4}+32b=0$$

$\Rightarrow \quad b = -128$

54. Answer (3)

$2ae = 6$

$\frac{2a}{e} = 12$

Multiplying both;

$4a^2 = 72$

$a^2 = 18$

$$\Rightarrow \quad e = \frac{6}{2.3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$e^2 = 1 - \frac{b^2}{a^2} = \frac{1}{2}$$

$$\frac{18}{2} = b^2 \qquad \Rightarrow \quad b^2 = 9$$

$$\therefore \text{ Latus rectum} = \frac{2b^2}{a} = \frac{2 \times 9}{3\sqrt{2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}$$

55. Answer (2)

A line $y = mx + c$ be a tangent to ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $c^2 = a^2m^2 + b^2$

Here, eq. of tangent is : $4y = -3x + 12\sqrt{2}$

$$\therefore \quad y = -\frac{3}{4}x + 3\sqrt{2}$$

$$\therefore \quad \left(3\sqrt{2}\right)^2 = a^2.\left(-\frac{3}{4}\right)^2 + 9$$

$$\therefore \quad a^2 = 9 \times \frac{16}{9} = 16$$

$\therefore$ Eccentricity of ellipse = $e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

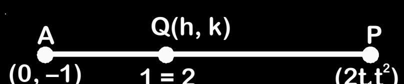
$\therefore$ Distance between foci = 2ac = $2.4.\frac{\sqrt{7}}{4}$

$= 2\sqrt{7}$

56. Answer (1)

Let $P \equiv (2t, t^2)$

given



by section formula

$$\frac{(0)(2) + 2t(1)}{3} = h \qquad \text{...(i)}$$

$$\text{and} \quad \frac{(-1)(2) + t^2(1)}{3} = k \qquad \text{...(ii)}$$

by (i) and (ii)

$$\Rightarrow \quad 3k + 2 = \left(\frac{3h}{2}\right)^2$$

$$\Rightarrow \quad 12y + 8 = 9x^2$$

57. Answer (1)

Let $P(a, b)$ then equation of normal at P is

$$\frac{x}{2a} - \frac{y}{b} = \frac{-1}{2}$$

$$\downarrow \left(\frac{-1}{3\sqrt{2}}, 0\right)$$

$$\Rightarrow \quad \frac{-1}{6\sqrt{2}a} = \frac{-1}{2} \Rightarrow a = \frac{1}{3\sqrt{2}}$$

also $2a^2 + b^2 = 1$ we get

$$b^2 = 1 - 2 \times \frac{1}{18} = \frac{8}{9} \Rightarrow b = \frac{2\sqrt{2}}{3}$$

Hence normal is $\frac{3\sqrt{2}x}{2} - \frac{3y}{2\sqrt{2}} = \frac{-1}{2}$

$$\downarrow (0, \beta)$$

$$\beta = \frac{\sqrt{2}}{3}$$

58. Answer (1)

Given curve $x^2 + 3xy - xy - 3y^2 = 0$

$$\Rightarrow \quad (x - y)(x + 3y) = 0 \Rightarrow \ y = x \text{ and } \boxed{y = -\frac{x}{3}}$$

$\therefore$ Normal pass through (2, 2) and is perpendicular to line $x - y = 0$

Let normal is $x + y + \lambda = 0 \Rightarrow \boxed{\lambda = -4}$

$$\therefore \quad \text{Perpendicular distance} = \left|\frac{-4}{\sqrt{2}}\right| = 2\sqrt{2}$$

59. Answer (2)

Let hyperbola = $\frac{x^2}{36}-\frac{y^2}{b^2}=1$ $\therefore$ (10, 16) lies on it

$\Rightarrow \frac{100}{36}-\frac{256}{b^2}=1 \Rightarrow \frac{64}{36}=\frac{256}{b^2}$

$\Rightarrow \boxed{b^2=144} \quad \Rightarrow \boxed{b=12}$

$\therefore$ Equation of normal $\frac{x-10}{\frac{10}{36}}=\frac{y-16}{(-)\frac{16}{144}}$

$\Rightarrow$ 2x – 20 = –5y + 80

$\Rightarrow \boxed{2x+5y=100}$

60. Answer (4)

$\because \quad e_1^2=1-\frac{b^2}{a^2}=1-\frac{4}{18}=\frac{7}{9}$

and $e_2^2=1+\frac{b^2}{a^2}=1+\frac{4}{9}=\frac{13}{9}$

$\because$ (e_1, e_2) lies on $15x^2 + 3y^2 = k$

So $k=15e_1^2+3e_2^2=\frac{35}{3}+\frac{13}{3}=16$

61. Answer (4)

2b = $\frac{4}{\sqrt{3}} \Rightarrow b^2=\frac{4}{3}$

Equation of tangent to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 is

$y = mx \pm \sqrt{a^2m^2+b^2}$, here $m = -\frac{1}{6}$

so equation of tangent is $y = -\frac{x}{6}\pm\sqrt{\frac{a^2}{36}+\frac{4}{3}}$

But $x + 6y = 8$ is given to be a tangent

So after comparing we get $a = 4$

Now $e = \sqrt{1-\frac{b^2}{a^2}}=\sqrt{\frac{11}{12}}$

62. Answer (1)

Given parabola is $y^2 = 8x$

One end of the focal chord is $\left(\frac{1}{2}, -2\right)$

Let $\left(\frac{1}{2}, -2\right)=\left(2t_1^2, 2at_1\right) \quad \Rightarrow t_1=-\frac{1}{2}$

As $t_2=-\frac{1}{t_1} \quad \Rightarrow t_2 = 2$

So, coordinates of the other end are (8, 8)

equation of tangent at (8, 8) is

$y(8) = 4(x + 8)$

$\Rightarrow$ $2y = x + 8$

63. Answer (2)

Hyperbola : $\frac{x^2}{4}-\frac{y^2}{2}=1$

So, let the point on it be

$(2\sec\theta, \sqrt{2}\tan\theta)=(x_1, y_1)$

So, equation of tangent = $\frac{2x\sec\theta}{4}-\frac{\sqrt{2}y\tan\theta}{2}=1$

$\Rightarrow$ slope = $\frac{1}{\sqrt{2}\sin\theta}$

Which will be equal to the slope of given line $2x - y = 0$

$\Rightarrow \frac{1}{\sqrt{2}\sin\theta}=2 \Rightarrow \sin\theta=\frac{1}{2\sqrt{2}}$

$\Rightarrow \cos^2\theta=1-\sin^2\theta=1-\frac{1}{8}=\frac{7}{8}$

$\Rightarrow \sec^2\theta=\frac{8}{7}$

$\Rightarrow \tan^2\theta=\frac{8}{7}-1=\frac{1}{7}$

Now, $x_1^2+5y_1^2=4\sec^2\theta+10\tan^2\theta$

$= 4\times\frac{8}{7}+10\times\frac{1}{7}=\frac{42}{7}=6$

64. Answer (3)

$$E:\frac{x^2}{\cos^2\theta}+\frac{y^2}{1}=5\ \Rightarrow e_e=\sqrt{1-\frac{\cos^2\theta}{1}}=\sin\theta$$

$$H:\frac{x^2}{1}-\frac{y^2}{\cos^2\theta}=10\Rightarrow e_H=\sqrt{1+\cos^2\theta}$$

$$\because\quad e_H=\sqrt{5}e_e\ \Rightarrow 1+\cos^2\theta=5\sin^2\theta\Rightarrow \sin\theta=\frac{1}{\sqrt{3}}$$

Length of latus rectum of ellipse $=\dfrac{2a^2}{b}$

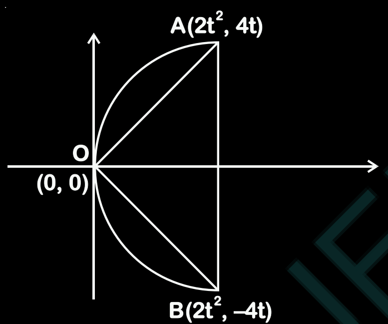
$$=\frac{2(5\cos^2\theta)}{\sqrt{5}}$$

$$=2\sqrt{5}\left(\frac{2}{3}\right)=\frac{4\sqrt{5}}{3}$$

65. Answer (4)

Let $A=(2t^2, 4t)$

$B\equiv(2t^2, 4t)$ (by symmetry as equilateral triangle)

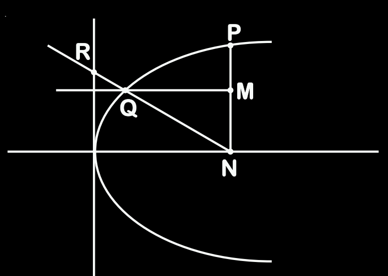


for equilateral triangle (angle at O is 60º)

$$\frac{4t}{2t^2}=\frac{1}{\sqrt{3}}\Rightarrow t=2\sqrt{3}$$

$$\text{Area }=\frac{1}{2}.8(2\sqrt{3}).2.24=192\sqrt{3}$$

66. Answer (1)



Let $P(at^2, 2at)$ where $a = 3$

$\Rightarrow\ N(at^2, 0)\qquad\Rightarrow\ M(at^2, at)$

$\because\ QM\equiv y=at$

So $y^2 = 4ax\qquad\Rightarrow\ x=\dfrac{at^2}{4}$

$$\Rightarrow\ Q\left(\frac{at^2}{4},at\right)$$

$$\Rightarrow\ QN\equiv y=\frac{-4}{3t}\left(x-at^2\right)$$

$\because$ QN passes through $\left(0,\dfrac{4}{3}\right)$, then

$$\frac{4}{3}=-\frac{4}{3t}\left(-at^2\right)\qquad\Rightarrow\ \text{at}=1\qquad\Rightarrow\ t=\frac{1}{3}$$

Now, $MQ=\dfrac{3}{4}at^2=\dfrac{1}{4}$ and $PN = 2at = 2$

67. Answer (2)

Here ellipse : $3x^2 + 4y^2 = 12$

$$\Rightarrow\ \frac{x^2}{4}+\frac{y^2}{3}=1$$

$$\Rightarrow\ b^2=a^2(1-e^2)$$

$$\Rightarrow\ e=\frac{1}{2}$$

Foci = $(\pm1, 0)$

Now for Hyperbola –

$$2a=\sqrt{2}\Rightarrow a=\frac{1}{\sqrt{2}}\ \&\ ae=1\Rightarrow e=\sqrt{2}\Rightarrow b=\frac{1}{\sqrt{2}}$$

So equation of hyperbola : $\dfrac{x^2}{\frac{1}{2}}-\dfrac{y^2}{\frac{1}{2}}=1$

$$\Rightarrow\ \boxed{x^2-y^2=\frac{1}{2}}$$

So option (2) does not satisfy it.

68. Answer (1)

Equation of ellipse : $\dfrac{x^2}{25}+\dfrac{y^2}{b^2}=1$

$$\therefore\ e_1=\sqrt{1-\frac{b^2}{25}}$$

Equation of hyperbola : $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ then

$e_2=\sqrt{1+\frac{b^2}{16}}$

$\because \quad e_1 \cdot e_2 = 1$

$$\left(\frac{25-b^2}{25}\right)\left(\frac{16+b^2}{16}\right) = 1$$

$\Rightarrow \quad b^2(b^2-9) = 0$

$\Rightarrow \quad b = 3$

$\therefore \quad e_1 = \frac{4}{5}$ and $e_2 = \frac{5}{4}$

$\because$ Distance between focii of ellipse = α = 8

Distance between focii of hyperbola = β = 10

69. Answer (2)

$\because$ equation of hyperbola is : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\because$ it passes through (3, 3) : $\frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{9}$ ...(1)

equation of normal at point (3, 3) is :

$$\frac{x-3}{\frac{1}{a^2}\cdot 3}=\frac{y-3}{-\frac{1}{b^2}\cdot 3}$$

$\because$ It passes through (9, 0) : $\frac{6}{\frac{1}{a^2}}=\frac{-3}{-\frac{1}{b^2}}$

$\therefore \quad \frac{1}{b^2}=\frac{1}{2a^2}$ ...(2)

From equation (1) and equation (2)

$$a^2=\frac{9}{2},\ b^2=9$$

$\therefore$ Eccentricity = e, then $e^2 = 1+\frac{b^2}{a^2}=3$

$$\therefore \quad (a^2,e^2)=\left(\frac{9}{2},3\right)$$

70. Answer (3)

$\because$ Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>b)$

Given $\frac{2b^2}{a}=10 \Rightarrow b^2=5a$ ...(1)

Now $\phi(t)=\frac{5}{12}+t-t^2$

$\phi'(t)=1-2t=0 \Rightarrow t=\frac{1}{2}$

$\phi''(t) = -2 < 0 \Rightarrow$ maximum

$\Rightarrow \quad \phi(t)_{max}=\frac{5}{12}+\frac{1}{2}-\frac{1}{4}=\frac{8}{12}=\frac{2}{3}=e$ (given)

