Chapter 6

Gravitation

1. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is **[AIEEE-2009]**

   (1) $\frac{R}{\sqrt{2}}$ (2) $\frac{R}{2}$

   (3) $\sqrt{2}R$ (4) $2R$

2. Two particles of equal mass m go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is **[AIEEE-2011]**

   (1) $\sqrt{\frac{Gm}{2R}}$ (2) $\sqrt{\frac{Gm}{R}}$

   (3) $\sqrt{\frac{Gm}{4R}}$ (4) $\sqrt{\frac{Gm}{3R}}$

3. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be **[AIEEE-2012]**

   (1) 6.4×10^8 Joules

   (2) 6.4×10^9 Joules

   (3) 6.4×10^{10} Joules

   (4) 6.4×10^{11} Joules

4. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$? **[JEE (Main)-2013]**

   (1) $\frac{5GmM}{6R}$ (2) $\frac{2GmM}{3R}$

   (3) $\frac{GmM}{2R}$ (4) $\frac{GmM}{3R}$

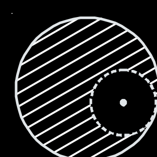
5. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is **[JEE (Main)-2014]**

   (1) $\sqrt{\frac{GM}{R}}$ (2) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

   (3) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (4) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

6. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is

   (G = gravitational constant) **[JEE (Main)-2015]**



   (1) $\frac{-GM}{2R}$ (2) $\frac{-GM}{R}$

   (3) $\frac{-2GM}{3R}$ (4) $\frac{-2GM}{R}$

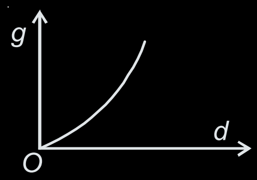
7. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; $h \ll R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.) **[JEE (Main)-2016]**

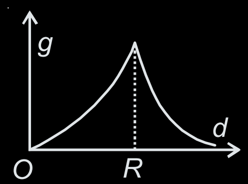
   (1) $\sqrt{gR}$ (2) $\sqrt{gR/2}$

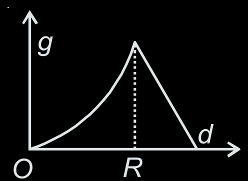
   (3) $\sqrt{gR}(\sqrt{2}-1)$ (4) $\sqrt{2gR}$

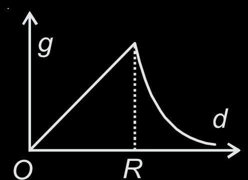
8. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) :

[JEE (Main)-2017]

(1) 

(2) 

(3) 

(4) 

9. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is

[JEE (Main)-2019]

(1) $\frac{L}{m}$ (2) $\frac{4L}{m}$

(3) $\frac{L}{2m}$ (4) $\frac{2L}{m}$

10. The energy required to take a satellite to a height 'h' above Earth surface (radius of Earth $= 6.4 \times 10^3$ km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is

[JEE (Main)-2019]

(1) 3.2×10^3 km (2) 1.6×10^3 km

(3) 1.28×10^4 km (4) 6.4×10^3 km

11. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is **[JEE (Main)-2019]**

(1) $2mv^2$ (2) mv^2

(3) $\frac{1}{2}mv^2$ (4) $\frac{3}{2}mv^2$

12. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is

(Take Gravitational constant **[JEE (Main)-2019]**

$G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2})

(1) 2.8×10^5 m/s (2) 1.4×10^5 m/s

(3) 2.4×10^4 m/s (4) 3.8×10^4 m/s

13. A satellite is revolving in a circular orbit at a height h from the earth surface, such that $h << R$ where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of eath is **[JEE (Main)-2019]**

(1) $\sqrt{2gR}$ (2) $\sqrt{gR}(\sqrt{2}-1)$

(3) $\sqrt{gR}$ (4) $\sqrt{\frac{gR}{2}}$

14. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2 s. The period of oscillation of the same pendulum on the planet would be

[JEE (Main)-2019]

(1) $\frac{2}{\sqrt{3}}$ s (2) $\frac{3}{2}$ s

(3) $\frac{\sqrt{3}}{2}$ s (4) $2\sqrt{3}$ s

15. A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force it exerts on a point mass 'm' at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by

[JEE (Main)-2019]

(1) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$

(2) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$

(3) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

(4) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$

16. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be **[JEE (Main)-2019]**

(1) Such that it escapes to infinity

(2) In a circular orbit of a different radius

(3) In an elliptical orbit

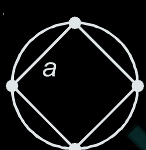
(4) In the same circular orbit of radius R

17. Two satellites, A and B, have masses m and 2 m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2 R around the earth. The ratio of their kinetic energies, $\frac{T_A}{T_B}$ is **[JEE (Main)-2019]**

(1) 1 (2) $\frac{1}{2}$

(3) 2 (4) $\sqrt{\frac{1}{2}}$

18. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of others' gravitational field in a circular orbit circumscribing the square? **[JEE (Main)-2019]**



(1) $1.41\sqrt{\frac{GM}{a}}$ (2) $1.16\sqrt{\frac{GM}{a}}$

(3) $1.21\sqrt{\frac{GM}{a}}$ (4) $1.35\sqrt{\frac{GM}{a}}$

19. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon. **[JEE (Main)-2019]**

(1) $\frac{E}{64}$ (2) $\frac{E}{4}$

(3) $\frac{E}{16}$ (4) $\frac{E}{32}$

20. A solid sphere of mass M and radius a is surrounded by a uniform concentric spherical shell of thickness $2a$ and mass $2M$. The gravitational field at distance $3a$ from the centre will be

[JEE (Main)-2019]

(1) $\frac{GM}{9a^2}$ (2) $\frac{2GM}{9a^2}$

(3) $\frac{GM}{3a^2}$ (4) $\frac{2GM}{3a^2}$

21. A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r)=\frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T

[JEE (Main)-2019]

(1) T/R is a constant (2) TR is a constant

(3) T/R^2 is a constant (4) T^2/R^3 is a constant

22. The value of acceleration due to gravity at Earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to : (Radius of earth = 6.4×10^6 m) **[JEE (Main)-2019]**

(1) 9.0×10^6 m (2) 6.4×10^6 m

(3) 1.6×10^6 m (4) 2.6×10^6 m

23. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet? **[JEE (Main)-2019]**

[Given: Mass of planet = 8×10^{22} kg,

Radius of planet = 2×10^6 m,

Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2]

(1) 17 (2) 13

(3) 11 (4) 9

24. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density) **[JEE (Main)-2019]**

(1) $\frac{R}{4}$ (2) $\frac{R}{3}$

(3) $\frac{R}{9}$ (4) $\frac{R}{2}$

25. A stellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R(R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth) **[JEE (Main)-2020]**

(1) $5m\left(u^2 - \frac{119}{200}\frac{GM}{R}\right)$

(2) $\frac{3m}{8}\left(u + \sqrt{\frac{5GM}{6R}}\right)^2$

(3) $\frac{m}{20}\left(u - \sqrt{\frac{2GM}{3R}}\right)^2$

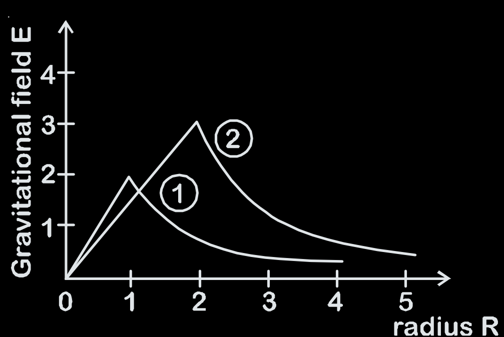
(4) $\frac{m}{20}\left(u^2 + \frac{113}{200}\frac{GM}{R}\right)$

26. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take g = 10 ms^{-2} at the north pole and the radius of the earth = 6400 km) **[JEE (Main)-2020]**

(1) 194.66 N (2) 195.66 N

(3) 195.32 N (4) 194.32 N

27. Consider two solid spheres of radii R_1 = 1 m, R_2 = 2 m and masses M_1 and M_2, respectively. The gravitational field due to sphere 1 and 2 are shown. The value of $\frac{M_1}{M_2}$ is



[JEE (Main)-2020]

(1) $\frac{1}{6}$ (2) $\frac{1}{2}$

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

28. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A. The collision is completely inelastic. Then the combined body **[JEE (Main)-2020]**

