Chapter 14

Sets

1. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[AIEEE-2009]**

   (1) $A = C$ (2) $B = C$

   (3) $A \cap B = \phi$ (4) $A = B$

2. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is **[AIEEE-2012]**

   (1) 3^5 (2) 2^5

   (3) 5^3 (4) 5^2

3. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is **[JEE (Main)-2019]**

   (1) 1 (2) 38

   (3) 102 (4) 42

4. Let $S = \{1, 2, 3, ... , 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is **[JEE (Main)-2019]**

   (1) $2^{100} - 1$

   (2) $2^{50} + 1$

   (3) $2^{50}(2^{50} - 1)$

   (4) $2^{50} - 1$

5. Let Z be the set of integers.

   If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and

   $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$, is **[JEE (Main)-2019]**

   (1) 2^{15} (2) 2^{12}

   (3) 2^{18} (4) 2^{10}

6. Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is **[JEE (Main)-2019]**

   (1) 13.9 (2) 13

   (3) 12.8 (4) 13.5

7. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true? **[JEE (Main)-2019]**

   (1) $B \cap C \neq \phi$

   (2) $(C \cup A) \cap (C \cup B) = C$

   (3) If $(A - C) \subseteq B$, then $A \subseteq B$

   (4) If $(A - B) \subseteq C$, then $A \subseteq C$

8. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is **[JEE (Main)-2020]**

   (1) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

   (2) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$

   (3) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

   (4) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

9. If $A = \{x \in R : |x| < 2\}$ and $B = \{x \in R : |x - 2| \geq 3\}$; then **[JEE (Main)-2020]**

   (1) $A - B = [-1, 2)$

   (2) $A \cup B = R - (2, 5)$

   (3) $B - A = R - (-2, 5)$

   (4) $A \cap B = (-2, -1)$

10. Consider the two sets :

    $A = \{m \in R$: both the roots of $x^2 - (m + 1)x + m + 4 = 0$ are real$\}$ and $B = [-3, 5)$.

    Which of the following is not true? **[JEE (Main)-2020]**

    (1) $A \cap B = \{-3\}$

    (2) $B - A = (-3, 5)$

    (3) $A \cup B = R$

    (4) $A - B = (-\infty, -3) \cup (5, \infty)$

11. A survey shows that 63% of the people in a city read newspaper *A* whereas 76% read newspaper *B*. If *x*% of the people read both the newspapers, then a possible value of *x* can be

[JEE (Main)-2020]

(1) 37 (2) 55

(3) 29 (4) 65

12. Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to **[JEE (Main)-2020]**

(1) 50 (2) 15

(3) 30 (4) 45

13. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If *x* denotes the percentage of them, who like both coffee and tea, then x cannot be

[JEE (Main)-2020]

(1) 63 (2) 36

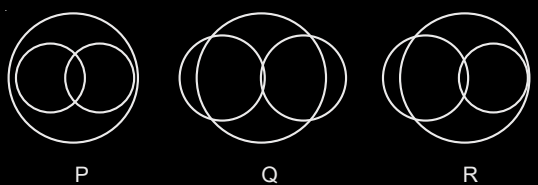
(3) 38 (4) 54

14. Set *A* has *m* elements and Set *B* has *n* elements. If the total number of subsets of *A* is 112 more than the total number of subsets of *B*, then the value of $m \cdot n$ is _____. **[JEE (Main)-2020]**

15. Let $X = \{n \in N : 1 \le n \le 50\}$. If $A = \{n \in X : n$ is a multiple of 2$\}$ and $B = \{n \in X : n$ is a multiple of 7$\}$, then the number of elements in the smallest subset of X containing both A and B is _____.

[JEE (Main)-2020]

16. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement?



(1) Q and R (2) P and Q

(3) P and R (4) None of these

17th Mar (M)

17. The sum of all the elements in the set $\{n \in \{1, 2, ..., 100\} \mid$ H.C.F. of n and 2040 is 1$\}$ is equal to _____.