Now $b^2 = a^2(1-e^2)$

$$5a = a^2\left(1-\frac{4}{9}\right)$$

$$5a=\frac{5a^2}{9}\Rightarrow a^2-9a=0$$

$\Rightarrow \quad a = 9 \Rightarrow a^2 = 81 \Rightarrow b^2 = 45$

$\Rightarrow \quad a^2 + b^2 = 81 + 45 = 126$

71. Answer (2)

Given $x = 4$

$\Rightarrow \quad \pm\frac{a}{e} = 4$

$\Rightarrow \quad a = 4e$

$\Rightarrow \quad a=4\times\frac{1}{2}=2$

$\Rightarrow \quad \boxed{a = 2}$

Now, $b^2 = a^2(1-e^2)$

$\Rightarrow \quad b^2=4\left(1-\frac{1}{4}\right)=4\times\frac{3}{4}=3$

So, equation $\frac{x^2}{4}+\frac{y^2}{3}=1$

$\Rightarrow \quad 3x^2 + 4y^2 = 12$ ...(i)

Now $P(1, \beta)$ lies on it

$\Rightarrow \quad 3 + 4\beta^2 = 12$

$\Rightarrow \quad 4\beta^2 = 9$

$\Rightarrow \quad \beta=\frac{3}{2}$

So, $P\left(1, \frac{3}{2}\right)$

Now differentiating (i)

$$6x + 8y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-3x}{4y}$$

$$\Rightarrow \left.\frac{dy}{dx}\right]_{\left(1, \frac{3}{2}\right)} = -\frac{1}{2}$$

So, slope of normal = 2

So, equation of normal : $y - \frac{3}{2} = 2(x - 1)$

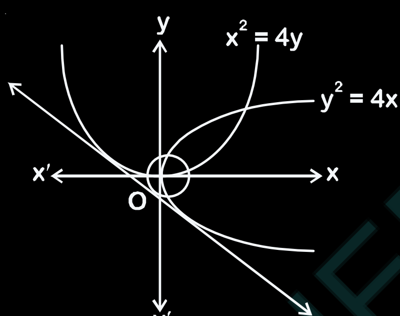
$\Rightarrow \quad 2y - 3 = 4x - 4$

$\Rightarrow \quad 4x - 2y = 1$

72. Answer (4)

Equation tangent to parabola $y^2 = 4x$

with given slope m is :



$$y = mx + \frac{1}{m} \qquad \text{...(i)}$$

$\because$ Line (i) is tangent to $x^2 = 4y$.

$$\therefore \quad x^2 = 4mx + \frac{4}{m}$$

$\Rightarrow \quad mx^2 - 4m^2x - 4 = 0$

For tangent : $16m^4 + 16m = 0$

$16m\,(m^3 + 1) = 0$

$m = 0, -1$

$\therefore$ Equation tangent : $x + y + 1 = 0$

It is tangent to circle $x^2 + y^2 = c^2$

$$\Rightarrow \quad c = \frac{1}{\sqrt{2}}$$

73. Answer (4)

Curve is $C \equiv \frac{x^2}{5} + \frac{y^2}{4} = 1$

Let a point on curve be $\left(\sqrt{5}\cos\theta,\ 2\sin\theta\right)$

$$PQ^2 = \left(\sqrt{5}\cos\theta\right)^2 + (-4 - 2\sin\theta)^2$$

$$= 5\cos^2\theta + 4\sin^2\theta + 16 + 16\sin\theta$$

$\Rightarrow \quad PQ^2 = 21 + 16\sin\theta - \sin^2\theta$

$= 21 + 64 - (\sin\theta - 8)^2$

$= 85 - (\sin\theta - 8)^2$

For PQ^2 to be maximum $\sin\theta = 1$

$$PQ^2_{max} = 85 - 49 = 36$$

74. Answer (4)

General tangent to hyperbola in slope form is

$$y = mx \pm \sqrt{100m^2 - 64}$$

and that of cirlce is

$$y = mx \pm 6\sqrt{1 + m^2}$$

For common tangent

$36(1 + m^2) = 100m^2 - 64$

$100 = 64m^2$

$$\Rightarrow \quad m^2 = \frac{100}{64}$$

$$\therefore \quad c^2 = 36\left(1 + \frac{100}{64}\right) = \frac{164 \times 36}{64} = \frac{369}{4}$$

$\Rightarrow \quad 4c^2 = 369$

75. Answer (2)

$$L_1 : y = m_1(x + 1) + \frac{1}{m_1}$$

$$L_2 : y = m_2(x + 2) + \frac{2}{m_2}$$

If L_1 and L_2 intersects at (h, k) then

$$m_1^2(h + 1) - km_1 + 1 = 0 \qquad \text{...(1)}$$

$$m_2^2(h + 2) - km_2 + 1 = 0 \qquad \text{...(2)}$$

$$\because \quad m_2 = -\frac{1}{m_1}$$

$\Rightarrow \quad m_1^2 + km_1 + (h+2) = 0 \qquad ...(3)$

from (1) and (3)

$$\frac{h+1}{1} = \frac{-k}{k} = \frac{1}{h+2}$$

$\Rightarrow \quad h + 3 = 0$

$\Rightarrow \quad x + 3 = 0$

76. Answer (3)

Locus will be $x^2 + y^2 = 4$ (auxiliary circle)

$\therefore \quad \left(-1, \sqrt{3}\right)$ satisfies the given equation

(Property of ellipse is that locus of foot of perpendicular from any foci to tangent lies on auxiliary circle of ellipse)

77. Answer (4)

Normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $\left(ae_1 \frac{b^2}{a}\right)$ is

$$x - ey = \frac{e(a^2 - b^2)}{a}$$

$\because \quad (0, -b)$ lies on it, so

$$be = \frac{e(a^2 - b^2)}{a}$$

$\Rightarrow \quad ab = a^2e^2$

$\Rightarrow \quad b = ae^2 \Rightarrow \frac{b^2}{a^2} = e^4$

$1 - e^2 = e^4 \Rightarrow e^4 + e^2 - 1 = 0$

78. Answer (04.00)

$$2y\frac{dy}{dx} - 6x + \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{6x}{2y+1}$$

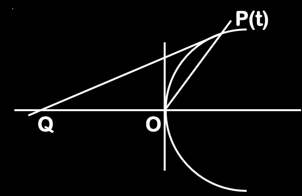
$$m_N = -\left(\frac{2y_1 + 1}{6x_1}\right) = \frac{\frac{3}{2} - y_1}{-x_1}$$

$\Rightarrow \quad 2y_1 + 1 = 9 - 6y'$

$\Rightarrow \quad y_1 = 1$ and $x_1 = \pm 2$

$$\therefore \quad m_T = \left(\frac{6(\pm 2)}{2(1)+1}\right) = 4$$

79. Answer (.50)



Tangent at Point $P(t)$ is

$$yt = x + \frac{1}{4}t^2$$

$$\therefore \quad \text{Point } Q \text{ is } \left(-\frac{t^2}{4}, 0\right)$$

$$P\left(\frac{t^2}{4}, \frac{t}{2}\right)$$

$$\therefore \quad \text{Area of } \Delta OPQ = \frac{1}{2}\left|\frac{t^3}{8}\right| = 4$$

$\Rightarrow \quad |t^3| = 64 \Rightarrow \boxed{t = 4}$

$$\text{Now, } m = \frac{t \times 4}{2 \times t^2} = \frac{2}{t} = \frac{2}{4}$$

$$\boxed{= \frac{1}{2}} = \boxed{.50}$$

80. Answer (36)

The given circles are $(x - 0)^2 + (y - 4)^2 = \left(\sqrt{k}\right)^2$ and $(x - 3)^2 + (y - 0)^2 = 1^2$

Let 'd' denote the distance between their centres

$$\therefore \quad d = \sqrt{(3-0)^2 + (4-0)^2} = 5$$

Circles will touch if $d = |r_1 \pm r_2|$

i.e., $\left|\sqrt{k} \pm 1\right| = 5$

Clearly the maximum value of $k = 36$

81. Answer (3)

Let the moving point be $P(at^2, 2at)$

Focus of given parabola is $(a, 0)$

Let point of required locus (h, k)

$$\therefore \quad \frac{at^2 + a}{2} = h \qquad ...(i)$$

$$\text{and } \frac{2at + 0}{2} = k \qquad ...(ii)$$

$$\Rightarrow \quad \frac{a}{2}(t^2+1)=h \qquad ...(iii)$$

$$\text{and } t=\frac{k}{a} \qquad ...(iv)$$

By (iii) and (iv) we have

$$\frac{a}{2}\left(\frac{k^2}{a^2}+1\right)=h$$

Locus is $k^2 + a^2 = 2ah$

$$\Rightarrow \quad y^2=2a\left(x-\frac{a}{2}\right)$$

Equation of directrix $x-\frac{a}{2}+\frac{a}{2}=0$

$\Rightarrow$ $x = 0$

82. Answer (4)

Closest point will be point of tangency of tangent of same slope i.e. 4

Let equation of tangent $y = 4x + c$

$\Rightarrow$ $4x + c = x^2 + 4$ have $D = 0$

i.e. $x^2 - 4x + (4 - c) = 0$

$D = 0 \;\; \Rightarrow \;\; 16 - 4(4 - c) = 0 \;\; \Rightarrow \;\; c = 0$

Tangent is $y = 4x$ gives $x = 2$ and $y = 8$ as point of tangency

$\therefore$ Nearest point (2, 8)

83. Answer (3)

Given, Curves are $\frac{x^2}{a}+\frac{y^2}{b}=1$ [Ellipse]

and other curves can be written as

$\frac{x^2}{c}-\frac{y^2}{(-d)}=1$, Which is a hyperbola

Since these both are orthogonal

So, $\sqrt{a-b}=\sqrt{c-d}$

$\Rightarrow$ $a - b = c - d$

84. Answer (4)

Slope of line : $2x + y = 1$ is -2

Slope of line perpendicular to given line is $\frac{1}{2}$

$\therefore$ Equation of tangents to parabola $y^2 = 6x$ is

$$y=\frac{1}{2}x+\frac{\frac{6}{4}}{\frac{1}{2}}$$

$$y=\frac{1}{2}x+3$$

$x - 2y + 6 = 0$

$\therefore$ (5, 4) does not lies on $x - 2y + 6 = 0$

85. Answer (2)

$$L_1 : \sqrt{3x}+y=\frac{4\sqrt{3}}{k}$$

and $L_2 : \sqrt{3x}-y=4\sqrt{3}k$

So point of intersection will always satisfy

$$\left(\sqrt{3}x-y\right)\left(\sqrt{3}x-y\right)=48$$

$$\Rightarrow \quad \frac{x^2}{16}-\frac{y^2}{48}=1$$

$$e=\sqrt{1+\frac{48}{16}}=2$$

86. Answer (1)

Eccentricity of Ellipse $e_1=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

Foci = $(\pm ae, 0) = (\pm 3, 0)$

For Hyperbola

Eccentricity $e_2=\frac{5}{3}$

Semi-transverse axis $\rightarrow a = 3$

$$b^2=a^2(e^2-1)=9\left(\frac{25}{9}-1\right)=16$$

Equation of Hyperbola

$$\frac{x^2}{9}-\frac{y^2}{16}=1$$

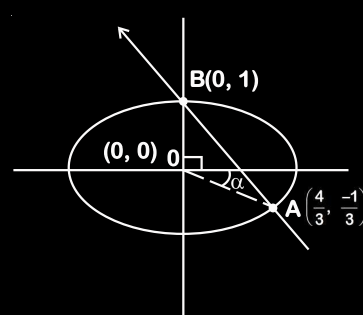
87. Answer (4)

$y = 1 - x \qquad ...(i)$

$x^2 + 2y^2 = 2 \qquad ...(ii)$

$\Rightarrow$ $x^2 + 2(1 - x)^2 = 2$

$3x^2 - 4x = 0$



$$x=0, \frac{4}{3}$$

$$y=1, \frac{-1}{3}$$

$B(0,1),\ A\left(\frac{4}{3}, \frac{-1}{3}\right)$

$$\tan\alpha = \frac{\frac{1}{3}}{\frac{4}{3}} = \frac{1}{4} \Rightarrow \alpha = \tan^{-1}\frac{1}{4}$$

$$\angle AOB = \frac{\pi}{2} + \tan^{-1}\frac{1}{4}$$

88. Answer (4)

Equation of line parallel to $x - y = 1$ is

$x - y = c$ ...(i)

If line $x - y = c$ is tangent to parabola $x^2 = 2y$ then $x^2 = 2(x - c)$ has unique roots

$x^2 - 2x + 2c = 0$

$\therefore$ $D = 0 \Rightarrow 4 - 4 \times 1 \times 2c = 0$

$\therefore$ $c = \frac{1}{2}$

$\therefore$ Tangent of parabola is $x - y = \frac{1}{2}$

$\therefore$ Shortest distance $= \frac{\left|1 - \frac{1}{2}\right|}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$ units

89. Answer (9)

Let equation of tangent to $y^2 = 4x$ as

$y = mx + \frac{1}{m}$

If it is a common tangent, then

$$\left|\frac{3m + \frac{1}{m}}{\sqrt{1+m^2}}\right| = 3 \Rightarrow m = \pm\frac{1}{\sqrt{3}}$$

Equation of common tangent having point of contact in first quadrant ; $y = \frac{x+3}{\sqrt{3}}$.