(1) Escapes from the Planet's Gravitational field

(2) Continues to move in a circular orbit

(3) Falls vertically downwards towards the planet

(4) Starts moving in an elliptical orbit around the planet

29. Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is **[JEE (Main)-2020]**

(1) 1 (2) 4

(3) 3 (4) 2

30. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as **[JEE (Main)-2020]**

(1) $T^2 \propto \frac{1}{R^3}$ (2) $T^2 \propto R$

(3) $T \propto R$ (4) $T^2 \propto R^3$

31. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected) **[JEE (Main)-2020]**

(1) $\frac{\sqrt{5}R - R}{2}$

(2) $\frac{\sqrt{3}R - R}{2}$

(3) $\frac{R}{2}$

(4) $\frac{\sqrt{5}}{2}R - R$

32. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is **[JEE (Main)-2020]**

(1) $3R_e$ (2) $4R_e$

(3) $2.5R_e$ (4) $2R_e$

33. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at **[JEE (Main)-2020]**

(1) $r = \frac{1}{\sqrt{3}}R$ (2) $r = \sqrt{\frac{3}{4}}R$

(3) $r = \sqrt{\frac{5}{9}}R$ (4) r = R

34. On the x-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is **[JEE (Main)-2020]**

(1) $A(x^2+a^2)^{3/2}$ (2) $\frac{A}{(x^2+a^2)^{3/2}}$

(3) $\frac{A}{(x^2+a^2)^{1/2}}$ (4) $A(x^2+a^2)^{1/2}$

35. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is **[JEE (Main)-2020]**

(1) 1 (2) $\sqrt{2}$

(3) 2 (4) $\frac{1}{\sqrt{2}}$

36. The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals **[JEE (Main)-2020]**

(1) $\frac{5}{9}$ (2) $\frac{1}{3}$

(3) $\frac{7}{9}$ (4) $\frac{4}{9}$

37. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is ($h << R$, where R is the radius of the earth) **[JEE (Main)-2020]**

(1) $\frac{R^2\omega^2}{2g}$ (2) $\frac{R^2\omega^2}{g}$

(3) $\frac{R^2\omega^2}{8g}$ (4) $\frac{R^2\omega^2}{4g}$

38. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is **[JEE (Main)-2020]**

(1) 1 : 6 (2) 1 : 2

(3) 3 : 4 (4) 1 : 3

39. Two planets have masses M and $16M$ and their radii are a and $2a$, respectively. The separation between the centres of the planets is $10a$. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is **[JEE (Main)-2020]**

(1) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$ (2) $\sqrt{\frac{GM^2}{ma}}$

(3) $2\sqrt{\frac{GM}{a}}$ (4) $4\sqrt{\frac{GM}{a}}$

40. An asteroid is moving directly towards the centre of the earth. When at a distance of 10 R (R is the radius of the earth) from the earths centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/s)? Give your answer to the nearest integer in kilometer/s _______. **[JEE (Main)-2020]**

41. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8 hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is: **[JEE (Main)-2021]**

(1) 1 : 4 (2) 1 : 8

(3) 2 : 1 (4) 8 : 1

42. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be : **[JEE (Main)-2021]**

(1) $\frac{\sqrt{(1+2\sqrt{2})G}}{2}$ (2) $\sqrt{G(1+2\sqrt{2})}$

(3) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$ (4) $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$

43. Two stars of masses m and 2m at a distance d rotate about their common centre of mass in free space. The period of revolution is:

[JEE (Main)-2021]

(1) $2\pi\sqrt{\frac{d^3}{3Gm}}$ (2) $2\pi\sqrt{\frac{3Gm}{d^3}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$

44. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator? **[JEE (Main)-2021]**

[Use $g = \frac{GM}{R^2}$ = 9.8 ms^{-2} and radius of earth, R = 6400 km.]

(1) 49.83 N

(2) 49 N

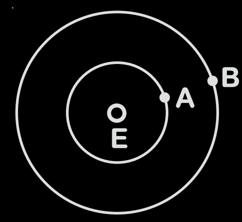
(3) 48.83 N

(4) 49.17 N

45. Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively.

If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:

[JEE (Main)-2021]



[Given : radius of earth = 6400 km, mass of earth = 6 × 10^{24} kg]

(1) 4.24 × 10^2 s

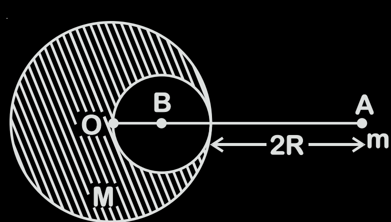
(2) 1.33 × 10^3 s

(3) 4.24 × 10^3 s

(4) 3.33 × 10^2 s

46. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is :

[JEE (Main)-2021]



(1) 36 : 25

(2) 50 : 41

(3) 41 : 50

(4) 25 : 36

47. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R : The product of their mass and radius must be same. $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below : **[JEE (Main)-2021]**

(1) A is not correct but R is correct

(2) Both A and R are correct but R is NOT the correct explanation of A

(3) A is correct but R is not correct

(4) Both A and R are correct and R is the correct explanation of A

48. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be ______. **[JEE (Main)-2021]**

49. A planet revolving in elliptical orbit has :

A. a constant velocity of revolution.

B. has the least velocity when it is nearest to the sun.

C. its areal velocity is directly proportional to its velocity.

D. areal velocity is inversely proportional to its velocity.

E. to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below : **[JEE (Main)-2021]**

(1) A only

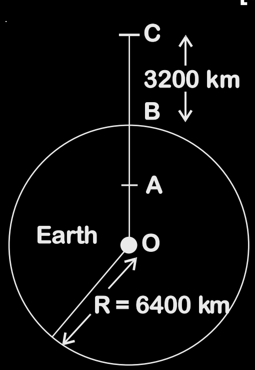
(2) E only

(3) D only

(4) C only

50. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be x : y. The value of x is________.

[JEE (Main)-2021]



51. The maximum and minimum distances of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m respectively. If the speed of the comet at the nearest point is 6×10^4 ms $^{-1}$, the speed at the farthest point is: **[JEE (Main)-2021]**

(1) 6.0×10^3 m/s

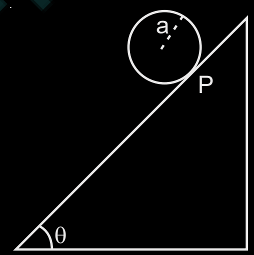
(2) 3.0×10^3 m/s

(3) 4.5×10^3 m/s

(4) 1.5×10^3 m/s

52. A solid disc of radius 'a' and mass 'm' rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b} g \sin\theta$ where b is ______. (Round off to the Nearest Integer)

(g = acceleration due to gravity, θ = angle as shown in figure.) **[JEE (Main)-2021]**



53. The radius in kilometer to which the present radius of earth (R = 6400 km) to be compressed so that the escape velocity is increased 10 times is ______. **[JEE (Main)-2021]**

54. A geostationary satellite is orbiting around an arbitrary planet 'P' at a height of 11R above the surface of 'P', R being the radius of 'P'. The time period of another satellite in hours at a height of 2R from the surface of 'P' is _____. 'P' has the time period of 24 hours. **[JEE (Main)-2021]**

(1) 5 (2) 3

(3) $6\sqrt{2}$ (4) $\frac{6}{\sqrt{2}}$

55. The time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 9R is : **[JEE (Main)-2021]**

(1) 12 T

(2) 27 T

(3) 9 T

(4) 3 T

56. If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately:

[Take g = 10 ms^{-2}, the radius of earth, R = 6400 × 10^3 m, Take π = 3.14] **[JEE (Main)-2021]**

(1) 60 minutes

(2) Does not change

(3) 84 minutes

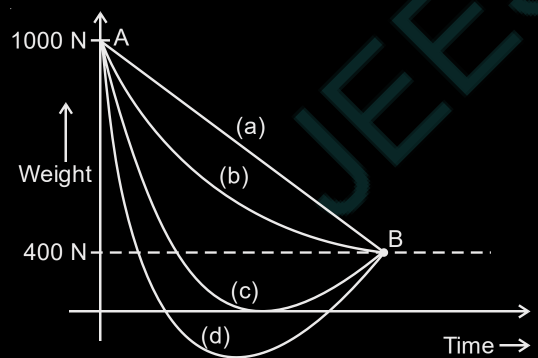
(4) 1200 minutes

57. The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is: **[JEE (Main)-2021]**

(1) $\frac{L}{2M}$ (2) $\frac{L}{M}$

(3) $\frac{4L}{M}$ (4) $\frac{2L}{M}$

58. A person whose mass is 100 kg travels from Earth to Mars in a spaceship. Neglect all other objects in sky and take acceleration due to gravity on the surface of the Earth and Mars as 10 m/s^2 and 4 m/s^2 respectively. Identify from the below figures, the curve that fits best for the weight of the passenger as a function of time. **[JEE (Main)-2021]**



