Chapter 1

# Complex Numbers

1. If $\left|Z-\frac{4}{Z}\right|=2$, then the maximum value of $|Z|$ is equal to **[AIEEE-2009]**

(1) $\sqrt{5}+1$ (2) 2

(3) $2+\sqrt{2}$ (4) $\sqrt{3}+1$

2. The number of complex numbers $z$ such that $|z-1|=|z+1|=|z-i|$ equals **[AIEEE-2010]**

(1) 0 (2) 1

(3) 2 (4) $\infty$

3. If $z\neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies **[AIEEE-2012]**

(1) On a circle with centre at the origin.

(2) Either on the real axis or on a circle not passing through the origin

(3) On the imaginary axis

(4) Either on the real axis or on a circle passing through the origin

4. If z is a complex number of unit modulus and argument $\theta$, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals **[JEE (Main)-2013]**

(1) $-\theta$ (2) $\frac{\pi}{2}-\theta$

(3) $\theta$ (4) $\pi-\theta$

5. If $z$ is a complex number such that $|z|\geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$ **[JEE (Main)-2014]**

(1) Is strictly greater than $\frac{5}{2}$

(2) Is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

(3) Is equal to $\frac{5}{2}$

(4) Lies in the interval (1, 2)

6. A complex number $z$ is said to be unimodular if $|z|=1$. Suppose $z_1$ and $z_2$ are complex numbers such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and $z_2$ is not unimodular. Then the point $z_1$ lies on a **[JEE (Main)-2015]**

(1) Straight line parallel to $x$-axis

(2) Straight line parallel to $y$-axis

(3) Circle of radius 2

(4) Circle of radius $\sqrt{2}$

7. A value of $\theta$ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is **[JEE (Main)-2016]**

(1) $\frac{\pi}{6}$ (2) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(3) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $\frac{\pi}{3}$

8. Let $A=\left\{\theta\in\left(-\frac{\pi}{2},\pi\right):\frac{3+2i\sin\theta}{1-2i\sin\theta}\text{ is purely imaginary}\right\}$.

Then the sum of the elements in $A$ is **[JEE (Main)-2019]**

(1) $\frac{5\pi}{6}$ (2) $\pi$

(3) $\frac{3\pi}{4}$ (4) $\frac{2\pi}{3}$

9. Let $z_0$ be a root of the quadratic equation, $x^2+x+1=0$. If $z=3+6iz_0^{81}-3iz_0^{93}$, then arg $z$ is equal to **[JEE (Main)-2019]**

(1) 0 (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

10. Let $z_1$ and $z_2$ be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then **[JEE (Main)-2019]**

(1) $\text{Im}(z) = 0$ (2) $\frac{3}{2} \le |z| \le \frac{5}{2}$

(3) $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$ (4) $\text{Re}(z) = 0$

11. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of $z$, then **[JEE (Main)-2019]**

(1) $I(z) = 0$

(2) $R(z) > 0$ and $I(z) > 0$

(3) $R(z) < 0$ and $I(z) > 0$

(4) $R(z) = -3$

12. Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x+iy}{27}\ (i = \sqrt{-1})$, where $x$ and $y$ are real numbers, then $y - x$ equals

**[JEE (Main)-2019]**

(1) –85 (2) –91

(3) 85 (4) 91

13. Let $z$ be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$). Then $|z|$ is equal to

**[JEE (Main)-2019]**

(1) $\frac{\sqrt{41}}{4}$ (2) $\frac{5}{4}$

(3) $\frac{5}{3}$ (4) $\frac{\sqrt{34}}{3}$

14. If $\frac{z-\alpha}{z+\alpha}\ (\alpha \in R)$ is a purely imaginary number and $|z| = 2$, then a value of $\alpha$ is **[JEE (Main)-2019]**

(1) $\sqrt{2}$ (2) 2

(3) $\frac{1}{2}$ (4) 1

15. Let $z_1$ and $z_2$ be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then the minimum value of $|z_1 - z_2|$ is **[JEE (Main)-2019]**

(1) 0 (2) $\sqrt{2}$

(3) 1 (4) 2

16. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\ (i = \sqrt{-1})$, then **[JEE (Main)-2019]**

(1) 0 (2) $(-1 + 2i)^9$

(3) $-1$ (4) 1

17. All the points in the set

$$S = \left\{\frac{\alpha + i}{\alpha - i} : \alpha \in R\right\} (i = \sqrt{-1})$$

lie on a **[JEE (Main)-2019]**

(1) Straight line whose slope is 1

(2) Circle whose radius is $\sqrt{2}$

(3) Circle whose radius is 1

(4) Straight line whose slope is –1

18. Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5+3z}{5(1-z)}$, then **[JEE (Main)-2019]**

(1) $5\,\text{Re}(\omega) > 4$ (2) $5\,\text{Re}(\omega) > 1$

(3) $4\,\text{Im}(\omega) > 5$ (4) $5\,\text{Im}(\omega) < 1$

19. If $a > 0$ and $z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to : **[JEE (Main)-2019]**

(1) $-\frac{1}{5} + \frac{3}{5}i$ (2) $-\frac{3}{5} - \frac{1}{5}i$

(3) $\frac{1}{5} - \frac{3}{5}i$ (4) $-\frac{1}{5} - \frac{3}{5}i$

20. If $z$ and $w$ are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then :

**[JEE (Main)-2019]**

(1) $z\bar{w} = \frac{1-i}{\sqrt{2}}$ (2) $\bar{z}w = i$

(3) $z\bar{w} = \frac{-1+i}{\sqrt{2}}$ (4) $\bar{z}w = -i$

21. The equation $|z - i| = |z - 1|$, $i = \sqrt{-1}$, represents :

**[JEE (Main)-2019]**

(1) The line through the origin with slope –1

(2) A circle of radius $\frac{1}{2}$

(3) A circle of radius 1

(4) The line through the origin with slope 1

22. Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number $n$. Then

**[JEE (Main)-2019]**

(1) $n = 20$ and $Re(z) = 10$

(2) $n = 20$ and $Re(z) = -10$

(3) $n = 40$ and $Re(z) = -10$

(4) $n = 40$ and $Re(z) = 10$

23. If $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point $(x, y)$ lies on a **[JEE (Main)-2020]**

(1) Circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$

(2) Straight line whose slope is $-\frac{2}{3}$

(3) Circle whose diameter is $\frac{\sqrt{5}}{2}$

(4) Straight line whose slope is $\frac{3}{2}$

24. If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta \in [0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is

**[JEE (Main)-2020]**

(1) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$ (2) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$

(3) $-\tan^{-1}\left(\frac{3}{4}\right)$ (4) $\tan^{-1}\left(\frac{4}{3}\right)$

25. If the equation, $x^2 + bx + 45 = 0$ $(b \in R)$ has conjugate complex roots and they satisfy $|z+1| = 2\sqrt{10}$, then **[JEE (Main)-2020]**

(1) $b^2 - b = 42$ (2) $b^2 - b = 30$

(3) $b^2 + b = 12$ (4) $b^2 + b = 72$

26. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$ and $b = \sum_{k=0}^{100}\alpha^{3k}$, then $a$ and $b$ are the roots of the quadratic equation **[JEE (Main)-2020]**

(1) $x^2 - 101x + 100 = 0$

(2) $x^2 - 102x + 101 = 0$

(3) $x^2 + 101x + 100 = 0$

(4) $x^2 + 102x + 101 = 0$

27. Let $z$ be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z + 3i|$ is

**[JEE (Main)-2020]**

(1) $2\sqrt{3}$ (2) $\frac{7}{2}$

(3) $\sqrt{10}$ (4) $\frac{15}{4}$

28. If $z$ be a complex number satisfying $|\text{Re}(z)| + |\text{Im}(z)| = 4$, then $|z|$ cannot be

**[JEE (Main)-2020]**

(1) $\sqrt{10}$ (2) $\sqrt{8}$

(3) $\sqrt{\frac{17}{2}}$ (4) $\sqrt{7}$

29. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is

**[JEE (Main)-2020]**

(1) $-\frac{1}{2}(1-i\sqrt{3})$ (2) $\frac{1}{2}(1-i\sqrt{3})$

(3) $\frac{1}{2}(\sqrt{3}-i)$ (4) $-\frac{1}{2}(\sqrt{3}-i)$

30. The imaginary part of

$\left(3+2\sqrt{-54}\right)^{1/2} - \left(3-2\sqrt{-54}\right)^{1/2}$ can be

**[JEE (Main)-2020]**

(1) $\sqrt{6}$ (2) $-\sqrt{6}$

(3) $-2\sqrt{6}$ (4) 6

31. If $z_1$, $z_2$ are complex numbers such that $\text{Re}(z_1) = |z_1 - 1|$, $\text{Re}(z_2) = |z_2 - 1|$, and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\text{Im}(z_1 + z_2)$ is equal to

**[JEE (Main)-2020]**

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{3}}$

(3) $\frac{2}{\sqrt{3}}$ (4) $2\sqrt{3}$

32. Let $u = \frac{2z+i}{z-ki}$, $z = x + iy$ and $k > 0$. If the curve represented by Re($u$) + Im($u$) = 1 intersects the $y$-axis at the point $P$ and $Q$ where $PQ = 5$, then the value of $K$ is **[JEE (Main)-2020]**

(1) 1/2 (2) 3/2

(3) 2 (4) 4

33. If $a$ and $b$ are real numbers such that $(2 + \alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$, then $a + b$ is equal to **[JEE (Main)-2020]**

(1) 33 (2) 9

(3) 24 (4) 57

34. If the four complex numbers $z$, $\bar{z}$, $\bar{z} - 2\text{Re}(\bar{z})$ and $z - 2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to **[JEE (Main)-2020]**

(1) $4\sqrt{2}$ (2) 2

(3) $2\sqrt{2}$ (4) 4

35. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is **[JEE (Main)-2020]**

(1) $-2^{15}i$ (2) $-2^{15}$

(3) $2^{15}i$ (4) 65

36. The region represented by $\{z = x + iy \in C : |z| - \text{Re}(z) \le 1\}$ is also given by the inequality **[JEE (Main)-2020]**

(1) $y^2 \le x + \frac{1}{2}$ (2) $y^2 \le 2\left(x + \frac{1}{2}\right)$

(3) $y^2 \ge x + 1$ (4) $y^2 \ge 2(x + 1)$

37. Let $z = x + iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then $z$ lies on the **[JEE (Main)-2020]**

(1) Line, $y = x$ (2) Imaginary axis

(3) Real axis (4) Line, $y = -x$

38. If $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$, $(m, n \in N)$ then the greatest common divisor of the least values of $m$ and $n$ is ________. **[JEE (Main)-2020]**

39. Let p and q be two positive numbers such that p + q = 2 and $p^4 + q^4 = 272$. Then p and q are roots of the equation : **[JEE (Main)-2021]**

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

40. If range of real values of $\alpha$, for which the equation $z + \alpha|z - 1| + 2i = 0$ $\left(z \in C \text{ and } i = \sqrt{-1}\right)$ has a solution, is [p, q) then $4\left(p^2 + q^2\right)$ is equal to ______.

