Chapter 9

Oscillations

1. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time ? **[AIEEE-2009]**

(1) aT/x (2) $aT + 2\pi v$

(3) aT/v (4) $a^2T^2 + 4\pi^2 v^2$

2. A wooden cube (density of wood d) of side l floats in a liquid of density ρ with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period T. Then, T is equal to **[AIEEE-2011]**

(1) $2\pi\sqrt{\frac{ld}{(\rho-d)g}}$ (2) $2\pi\sqrt{\frac{l\rho}{(\rho-d)g}}$

(3) $2\pi\sqrt{\frac{ld}{\rho g}}$ (4) $2\pi\sqrt{\frac{l\rho}{dg}}$

3. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0 s to t = τ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds **[AIEEE-2012]**

(1) b (2) $\frac{1}{b}$

(3) $\frac{2}{b}$ (4) $\frac{0.693}{b}$

4. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5 s. In another 10 s it will decrease to α times its original magnitude, where α equals **[JEE (Main)-2013]**

(1) 0.7 (2) 0.81

(3) 0.729 (4) 0.6

5. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels $2a$ in same direction then **[JEE (Main)-2014]**

(1) Amplitude of motion is $3a$

(2) Time period of oscillations is 8τ

(3) Amplitude of motion is $4a$

(4) Time period of oscillations is 6τ

6. A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to

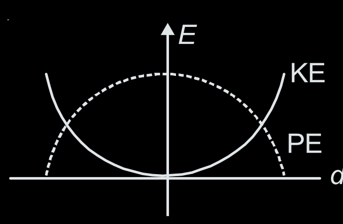
(g = gravitational acceleration) **[JEE (Main)-2015]**

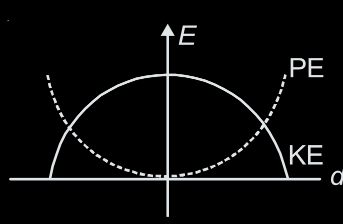
(1) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ (2) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

(3) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (4) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

7. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? **[JEE (Main)-2015]**

(Graphs are schematic and not drawn to scale)

(1) 

(2) 

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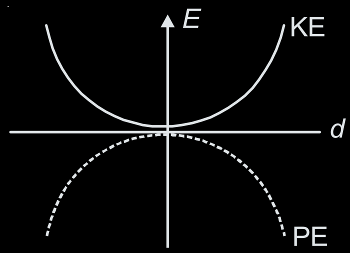

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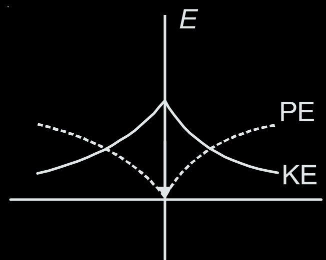


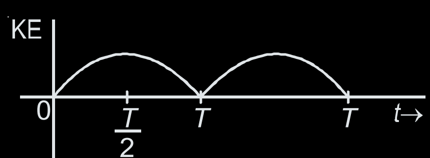
8. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is **[JEE (Main)-2016]**

(1) $3A$ (2) $A\sqrt{3}$

(3) $\frac{7A}{3}$ (4) $\frac{A}{3}\sqrt{41}$

9. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like : **[JEE (Main)-2017]**

(1)



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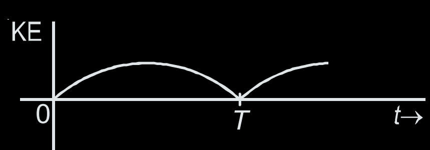

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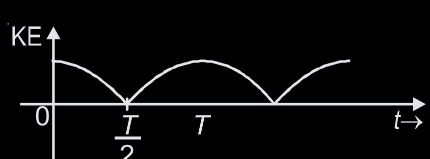

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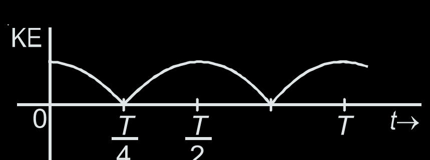


10. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/second. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = 6.02×10^{23} gm mole^{-1}) **[JEE (Main)-2018]**

(1) 6.4 N/m (2) 7.1 N/m

(3) 2.2 N/m (4) 5.5 N/m

11. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about x = 0. When its potential energy (PE) equals kinetic energy (KE), the position of the particle will be **[JEE (Main)-2019]**

(1) $\frac{A}{\sqrt{2}}$ (2) A

(3) $\frac{A}{2\sqrt{2}}$ (4) $\frac{A}{2}$

12. A rod of mass 'M' and length '$2L$' is suspended at its middle by a wire. It exhibits torsional oscillations; if two masses each of 'm' are attached at distance '$L/2$' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to **[JEE (Main)-2019]**

(1) 0.77 (2) 0.17

(3) 0.37 (4) 0.57

13. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to: (density fo water = 10^3 kg/m^3) **[JEE (Main)-2019]**

(1) 2.50 rad s^{-1} (2) 3.75 rad s^{-1}

(3) 8.00 rad s^{-1} (4) 1.25 rad s^{-1}

14. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is **[JEE (Main)-2019]**

(1) $\frac{4\pi}{3}$ (2) $\frac{3}{8}\pi$

(3) $\frac{8\pi}{3}$ (4) $\frac{7}{3}\pi$

15. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210 s will be

[JEE (Main)-2019]

(1) 1 (2) 3

(3) 2 (4) $\frac{1}{3}$

16. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by

[JEE (Main)-2019]

(1) 10^{-3} rad/s (2) 10^{-1} rad/s

(3) 10^{-5} rad/s (4) 1 rad/s

17. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then

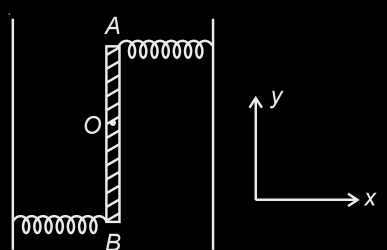
[JEE (Main)-2019]

(1) $K_2 = 2K_1$ (2) $K_2 = \frac{K_1}{4}$

(3) $K_2 = K_1$ (4) $K_2 = \frac{K_1}{2}$

18. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is

[JEE (Main)-2019]



(1) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

19. A simple harmonic motion is represented by :

$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$cm

The amplitude and time period of the motion are

[JEE (Main)-2019]

(1) 10 cm, $\frac{2}{3}$s (2) 5 cm, $\frac{2}{3}$s

(3) 10 cm, $\frac{3}{2}$s (4) 5 cm, $\frac{3}{2}$s

20. The bob of a simple pendulum has mass 2 g and a charge of 5.0 μC. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is **[JEE (Main)-2019]**

(Take, g = 10 m/s^2)

(1) $\tan^{-1}$ (0.2) (2) $\tan^{-1}$ (5.0)

(3) $\tan^{-1}$ (2.0) (4) $\tan^{-1}$ (0.5)

21. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to: **[JEE (Main)-2019]**

(1) 100 s (2) 10 s

(3) 50 s (4) 20 s

22. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is **[JEE (Main)-2019]**

(1) $2T\sqrt{\frac{1}{14}}$ (2) $4T\sqrt{\frac{1}{15}}$

(3) $4T\sqrt{\frac{1}{14}}$ (4) $2T\sqrt{\frac{1}{10}}$

23. The displacement of a damped harmonic oscillator is given by

$x(t) = e^{-0.1t}\cos(10\pi t + \phi)$. Here t is in seconds.

