This Question Paper contains 20 printed pages.
(Part - A & Part - B)

Sl.No. 1600595

050 (E)

(MARCH/APRIL 2022)
(SCIENCE STREAM)
(CLASS - XII)

પ્રશ્ન પેપરનો સેટ નંબર જેની સામેનું વર્તુળ **OMR** શીટમાં ઘટ્ટ કરવાનું રહે છે.
Set No. of Question Paper, circle against which is to be darken in OMR sheet.

16

Part - A : Time : 1 Hour / Marks : 50
Part - B : Time : 2 Hours / Marks : 50

(Part - A)

Time : 1 Hour] ***[Maximum Marks : 50***

Instructions :

1) **There are 50 objective type (M.C.Q.) questions in Part - A and all questions are compulsory.**
2) **The questions are serially numbered from 1 to 50 and each carries 1 mark.**
3) **Read each question carefully, select proper alternative and answer in the O.M.R. sheet.**
4) **The OMR Sheet is given for answering the questions. The answer of each question is represented by (A) O, (B) O, (C) O and (D) O. Darken the circle ● of the correct answer with ball-pen.**
5) **Rough work is to be done in the space provided for this purpose in the Test Booklet only.**
6) **Set No. of Question Paper printed on the upper- most right side of the Question Paper is to be written in the column provided in the OMR sheet.**
7) **Use of simple calculator and log table is allowed, if required.**
8) **Notations used in this question paper have proper meaning.**

1) The point on the curve $x^2 = 2y$ which is nearest to the point (0, 5) is _______ .

(A) $(2\sqrt{2},4)$

(B) $(2\sqrt{2},0)$

(C) $(-2\sqrt{2},4)$

(D) $(-2\sqrt{2},0)$

Rough Work

2) The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is ______ .

(A) $-\frac{7}{6}$

(B) $\frac{7}{6}$

(C) $\frac{6}{7}$

(D) $-\frac{6}{7}$

3) The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is

(A) $0.06\, x^3\, m^3$

(B) $0.6\, x^3\, m^3$

(C) $0.09\, x^3\, m^3$

(D) $0.9\, x^3\, m^3$

4) The interval in which $y = x^2 e^{-x}$ is increasing is

(A) $(-\infty, \infty)$

(B) $(-2, 0)$

(C) $(2, \infty)$

(D) $(0, 2)$

5) $\int_0^{\pi} \sin x\, dx =$ ______ .

(A) 1

(B) 2

(C) 0

(D) -2

6) $\int \frac{1}{\sqrt{(x-1)(x-2)}}dx =$ ________ + C.

(A) $\log\left|\left(x-\frac{3}{2}\right)-\sqrt{x^2-3x+2}\right|$

(B) $\log\left|x+\sqrt{x^2-3x+2}\right|$

(C) $\log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|$

(D) $\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|$

7) $\int e^{3x}\cdot\sin(4x-5)\,dx =$ ______ + C.

(A) $\frac{e^{3x}}{25}[3\sin(4x-5)+4\cos(4x-5)]$

(B) $\frac{e^{3x}}{25}[3\cos(4x-5)-4\sin(4x-5)]$

(C) $\frac{e^{3x}}{25}[3\sin(4x-5)-4\cos(4x-5)]$

(D) $\frac{e^{3x}}{25}[4\sin(4x-5)-3\cos(4x-5)]$

8) $\int \sqrt{x^2+4x+1}\, dx =$ ______ $+ C$.

(A) $\frac{x+2}{2}\sqrt{x^2+4x+1}+\frac{3}{2}\log\left|x+2+\sqrt{x^2+4x+1}\right|$

(B) $\frac{x+2}{2}\sqrt{x^2+4x+1}-\frac{3}{2}\log\left|x+2+\sqrt{x^2+4x+1}\right|$

(C) $\frac{x+2}{2}\sqrt{x^2+4x+1}-9\log\left|x+2+\sqrt{x^2+4x+1}\right|$

(D) $\frac{x+2}{2}\sqrt{x^2+4x+1}+9\log\left|x+2+\sqrt{x^2+4x+1}\right|$

9) The value of $\int_{-\pi/2}^{\pi/2} (x^3+\cos x+\tan^5 x)\,dx$ is ______.

(A) 0 (B) 2

(C) π (D) 1

10) $\int_{-1}^{1} \sin^5 x \cos^4 x\, dx =$ ______.

(A) 0 (B) 2

(C) -2 (D) 3

11) The value of $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ is ______

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

12) $\int x^2 e^{x^3} dx$ equals

(A) $\frac{1}{3}e^{x^2}+C$

(B) $\frac{1}{3}e^{x^3}+C$

(C) $\frac{1}{2}e^{x^3}+C$

(D) $\frac{1}{2}e^{x^2}+C$

13) The area of the parabola $y^2 = 12x$ bounded by its latus rectum is ________ .