The tangent intersects the parabola at $\left(3, 2\sqrt{3}\right)$

and circle at $\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

So, $2(a + c) = 9$

90. Answer (4)

Let $P(h, k)$

Required laws $\frac{3+\cos\theta}{2} = h$ and $\frac{2+\sin\theta}{2} = k$

$\cos\theta = 2h - 3$ and $\sin\theta = 2h - 2$

Squaring and adding we get

$(2h - 3)^2 + (2h - 2)^2 = 1$

$\Rightarrow$ $4x^2 - 12x + 9 + 4y^2 - 8y + 4 = 1$

$\Rightarrow$ $4x^2 + 4y^2 - 12x - 8y + 12 = 0$

$\Rightarrow$ $x^2 + y^2 - 3x - 2y + 3 = 0$

Radius $= \sqrt{\frac{9}{4} + 1 - 3} = \frac{1}{2}$

91. Answer (3)

Tangent to the curve $\frac{x^2}{9} + \frac{y^2}{14} = 1$ is

$y = mx + \sqrt{9m^2 + 4}$

and equation of tangent to the curve $x^2 + y^2 = \frac{31}{4}$ is

$y = mx + \sqrt{\frac{31}{4}\left(1 + m^2\right)}$

for common tangent $9m^2 + 4 = \frac{31}{4} + \frac{31}{4}m^2$

$\Rightarrow$ $\frac{5}{4}m^2 = \frac{15}{4}$

$\Rightarrow$ $m^2 = 3$

92. Answer (3)

Three normals can be drawn to the parabola $y^2 = 4bx$ from $(a, 0)$ if $a > 2b$.

So, $a > 1$

93. Answer (4)

Let mid-point be (h, k)

$\therefore$ Chord of circle is $hx + ky = h^2 + k^2$

$\Rightarrow$ $y = \frac{-h}{k}x + \left(\frac{h^2 + k^2}{k}\right)$ ...(i)

$\Rightarrow$ Tangent to hyperbola (in slope form)

$y = mx \pm \sqrt{9m^2 - 16}$ ...(ii)

Comparing (i) and (ii) we get,

$$\left(\frac{h^2 + k^2}{k}\right) = 9\left(\frac{h^2}{k^2}\right) - 16$$

$\Rightarrow$ $(h^2 + k^2)^2 = 9h^2 - 16k^2$

$\Rightarrow$ $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

94. Answer (3)

Equation of C $x^2 = 4y$

Tangent at (2, 1) is

$2x = 2(y + 1)$

$x - y = 1$

95. Answer (3)

$$\frac{x^2}{16}+\frac{y^2}{b^2}=1 \quad ...(i)$$

$x^2 + y^2 = 4b,\ b > 4$ ...(ii)

$y^2 = 3x^2$ ...(iii)

Solving (ii) & (iii), $x^2 + 3x^2 = 4b$

$x^2 = b$ ...(iv)

(iii) & (i) $\Rightarrow \dfrac{x^2}{16}+\dfrac{3x^2}{b^2}=1$

$(48 + b^2)x^2 = 16b^2$

$(48 + b^2)b = 16b^2$ (using (iv))

$b^2 - 16b + 48 = 0$

$b = 12$ ($\because$ $b > 4$)

96. Answer (1)

Any line perpendicular to given tangent is

$x + 2y + \lambda = 0$

Passes through (2, 5) $\Rightarrow \lambda = -12$

Hence line in $x + 2y - 12 = 0$

Solving with $x - 2y - 4 = 0$ gives

Centre $\equiv$ (8, 2)

Radius = $\sqrt{(8-2)^2+(2-5)^2}$

$= 3\sqrt{5}$

97. Answer (3)

$y^2 = 4x - 20 \Rightarrow \dfrac{dy}{dx}=\dfrac{2}{y}\Rightarrow \dfrac{dy}{dx_{(6,2)}}=1$

Equation of tangent,

$T : y - 2 = 1(x - 6) \Rightarrow y = x - 4$

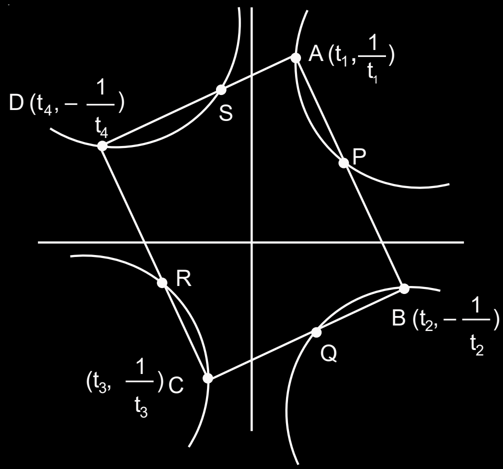
$\because$ T is tangent to given ellipse,

$T : y = x \pm \sqrt{2+b}$

Clearly $\sqrt{2+b}=4 \Rightarrow b=14$

98. Answer (80)

Refer to diagram



$(t_1+t_2)\left(\dfrac{1}{t_1}-\dfrac{1}{t_2}\right)=4$

$\Rightarrow \dfrac{t_2}{t_1}-\dfrac{t_1}{t_2}-4=0$

$\dfrac{t_2}{t_1}=\sqrt{5}+2$

Similarly, $\dfrac{t_4}{t_1}=-\left(\sqrt{5}+2\right)$

$\because$ AB $\perp$ AD, then

Slope of AB $=-\left(\dfrac{\sqrt{5}+1}{\sqrt{5}+3}\right)t_1t_2$

Slope of AD $=-\left(\dfrac{3+\sqrt{5}}{\sqrt{5}+1}\right)t_1t_4$

$\Rightarrow t_1^2t_2t_4=-1 \Rightarrow t_1^2=\sqrt{5}-2,$

then, $t_2^2=\sqrt{5}+2$

area of require triangle

$=(t_1-t_2)^2+\left(\dfrac{1}{t_1}+\dfrac{1}{t_2}\right)^2=4\sqrt{5}$

$\Rightarrow \Delta^2 = 80$

99. Answer (3)

Tangent $=\dfrac{x}{3\sqrt{3}}\cos\theta+y\sin\theta=1$

x-intercept $=3\sqrt{3}\sec\theta$

y-intercept = cosecθ

sum $= 3\sqrt{3}\sec\theta + \operatorname{cosec}\theta = f(\theta) \quad \theta \in \left(0, \dfrac{\pi}{2}\right)$

$\Rightarrow \quad f'(\theta) = 3\sqrt{3}\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta = 0$

$\Rightarrow \quad \dfrac{3\sqrt{3}\sin\theta}{\cos^2\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\Rightarrow \quad \tan^3\theta = \left(\dfrac{1}{\sqrt{3}}\right)^3$

$\Rightarrow \quad \tan\theta = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = \dfrac{\pi}{6}$

100. Answer (4)

Hyperbola $x^2 - 2y^2 = 4 \quad \Rightarrow \quad \dfrac{x^2}{4} - \dfrac{y^2}{2} = 1$

$e = \sqrt{1 + \dfrac{2}{4}} = \dfrac{\sqrt{3}}{\sqrt{2}} \qquad S \equiv (\pm\sqrt{6}, 0)$

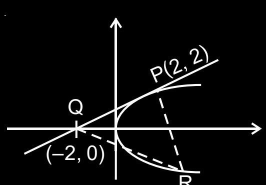
Directrix $\equiv x = \sqrt{6}$

Tangent at $(4, \sqrt{6})$ is $4x - 2\sqrt{6}y = 4$

$\therefore \quad Q \equiv (1, 0),\ R \equiv \left(\sqrt{6}, \dfrac{4\sqrt{6} - 4}{2\sqrt{6}}\right)$

Area of $\Delta QFR = \dfrac{1}{2}(\sqrt{6} - 1)\dfrac{4(\sqrt{6} - 1)}{2\sqrt{6}} = \dfrac{7}{\sqrt{6}} - 2$

101. Answer (1)



Tangent to $y^2 = 2x$ at P (2, 2)

$2y = x + 2$

$\therefore \quad Q\ (-2, 0)$

Normal at P (2, 2) meet at R $\left(\dfrac{9}{2}, -3\right)$

Area of $\Delta PQR = \left|\dfrac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -2 & 0 & 1 \\ \dfrac{9}{2} & 3 & 1 \end{vmatrix}\right|$

$= \dfrac{25}{2}$ sq. units

102. Answer (34)

$\because \quad y = mx + c$ passes through (–16, 0)

then c = 16 m ...(1)

also y = mx + c touches the given circle

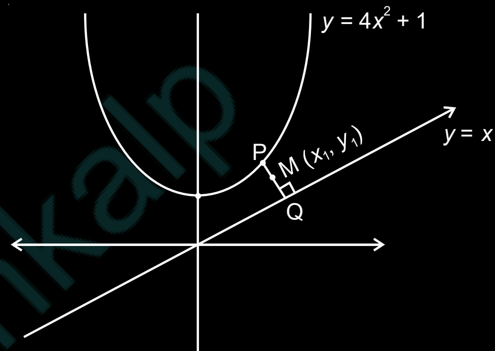
So, $\left|\dfrac{-10\,m + c}{\sqrt{1 + m^2}}\right| = 2 \qquad \ldots(2)$

$\Rightarrow \quad |3\,m| = \sqrt{1 + m^2}$

$\Rightarrow \quad m = \dfrac{1}{2\sqrt{2}}$ and $c = 4\sqrt{2}$

then $4\sqrt{2}(m + c) = 2 + 32 = 34$

103. Answer (2)



Let coordinate of mid-point M of PQ be (x_1, y_1)

Let coordinate of Q be (α, β).

$\therefore \quad \dfrac{\alpha - x_1}{1} = \dfrac{\beta - y_1}{-1} = \dfrac{-(x_1 - y_1)}{2}$

$\therefore \quad Q = (\alpha, \beta) = \left(\dfrac{x_1 + y_1}{2}, \dfrac{x_1 + y_1}{2}\right)$

and coordinate of $P = \left(\dfrac{3x_1 - y_1}{2}, \dfrac{3y_1 - x_1}{2}\right)$

$\therefore$ P lies on parabola

$\therefore \quad \dfrac{3y_1 - x_1}{2} = 4\left(\dfrac{3x_1 - y_1}{2}\right)^2 + 1$

$\therefore$ Required locus is $2(3x - y)^2 + (x - 3y) + 2 = 0$

104. Answer (4)

Let $E_2 = \dfrac{x^2}{A^2} + \dfrac{y^2}{B^2} = 1 \ (B > A)$

focus (O, Be)

$\therefore$ Be = b and A = a

$e = \sqrt{1 - \dfrac{b^2}{a^2}} = \sqrt{1 - \dfrac{A^2}{B^2}}$

$$\therefore \quad 1-\frac{a^2e^2}{b^2}=1-\frac{b^2}{a^2} \Rightarrow e^2=\frac{b^4}{a^4}$$

$$\therefore \quad e=\sqrt{1-e} \Rightarrow e^2+e-1=0$$

$$e=\frac{-1\pm\sqrt{5}}{2}$$

$$e=\frac{\sqrt{5}-1}{2}$$

105. Answer (1)

$\because$ Line L: $2x + y = k, k > 0$

$\Rightarrow$ L: $y = -2x + k$ is tangent to $\frac{x^2}{3}-\frac{y^2}{3}=1$

$\therefore$ $k^2 = 3 \cdot 4 - 3 = 9$

$\because$ L = 0 is also tangent to $y^2 = \alpha x$

$$\therefore \quad k = \frac{\alpha/4}{-2}$$

$\therefore$ $\alpha = -8k$

$\alpha = -24$

106. Answer (1)

$H \equiv 16(x^2 + 2x + 1) - 9(y^2 - 4y + 4) = 144$

$$\equiv \frac{(x+1)^2}{9}-\frac{(y-2)^2}{16}=1$$

Foci $\frac{x+1}{9}=\pm 3\left(\frac{5}{3}\right) \Rightarrow x+1=\pm 45$

i.e. (44, 2) and (–46, 2)

Let a general point $(-1 + 3\sec\theta, 2 + 4\tan\theta)$

Let centroid be (h, k)

$\therefore$ $3h = 44 + (-46) - 1 + 3\sec\theta$ &

$3k = 2 + 4\tan\theta + 2 + 2$

$\Rightarrow$ $\sec\theta = (h + 1)$ & $\frac{3}{4}(k-2) = \tan\theta$

Now $\sec^2\theta - \tan^2\theta = 1$

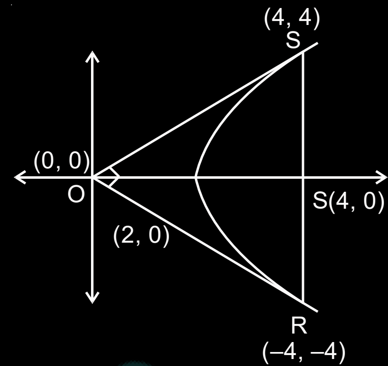
$\Rightarrow$ $16(h + 1)^2 - 9(k - 2)^2 = 16$

$\Rightarrow$ $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

107. Answer (1)

From given condition y axis is directrix and equation of parabola is

$y^2 = 8(x - 2)$



Let $y = mx$ be tangent

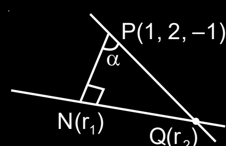
$\therefore$ $m^2x^2 - 8x + 16 = 0 \Rightarrow m = \pm 1$

So that lines $y = x$ and $y = -x$ are tangents.

