Chapter 8

Trigonometric Functions

1. Let A and B denote the statements :

   A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

   B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

   If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then **[AIEEE-2009]**

   (1) A is false and B is true

   (2) Both A and B are true

   (3) Both A and B are false

   (4) A is true and B is false

2. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and let $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha$ = **[AIEEE-2010]**

   (1) $\frac{25}{16}$ (2) $\frac{56}{33}$

   (3) $\frac{19}{12}$ (4) $\frac{20}{7}$

3. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A ***false*** statement among the following is **[AIEEE-2010]**

   (1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

   (2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

   (3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

   (4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

4. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are **[AIEEE-2011]**

   (1) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

   (2) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

   (3) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

   (4) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

5. In a ΔPQR, if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to **[AIEEE-2012]**

   (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$

   (3) $\frac{3\pi}{4}$ (4) $\frac{5\pi}{6}$

6. $ABCD$ is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to **[JEE (Main)-2013]**

   (1) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (2) $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$

   (3) $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$ (4) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

7. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as **[JEE (Main)-2013]**

   (1) $\sin A \cos A + 1$

   (2) $\sec A \operatorname{cosec} A + 1$

   (3) $\tan A + \cot A$

   (4) $\sec A + \operatorname{cosec} A$

8. Let $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$ where $x\in R$ and $k\geq 1$. Then $f_4(x)-f_6(x)$ equals **[JEE (Main)-2014]**

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$

(3) $\frac{1}{6}$ (4) $\frac{1}{3}$

9. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point *O* on the ground is 45°. It flies off horizontally straight away from the point *O*. After one second, the elevation of the bird from *O* is reduced to 30°. Then the speed (in m/s) of the bird is **[JEE (Main)-2014]**

(1) $20\sqrt{2}$ (2) $20\left(\sqrt{3}-1\right)$

(3) $40\left(\sqrt{2}-1\right)$ (4) $40\left(\sqrt{3}-\sqrt{2}\right)$

10. If the angles of elevation of the top of a tower from three collinear points *A*, *B* and *C*, on a line leading to the foot of the tower, are 30º, 45º and 60º respectively, then the ratio, *AB* : *BC*, is **[JEE (Main)-2015]**

(1) $\sqrt{3}:1$ (2) $\sqrt{3}:\sqrt{2}$

(3) $1:\sqrt{3}$ (4) 2 : 3

11. If $0\leq x<2\pi$, then the number of real values of *x*, which satisfy the equation $\cos x+\cos 2x+\cos 3x+\cos 4x=0$, is **[JEE (Main)-2016]**

(1) 5 (2) 7

(3) 9 (4) 3

12. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point *A* on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from *A* in the same direction, at a point *B*, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from *B* to reach the pillar, is **[JEE (Main)-2016]**

(1) 10 (2) 20

(3) 5 (4) 6

13. If 5 $(\tan^2 x-\cos^2 x)=2\cos 2x+9$, then the value of $\cos 4x$ is **[JEE (Main)-2017]**

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$

(3) $-\frac{7}{9}$ (4) $-\frac{3}{5}$

14. Let a vertical tower *AB* have its end *A* on the level ground. Let *C* be the mid-point of *AB* and *P* be a point on the ground such that *AP* = 2*AB*. If $\angle BPC=\beta$ then $\tan\beta$ is **[JEE (Main)-2017]**

(1) $\frac{1}{4}$ (2) $\frac{2}{9}$

(3) $\frac{4}{9}$ (4) $\frac{6}{7}$

15. If sum of all the solutions of the equation $8\cos x\cdot\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0,\pi]$ is $k\pi$, then *k* is equal to : **[JEE (Main)-2018]**

(1) $\frac{2}{3}$ (2) $\frac{13}{9}$

(3) $\frac{8}{9}$ (4) $\frac{20}{9}$

16. *PQR* is a triangular park with *PQ* = *PR* = 200 m. A T.V. tower stands at the mid-point of *QR*. If the angles of elevation of the top of the tower at *P*, *Q* and *R* are respectively 45º, 30º and 30º, then the height of the tower (in m) is **[JEE (Main)-2018]**

(1) 100 (2) 50

(3) $100\sqrt{3}$ (4) $50\sqrt{2}$

17. For any $\theta\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$, the expression $3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4\sin^6\theta$ equals **[JEE (Main)-2019]**

(1) $13-4\cos^2\theta+6\sin^2\theta\cos^2\theta$

(2) $13-4\cos^2\theta+6\cos^4\theta$

(3) $13-4\cos^6\theta$

(4) $13-4\cos^4\theta+2\sin^2\theta\cos^2\theta$

18. If $0\leq x<\frac{\pi}{2}$, then the number of values of *x* for which $\sin x-\sin 2x+\sin 3x=0$, is **[JEE (Main)-2019]**

(1) 2 (2) 3

(3) 1 (4) 4

19. Consider a triangular plot *ABC* with sides *AB* = 7 m, *BC* = 5 m and *CA* = 6 m. A vertical lamp-post at the midpoint *D* of *AC* subtends an angle 30° at *B*. The height (in m) of the lamp-post is **[JEE (Main)-2019]**

(1) $2\sqrt{21}$ (2) $7\sqrt{3}$

(3) $\frac{2}{3}\sqrt{21}$ (4) $\frac{3}{2}\sqrt{21}$

20. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is **[JEE (Main)-2019]**

(1) $\frac{5\pi}{4}$ (2) $\frac{\pi}{2}$

(3) $\frac{3\pi}{8}$ (4) π

21. If 5, $5r$, $5r^2$ are the lengths of the sides of a triangle, then r **cannot** be equal to

[JEE (Main)-2019]

(1) $\frac{3}{2}$ (2) $\frac{7}{4}$

(3) $\frac{3}{4}$ (4) $\frac{5}{4}$

22. With the usual notation, in ΔABC, if $\angle A + \angle B = 120°$, $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, then the ratio $\angle A : \angle B$, is **[JEE (Main)-2019]**

(1) 7 : 1 (2) 3 : 1

(3) 9 : 7 (4) 5 : 3

23. The value of

$\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}. \ldots .\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$ is

[JEE (Main)-2019]

(1) $\frac{1}{512}$ (2) $\frac{1}{256}$

(3) $\frac{1}{2}$ (4) $\frac{1}{1024}$

24. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{-1}{12}$ (2) $\frac{1}{12}$

(3) $\frac{5}{12}$ (4) $\frac{1}{4}$

25. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triplet (α, β, γ) has a value

[JEE (Main)-2019]

(1) (3, 4, 5) (2) (7, 19, 25)

(3) (19, 7, 25) (4) (5, 12, 13)

26. The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is **[JEE (Main)-2019]**

(1) $\sqrt{34}$ (2) $\sqrt{19}$

(3) $\frac{\sqrt{79}}{2}$ (4) $\sqrt{31}$

27. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in meters) from the surface of the lake is

[JEE (Main)-2019]

(1) 45 (2) 50

(3) 42 (4) 60

28. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to

[JEE (Main)-2019]

(1) $\sqrt{2}$ (2) $-\sqrt{2}$

(3) –1 (4) 0

29. If $\cos(\alpha+\beta) = \frac{3}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to

[JEE (Main)-2019]

(1) $\frac{21}{16}$ (2) $\frac{63}{52}$

(3) $\frac{33}{52}$ (4) $\frac{63}{16}$

30. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is **[JEE (Main)-2019]**

(1) 16 (2) 18

(3) 15 (4) 12

31. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is

[JEE (Main)-2019]

(1) 4 : 5 : 6 (2) 3 : 4 : 5

(3) 5 : 9 : 13 (4) 5 : 6 : 7

32. Let $S = \{\theta \in [-2\pi, 2\pi] ; 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is

[JEE (Main)-2019]

(1) π (2) 2π

(3) $\frac{13\pi}{6}$ (4) $\frac{5\pi}{3}$

33. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is **[JEE (Main)-2019]**

(1) $\frac{3}{4}$ (2) $\frac{3}{2}(1+\cos 20°)$

(3) $\frac{3}{2}$ (4) $\frac{3}{4}+\cos 20°$

34. Two poles standing on a horizontal ground are of heights 5 m and 10 m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is

[JEE (Main)-2019]

(1) $5\left(2+\sqrt{3}\right)$ (2) $10\left(\sqrt{3}-1\right)$

(3) $5\left(\sqrt{3}+1\right)$ (4) $\frac{5}{2}\left(2+\sqrt{3}\right)$

35. The value of sin10° sin30° sin50° sin70° is

[JEE (Main)-2019]

(1) $\frac{1}{18}$ (2) $\frac{1}{32}$

(3) $\frac{1}{16}$ (4) $\frac{1}{36}$

36. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \le 1$ also satisfy the equation

[JEE (Main)-2019]

(1) $\sin x = |\sin y|$ (2) $\sin x = 2 \sin y$

(3) $2 \sin x = \sin y$ (4) $2|\sin x| = 3 \sin y$

37. ABC is a triangular park with $AB = AC = 100$ metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\operatorname{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is : **[JEE (Main)-2019]**

(1) 20 (2) $10\sqrt{5}$

(3) 25 (4) $\frac{100}{3\sqrt{3}}$

38. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq.cm) of this triangle is :

[JEE (Main)-2019]

(1) $\frac{2}{\sqrt{3}}$ (2) $4\sqrt{3}$

(3) $2\sqrt{3}$ (4) $\frac{4}{\sqrt{3}}$

39. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/s, then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is

[JEE (Main)-2019]

(1) $\frac{25}{3}$ (2) $25\sqrt{3}$

(3) $\frac{25}{\sqrt{3}}$ (4) 25

40. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is

[JEE (Main)-2019]

(1) 3 (2) 5

(3) 4 (4) 7

41. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is **[JEE (Main)-2019]**

(1) $15(3+\sqrt{3})$ (2) $15(1+\sqrt{3})$

(3) $15(3-\sqrt{3})$ (4) $15(5-\sqrt{3})$

42. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to **[JEE (Main)-2019]**

(1) [1, 4] (2) R

(3) [2, 6] (4) [3, 7]

43. The value of

$$\cos^3\left(\frac{\pi}{8}\right).\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right).\sin\left(\frac{3\pi}{8}\right) \text{ is}$$

[JEE (Main)-2020]

(1) $\frac{1}{4}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{2}$ (4) $\frac{1}{\sqrt{2}}$

44. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval

[JEE (Main)-2020]

(1) $\left[-1, -\frac{1}{2}\right]$ (2) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$

(3) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$ (4) $\left(-\frac{5}{4}, -1\right)$

45. Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is

[JEE (Main)-2020]

(1) 6 (2) 20/3

(3) 10/3 (4) 5

46. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and

$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then

[JEE (Main)-2020]

(1) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(2) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(3) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$

(4) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

47. The angle of elevation of a cloud C from a point P, 200 m above a still lake is 30°. If the angle of depression of the image of C in the lake from the point P is 60°, then PC (in m) is equal to

[JEE (Main)-2020]

(1) 400 (2) $400\sqrt{3}$

(3) 100 (4) $200\sqrt{3}$

48. The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is

[JEE (Main)-2020]

(1) $\frac{1}{\sqrt{3}+1}$ (2) $\frac{1}{\sqrt{3}-1}$

(3) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (4) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

49. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$, $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to ________.

[JEE (Main)-2020]

50. The number of distinct solutions of the equation, $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$, is ____________. **[JEE (Main)-2020]**

51. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is _____. **[JEE (Main)-2020]**

52. If $e^{\left(\cos^2 x+\cos^4 x+\cos^6 x+...\infty\right)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0 < x < \frac{\pi}{2}\right)$ is :

[JEE (Main)-2021]

(1) $\frac{3}{2}$ (2) $\sqrt{3}$

(3) $2\sqrt{3}$ (4) $\frac{1}{2}$

53. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is : **[JEE (Main)-2021]**

(1) $25\sqrt{3}$ (2) 30

(3) 25 (4) $20\sqrt{3}$

54. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is :

[JEE (Main)-2021]

(1) $3600\sqrt{3}$ m (2) $2400\sqrt{3}$ m

(3) $1800\sqrt{3}$ m (4) $1200\sqrt{3}$ m

55. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in : **[JEE (Main)-2021]**

(1) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(2) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(3) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(4) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

56. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At the point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is : **[JEE (Main)-2021]**

(1) $10\sqrt{3}$ (2) 10

(3) $10\left(\sqrt{3}+1\right)$ (4) $10\left(\sqrt{3}-1\right)$

57. If 0 < x, y < π and cos x + cos y – cos (x + y) = $\frac{3}{2}$, then sin x + cos y is equal to :

[JEE (Main)-2021]

(1) $\frac{1+\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{2}$ (4) $\frac{1-\sqrt{3}}{2}$

58. The number of integral values of 'k' for which the equation 3sinx + 4cosx = k+1 has a solution, k $\in$ R is __________. **[JEE (Main)-2021]**

59. If $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3}-1\right)$ cos x + 1, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is __________. **[JEE (Main)-2021]**

60. Let A(1, 4) and B(1, –5) be two points. Let P be a point on the circle $(x-1)^2 + (y-1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points, P, A and B lie on :

[JEE (Main)-2021]

(1) an ellipse (2) a parabola

(3) a straight line (4) a hyperbola

61. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval [0, π] is equal to **[JEE (Main)-2021]**

(1) 4 (2) 2

(3) 8 (4) 3

62. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\beta$, where α, β are integers, then $\alpha + \beta$ is equal to ______. **[JEE (Main)-2021]**

63. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to ________. **[JEE (Main)-2021]**

64. The number of solutions of the equation x + 2 tanx = $\frac{\pi}{2}$ in the interval [0, 2π] is :

[JEE (Main)-2021]

(1) 2 (2) 4

(3) 3 (4) 5

65. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the area of ΔPAB and ΔCAB is : **[JEE (Main)-2021]**

(1) 2 : 1 (2) 3 : 1

(3) 11 : 4 (4) 9 : 4

66. The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is ______. **[JEE (Main)-2021]**

67. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then $(R + r)$ is equal to : **[JEE (Main)-2021]**

(1) $3\sqrt{2}$ (2) $2\sqrt{2}$

(3) $\frac{9}{\sqrt{2}}$ (4) $7\sqrt{2}$

68. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to :

[JEE (Main)-2021]

(1) $\frac{2\sqrt{3}}{3}$ (2) $\frac{1}{\sqrt{3}}$

(3) $2\sqrt{3}$ (4) $\sqrt{3}$

69. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in \mathbf{R}$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to :

[JEE (Main)-2021]

(1) 350 (2) 250

(3) 400 (4) 500

70. If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :

[JEE (Main)-2021]

(1) $10 + 6\sqrt{2}$ (2) $6 + 8\sqrt{3}$

(3) $8 + 2\sqrt{2}$ (4) $4 + 2\sqrt{3}$

71. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to : **[JEE (Main)-2021]**

