Chapter 17

Functions

1. Let $f(x) = (x + 1)^2 - 1$, $x \geq -1$.

   Statement-1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$.

   Statement-2 : f is a bijection.

   [AIEEE-2009]

   (1) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

   (2) Statement-1 is true, Statement-2 is false

   (3) Statement-1 is false, Statement-2 is true

   (4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

2. For real x, let $f(x) = x^3 + 5x + 1$, then

   [AIEEE-2009]

   (1) f is onto **R** but not one-one

   (2) f is one-one and onto **R**

   (3) f is neither one-one nor onto **R**

   (4) f is one-one but not onto **R**

3. Let y be an implict function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals

   [AIEEE-2009]

   (1) 1 (2) log 2

   (3) –log 2 (4) –1

4. Let f be a function defined by

   $f(x) = (x-1)^2 + 1, (x \geq 1)$.

   Statement - 1 : The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$.

   Statement - 2 : f is a bijection and $f^{-1}(x) = 1 + \sqrt{x-1}, x \geq 1$.

   [AIEEE-2011]

   (1) Statement-1 is true, Statement-2 is false

   (2) Statement-1 is false, Statement-2 is true

   (3) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1

   (4) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

5. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

   [AIEEE-2012]

   (1) No real roots

   (2) Exactly one real root

   (3) Exactly four real roots

   (4) Infinite number of real roots

6. If $a \in R$ and the equation

   $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$

   (where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval **[JEE (Main)-2014]**

   (1) $(-2, -1)$ (2) $(-\infty, -2) \cup (2, \infty)$

   (3) $(-1, 0) \cup (0, 1)$ (4) $(1, 2)$

7. If g is the inverse of a function f and $f'(x) = \dfrac{1}{1+x^5}$, then $g'(x)$ is equal to

   [JEE (Main)-2014]

   (1) $\dfrac{1}{1+\{g(x)\}^5}$ (2) $1 + \{g(x)\}^5$

   (3) $1 + x^5$ (4) $5x^4$

8. If $f(x) + 2f\left(\dfrac{1}{x}\right) = 3x, x \neq 0,$ and

   $S = \{x \in R : f(x) = f(-x)\}$; then S

   [JEE (Main)-2016]

   (1) Contains exactly one element

   (2) Contains exactly two elements

   (3) Contains more than two elements

   (4) Is an empty set

9. The function $f : R \to \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ defined as $f(x) = \dfrac{x}{1+x^2}$, is **[JEE (Main)-2017]**

   (1) Injective but not surjective

   (2) Surjective but not injective

   (3) Neither injective nor surjective

   (4) Invertible

10. For $x \in R-\{0,1\}$, let $f_1(x)=\frac{1}{x}, f_2(x)=1-x$ and $f_3(x)=\frac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then $J(x)$ is equal to **[JEE (Main)-2019]**

(1) $f_1(x)$ (2) $\frac{1}{x}f_3(x)$

(3) $f_2(x)$ (4) $f_3(x)$

11. Let $A = \{x \in R : x$ is not a positive integer$\}$. Define a function $f : A \rightarrow R$ as $f(x)=\frac{2x}{x-1}$, then f is **[JEE (Main)-2019]**

(1) Injective but not surjective

(2) Neither injective nor surjective

(3) Surjective but not injective

(4) Not injective

12. Let N be the set of natural numbers and two functions f and g be defined as

$f, g : N \rightarrow N$ such that

$$f(n)=\begin{cases}\frac{n+1}{2} \text{ if } n \text{ is odd}\\ \frac{n}{2} \text{ if } n \text{ is even}\end{cases}$$

and $g(n) = n - (-1)^n$. Then fog is **[JEE (Main)-2019]**

(1) One-one but not onto.

(2) Onto but not one-one.

(3) Neither one-one nor onto.

(4) Both one-one and onto.

13. Let $f : R \rightarrow R$ be defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is **[JEE (Main)-2019]**

(1) $R-\left[-\frac{1}{2},\frac{1}{2}\right]$ (2) $\left[-\frac{1}{2},\frac{1}{2}\right]$

(3) $(-1, 1) - \{0\}$ (4) $R - [-1, 1]$

14. Let a function $f : (0, \infty) \rightarrow [0, \infty)$ be defined by $f(x)=\left|1-\frac{1}{x}\right|$. Then f is **[JEE (Main)-2019]**

(1) Injective only

(2) Both injective as well as surjective

(3) Not injective but it is surjective

(4) Neither injective nor surjective

15. If $f(x)=\log_e\left(\frac{1-x}{1+x}\right), |x|<1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to : **[JEE (Main)-2019]**

(1) $2f(x)$ (2) $2f(x^2)$

(3) $-2f(x)$ (4) $(f(x))^2$

16. Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals **[JEE (Main)-2019]**

(1) $2f_1(x)f_1(y)$

(2) $2f_1(x + y)f_1(x - y)$

(3) $2f_1(x + y)f_2(x - y)$

(4) $2f_1(x)f_2(y)$

17. Let $\sum_{k=1}^{10} f(a+k)=16(2^{10}-1)$, where the function f satisfies $f(x + y) = f(x) f(y)$ for all natural numbers x, y and $f(1) = 2$. Then the natural number a is **[JEE (Main)-2019]**

(1) 2 (2) 3

(3) 16 (4) 4

18. If the function $f : R - \{1, -1\} \rightarrow A$ defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to **[JEE (Main)-2019]**

(1) $[0, \infty)$ (2) $R - \{-1\}$

(3) $R - (-1, 0)$ (4) $R - [-1, 0)$

19. The domain of the definition of the function $f(x)=\frac{1}{4-x^2}+\log_{10}(x^3-x)$ is **[JEE (Main)-2019]**

(1) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

(2) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(3) $(1, 2) \cup (2, \infty)$

(4) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

20. Let $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R : f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true ? **[JEE (Main)-2019]**

(1) $f(g(S)) = S$ (2) $g(f(S)) = g(S)$

(3) $g(f(S)) \neq S$ (4) $f(g(S)) \neq f(S)$

21. The number of real roots of the equation $5+|2^x-1|=2^x(2^x-2)$ is **[JEE (Main)-2019]**

(1) 4 (2) 2

(3) 1 (4) 3

22. For $x \in R$, let $[x]$ denote the greatest integer $\le x$, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+....+\left[-\frac{1}{3}-\frac{99}{100}\right]$$

is: **[JEE (Main)-2019]**

(1) – 135 (2) –153

(3) –133 (4) –131

23. For $x \in \left(0, \frac{3}{2}\right)$, let $f(x)=\sqrt{x}, g(x)=\tan x$ and $h(x)=\frac{1-x^2}{1+x^2}$. If $\phi(x)$ = $((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to : **[JEE (Main)-2019]**

(1) $\tan\frac{5\pi}{12}$ (2) $\tan\frac{\pi}{12}$

(3) $\tan\frac{11\pi}{12}$ (4) $\tan\frac{7\pi}{12}$

24. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to **[JEE (Main)-2020]**

(1) $-\frac{1}{2}$ (2) $\frac{3}{2}$

(3) $\frac{1}{2}$ (4) $-\frac{3}{2}$

25. The inverse function of

$$f(x)=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}, x \in (-1, 1), \text{ is \_\_\_\_\_\_\_}$$

[JEE (Main)-2020].

(1) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(2) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$

(3) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

(4) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$

26. Let $f : R \to R$ be a function which satisfies $f(x + y) = f(x) + f(y)\ \forall\, x, y \in R$. If $f(1) = 2$ and $g(n)=\sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of n, for which $g(n) = 20$, is **[JEE (Main)-2020]**

(1) 20 (2) 9

(3) 5 (4) 4

27. Let $[t]$ denote the greatest integer $\le t$. Then the equation in x, $[x]^2 + 2[x + 2] - 7 = 0$ has **[JEE (Main)-2020]**

(1) Exactly two solutions

(2) Infinitely many solutions

(3) Exactly four integral solutions

(4) No integral solution

28. If $f(x + y) = f(x)\, f(y)$ and $\sum_{x=1}^{\infty} f(x)=2$, $x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is **[JEE (Main)-2020]**

(1) $\frac{1}{9}$ (2) $\frac{4}{9}$

(3) $\frac{1}{3}$ (4) $\frac{2}{3}$

29. For a suitably chosen real constant a, let a function, $f : R - \{-a\} \to R$ be defined by $f(x)=\frac{a-x}{a+x}$. Further suppose that for any real number $x \ne -a$ and $f(x) \ne -a$, $(fof)(x) = x$. Then $f\left(-\frac{1}{2}\right)$ is equal to **[JEE (Main)-2020]**

(1) –3 (2) $\frac{1}{3}$

(3) $-\frac{1}{3}$ (4) 3

30. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f : A \to B \mid 2 \in f(A)$ and f is not one-one$\}$ is ______.

[JEE (Main)-2020]

31. Suppose that a function $f : R \to R$ satisfies $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(1) = 3$. If $\sum_{i=1}^{n} f(i)=363$, then n is equal to ________.

