Chapter 10

Straight Lines

1. The lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line for **[AIEEE-2009]**
   (1) Exactly one value of p
   (2) Exactly two values of p
   (3) More than two values of p
   (4) No value of p

2. Three distinct points A, B and C are given in the 2 - dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point **[AIEEE-2009]**
   (1) $\left(\frac{5}{4}, 0\right)$ (2) $\left(\frac{5}{2}, 0\right)$
   (3) $\left(\frac{5}{3}, 0\right)$ (4) (0, 0)

3. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1.$ Then the distance between L and K is **[AIEEE-2010]**
   (1) $\frac{23}{\sqrt{15}}$ (2) $\sqrt{17}$
   (3) $\frac{17}{\sqrt{15}}$ (4) $\frac{23}{\sqrt{17}}$

4. The lines $x + y = |a|$ and $ax - y = 1$ intersect each other in the first quadrant. Then the set of all possible values of a is the interval **[AIEEE-2011]**
   (1) $(-1, \infty)$ (2) $(-1, 1]$
   (3) $(0, \infty)$ (4) $[1, \infty)$

5. If $A(2, -3)$ and $B(-2, 1)$ are two vertices of a triangle and third vertex moves on the line $2x + 3y = 9$, then the locus of the centroid of the triangle is: **[AIEEE-2011]**
   (1) $2x + 3y = 3$ (2) $2x - 3y = 1$
   (3) $x - y = 1$ (4) $2x + 3y = 1$

6. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals **[AIEEE-2012]**
   (1) 5 (2) 6
   (3) 11/5 (4) 29/5

7. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it form a triangle OPQ where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is **[AIEEE-2012]**
   (1) –4 (2) –2
   (3) –1/2 (4) –1/4

8. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is **[JEE (Main)-2013]**
   (1) $y=x+\sqrt{3}$ (2) $\sqrt{3}y=x-\sqrt{3}$
   (3) $y=\sqrt{3}x-\sqrt{3}$ (4) $\sqrt{3}y=x-1$

9. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1), (1, 1) and (1, 0) is **[JEE (Main)-2013]**
   (1) $2+\sqrt{2}$ (2) $2-\sqrt{2}$
   (3) $1+\sqrt{2}$ (4) $1-\sqrt{2}$

10. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through (1, –1) and parallel to PS is **[JEE (Main)-2014]**
    (1) $4x + 7y + 3 = 0$ (2) $2x - 9y - 11 = 0$
    (3) $4x - 7y - 11 = 0$ (4) $2x + 9y + 7 = 0$

11. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then **[JEE (Main)-2014]**

(1) $3bc - 2ad = 0$ (2) $3bc + 2ad = 0$

(3) $2bc - 3ad = 0$ (4) $2bc + 3ad = 0$

12. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0), is **[JEE (Main)-2015]**

(1) 901 (2) 861

(3) 820 (4) 780

13. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a **[JEE (Main)-2015]**

(1) Straight line parallel to x-axis

(2) Straight line parallel to y-axis

(3) Circle of radius $\sqrt{2}$

(4) Circle of radius $\sqrt{3}$

14. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus? **[JEE (Main)-2016]**

(1) $(-3, -8)$ (2) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

(3) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (4) $(-3, -9)$

15. Let k be an integer such that the triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq. units. Then the orthocentre of this triangle is at the point **[JEE (Main)-2017]**

(1) $\left(1, \frac{3}{4}\right)$ (2) $\left(1, -\frac{3}{4}\right)$

(3) $\left(2, \frac{1}{2}\right)$ (4) $\left(2, -\frac{1}{2}\right)$

16. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle $OPRQ$ is completed, then the locus of R is **[JEE (Main)-2018]**

(1) $3x + 2y = 6$ (2) $2x + 3y = xy$

(3) $3x + 2y = xy$ (4) $3x + 2y = 6xy$

17. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true? **[JEE (Main)-2019]**

(1) The lines are all parallel

(2) The lines are not concurrent

(3) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

(4) Each line passes through the origin

18. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50 sq. units, then the number of elements in the set S is: **[JEE (Main)-2019]**

(1) 9 (2) 32

(3) 36 (4) 18

19. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1, 1), then the equation of its third side is **[JEE (Main)-2019]**

(1) $26x - 122y - 1675 = 0$

(2) $122y - 26x - 1675 = 0$

(3) $122y + 26x + 1675 = 0$

(4) $26x + 61y + 1675 = 0$

20. If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin is **[JEE (Main)-2019]**

(1) (4, 3) (2) (3, 4)

(3) (4, 4) (4) (2, 2)

21. A point P moves on the line $2x - 3y + 4 = 0$. If $Q(1, 4)$ and $R(3, -2)$ are fixed points, then the locus of the centroid of ΔPQR is a line **[JEE (Main)-2019]**

(1) Parallel to y-axis (2) With slope $\frac{3}{2}$

(3) With slope $\frac{2}{3}$ (4) Parallel to x-axis

22. Two vertices of a triangle are (0, 2) and (4, 3). If its orthocentre is at the origin, then its third vertex lies in which quadrant? **[JEE (Main)-2019]**

(1) Fourth

(2) Third

(3) First

(4) Second

23. The straight line $x + 2y = 1$ meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is **[JEE (Main)-2019]**

(1) $\frac{\sqrt{5}}{4}$ (2) $\frac{\sqrt{5}}{2}$

(3) $4\sqrt{5}$ (4) $2\sqrt{5}$

24. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is **[JEE (Main)-2019]**

(1) $\frac{c}{\sqrt{3}}$ (2) $\frac{3}{2}y$

(3) $\frac{c}{3}$ (4) $\frac{y}{\sqrt{3}}$

25. If in a parallelogram $ABDC$, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is

[JEE (Main)-2019]

(1) $5x + 3y - 11 = 0$

(2) $3x + 5y - 13 = 0$

(3) $3x - 5y + 7 = 0$

(4) $5x - 3y + 1 = 0$

26. If the straight line, $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals

[JEE (Main)-2019]

(1) $-\frac{35}{3}$ (2) –5

(3) 5 (4) $\frac{35}{3}$

27. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in **[JEE (Main)-2019]**

(1) 4th quadrant

(2) 1st quadrant

(3) 1st, 2nd and 4th quadrants

(4) 1st and 2nd quadrants

28. Suppose that the points (h, k), (1, 2) and (–3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4, 3) is perpendicular to L_1, then $\frac{k}{h}$ equals **[JEE (Main)-2019]**

(1) 3 (2) $-\frac{1}{7}$

(3) 0 (4) $\frac{1}{3}$

29. Slope of a line passing through $P(2, 3)$ and intersecting the line, $x + y = 7$ at a distance of 4 units from P, is **[JEE (Main)-2019]**

(1) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (2) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

(3) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ (4) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

30. A rectangle is inscribed in a circle with a diameter lying along the line $3y = x + 7$. If the two adjacent vertices of the rectangle are (–8, 5) and (6, 5), then the area of the rectangle (in sq. units) is

[JEE (Main)-2019]

(1) 56 (2) 84

(3) 72 (4) 98

31. If the two lines $x + (a - 1)y = 1$ and $2x + a^2y = 1$ ($a \in R - \{0, 1\}$) are perpendicular, then the distance of their point of intersection from the origin is **[JEE (Main)-2019]**

(1) $\sqrt{\frac{2}{5}}$ (2) $\frac{\sqrt{2}}{5}$

(3) $\frac{2}{5}$ (4) $\frac{2}{\sqrt{5}}$

32. Lines are drawn parallel to the line $4x - 3y + 2 = 0$, at a distance $\frac{3}{5}$ from the origin. Then which one of the following points lies on any of these lines? **[JEE (Main)-2019]**

(1) $\left(\frac{1}{4}, -\frac{1}{3}\right)$ (2) $\left(\frac{1}{4}, \frac{1}{3}\right)$

(3) $\left(-\frac{1}{4}, \frac{2}{3}\right)$ (4) $\left(-\frac{1}{4}, -\frac{2}{3}\right)$

33. The equation $y = \sin x \sin(x + 2) - \sin^2(x + 1)$ represents a straight line lying in

[JEE (Main)-2019]

(1) Third and fourth quadrants only

(2) First, third and fourth quadrants

(3) First, second and fourth quadrants

(4) Second and third quadrants only

34. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point

[JEE (Main)-2019]

(1) (1, –4, 1) (2) (2, –4, 1)

(3) (1, 4, –1) (4) (2, 4, 1)

35. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2, 3). Then the centroid of this triangle is

[JEE (Main)-2019]

(1) $\left(\frac{1}{3}, 2\right)$ (2) $\left(\frac{1}{3}, \frac{5}{3}\right)$

(3) $\left(\frac{1}{3}, 1\right)$ (4) $\left(1, \frac{7}{3}\right)$

36. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line L is

[JEE (Main)-2019]

(1) $\left(\sqrt{3}+1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$

(2) $\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$

(3) $\sqrt{3}x + y = 8$

(4) $x + \sqrt{3}y = 8$

37. Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at (2, 4) then one of its vertex is

[JEE (Main)-2019]

(1) (2, 1) (2) (3, 5)

(3) (2, 6) (4) (3, 6)

38. The locus of the mid-points of the perpendiculars drawn from points on the line, $x = 2y$ to the line $x = y$ is **[JEE (Main)-2020]**

(1) $5x - 7y = 0$ (2) $2x - 3y = 0$

(3) $3x - 2y = 0$ (4) $7x - 5y = 0$

39. Let two points be $A(1, -1)$ and $B(0, 2)$. If a point $P(x', y')$ be such that the area of $\Delta PAB = 5$ sq. units and it lies on the line, $3x + y - 4\lambda = 0$, then a value of λ is **[JEE (Main)-2020]**

(1) 3 (2) 4

(3) 1 (4) –3

40. Let C be the centroid of the triangle with vertices (3, –1), (1, 3) and (2, 4). Let P be the point of intersection of the lines $x + 3y - 1 = 0$ and $3x - y + 1 = 0$. Then the line passing through the points C and P also passes through the point

[JEE (Main)-2020]

(1) (–9, – 6) (2) (–9, – 7)

(3) (9, 7) (4) (7, 6)

41. The set of all possible values of θ in the interval $(0, \pi)$ for which the points (1, 2) and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y = 1$ is

[JEE (Main)-2020]

(1) $\left(0, \frac{\pi}{2}\right)$ (2) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

(3) $\left(0, \frac{\pi}{4}\right)$ (4) $\left(0, \frac{3\pi}{4}\right)$

42. If a ΔABC has vertices $A(-1, 7)$, $B(-7, 1)$ and $C(5, -5)$, then its orthocentre has coordinates

[JEE (Main)-2020]

(1) (–3, 3) (2) $\left(-\frac{3}{5}, \frac{3}{5}\right)$

(3) (3, –3) (4) $\left(\frac{3}{5}, -\frac{3}{5}\right)$

43. A triangle ABC lying in the first quadrant has two vertices as $A(1, 2)$ and $B(3, 1)$. If $\angle BAC = 90°$, and $\text{ar}(\Delta ABC) = 5\sqrt{5}$ sq. units, then the abscissa of the vertex C is **[JEE (Main)-2020]**

(1) $1+\sqrt{5}$ (2) $1+2\sqrt{5}$

(3) $2\sqrt{5}-1$ (4) $2+\sqrt{5}$

44. If the perpendicular bisector of the line segment joining the points $P(1, 4)$ and $Q(k, 3)$ has y-intercept equal to –4, then a value of k is