(A) 24

(B) 12

(C) 18

(D) 30

14) Smaller area enclosed by the circle $x^2 + y^2 = 16$ and the line $x + y = 4$ is ________ .

(A) $4(\pi - 1)$

(B) $8\pi - 4$

(C) $4(\pi - 4)$

(D) $4(\pi - 2)$

15) The area of the region bounded by the ellipse $9x^2 + 4y^2 = 36$ is ____________ .

(A) 12π

(B) 36π

(C) 6π

(D) 72π

16) The order and degree of the differential equation $1+\left(\frac{dy}{dx}\right)^2=\sqrt{\frac{d^2y}{dx^2}}$ respectively are

(A) 1, 2

(B) 2, 2

(C) 2, 1

(D) 4, 2

17) The integrating factor of the differential equation $x\frac{dy}{dx}+2y=x^2\log x$ is

(A) e^{2x}

(B) x^2

(C) e^x

(D) x

18) The general solution of the differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ is ________ .

(A) $\tan^{-1}y=\tan^{-1}x+C$

(B) $\sin^{-1}y=\sin^{-1}x+C$

(C) $\log|y^2+1|=\log|1+x^2|+C$

(D) $\cos^{-1}y=\cos^{-1}x+C$

19) The vector in the direction of vector $5\hat{i}-\hat{j}+2\hat{k}$ which has magnitude 8 units is ________ .

(A) $\frac{40}{\sqrt{30}}\hat{i}-\frac{8}{\sqrt{30}}\hat{j}+\frac{16}{\sqrt{30}}\hat{k}$

(B) $40\hat{i}-8\hat{j}+16\hat{k}$

(C) $\frac{4}{3}\hat{i}-\frac{8}{30}\hat{j}+\frac{16}{30}\hat{k}$

(D) None

20) If $\vec{a}$ is unit vector and $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=8$, then $|\vec{x}|$ is ______.

(A) 4

(B) 3

(C) 7

(D) None

21) If $|\vec{a}|=10, |\vec{b}|=2$ and $\vec{a}\cdot\vec{b}=12$, then value of $|\vec{a}\times\vec{b}|$ is ______.

(A) 5

(B) 10

(C) 16

(D) 14

22) The area of a parallelogram whose adjacent sides are given by the vectors $\vec{a}=3\hat{i}+\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ is ______.

(A) $\sqrt{21}$

(B) 42

(C) $\sqrt{42}$

(D) 21

23) The angle 'θ' between the vectors $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ is ______.

(A) $\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

(B) $\cos^{-1}\left(-\frac{1}{3}\right)$

(C) $-\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

(D) None of the above

24) The value of $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})$ is ______.

(A) 0

(B) -1

(C) 1

(D) 3

25) The angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x+2y-11z=3$ is ______.

(A) $\cos^{-1}\left(\frac{8}{21}\right)$ (B) $\sin^{-1}\left(\frac{8}{21}\right)$

(C) $\sin^{-1}\left(-\frac{8}{21}\right)$ (D) $\cos^{-1}\left(-\frac{8}{21}\right)$

26) The equation of a line parallel to Y- axis and passing through the point (2, 3, 4) is ______.

(A) $\frac{x-2}{1}=\frac{y-3}{0}=\frac{z-4}{1}$

(B) $\frac{x-2}{0}=\frac{y+3}{1}=\frac{z-4}{0}$

(C) $\frac{x+2}{1}=\frac{y+3}{0}=\frac{z+4}{1}$

(D) $\frac{x-2}{0}=\frac{y-3}{1}=\frac{z-4}{0}$

27) Distance between the two planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is ______.

(A) $-\frac{2}{\sqrt{29}}$ (B) $\frac{1}{\sqrt{29}}$

(C) $\frac{2}{\sqrt{29}}$ (D) None

28) The corner points of the feasible region determined by the following system of linear inequalities $2x+y \le 10$, $x+3y \le 15$, $x, y \ge 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = qx + py$ where $p, q > 0$, condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is

(A) $q = p$ (B) $q = 2p$

(C) $q = 3p$ (D) $p = 3q$

29) The corner points of feasible region determined by the system of linear constraints are (2, 72), (15, 20), (40, 15). Let $Z = 6x + 3y$ be the objective function. Minimum of Z occurs at

(A) (2, 72)   (B) (15, 20)

(C) (40, 15)   (D) (0, 0)

30) If for a linear programming problem feasible region is bounded, then the objective function has ________ .