22nd Jul (E)

18. Let $A = \{n \in N \mid n^2 \le n + 10{,}000\}$, $B = \{3k + 1 \mid k \in N\}$ and $C = \{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to _____.

27nd Jul (E)

19. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K cannot belong to the set:

(1) {84, 87, 90, 93}

(2) {84, 86, 88, 90}

(3) {79, 81, 83, 85}

(4) {80, 83, 86, 89}

20. If $A = \{x \in \mathbf{R} : |x - 2| > 1\}$, $B = \{x \in \mathbf{R} : \sqrt{x^2 - 3} > 1\}$, $C = \{x \in \mathbf{R} : |x - 4| \ge 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^C \cap Z$ is _____.

27th Aug (M)

21. Let S = {1, 2, 3, 4, 5, 6, 9}. Then the number of elements in the set $T = \{A \subseteq S : A \ne \phi$ and the sum of all the elements of A is not a multiple of 3$\}$ is _____.

27th Aug (E)

22. The number of elements in the set $\{x \in R : (|x| - 3)|x + 4| = 6\}$ is equal to : **[JEE (Main)-2021]**

(1) 4 (2) 2

(3) 1 (4) 3

23. Let $A = \{x \in R : |x + 1| < 2\}$ and $B = \{x \in R : |x - 1| \ge 2\}$. Then which one of the following statements is **NOT** true? **[JEE (Main)-2022]**

(1) $A - B = (-1, 1)$

(2) $B - A = R - (-3, 1)$

(3) $A \cap B = (-3, -1]$

(4) $A \cup B = R - [1, 3)$

24. Let $A = \{n \in \mathbf{N} : \text{H.C.F. } (n, 45) = 1\}$ and Let $B = \{2k : k \in \{1, 2, \ldots, 100\}\}$. Then the sum of all the elements of $A \cap B$ is _____. **[JEE (Main)-2022]**

25. Let $A = \{1, a_1, a_2 \ldots a_{18}, 77\}$ be a set of integers with $1 < a_1 < a_2 < \ldots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \ldots + a_{18}$ is equal to _____.

[JEE (Main)-2022]

26. Let $S = \left\{x \in [-6, 3] - \{-2, 2\} : \frac{|x+3|-1}{|x|-2} \geq 0\right\}$ and $T = \left\{x \in \mathbb{Z} : x^2 - 7|x| + 9 \leq 0\right\}$. Then the number of elements in $S \cap T$ is **[JEE (Main)-2022]**

(1) 7 (2) 5

(3) 4 (4) 3

27. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A : \text{either } 1 \notin T \text{ or } 2 \in T\}$ and $C = \{T \subseteq A : T$ the sum of all the elements of T is a prime number$\}$. Then the number of elements in the set $B \cup C$ is _____. **[JEE (Main)-2022]**

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Chapter 14

Sets

1. Answer (2)

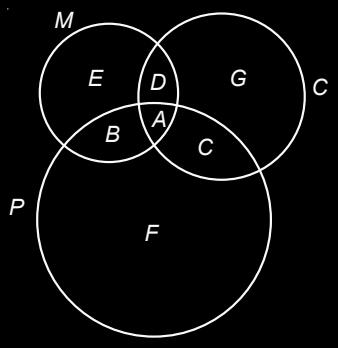
   $A \cap B = A \cap C$ and $A \cup B = A \cup C$

   $\Rightarrow \quad B = C$

2. Answer (1)

   No. of ways = $3 \times 3 \times 3 \times 3 \times 3 = 3^5$

3. Answer (2)



   $A = \{30, 60, 90, 120\} \quad \Rightarrow \quad n(A) = 4$

   $B = \{6n: n \in N, 1 \le n \le 23\} - A \quad \Rightarrow \quad n(B) = 19$

   $C = \{15n: n \in N, 1 \le n \le 9\} - A \quad \Rightarrow \quad n(C) = 5$

   $D = \{10n: n \in N, 1 \le n \le 14\} - A \quad \Rightarrow \quad n(D) = 10$

   $n(E) = 70 - n(A) - n(B) - n(D) = 70 - 33 = 37$

   $n(F) = 46 - n(A) - n(B) - n(C) = 46 - 28 = 18$

   $n(G) = 28 - n(A) - n(C) - n(D) = 28 - 19 = 9$

   $\Rightarrow$ Number of required students

   $= 140 - (4 + 19 + 5 + 10 + 37 + 18 + 9)$

   $= 140 - 102 = 38$

4. Answer (3)

   Number of required subsets = Total number of subsets – Total number of subsets having only odd numbers