**[JEE (Main)-2021]**

41. Let $i = \sqrt{-1}$. If $\frac{\left(-1+i\sqrt{3}\right)^{21}}{(1-i)^{24}} + \frac{\left(1+i\sqrt{3}\right)^{21}}{(1+i)^{24}} = k$, and $n = [|k|]$ be the greatest integral part of $|k|$. Then $\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to ________.

**[JEE (Main)-2021]**

42. Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is : **[JEE (Main)-2021]**

(1) $3\sqrt{2}$ (2) $\frac{3}{\sqrt{2}}$

(3) $\frac{3}{2\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

43. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to:

**[JEE (Main)-2021]**

(1) –3 (2) –7

(3) 7 (4) 3

44. The sum of 162$^{\text{th}}$ power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is __________.

**[JEE (Main)-2021]**

45. Let z be those complex numbers which satisfy $|z + 5| \le 4$ and $z(1+i) + \bar{z}(1-i) \ge -10$, $i = \sqrt{-1}$. If the maximum value of $|z + 1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is ______.

**[JEE (Main)-2021]**

46. Let a complex number z, $|z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to _____. **[JEE (Main)-2021]**

(1) 8 (2) 7
(3) 6 (4) 5

47. Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2, \left|\frac{z+i}{z-3i}\right| = 1$ and Re(w) has minimum value. Then, the minimum value of $n \in N$ for which $w^n$ is real, is equal to **[JEE (Main)-2021]**

48. The least value of |z| where z is complex number which satisfies the inequality exp

$$\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|, i = \sqrt{-1},$$

is equal to : **[JEE (Main)-2021]**

(1) 2 (2) 8
(3) 3 (4) $\sqrt{5}$

49. The area of the triangle with vertices A(z), B(iz) and C(z + iz) is **[JEE (Main)-2021]**

(1) $\frac{1}{2}|z|^2$ (2) $\frac{1}{2}|z+iz|^2$
(3) $\frac{1}{2}$ (4) 1

50. Let $S_1$, $S_2$ and $S_3$ be three sets defined as

$S_1 = \{z \in \mathbb{C} : |z-1| \leq \sqrt{2}\}$

$S_2 = \{z \in \mathbb{C} : \text{Re}((1-i)z) \geq 1\}$

$S_3 = \{z \in \mathbb{C} : \text{Im}(z) \leq 1\}$

Then the set $S_1 \cap S_2 \cap S_3$ **[JEE (Main)-2021]**

(1) Has infinitely many elements
(2) Is a singleton
(3) Has exactly three elements
(4) Has exactly two elements

51. If the equation $a|z|^2 + \overline{\alpha}z + \alpha\overline{z} + d = 0$ represents a circle where a, d are real constants, then which of the following condition is correct?

**[JEE (Main)-2021]**

(1) $|\alpha|^2 - ad > 0$ and $a \in \mathbf{R} - \{0\}$
(2) $|\alpha|^2 - ad \neq 0$
(3) $\alpha = 0$, a, d $\in \mathbf{R}^+$
(4) $|\alpha|^2 - ad \geq 0$ and $a \in \mathbf{R}$

52. Let $z_1$, $z_2$ be the roots of the equation $z^2 + az + 12 = 0$ and $z_1$, $z_2$ form an equilateral triangle with origin. Then, the value of |a| is ________. **[JEE (Main)-2021]**

53. Let a complex number be $w = 1 - \sqrt{3}\,i$. Let another complex number z be such that |zw| = 1 and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

**[JEE (Main)-2021]**

(1) $\frac{1}{2}$ (2) 2
(3) 4 (4) $\frac{1}{4}$

54. If f(x) and g(x) are two polynomials such that the polynomial $P(x) = f(x^3) + x\,g(x^3)$ is divisible by $x^2 + x + 1$, then P(1) is equal to _______. **[JEE (Main)-2021]**

55. If $\alpha$ and $\beta$ are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to

**[JEE (Main)-2021]**

(1) $28 \times 3^{25}$ (2) $56 \times 3^{24}$
(3) $52 \times 3^{24}$ (4) $56 \times 3^{25}$

56. If z and $\omega$ are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is

(Here arg(z) denotes the principal argument of complex number z) **[JEE (Main)-2021]**

(1) $\frac{3\pi}{4}$ (2) $-\frac{3\pi}{4}$
(3) $\frac{\pi}{4}$ (4) $-\frac{\pi}{4}$

57. Let n denotes the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty}\frac{1}{n^k}$ is equal to

**[JEE (Main)-2021]**

(1) 1 (2) 2
(3) $\frac{4}{3}$ (4) $\frac{3}{2}$

58. Let C be the set of all complex numbers. Let

$S_1 = \{z \in C \mid |z - 3 - 2i|^2 = 8\}$,

$S_2 = \{z \in C \mid \text{Re}(z) \geq 5\}$ and

$S_3 = \{z \in C \mid |z - \bar{z}| \geq 8\}$.

Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to **[JEE (Main)-2021]**

(1) 0 (2) 1

(3) 2 (4) Infinite

59. Let $\mathbb{C}$ be the set of all complex numbers. Let

$S_1 = \{z \in \mathbb{C} : |z - 2| \leq 1\}$ and

$S_2 = \{z \in \mathbb{C} : z(1+i) + \bar{z}(1-i) \geq 4\}$.

Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to **[JEE (Main)-2021]**

(1) $\frac{5+2\sqrt{2}}{2}$ (2) $\frac{5+2\sqrt{2}}{4}$

(3) $\frac{3+2\sqrt{2}}{4}$ (4) $\frac{3+2\sqrt{2}}{2}$

60. If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to ________. **[JEE (Main)-2021]**

61. The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with : **[JEE (Main)-2021]**

(1) Centre at (0, 0) and radius $\sqrt{2}$

(2) Centre at (0, 1) and radius $\sqrt{2}$

(3) Centre at (0, –1) and radius $\sqrt{2}$

(4) Centre at (0, 1) and radius 2

62. Let $z = \frac{1-i\sqrt{3}}{2}, i = \sqrt{-1}$. Then the value of

$$21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \ldots\ldots + \left(z^{21} + \frac{1}{z^{21}}\right)^3$$ is _____.

**[JEE (Main)-2021]**

63. If $(\sqrt{3}+i)^{100} = 2^{99}(p + iq)$, then p and q are roots of the equation **[JEE (Main)-2021]**

(1) $x^2 - (\sqrt{3}-1)x - \sqrt{3} = 0$

(2) $x^2 + (\sqrt{3}-1)x - \sqrt{3} = 0$

(3) $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

(4) $x^2 + (\sqrt{3}+1)x + \sqrt{3} = 0$

64. The least positive integers n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$, is a positive integer, is ________. **[JEE (Main)-2021]**

65. If $S = \left\{z \in \mathbf{C} : \frac{z-i}{z+2i}, \in \mathbf{R}\right\}$, then

(1) S contains exactly two elements

(2) S is a circle in the complex plane

(3) S is a straight line in the complex plane

(4) S contains only one element

**[JEE (Main)-2021]**

66. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $\left|z - 9\sqrt{2} - 2i\right|^2$ is equal to ______.

**[JEE (Main)-2021]**

67. If $z$ is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is **[JEE (Main)-2021]**

(1) $6\sqrt{2}$ (2) $2\sqrt{2}$

(3) $3\sqrt{2}$ (4) $2\sqrt{2} - 1$

68. If for the complex number $z$ satisfying $|z - 2 - 2i| \leq 1$, the maximum value of $|3iz + 6|$ is attained at $a + ib$, then $a + b$ is equal to______.

**[JEE (Main)-2021]**

69. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to

**[JEE (Main)-2021]**

(1) –1 (2) 0

(3) 1 (4) 2

70. Let $\alpha$ and $\beta$ be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to

**[JEE (Main)-2021]**

(1) –512 (2) 512

(3) 256 (4) –256

71. Let $S = \{z \in \mathbb{C} : |z - 3| \le 1$ and $z(4 + 3i) + \bar{z}(4 - 3i) \le 24\}$. If $\alpha + i\beta$ is the point in $S$ which is closest to $4i$, then $25(\alpha + \beta)$ is equal to ______.