(A) only maximum value

(B) only minimum value

(C) both maximum and minimum value

(D) neither maximum nor minimum value

31) If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

(A) $P(A/B) = \frac{P(B)}{P(A)}$ (B) $P(A/B) < P(A)$

(C) $P(A/B) \geq P(A)$ (D) None of these

32) The probability that a student is not a swimmer is $\frac{1}{5}$ then the probability that out of five students, four are swimmers is _____.

(A) ${}^{5}C_{1}\left(\frac{4}{5}\right)^{4}\frac{1}{5}$ (B) $\left(\frac{4}{5}\right)^{4}\frac{1}{5}$

(C) $5\left(\frac{4}{5}\right)^{4}$ (D) None of these

33) A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Then the value of k is ______.

(A) -1

(B) 1

(C) $\frac{1}{10}$

(D) $-\frac{1}{10}$

34) If f: R→R be given by $f(x)=(3-x^3)^{1/3}$, then $fo\ (fof)\ (x)$ is

(A) $x^{1/3}$

(B) x^3

(C) x

(D) $(3-x^3)^{1/3}$

35) Let R be the relation in the set {1, 2, 3} given by R = { (1, 1), (2, 2), (3, 3) }. Choose the correct answer.

(A) R is an equivalence relation

(B) R is reflexive and symmetric but not transitive

(C) R is reflexive and transitive but not symmetric

(D) R is symmetric and transitive but not reflexive

36) Consider a binary operation $*$ on N defined as $a * b = |a - b|$. Choose the correct answer.

(A) $*$ is both associative and commutative

(B) $*$ is commutative but not associative

(C) $*$ is associative but not commutative

(D) $*$ is neither commutative nor associative

37) If $\tan^{-1} x = y$, then ______ .

(A) $0 \le y \le \pi$

(B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$

(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

38) The value of $\sin^{-1}\left(\sin \frac{7\pi}{6}\right)$ is ______ .

(A) $-\frac{7\pi}{6}$

(B) $\frac{7\pi}{6}$

(C) $-\frac{\pi}{6}$

(D) $\frac{\pi}{6}$

39) The value of $\cot^{-1}\left(-\sqrt{3}\right) - \tan^{-1}\sqrt{3}$ is ______ .

(A) $-\frac{\pi}{2}$

(B) 0

(C) $\frac{\pi}{2}$

(D) π

40) $\cos(\tan^{-1}x)$, $|x| < 1$ is equal to

(A) $\dfrac{1}{\sqrt{1-x^2}}$

(B) $\dfrac{1}{\sqrt{1+x^2}}$

(C) $\dfrac{x}{\sqrt{1-x^2}}$

(D) $\dfrac{x}{\sqrt{1+x^2}}$

41) If $A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$ and $A + A' = I$, then value of α is ____.

(A) $\dfrac{\pi}{6}$

$\begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$

$A + A' = I$

$A' = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

(B) $\dfrac{\pi}{3}$

(C) π

(D) $\dfrac{3\pi}{2}$

42) If $[x \ \ -5 \ \ -1] \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$, then value of x is ________.

(A) 0

(B) $\pm 2\sqrt{3}$

(C) $\pm 4\sqrt{3}$

(D) $\pm 6\sqrt{3}$

R

43) If $A=\begin{bmatrix} 8 & -2 \\ -4 & 1 \end{bmatrix}$, then what is value of A^{-1}?

(A) $\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$

(B) $\begin{bmatrix} \frac{1}{16} & \frac{2}{16} \\ \frac{4}{16} & \frac{8}{16} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{1}{16} & -\frac{1}{8} \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$

(D) Does not exist

44) If A is square matrix such that $A^2=A$, then $(I+A)^3-8A$ is equal to ________.

(A) A

(B) I – A

(C) I

(D) 3 A

45) Let $A = \begin{bmatrix} 1 & -\cos\theta & -1 \\ \cos\theta & 1 & -\cos\theta \\ 1 & \cos\theta & 1 \end{bmatrix}$ where $0 < \theta < \frac{\pi}{2}$. Then

(A) $\det(A) = 0$

(B) $\det(A) \in (2, \infty)$

(C) $\det(A) \in (2, 4)$

(D) $\det(A) \in [2, 4]$

46) Let A be a nonsingular square matrix of order 3×3. Then |adj A| is equal to

(A) $|A|$

(B) $|A|^2$

(C) $|A|^3$

(D) $3|A|$

47) If area of triangle is 4 sq. units with vertices (–2, 0), (0, 4) and (0, k), then k is ______.

(A) 0

(B) –8

(C) 0, 8

(D) 0, –8

48) If $f(x)=\begin{cases} \dfrac{k\cos x}{\pi-2x}, & \text{if } x\neq\dfrac{\pi}{2} \\ 3, & \text{if } x=\dfrac{\pi}{2} \end{cases}$ for which value of k, f is continuous at $x=\dfrac{\pi}{2}$.

