Chapter 3

Binomial Theorem and Principle of Mathematical Induction

1. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is **[AIEEE-2009]**

(1) 2 (2) 7

(3) 8 (4) 0

2. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j,\ S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and

$S_3 = \sum_{j=1}^{10} j^2\ {}^{10}C_j$

Statement-1 : $S_3 = 55 \times 2^9$

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$ **[AIEEE-2010]**

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

3. **Statement-1:** For each natural number *n*, $(n + 1)^7 - n^7 - 1$ is divisible by 7.

Statement-2: For each natural number *n*, $n^7 - n$ is divisible by 7. **[AIEEE-2011]**

(1) Statement-1 is true, statement-2 is false.

(2) Statement-1 is false, statement-2, is true.

(3) Statement-1 is true, statement-2 is true, statement-2 is a correct explanation for statement-1

(4) Statement-1 is true, statement-2 is true; statement-2 is ***not*** a correct explanation for statement-1

4. If *n* is a positive integer, then $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$ is **[AIEEE-2012]**

(1) An odd positive integer

(2) An even positive integer

(3) A rational number other than positive integer

(4) An irrational number

5. Let *A* and *B* be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is **[JEE (Main)-2013]**

(1) 256 (2) 220

(3) 219 (4) 211

6. The term independent of *x* in expansion of

$$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$$ is **[JEE (Main)-2013]**

(1) 4 (2) 120

(3) 210 (4) 310

7. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, where *N* is the set of natural numbers, then $X \cup Y$ is equal to **[JEE (Main)-2014]**

(1) X (2) Y

(3) N (4) $Y - X$

8. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of *x* are both zero, then (a, b) is equal to **[JEE (Main)-2014]**

(1) $\left(14, \frac{272}{3}\right)$ (2) $\left(16, \frac{272}{3}\right)$

(3) $\left(16, \frac{251}{3}\right)$ (4) $\left(14, \frac{251}{3}\right)$

9. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is

[JEE (Main)-2015]

(1) $\frac{1}{2}(3^{50}+1)$ (2) $\frac{1}{2}(3^{50})$

(3) $\frac{1}{2}(3^{50}-1)$ (4) $\frac{1}{2}(2^{50}+1)$

10. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is

[JEE (Main)-2016]

(1) 2187 (2) 243

(3) 729 (4) 64

11. The value of

$({}^{21}C_1 - {}^{10}C_1)+({}^{21}C_2 - {}^{10}C_2)+({}^{21}C_3 - {}^{10}C_3)+$

$({}^{21}C_4 - {}^{10}C_4)+...+({}^{21}C_{10} - {}^{10}C_{10})$ is

[JEE (Main)-2017]

(1) $2^{21} - 2^{10}$ (2) $2^{20} - 2^{9}$

(3) $2^{20} - 2^{10}$ (4) $2^{21} - 2^{11}$

12. The sum of the co-efficients of all odd degree terms in the expansion of

$\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is

[JEE (Main)-2018]

(1) –1 (2) 0

(3) 1 (4) 2

13. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to

[JEE (Main)-2019]

(1) 8 (2) 4

(3) 6 (4) 14

14. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is

[JEE (Main)-2019]

(1) 15 (2) 14

(3) 12 (4) 10

15. If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$, then k equals

[JEE (Main)-2019]

(1) 400 (2) 100

(3) 200 (4) 50

16. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is

[JEE (Main)-2019]

(1) 1465 (2) 1356

(3) 1365 (4) 1256

17. If the third term in the binomial expansion of $(1+x^{\log_2 x})^5$ equals 2560, then a possible value of x is

[JEE (Main)-2019]

(1) $2\sqrt{2}$ (2) $\frac{1}{4}$

(3) $4\sqrt{2}$ (4) $\frac{1}{8}$

18. Consider the statement : "$P(n)$: $n^2 - n + 41$" is prime." Then which one of the following is true?

[JEE (Main)-2019]

(1) $P(5)$ is false but $P(3)$ is true

(2) $P(3)$ is false but $P(5)$ is true

(3) Both $P(3)$ and $P(5)$ are false

(4) Both $P(3)$ and $P(5)$ are true

19. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is

[JEE (Main)-2019]

(1) 3 (2) 4

(3) $\sqrt{5}$ (4) $2\sqrt{2}$

20. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is

[JEE (Main)-2019]

(1) 4 (2) 8

(3) 0 (4) 6

21. Let $(x + 10)^{50} + (x - 10)^{50}$

$= a_0 + a_1x + a_2x^2 + ... + a_{50}x^{50}$, for all

$x \in R$; then $\frac{a_2}{a_0}$ is equal to

[JEE (Main)-2019]

(1) 12.25 (2) 12.75

(3) 12.00 (4) 12.50

22. A ratio of the 5th term from the beginning to the 5th term from the end in the binomial expansion of $\left(2^{\frac{1}{3}}+\frac{1}{2(3)^{\frac{1}{3}}}\right)^{10}$ is **[JEE (Main)-2019]**

(1) $1:4(16)^{\frac{1}{3}}$ (2) $1:2(6)^{\frac{1}{3}}$

(3) $2(36)^{\frac{1}{3}}:1$ (4) $4(36)^{\frac{1}{3}}:1$

23. The total number of irrational terms in the binomial expansion of $\left(7^{1/5}-3^{1/10}\right)^{60}$ is **[JEE (Main)-2019]**

(1) 48 (2) 49

(3) 54 (4) 55

24. The sum of the co-efficients of all even degree terms in x in the expansion of $(x+\sqrt{x^3-1})^6+(x-\sqrt{x^3-1})^6, (x>1)$ is equal to : **[JEE (Main)-2019]**

(1) 24 (2) 32

(3) 26 (4) 29

25. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}}+x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x > 1$, then the value of x is : **[JEE (Main)-2019]**

(1) 10 (2) 10^3

(3) 100 (4) 10^4

26. If the fourth term in the Binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ $(x > 0)$ is 20×8^7, then a value of x is **[JEE (Main)-2019]**

(1) 8^3 (2) 8

(3) 8^{-2} (4) 8^2

27. If some three consecutive coefficients in the binomial expansion of $(x + 1)^n$ in powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficients is **[JEE (Main)-2019]**

(1) 625 (2) 964

(3) 232 (4) 227

28. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to : **[JEE (Main)-2019]**

(1) (– 54, 315) (2) (28, 861)

(3) (–21, 714) (4) 28, 315

29. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is **[JEE (Main)-2019]**

(1) 58 (2) 35

(3) 38 (4) 23

30. The coefficient of x^{18} in the product $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$ is **[JEE (Main)-2019]**

(1) 84 (2) –126

(3) –84 (4) 126

31. If ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + ... + (20^2)\,{}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to **[JEE (Main)-2019]**

(1) (420, 19) (2) (380, 19)

(3) (420, 18) (4) (380, 18)

32. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to **[JEE (Main)-2019]**

(1) –108 (2) –36

(3) –72 (4) 36

33. The number of ordered pairs (r, k) for which $6\cdot{}^{35}C_r = (k^2 - 3)\cdot{}^{36}C_{r+1}$, where k is an integer, is **[JEE (Main)-2020]**

(1) 3 (2) 6

(3) 2 (4) 4

34. The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots + x^{10}$ is **[JEE (Main)-2020]**

(1) 120 (2) 330

(3) 420 (4) 210

35. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$, then **[JEE (Main)-2020]**

(1) $\alpha - \beta = 60$ (2) $\alpha + \beta = 60$

(3) $\alpha - \beta = -132$ (4) $\alpha + \beta = -30$

36. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $l_2 : l_1$ is equal to **[JEE (Main)-2020]**

(1) 1 : 16 (2) 16 : 1

(3) 1 : 8 (4) 8 : 1

37. Let $\alpha > 0$, $\beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$ is $10k$, then k is equal to **[JEE (Main)-2020]**

(1) 84 (2) 176

(3) 336 (4) 352

38. If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}}+5^{\frac{1}{8}}\right)^{n}$ is exactly 33, then the least value of n is **[JEE (Main)-2020]**

(1) 264 (2) 128

(3) 256 (4) 248

39. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is k, then $18k$ is equal to **[JEE (Main)-2020]**

(1) 9 (2) 11

(3) 5 (4) 7

40. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to **[JEE (Main)-2020]**

(1) ${}^{51}C_7 - {}^{30}C_7$ (2) ${}^{50}C_7 - {}^{30}C_7$

(3) ${}^{51}C_7 + {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

41. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is **[JEE (Main)-2020]**

(1) 252 (2) 462

(3) 792 (4) 330

42. If $\{p\}$ denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to **[JEE (Main)-2020]**

(1) $\frac{5}{8}$ (2) $\frac{1}{8}$

(3) $\frac{7}{8}$ (4) $\frac{3}{8}$

43. If the constant term in the binomial expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals **[JEE (Main)-2020]**

(1) 2 (2) 3

(3) 9 (4) 1

44. If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then **[JEE (Main)-2020]**

(1) $\frac{a}{10}=\frac{b}{11}=\frac{c}{42}$ (2) $\frac{a}{11}=\frac{b}{22}=\frac{c}{42}$

(3) $\frac{a}{10}=\frac{b}{11}=\frac{c}{21}$ (4) $\frac{a}{11}=\frac{b}{22}=\frac{c}{21}$

45. If the sum of the coefficients of all even powers of x in the product $\left(1+x+x^2+...+x^{2n}\right)\left(1-x+x^2-x^3+...+x^{2n}\right)$ is 61, then n is equal to ________. **[JEE (Main)-2020]**

46. The coefficient of x^4 in the expansion of $(1 + x + x^2)^{10}$ is ________. **[JEE (Main)-2020]**

47. If $C_r \equiv {}^{25}C_r$ and $C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.\,k$, then k is equal to ________. **[JEE (Main)-2020]**

48. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to ____. **[JEE (Main)-2020]**

49. Let $\left(2x^2+3x+4\right)^{10}=\sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to ______. **[JEE (Main)-2020]**

50. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m+\frac{1}{x^2}\right)^{22}$ is 1540, is ________. **[JEE (Main)-2020]**

51. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is ______.

[JEE (Main)-2020]

52. The value of

$$-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... - 15.{}^{15}C_{15}$$
$$+{}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ... + {}^{14}C_{11} \text{ is}$$

[JEE (Main)-2021]

(1) 2^{14}
(2) $2^{16} - 1$
(3) $2^{13} - 13$
(4) $2^{13} - 14$

53. If $n \geq 2$ is a positive integer, then the sum of the series

$${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2\right) \text{ is :}$$

[JEE (Main)-2021]

(1) $\dfrac{n(2n+1)(3n+1)}{6}$
(2) $\dfrac{n(n-1)(2n+1)}{6}$
(3) $\dfrac{n(n+1)^2(n+2)}{12}$
(4) $\dfrac{n(n+1)(2n+1)}{6}$

54. For integers n and r, let

$$\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

The maximum value of k for which the sum

$$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+i-i}$$ exists, is equal to _______.

[JEE (Main)-2021]

55. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is __________. **[JEE (Main)-2021]**

56. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is ________.

[JEE (Main)-2021]

57. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}} + \dfrac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x \in (0,1)$ is : **[JEE (Main)-2021]**

(1) $\dfrac{2.10!}{3(5!)^2}$
(2) $\dfrac{2.10!}{3\sqrt{3}(5!)^2}$
(3) $\dfrac{10!}{\sqrt{3}(5!)^2}$
(4) $\dfrac{10!}{2(5!)^2}$

58. Let $m, n \in N$ and gcd (2, n) = 1, If

$$30\binom{30}{0} + 29\binom{30}{1} + ... + 2\binom{30}{28} + 1\binom{30}{29} = n \cdot 2^m,$$

then n + m is equal to ____________.

$$\left(\text{Here}\binom{n}{k} = {}^nC_k\right)$$

[JEE (Main)-2021]

59. If n is the number of irrational terms in the expansion of $\left(3^{\frac{1}{4}} + 5^{\frac{1}{8}}\right)^{60}$, then (n – 1) is divisible by :

[JEE (Main)-2021]

(1) 7
(2) 26
(3) 8
(4) 30

60. Let [x] denote greatest integer less than or equal to x. If for $n \in N$,

$$\left(1 - x + x^3\right)^n = \sum_{j=0}^{3n} a_j x^j, \text{ then}$$

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} \text{ is equal to :}$$

[JEE (Main)-2021]

(1) 1
(2) 2^{n-1}
(3) n
(4) 2

61. Let n be a positive integer. Let

$$A = \sum_{k=0}^{n}(-1)^k\, {}^nC_k\left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$$

If $63A = 1 - \dfrac{1}{2^{30}}$, then n is equal to _______.