(1) $\frac{\sqrt{5}+1}{4}$ (2) $\frac{\sqrt{2}-1}{2}$

(3) $\frac{\sqrt{5}-1}{2}$ (4) $\frac{\sqrt{5}-1}{4}$

72. Consider a triangle having vertices A(–2, 3), B(1, 9) and C(3, 8). If a line L passing through the circumcenter of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is _____. **[JEE (Main)-2021]**

73. The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to : **[JEE (Main)-2021]**

(1) 5 (2) 7

(3) 11 (4) 9

74. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to _______. **[JEE (Main)-2021]**

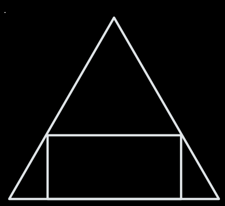
75. The value of $\cot\frac{\pi}{24}$ is

[JEE (Main)-2021]

(1) $3\sqrt{2} - \sqrt{3} - \sqrt{6}$ (2) $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$

(3) $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$ (4) $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$

76. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is _________. **[JEE (Main)-2021]**



77. If $\sin\theta + \cos\theta = \frac{1}{2}$, then $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to **[JEE (Main)-2021]**

(1) 23 (2) –23

(3) 27 (4) –27

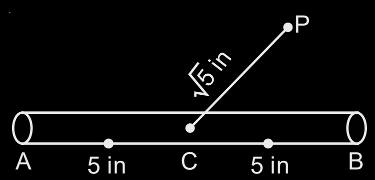
78. The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, \frac{\pi}{4}\right\}$ is :

[JEE (Main)-2021]

(1) $\frac{\pi}{10}$ (2) $-\frac{7\pi}{30}$

(3) $-\frac{11\pi}{30}$ (4) $-\frac{\pi}{15}$

79. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that $PC = \sqrt{5}$ inches and $\angle PCB = \tan^{-1}(2)$. The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is **[JEE (Main)-2021]**



(1) $\tan^{-1}\left(\frac{1}{2}\right)$ (2) $\tan^{-1}\left(\frac{3}{4}\right)$

(3) $\tan^{-1}(1)$ (4) $\tan^{-1}\left(\frac{4}{3}\right)$

80. The value of

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

is **[JEE (Main)-2021]**

(1) $\frac{1}{8\sqrt{2}}$ (2) $\frac{1}{4\sqrt{2}}$

(3) $\frac{1}{8}$ (4) $\frac{1}{4}$

81. Let $A(a, 0)$, $B(b, 2b + 1)$ and $C(0, b)$, $b \neq 0$, $|b| \neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is **[JEE (Main)-2021]**

(1) $\frac{2b}{b+1}$ (2) $\frac{-2b^2}{b+1}$

(3) $\frac{2b^2}{b+1}$ (4) $\frac{-2b}{b+1}$

82. Two poles, AB of length a metres and CD of length $a + b$ $(b \neq a)$ metres are erected at the same horizontal level with bases at B and D. If $BD = x$ and $\tan\underline{|ACB} = \frac{1}{2}$, then **[JEE (Main)-2021]**

(1) $x^2 - 2ax + a(a + b) = 0$

(2) $x^2 + 2(a + 2b)x - b(a + b) = 0$

(3) $x^2 + 2(a + 2b)x + a(a + b) = 0$

(4) $x^2 - 2ax + b(a + b) = 0$

83. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then $\frac{8S}{\pi}$ is equal to _______.

[JEE (Main)-2021]

84. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

[JEE (Main)-2021]

(1) $8\sqrt{10}$ (2) $12\sqrt{10}$

(3) $12\sqrt{15}$ (4) $6\sqrt{10}$

85. The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81$, $0 \le x \le \frac{\pi}{4}$ is **[JEE (Main)-2021]**

(1) 3 (2) 1

(3) 2 (4) 0

86. If n is the number of solutions of the equation

$$2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1,\ x \in [0, \pi]$$

and S is the sum of all these solutions, then the ordered pair (n, S) is : **[JEE (Main)-2021]**

(1) $\left(3, \frac{5\pi}{3}\right)$ (2) $\left(3, \frac{13\pi}{9}\right)$

(3) $\left(2, \frac{2\pi}{3}\right)$ (4) $\left(2, \frac{8\pi}{9}\right)$

87. Let the points of intersections of the lines $x - y + 1 = 0$, $x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is _____.

[JEE (Main)-2021]

88. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$f(x+y)+f(x-y)=2f(x)f(y),\ f\left(\frac{1}{2}\right)=-1$. Then, the value of $\sum_{k=1}^{20}\frac{1}{\sin(k)\sin(k+f(k))}$ is equal to

[JEE (Main)-2021]

(1) $\operatorname{cosec}^2(21)\cos(20)\cos(2)$

(2) $\sec^2(21)\sin(20)\sin(2)$

(3) $\operatorname{cosec}^2(1)\operatorname{cosec}(21)\sin(20)$

(4) $\sec^2(1)\sec(21)\cos(20)$

89. If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n} \theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$,

for $0 < \theta < \frac{\pi}{4}$, then **[JEE (Main)-2021]**

(1) $x(1 - y) = 1$ (2) $y(1 + x) = 1$

(3) $y(1 - x) = 1$ (4) $x(1 + y) = 1$

90. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is **[JEE (Main)-2021]**

(1) $8\left(2+2\sqrt{3}+\sqrt{2}\right)$ (2) $8\left(\sqrt{6}-\sqrt{2}+2\right)$

(3) $8\left(\sqrt{2}+2+\sqrt{3}\right)$ (4) $8\left(\sqrt{6}+\sqrt{2}+2\right)$

91. The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to : **[JEE (Main)-2021]**

(1) 11π (2) 9π

(3) 8π (4) 12π

92. Let

$$S = \left\{\theta \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}.$$

If $T = \sum_{\theta \in S} \cos 2\theta$, then $T + n(S)$ is equal to

[JEE (Main)-2022]

(1) $7+\sqrt{3}$ (2) 9

(3) $8+\sqrt{3}$ (4) 10

93. The number of solutions of the equation

$$\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{4}\cos^2 2x, \quad x \in [-3\pi, 3\pi]$$

is **[JEE (Main)-2022]**

(1) 8 (2) 5

(3) 6 (4) 7

94. Let a, b and c be the length of sides of a triangle ABC such that $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, then the value of $\frac{R}{r}$ is equal to **[JEE (Main)-2022]**

(1) $\frac{5}{2}$ (2) 2

(3) $\frac{3}{2}$ (4) 1

95. The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\operatorname{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$ holds, is ______________. **[JEE (Main)-2022]**

96. The value of $2\sin(12°) - \sin(72°)$ is

[JEE (Main)-2022]

(1) $\frac{\sqrt{5}\left(1-\sqrt{3}\right)}{4}$ (2) $\frac{1-\sqrt{5}}{8}$

(3) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{2}$ (4) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

97. If $\sin^2(10°)\sin(20°)\sin(40°)\sin(50°)\sin(70°)$

$= \alpha - \frac{1}{16}\sin(10°)$, then $16 + \alpha^{-1}$ is equal to _______. **[JEE (Main)-2022]**

98. $16\sin(20°)\sin(40°)\sin(80°)$ is equal to

[JEE (Main)-2022]

(1) $\sqrt{3}$ (2) $2\sqrt{3}$

(3) 3 (4) $4\sqrt{3}$

99. The value of $\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$ is equal to **[JEE (Main)-2022]**

(1) –1 (2) $-\frac{1}{2}$

(3) $-\frac{1}{3}$ (4) $-\frac{1}{4}$

100. $\alpha = \sin 36°$ is a root of which of the following equation? **[JEE (Main)-2022]**

(1) $16x^4 - 10x^2 - 5 = 0$ (2) $16x^4 + 20x^2 - 5 = 0$

(3) $16x^4 - 20x^2 + 5 = 0$ (4) $16x^4 - 10x^2 + 5 = 0$

101. Let AB and PQ be two vertical poles, 160 m apart from each other. Let C be the middle point of B and Q, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation from C to P and A, respectively. If the height of pole PQ is twice the height of pole AB, then $\tan^2\theta$ is equal to

[JEE (Main)-2022]

(1) $\frac{3-2\sqrt{2}}{2}$ (2) $\frac{3+\sqrt{2}}{2}$

(3) $\frac{3-2\sqrt{2}}{4}$ (4) $\frac{3-\sqrt{2}}{4}$

102. If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are **[JEE (Main)-2022]**

(1) $-\frac{1}{7}$ and IV$^{\text{th}}$ quadrant

(2) 7 and I$^{\text{st}}$ quadrant

(3) –7 and IV$^{\text{th}}$ quadrant

(4) $\frac{1}{7}$ and I$^{\text{st}}$ quadrant

103. The number of elements in the set

$S = \{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\}$ is __________. **[JEE (Main)-2022]**

104. The number of solutions of the equation $2\theta - \cos^2\theta + \sqrt{2} = 0$ in **R** is equal to __________. **[JEE (Main)-2022]**

105. From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60°. The pole subtends an angle 30° at the top of the tower. Then the height of the tower is **[JEE (Main)-2022]**

(1) $15\sqrt{3}$ (2) $20\sqrt{3}$

(3) $20+10\sqrt{3}$ (4) 30

106. A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that QR = 15 m. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is :

(1) $5\left(2\sqrt{3}+3\right)$ m (2) $5\left(\sqrt{3}+3\right)$ m

(3) $10\left(\sqrt{3}+1\right)$ m (4) $10\left(2\sqrt{3}+1\right)$ m

[JEE (Main)-2022]

107. Let a vertical tower AB of height $2h$ stands on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation 2α. When from P, he moves a distance d in the direction of $\overrightarrow{AP}$, he can see the top B of the tower with an angle of elevation α. if $d = \sqrt{7}h$, then $\tan\alpha$ is equal to

(1) $\sqrt{5}-2$ (2) $\sqrt{3}-1$

(3) $\sqrt{7}-2$ (4) $\sqrt{7}-\sqrt{3}$

[JEE (Main)-2022]

108. The angle of elevation of the top P of a vertical tower PQ of height 10 from a point A on the horizontal ground is 45°, Let R be a point on AQ and from a point B, vertically above R, the angle of elevation of P is 60°. If $\angle BAQ = 30°$, $AB = d$ and the area of the trapezium $PQRB$ is α, then the ordered pair (d, α) is :

(1) $\left(10(\sqrt{3}-1), 25\right)$ (2) $\left(10(\sqrt{3}-1), \frac{25}{2}\right)$

(3) $\left(10(\sqrt{3}+1), 25\right)$ (4) $\left(10(\sqrt{3}+1), \frac{25}{2}\right)$

[JEE (Main)-2022]

109. Let $S = \left\{\theta \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right)\sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}$.

Then **[JEE (Main)-2022]**

(1) $S = \left\{\frac{\pi}{12}\right\}$ (2) $S = \left\{\frac{2\pi}{3}\right\}$

(3) $\sum_{\theta \in S} \theta = \frac{\pi}{2}$ (4) $\sum_{\theta \in S} \theta = \frac{3\pi}{4}$

110. A horizontal park is in the shape of a triangle OAB with $AB = 16$. A vertical lamp post OP is erected at the point O such that $\angle PAO = \angle PBO = 15°$ and $\angle PCO = 45°$, where C is the midpoint of AB. Then $(OP)^2$ is equal to

(1) $\frac{32}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (2) $\frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$

(3) $\frac{16}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (4) $\frac{16}{\sqrt{3}}\left(2-\sqrt{3}\right)$

[JEE (Main)-2022]

111. Let $S=\left[-\pi, \frac{\pi}{2}\right)-\left[-\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}\right]$. Then the number of elements in the set $A=\left\{\theta \in S : \tan\theta\left(1+\sqrt{5}\tan(2\theta)\right)=\sqrt{5}-\tan(2\theta)\right\}$ is ________.

[JEE (Main)-2022]

112. The number of elements in the set

$S=\left\{x \in \mathbb{R} : 2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}\right\}$ is :

(1) 1 (2) 3

(3) 0 (4) Infinite

[JEE (Main)-2022]

113. If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to _______.

[JEE (Main)-2022]

114. Let $S=\left\{\theta \in [0, 2\pi] : 8^{2\sin^2\theta}+8^{2\cos^2\theta}=16\right\}$. Then $n(S)+\sum_{\theta \in S}\left(\sec\left(\frac{\pi}{4}+2\theta\right)\text{cosec}\left(\frac{\pi}{4}+2\theta\right)\right)$ is equal to

[JEE (Main)-2022]

(1) 0 (2) – 2

(3) – 4 (4) 12

115. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to :

[JEE (Main)-2022]

(1) $\frac{3}{16}$ (2) $\frac{1}{16}$

(3) $\frac{1}{32}$ (4) $\frac{9}{32}$

116. Let $S = \{\theta \in (0, 2\pi) : 7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2\}$. Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$, $\theta \in S$, is _______.

[JEE (Main)-2022]

117. The angle of elevation of the top of a tower from a point A due north of it is α and from a point B at a distance of 9 units due west of A is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point B from the tower is 15 units, then $\cot\alpha$ is equal to :

(1) $\frac{6}{5}$ (2) $\frac{9}{5}$

(3) $\frac{4}{3}$ (4) $\frac{7}{3}$

[JEE (Main)-2022]

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Chapter 8

Trigonometric Functions

1. Answer (2)

$2(\cos\beta\cos\gamma + \sin\beta\sin\gamma) + 2(\cos\gamma\cos\alpha + \sin\gamma\sin\alpha) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$+ \sin^2\alpha + \cos^2\alpha + \sin^2\beta + \cos^2\beta + \sin^2\gamma + \cos^2\gamma = 0$

$\Rightarrow (\sin\alpha + \sin\beta + \sin\gamma)^2 + (\cos\alpha + \cos\beta + \cos\gamma)^2 = 0$

$\Rightarrow \sin\alpha + \sin\beta + \sin\gamma = 0 = \cos\alpha + \cos\beta + \cos\gamma$

$\therefore$ Both A and B are true.