[JEE (Main)-2020]

32. The number of function f from $\{1, 2, 3, ...,20\}$ onto $\{1, 2, 3, ..., 20\}$ such that $f(k)$ is a multiple of 3, whenever k is a multiple of 4, is

[JEE (Main)-2019]

(1) $5^6 \times 15$ (2) $6^5 \times (15)!$

(3) $5! \times 6!$ (4) $(15)! \times 6!$

33. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is ________.

[JEE (Main)-2021]

34. If $a + \alpha = 1$, $b + \beta = 2$ and

$af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x},\ x\neq 0$, then the value of the expression $\dfrac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is________.

[JEE (Main)-2021]

35. Let $f, g : N \to N$ such that $f(n+1)=f(n)+f(1)\forall n\in N$ and g be any arbitrary function. Which of the following statements is NOT true ? **[JEE (Main)-2021]**

(1) If g is onto, then fog is one-one

(2) If f is onto, then $f(n)=n\forall n\in N$

(3) f is one-one

(4) If fog is one-one, then g is one-one

36. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then : **[JEE (Main)-2021]**

(1) $2y = 273x$ (2) $2y = 91x$

(3) $y = 273x$ (4) $y = 91x$

37. A function $f(x)$ is given by $f(x)=\dfrac{5^x}{5^x+5}$, then the sum of the series

$f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+....+f\left(\frac{39}{20}\right)$ is equal to:

[JEE (Main)-2021]

(1) $\frac{19}{2}$ (2) $\frac{29}{2}$

(3) $\frac{49}{2}$ (4) $\frac{39}{2}$

38. Let $A = \{1, 2, 3, ..., 10\}$ and $f : A \to A$ be defined as

$$f(k)=\begin{cases}k+1 & \text{if k is odd}\\ k & \text{if k is even}\end{cases}$$

Then the number of possible functions $g : A \to A$ such that $gof = f$ is :

[JEE (Main)-2021]

(1) 5^5 (2) 10^5

(3) $5!$ (4) ${}^{10}C_5$

39. Let $f(x) = \sin^{-1} x$ and $g(x) = \dfrac{x^2-x-2}{2x^2-x-6}$. If $g(2) = \lim\limits_{x\to 2} g(x)$, then the domain of the function fog is : **[JEE (Main)-2021]**

(1) $(-\infty,-2]\cup\left[-\frac{4}{3},\infty\right)$

(2) $(-\infty,-2]\cup\left[-\frac{3}{2},\infty\right)$

(3) $(-\infty,-1]\cup[2,\infty)$

(4) $(-\infty,-2]\cup[-1,\infty)$

40. If for $x\in\left(0,\frac{\pi}{2}\right)$, $\log_{10}\sin x+\log_{10}\cos x=-1$ and $\log_{10}(\sin x+\cos x)=\frac{1}{2}(\log_{10}n-1),\ n>0$, then the value of n is equal to **[JEE (Main)-2021]**

(1) 9 (2) 16

(3) 12 (4) 20

41. The inverse of $y = 5^{\log x}$ is

[JEE (Main)-2021]

(1) $x = y^{\log 5}$ (2) $x = 5^{\log y}$

(3) $x = y^{\frac{1}{\log 5}}$ (4) $x = 5^{\frac{1}{\log y}}$

42. If the functions are defined as $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$, then what is the common domain of the following functions :

$f + g$, $f - g$, f/g, g/f, $g - f$

where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x)=\dfrac{f(x)}{g(x)}$

[JEE (Main)-2021]

(1) $0 < x < 1$ (2) $0 < x \le 1$

(3) $0 \le x \le 1$ (4) $0 \le x < 1$

43. Let f : **R** – {3} → **R** – {1} be defined by $f(x)=\frac{x-2}{x-3}$.

Let g : **R** → **R** be given as g(x) = 2x – 3. Then, the sum of all the values of x for which $f^{-1}(x)+g^{-1}(x)=\frac{13}{2}$ is equal to

[JEE (Main)-2021]

(1) 3 (2) 5

(3) 7 (4) 2

44. Let [x] denote the greatest integer ≤ x, where x ∈ R. If the domain of the real valued function $f(x)=\sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a) \cup [b, c) \cup (4, \infty)$, a < b < c, then the value of a + b + c is

[JEE (Main)-2021]

(1) –2 (2) 1

(3) 8 (4) –3

45. If sum of the first 21 terms of the series $\log_{9^{1/2}} x+\log_{9^{1/3}} x+\log_{9^{1/4}} x+\ldots$, where $x > 0$ is 504, then x is equal to **[JEE (Main)-2021]**

(1) 81 (2) 243

(3) 9 (4) 7

46. Let $f:\mathbf{R}-\left\{\frac{\alpha}{6}\right\}\to\mathbf{R}$ be defined by $f(x)=\frac{5x+3}{6x-\alpha}$. Then the value of α for which (*fof*)(*x*) = *x*, for all $x\in\mathbf{R}-\left\{\frac{\alpha}{6}\right\}$, is **[JEE (Main)-2021]**

(1) 5 (2) 8

(3) No such α exists (4) 6

47. The number of solutions of the equation $\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0$, $x > 0$, is _____. **[JEE (Main)-2021]**

48. If the domain of the function

$$f(x)=\frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$$ is the interval (α, β], then α + β is equal to **[JEE (Main)-2021]**

(1) 1 (2) $\frac{3}{2}$

(3) $\frac{1}{2}$ (4) 2

49. Let [*x*] denote the greatest integer less than or equal to *x*. Then, the values of *x*∈**R** satisfying the equation $[e^x]^2+[e^x+1]-3=0$ lie in the interval.

[JEE (Main)-2021]

(1) [0, 1/e) (2) [1, e)

(3) $[0, \log_e 2)$ (4) $[\log_e 2, \log_e 3)$

50. Let A = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions $f: A\to A$ such that *f*(1) + *f*(2) = 3 – *f*(3) is equal to _______. **[JEE (Main)-2021]**

51. Let *g* : N → N be defined as

g(3n + 1) = 3n + 2,

g(3n + 2) = 3n + 3,

g(3n + 3) = 3n + 1, for all n ≥ 0.

Then which of the following statements is true?

[JEE (Main)-2021]

(1) *gogog* = *g*

(2) There exists an onto function *f* : N → N such that *fog* = *f*

(3) There exists a one-one function *f* : N → N such that *fog* = *f*

(4) There exists a function *f* : N → N such that *gof* = *f*

52. Consider functions *f* : A → B and *g* : B → C(A, B, C ⊆ R) such that $(gof)^{-1}$ exists, then

[JEE (Main)-2021]

(1) *f* is one-one and *g* is onto

(2) *f* is onto and *g* is one-one

(3) *f* and *g* both are one-one

(4) *f* and *g* both are onto

53. If for *x*, *y* ∈ **R**, *x* > 0, $y=\log_{10}x+\log_{10}x^{1/3}+\log_{10}x^{1/9}$ +...upto ∞ terms and $\frac{2+4+6+\ldots+2y}{3+6+9+\ldots+3y}=\frac{4}{\log_{10}x}$, then the ordered pair (*x*, *y*) is equal to :

[JEE (Main)-2021]

(1) $(10^6, 9)$ (2) $(10^6, 6)$

(3) $(10^4, 6)$ (4) $(10^2, 3)$

54. If $x^2+9y^2-4x+3=0$, *x*, *y* ∈ **R**, then *x* and *y* respectively lie in the intervals

[JEE (Main)-2021]

(1) [1, 3] and [1, 3]

(2) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and $\left[-\frac{1}{3},\frac{1}{3}\right]$

(3) $\left[-\frac{1}{3},\frac{1}{3}\right]$ and [1, 3]

(4) [1, 3] and $\left[-\frac{1}{3},\frac{1}{3}\right]$

55. Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the probability that a randomly chosen onto function g from S to S satisfies $g(3) = 2g(1)$ is **[JEE (Main)-2021]**

(1) $\frac{1}{30}$ (2) $\frac{1}{10}$

(3) $\frac{1}{15}$ (4) $\frac{1}{5}$

56. The sum of the roots of the equation,

$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is

[JEE (Main)-2021]

(1) $\log_2 14$ (2) $\log_2 12$

(3) $\log_2 11$ (4) $\log_2 13$

57. Let $f : N \to N$ be a function such that $f(m + n) = f(m) + f(n)$ for every $m, n \in N$. If $f(6) = 18$, then $f(2)\cdot f(3)$ is equal to **[JEE (Main)-2021]**

(1) 54 (2) 18

(3) 6 (4) 36

58. The range of the function

$$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$$

is **[JEE (Main)-2021]**

(1) $[0, 2]$ (2) $[-2, 2]$

(3) $(0, \sqrt{5})$ (4) $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$

59. The number of one-one functions $f : \{a, b, c, d\} \to \{0, 1, 2, ..., 10\}$ such that $2f(a) - f(b) + 3f(c) + f(d) = 0$ is ________. **[JEE (Main)-2022]**

60. Let $f : \mathbf{N} \to \mathbf{R}$ be a function such that $f(x + y) = 2f(x)\, f(y)$ for natural numbers x and y. If $f(1) = 2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha + k) = \frac{512}{3}(2^{20} - 1)$$ holds, is :

[JEE (Main)-2022]

(1) 2 (2) 3

(3) 4 (4) 6

61. Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$f(x) = x^3 + x - 5$

If $g(x)$ is a function such that $f(g(x)) = x, \forall' x' \in \mathbf{R}$, then $g'(63)$ is equal to______. **[JEE (Main)-2022]**

(1) $\frac{1}{49}$ (2) $\frac{3}{49}$

(3) $\frac{43}{49}$ (4) $\frac{91}{49}$

62. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined by

$$f(x) = \left(2\left(1 - \frac{x^{25}}{2}\right)\left(2 + x^{25}\right)\right)^{\frac{1}{50}}$$. If the function $g(x) = f(f(f(x))) + f(f(x))$, then the greatest integer less than or equal to $g(1)$ is ____________.