(1) (a) (2) (c)

(3) (d) (4) (b)

59. Consider a binary star system of star A and star B with masses m_A and m_B revolving in a circular orbit of radii r_A and r_B, respectively. If T_A and T_B are the time period of star A and star B, respectively, then: **[JEE (Main)-2021]**

(1) $T_A = T_B$ (2) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{\frac{3}{2}}$

(3) $T_A > T_B$ (if $r_A > r_B$) (4) $T_A > T_B$ (if $m_A > m_B$)

60. A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02R. The percentage difference in the time periods of the two satellites is **[JEE (Main)-2021]**

(1) 1.5 (2) 2.0

(3) 0.7 (4) 3.0

61. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height h is ________s. **[JEE (Main)-2021]**

(1) $\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}}-1\right]$

(2) $\frac{1}{3}\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}}-1\right]$

(3) $\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}}-1\right]$

(4) $\frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}}-1\right]$

62. The minimum and maximum distances of a planet revolving around the Sun are x_1 and x_2. If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be: **[JEE (Main)-2021]**

(1) $\frac{v_0 x_2^2}{x_1^2}$ (2) $\frac{v_0 x_1}{x_2}$

(3) $\frac{v_0 x_1^2}{x_2^2}$ (4) $\frac{v_0 x_2}{x_1}$

63. Consider a planet in some solar system which has a mass double the mass of earth and density equal to the average density of earth. If the weight of an object on earth is W, the weight of the same object on that planet will be: **[JEE (Main)-2021]**

(1) 2 W (2) $2^{\frac{1}{3}}$ W

(3) $\sqrt{2}$ W (4) W

64. Two identical particles of mass 1 kg each go round a circle of radius R, under the action of their mutual gravitational attraction. The angular speed of each particle is: **[JEE (Main)-2021]**

(1) $\sqrt{\dfrac{G}{2R^3}}$ (2) $\sqrt{\dfrac{2G}{R^3}}$

(3) $\dfrac{1}{2R}\sqrt{\dfrac{1}{G}}$ (4) $\dfrac{1}{2}\sqrt{\dfrac{G}{R^3}}$

65. The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital radius of 9.0×10^3 km. Find the mass of Mars.

$$\left\{\text{Given}\ \frac{4\pi^2}{G} = 6\times10^{11}\ N^{-1}m^{-2}\ kg^2\right\}$$

[JEE (Main)-2021]

(1) 3.25×10^{21} kg (2) 5.96×10^{19} kg

(3) 6.00×10^{23} kg (4) 7.02×10^{25} kg

66. A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is:

[JEE (Main)-2021]

(1) +2G (2) –60G

(3) –20G (4) –4G

67. The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle to these two masses.

[JEE (Main)-2021]

(1) $V = \dfrac{1}{2}\sqrt{\dfrac{4G(M_1+M_2)}{r}}$

(2) $V = \dfrac{\sqrt{2G}\ (M_1+M_2)}{r}$

(3) $V = \sqrt{\dfrac{4G(M_1+M_2)}{r}}$

(4) $V = \dfrac{1}{2}\sqrt{\dfrac{2G(M_1+M_2)}{r}}$

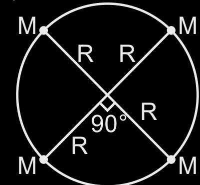
68. If R_E be the radius of Earth, then the ratio between the acceleration due to gravity at a depth 'r' below and a height 'r' above the earth surface is:

(Given : $r < R_E$) **[JEE (Main)-2021]**

(1) $1-\dfrac{r}{R_E}-\dfrac{r^2}{R_E^2}-\dfrac{r^3}{R_E^3}$ (2) $1+\dfrac{r}{R_E}+\dfrac{r^2}{R_E^2}+\dfrac{r^3}{R_E^3}$

(3) $1+\dfrac{r}{R_E}-\dfrac{r^2}{R_E^2}-\dfrac{r^3}{R_E^3}$ (4) $1+\dfrac{r}{R_E}-\dfrac{r^2}{R_E^2}+\dfrac{r^3}{R_E^3}$

69. Four particles each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction as shown in figure. The speed of each particle is: **[JEE (Main)-2021]**



(1) $\dfrac{1}{2}\sqrt{\dfrac{GM}{R(2\sqrt{2}+1)}}$ (2) $\dfrac{1}{2}\sqrt{\dfrac{GM}{R}(2\sqrt{2}-1)}$

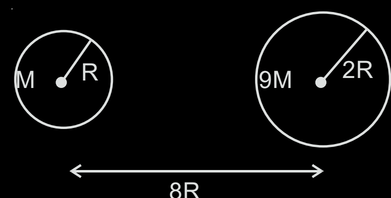
(3) $\sqrt{\dfrac{GM}{R}}$ (4) $\dfrac{1}{2}\sqrt{\dfrac{GM}{R}(2\sqrt{2}+1)}$

70. Two satellites revolve around a planet in coplanar circular orbits in anticlockwise direction. Their period of revolutions are 1 hour and 8 hours respectively. The radius of the orbit of nearer satellite is 2×10^3 km. The angular speed of the farther satellite as observed from the nearer satellite at the instant when both the satellites are closest is $\dfrac{\pi}{x}$ rad h^{-1} where x is _____.

[JEE (Main)-2021]

71. Suppose two planets (spherical in shape) of radii R and 2R, but mass M and 9 M respectively have a centre to centre separation 8 R as shown in the figure. A satellite of mass '*m*' is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed '*v*' required for the satellite to reach the surface of the second planet is $\sqrt{\dfrac{a}{7}\dfrac{GM}{R}}$ then the value of 'a' is ______. **[JEE (Main)-2021]**

[Given : The two planets are fixed in their position]



72. The approximate height from the surface of earth at which the weight of the body becomes $\frac{1}{3}$ of its weight on the surface of earth is

[Radius of earth $R = 6400$ km and $\sqrt{3} = 1.732$]

[JEE (Main)-2022]

(1) 3840 km

(2) 4685 km

(3) 2133 km

(4) 4267 km

73. The height of any point P above the surface of earth is equal to diameter of earth. The value of acceleration due to gravity at point P will be (Given g = acceleration due to gravity at the surface of earth).

[JEE (Main)-2022]

(1) $\frac{g}{2}$ (2) $\frac{g}{4}$

(3) $\frac{g}{3}$ (4) $\frac{g}{9}$

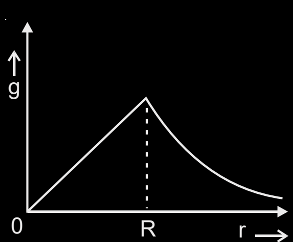
74. Two satellites S_1 and S_2 are revolving in circular orbits around a planet with radius R_1 = 3200 km and R_2 = 800 km respectively. The ratio of speed of satellite S_1 to the speed of satellite S_2 in their respective orbits would be $\frac{1}{x}$ where x =

[JEE (Main)-2022]

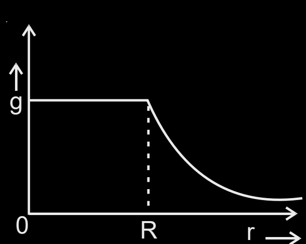
75. The variation of acceleration due to gravity (g) with distance (r) from the center of the earth is correctly represented by

(Given R = radius of earth) **[JEE (Main)-2022]**

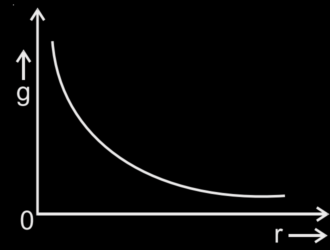
(1)



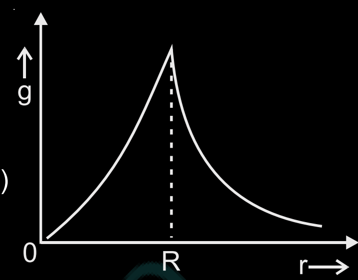
(2)



(3)



(4)



76. Given below are two statements

Statement-I: The law of gravitation holds good for any pair of bodies in the universe.

Statement-II: The weight of any person becomes zero when the person is at the centre of the earth.