The time taken for its amplitude of vibration to drop to half of its initial value is close to :

[JEE (Main)-2019]

(1) 7 s (2) 27 s

(3) 13 s (4) 4 s

24. A particle moves such that its position vector $\vec{r}(t) = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ where ω is a constant and *t* is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle **[JEE (Main)-2020]**

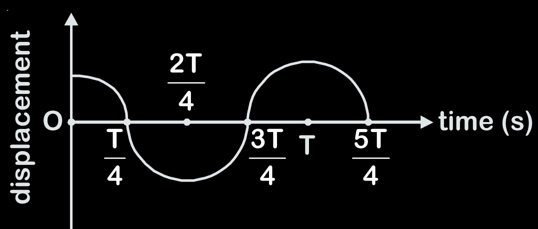
(1) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$

(2) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

(3) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$

(4) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin

25. The displacement time graph of a particle executing S.H.M is given in figure (sketch is schematic and not to scale) **[JEE (Main)-2020]**



Which of the following statements is/are true for this motion?

(A) The force is zero at $t = \frac{3T}{4}$

(B) The acceleration is maximum at $t = T$

(C) The speed is maximum at $t = \frac{T}{4}$

(D) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(1) (B), (C) and (D) (2) (A) and (D)

(3) (A), (B) and (C) (4) (A), (B) and (D)

26. A block of mass *m* attached to a massless spring is performing oscillatory motion of amplitude '*A*' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become *fA*. The value of *f* is **[JEE (Main)-2021]**

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$

(3) 1 (4) $\frac{1}{2}$

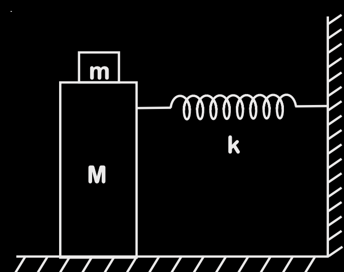
27. When a particle of mass *m* is attached to a vertical spring of spring constant *k* and released, its motion is described by $y(t) = y_0\sin^2\omega t$, where '*y*' is measured from the lower end of unstretched spring. Then ω is **[JEE (Main)-2021]**

(1) $\sqrt{\frac{2g}{y_0}}$ (2) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$

(3) $\sqrt{\frac{g}{2y_0}}$ (4) $\sqrt{\frac{g}{y_0}}$

28. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be:

[JEE (Main)-2021]



(1) $A\sqrt{\frac{M-m}{M}}$

(2) $A\sqrt{\frac{M}{M-m}}$

(3) $A\sqrt{\frac{M}{M+m}}$

(4) $A\sqrt{\frac{M+m}{M}}$

29. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is: **[JEE (Main)-2021]**

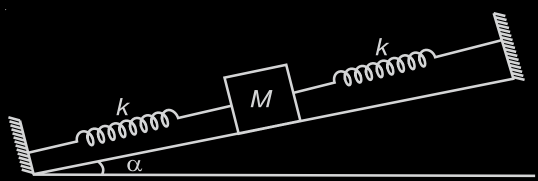
(1) parabolic

(2) straight line

(3) circular

(4) elliptical

30. In the given figure, a body of mass M is held between two massless spring, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, the frequency of oscillation of given body is : **[JEE (Main)-2021]**



(1) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$

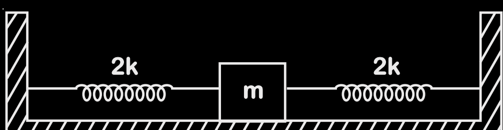
31. $Y = A\sin(\omega t + \phi_0)$ is the time-displacement equation of a SHM. At t = 0 the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then the initial phase angle ϕ_0 will be:

[JEE (Main)-2021]

(1) $\frac{\pi}{6}$ (2) $\frac{2\pi}{3}$

(3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{3}$

32. Two identical springs of spring constant '2k' are attached to a block of mass m and to fixed support (see figure). When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. The time period of oscillations of this system is : **[JEE (Main)-2021]**



(1) $\pi\sqrt{\frac{m}{k}}$ (2) $\pi\sqrt{\frac{m}{2k}}$

(3) $2\pi\sqrt{\frac{m}{2k}}$ (4) $2\pi\sqrt{\frac{m}{k}}$

33. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance (R/2) from the earth's centre, where 'R' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period :

[JEE (Main)-2021]

(1) $\frac{g}{2\pi R}$ (2) $\frac{2\pi R}{g}$

(3) $2\pi\sqrt{\frac{R}{g}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

34. If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor : **[JEE (Main)-2021]**

(1) $\frac{1}{4}, 2\sqrt{2}$ (2) $\frac{1}{2}, 2\sqrt{2}$

(3) $\frac{1}{2}, \sqrt{2}$ (4) $\frac{1}{4}, \sqrt{2}$

35. A particle executes S.H.M., the graph of velocity as a function of displacement is

[JEE (Main)-2021]

(1) a parabola (2) a helix

(3) a circle (4) an ellipse

36. Given below are two statements :

Statement I : A second's pendulum has a time period of 1 second.

Statement II : It takes precisely one second to move between the two extreme positions.

In the light of the above Statement, choose the correct answer from the options given

[JEE (Main)-2021]

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

37. Time period of a simple pendulum is T. The time taken to complete $\frac{5}{8}$ oscillations starting from mean position is $\frac{\alpha}{\beta}T$. The value of α is______.

[JEE (Main)-2021]

38. A particle executes S.H.M. with amplitude 'a' and time period 'T'. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{x}a}{2}$. The value of x is ______.[JEE (Main)-2021]

39. Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration g/2, the time period of pendulum will be : [JEE (Main)-2021]

(1) $\sqrt{\frac{2}{3}}T$ (2) $\sqrt{3}T$

(3) $\sqrt{\frac{3}{2}}T$ (4) $\frac{T}{\sqrt{3}}$

40. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500 g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to half of its initial value?