2. Answer (2)

$$\cos(\alpha+\beta) = \frac{4}{5} \Rightarrow \alpha + \beta \in 1^{st} \text{ quadrant}$$

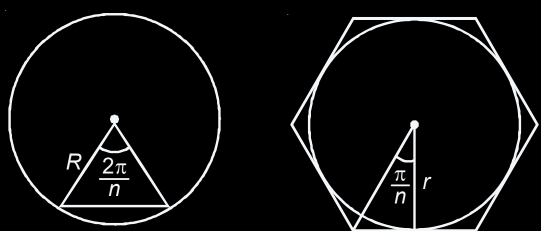
$$\sin(\alpha-\beta) = \frac{5}{13} \Rightarrow \alpha - \beta \in 1^{st} \text{ quadrant}$$

$2\alpha = (\alpha + \beta) + (\alpha - \beta)$

$$\therefore \tan 2\alpha = \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4}\cdot\frac{5}{12}} = \frac{56}{33}$$

3. Answer (3)



$$\frac{a}{2R} = \sin\frac{\pi}{n}$$

$$\frac{a}{2r} = \tan\frac{\pi}{n} \qquad \therefore \qquad \frac{r}{R} = \cos\frac{\pi}{n}$$

$n = 3$ gives $\dfrac{r}{R} = \dfrac{1}{2}$

$n = 4$ gives $\dfrac{r}{R} = \dfrac{1}{\sqrt{2}}$

$n = 6$ gives $\dfrac{r}{R} = \dfrac{\sqrt{3}}{2}$

4. Answer (2)

$\sin\theta + \sin 4\theta + \sin 7\theta = 0$

$2\sin 4\theta \cos 3\theta + \sin 4\theta = 0$

$\sin 4\theta (2\cos 3\theta + 1) = 0$

$$\sin 4\theta = 0 \text{ or } \cos 3\theta = -\frac{1}{2}$$

$$\Rightarrow 3\theta = 2n\pi \pm \left(\frac{2\pi}{3}\right)$$

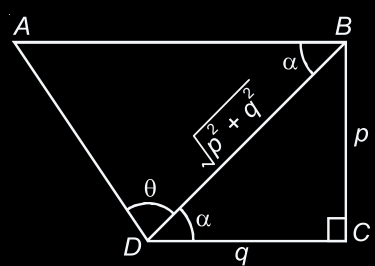
$$\theta = \frac{2n\pi}{3} \pm \frac{2\pi}{9}$$

$$\text{or } 4\theta = n\pi, \ \theta = \frac{n\pi}{4}$$

$$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$$

5. Answer (1)

6. Answer (1)



$$\frac{AB}{\sin\theta} = \frac{BD}{\sin(\theta+\alpha)}$$

$$\Rightarrow AB = \frac{\sqrt{p^2+q^2}\cdot\sin\theta}{\sin\theta\cdot\cos\alpha + \cos\theta\cdot\sin\alpha}$$

$$= \frac{\sqrt{p^2+q^2}\cdot\sin\theta}{\sin\theta\cdot\frac{q}{\sqrt{p^2+q^2}} + \cos\theta\cdot\frac{p}{\sqrt{p^2+q^2}}}$$

$$= \frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$$

7. Answer (2)

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$$

$$=\frac{\tan^2 A}{\tan A-1}+\frac{\cot A}{1-\tan A}$$

$$=\frac{\tan^2 A-\cot A}{\tan A-1}$$

$$=\frac{\tan^3 A-1}{\tan A(\tan A-1)}$$

$$=\frac{\tan^2 A+\tan A+1}{\tan A}$$

$= \tan A + 1 + \cot A$

$$=\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}+1$$

$$=\frac{1+\sin A\cos A}{\sin A\cdot\cos A}$$

$= 1 + \sec A\cdot\operatorname{cosec} A$

8. Answer (2)

$$f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$$

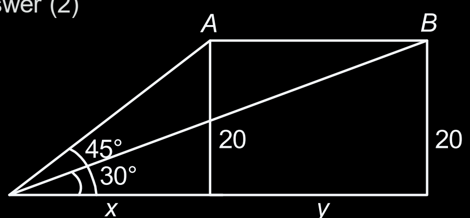
$$f_4(x)-f_6(x)$$

$$=\frac{1}{4}(\sin^4 x+\cos^4 x)-\frac{1}{6}(\sin^6 x+\cos^6 x)$$

$$=\frac{1}{4}[1-2\sin^2 x\cos^2 x]-\frac{1}{6}[1-3\sin^2 x\cos^2 x]$$

$$=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$$

9. Answer (2)



$t = 1$ s

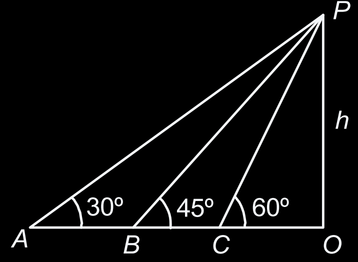
From figure $\tan 45^\circ=\dfrac{20}{x}$

and $\tan 30^\circ=\dfrac{20}{x+y}$

so, $y=20(\sqrt{3}-1)$

i.e., speed = $20(\sqrt{3}-1)$ m/s.

10. Answer (1)



$AO = h\cot 30^\circ$

$=h\sqrt{3}$

$BO = h$

$$CO=\frac{h}{\sqrt{3}}$$

$$\therefore\ \frac{AB}{BC}=\frac{AO-BO}{BO-CO}=\frac{h\sqrt{3}-h}{h-\frac{h}{\sqrt{3}}}=\sqrt{3}$$

11. Answer (2)

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$$2\cos\frac{5x}{2}\cdot\cos\frac{3c}{2}+2\cos\frac{5x}{2}\cdot\cos\frac{x}{2}=0$$

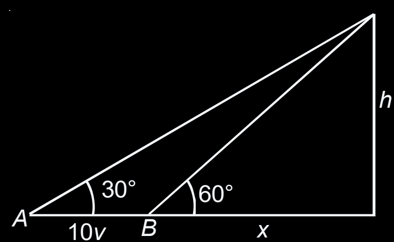
$$2\cos\frac{5x}{2}\times 2\cos\cdot x\cos\frac{x}{2}=0$$

$$x=\frac{(2n+1)\pi}{5},\frac{(2k+1)\pi}{2},(2r+1)\pi,$$

where $n, k, \in \mathbb{Z}$ $0 \le x < 2\pi$

Hence $x=\dfrac{\pi}{5},\dfrac{3\pi}{5},\pi,\dfrac{2\pi}{5},\dfrac{9\pi}{5},\dfrac{\pi}{2},\dfrac{3\pi}{2}$

12. Answer (3)



let speed = v units/min

$$\frac{h}{10v+x}=\tan 30^\circ$$

$$\frac{h}{x}=\tan 60^\circ$$

$$\Rightarrow\ \frac{x}{10v+x}=\frac{1}{3}\Rightarrow x=5v$$

So, time = 5 minutes.

13. Answer (3)

$5\tan^2 x = 9\cos^2 x + 7$

$5\sec^2 x - 5 = 9\cos^2 x + 7$

Let $\cos^2 x = t$

$$\frac{5}{t}=9t+12$$

$9t^2 + 12t - 5 = 0$

$t = \dfrac{1}{3}$ as $t \neq -\dfrac{5}{3}$

$\cos^2 x = \dfrac{1}{3}$, $\cos 2x = 2\cos^2 x - 1$

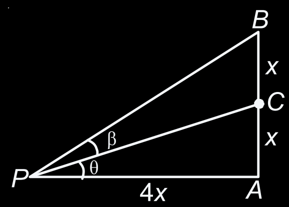
$= -\dfrac{1}{3}$

$\cos 4x = 2\cos^2 2x - 1$

$= \dfrac{2}{9} - 1$

$= -\dfrac{7}{9}$

14. Answer (2)



$\tan\theta = \dfrac{1}{4}$

$\tan(\theta + \beta) = \dfrac{1}{2}$

$\therefore \dfrac{\dfrac{1}{4} + \tan\beta}{1 - \dfrac{1}{4}\tan\beta} = \dfrac{1}{2}$

Solving $\tan\beta = \dfrac{2}{9}$

15. Answer (2)

$8\cos x \cdot \left(\cos^2\dfrac{\pi}{6} - \sin^2 x - \dfrac{1}{2}\right) = 1$

$\Rightarrow 8\cos x\left(\dfrac{3}{4} - \dfrac{1}{2} - 1 + \cos^2 x\right) = 1$

$\Rightarrow 8\cos x\left(\dfrac{-3 + 4\cos^2 x}{4}\right) = 1$

$\Rightarrow \cos 3x = 1$

$\Rightarrow \cos 3x = \dfrac{1}{2}$

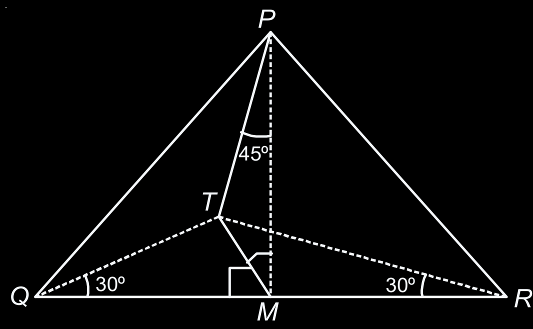
$\Rightarrow 3x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \dfrac{7\pi}{3}$

$\Rightarrow x = \dfrac{\pi}{9}, \dfrac{5\pi}{9}, \dfrac{7\pi}{9}$

$\Rightarrow$ Sum $= \dfrac{13\pi}{9}$

$\Rightarrow k = \dfrac{13}{9}$

16. Answer (1)



Let height of tower TM be h

$\therefore PM = h$

In ΔTQM, $\tan 30° = \dfrac{h}{QM}$

$QM = \sqrt{3}\,h$

In ΔPMQ, $PM^2 + QM^2 = PQ^2$

$h^2 + (\sqrt{3}h)^2 = 200^2$

$\Rightarrow 4h^2 = 200^2$

$\Rightarrow h = 100$ m

17. Answer (3)

$3(1 - 2\sin\theta\cos\theta)^2 + 6(1 + 2\sin\theta\cos\theta) + 4\sin^6\theta$

$= 3(1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta) + 6 + 12\sin\theta\cos\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta\cos^2\theta + 4\sin^6\theta$

$= 9 + 12\cos^2\theta(1 - \cos^2\theta) + 4(1 - \cos^2\theta)^3$

$= 9 + 12\cos^2\theta - 12\cos^4\theta + 4(1 - \cos^6\theta - 3\cos^2\theta + 3\cos^4\theta)$

$= 9 + 4 - 4\cos^6\theta$

$= 13 - 4\cos^6\theta$

18. Answer (1)

$\sin x - \sin 2x + \sin 3x = 0$

$\sin x - 2\sin x.\cos x + 3\sin x - 4\sin^3 x = 0$

$4\sin x - 4\sin^3 x - 2\sin x.\cos x = 0$

$2\sin x(1 - \sin^2 x) - \sin x.\cos x = 0$

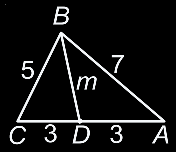
$2\sin x.\cos^2 x - \sin x.\cos x = 0$

$\sin x.\cos x(2\cos x - 1) = 0$

$\therefore \sin x = 0, \cos x = 0, \cos x = \dfrac{1}{2}$

$\therefore x = 0, \dfrac{\pi}{3} \quad \because x \in \left[0, \dfrac{\pi}{2}\right)$

19. Answer (3)



By Appolonius Theorem,

$$2\left(BD^2+\left(\frac{AC}{2}\right)^2\right)=BC^2+AB^2$$

$$\Rightarrow 2\left(m^2+3^2\right)=25+9 \Rightarrow m=2\sqrt{7}$$

$$\tan 30^\circ=\frac{\text{height of lamppost}}{\text{BD}}$$

$$\Rightarrow \text{Height}=2\sqrt{7}\times\frac{1}{\sqrt{3}}=\frac{2\sqrt{21}}{3}$$

20. Answer (2)

$$\sin^2 2\theta+\cos^4 2\theta=\frac{3}{4}$$

$$\Rightarrow 1-\cos^2 2\theta+\cos^4 2\theta=\frac{3}{4}$$

$$\Rightarrow \cos^2 2\theta\left(1-\cos^2 2\theta\right)=\frac{1}{4} \qquad ...(1)$$

$$\text{But } \left(\cos^2 2\theta\right)\left(1-\cos^2 2\theta\right)\le\left(\frac{\cos^2 2\theta+\left(1-\cos^2 2\theta\right)}{2}\right)^2$$

$$=\frac{1}{4} \qquad ...(2)$$

Equations (1) and (2)

$\Rightarrow \cos^2 2\theta = 1 - \cos^2 2\theta$

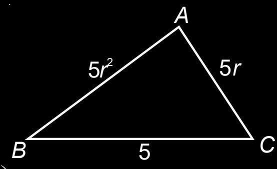
$\Rightarrow \cos^2 2\theta = \frac{1}{2}$

$\Rightarrow \cos 2\theta = \pm\frac{1}{\sqrt{2}}$

$$\Rightarrow \theta=\frac{\pi}{8},\ \frac{3\pi}{8}$$

$$\text{Sum}=\frac{\pi}{8}+\frac{3\pi}{8}=\frac{\pi}{2}$$

21. Answer (2)



For ΔABC is possible if

$5 + 5r > 5r^2$

$1 + r > r^2$

$r^2 - r - 1 < 0$

$$\left(r-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\left(r-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)<0$$

$$\therefore\ r\in\left(\frac{-\sqrt{5}+1}{2},\frac{\sqrt{5}+1}{2}\right)$$

$$\therefore\ r\ne\frac{7}{4}$$

22. Answer (1)

$\because\ A + B = 120°$

$\Rightarrow\ C = 180° - 120° = 60°$

$$\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$=\frac{2}{2\sqrt{3}}(\cot 30^\circ)=\frac{1}{\sqrt{3}}\times\sqrt{3}=1$$

$$\Rightarrow\ \frac{A-B}{2}=\frac{\pi}{2}$$

$\Rightarrow\ A - B = 90°$

$\Rightarrow\ A = 105°,\qquad B = 15°$

$\Rightarrow\ \angle A : \angle B :: 7 : 1$

Option (1) is correct.

23. Answer (1)

$$E = \cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}.....\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$$

$$= \frac{1}{2}\left(\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}....\cos\frac{\pi}{2^9}\sin\frac{\pi}{2^9}\right)$$

$$= \frac{1}{2^8}\left(\cos\frac{\pi}{2^2}.\sin\frac{\pi}{2^2}\right)=\frac{1}{2^9}\sin\frac{\pi}{2}$$

$$= \frac{1}{512}$$

24. Answer (2)

$$f_k(x)=\frac{1}{k}\left(\sin^k x+\cos^k x\right)$$

$$f_4(x)=\frac{1}{4}[\sin^4 x+\cos^4 x]=\frac{1}{4}\left[1-\frac{(\sin 2x)^2}{2}\right]$$

$$f_6(x)=\frac{1}{6}[\sin^6 x+\cos^6 x]=\frac{1}{6}\left[1-\frac{3}{4}(\sin 2x)^2\right]$$

$$\text{Now } f_4(x)-f_{(6)}(x)=\frac{1}{4}-\frac{1}{6}-\frac{(\sin 2x)^2}{8}+\frac{1}{8}(\sin 2x)^2$$

$$=\frac{1}{12}$$

25. Answer (2)

$\because \quad \frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = k \text{ (Say)}.$

$\therefore \quad b + c = 11k,\ c + a = 12k,\ a + b = 13k$

$\therefore \quad a + b + c = 18k$

$\therefore \quad a = 7k,\ b = 6k$ and $c = 5k$

$$\therefore \quad \cos A = \frac{36k^2 + 25k^2 - 49k^2}{2.30k^2} = \frac{1}{5}$$

$$\text{and } \cos B = \frac{49k^2 + 25k^2 - 36k^2}{2.35k^2} = \frac{19}{35}$$

$$\text{and } \cos C = \frac{49k^2 + 36k^2 - 25k^2}{2.42k^2} = \frac{5}{7}$$

$\therefore \quad \cos A : \cos B : \cos C = 7 : 19 : 25$

$$\therefore \quad \frac{\cos A}{7} = \frac{\cos B}{19} = \frac{\cos C}{25}$$

26. Answer (2)

$$f(\theta) = 3\cos\theta + 5\sin\theta\cdot\cos\frac{\pi}{6} - 5\sin\frac{\pi}{6}\cos\theta$$

$$= \left(3 - \frac{5}{2}\right)\cos\theta + 5\times\frac{\sqrt{3}}{2}\sin\theta$$

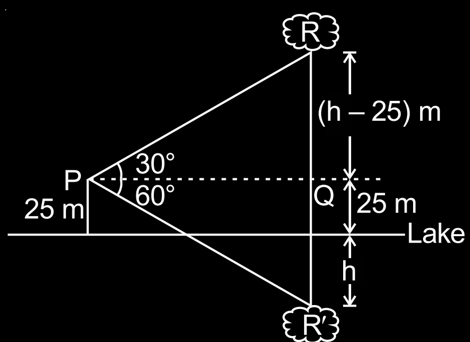
$$= \frac{1}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$$

$$\max f(\theta) = \sqrt{\frac{1}{4} + \frac{25}{4}\times 3}$$

$$= \sqrt{\frac{76}{4}} = \sqrt{19}$$

27. Answer (2)

Let height of the cloud from the surface of the lane is h meters.