[JEE (Main)-2021]

62. If the fourth term in the expansion of $(x + x^{\log_2 x})^7$ is 4480, then the value of x where $x \in N$ is

[JEE (Main)-2021]

(1) 2
(2) 1
(3) 3
(4) 4

63. The value of $\sum_{r=0}^{6}\left({}^6C_r \cdot {}^6C_{6-r}\right)$ is equal to :

[JEE (Main)-2021]

(1) 924
(2) 1124
(3) 1024
(4) 1324

64. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x+\frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to ________. **[JEE (Main)-2021]**

65. Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$. Then, $a_1 + a_3 + a_5 + ... + a_{37}$ is equal to **[JEE (Main)-2021]**

(1) $2^{19}(2^{20} - 21)$ (2) $2^{19}(2^{20} + 21)$

(3) $2^{20}(2^{20} - 21)$ (4) $2^{20}(2^{20} + 21)$

66. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$.

If $\sum\limits_{k=0}^{10}(2^2+3k)\,{}^{10}C_k = \alpha 3^{10} + \beta.2^{10}$, $\alpha, \beta \in R$, then $\alpha + \beta$ equal to ________. **[JEE (Main)-2021]**

67. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to ______. **[JEE (Main)-2021]**

68. The coefficient of x^{256} in the expansion of $(1 - x)^{101} (x^2 + x + 1)^{100}$ is : **[JEE (Main)-2021]**

(1) ${}^{100}C_{15}$ (2) ${}^{100}C_{16}$

(3) $-{}^{100}C_{16}$ (4) $-{}^{100}C_{15}$

69. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}}+5^{\frac{1}{6}}\right)^{120}$ is ________. **[JEE (Main)-2021]**

70. For the natural numbers m, n, if $(1 - y)^m (1 + y)^n = 1 + a_1y + a_2y^2 + ... + a_{m+n}y^{m+n}$ and $a_1 = a_2 = 10$, then the value of (m + n) is equal to **[JEE (Main)-2021]**

(1) 64 (2) 80

(3) 88 (4) 100

71. The number of elements in the set $\{n \in \{1, 2, 3, ..., 100\} \mid (11)^n > (10)^n + (9)^n\}$ is ________. **[JEE (Main)-2021]**

72. If the constant term, in binomial expansion of $\left(2x^r+\frac{1}{x^2}\right)^{10}$ is 180, then r is equal to ________. **[JEE (Main)-2021]**

73. If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity

$$\frac{1}{a-b}+\frac{1}{a-2b}+\frac{1}{a-3b}+....+\frac{1}{a-nb}=\alpha n+\beta n^2+\gamma n^3$$

then the value of γ is **[JEE (Main)-2021]**

(1) $\frac{a+b^2}{3a^3}$ (2) $\frac{a+b}{3a^2}$

(3) $\frac{a^2+b}{3a^3}$ (4) $\frac{b^2}{3a^3}$

74. The ratio of the coefficient of the middle term in the expansion of $(1 + x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1 + x)^{19}$ is ________. **[JEE (Main)-2021]**

75. The term independent of '*x*' in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x \neq 0, 1$ is equal to ________. **[JEE (Main)-2021]**

76. If the greatest value of the term independent of '*x*' in the expansion of $\left(x\sin\alpha+a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of 'a' is equal to **[JEE (Main)-2021]**

(1) –1 (2) –2

(3) 2 (4) 1

77. The lowest integer which is greater than $\left(1+\frac{1}{10^{100}}\right)^{10^{100}}$ is ________. **[JEE (Main)-2021]**

(1) 1 (2) 4

(3) 3 (4) 2

78. The sum of all those terms which are rational numbers in the expansion of $\left(2^{1/3}+2^{1/4}\right)^{12}$ is **[JEE (Main)-2021]**

(1) 43 (2) 27

(3) 35 (4) 89

79. If the co-efficients of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^n$ are equal, then the value of n is equal to ______. **[JEE (Main)-2021]**

80. If the coefficients of x^7 in $\left(x^2+\dfrac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x-\dfrac{1}{bx^2}\right)^{11}$, $b \neq 0$, are equal, then the value of b is equal to **[JEE (Main)-2021]**

(1) 1 (2) –2

(3) –1 (4) 2

81. A possible value of 'x', for which the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}$ is equal to 180, is **[JEE (Main)-2021]**

(1) –1 (2) 0

(3) 1 (4) 2

82. If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of $(1 + x)^{20}$, then the value of $\sum_{r=0}^{20} r^2\ {}^{20}C_r$ is equal to : **[JEE (Main)-2021]**

(1) 420×2^{19} (2) 420×2^{18}

(3) 380×2^{19} (4) 380×2^{18}

83. Let $\binom{n}{k}$ denote ${}^nC_k$ and

$$\begin{bmatrix} n \\ k \end{bmatrix} = \begin{cases} \binom{n}{k}, & \text{if } 0 \le k \le n \\ 0, & \text{otherwise.} \end{cases}$$

If $A_k = \sum_{i=0}^{9}\binom{9}{i}\begin{bmatrix} 12 \\ 12-k+i \end{bmatrix} + \sum_{i=0}^{8}\binom{8}{i}\begin{bmatrix} 13 \\ 13-k+i \end{bmatrix}$ and $A_4 - A_3 = 190$ p, then p is equal to ________. **[JEE (Main)-2021]**

84. $\sum_{k=0}^{20}\left({}^{20}C_K\right)^2$ is equal to **[JEE (Main)-2021]**

(1) ${}^{41}C_{20}$ (2) ${}^{40}C_{19}$

(3) ${}^{40}C_{21}$ (4) ${}^{40}C_{20}$

85. If $\left(\dfrac{3^6}{4^4}\right)k$ is the term, independent of x, in the binomial expansion of $\left(\dfrac{x}{4}-\dfrac{12}{x^2}\right)^{12}$, then k is equal to ______. **[JEE (Main)-2021]**

86. If the coefficient of a^7b^8 in the expansion of $(a + 2b + 4ab)^{10}$ is $K \cdot 2^{16}$, then K is equal to **[JEE (Main)-2021]**

87. If the sum of the coefficients in the expansion of $(x + y)^n$ is 4096, then the greatest coefficient in the expansion is_____. **[JEE (Main)-2021]**

88. If $(2021)^{3762}$ is divided by 17, then the remainder is _____. **[JEE (Main)-2021]**

89. $3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder ______. **[JEE (Main)-2021]**

90. Let $n \in N$ and $[x]$ denote the greatest integer less than or equal to x. If the sum of $(n + 1)$ terms ${}^nC_0$, $3 \cdot {}^nC_1$, $5 \cdot {}^nC_2$, $7 \cdot {}^nC_3$, ... is equal to $2^{100} \cdot 101$, then $2\left[\dfrac{n-1}{2}\right]$ is equal to _________. **[JEE (Main)-2021]**

91. The remainder on dividing $1 + 3 + 3^2 + 3^3 + \ldots + 3^{2021}$ by 50 _____ is **[JEE (Main)-2022]**

92. Let C_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^{10}$. If for $\alpha, \beta \in \mathbf{R}$, $C_1 + 3\times2\ C_2 + 5\times3\ C_3 + \ldots$ upto 10 terms

$$= \frac{\alpha \times 2^{11}}{2^{\beta}-1}\left(C_0+\frac{C_1}{2}+\frac{C_2}{3}+\ldots \text{ upto 10 terms}\right)$$ then the value of $\alpha + \beta$ is equal to _______. **[JEE (Main)-2022]**

93. The coefficient of x^{101} in the expression $(5 + x)^{500} + x(5 + x)^{499} + x^2(5 + x)^{498} + \ldots\ldots + x^{500}$, $x > 0$, is **[JEE (Main)-2022]**

(1) ${}^{501}C_{101}(5)^{399}$ (2) ${}^{501}C_{101}(5)^{400}$

(3) ${}^{501}C_{100}(5)^{400}$ (4) ${}^{500}C_{101}(5)^{399}$

94. If the sum of the co-efficients of all the positive even powers of x in the binomial expansion of $\left(2x^3+\dfrac{3}{x}\right)^{10}$ is $5^{10} - \beta \cdot 3^9$, the β is equal to ____. **[JEE (Main)-2022]**

95. The remainder when $(2021)^{2023}$ is divided by 7 is :

(1) 1 (2) 2

(3) 5 (4) 6

[JEE (Main)-2022]

96. If $\left({}^{40}C_0\right)+\left({}^{41}C_1\right)+\left({}^{42}C_2\right)+...+\left({}^{60}C_{20}\right)=\frac{m}{n}\,{}^{60}C_{20}$ m and n are coprime, then $m + n$ is equal to ______.

[JEE (Main)-2022]

97. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}}+\frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k \cdot l$, where $l, k \in \mathbf{N}$ and l is co-prime to 5, then k is equal to ___________.

[JEE (Main)-2022]

98. If the sum of the coefficients of all the positive powers of x, in the Binomial expansion of $\left(x^n+\frac{2}{x^5}\right)^7$ is 939, then the sum of all the possible integral values of n is ____________.

[JEE (Main)-2022]

99. If $\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right)-\sum_{k=1}^{30}\left({}^{30}C_k\right)\left({}^{30}C_{k-1}\right)=\frac{\alpha(60!)}{(30!)(31!)}$, where $\alpha \in \mathbf{R}$, then the value of 16α is equal to

(1) 1411 (2) 1320

(3) 1615 (4) 1855

[JEE (Main)-2022]

100. The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3+\frac{3}{x^k}\right)^{12}, x \neq 0$ is $2^8 \cdot \ell$, where ℓ is an odd integer, is______.

[JEE (Main)-2022]

101. The term independent of x in the expansion of $\left(1-x^2+3x^3\right)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}, x \neq 0$ is:

(1) $\frac{7}{40}$ (2) $\frac{33}{200}$

(3) $\frac{39}{200}$ (4) $\frac{11}{50}$

[JEE (Main)-2022]

102. If the constant term in the expansion of $\left(3x^3-2x^2+\frac{5}{x^5}\right)^{10}$ is $2^k \cdot l$, where l is an odd integer, then the value of k is equal to

(1) 6 (2) 7

(3) 8 (4) 9

[JEE (Main)-2022]

103. Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\alpha$ and $6^n - 5n - 1 = 25\beta$, then $\alpha - \beta$ is equal to

(1) $1+{}^nC_2(8-5)+{}^nC_3(8^2-5^2)+...+{}^nC_n(8^{n-1}-5^{n-1})$

(2) $1+{}^nC_3(8-5)+{}^nC_4(8^2-5^2)+...+{}^nC_n(8^{n-2}-5^{n-2})$

(3) ${}^nC_3(8-5)+{}^nC_4(8^2-5^2)+...+{}^nC_n(8^{n-2}-5^{n-2})$

(4) ${}^nC_4(8-5)+{}^nC_5(8^2-5^2)+...+{}^nC_n(8^{n-3}-5^{n-3})$

[JEE (Main)-2022]

104. Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}}-\frac{1}{x^{\frac{1}{5}}}\right)^{15}, x>0,$ be m and n respectively.

If r is a positive integer such that $mn^2={}^{15}C_r \cdot 2^r$, then the value of r is equal to _______.

[JEE (Main)-2022]

105. If the maximum value of the term independent of t in the expansion of $\left(t^2x^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right), x \geq 0$ is K, then $8K$ is equal to __________.

[JEE (Main)-2022]

106. If the coefficients of x and x^2 – in the expansion of $(1+x)^p(1-x)^q$, $p, q \leq 15$, are -3 and -5 respectively, then coefficient of x^3 is equal to _______.

[JEE (Main)-2022]

107. The remainder when $(2021)^{2022} + (2022)^{2021}$ is divided by 7 is

[JEE (Main)-2022]

(1) 0 (2) 1

(3) 2 (4) 6

108. Let for the 9th term in the binomial expansion of $(3 + 6x)^n$, in the increasing powers of $6x$, to be the greatest for $x = \frac{3}{2}$, the least value of n is n_0. If k is the ratio of the coefficient of x^6 to the coefficient of x^3, then $k + n_0$ is equal to :

[JEE (Main)-2022]

109. The remainder when $7^{2022} + 3^{2022}$ is divided by 5 is:

[JEE (Main)-2022]

(1) 0 (2) 2

(3) 3 (4) 4

110. Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}}+\beta x\right)^4$, $(1 - 3\beta x)^2$ and $\left(1-\frac{\beta}{2}x\right)^6$, $\beta > 0$, respectively form the first three terms of an A.P. If d is the common difference of this A.P., then $50-\frac{2d}{\beta^2}$ is equal to _______.