In the light of the above statements, choose the **correct** answer from the options given below

[JEE (Main)-2022]

(1) Both **Statement I** and **Statement II** are true

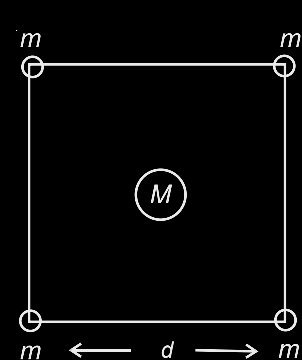
(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is true but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

77. Four spheres each of mass m form a square of side d (as shown in figure). A fifth sphere of mass M is situated at the centre of square. The total gravitational potential energy of the system is:

[JEE (Main)-2022]



(1) $-\frac{Gm}{d}\left[\left(4+\sqrt{2}\right)m+4\sqrt{2}M\right]$

(2) $-\frac{Gm}{d}\left[\left(4+\sqrt{2}\right)M+4\sqrt{2}m\right]$

(3) $-\frac{Gm}{d}\left[3m^2+4\sqrt{2}M\right]$

(4) $-\frac{Gm}{d}\left[6m^2+4\sqrt{2}M\right]$

78. Two planets *A* and *B* of equal mass are having their period of revolutions T_A and T_B such that $T_A = 2T_B$. These planets are revolving in the circular orbits of radii r_A and r_B respectively. Which out of the following would be the correct relationship of their orbits? **[JEE (Main)-2022]**

(1) $2r_A^2 = r_B^3$

(2) $r_A^3 = 2r_B^3$

(3) $r_A^3 = 4r_B^3$

(4) $T_A^2 - T_B^2 = \frac{\pi^2}{GM}\left(r_B^3 - 4r_A^3\right)$

79. Two objects of equal masses placed at certain distance from each other attracts each other with a force of *F*. If one-third mass of one object is transferred to the other object, then the new force will be **[JEE (Main)-2022]**

(1) $\frac{2}{9}F$ (2) $\frac{16}{9}F$

(3) $\frac{8}{9}F$ (4) F

80. The escape velocity of a body on a planet '*A*' is 12 kms^{-1}. The escape velocity of the body on another planet '*B*', whose density is four times and radius is half of the planet '*A*', is: **[JEE (Main)-2022]**

(1) 12 kms^{-1}

(2) 24 kms^{-1}

(3) 36 kms^{-1}

(4) 6 kms^{-1}

81. The time period of a satellite revolving around earth in a given orbit is 7 hours. If the radius of orbit is increased to three times its previous value, then approximate new time period of the satellite will be

[JEE (Main)-2022]

(1) 40 hours (2) 36 hours

(3) 30 hours (4) 25 hours

82. Three identical particles *A*, *B* and *C* of mass 100 kg each are placed in a straight line with *AB* = *BC* = 13 m. The gravitational force on a fourth particle *P* of the same mass is *F*, when placed at a distance 13 m from the particle *B* on the perpendicular bisector of the line *AC*. The value of *F* will be approximately

[JEE (Main)-2022]

(1) 21*G* (2) 100*G*

(3) 59*G* (4) 42*G*

83. The length of a seconds pendulum at a height *h* = 2*R* from earth surface will be

(Given *R* = Radius of earth and acceleration due to gravity at the surface of earth, $g = \pi^2$ ms^{-2})

[JEE (Main)-2022]

(1) $\frac{2}{9}$ m (2) $\frac{4}{9}$ m

(3) $\frac{8}{9}$ m (4) $\frac{1}{9}$ m

84. An object is taken to a height above the surface of earth at a distance $\frac{5}{4}R$ from the centre of the earth. Where radius of earth, *R* = 6400 km. The percentage decrease in the weight of the object will be

[JEE (Main)-2022]

(1) 36% (2) 50%

(3) 64% (4) 25%

85. The percentage decrease in the weight of a rocket, when taken to a height of 32 km above the surface of earth will, be:

(Radius of earth = 6400 km) **[JEE (Main)-2022]**

(1) 1% (2) 3%

(3) 4% (4) 0.5%

86. Two satellites A and B, having masses in the ratio 4 : 3, are revolving in circular orbits of radii $3r$ and $4r$ respectively around the earth. The ratio of total mechanical energy of A to B is **[JEE (Main)-2022]**

(1) 9 : 16 (2) 16 : 9

(3) 1 : 1 (4) 4 : 3

87. A body of mass m is projected with velocity $\lambda\ v_e$ in vertically upward direction from the surface of the earth into space. It is given that v_e is escape velocity and $\lambda < 1$. If air resistance is considered to be negligible, then the maximum height from the centre of earth, to which the body can go, will be:

(R : radius of earth) **[JEE (Main)-2022]**

(1) $\frac{R}{1+\lambda^2}$ (2) $\frac{R}{1-\lambda^2}$

(3) $\frac{R}{1-\lambda}$ (4) $\frac{\lambda^2 R}{1-\lambda^2}$

88. Assume there are two identical simple pendulum clocks. Clock-1 is placed on the earth and Clock-2 is placed on a space station located at a height h above the earth surface. Clock – 1 and Clock-2 operate at time periods 4 s and 6 s respectively. Then the value of h is

(consider radius of earth R_E = 6400 km and g on earth 10 m/s^2) **[JEE (Main)-2022]**

(1) 1200 km (2) 1600 km

(3) 3200 km (4) 4800 km

89. If the acceleration due to gravity experienced by a point mass at a height h above the surface of earth is same as that of the acceleration due to gravity at a depth αh ($h << R_e$) from the earth surface. The value of α will be ________.

(Use R_e = 6400 km) **[JEE (Main)-2022]**

90. An object of mass 1 kg is taken to a height from the surface of earth which is equal to three times the radius of earth. The gain in potential energy of the object will be

[If, g = 10 ms^{-2} and radius of earth = 6400 km]

[JEE (Main)-2022]

(1) 48 MJ (2) 24 MJ

(3) 36 MJ (4) 12 MJ

91. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: If we move from poles to equator, the direction of acceleration due to gravity of earth always points towards the center of earth without any variation in its magnitude.

Reason R: At equator, the direction of acceleration due to the gravity is towards the center of earth.

In the light of above statements, choose the correct answer from the options given below

[JEE (Main)-2022]

(1) Both A and R are true and R is the correct explanation of A.

(2) Both A and R are true but R is NOT the correct explanation of A.

(3) A is true but R is false.

(4) A is false but R is true.

92. A body is projected vertically upwards from the surface of earth with a velocity equal to one third of escape velocity. The maximum height attained by the body will be:

(Take radius of earth = 6400 km and g = 10 ms^{-2})

[JEE (Main)-2022]

(1) 800 km (2) 1600 km

(3) 2133 km (4) 4800 km

93. The distance between Sun and Earth is R. The duration of year if the distance between Sun and Earth becomes $3R$ will be : **[JEE (Main)-2022]**

(A) $\sqrt{3}$ years (B) 3 years

(C) 9 years (D) $3\sqrt{3}$ years

❑ ❑ ❑

Chapter 6

Gravitation

1. Answer (4)

As, $g(h) = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$

$\Rightarrow \dfrac{g}{9} = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$

$\Rightarrow \left(1+\dfrac{h}{R}\right) = 3$

$\Rightarrow \dfrac{h}{R} = 2 \Rightarrow \boxed{h = 2R}$

2. Answer (3)

$$\frac{mv^2}{R} = \frac{Gm^2}{(2R)^2}$$

$$v = \sqrt{\frac{Gm}{4R}}$$

3. Answer (3)

4. Answer (1)

At surface, $E = -\dfrac{GMm}{R}$

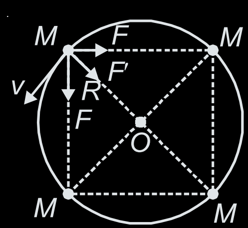
In orbit, $E = -\dfrac{GMm}{2(3R)} = -\dfrac{GMm}{6R}$

$\Rightarrow$ Required energy = $\dfrac{5\,GMm}{6\,R}$

5. Answer (4)

$$\frac{F}{\sqrt{2}} + \frac{F}{\sqrt{2}} + F' = \frac{Mv^2}{R}$$

$$\frac{2\times GM^2}{\sqrt{2}(R\sqrt{2})^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$$



$$\frac{GM^2}{R}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right] = Mv^2$$

$$v = \sqrt{\frac{Gm}{R}\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right)} = \frac{1}{2}\sqrt{\frac{Gm}{R}\left(1+2\sqrt{2}\right)}$$