   $= 2^{100} - 2^{50}$

   $= 2^{50}(2^{50} - 1)$

5. Answer (1)

   $2^{(x+2)(x^2-5x+6)} = 1$

   $\Rightarrow \quad (x + 2)(x - 2)(x - 3) = 0$

   $x = -2, 2, 3$

   $A = \{-2, 2, 3\}$

   $\Rightarrow \quad n(A) = 3$

   $B : -3 < 2x - 1 < 9$

   $-1 < x < 5$ and $x \in Z$

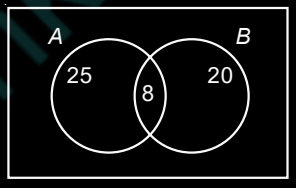
   $\therefore \quad B = \{0, 1, 2, 3, 4\}$

   $n(B) = 5$

   $n(A \times B) = 3 \times 5 = 15$

   $\therefore$ Number of subsets of $A \times B = 2^{15}$

6. Answer (1)



   $n(A \text{ only}) = 25 - 8 = 17\%$

   $n(B \text{ only}) = 20 - 8 = 12\%$

   % of people from A only who read advertisement

   $= 17 \times 0.3 = 5.1\%$

   % of people from B only who read advertisement

   $= 12 \times 0.4 = 4.8\%$

   % of people from A & B both who read advertisement = $8 \times 0.5 = 4\%$

   Total % of people who read advertisement

   $= 5.1 + 4.8 + 4 = 13.9\%$

7. Answer (3)

   $\because \quad A \cap B \subseteq C$ and $A \cap B \neq \phi$

   (1) $B \cap C \neq \phi$ is correct

   (2) $(C \cup A) \cap (C \cup B) = C \cup (A \cap B) = C$ (correct) (becasue $A \cap B \subseteq C$)

   (3) If $A = C$ then $A - C = \phi$

   Clearly $\phi \subseteq B$ but $A \subseteq B$ is not always true.

   (4) $\because \quad A - B \subseteq C$ and $A \cap B \subseteq C$ so $A \subseteq C$ (correct)

8. Answer (3)

Consider the statements;

p : $A \subseteq B$ and $B \subseteq D$

q : $A \subseteq C$

Given statement is "If p then q". It's contrapositive will be "If not q then not p"

$\Rightarrow$ If $A \not\subseteq C$ then $A \not\subseteq B$ or $B \not\subseteq D$.

9. Answer (3)

$A = \{x : |x| < 2\}$, $B = \{x : |x - 2| \le 3\}$

$$A \cap B = \{x : x \in [-2, -1)$$

$$A \cup B = R - \{x : x \in [2, 5)\}$$

$$A - B = \{x : x \in (-1, 2)\}$$

$$B - A = (-\infty, -2] \cup [5, \infty)$$

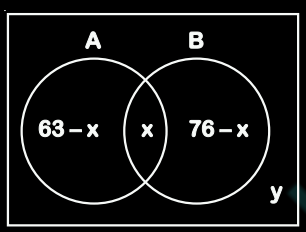
10. Answer (4)

$A = \{m \in R : x^2 - (m + 1)x + m + 4 = 0$ has real roots$\}$

$A = \{(-\infty, -3] \cup [5, \infty)\}$ $\{D \ge 0\}$

$B = [-3, 5) \Rightarrow A - B = (-\infty, -3) \cup [5, \infty)$

11. Answer (2)



Here $63 - x + x + 76 - x + y = 100$

$39 + y = x$

$\therefore$ $39 \le x \le 63$

$\therefore$ Possible value of x is 55.