**[JEE (Main)-2022]**

72. Let a circle $C$ in complex plane pass through the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on $C$ such that the line through $z$ and $z_1$ is perpendicular to the line through $z_2$ and $z_3$, then $\arg(z)$ is equal to:

(1) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \pi$ (2) $\tan^{-1}\left(\frac{24}{7}\right) - \pi$

(3) $\tan^{-1}(3) - \pi$ (4) $\tan^{-1}\left(\frac{3}{4}\right) - \pi$

**[JEE (Main)-2022]**

73. Let $z_1$ and $z_2$ be two complex numbers such that $\overline{z_1} = i\overline{z_2}$ and $\arg\left(\frac{z_1}{\overline{z_2}}\right) = \pi$. Then

(1) $\arg z_2 = \left(\frac{\pi}{4}\right)$ (2) $\arg z_2 = -\frac{3\pi}{4}$

(3) $\arg z_1 = \frac{\pi}{4}$ (4) $\arg z_1 = -\frac{3\pi}{4}$

**[JEE (Main)-2022]**

74. If $z^2 + z + 1 = 0$, $z \in \mathbb{C}$, then $\left|\sum_{n=1}^{15}\left(z^n + (-1)^n \frac{1}{z^n}\right)^2\right|$ is equal to ________.

**[JEE (Main)-2022]**

75. The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z} = iz^2$ is :

(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$

(C) $\frac{3}{2}$ (D) $\frac{3}{4}$

**[JEE (Main)-2022]**

76. The number of points of intersection of $|z - (4 + 3i)| = 2$ and $|z| + |z - 4| = 6$, $z \in C$, is

(1) 0 (2) 1

(3) 2 (4) 3

**[JEE (Main)-2022]**

77. Let for some real numbers $\alpha$ and $\beta$, $a = \alpha - i\beta$. If the system of equations $4ix + (1 + i)y = 0$ and $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$ has more than one solution, then $\frac{\alpha}{\beta}$ is equal to

(1) $-2+\sqrt{3}$ (2) $2-\sqrt{3}$

(3) $2+\sqrt{3}$ (4) $-2-\sqrt{3}$

**[JEE (Main)-2022]**

78. The number of elements in the set $\{z = a + ib \in \mathbb{C} : a, b \in \mathbb{Z}$ and $1 < |z - 3 + 2i| < 4\}$ is ________.

**[JEE (Main)-2022]**

79. Let $\alpha$ and $\beta$ be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to:

(1) 50 (2) 250

(3) 1250 (4) 1500

**[JEE (Main)-2022]**

80. Let $S = \{z \in \mathbf{C} : |z - 2| \le 1, z(1 + i) + \bar{z}(1 - i) \le 2\}$. Let $|z - 4i|$ attains minimum and maximum values, respectively, at $z_1 \in S$ and $z_2 \in S$. If $5\left(|z_1|^2 + |z_2|^2\right) = \alpha + \beta\sqrt{5}$, where $\alpha$ and $\beta$ are integers, then the value of $\alpha + \beta$ is equal to ______.

**[JEE (Main)-2022]**

81. Let $\alpha$ be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to

(1) 1 (2) $\alpha$

(3) $1 + \alpha$ (4) $1 + 2\alpha$

**[JEE (Main)-2022]**

82. Let arg(z) represent the principal argument of the complex number $z$.

Then, $|z| = 3$ and $\arg(z - 1) - \arg(z + 1) = \frac{\pi}{4}$ intersect

(1) exactly at one point

(2) exactly at two points

(3) nowhere

(4) at infinitely many points

**[JEE (Main)-2022]**

83. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to **[JEE (Main)-2022]**

(1) –4 (2) –1

(3) 1 (4) 4

84. Let $O$ be the origin and $A$ be the point $z_1 = 1 + 2i$. If $B$ is the point $z_2$, $\text{Re}(z_2) < 0$, such that $OAB$ is a right angled isosceles triangle with $OB$ as hypotenuse, then which of the following is NOT true? **[JEE (Main)-2022]**

(1) $\arg z_2 = \pi - \tan^{-1}3$

(2) $\arg (z_1 - 2z_2) = -\tan^{-1}\left(\frac{4}{3}\right)$

(3) $|z_2| = \sqrt{10}$

(4) $|2z_1 - z_2| = 5$

85. Let the minimum value $v_0$ of $v = |z|^2 + |z - 3|^2 + |z - 6i|^2$, $z \in \mathbb{C}$ is attained at $z = z_0$. Then $\left|2z_0^2 - \bar{z}_0^3 + 3\right|^2 + v_0^2$ is equal to

**[JEE (Main)-2022]**

(1) 1000 (2) 1024

(3) 1105 (4) 1196

86. Let $S = \left\{z \in \mathbb{C} : z^2 + \bar{z} = 0\right\}$. Then $\sum_{z \in S} (\text{Re}(z) + \text{Im}(z))$ is equal to ________.

**[JEE (Main)-2022]**

87. Let $S$ be the set of all $(\alpha, \beta)$, $\pi < \alpha, \beta < 2\pi$, for which the complex number $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely imaginary and $\frac{1 + i\cos\beta}{1 - 2i\cos\beta}$ is purely real, Let $Z_{\alpha\beta} = \sin 2\alpha + i \cos 2\beta$, $(\alpha, \beta) \in S$. Then $\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta} + \frac{1}{i\bar{Z}_{\alpha\beta}}\right)$ is equal to

**[JEE (Main)-2022]**

(1) 3 (2) $3i$

(3) 1 (4) $2 - i$

88. Let $S_1 = \left\{z_1 \in C : |z_1 - 3| = \frac{1}{2}\right\}$ and $S_2 = \left\{z_2 \in C : \left|z_2 - |z_2 + 1|\right| = \left|z_2 + |z_2 - 1|\right|\right\}$. Then, for $z_1 \in S_1$ and $z_2 \in S_2$, the least value of $|z_2 - z_1|$ is : **[JEE (Main)-2022]**

(1) 0 (2) $\frac{1}{2}$

(3) $\frac{3}{2}$ (4) $\frac{5}{2}$

89. Let $z = a + ib$, $b \neq 0$ be complex numbers satisfying $z^2 = \bar{z} \cdot 2^{1-|z|}$. Then the least value of $n \in N$, such that $z^n = (z + 1)^n$, is equal to _____.

**[28-07-2022 Evening]**

90. If $z \neq 0$ be a complex number such that $\left|z - \frac{1}{z}\right| = 2$, then the maximum value of $|z|$ is

**[JEE (Main)-2022]**

(1) $\sqrt{2}$ (2) 1

(3) $\sqrt{2} - 1$ (4) $\sqrt{2} + 1$

91. Let $S = \{z = x + iy : |z - 1 + i| \geq |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of $x$, for which $w = 2x + iy \in S$ for some $y \in$ R, is

**[JEE (Main)-2022]**

(1) $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$ (2) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

(3) $\left(-\sqrt{2}, \frac{1}{2}\right]$ (4) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

92. If $z = 2 + 3i$, then $z^5 + (\bar{z})^5$ is equal to :

(1) 244 (2) 224

(3) 245 (4) 265

**[JEE (Main)-2022]**

93. For $z \in \mathbb{C}$ if the minimum value of $\left(|z - 3\sqrt{2}| + |z - p\sqrt{2}i|\right)$ is $5\sqrt{2}$, then a value of $p$ is ________. **[JEE (Main)-2022]**

(1) 3 (2) $\frac{7}{2}$

(3) 4 (4) $\frac{9}{2}$

94. If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z - i| - |z + 5i| = 0$, then [JEE (Main)-2022]

(1) $x + 2y - 4 = 0$ (2) $x^2 + y - 4 = 0$

(3) $x + 2y + 4 = 0$ (4) $x^2 - y + 3 = 0$

95. Let $A = \{z \in \mathbf{C} : 1 \le |z - (1 + i)| \le 2\}$ and $B = \{z \in A : |z - (1 - i)| = 1\}$. Then, $B$ : **[JEE (Main)-2022]**

(1) Is an empty set

(2) Contains exactly two elements

(3) Contains exactly three elements

(4) Is an infinite set

96. Let $A = \left\{z \in \mathbf{C} : \left|\frac{z+1}{z-1}\right| < 1\right\}$

and $B = \left\{z \in \mathbf{C} : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}\right\}$.

Then $A \cap B$ is : **[JEE (Main)-2022]**

(1) A portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

(2) A portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

(3) An empty set

(4) A portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

97. Sum of squares of modulus of all the complex numbers $z$ satisfying $\bar{z} = iz^2 + z^2 - z$ is equal to ________. **[JEE (Main)-2022]**

❑ ❑ ❑

# Chapter 1

# Complex Numbers

1. Answer (1)

$$\left|z - \frac{4}{z}\right| = 2$$

$$\Rightarrow \left|z - \frac{4}{z}\right| \geq \left| |z| - \frac{4}{|z|} \right|$$

$$\Rightarrow |z| - \frac{4}{|z|} \leq 2$$

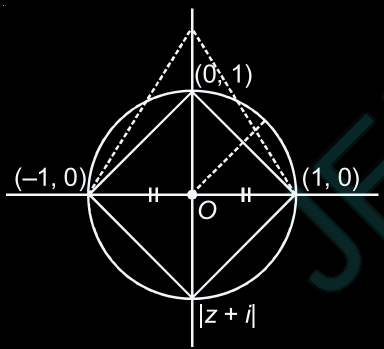
$$\Rightarrow |Z|^2 - 4 - 2|Z| \leq 0$$

$$\Rightarrow |Z|^2 - 2|Z| - 4 \leq 0$$

$$1 - \sqrt{5} \leq |Z| \leq 1 + \sqrt{5}$$

Hence maximum value = $1 + \sqrt{5}$

2. Answer (2)




We have,

$|z - 1| = |z + 1| = |z - i|$

Clearly $z$ is the circumcentre of the triangle formed by the vertices (1, 0) and (0, 1) and (–1, 0), which is unique.