[JEE (Main)-2022]

63. Let $f(x) = \frac{x - 1}{x + 1}, x \in R - \{0, -1, 1\}$. If $f^{n+1}(x) = f(f^n(x))$ for all $n \in \mathbf{N}$, then $f^6(6) + f^7(7)$ is equal to :

[JEE (Main)-2022]

(1) $\frac{7}{6}$ (2) $-\frac{3}{2}$

(3) $\frac{7}{12}$ (4) $-\frac{11}{12}$

64. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = x - 1$ and $g : \mathbb{R} - \{1, -1\} \to \mathbb{R}$ be defined as $g(x) = \frac{x^2}{x^2 - 1}$.

Then the function fog is: **[JEE (Main)-2022]**

(1) One-one but not onto

(2) Onto but not one-one

(3) Both one-one and onto

(4) Neither one-one nor onto

65. Let $f : \mathbb{R} \to \mathbb{R}$ satisfy $f(x + y) = 2^x f(y) + 4^y f(x)$, $\forall x, y \in \mathbb{R}$. If $f(2) = 3$, then $14 \cdot \frac{f'(4)}{f'(2)}$ is equal to ____.

[JEE (Main)-2022]

66. Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined by

$$f(x) = \frac{2e^{2x}}{e^{2x}+e}.$$

Then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+...+f\left(\frac{99}{100}\right)$ is equal to ________. **[JEE (Main)-2022]**

67. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Define $f : S\,!\,S$ as

$$f(n) = \begin{cases} 2n & \text{, if } n = 1, 2, 3, 4, 5 \\ 2n-11 & \text{, if } n = 6, 7, 8, 9, 10 \end{cases}.$$

Let $g : S\,!\,S$ be a function such that

$$fog(n) = \begin{cases} n+1 & \text{, if } n \text{ is odd} \\ n-1 & \text{, if } n \text{ is even} \end{cases},$$

Then $g(10)\,(g(1) + g(2) + g(3) + g(4) + g(5))$ is equal to __________. **[JEE (Main)-2022]**

68. Let a function $f : \mathrm{N} \to \mathrm{N}$ be defined by

$$f(n) = \begin{bmatrix} 2n, & n = 2, 4, 6, 8, ... \\ n-1, & n = 3, 7, 11, 15, ... \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, ... \end{bmatrix}$$

then, f is **[JEE (Main)-2022]**

(1) One-one but not onto

(2) Onto but not one-one

(3) Neither one-one nor onto

(4) One-one and onto

69. Let $\mathrm{S} = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : \mathrm{S} \times \mathrm{S} \to \mathrm{S} : f$ is onto and $f(a, b) = f(b, a) \geq a \,\forall (a, b) \in \mathrm{S} \times \mathrm{S}\}$ is _____.

[JEE (Main)-2022]

70. Let $c, k \in \mathbf{R}$. If $f(x) = (c + 1)x^2 + (1 - c^2)x + 2k$ and $f(x + y) = f(x) + f(y) - xy$, for all $x, y \in \mathbf{R}$, then the value of $\left|2(f(1)+f(2)+f(3)+.....+f(20))\right|$ is equal to __________. **[JEE (Main)-2022]**

71. The number of solutions of the equation $\sin x = \cos^2 x$ in the interval $(0, 10)$ is _____.

72. The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \leq x \leq 4\pi$ is : **[JEE (Main)-2022]**

(1) 4 (2) 6

(3) 8 (4) 12

73. Let $f, g : \mathbb{N} - \{1\} \to \mathbb{N}$ be functions defined by $f(a) = \alpha$, where α is the maximum of the powers of those primes p such that p^α divides a, and $g(a) = a + 1$, for all $a \in \mathbb{N} - \{1\}$. Then, the function $f + g$ is

[JEE (Main)-2022]

(1) one-one but not onto

(2) onto but not one-one

(3) both one-one and onto

(4) neither one-one nor onto

74. The number of functions f, from the set $A = \{x \in N : x^2 - 10x + 9 \leq 0\}$ to the set $B = \{n^2 : n \in N\}$ such that $f(x) \leq (x - 3)^2 + 1$, for every $x \in A$, is __________.

[JEE (Main)-2022]

75. Let α, β and γ be three positive real numbers. Let $f(x) = \alpha x^5 + \beta x^3 + \gamma x$, $x \in \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be such that $g(f(x)) = x$ for all $x \in \mathbb{R}$. If $a_1, a_2, a_3, \ldots, a_n$ be in arithmetic progression with mean zero, then the value of $f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right)$ is equal to

[JEE (Main)-2022]

(1) 0 (2) 3

(3) 9 (4) 27

❑ ❑ ❑

Chapter 17

Functions

1. Answer (2)

We have, $f(x) = (x + 1)^2 - 1,\ x \geq -1$

$\Rightarrow\ f'(x) = 2\,(x + 1) \geq 0$ for $x \geq -1$

$\Rightarrow\ f(x)$ is one-one

Since co-domain of the given function is not given, hence it can be considered as R, the set of reals and consequently R is not onto.

Hence f is not bijective statement-2 is false.

Also $\quad f(x) = (x + 1)^2 - 1 \quad \geq \quad -1$ for $x \geq -1$

$\Rightarrow \qquad R_f = [-1, \infty)$

Clearly $f(x) = f^{-1}(x)$ at $x = 0$ and $x = -1$.

Statement-1 is true.

2. Answer (2)

$f(x) = x^3 + 5x + 1$

$f'(x) = 3x^2 + 5 > 0\ \forall\ x \in R$

Hence $f(x)$ is monotonic increasing. Therefore it is one-one.

Also it onto on R

Hence it one-one and onto R.

3. Answer (4)

$\because\ (x^x)^2 - 2.x^x \cot y = 1,$

$\therefore$ when $x = 1$, $y = \dfrac{\pi}{2}$

Differentiating,

$$2.x^x.x^x(1+\log_e x)$$
$$-2\left[-x^x\operatorname{cosec}^2 y\frac{dy}{dx} + \cot y.x^x(1+\log x)\right] = 0$$

Put $x = 1$ and $y = \dfrac{\pi}{2}$

$$2 + 2.\frac{dy}{dx} - 2\times 0 = 0$$

$$\frac{dy}{dx} = -1$$

4. Answer (3)

Given $f(x) = (x - 1)^2 + 1$

$\Rightarrow\ y = (x-1)^2 + 1$

$\Rightarrow\ (x-1)^2 = y - 1$

$\Rightarrow\ x = 1 + \sqrt{y-1}$

$\Rightarrow\ f^{-1}(x) = 1 + \sqrt{x-1}$

Statement-1 :

$f(x) = f^{-1}(x)$

$\Rightarrow\ (x-1)^2 + 1 = 1 + \sqrt{x-1}$

$\Rightarrow\ (x-1)^4 = (x-1)$

$\Rightarrow\ (x-1)\,((x-1)^3 - 1) = 0$

After solving

$\Rightarrow\ x = 1, 2$

$\Rightarrow$ Statement-1 is true.

Statement-2 :

$f^{-1}(x) = 1 + \sqrt{x-1}$

$\Rightarrow$ Statement-2 is also true.

But statement-2 is a correct explanation of statement 1.