[JEE (Main)-2020]

(1) $\sqrt{14}$ (2) $\sqrt{15}$

(3) –4 (4) –2

45. A ray of light coming from the point $\left(2, 2\sqrt{3}\right)$ is incident at an angle 30° on the line $x = 1$ at the point A. The ray gets reflected on the line $x = 1$ and meets x-axis at the point B. Then, the line AB passes through the point **[JEE (Main)-2020]**

(1) $\left(3, -\frac{1}{\sqrt{3}}\right)$ (2) $\left(3, -\sqrt{3}\right)$

(3) $\left(4, -\sqrt{3}\right)$ (4) $\left(4, -\frac{\sqrt{3}}{2}\right)$

46. Let L denote the line in the xy-plane with x and y intercepts as 3 and 1 respectively. Then the image of the point (–1, –4) in this line is

[JEE (Main)-2020]

(1) $\left(\frac{29}{5}, \frac{8}{5}\right)$ (2) $\left(\frac{29}{5}, \frac{11}{5}\right)$

(3) $\left(\frac{8}{5}, \frac{29}{5}\right)$ (4) $\left(\frac{11}{5}, \frac{28}{5}\right)$

47. Let $A(1, 0)$, $B(6, 2)$ and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is________. **[JEE (Main)-2020]**

48. If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible values of α and β is ______.

[JEE (Main)-2020]

49. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man? **[JEE (Main)-2021]**

(1) C only (2) B only

(3) All the three (4) A only

50. The image of the point (3, 5) in the line $x - y + 1 = 0$, lies on : **[JEE (Main)-2021]**

(1) $(x - 4)^2 + (y + 2)^2 = 16$

(2) $(x - 4)^2 + (y - 4)^2 = 8$

(3) $(x - 2)^2 + (y - 2)^2 = 12$

(4) $(x - 2)^2 + (y - 4)^2 = 4$

51. The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a :

[JEE (Main)-2021]

(1) None of the above (2) Isosceles triangle

(3) Right angled triangle (4) Equilateral triangle

52. Let A(–1, 1), B(3, 4) and C(2, 0) be given three points. A line $y = mx$, $m > 0$, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

[JEE (Main)-2021]

(1) 2 (2) 3

(3) $\frac{4}{15}$ (4) 1

53. In a triangle PQR, the co-ordinates of the points P and Q are (–2, 4) and (4, –2) respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the ΔPQR is **[JEE (Main)-2021]**

(1) (–2, –2) (2) (0, 2)

(3) (1, 4) (4) (–1, 0)

54. Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin. If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cos\beta \cos\gamma}\right)^2$ is equal to ________.

[JEE (Main)-2021]

55. The equation of one of the straight lines which passes through the point (1, 3) and makes an angle $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line, $y + 1 = 3\sqrt{2}x$ is :

[JEE (Main)-2021]

(1) $4\sqrt{2}x - 5y - \left(5 + 4\sqrt{2}\right) = 0$

(2) $4\sqrt{2}x + 5y - 4\sqrt{2} = 0$

(3) $4\sqrt{2}x + 5y - \left(15 + 4\sqrt{2}\right) = 0$

(4) $5\sqrt{2}x + 4y - \left(15 + 4\sqrt{2}\right) = 0$

56. The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is : **[JEE (Main)-2021]**

(1) 0 (2) 3

(3) 1 (4) 2

57. Let the equation of the pair of lines, $y = px$ and $y = qx$, can be written as $(y - px)(y - qx) = 0$. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is **[JEE (Main)-2021]**

(1) $x^2 - 3xy + y^2 = 0$ (2) $x^2 + 3xy - y^2 = 0$

(3) $x^2 - 3xy - y^2 = 0$ (4) $x^2 + 4xy - y^2 = 0$

58. Two sides of a parallelogram are along the lines $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals of the parallelogram is $11x + 7y = 9$, then other diagonal passes through the point **[JEE (Main)-2021]**

(1) (2, 2) (2) (2, 1)

(3) (1, 3) (4) (1, 2)

59. Let ABC be a triangle with A(–3, 1) and $\angle ACB = \theta$, $0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is $2x + y - 3 = 0$ and the equation of angle bisector of C is $7x - 4y - 1 = 0$, then $\tan\theta$ is equal to **[JEE (Main)-2021]**

(1) 2 (2) $\frac{3}{4}$

(3) $\frac{4}{3}$ (4) $\frac{1}{2}$

60. Let A be a fixed point (0, 6) and B be a moving point $(2t, 0)$. Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is **[JEE (Main)-2021]**

(1) $3x^2 + 2y - 6 = 0$ (2) $2x^2 + 3y - 9 = 0$

(3) $3x^2 - 2y - 6 = 0$ (4) $2x^2 - 3y + 9 = 0$

61. Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is $4x + 3y = 10$, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \neq C_2$ are their centres, then $|(\alpha + \beta)(\gamma + \delta)|$ is equal to _______. **[JEE (Main)-2021]**

62. If p and q are the lengths of the perpendiculars from the origin on the lines, $x\text{cosec}\alpha - y\sec\alpha = k\cot 2\alpha$ and $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$ respectively, then k^2 is equal to : **[JEE (Main)-2021]**

(1) $2p^2 + q^2$ (2) $p^2 + 4q^2$

(3) $4p^2 + q^2$ (4) $p^2 + 2q^2$

63. Let A be the set of all points (α, β) such that the area of triangle formed by the points (5, 6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is **[JEE (Main)-2021]**

(1) $\frac{8}{\sqrt{5}}$ (2) $\frac{16}{\sqrt{5}}$

(3) $\frac{4}{\sqrt{5}}$ (4) $\frac{12}{\sqrt{5}}$

64. A man starts walking from the point P(–3, 4), touches the x-axis at R, and then turns to reach at the point Q(0, 2). The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $50(PR)^2 + (RQ)^2$ is equal to ________. **[JEE (Main)-2021]**

65. If a straight line passing through the point $P(-3, 4)$ is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is **[JEE (Main)-2021]**

(1) $3x - 4y + 25 = 0$ (2) $4x - 3y + 24 = 0$

(3) $x - y + 7 = 0$ (4) $4x + 3y = 0$

66. Let the area of the triangle with vertices $A(1, \alpha)$, $B(\alpha, 0)$ and $C(0, \alpha)$ be 4 sq. units. If the points $(\alpha, -\alpha)$, $(-\alpha, \alpha)$ and (α^2, β) are collinear, then β is equal to **[JEE (Main)-2022]**

(1) 64 (2) –8

(3) –64 (4) 512

67. Let R be the point (3, 7) and let P and Q be two points on the line $x + y = 5$ such that PQR is an equilateral triangle, Then the area of ΔPQR is **[JEE (Main)-2022]**

(1) $\frac{25}{4\sqrt{3}}$ (2) $\frac{25\sqrt{3}}{2}$

(3) $\frac{25}{\sqrt{3}}$ (4) $\frac{25}{2\sqrt{3}}$

68. In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is $2x + y = 4$. Let the point B lie on the line $x + 3y = 7$. If (α, β) is the centroid of ΔABC, then $15(\alpha + \beta)$ is equal to **[JEE (Main)-2022]**

(1) 39 (2) 41

(3) 51 (4) 63

69. Let a triangle be bounded by the lines $L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ and the line L_3, which passes through the point $P(2, 3)$, intersects L_2 at A and L_1 at B. If the point P divides the line-segment AB, internally in the ratio 1 : 3, then the area of the triangle is equal to **[JEE (Main)-2022]**

(1) $\frac{110}{13}$ (2) $\frac{132}{13}$

(3) $\frac{142}{13}$ (4) $\frac{151}{13}$

70. The distance between the two points A and A' which lie on $y = 2$ such that both the line segments AB and $A'B$ (where B is the point (2, 3)) subtend angle $\frac{\pi}{4}$ at the origin, is equal to

[JEE (Main)-2022]

(1) 10 (2) $\frac{48}{5}$

(3) $\frac{52}{5}$ (4) 3

71. A line, with the slope greater than one, passes through the point $A(4, 3)$ and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line

[JEE (Main)-2022]

(1) $2x + y = 9$ (2) $3x - 2y = 7$

(3) $x + 2y = 6$ (4) $2x - 3y = 3$

72. The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 15a$ and $x - y = 3$ respectively. If its orthocentre is $(2, a)$, $-\frac{1}{2} < a < 2$, then p is equal to ________.

[JEE (Main)-2022]

73. Let $A(1, 1)$, $B(-4, 3)$, $C(-2, -5)$ be vertices of a triangle ABC, P be a point on side BC, and Δ_1 and Δ_2 be the areas of triangles APB and ABC, respectively. If $\Delta_1 : \Delta_2 = 4 : 7$, then the area enclosed by the lines AP, AC and the x-axis is **[JEE (Main)-2022]**

(1) $\frac{1}{4}$ (2) $\frac{3}{4}$

(3) $\frac{1}{2}$ (4) 1

74. The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 39$ and $x - y = 3$ respectively and $P(2, 3)$ is its circumcentre. Then which of the following is **NOT** true?

[JEE (Main)-2022]

(1) $(AC)^2 = 9p$

(2) $(AC)^2 + p^2 = 136$

(3) $32 <$ area$(\Delta ABC) < 36$

(4) $34 <$ area$(\Delta ABC) < 38$

75. Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2 + 11a + 3(m_1^2 + m_2^2) = 220$. If one vertex of the square is $(10(\cos\alpha - \sin\alpha), 10(\sin\alpha + \cos\alpha))$, where $\alpha \in \left(0, \frac{\pi}{2}\right)$ and the equation of one diagonal is $(\cos\alpha - \sin\alpha)x + (\sin\alpha + \cos\alpha)y = 10$, then $72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$ is equal to

[JEE (Main)-2022]

(1) 119 (2) 128

(3) 145 (4) 155

76. Let $A(\alpha, -2)$, $B(\alpha, 6)$ and $C\left(\frac{\alpha}{4}, -2\right)$ be vertices of a ΔABC. If $\left(5, \frac{\alpha}{4}\right)$ is the circumcentre of ΔABC, then which of the following is **NOT** correct about ΔABC. **[JEE (Main)-2022]**

(1) area is 24 (2) perimeter is 25

(3) circumradius is 5 (4) inradius is 2

77. The distance of the origin from the centroid of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3}, \frac{7}{3}\right)$ is **[JEE (Main)-2022]**

(1) $\sqrt{2}$ (2) 2

(3) $2\sqrt{2}$ (4) 4

78. Let the point $P(\alpha, \beta)$ be at a unit distance from each of the two lines $L_1 : 3x - 4y + 12 = 0$, and $L_2 : 8x + 6y + 11 = 0$. If P lies below L_1 and above L_2, then $100(\alpha + \beta)$ is equal to

(1) –14 (2) 42

(3) –22 (4) 14

[JEE (Main)-2022]

79. Let the circumcentre of a triangle with vertices $A(a, 3)$, $B(b, 5)$ and $C(a, b)$, $ab > 0$ be $P(1, 1)$. If the line AP intersects the line BC at the point $Q(k_1, k_2)$, then $k_1 + k_2$ is equal to : [JEE (Main)-2022]

(A) 2 (B) $\frac{4}{7}$

(C) $\frac{2}{7}$ (D) 4

80. A ray of light passing through the point $P(2, 3)$ reflects on the x-axis at point A and the reflected ray passes through the point $Q(5, 4)$. Let R be the point that divides the line segment AQ internally into the ratio 2 : 1. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of $7\alpha + 3\beta$ is equal to ___________. **[JEE (Main)-2022]**

❑ ❑ ❑

Chapter 10

Straight Lines

1. Answer (1)

Lines perpendicular to same line are parallel to each other.