12. Answer (3)

$$\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T,$$

$$\Rightarrow \quad \frac{10 \times 50}{20} = \frac{5n}{6}$$

$$\left\{ \begin{array}{l} \because n(X_i) = 10, n(Y_i) = 5 \\ \text{So, } \bigcup_{i=1}^{50} X_i = 500, \bigcup_{i=1}^{n} Y_i = 5n \end{array} \right\}$$

$\Rightarrow$ $n = 30$

13. Answer (2)

$n(C) = 73$, $n(T) = 65$

$\therefore$ $65 \ge n(C \cap T) \ge 65 + 73 - 100$

$65 \ge x \ge 38$

$x \ne 36$

14. Answer (28.00)

Number of subsets of $A = 2^m$

Number of subsets of $B = 2^n$

Given $= 2^m - 2^n = 112$

$\therefore$ $(m, n) = (7, 4)$

$\Rightarrow$ $mn = 28$

15. Answer (29)

$\because$ $X = \{1, 2, 3, 4, \ldots, 50\}$

$A = \{2, 4, 6, 8, \ldots, 50\}$

$B = \{7, 14, 21, 28, 35, 42, 49\}$

Here $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 29.$

$\therefore$ Number of elements in smallest subset of X containing both A and B is 29.

16. Answer (4)

As none play all three games, the intersection of all three circles must be zero

Hence none of P, Q, R justify the given statement.

17. Answer (1251)

$\because$ $2040 = 2^3 \cdot 3 \cdot 5 \cdot 17$

Let

A = Sum of all numbers which are divisible by 2 upto 100

B = Sum of all numbers which are divisible by 3 upto 100

C = Sum of all numbers which are divisible by 5 upto 100

D = Sum of all numbers which are divisible by 17 upto 100

$A \cup B \cup C \cup D = (A + B + C + D) - (A \cap B + A \cap C + A \cap D + B \cap C + B \cap D + C \cap D) + (A \cap B \cap C + A \cap B \cap D + A \cap C \cap D + B \cap C \cap D) - (A \cap B \cap C \cap D)$

$= (50 \times 51 + 33 \times 51 + 1050 + 51 \times 5) - (51 \times 16 + 550 + 102 + 315 + 51 + 85) + (180 + 0 + 0 + 0) - 0 = 3799$

Required sum $= 5050 - 3799 = 1251$

18. Answer (832)

$A = \{1, 2, 3, ..., 100\}$

and $B - C = \{3k + 1 \mid k \in \text{even}\}$

$\Rightarrow$ $B - C = \{7, 13, 19, ..., 97\}$

Sum of all elements $= \frac{16}{2}[7 + 97] = 832$

19. Answer (3)

Let A denotes the set of patients suffering from heart ailment and B denotes the set of patients suffering from lungs infection.

$n(A) = 89$, $n(B) = 98$ and $n(A \cap B) = K$

$\because$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow$ $89 + 98 - K \leq 100$

$\Rightarrow$ $K \geq 87$

20. Answer (256)

$A = \{x \in \mathbf{R} : |x - 2| > 1\}$

$|x - 2| > 1 \Rightarrow$ $x - 2 > 1$ or $x - 2 < -1$

$x > 3$ or $x < 1$ ...(i)

$B = \{x \in \mathbf{R} : \sqrt{x^2 - 3} > 1\}$

$x^2 - 3 \geq 0$ or $x^2 - 3 > 1$

$x^2 - 4 > 0$ $\Rightarrow$ $x > 2$ or $x < -2$ ...(ii)

$C = \{x \in \mathbf{R} : |x - 4| \geq 2\}$

$x - 4 \geq 2$ or $x - 4 \leq -2$

$x \geq 6$ or $x \leq 2$ ...(iii)

$(A \cap B \cap C)$ $= x < -2$ or $x \geq 6$

$(A \cap B \cap C)^C \cap Z = -2 \leq x < 6$

$n\{(A \cap B \cap C)^C \cap Z\} = 8$

No. of subsets $= 2^8 = 256$

21. Answer (80)

There 2 numbers of the type $3\lambda + 1$, 2 numbers of the type $3\lambda - 1$ and 3 numbers of the type 3λ.