5. Answer (1)

$e^{\sin x} - e^{-\sin x} = 4$

$\Rightarrow\ e^{2\sin x} = 4e^{\sin x} + 1$

As no intersection in $[0, 2\pi)$

$\therefore$ by perodicity no solution

6. Answer (3)

$-3(x - [x])^2 + 2[x - [x]) + a^2 = 0$

$3\,\{x\}^2 - 2\{x\} - a^2 = 0$

$$a \neq 0,\ 3\left(\{x\}^2 - \frac{2}{3}\{x\}\right) = a^2$$

$$a^2 = 3\left(\{x\} - \frac{1}{3}\right)^2 - \frac{1}{3}$$

$$0 \leq \{x\} < 1 \text{ and } -\frac{1}{3} \leq \{x\} - \frac{1}{3} < \frac{2}{3}$$

$$0 \le 3\left(\{x\}-\frac{1}{3}\right)^2 < \frac{4}{3}$$

$$-\frac{1}{3} \le 3\left(\{x\}-\frac{1}{3}\right)^2 - \frac{1}{3} < 1$$

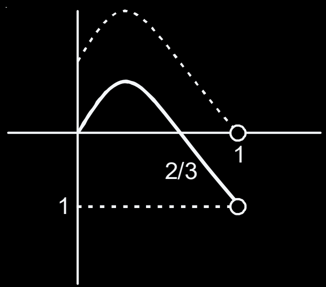
For non-integral solution

$0 < a^2 < 1$ and $a \in (-1, 0) \cup (0, 1)$

Alternative

$-3\{x\}^2 + 2\{x\} + a^2 = 0$

Now, $-3\{x\}^2 + 2\{x\}$



to have no integral roots $0 < a^2 < 1$

$\therefore \quad a \in (-1, 0) \cup (0, 1)$

7. Answer (2)

$$f'(x) = \frac{1}{1+x^5} = f(g(x)) = x \quad \rightarrow \quad f'(g(x))\, g'(x) = 1$$

$$g'(x) = \frac{1}{f'(g(x))} = 1 + (g(x))^5$$

8. Answer (2)

$$f(x) + 2f\left(\frac{1}{x}\right) = 3x$$

$$\Rightarrow \quad f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$$

$$\therefore \quad 3f(x) = \frac{6}{x} - 3x$$

$$\therefore \quad f(x) = \left(\frac{2}{x} - x\right)$$

$$f(-x) = -\frac{2}{x} + x$$

$f(x) = f(-x)$

$$\Rightarrow \quad \frac{2}{x} - x = -\frac{2}{x} + x$$

$$\Rightarrow \quad 2x - \frac{4}{x} = 0$$

$$\Rightarrow \quad x = \pm\sqrt{2}$$

9. Answer (2)

$$f(x) = \frac{x}{1+x^2}$$

$$f'(x) = \frac{(1+x^2)\cdot 1 - x \cdot 2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$f'(x)$ changes sign in different intervals.

$\therefore$ Not injective.

$$y = \frac{x}{1+x^2}$$

$$yx^2 - x + y = 0$$

For $y \ne 0$

$$D = 1 - 4y^2 \ge 0 \Rightarrow y \in \left[-\frac{1}{2}, \frac{1}{2}\right] - \{0\}$$

For, $y = 0 \Rightarrow x = 0$

$\therefore$ Part of range

$\therefore$ Range : $\left[-\frac{1}{2}, \frac{1}{2}\right]$

$\therefore$ Surjective but not injective.

10. Answer (4)

$$(f_2 o\, J o\, f_1)(x) = f_3(x) = \frac{1}{1-x}$$

$$\Rightarrow (f_2 o\, J) f_1(x) = \frac{1}{1-x}$$

$$\Rightarrow (f_2 o\, J)\left(\frac{1}{x}\right) = \frac{1}{1-\frac{1}{\frac{1}{x}}} = \frac{\frac{1}{x}}{\frac{1}{x}-1}$$

$$\Rightarrow (f_2 o\, J)(x) = \frac{x}{x-1}$$

$$\Rightarrow f_2(J(x)) = \frac{x}{x-1}$$

$$\Rightarrow 1 - J(x) = \frac{x}{x-1} = 1 + \frac{1}{x-1} = 1 - \frac{1}{1-x}$$

$$\therefore J(x) = \frac{1}{1-x} = f_3(x)$$

11. Answer (1)

As $A = \{x \in R : x$ is not a positive integer$\}$

$$f : A \to R \text{ given by } f(x) = \frac{2x}{x-1}$$

$$f(x_1) = f(x_2) \;\Leftrightarrow\; x_1 = x_2$$

So, f is one-one.

As $f(x) \neq 2$ for any $x \in A \Rightarrow f$ is not onto.

$\therefore$ f is injective but not surjective.

12. Answer (2)

$$f(n) = \begin{cases} \dfrac{n+1}{2}, \text{ if } n \text{ is odd} \\ \dfrac{n}{2}, \text{ if } n \text{ is even} \end{cases}$$

$$g(n) = \begin{cases} 2, & n=1 \\ 1, & n=2 \\ 4, & n=3 \\ 3, & n=4 \\ 6, & n=5 \\ 5, & n=6 \end{cases}$$

$$\Rightarrow \quad f(g(n)) = \begin{cases} \dfrac{g(n)+1}{2}, \text{ if } g(n) \text{ is odd} \\ \dfrac{g(n)}{2}, \text{ if } g(n) \text{ is even} \end{cases}$$

$$f(g(n)) = \begin{cases} 1, & n=1 \\ 1, & n=2 \\ 2, & n=3 \\ 2, & n=4 \\ 3, & n=5 \\ 3, & n=6 \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \end{cases} \Rightarrow fog \text{ is onto but not one - one}$$

$\Rightarrow$ Option (2) is correct.

13. Answer (2)

$$f(x) = \frac{x}{1+x^2}, x \in R$$

$$y = \frac{x}{1+x^2}$$

$yx^2 - x + y = 0$

$D \geq 0$

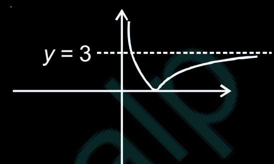
$1 \geq 4y^2$

$$|y| \leq \frac{1}{2}$$

$$-\frac{1}{2} \leq y \leq \frac{1}{2}$$

$\Rightarrow$ Option (2) is correct.

14. Answer (3)



Graphically $f(x)$ is not injective but surjective.

15. Answer (1)

$$\because \quad f(x) = \ln\left(\frac{1-x}{1+x}\right)$$

$$f\left(\frac{2x}{1+x^2}\right) = \ln\left(\frac{1-\dfrac{2x}{1+x^2}}{1+\dfrac{2x}{1+2x^2}}\right) = \ln\left(\frac{1+x^2-2x}{1+x^2+2x}\right)$$

$$= \ln\left(\frac{1-x}{1+x}\right)^2 = 2\ln\left(\frac{1-x}{1+x}\right)$$

$$= 2f(x)$$

16. Answer (1)

$$f(x) = a^x = \left(\frac{a^x + a^{-x}}{2}\right) + \left(\frac{a^x - a^{-x}}{2}\right)$$

where $\quad f_1(x) = \dfrac{a^x + a^{-x}}{2}$ is even function

$\quad f_2(x) = \dfrac{a^x - a^{-x}}{2}$ is odd function

$\Rightarrow \quad f_1(x+y) + f_1(x-y)$

$$= \left(\frac{a^{x+y} + a^{-x-y}}{2}\right) + \left(\frac{a^{x-y} + a^{-x+y}}{2}\right)$$

$$= \frac{1}{2}\left[a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})\right]$$

$$= \frac{(a^x + a^{-x})(a^y + a^{-y})}{2}$$

$= 2f_1(x).f_1(y)$

17. Answer (2)

$\because$ $f(x + y) = f(x) \cdot f(y)$

$\therefore$ Let $\boxed{f(x) = b^x}$

$\because$ $f(1) = 2$

$\therefore$ $b' = 2$

$\Rightarrow$ $\boxed{f(x) = 2^x}$

Now, $\sum_{k=1}^{10} 2^{a+k} = 16(2^{10} - 1)$

$\Rightarrow$ $2^a \sum_{k=1}^{10} 2^k = 16(2^{10} - 1)$

$\Rightarrow$ $2^a \times \dfrac{((2^{10}) - 1) \times 2}{(2-1)} = 16 \times (2^{10} - 1)$

$\Rightarrow$ $2^a = 8$

$\Rightarrow$ $\boxed{a = 3}$

18. Answer (4)

$f(x) = \dfrac{x^2}{1 - x^2}$

$\Rightarrow$ $f(-x) = \dfrac{x^2}{1 - x^2} = f(x)$

$\Rightarrow$ $f'(x) = \dfrac{2x}{(1 - x^2)^2}$

$\therefore$ $f(x)$ increases in $x \in (0, \infty)$

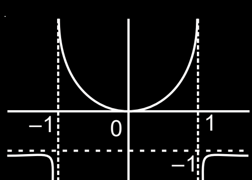
Also, $f(0) = 0$

$\lim_{x \to \pm\infty} f(x) = -1$

and $F(x)$ is even function

$\therefore$ Set $A \to R - [-1, 0)$

$\therefore$ Graph of function



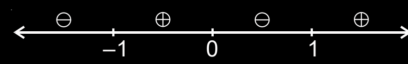
19. Answer (2)

For domain denominator $\neq 0$

$4 - x^2 \neq 0 \quad \Rightarrow \quad x \neq \pm 2 \quad \ldots(1)$

and $x^3 - x > 0$

$\Rightarrow$ $x(x - 1)(x + 1) > 0$



$x \in (-1, 0) \cup (1, \infty) \quad \ldots(2)$

Hence domain is intersection of (1) & (2) i.e.