6. Answer (2)

$V = V_1 - V_2$

$$V_1 = -\frac{GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$V_2 = -\frac{3G\left(\dfrac{M}{8}\right)}{2\left(\dfrac{R}{2}\right)}$$

$$\Rightarrow V = \frac{-GM}{R}$$

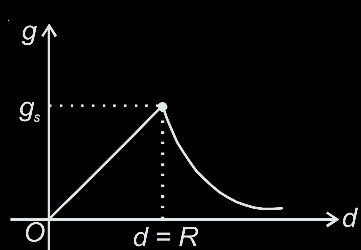
7. Answer (3)

$V_0 = \sqrt{gR}$

$V_e = \sqrt{2gR}$

$\Delta V = (\sqrt{2}-1)\sqrt{gR}$

8. Answer (4)



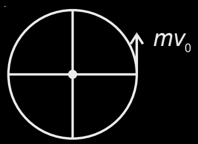
Variation of g inside earth surface

$d < R = g = \dfrac{Gm}{R^2}\cdot d$

$d = R = g_s = \dfrac{Gm}{R^2}$

$d > R = g = \dfrac{Gm}{d^2}$

9. Answer (3)



$\therefore\ mv_0R = L$

$\therefore\ v_0 = \dfrac{L}{mR}$

$\therefore\ T = \dfrac{2\pi R}{v_0}$

Areal velocity = $\dfrac{\pi R^2}{T}$

$\Rightarrow \dfrac{dA}{dt} = \dfrac{\pi R^2 v_0}{2\pi R} = \dfrac{Rv_0}{2} = \dfrac{L}{2m}$

10. Answer (1)

$$E_1 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}$$

$$E_1 = \frac{GMm}{R} - \frac{GMm}{(R+h)}$$

$$E_1 = \frac{GMmh}{R(R+h)}$$

$$E_2 = \frac{1}{2}\frac{GMm}{(R+h)}$$

Given $E_1 = E_2$

$\dfrac{h}{R} = \dfrac{1}{2}, h = \dfrac{R}{2}$

11. Answer (2)

$$U = -2\times\frac{1}{2}mv^2$$

In order to escape $U + K = 0$

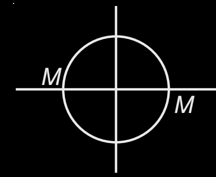
$\Rightarrow K = mv^2$

12. Answer (1)

$2R = d = 2 \times 10^{11}$ m

$\Rightarrow\ R = 10^{11}$ m

$\therefore\ V(\text{at centre}) = -\dfrac{GM}{R}\times 2$



$\therefore$ Let v_0 be the minimum speed of meteorite

then $\dfrac{1}{2}mv_0^2 - \dfrac{2GMm}{R} = 0$

$$v_0 = \sqrt{\frac{4GM}{R}} = \sqrt{\frac{4\times 6.67\times 10^{-11}\times 3\times 10^{31}}{10^{11}}}$$

$\Rightarrow\ v_0 = 2.83 \times 10^5$ m/s $\approx 2.8 \times 10^5$ m/s

13. Answer (2)

$v_0 = \sqrt{2gR}$

$v_e = \sqrt{2gR}$

$\Delta v = \sqrt{gR}\,(\sqrt{2}-1)$

14. Answer (4)

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

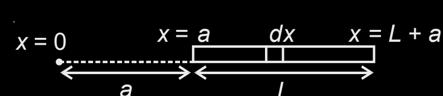
$$\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$$

$$T_2 = 2\sqrt{\frac{g_1}{g_2}}$$

$$g_2 = \frac{3GM}{(3R)^2} = \frac{g}{3}$$

$T_2 = 2\sqrt{3}$ s

15. Answer (3)

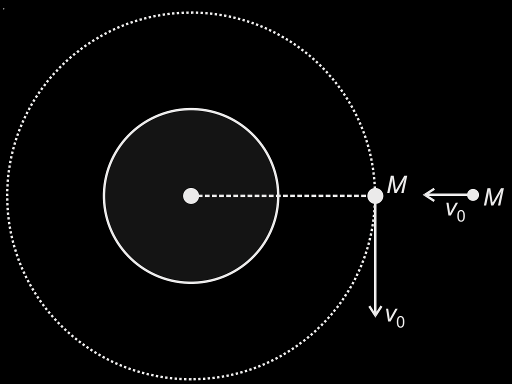


$$dF = -Gm\int_a^{L+a}\frac{(A+Bx^2)dx}{x^2}$$

$$F = -Gm\left[-A\left[\frac{1}{L+a}-\frac{1}{a}\right]+BL\right]$$

$$= -Gm\left[A\left[\frac{1}{a}-\frac{1}{a+L}\right]+BL\right]$$

16. Answer (3)



$$v_0 = \sqrt{\frac{GM}{R}}$$

After collision

$$mv_0(-\hat{j}) + mv_0(-\hat{i}) = 2m\vec{v}$$

$$\vec{v} = -\frac{v_0}{2}\hat{i} - \frac{v_0}{2}\hat{j}$$

$$|\vec{v}| = \frac{v_0}{\sqrt{2}} = 0.7\, v_0$$

$\because$ v < v_0

$\therefore$ The path will be elliptical.

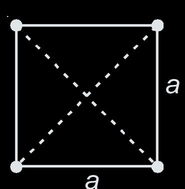
17. Answer (1)

$$\frac{V^2}{r} = \frac{GM}{r^2}$$

$$\Rightarrow \quad V^2 = \frac{GM}{r}$$

$$\frac{T_A}{T_B} = \frac{\frac{1}{2}mV_A^2}{\frac{1}{2}2mV_B^2} = \frac{1}{2}\left(\frac{V_A}{V_B}\right)^2 = \frac{1}{2}\frac{R_B}{R_A} = 1$$

18. Answer (2)



$$r = \frac{a}{\sqrt{2}}, F = \frac{GM^2}{a^2}(\sqrt{2}) + \frac{GM^2}{2a^2}$$

$$\therefore \quad \frac{Mv^2}{\left(\frac{a}{\sqrt{2}}\right)} = \frac{GM^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$\Rightarrow \quad v^2 = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right)} = 1.16\sqrt{\frac{GM}{a}}$$

19. Answer (3)

$$E = \frac{GM_E m}{R_E}$$

$$E' = \frac{GM_m m}{R_M}$$

$$\rho R_E^3 = 64\,\rho R_M^3$$

$$\Rightarrow \quad R_E = 4\, R_M$$

$$\frac{E'}{E} = \frac{M_M}{M_E}.\frac{R_E}{R_M} = \frac{1}{64}.4 = \frac{1}{16}$$

$$\Rightarrow \quad E' = \frac{E}{16}$$

20. Answer (3)

$$E = \frac{GM}{(3a)^2} + \frac{2GM}{(3a)^2}$$

$$E = \frac{GM}{3a^2}$$

21. Answer (1)

$$M_{(\text{in})} = \int_0^R 4\pi r^2 dr \cdot \frac{k}{r^2}$$

$$M_{(\text{in})} = 4\pi KR$$

$$\therefore \quad G \cdot \frac{4\pi KR}{R^2} = \frac{V^2}{R} \quad \Rightarrow \quad v = \sqrt{4\pi GK}$$

$$\therefore \quad T = \frac{2\pi R}{v} = \frac{2\pi \cdot R}{\sqrt{4\pi GK}}$$

$$\therefore \quad \frac{T}{R} = \text{constant}$$

22. Answer (4)

Given

$$g_{\text{height}} = \frac{g_{\text{surface}}}{2} = 4.9 \text{ m/s}^2$$

$$\text{As} \quad g_h = g\left(1 + \frac{h}{R_e}\right)^{-2}$$

$$h = R_e\left(\sqrt{2}-1\right)$$

$h = 6400 \times 0.414$

$h = 2649.6$ km

$h = 2.6 \times 10^6$ m

23. Answer (3)

$$T = \frac{2\pi r}{v}, v = \sqrt{\frac{GM}{r}}$$

$$T = 2\pi r\sqrt{\frac{r}{GM}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$T = 2\pi\sqrt{\frac{(202)^3 \times 10^{12}}{6.67 \times 10^{-11} \times 8 \times 10^{22}}} \quad \text{sec}$$