[JEE (Main)-2022]

111. If $1 + (2 + {}^{49}C_1 + {}^{49}C_2 + \dots {}^{49}C_{49})({}^{50}C_2 + {}^{50}C_4 + \dots {}^{50}C_{50})$ is equal to $2^n \cdot m$, where m is odd, then $n + m$ is equal to ______. **[JEE (Main)-2022]**

112. If $\sum_{k=1}^{10} K^2 \left({}^{10}C_K\right)^2 = 22000L$, then L is equal to _____.

[JEE (Main)-2022]

113. The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is

[JEE (Main)-2022]

(1) 1 (2) 4

(3) 6 (4) 8

114. $\sum_{\substack{i,j=0 \\ i\neq j}}^{n} {}^nC_i \, {}^nC_j$ is equal to **[JEE (Main)-2022]**

(1) $2^{2n} - {}^{2n}C_n$ (2) $2^{2n-1} - {}^{2n-1}C_{n-1}$

(3) $2^{2n} - \frac{1}{2}\,{}^{2n}C_n$ (4) $2^{n-1} + 2^{2n-1}C_n$

115. The remainder when 3^{2022} is divided by 5 is :

(1) 1 (2) 2

(3) 3 (4) 4

[JEE (Main)-2022]

❑ ❑ ❑

Chapter 3

Binomial Theorem and Principle of Mathematical Induction

1. Answer (1)

   Put $n = 0$

   Then when $1 - 62$ is divided by 9 then remainder is same as when $63–61$ is divided by 9 which is 2.

2. Answer (3)

   $$S_2 = \sum_{j=1}^{10} j\ {}^{10}C_j = 10.2^9$$

   $\therefore$ Statement-2 is false.

   Only choice is (3).

3. Answer (3)

   Statement 1 : $(n + 1)^7 - n^7 - 1$

   $= n^7 + {}^7C_1 n^6 + {}^7C_2 n^5 + \ldots + {}^7C_6\, n + {}^7C_7 - n^7 - 1$

   $= {}^7C_1 n^6 + {}^7C_2 n^5 + \ldots + {}^7C_6 n$

   $= 7m\ \ m \in I.$

   Statement 1 is true.

   Statement 2 : By mathematical induction

   $n^7 - n$ is divisible by 7 (true)

   Let $n^7 - n = 7p\ \ p \in I$

   $\Rightarrow\ n^7 = 7p + n$ ...(i)

   $(n + 1)^7 - n^7 - 1 = (n + 1)^7 - (7p + n) - 1$

   $= (n + 1)^7 - (n + 1) - 7p$

   $= 7l + 7p\ \ \ l, p \in I$

   Statement 2 is a correct explanation of statement 1.

4. Answer (4)

   $$\left(\sqrt{3} + 1\right)^{2n} - \left(\sqrt{3} - 1\right)^{2n}$$

   $$= 2\left({}^{2n}C_1\left(\sqrt{3}\right)^{2n-1} + {}^{2n}C_3\left(\sqrt{3}\right)^{2n-3} + \ldots\right)$$

   $\therefore$ An irrational number

5. Answer (3)

   $n(A \times B) = 2 \times 4 = 8$

   The number of subsets of $A \times B$ having 3 or more elements.

   $= {}^8C_3 + {}^8C_4 + \ldots + {}^8C_8$

   $= 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$

   $= 256 - 1 - 8 - 28$

   $= 219$

6. Answer (3)

   Given expression can be written as

   $$\left\{\left(x^{1/3} + 1\right) - \left(\frac{x^{1/2} + 1}{x^{1/2}}\right)\right\}^{10} = \left\{x^{1/3} - x^{-1/2}\right\}^{10}$$

   General term $= {}^{10}C_r \cdot \left(x^{1/3}\right)^{10-r} \cdot \left(x^{-1/2}\right)^r$

   From question,

   $$\frac{10}{3} - \frac{r}{3} - \frac{r}{2} = 0$$

   $\Rightarrow\ r = 4$

   i.e., constant term $= {}^{10}C_4 = 210$

7. Answer (2)

   $X = \{(1+3)^n - 3n - 1, n \in N\}$

   $= 3^2({}^nC_2 + {}^nC_3.3 + \ldots + 3^{n-2}), n \in N\}$

   = {Divisible by 9}

   $Y = \{9(n - 1),\ n \in N\}$

   = (All multiples of 9}

   So, $X \subseteq Y$

   i.e., $\boxed{X \cup Y = Y}$

8. Answer (2)

   $(1 + ax + bx^2)\ (1 - 2x)^{18}$

   $(1 + ax + bx^2)[{}^{18}C_0 - {}^{18}C_1(2x) + {}^{18}C_2(2x)^2 - {}^{18}C_3(2x)^3 + {}^{18}C_4(2x)^4 - \ldots\ldots.]$

   Coeff. of $x^3 = -{}^{18}C_3.8 + a \times 4.{}^{18}C_2 - 2b \times 18 = 0$

   $$= -\frac{18 \times 17 \times 16}{6}.8 + \frac{4a + 18 \times 17}{2} - 36b = 0$$

$= -51 \times 16 \times 8 + a \times 36 \times 17 - 36b = 0$

$= -34 \times 16 + 51a - 3b = 0$

$= 51a - 3b = 34 \times 16 = 544$

$= 51a - 3b = 544$ ... (i)

Only option number (2) satisfies the equation number (i).

9. Answer (1)

$$\left(1-2\sqrt{x}\right)^{50} = {}^{50}C_0 - {}^{50}C_1\left(2\sqrt{x}\right)^1 + {}^{50}C_2(2\sqrt{x})^2 + \ldots\ldots + {}^{50}C_{50}(-2\sqrt{x})^{50}$$

Sum of coefficient of integral power of x

$$= {}^{50}C_0 2^0 + {}^{50}C_2 \cdot 2^2 + {}^{50}C_4 \cdot 2^4 + \cdots + {}^{50}C_{50} \cdot 2^{50}$$

We know that

$$(1 + 2)^{50} = {}^{50}C_0 + {}^{50}C_1 \cdot 2 + \ldots\ldots + {}^{50}C_{50} \cdot 2^{50}$$

Then,

$$ {}^{50}C_0 + {}^{50}C_2 \cdot 2^2 + \ldots\ldots + {}^{50}C_{50} \cdot 2^{50} = \frac{3^{50}+1}{2}$$

10. Answer (3)

Number to terms is $2n + 1$ which is odd but it is given 28. If we take $(x + y + z)^n$ then number of terms is ${}^{n+2}C_2 = 28$

Hence $n = 6$

$$\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^6 = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_6x^6$$

Sum of coefficients can be obtained by $x = 1$

$(1 - 2 + 4)^6 = 3^6 = 729$

So according to what the examiner is trying to ask option 3 can be correct.

11. Answer (3)

$${}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10} = \frac{1}{2}\left\{{}^{21}C_0 + {}^{21}C_1 + \ldots + {}^{21}C_{21}\right\} - 1$$

$$= 2^{20} - 1$$

$$\left({}^{10}C_1 + {}^{10}C_2 + \ldots + {}^{10}C_{10}\right) = 2^{10} - 1$$

$\therefore$ Required sum $= (2^{20} - 1) - (2^{10} - 1)$

$= 2^{20} - 2^{10}$

12. Answer (4)

$$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$$

$$= 2\left[{}^5C_0\, x^5 + {}^5C_2\, x^3\,(x^3-1) + {}^5C_4\, x(x^3-1)^2\right]$$

$$= 2\left[x^5 + 10(x^6 - x^3) + 5x(x^6 - 2x^3 + 1)\right]$$

$$= 2\left[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x\right]$$

$$= 2\left[5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x\right]$$

Sum of odd degree terms coefficients

$= 2(5 + 1 - 10 + 5)$

$= 2$

13. Answer (1)

$$2^{403} = 8\left(2^4\right)^{100} = 8(16)^{100}$$

$= 8\,(1 + 15)^{100}$

$= 8 + 15\,\lambda$

When divided by 15, remainder is 8.

Hence fractional part is $\frac{8}{15}$

$\therefore$ Value of K is 8

14. Answer (1)

$$\left(\frac{1-t^6}{1-t}\right)^3 = (1-t^6)^3(1-t)^{-3}$$

$$= (1-3t^6+3t^{12}-t^{18})\left(1+3t+\frac{3\cdot 4}{2!}t^2 + \frac{3\cdot 4\cdot 5}{3!}t^3 + \frac{3\cdot 4\cdot 5\cdot 6}{4!}t^4 + \ldots\infty\right)$$

$\therefore$ Coefficient of $t^4 = 1\cdot\frac{3\cdot 4\cdot 5\cdot 6}{4!}$

$$= \frac{3\times 4\times 5\times 6}{4\times 3\times 2\times 1}$$

$= 15$

15. Answer (2)

$$\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}} = \frac{{}^{20}C_{i-1}}{{}^{20}C_i} = \frac{20!}{(i-1)!(21-i)!} \times \frac{i!(21-i)!}{21!}$$

$$\therefore \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \sum_{i=1}^{20}\left(\frac{i}{21}\right)^3 = \frac{(1)}{(21)^3}\sum_{i=1}^{20} i^3$$

$$= \frac{1}{(21)^3} \times \left(\frac{20\times 21}{2}\right)^2 = \frac{100}{21}$$

$\therefore k = 100$

16. Answer (2)

$S = 16 + 23 + 30 + ... + 93 = 654$

$S' = 12 + 19 + 26 + ... + 96 = 702$

Required Sum $= 654 + 702$

$= 1356$

17. Answer (2)

Third term of $(1+x^{\log_2 x})^5 = {}^5C_2(x^{\log_2 x})^2$

given, ${}^5C_2(x^{\log_2 x})^2 = 2560$

$\Rightarrow \quad (x^{\log_2 x})^2 = 256 = (\pm 16)^2$

$\Rightarrow \quad x^{\log_2 x} = 16$ or $x^{\log_2 x} = -16$ (rejected)

$\Rightarrow \quad x^{\log_2 x} = 16 \Rightarrow \log_2 x \log_2 x = \log_2 16 = 4$

$\Rightarrow \quad \log_2 x = \pm 2 \Rightarrow x = 2^2$ or 2^{-2}

$\Rightarrow \quad x = 4$ or $\frac{1}{4}$

18. Answer (4)

$P(n) = n^2 - n + 41$

$\Rightarrow \quad P(3) = 9 - 3 + 41 = 47$

$P(5) = 5^2 - 5 + 41 = 61$

Both 47 and 61 are prime

19. Answer (2)

Coefficient of x^2 in $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

= co-efficient of x° in $\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

General term in $\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10} = {}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{\lambda}{x^2}\right)^r$

for constant term

$\frac{10-r}{2} - 2r = 0$

$\Rightarrow \quad r = 2$

$\Rightarrow$ Co-efficient of x^2 in expression

$= {}^{10}C_2\lambda^2 = 720$

$\Rightarrow \quad \lambda^2 = \frac{720}{5\times 9} = 16$

$\lambda = 4$

Option (2) is correct.

20. Answer (3)

Middle term, $\left(\frac{n}{2}+1\right)^{\text{th}}$

$T_{4+1} = {}^8C_4\left(\frac{x^3}{3}\right)^4\left(\frac{3}{x}\right)^4 = 5670$

$\Rightarrow \quad \frac{8\times 7\times 6\times 5}{4\times 3\times 2}\times x^8 = 5670$

$x^8 = 81$

$x^8 - 81 = 0$

Now sum of all values of x = zero

21. Answer (1)

$(x + 10)^{50} + (x - 10)^{50}$

$= a_0 + a_1x + a_2x^2 + ... + a_{50}x^{50}$

$\therefore \; a_0 + a_1x + a_2x^2 + ... + a_{50}x^{50}$

$= 2({}^{50}C_0x^{50} + {}^{50}C_2x^{48}\cdot 10^2 + {}^{50}C_4x^{46}\cdot 10^4 + ...)$

$\therefore \quad a_0 = 2.{}^{50}C_{50}10^{50}$

$a_2 = 2.{}^{50}C_2\cdot 10^{48}$

$\therefore \quad \frac{a_2}{a_0} = \frac{{}^{50}C_2\times 10^{48}}{{}^{50}C_{50}10^{50}}$

$= \frac{50\times 49}{2\times 100} = \frac{49}{4}$

$= 12.25$

22. Answer (4)

$\left(2^{\frac{1}{3}}+\frac{1}{2(3)^{\frac{1}{3}}}\right)^{10}$

5^{th} term from beginning $T_5 = {}^{10}C_4\left(2^{\frac{1}{3}}\right)^6\frac{1}{\left(2.3^{\frac{1}{3}}\right)^4}$

5^{th} term from end $T_{11-5+1} = {}^{10}C_6\left(2^{\frac{1}{3}}\right)^4\left(\frac{1}{2.3^{\frac{1}{3}}}\right)^6$

Now $T_5 : T_7$

${}^{10}C_4\left(2^{\frac{1}{3}}\right)^6\left(\frac{1}{2.3^{\frac{1}{3}}}\right)^4 : {}^{10}C_6\left(2^{\frac{1}{3}}\right)^4\left(\frac{1}{2.3^{\frac{1}{3}}}\right)^6$

$\left(2^{\frac{1}{3}}\right)^2 : \left(\frac{1}{2.3^{\frac{1}{3}}}\right)^2$

$= \frac{2^{\frac{2}{3}}\cdot 2^2\cdot 3^{\frac{2}{3}}}{1} = 4.6^{\frac{2}{3}} : 1 = 4\cdot(36)^{\frac{1}{3}} : 1$

23. Answer (3)

$$T_{r+1} = {}^{60}C_r\left(7^{\frac{1}{5}}\right)^{60-r}\left(-3^{\frac{1}{10}}\right)^r$$

$$= {}^{60}C_r.(7)^{12-\frac{r}{5}}(-1)^r.(3)^{\frac{r}{10}}$$

So for getting rational terms, r should be multiple of L.C.M. of (5, 10)

So r can be 0, 10, 20, 30, 40, 50, 60.