$x \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

20. Answer (2)

$f(x) = x^2 x \in R$

$g(A) = \{x \in R : f(x) \in A\} \quad S \equiv [0, 4]$

$g(S) = \{x \in R : f(x) \in I\}$

$= \{x \in R : 0 \leq x^2 \leq 4\}$

$= \{x \in R : -2 \leq x \leq 2\}$

$\therefore$ $g(S) \neq S$

$\therefore$ $f(g(S)) \neq f(S)$

$g(f(S)) = \{x \in R : f(x) \in f(S)\}$

$= \{x \in R : x^2 \in S^2\}$

$= \{x \in R : 0 \leq x^2 \leq 16\}$

$= \{x \in R : -4 \leq x \leq 4\}$

$\therefore$ $g(f(S)) \neq g(S)$

$\therefore$ $g(f(S)) = g(S)$ is incorrect

21. Answer (3)

Let $2^x - 1 = t$

$5 + |t| = (t + 1)(t - 1)$

$\Rightarrow$ $|t| = t^2 - 6$

For $t > 0$, $t^2 - t - 6 = 0$

i.e., $t = 3$ or -2 (rejected)

For $t < 0$, $t^2 + t - 6 = 0$

i.e., $t = -3$ or 2 (both rejected)

$\therefore$ $2^x - 1 = 3$

$\Rightarrow$ $x = 2$

22. Answer (3)

As $[x] + \left[x + \dfrac{1}{n}\right] + \left[x + \dfrac{2}{n}\right] \ldots \left[x + \dfrac{n-1}{n}\right] = [nx]$

As $[x] + [-x] = -1$ $(x \notin z)$

Required value

$= -100 - \left\{\left[\dfrac{1}{3}\right] + \left[\dfrac{1}{3} + \dfrac{1}{100}\right] + \ldots \left[\dfrac{1}{3} + \dfrac{99}{100}\right]\right\}$

$= -100 - \left[\dfrac{100}{3}\right]$

$= -133$

23. Answer (3)

$$\phi\left(\frac{\pi}{3}\right) = h\left(f\left(g\left(\frac{\pi}{3}\right)\right)\right) = h\left(f\left(\sqrt{3}\right)\right) = h(3^{\frac{1}{4}})$$

$$= \frac{1-\sqrt{3}}{1+\sqrt{3}} = -\frac{1}{2}(1+3-2\sqrt{3}) = \sqrt{3}-2 = -(-\sqrt{3}+2)$$

$$= -\tan 15° = \tan(180°-15°) = \tan\left(\pi - \frac{\pi}{12}\right)$$

$$= \tan\frac{11\pi}{12}$$

24. Answer (1)

$$g\left(f\left(\frac{5}{4}\right)\right) = 4\left(\frac{5}{4}\right)^2 - 10\left(\frac{5}{4}\right) + 5 = \frac{-5}{4}$$

Now, $g\left(f\left(\frac{5}{4}\right)\right) = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$

Let $f\left(\frac{5}{4}\right) = t$

$\Rightarrow \quad t^2 + t - 1 = \frac{-5}{4}$

$t^2 + t + \frac{1}{4} = 0$

$\therefore \quad \left(t + \frac{1}{2}\right)^2 = 0$

i.e., $f\left(\frac{5}{4}\right) = \frac{-1}{2}$

25. Answer (2)

Let $y = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$

$\Rightarrow \quad y = \frac{8^{4x} - 1}{8^{4x} + 1}$

$\Rightarrow \quad 8^{4x}. y + y = 8^{4x} - 1$

$\Rightarrow \quad 1 + y = 8^{4x}(1 - y)$

$\Rightarrow \quad 8^{4x} = \frac{1+y}{1-y}$

$\Rightarrow \quad 4x = \log_8\left(\frac{1+y}{1-y}\right)$

$\therefore \quad f^{-1}(x) = \frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right) = \frac{1}{4}\log_8 e \log_e\left(\frac{1+x}{1-x}\right)$

26. Answer (3)

$f(x + y) = f(x) + f(y), \ \forall x, y \in R, f(1) = 2$

$\Rightarrow \quad f(x) = 2x$

Now, $g(n) = \sum_{k=1}^{n-1} f(k)$

$= f(1) + f(2) + f(3) + ...f(n - 1)$

$= 2 + 4 + 6 + ... +2(n - 1)$

$= 2[1 + 2 + 3 + ... +(n - 1)]$

$= 2 \times \frac{(n-1)(n)}{2} = n^2 - n$

So, $n^2 - n = 20$ (given)

$\Rightarrow \quad n^2 - n - 20 = 0$

$(n - 5)(n + 4) = 0$

$\Rightarrow \quad \boxed{n = 5}$

27. Answer (2)

$[x]^2 + 2[x] + 4 - 7 = 0$

$\Rightarrow \quad [x]^2 + 2[x] - 3 = 0$

$\Rightarrow \quad [x]^2 + 3[x] - [x] - 3 = 0$

$\Rightarrow \quad ([x] + 3)([x] - 1) = 0$

$\Rightarrow \quad [x] = 1$ or -3

$\Rightarrow \quad x \in [-3, -2) \cup [1, 2)$

infinitely many solutions.

28. Answer (2)

Let $f(1) = a$

then $f(1 + 1) = a^2$

$f(2 + 1) = a^3$

and so on.

$\sum_{x=1}^{\infty} f(x) = 2 \Rightarrow \quad a + a^2 + a^3 + ... \infty = 2$

$\Rightarrow \quad \frac{a}{1-a} = 2$

$\Rightarrow \quad a = \frac{2}{3}$

Now, $\frac{f(4)}{f(2)} = \frac{a^4}{a^2} = a^2 = \frac{4}{9}$

29. Answer (4)

$$f(f(x)) = \frac{a - \left(\frac{a-x}{a+x}\right)}{a + \left(\frac{a-x}{a+x}\right)} = x$$

$$\Rightarrow \frac{a^2 + ax - a + x}{a^2 + ax + a - x} = x$$

$\Rightarrow$ $a^2 + (a + 1)x - a = a^2x + (a - 1)x^2 + ax$

$\Rightarrow$ $(a - 1)x^2 + (a^2 - 1)x + (a - a^2) = 0$

$\forall x \in R - \{-a\}$

Hence a = 1

$\therefore \quad f(x) = \frac{1-x}{1+x} \quad \Rightarrow f\left(-\frac{1}{2}\right) = 3$

30. Answer (19)

The desired functions will contain either one element or two elements in its codomain of which '2' always belongs to f(A).

$\therefore$ The set B can be

{2}, {1, 2}, {2, 3}, {2, 4}

Total number of functions

$= 1 + (2^3 - 2)3$

$= 19$

31. Answer (05.00)

$\because$ $f(x + y) = f(x).f(y) \quad \forall x \in R \quad f(1) = 3$

$\Rightarrow$ $f(x) = 3^x \Rightarrow f(i) = 3^i$

$$\Rightarrow \sum_{i=1}^{n} f(i) = 363$$

$\Rightarrow$ $3 + 3^2 + 3^3 + \ldots + 3^n = 363$

$$\frac{3(3^n - 1)}{3 - 1} = 363$$

$$3^n - 1 = \frac{363 \times 2}{3} = 242$$

$3^n = 243 = 3^5$

$n = 5$

32. Answer (4)

Domain and codomain = {1, 2, 3, ..., 20}.

There are five multiple of 4 as 4, 8, 12, 16 and 20.

and there are 6 multiple of 3 as 3, 6, 9, 12, 15, 18.

when ever k is multiple of 4 then $f(k)$ is multiple of 3 then total number of arrangement

$= {}^6C_5 \times 5! = 6!$

Remaining 15 are arrange is 15! ways.

$\therefore$ given function in onto

$\therefore$ Total number of arrangement = $15! \cdot 6!$

33. Answer (1)

Domain : $x - 1 > 0$ and $x - 3 > 0$

$\Rightarrow$ $x \in (3, \infty)$

$\because$ $\log_4(x - 1) = \log_2(x - 3)$

$\Rightarrow$ $x - 1 = (x - 3)^2$

$\Rightarrow$ $x^2 - 7x + 8 = 0$

$$\Rightarrow x = \frac{7 \pm \sqrt{17}}{2}$$

but only $\frac{7 + \sqrt{17}}{2}$ is the correct answer.

34. Answer (2)

$$af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x} \qquad \ldots(i)$$

Replace x by $\frac{1}{x}$

$$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x \qquad \ldots(ii)$$

(i) + (ii)

$$\Rightarrow (a + \alpha)\left(f(x) + f\left(\frac{1}{x}\right)\right) = (b + \beta)\left(x + \frac{1}{x}\right)$$

$$\Rightarrow \frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{b + \beta}{a + \alpha} = \frac{2}{1} = 2$$

35. Answer (1)

Given f, g : $N \to N$

& $f(n + 1) = f(n) + 1$

$$\left.\begin{array}{l} \Rightarrow f(2) = 2f(1) \\ \Rightarrow f(3) = 3f(1) \\ \quad f(4) = 4f(4) \\ \quad \ldots\ldots\ldots\ldots \\ \quad f(n) = nf(1) \end{array}\right\} \Rightarrow f \text{ is one} - \text{one.}$$

Now if f is onto $\Rightarrow$ f(1) = 1

$\Rightarrow \boxed{f(n) = n}$

Also it is clear if fog is one-one $\Rightarrow$ g will be one-one.