(ln2 = 0.693) [JEE (Main)-2021]

(1) 15.01 s (2) 0.034 s

(3) 34.65 s (4) 17.32 s

41. For what value of displacement the kinetic energy and potential energy of a simple harmonic oscillation become equal? [JEE (Main)-2021]

(1) $x = \pm A$ (2) $x = 0$

(3) $x = \pm\frac{A}{\sqrt{2}}$ (4) $x = \frac{A}{2}$

42. Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown Fig. 1 shows one of them and Fig. 2 shows their series combination. The ratio of time period of oscillation of the two SHM is $T_b / T_a = \sqrt{x}$, where value of x is _____.

(Round off to the Nearest Integer).

[JEE (Main)-2021]

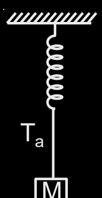


Fig. 1

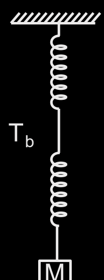


Fig. 2

43. Two particles A and B of equal masses are suspended from two massless springs of spring constants K_1 and K_2 respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is [JEE (Main)-2021]

(1) $\sqrt{\frac{K_2}{K_1}}$ (2) $\frac{K_2}{K_1}$

(3) $\frac{K_1}{K_2}$ (4) $\sqrt{\frac{K_1}{K_2}}$

44. A block of mass 1 kg attached to a spring is made to oscillate with an initial amplitude of 12 cm. After 2 minutes the amplitude decreases to 6 cm. Determine the value of the damping constant for this motion. (take ln 2 = 0.693)

[JEE (Main)-2021]

(1) 0.69×10^{-2} kg s^{-1}

(2) 3.3×10^{-2} kg s^{-1}

(3) 5.7×10^{-3} kg s^{-1}

(4) 1.16×10^{-2} kg s^{-1}

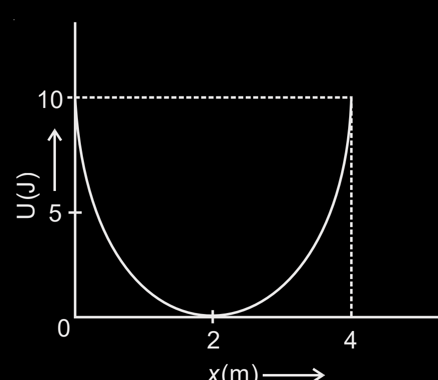
45. A particle performs simple harmonic motion with a period of 2 second. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $\frac{1}{a}$ s.

The value of 'a' to the nearest integer is ______.

[JEE (Main)-2021]

46. A mass of 5 kg is connected to a spring. The potential energy curve of the simple harmonic motion executed by the system is shown in the figure. A simple pendulum of length 4 m has the same period of oscillation as the spring system. What is the value of acceleration due to gravity on the planet where these experiments are performed?

[JEE (Main)-2021]



(1) 9.8 m/s^2 (2) 4 m/s^2

(3) 10 m/s^2 (4) 5 m/s^2

47. The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is:

[JEE (Main)-2021]

(1) $3\cos\left(\frac{\pi}{4}-2\omega t\right)$

(2) $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$

(3) $\sin^2(\omega t)$

(4) $\sin(\omega t) + \cos(\omega t)$

48. A particle is making simple harmonic motion along the X-axis. If at a distances x_1 and x_2 from the mean position the velocities of the particle are v_1 and v_2 respectively. The time period of its oscillation is given as: **[JEE (Main)-2021]**

(1) $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 + v_2^2}}$ (2) $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$

(3) $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 - v_2^2}}$ (4) $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{v_1^2 + v_2^2}}$

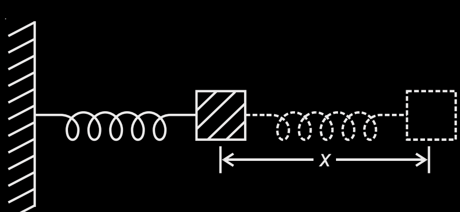
49. T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\frac{1}{16}$ times of its initial value, the modified time period is : **[JEE (Main)-2021]**

(1) $4\,T_0$ (2) $\frac{1}{4}T_0$

(3) T_0 (4) $8\pi\,T_0$

50. The motion of a mass on a spring, with spring constant K is as shown in figure.

[JEE (Main)-2021]



The equation of motion is given by $x(t) = A\sin\omega t + B\cos\omega t$ with $\omega = \sqrt{\frac{K}{m}}$

Suppose that at time $t = 0$, the position of mass is $x(0)$ and velocity $v(0)$, then its displacement can also be represented as $x(t) = C\cos(\omega t - \phi)$, where C and ϕ are: **[JEE (Main)-2021]**

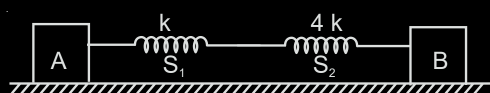
(1) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2},\ \phi = \tan^{-1}\left(\frac{x(0)\omega}{2v(0)}\right)$

(2) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2},\ \phi = \tan^{-1}\left(\frac{x(0)\omega}{v(0)}\right)$

(3) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2},\ \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

(4) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2},\ \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

51. In the reported figure, two bodies A and B of masses 200 g and 800 g are attached with the system of springs. Springs are kept in a stretched position with some extension when the system is released. The horizontal surface is assumed to be frictionless. The angular frequency will be ___ rad/s when k = 20 N/m. **[JEE (Main)-2021]**



52. A pendulum bob has a speed of 3 m/s at its lowest position. The pendulum is 50 cm long. The speed of bob, when the length makes an angle of 60° to the vertical will be (g = 10 m/s^2) _______ m/s.

[JEE (Main)-2021]

53. In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is midway between mean and extreme position? **[JEE (Main)-2021]**

(1) $\frac{3}{4}$ (2) $\frac{1}{4}$

(3) $\frac{1}{3}$ (4) $\frac{1}{2}$

54. A particle starts executing simple harmonic motion (SHM) of amplitude 'a' and total energy E. At any instant, its kinetic energy is $\frac{3E}{4}$ then its displacement 'y' is given by **[JEE (Main)-2021]**

(1) $y = \frac{a}{\sqrt{2}}$ (2) $y = a$

(3) $y = \frac{a}{2}$ (4) $y = \frac{a\sqrt{3}}{2}$

55. An object of mass 0.5 kg is executing simple harmonic motion. Its amplitude is 5 cm and time period (T) is 0.2 s. What will be the potential energy of the object at an instant $t = \frac{T}{4}$ s starting from mean position. Assume that the initial phase of the oscillation is zero **[JEE (Main)-2021]**

(1) 6.2×10^{-3} J (2) 6.2×10^{3} J

(3) 0.62 J (4) 1.2×10^{3} J

56. A particle executes simple harmonic motion represented by displacement function as

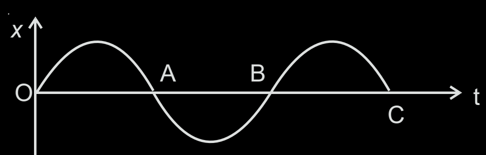
$x(t) = A \sin(\omega t + \phi)$

If the position and velocity of the particle at $t = 0$ s are 2 cm and 2ω cm s^{-1} respectively, then its amplitude is $x\sqrt{2}$ cm where the value of x is _____.