$T = 7812.2$ s

$T \simeq 2.17$ hr $\Rightarrow$ 11 revolutions

24. Answer (4)

$$\frac{g_e}{g_p} = \frac{9}{4}$$

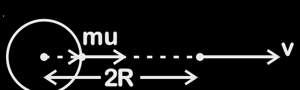
$$\Rightarrow \quad \frac{M_e \times R_P^2}{R_e^2 \times M_p} = \frac{9}{4}$$

$$\Rightarrow \quad 9 \times \left(\frac{R_p}{R_e}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \quad R_p = \frac{R_e}{2}$$

$$\therefore \quad R_p = \frac{R}{2}$$

25. Answer (1)



$$\frac{1}{2}mu^2 + \frac{-GMm}{R} = \frac{1}{2}mv^2 + \frac{-GMm}{2R}$$

$$\Rightarrow \frac{1}{2}m\left(v^2 - u^2\right) = \frac{-GMm}{2R}$$

$$\Rightarrow v = \sqrt{u^2 - \frac{GM}{R}} \qquad \text{...(i)}$$

$$v_0 = \sqrt{\frac{GM}{2R}} \qquad \therefore \; v_{\text{rad}} = \frac{m \times v}{\left(\frac{m}{10}\right)} = 10\,v$$

$$\therefore \; \frac{9m}{10} \times \sqrt{\frac{GM}{2R}} = \frac{m}{10} \times v_\tau \;\Rightarrow v_\tau^2 = 81\frac{GM}{2R}$$

$$\therefore \; KE_{\text{rocket}} = \frac{1}{2} \times \frac{m}{10} \times \left((u^2 - \frac{GM}{R})100 + 81\frac{GM}{2R}\right)$$

$$= \frac{m}{20} \times 100\left(u^2 - \frac{GM}{R} + \frac{81}{200}\frac{GM}{R}\right)$$

$$= 5m\left(u^2 - \frac{119}{200}\frac{GM}{R}\right)$$

26. Answer (3)

Weight at equator = mg′

$= m(g - \omega^2 R)$

mg = 196 N $\Rightarrow$ m = 19.6 kg

$\Rightarrow$

$$mg' = 19.6\left[10 - \left(\frac{2\pi}{24 \times 3600}\right)^2 \times 6400 \times 10^3\right] \text{N}$$

= 19.6[10 – 0.034] = 195.33 N

27. Answer (1)

From the diagram

Gravitation field at the surface

$$E = \frac{Gm}{r^2}$$

$$\therefore \quad E_1 = \frac{Gm_1}{r_1^2}$$

and $E_2 = \dfrac{Gm_2}{r_2^2}$

$$\therefore \quad \frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2\left(\frac{m_1}{m_2}\right)$$

$$\therefore \quad \frac{2}{3} = \left(\frac{2}{1}\right)^2\left(\frac{m_1}{m_2}\right)$$

$$\Rightarrow \quad \left(\frac{m_1}{m_2}\right) = \frac{1}{6}$$

28. Answer (4)

$$\vec{p}_i = \vec{p}_f$$

$$\Rightarrow \quad v_f = \frac{\left(mv + \frac{mv}{4}\right)}{\frac{3m}{2}} = \frac{5v}{6}$$

$v_f < v_i \Rightarrow$ Path will be elliptical

29. Answer (2)

$$V = \sqrt{\frac{2GM}{R}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{M_1R_2}{M_2R_1}}$$

30. Answer (2)

$$m\omega^2 R = \frac{GMm}{R^2}$$

$$M = \int_0^R 4\pi r^2 dr\left(\frac{K}{r}\right) = 2\pi K R^2$$

$\omega^2 R = 2G\pi K$

$T^2 \propto R$

31. Answer (1)

$$\frac{GM}{(h+R)^2} = \frac{GM}{R^2}\left(1 - \frac{h}{R}\right)$$

$$R^3 = (h+R)^2(R-h)$$

Solving $h = \frac{\sqrt{5}-1}{2}R$

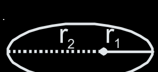
32. Answer (1)

Total energy $E = \frac{-GMm}{2a}$

$$\therefore \quad \frac{1}{2}m \times \left(\frac{2}{3}\frac{GM}{R}\right) - \frac{GMm}{R} = \frac{-GMm}{2a}$$

$\Rightarrow$ a = 2R

Major axis = 4R



$r_1 = R_1 \Rightarrow r_2 = 3R$

33. Answer (3)

$$\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$$

$$\therefore \quad m = \int_0^r \rho(r) \times 4\pi r^2 dr$$

$$= 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$$

$$\therefore \quad E = \frac{Gm}{r^2} = 4\pi\rho_0\left[\frac{r}{3} - \frac{r^3}{5R^2}\right]$$

$$\therefore \quad \frac{dE}{dr} = 0 \quad \Rightarrow r = \sqrt{\frac{5}{9}}\,R$$

34. Answer (3)

$$\int_0^V dV = -\int_\infty^x E \cdot dx$$

$$V = -A\int_\infty^x \frac{x\,dx}{(x^2+a^2)^{3/2}}$$

$$V = \frac{A}{(x^2+a^2)^{1/2}}$$

35. Answer (4)

$$v_o = \sqrt{\frac{GM}{R}}$$

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$\frac{u_o}{u_e} = \frac{1}{\sqrt{2}}$$

36. Answer (1)

$$g\left(h = \frac{R}{2}\right) = \frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{4g}{9}$$

$$g\left(1 - \frac{d}{R}\right) = \frac{4g}{9}$$

$$d = \frac{5R}{9}$$

37. Answer (1)

$g_{equator} = g - \omega^2 R$

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

$$g\left(\frac{2h}{R}\right) = \omega^2 R$$

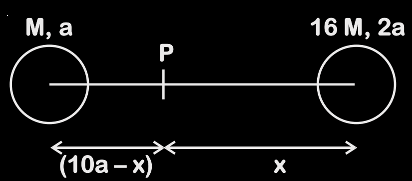
$$h = \frac{\omega^2 R^2}{2g}$$

38. Answer (1)

$$\frac{V_{max}}{V_{min}} = \frac{(1+e)}{(1-e)}$$

$$\frac{r_{max}}{r_{min}} = \frac{(1+e)}{1-e} = 6$$

39. Answer (1)



Velocity should be given so as to reach a point where field is zero.

$$\frac{16\,M}{x^2} = \frac{M}{(10a - x)^2}$$

x = 8a

By CoE

$$-\frac{GMm}{8a} - \frac{16\,GMm}{2a} + \frac{1}{2}mv^2 = -\frac{16\,GMm}{8a} - \frac{GMm}{2a}$$

$$v = \frac{3}{2}\sqrt{\frac{5\,Gm}{a}}$$

40. Answer (16.00)

$$-\frac{GM_E m}{10R} + \frac{1}{2}mV_0^2 = -\frac{GM_E m}{R} + \frac{1}{2}mV^2$$

$$\Rightarrow \frac{GM_E}{R}\left(1 - \frac{1}{10}\right) + \frac{V_0^2}{2} = \frac{V^2}{2}$$

$$V^2 = V_0^2 + \frac{9}{5}gR$$

$$\Rightarrow V = \sqrt{V_0^2 + \frac{9}{5}gR} \approx 16 \text{ km/s}$$

41. Answer (4)

$$\frac{\omega_1}{\omega_2} = \left(\frac{2\pi}{T_1}\right) \times \left(\frac{T_2}{2\pi}\right) = \frac{T_2}{T_1} = \frac{8}{1}$$

42. Answer (1)

Centripetal force is being provided by gravitational force, F_g

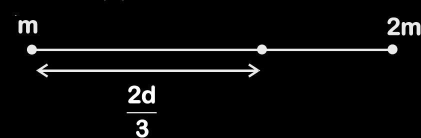
$$F_g = \frac{\sqrt{2}\,Gm^2}{(\sqrt{2}\,r)^2} + \frac{Gm^2}{4r^2}$$

$$\frac{mv^2}{r} = \frac{Gm^2}{\sqrt{2}\,r^2} + \frac{Gm^2}{4r^2}$$

$$v^2 = \frac{G}{\sqrt{2}} + \frac{G}{4}$$

$$v = \frac{\sqrt{(1 + 2\sqrt{2})G}}{2}$$

43. Answer (1)



$$m\omega^2\left(\frac{2d}{3}\right) = \frac{2Gm^2}{d^2}$$

$$\omega^2 = \frac{3Gm}{d^3}$$

$$\omega = \sqrt{\frac{3Gm}{d^3}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{3Gm}{d^3}}$$

$$T = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

44. Answer (3)

$g' = g - \omega^2 R$

Weight at equator will decrease.