Now total number of terms = 61

Total irrational terms = 61 – 7 = 54

24. Answer (1)

$$\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6$$

$$= 2[{}^6C_0x^6 + {}^6C_2x^4(x^3-1) + {}^6C_4x^2(x^3-1)^2 + {}^6C_6(x^3-1)^3]$$

$$= 2[x^6 + 15x^7 - 15x^4 + 15x^8 - 30x^5 + 15x^2 + x^9 - 3x^6 + 3x^3 - 1]$$

Sum of coefficients of even powers of $x = 2[1 - 15 + 15 + 15 - 3 - 1] = 24$

25. Answer (1)

$$T_4 = {}^6C_3\left(\sqrt{x^{\left(\frac{1}{1+\log_{10}x}\right)}}\right)^3\left(x^{\frac{1}{12}}\right)^3 = 200$$

$$\Rightarrow \quad 20x^{\frac{3}{2(1+\log_{10}x)}}\cdot x^{\frac{1}{4}} = 200$$

$$x^{\frac{1}{4}+\frac{3}{2(1+\log_{10}x)}} = 10$$

Taking $\log_{10}$ on both sides and put $\log_{10}x = t$

$$\left(\frac{1}{4}+\frac{3}{2(1+t)}\right)t = 1$$

$$\left(\frac{(1+t)+6}{4(1+t)}\right)\times t = 1 \quad \Rightarrow \quad t^2 + 7t = 4 + 4t$$

$$t^2 + 3t - 4 = 0 \quad \Rightarrow \quad t^2 + 4t - t - 4 = 0$$

$\Rightarrow$ $t(t + 4) - 1(t + 4) = 0$

$\Rightarrow$ $t = 1$ or $t = -4$

$\log_{10}x = 1$

$\Rightarrow$ $x = 10$ or if $\log_{10}x = -4$

$\Rightarrow$ $x = 10^{-4}$

Note: There seems a printing error in this question in the original question paper.

26. Answer (4)

$$T_4 = 20\times 8^7 = {}^6C_3\left(\frac{2}{x}\right)^3\times\left(x^{\log_8 x}\right)^3$$

$$\Rightarrow \quad 8\times 20\times\left(\frac{x^{\log_8 x}}{x}\right)^3 = 20\times 8^7$$

$$\Rightarrow \quad \left(\frac{x^{\log_8 x}}{x}\right)^3 = \left(8^2\right)^3$$

$$\Rightarrow \quad \frac{x^{\log_8 x}}{x} = 64 \qquad \text{Take } \log_8 \text{ both side}$$

$\Rightarrow$ $(\log_8x)^2 - (\log_8x) = 2$

$\Rightarrow$ $\log_8x = -1$ or $\log_8x = 2$

$$\Rightarrow \quad x = \frac{1}{8} \quad \text{or} \quad x = 8^2$$

27. Answer (3)

Given ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:15:70$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15} \text{ and } \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$$

$$\Rightarrow \quad \frac{r}{n-r+1} = \frac{2}{15} \text{ and } \frac{r+1}{n-r} = \frac{3}{14}$$

$\Rightarrow$ $15r = 2n - 2r + 2$ and $14r + 14 = 3n - 3r$

$\Rightarrow$ $17r = 2n + 2$ and $17r = 3n - 14$

i.e., $2n + 2 = 3n - 14 \Rightarrow n = 16$ and $r = 2$

$$\text{Mean} = \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3}$$

$$= \frac{16+120+560}{3}$$

$$= \frac{696}{3} = 232$$

28. Answer (4)

$(1 + ax + bx^2)(1 - 3x)^{15}$

Co-eff. of $x^2 = 1.{}^{15}C_2(-3)^2 + a.{}^{15}C_1(-3) + b.{}^{15}C_0$

$$= \frac{15\times 14}{2}\times 9 - 15\times 3a + b = 0 \text{ (Given)}$$

$\Rightarrow$ $945 - 45a + b = 0$ ...(i)

Now co-eff. of $x^3 = 0$

$\Rightarrow$ ${}^{15}C_3(-3)^3 + a.{}^{15}C_2(-3)^2 + b.{}^{15}C_1(-3) = 0$

$$\Rightarrow \quad \frac{15\times 14\times 13}{3\times 2}\times(-3\times 3\times 3) + a\times\frac{15\times 14\times 9}{2} - b\times 3\times 15 = 0$$

$\Rightarrow$ $15 \times 3[-3 \times 7 \times 13 + a \times 7 \times 3 - b] = 0$

$\Rightarrow$ $21a - b = 273$ ...(ii)

From (i) and (ii),

$a = +28$, $b = 315 \equiv (a, b) \equiv (28, 315)$

29. Answer (3)

$$\left(x^2+\frac{1}{x^3}\right)^n$$

General term $T_{r+1} = {}^nC_r\left(x^2\right)^{n-r}\left(\frac{1}{x^3}\right)^r$

${}^nC_r \cdot x^{2n-5r}$

for coefficiant of x, $2n - 5r = 1$

Given ${}^nC_r = {}^nC_{23}$

$r = 23$ or $n - r = 23$

$\Rightarrow$ $n = 58$ or $n = 38$

Minimum value is $n = 38$

30. Answer (1)

$$(1-x)^{10}(1+x+x^2)^9(1+x)$$

$$= (1-x^3)^9(1-x^2)$$

$$= (1-x^3)^9 - x^2(1-x^3)^9$$

$\Rightarrow$ Coefficient of x^{18} in $(1-x^3)^9$ – coeff. of x^{16} in $(1-x^3)^9$

$$= {}^9C_6 = \frac{9!}{6!3!} = \frac{7\times8\times9}{6} = 84$$

31. Answer (3)

${}^{20}C_1 + 2^2 \cdot {}^{20}C_2 + 3^2 \cdot {}^{20}C_3 + ... + 20^2 \cdot {}^{20}C_{20}$

$$= \sum_{r=1}^{20} r^2 \cdot {}^{20}C_r$$

$$= 20\sum_{r=1}^{20} r \cdot {}^{19}C_{r-1}$$

$$= 20\left[\sum_{r=1}^{20}(r-1)\,{}^{19}C_{r-1} + \sum_{r=1}^{20}{}^{19}C_{r-1}\right]$$

$$= 20\left[19\sum_{r=2}^{20}{}^{18}C_{r-2} + 2^{19}\right]$$

$= 20[19 \cdot 2^{18} + 2^{19}]$

$= 20 \times 21 \times 2^{18}$

$= 420 \times 2^{18}$

So; $A = 420$ and $\beta = 18$

32. Answer (2)

$$\left(\frac{1}{60}-\frac{x^8}{81}\right)\left(2x^2-\frac{3}{x^2}\right)^6$$

$$= \frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6 - \frac{x^8}{81}\left(2x^2-\frac{3}{x^2}\right)^6$$

Coefficient of x° in $\frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6 - \frac{1}{81}$

coefficient of x^{-8} in $\left(2x^2-\frac{3}{x^2}\right)^6$

$$= \frac{-1}{60}\,{}^6C_3(2)^3(3)^3 + \frac{1}{81}\,{}^6C_5\,(2)(3)^5$$

$= -72 + 36 = -36$

33. Answer (4)

$$\because \quad {}^{36}C_{r+1}\cdot\left(k^2-3\right) = {}^{35}C_r \times 6$$

$$\frac{36!}{(r+1)!(35-r)!}\cdot\left(k^2-3\right) = \frac{35!}{r!(35-r)!}\times 6$$

$6(k^2 - 3) = r + 1$

$$\therefore \quad k^2 = 3 + \frac{r+1}{6}$$

$\therefore$ r can be 5 and 35

When $r = 5$ then $k = \pm2$

and when $r = 35$, then $k = \pm3$.

$\therefore$ Total number of ordered pairs = 4

34. Answer (2)

$(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots + x^{10}$

$$= \frac{(1+x)^{10}\left(1-\left(\frac{x}{1+x}\right)^{11}\right)}{1-\frac{x}{1+x}}$$

$= (1 + x)^{11} - x^{11}$

$\therefore$ Coeff. of $x^7 = {}^{11}C_7 = {}^{11}C_4$

$$= \frac{11\times10\times9\times8}{4\times3\times2\times1}$$

$= 330$

35. Answer (3)

$$\because \quad \left(x+\sqrt{x^2-1}\right)^6 + \left(x-\sqrt{x^2-1}\right)^6 = 2$$

$$\left[{}^6C_0x^6 + {}^6C_2x^4\left(x^2-1\right) + {}^6C_4x^2\left(x^2-1\right)^2 + {}^6C_6\left(x^2-1\right)^3\right]$$

$= 2[32x^6 - 48x^4 + 18x^2 - 1]$

$\therefore$ α = coefficient of $x^4 = -96$

β = coefficient of $x^2 = 36$

$\Rightarrow$ $\alpha - \beta = -96 - 36 = -132$

36. Answer (2)

$$T_{r+1} = {}^{16}C_r \cdot \left(\frac{x}{\cos\theta}\right)^{16-r} \cdot \left(\frac{1}{x\sin\theta}\right)^r$$

$$= {}^{16}C_r \cdot \frac{x^{16-2r}}{(\cos\theta)^{16-r} \cdot (\sin\theta)^r}$$

$\because$ $16 - 2r = 0 \Rightarrow r = 8$

$$T_9 = \frac{{}^{16}C_8 \cdot 2^8}{\sin^8 2\theta}$$

$\because$ $\sin 2\theta$ is increasing in $\left[\frac{\pi}{16}, \frac{\pi}{4}\right]$

Hence, $I_1 = \frac{{}^{16}C_8 \cdot 2^8}{\sin^8\left(\frac{\pi}{2}\right)}$ and $I_2 = \frac{{}^{16}C_8 \cdot 2^8}{\sin^8\left(\frac{\pi}{4}\right)}$

$$\frac{I_2}{I_1} = \frac{\sin^8\left(\frac{\pi}{2}\right)}{\sin^8\left(\frac{\pi}{4}\right)} = \frac{16}{1}$$

37. Answer (3)

General term of

$$\left(\alpha x^{1/9} + \beta x^{-1/6}\right)^{10} = {}^{10}C_r\left(\alpha x^{1/9}\right)^{10-r}\left(\beta x^{-1/6}\right)^2$$

for term independent of 'x' $r = 4$

$\therefore$ Term independent of $x = {}^{10}C_4\alpha^6\beta^4$

Also $\alpha^3 + \beta^2 = 4$

By AM-GM inequality

$$\frac{\alpha^3 + \beta^2}{2} \ge \left(\alpha^3\beta^2\right)^{1/2}$$

$\Rightarrow$ $(2)^2 \ge \alpha^3\beta^2$

$\Rightarrow$ $\alpha^6\beta^4 \le 16$

$\therefore$ $10k = {}^{10}C_4 \cdot 16$

$\Rightarrow$ $k = 336$

38. Answer (3)

$$\left(3^{\frac{1}{2}} + 5^{\frac{1}{8}}\right)^n$$

Let $T_{r+1} = {}^nC_r\,(3)^{\frac{n-r}{2}}\,(5)^{\frac{r}{8}}$

So r must be 0, 8, 16, 24 ......

Now $n = t_{33} = 0 + 32 \times 8 = 256$

$\Rightarrow$ $n = 256$

39. Answer (4)

General term $= T_{r+1} = {}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$$= {}^9C_r\,(-1)^r \cdot \frac{3^{9-2r}}{2^{9-r}}\,x^{18-3r}$$

If term is independent of x then $r = 6$

$\therefore$ $k = {}^9C_6 \cdot \frac{3^{-3}}{2^3} = \frac{7}{18}$

$\therefore$ $18k = 7$

40. Answer (1)

Here $\sum_{r=0}^{20} {}^{50-r}C_6$

$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + {}^{47}C_6 + \ldots {}^{32}C_6 + {}^{31}C_6 + {}^{30}C_6$

$\Rightarrow$ $({}^{30}C_7 + {}^{30}C_6) + {}^{31}C_6 + {}^{32}C_6 + \ldots + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

$\Rightarrow$ $({}^{31}C_7 + {}^{31}C_6) + {}^{32}C_6 + \ldots + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

$= ({}^{32}C_7 + {}^{32}C_6) + \ldots + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

....

....

....