$\therefore \quad -p(p^2 + 1) = p^2 + 1$

$\Rightarrow \quad p = -1$

$\therefore$ There is exactly one value of p.

2. Answer (1)

Let (x, y) denote the coordinates of A, B and C.

Then, $\dfrac{(x-1)^2 + y^2}{(x+1)^2 + y^2} = \dfrac{1}{9}$

$\Rightarrow \quad 9x^2 + 9y^2 - 18x + 9 = x^2 + y^2 + 2x + 1$

$\Rightarrow \quad 8x^2 + 8y^2 - 20x + 8 = 0$

$$x^2 + y^2 - \frac{5}{2}x + 1 = 0$$

$\therefore$ A, B, C lie on a circle with $C\left(\dfrac{5}{4}, 0\right)$.

3. Answer (4)

$$\frac{13}{5} + \frac{32}{b} = 1 \Rightarrow \frac{32}{b} = -\frac{8}{5} \therefore b = -20$$

The line K must have equation

$$\frac{x}{5} - \frac{y}{20} = a \text{ or } \frac{x}{5a} - \frac{y}{20a} = 1$$

Comparing with $\dfrac{x}{c} + \dfrac{y}{3} = 1$

$$\left(\text{Given } 20a = -3,\ c = 5a = -\frac{3}{4}\right)$$

Distance between lines is

$$= \frac{|a-1|}{\sqrt{\frac{1}{25} + \frac{1}{400}}} = \frac{\left|\frac{-3}{20} - 1\right|}{\sqrt{\frac{17}{400}}} = \frac{23}{\sqrt{17}}$$

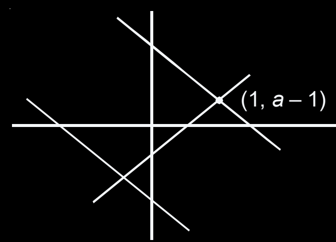
4. Answer (4)

Given

$x + y = |a|$

$ax - y = 1$

$$\frac{x}{\frac{1}{a}} + \frac{y}{-1} = 1$$



$\therefore \quad x + y = a.$ ...(i) for I[st] quadrant

$ax - y = 1.$ ...(ii)

After solving (i) & (ii)

$\Rightarrow \quad x = 1$

$\therefore \quad y = a - 1$

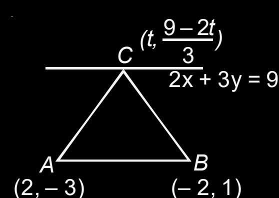
Clearly $a - 1 \geq 0$

$\Rightarrow \quad a \geq 1$

$\Rightarrow \quad a \in [1, \infty)$

5. Answer (4)

Let vertex of C be $\left(t, \dfrac{9-2t}{3}\right)$



Let (h, k) be centroid

$$\therefore \quad h = \frac{t+2-2}{3},\ k = \frac{-3+1+\frac{9-2t}{3}}{3}$$

$$h = \frac{t}{3} \quad \text{...(i)}$$

$$k = \frac{-6+9-2t}{9} \quad \text{...(ii)}$$

from (i) and (ii)

$$k = \frac{3-2(3h)}{9}$$

$$9k = 3 - 6h$$

$$6h + 9k = 3$$

$$2h + 3k = 1$$

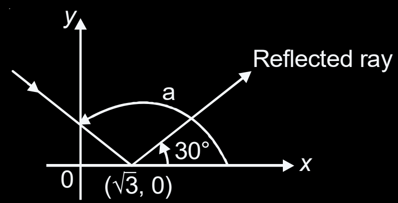
Required locus is

$$2x + 3y = 1$$

6. Answer (2)

7. Answer (2)

8. Answer (2)



Slope of incident ray = $-\frac{1}{\sqrt{3}}$

$\Rightarrow$ $\alpha = 150°$

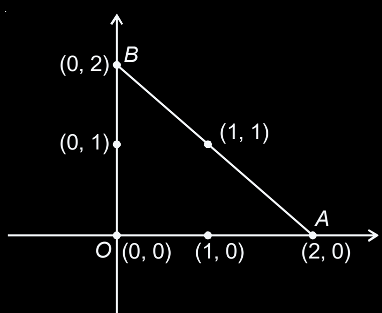
$\therefore$ Slope of reflected ray = tan30°

$$= \frac{1}{\sqrt{3}}$$

$\therefore$ Reflected ray is $y = \frac{1}{\sqrt{3}}(x-\sqrt{3})$

$\Rightarrow$ $\sqrt{3}y = x - \sqrt{3}$

9. Answer (2)



Required triangle is ΔOAB

So, x co-ordinate of incentre

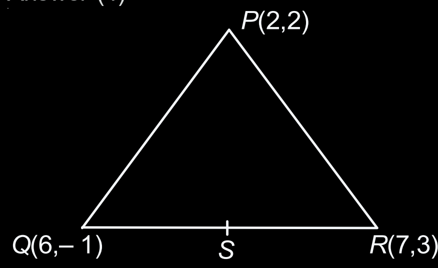
$$= \frac{2\times 0 + 2\times 2 + 2\sqrt{2}\times 0}{2+2+2\sqrt{2}}$$

$$= \frac{4}{4+2\sqrt{2}}$$

$$= \frac{2}{2+\sqrt{2}}$$

$$= 2-\sqrt{2}$$

10. Answer (4)



S is mid-point of QR

So $S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$

Slope of PS = $\frac{2-1}{2-\frac{13}{2}} = -\frac{2}{9}$

Equation of line $\Rightarrow$ $y-(-1) = -\frac{2}{9}(x-1)$

$9y + 9 = -2x + 2 \Rightarrow 2x + 9y + 7 = 0$

11. Answer (1)

Let $(\alpha, -\alpha)$ be the point of intersection

$\therefore$ $4a\alpha - 2a\alpha + c = 0 \Rightarrow \alpha = -\frac{c}{2a}$

and $5b\alpha - 2b\alpha + d = 0 \Rightarrow \alpha = -\frac{d}{3b}$

$\Rightarrow$ $3bc = 2ad$

$\Rightarrow$ $3bc - 2ad = 0$

Alternative method :

The point of intersection will be

$$\frac{x}{2ad-2bc} = \frac{-y}{4ad-5bc} = \frac{1}{8ab-10ab}$$

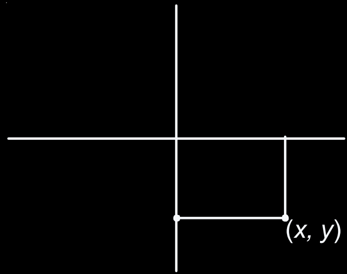
$$\Rightarrow \quad x = \frac{2(ad-bc)}{-2ab}$$

$$\Rightarrow \quad y = \frac{5bc-4ad}{-2ab}$$

$\because$ Point of intersection is in fourth quadrant so x is positive and y is negative.

Also distance from axes is same

So $x = -y$ ($\because$ distance from x-axis is $-y$ as y is negative)

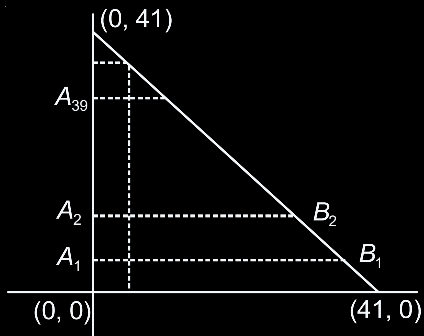


$$\frac{2(ad-bc)}{-2ab}=\frac{-(5bc-4ad)}{-2ab}$$

$2ad - 2bc = -5bc + 4ad$

$\Rightarrow\ 3bc - 2ad = 0$ ...(i)

12. Answer (4)

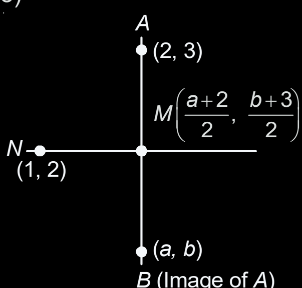


Total number of integral coordinates as required

$= 39 + 38 + 37 + ... + 1$

$$=\frac{39\times 40}{2}=780$$

13. Answer (3)



After solving equation (i) & (ii)

$2x - 3y + 4 = 0$ ...(i)

$2x - 4y + 6 = 0$ ...(ii)

$x = 1$ and $y = 2$

Slope of AB × Slope of $MN = -1$

$$\frac{b-3}{a-2}\times\frac{\frac{b+3}{2}-2}{\frac{a+2}{2}-1}=-1$$

$(y-3)(y-1) = -(x-2)x$

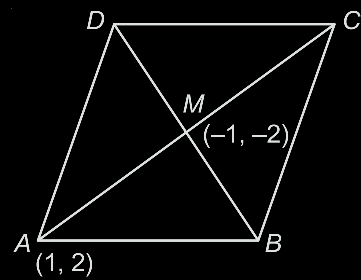
$y^2 - 4y + 3 = -x^2 + 2x$

$x^2 + y^2 - 2x - 4y + 3 = 0$

Circle of radius $= \sqrt{2}$

14. Answer (2)

Point of intersection of sides



$x - y + 1 = 0$

and $7x - y - 5 = 0$

$\therefore\ x = 1, y = 2$

Slope of $AM = \dfrac{4}{2} = 2$

$\therefore$ Equation of BD : $y + 2 = -\dfrac{1}{2}(x+1)$

$\Rightarrow\ x + 2y + 5 = 0$

Solving $x + 2y + 5 = 0$ and $7x - y - 5 = 0$

$$x=\frac{1}{3},\ y=-\frac{8}{3}\Rightarrow\left(\frac{1}{3},-\frac{8}{3}\right)$$

15. Answer (3)

$$\text{Area}=\left|\frac{1}{2}\begin{vmatrix}k & -3k & 1\\ 5 & k & 1\\ -k & 2 & 1\end{vmatrix}\right|=28$$

$$\begin{vmatrix}k-5 & -4k & 0\\ 5+k & k-2 & 0\\ -k & 2 & 1\end{vmatrix}=\pm 56$$

$(k^2-7k+10)+4k^2+20k=\pm 56$

$5k^2+13k+10=\pm 56$

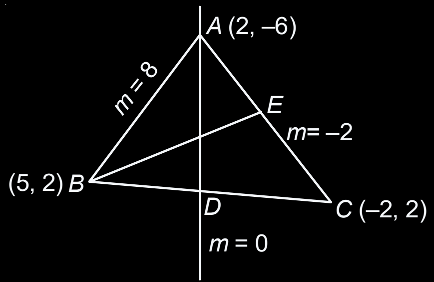
$5k^2+13k-46=0$ | $5K^2+13K+66=0$

$5k^2+13k-46=0$

$$k=\frac{-13\pm\sqrt{169+920}}{10}$$

$= 2, -4.6$
reject

For $k = 2$



Equation of AD,

$x = 2$ ...(i)

Also equation of BE,

$$y-2=\frac{1}{2}(x-5)$$

$$2y-4=x-5$$

$$x-2y-1=0 \quad \text{...(ii)}$$

Solving (i) & (ii), $2y = 1$

$$y=\frac{1}{2}$$

Orthocentre is $\left(2, \frac{1}{2}\right)$

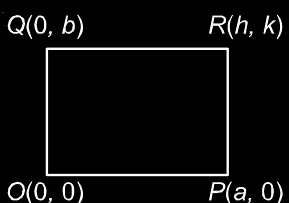
16. Answer (3)