$\therefore \quad$ In ΔPQR :

$$\tan 30° = \frac{h - 25}{PQ}$$

$\therefore \quad PQ = (h - 25)\sqrt{3} \qquad ...(1)$

and in $\Delta PQR'$: $\tan 60° = \frac{h + 25}{PQ}$

$$PQ = \frac{h + 25}{\sqrt{3}} \qquad ...(2)$$

From Eq. (1) and (2),

$$(h - 25)\sqrt{3} = \frac{h + 25}{\sqrt{3}}$$

$\therefore \quad h = 50$ m

28. Answer (2)

$\because \quad \sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}$

$\sin\alpha\cdot\cos\beta,\ \alpha, \beta \in [0, \pi]$

By A.M., G.M. ineqality;

$$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq \left(\sin^4\alpha\cdot 4\cos^4\beta\cdot 1\cdot 1\right)^{\frac{1}{4}}$$

$$\sin^4\alpha + 4\cos^4\beta + 1 + 1 \geq 4\sqrt{2}\sin\alpha\cdot|\cos\beta|$$

When $\cos\beta < 0$ then inequality still holds but L.H.S. is positive than $\cos\beta > 0$

Here, L.H.S. = R.H.S

$\therefore \quad \sin^4\alpha = 1$ and $\cos^4\beta = \frac{1}{4}$

$\therefore \quad \alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{4}$

$\therefore \quad \cos(\alpha + \beta) - \cos(\alpha - \beta)$

$$= \cos\left(\frac{\pi}{2} + \beta\right) - \cos\left(\frac{\pi}{2} - \beta\right)$$

$$= -\sin\beta - \sin\beta$$

$$= -2\sin\frac{\pi}{4} = -\sqrt{2}$$

29. Answer (4)

$\because \quad \alpha + \beta$ and $\alpha - \beta$ both are acute angles.

$$\cos(\alpha + \beta) = \frac{3}{5}$$

$$\Rightarrow \quad \tan(\alpha + \beta) = \frac{4}{3}$$

$$\text{And } \sin(\alpha - \beta) = \frac{5}{13}$$

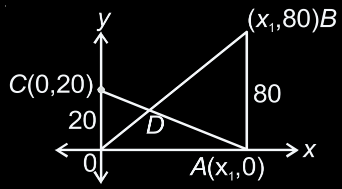
$$\Rightarrow \quad \tan(\alpha - \beta) = \frac{5}{12}$$

Now, $\tan 2\alpha = \tan\left((\alpha + \beta) + (\alpha - \beta)\right)$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\cdot\tan(\alpha - \beta)}$$

$$= \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}\cdot\frac{5}{12}} = \frac{63}{16}$$

30. Answer (1)



equation of line *OB* and *AC* are respectively

$$y = \frac{80}{x_1}x \qquad \text{...(i)}$$

$$\frac{x}{x_1} + \frac{y}{20} = 1 \qquad \text{...(ii)}$$

For intersection point, from equations (i) and (ii)

$$\frac{y}{80} + \frac{y}{20} = 1$$

$\Rightarrow$ $y + 4y = 80$

$\Rightarrow$ $y = 16$ m

$\Rightarrow$ Height of intersection point is 16 m

31. Answer (1)

Let $a > b > c$

$\because$ $A = 2C$

$\Rightarrow$ $A + B + C = \pi$

$\Rightarrow$ $B = \pi - 3C$ …(i)

$\because$ $a + c = 2b$

$\Rightarrow$ $\sin A + \sin C = 2\sin B$ …(ii)

$\Rightarrow$ $\sin A = \sin(2C)$, $\sin B = \sin 3C$

$\Rightarrow$ From (ii),

$\sin 2C + \sin C = 2\sin 3C$

$(2\cos C + 1)\sin C = 2\sin C\,(3 - 4\sin^2 C)$

$\Rightarrow$ $2\cos C + 1 = 6 - 8(1 - \cos^2 C)$

$\Rightarrow$ $8\cos^2 C - 2\cos C - 3 = 0$

$\Rightarrow$ $\cos C = \frac{3}{4}$ or $\cos C = -\frac{1}{2}$

$\because$ C is acute angle.

$\Rightarrow$ $\cos C = \frac{3}{4}, \sin A = 2\sin C \cos C = 2 \times \frac{\sqrt{7}}{4} \times \frac{3}{4}$

$\Rightarrow$ $\sin C = \frac{\sqrt{7}}{4}, \sin B = \frac{3\sqrt{7}}{4} - \frac{4\sqrt{7}}{4} \times \frac{7}{16} = \frac{5\sqrt{7}}{16}$

$\Rightarrow$ $\sin A : \sin B : \sin C :: a : b : c$ is $6 : 5 : 4$

32. Answer (2)

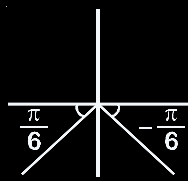
$\because$ $2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow$ $2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow$ $(2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow$ $\sin\theta = -\frac{1}{2}$; $\boxed{\sin\theta = 2} \rightarrow$ Not Possible

$\Rightarrow$



$\therefore$ Sum of all solutions in $[-2\pi, 2\pi]$ is

$$= \left(\pi + \frac{\pi}{6}\right) + \left(2\pi - \frac{\pi}{6}\right) + \left(-\frac{\pi}{6}\right) + \left(-\pi + \frac{\pi}{6}\right) = 2\pi$$

33. Answer (1)

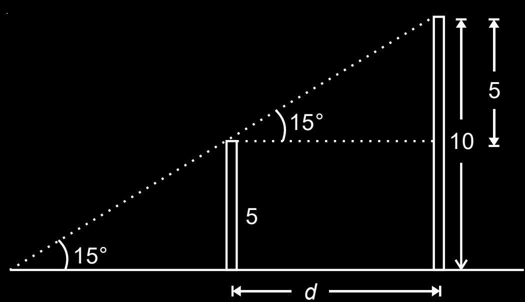
$$\left(\frac{1+\cos 20^\circ}{2}\right) + \left(\frac{1+\cos 100^\circ}{2}\right) - \frac{1}{2}(2\cos 10^\circ \cos 50^\circ)$$

$$= 1 + \frac{1}{2}(\cos 20^\circ + \cos 100^\circ) - \frac{1}{2}[\cos 60^\circ + \cos 40^\circ]$$

$$= \left(1 - \frac{1}{4}\right) + \frac{1}{2}[\cos 20^\circ + \cos 100^\circ - \cos 40^\circ]$$

$$= \frac{3}{4} + \frac{1}{2}[2\cos 60^\circ \times \cos 40^\circ - \cos 40^\circ] = \frac{3}{4}$$

34. Answer (1)



$$\tan 15^\circ = \frac{5}{d} \Rightarrow d = \frac{5}{\tan 15^\circ} = \frac{5(\sqrt{3}+1)}{\sqrt{3}-1}$$

$$= \frac{5(4+2\sqrt{3})}{2}$$

$$= 5(2+\sqrt{3})$$

35. Answer (3)

$\sin(60° + A) \cdot \sin(60° - A) \sin A = \frac{1}{4} \sin 3A$

Hence, $\sin 10° \sin 50° \sin 70°$

$= \sin 10° \sin(60° - 10°) \sin(60° + 10°) = \frac{1}{4} \sin 30°$

Hence, $\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{4} \sin^2 30° = \frac{1}{16}$

36. Answer (1)

$2^{\sqrt{\sin^2 x - 2\sin x + 5}} \le 2^{2\sin^2 y}$

$\Rightarrow \sqrt{\sin^2 x - 2\sin x + 5} \le 2\sin^2 y$

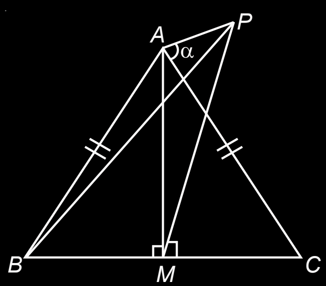
$\Rightarrow \sqrt{(\sin x - 1)^2 + 4} \le 2\sin^2 y$

it is true when $\sin x = 1$

$|\sin y| = 1$

so $\sin x = |\sin y|$

37. Answer (1)



ΔAPM,

$\frac{h}{AM} = \frac{1}{3\sqrt{2}}$

ΔBPM,

$\frac{h}{BM} = \frac{1}{\sqrt{7}}$

ΔABM

$\therefore AM^2 + MB^2 = (100)^2$

$\Rightarrow 18h^2 + 7h^2 = 100 \times 100$

$\Rightarrow h^2 = 4 \times 100$

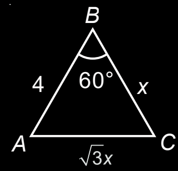
$\Rightarrow h = 20$

38. Answer (3)

$\because$ A, B, C, are in A.P

$\Rightarrow 2B = A + C$

$\Rightarrow B = \frac{\pi}{3}$



$\text{Area} = \frac{1}{2}(4x)\sin 60°$

$= \sqrt{3}x$

Now $\cos 60° = \frac{16 + x^2 - 3x^2}{8x}$

$\Rightarrow 4x = 16 - 2x^2$

$x = 2$ (as -4 is rejected)

Hence, area $= 2\sqrt{3}$ sq. cm

39. Answer (3)

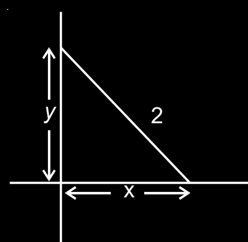
Given, $\frac{dy}{dt} = -25$ at $y = 1$

$x^2 + y^2 = 4$

When $y = 1$, $x = \sqrt{3}$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\Rightarrow x\frac{dx}{dt} + y\frac{dy}{dt} = 0$



$\Rightarrow \sqrt{3}\frac{dx}{dt} + (-25) = 0$

$\Rightarrow \frac{dx}{dt} = \frac{25}{\sqrt{3}}$ cm/s

40. Answer (2)

$1 + \sin^4 x = \cos^2 3x$

L.H.S $= 1 + \sin^4 x$, R.H.S $= \cos^2 3x$

L.H.S ≥ 1 R.H.S ≤ 1

$\Rightarrow$ L.H.S. = R.H.S. = 1

$\sin^4 x = 0$ and $\cos^2 3x = 1$

$\sin x = 0$ and $(4\cos^2 x - 3)^2 \cos^2 x = 1$

$\Rightarrow \sin x = 0$ and $\cos^2 x = 1$

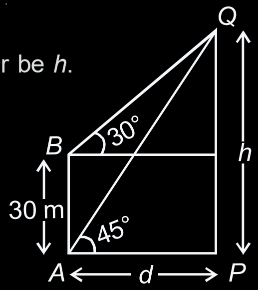
$\Rightarrow x = 0, \pm\pi, \pm 2\pi$

$\Rightarrow$ Total number of solutions is 5

41. Answer (1)

Let the height of the tower be h.

Refer to diagram ;



$$\tan 45^\circ = \frac{h}{d} = 1$$

$h = d$ ...(i)

$$\tan 30^\circ = \frac{h-30}{d}$$

$\sqrt{3}(h-30) = d$ ...(ii)

from (i) and (ii),

$$\sqrt{3}d = d + 30\sqrt{3}$$

$$d = \frac{30\sqrt{3}}{\sqrt{3}-1} = 15\sqrt{3}\left(\sqrt{3}+1\right) = 15\left(3+\sqrt{3}\right)$$

42. Answer (3)

$\cos 2x + \alpha \sin x = 2\alpha - 7$

$1 - 2\sin^2 x + \alpha \sin x = 2\alpha - 7$

$\Rightarrow\ 2\sin^2 x - \alpha \sin x + (2\alpha - 8) = 0$

$$\Rightarrow\ \sin x = \frac{\alpha \pm \sqrt{\alpha^2 - 8(2\alpha - 8)}}{4}$$

$$\Rightarrow\ \sin x = \frac{\alpha \pm (\alpha - 8)}{4}$$

$\Rightarrow\ \sin x = 2$ (rejected) or $\sin x = \dfrac{\alpha - 4}{2}$

$\because$ Equation has solution, then $\dfrac{\alpha - 4}{2} \in [1, 1]$

$\Rightarrow\ \alpha \in [2, 6]$

43. Answer (2)

$$\because\ \frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2}$$

$$\cos^3\frac{\pi}{8}\cdot\cos\frac{3\pi}{8} + \sin^3\frac{\pi}{8}\cdot\sin\frac{3\pi}{8}$$

$$= \cos^3\frac{\pi}{8}\cdot\sin\frac{\pi}{8} + \sin^3\frac{\pi}{8}\cdot\cos\frac{\pi}{8}$$

$$= \sin\frac{\pi}{8}\cdot\cos\frac{\pi}{8} \qquad \left(\because \sin\theta\cdot\cos\theta = \frac{1}{2}\sin 2\theta\right)$$

$$= \frac{1}{2}\sin\frac{\pi}{4}$$

$$= \frac{1}{2\sqrt{2}}$$

44. Answer (1)

$\sin^4\theta + \cos^4\theta = -\lambda$

$\Rightarrow\ 1 - 2\sin^2\theta\cos^2\theta = -\lambda$

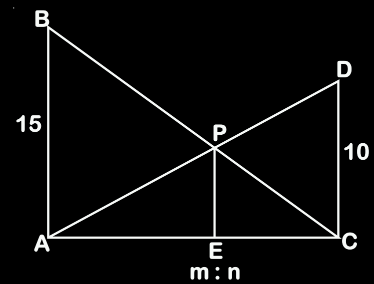
$$\Rightarrow\ \lambda = \frac{(\sin 2\theta)^2}{2} - 1$$