$= {}^{51}C_7 - {}^{30}C_7$

41. Answer (2)

Consider the three consecutive coefficients as

${}^{n+5}C_r,\ {}^{n+5}C_{r+1},\ {}^{n+5}C_{r+2}$

$\because$ $\frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \frac{1}{2}$

$\Rightarrow$ $\frac{r+1}{n+5-r} = \frac{1}{2}$ $\Rightarrow$ $3r = n + 3$ ...(i)

and $\frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_{r+2}} = \frac{5}{7}$

$\Rightarrow$ $\frac{r+2}{n+4-r} = \frac{5}{7}$ $\Rightarrow$ $12r = 5n + 6$ ...(ii)

From (i) and (ii) $n = 6$

Largest coefficient in the expansion is ${}^{11}C_6 = 462$

42. Answer (2)

$$\frac{3^{200}}{8} = \frac{1}{8}\left(9^{100}\right) = \frac{1}{8}(1+8)^{100} = \frac{1}{8} + \text{Integer}$$

$\therefore$ $\left\{\frac{3^{200}}{8}\right\} = \left\{\frac{1}{8} + \text{ integer}\right\} = \frac{1}{8}$

43. Answer (2)

General term = $T_{r+1} = {}^{10}C_r\left(\sqrt{x}\right)^{10-r} . \left(-\frac{k}{x^2}\right)^r$

$= {}^{10}C_r\,(-k)^r\,.\,x^{\frac{10-r}{2}-2r}$

$= {}^{10}C_r\,(-k)^r\,.\,x^{\frac{10-5r}{2}}$

If it is constant term then $x = 2$

$\because\ {}^{10}C_2\,(-k)^2 = 405$

$\Rightarrow\ k^2 = \frac{405\times 2}{10\times 9} = \frac{81}{9} = 9$

$|k| = 3$

44. Answer (2)

$a = {}^{19}C_{10}$ or ${}^{19}C_9$

$b = {}^{20}C_{10}$

$c = {}^{21}C_{10}$ or ${}^{21}C_{11}$

$\Rightarrow\ 1 = \frac{a}{{}^{19}C_{10}} = \frac{b}{{}^{20}C_{10}} = \frac{c}{{}^{21}C_{10}}$

$\Rightarrow\ \frac{a}{11} = \frac{b}{22} = \frac{c}{42}$

45. Answer (30)

Let $(1 - x + x^2 - x^3\ .....\ x^{2n})$

$(1 + x + x^2 + x^3 + ... + x^{2n})$

$= a_0 + a_1x + a_2x^2 + ... + a_{4n}x^{4n}$

Put $x = 1$

$\Rightarrow\ (1)(2n + 1) = a_0 + a_1 + a_2 + ... + a_{4n}$ ...(i)

Put $x = -1$

$\Rightarrow\ (2n + 1)(1) = a_0 - a_1 + a_2 + ... + a_{4n}$ ...(ii)

Adding (i) and (ii)

$\therefore\ \frac{4n+2}{2} = a_0 + a_2 + a_4 + ...a_{4n}$

As $61 = 2n + 1$

$\therefore\ n = 30$

46. Answer (615.00)

$(1 + x + x^2)^{10} = {}^{10}C_0(1 + x)^{10} + {}^{10}C_1(1 + x)^9\cdot x^2 + {}^{10}C_2\cdot(1 + x)^8\cdot x^4 + ...$

Coeff. of $x^4 = {}^{10}C_0\cdot{}^{10}C_4 + {}^{10}C_1\cdot{}^{9}C_2 + {}^{10}C_2\cdot{}^{8}C_0$

$= 210 + 360 + 45$

$= 615$

47. Answer (51)

$C_0 + 5C_1 + 9C_2 + ... + 101.C_{25} = 2^{25}k$

$\Rightarrow\ \sum_{r=0}^{25}(4r+1)\,C_r = 2^{25}k$

$\Rightarrow\ 4\sum_{r=0}^{25} r.C_r + \sum_{r=0}^{25} C_r = 2^{25}k$

$\Rightarrow\ 4\sum_{r=1}^{25} r.\frac{25}{r}\ {}^{24}C_{r-1} + 2^{25} = 2^{25}.k$

$\Rightarrow\ 4\sum_{r=1}^{25} {}^{24}C_{r-1}(25) + 2^{25} = 2^{25}.k$

$\Rightarrow\ 100.2^{24} + 2^{25} = 2^{25}\ k$

$\Rightarrow\ 2^{25}(50 + 1) = 2^{25}\ k$

$\Rightarrow\ k = 51$

48. Answer (118)

${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2 : 5 : 12$

$\Rightarrow\ \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{5}{2}$ and $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{12}{5}$

$\Rightarrow\ \frac{n-r+1}{r} = \frac{5}{2}$ and $\frac{n-r}{r+1} = \frac{12}{5}$

$\Rightarrow\ 2n - 7r + 2 = 0$ and $5n - 17r - 12 = 0$

Solving; $x = 118$, $r = 34$

49. Answer (8)

$(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$

General term = $\frac{10!}{r_1!r_2!r_3!}(2x^2)^{r_1}(3x)^{r_2}(4)^{r_3}$

As a_7 = coeff of x^7

$2r_1 + r_2 = 7$ and $r_1 + r_2 + r_3 = 10$

Possibilities are

r_1	r_2	r_3
0	7	3
1	5	4
2	3	5
3	1	6

$$a_7 = \frac{10!3^7 4^3}{7!3!} + \frac{10!(2)(3)^5(4)^4}{5!4!} + \frac{10!(2)^2(3)^3(4)^5}{2!3!5!} + \frac{10!(2)^3(3)(4)^6}{3!6!}$$

a_{13} = Coeff of x^{13}

$2r_1 + r_2 = 13$ and $r_1 + r_2 + r_3 = 10$

Possibilities are

r_1	r_2	r_3
3	7	0
4	5	1
5	3	2
6	1	3

$$a_{13} = \frac{10!(2^3)(3^7)}{3!7!} + \frac{10!(2^4)(3^5)(4)}{4!5!} + \frac{10!(2^5)(3^3)(4^2)}{5!3!2!} + \frac{10!(2^6)(3)(4^3)}{6!1!3!}$$

Clearly $\frac{a_7}{a_{13}} = 2^3 = 8$

50. Answer (13)

$$T_{r+1} = {}^{22}C_r \cdot (x^m)^{22-r} \cdot \left(\frac{1}{x^2}\right)^r$$

$$T_{r+1} = {}^{22}C_r \cdot x^{22m-mr-2r}$$

$\because$ $22m - mr - 2r = 1$ and ${}^{22}C_r = 1540$

$\because$ ${}^{22}C_3 = 1540 \Rightarrow r = 3$ or 19

Now, for $r = 3$; $22m - 3m - 6 = 1$

$\Rightarrow 19m = 7 \quad \Rightarrow m = \frac{7}{19}$ (not acceptable)

for $r = 19$; $22m - 19m = 39$

$\Rightarrow m = 13$

51. Answer (120)

$$(1 + x + x^2 + x^3)^6 = \left(\frac{1-x^4}{1-x}\right)^6$$

Coefficient of x^4 in $\left(\frac{1-x^4}{1-x}\right)^6$ = coefficient of x^4 in

$(1 - 6x^4)(1 - x)^{-6}$

= coefficient of x^4 in $(1 - 6x^4)$ $\left[1 + {}^6C_1 x + {}^7C_2 x^2 + ...\right]$

$= {}^9C_4 - 6.1 = 126 - 6 = 120$

52. Answer (4)

$$\because \quad (1+x)^{14} = {}^{14}C_0 + {}^{14}C_1 x + {}^{14}C_2 x^2 + ... + {}^{14}C_{14} x^{14} \quad ...(i)$$

$$\therefore \quad 2^{14} = {}^{14}C_0 + {}^{14}C_1 + {}^{14}C_2 + ... + {}^{14}C_{14} \quad ...(ii)$$

$$0 = {}^{14}C_0 - {}^{14}C_1 + {}^{14}C_2 .... + {}^{14}C_{14} \quad ...(iii)$$

$$\therefore \quad {}^{14}C_1 + {}^{14}C_3 + ... + {}^{14}C_{13} = 2^{13} \quad ...(iv)$$

and $(1-x)^{15} = {}^{15}C_0 - {}^{15}C_1 x + {}^{15}C_2 x^2 + ... {}^{15}C_{15} x^{15}$

Differentiate w.r.t. x we get

$$-15(1-x)^{14} = -{}^{15}C_1 + 2 . {}^{15}C_2 x + ... - 15 . {}^{15}C_{15} x^{14}$$

Put x = 1, we get

$$-{}^{15}C_1 + 2 . {}^{15}C_2 - 3 . {}^{15}C_3 + ... - 15 . {}^{15}C_{15} = 0 \quad ...(v)$$

From equation (iv) + equation (v) we get

$$-{}^{15}C_1 + 2 . {}^{15}C_2 + ... - 15 {}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_r + ... + {}^{14}C_{11}$$

$$= 2^{13} - {}^{14}C_{13}$$

$= 2^{13} - 14$

53. Answer (4)

Sum of ${}^2C_2 + {}^3C_2 + ... + {}^nC_2$ is coefficient of x^2 in $(1 + x)^2 + (1 + x)^3 + ... + (1 + x)^n$

i.e. coefficient of x^2 in

$$(1+x)^2 \frac{\left((1+x)^{n-1} - 1\right)}{(1+x-1)} = {}^{n+1}C_3$$

Hence required sum = ${}^{n+1}C_2 + 2.{}^{n+1}C_3$

$$= \frac{(n+1)(n)}{2} + \frac{2(n+1)n(n-1)}{6}$$

$$= \frac{n(n+1)}{2}\frac{(3+2n-2)}{3} = \frac{n(n+1)(2n+1)}{6}$$

54. Answer (12)

$$\sum_{i=0}^{k} {}^{10}C_1 . {}^{15}C_{k-i} + \sum_{i=0}^{k+1} {}^{12}C_1 . {}^{13}C_{k+1-i}$$

$$= {}^{25}C_k + {}^{25}C_{k+1}$$

$$= {}^{26}C_{k+1}$$

$0 \le k + 1 \le 25$

$-1 \le k \le 24$

But ${}^{13}C_{k+1-i}$ exists for $0 \le i \le k+1$

then $0 \le i \le k+1$

$\Rightarrow \quad k \le 12$

Hence $k_{max} = 12$

55. Answer (1)

$\because\ x = 4y + 3$

then $(2020 + x)^{2022} = (2023 + 4y)^{2022}$

$= (4\lambda - 1)^{2022}$

$= (16\lambda^2 - 8\lambda + 1)^{2022}$

$= (8\mu + 1)^{1011}$

$= 8\gamma + 1$ where $\lambda, \mu, \gamma \in \mathbf{N}$

56. Answer (45)

$3^n + 7^n$ is divisible by $(3 + 7)$ if n is odd.

So, number of two digit odd numbers = 45

57. Answer (2)

$$T_{r+1} = {}^{10}C_r (tx^{\frac{1}{5}})^{10-r}\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^r$$

For term independent of f

$10 - r - r = 0 \Rightarrow r = 5$

$$T_6 = {}^{10}C_5 \ x(1-x)^{\frac{1}{2}} = f(x) \qquad \text{(Let)}$$

$$\therefore \quad f'(x) = {}^{10}C_5\left((1-x)^{\frac{1}{2}} - \frac{x}{2(1-x)^{\frac{1}{2}}}\right) = 0$$

$$2 - 2x = x \Rightarrow x = \frac{2}{3}$$

$$f''(x) < 0 \text{ at } x = \frac{2}{3}$$

$$T_{6(max)} = {}^{10}C_5 \cdot \frac{2}{3}\left(\frac{1}{3}\right)^{\frac{1}{2}} = \frac{2.10!}{(5!)^2 3\sqrt{3}}$$

58. Answer (45)

$$\sum_{r=0}^{29}(30-r)\cdot {}^{30}C_r$$

$$= \sum_{r=1}^{30} r \cdot {}^{30}C_{30-r} = \sum_{r=1}^{30} r \cdot {}^{30}C_r$$

$$= 30\sum_{r=1}^{30} {}^{29}C_{r-1} = 30 \cdot 2^{29}$$

$$= 15 \cdot 2^{30}$$

Clearly n = 15, m = 30

and m + n = 45

59. Answer (2)

$$T_{r+1} = {}^{60}C_r \cdot \left(3^{\frac{1}{4}}\right)^{60-r} \cdot \left(5^{\frac{1}{8}}\right)^r$$

$$= {}^{60}C_r \cdot 3^{15-\frac{r}{4}} \cdot 5^{\frac{r}{8}}$$

Term will be rational is r is divisible by 8.

r = 0, 8, 16, 24, 32, 40, 48, 56

Total number of irrational terms = n = 61 – 8 = 53
hence n – 1 is divisible by 26.