[JEE (Main)-2021]

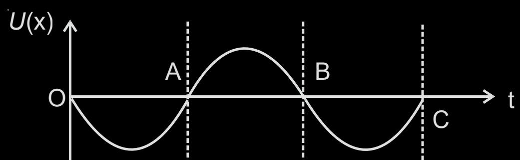
57. Two simple harmonic motions are represented by the equations $x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$ and $x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$. The amplitude of second motion is ______ times the amplitude in first motion. **[JEE (Main)-2021]**

58. The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure. **[JEE (Main)-2021]**

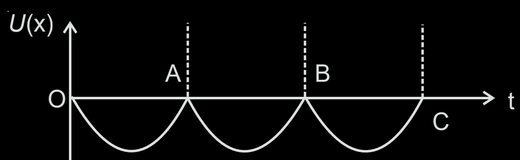


The potential energy U(x) versus time (t) plot of the particle is correctly shown in figure.

(1)



(2)



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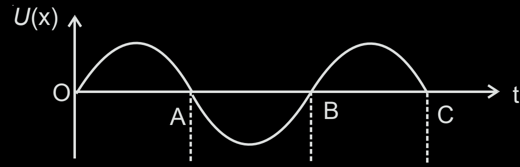
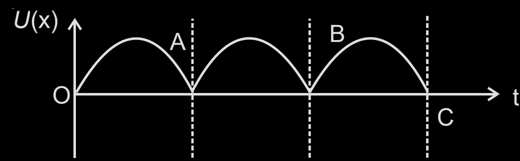


(4)



59. Two simple harmonic motion, are represented by the equations **[JEE (Main)-2021]**

$$y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

$$y_2 = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$$

Ratio of amplitude of y_1 to y_2 = x : 1. The value of x is ____

60. A particle of mass 1 kg is hanging from a spring of force constant 100 Nm^{-1}. The mass is pulled slightly downward and released so that it executes free simple harmonic motion with time period T. The time when the kinetic energy and potential energy of the system will become equal, is $\frac{T}{x}$. The value of x is______. **[JEE (Main)-2021]**

61. A bob of mass 'm' suspended by a thread of length l undergoes simple harmonic oscillations with time period T. If the bob is immersed in a liquid that has density $\frac{1}{4}$ times that of the bob and the length of the thread is increased by 1/3rd of the original length, then the time period of the simple harmonic oscillations will be: **[JEE (Main)-2021]**

(1) $\frac{3}{4}$T (2) T

(3) $\frac{4}{3}$T (4) $\frac{3}{2}$T

62. For a body executing S.H.M.: **[JEE (Main)-2021]**

(a) Potential energy is always equal to its K.E.

(b) Average potential and kinetic energy over any given time interval are always equal.

(c) Sum of the kinetic and potential energy at any point of time is constant.

(d) Average K.E. in one time period is equal to average potential energy in one time period.

Choose the **most appropriate** option from the options given below:

(1) Only (b) (2) (b) and (c)

(3) Only (c) (4) (c) and (d)

63. Time period of a simple pendulum in a stationary lift is 'T'. If the lift accelerates with $\frac{g}{6}$ vertically upwards then the time period will be

(Where g = acceleration due to gravity)

[JEE (Main)-2022]

(1) $\sqrt{\frac{6}{5}}T$ (2) $\sqrt{\frac{5}{6}}T$

(3) $\sqrt{\frac{6}{7}}T$ (4) $\sqrt{\frac{7}{6}}T$

64. The displacement of simple harmonic oscillator after 3 seconds starting from its mean position is equal to half of its amplitude. The time period of harmonic motion is : **[JEE (Main)-2022]**

(1) 6 s

(2) 8 s

(3) 12 s

(4) 36 s

65. The equation of a particle executing simple harmonic motion is given by $x = \sin\pi\left(t + \frac{1}{3}\right)m$. At t = 1 s, the speed of particle will be

[JEE (Main)-2022]

(1) 0 cm s^{-1}

(2) 157 cm s^{-1}

(3) 272 cm s^{-1}

(4) 314 cm s^{-1}

66. A particle executes simple harmonic motion. Its amplitude is 8 cm and time period is 6 s. The time it will take to travel from its position of maximum displacement to the point corresponding to half of its amplitude, is _________ s. **[JEE (Main)-2022]**

67. A body is performing simple harmonic with an amplitude of 10 cm. The velocity of the body was tripled by air Jet when it is at 5 cm from its mean position. The new amplitude of vibration is $\sqrt{x}$ cm. The value of x is ____. **[JEE (Main)-2022]**

68. The motion of a simple pendulum executing S.H.M. is represented by the following equation.

$y = A\sin(\pi t + \phi)$, where time is measured in second.

The length of pendulum is **[JEE (Main)-2022]**

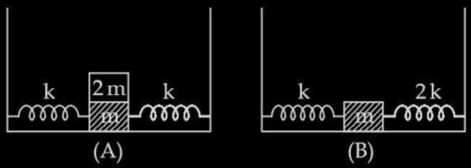
(1) 97.23 cm

(2) 25.3 cm

(3) 99.4 cm

(4) 406.1 cm

69. In figure (A), mass '2 m' is fixed on mass 'm' which is attached to two springs of spring constant k. In figure (B), mass 'm' is attached to two springs of spring constant 'k' and '$2k$'. If mass 'm' in (A) and in (B) are displaced by distance' x' horizontally and then released, then time period T_1 and T_2 corresponding to (A) and (B) respectively follow the relation. **[JEE (Main)-2022]**



(1) $\frac{T_1}{T_2} = \frac{3}{\sqrt{2}}$

(2) $\frac{T_1}{T_2} = \sqrt{\frac{3}{2}}$

(3) $\frac{T_1}{T_2} = \sqrt{\frac{2}{3}}$

(4) $\frac{T_1}{T_2} = \frac{\sqrt{2}}{3}$

70. Two waves executing simple harmonic motions travelling in the same direction with same amplitude and frequency are superimposed. The resultant amplitude is equal to the $\sqrt{3}$ times of amplitude of individual motions. The phase difference between the two motions is____ (degree).

[JEE (Main)-2022]

71. When a particle executes Simple Harmonic Motion, the nature of graph of velocity as a function of displacement will be :

[JEE (Main)-2022]

(1) Circular

(2) Elliptical

(3) Sinusoidal

(4) Straight line

72. The potential energy of a particle of mass 4 kg in motion along the x-axis is given by $U = 4\,(1 - \cos 4x)$ J. The time period of the particle for small oscillation $(\sin\theta \simeq \theta)$ is $\left(\frac{\pi}{K}\right)$ s. The value of K is ______.