$\Rightarrow$ as $\sin^2 2\theta \in [0, 1]$

$$\Rightarrow\ \lambda \in \left[-1, \frac{-1}{2}\right]$$

45. Answer (1)

Refer to diagram,



Let $PE \perp AC$

and $\dfrac{AE}{EC} = \dfrac{m}{n}$

So; $PE = \dfrac{10m}{m+n}$ ...(1)

(because ΔACD and ΔAEP are similar)

Similarly $PE = \dfrac{15n}{m+n}$ ...(2)

From (1) and (2)

$10m = 15\,n \quad \Rightarrow \quad m = \dfrac{3}{2}n$

So $PE = 6$

46. Answer (2)

$$L + M = 1 - 2\sin^2\frac{\pi}{8} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \quad \text{...(i)}$$

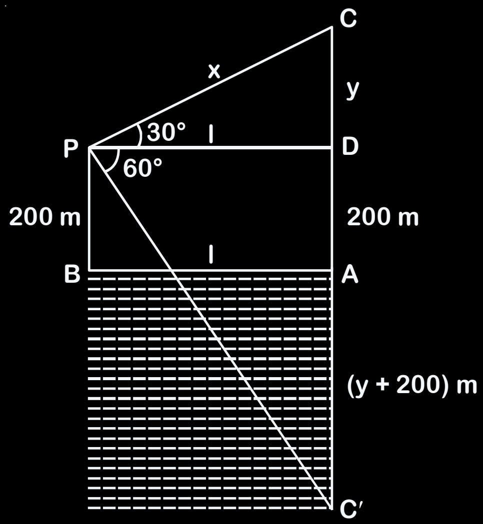
and $L - M = -\cos\dfrac{\pi}{8}$ ...(ii)

By (i) and (ii)

$$L = \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\frac{\pi}{8}$$

$$\&\ M = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$$

47. Answer (1)



Here in ΔPCD

$$\sin 30° = \frac{y}{x}$$

$$\Rightarrow \quad x = 2y$$

$$\Rightarrow \quad \boxed{y = \frac{x}{2}} \qquad \ldots(i)$$

$$\cos 30° = \frac{l}{x}$$

$$\Rightarrow \quad l = \frac{\sqrt{3}x}{2}$$

Now, in right $\Delta PC'D$

$$\tan 60° = \frac{y + 400}{l}$$

$$\Rightarrow \quad y + 400 = \sqrt{3}l = \sqrt{3} \times \frac{\sqrt{3}x}{2}$$

$$\Rightarrow \quad \frac{x}{2} - \frac{3x}{2} = -400$$

$$\Rightarrow \quad x = 400 \text{ m}$$

So, $PC = 400$ m

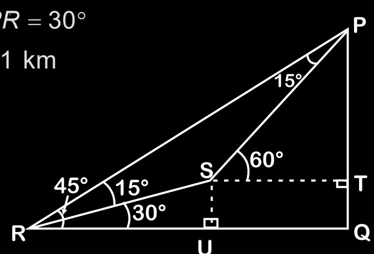
48. Answer (2)

$\because \angle SRP = \angle SPR = 30°$

$\therefore \quad SP = SR = 1$ km

In ΔSPT,

$$\frac{PT}{SP} = \sin 60°$$



$$PT = \frac{\sqrt{3}}{2} \text{ km}$$

In ΔRSU, $\sin 30° = \dfrac{SU}{RS} \Rightarrow SU = \dfrac{1}{2}$ km

$$\therefore \text{ Height of mountain } = \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \left(\frac{\sqrt{3}+1}{2}\right) \text{ km}$$

$$= \frac{1}{\sqrt{3}-1} \text{ km}$$

49. Answer (01)

$$\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\,|\cos\alpha|} = \frac{1}{7} \quad \Rightarrow \quad \tan\alpha = \frac{1}{7} \quad \because \ \alpha \in \left(0, \frac{\pi}{2}\right)$$

$$\therefore \ |\cos\alpha| = \cos\alpha.$$

$$\text{Now, } |\sin\beta| = \frac{1}{\sqrt{10}} \ \Rightarrow \ \sin\beta = \frac{1}{\sqrt{10}} \ \because \ \beta \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow \boxed{\tan\beta = \frac{1}{3}}$$

$$\Rightarrow \quad \tan(2\beta) = \frac{2\tan\beta}{1 - \tan^2\beta} = \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{2}{3} \times \frac{9}{8} = \frac{3}{4}.$$

$$\therefore \quad \tan(\alpha + 2\beta) = \frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \times \frac{3}{4}} = \frac{25}{25} = 1$$

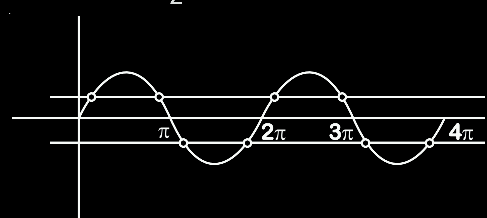
50. Answer (8.00)

$$\log_{\frac{1}{2}}|\sin x| + \log_{\frac{1}{2}}|\cos x| = 2$$

$$\Rightarrow \quad \log_{\frac{1}{2}}|\sin x \cdot \cos x| = 2$$

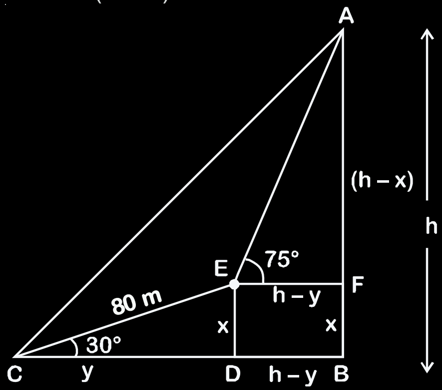
$$\Rightarrow \quad |\sin 2x| = \frac{1}{2}$$

$$\Rightarrow \quad \sin 2x = \pm\frac{1}{2}, \quad \text{where } 2x \in [0, 4\pi]$$



Total 8 solutions

51. Answer (80.00)



In rt ΔCDE–

Let height = h m

$$\sin 30° = \frac{x}{80} \Rightarrow x = 40$$

$$\cos 30° = \frac{y}{80} \Rightarrow y = 40\sqrt{3}$$

Now, In ΔAEF

$$\tan 75° = \frac{h-x}{h-y}$$

$$(2+\sqrt{3}) = \frac{h-40}{h-40\sqrt{3}}$$

$$(2+\sqrt{3})(h-40\sqrt{3}) = h-40$$

$$\Rightarrow \quad 2h - 80\sqrt{3} + \sqrt{3}h - 120 = h - 40$$

$$\Rightarrow \quad h + \sqrt{3}h = 80 + 80\sqrt{3}$$

$$\Rightarrow \quad (\sqrt{3}+1)h = 80(\sqrt{3}+1)$$

h = 80 m

52. Answer (4)

$$e^{\left(\frac{\cos^2 x}{1-\cos^2 x}\right)\ln 2} = 2^{\cot^2 x}$$

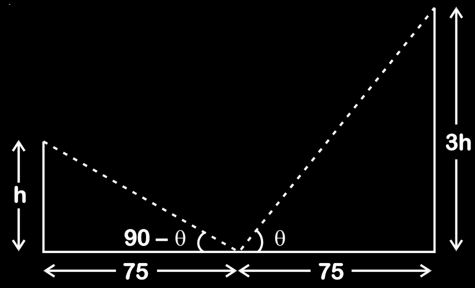
$\because$ t = 1 or 8

So, $2^{\cot^2 x} = 2^0$ or $2^3 \Rightarrow \cot^2 x = 0$ or 3

$$\because \quad x \in \left(0, \frac{\pi}{2}\right) \text{ then } \cot x = \sqrt{3} \Rightarrow x = \frac{\pi}{6}$$

$$\frac{2\sin x}{\sin x + \sqrt{3}\cos x} = \frac{2\left(\frac{1}{2}\right)}{\frac{1}{2} + \sqrt{3}\cdot\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{2}\sqrt{2}$$

53. Answer (1)



Given $\tan\theta = \dfrac{3h}{75}$ ...(i)

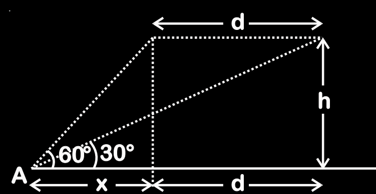
and $\tan(90-\theta) = \dfrac{h}{75}$ ...(ii)

$\Rightarrow$ Multiplying (i) and (ii) we get,

$$1 = \frac{3h^2}{(75)^2}$$

$$\Rightarrow \quad h = 25\sqrt{3}$$

54. Answer (4)



Given $\tan 30° = \dfrac{h}{x+d}$ and $\tan 60° = \dfrac{h}{x}$

$$\Rightarrow \quad x + d = \sqrt{3}h \text{ and } x = \frac{h}{\sqrt{3}}$$

$$\Rightarrow \quad d = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)h = \frac{2h}{\sqrt{3}}$$

Given $\dfrac{d}{20} = \dfrac{432 \times 1000}{3600}$

$\Rightarrow$ d = 2400 m

$$\Rightarrow \quad 2400 = \frac{2h}{\sqrt{3}} \Rightarrow h = 1200\sqrt{3} \text{ m}$$

55. Answer (4)

$\sin 2\theta + \tan 2\theta > 0$ $\quad \dfrac{\sin 2\theta + \cos 2\theta + \sin 2\theta}{\cos 2\theta} > 0$

$\tan 2\theta(1 + \cos 2\theta) > 0$

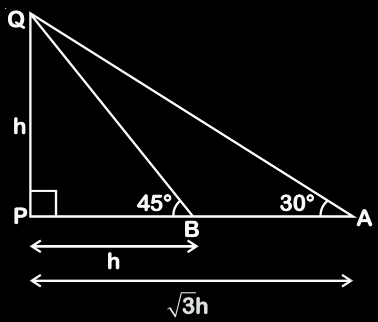
$\Rightarrow$ $\tan 2\theta > 0$ and $\cos 2\theta \neq -1$

$\Rightarrow \quad 2\theta \in \left(n\pi,\ n\pi + \frac{\pi}{2}\right)$	$2\theta \neq (2n+1)\pi$
$\Rightarrow \quad \theta \in \left(\frac{n\pi}{2},\ (2n+1)\frac{\pi}{4}\right)$ ...(i)	$\theta \neq (2n+1)\frac{\pi}{2}$ ...(ii)

$\Rightarrow \quad \theta \in [0, 2\pi]$

$\therefore \quad \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

56. Answer (3)



Let PQ = h

PB = h cot45° = h

PA = h cot30 = $\sqrt{3}$**h**

AB = PA – PB

$= \left(\sqrt{3} - 1\right)h$

$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

$\frac{AB}{20} = \frac{PB}{t}$

$\frac{\left(\sqrt{3}-1\right)h}{20} = \frac{h}{t} \Rightarrow t = \frac{20}{\sqrt{3}-1} = 10\left(\sqrt{3}+1\right)$

57. Answer (1)

LHS = cosx + cosy – cos(x + y)

$= 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - \left(2\cos^2\frac{x+y}{2} - 1\right)$

$\leq 2\cos\frac{x+y}{2} - 2\cos^2\frac{x+y}{2} + 1$

$\because \quad \left[\frac{x-y}{2} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow 0 < \cos\left(\frac{x-y}{2}\right) \leq 1\right]$

$= 1 - 2\left(\cos^2\left(\frac{x+y}{2}\right) - \cos\left(\frac{x+y}{2}\right)\right)$

$= 1 - 2\left[\left(\cos\left(\frac{x+y}{2}\right) - \frac{1}{2}\right)^2 - \frac{1}{4}\right]$

$= \frac{3}{2} - 2\left(\cos\left(\frac{x+y}{2}\right) - \frac{1}{2}\right)^2 \leq \frac{3}{2}$

But given that LHS $= \frac{3}{2}$

$\therefore \quad \cos\frac{x-y}{2} = 1$ and $\cos\left(\frac{x+y}{2}\right) = \frac{1}{2}$

$\Rightarrow \quad x - y = 0$ and $x + y = \frac{2\pi}{3}$

$\Rightarrow \quad x = y = \frac{\pi}{3}$

$\Rightarrow \quad \sin x + \cos y = \frac{\sqrt{3}+1}{2}$

58. Answer (11)

3sinx + 4cosx = k + 1 has a solution then

$k + 1 \in [-5, 5]$

$\therefore \quad k \in [-6, 4]$

$\therefore$ Number of possible integral values of k = 11.

59. Answer (01)

$\sqrt{3}\cos^2 x = (\sqrt{3} - 1)\cos x + 1$

$\sqrt{3}\cos^2 x = \sqrt{3}\cos x + \cos x - 1 = 0$

$\sqrt{3}\cos x(\cos x - 1) + (\cos x - 1) = 0$

$(\cos x - 1)(\sqrt{3}\cos x + 1) = 0$

$\therefore \quad \cos x = 1$ or $-\frac{1}{\sqrt{3}}$

$\therefore$ Number of solution in $x \in \left[0, \frac{\pi}{2}\right]$ is 1.

60. Answer (3)

Let P be (1 + cosθ, 1 + sinθ)

$(PA)^2 + (PB)^2 = (\cos\theta)^2 + (\sin\theta - 3)^2 + (\cos\theta)^2 + (\sin\theta + 6)^2$

$= 1 - 6\sin\theta + 9 + 1 + 12\sin\theta + 36$

$= 45 + 6\sin\theta$ maximum at $\theta = \frac{\pi}{2}$

$\therefore$ P (1, 2)

$\therefore$ P, A and B are colinear

61. Answer (1)

$81^{\sin^2 x} + 81^{1-\sin^2 x} = 30$

Let $81^{\sin^2 x} = t$

$\Rightarrow \quad t + \frac{81}{t} = 30 \Rightarrow t^2 - 30t + 81 = 0$

$\Rightarrow t = 3$ or 27

i.e. $81^{\sin^2 x} = 3 \text{ or } 3^3$

$\Rightarrow \quad 3^{4\sin^2 x} = 3^1 \text{ or } 3^3$

$\Rightarrow \quad \sin^2 x = \frac{1}{4} \text{ or } \frac{3}{4}$

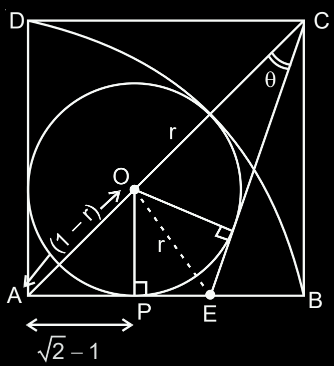
$\Rightarrow \quad \sin x = \pm\frac{1}{2},\ \pm\frac{\sqrt{3}}{2}$

If $x \in (0, \pi)$ then $\sin x = \frac{1}{2}$ or $\frac{\sqrt{3}}{2}$ only

Hence 4 solutions.