Let the equation of line be $\frac{x}{a}+\frac{y}{b}=1$ ...(i)

(i) passes through the fixed point (2, 3)

$\Rightarrow \frac{2}{a}+\frac{3}{b}=1$ ...(ii)

$P(a, 0)$, $Q(0, b)$, $O(0, 0)$, Let $R(h, k)$,



Midpoint of OR is $\left(\frac{h}{2},\frac{k}{2}\right)$

Midpoint of PQ is $\left(\frac{a}{2},\frac{b}{2}\right) \Rightarrow h=a, \quad k=b$ ... (iii)

From (ii) & (iii),

$\frac{2}{h}+\frac{3}{k}=1 \quad \Rightarrow$ locus of $R(h, k)$

$\frac{2}{x}+\frac{3}{y}=1 \quad \Rightarrow 3x + 2y = xy$

17. Answer (3)

$px + qy + r = 0$

$\Rightarrow 4px + 4qy + 4r = 0$

$\Rightarrow 4px - 3p + 4qy - 2q + 3p + 2q + 4r = 0$

$\Rightarrow 4px - 3p + 4qy - 2q = 0$

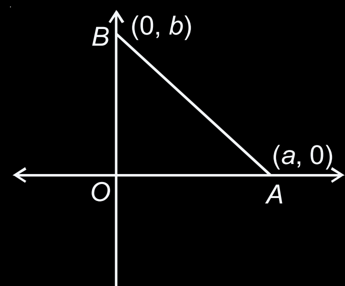
$\Rightarrow p(4x - 3) + q(4y - 2) = 0$

i.e. $(4x - 3) + \lambda (4y - 2) = 0 \quad \left(\text{Where } \lambda=\frac{q}{p}\right)$

$\therefore$ Set of lines are passing through $x=\frac{3}{4}, y=\frac{1}{2}$

18. Answer (3)

One of the possible ΔOAB is $A(a, 0)$ and $B(0, b)$.



Area of $\Delta OAB=\frac{1}{2}|ab|$.

$\therefore \quad |ab| = 100$

$|a|\,|b| = 100$

but $100 = 1 \times 100, 2 \times 50, 4 \times 25, 5 \times 20$ or 10×10

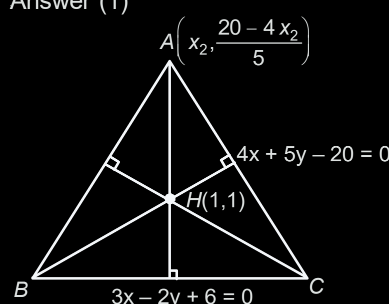
$\therefore$ For 1×100, $a = 1$ or -1 and $b = 100$ or -100

$\therefore$ Total possible pairs are 8.

and for 10×10 total possible pairs are 4.

$\therefore$ Total number of possible triangles with integral coordinates are $4 \times 8 + 4 = 36$.

19. Answer (1)



$\left(x_1, \frac{3x_1+6}{2}\right)$

$\because m_{AH}.m_{BC}=-1$

$$\left(\frac{\frac{20-4x_2}{5}-1}{x_2-1}\right)\times\frac{3}{2}=-1$$

$$\frac{15-4x_2}{5(x_2-1)}=-\frac{2}{3}$$

$$45-12x_2=-10x_2+10$$

$$2x_2=35\Rightarrow x_2=\frac{35}{2}$$

$$\Rightarrow A\left(\frac{35}{2},-10\right)$$

$$\because m_{BH}.m_{CA}=-1$$

$$\left(\frac{\frac{3x_1}{2}+3-1}{x_1-1}\right)\left(-\frac{4}{5}\right)=-1$$

$$\frac{(3x_1+4)}{2(x_1-1)}\times 4=5$$

$\Rightarrow$ $6x_1+8=5x_1-5\Rightarrow x_1=-13\Rightarrow\left(-13,\frac{-33}{2}\right)$

$\Rightarrow$ Equation of line AB is

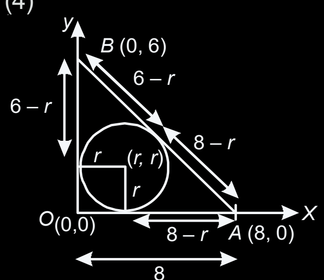
$$y+10=\left(\frac{-\frac{33}{2}+10}{-13-35}\right)\left(x-\frac{35}{2}\right)$$

$$\Rightarrow -61y-610=-13x+\frac{455}{2}$$

$$\Rightarrow -122y-1220=-26x+455$$

$$\Rightarrow 26x-122y-1675=0$$

20. Answer (4)



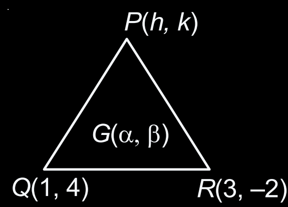
$8-r+6-r=10$

$\Rightarrow$ $2r=4$

$\Rightarrow$ $r=4$

$\therefore$ Incentre = (2, 2)

21. Answer (3)



Let centroid G be (α, β)

we have $3\alpha = 1 + 3 + h \quad \Rightarrow h = 3\alpha - 4$

$3\beta = 4 - 2 + k \;\Rightarrow k = 3\beta - 2$

but $P(h, k)$ lies on $2x - 3y + 4 = 0$

$\Rightarrow$ $2(3\alpha - 4) - 3(3\beta - 2) + 4 = 0$

$\Rightarrow$ $6\alpha - 9\beta - 8 + 6 + 4 = 0$

$\Rightarrow$ $6\alpha - 9\beta + 2 = 0$

Locus: $6x - 9y + 2 = 0$

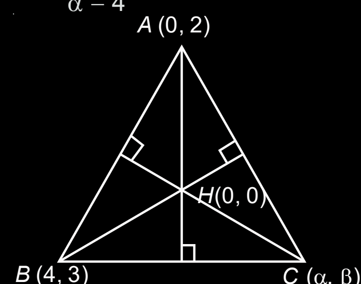
$$\text{Slope }=\frac{6}{9}=\frac{2}{3}$$

22. Answer (4)

$m_{BC}\times m_{AH}=-1$

$$\Rightarrow\ m_{BC}=-\frac{1}{m_{AH}}$$

$$\Rightarrow\ m_{BC}=\frac{\beta-3}{\alpha-4}=0$$



$\Rightarrow$ $\beta = 3$

$m_{AB}\times m_{CH}=-1$

$$\Rightarrow\ \frac{1}{4}\times\frac{\beta}{\alpha}=-1$$

$\Rightarrow$ $\beta=-4\alpha$

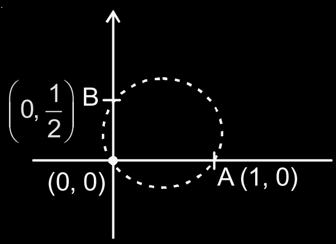
$$\Rightarrow\ \alpha=-\frac{3}{4}$$

Vertex C Is $\left(\frac{-3}{4},3\right)$

$\Rightarrow$ Vertex C lies in second quadrant.

$\Rightarrow$ Option (4) is correct.

23. Answer (2)



Let equation of circle be $x^2 + y^2 + 2gx + 2fy = 0$

As length of intercept on x axis is $1 = 2\sqrt{g^2 - c}$

$\Rightarrow\ |g| = \frac{1}{2}$

length of intercept on y-axis = $\frac{1}{2} = 2\sqrt{f^2 - c}$

$\Rightarrow\ |f| = \frac{1}{4}$

Equation of circle that passes through given points is $x^2 + y^2 - x - \frac{y}{2} = 0$

Tangent at (0, 0) is $\frac{x}{2} + \frac{y}{4} = 0$

$\Rightarrow\ 2x + y = 0$

Sum of perpendicular distance = $\dfrac{\frac{1}{2} + 2}{\sqrt{5}} = \dfrac{\sqrt{5}}{2}$.

24. Answer (1)

$a + b = x$

$ab = y$

$x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$

$\Rightarrow (a + b - c)(a + b + c) = ab$

$\Rightarrow 2(s - c)(2s) = ab$

$\Rightarrow 4s(s - c) = ab$

$\Rightarrow \dfrac{s(s-c)}{ab} = \dfrac{1}{4}$

$\Rightarrow \cos^2 \dfrac{c}{2} = \dfrac{1}{2}$

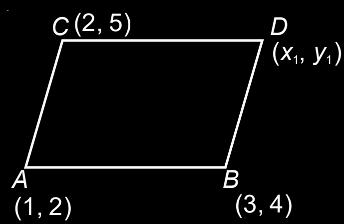
$\Rightarrow \cos c = -\dfrac{1}{2} \Rightarrow c = 120°$

$\Rightarrow \Delta = \dfrac{1}{2} ab (\sin 120°) = \dfrac{\sqrt{3}}{4} ab$

$\Rightarrow R = \dfrac{abc}{\sqrt{3}\, ab} = \dfrac{c}{\sqrt{3}}$

25. Answer (4)

Mid-point of AD = mid-point of BC



$$\left(\frac{x_1 + 1}{2}, \frac{y_1 + 2}{2}\right) = \left(\frac{3+2}{2}, \frac{4+5}{2}\right)$$

$\therefore (x_1, y_1) = (4, 7)$

$\therefore$ Equation of AD : $y - 7 = \dfrac{2-7}{1-4}(x - 4)$

$$y - 7 = \frac{5}{3}(x - 4)$$

$$3y - 21 = 5x - 20$$

$$5x - 3y + 1 = 0$$

26. Answer (3)

Slope of straight line = $\dfrac{-2}{-3} = \dfrac{2}{3}$

Slope of line passing through two points = $\dfrac{\beta - 17}{15 - 7}$

$= \dfrac{\beta - 17}{8}$

$m_1 m_2 = -1$

$\left(\dfrac{2}{2}\right)\left(\dfrac{\beta - 17}{2}\right) = -1$

$\Rightarrow\ \beta = 5$

27. Answer (4)

A point which is equidistant from both the axes lies on either $y = x$ and $y = -x$.