45. Answer (2)

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$\therefore \quad T_B - T_A = \frac{2\pi}{\sqrt{GM}}\left[\left(8\times10^6\right)^{3/2} - \left(7\times10^6\right)^{3/2}\right]$$

$$= \frac{2\pi}{\sqrt{6.67\times10^{-11}\times6\times10^{24}}}\times10^9\left[8^{3/2} - 7^{3/2}\right]$$

≈ 1300 s

46. Answer (2)

$$F_1 = \frac{GMm}{(3R)^2} = \frac{GMm}{9R^2}$$

$$F_2 = \frac{GMm}{(3R)^2} - \frac{G\left(\frac{M}{8}\right)m}{\left(\frac{5R}{2}\right)^2} = \frac{41}{450}\frac{GMm}{R^2}$$

$$\therefore \quad \frac{F_1}{F_2} = \frac{1\times450}{9\times41} = \frac{50}{41}$$

47. Answer (3)

$$V_{es} \propto \sqrt{\frac{GM}{R}}$$

48. Answer (10)

$$-\frac{GM_e m}{R} + \frac{1}{2}mv^2 = -\frac{Gm_e m}{11R}$$

$$v = \sqrt{\frac{20}{11}\frac{6M_e}{R}}$$

$$v_e = \sqrt{\frac{26M_e}{R}}$$

$$v = \sqrt{\frac{10}{11}}.V_e$$

49. Answer (2)

$\frac{dA}{dt}$ = constant according to Kepler's law

50. Answer (04.00)

$$g_C = \frac{GM}{\left(R + \frac{R}{2}\right)^2} = \frac{4}{9}g_0$$

$$g_A = g_C \Rightarrow \frac{4}{9}g_0 = g_0\left(1 - \frac{AB}{R}\right)$$

$$\Rightarrow AB = \frac{5R}{9}$$

$$\Rightarrow OA = \frac{4R}{9}$$

$$\Rightarrow \frac{OA}{AB} = \frac{4}{5}$$

$\Rightarrow x = 04.00$

51. Answer (2)

$\therefore \quad mv_1r_1 = mv_2r_2$

$\Rightarrow 6 \times 10^4 \times 8 \times 10^{10} = v_2 \times 1.6 \times 10^{12}$

$$\Rightarrow v_2 = \frac{6\times8}{1.6}\times10^2$$

$= 3 \times 10^3$ m/s

52. Answer (3)

$$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}} = \frac{2}{3}g\sin\theta$$

53. Answer (64)

$$v_e = \sqrt{\frac{2GM}{R}} \quad 10v_e = \sqrt{\frac{2GM}{R'}}$$

$$\Rightarrow R' = \frac{R}{100} = 64 \text{ km}$$

54. Answer (2)

$T^2 \propto R^3$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} = \frac{1}{64}$$

$$T_1 = \frac{(24)}{(64)^{1/2}} = 3 \text{ hrs.}$$

55. Answer (2)

$T^2 \propto R^3$

$$\Rightarrow \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

$$\Rightarrow \frac{T^2}{T_2^2} = \frac{R^3}{729R^3}$$

$\Rightarrow T_2 = 27$ T

56. Answer (3)

$g = g_0 - R\omega^2$

$0 = g_0 - R\omega^2$

$$\Rightarrow \omega = \sqrt{\frac{g_0}{R}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g_0}}$$

$= 83.73$ minutes

57. Answer (1)

$$\frac{dA}{dt} = \frac{L}{2M} \text{ standard result}$$

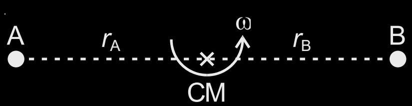
58. Answer (2)

Weight of the person becomes zero only once.

Hence, option (2) is correct

59. Answer (1)

$$T_A = \frac{2\pi}{\omega}$$



$$T_B = \frac{2\pi}{\omega}$$

$\Rightarrow \ T_A = T_B$

60. Answer (4)

$T^2 = CR^3$

$$2\frac{dT}{T} = 3\frac{dR}{R}$$

$$\frac{dT}{T} = \frac{3}{2}\times\frac{0.02R}{R}$$

$= 0.03$

So % difference in the time period

$= 0.03 \times 100 = 3\%$

61. Answer (4)

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0 \Rightarrow v = \sqrt{\frac{2GM}{r}}$$

$$\frac{dr}{dt} = \sqrt{\frac{2GM}{r}}$$

$$\Rightarrow \int_{R_e}^{(R_e+h)} \sqrt{r}\, dr = \int_0^t \sqrt{2GM}\, dt$$

$$\Rightarrow \frac{2}{3}\left[(R_e+h)^{3/2} - R_e^{3/2}\right] = (t)\sqrt{2GM}$$

$$\Rightarrow t = \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$$

62. Answer (4)

$\vec{L}_i = \vec{L}_f$

$v_0x_2 = vx_1$

$$\Rightarrow \quad v = \frac{v_0x_2}{x_1}$$

63. Answer (2)

$$g_{\text{eff}} = \frac{GM}{R^2}$$

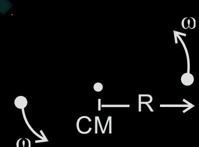
$$2\times\rho\times\frac{4}{3}\pi R_1^3 = \frac{4}{3}\pi R_2^3\times\rho$$

$$R_2 = 2^{1/3}R$$

$$g_1 = \frac{GM}{R^2}$$

$$g_2 = \frac{2GM}{[2^{1/3}R]^2} = \frac{2^{1/3}GM}{R^2}$$

64. Answer (4)



$$\frac{GM^2}{(2R)^2} = M\omega^2R$$

$$\frac{GM}{4R^3} = \omega^2$$

$$\Rightarrow \quad \omega = \frac{1}{2}\sqrt{\frac{GM}{R^3}}$$

$$= \frac{1}{2}\sqrt{\frac{G}{R^3}}$$

Option 4 is correct.

65. Answer (3)

$$m\omega^2r = \frac{GMm}{r^2}$$

$$\omega^2 = \frac{GM}{r^3}$$

$$\frac{4\pi^2}{T^2} = \frac{GM}{r^3}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{6\times10^{11}\times729\times10^{18}}{T^2}$$

$M = 6.0 \times 10^{23}$ kg

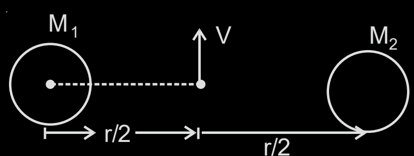
66. Answer (4)

$$V = -\frac{GM}{R} - \frac{Gm}{r}$$

$$= \frac{(-G)50}{25} - \frac{(G)(100)}{50}$$

$= -4G$

67. Answer (3)



$$-\frac{GM_1 m}{\frac{r}{2}} + \frac{-GM_2 m}{\frac{r}{2}} + \frac{1}{2}mV^2 = 0$$

$$\frac{V^2}{2} = \frac{2G}{r}(M_1 + M_2)$$

$$V = \sqrt{\frac{4G}{r}(M_1 + M_2)}$$

68. Answer (3)

at a depth r

$$a_1 = \frac{Gm}{R_E^3}\times(R_E - r) = \frac{GM}{R_E^2}\left(1 - \frac{r}{R_E}\right)$$