60. Answer (1)

$$f(x) = (1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$$

$$\because \quad = \sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} = \frac{1}{2}\left(f(1) + f(-1)\right) = \frac{1}{2}(1+1) = 1$$

$$\because \quad = \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = \frac{1}{2}\left(f(1) - f(-1)\right) = 0$$

$$\text{Clearly } = \sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = 1$$

61. Answer (6)

$$\sum(-1)^k\ {}^nC_k\left(\frac{1}{2}\right)^k + \sum(-1)^k\ {}^nC_k\left(\frac{3}{4}\right)^k + \ldots$$

$$= \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \ldots + \left(1-\frac{31}{32}\right)^n$$

$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{2n} + \left(\frac{1}{2}\right)^{3n} + \ldots + \left(\frac{1}{2}\right)^{5n}$$

$$= \left(\frac{1}{2}\right)^n\left(\frac{1-\left(\frac{1}{2}\right)^{5n}}{1-\left(\frac{1}{2}\right)^n}\right) = \frac{2^{5n}-1}{2^{5n}\left(2^n-1\right)}$$

Given, $A = \frac{1}{63}\left(1 - \frac{1}{2^{30}}\right)$

$\Rightarrow \ n = 6$

62. Answer (1)

$$T_4 = {}^7C_3 \cdot (x^{\log_2 x})^3 \cdot x^4 = 4480$$

$$\Rightarrow \left(x^{\log_2 x}\right)^3 \cdot x^4 = 128$$

x = 2 is the only solution for $x \in N$

63. Answer (1)

$$\sum_{r=0}^{6} {}^6C_r \cdot {}^6C_{6-r} = \text{Coeff. of } x^6 \text{ in the expansion of}$$

$(1 + x)^6(x + 1)^6$

$= {}^{12}C_6$

$= 924$

64. Answer (60)

$$\frac{{}^nC_2}{{}^nC_3}\cdot a = \frac{12}{8} \Rightarrow \frac{3a}{n-2} = \frac{3}{2} \qquad \ldots(i)$$

Similarly, $\dfrac{4a}{n-3} = \dfrac{8}{3}$ $\qquad \ldots(ii)$

From (i) and (ii), n = 6 and a = 2

The term independent of x = ${}^6C_4a^2$

$= 15 \times 4 = 60$

65. Answer (1)

$(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{40}x^{40}$

Put x = 1

$\Rightarrow 4^{20} = a_0 + a_1 + \ldots\ldots\ldots + a_{40}$ ...(i)

Put x = –1

$\Rightarrow 2^{20} = a_0 - a_1 + \ldots\ldots\ldots - a_{39} + a_{40}$ ...(ii)

by (i) – (ii) we get,

$4^{20} - 2^{20} = 2(a_1 + a_3 + \ldots\ldots + a_{37} + a_{39})$

$\Rightarrow a_1 + a_3 + \ldots\ldots + a_{37} = 2^{39} - 2^{19} - a_{39}$ ...(iii)

a_{39} = coeff. x^{39} in $(1 + x + 2x^2)^{20}$

$$= \frac{20!}{0!\,1!\,19!}(1)^0(1)^1\ (2)^{19}$$

$= 20.2^{19}$

$\therefore a_1 + a_3 + \ldots\ldots + a_{37} = 2^{39} - 2^{19}.21$

$\Rightarrow 2^{19}(2^{20} - 21)$

66. Answer (19)

$$\sum_{k=0}^{10}(3k+4)\,{}^{10}C_k = 30\sum_{k=1}^{10}{}^9C_{k-1} + 4\sum_{k=0}^{10}{}^{10}C_k$$

$= 30.2^9 + 4.2^{10}$

$= 19.2^{10}$

$\Rightarrow \alpha = 0$ and $\beta = 19$

67. Answer (210)

$$\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{10}$$

$$= \left(\frac{(x^{\frac{1}{3}}+1)(x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1)}{(x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1)} - \frac{(x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}+1)}{x^{\frac{1}{2}}(x^{\frac{1}{2}}-1)}\right)^{10}$$

$$= \left((x^{\frac{1}{3}}+1) - (1 + x^{-\frac{1}{2}})\right)^{10}$$

$$= \left(x^{\frac{1}{3}} - x^{-\frac{1}{2}}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r\left(x^{\frac{1}{3}}\right)^{(10-r)}\left(x^{-\frac{1}{2}}\right)^r$$

For being independent of x : $\dfrac{10-r}{3} - \dfrac{r}{2} = 0 \Rightarrow r = 4$

Term independent of $x = {}^{10}C_4 = 210$

68. Answer (1)

$(1 - x)((1 - x)(1 + x + x^2))^{100}$

$\Rightarrow (1 - x)(1 - x^3)^{100}$

General term in $(1 - x^3)^{100}$ is ${}^{100}C_r\left(-x^3\right)^r$

$\therefore$ x^{256} occur if $3r = 256$ or $3r + 1 = 256$

$r = \dfrac{256}{3}$ (not valid) $\qquad r = \dfrac{255}{3} = 85$

$\therefore$ Coefficient of x^{256} $\qquad {}^{100}C_{85} = {}^{100}C_{15}$

69. Answer (21)

$$T_{r+1} = {}^{120}C_r.\left(4^{\frac{1}{4}}\right)^{120-r}.\left(5^{\frac{1}{6}}\right)^r$$

$$= {}^{120}C_r.2^{60-\frac{r}{2}}.5^{\frac{r}{6}}$$

For T_{r+1} to be rational, r must be divisible by 6.

r = 0, 6, 12, …, 120

Number of rational terms = 21

70. Answer (2)

$(1 - y)^n (1 + y)^n = 1 + a_1y + a_2y^2 + \ldots\ldots + a_{m+n}y^{m+n}$

Given $(a_1 = a_2 = 10)$

$(1 - my + {}^mC_2y^2 + \ldots\ldots)(1 + ny + {}^nC_2y^2 + \ldots\ldots)$

$= 1 + a_1y + a_2y^2 + \ldots\ldots$

$\Rightarrow n - m = 10$ ...(i)

$\Rightarrow {}^mC_2 + {}^nC_2 - mn = 10$ ...(ii)

$$\frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn = 10$$

$$\Rightarrow \frac{m^2 - m}{2} + \frac{(10+m)(9+m)}{2} - m(10+m) = 10$$

$\Rightarrow m^2 - m + m^2 + 19m + 90 - 2(m^2 + 10m) = 20$

$\Rightarrow 18m + 90 - 20m = 20$

$\Rightarrow 2m = 70$

$\Rightarrow m = 35$ & $n = 45$

$m + n = 80$

71. Answer (96)

$11^n - 9^n > 10^n$

$$\Rightarrow \left(1+\frac{1}{10}\right)^n - \left(1-\frac{1}{10}\right)^n > 1$$

$$\Rightarrow {}^nC_1.\frac{1}{10} + \underbrace{{}^nC_3.\frac{1}{10^3} + {}^nC_5.\frac{1}{10^5} + \ldots\ldots}_{\text{Neglecting these terms}} > \frac{1}{2}$$

$\Rightarrow n \geq 5$

Possible values of n = 5, 6, 7, 8, …, 100

72. Answer (8)

$$T_{k+1} = {}^{10}C_k.(2x^r)^{10-k}.\left(\frac{1}{x^2}\right)^k = {}^{10}C_k.2^{10-k}.x^{(10-k)r-2k}$$

$\because$ $(10-k)r - 2k = 0$ and ${}^{10}C_k.2^{10-k} = 180$

$\Rightarrow$ ${}^{10}C_k.2^{10-k} = 45.2^2$

$\Rightarrow$ $k = 8$

and $r = 8$

73. Answer (4)

$$\frac{1}{a}\left(\frac{1}{1-b/a} + \frac{1}{1-2b/a} + \frac{1}{1-3b/a} + \ldots.. + \frac{1}{1-nb/a}\right)$$

$$= \alpha n + \beta n^2 + \gamma n^3$$

Let $\frac{b}{a} = x$

$$\frac{1}{a}\left[(1-x)^{-1} + (1-2x)^{-1} + (1-3x)^{-1} + \ldots. + (1-nx)^{-1}\right]$$

$$\Rightarrow \frac{1}{a}\left[\left(1+x+x^2\right) + \left(1+2x+(2x)^2\right) + \left(1+3x+(3x)^2\right) + \ldots. + \left(1+nx+(nx)^2\right)\right]$$

$$\Rightarrow \frac{1}{a}\left(n + \frac{n(n+1)}{2}x + \frac{n(n+1)(2n+1)}{6}x^2\right) = \alpha n + \beta n^2 + \gamma n^3$$

back substituting $x = \frac{b}{a}$, we get

$$\frac{n}{a} + \frac{n^2+n}{2}\frac{b}{a^2} + \frac{n(n+1)(2n+1)}{6}\frac{b^2}{a^3} = \alpha n + \beta n + \gamma n^3$$

comparing coefficient of n^3, we get

$$\frac{1}{3}\frac{b^2}{a^3} = \gamma$$

74. Answer (1)

Coefficient of middle term in $(1+x)^{20} = {}^{20}C_{10}$

Sum of coefficient of two middle terms in $(1+x)^{19} = {}^{19}C_9 + {}^{19}C_{10} = {}^{20}C_{10}$

75. Answer (210)

$$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10} = \left(\left(x^{1/3}+1\right) - \frac{\sqrt{x}+1}{\sqrt{x}}\right)^{10}$$

$$= \left(x^{1/3} - x^{-1/2}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r \cdot x^{\frac{10-r}{3}} \cdot x^{-\frac{r}{2}}$$

$$\because \frac{10-r}{3} = \frac{r}{2} \Rightarrow r = 4$$

put $r = 4$, $T_5 = {}^{10}C_4 = 210$

76. Answer (3)

$$T_{r+1} = {}^{10}C_r(x\sin\alpha)^{10-r}\left(\frac{a\cos\alpha}{x}\right)^r$$

r = 5 for term independent of x

$$\therefore \quad {}^{10}C_5 \sin^5\alpha\, a^5 \cos^5\alpha = \frac{10!}{(5!)^2}$$

$$\therefore \quad \frac{a^5}{2^5}\sin^5 2\alpha = 1$$

For greatest value to occur $\sin 2\alpha = 1$

and $\frac{a^5}{2^5} = 1 \Rightarrow a = 2$

77. Answer (3)

Let $10^{100} = x$

$$\left(1+\frac{1}{x}\right)^x \in (2, e)$$

Hence lowest integer 3

78. Answer (1)

The general term of $\left(2^{1/3}+3^{1/4}\right)^{12}$ is

$$T_{r+1} = {}^{12}C_r 2^{\frac{12-r}{3}} \cdot 3^{\frac{r}{4}}$$

For rational terms r = 0 and 12.

$\therefore$ Sum of rational terms

$$= {}^{12}C_0 \cdot 2^4 \cdot 3^0 + {}^{12}C_{12} \cdot 2^0 \cdot 3^3$$

$= 16 + 27$

$= 43$

79. Answer (55)

$${}^nC_7 \cdot 2^{n-7} \times \frac{1}{3^7} = {}^nC_8 \cdot 2^{n-8} \times \frac{1}{3^8}$$

$$\frac{{}^nC_8}{{}^nC_7} = 6$$

$$\frac{n-8+1}{8} = 6$$

n = 55

80. Answer (1)

General term of $\left(x^2+\frac{1}{bx}\right)^{11}$

$$T_{r+1} = {}^{11}C_r \left(x^2\right)^{11-r}\left(\frac{1}{bx}\right)^r = \frac{{}^{11}C_r}{b^r} x^{22-3r}$$

Coeff. of $x^7 = \frac{{}^{11}C_5}{b^5}$

Similarly general term of $\left(x-\frac{1}{bx^2}\right)^{11}$

$$T_{r+1} = {}^{11}C_r (x)^{11-r}\left(-\frac{1}{bx^2}\right)^r = \frac{{}^{11}C_r}{(-b)^r} x^{11-2r}$$

Coeff. of $x^{-7} = \frac{{}^{11}C_6}{b^6}$

$$\Rightarrow \quad b = \frac{{}^{11}C_6}{{}^{11}C_5} = 1$$

81. Answer (3)

Given expression reduces to

$$\left[\left(5^{2(x-1)}+7\right)^{\frac{1}{2}} + \left(5^{x-1}+1\right)^{-\frac{1}{8}}\right]^{10}$$

$${}^{10}C_8\left(5^{2(x-1)}+7\right)\left(5^{x-1}+1\right)^{-1} = 180$$

Let $5^{x-1} = t$

$(t^2 + 7)(t + 1)^{-1} = 4$

$t^2 + 7 = 4t + 4$

$t^2 - 4t + 3 = 0$

$(t - 3)(t - 1) = 0$

$5^{x-1} = 1$ or 3

$x = 1$ or $x = 1 + \log_5 3$

82. Answer (2)

$\because$ $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2 + \ldots + {}^{20}C_{20}x^{20}$