$\therefore$ Coordinate of S and R be (4, 4) and (–4, 4)

$$\therefore \quad \text{area of } \Delta SOR = \left(\frac{1}{2}\times 4\times 4\right)\times 2 = 16$$

108. Answer (3)



$$L : \frac{x}{1}=\frac{y}{0}=\frac{z}{-1}=r$$

$N \equiv (r_1, 0, -r_1)$

P(1, 2, –1)

$NP(r_1 - 1, -2, -r_1 + 1) \perp NQ(1, 0, -1)$

$\Rightarrow$ $r_1 = 1$

$\Rightarrow$ N(1, 0, –1)

$Q(r_2, 0, -r_2)$

$PQ(r_2 - 1, -2, -r_2 + 1) \perp (1, 1, 2)$

$\Rightarrow$ $r_2 = -1$

$\Rightarrow$ Q(–1, 0, 1)

$$\cos\alpha = \frac{NP}{PQ} = \frac{1}{\sqrt{3}}$$

109. Answer (2)

Tangent at $(a\cos\theta, b\sin\theta)$ is

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1, (a=2, b=1)$$

$$x=a\to y=b\tan\frac{\theta}{2}, A\left(a, b\tan\frac{\theta}{2}\right)$$

$$x=-a\to y=b\cot\frac{\theta}{2}, B\left(-a, b\cot\frac{\theta}{2}\right)$$

Equations of circle

$$(x-a)(x+a)+\left(y-b\tan\frac{\theta}{2}\right)\left(y-b\cot\frac{\theta}{2}\right)=0$$

This passes through $(\sqrt{3}, 0)$

110. Answer (1)

Eqn. of circle ; $(x^2 - y^2) + 2(y^2) + 2x = 0$

$\Rightarrow$ $x^2 + y^2 + 2x = 0$ has centre A(–1, 0)

Vertex of parabola, $(x - 3)^2 = (y - 4)$ is B(3, 4)

AB = $y = (x + 1)$ has y-intercept 1.

111. Answer (1)

Image of (2, 1) w.r.t. y axis is (–2, 1)

$\therefore$ equation of reflected ray is

$$y-1=\frac{3-1}{5+2}(x+2)$$

$\therefore$ $2x - 7y + 11 = 0$ ...(i)

$\because$ $\frac{a}{e}-ae=\frac{8}{\sqrt{53}} \Rightarrow a=\frac{3}{\sqrt{53}}$

Now $\frac{2a}{e}-2\cdot\frac{3}{\sqrt{53}}\times=\frac{18}{\sqrt{53}}$

The equation of other directrix is : $2x - 7y + k = 0$

$\therefore$ $\left|\frac{k-11}{\sqrt{53}}\right|=\frac{18}{\sqrt{53}} \Rightarrow |k-11|=18$

$\therefore$ k = 29 or –7

$\therefore$ equation of directrix may be: $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$

112. Answer (3)

$\because$ ae = 1 and a = 2 so $b=\sqrt{3}$

$$E : \frac{(x-3)^2}{4}+\frac{(y+4)^2}{3}=1$$

Equation of tangent

$$y+4=m(x-3)\pm\sqrt{4m^2+3}$$

$$\Rightarrow \quad y=mx-3m-4\pm\sqrt{4m^2+3}$$

Comparing with $y = mx - 4$

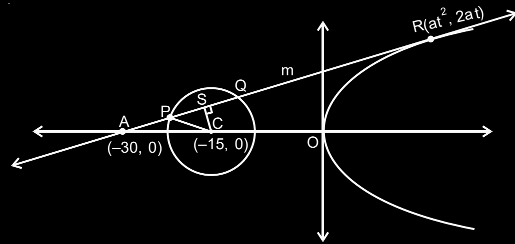
we get $-3\pm\sqrt{4m^2+3}=0$

$\Rightarrow$ $9m^2 = 4m^2 + 3$

$\Rightarrow$ $5m^2 = 3$

113. Answer (1)

Circle : $(x+15)^2+y^2=\left(\frac{15}{2}\right)^2$



Equation of tangent is

$$ty=x+\frac{15}{2}t^2 \qquad ...(i)$$

(i) passes through (–30, 0)

$\Rightarrow$ $0=-30+\frac{15}{2}t^2$

$\Rightarrow$ $t = \pm 2$

(i) $\Rightarrow$ $\pm 2y = x + 30$

$\Rightarrow$ $x - 2y + 30 = 0$ or $x + 2y + 30 = 0$

Consider $x - 2y + 30 = 0$

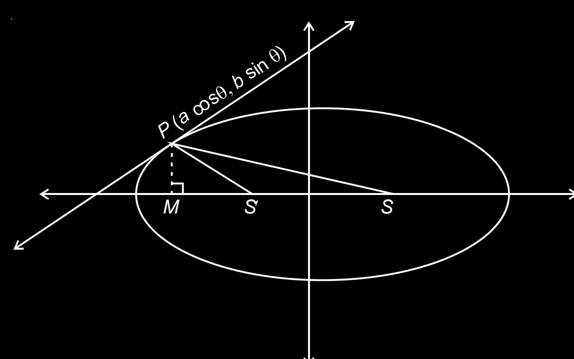
$$CS=\left|\frac{-15-0+30}{\sqrt{5}}\right|=3\sqrt{5}$$

$$PS=\sqrt{CP^2-CS^2}=\sqrt{\left(\frac{15}{2}\right)^2-\left(3\sqrt{5}\right)^2}=\frac{3\sqrt{5}}{2}$$

$$PQ=2PS=3\sqrt{5}$$

114. Answer (3)

Slope of line = $\frac{-1}{2}$



Slope of tangent = 2

Equation of tangent

$$\frac{x}{2\sqrt{2}}\cos\theta+\frac{y}{2}\sin\theta=1$$

$$\text{Slope}=-\frac{\cos\theta}{\sqrt{2}\sin\theta}=2$$

$$\tan\theta=\frac{-1}{2\sqrt{2}}\Rightarrow\cos\theta=\frac{-2\sqrt{2}}{3},\ \sin\theta=\frac{1}{3}$$

$$P(2\sqrt{2}\cos\theta,2\sin\theta)\equiv\left(\frac{-8}{3},\frac{2}{3}\right)$$

$$PM=\frac{2}{3}$$

$$A=\frac{1}{2}\times SS'\times PM$$

$$=\frac{1}{2}\times 2ae\times\frac{2}{3}$$

$$=\sqrt{a^2-b^2}\times\frac{2}{3}$$

$$=\sqrt{8-4}\times\frac{2}{3}$$

$$=\frac{4}{3}$$

$$e=\sqrt{1-\frac{4}{8}}=\frac{1}{\sqrt{2}}$$

$$\left(5-e^2\right)\cdot A=\left(5-\left(\frac{1}{\sqrt{2}}\right)^2\right)\times\frac{4}{3}=6$$

115. Answer (3)

Let P (asecθ, btanθ)

$$T:\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1\quad\Rightarrow\quad Q\left(0,-\frac{b}{\tan\theta}\right)$$

$$N:\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2\Rightarrow\quad R\left(0,\frac{a^2+b^2}{b}\tan\theta\right)$$

$$QR=\left|\left(\frac{a^2+b^2}{b}\right)\tan\theta+\frac{b}{\tan\theta}\right|$$

$\because\ \dfrac{b^2}{a^2}=e^2-1=\dfrac{1}{4}\Rightarrow$ $4b^2 = a^2$, so $b^2 = 3$ and $a^2 = 12$

$$\sec\theta=-\sqrt{2}\text{ and }\tan\theta=1$$

$$\text{Now, }QR=\left|\left(\frac{12+3}{\sqrt{3}}\right)1+\frac{\sqrt{3}}{1}\right|=6\sqrt{3}$$

116. Answer (1)

Let mid point of chord of hyperbola $x^2 - y^2 = 4$ be (x_1, y_1)

$\therefore$ Equation of chord is :

$$xx_1-yy_1-4=x_1^2-y_1^2-4$$

$$\therefore\quad yy_1=xx_1-x_1^2+y_1^2$$

$$\therefore\quad y=\frac{x_1}{y_1}x+\frac{y_1^2-x_1^2}{y_1}\quad\text{...(i)}$$

$\because$ Equation (i) is tangent to parabola $y^2 = 8x$ then

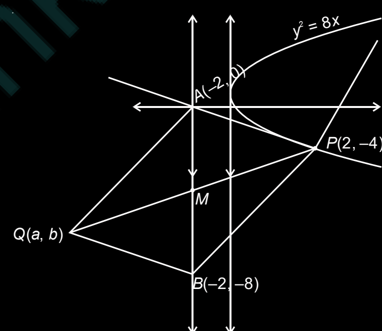
$$\frac{y_1^2-x_1^2}{y_1}=\frac{2}{\frac{x_1}{y_1}}$$

$$\therefore\quad (y_1^2-x_1^2)x_1=2y_1^2$$

$$\therefore\quad y_1^2(x_1-2)=x_1^3$$

$\therefore$ Required locus is : $y^2(x - 2) = x^3$

117. Answer (3)



Directrix: $x = -2$

Tangent at (2, –4)

$-4y = 4(x + 2)$

$x + y + 2 = 0$

if $x = -2 \Rightarrow y = 0$

$A \equiv (-2, 0)$

Normal at (2, –4)

$x - y = 6$

$x = -2$

$\Rightarrow\ y = -8$

$B \equiv (-2, -8)$

$$\frac{a+2}{2}=\frac{(-2)+(-2)}{2}$$

$\Rightarrow\ a = -6$

$$\frac{b+(-4)}{2}=\frac{0+(-8)}{2}$$

$\Rightarrow\ b = -4$

$2a + b = -16$

118. Answer (2)

Any point on ellipse is $P(b\cos\theta, 2a\sin\theta)$

tangent at $P : \frac{x}{b}\cos\theta + \frac{y}{2a}\sin\theta = 1$

$x = 0 \Rightarrow y = \frac{2a}{\sin\theta}$ and $y = 0 \Rightarrow x = \frac{b}{\cos\theta}$

Area of triangle $= \frac{1}{2}\left|\frac{2a}{\sin\theta} \times \frac{b}{\cos\theta}\right| = \left|\frac{2ab}{\sin 2\theta}\right|$

Minimum area = $2ab$ where $\sin 2\theta = \pm 1$

$\Rightarrow$ $k = 2$.