[JEE (Main)-2022]

73. The time period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle, which moves without friction down an inclined plane of inclination α, is given by: **[JEE (Main)-2022]**

(1) $2\pi\sqrt{L/(g\cos\alpha)}$

(2) $2\pi\sqrt{L/(g\sin\alpha)}$

(3) $2\pi\sqrt{L/g}$

(4) $2\pi\sqrt{L/(g\tan\alpha)}$

74. The metallic bob of simple pendulum has the relative density 5. The time period of this pendulum is 10 s. If the metallic bob is immersed in water, then the new time period becomes $5\sqrt{x}$ s. The value of x will be _____. **[JEE (Main)-2022]**

75. Two massless springs with spring constant 2 k and 9 k, carry 50 g and 100 g masses at their free ends. These two masses oscillate vertically such that their maximum velocities are equal. Then, the ratio of their respective amplitude will be **[JEE (Main)-2022]**

(1) 1 : 2

(2) 3 : 2

(3) 3 : 1

(4) 2 : 3

76. A mass 0.9 kg, attached a horizontal spring, executes SHM with an amplitude A_1. When this mass passes through its mean position, then a smaller mass of 124 g is placed over it and both masses move together with amplitude A_2. If the ratio $\frac{A_1}{A_2}$ is $\frac{\alpha}{\alpha - 1}$, then the value of α will be _____. **[JEE (Main)-2022]**

❑ ❑ ❑

Chapter 9

Oscillations

1. Answer (1)

$x = A\sin(\omega t + \phi)$

$a = -A\omega^2 \sin(\omega t + \phi)$

So $\frac{aT}{x} = -\omega^2 T$ (which is constant)

2. Answer (3)

$F_{restoring} = -(Ax)\rho g$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l^3 d}{A\rho g}}$$

Now, $A = l^2 \Rightarrow T = 2\pi\sqrt{\frac{ld}{\rho g}}$

3. Answer (3)

4. Answer (3)

In 10 seconds it will become $(0.9)^3 = 0.729$ times.

5. Answer (4)

As it starts from rest, we have

$x = A\cos\omega t$. At $t = 0$, $x = A$

when $t = \tau$, $x = A - a$

when $t = 2\tau$, $x = A - 3a$

$\Rightarrow A - a = A\cos\omega\tau$

$A - 3a = A\cos 2\omega\tau$

As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$

$$\Rightarrow \frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

$$\frac{A-3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$A^2 - 3aA = A^2 + 2a^2 - 4Aa$

$a^2 = 2aA$

$A = 2a$

Now, $A - a = A\cos\omega\tau$

$$\Rightarrow \cos\omega\tau = \frac{1}{2}$$

$$\frac{2\pi}{T}\tau = \frac{\pi}{3}$$

$\Rightarrow T = 6\tau$

6. Answer (1)

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{...(i)}$$

$$T_M = 2\pi\sqrt{\frac{l + \Delta l}{g}} \quad \text{...(ii)}$$

$$Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l = \frac{Mgl}{AY} \quad \text{...(iii)}$$

$$\Rightarrow \frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

7. Answer (2)

$$\text{KE} = \frac{1}{2}m\omega^2(A^2 - d^2)$$

$$\text{PE} = \frac{1}{2}m\omega^2 d^2$$

At $d = \pm A$,

PE = maximum while KE = 0.

8. Answer (3)

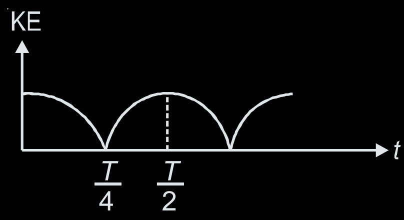
$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \frac{\omega\sqrt{5}A}{3}$$

Now, $V' = 3 \times \frac{\omega\sqrt{5}A}{3}$

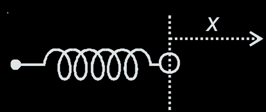
Now, $V' = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2} \Rightarrow A' = \frac{7A}{3}$

9. Answer (4)

$$K.E = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$$



10. Answer (2)



$Kx = ma \Rightarrow a = (K/m)x$

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = 10^{12}$$

$$= \frac{1}{4\pi^2} \times \frac{K}{m} = 10^{24}$$

$$K = 4\pi^2 m \times 10^{24} = \frac{4 \times 10 \times 108 \times 10^{-3}}{6.02 \times 10^{23}} \times 10^{24}$$

$= 7.1$ N/m

11. Answer (1)

KE = potential energy

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}K\,x^2$$

$$\frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}K\,x^2$$

$A^2 - x^2 = x^2$

$$x = \frac{A}{\sqrt{2}}$$

12. Answer (3)

$$I_1 = \frac{M(2L)^2}{12} = \frac{ML^2}{3}$$

$$I_2 = I_1 + 2\frac{mL^2}{4} = \frac{ML^2}{3} + \frac{mL^2}{2}$$

$$\omega \propto \frac{1}{\sqrt{I}}$$

$$\frac{\omega_1}{\omega_2} = \frac{1}{0.8} = \sqrt{\frac{\frac{M}{3} + \frac{m}{2}}{\frac{M}{3}}}$$

$$\Rightarrow \frac{m}{M} = 0.375$$

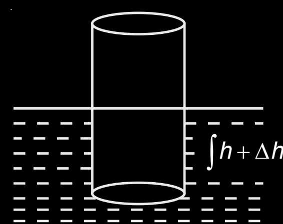
13. Answer (3)

$mg = B_o = \rho Ah.g$

$ma = B - mg$

$-ma = \rho A(h + \Delta h)g - \rho Ahg$

$-ma = \rho Ag\Delta h$



$$a = \frac{-\rho Ag\Delta h}{m}$$

$$\omega = \sqrt{\frac{10^3 \times A \times g}{310 \times 10^{-3}}}$$

$$= \sqrt{\frac{10^3 \times \pi \times 6.25 \times 10^{-4} \times 10}{310 \times 10^{-3} \times 100}}$$

$= \sqrt{62.5} \cong 8$ rad/s

14. Answer (3)

$a = -\omega^2 x = -\omega^2 \times 4$

$V = \omega\sqrt{A^2 - x^2} = \omega \times \sqrt{5^2 - 4^2}$

$\because \omega^2 \times 4 = \omega \times 3$

$$\Rightarrow \omega = \frac{3}{4}$$

$$\Rightarrow T = \frac{2\pi}{3/4} = \frac{8\pi}{3}$$

15. Answer (4)

$$\frac{KE}{PE} = \frac{\frac{1}{2}kA^2 - \frac{1}{2}kA^2 \sin^2\frac{\pi t}{90}}{\frac{1}{2}kA^2 \sin^2\frac{\pi t}{90}} = \frac{1}{3}$$