On differentiating both sides w.r.t. x we get :

$20(1+x)^{19} = 1 \cdot {}^{20}C_1 + 2 \cdot {}^{20}C_2 x + \ldots + 20 \cdot {}^{20}C_{20}x^{19}$

$\Rightarrow$ $20(1+x)^{19} x = 1 \cdot {}^{20}C_1 x + 2 \cdot {}^{20}C_2 x^2 + \ldots + 20 \cdot {}^{20}C_{20}x^{20}$

Again on differentiating w.r.t. x we get :

$20(1+x)^{19} + 20 \times 19 \cdot x(1+x)^{18} = 1^2 \cdot {}^{20}C_1 + 2^2 \cdot {}^{20}C_2 x + \ldots + 20^2 \cdot {}^{20}C_{20}x^{19}$

Replace x by 1 we get :

$1^2 \cdot {}^{20}C_1 + 2^2 \cdot {}^{20}C_2 + 3^2 \cdot {}^{20}C_3 + \ldots + 20^2 \cdot {}^{20}C_{20} = 20 \cdot 2^{19} + 20 \times 19 \cdot 2^{18}$

$$\therefore \quad \sum_{r=0}^{20} r^2\, {}^{20}C_r = 420 \times 2^{18}$$

83. Answer (49)

$$A_k = \sum_{k=0}^{9} {}^9C_i\, {}^{12}C_{12-k+i} + \sum_{k=0}^{8} {}^8C_i \cdot {}^{13}C_{13-k+i}$$

$$A_k = \sum_{k=0}^{9} {}^9C_i \cdot {}^{12}C_{k-i} + \sum_{i=0}^{8} {}^8C_i \cdot {}^{13}C_{k-i}$$

A_k = Coeff of x^k in $(1+x)^9 \cdot (1+x)^{12} + (1+x)^8 \cdot (1+x)^{13}$

$A_k = 2 \cdot {}^{21}C_k$

$$A_4 - A_3 = 2\left[{}^{21}C_4 - {}^{21}C_3\right]$$

$$= 2\left[\frac{21\times 20\times 19\times 18}{24} - \frac{21\times 20\times 19}{6}\right]$$

$$= 2\times 21\times 20\times 19\left[\frac{18}{24}-\frac{1}{6}\right] = 190\times 49$$

p = 49

84. Answer (4)

$$\sum_{k=0}^{20}\left({}^{20}C_k\right)^2=\left({}^{20}C_0\right)^2+\left({}^{20}C_1\right)^2+\left({}^{20}C_2\right)^2+.....$$

$$....+\left({}^{20}C_{20}\right)^2={}^{40}C_{20}$$

(Using bino-binomial series

$${}^nC_0^2+{}^nC_1^2+.....+{}^nC_n^2=2nC_n)$$

85. Answer (55)

In expansion of $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$,

General term

$$\Rightarrow\ T_{r+1}={}^{12}C_r\cdot\left(\frac{x}{4}\right)^{12-r}\left(-\frac{12}{x^2}\right)^r$$

$$T_{r+1}={}^{12}C_r\cdot\left(\frac{1}{4}\right)^{12-r}(-12)^r\cdot x^{12-3r}$$

term independent of x

$\Rightarrow\ 12-3r=0$

$\Rightarrow\ r=4$

$$\left(\frac{3^6}{4^4}\right)k={}^{12}C_4\left(\frac{1}{4}\right)^8(-12)^4$$

$$\left(\frac{3^6}{4^4}\right)k=\frac{12\times11\times10\times9}{24}\left(\frac{3}{4}\right)^4$$

$k=55$

86. Answer (315)

General term in expansion of $(a+2b+4ab)^{10}$

$$=\frac{10!a^p\cdot(2b)^q\cdot(4ab)^{10-p-q}}{p!q!(10-p-q)!}$$

$$=\frac{10!\cdot2^{q+20-2p-2q}}{p!\cdot q!(10-p-q)!}\cdot a^{10-q}\cdot b^{10-p}$$

for a^7b^8, $p=2$ & $q=3$

$$\Rightarrow\ \text{Coefficient of } a^7b^8=\frac{10!}{2!3!\cdot5!}\cdot2^{13}=K\cdot2^{16}$$

$\Rightarrow\ K=315$

87. Answer (924)

Sum of coeff. in $(x+y)^n=4096$

Put $x=y=1$ $\Rightarrow$ $2^n=2^{12}$ $\Rightarrow$ $n=12$

Greatest coeff. in $(x+y)^{12}$ = coeff. of middle term $={}^{12}C_6$

$$=\frac{12!}{6!\times6!}$$

$$=\frac{12\times11\times10\times9\times8\times7}{6\times5\times4\times3\times2\times1}$$

$=924$

88. Answer (4)

$(2021)^{3762}$

$=(2023-2)^{3762}=m(17)+2^{3762}$

$\{\because\ 2023=17\times119\}$

Where m(17) denotes "multiple of 17"

Required remainder = remainder on dividing 2^{3762} by 17.

Now $2^{3762}=4.16^{940}=4.(1-17)^{940}=m(17)+4$

Here required remainder is 4.

89. Answer (15)

$3\times7^{22}+2\times10^{22}-44$

$=3\times(1+6)^{22}+2(1+9)^{22}-44$

$$=3\left[{}^{22}C_0+{}^{22}C_1(6)+{}^{22}C_2(6)^2+....+{}^{22}C_{22}(6)^{22}\right]$$

$$+2\left[{}^{22}C_0+{}^{22}C_1(9)+.....+{}^{22}C_{22}(9)^{22}\right]-44$$

$=3.{}^{22}C_0+18k_1+2.{}^{22}C_0\cdot18k_2-44$

Remainder when divided by 18 = 3 + 2 – 44 = –39

Remainder = (–39 + 54) – 54 $\Rightarrow$ 15 – 54

= 15

90. Answer (98)

$$\because\ \sum_{r=0}^{n}(2r+1)\,{}^nC_r=2^{100}\cdot101$$

$$\Rightarrow\ 2n\sum_{r=1}^{n}{}^{n-1}C_{r-1}+\sum_{r=0}^{n}{}^nC_r=101\cdot2^{100}$$

$$\Rightarrow\ 2n\cdot2^{n-1}+2^n=101\cdot2^{100}$$

$$\Rightarrow\ (n+1)\cdot2^n=101\cdot2^{100}$$

$\Rightarrow$ n = 100

91. Answer (4)

$$1 + 3 + 3^2 + \dots + 3^{2021} = \frac{3^{2022}-1}{2}$$

$$= \frac{1}{2}\left\{(10-1)^{1011}-1\right\}$$

$$= \frac{1}{2}\{100k + 10110 - 1 - 1\}$$

$$= 50k_1 + 4$$

$\therefore$ Remainder = 4

92. Answer (286)

Given that $C_1 + 2\times3C_2 + 5\times3C_3 + \dots$ 10 terms

$$= \frac{\alpha.2^{11}}{2^{\beta}-1}\left(C_1 + \frac{C_2}{2} + \dots\right)$$

$$= \sum_{r=1}^{10} r(2r-1)C_r = \frac{\alpha.2^{11}}{2^{\beta}-1}\left(\sum_{r=1}^{10}\frac{C_r}{r}\right)$$

Using $C_1 + 2C_2 + \dots + nC_n = n.2^{n-1}$,

$1^2C_1 + 2^2C_2 + \dots + n^2C_n = n.2^{n-1} + n(n-1)2^{n-2}$

and $C_0 + \frac{C_1}{2} + \dots \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$ we get

$$\Rightarrow 2(10.2^9 + 10.9.2^8) - 10.2^9 = \frac{\alpha.2^{11}}{2^{\beta}-1}\frac{(2^{11}-1)}{11}$$

Comparing both side we get

$$2^{11}.25 = \frac{\alpha.2^{11}}{2^{\beta}-1}\frac{(2^{11}-1)}{11}$$

$\Rightarrow$ $\alpha = 25 \times 11 = 275$ & $\beta = 11$

$\Rightarrow$ $\alpha + \beta = 286$

93. Answer (1)

$$\text{Coeff. of } x^{101} \text{ in} \frac{x^{500}\left[\left(\frac{x+5}{x}\right)^{501}-1\right]}{\frac{x+5}{x}-1}$$

$$= \text{Coeff. of } x^{101} \text{ in } \frac{1}{5}\left[(x+5)^{501} - x^{501}\right]$$

$$= \frac{1}{5}\,{}^{501}C_{101}\cdot 5^{400}$$

$$= {}^{501}C_{101}\cdot 5^{399}$$

94. Answer (83)

$$T_{r+1} = 10_{C_r}\ (2x^3)^{10-r}\left(\frac{3}{x}\right)^r$$

$$= 10_{C_r}\ 2^{10-r}\ 3^r\ x^{30-4r}$$

So, $r \neq 8, 9, 10$

$$\text{Sum of required Coeff.} = \left(2.1^3 + \frac{3}{1}\right)^{10}$$

$$\left({}^{10}C_8 2^2 3^8 + {}^{10}C_9\, 2^1 3^9 + {}^{10}C_{10} 2^0 3^{10}\right)$$

$$= 5^{10} - 3^9\left(\frac{{}^{10}C_8\cdot 2^2}{3} + {}^{10}C_9\cdot 2^1 + {}^{10}C_{10}\cdot 3\right)$$

$$\beta = \frac{4}{3}\cdot{}^{10}C_8 + 20 + 3 = 83$$

95. Answer (3)

$2021 \equiv -2 \pmod 7$

$\Rightarrow$ $(2021)^{2023} \equiv -(2)^{2023} \pmod 7$

$\equiv -2(8)^{674} \pmod 7$

$\equiv -2(1)^{674} \pmod 7$

$\equiv -2 \pmod 7$

$\equiv 5 \pmod 7$

So when $(2021)^{2023}$ is divided by 7, remainder is 5.

96. Answer (102)

$${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \dots + {}^{60}C_{20}$$

$$= {}^{40}C_{40} + {}^{41}C_{40} + {}^{42}C_{40} + \dots + {}^{60}C_{40}$$

$$= {}^{61}C_{41}$$

$$= \frac{61}{41}\cdot{}^{60}C_{40}$$

$\therefore m = 61, n = 41$

$\therefore m + n = 102$

97. Answer (5)

$$T_{r+1} = {}^{60}C_r \left(x^{\frac{1}{2}}\right)^{60-r} \left(x^{-\frac{1}{3}}\right)^r \left(5^{\frac{-1}{4}}\right)^{60-r} \left(5^{\frac{1}{2}}\right)^r$$

for x^{10} $\dfrac{60-r}{2} - \dfrac{r}{3} = 10$

$\Rightarrow$ $180 - 3r - 2r = 60$

$\Rightarrow$ $r = 24$

$\therefore$ Coeff. of $x^{10} = \dfrac{{}^{60}C_{24}}{5^9} 5^{12} = 5^k\ l$

as l and 5 are coprime

$k = 3 +$ exponent of 5 in ${}^{60}C_{24}$

$$= 3 + \left(\left[\frac{60}{5}\right] + \left[\frac{60}{5^2}\right] - \left[\frac{24}{5}\right] - \left[\frac{24}{5^2}\right] - \left[\frac{36}{5}\right] - \left[\frac{36}{5^2}\right]\right)$$

$= 3 + (12 + 2 - 4 - 0 - 7 - 1)$

$= 3 + 2 = 5$

98. Answer (57)

$$\left(x^n + \frac{2}{x^5}\right)^7 = \sum_{r=0}^{7} {}^7C_r (x^n)^{7-r} \cdot \left(\frac{2}{x^5}\right)^r$$

$$= \sum_{r=0}^{7} {}^7C_r \cdot 2^r \cdot x^{7n-nr-5r}$$

$\therefore$ ${}^7C_0 \cdot 2^0 + {}^7C_1 \cdot 2^1 + {}^7C_2 \cdot 2^2 + {}^7C_3 \cdot 2^3 + {}^7C_4 \cdot 2^4$

$= 939$

$\therefore$ $r = 4$

$\because$ $7n - nr - 5r = 0$

and $r = 4$ then $n > \dfrac{20}{3}$

and r should not be 5

$\therefore$ $n < \dfrac{25}{2}$

$\therefore$ Possible values of n are 7, 8, 9, 10, 11, 12

$\therefore$ Sum of integral value of $n = 57$

99. Answer (1)

$$\sum_{k=1}^{31} {}^{31}C_k\ {}^{31}C_{k-1} - \sum_{k=1}^{30} {}^{30}C_k\ {}^{30}C_{k-1}$$

$$= \sum_{k=1}^{31} {}^{31}C_k \cdot {}^{31}C_{32-k} - \sum_{k=1}^{30} {}^{30}C_k \cdot {}^{30}C_{31-k}$$