3. Answer (4)

$$\frac{z^2}{z-1} = \frac{(x^2 - y^2 + 2ixy)}{((x-1) + iy)} \times \frac{((x-1) - iy)}{((x-1) - iy)}$$

$$\text{Im}\left(\frac{z^2}{z-1}\right) = \frac{2xy(x-1) - y(x^2 - y^2)}{(x-1)^2 + y^2} = 0$$

$$\Rightarrow y(x - y)^2 = 0$$

4. Answer (3)

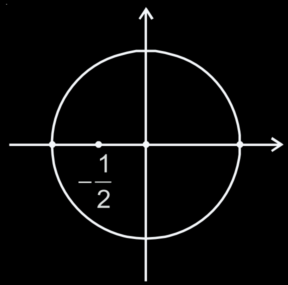
$$\arg\left(\frac{1+z}{1+\bar{z}}\right) = \arg\left(\frac{z\bar{z} + z}{1+\bar{z}}\right)$$

$$= \arg(z)$$

$$= \theta$$

5. Answer (4)

$$\left|z + \frac{1}{2}\right|$$




So, $\left| |z| - \frac{1}{2} \right| \leq \left| z + \frac{1}{2} \right|$

$$\Rightarrow \left|z + \frac{1}{2}\right| \geq \left|2 - \frac{1}{2}\right|$$

$$\Rightarrow \left|z_{\text{min.}} = \frac{3}{2}\right|$$

6. Answer (3)

$$\left(\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right) = 1$$

$$\left(\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right)\left(\frac{\bar{z}_1 - 2\bar{z}_2}{2 - \bar{z}_1 z_2}\right) = 1$$

$$z_1\bar{z}_1 - 2z_1\bar{z}_2 - 2z_2\bar{z}_1 + 4z_2\bar{z}_2$$

$$= 4 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2 + z_1\bar{z}_1 z_2\bar{z}_2$$

$$z_1\bar{z}_1 + 4z_2\bar{z}_2 = 4 + z_1\bar{z}_1 z_2\bar{z}_2$$

$$z\bar{z}_1(1 - z_2\bar{z}_2) - 4(1 - z_2\bar{z}_2) = 0$$

$$(z_1\bar{z}_1 - 4)(1 - z_2\bar{z}_2) = 0$$

$$\Rightarrow z_1\bar{z}_1 = 4$$

$|z| = 2$ *i.e.* $z$ lies on circle of radius 2.

7. Answer (3)

$$\frac{2+3i\sin\theta}{1-2i\sin\theta}\times\frac{1+2i\sin\theta}{1+2i\sin\theta} = \text{purely in imaginary}$$

$$\Rightarrow \; 2-6\sin^2\theta = 0 \Rightarrow \sin^2\theta = \frac{1}{3}$$

$$\therefore \; \sin\theta = \pm\frac{1}{\sqrt{3}}$$

8. Answer (4)

Let $z=\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$

As $z$ is purely imaginary, $z+\bar{z}=0$

$$\frac{3+2i\sin\theta}{1-2i\sin\theta}+\frac{3-2i\sin\theta}{1+2i\sin\theta}=0$$

$$\Rightarrow \frac{(3+2i\sin\theta)(1+2i\sin\theta)+(3-2i\sin\theta)(1-2i\sin\theta)}{1+4\sin^2\theta}=0$$

$$\Rightarrow \sin^2\theta=\frac{3}{4}$$

$$\Rightarrow \theta = -\frac{\pi}{3},\frac{\pi}{3},\frac{2\pi}{3}$$

9. Answer (3)

$\because$ $z_0$ is a root of quadratic equation

$x^2+x+1=0$

$\therefore$ $z_0 = \omega$ or $\omega^2 \Rightarrow z_0^3 = 1$

$\therefore$ $z = 3+6i\,z_0^{81} - 3i\,z_0^{93}$

$= 3+6i-3i$

$= 3+3i$

$$\therefore \; \arg(z)=\tan^{-1}\left(\frac{3}{3}\right)=\frac{\pi}{4}$$

10. Answer (2)

Let $z_1 = r_1e^{i\theta}$ and $z_2 = r_2e^{i\phi}$

$3|z_1| = 4|z_2| \;\Rightarrow\; 3r_1 = 4r_2$

$$z=\frac{3z_1}{2z_2}+\frac{2z_2}{3z_1}=\frac{3}{2}\frac{r_1}{r_2}\;e^{i(\theta-\phi)}+\frac{2}{3}\frac{r_2}{r_1}r^{i(\phi-\theta)}$$

$$=\frac{3}{2}\times\frac{4}{3}(\cos(\theta-\phi)+i\sin(\theta-\phi))+$$

$$\frac{2}{3}\times\frac{3}{4}[\cos(\theta-\phi)+i\sin(\phi-\theta)]$$

$$z=\left(2+\frac{1}{2}\right)\cos(\theta-\phi)+i\left(2-\frac{1}{2}\right)\sin(\theta-\phi)$$

$$\therefore \; |z|=\sqrt{\frac{25}{4}\cos2(\theta-\phi)+\frac{9}{4}\sin^2(\theta-\phi)}$$

$$=\sqrt{\frac{34}{8}+2\cos2(\theta-\phi)}$$

$$\Rightarrow \; \frac{3}{2}\le|z|\le\frac{5}{2}$$

11. Answer (1)

$$z=(e^{i\frac{\pi}{6}})^5+(e^{-i\frac{\pi}{6}})^5=2\cos\frac{\pi}{6}=\sqrt{3}$$

$$\Rightarrow \; I(z)=0,\ \text{Re}(z)=\sqrt{3}$$

$\Rightarrow$ Option (1) is correct

12. Answer (4)

$-(6+i)^3 = x+iy$

$-[216-i+18i(6+i)] = x+iy$

$\Rightarrow$ $-[216-i+108i-18] = x+iy$

$\Rightarrow$ $-216+i-108i+18 = x+iy$

$\Rightarrow$ $-198-107i = x+iy$

$\Rightarrow$ $x = -198$, $y = -107$

$\Rightarrow$ $y-x = -107+198 = 91$

13. Answer (3)

Given, $|z|+z = 3+i$

Let $z = a+ib$

$$\Rightarrow \; \sqrt{a^2+b^2}+a+ib=3+i$$

$$\Rightarrow \; b=1, \sqrt{a^2+b^2}+a=3$$

$$\sqrt{a^2+1}=3-a$$

$a^2+1 = a^2+9-6a$

$6a = 8$

$$a=\frac{4}{3}$$

$$|z|=\sqrt{\left(\frac{4}{3}\right)^2+1}=\sqrt{\frac{16}{9}+1}=\frac{5}{3}$$

14. Answer (2)

Let $t = \dfrac{z-\alpha}{z+\alpha}$

$t+\bar{t}=0$

$\Rightarrow \dfrac{z-\alpha}{z+\alpha}+\dfrac{\bar{z}-\alpha}{\bar{z}+\alpha}=0$

$\Rightarrow (z-\alpha)(\bar{z}+\alpha)+(\bar{z}-\alpha)(z+\alpha)=0$

$\Rightarrow z\bar{z}-\alpha^2+z\bar{z}-\alpha^2=0$

$\Rightarrow z\bar{z}-\alpha^2=0$

$\Rightarrow |z|^2-\alpha^2=0$

$\Rightarrow \alpha^2=4$

$\alpha=\pm 2$

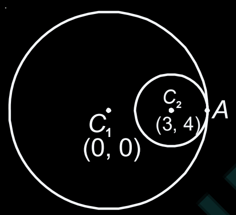
15. Answer (1)

$|z_1|=9,\ |z_2-3-4i|=4$

$z_1$ lies on a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$

$z_2$ lies on a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$

Minimum value of $|z_1 - z_2|$ is zero at point of contact (i.e. $A$)




16. Answer (3)

$z=\dfrac{\sqrt{3}}{2}+\dfrac{i}{2}=-i\left(-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)=-i\omega$

where $\omega$ is not real cube root of unity

$\Rightarrow (1+iz+z^5+iz^8)^9=(1+\omega-i\omega^2+i\omega^2)^9$

$=(1+\omega)^9$

$=(-\omega^2)^9$

$=-\omega^{18}$

$=-1$

17. Answer (3)

$\because\ S=\dfrac{\alpha+i}{\alpha-i}$ Let $S = x + iy$

$\Rightarrow x+iy=\dfrac{(\alpha+i)^2}{\alpha^2+1}$ (by rationalisation)

$\Rightarrow x+iy=\dfrac{(\alpha^2-1)}{\alpha^2+1}+\dfrac{i(2\alpha)}{\alpha^2+1}$

(On comparing both sides)

$\Rightarrow x=\dfrac{\alpha^2-1}{\alpha^2+1}$ ... (i) $y=\dfrac{2\alpha}{\alpha^2+1}$ ... (ii)

By squaring and adding,

$x^2+y^2=1$

18. Answer (2)

$\omega=\dfrac{5+3z}{5-5z} \Rightarrow 5w-5wz=5+3z$

$\Rightarrow 5\omega-5=z(3+5\omega)$

$\Rightarrow z=\dfrac{5(\omega-1)}{3+5\omega}$

Given $|z| < 1$

$\Rightarrow 5|\omega-1|<|3+5\omega|$

$\Rightarrow 25(\omega\bar{\omega}-\omega-\bar{\omega}+1)<9+25\omega\bar{\omega}+15\omega+15\bar{\omega}$

(using $|z|^2 = z\bar{z}$)

$\Rightarrow 16<40\omega+40\bar{\omega}$

$\Rightarrow \omega+\bar{\omega}>\dfrac{2}{5}$

$\Rightarrow 2\,\mathrm{Re}(\omega)>\dfrac{2}{5} \Rightarrow \mathrm{Re}(\omega)>\dfrac{1}{5}$