16. Answer (1)

$$T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}}$$

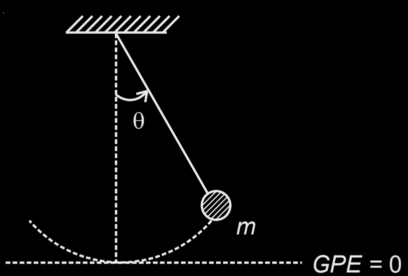
$$\omega = \sqrt{\frac{g_{\text{eff}}}{l}}$$

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{\text{eff}}}{g_{\text{eff}}}$$

$$= \frac{1}{2}\frac{(2\omega^2 A)}{g}$$

$$\frac{\Delta\omega}{\omega} = 10^{-3} \text{ rad/s}$$

17. Answer (1)



$U = mgL(1-\cos\theta)$

$GPE = 0$

$K_1 = mgL(1-\cos\theta)$

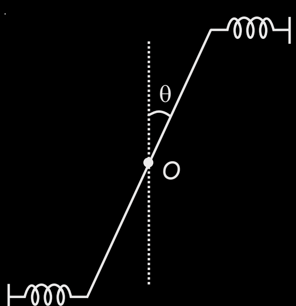
$K_2 = mg(2L)(1-\cos\theta)$

$K_2 = 2K_1$

18. Answer (4)

$\tau = I\alpha$

$$\frac{Ml^2}{12}\alpha = 2k\left(\frac{l}{2}\right)\left(\frac{l}{2}\right)\theta$$



$$\frac{Ml^2}{12}\alpha = \frac{-kl^2}{2}\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6k}{m}}$$

$$\Rightarrow \quad \nu = \frac{1}{2\pi}\sqrt{\frac{6K}{m}}$$

19. Answer (1)

$$y = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$$

$$y = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

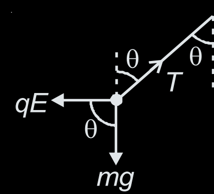
A = 10 cm

$$\frac{2\pi}{T} = 3\pi$$

$$T = \frac{2}{3}\text{ s}$$

20. Answer (4)

$T\cos\theta = mg$



$T\sin\theta = qE$

$$\tan\theta = \frac{qE}{mg}$$

$$\tan\theta = \frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10} = \frac{1}{2}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{1}{2}\right)$$

21. Answer (4)

$$\text{Time for 10 oscillations} = \frac{10}{5} = 2\text{ s}$$

$A = A_0\, e^{-kt}$

$$\frac{1}{2} = e^{-2k} \quad \Rightarrow \quad \ln 2 = 2k$$

$$10^{-3} = e^{-kt} \quad \Rightarrow \quad 3\ln 10 = kt$$

$$t = \frac{3\ln 10}{k} = \frac{3\ln 10}{\ln 2}\times 2$$

$$= 6\times\frac{2.3}{0.69} \approx 20\text{ s}$$

22. Answer (2)

$$T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}}$$

$$\frac{T'}{T} = \sqrt{\frac{g_{\text{eff}}}{g'_{\text{eff}}}}$$

$$g'_{\text{eff}} = g - \frac{g}{16} = \frac{15}{16}$$

$$\frac{T'}{T} = \sqrt{\frac{16}{15}}$$

23. Answer (1)

Amplitude at (t = 0) $A_0 = e^{-0.1\times 0} = 1$

$\therefore$ at $t = t$ if $A = \frac{A_0}{2}$

$\Rightarrow \frac{1}{2} = e^{-0.1t}$

$t = 10 \ln 2 \simeq 7$ s

24. Answer (2)

$$\vec{r} = \cos\omega t\ \hat{i} + \sin\omega t\ \hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \omega(-\sin\omega t\ \hat{i} + \cos\omega t\ \hat{j})$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2\ (\cos\omega t\ \hat{i} + \sin\omega t\ \hat{j})$$

$\vec{a} = -\omega^2\vec{r}$ $\therefore$ $\vec{a}$ is antiparallel to $\vec{r}$

Also $\vec{v}\cdot\vec{r} = 0$ $\therefore$ $\vec{v} \perp \vec{r}$

Actually particle is in state of uniform circular motion.

25. Answer (3)

At $t = \frac{3T}{4}$, particle is at mean position

$\Rightarrow$ Force = 0

At t = T, particle is at extreme position

$\Rightarrow$ Acceleration is maximum

At $t = \frac{T}{4}$, particle is at mean position

$\Rightarrow$ Speed is maximum

26. Answer (1)

$A'\omega = A_0\omega_0$

$$\Rightarrow A' = A_0\frac{\omega_0}{\omega}$$

$$A' = \frac{A_0}{\sqrt{2}}$$

27. Answer (3)

$$y(t) = \frac{y_0}{2}(1-\cos 2\omega t)$$

$\frac{Mg}{K} = \frac{y_0}{2}$ [Amplitude]

$$2\omega = \sqrt{\frac{K}{m}}$$

$$\omega = \frac{1}{2}\sqrt{\frac{K}{m}} = \frac{1}{2}\sqrt{\frac{2g}{y_0}} = \sqrt{\frac{g}{2y_0}}$$

28. Answer (3)

On putting m on M Let velocity becomes V

$(m + M)\ V = MV_0$

Now Kinetic Energy = $\frac{1}{2}(m + M)V^2$

$$K' = \frac{1}{2}(m + M)\frac{M^2V_0^2}{(m + M)^2}$$

$$K' = \frac{1}{2}\frac{M^2V_0^2}{(M + m)}$$

$$\frac{1}{2}MV_0^2 = \frac{1}{2}KA^2$$

$$K' = \frac{1}{2}K(A')^2$$

Hence, $A' = \sqrt{\frac{M}{(M + m)}}A$

29. Answer (4)

$$v = \omega\sqrt{A^2 - x^2}$$

$$\frac{v^2}{\omega^2} + x^2 = A^2$$

30. Answer (1)

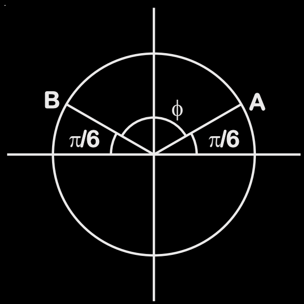
$$T = 2\pi\sqrt{\frac{M}{k_{eq}}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{M}}$$

$$k_{eq} = 2k$$

31. Answer (3)

Draw phasor :



At B particle is going toward mean position $\Rightarrow$ negative x-direction

$$\phi = \frac{5\pi}{6}$$

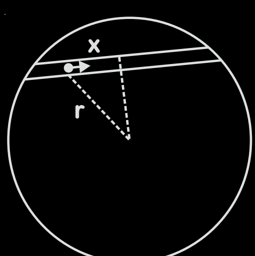
32. Answer (1)