(A) 6

(B) 3

(C) $\dfrac{3}{2}$

(D) 0

49) What is differentiation of $\cos^{-1}(\sin x)$ with respect to x?

(A) 1

(B) –1

(C) $\dfrac{\pi}{2}-1$

(D) $\dfrac{\pi}{2}$

50) If $e^y(x+1)=1$, then what is the value of $\dfrac{d^2y}{dx^2}$?

(A) $-\left(\dfrac{dy}{dx}\right)$

(B) $\left(\dfrac{dy}{dx}\right)$

(C) $-\left(\dfrac{dy}{dx}\right)^2$

(D) $\left(\dfrac{dy}{dx}\right)^2$

050 (E)

(MARCH/APRIL 2022)
(SCIENCE STREAM)
(CLASS - XII)

(Part - B)

Time : 2 Hours] *[Maximum Marks : 50*

Instructions :

1) Write in a clear legible handwriting.
2) There are three sections in Part - B of the question paper and total 1 to 27 questions are there.
3) All the Sections are compulsory and general options are given in each Section.
4) The numbers at right side represent the marks of the question.
5) Start new section on new page.
6) Maintain sequence.
7) Use of simple calculator and log table is allowed, if required.
8) Use the graph paper to solve the problem of L.P.

SECTION - A

■ Answer any eight questions from question number 1 to 12. (Each of 2 marks) [16]

1) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form. [2]

2) Prove that $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$. [2]

3) Verify Mean Value Theorem if $f(x) = x^3 - 5x^2 - 3x$ in the interval $[a, b]$, where $a = 1$ and $b = 3$. Find all $c \in (1, 3)$ for which $f'(c) = 0$. [2]

4) Evaluate $\int_0^2 e^x\, dx$ as the limit of a sum. [2]

5) Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the X- axis. [2]

6) Find the area of the region in the first quadrant enclosed by the X- axis, the line $y = x$ and the circle $x^2 + y^2 = 32$. [2]

7) Find the intervals in which the function f given by $f(x) = -2x^3 - 9x^2 - 12x + 1$ is increasing or decreasing. [2]

8) Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle. [2]

9) Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. [2]

10) Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. [2]

11) Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that one of them is black and other is red. [2]

12) If a fair coin is tossed 10 times, find the probability of exactly six heads appears. [2]

SECTION - B

- **Answer any six questions from question number 13 to 21. (Each of 3 marks) [18]**

13) Let $f : \mathrm{N} \to \mathrm{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$ show that $f : \mathrm{N} \to \mathrm{S}$, where S in the range of f, is invertible. Find the inverse of f. **[3]**

14) Express the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. **[3]**

$$\begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ 1 & 10 & -1 \end{bmatrix}$$

15) Obtain the inverse of the following matrix. **[3]**

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$A_{11} = (-1)^{1+1}(-1) = -1$
$A_{12} = (-1)^{1+2}(-8) = 8$
$A_{13} = (-1)^{1+3}(-5) = -5$
$A_{21} = (-1)^{2+1}(-1) = 1$
$A_{22} = (-1)^{2+2}(-6) = -6$
$A_{23} = (-1)^{2+3}(-3) = 3$
$A_{31} = (-1)^{3+1}(1) = 1$
$A_{32} = (-1)^{3+2}(-2) = 10$
$A_{33} = (-1)^{3+3}(-1) = -1$

16) If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\dfrac{d^2y}{dx^2}$. **[3]**

17) Find the equation of the normal to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$. **[3]**

18) Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 15$. **[3]**

19) Find the coordinates of the point where the line through (3, –4, –5) and (2, –3, 1) crosses the plane $2x + y + z = 7$. **[3]**

20) Solve the following problem graphically : **[3]**

Minimise and Maximise

$Z = 3x + 9y$

Subject to the constraints :

$x + 3y \leq 60,\ x + y \geq 10,\ x \leq y,\ x \geq 0, y \geq 0$

$x + 3y = 60$
$x + y = 10$
$2y = 50$
$y = 25$

21) A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B? [3]

SECTION - C

■ **Answer any four questions from question number 22 to 27. (Each of 4 marks) [16]**

22) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$. [4]

23) Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$. [4]

24) If $y = (\tan^{-1} x)^2$, show that $(x^2+1)^2 y_2 + 2x(x^2+1) y_1 = 2$. [4]

25) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume. [4]

26) Find $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$. [4]

27) Find the particular solution of the differential equation: [4]

$$(x^3+x^2+x+1)\frac{dy}{dx} = 2x^2 + x;\; y = 1 \text{ when } x = 0.$$