So number of subsets whose sum is divisible by 3

$= 2^3 \cdot ({}^2C_0^2 + {}^2C_1^2 + {}^2C_2^2)$

$= 48$

Required number of subsets $= 2^7 - 48 = 80$

22. Answer (2)

If $x \in (-\infty, -4)$

$(x + 3)(x + 4) = 6 \Rightarrow x^2 + 7x + 6 = 0 \Rightarrow x = -6$ only

If $x \in (-4, 0)$

$(x + 3)(x + 4) = -6 \Rightarrow x^2 + 7x + 18 = 0 \Rightarrow$ No solution.

If $x \in (0, \infty)$

$(x - 3)(x + 4) = 6 \Rightarrow x^2 + x - 18 = 0$

$\Rightarrow$ $x = \frac{\sqrt{73} - 1}{2}$ only.

So the given set contains only 2 elements.

23. Answer (2)

$A = (-3, 1)$ and $B = (-\infty, -1] \cup [3, \infty)$

So, $A - B = (-1, 1)$

$B - A = (-\infty, -3] \cup [3, \infty) = R - (-3, 3)$

$A \cap B = (-3, -1]$

and $A \cup B = (-\infty, 1) \cup [3, \infty) = R - [1, 3)$

24. Answer (5264)

Sum of all elements of $A \cap B = 2$ [Sum of natural numbers upto 100 which are neither divisible by 3 nor by 5]

$= 2\left[\frac{100 \times 101}{2} - 3\left(\frac{33 \times 34}{2}\right) - 5\left(\frac{20 \times 21}{2}\right) + 15\left(\frac{6 \times 7}{2}\right)\right]$

$= 10100 - 3366 - 2100 + 630$

$= 5264$

25. Answer (702)

If we write the elements of $A + A$, we can certainly find 39 distinct elements as $1 + 1$, $1 + a_1$, $1 + a_2$,.....$1 + a_{18}$, $1 + 77$, $a_1 + 77$, $a_2 + 77$,.....$a_{18} + 77$, $77 + 77$.

It means all other sums are already present in these 39 values, which is only possible in case when all numbers are in A.P.

Let the common difference be 'd'.

$77 = 1 + 19d \Rightarrow d = 4$

So, $\sum_{i=1}^{18} a_1 = \frac{18}{2}[2a_1 + 17d] = 9[10 + 68] = 702$

26. Answer (4)

$|x|^2 - 7|x| + 9 \le 0$

$\Rightarrow \quad |x| \in \left[\frac{7-\sqrt{13}}{2}, \frac{7+\sqrt{13}}{2}\right]$

As $x \in Z$

So, x can be $\pm 2, \pm 3, \pm 4, \pm 5$

Out of these values of x,

$x = 3, -4, -5$

satisfy S as well

$n(S \cap T) = 3$

27. Answer (107)

$\because \quad (B \cup C)' = B' \cap C'$

B' is a set containing sub sets of A containing element 1 and not containing 2.

And C' is a set containing subsets of A whose sum of elements is not prime.

So, we need to calculate number of subsets of {3, 4, 5, 6, 7} whose sum of elements plus 1 is composite.

Number of such 5 elements subset = 1

Number of such 4 elements subset = 3 (except selecting 3 or 7)

Number of such 3 elements subset = 6 (except selecting {3, 4, 5}, {3, 6, 7}, {4, 5, 7} or {5, 6, 7})

Number of such 2 elements subset = 7 (except selecting {3, 7}, {4, 6}, {5, 7})

Number of such 1 elements subset = 3 (except selecting {4} or {6})

Number of such 0 elements subset = 1

$n(B' \cap C') = 21 \Rightarrow n(B \cup C) = 2^7 - 21 = 107$

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