As it is given that the point lies on the line $3x + 5y = 15$

So the required point is :

$$\begin{array}{l} 3x + 5y = 15 \\ \underline{x + y = 0} \\ x = -\frac{15}{2},\quad y = \frac{15}{2} \Rightarrow \left(\frac{15}{2}, \frac{15}{2}\right) \{2^{nd} \text{ quadrant}\} \end{array}$$

$$\begin{array}{l} 3x + 5y = 15 \\ \text{or } \underline{x = y} \\ x = \frac{15}{8},\quad y = \frac{15}{8} \Rightarrow \left(\frac{15}{8}, \frac{15}{8}\right) \{1^{st} \text{ quadrant}\} \end{array}$$

28. Answer (4)

(h, k), $(1, 2)$ and $(-3, 4)$ and collinear

$$\begin{vmatrix} h & k & 1 \\ 1 & 2 & 1 \\ -3 & 4 & 1 \end{vmatrix} = 0 \Rightarrow -2h - 4k + 10 = 0$$

$\Rightarrow$ h + 2k = 5 ...(i)

$$m_{L_1} = \frac{4-2}{-3-1} = \frac{2}{-4} = -\frac{1}{2} \Rightarrow m_{L_2} = 2$$

$$\Rightarrow m_{L_2} = \frac{k-3}{h-4} = 2 \Rightarrow k - 3 = 2h - 8$$

$2h - k = 5$ ...(ii)

from (i) and (ii),

$$h = 3,\ k = 1 \Rightarrow \frac{k}{h} = \frac{1}{3}$$

29. Answer (2)

Point at 4 units from $P(2, 3)$ will be

$A(4\cos\theta + 2, 4\sin\theta + 3)$ will satisfy $x + y = 7$

$\Rightarrow \cos\theta + \sin\theta = \frac{1}{2}$ on squaring

$$\Rightarrow \boxed{\sin 2\theta = \frac{-3}{4}} \Rightarrow \frac{2\tan\theta}{1+\tan^2\theta} = -\frac{3}{4}$$

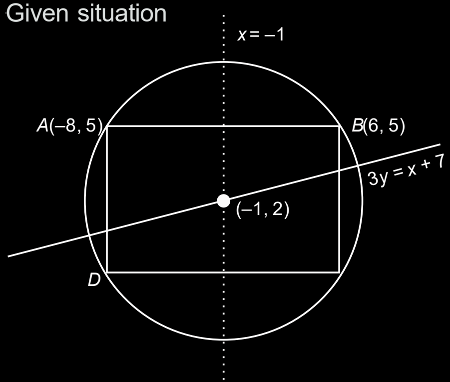
$\Rightarrow 3\tan^2\theta + 8\tan\theta + 3 = 0$

$\Rightarrow \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6}$ (Ignoring –ve sign)

$$\Rightarrow \tan\theta = \frac{-8+2\sqrt{7}}{6} = \frac{1-\sqrt{7}}{1+\sqrt{7}}$$

30. Answer (2)

Given situation



Perpendicular bisector of AB will pass from centre.

$\therefore$ Equation of perpendicular bisector $x = -1$

Hence centre $(-1, 2)$

Let

$$D = (\alpha, \beta) \Rightarrow \frac{\alpha+6}{2} = -1 \ \&\ \frac{\beta+5}{2} = 2$$
$$\alpha = -8 \quad \&\ \beta = -1 \quad D \equiv (-8, -1)$$

$|AD| = 6$ and $|AB| = 14$

Area = 6 × 14 = 84

31. Answer (1)

For perpendicular $m_1 m_2 = -1$

$$\left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$$

$\Rightarrow 2 = a^2(1 - a)$

$\Rightarrow a^3 - a^2 + 2 = 0$

$\Rightarrow (a + 1)(a^2 + 2a + 2) = 0$

$\boxed{a = -1}$

Hence, lines are $x - 2y = 1$ and $2x + y = 1$

$\therefore$ Intersection point $\left(\frac{3}{5}, \frac{-1}{5}\right)$

Distance from origin $= \sqrt{\frac{9}{25} + \frac{1}{25}} = \sqrt{\frac{10}{25}} = \sqrt{\frac{2}{5}}$

32. Answer (3)

Let straight line be $4x - 3y + \alpha = 0$

Given $\frac{3}{5} = \left|\frac{\alpha}{5}\right|$

$\Rightarrow \alpha = \pm 3$

Line is $4x - 3y + 3 = 0$ or $4x - 3y - 3 = 0$

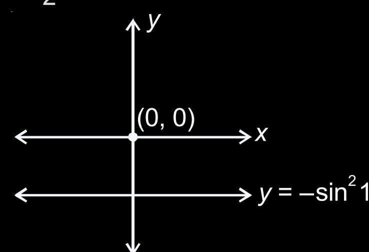
Clearly $\left(-\frac{1}{4}, \frac{2}{3}\right)$ satisfies $4x - 3y + 3 = 0$

33. Answer (1)

$y = \sin x \cdot \sin(x + 2) - \sin^2(x + 1)$

$$= \frac{1}{2}\cos(-2) - \frac{\cos(2x+2)}{2} - \left[\frac{1-\cos(2x+2)}{2}\right]$$

$$= \frac{(\cos 2) - 1}{2} = -\sin^2 1$$



Graph of y lies in

III and IV Quadrant

34. Answer (2)

Equation of angle bisectors;

$$\frac{x+2y+2z-2}{3} = \pm\frac{2x-y+2z-4}{3}$$

$\Rightarrow x - 3y - 2 = 0$ or $3x + y + 4z - 6 = 0$

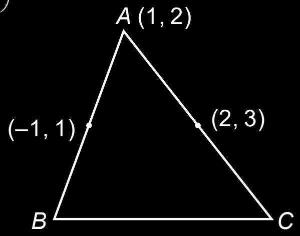
Only $(2, -4, 1)$ lies on the second plane

35. Answer (1)

Co-ordinates of vertex B and C are $B(-3, 0)$ and $C(3, 4)$

Centroid $G\left(\frac{3-3+1}{3}, \frac{0+4+2}{3}\right)$

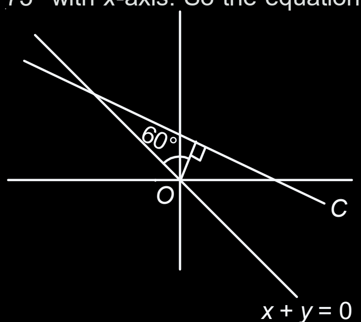
$G\left(\frac{1}{3}, 2\right)$



36. Answer (2)

If perpendicular makes an angle of 60° with the line $x + y = 0$.

Then the perpendicular makes an angle of 15° or 75° with x-axis. So the equation of line will be



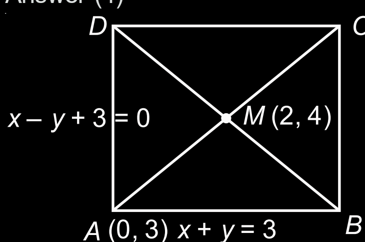
$x\cos75° + y\sin75° = 4$ or $x\cos15° + y\sin15° = 4$

$$\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$$

or $3\left(\sqrt{3}+1\right)x + \left(\sqrt{3}-1\right)y = 8\sqrt{2}$

By rotating the normal towards the line $x + y = 0$ in anticlockwise sense we get the answer (2).

37. Answer (4)



$x - y + 3 = 0$ and $x + y = 3$ are perpendicular lines intersection point of $x - y + 3 = 0$ and $x + y = 3$ is $A(0, 3)$.

$\Rightarrow$ M is mid-point of AC $\qquad \Rightarrow C(4, 5)$

Let $D(x_1, x_1 + 3)$ and $B(x_2, 3 - x_2)$

M is mid-point of DB

$\Rightarrow$ $x_1 + x_2 = 4$, $\qquad x_1 + 3 + 3 - x_2 = 8$

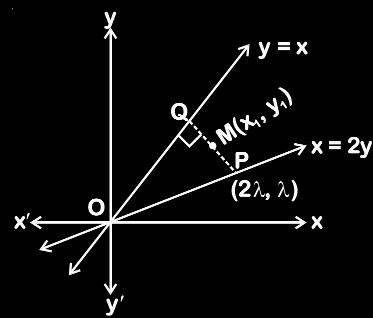
$\Rightarrow$ $x_1 = 3$, $x_2 = 1$

Option (4) is correct.

38. Answer (1)

Let coordinate of P is $(2\lambda, \lambda)$

and coordinate of mid-point M is (x_1, y_1).



$\therefore$ Coordinate of Q

$= (2x_1 - 2\lambda, 2y_1 - \lambda)$

$\because$ Q lies on line $y = x$

$\therefore$ $\lambda = 2x_1 - 2y_1$ ...(i)

(Slope of line PQ) · (Slope of line $y = x$) = –1

$\therefore$ $\frac{\lambda - y_1}{2\lambda - x_1} = -1$

$\therefore$ $\lambda = \frac{x_1 + y_1}{3}$ ...(ii)

From equation (i) and (ii) : $5x_1 = 7y_1$

$\therefore$ Required locus is $5x = 7y$.

39. Answer (1)

Given $3x' + y' = 4\lambda$

and $\left\|\begin{matrix} x' & y' & 1 \\ 1 & -1 & 1 \\ 0 & 2 & 1 \end{matrix}\right\| = 10$

$\Rightarrow$ $|x'(-3) - y'(1) + 1(2)| = 10$

$\Rightarrow$ $|-4\lambda + 2| = 10$

$\Rightarrow$ $2 - 4\lambda = +10$ or $2 - 4\lambda = -10$

$\Rightarrow$ $\lambda = -2$ or $\lambda = 3$

40. Answer (1)

Centroid $D(2, 2)$

Point of intersection of given lines; $P\left(-\frac{1}{5}, \frac{2}{5}\right)$

Equation of line joining D and P;

$$y - 2 = \frac{8}{11}(x - 2)$$

$\Rightarrow$ $11y = 8x + 6$

Only $(-9, -6)$ satisfy this equation

41. Answer (1)

Let $f(x, y) = x + y - 1$

$\because\ f(1, 2).f(\sin\theta, \cos\theta) > 0$

$\Rightarrow\ 2[\sin\theta + \cos\theta - 1] > 0$

$\Rightarrow\ \sin\theta + \cos\theta > 1$

$\Rightarrow\ \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$

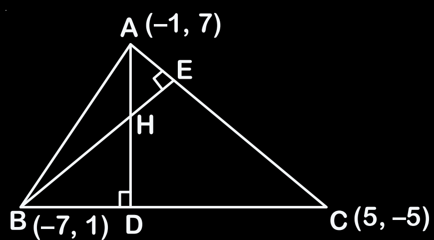
$\Rightarrow\ \theta + \frac{\pi}{4} \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\Rightarrow\ \theta \in \left(0, \frac{\pi}{2}\right)$

42. Answer (1)

Here $m_{BC} = \frac{-6}{12} = \frac{-1}{2}$

So $m_{AD} = 2$



So equation of $AD \Rightarrow y - 7 = 2(x + 1)$

$2x - y + 9 = 0$ ...(i)

Now

$m_{AC} = \frac{12}{-6} = -2 \Rightarrow m_{BE} = \frac{1}{2}$

So equation of $BE \Rightarrow (y - 1) = \frac{1}{2}(x + 7)$

$\Rightarrow\ 2y - 2 = x + 7 \Rightarrow x - 2y + 9 = 0$ ...(ii)

On solving (i) and (ii)

$2x - y + 9 = 0$

$x - 2y + 9 = 0$

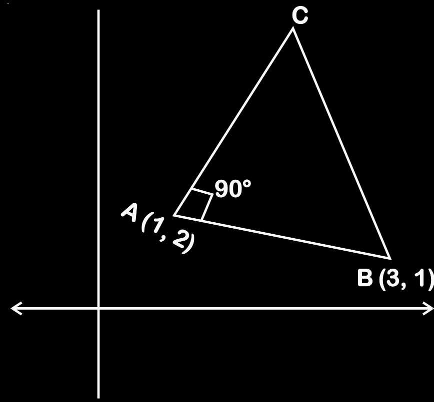
$$\begin{array}{r} 2x - y + 9 = 0 \\ \underset{-}{2x} \underset{+}{-} 4y \underset{-}{+} 18 = 0 \\ \hline 3y - 9 = 0 \end{array}$$

$\Rightarrow$ **y = 2**

$\Rightarrow$ **x = –3**

So $H = (-3, 3)$

43. Answer (2)



Slope of line $AB = -\frac{1}{2}$

Slope of line $AC = 2$

Length of $AB = \sqrt{5}$

$\because\ \frac{1}{2} AB \cdot AC = 5\sqrt{5}$

$\therefore\ AC = 10$

$\therefore$ Coordinate of $C = (1 + 10\cos\theta, 2 + 10\sin\theta)$

Here $\tan\theta = 2$ $\Rightarrow \cos\theta = \frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}$