62. Answer (1)

Let the centre of C_2 be O.



$\because \quad OC = \sqrt{2} - (1 - r)$

where r is the radius of C_2.

$\because \quad 1 - r = \sqrt{r^2 + r^2}$

$\Rightarrow \quad r = \sqrt{2} - 1$

hence $OC = \sqrt{2} - 1 + r = 2r$

$\because \quad \sin\theta = \frac{r}{2r} = \frac{1}{2} \Rightarrow \theta = 30°$

then $\angle BCE = 15°$

So, $EB = \tan 15° = 2 - \sqrt{3}$

63. Answer (15)

$\Delta = \frac{1}{2} \times 12 \times 5 \sin\theta = 30$

$\theta = 90°$

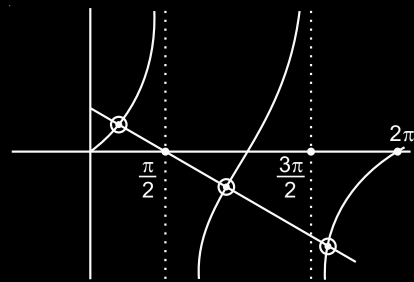
2R = hypotenuse = 13

$r = \frac{\Delta}{s} = \frac{30}{\left(\frac{5+12+13}{2}\right)} = 2$

2R + r = 15

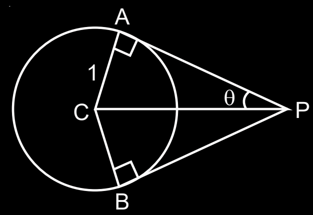
64. Answer (3)

$\because \quad 2\tan x = \frac{\pi}{2} - x$



3 solutions.

65. Answer (4)



$\tan 2\theta = \frac{12}{5}$

$\Rightarrow \quad \tan\theta = \frac{2}{3}$

$$\frac{[\Delta PAB]}{[\Delta CAB]} = \frac{\frac{1}{2}PA^2 \cdot \sin 2\theta}{\frac{1}{2}CA^2 \sin(\pi - 2\theta)}$$

$\left(\frac{PA}{CA}\right)^2 = \cot^2\theta = \frac{9}{4}$

66. Answer (01)

If $x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, 3\frac{\pi}{2}\right)$

$\Rightarrow \quad \cot x = \cot x + \frac{1}{\sin x}$

$\Rightarrow$ No solution

if $x \in \left(\frac{\pi}{2}, \pi\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

$\Rightarrow \quad \frac{2\cos x + 1}{\sin x} = 0$

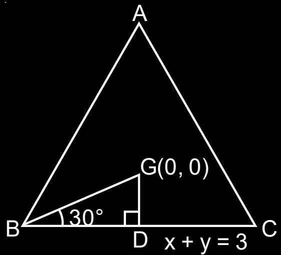
$\Rightarrow \quad \cos x = \frac{-1}{2}$

$\Rightarrow \quad x = \frac{2\pi}{3}$

$\Rightarrow$ One solution

67. Answer (3)

Here $GD = \dfrac{3}{\sqrt{2}}$



$\therefore \quad \dfrac{GD}{BD} = \tan 30°$

$\therefore \quad BD = \dfrac{3\sqrt{3}}{\sqrt{2}}$

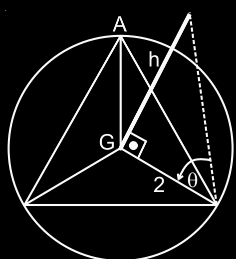
$\therefore$ side length = a = $3\sqrt{6}$

Circumradius = $R = \dfrac{a}{2\sin A} = \dfrac{3\sqrt{6}}{2\cdot\dfrac{\sqrt{3}}{2}} = 3\sqrt{2}$

and inradius = r = $GD = \dfrac{3}{\sqrt{2}}$

$\therefore R + r = 3\sqrt{2} + \dfrac{3}{\sqrt{2}} = \dfrac{9}{\sqrt{2}}$

68. Answer (3)



Triangle is equilateral

Pole is at circumcenter

$h = 2\tan\dfrac{\pi}{3}$

$h = 2\sqrt{3}$

69. Answer (2)

$15\sin^2\alpha + 10(1 - \sin^2\alpha)^2 = 6$

$\Rightarrow \; 25\sin^2\alpha - 20\sin^2\alpha + 4 = 0$

$\Rightarrow \; 25\sin^2\alpha - 10\sin^2\alpha - 10\sin^2\alpha + 4 = 0$

$\Rightarrow \; (5\sin^2\alpha - 2)^2 = 0 \qquad \Rightarrow \sin^2\alpha = \dfrac{2}{5}$

$\therefore \quad \cos^2\alpha = \dfrac{3}{5}$

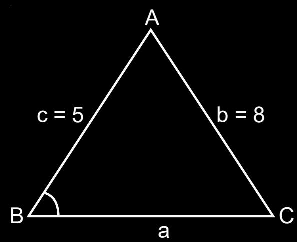
$\therefore \quad 27\sec^6\alpha + 8\mathrm{cosec}^6\alpha = 27\left(\dfrac{5}{3}\right)^3 + 8\left(\dfrac{5}{2}\right)^3$

$= 125 + 125 = 250$

70. Answer (2)

In $\Delta ABC,\ \cos B = \dfrac{3}{5}$

$\therefore \quad \sin B = \dfrac{4}{5}$



$\because \quad \dfrac{5}{\sin C} = \dfrac{a}{\sin A} = \dfrac{b}{\dfrac{4}{5}} = 10 \qquad$ [Sine rule]

$\Rightarrow$ C = 30° and b = 8

$\because \quad \sin A = \sin(B + C) = \dfrac{4}{5}\cdot\dfrac{\sqrt{3}}{2} + \dfrac{3}{5}\cdot\dfrac{1}{2} = \dfrac{3 + 4\sqrt{3}}{10}$

$\because \quad \Delta = \dfrac{1}{2}bc\sin A = \dfrac{1}{2}\cdot 8\cdot 5\cdot\left(\dfrac{3+4\sqrt{3}}{10}\right) = 6 + 8\sqrt{3}$

71. Answer (3)

Let a ΔABC having C = 90° and A = θ

$\dfrac{\sin\theta}{a} = \dfrac{\cos\theta}{b} = \dfrac{1}{c} \qquad$ ...(i)

Also for triangle of reciprocals

$$\cos A = \dfrac{\left(\dfrac{1}{c}\right)^2 + \left(\dfrac{1}{b}\right)^2 - \left(\dfrac{1}{a}\right)^2}{2\left(\dfrac{1}{c}\right)\left(\dfrac{1}{b}\right)}$$

$\dfrac{1}{c^2} + \dfrac{1}{(c\cos\theta)^2} = \dfrac{1}{(c\sin\theta)^2}$

$\Rightarrow \; 1 + \sec^2\theta = \mathrm{cosec}^2\theta$

$$\Rightarrow \quad \frac{1}{4} = \frac{\cos^2\theta}{4\sin^2\theta\cos^2\theta}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{\cos^2\theta}{\sin^2 2\theta}$$

$$\Rightarrow \quad 1-\cos^2 2\theta = 4\cos 2\theta$$

$\cos^2 2\theta + 4\cos 2\theta - 1 = 0$

$$\cos 2\theta = \frac{-4 \pm \sqrt{16+4}}{2}$$

$$\cos 2\theta = -2 \pm \sqrt{5}$$

$$\cos 2\theta = \sqrt{5} - 2 = 1 - 2\sin^2\theta$$

$$\Rightarrow \quad 2\sin^2\theta = 3 - \sqrt{5}$$

$$\Rightarrow \quad \sin^2\theta = \frac{3-\sqrt{5}}{2}$$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{5}-1}{2}$$

72. Answer (9)

Line L is perpendicular bisector of BC, which is

L : $4x - 2y + 9 = 0$

L cuts the y-axis at $\left(0, \frac{9}{2}\right)$

Clearly $\alpha = 9$

73. Answer (1)

$\sin^7 x + \cos^7 x = 1$

As $\sin^7 x + \cos^7 x \le \sin^2 x + \cos^2 x$

≤ 1

The equation gives solution only when one of $\sin x$, $\cos x$ is unity and other vanishes

i.e., $x = 0, \frac{\pi}{2}, 2\pi, \frac{5\pi}{2}, 4\pi$

74. Answer (238)

5	6	8		
10th	11th	12th		
2	2	6	$\rightarrow$	10 × 15 × 25 = 4200
2	3	5	$\rightarrow$	10 × 20 × 56 = 11200
3	2	5	$\rightarrow$	10 × 15 × 56 = 8400
				23800 = 100 k

75. Answer (4)

$$\cot\frac{\pi}{24} = \frac{2\cos\frac{\pi}{24}\cdot\sin\frac{\pi}{24}}{2\sin\frac{\pi}{24}\cdot\sin\frac{\pi}{24}}$$

$$= \frac{\sin\frac{\pi}{12}}{1-\cos\frac{\pi}{12}} = \frac{\frac{\sqrt{3}-1}{2\sqrt{2}}}{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}-\sqrt{3}-1} \times \frac{2\sqrt{2}+\sqrt{3}+1}{2\sqrt{2}+\sqrt{3}+1}$$

$$= \frac{2\sqrt{6}+3+\sqrt{3}-2\sqrt{2}-\sqrt{3}-1}{8-\left(3+1+2\sqrt{3}\right)}$$

$$= \frac{\sqrt{6}-\sqrt{2}+1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

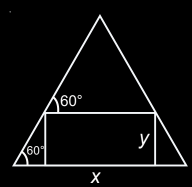
$$= 2\sqrt{6}+3\sqrt{2}-2\sqrt{2}-\sqrt{6}+2+\sqrt{3}$$

$$= \sqrt{6}+\sqrt{2}+2+\sqrt{3}$$

$$= \sqrt{2}+\sqrt{3}+2+\sqrt{6}$$

76. Answer (3)

Let the sides of rectangle be x and y.



$$\because \quad x + \frac{2y}{\sqrt{3}} = 2\sqrt{2}$$

and $\Delta = xy$

$$\Rightarrow \quad \Delta = y\left(2\sqrt{2} - \frac{2y}{\sqrt{3}}\right)$$

$\Rightarrow \quad \Delta = \frac{2}{\sqrt{3}}y\left(\sqrt{6}-y\right)$ will be maximum if $y = \frac{\sqrt{6}}{2}$

$$\Rightarrow \quad \Delta_{max} = \frac{2}{\sqrt{3}}\frac{\sqrt{6}}{2}\cdot\frac{\sqrt{6}}{2} = \sqrt{3}$$

$$\Rightarrow \quad \left(\Delta_{max}\right)^2 = 3$$

77. Answer (2)

$$\because \quad \sin\theta + \cos\theta = \frac{1}{2}$$

$$\therefore \quad \sin 2\theta = \frac{1}{4} - 1 = -\frac{3}{4}$$

$\therefore$ 16(sin2θ + cos4θ + sin6θ)

= 16{sin2θ + 1 – 2 $\sin^2 2\theta$ + 3sin2θ – 4$\sin^3 2\theta$}

$$= 16\left\{-\frac{4\times 3}{4} + 1 - 2\cdot\frac{9}{16} - 4\times -\frac{27}{64}\right\}$$

$$= 16\left\{-2 - \frac{9}{8} + \frac{27}{16}\right\}$$

= –23

78. Answer (3)

$$\frac{\cos x}{1+\sin x} = |\tan 2x|,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$$

If $x \in \left(0, \frac{\pi}{4}\right)$ then $\dfrac{\cos x}{1+\sin x} = \dfrac{2\tan x}{1-\tan^2 x}$

$$\Rightarrow \quad \frac{\cos x}{1+\sin x} = \frac{2\sin x\cdot\cos x}{\cos^2 x - \sin^2 x}$$

$\Rightarrow$ $\cos^2 x - \sin^2 x = 2\sin x + 2\sin^2 x$ ($\because$ $\cos x \neq 0$)

$\Rightarrow$ $4\sin^2 x + 2\sin x - 1 = 0$

$$\therefore \quad \sin x = \frac{\sqrt{5}-1}{4},\ \frac{-\sqrt{5}-1}{4} \quad \text{(Not acceptable)}$$

$$\therefore \quad x = \frac{\pi}{10} \qquad \text{...(i)}$$

If $x \in \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$ then again

$$\sin x = \frac{\sqrt{5}-1}{4}, -\left(\frac{\sqrt{5}+1}{4}\right).$$

$$\therefore \quad x = -\frac{3\pi}{10} \qquad \text{...(ii)}$$

If $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(-\frac{\pi}{4}, 0\right)$ then $1 - 2\sin^2 x = -2\sin x - 2\sin^2 x$

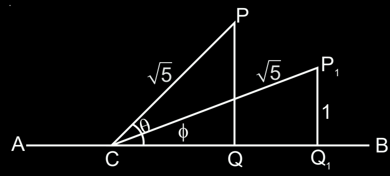
$$\therefore \quad \sin x = -\frac{1}{2}$$

$$\therefore \quad x = -\frac{\pi}{6} \qquad \text{...(iii)}$$

$$\therefore \quad \text{Sum of solution} = \frac{\pi}{10} - \frac{3\pi}{10} - \frac{\pi}{6}$$

$$= -\frac{11\pi}{30}$$

79. Answer (2)



$$\theta = \tan^{-1}\sqrt{2}$$

$CQ_1 = 2$

$$\tan\phi = \frac{1}{2}$$

$$\text{Required angle} = \theta - \phi = \tan^{-1}2 - \tan^{-1}\frac{1}{2}$$

$$= \tan^{-1}\left(\frac{2-\frac{1}{2}}{1+2\times\frac{1}{2}}\right) = \tan^{-1}\frac{3}{4}$$

80. Answer (3)

$$\sin\frac{5\pi}{8} = \sin\left(\pi - \frac{3\pi}{8}\right) = \sin\left(\frac{3\pi}{8}\right)$$

$$\sin\left(\frac{7\pi}{8}\right) = \sin\left(\pi - \frac{\pi}{8}\right) = \sin\frac{\pi}{8}$$

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

$$= 2\sin^2\frac{\pi}{8}\times\sin^2\left(\frac{3\pi}{8}\right)\times\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \left(\sin\frac{\pi}{8}\cdot\cos\frac{\pi}{8}\right)^2$$

$$\left[\because\ \sin\frac{3\pi}{8} = \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \cos\frac{\pi}{8}\right]$$

$$= \left(\frac{1}{2}\sin\frac{\pi}{4}\right)^2 = \left(\frac{1}{2\sqrt{2}}\right)^2 = \frac{1}{8}$$