19. Answer (4)

$z=\dfrac{(1+i)^2}{a-i}\times\dfrac{a+i}{a+i}$

$z=\dfrac{(1-1+2i)(a+i)}{a^2+1}=\dfrac{2ai-2}{a^2+1}$ ...(i)

$|z|=\sqrt{\left(\dfrac{-2}{a^2+1}\right)^2+\left(\dfrac{2a}{a^2+1}\right)^2}$

$=\sqrt{\dfrac{4+4a^2}{(a^2+1)^2}}=\sqrt{\dfrac{4(1+a^2)}{(1+a^2)^2}}$

$=\dfrac{2}{\sqrt{1+a^2}}$

given $|z| = \sqrt{\frac{2}{5}}$

so $\sqrt{\frac{2}{5}} = \frac{2}{\sqrt{1+a^2}}$ from equation (i)

(square both side)

$\Rightarrow \frac{2}{5} = \frac{4}{1+a^2}$

$\Rightarrow 1 + a^2 = 10$

$a^2 = 9$

$\Rightarrow a \pm 3 \because (a > 0) \therefore a = 3$

Hence $z = \frac{1+i^2+2i}{3-i} = \frac{2i}{3-i} = \frac{2i(3+i)}{10} = \frac{-1+3i}{5}$

$\bar{z} = \frac{-1}{5} - \frac{3}{5}i$

20. Answer (4)

$|zw| = 1$ ...(i)

$\arg\left(\frac{z}{w}\right) = \frac{\pi}{2}$ ...(ii)

$\therefore \frac{z}{w} + \frac{\bar{z}}{\bar{w}} = 0 \Rightarrow z\bar{w} = -\bar{z}w$

from (i),

$z\bar{z}\,w\bar{w} = 1$

$(\bar{z}w)^2 = -1 \Rightarrow \bar{z}w = \pm i$

from (ii),

$-\arg(\bar{z}) - \arg w = \frac{\pi}{2}$

$\Rightarrow \arg(\bar{z}w) = \frac{-\pi}{2}$

Hence, $\bar{z}w = -i$

21. Answer (4)

$|z - 1| = |z - i|$

Let $z = x + iy$

$(x - 1)^2 + y^2 = x^2 + (y - 1)^2$

$1 - 2x = 1 - 2y$

$\Rightarrow x - y = 0$

Locus is straight line with slope 1

22. Answer (3)

Let $z = x + 10i$

$2z - n = (2i - 1)(2z + n)$

$(2x - n) + 20i = (2i - 1)((2x + n) + 20i)$

Comparing real and imaginary part

$-(2x + n) - 40 = 2x - n$ and $20 = 4x + 2n - 20$

$\Rightarrow 4x = -40$ $\qquad 40 = -40 + 2n$

$\Rightarrow x = -10$ $\qquad n = 40$

$\Rightarrow Re(z) = -10$

23. Answer (3)

$\left(\frac{z-1}{2z+i}\right) = \left(\frac{x-1+iy}{2x+i(1+2y)}\right)$

$= \frac{(x-1+iy)(2x-i(1+2y))}{4x^2+(2y+1)^2}$

As its real part is 1

$\Rightarrow \frac{2x^2-2x+y+2y^2}{4x^2+4y^2+1+4y} = 1$

$\Rightarrow 2x^2 + 2y^2 + 2x + 3y + 1 = 0$

i.e. $x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$

$\Rightarrow \left(x+\frac{1}{2}\right)^2 + \left(y+\frac{3}{4}\right)^2 = \frac{5}{16}$

i.e. circle with diameter $\frac{\sqrt{5}}{2}$

24. Answer (2)

$\because Z = \frac{3+i\sin\theta}{4-i\cos\theta} \times \frac{4+i\cos\theta}{4+i\cos\theta}$

$= \frac{(12-\sin\theta\cos\theta)+i(4\sin\theta+3\cos\theta)}{16+\cos^2\theta}$

$\because$ $Z$ is purely real

$\therefore 4\sin\theta + 3\cos\theta = 0$

$\tan\theta = -\frac{3}{4}$

if $\theta \in \left(\frac{\pi}{2}, \pi\right)$, then

$$\arg(\sin\theta + i\cos\theta) = -\left(\tan^{-1}(\cot\theta)\right)$$

$$= \pi - \tan^{-1}\left(\frac{4}{3}\right)$$

if $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$, then

$$\arg\ (\sin\theta + i\cos\theta) = -\tan^{-1}\frac{4}{3}$$

25. Answer (2)

As $b \in R$ let roots be $\alpha \pm i\beta$

$\Rightarrow$ $2\alpha = -b$ and $\alpha^2 + \beta^2 = 45$

Also $|\alpha + i\beta + 1| = 2\sqrt{10}$

$\Rightarrow$ $(\alpha + 1)^2 + \beta^2 = 40$

$\Rightarrow$ $45 + 2\alpha + 1 = 40$

$\Rightarrow$ $\alpha = -3$ and $b = 6$

$\Rightarrow$ $b^2 - b = 30$

26. Answer (2)

$\because$ $\alpha = w$ (complet non real cube of unity)

$\Rightarrow$ $a = (1 + w)(1 + w^2 + w^4 + w^6 + \ldots\ldots\ w^{200})$

$$\Rightarrow \quad a = (1+w)\frac{\left(1-\left(w^2\right)^{101}\right)}{\left(1-w^2\right)} = \frac{(1-w)(1+w)}{1-w^2} = 1$$

and $b = 1 + w^3 + w^6 + \ldots\ldots\ w^{300} = 101$

Equation $x^2 - (102)x + 101 = 0$.

27. Answer (2)

$\therefore$ $|z - i| = |z + 2i|$ is perpendicular bisector of line segment joining (0, 1) and (0, –2) that is

$$y = -\frac{1}{2} \qquad \ldots\text{(i)}$$

$|z| = \frac{5}{2}$ represents a circle having equation

$$x^2 + y^2 = \frac{25}{4} \qquad \ldots\text{(ii)}$$

From (i) and (ii) $x = \pm\sqrt{6},\ y = -\frac{1}{2}$

$$\Rightarrow \quad z = \pm\sqrt{6} - \frac{1}{2}i$$

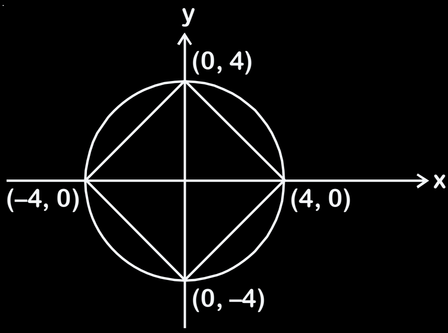
So $|z + 3i| = \sqrt{\left(\pm\sqrt{6}\right)^2 + \left(\frac{5}{2}\right)^2} = \frac{7}{2}$

28. Answer (4)

$z = x + iy$

$A/Q$ $|x| + |y| = 4$

Min. $|z| = 2\sqrt{2}$




Max. $|z| = 4$

so $|z| \in \left[2\sqrt{2}, 4\right]$

$\Rightarrow |z| \neq \sqrt{7}$

29. Answer (4)

Let $\theta = \frac{2\pi}{9}$

$$\therefore \quad \left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$$

$$= \left(\frac{1+\cos\left(\frac{\pi}{2}-\theta\right)+i\sin\left(\frac{\pi}{2}-\theta\right)}{1+\cos\left(\frac{\pi}{2}-\theta\right)-i\sin\left(\frac{\pi}{2}-\theta\right)}\right)^3$$

$$= \left(\frac{\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)+i\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)-i\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}\right)^3$$

$$= \left(e^{i\left(\frac{\pi}{4}-\frac{\theta}{2}\right)} \cdot e^{i\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}\right)^3$$

$$= \left(e^{i\left(\frac{\pi}{2}-\theta\right)}\right)^3 = e^{i\left(\frac{3\pi}{2}-3\theta\right)}$$

$= -\sin3\theta - i\cos3\theta$

$$= -\sin\frac{2\pi}{3} - i\cos\frac{2\pi}{3}$$

$$= -\frac{\sqrt{3}}{2} + \frac{i}{2} = -\frac{1}{2}\left(\sqrt{3} - 1\right)$$

30. Answer (3)

Let $\sqrt{3+6\sqrt{6}\,i} = a+ib$

$\Rightarrow$ $a^2 - b^2 = 3$ and $ab = 3\sqrt{6}$

$\Rightarrow$ $a^2 + b^2 = 15$

So, $a = \pm 3$ and $b = \pm\sqrt{6}$

$$\sqrt{3+6\sqrt{6}\,i} = \pm\left(3+\sqrt{6}\,i\right)$$

Similarly, $\sqrt{3-6\sqrt{6}\,i} = \pm\left(3-\sqrt{6}\,i\right)$

$$\text{Im}\left(\sqrt{3+6\sqrt{6}\,i} - \sqrt{3-6\sqrt{6}\,i}\right) = \pm 2\sqrt{6}$$

31. Answer (4)

Let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

$\Rightarrow$ $x_1 = \sqrt{(x_1-1)^2 + y_1}$

$\Rightarrow$ $x_1^2 = x_1^2 + 1 - 2x_1 + y_1^2$

$\Rightarrow$ $y_1^2 - 2x_1 + 1 = 0$ ...(i)

Similarly

$y_2^2 - 2x_2 + 1 = 0$ ...(ii)

Now

(i) – (ii), $(y_1 - y_2)(y_1 + y_2) = 2(x_1 - x_2)$

$\Rightarrow$ $y_1 - y_2 = \dfrac{2(x_1-x_2)}{y_1+y_2} \Rightarrow \dfrac{y_1-y_2}{x_1-x_2} = \dfrac{2}{y_1+y_2}$