$\therefore$ Coordinate of $C = \left(1 + 2\sqrt{5}, 2 + 4\sqrt{5}\right)$

$\therefore$ abscissa of vertex C is $1 + 2\sqrt{5}$

44. Answer (3)

Mid point of line segment PQ is $\left(\frac{k+1}{2}, \frac{7}{2}\right)$

Slope of PQ is $\frac{1}{1-k}$

So equation of perpendicular bisector of PQ is

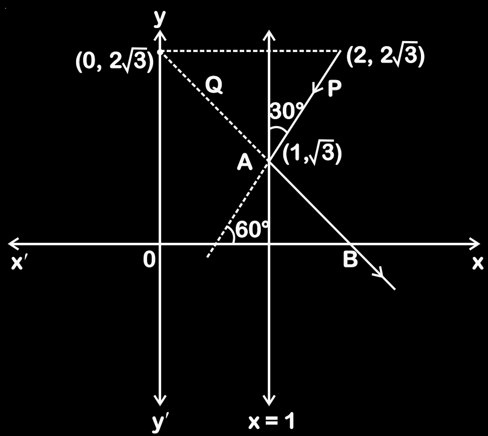
$y - \frac{7}{2} = (k - 1)\left[x - \frac{k+1}{2}\right]$

$\because$ Line passes through (0, –4); then

$-\frac{15}{2} = -\frac{(k^2 - 1)}{2} \quad \Rightarrow \quad k = \pm 4$

45. Answer (2)

Equation of incident line AP is



$$y - 2\sqrt{3} = \sqrt{3}(x-2)$$

$$\sqrt{3}x - y = 0 \qquad ...(1)$$

Image of P w.r.t. line x = 1

is point $Q = \left(0,\ 2\sqrt{3}\right)$.

Equation of reflected Ray AB :

$$y - \sqrt{3} = \frac{2\sqrt{3}-\sqrt{3}}{0-1}(x-1)$$

$$\sqrt{3}x + y = 2\sqrt{3}$$

$\therefore$ Point $\left(3, -\sqrt{3}\right)$ lies on line AB.

46. Answer (4)

Line is $\dfrac{x}{3} + y = 1$

$x + 3y - 3 = 0$

Let image be (α, β)

Hence, $\dfrac{\alpha+1}{1} = \dfrac{\beta+y}{3} = -\dfrac{2(-1-12-3)}{10}$

$$\alpha + 1 = \frac{\beta+4}{3} = \frac{16}{5}$$

$$\Rightarrow \alpha = \frac{11}{5}, \beta = \frac{28}{5}$$

47. Answer (05)

P is centroid of ΔABC

$$\therefore \quad P\left(\frac{1+6+\frac{3}{2}}{3}, \frac{0+6+2}{3}\right) = \left(\frac{17}{6}, \frac{8}{3}\right)$$

$$\Rightarrow \quad PQ = \sqrt{\left(\frac{17}{6}+\frac{7}{6}\right)^2 + \left(\frac{8}{3}+\frac{1}{3}\right)^2} = 5$$

48. Answer (30)

$L_1 : 2x - y + 3 = 0$

$L_1 : 4x - 2y + \alpha = 0$

$L_1 : 6x - 3y + \beta = 0$

Distance between L_1 and L_2;

$$\left|\frac{\alpha-6}{2\sqrt{5}}\right| = \frac{1}{\sqrt{5}} \Rightarrow |\alpha - 6| = 2$$

$\Rightarrow$ α = 4, 8

Distance between L_1 and L_3;

$$\left|\frac{\beta-9}{3\sqrt{5}}\right| = \frac{2}{\sqrt{5}} \Rightarrow |\beta - 9| = 6$$

$\Rightarrow$ β = 15, 3

Sum of all values = 4 + 8 + 15 + 3 = 30

49. Answer (2)

Let line be $\dfrac{x}{a} + \dfrac{y}{b} = 1 \qquad ...(i)$

given $\dfrac{\frac{1}{a}+\frac{1}{b}}{2} = \dfrac{1}{4}$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{1}{2} \qquad ...(ii)$$

By (i) and (ii), we get

$$\frac{x}{a} + \left(\frac{1}{2} - \frac{1}{a}\right)y = 1$$

$$\Rightarrow \lambda(x-y) + \left(\frac{y}{2} - 1\right) = 0$$

$\therefore$ Represents family of line passing through (2, 2)

50. Answer (4)

Given the point (3, 5)

and the line $x - y + 1 = 0$

So, let the image is (x, y)

So, we have

$$\frac{x-3}{1}=\frac{y-5}{-1}=-\frac{2(3-5+1)}{1+1}$$

$\Rightarrow$ x = 4, y = 4

$\Rightarrow$ Point (4, 4)

Which will satisfy the curve

$(x - 2)^2 + (y - 4)^2 = 4$

as $(4 - 2)^2 + (4 - 4)^2$

= 4 + 0 = 4

51. Answer (2)

The given three lines are $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ then point of intersection

lines $x - y = 0$ and $x + 2y = 3$ is (1, 1)

lines $x - y = 0$ and $2x + y = 6$ is (2, 2)

and lines $x + 2y = 3$ and $2x + y = 0$ is (3, 0)

The triangle ABC has vertices A(1, 1), B(2, 2) and C(3, 0)

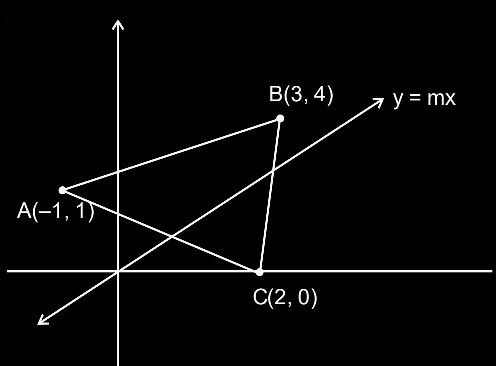
$\therefore$ $AB=\sqrt{2}, BC=\sqrt{5}$ and $AC=\sqrt{5}$

$\therefore$ ΔABC is isosceles

52. Answer (4)

$y = mx$ ...(i)

Equation of AC



$x + 3y = 2$ ...(ii)

(i) and (ii)

$$\Rightarrow P\left(\frac{2}{3m+1},\frac{2m}{3m+1}\right)$$

Equation of BC is

$y = 4x - 8$ ...(iii)

(i) and (iii)

$$\Rightarrow Q\equiv\left(\frac{8}{4-m},\frac{8m}{4-m}\right)$$

$$A_1=\frac{1}{2}\begin{vmatrix}-1 & 1 & 1\\ 2 & 0 & 1\\ 3 & 4 & 1\end{vmatrix}=\frac{13}{2}$$

$$A_2=\frac{1}{3}A_1=\frac{13}{6}$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix}2 & 0 & 1\\ \frac{8}{4-m} & \frac{8m}{4-m} & 1\\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1\end{vmatrix}=\frac{13}{6}$$

[Taking points anticlockwise]

$15m^2 - 11m - 4 = 0$

$m = 1, \frac{-4}{15}$ But$(m>0)$

m = 1

53. Answer (1)

$$\text{Mid point of } PQ\equiv\left(\frac{-2+4}{2},\frac{4-2}{2}\right)\equiv(1,1)$$

$$\text{Slope of } PQ=\frac{4+2}{-2-4}=-1$$

Slope of perpendicular bisector of PQ = 1

Equation of perpendicular bisector of PQ

$y-1=1(x-1)$

$\Rightarrow$ y = x

Solving with perpendicular bisector of PR

Circumcentre is (–2, –2)

54. Answer (144)

$\because$ Origin is circumcentre, then let $A(r\cos\alpha, r\sin\alpha)$ $B(r\cos\beta, r\sin\beta)$ and $C(r\cos\gamma, r\sin\gamma)$

$\because$ Orthocentre lies on y-axis, then

$\cos\alpha + \cos\beta + \cos\gamma = 0$

$\Rightarrow \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cdot\cos\beta\cdot\cos\gamma$

Now, $\cos3\alpha + \cos3\beta + \cos3\gamma$

$= 4(\cos^3\alpha + \cos^3\beta + \cos^3\gamma)$

$$\frac{\cos3\alpha+\cos3\beta+\cos3\gamma}{\cos\alpha\cos\beta\cos\gamma}=12$$

55. Answer (3)

Let slope of line be m

$$\therefore \quad \left|\frac{m-3\sqrt{2}}{1+3\sqrt{2}\ m}\right| = \sqrt{2}$$

$$\Rightarrow \quad m - 3\sqrt{2} = \pm\sqrt{2} \pm 6\ m$$

$$\Rightarrow \quad m \mp 6\ m = \pm\sqrt{2} + 3\sqrt{2}$$

$$\Rightarrow \quad m = -\frac{4\sqrt{2}}{5} \ \text{ or } \ \frac{2\sqrt{2}}{7}$$

Hence line can be

$$y - 3 = \frac{-4\sqrt{2}}{5}(x-1)$$

$$\Rightarrow \quad 5y - 15 = -4\sqrt{2}x + 4\sqrt{2}$$

$$\Rightarrow \quad 4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$$

56. Answer (4)

$$\begin{aligned} 3x + 4y &= 9 \quad \times 1 \\ mx - y &= -1 \quad \times 4 \\ \hline (3+4m)x &= 5 \end{aligned}$$

$$\Rightarrow \quad x = \frac{5}{3+4m}$$

m = –1 and –2 only gives x-coordinate as integer

57. Answer (2)

Pair of bisector for $ax^2 + 2hxy + by^2 = 0$ are

$$\frac{x^2 - y^2}{a-b} = \frac{xy}{h}$$

$\therefore$ for $x^2 - 4xy - 5y^2 = 0$ are

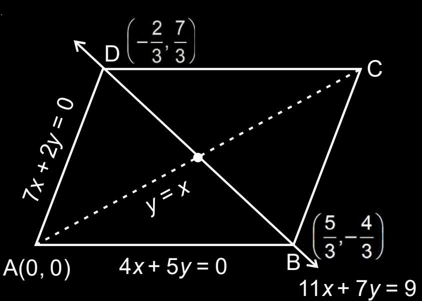
$$\frac{x^2 - y^2}{1+5} = \frac{xy}{-2}$$

$\Rightarrow$ $-x^2 + y^2 - 3xy = 0$

$\Rightarrow$ $x^2 - y^2 + 3xy = 0$

58. Answer (1)

On solving equation $4x + 5y = 0$



and $11x + 7y = 9$ we get

$$B = \left(\frac{5}{3}, -\frac{4}{3}\right)$$

and on solving equation

$7x + 2y = 0$ and $11x + 7y = 9$, we get

$$\text{Coordinate of } D = \left(-\frac{2}{3}, \frac{7}{3}\right)$$

$\therefore$ Mid point of $BD = M = \left(\frac{1}{2}, \frac{1}{2}\right)$

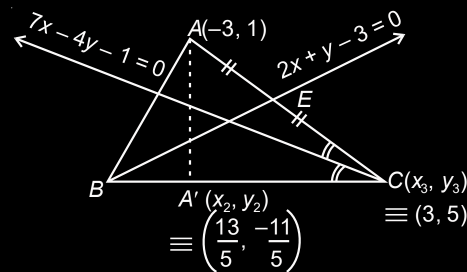
$\therefore$ Equation of other diagonal is $y = x$

$\therefore$ Point (2, 2) lies on other diagonal.