119. Answer (3)

From directrix of parabola perpendicular tangent are drawn.

and directrix of parabola $y^2 = 16(x - 3)$ is $x = -1$

$\therefore$ Required locus is $x + 1 = 0$

120. Answer (6)

$z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ and $z_1 - z_2 = (x_1 - x_2) + 2(y_1 - y_2)$

$\arg(z_1 - z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{4}$

$y_1 - y_2 = x_1 - x_2$ ...(i)

$|z - 3| = \text{Re}(z) \Rightarrow |[x - 3) + 2y| = x$

$(x - 3)^2 + (y)^2 = x^2$

$y^2 = 6\left(x - \frac{3}{2}\right)$

Let point on this parabola

$\left(\frac{3}{2} + at_1^2,\ 2at_1\right)$ and $\left(\frac{3}{2} + at_2^2,\ 2at_2\right)$, where $a = \frac{6}{4}$

$y_1 - y_2 = x_1 - x_2$

$2a(t_1 - t_2) = a(t_1^2 - t_2^2)$

$t_1 + t_2 = 2$

Now, $\text{img}(z_1 + z_2) = y_1 + y_2$

$= 2a(t_1 + t_2)$

$= 2 \times \frac{6}{4}(2) = 6$

121. Answer (2)

$x\left(\frac{\cos\theta}{5}\right) + y\left(\frac{\sin\theta}{12}\right) = 1$ ...(i)

Let tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

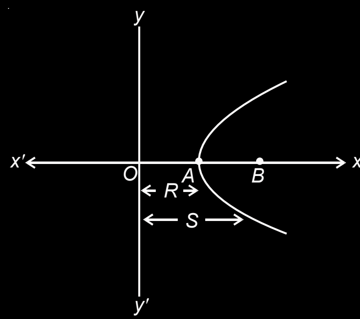
i.e., $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$ ...(ii)

Comparing (i) and (ii) we get $a = 5$ & $b = 12$

Curve is $\frac{x^2}{25} + \frac{y^2}{144} = 1$

$\Rightarrow$ $144x^2 + 25y^2 = 3600$

122. Answer (2)



$\therefore$ OA = R and = OB = S

$\therefore$ AB = a = $S - R$

$\therefore$ Length of latus rectum = 4(S – R)

123. Answer (4)

Let general point on ellipse $(2\cos\theta, 3\sin\theta)$

Let mid-point of $(2\cos\theta, 3\sin\theta)$ & $(-3, -5)$ be (h, k)

$\therefore$ $2\cos\theta - 3 = 2h$ & $3\sin\theta - 5 = 2k$

$\Rightarrow$ $\cos\theta = \frac{2h+3}{2}$ & $\sin\theta = \frac{2k+5}{3}$

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow$ $\left(\frac{2y+5}{3}\right)^2 + \left(\frac{2x+3}{2}\right)^2 = 1$

$\Rightarrow$ $4(2y + 5)^2 + 9(2x + 3)^2 = 36$

$\Rightarrow$ $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

124. Answer (2)

Equation of tangent $yy_1 = 4(x + x_1)$

$y(-4) = 4(x + 2)$

$-y = x + 2$

$\Rightarrow$ $x + y + z = 0$

Length of perpendicular from centre (0, 0) should be equal to radius

Length of perpendicular $= \frac{2}{\sqrt{2}} = \sqrt{a} \Rightarrow a = 2$

125. Answer (1)

Ellipse : $x^2 + 9y^2 = 9$

Circle : $x^2 + y^2 = 3$

$$x^2 = \frac{9}{4}, y^2 = \frac{3}{4} \Rightarrow x = \frac{\pm 3}{2}, y = \frac{\pm\sqrt{3}}{2}$$

Point of intersection $\left(\frac{\pm 3}{2}, \frac{\pm\sqrt{3}}{2}\right)$

Consider one point, say $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$

Tangent to ellipse $\frac{3}{2}x + \frac{9\sqrt{3}}{2}y = 9$

$$m_1 = \frac{-1}{3\sqrt{3}}$$

Tangent to circle $\frac{3}{2}x + \frac{\sqrt{3}}{2}y = 3$

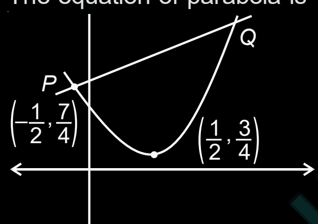
$$m_2 = -\sqrt{3}$$

$$\tan\theta = \frac{\frac{-1}{3\sqrt{3}} + \sqrt{3}}{1 + \left(\frac{-1}{3\sqrt{3}}\right) \times \left(-\sqrt{3}\right)}$$

$$\therefore \quad \frac{8}{4\sqrt{3}} = \frac{2}{\sqrt{3}}$$

126. Answer (2)

The equation of parabola is



$$\left(x - \frac{1}{2}\right)^2 = \left(y - \frac{3}{4}\right)$$

$\therefore$ $y = x^2 - x + 1$

$\therefore$ Point $P = \left(-\frac{1}{2}, \frac{7}{4}\right)$

$$\therefore \quad \frac{dy}{dx} = 2x - 1$$

Slope of normal at $x = -\frac{1}{2}$ is $\frac{1}{2}$.

Equation of normal is : $y - \frac{7}{4} = \frac{1}{2}\left(x + \frac{1}{2}\right)$

$\therefore$ $2x - 4y + 8 = 0$

$\therefore$ $x - 2y + 4 = 0$

$\therefore$ Coordinate of $Q = (2, 3)$

$$\therefore \quad PQ^2 = \left(2 + \frac{1}{2}\right)^2 + \left(3 - \frac{7}{4}\right)^2 = \frac{125}{16}$$

127. Answer (2)

$$\sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $2a^2 = 3b^2$ ...(i)

$\left(\sqrt{\frac{3}{2}}, 1\right)$ lies on E $\Rightarrow$ $\frac{3}{2a^2} + \frac{1}{b^2} = 1$

$\Rightarrow$ $b^2 = 2$ and $a^2 = 3$ (using (i))

S (ae, 0) = (1, 0)

Circle: $(x - 1)^2 + y^2 = \frac{4}{3}$ ...(ii)

Ellipse: $\frac{x^2}{3} + \frac{y^2}{2} = 1$ ...(iii)

Solving (ii) & (iii)

x = 1, 5 (rejected)

$$x = 1, \ y = \pm\frac{2}{\sqrt{3}}, PQ^2 = \frac{16}{3}$$

128. Answer (1)

A tangent to parabola $y^2 = 4x$ is

$y = mx + \frac{1}{m}$ ...(1)

This line is also the tangent to circle

$x^2 + y^2 - 6x = 0$

$\therefore$ Centre of circle = (3, 0)

radius of circle = 3

$$\therefore \quad \frac{\left|3m + \frac{1}{m}\right|}{\sqrt{1 + m^2}} = 3$$

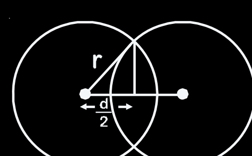
$$\therefore \quad m = \pm\frac{1}{\sqrt{3}}$$

Equation of common tangents are: $y = \pm\frac{1}{\sqrt{3}}x \pm \sqrt{3}$

$\therefore$ $\sqrt{3}y = x + 3$ is one of the common tangent

129. Answer (1)

Let the distance between centres be '*d*'.



Length of common chord $= 2\sqrt{r^2 - \left(\frac{d}{2}\right)^2} = 4$

$$\Rightarrow \quad r^2 - \frac{d^2}{4} = 4 \qquad \left(\text{given } r = 2\sqrt{5}\right)$$

$$\Rightarrow \quad \frac{d^2}{4} = 16$$

$\Rightarrow$ $d = 8$

130. Answer (3)

Circle passes through $A(0, 1)$ and $B(2, 4)$. So its centre is the point of intersection of perpendicular bisector of AB and normal to the parabola at (2, 4).

Perpendicular bisector of AB;

$$y - \frac{5}{2} = -\frac{2}{3}(x-1) \Rightarrow 4x + 6y = 19$$

Normal to the parabola at (2, 4),

$$y - 4 = -\frac{1}{4}(x-2) \Rightarrow x + 4y = 18$$

Centre of the circle is $\left(-\frac{16}{5}, \frac{53}{10}\right)$

131. Answer (4)

$$E : \frac{x^2}{16} + \frac{y^2}{9} = 1$$

$\because$ $\left(\pm\sqrt{7}, 0\right)$ are the foci of given ellipse. So for any point P on it; $PA + PB = 2a$

$\Rightarrow$ $PA + PB = 2(4) = 8$

132. Answer (1)

For tangent to parabola $y = x^2$ at (2, 4)

$$\left.\frac{dy}{dx}\right|_{(2,4)} = 4$$

Equation of tangent is

$y - 4 = 4(x - 2)$

$\Rightarrow$ $4x - y - 4 = 0$

Family of circle can be given by

$(x - 2)^2 + (y - 4)^2 + \lambda(4x - y - 4) = 0$

As it passes through (0, 6)

$2^2 + 2^2 + \lambda(-10) = 0$

$$\Rightarrow \quad \lambda = \frac{4}{5}$$

Equation of circle is

$$(x-2)^2 + (y-4)^2 + \frac{4}{5}(4x - y - 4) = 0$$

$$\Rightarrow \quad (x^2 + y^2 - 4x - 8y + 20) + \left(\frac{16}{5}x - \frac{4}{5}y - \frac{16}{5}\right) = 0$$

$$A = -4 + \frac{16}{5},\ C = 20 - \frac{16}{5}$$

So, $A + C = 16$

133. Answer (4)

$$\frac{x^2}{\left(\frac{b^2}{a^2}\right)} - \frac{y^2}{b^2} = 1$$

Tangent in slope form $\Rightarrow$ $y = mx \pm \sqrt{\frac{b^2}{a^2}m^2 - b^2}$

i.e., same as $y = \frac{\lambda x}{2} - \frac{\mu}{2}$

Comparing coefficients,

$$m = \frac{\lambda}{2}, \frac{b^2}{a^2}m^2 - b^2 = \frac{\mu^2}{4}$$

Eliminating m, $\frac{b^2}{a^2}.\frac{\lambda^2}{4} - b^2 = \frac{\mu^2}{4}$

$$\Rightarrow \quad \frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2} = 4$$

134. Answer (2929)

$\because$ (α, β) lies on the given ellipse, $25\alpha^2 + 4\beta^2 = 1$ ...(1)

Tangent to the parabola, $y = mx + \frac{1}{m}$ passes through (α, β). So, $\alpha m^2 - \beta m + 1 = 0$ has roots m_1 and $4m_1$,

$m_1 + 4m_1 = \frac{\beta}{\alpha}$ and $m_1 \cdot 4m_1 = \frac{1}{\alpha}$

Gives that $4\beta^2 = 25\alpha$ ...(2)

from (1) and (2)

$25(\alpha^2 + \alpha) = 1$ ...(3)

Now, $(10\alpha + 5)^2 + (16\beta^2 + 50)^2$

$= 25(2\alpha + 1)^2 + 2500\,(2\alpha + 1)^2$

$= 2525\,(4\alpha^2 + 4\alpha + 1)$ from equation (3)

$= 2525\left(\frac{4}{25}+1\right)$

$= 2929$

135. Answer (1)

According to the question $\left(\text{Let } P(x,y)\right)$

$2x - y\frac{dx}{dy} = 0$ $\left(\begin{array}{l}\because \text{ equation of tangent at} \\ P : y - y = \frac{dy}{dx}(y - x)\end{array}\right)$

$\therefore \quad 2\frac{dy}{y} = \frac{dx}{x}$

$\Rightarrow \quad 2\ln y = \ln x + \ln c$

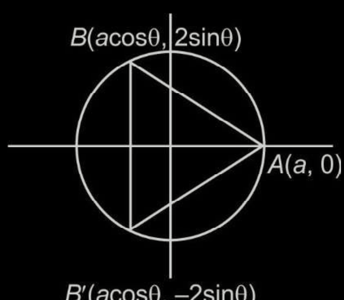
$\Rightarrow \quad y^2 = cx$ $\quad\because$ this curve passes

through (3, 3) $\therefore$ $\boxed{c = 3}$ $\quad\therefore$ required parabola

$y^2 = 3x$ and L.R = 3

136. Answer (1)

Given ellipse $\frac{x^2}{a^2} + \frac{y^2}{4} = 1,\ a > 2$



$\therefore$ Let $A(\theta)$ be the area of $\Delta ABB'$

Then $A(\theta) = \frac{1}{2}4\sin\theta(a + a\cos\theta)$

$A'(\theta) = a\left(2\cos\theta + 2\cos^2\theta\right)$

For maxima $A'(\theta) = 0$

$\Rightarrow \quad \cos\theta = -1,\ \cos\theta = \frac{1}{2}$

But for maximum area $\cos\theta = \frac{1}{2}$

$\therefore \quad A(\theta) = 6\sqrt{3}$

$\Rightarrow \quad 2\frac{\sqrt{3}}{2}\left(a + \frac{a}{2}\right) = 6\sqrt{3}$

$\Rightarrow \quad a = 4$

$\therefore \quad e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4}{16}} = \frac{\sqrt{3}}{2}$

137. Answer (42)

$\because \quad H : \frac{x^2}{a^2} - \frac{y^2}{1} = 1$

$\therefore$ Length of latus rectum $= \frac{2}{a}$

$E : \frac{x^2}{4} + \frac{y^2}{3} = 1$

Length of latus rectum $= \frac{6}{2} = 3$

$\because \quad \frac{2}{a} = 3 \Rightarrow a = \frac{2}{3}$

$\therefore \quad 12\left(e_H^2 + e_E^2\right) = 12\left(1 + \frac{9}{4}\right) + \left(1 - \frac{3}{4}\right) = 42$

138. Answer (10)

Focus = (4, 4) and vertex = (3, 2)

$\therefore$ Point of intersection of directrix with axis of parabola = A = (2, 0)

Image of $A(2, 0)$ with respect to line

$x + 2y = 6$ is $B(x_2, y_2)$

$\therefore \quad \frac{x_2 - 2}{1} = \frac{y_2 - 0}{2} = \frac{-2(2 + 0 - 6)}{5}$

$\therefore \quad B(x_2, y_2) = \left(\frac{18}{5}, \frac{16}{5}\right)$.

Point B is point of intersection of direction with axes of parabola P_2.