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

$$k_{eq} = 4k$$

$$T = \pi\sqrt{\frac{m}{k}}$$

33. Answer (3)



$$\Delta F = \frac{GMr}{R^3} \times m \times \left(\frac{x}{r}\right)$$

$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{GM}{R^3}x$$

$$\therefore \quad T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$$

34. Answer (3)

$$\frac{1}{K_1} = \frac{1}{K} + \frac{1}{K}$$

$$K_1 = \frac{K}{2}$$

$$T \propto \sqrt{\frac{1}{K}}$$

35. Answer (4)

$$v = \omega\sqrt{A^2 - x^2}$$

$$\Rightarrow \frac{v^2}{\omega^2} + x^2 = A^2$$

$$\Rightarrow \frac{x^2}{A^2} + \frac{v^2}{(A\omega)^2} = 1 \text{ .... Ellipse equation}$$

36. Answer (4)

Time period of second pendulum is 2 sec

$$\Delta t = \frac{T}{2} = 1$$

37. Answer (07.00)

$$\frac{5}{8}\text{oscillation} = \frac{1}{2}\text{ oscillation} + \frac{1}{8}\text{ oscillation}$$

$$\Delta t = \frac{T}{2} + \frac{T}{12} = \frac{7T}{12}$$

$\Rightarrow \alpha = 7$

38. Answer (03.00)

$$V = \omega\sqrt{a^2 - x^2}$$

$$\Rightarrow \frac{a\omega}{2} = \omega\sqrt{a^2 - x^2}$$

$$\Rightarrow \frac{a^2}{4} = a^2 - x^2$$

$$\Rightarrow x^2 = \frac{3a^2}{4}$$

$$\Rightarrow x = \frac{a\sqrt{3}}{2}$$

39. Answer (1)

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$$T' = \sqrt{\frac{2}{3}}T$$

40. Answer (3)

$$\because \quad A = A_0 e^{-\frac{bt}{2m}}$$

$$\Rightarrow \quad A = A_0 \times e^{-\frac{20\times10^{-3}}{2\times0.5}t}$$

$$\Rightarrow \quad \frac{A_0}{2} = A_0 \times e^{-20\times10^{-3}t}$$

$$\Rightarrow \quad 20 \times 10^{-3}\ t = \ln 2$$

$$\Rightarrow \quad t = \frac{\ln 2}{20\times10^{-3}} = 34.65\ \text{s}$$

41. Answer (3)

$$\frac{1}{2}k\left(A^2 - x^2\right) = \frac{1}{2}kx^2$$

$$\Rightarrow x = \pm\frac{A}{\sqrt{2}}$$

42. Answer (2)

$$T_a = 2\pi\sqrt{\frac{m}{k}} \qquad T_b = 2\pi\sqrt{\frac{m}{k/2}} \qquad \frac{T_b}{T_a} = \sqrt{2}$$

43. Answer (1)

$$V_1 = W_1 \times A_1$$

$$V_2 = W_2 \times A_2$$

$$\Rightarrow \frac{A_1}{A_2} = \frac{V_1}{V_2} \times \frac{W_2}{W_1}$$

$$= 1 \times \sqrt{\frac{K_2}{m} \times \frac{m}{K_1}}$$

$$= \sqrt{\frac{K_2}{K_1}}$$

44. Answer (4)

$A = A_0 e^{-bt/2m}$

$$b = \frac{2m\ln(2)}{t} = 2 \times \frac{0.693}{120} = 1.16 \times 10^{-2}$$

45. Answer (6)

$$\frac{A}{2} = A\sin\left(\frac{2\pi t}{T}\right)$$

$$\Rightarrow \frac{2\pi t}{T} = \frac{\pi}{6}$$

$$\Rightarrow t = \frac{T}{12} = \frac{1}{6}\ \text{seconds}$$

$x = 6$

46. Answer (2)

$$U = \frac{1}{2}k(x-2)^2$$

$$\frac{1}{2} \times k \times 4 = 10$$

$k = 5$

$$\sqrt{\frac{k}{m}} = \sqrt{\frac{g}{l}}$$

$$\Rightarrow \quad g = 4$$

47. Answer (1)

$$T = \frac{2\pi}{\omega'} = \frac{\pi}{\omega}$$

$$\Rightarrow \quad \omega' = 2\omega$$

$$\Rightarrow \quad y = 3\cos\left(\frac{\pi}{4} - 2\omega t\right)$$

48. Answer (2)

$$v_1^2 = \omega^2(A^2 - x_1^2)$$

$$v_2^2 = \omega^2(A^2 - x_2^2)$$

$$v_1^2 - v_2^2 = \omega^2(x_2^2 - x_1^2)$$

$$v_1^2 - v_2^2 = \left[\frac{2\pi}{T}\right]^2(x_2^2 - x_1^2)$$

$$T = 2\pi\sqrt{\frac{(x_2^2 - x_1^2)}{(v_1^2 - v_2^2)}}$$

49. Answer (2)

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T' = \frac{T_0}{4}$$

50. Answer (3)

$C\cos\phi = x(0)$

$tC\omega\sin\phi = v(0)$

$$\left[\frac{v(0)}{\omega}\right]^2 + [x(0)]^2 = C^2$$

$$\tan\phi = \frac{v(0)}{x(0)\omega}$$

51. Answer (10)

μ = 160 g = 0.16 kg

k_{eff} = 16

$$\omega = \sqrt{\frac{k_{eff}}{\mu}} = \sqrt{100} = 10$$

52. Answer (2)

$$mgl(1-\cos\theta) + \frac{1}{2}mv^2 = \frac{1}{2}mu^2$$

$$\frac{mg(0.5)}{2} + \frac{1}{2}mv^2 = \frac{1}{2}m(9)$$

$5 + v^2 = 9$

$v = 2$

53. Answer (1)

$$U = \frac{1}{2}m\omega^2 A^2$$

$$v = \omega\sqrt{A^2 - \frac{A^2}{4}}$$

$$k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 \times \frac{3A^2}{4}$$

54. Answer (3)

$$E = \frac{1}{2}m\omega^2 a^2$$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2[a^2 - y^2]$$

$$a^2 - y^2 = \frac{3}{4}a^2$$

$$y = \frac{a}{2}$$

55. Answer (3)

$m = \frac{1}{2}\text{kg}$ A = 5 cm, T = 0.2 s

$x = A\sin\omega t$

$$\text{P.E} = \frac{1}{2}m\omega^2 A^2 \sin\omega t$$

$$= \frac{1}{2}m\omega^2 A^2 \sin\frac{2\pi}{T} \times \frac{T}{4}$$

$$= \frac{1}{2}m\omega^2 A^2$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{4\pi^2}{(0.2)^2} \times \left(\frac{5}{100}\right)^2$$

$$= \frac{\pi^2}{16}$$

≈ 0.62 J

56. Answer (2)

$|A\sin\phi| = 2$

$|\omega A\cos\phi| = 2\omega$

$$\Rightarrow \quad |\tan\phi| = 1 \quad \Rightarrow \quad \phi = \frac{\pi}{4}$$

$$x(0) = A\sin\left(\frac{\pi}{4}\right) = 2 \text{ cm}$$

$$\Rightarrow \quad A = 2\sqrt{2} \text{ cm}$$

$\Rightarrow$ $x = 2$

57. Answer (2)

$$x_1 = 5\sin\left(2\pi t + \frac{\pi}{4}\right) \quad \Rightarrow A_1 = 5$$

$$x_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$$

$$= 10\sin\left(2\pi t + \frac{\pi}{4}\right) \Rightarrow A_2 = 10$$

$$\frac{A_2}{A_1} = 2$$

58. Answer (4)

$$U = \frac{1}{2}kx^2$$

So option (4)