So only option (1) is not correct.

36. Answer (2)

$n(A) = 3,\ n(B) = 5$

$x = {}^5C_3 \times 3! = 5 \times 4 \times 3$

$n(A \times B) = 15$

$y = {}^{15}C_3 \times 3! = 15 \times 14 \times 13$

$$\frac{y}{x} = \frac{15 \times 14 \times 13}{5 \times 4 \times 3} = \frac{91}{2}$$

$2y = 91x$

37. Answer (4)

$$f(2-x) = \frac{5^{2-x}}{5^{2-x}+5} = \frac{5}{5+5^x}$$

So $f(x) + f(2 - x) = 1$

$$\sum_{r=1}^{39} f\left(\frac{r}{20}\right) = \sum_{r=1}^{19}\left(f\left(\frac{r}{20}\right) + f\left(2-\frac{r}{20}\right)\right) + f(1)$$

$$= 19 + \frac{1}{2} = \frac{39}{2}$$

38. Answer (2)

Note that $f(1) = f(2) = 2$

$f(3) = f(4) = 4$

$f(5) = f(6) = 6$

$f(7) = f(8) = 8$

$f(9) = f(10) = 10$

$gof(1) = f(1) \Rightarrow g(2) = f(1) = 2$

$gof(2) = f(2) \Rightarrow g(2) = f(2) = 2$

$gof(3) = f(3) \Rightarrow g(4) = f(3) = 4$

$\therefore$ In function g(x), 2, 4, 6, 8, 10 should be mapped to 2, 4, 6, 8, 10 respectively. Each of remaining elements can be mapped to any of 10 elements.

Number of possible g(x) is 10^5

39. Answer (1)

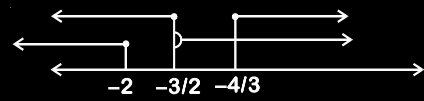
$$g(2) = \lim_{x\to 2} g(x) = \frac{(x-2)(x+1)}{(2x+3)(x-2)} = \frac{3}{7}$$

$$\log(x) = \sin^{-1}\left(\frac{x+1}{2x+3}\right)$$

for domain $-1 \le \dfrac{x+1}{2x+3} \le 1$

$$\Rightarrow \frac{3x+4}{2x+3} \ge 0 \text{ and } \frac{x+2}{2x+3} \ge 0$$

$x \in (-\infty, -3/2) \cup [-4/3, \infty]$ and $x \in (-\infty, -2] \cup (-3/2, \infty)$



Hence $x \in (-\infty, -2]\ u\ (-4/3, \infty]$

40. Answer (3)

Given, $\log_{10}(\sin x \cos x) = -1$

$$\Rightarrow \sin 2x = \frac{2}{10} \Rightarrow 1 + \sin 2x = \frac{6}{5}$$

Also $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

$$\Rightarrow \frac{1}{2}\log_{10}(1+\sin 2x) = \frac{1}{2}(\log_{10} n - \log_{10} 10)$$

$$\Rightarrow \frac{6}{5} = \frac{n}{10} \Rightarrow n = 12$$

41. Answer (3)

$y = 5^{\log x}$

$\Rightarrow \log y = \log x \cdot \log 5$

$$\Rightarrow \log x = \frac{\log y}{\log 5} = \log_5 y$$

$\Rightarrow x = e^{\log_5 y}$

$\Rightarrow x = y^{\log_5 e}$

$\Rightarrow x = y^{\frac{1}{\log 5}}$

42. Answer (1)

For common domain $\equiv$ (domain of) $\cap$ domain of g – {Points where either or both of f, g vanishes}

$\Rightarrow x > 0$ and $1 - x > 0$

$\Rightarrow x \in (0, 1)$

43. Answer (2)

Finding inverse of f(x)

$$y = \frac{x-2}{x-3} \Rightarrow xy - 3y = x - 2 \Rightarrow x(y-1) = 3y - 2$$

$$\therefore f^{-1}(x) = \frac{3x-2}{x-1}$$

Similarly for $g^{-1}(x)$

$$y = 2x - 3 \Rightarrow x = \frac{y+3}{2} \Rightarrow g^{-1}(x) = \frac{x+3}{2}$$

$$\therefore \frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$$

$\Rightarrow 6x - 4 + x^2 + 2x - 3 = 13x - 13$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow (x - 2)(x - 3) = 0$

$\Rightarrow x = 2$ or 3

44. Answer (1)

$$\frac{|\,[x]\,|-2}{|\,[x]\,|-3} \ge 0 \Rightarrow |\,[x]\,| \le 2 \text{ Or } |\,[x]\,| > 3$$

$\Rightarrow$ $-2 \le [x] \le 2$ Or $[x] < -3$ Or $[x] > 3$

$\Rightarrow$ $-2 \le x < 3$ Or $x < -3$ or $x \ge 4$

$\Rightarrow$ $x \in (-\infty, -3) \cup [-2, 3) \cup [4, \infty)$

$a = -3,\ b = -2,\ c = 3$

45. Answer (1)

$$\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + \ldots$$

$\Rightarrow$ $\log_9 x^2 + \log_9 x^3 + \log_9 x^4 + \ldots$

$\Rightarrow$ $\log_9\left(x^{2+3+\ldots\ldots.21\text{–terms}}\right) = 504$

$\Rightarrow$ $252 \log_9 x = 504$

$\Rightarrow$ $x = 9^2 = 81$

46. Answer (1)

For $f(f(x)) = x$

$f(x) = f^{-1}(x)$

finding $f^{-1}(x)$

$$y = \frac{3x+3}{6x-\alpha}$$

$\Rightarrow$ $f^{-1}(x) = \dfrac{3+\alpha x}{6x-5}$

$\therefore$ $f(x) = f^{-1}(x)$ gives

$$\frac{3+\alpha x}{6x-5} = \frac{5x+3}{6x-\alpha}$$

$\Rightarrow$ $(30 - 6\alpha)x^2 + (\alpha^2 - 25)x + (3\alpha - 15) = 0$

$\therefore$ $\alpha = 5$

47. Answer (1)

$\log_{(x+1)}(x + 1)(2x + 5) + \log_{(2x+5)}(x + 1)^2 = 4$

$\Rightarrow$ $1 + \log_{(x+1)}(2x + 5) + 2\log_{(2x+5)}(x + 1) = 4$

Let $\log_{(x+1)}(2x + 5) = t$

then $t + \dfrac{2}{t} = 3 \Rightarrow t = 1, 2$

$\Rightarrow$ $2x + 5 = x + 1$ or $2x + 5 = (x + 1)^2$

$\Rightarrow$ $x = -4, +2, -2$ out of which only $x = 2$ is acceptable.

48. Answer (2)

$$f(x) = \frac{\cos^{-1}\left(\sqrt{x^2-x+1}\right)}{\sqrt{\sin^{-1}\left(\dfrac{2x-1}{2}\right)}}$$

$0 < \dfrac{2x-1}{2} \le 1$ ...(i)

$\Rightarrow$ $0 < 2x - 1 \le 2$

$\Rightarrow$ $1 < 2x \le 3$

$\Rightarrow$ $\dfrac{1}{2} < x \le \dfrac{3}{2}$

and $0 \le x^2 - x + 1 \le 1$ ...(ii)

$x^2 - x \le 0$

$x(x - 1) \le 0$

$0 \le x \le 1$

$\therefore$ domain $x \in \left(\dfrac{1}{2}, 1\right] = (\alpha, \beta] \Rightarrow \alpha + \beta = \dfrac{3}{2}$

49. Answer (3)

$\because$ $[e^x]^2 + [e^x+1] - 3 = 0$

$\Rightarrow$ $[e^x]^2 + [e^x] - 2 = 0$

$\Rightarrow$ $([e^x] + 2)([e^x] - 1) = 0$

$[e^x] = -2$ not possible

and $[e^x] = 1$

$\therefore$ $e^x \in [1, 2)$

$\therefore$ $x \in [0, \ln 2)$

50. Answer (720)

Clearly $f(1)$, $f(2)$ and $f(3)$ are the permutations of 0, 1, 2; and $f(0)$, $f(4)$, $f(5)$, $f(6)$ and $f(7)$ are the permutations of 3, 4, 5, 6 and 7.

Total number of bijective functions $= \underline{|5} \cdot \underline{|3} = 720$

51. Answer (2)

$\because$ $g(3n + 1) = 3n + 2$, $g(3n + 2) = 3n + 3$ and $g(3n + 3) = 3n + 1$

$\because$ $gogog\,(3n + 1) = g(g(g(3n + 1))) = g(g(3n + 2)) = g(3n + 3) = 3n + 1$

Similarly we can see that $gogog = x$ (identity)

For $fog = f$ to hold

'f' must be an onto function

52. Answer (1)

$\because$ $f : A \to B$ and $g : B \to C$ then $(gof)^{-1} = f^{-1}\ o\ g^{-1}$

$\therefore$ $f^{-1} : B \to A$ and $g^{-1} : C \to B$

$\therefore$ $(gof)^{-1} : C \to A$

$\therefore$ f must be one-one and g will be onto function.