81. Answer (2)

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix}\right| = 1$$

$\Rightarrow$ $a(2b + 1 - b) - b(-b) = \pm 2$

$\Rightarrow$ $a(b + 1) = \pm 2 - b^2$

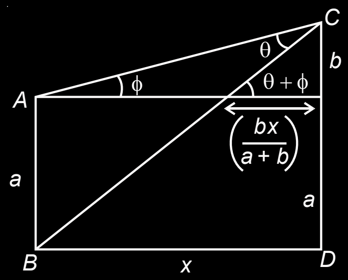
$$\Rightarrow \quad a = \frac{2-b^2}{b+1} \text{ or } \frac{-2-b^2}{b+1}$$

$$\text{Sum of values of } a = \frac{-2-b^2+2-b^2}{b+1} = \frac{-2b^2}{b+1}$$

82. Answer (4)

$\because \tan\theta = \dfrac{1}{2}$

$\tan\phi = \dfrac{b}{x}$



and $\tan(\theta+\phi) = \dfrac{a+b}{x}$

$\Rightarrow \quad \dfrac{\frac{1}{2}+\frac{b}{x}}{1-\frac{b}{2x}} = \dfrac{a+b}{x}$

$\Rightarrow \quad 2bx = x^2 = (a+b)2x - b(a+b)$

$\Rightarrow \quad x^2 - 2ax + b(a+b) = 0$

83. Answer (56)

$(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$

Let $\sin\theta\cdot\cos\theta = t$, $1 - 2t^2 - t = 0$

$2t^2 + t - 1 = 0 \qquad \Rightarrow \quad t = \dfrac{1}{2}$ OR -1

$\sin\theta\cdot\cos\theta = \dfrac{1}{2}$	$\sin\theta\cos\theta = -1$
$\sin 2\theta = 1$	$\sin 2\theta = -2$
$\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \dfrac{13\pi}{4},$	(Not Possible)

$S = 7\pi$

$\dfrac{8s}{\pi} = 56$

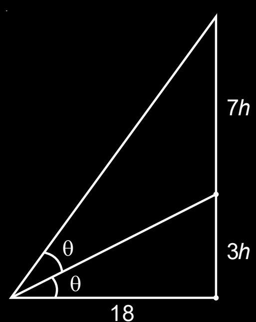
84. Answer (2)

$\tan\theta = \dfrac{h}{6}$

and $\tan 2\theta = \dfrac{5h}{9}$

$\because \quad \dfrac{2\left(\frac{h}{6}\right)}{1-\frac{h^2}{36}} = \dfrac{5h}{9}$

$\Rightarrow \quad h^2 = \dfrac{72}{5}$



Height of the pole $= 10h = 10\sqrt{\dfrac{72}{5}} = 12\sqrt{10}$

85. Answer (2)

$32^{\tan^2 x} + 32^{1+\tan^2 x} = 81$

$32^{\tan^2 x}(33) = 81$

$32^{\tan^2 x} = \dfrac{81}{33}$

If $x \in \left[0, \dfrac{\pi}{4}\right]$ then $32^{\tan^2 x} \in [1, 32]$ & is always increasing.

Hence one solution only

86. Answer (2)

$2\cos x\left(4\sin\left(\dfrac{\pi}{4}+x\right)\sin\left(\dfrac{\pi}{4}-x\right)-1\right) = 1$

$2\cos x\left(4\left(\sin^2\dfrac{\pi}{4}-\sin^2 x\right)-1\right) = 1$

$2\cos x\left(4\times\dfrac{1}{2}-4\sin^2 x-1\right) = 1$

$2\cos x(1-2(1-\cos 2x)) = 1$

$4\cos x\cos 2x - 2\cos x = 1$

$2[\cos 3x + \cos x] - 2\cos x = 1$

$2\cos 3x = 1$

$\cos 3x = \dfrac{1}{2} = \cos\dfrac{\pi}{3}$

$3x = 2n\pi \pm \dfrac{\pi}{3} = (6n\pm 1)\dfrac{\pi}{3}$

$x = (6n\pm 1)\dfrac{\pi}{9}$

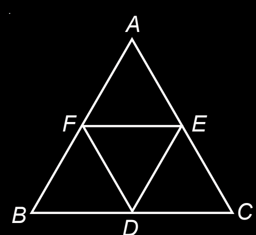
$x = \dfrac{\pi}{9}, \dfrac{5\pi}{9}, \dfrac{7\pi}{9}$

Sum $= \dfrac{\pi}{9}+\dfrac{5\pi}{9}+\dfrac{7\pi}{9} = \dfrac{13\pi}{9}$

87. Answer (6)

Let P.O.I of lines are D, E, F

$x-y+1=0$	$x-y+1=0$	$x-2y+3=0$
$x-2y+3=0$	$2x-5y+11=0$	$2x-5y+11=0$
$x=1,\ y=2$	$x=2,\ y=3$	$x=7,\ y=5$



Area of ΔABC = 4. (Area of ΔDEF)

$$\Delta ABC = 4\times\frac{1}{2}\begin{vmatrix}1 & 2 & 1\\ 2 & 3 & 1\\ 7 & 5 & 1\end{vmatrix}$$

= |2[1(3 – 5) + 2(7 – 2) + 1(10 – 21)]|

= |2 × [–2 + 10 – 11]|

= 6 sq. units

88. Answer (3)

$\because$ $f(x + y) + f(x - y) = 2f(x).f(y)$

$\therefore$ $f(x) = \cos(\lambda x)$

$\therefore$ $f\left(\frac{1}{2}\right) = -1$, then $\lambda = 2n\pi,\ n\in I$

$\therefore$ $f(x) = \cos(2\pi x) \Rightarrow f(k) = 1,\ k\in I$

$$\sum_{k=1}^{20}\frac{1}{\sin k\sin(k+f(k))} = \sum_{k=1}^{20}\frac{1}{\sin k.\sin(k+1)}$$

$$= \sum_{k=1}^{20}\frac{1}{\sin 1}\frac{\sin\{(k+1)-k\}}{\sin k.\sin(k+1)}$$

$$= \frac{1}{\sin 1}\sum_{k=1}^{20}(\cot k - \cot(k+1))$$

$$= \frac{1}{\sin 1}(\cot 1 - \cot 21)$$

$$= \frac{1}{\sin 1}.\frac{\sin(21-1)}{\sin 1.\sin 21} = \text{cosec}^2(1).\text{cosec}(21).\sin(20)$$

89. Answer (3)

$$\because\quad x = \sum_{n=0}^{\infty}(-\tan^2\theta)^n = \frac{1}{1-(-\tan^2\theta)}$$

$$= \frac{1}{\sec^2\theta} = \cos^2\theta$$

and $y = \sum_{n=0}^{\infty}(\cos^2\theta)^n = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$

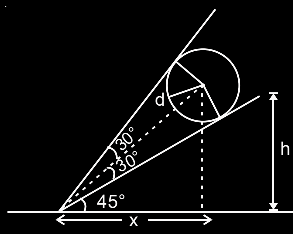
$\because$ $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow$ $\frac{1}{y} + x = 1$

$\Rightarrow$ $1 + xy = y$

$\Rightarrow$ $y(1 - x) = 1$

90. Answer (4)



$$\tan 45^\circ = \frac{h}{x}$$

Also $\sin 75^\circ = \frac{h+16}{d}$ ...(i)

and $\sin 30^\circ = \frac{16}{d}$ ...(ii)

$$32\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = h + 16$$

$\Rightarrow$ $8\sqrt{6} + 8\sqrt{2} - 16 = h$

Height of topmost point $= h + 16 = 8\left(\sqrt{6}+\sqrt{2}+2\right)$

91. Answer (2)

$\sin x + \sin 4x + \sin 2x + \sin 3x = 0$

$$2\sin\frac{5x}{2}\left(\cos\frac{3x}{2} + \cos\frac{x}{2}\right) = 0$$

$$4\sin\frac{5x}{2}\cos x\cos\frac{x}{2} = 0$$

$$\sin\frac{5x}{2} = 0 \Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos\frac{x}{2} = 0 \Rightarrow x = \pi$$

Sum = 9π

92. Answer (2)

$\tan\theta(\sin\theta + 1) - \sin 2\theta = 0$

$$\tan\theta(\sin\theta + 1 - 2\cos^2\theta) = 0$$

$\Rightarrow$ $\tan\theta = 0$ or $2\sin^2\theta + \sin\theta - 1 = 0$

$\Rightarrow$ $(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\Rightarrow$ $\sin\theta = \frac{-1}{2}$ or 1

But, $\sin\theta = 1$ not possible

$$\theta = 0, \pi, -\pi, -\frac{\pi}{6}, \frac{-5\pi}{6}$$

$n(S) = 5$

$$T = \sum \cos 2\theta = \cos 0^\circ + \cos 2\pi + \cos(-2\pi)$$

$$+\cos\left(-\frac{5\pi}{3}\right) + \cos\left(-\frac{\pi}{3}\right)$$

$= 4$

93. Answer (4)

$$\cos\left(x + \frac{\pi}{3}\right)\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4}\cos^2 2x,\ x \in [-3\pi, 3\pi]$$

$$\Rightarrow \cos 2x + \cos\frac{2\pi}{3} = \frac{1}{2}\cos^2 2x$$

$$\Rightarrow \cos^2 2x - 2\cos 2x - 1 = 0$$

$\Rightarrow$ $\cos 2x = 1$

$\therefore$ $x = -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi$

$\therefore$ Number of solutions = 7

94. Answer (1)

$$\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9} = \lambda$$

$$\begin{array}{l} a+b = 7\lambda \\ b+c = 8\lambda \\ c+a = 9\lambda \\ \hline a+b+c = 12\lambda \end{array}$$

$\therefore$ $a = 4\lambda$, $b = 3\lambda$, $c = 5\lambda$

$$S = \frac{4\lambda + 3\lambda + 5\lambda}{2} = 6\lambda$$

$$\Delta = \sqrt{S(s-a)(s-b)(s-c)} = \sqrt{(6\lambda)(2\lambda)(3\lambda)(\lambda)}$$

$$= 6\lambda^2$$

$$R = \frac{abc}{4\Delta} = \frac{(4\lambda)(3\lambda)(5\lambda)}{4(6\lambda^2)} = \frac{5}{2}\lambda$$

$$r = \frac{\Delta}{s} = \frac{6\lambda^2}{6\lambda} = \lambda$$

$$\frac{R}{r} = \frac{\frac{5}{2}\lambda}{\lambda} = \frac{5}{2}$$

95. Answer (4)

$$\frac{14}{\sin^2 x} - 2\sin^2 x = 21 - 4(1 - \sin^2 x)$$

Let $\sin^2 x = t$

$\Rightarrow$ $14 - 2t^2 = 21t - 4t + 4t^2$

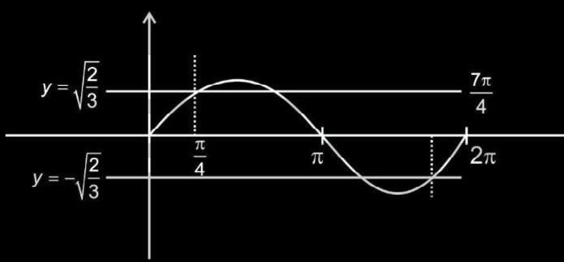
$\Rightarrow$ $6t^2 + 17t - 14 = 0$

$\Rightarrow$ $6t^2 + 21t - 4t - 14 = 0$

$\Rightarrow$ $3t(2t + 7) - 2(2t + 7) = 0$

$\Rightarrow$ $\sin^2 x = \frac{2}{3}$ or $-\frac{7}{3}$ (rejected)

$$\Rightarrow \sin x = \pm\sqrt{\frac{2}{3}}$$



$\therefore$ $\sin x = \pm\sqrt{\frac{2}{3}}$ has 4 solutions in $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$.

96. Answer (4)

$$2\sin 12^\circ - \sin 72^\circ$$

$$= \sin 12^\circ + (-2\cos 42^\circ \cdot \sin 30^\circ)$$

$$= \sin 12^\circ - \cos 42^\circ$$

$$= \sin 12^\circ - \sin 48^\circ$$

$$= -2\left(\frac{\sqrt{5}-1}{4}\right)\cdot\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$$

97. Answer (80)

$(\sin 10^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ) \cdot (\sin 10^\circ \cdot \sin 20^\circ \cdot \sin 40^\circ)$

$$= \left(\frac{1}{4}\sin 30^\circ\right)\cdot\left[\frac{1}{2}\sin 10^\circ(\cos 20^\circ - \cos 60^\circ)\right]$$

$$= \frac{1}{16}\left[\sin 10^\circ\left(\cos 20^\circ - \frac{1}{2}\right)\right]$$

$$= \frac{1}{32}[2\sin 10^\circ \cdot \cos 20^\circ - \sin 10^\circ]$$

$$= \frac{1}{32}[\sin 30^\circ - \sin 10^\circ - \sin 10^\circ]$$

$$= \frac{1}{64} - \frac{1}{16}\sin 10^\circ$$

Clearly $\alpha = \frac{1}{64}$

Hence, $16 + \alpha^{-1} = 80$

98. Answer (2)

$16\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ$

$= 4\sin 60^\circ$

$\{\because 4\sin\theta \times \sin(60^\circ - \theta) \times \sin(60^\circ + \theta) = \sin 3\theta\}$

$= 2\sqrt{3}$

99. Answer (2)

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$$

$$= \frac{\sin 3\left(\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \cos\frac{\left(\frac{2\pi}{7} + \frac{6\pi}{7}\right)}{2}$$

$$= \frac{\sin\left(\frac{3\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)}$$

$$= \frac{2\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}}{2\sin\frac{\pi}{7}}$$

$$= \frac{\sin\left(\frac{8\pi}{7}\right)}{2\sin\frac{\pi}{7}} = \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}} = \frac{-1}{2}$$

100. Answer (3)

$\alpha = \sin 36^\circ = x$(say)

$$\therefore \quad x = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$$

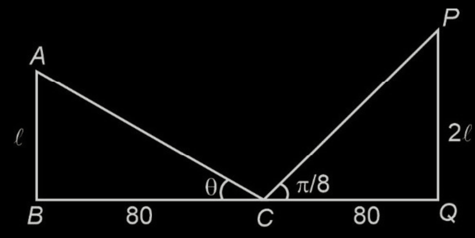
$\Rightarrow \quad 16x^2 = 10 - 2\sqrt{5}$

$\Rightarrow \quad (8x^2 - 5)^2 = 5$

$\Rightarrow \quad 16x^4 - 80x^2 + 20 = 0$

$\therefore \quad 4x^4 - 20x^2 + 5 = 0$

101. Answer (3)



$$\frac{\ell}{80} = \tan\theta \qquad \ldots(i)$$

$$\frac{2\ell}{80} = \tan\frac{\pi}{8} \qquad \ldots(ii)$$

From (i) and (ii),

$$\frac{1}{2} = \frac{\tan\theta}{\tan\frac{\pi}{8}} \Rightarrow \tan^2\theta = \frac{1}{4}\tan^2\frac{\pi}{8}$$

$$\Rightarrow \quad \tan^2\theta = \frac{\sqrt{2} - 1}{4\left(\sqrt{2} + 1\right)} = \frac{3 - 2\sqrt{2}}{4}$$

102. Answer (1)

$\because \quad \cot\alpha = 1, \qquad \alpha \in \left(\pi, \frac{3\pi}{2}\right)$

then $\tan\alpha = 1$

and $\sec\beta = -\frac{5}{3}, \qquad \beta \in \left(\frac{\pi}{2}, \pi\right)$

then $\tan\beta = -\frac{4}{3}$

$$\therefore \quad \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$$

$$= \frac{1 - \frac{4}{3}}{1 + \frac{4}{3}}$$

$$= -\frac{1}{7}$$

$\alpha + \beta \in \left(\frac{3\pi}{2}, 2\pi\right)$ *i.e.* fourth quadrant.