$\therefore \quad x + 2y = \lambda$ must have point $\left(\frac{18}{5}, \frac{16}{5}\right)$

$\therefore \quad x + 2y = 10$

139. Answer (3)

Let tangent to $y^2 = x$ be

$$y = mx + \frac{1}{4m}$$

For it being tangent to circle.

$$\left|\frac{\frac{1}{4}m}{\sqrt{1+m^2}}\right| = \sqrt{2}$$

$\Rightarrow \quad 32m^4 + 32m^2 - 1 = 0$

$$\Rightarrow \quad m^2 = \frac{-32 \pm \sqrt{(32)^2 + 4(32)}}{64}$$

$$\Rightarrow \quad 8m_1m_2 = -4 + 3\sqrt{2}$$

140. Answer (4)

$$x = 2t,\ y = \frac{2}{3}$$

$$t \to 1 \quad A \equiv \left(2, \frac{1}{3}\right)$$

Given conic is $x^2 = 12y \Rightarrow S \equiv (0, 3)$

Let $B \equiv (0, \beta)$

Given $SA \perp BA$

$$\left(\frac{\frac{1}{3}}{2-3}\right)\left(\frac{\beta - \frac{1}{3}}{-2}\right) = -1$$

$$\Rightarrow \quad \left(\beta - \frac{1}{3}\right)\frac{1}{3} = -2$$

$$\Rightarrow \quad \beta = \frac{1}{3}\left(\frac{-17}{3}\right)$$

$$\underset{(\text{as } t \to 1)}{\text{Ordinate of centroid}} = k = \frac{\beta + \frac{1}{3} + 3}{3}$$

$$= \frac{\frac{-17}{9} + \frac{10}{3}}{3} = \frac{13}{18}$$

141. Answer (3)

$\because \quad$ Line $y = kx + 4$ touches the parabola $y = x - x^2$.

So, $kx + 4 = x - x^2 \Rightarrow x^2 + (k - 1)x + 4 = 0$ has only one root

$(k - 1)^2 = 16 \Rightarrow k = 5$ or -3 but $k > 0$

So, $k = 5$.

And hence $x^2 + 4x + 4 = 0 \Rightarrow x = -2$

So, $P(-2, -6)$ and V is $\left(\frac{1}{2}, \frac{1}{4}\right)$

$$\text{Slope of } PV = \frac{\frac{1}{4} + 6}{\frac{1}{2} + 2} = \frac{5}{2}$$

142. Answer (1)

Let point $(a, a + 1)$ as the point of intersection of line and ellipse.

$$\text{So, } \frac{a^2}{4} + \frac{(a+1)^2}{2} = 1 \Rightarrow a^2 + 2(a^2 + 2a + 1) = 4$$

$\Rightarrow \quad 3a^2 + 4a - 2 = 0$

If roots of this equation are α and β.

So, $P(\alpha, \alpha + 1)$ and $Q(\beta, \beta + 1)$

$PQ^2 = 4r^2 = (\alpha - \beta)^2 + (\alpha - \beta)^2$

$$\Rightarrow \quad 9r^2 = \frac{9}{4}(2(\alpha - \beta)^2)$$

$$= \frac{9}{2}\left[(\alpha + \beta)^2 - 4\alpha\beta\right]$$

$$= \frac{9}{2}\left[\left(-\frac{4}{3}\right)^2 + \frac{8}{3}\right]$$

$$= \frac{1}{2}[16 + 24] = 20$$

143. Answer (85)

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\left(e=\frac{5}{4}\right)$$

So, $b^2=a^2\left(\frac{25}{16}-1\right)\Rightarrow b=\frac{3}{4}a$

Also $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ lies on the given hyperbola

So, $\frac{64}{5a^2}-\frac{144}{25\left(\frac{9a^2}{16}\right)}=1\Rightarrow a=\frac{8}{5}$ and $b=\frac{6}{5}$

Equation of normal

$$\frac{64}{25}\left(\frac{x}{8/\sqrt{5}}\right)+\frac{36}{25}\left(\frac{y}{12/5}\right)=4$$

$\Rightarrow\ \frac{8}{5\sqrt{5}}x+\frac{3}{5}y=4$

$\Rightarrow\ 8\sqrt{5}x+15y=100$

So, $\beta=15$ and $\lambda=100$

Gives $\lambda-\beta=85$

144. Answer (2)

Let $P(at^2, 2at)$ where $a=\frac{3}{2}$

$T: yt = x + at^2$ So point Q is $\left(-a, at-\frac{a}{t}\right)$

$N: y = -tx + 2at + at^3$ passes through $(5, -8)$

$$-8=-5t+3t+\frac{3}{2}t^3$$

$\Rightarrow\ 3t^3 - 4t + 16 = 0$

$\Rightarrow\ (t + 2)(3t^2 - 6t + 8) = 0$

$\Rightarrow\ t = -2$

So ordinate of point Q is $-\frac{9}{4}$.

145. Answer (4)

Let $y = mx + c$ is the common tangent

So $c=\frac{1}{m}=\pm\frac{3}{2}\sqrt{1+m^2}\Rightarrow m^2=\frac{1}{3}$

So equation of common tangents will be

$y=\pm\frac{1}{\sqrt{3}}x\pm\sqrt{3}$, which intersects at $Q(-3, 0)$

Major axis and minor axis of ellipse are 12 and 6. So eccentricity

$e^2=1-\frac{1}{4}=\frac{3}{4}$ and length of latus rectum

$=\frac{2b^2}{a}=3$

Hence $\frac{\ell}{e^2}=\frac{3}{3/4}=4$

146. Answer (2)

$C_1:\frac{x^2}{16}+\frac{y^2}{9}=1$ and $C_2: x^2+y^2=12$

Let $y=mx\pm\sqrt{16m^2+9}$ be any tangent to C_1 and if this is also tangent to C_2 then

$$\left|\frac{\sqrt{16m^2+9}}{\sqrt{m^2+1}}\right|=\sqrt{12}$$

$\Rightarrow\ 16m^2+9=12m^2+12$

$\Rightarrow\ 4m^2=3\ \Rightarrow 12m^2=9$

147. Answer (3)

Let $P(2\cos\theta, \sqrt{2}\sin\theta)$ be any point on ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$ and $Q(4, 3)$ and let (h, k) be the mid point of PQ

then $h=\frac{2\cos\theta+4}{2},\ k=\frac{\sqrt{2}\sin\theta+3}{2}$

$\therefore\ \cos\theta=h-2,\ \sin\theta=\frac{2k-3}{\sqrt{2}}$

$$\therefore \quad (h-2)^2+\left(\frac{2k-3}{\sqrt{2}}\right)^2=1$$

$$\Rightarrow \quad \frac{(x-2)^2}{1}+\frac{\left(y-\frac{3}{2}\right)^2}{\frac{1}{2}}=1$$

$$\therefore \quad e=\sqrt{1-\frac{1}{2}}=\frac{1}{\sqrt{2}}$$

148. Answer (3)

Given hyperbola : $\dfrac{x^2}{a^2}-\dfrac{y^2}{9}=1$

$\because$ It passes through $(8, 3\sqrt{3})$

$$\because \quad \frac{64}{a^2}-\frac{27}{9}=1 \Rightarrow a^2=16$$

Now, equation of normal to hyperbola

$$\frac{16x}{8}+\frac{9y}{3\sqrt{3}}=16+9$$

$$\Rightarrow \quad 2x+\sqrt{3}y=25 \qquad \ldots\text{(i)}$$

$\left(-1, 9\sqrt{3}\right)$ satisfies (i)

149. Answer (12)

Equation of L_1 is

$$\frac{x\sec\theta}{4}-\frac{y\tan\theta}{2}=1 \qquad \ldots\text{(i)}$$

Equation of line L_2 is

$$\frac{x\tan\theta}{2}+\frac{y\sec\theta}{4}=0 \qquad \ldots\text{(ii)}$$

$\because$ Required point of intersection of L_1 and L_2 is (x_1, y_1) then

$$\frac{x_1\sec\theta}{4}-\frac{y_1\tan\theta}{2}-1=0 \qquad \ldots\text{(iii)}$$

$$\text{and } \frac{y_1\sec\theta}{4}+\frac{x_1\tan\theta}{2}=0 \qquad \ldots\text{(iv)}$$

From equations (iii) and (iv)

$$\sec\theta=\frac{4x_1}{x_1^2+y_1^2} \text{ and } \tan\theta=\frac{-2y_1}{x_1^2+y_1^2}$$

$\therefore$ Required locus of (x_1, y_1) is

$$(x^2+y^2)^2=16x^2-4y^2$$

$$\therefore \quad \alpha=16, \beta=-4$$

$$\therefore \quad \alpha+\beta=12$$

150. Answer (2)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\Rightarrow \quad \frac{\left(-4\sqrt{\frac{2}{5}}\right)^2}{a^2}+\frac{32}{b^2}=1$$

$$\Rightarrow \quad \frac{32}{5a^2}+\frac{9}{b^2}=1 \qquad \ldots\text{(i)}$$

$a^2(1-e^2)=b^2$

$$a^2\left(1-\frac{1}{16}\right)=b^2$$

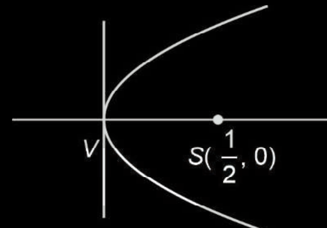
$$15a^2=16b^2 \Rightarrow a^2=\frac{16b^2}{15}$$

From (i)

$$\frac{6}{b^2}+\frac{9}{b^2}=1 \Rightarrow b^2=15 \;\;\&\; a^2=16$$

$a^2+b^2=15+16=31$

151. Answer (63)



Let the equation of circle be

$$x\left(x-\frac{1}{2}\right)+y^2+\lambda y=0$$

$\Rightarrow \quad x^2 + y^2 - \frac{1}{2}x + \lambda y = 0$

Radius = $\sqrt{\frac{1}{16} + \frac{\lambda^2}{4}} = 2$

$\Rightarrow \quad \lambda^2 = \frac{63}{4}$

$\Rightarrow \quad \left(x - \frac{1}{4}\right)^2 + \left(y + \frac{\lambda}{2}\right)^2 = 4$

$\because$ This circle and parabola $y - \alpha = \left(x - \frac{1}{4}\right)^2$ touch each other, so

$\alpha = -\frac{\lambda}{2} + 2$

$\Rightarrow \alpha - 2 = -\frac{\lambda}{2}$

$\Rightarrow (\alpha - 2)^2 = \frac{\lambda^2}{4} = \frac{63}{16}$

$\Rightarrow (4\alpha - 8)^2 = 63$

152. Answer (4)

Given vertex is (5, 4) and directrix $3x + y - 29 = 0$

Let foot of perpendicular of (5, 4) on directrix is (x_1, y_1)

$$\frac{x_1 - 5}{3} = \frac{y_1 - 4}{1} = \frac{-(-10)}{10}$$

$\therefore \quad (x_1, y_1) \equiv (8, 5)$

So, focus of parabola will be $S = (2, 3)$

Let $P(x, y)$ be any point on parabola, then

$$(x-2)^2 + (y-3)^2 = \frac{(3x + y - 29)^2}{10}$$

$$\Rightarrow \quad 10(x^2 + y^2 - 4x - 6y + 13) = 9x^2 + y^2 + 841 + 6xy - 58y - 174x$$

$\Rightarrow \quad x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$

and given parabola

$x^2 + ay^2 + bxy + cx + dy + k = 0$

$\therefore \quad a = 9, b = -6, c = 134, d = -2, k = -711$

$\therefore \quad a + b + c + d + k = -576$

153. Answer (2)

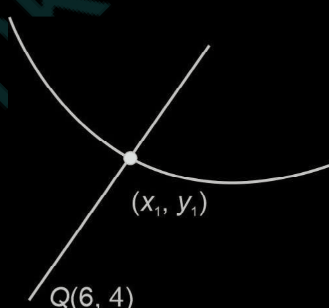
$1 + \frac{b^2}{a^2} = \frac{5}{2} \Rightarrow \frac{b^2}{a^2} = \frac{3}{2}$

$\frac{2b^2}{a} = 6\sqrt{2} \Rightarrow 2.\frac{3}{2}.a = 6\sqrt{2}$

$\Rightarrow \quad a = 2\sqrt{2}, b^2 = 12$

$c^2 = a^2m^2 - b^2 = 8.4 - 12 = 20$

154. Answer (13)



$\frac{y_1 - 4}{x_1 - 6} = -\frac{1}{4x_1 + 1}$

$\Rightarrow \quad \frac{2x_1^2 + x_1 - 2}{x_1 - 6} = -\frac{1}{4x_1 + 1}$

$\Rightarrow \quad 6 - x_1 = 8x_1^3 + 6x_1^2 - 7x_1 - 2$

$\Rightarrow \quad 8x_1^3 + 6x_1^2 - 6x_1 - 8 = 0$

So $x_1 = 1 \Rightarrow y_1 = 5$

Area = $\left|\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 4 & 1 \\ 1 & 5 & 1 \end{vmatrix}\right| = 13$

155. Answer (4)

H : $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1,$ then

$e^2=\dfrac{11}{14}l$ (l be the length of LR)

$\Rightarrow\ a^2+b^2=\dfrac{11}{7}b^2a$ ...(i)

and $e'^2=\dfrac{11}{8}l'$

(l' be the length of LR of conjugate hyperbola)

$\Rightarrow\ a^2+b^2=\dfrac{11}{4}a^2b$ ...(ii)

By (i) and (ii)

$\boxed{7a=4b}$

then by (i)