53. Answer (1)

$$y = \log_{10} x + \log_{10} x^{1/3} + \log_{10} x^{1/9} + \ldots\infty$$

$$= \log_{10}(x \cdot x^{1/3} \cdot x^{1/9} \ldots \infty)$$

$$= \log_{10}\left(x^{1+\frac{1}{3}+\frac{1}{9}+\ldots\infty}\right)$$

$$y = \log_{10}\left(x^{\frac{1}{1-\frac{1}{3}}}\right) = \log_{10} x^{3/2} = \frac{3}{2}\log_{10} x$$

$$\frac{2+4+6\ldots+2y}{3+6+9+\ldots+3y} = \frac{4}{\log_{10} x}$$

$$\Rightarrow \frac{2(1+2+3+\ldots+y)}{3(1+2+3+\ldots+y)} = \frac{4}{\log_{10} x}$$

$$\Rightarrow \frac{2}{3} = \frac{4}{\log_{10} x}$$

$$\Rightarrow \log_{10} x = 6$$

$$\Rightarrow x = 10^6$$

$$\Rightarrow y = \frac{3}{2} \times 6 = 9$$

54. Answer (4)

$9y^2 = -x^2 + 4x - 3$ ...(i)

$9y^2 \geq 0$

$\Rightarrow -x^2 + 4x - 3 \geq 0$

$x^2 - 4x + 3 \leq 0$

$(x - 1)(x - 3) \leq 0$

$x \in [1, 3]$

Let $f(x) = -x^2 + 4x - 3$

$(f(x))_{max} = f(2) = 1$

$(f(x))_{min} = f(1)$ or $f(3) = 0$,

$0 \leq -x^2 + 4x - 3 \leq 1$

$0 \leq 9y^2 \leq 1$

$$0 \leq y^2 \leq \frac{1}{9}$$

$$0 \leq |y| \leq \frac{1}{3}$$

$$-\frac{1}{3} \leq y \leq \frac{1}{3}$$

55. Answer (2)

Total number of onto functions = $|\underline{6}$

$\because g(3) = 2g(1)$ then $(g(1), g(3))$

$= (1, 2)$ or $(2, 4)$ or $(3, 6)$

In each case number of onto functions = $|\underline{4}$

Required probability = $\dfrac{3|\underline{4}}{|\underline{6}} = \dfrac{1}{10}$

56. Answer (3)

$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$

$\Rightarrow x + 1 + \log_2(10 - 2^{-x}) - \log_2(3 + 2^x)^2 = 0$

$$\Rightarrow x + 1 = \log_2\left(\frac{(3+2^x)^2}{(10-2^{-x})}\right)$$

$$\Rightarrow 2^{x+1} = \frac{9+6\cdot 2^x + 2^{2x}}{10-2^{-x}}$$

$$\Rightarrow 20\cdot 2^x - 2 = 9 + 6\cdot 2^x + 2^{2x}$$

$\Rightarrow (2^x)^2 - 14(2^x) + 11 = 0$

Let two roots are 2^{x_1} and 2^{x_2}

Then $2^{x_1} \cdot 2^{x_2} = 11 \Rightarrow x_1 + x_2 = \log_2 11$

$\therefore$ Sum of roots = $\log_2 11$

57. Answer (1)

$\because$ $f(m + n) = f(m) + f(n)$, $f : N \to N$

then $f(x) = kx$

$\because$ $f(6) = 18 \Rightarrow 18 = k\cdot 6 \Rightarrow k = 3$

$\therefore$ $f(x) = 3x$

$\therefore$ $f(2)\cdot f(3) = 6 \times 9$

$= 54$

58. Answer (1)

$$f(x) = \log_{\sqrt{5}}\left(3 + 2\sin\left(\frac{3\pi}{4}\right)\sin(-x) + 2\cos\left(\frac{\pi}{4}\right)\cdot\cos(x)\right)$$

$$= \log_{\sqrt{5}}\left(3 + \sqrt{2}(\cos x - \sin x)\right)$$

$\because$ Range of $\cos x - \sin x$ is $\left[-\sqrt{2}, \sqrt{2}\right]$

Then range of $f(x)$ is $[0, 2]$

59. Answer (31)

$\because$ $3f(c) + 2f(a) + f(d) = f(b)$

Value of $f(c)$	**Value of $f(a)$**	**Number of functions**
0	1	7
	2	5
	3	3
	4	2

1	0	6
	2	2
	3	1
2	0	3
	1	1
3	0	1
	Total Number of functions =	**31**

60. Answer (3)

$f(x+y) = 2f(x)f(y)$ & $f(1) = 2$

$x = y = 1$

$$\left.\begin{aligned} \Rightarrow \quad & f(2) = 2^3 \\ & x = 2, y = 1 \\ \Rightarrow \quad & f(3) = 2^5 \end{aligned}\right\} f(x) = 2^{(2x-1)}$$

Now,

$$\sum_{k=1}^{10} f(\alpha+k) = \frac{512}{3}(2^{20}-1)$$

$$2\sum_{k=1}^{10} f(\alpha)f(k) = \frac{512}{3}(2^{20}-1)$$

$$2f(\alpha)[f(1)+f(2)+.....+f(10)] = \frac{512}{3}(2^{20}-1)$$

$$= 2f(\alpha)\left[2+2^3+2^5+.....\text{upto 10 terms}\right] = \frac{512}{3}(2^{20}-1)$$

$$= 2f(\alpha)\cdot 2\left(\frac{2^{20}-1}{4-1}\right) = \frac{512}{3}(2^{20}-1)$$

$f(\alpha) = 128 = 2^{2\alpha} - 1$

$= 2\alpha - 1 = 7$

$\Rightarrow \quad \alpha = 4$

61. Answer (1)

$f(x) = 3x^2 + 1$

$f'(x)$ is bijective function

and $f(g(x)) = x \Rightarrow g(x)$ is inverse of $f(x)$

$g(f(x)) = x$

$$g'(f(x))\cdot f'(x) = 1$$

$$g'(f(x)) = \frac{1}{3x^2+1}$$

Put $x = 4$ we get

$$g'(63) = \frac{1}{49}$$

62. Answer (2)

$$f(x) = \left(2\left(\frac{2-x^{25}}{2}\right)\left(2+x^{25}\right)\right)^{\frac{1}{50}}$$

$$= \left(4-x^{50}\right)^{\frac{1}{50}}$$

$$f(f(x)) = \left(4-\left(\left(4-x^{50}\right)^{\frac{1}{50}}\right)^{50}\right)^{\frac{1}{50}} = x$$

As $f(f(x)) = x$ we have

$g(x) = f(f(f(x))) + f(f(x)) = f(x) + x$

$\Rightarrow \quad g(x) = (4 - x^{50})^{1/50} + x$

$\Rightarrow \quad g(1) = 3^{1/50} + 1$

$\Rightarrow \quad [g(1)] = 2$

63. Answer (2)

$$f(x) = \frac{x-1}{x+1} \Rightarrow f(f(x)) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} = -\frac{1}{x}$$

$$\Rightarrow \quad f^3(x) = -\frac{x+1}{x-1} \Rightarrow f^4(x) = -\frac{\frac{x-1}{x+1}+1}{\frac{x-1}{x+1}-1} = x$$

So, $f^6(6) + f^7(7) = f^2(6) + f^3(7)$

$$= -\frac{1}{6} - \frac{7+1}{7-1} = -\frac{9}{6} = -\frac{3}{2}$$

64. Answer (4)

$f : \mathbb{R} \to \mathbb{R}$ defined as

$f(x) = x - 1$ and $g : \mathbb{R} \to \{1, -1\} \to \mathbb{R}$, $g(x) = \dfrac{x^2}{x^2-1}$

Now $fog(x)$ $\dfrac{x^2}{x^2-1} - 1 = \dfrac{1}{x^2-1}$

$\therefore$ Domain of $fog(x) = \mathbb{R} - \{-1, 1\}$

And range of $fog(x) = (-\infty, -1] \cup (0, \infty)$

Now, $\frac{d}{dx}(fog(x)) = \frac{-1}{(x^2-1)^2} \cdot 2x = \frac{2x}{(1-x^2)^2}$

$\therefore \frac{d}{dx}(fog(x)) > 0$ for $\frac{2x}{((1-x)(1+x))^2} > 0$

$\Rightarrow \frac{x}{((x-1)(x+1))^2} < 0$

$\therefore x \in (-\infty, 0)$

and $\frac{d}{dx}(fog(x)) < 0$ for $x \in (0, \infty)$

$\therefore$ $fog(x)$ is neither one-one nor onto.