103. Answer (32)

$$3\cos^2 2\theta + 6\cos 2\theta - \frac{10(1+\cos 2\theta)}{2} + 5 = 0$$

$\Rightarrow$ $3\cos^2 2\theta + \cos 2\theta = 0$

$\Rightarrow$ $\cos 2\theta = 0$ or $\cos 2\theta = \frac{-1}{3}$

As $\theta \in [0, \pi]$, $\cos 2\theta = \frac{-1}{3} \Rightarrow 2$ times

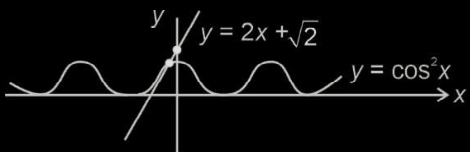
$\Rightarrow$ $\theta \in [-4\pi, 4\pi]$, $\cos 2\theta = \frac{-1}{3} \Rightarrow 16$ times

Similarly, $\cos 2\theta = 0 \Rightarrow 16$ times

$\therefore$ Total 32 solutions

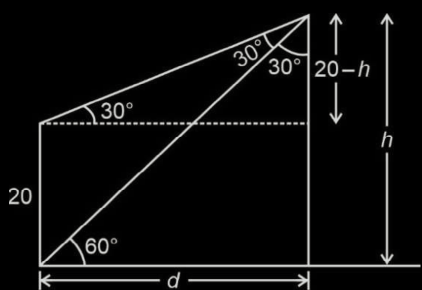
104. Answer (1)

$$\cos^2\theta = 2\theta + \sqrt{2}$$



1 point of intersection = 1 solution

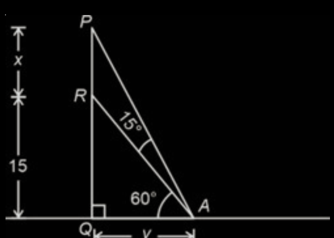
105. Answer (4)



$$\tan 60^\circ = \frac{h}{d} \quad \& \quad \tan 30^\circ = \frac{20-h}{d}$$

$$\Rightarrow \frac{20-h}{h/\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \frac{2h}{3} = 20 \Rightarrow h = 30 \text{ m}$$

106. Answer (1)



From ΔAPQ

$$\frac{x+15}{y} = \tan 75^\circ \quad \ldots\text{(i)}$$

From ΔRQA,

$$\frac{15}{y} = \tan 60^\circ \quad \ldots\text{(ii)}$$

From (i) and (ii)

$$\frac{x+15}{15} = \frac{\tan 75^\circ}{\tan 60^\circ} = \frac{\tan(45^\circ+30^\circ)}{\tan 60^\circ} = \frac{\sqrt{3}+1}{(\sqrt{3}-1).\sqrt{3}}$$

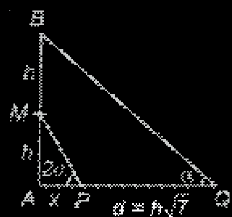
On simplification,

$$x = 10\sqrt{3} \text{ m}$$

Hence height of the tower $= (15 + 10\sqrt{3})$ m

$= 5(2\sqrt{3}+3)$ m

107. Answer (3)



ΔAPM gives

$$\tan 2\alpha = \frac{h}{x} \quad \ldots\text{(i)}$$

ΔAQB gives

$$\tan\alpha = \frac{2h}{x+d} = \frac{2h}{x+h\sqrt{7}} \quad \ldots\text{(ii)}$$

From (i) and (ii)

$$\tan\alpha = \frac{2\cdot\tan 2\alpha}{1+\sqrt{7}\cdot\tan 2\alpha}$$

Let $t = \tan\alpha$

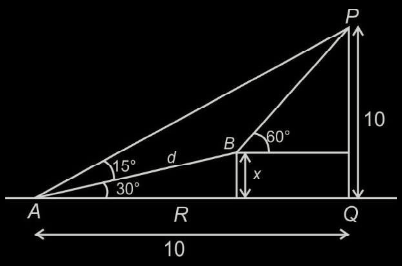
$$\Rightarrow \quad t = \frac{2\frac{2t}{1-t^2}}{1+\sqrt{7}\cdot\frac{2t}{1-t^2}}$$

$$\Rightarrow \quad t^2 - 2\sqrt{7}t + 3 = 0$$

$$t = \sqrt{7} - 2$$

108. Answer (1)

Let $BR = x$



$$\frac{x}{d} = \frac{1}{2} \Rightarrow x = \frac{d}{2}$$

$$\frac{10 - x}{10 - x\sqrt{3}} = \sqrt{3} \Rightarrow 10 - x = 10\sqrt{3} - 3x$$

$$2x = 10(\sqrt{3} - 1)$$

$$x = 5(\sqrt{3} - 1)$$

$$d = 2x = 10(\sqrt{3} - 1)$$

$$\alpha = \frac{1}{2}(x + 10)\left(10 - x\sqrt{3}\right) = \text{Area}(PQRB)$$

$$= \frac{1}{2}\left(5\sqrt{3} - 5 + 10\right)\left(10 - 5\sqrt{3}(\sqrt{3} - 1)\right)$$

$$= \frac{1}{2}\left(5\sqrt{3} + 5\right)\left(10 - 15 + 5\sqrt{3}\right) = \frac{1}{2}(75 - 25) = 25$$

109. Answer (3)

$$S = \left\{\theta \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right)\sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}.$$

$$\sum_{m=1}^{9} \frac{1}{\cos\left(\theta + (m-1)\frac{\pi}{6}\right)\cos\left(\theta + m\frac{\pi}{6}\right)}$$

$$\frac{1}{\sin\left(\frac{\pi}{6}\right)} \sum_{m=1}^{9} \frac{\sin\left[\left(\theta + \frac{m\pi}{6}\right) - \left(\theta + (m-1)\frac{\pi}{6}\right)\right]}{\cos\left(\theta + (m-1)\frac{\pi}{6}\right)\cos\left(\theta + m\frac{\pi}{6}\right)}$$

$$= 2\sum_{m=1}^{9}\left[\tan\left(\theta + \frac{m\pi}{6}\right) - \tan\left(\theta + (m-1)\frac{\pi}{6}\right)\right]$$

Now, $m = 1 \quad 2\left[\tan\left(\theta + \frac{\pi}{6}\right) - \tan(\theta)\right]$

$m = 2 \quad 2\left[\tan\left(\theta + \frac{2\pi}{6}\right) - \tan\left(\theta + \frac{\pi}{6}\right)\right]$

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$m = 9 \quad 2\left[\tan\left(\theta + \frac{9\pi}{6}\right) - \tan\left(\theta + 8\frac{\pi}{6}\right)\right]$

$$\therefore \quad = 2\left[\tan\left(\theta + \frac{3\pi}{2}\right) - \tan\theta\right] = \frac{-8}{\sqrt{3}}$$

$$= -2[\cot\theta + \tan\theta] = \frac{-8}{\sqrt{3}}$$

$$= -\frac{2 \times 2}{2\sin\theta\cos\theta} = \frac{-8}{\sqrt{3}}$$

$$= \frac{1}{\sin 2\theta} = \frac{2}{\sqrt{3}}$$

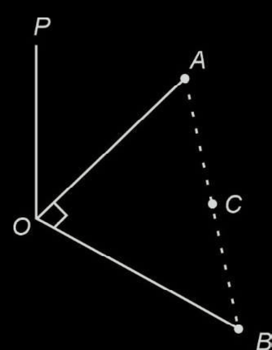
$$\Rightarrow \quad \sin 2\theta = \frac{\sqrt{3}}{2}$$

$$2\theta = \frac{\pi}{3} \qquad 2\theta = \frac{2\pi}{3}$$

$$\theta = \frac{\pi}{6} \qquad \theta = \frac{\pi}{3}$$

$$\sum \theta_i = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$$

110. Answer (2)



$OP = OA \tan 15 = OB \tan 15$ ...(i)

$OP = OC \tan 45 \Rightarrow OP = OC$ ...(ii)

$OA = OB$ ...(iii)

$OC^2 + 8^2 = OA^2$

$$OP^2 + 64 = OP^2\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$$

$$64 = OP^2\left[\frac{(\sqrt{3}+1)^2 - (\sqrt{3}-1)^2}{(\sqrt{3}-1)^2}\right]$$

$$= OP^2\left(\frac{4\sqrt{3}}{(\sqrt{3}-1)^2}\right)$$

$$OP^2 = \frac{64(\sqrt{3}-1)^2}{4\sqrt{3}} = \frac{32}{\sqrt{3}}(2-\sqrt{3})$$

111. Answer (5)

Let tan $\alpha = \sqrt{5}$

$\therefore \quad \tan\theta = \dfrac{\tan\alpha - \tan 2\theta}{1+\tan\alpha\tan 2\theta}$

$\therefore \quad \tan\theta = \tan(\alpha - 2\theta)$

$\alpha - 2\theta = n\pi + \theta$

$\Rightarrow \quad 3\theta = \alpha - n\pi$

$\Rightarrow \quad \theta = \dfrac{\alpha}{3} - \dfrac{n\pi}{3} \quad ; n \in Z$

If $\theta \in [-\pi, \pi/2)$ then

$n = 0, 1, 2, 3, 4$ are acceptable

5 solutions.

112. Answer (1)

$$S = \left\{x \in \mathbb{R} : 2\cos\left(\frac{x^2+x}{6}\right) = 4^x + 4^{-x}\right\}$$

LHS is less than or equal to 2 and RHS is greater than or equal to 2.

So equality holds only if LHS = RHS = 2

RHS is 2 when $x = 0$

and at $x = 0$, LHS is also 2.

So, only one solution exist.

113. Answer (3)

Equation (1) $\quad 2\sin^2\theta = 1 - 2\sin^2\theta$

$\Rightarrow \quad \sin^2\theta = \dfrac{1}{4}$

$\Rightarrow \quad \sin\theta = \pm\dfrac{1}{2}$

$\Rightarrow \quad \theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

Equation (2) $\quad 2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow \quad 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow \quad 2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$

$\Rightarrow \quad (\sin\theta - 2)(2\sin\theta + 1) = 0$

$\Rightarrow \quad \sin\theta = \dfrac{-1}{2} \qquad \Rightarrow \quad \theta = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$\therefore$ Common solutions = $\dfrac{7\pi}{6}; \dfrac{11\pi}{6}$

Sum of solutions = $\dfrac{7\pi + 11\pi}{6} = \dfrac{18\pi}{6} = 3\pi$

$\therefore \quad k = 3$

114. Answer (3)

$$S = \left\{\theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16\right\}$$

Now, apply AM $\geq$ GM for $8^{2\sin^2\theta}, 8^{2\cos^2\theta}$

$$\frac{8^{2\sin^2\theta} + 8^{2\cos^2\theta}}{2} \geq \left(8^{2\sin^2\theta + 2\cos^2\theta}\right)^{\frac{1}{2}}$$

$8 \geq 8$

$\Rightarrow \quad 8^{2\sin^2\theta} = 8^{2\cos^2\theta}$

or $\quad \sin^2\theta = \cos^2\theta$

$\therefore \quad \theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

$$n(S) + \sum_{\theta\in S} \sec\left(\frac{\pi}{4} + 2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4} + 2\theta\right)$$

$$4 + \sum_{\theta\in S} \frac{2}{2\sin\left(\frac{\pi}{4}+2\theta\right)\cos\left(\frac{\pi}{4}+2\theta\right)}$$

$$= 4 + \sum_{\theta \in S} \frac{2}{\sin\left(\frac{\pi}{2} + 4\theta\right)} = 4 + 2\sum_{\theta \in S} \operatorname{cosec}\left(\frac{\pi}{2} + 4\theta\right)$$

$$= 4 + 2\left[\operatorname{cosec}\left(\frac{\pi}{2} + \pi\right) + \operatorname{cosec}\left(\frac{\pi}{2} + 3\pi\right) + \operatorname{cosec}\left(\frac{\pi}{2} + 5\pi\right) + \operatorname{cosec}\left(\frac{\pi}{2} + 7\pi\right)\right]$$

$$= 4 + 2\left[-\operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2}\right]$$

$= 4 - 2(4)$

$= 4 - 8$

$= -4$

115. Answer (2)

$$2\sin\frac{\pi}{22}\sin\frac{3\pi}{22}\sin\frac{5\pi}{22}\sin\frac{7\pi}{22}\sin\frac{9\pi}{22}$$

$$= 2\sin\left(\frac{11\pi - 10\pi}{22}\right)\sin\left(\frac{11\pi - 8\pi}{22}\right)\sin\left(\frac{11\pi - 6\pi}{22}\right)\sin\left(\frac{11\pi - 4\pi}{22}\right)\sin\left(\frac{11\pi - 2\pi}{22}\right)$$

$$= 2\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}$$

$$= \frac{2\sin\frac{32\pi}{11}}{2^5\sin\frac{\pi}{11}}$$

$$= \frac{1}{16}$$

116. Answer (16)

$7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$

$$\Rightarrow \quad 4\left(\frac{1 + \cos 2\theta}{2}\right) + 3\cos 2\theta - 2\cos^2 2\theta = 2$$

$\Rightarrow \quad 2 + 5\cos^2\theta - 2\cos^2 2\theta = 2$

$$\Rightarrow \quad \cos 2\theta = 0 \text{ or } \frac{5}{2} \text{ (rejected)}$$

$$\Rightarrow \quad \cos 2\theta = 0 = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} \Rightarrow \tan^2\theta = 1$$

$\therefore$ Sum of roots = $2(\tan^2\theta + \cot^2\theta) = 2 \times 2 = 4$

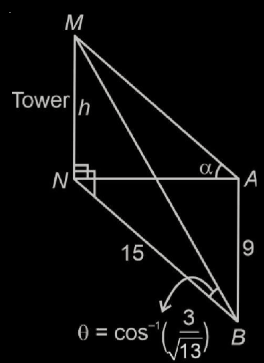
But as $\tan\theta = \pm 1$ for $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ in the interval $(0, 2\pi)$

$\therefore$ Four equations will be formed

Hence sum of roots of all the equations

$= 4 \times 4 = 16.$

117. Answer (1)



$$NA = \sqrt{15^2 - 9^2} = 12$$

$$\frac{h}{15} = \tan\theta = \frac{2}{3}$$

$h = 10$ units

$$\cot\alpha = \frac{12}{10} = \frac{6}{5}$$

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