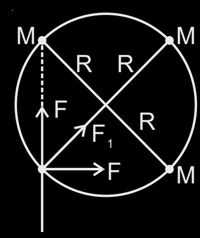
at a height r

$$a_2 = \frac{Gm}{(R_E + r)^2} = \frac{GM}{R_E^2\left(1 + \frac{r}{R_E}\right)^2}$$

$$\therefore \quad \frac{a_1}{a_2} = \left(1 - \frac{r}{R_E}\right)\left(1 + \frac{r}{R_E}\right)^2$$

$$= 1 + \frac{r}{R_E} - \frac{r^2}{R_E^2} - \frac{r^3}{R_E^3}$$

69. Answer (4)



$$F = \frac{GM^2}{(R\sqrt{2})^2},\ F_1 = \frac{GM^2}{(2R)^2}$$

$F_R = (2F\cos45° + F_1)$

$$\left(\frac{2F}{\sqrt{2}} + F_1\right) = \frac{mv^2}{R}$$

$$\Rightarrow v = \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}$$

70. Answer (3)

$$\omega = \frac{v_{rel}}{r_{rel}}$$

$$= \frac{2\pi\left(\frac{R}{T}\right) - 2\pi\left(\frac{4R}{8T}\right)}{3R}$$

$$\omega = \frac{\pi}{3T}$$

$$\omega = \frac{\pi}{3}\ \text{rad h}^{-1}$$

71. Answer (4)

$$-\frac{GMm}{2R} - \frac{9GMm}{6R} = \frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{9GMm}{7R}$$

$$\frac{1}{2}mv^2 = \frac{16\ GM}{7R} - \frac{2GM}{R}$$

$$\frac{1}{2}mv^2 = \frac{2\ GM}{7R}$$

$$v = \sqrt{\frac{4GM}{7R}}$$

72. Answer (2)

According to the given information

$$\frac{GM}{(R+h)^2}=\frac{1}{3}\times\frac{GM}{R^2}$$

$$\Rightarrow \quad R+h=\sqrt{3}\,R$$

$$\Rightarrow \quad h=\left(\sqrt{3}-1\right)R \simeq 4685 \text{ km}$$

73. Answer (4)

$h = 2R$

$$\therefore \quad g'=\frac{GM}{(R+h)^2}$$

$$=\frac{GM}{9R^2}$$

$$=\frac{g}{9}$$

74. Answer (2)

$$v=\sqrt{\frac{GM}{R}}$$

$$\Rightarrow \quad \frac{v_1}{v_2}=\sqrt{\frac{R_2}{R_1}}$$

$$\frac{v_2}{v_1}=\sqrt{\frac{3200}{800}}=2$$

$$\Rightarrow \quad \frac{v_1}{v_2}=\frac{1}{2}$$

$x = 2$

75. Answer (1)

For $r < R$ $g=\frac{Gmr}{R^3}=Cr$ (C = Constant)

For $r > R$ $g=\frac{Gm}{r^2}=\frac{C'}{r^2}$ (C' = Constant)

For the above equations the best suited graph is as given in option (1)

76. Answer (1)

Statement-I is true as law of gravitation is a universal law

Statement-II is also true as gravitational field at centre of earth is zero.

77. Answer (1)

Total gravitational potential energy

$$=-\left\{\frac{4GMm}{d/\sqrt{2}}+\frac{4Gm^2}{d}+\frac{2Gm^2}{\sqrt{2}d}\right\}$$

$$=-\frac{Gm}{d}\left\{M4\sqrt{2}+(4+\sqrt{2})m\right\}$$

$$=-\frac{Gm}{d}\left\{4\sqrt{2}M+(4+\sqrt{2})m\right\}$$

78. Answer (3)

$T_A = 2T_B$

Now $T_A^2 \propto r_A^3$

$$\Rightarrow \quad \left(\frac{r_A}{r_B}\right)^3=\left(\frac{T_A}{T_B}\right)^2$$

$$\Rightarrow \quad r_A^3=4r_B^3$$

79. Answer (3)

Let the masses are m and distance between them is l, then $F=\frac{Gm^2}{l^2}$.

When 1/3rd mass is transferred to the other then masses will be $\frac{4m}{3}$ and $\frac{2m}{3}$. So new force will be

$$F'=\frac{G\frac{4m}{3}\times\frac{2m}{3}}{l^2}=\frac{8}{9}\frac{Gm^2}{l^2}=\frac{8}{9}F$$

80. Answer (1)

$$v_{esc}=\sqrt{\frac{2GM}{R}}=\sqrt{\frac{2G}{R}\times\rho\times\frac{4}{3}\pi R^3}$$

$$\Rightarrow \quad v_{esc}\propto R\sqrt{\rho}$$

$$\Rightarrow \quad \frac{(v_{esc})_B}{(v_{esc})_A}=1$$

$$\Rightarrow \quad (v_{esc})_B = 12 \text{ km/s}$$

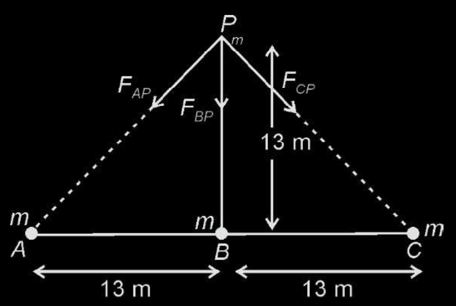
81. Answer (2)

$$T_2^2 = \left(\frac{R_2}{R_1}\right)^3 T_1^2$$

$$\Rightarrow \quad T_2 = (3)^{3/2} \times 7 \approx 5.2 \times 7$$

$$T_2 \cong 36 \text{ hrs.}$$

82. Answer (2)



m = 100 kg

$$F_{AP} = \frac{Gm^2}{(13\sqrt{2})^2}$$

$$F_{BP} = \frac{Gm^2}{13^2}$$

$$F_{CP} = \frac{Gm^2}{(13\sqrt{2})^2}$$

$F_{\text{net}} = F_{BP} + F_{AP}\cos 45° + F_{CP}\cos 45°$

$$= \frac{Gm^2}{13^2}\left(1 + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{G100^2}{169}(1 + 0.707)$$

$$\simeq 100G$$

83. Answer (4)

$$g = \frac{GM}{(R+h)^2} = \frac{GM}{9R^2} = \frac{g_0}{9}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{\frac{g_0}{9}}}$$

$$\Rightarrow \quad 2 = 2\pi\sqrt{\frac{9\ell}{g_0}}$$

$$\Rightarrow \quad \ell = \frac{g_0}{9\pi^2} = \frac{1}{9} \text{ m}$$

84. Answer (1)

$w = mg$

$$w' = \frac{mg}{\left(1 + \frac{h}{R}\right)^2} = \frac{mg}{\left(\frac{5}{4}\right)^2} = \frac{16}{25} mg$$

$\therefore$ % decrease in weight

$$= \left(1 - \frac{16}{25}\right) \times 100\%$$

$= 36\%$

85. Answer (1)

$$\because \quad g = \frac{GM}{r^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = 2\frac{\Delta r}{r}$$

$$\Rightarrow \quad \frac{\Delta g}{g} \times 100 = 2 \times \frac{32}{6400} \times 100\% = 1\%$$

$\Rightarrow$ % decrease in weight = 1%

86. Answer (2)

$$U = -\frac{GM_e m}{2r}$$

So, $\dfrac{U_A}{U_B} = \dfrac{m_A}{m_B} \times \dfrac{r_B}{r_A}$

$$= \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$$

87. Answer (2)

Using energy conservation

$$-\frac{GM_e m}{R_e} + \frac{1}{2} m\left(\lambda\sqrt{\frac{2GM_e}{R_e}}\right)^2 = -\frac{GM_e m}{r}$$

$$\frac{GM_e m}{r} = \frac{GM_e m}{R_e} - \frac{GM_e m}{R_e}\lambda^2$$

$$r = \frac{R_e}{1 - \lambda^2}$$

88. Answer (3)

$T \propto \sqrt{1/g}$

$$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \frac{R}{R+h}$$

$$\frac{4}{6} = \frac{R}{R+h}$$

$\Rightarrow h = R/2$

$= 3200$ km

89. Answer (2)

$$g\left(1-\frac{2h}{R}\right) = g\left(1-\frac{d}{R}\right)$$

$\Rightarrow \quad 2h = d$

$\Rightarrow \quad \alpha = 2$

90. Answer (1)

$\Delta U = U_f - U_i$

$$= -\frac{GMm}{4R} + \frac{GMm}{R}$$

$$= \frac{3GMm}{4R} = \frac{3}{4}mgR$$

$= 48$ MJ

91. Answer (4)

$g' = g_0 - \omega^2 R\cos^2\theta$

θ = latitude.

92. Answer (1)

Applying conservation of energy

$$-\frac{GM_e m}{R_e} + \frac{1}{2}m\left(\frac{1}{3}\sqrt{\frac{2Gm_e}{R_e}}\right)^2 = -\frac{GM_e m}{R_e + h}$$

$$-\frac{GM_e m}{R_e} + \frac{GM_e m}{9R_e} = -\frac{GM_e m}{R_e + h}$$

$$\frac{8}{9R_e} = \frac{1}{R_e + h}$$

$$\Rightarrow h = \frac{R_e}{8} = \frac{6400}{8} = 800 \text{ km}$$

93. Answer (4)

We know that

$T^2 \propto R^3$

$$\Rightarrow \left(\frac{T'}{T}\right)^2 = \left(\frac{3R}{R}\right)^3$$

$$\Rightarrow \frac{T'}{T} = 3\sqrt{3}$$

$\Rightarrow \quad T' = 3\sqrt{3}$ years

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