59. Answer (3)

$E\left(\frac{x_3 - 3}{2}, \frac{y_3 + 1}{2}\right)$ lies on $2x + y - 3 = 0$

$$\Rightarrow \quad 2\left(\frac{x_3 - 3}{2}\right) + \frac{y_3 + 1}{2} - 3 = 0$$



$\Rightarrow$ $2x_3 + y_3 - 11 = 0$ ...(i)

$C(x_3, y_3)$ lies on $7x - 4y - 1 = 0$

$\therefore$ $7x_3 - 4y_3 - 1 = 0$ ...(ii)

(i) & (ii) $\Rightarrow$ $C \equiv (x_3, y_3) \equiv (3, 5)$

A' (x_2, y_2) is image of A and is given by

$$\frac{x_2 + 3}{7} = \frac{y_2 - 1}{-4} = -2\left(\frac{7(-3) - 4(1) - 1}{7^2 + (-4)^2}\right) = \frac{4}{5}$$

$$\Rightarrow \quad A' \equiv (x_2, y_2) \equiv \left(\frac{13}{5}, \frac{-11}{5}\right)$$

$$m_{AC} = m_1 = \frac{5-1}{3-(-3)} = \frac{2}{3}$$

$$m_{BC} = m_2 = \frac{5 + \frac{11}{5}}{3 - \frac{13}{5}} = 18$$

$$\tan\theta = \frac{18 - \frac{2}{3}}{1 + 18 \times \frac{2}{3}} = \frac{4}{3}$$

60. Answer (2)

$A(0, 6)$ and $B(2t, 0)$

Let mid point AB be $m = (t, 3)$

and $m_{AB} = \frac{-6}{2t} = \frac{-3}{t}$

$\therefore$ Equation of perpendicualr bisector is

$$y - 3 = \frac{t}{3}(x - t)$$

$\Rightarrow$ $3y - 9 = tx - t^2$

$\therefore$ $C \equiv \left(0, \frac{9 - t^2}{3}\right)$

Let mid point of MC be (h, k)

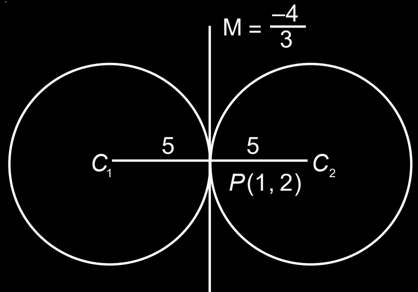
$\therefore$ $\frac{t + 0}{2} = h$ and $\frac{3 + 9 - t^2}{3} = 2k$

$\Rightarrow$ $9 + 9 - (2h)^2 = 6k$

$\Rightarrow$ $18 - 4x^2 = 6y$

$\Rightarrow$ $2x^2 + 3y - 9 = 0$

61. Answer (40)



$M_{C_1C_2} = \frac{3}{4} \Rightarrow \cos\theta = \frac{4}{5}, \sin\theta = \frac{3}{5}$

Point $\frac{x - 1}{\frac{4}{5}} = \frac{y - 2}{\frac{3}{5}} = \pm 5$ (by parametric form of line)

$x - 1 = \pm 4$ or $y - 2 = \pm 3$

$x = 5, y = 5$ or $x = -3, y = -1$

$C_1(5, 5)$ and $C_2(-3, -1)$

$|(\alpha + \beta)(\gamma + \delta)| = |(5 + 5)(-3 - 1)| = 40$

62. Answer (3)

$$p = \left|\frac{k\cot 2\alpha}{\sqrt{\sec^2\alpha + \operatorname{cosec}^2\alpha}}\right| \Rightarrow p^2 = \frac{k^2}{4}\cos^2 2\alpha$$

and $q = \left|\frac{k\sin 2\alpha}{\sqrt{\sin^2\alpha + \cos^2\alpha}}\right| \Rightarrow q^2 = k^2\sin^2 2\alpha$

So, $4p^2 + q^2 = k^2$

63. Answer (1)

$$\frac{1}{2}\left\|\begin{matrix} \alpha & \beta & 1 \\ 5 & 6 & 1 \\ 3 & 2 & 1 \end{matrix}\right\| = 12$$

$|4\alpha - 2\beta - 8| = 24$

$|2\alpha - \beta - 4| = 12$

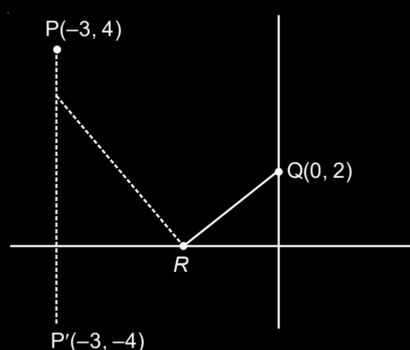
Locus = $2x - y - 4 = 12$, $2x - y - 4 = -12$

$2x - y - 16 = 0$, $2x - y + 8 = 0$

Required length = minimum perpendicular distance from origin

$$= \min\left\{\frac{16}{\sqrt{5}}, \frac{8}{\sqrt{5}}\right\} = \frac{8}{\sqrt{5}}$$

64. Answer (1250)



To minimize distance PR + RQ

Take mirror image of P in $y = 0$

P′ = (–3, –4)

If we join P′Q we will get required R

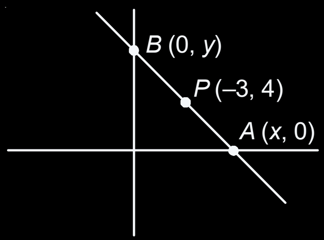
Equation of P′Q $\Rightarrow$ $y = 2x + 2$ So R = (–1, 0)

P = (–3, 4) R(–1, 0) Q(0, 2)

$PR^2 + RQ^2 = 20 + 5 = 25$

65. Answer (2)

P is midpoint of AB



So $x = -3 \times 2$

$x = -6$

and $y + 0 = 2 \times 4$

$y = 8$

Now equation AB is

$$\frac{x}{-6} + \frac{y}{8} = 1$$

$$\Rightarrow \quad 4x - 3y + 24 = 0$$

66. Answer (3)

$\because$ $A(1, \alpha)$, $B(\alpha, 0)$ and $C(0, \alpha)$ are the vertices of ΔABC and area of $\Delta ABC = 4$

$$\therefore \quad \left| \frac{1}{2} \begin{vmatrix} 1 & \alpha & 1 \\ \alpha & 0 & 1 \\ 0 & \alpha & 1 \end{vmatrix} \right| = 4$$

$$\Rightarrow \quad \left| 1(1-\alpha) - \alpha(\alpha) + \alpha^2 \right| = 8$$

$$\Rightarrow \quad \alpha = \pm 8$$

Now, $(\alpha, -\alpha)$, $(-\alpha, \alpha)$ and (α^2, β) are collinear.

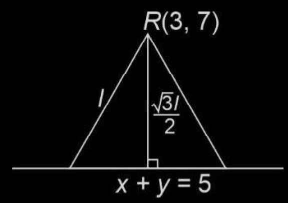
$$\therefore \quad \begin{vmatrix} 8 & -8 & 1 \\ -8 & 8 & 1 \\ 64 & \beta & 1 \end{vmatrix} = 0 = \begin{vmatrix} -8 & 8 & 1 \\ 8 & -8 & 1 \\ 64 & \beta & 1 \end{vmatrix}$$

$$\Rightarrow \quad 8(8-\beta) + 8(-8-64) + 1(-8\beta - 8 \times 64) = 0$$

$$\Rightarrow \quad 8 - \beta - 72 - \beta - 64 = 0$$

$$\Rightarrow \quad \beta = -64$$

67. Answer (4)



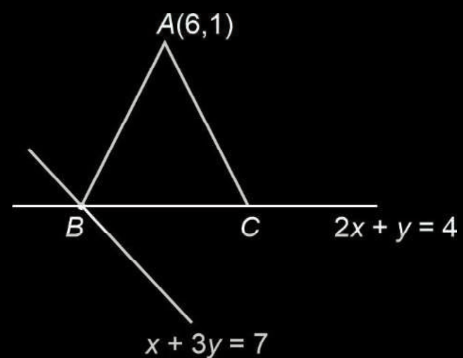
Altitude of equilateral triangle,

$$\frac{\sqrt{3}l}{2} = \frac{5}{\sqrt{2}}$$

$$l = \frac{5\sqrt{2}}{\sqrt{3}}$$

Area of triangle $= \frac{\sqrt{3}}{4} l^2 = \frac{\sqrt{3}}{4} \cdot \frac{50}{3} = \frac{25}{2\sqrt{3}}$

68. Answer (3)



$$\left.\begin{array}{l} 2x + y = 4 \\ 2x + 6y = 14 \end{array}\right\} y = 2,\ x = 3$$

$B(1, 2)$

Let $C(k, 4 - 2k)$

Now $AB^2 = AC^2$

$5^2 + (-1)^2 = (6 - k)^2 + (-3 + 2k)^2$

$\Rightarrow \ 5k^2 - 24k + 19 = 0$

$$(5k - 19)(k - 1) = 0 \Rightarrow k = \frac{19}{5}$$

$$C\left(\frac{19}{5}, -\frac{18}{5}\right)$$

Centroid (α, β)

$$\alpha = \frac{6+1+\frac{19}{5}}{3} = \frac{18}{5}$$

$$\beta = \frac{1+2-\frac{18}{5}}{3} = -\frac{1}{5}$$

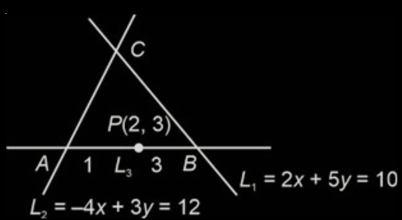
Now, 15(α + β)

$$15\left(\frac{17}{5}\right) = 51$$

69. Answer (2)

$L_1 : 2x + 5y = 10$

$L_2 : -4x + 3y = 12$



Solving L_1 and L_2 we get

$$C \equiv \left(\frac{-15}{13}, \frac{32}{13}\right)$$

Now, let $A\left(x_1, \frac{1}{3}(12+4x_1)\right)$

and $B\left(x_2, \frac{1}{5}(10-2x_2)\right)$

$$\therefore \quad \frac{3x_1+x_2}{4} = 2$$

and $\dfrac{(12+4x_1)+\frac{10-2x_2}{5}}{4} = 3$

So, $3x_1 + x_2 = 8$ and $10x_1 - x_2 = -5$

So, $(x_1, x_2) = \left(\frac{3}{13}, \frac{95}{13}\right)$

$$A = \left(\frac{3}{13}, \frac{56}{13}\right) \text{ and } B = \left(\frac{95}{13}, \frac{-12}{13}\right)$$

$$= \left|\frac{1}{2}\left(\frac{3}{13}\left(\frac{-44}{13}\right)\frac{-56}{13}\left(\frac{110}{13}\right)+1\left(\frac{2860}{169}\right)\right)\right|$$

$$= \frac{132}{13} \text{ sq. units}$$

70. Answer (3)

Let $A(\alpha, 2)$ Given $B(2, 3)$

$$m_{OA} = \frac{2}{\alpha} \;\&\; m_{OB} = \frac{3}{2}$$

$$\tan\frac{\pi}{4} = \left|\frac{\frac{2}{\alpha}-\frac{3}{2}}{1+\frac{2}{\alpha}\cdot\frac{3}{2}}\right| \Rightarrow \frac{4-3\alpha}{2\alpha+6} = \pm 1$$

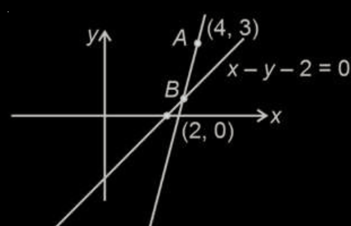
$4 - 3\alpha = 2\alpha + 6$ & $4 - 3\alpha = -2\alpha - 6$

$$\alpha = \frac{-2}{5} \;\&\; \alpha = 10$$

$$A\left(-\frac{2}{5}, 2\right) \;\&\; A'(10,2) \text{ and } B(2, 3)$$

$$AA' = 10 + \frac{2}{5} = \frac{52}{5}$$

71. Answer (3)



Let inclination of required line is θ.