$\dfrac{16}{49}b^2+b^2=\dfrac{11}{7}b^2\cdot\dfrac{4b}{7}$

$\Rightarrow$ $44b = 65$ and $77a = 65$

$\therefore$ $77a + 44b = 130$

156. Answer (2)

Vertex of Parabola : (2, –1)

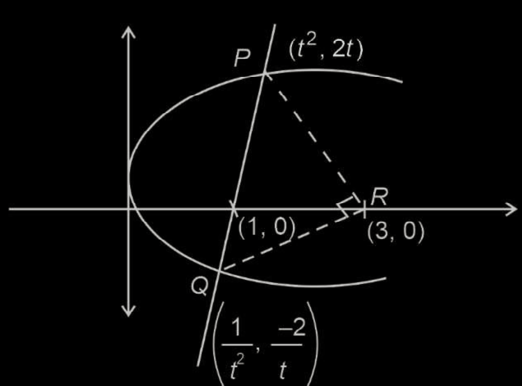
and directrix : $4x - 3y = 21$

Distance of vertex from the directrix

$a=\left|\dfrac{8+3-21}{\sqrt{25}}\right|=2$

$\therefore$ length of latus rectum = $4a$ = 8

157. Answer (2)



As $\angle PRQ=\dfrac{\pi}{2}$

$\left(\dfrac{\frac{2}{t}}{3-\frac{1}{t^2}}\right)\cdot\left(\dfrac{-2t}{3-t^2}\right)=-1$

$\Rightarrow$ $t = \pm 1$

$\therefore$ $P \equiv (1, 2)$ & $Q(1, -2)$

$\therefore$ for ellipse $\dfrac{1}{a^2}+\dfrac{4}{b^2}=1$ and $ae=1$

$\Rightarrow\ \dfrac{1}{a^2}+\dfrac{4}{a^2(1-e^2)}=1$

$\Rightarrow\ 1+\dfrac{4}{(1-e^2)}=\dfrac{1}{e^2}$

$\Rightarrow$ $(5 - e^2)e^2 = 1 - e^2$

$\Rightarrow$ $e^4 - 6e^2 + 1 = 0$

$\Rightarrow\ e^2=\dfrac{1}{3-2\sqrt{2}}\Rightarrow\dfrac{1}{e^2}=3+2\sqrt{2}$ 8. Answer (88)

$2a+2b=4\left(2\sqrt{2}+\sqrt{14}\right)$ ...(1)

$1+\dfrac{b^2}{a^2}=\dfrac{11}{4}$ ....(2)

$\Rightarrow\ \dfrac{b^2}{a^2}=\dfrac{7}{4}$ ...(3)

and $a+b=4\sqrt{2}+2\sqrt{14}$ ...(4)

By (3) and (4)

$\Rightarrow\ a+\dfrac{\sqrt{7}}{2}a=4\sqrt{2}+2\sqrt{14}$

$\Rightarrow\ \dfrac{a\left(2+\sqrt{7}\right)}{2}=2\sqrt{2}\left(2+\sqrt{7}\right)$

$\Rightarrow\ a=4\sqrt{2}$ $\Rightarrow a^2 = 32$ and $b^2 = 56$

$\Rightarrow$ $a^2 + b^2 = 32 + 56 = 88$

158. Answer (4)

$$P: y^2 = 4ax,\ a > 0 \qquad S(a,\ 0)$$

Equation of tangent on parabola $y = mx + \dfrac{a}{m}$

$$y = 3x + 5$$

$$\tan\frac{\pi}{4} = \left|\frac{m-3}{1+3m}\right| \Rightarrow m - 3 = \pm(1+3m)$$

$$m - 3 = 1 + 3m$$

$$\boxed{m = -2}$$

$$m - 3 = -1 - 3m$$

$$\boxed{m = \frac{1}{2}}$$

Equation of one tangent : $y = -2x - \dfrac{a}{2}$

Equation of other tangent : $y = \dfrac{x}{2} + 2a$

Point of contact are

$$\left(\frac{a}{(-2)^2}, \frac{-2a}{(-2)}\right) \text{ and } \left(\frac{a}{\left(\frac{1}{2}\right)^2}, \frac{-2a}{\frac{1}{2}}\right)$$

$$A\left(\frac{a}{4}, a\right) \text{ and } B(4a,\ -4a)$$

Now or (ΔABS) = 0 [S is the focus]

$$\frac{1}{2}\begin{vmatrix} \frac{a}{4} & a & 1 \\ 4a & -4a & 1 \\ a & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{a}{4}(-4a - 0) - a(4a - a) + 1\left(0 - \left(-4a^2\right)\right) = 0$$

$$= -a^2 - 3a^2 + 4a^2 = 0$$

Always true

159. Answer (2)

$x^2 + y^2 - 2x + 2fy + 1 = 0$ [entre = (1, $-f$]

Diameter $2px - y = 1$ ...(i)

$2x + py = 4p$ ...(ii)

$$x = \frac{5P}{2P^2 + 2} \qquad y = \frac{4P^2 - 1}{1 + P^2}$$

$\because$ $x = 1$ $f = 0$ [for $P = \dfrac{1}{2}$]

$\dfrac{5P}{2P^2 + 2} = 1$ $f = 3$ [for $P = 2$]

$\therefore$ $P = \dfrac{1}{2}, 2$

Centre can be $\left(\dfrac{1}{2}, 0\right)$ or $(1, 3)$

$\left(\dfrac{1}{2}, 0\right)$ will not satisfy

$\therefore$ Tangent should pass through

$(2, 3)$ for $3x^2 - y^2 = 3$

$$\frac{x^2}{1} - \frac{y^2}{3} = 1$$

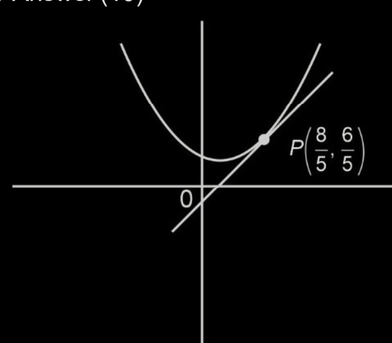
$$y = mx \pm \sqrt{m^2 - 3}$$

substitute (2, 3)

$$3 = m \pm \sqrt{m^2 - 3}$$

$\therefore$ $\boxed{m = 2}$

160. Answer (10)



Equation of tangent to the parabola at $P\left(\dfrac{8}{5}, \dfrac{6}{5}\right)$

$$75x \cdot \frac{8}{5} = 160\left(y + \frac{6}{5}\right) - 192$$

$\Rightarrow$ $120x = 160y$

$\Rightarrow$ $3x = 4y$

Equation of circle touching the given parabola at P can be taken as

$$\left(x - \frac{8}{5}\right)^2 + \left(y - \frac{6}{5}\right)^2 + \lambda(3x - 4y) = 0$$

If this circle touches y-axis then

$$\frac{64}{25} + \left(y - \frac{6}{5}\right)^2 + \lambda(-4y) = 0$$

$$\Rightarrow \quad y^2 - 2y\left(2\lambda + \frac{6}{5}\right) + 4 = 0$$

$\Rightarrow$ $D = 0$

$$\Rightarrow \quad \left(2\lambda + \frac{6}{6}\right)^2 = 4$$

$$\Rightarrow \quad \lambda = \frac{2}{5} \text{ or } -\frac{8}{5}$$

Radius = 1 or 4

Sum of diameter = 10

161. Answer (4)

Any tangent to $y^2 = 24x$ at (α, β)

$\beta y = 12(x + \alpha)$

Slope $= \frac{12}{\beta}$ and perpendicular to $2x + 2y = 5$

$$\Rightarrow \quad \frac{12}{\beta} = 1 \quad \Rightarrow \quad \beta = 12, \alpha = 6$$

Hence hyperbola is $\frac{x^2}{36} - \frac{y^2}{144} = 1$ and normal is drawn at (10, 16)

Equation of normal $\frac{36 \cdot x}{10} + \frac{144 \cdot y}{16} = 36 + 144$

$$\Rightarrow \quad \frac{x}{50} + \frac{y}{20} = 1$$

This does not pass though (15, 13) out of given option

162. Answer (3)

$y = mx + 2a + \frac{1}{m^2}$ (Equation of normal to $x^2 = 4ay$ in slope form) through (1, –1).

$4m^3 + 6m^2 + 1 = 0$

$\Rightarrow$ $m \simeq -1.6$

Slope of normal $\simeq \frac{-8}{5} = \tan\theta$

$$\Rightarrow \quad \cos\theta \simeq \frac{-5}{\sqrt{89}}, \ \sin\theta \simeq \frac{8}{\sqrt{89}}$$

$$x_p = 1 + \cos\theta \simeq 1 - \frac{5}{\sqrt{89}} \in \left(\frac{1}{4}, \frac{1}{2}\right)$$

163. Answer (4)

Centre of circle $x^2 + y^2 - 10x - 14y + 65 = 0$ is at (5, 7).

Let the equation of tangent to $y^2 = 8x$ is

$yt = x + 2t^2$

which passes through (5, 7)

$7t = 5 + 2t^2$

$\Rightarrow$ $2t^2 - 7t + 5 = 0$

$$t = 1, \frac{5}{2}$$

$$A = 2 \times 1^2 \times 2 \times \left(\frac{5}{2}\right)^2 = 25$$

$$B = 2 \times 2 \times 1 \times 2 \times 2 \times \frac{5}{2} = 40$$

$A + B = 65$

164. Answer (1552)

Vertices of hyperbola = (0, ± 8)

As ellipse pass through it i.e.,

$$0 + \frac{64}{b^2} = 1 \Rightarrow b^2 = 64 \qquad \ldots(1)$$

As major axis of ellipse coincide with transverse axis of hyperbola we have $b > a$ i.e.

$$e_E = \sqrt{1 - \frac{a^2}{64}} = \frac{\sqrt{64 - a^2}}{8}$$

and $e_H = \sqrt{1 + \frac{49}{64}} = \frac{\sqrt{113}}{8}$

$$\therefore \quad e_E \cdot e_H = \frac{1}{2} = \frac{\sqrt{64-a^2}\sqrt{113}}{64}$$

$\Rightarrow$ $(64 - a^2)(113) = 32^2$

$$\Rightarrow \quad a^2 = 64 - \frac{1024}{113}$$

$$\text{L.R of ellipse } = \frac{2a^2}{b} = \frac{2}{8}\left(\frac{113 \times 64 - 1024}{113}\right)$$

$$= l = \frac{1552}{113}$$

$\therefore$ $113l = 1552$

165. Answer (75)

Equation of ellipse is: $x^2 + 4y^2 + 2x + 8y - \lambda = 0$

$(x + 1)^2 + 4(y + 1)^2 = \lambda + 5$

$$\frac{(x+1)^2}{\lambda+5} + \frac{(y+1)^2}{\left(\frac{\lambda+5}{4}\right)} = 1$$

$$\text{Length of latus rectum } = \frac{2\cdot\left(\frac{\lambda+5}{4}\right)}{\sqrt{\lambda+5}} = 4.$$

$\therefore$ $\lambda = 59.$

Length of major axis = $2\cdot\sqrt{\lambda+5} = 16 = l$

$\therefore$ $\lambda + l = 75.$

166. Answer (3)

Equation of tangent at vertex : $L \equiv x + y - a = 0$

Focus : $F \equiv (a,\ a)$

Perpendicular distance of L from F

$$= \left|\frac{a+a-a}{\sqrt{2}}\right| = \left|\frac{a}{\sqrt{2}}\right|$$

$$\text{Length of latus rectum } = 4\left|\frac{a}{\sqrt{2}}\right|$$

$$\text{Given } 4\cdot\left|\frac{a}{\sqrt{2}}\right| = 16$$

$\Rightarrow |a| = 4\sqrt{2}$

167. Answer (20)

Equation of tangent to ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and given slope m is :

$y = mx + \sqrt{4m^2 + 9}$ ...(i)

For slope m equation of tangent to hyperbola is :

$y = mx + \sqrt{42m^2 - 143}$ ...(ii)

Tangents from (i) and (ii) are identical then

$$4m^2 + 9 = 42m^2 - 143$$

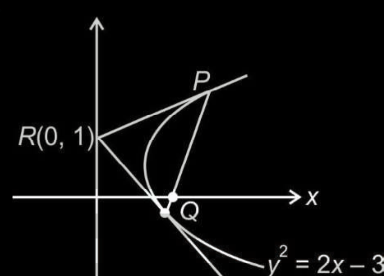
$\therefore$ $m = \pm 2$ (+2 is not acceptable)

$\therefore$ $m = -2.$

Hence $x_1 = \frac{8}{5}$ and $x_2 = \frac{84}{5}$

$$\therefore \quad |2x_1 + x_2| = \left|\frac{16}{5} + \frac{84}{5}\right| = 20$$

168. Answer (2)



Equation of chord PQ

$\Rightarrow$ $y \times 1 = x - 3$

$\Rightarrow$ $x - y = 3$

For point P & Q

Intersection of PQ with parabola $P : (6, 3)$ $Q : (2, -1)$

Slope of $RQ = -1$ & Slope of $PQ = 1$

Therefore $\angle PQR = 90° \Rightarrow$ Orthocentre is at $Q : (2, -1)$

169. Answer (03.00)

The equation of tangent to hyperbola $x^2 - y^2 = 1$ within slope m is equal to

$y = mx \pm \sqrt{m^2 - 1}$ ...(i)

And for same slope m, equation of tangent to

ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $y=mx\pm\sqrt{a^2m^2+b^2}$ ...(ii)

$\because$ Equation (i) and (ii) are identical

$\therefore$ $a^2m^2 + b^2 = m^2 - 1$

$\therefore$ $m^2=\frac{1+b^2}{1-a^2}$

But equation of common tangent is $y=\sqrt{\frac{5}{2}}x+k$

$\therefore$ $m=\sqrt{\frac{5}{2}}\Rightarrow\frac{5}{2}=\frac{1+b^2}{1-a^2}$

$\therefore$ $5a^2 + 2b^2 = 3$ ...(i)

eccentricity of ellipse $=\frac{1}{\sqrt{2}}$

$\therefore$ $1-\frac{b^2}{a^2}=\frac{1}{2}$

$\Rightarrow$ $a^2 = 2b^2$ ...(ii)

From equation (i) and (ii): $a^2=\frac{1}{2},\ b^2=\frac{1}{4}$

$\therefore$ $4(a^2 + b^2) = 3$

170. Answer (2)

$H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Focus of parabola: $(ae, 0)$

Directrix: $x = -ae$.