Now

$$\arg(z_1 - z_2) = \frac{\pi}{6}$$

$$\tan^{-1}\left(\frac{y_1-y_2}{x_1-x_2}\right) = \frac{\pi}{6}$$

$$\frac{y_1-y_2}{x_1-x_2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{2}{y_1+y_2} = \frac{1}{\sqrt{3}}$$

$$y_1 + y_2 = 2\sqrt{3}$$

So $I_M = (z_1 + z_2) = y_1 + y_2 = 2\sqrt{3}$

32. Answer (3)

$$u = \frac{2(x+iy)+i}{(x+iy)-ki} = \frac{2x+i(2y+1)}{x+i(y-k)}$$

$\therefore$ $\text{Re}(u) = \dfrac{2x^2 + (y-K)(2y+1)}{x^2+(y-K)^2}$

and $\text{Im}(u) = \dfrac{-2x(y-K)+x(2y+1)}{x^2+(y-K)^2}$

Also Re($u$) + Im($u$) = 1

$\Rightarrow$ $2x^2 + 2y^2 - 2Ky + y - K - 2xy + 2Kx + 2xy + x$

$= x^2 + y^2 + K^2 - 2Ky$

Let $y_1$ & $y_2$ are roots of equations if $x = 0$

$\Rightarrow$ $\mathbf{y^2 + y - K(K+1) = 0} \begin{matrix} \mathbf{y_1} \\ \mathbf{y_2} \end{matrix}$

Given $PQ = 5$ $\Rightarrow |y_1 - y_2| = 5$

$\Rightarrow$ $4k^2 + 4k - 24 = 0 \Rightarrow k = 2$ or $-3$

as $k > 0$ $k = 2$

33. Answer (2)

Here $\alpha = \dfrac{-1+\sqrt{3}i}{2} = \omega$

Now, $(2 + \omega)^4 = a + b\omega$

$\Rightarrow$ $(4 + \omega^2 + 4\omega)^2 = a + b\omega$

$\Rightarrow$ $(\omega^2 + 4(1 + \omega))^2 = a + b\omega$

$\Rightarrow$ $(\omega^2 - 4\omega^2)^2 = a + b\omega$

$\Rightarrow$ $(-3\omega^2)^2 = a + b\omega$

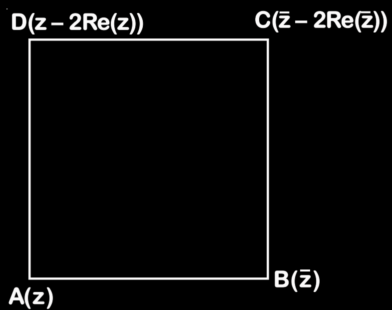
$\Rightarrow$ $9\omega^4 = a + b\omega$

$\Rightarrow$ $9\omega = a + b\omega$, $\{\because \omega^3 = 1\}$

$\Rightarrow$ $a = 0$, $b = 9$

$\Rightarrow$ $a + b = 0 + 9 = 9$

34. Answer (3)




Let $z = x + iy$

$\because$ Length of side of square = 4 units

$\Rightarrow$ $|z - \bar{z}| = 4 \Rightarrow |2iy| = 4$

$\Rightarrow$ $|y| = 2$

Also $|z - (z - 2\text{Re}(z))| = 4$

$\Rightarrow$ $|2\text{Re}(z)| = 4 \Rightarrow |2x| = 4 \Rightarrow |x| = 2$

Now $|z| = \sqrt{x^2+y^2} = \sqrt{4+4} = 2\sqrt{2}$

35. Answer (1)

$\because \quad -1+\sqrt{3}\,i = 2.e^{\frac{2\pi}{3}i}$

and $1-i = \sqrt{2}.e^{-\frac{i\pi}{4}}$

So, $\left(\frac{-1+\sqrt{3}\,i}{1-i}\right)^{30} = \left(\sqrt{2}\,e^{\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)i}\right)^{30}$

$= 2^{15}.e^{-\frac{\pi}{2}i} = -2^{15}.i$

36. Answer (2)

$\because \quad |z| - \text{Re}\,(z) \le 1$

$\sqrt{x^2+y^2} - x \le 1$

$\sqrt{x^2+y^2} \le 1+x$

$x^2+y^2 \le 1+x^2+2x$

$y^2 \le 1+2x$

$y^2 \le 2\left(x+\frac{1}{2}\right)$

37. Answer (1)

$z = x + iy$

$x^2 - y^2 + 2ixy = i(x^2 + y^2)$

$\Rightarrow \quad x^2 - y^2 = 0$ and $2xy = x^2 + y^2$

$\Rightarrow \quad (x - y)(x + y) = 0$ and $(x - y)^2 = 0$

$\Rightarrow \quad x = y$

38. Answer (4)

$\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$

$\left(\frac{(1+i)^2}{2}\right)^{m/2} = \left(\frac{(1+i)^2}{-2}\right)^{n/3} = 1$

$\Rightarrow \quad i^{m/2} = (-i)^{n/3} = 1$

$m_{\text{least}} = 8,\ n_{\text{least}} = 12$

GCD (8, 12) = 4

39. Answer (4)

$\because \quad p^4 + q^4 = (p + 4)^4 - 4pq(p^2 + q^2) - 6p^2q^2$

$\Rightarrow \quad 272 = 16 - 4pq(4 - 2pq) - 6p^2q^2$

$\Rightarrow \quad 2p^2q^2 - 16pq - 256 = 0$

$\Rightarrow \quad pq = -8$ or $16$

$\because \quad p, q > 0$, so $pq = 16$

Required quadratic equation is

$x^2 - 2x + 16 = 0$

40. Answer (9)

Let $z = x + iy$

$x + \alpha\sqrt{(x-1)^2+y^2} + i(y+2) = 0$

$\Rightarrow \quad y = -2$ and $\alpha = \frac{-x}{\sqrt{x^2-2x+5}}$

$\frac{d\alpha}{dx} = \frac{(x^2-x)-(x^2-2x+5)}{(x^2-2x+5)^{3/2}} = \frac{x-5}{(x^2-2x+5)^{3/2}}$

So $\alpha$ is decreasing in $(-\infty, 5)$ and increasing in $(5, \infty)$

$\alpha_{\text{min}} = -\frac{5}{\sqrt{20}} = 1\sqrt{\frac{5}{4}} = p(\text{at } x = 5)$

and $\alpha_{\text{max}} = \lim_{x\to-\infty} \frac{-x}{\sqrt{x^2-2x+5}} = 1$, $q = 1$(however this value is not achievable.)

41. Answer (310)

$k = \frac{\left(-1+i\sqrt{3}\right)^{21}}{(1-i)^{24}} + \frac{\left(1+i\sqrt{3}\right)^{21}}{(1+i)^{24}}$

$= \frac{\left(2e^{\frac{2\pi}{3}i}\right)^{21}}{\left(\sqrt{2}e^{-\frac{\pi}{4}i}\right)^{24}} + \frac{\left(2e^{\frac{\pi}{3}i}\right)^{21}}{\left(\sqrt{2}e^{\frac{\pi}{4}i}\right)^{24}}$

$= \mathbf{2^9}\left[\mathbf{e^{(14\pi+6\pi)i}} + \mathbf{e^{(7\pi-6\pi)i}}\right]$

$= 2^9[0] = 0$

Now $\sum_{j=0}^{5}\left\{(j+5)^2-(j+5)\right\} = \sum_{j=0}^{5}(j+4)(j+5)$

$= \sum_{j=0}^{5}\frac{1}{3}\left\{(j+4)(j+5)(j+6)-(j+3)(j+4)(j+5)\right\}$

$= \frac{1}{3}\left[9\times10\times11-3\times4\times5\right] = 310$

42. Answer (3)

Given lines are

$(2 - i)\ z = (2 + i)\ \bar{z}$ ...(1)

and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$

or $-i(2 + i)z - i\ (i - 2)\bar{z} - 4 = 0$

$\Rightarrow\ (1 - 2i)z + (1 + 2i)\bar{z} - 4 = 0$ ...(2)

Let $z = x + iy$

So from (1) we get the line $y = \dfrac{x}{2}$ ...(3)

and from (2) $(1 - 2i)(x + iy) + (1 + 2i)(x - iy) - 4 = 0$

$\Rightarrow\ x + 2y - 2 = 0$ ...(4)

On solving (3) and (4) we get $x = 1,\ y = \dfrac{1}{2}$

$\because$ These lines were normal to the circle.

So centre $= \left(1, \dfrac{1}{2}\right)$

Now the line $iz + \bar{z} + 1 + i = 0$

or $i(1-i)z + (1-i)\bar{z} + (1+1) = 0$

$\Rightarrow\ (1+i)z + (1-i)\bar{z} + 2 = 0$

$\Rightarrow\ (z+\bar{z}) + i\,(z-\bar{z}) + 2 = 0 \quad \Rightarrow \quad 2x - 2y + 2 = 0$

$x - y + 1 = 0$

$\because$ This line is tangent to circle

So, $r = \dfrac{\left|1 - \dfrac{1}{2} + 1\right|}{\sqrt{1+1}} = \dfrac{\left|\dfrac{3}{2}\right|}{\sqrt{2}}$

$r = \dfrac{3}{2\sqrt{2}}$

43. Answer (2)

As $\alpha, \beta \in R$ roots are $1 - 2i$ and $1 + 2i$

$-\alpha = 2 \Rightarrow \alpha = -2$

and $\beta = (1)^2 - (2i)^2 = 5 \Rightarrow \alpha - \beta = -7$

44. Answer (3)

$x^3 - 1 + 2x - 2x^2 = 0$

$\Rightarrow\ (x - 1)\ [x^2 - x + 1] = 0$

$\Rightarrow\ x = 1, -\omega, -\omega^2$

$S = 1^{162} + (-\omega)^{162} + (-\omega^2)^{162}$

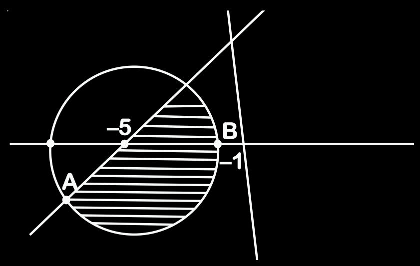
$= 1 + 1 + 1 = 3$

45. Answer (48)

$z(1+i) + \bar{z}(1+i) \geq -10 \Rightarrow x - y + 5 \geq 0$

and $|z+5| \leq 4$ is interior of a circle with centre $-5$ and radius 4.