59. Answer (1)

$$y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

$$y_2 = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$$

$$= 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

$$\frac{A_1}{A_2} = 1$$

60. Answer (8)

K.E = P.E.

$$\Rightarrow \quad x = \frac{A}{\sqrt{2}}$$

$$x = A\cos\frac{2\pi}{T}\cdot t = \frac{A}{\sqrt{2}}$$

$$\Rightarrow \quad t = \frac{T}{8}$$

61. Answer (3)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T' = 2\pi\sqrt{\frac{\frac{4l}{3}}{g - \frac{1}{4}g}} = 2\pi\sqrt{\frac{\frac{4}{3}l}{\frac{3}{4}g}} = \frac{4}{3}T$$

62. Answer (4)

$$v = \omega\sqrt{A^2 - x^2}$$

$$KE = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$PE = \frac{1}{2}m\omega^2 x^2$$

E_{Total} = KE + PE

$$= \frac{1}{2}m\omega^2 A^2$$

Also $(KE)_{avg}$ = $(PE)_{avg}$

63. Answer (3)

$$T' = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$$T' = 2\pi\sqrt{\frac{l}{g + \frac{g}{6}}} = 2\pi\sqrt{\frac{6l}{7g}}$$

$$\Rightarrow \quad T' = \sqrt{\frac{6}{7}}T$$

64. Answer (4)

Time taken by the harmonic oscillator to move from mean position to half of amplitude is $\frac{T}{12}$

So, $\frac{T}{12} = 3$

T = 36 sec

65. Answer (2)

$$x = \sin\left(\pi t + \frac{\pi}{3}\right)\text{m}$$

$$\Rightarrow \quad \frac{dx}{dt} = \pi\cos\left(\pi t + \frac{\pi}{3}\right)$$

$$= \pi\cos\left(\pi + \frac{\pi}{3}\right) \text{ at } t = 1 \text{ s}$$

$$= -\frac{\pi}{2} \text{ m/s}$$

or $\left|\frac{dx}{dt}\right| = 157$ cm/s

66. Answer (1)

A = 8 cm

T = 6 s

$$A\cos\left(\frac{2\pi t}{T}\right) = \frac{A}{2}$$

$$\Rightarrow \quad \frac{2\pi t}{T} = \frac{\pi}{3}$$

or $t = \frac{T}{6} = 1\,\text{s}$

67. Answer (700)

$$v = \omega\sqrt{A^2 - y^2}$$

$$\Rightarrow \quad 3\omega\sqrt{10^2 - 5^2} = \omega\sqrt{(A')^2 - 5^2}$$

$\Rightarrow\ 9 \times 75 = (A')^2 - 25$

$\Rightarrow\ A' = \sqrt{28 \times 25}$ cm

$\Rightarrow\ x = 700$

68. Answer (3)

$$\omega = \pi = \sqrt{\frac{g}{\ell}}$$

So $\ell = \dfrac{g}{\pi^2}$

$\simeq$ 99.4 cm (Nearest value)

69. Answer (1)

Both the springs are in parallel combination in both the diagrams so

$$T_1 = 2\pi\sqrt{\frac{3m}{2k}}$$

and $T_2 = 2\pi\sqrt{\dfrac{m}{3k}}$

So, $\dfrac{T_1}{T_2} = \dfrac{3}{\sqrt{2}}$

70. Answer (60)

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\sqrt{3}A = \sqrt{A^2 + A^2 + 2A^2\cos\phi}$$

$$3A^2 = 2A^2 + 2A^2\cos\phi$$

$$\cos\phi = \frac{1}{2}$$

$$\phi = 60^\circ$$

71. Answer (2)

Let $x = A\sin\omega t$

$\Rightarrow\ v = A\omega\cos\omega t$

$\Rightarrow\ v = \pm\omega\sqrt{A^2 - x^2}$

$\Rightarrow\ \dfrac{v^2}{\omega^2} + x^2 = A^2$

$\Rightarrow$ Ellipse

72. Answer (2)

$U = 4(1 - \cos 4x)$

$\Rightarrow\ F = -\dfrac{dU}{dx} = -(4)(4\sin 4x)$

$= -16\sin 4x$

as small x

$F = -16(4x) = -64x \equiv -kx$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4}{64}} = \frac{\pi}{2}$$

$\Rightarrow\ K = 2$

73. Answer (1)

$$|g_{eff}| = |\bar{g} - \bar{a}|$$

$\Rightarrow\ g_{eff} = g\cos\theta$

$\Rightarrow\ T = 2\pi\sqrt{\dfrac{l}{g_{eff}}}$

$$= 2\pi\sqrt{\frac{L}{g\cos\theta}}$$

74. Answer (5)

$$T = 2\pi\sqrt{\frac{\ell}{g}} = 10$$

$$T' = 2\pi\sqrt{\frac{\ell}{g\left(1 - \frac{1}{\rho}\right)}}$$

$$= 2\pi\sqrt{\frac{\ell}{g} \times \frac{5}{4}} = 10\sqrt{\frac{5}{4}} = 5\sqrt{5}$$

75. Answer (2)

$$\omega_1 A_1 = \omega_2 A_2$$

$$\Rightarrow\ \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{k_2}{m_2}} \times \sqrt{\frac{m_1}{k_1}} = \sqrt{\frac{9k}{100} \times \frac{50}{2k}} = \frac{3}{2}$$

76. Answer (16)

$$(0.9)A_1\sqrt{\frac{K}{0.9}} = (0.9 + 0.124)A_2\sqrt{\frac{K}{0.9 + 0.124}}$$

$$\frac{A_1}{A_2} = \sqrt{\frac{0.9 + 0.124}{0.9}}$$

$$= \sqrt{\frac{1.024}{0.9}}$$

$$= \frac{\alpha}{\alpha - 1}$$

$\alpha = 16$

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