65. Answer (248)

$f(x + y) = 2^x f(y) + 4^y f(x)$ ...(1)

Now, $f(y + x)\ 2^y f(x) + 4^x f(y)$ ...(2)

$\therefore 2^x f(y) + 4^y f(x) = 2^y f(x) + 4^x f(y)$

$(4^y - 2^y) f(x) = (4^x - 2^x) f(y)$

$\frac{f(x)}{4^x - 2^x} = \frac{f(y)}{4^y - 2^y} = k$ (Say)

$\therefore f(x) = k(4^x - 2^x)$

$\because f(2) = 3$ then $k = \frac{1}{4}$

$\therefore f(x) = \frac{4^x - 2^x}{4}$

$\therefore f'(x) = \frac{4^x \ln 4 - 2^x \ln 2}{4}$

$f'(x) = \frac{(2.4^x - 2^x) \ln 2}{4}$

$\therefore \frac{f'(4)}{f'(2)} = \frac{2.256 - 16}{2.16 - 4}$

$\therefore 14\frac{f'(4)}{f'(2)} = 248$

66. Answer (99)

$f(x) = \frac{2e^{2x}}{e^{2x} + e^x}$ and $f(1-x) = \frac{2e^{2-2x}}{e^{2-2x} + e^{1-x}}$

$\therefore \frac{f(x) + f(1-x)}{2} = 1$

i.e. $f(x) + f(1 - x) = 2$

$\therefore f\left(\frac{1}{100}\right) + f\left(\frac{2}{100}\right) + ... + f\left(\frac{99}{100}\right)$

$= \sum_{x=1}^{49} f\left(\frac{x}{100}\right) + f\left(1 - \frac{x}{100}\right) + f\left(\frac{1}{2}\right)$

$= 49 \times 2 + 1 = 99$

67. Answer (190)

$\because f(n) = \begin{cases} 2n & , n = 1, 2, 3, 4, 5 \\ 2n - 11, & n = 6, 7, 8, 9, 10 \end{cases}$

$\therefore f(1) = 2, f(2) = 4, \ldots, f(5) = 10$

and $f(6) = 1, f(7) = 3, f(8) = 5, \ldots, f(10) = 9$

Now, $f(g(n)) = \begin{cases} n+1, & \text{if } n \text{ is odd} \\ n-1, & \text{if } n \text{ is even} \end{cases}$

$\therefore f(g(10)) = 9 \quad \Rightarrow g(10) = 10$

$f(g(1)) = 2 \quad \Rightarrow g(1) = 1$

$f(g(2)) = 1 \quad \Rightarrow g(2) = 6$

$f(g(3)) = 4 \quad \Rightarrow g(3) = 2$

$f(g(4)) = 3 \quad \Rightarrow g(4) = 7$

$f(g(5)) = 6 \quad \Rightarrow g(5) = 3$

$\therefore g(10)\,(g(1) + g(2) + g(3) + g(4) + g(5)) = 190$

68. Answer (4)

When $n = 1, 5, 9, 13$ then $\frac{n+1}{2}$ will give all odd numbers.

When $n = 3, 7, 11, 15 \ldots$

$n - 1$ will be even but not divisible by 4

When $n = 2, 4, 6, 8, \ldots$

Then $2n$ will give all multiples of 4

So range will be N.

And no two values of n give same y, so function is one-one and onto.

69. Answer (37)

There are 16 ordered pairs in $S \times S$. We write all these ordered pairs in 4 sets as follows.

$A = \{(1, 1)\}$

$B = \{(1, 4), (2, 4), (3, 4) (4, 4), (4, 3), (4, 2), (4, 1)\}$

$C = \{(1, 3), (2, 3), (3, 3), (3, 2), (3, 1)\}$

$D = \{(1, 2), (2, 2), (2, 1)\}$

All elements of set B have image 4 and only element of A has image 1.

All elements of set C have image 3 or 4 and all elements of set D have image 2 or 3 or 4.

We will solve this question in two cases.

Case I : When no element of set C has image 3.

Number of onto functions = 2 (when elements of set D have images 2 or 3)

Case II : When atleast one element of set C has image 3.

Number of onto functions = $(2^3 - 1)(1 + 2 + 2)$

$= 35$

Total number of functions = 37

70. Answer (3395)

$f(x)$ is polynomial

Put $y = 1/x$ in given functional equation we get

$$f\left(x+\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)-1$$

$$\Rightarrow (c+1)\left(x+\frac{1}{x}\right)^2+\left(1-c^2\right)\left(x+\frac{1}{x}\right)+2K$$

$$= (c+1)x^2+\left(1-c^2\right)x+2K$$

$$+(c+1)\frac{1}{x^2}+\left(1-c^2\right)\frac{1}{x}+2K-1$$

$\Rightarrow \quad 2(c + 1) = 2K - 1 \qquad \ldots(1)$

and put $x = y = 0$ we get

$$f(0)=2+f(0)-0\Rightarrow f(0)=0\Rightarrow k=0$$

$\therefore \quad k = 0$ and $2c = -3 \Rightarrow c = -3/2$

$$f(x) = -\frac{x^2}{2}-\frac{5x}{4}=\frac{1}{4}\left(5x+2x^2\right)$$

$$\left|2\sum_{i=1}^{20}f(i)\right|=\left|\frac{-2}{4}\left(\frac{5.20.21}{2}+\frac{2.20.21.41}{6}\right)\right|$$

$$= \left|\frac{-1}{2}(2730+5740)\right|$$

$$= \left|-\frac{6790}{2}\right|=3395.$$

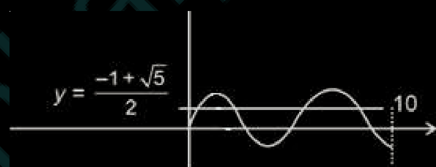
71. Answer (4)

$\sin x = 1 - \sin^2 x$

$\Rightarrow \quad \sin^2 x + \sin x - 1 = 0$

$$\Rightarrow \quad \sin x=\frac{-1\pm\sqrt{5}}{2}$$

$$\Rightarrow \quad \sin x=\frac{-1+\sqrt{5}}{2}$$

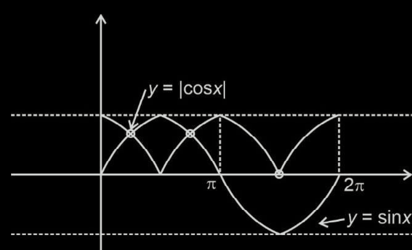


4 solutions

72. Answer (3)

Number of solutions of the equation $|\cos x| = \sin x$ for $x \in [-4\pi, 4\pi]$ will be equal to 4 times the number of solutions of the same equation for $x \in [0, 2\pi]$.

Graphs of $y = |\cos x|$ and $y = \sin x$ are as shown below.



Hence, two solutions of given equation in $[0, 2\pi]$

$\Rightarrow \quad$ Total of 8 solutions in $[-4\pi, 4\pi]$

73. Answer (4)

$f, g : N - \{1\} \rightarrow N$ defined as

$f(a) = \alpha$, where α is the maximum power of those primes p such that p^α divides a.

$g(a) = a + 1$,

Now, $\quad f(2) = 1, \qquad g(2) = 3 \quad \Rightarrow \quad (f + g)(2) = 4$

$f(3) = 1, \quad g(3) = 4 \quad \Rightarrow \quad (f+g)(3) = 5$

$f(4) = 2, \quad g(4) = 5 \quad \Rightarrow \quad (f+g)(4) = 7$

$f(5) = 1, \quad g(5) = 6 \quad \Rightarrow \quad (f+g)(5) = 7$

$\because \quad (f+g)(5) = (f+g)(4)$

$\therefore \quad f+g$ is not one-one

Now, $\because f_{min} = 1$, $g_{min} = 3$

So, there does not exist any $x \in N - \{1\}$ such that $(f+g)(x) = 1, 2, 3$

$\therefore \quad f+g$ is not onto

74. Answer (1440)

$$A = \{x \in N, \quad x^2 - 10x + 9 \le 0\}$$

$$= \{1, 2, 3, ..., 9\}$$

$$B = \{1, 4, 9, 16, ...\}$$

$$f(x) \le (x-3)^2 + 1$$

$$f(1) \le 5,\ f(2) \le 2,\ ....f(9) \le 37$$

$x = 1$ has 2 choices

$x = 2$ has 1 choice

$x = 3$ has 1 choice

$x = 4$ has 1 choice

$x = 5$ has 2 choices

$x = 6$ has 3 choices

$x = 7$ has 4 choices

$x = 8$ has 5 choices

$x = 9$ has 6 choices

$\therefore \quad$ Total functions $= 2 \times 1 \times 1 \times 1 \times 2 \times 3 \times 4 \times 5 \times 6$

$= 1440$

75. Answer (1)

$$f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right)$$

$$\frac{a_1 + a_2 + a_3 + ... + a_n}{n} = 0$$

$\therefore \quad$ First and last term, second and second last and so on are equal in magnitude but opposite in sign.

$$f(x) = \alpha x^5 + \beta x^3 + \gamma x$$

$$\sum_{i=1}^{n} f(a_i) = \alpha\left(a_1^5 + a_2^5 + a_3^5 + ... + a_n^5\right)$$

$$+ \beta\left(a_1^3 + a_2^3 + ... + a_n^3\right)$$

$$+ \gamma\left(a_1 + a_2 + ... + a_n\right)$$

$$= 0\alpha + 0\beta + 0\gamma$$

$$= 0$$

$$\therefore \quad f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right) = \frac{1}{n}\sum_{i=1}^{n} f(a_i) = 0$$

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