So, the coordinates of point B can be assumed as

$$\left(4 - \frac{\sqrt{29}}{3}\cos\theta, 3 - \frac{\sqrt{29}}{3}\sin\theta\right)$$

which satisfies $x - y - 2 = 0$.

$$4 - \frac{\sqrt{29}}{3}\cos\theta - 3 + \frac{\sqrt{29}}{3}\sin\theta - 2 = 0$$

$$\sin\theta - \cos\theta = \frac{3}{\sqrt{29}}$$

By squaring,

$$\sin 2\theta = \frac{20}{29} = \frac{2\tan\theta}{1+\tan^2\theta}$$

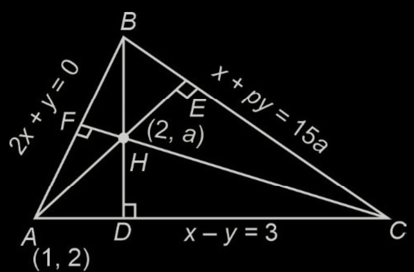
$$\tan\theta = \frac{5}{2} \text{ only(because slope is greater than 1)}$$

$$\sin\theta = \frac{5}{\sqrt{29}},\ \cos\theta = \frac{2}{\sqrt{29}}$$

$$\text{Point } B : \left(\frac{10}{3}, \frac{4}{3}\right)$$

which also satisfies $x + 2y = 6$.

72. Answer (3)



$$\text{Slope of } AH = \frac{a+2}{1}$$

$$\text{Slope of } BC = -\frac{1}{p}$$

$\therefore \quad p = a + 2 \qquad$ …(i)

$$\text{Coordinate of } C = \left(\frac{18p-30}{p+1}, \frac{15p-33}{p+1}\right)$$

Slope of HC

$$= \frac{\frac{15p-33}{p+1} - a}{\frac{18p-30}{p+1} - 2}$$

$$= \frac{15p - 33 - (p-2)(p+1)}{18p - 30 - 2p - 2}$$

$$= \frac{16p - p^2 - 31}{16p - 32}$$

$$\because \quad \frac{16p - p^2 - 31}{16p - 32} \times -2 = -1$$

$\therefore \quad p^2 - 8p + 15 = 0$

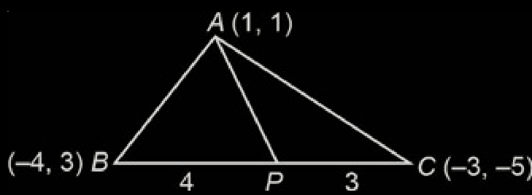
$\therefore \quad p = 3$ or 5

But if $p = 5$ then $a = 3$ not acceptable

$\therefore \quad p = 3$

73. Answer (3)

$$\frac{\Delta_1}{\Delta_2} = \frac{\frac{1}{2} \times BP \times AH}{\frac{1}{2} \times BC \times AH} = \frac{4}{7}$$



$$P\left(\frac{-20}{7}, \frac{-11}{7}\right)$$

Line AC : $y - 1 = 2(x - 1)$

$$\text{Intersection with } x\text{-axis} = \left(\frac{1}{2}, 0\right)$$

$$\text{Line } AP : y - 1 = \frac{2}{3}(x-1)$$

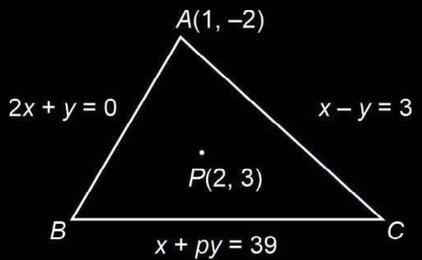
$$\text{Intersection with } x\text{-axis} = \left(\frac{-1}{2}, 0\right)$$

$$\text{Vertices are } (1, 1), \left(\frac{1}{2}, 0\right) \text{ and } \left(\frac{-1}{2}, 0\right)$$

$$\text{Area} = \frac{1}{2} \text{ sq. unit}$$

74. Answer (4)

Intersection of $2x + y = 0$ and $x - y = 3 : A(1, -2)$



Equation of perpendicular bisector of AB is

$x - 2y = -4$

Equation of perpendicular bisector of AC is

$x + y = 5$

Point B is the image of A in line $x - 2y + 4 = 0$, which can be obtained as $B\left(\frac{-13}{5}, \frac{26}{5}\right)$.

Similarly vertex $C : (7, 4)$.

Equation of line $BC : x + 8y = 39$

So, $p = 8$

$$AC = \sqrt{(7-1)^2 + (4+2)^2} = 6\sqrt{2}$$

Area of triangle $ABC = 32.4$

75. Answer (2)

One vertex of square is

$(10\,(\cos\alpha - \sin\alpha), 10(\sin\alpha + \cos\alpha))$

and one of the diagonal is

$(\cos\alpha - \sin\alpha)\,x + (\sin\alpha + \cos\alpha)y = 10$

So, the other diagonal can be obtained as

$(\cos\alpha + \sin\alpha)x - (\cos\alpha - \sin\alpha)y = 0$

So, point of intersection of diagonal will be

$(5(\cos\alpha - \sin\alpha), 5(\cos\alpha + \sin\alpha))$.

Therefore, the vertex opposite to the given vertex is $(0, 0)$.

So, the diagonal length $= 10\sqrt{2}$

Side length $(a) = 10$

It is given that

$$a^2 + 11a + 3\left(m_1^2 + m_2^2\right) = 220$$

$$m_1^2 + m_2^2 = \frac{220 - 100 - 110}{3} = \frac{10}{3}$$

and $m_1 m_2 = -1$

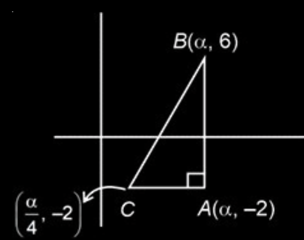
Slopes of the sides are $\tan\alpha$ and $-\cot\alpha$

$$\tan^2\alpha = 3 \text{ or } \frac{1}{3}$$

$$72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$$

$$= 72 \cdot \frac{\tan^4\alpha + 1}{\left(1 + \tan^2\alpha\right)^2} + a^2 - 3a + 13 = 128$$

76. Answer (2)



Circumcentre of ΔABC

$$= \left(\frac{\alpha + \frac{\alpha}{4}}{2}, \frac{6-2}{2}\right)$$

$$= \left(\frac{5\alpha}{8}, 2\right)$$

$$= \left(5, \frac{\alpha}{4}\right)$$

$\Rightarrow \alpha = 8$

$$\text{area}(\Delta ABC) = \frac{1}{2} \cdot \frac{3\alpha}{4} \times 8 = 24 \text{ sq. units}$$

$$\text{Perimeter} = 8 + \frac{3\alpha}{4} + \sqrt{8^2 + \left(\frac{3\alpha}{4}\right)^2}$$

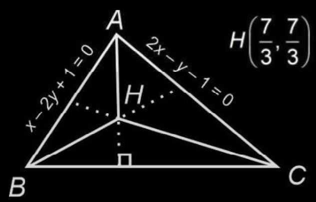
$$= 8 + 6 + 10 = 24$$

$$\text{Circumradius} = \frac{10}{2} = 5$$

$$r = \frac{\Delta}{s} = \frac{24}{12} = 2$$

77. Answer (3)

$$\left.\begin{array}{l} AB: x-2y+1=0 \\ AC: 2x-y-1=0 \end{array}\right\} A(1,1)$$

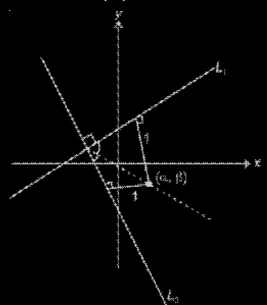


Altitude from B is $BH = x+2y-7=0 \Rightarrow B(3, 2)$

Altitude from C is $CH = 2x+y-7=0 \Rightarrow C(2, 3)$

Centroid of $\Delta ABC = G(2, 2)$ $OG = 2\sqrt{2}$

78. Answer (4)



$L_1 : 3x - 4y + 12 = 0$

$L_2 : 8x + 6y + 11 = 0$

Equation of angle bisector of L_1 and L_2 of angle containing origin

$2(3x - 4y + 12) = 8x + 6y + 11$

$2x + 14y - 13 = 0$ ...(i)

$$\frac{3\alpha-4\beta+12}{5}=1$$

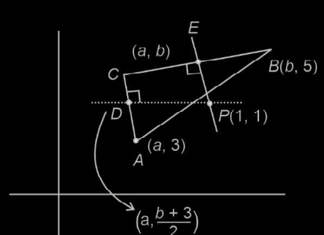
$\Rightarrow 3\alpha-4\beta+7=0$ ...(ii)

Solution of $2x + 14y - 13 = 0$ and $3x - 4y + 7 = 0$

gives the required point $P(\alpha, \beta)$, $\alpha = \frac{-23}{25}$, $\beta = \frac{53}{50}$

$100(\alpha+\beta)=14$

79. Answer (2)



Let D be mid-point of AC, then

$$\frac{b+3}{2}=1 \Rightarrow b=-1$$

Let E be mid-point of BC,

$$\frac{5-b}{b-a}\cdot\frac{\frac{(3+b)}{2}}{\frac{a+b}{2}-1}=-1$$

On Putting $b = -1$, we get $a = 5$ or -3

But $a = 5$ is rejected as $ab > 0$

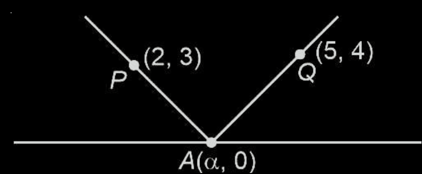
$A(-3, 3)$, $B(-1, 5)$, $C(-3, -1)$, $P(1, 1)$

Line $BC \Rightarrow y = 3x + 8$

Line $AP \Rightarrow y = \frac{3-x}{2}$

Point of intersection $\left(\frac{-13}{7}, \frac{17}{7}\right)$

80. Answer (31)



$$\frac{4}{5-\alpha}=\frac{3}{\alpha-2} \Rightarrow 4\alpha-8=15-3\alpha$$

$$\alpha=\frac{23}{7}$$

$$A=\left(\frac{23}{7},0\right) \quad Q=(5,4)$$

$$R=\left(\frac{10+\frac{23}{7}}{3},\frac{8}{3}\right)$$

$$=\left(\frac{31}{7},\frac{8}{3}\right)$$

Bisector of angle PAQ is $X = \frac{23}{7}$

$$\Rightarrow M=\left(\frac{23}{7},\frac{8}{3}\right)$$

So, $7\alpha + 3\beta = 31$

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