Equation of parabola $\equiv y^2 = 4aex$

Length of latus rectum of parabola = $4ae$

Length of latus rectum of hyperbola $=\frac{2.b^2}{a}$

as given, $4ae=\frac{2b^2}{a}\cdot e$

$2=\frac{b^2}{a^2}$ ...(i)

$\because$ H passes through $(2\sqrt{2},-2\sqrt{2})\Rightarrow\frac{8}{a^2}-\frac{8}{b^2}=1$

...(ii)

From (i) and (ii) $a^2 = 4$ and $b^2 = 8 \Rightarrow e=\sqrt{3}$

$\Rightarrow$ Equation of parabola is $y^2=8\sqrt{3}x$.

171. Answer (13)

$E\equiv\frac{x^2}{2}+\frac{y^2}{4}=1$

$T\equiv y=mx\pm\sqrt{2m^2+4}$

$\downarrow(\sqrt{2},\ 2\sqrt{2}-2)$

$\Rightarrow$ $(2\sqrt{2}-2-m\sqrt{2})=\pm\sqrt{2m^2+4}$

$\Rightarrow$ $2m^2-2m\sqrt{2}\ (2\sqrt{2}-2)+4(3-2\sqrt{2})=2m^2+4$

$\Rightarrow$ $-2\sqrt{2}\ m\ (2\sqrt{2}-2)=4-12+8\sqrt{2}$

$\Rightarrow$ $-4\sqrt{2}\ m\ (\sqrt{2}-1)=8(\sqrt{2}-1)$

$\Rightarrow$ $m=-\sqrt{2}$ and $m\to\infty$

$\therefore$ Tangents are $x=\sqrt{2}$ and $y=-\sqrt{2}x+\sqrt{8}$

$\therefore$ $P(\sqrt{2},0)$ and $Q(1,\sqrt{2})$

and $S=(0,-\sqrt{2})$

$\therefore$ $(PS)^2 + (QS)^2 = 4 + 9 = 13$

172. Answer (9)

Given: $y^2=\frac{-x}{2}$

$T\equiv y=mx-\frac{1}{8m}$

$\downarrow(2,0)$

$\Rightarrow$ $m^2=\frac{1}{16}\Rightarrow m=\pm\frac{1}{4}$

Tangents are $y=\frac{1}{4}x-\frac{1}{2},\ y=\frac{-x}{4}+\frac{1}{2}$

$4y = x - 2$ and $4y + x = 2$

If these are also tangent to circle then

$d_c = r$

$$\Rightarrow \left|\frac{5-2}{\sqrt{17}}\right| = \sqrt{r} \Rightarrow r = \left(\frac{3}{\sqrt{17}}\right)^2$$

$$\Rightarrow \quad 17r = 17\cdot\frac{9}{17} = 9$$

173. Answer (2)

$L_1 : bx + 10y - 8 = 0,\ L_2 : 2x - 3y = 0$

then $L : (bx + 10y - 8) + \lambda(2x - 3y) = 0$

$\because$ It passes through (1, 1)

$\therefore\ b + 2 - \lambda = 0 \Rightarrow \lambda = b + 2$

and touches the circle $x^2 + y^2 = \dfrac{16}{17}$

$$\left|\frac{8^2}{(2\lambda+b)^2+(10-3\lambda)^2}\right| = \frac{16}{17}$$

$$\Rightarrow 4\lambda^2 + b^2 + 4b\lambda + 100 + 9\lambda^2 - 60\lambda = 68$$

$$\Rightarrow 13(b+2)^2 + b^2 + 4b(b+2) - 60(b+2) + 32 = 0$$

$$\Rightarrow 18b^2 = 36 \ \therefore\ b^2 = 2$$

$\therefore$ Eccentricity of ellipse : $\dfrac{x^2}{5} + \dfrac{y^2}{b^2} = 1$ is

$$\therefore\ e = \sqrt{1-\frac{2}{5}} = \sqrt{\frac{3}{5}}$$

174. Answer (27)

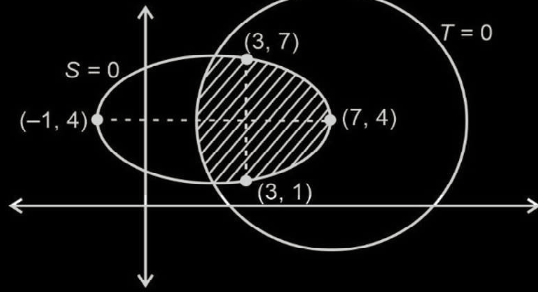
$$S = \left\{(x,y) \in \mathbb{N}\times\mathbb{N} : \frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} \le 1\right\}$$

represents all the integral points inside and on the ellipse $\dfrac{(x-3)^2}{16} + \dfrac{(y-4)^2}{9} = 1$, in first quadrant.

and $T = \left\{(x,y) \in \mathbb{R}\times\mathbb{R} : (x-7)^2 + (y-4)^2 \le 36\right\}$

represents all the points on and inside the circle

$(x-7)^2 + (y-4)^2 = 36$.



$\therefore\quad n(S \cap T) = \{(3, 1), (2, 2), (3, 2), (4, 2), (5, 2), (2, 3), ... (6, 5)\}$

Total number of points = 27

175. Answer (1)

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ meets the line $\dfrac{x}{7} + \dfrac{y}{2\sqrt{6}} = 1$ on the x-axis

So, $a = 7$

and $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ meets the line $\dfrac{x}{7} - \dfrac{y}{2\sqrt{6}} = 1$ on the y-axis

So, $b = 2\sqrt{6}$

Therefore, $e^2 = 1 - \dfrac{b^2}{a^2} = 1 - \dfrac{24}{49}$

$$e = \frac{5}{7}$$

176. Answer (3)

Given hyperbola : $\dfrac{x^2}{6/k} - \dfrac{y^2}{6} = 1$

Eccentricity = $e = \sqrt{1 + \dfrac{6}{6/k}} = \sqrt{1+k}$

Directrices : $x = \pm\dfrac{a}{e} \Rightarrow x = \pm\dfrac{\sqrt{6}}{\sqrt{k}\sqrt{k+1}}$

As given : $\dfrac{\sqrt{6}}{\sqrt{k}\sqrt{k+1}} = 1$

$\Rightarrow\ k = 2$

Here hyperbola is $\dfrac{x^2}{3} - \dfrac{y^2}{6} = 1$

Checking the option gives $(\sqrt{5}, -2)$ satisfies it.

177. Answer (4)

Ellipse : $\frac{x^2}{16}+\frac{y^2}{7}=1$

Eccentricity = $\sqrt{1-\frac{7}{16}}=\frac{3}{4}$

Foci $\equiv(\pm a\,e,0)\equiv(\pm 3,\ 0)$

Hyperbola : $\dfrac{x^2}{\left(\frac{144}{25}\right)}-\dfrac{y^2}{\left(\frac{\alpha}{25}\right)}=1$

Eccentricity = $\sqrt{1+\frac{\alpha}{144}}=\frac{1}{12}\sqrt{144+\alpha}$

Foci $\equiv(\pm ae,\,0)\equiv\left(\pm\frac{12}{5}.\frac{1}{12}\sqrt{144+\alpha},\,0\right)$

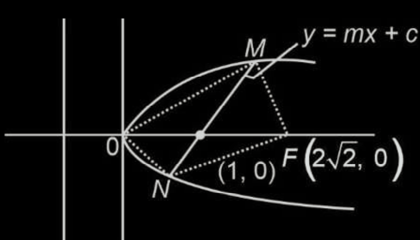
If foci coincide then $3=\frac{1}{5}\sqrt{144+\alpha}\Rightarrow\alpha=81$

Hence, hyperbola is $\dfrac{x^2}{\left(\frac{12}{5}\right)^2}-\dfrac{y^2}{\left(\frac{9}{5}\right)^2}=1$

Length of latus rectum = $2\cdot\dfrac{81/25}{12/5}=\dfrac{27}{10}$

178. Answer (2)

$H:\frac{x^2}{4}-\frac{y^2}{4}=1$



Focus $(ae,\,0)$

$F\left(2\sqrt{2},\ 0\right)$

$y = mx + c$ passes through (1, 0)

$0 = m + C$ ...(i)

L is tangent to hyperbola

$C=\pm\sqrt{4m^2-4}$

$-m=\pm\sqrt{4m^2-4}$

$m^2 = 4m^2 - 4$

$m=\frac{2}{\sqrt{3}}$

$C=\frac{-2}{\sqrt{3}}$

$T:y=\frac{2}{\sqrt{3}}x-\frac{2}{\sqrt{3}}$

$P:y^2 = 4x$

$y^2=4\left(\frac{\sqrt{3}y+2}{2}\right)$

$y^2-2\sqrt{3}y-4=0$

Area

$$\frac{1}{2}\begin{vmatrix}0 & 0\\ x_1 & y_1\\ 2\sqrt{2} & 0\\ x_2 & y_2\\ 0 & 0\end{vmatrix}$$

$=\left|\frac{1}{2}\left(-2\sqrt{2}y_1+2\sqrt{2}y_2\right)\right|$

$=\sqrt{2}\left|y_2-y_1\right|=\sqrt{2}\sqrt{\left(y_1+y_2\right)^2-4y_1y_2}$

$=\sqrt{56}$

$=2\sqrt{14}$

179. Answer (2)

Equation of tangent of slope m to $y = x^2$

$y=mx-\frac{1}{4}m^2$

Equation of tangent of slope m to $y = -(x-2)^2$

$y=m(x-2)+\frac{1}{4}m^2$

If both equation represent the same line

$$\frac{1}{4}m^2 - 2m = -\frac{1}{4}m^2$$

$m = 0, 4$

So, equation of tangent

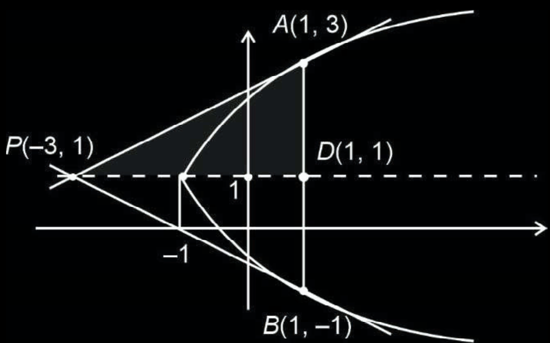
$$y = 4x - 4$$

180. Answer (4)

Given curve : $y^2 - 2x - 2y = 1$.

Can be written as

$$(y-1)^2 = 2(x+1)$$



And, the given information

Can be plotted as shown in figure

Tangent at A : $2y - x - 5 = 0$ {using $T = 0$}

Intersection with $y = 1$ is $x = -3$

Hence, point P is (–3, 1)

Taking advantage of symmetry

$$\text{Area of } \Delta PAB = 2 \times \frac{1}{2} \times (1-(-3)) \times (3-1)$$

= 8 sq. units

181. Answer (42)

$$\because\ H : \frac{x^2}{a^2} - \frac{y^2}{1} = 1$$

$\therefore$ Length of latus rectum = $\frac{2}{a}$

$$E : \frac{x^2}{4} + \frac{y^2}{3} = 1$$

Length of latus rectum = $\frac{6}{2} = 3$

$$\because\ \frac{2}{a} = 3 \Rightarrow a = \frac{2}{3}$$

$$\therefore\ 12\left(e_H^2 + e_E^2\right) = 12\left(1 + \frac{9}{4}\right) + \left(1 - \frac{3}{4}\right) = 42$$

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