$\because\ |z+1|$ represents the distance of z from $-1$.




$|z+1|$ is maximum is z is at A.

z is at A.

$AB^2 = |z+1|^2 = 4^2 + 4^2 - 2\cdot 4\cdot 4\cdot \cos 135° = 32 + 16\sqrt{2}$

$\Rightarrow\ \alpha = 32$ and $\beta = 16$

46. Answer (2)

$$\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{|z|^2 - 2|z| + 1}\right) \leq 2$$

$$\Rightarrow \frac{|z|+11}{|z|^2 - 2|z| + 1} \geq \frac{1}{2}$$

$\Rightarrow\ |z|^2 - 2|z| + 1 \leq 2|z| + 22$

$\Rightarrow\ (|z| - 7)(|z| + 3) \leq 0 \Rightarrow |z| \leq 7$

47. Answer (04)

Let $z = x + iy$

$\Rightarrow\ w = x^2 + y^2 - 2x - 2iy + 2$

$\Rightarrow\ Re(w) = (x - 1)^2 + y^2 + 1$ ...(i)

Also $|z + i| = |z - 3i|$

$(y + 1)^2 = (y - 3)^2$

$\Rightarrow\ 2y + 1 = -6y + 9$

$\Rightarrow\ y = 1$ ...(ii)

by (i) and (ii)

$Re(w)_{min} \Rightarrow x = 1$ and $y = 1$

$\Rightarrow w = 1 + i$

$(1 + i)^n =$ real $\Rightarrow n_{min} = 4$

48. Answer (3)

Let $|z| = t,\ t \geq 0$

$e^{\frac{(t+3)(t-1)}{t+1}\log_e 2} \geq \log_{\sqrt{2}} 16 = 8 \quad (\because t + 1 > 0)$

$$2^{\frac{(t+3)(t-1)}{t+1}} \geq 2^3$$

$$\frac{(t+3)(t-1)}{t+1} \geq 3$$

$$t^2 + 2t - 3 \geq 3t + 3$$

$$t^2 - t - 6 \geq 0$$

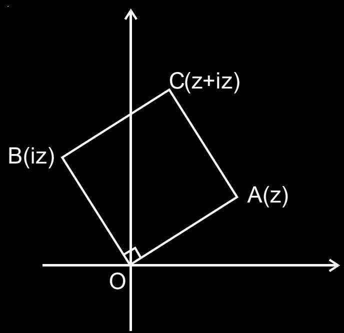
$t \in (-\infty, -2] \cup [3, \infty)$ But $t \geq 0$

$\therefore\ t \in [3, \infty)$

49. Answer (1)

Geometrically OABC form a square as shown

Each side length = $|z|$




Area of $\Delta ABC = \frac{1}{2}$(Area of square)

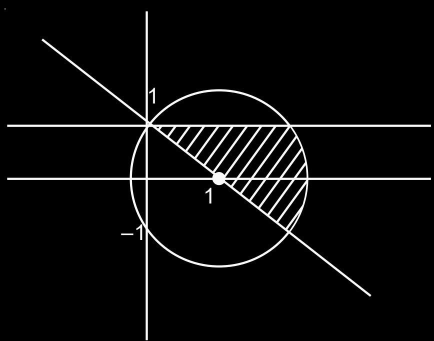
$$= \frac{1}{2}.|z|^2$$

50. Answer (1)

$S_1$ is interior of the circle having centre 1 and radius $\sqrt{2}$.

For $S_2$, let $z = x + iy$

$\Rightarrow\ x + y \geq 1$




Clearly there will be infinitely many elements in set $S_1 \cap S_2 \cap S_3$.

51. Answer (1)

$$az\bar{z} + \alpha\bar{z} + \bar{\alpha}z + d = 0$$

for equation of circle radius > 0

$$\Rightarrow\ z\bar{z} + \frac{\alpha}{a}\bar{z} + \frac{\bar{\alpha}}{a}z + \frac{d}{a} = 0$$

$$\text{Radius} = \sqrt{\frac{\alpha}{a}.\frac{\bar{\alpha}}{a} - \frac{d}{a}}$$

$$\Rightarrow\ \frac{\alpha\bar{\alpha}}{a^2} > \frac{d}{a}$$

$\Rightarrow\ |\alpha|^2 - ad > 0$ and $a \neq 0$

52. Answer (06)

$z_1^2 + z_2^2 = z_1 z_2$ (Condition for equilateral triangle)

$a^2 - 2(12) = 12$

$\Rightarrow\ |a| = 6$

53. Answer (1)

$|wz| = 1 \Rightarrow |w||z| = 1$ and $|w| = 2$

$$\Rightarrow |z| = \frac{1}{2}$$

Also $\arg(z) - \arg(w) = \frac{z}{2}$

$$\Rightarrow z = \frac{1}{2} \cdot \frac{\left(1 - i\sqrt{3}\right)}{2} \cdot i$$

Area of triangle $= \frac{1}{2} \cdot 2 \times \frac{1}{2} = \frac{1}{2}$

54. Answer (Zero)

$x^2 + x + 1 = 0 \Rightarrow (x - \omega)(x - \omega^2) = 0$

where $\omega$ is complex cube root of unity

$P(x)$ is divisible by $x^2 + x + 1$

Here $P(\omega) = 0$ and $P(\omega^2) = 0$

$\Rightarrow\ P(\omega) = f(\omega^3) + \omega\, g(\omega^3) = 0$

$0 = f(1) + \omega.g(1)$ ... eqn (i)

Also, $P(\omega^2) = f(\omega^6) + \omega^2.g(\omega^6) = 0$

$0 = f(1) + \omega^2 g(1)$ ... eqn (ii)

from (i) and (ii), $f(1) = g(1) = 0$

Here $P(1) = f(1) + 1g(1) = 0$

55. Answer (3)

$$x = \frac{-3^{1/4} \pm \sqrt{3^{1/2} - 4 \cdot 3^{1/2}}}{2}$$

$$= \frac{3^{1/4}\left(-1 \pm \sqrt{3}i\right)}{2} = 3^{1/4}\omega \text{ or } 3^{1/4}\omega^2$$

$\alpha^{108} + \beta^{108} - (\alpha^{96} + \beta^{96})$

$= 3^{108/4}(\omega^{108} + \omega^{216}) - 3^{96/4}(\omega^{96} + \omega^{192})$

$= 3^{27} \cdot 2 - 3^{24} \cdot 2$

$= 3^{24}(52) = 52 \times 3^{24}$

56. Answer (2)

$z = re^{i\theta} \qquad \therefore \quad \omega = \frac{1}{r}e^{i(\theta - 3\pi/2)}$

$$\frac{1 - 2\bar{z}\omega}{1 + 3\bar{z}\omega} = \frac{1 - 2e^{-i\theta} \cdot e^{i(-3\pi/2 + \theta)}}{1 + 3e^{-i\theta} \cdot e^{i(-3\pi/2 + \theta)}}$$

$$\therefore \quad \text{Arg}\left(\frac{1 - 2i}{1 + 3i}\right) = -\frac{3\pi}{4}$$

57. Answer (3)

$z^2 + 3\bar{z} = 0$

$x^2 - y^2 + 2ixy + 3x - 3iy = 0$

$x^2 - y^2 + 3x = 0$ & $(2x - 3)y = 0$

i.e. if $y = 0 \quad \Rightarrow x = 0$ or $-3$

if $x = \frac{3}{2} \Rightarrow y^2 = \frac{9}{4} + \frac{9}{2} = \frac{27}{4} \Rightarrow y = \pm\frac{3\sqrt{3}}{2}$

Number of solutions 4.

$$\therefore \quad \sum_{k=0}^{\infty} \frac{1}{n^k} = 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots..$$

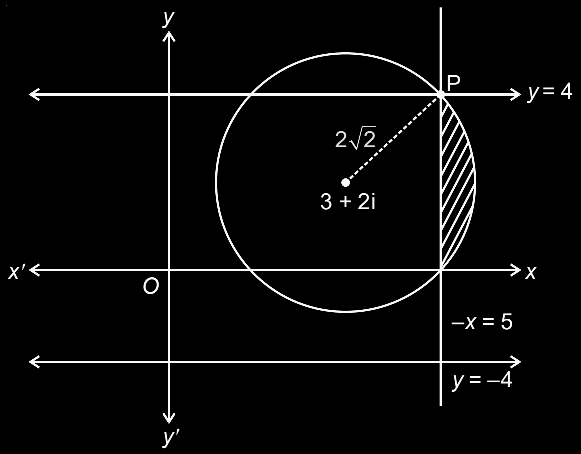
$$= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

58. Answer (2)

$\because$ $S_1$ be a circle of centre 3 + 2i and radius $2\sqrt{2}$

$S_3$ is half plane with real z more than S

and $S_3$ is plane with $y \in (-\infty, -4] \cup [4, \infty)$



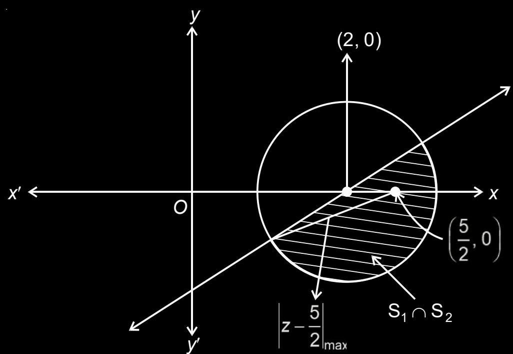

$\therefore$ Only one point P is the solution.