$= {}^{62}C_{32} - {}^{60}C_{31}$

$$= \frac{60!}{31!29!}\left(\frac{62 \cdot 61}{32 \cdot 30} - 1\right) = \frac{60!}{31!29!} \frac{2822}{32 \cdot 30}$$

$\alpha = \dfrac{2822}{32}$ $\Rightarrow$ $16\alpha = 1411$

100. Answer (2)

$$T_{r+1} = {}^{12}C_r \left(2x^3\right)^{12-r} \left(\frac{3}{x^k}\right)^r$$

$= {}^{12}C_r 2^{12-r} 3^r\ x^{36-3r-kr}$

For constant term $36 - 3r - kr = 0$

$r = \dfrac{36}{3+k}$

So, k can be 1, 3, 6, 9, 15, 33
In order to get 2^8, check by putting values of k and corresponding in general term. By checking, it is possible only where $k = 3$ or 6

101. Answer (2)

$$\left(1 - x^2 + 3x^3\right)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}, x \neq 0$$

General term of $\left(\dfrac{5}{2}x^3 - \dfrac{1}{5x^2}\right)^{11}$ is

$$T_{r+1} = {}^{11}C_r \left(\frac{5}{2}x^3\right)^{11-r} \left(\frac{-1}{5x^2}\right)^r$$

$$= {}^{11}C_r \left(\frac{5}{2}\right)^{11-r} \left(\frac{-1}{5}\right)^r x^{33-5r}$$

So, term independent from x in given expression

$$= -{}^{11}C_7 \left(\frac{5}{2}\right)^4 \left(\frac{-1}{5}\right)^7 = \frac{11 \times 10 \times 9 \times 8}{24} \times \frac{1}{16 \times 125}$$

$$= \frac{33}{200}$$

102. Answer (4)

Constant term in

$$\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10} \rightarrow x^0$$

$\Rightarrow$ $(3x^8 - 2x^7 + 5)^{10} \rightarrow x^{50}$

General term of $(3x^8 - 2x^7 + 5)^{10}$ is

$$\frac{10!}{p!q!r!}(3x^8)^p(-2x^7)^q(5)^r$$

Here $8p + 7q = 50$ and $p + q + r = 10$

$\Rightarrow$ $p = 1$, $q = 6$, $r = 3$ is only valid solution

$$\therefore \quad \frac{10!}{1!6!r!}3^1\,2^6\cdot 5^3 = 2^k\cdot l$$

$$\Rightarrow \quad 5\cdot 3\cdot 7\cdot 5^3\cdot 3\cdot 2^9 = 2^k\, l$$

$\therefore$ $k = 9$

103. Answer (3)

$(1 + 8)^n - 8n - 1 = 64\alpha$

$$\Rightarrow \quad 1+8n+{}^nC_2 8^2+{}^nC_3 8^3+\ldots+{}^nC_n 8^n-8n-1 = 64\alpha$$

$$\Rightarrow \quad \alpha = {}^nC_2+{}^nC_3 8+{}^nC_4 8^2+\ldots+{}^nC_n 8^{n-2} \qquad \ldots(1)$$

Similarly

$(1 + 5)^n - 5n - 1 = 25\beta$

$$\Rightarrow \quad 1+5n+{}^nC_2 5^2+{}^nC_3 5^3+\ldots+{}^nC_n 5^n-5n-1 = 25\beta$$

$$\Rightarrow \quad \beta = {}^nC_2+{}^nC_3.5+{}^nC_4.5^2+\ldots+{}^nC_n 5^{n-2} \quad \ldots(2)$$

$$\alpha-\beta = {}^nC_3(8-5)+{}^nC_4(8^2-5^2)+\ldots+{}^nC_n(8^{n-2}-5^{n-2})$$

104. Answer (5)

$$T_{r+1} = {}^{15}C_r\left(2x^{1/5}\right)^{15-r}\left(\frac{-1}{x^{1/5}}\right)^r$$

For coefficient of x^{-1}

$$\frac{15-r}{5}-\frac{r}{5} = -1 \Rightarrow 15-2r = -5 \Rightarrow r = 10$$

$\therefore$ $m = {}^{15}C_{10}.2^5$

& for coefficient of x^{-3}

$15 - 2r = -15 \Rightarrow r = 15$

$\therefore$ $n = -{}^{15}C_{15}$

Given $mn^2 = {}^{15}C_r.2^r$

$\Rightarrow$ ${}^{15}C_{10}.2^5 = {}^{15}C_r.2^r$

$\Rightarrow$ $r = 5$

105. Answer (6006)

$$\text{General Term} = {}^{15}C_r\left(t^2\, x^{\frac{1}{5}}\right)^{15-r}\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^r$$

for term independent on t

$2(15 - r) - r = 0$

$\Rightarrow$ $r = 10$

$\therefore$ $T_{11} = {}^{15}C_{10}\, x(1 - x)$

Maximum value of $x(1 - x)$ occur at $x = \frac{1}{2}$

i.e., $(x(1-x))_{\max} = \frac{1}{4}$

$\Rightarrow$ $K = {}^{15}C_{10} \times \frac{1}{4}$

$\Rightarrow$ $8K = 2({}^{15}C_{10}) = 6006$

106. Answer (23)

Coefficient of x in $(1 + x)^p (1 - x)^q$

$$-{}^pC_0\,{}^qC_1 + {}^pC_1\,{}^qC_0 = -3 \Rightarrow \boxed{p-q=-3}$$

Coefficient of x^2 in $(1 + x)^p (1 - x)^q$

$${}^pC_0\,{}^qC_2 - {}^pC_1\,{}^qC_1 + {}^pC_2\,{}^qC_0 = -5$$

$$\frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} = -5$$

$$\frac{q^2-q}{2} - (q-3)q + \frac{(q-3)(q-4)}{2} = -5$$

$\Rightarrow$ $q = 11$, $p = 8$

Coefficient of x^3 in $(1 + x)^8 (1 - x)^{11}$ is

$$= -{}^{11}C_3 + {}^8C_1\,{}^{11}C_2 - {}^8C_2\,{}^{11}C_1 + {}^8C_3 = 23$$

107. Answer (1)

$(2021)^{2022} + (2022)^{2021}$

$= (7k - 2)^{2022} + (7k_1 - 1)^{2021}$

$= \left[(7k-2)^3\right]^{674} + (7k_1)^{2021} - 2021(7k_1)^{2020} + \ldots.. - 1$

$= (7k_2 - 1)^{674} + (7m - 1)$

$= (7n + 1) + (7m - 1) = 7(m + n)$

(multiple of 7)

$\therefore$ Remainder = 0

108. Answer (24)

$(3+6x)^n = 3^n(1+2x)^n$

If T_9 is numerically greatest term

$\therefore \quad T_8 \le T_9 \ge T_{10}$

${}^nC_7\, 3^{n-7}(6x)^7 \le {}^nC_8\, 3^{n-8}(6x)^8 \ge {}^nC_9\, 3^{n-9}(6x)^9$

$$\Rightarrow \quad \frac{n!}{(n-7)!7!}9 \le \frac{n!}{(n-8)!8!}3.(6x) \ge \frac{n!}{(n-9)!9!}(6x)^2$$

$$\Rightarrow \quad \underbrace{\frac{9}{(n-7)(n-8)} \le \frac{18\left(\frac{3}{2}\right)}{(n-8)8}}\ \underbrace{\ge \frac{36}{9.8}\,\frac{9}{4}}$$

$72 \le 27(n-7)$ and $27 \ge 9(n-8)$

$\frac{29}{3} \le n$ and $n \le 11$

$\therefore \quad n_0 = 10$

For $(3+6x)^{10}$

$T_{r+1} = {}^{10}C_r \quad 3^{10-r}(6x)^r$

For coeff. of x^6

$r = 6 \Rightarrow {}^{10}C_6 3^4.6^6$

For coeff. of x^3

$r = 3 \Rightarrow {}^{10}C_3 3^7.6^3$

$$\therefore \quad k = \frac{{}^{10}C_6}{{}^{10}C_3}.\frac{3^4.6^6}{3^7.6^3} = \frac{10!\ 7!\ 3!}{6!\ 4!\ 10!}.8$$

$\Rightarrow \quad k = 14$

$\therefore \quad k + n_0 = 24$

109. Answer (3)

Let $E = 7^{2022} + 3^{2022}$

$= (15-1)^{1011} + (10-1)^{1011}$

$= -1 +$ (multiple of 15) $-1 +$ multiple of 10

$= -2 +$ (multiple of 5)

Hence remainder on dividing E by 5 is 3.

110. Answer (57)

Coefficients of middle terms of given expansions are ${}^4C_2 \frac{1}{6}\beta^2,\ {}^2C_1(-3\beta),\ {}^6C_3\left(\frac{-\beta}{2}\right)^3$ form an A.P.

$$\therefore \quad 2.2(-3\beta) = \beta^2 - \frac{5\beta^3}{2}$$

$\Rightarrow \ -24 = 2\beta - 5\beta^2$

$\Rightarrow \ 5\beta^2 - 2\beta - 24 = 0$

$\Rightarrow \ 5\beta^2 - 12\beta + 10\beta - 24 = 0$

$\Rightarrow \ \beta(5\beta - 12) + 2(5\beta - 12) = 0$

$\beta = \frac{12}{5}$

$d = -6\beta - \beta^2$

$$\therefore \ 50 - \frac{2d}{\beta^2} = 50 - 2\frac{(-6\beta - \beta^2)}{\beta^2} = 50 + \frac{12}{\beta} + 2 = 57$$

111. Answer (99)

$I = 1 + (1 + {}^{49}C_0 + {}^{49}C_1 + \ldots + {}^{49}C_{49})({}^{50}C_2 + {}^{50}C_4 + \ldots + {}^{50}C_{50})$

As ${}^{49}C_0 + {}^{49}C_1 + \ldots + {}^{49}C_{49} = 2^{49}$

and ${}^{50}C_0 + {}^{50}C_2 + \ldots + {}^{50}C_{50} = 2^{49}$

$\Rightarrow \ {}^{50}C_2 + {}^{50}C_4 + \ldots + {}^{50}C_{50} = 2^{49} - 1$

$\therefore \ I = 1 + (2^{49}+1)(2^{49}-1)$

$= 2^{98}$

$\therefore \ m = 1$ and $n = 98$

$m + n = 99$

112. Answer (221)

$$\sum_{K=1}^{10} K^2\,({}^{10}C_K)^2 = 1^2\ {}^{10}C_1^2 + 2^2\ {}^{10}C_2^2 + \ldots + 10^2\ {}^{10}C_{10}$$

Let $(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + \ldots + {}^{10}C_{10} x^{10}$

$\Rightarrow 10(1+x)^9 = {}^{10}C_1 + 2\cdot{}^{10}C_2 x + \ldots + 10\cdot{}^{10}C_{10} x^9 \ldots(1)$

Similarly, $10(x+1)^9 = 10\cdot{}^{10}C_0 x^9 + 9.{}^{10}C_1 x^8 + \ldots + 1.{}^{10}C_9$

$100(1+x)^{18}$ has required term with coefficient of x^9

i.e. ${}^{18}C_9\ 100 = 22000\ L$

$\Rightarrow \ L = 221$

113. Answer (4)

$$\text{Re}\left(\frac{(11)^{1011}+(1011)^{11}}{9}\right)=\text{Re}\left(\frac{2^{1011}+3^{11}}{9}\right)$$

For $\text{Re}\left(\frac{2^{1011}}{9}\right)$

$$\begin{aligned}2^{1011}=(9-1)^{337} &= {}^{337}C_0 9^{337}(-1)^0\\ &+{}^{337}C_1 9^{336}(-1)^1\\ &+{}^{337}C_2 9^{335}(-1)^2+\ldots\ldots\\ &+{}^{337}C_{337} 9^0(-1)^{337}\end{aligned}$$

so, remainder is 8

and $\text{Re}\left(\frac{3^{11}}{9}\right)=0$

So, remainder is 8

114. Answer (A*)

$$\sum_{\substack{i,j=0\\ i\neq j}}^{n} {}^nC_i\,{}^nC_j=\sum_{i,j=0}^{n} {}^nC_i\,{}^nC_j-\sum_{i=j}^{n} {}^nC_i\,{}^nC_j$$

$$=\sum_{j=0}^{n} {}^nC_i \sum_{j=0}^{n} {}^nC_j-\sum_{i=0}^{n} {}^nC_i\,C_i$$

$$=2^n\cdot 2^n-{}^{2n}C_n$$

$$=2^{2n}-{}^{2n}C_n$$

115. Answer (4)

$3^{2022}=(10-1)^{1011}={}^{1011}C_0(10)^{1011}(-1)^0+{}^{1011}C_1(10)^{1010}(-1)^1+\ldots..+{}^{1011}C_{1010}(10)^1(-1)^{1010}+{}^{1011}C_{1011}(10)^0(-1)^{1011}$

$=5k-1$, where $k\in I$

So when divided by 5, it leaves remainder 4.

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