59. Answer (2)

$S_1 \equiv |z - 2| \le 1 \Rightarrow (x - 2)^2 + y^2 \le 1$

$S_2 \equiv x - y \ge 2$

$S_1 \cap S_2$




Solving equation from (i) & (ii), we get

$$y^2 = \frac{1}{2} \Rightarrow y = -\frac{1}{2} \quad x = 2 - \frac{1}{\sqrt{2}}$$

$$\left|z - \frac{5}{2}\right|^2 = \left(x - \frac{5}{2}\right)^2 + y^2 = \left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}$$

$$= \frac{3 + 2\sqrt{2}}{4} + \frac{2}{4} = \frac{5 + 2\sqrt{2}}{4}$$

60. Answer (1)

$$z = \frac{(3+2i\cos\theta)(1+3\cos\theta)}{1+9\cos^2\theta}$$

$$\because \quad \text{Re}(z) = 0 = \frac{3-6\cos^2\theta}{1+9\cos^2\theta} = 0$$

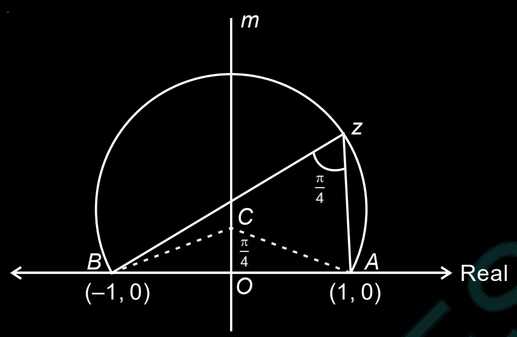
$$\Rightarrow \quad \cos^2\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4}$$

$$\sin^2 3\theta + \cos^2\theta = \frac{1}{2} + \frac{1}{2} = 1$$

61. Answer (2)

Here $OC = OA = 1$




$\therefore$ Centre of circle = (0, 1)

and radius of circle $= \sqrt{1^2+1^2} = \sqrt{2}$

62. Answer (13)

$$z = \frac{1-\sqrt{3}\,i}{2}$$

$$z = -\left[\frac{-1+\sqrt{3}\,i}{2}\right] = -\omega = z = -\frac{1+\sqrt{3}i}{2} = -\omega$$

$$21 + \left(-\omega - \frac{1}{\omega}\right)^3 + \left((-\omega)^2 + \left(-\frac{1}{\omega}\right)^2\right)^3 + \left((-\omega)^3 + \left(-\frac{1}{\omega^3}\right)^3\right)^3 + \ldots.$$

$$\ldots.. + \left((-\omega)^{21} + \frac{1}{(-\omega)^{21}}\right)^3$$

$= 21 - (\omega + \omega^2)^3 + (\omega + \omega^2)^3 + (-1 -1)^3 + (\omega + \omega^2)^3 - (\omega + \omega^2)^3 + (1 + 1)^3 + \ldots\ldots + (-\omega - \omega^2)^3 + (\omega^2 + \omega)^3 + (-\omega^3 - \omega^3)^3$

$= 21 + (1 - 1 - 8) + (-1 + 1 + 8) + (1 - 1 - 8) + (-1 + 1 + 8) + (1 - 1 - 8) + (-1 + 1 + 8) + (1 - 1 - 8)$

$= 21 - 8 = 13$

63. Answer (1)

$$\because \left(\sqrt{3}+i\right)^{100} = 2^{99}(p+iq)$$

$$\left(2e^{i\frac{\pi}{6}}\right)^{100} = 2^{99}(p+iq)$$

$$2e^{i\frac{50\pi}{3}} = p+iq$$

$$\Rightarrow \quad 2e^{i\left(16\pi+\frac{2\pi}{3}\right)} = p+iq$$

$$= 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = p+iq$$

$\therefore \quad p = -1,\ q = \sqrt{3}$

Equation with roots –1 and $\sqrt{3}$ is

$$x^2 - \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$$

64. Answer (16)

$$\frac{(2i)^n}{(1-i)^{n-2}} = \frac{\left(2e^{i\pi/2}\right)^n}{\left(\sqrt{2}e^{-i\pi/4}\right)^{n-2}}$$

$$= \left(\sqrt{2}\right)^{n+2} e^{i(3n-2)\frac{\pi}{4}}$$

For positive integer $n$ should be atleast 6

$$= \left(\sqrt{2}\right)^8 e^{i\cdot 4\pi} = \left(\sqrt{2}\right)^8 = 16$$

65. Answer (3)

Let $z = x + iy$

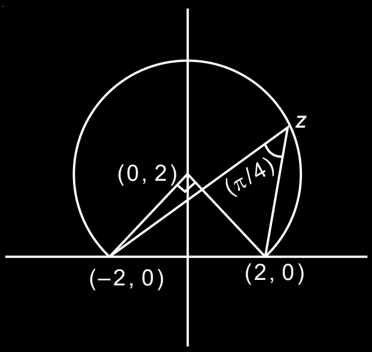
$$\because \quad \frac{x+(y-1)i}{x+(y+2)i} \text{ is real}$$

then $x(y - 1) - x(y + 2) = 0$

$\Rightarrow \quad -x - 2x = 0$

$\Rightarrow \quad x = 0$

66. Answer (98)




If $\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$ then $z$ lies on an arc of a circle as shown in figure.

Centre of this circle is (0, 2) and radius $=2\sqrt{2}$.

$\left|z-(9\sqrt{2}+2i)\right|=$ Distance of $z$ from $(9\sqrt{2},2)$.

Distance of $(9\sqrt{2},2)$ from centre (0, 2)

$=9\sqrt{2}$

Minimum value of $\left|z-9\sqrt{2}-2i\right|^2$

is equal to $(9\sqrt{2}-2\sqrt{2})^2=98$

67. Answer (2)

$\because\quad \frac{z-i}{z-1}$ is purely imaginary number

$\therefore\quad \frac{z-i}{z-1}+\frac{\bar{z}+i}{\bar{z}-1}=0$

$\Rightarrow\ (z-i)(\bar{z}-1)+(z-1)(\bar{z}+i)=0$

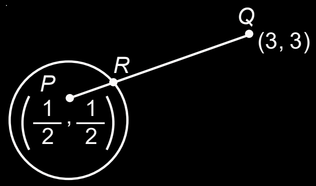
$\Rightarrow\ 2z\bar{z}-(z+\bar{z})+i(z-\bar{z})=0$

$\therefore\quad x^2+y^2-x-y=0$, let $z=x+iy$.

Which a circle of centre $\left(\frac{1}{2},\frac{1}{2}\right)$ and radius $\frac{1}{\sqrt{2}}$

$\therefore$ Minimum value of

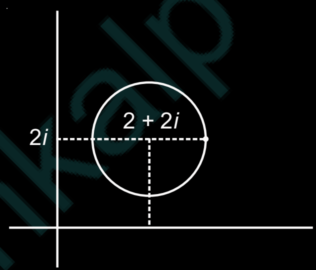
$\left|z-(3+3i)\right|=QR$




$=\sqrt{\left(3-\frac{1}{2}\right)^2+\left(3-\frac{1}{2}\right)^2}-\frac{1}{\sqrt{2}}$

$=\frac{5}{\sqrt{2}}-\frac{1}{\sqrt{2}}=2\sqrt{2}$

68. Answer (5)




$|z-2-2i|\le 1$

$\Rightarrow$ $z$ lies inside the circle with centre at $2+2i$ and radius = 1, as shown in figure.

$|3iz+6|=|3i|\left|z+\frac{6}{3i}\right|$

$=3|z-2i|$

This is distance of $z$ from $2i$

Hence for maximum value $z=3+2i$ (Refer figure)

Hence $a+b=5$

69. Answer (3)

$x^2-x+1=0$

Roots are $-\omega,-\omega^2$

Let $\alpha=-\omega$, $\beta=-\omega^2$

$\alpha^{101}+\beta^{107}=(-\omega)^{101}+(-\omega^2)^{107}$

$=-(\omega^{101}+\omega^{214})$

$=-(\omega^2+\omega)$

$=1$

70. Answer (4)

$$x = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$$

Let $\alpha = -1 + i$, $\beta = -1 - i$

$\alpha^{15} + \beta^{15} = (-1 + i)^{15} + (-1 - i)^{15}$

$$= \left(\sqrt{2}\, e^{i\frac{3\pi}{4}}\right)^{15} + \left(\sqrt{2}\, e^{-i\frac{3\pi}{4}}\right)^{15}$$

$$= \left(\sqrt{2}\right)^{15}\left[e^{\frac{i45\pi}{4}} + e^{\frac{-i45\pi}{4}}\right]$$

$$= \left(\sqrt{2}\right)^{15}\left[e^{\frac{i5\pi}{4}} + e^{\frac{-i5\pi}{4}}\right]$$

$$= \left(\sqrt{2}\right)^{15} .\ 2\cos\frac{5\pi}{4} = \frac{-2}{\sqrt{2}}\left(\sqrt{2}\right)^{15}$$

$$= -2\left(\sqrt{2}\right)^{14} = -256$$

71. Answer (80)

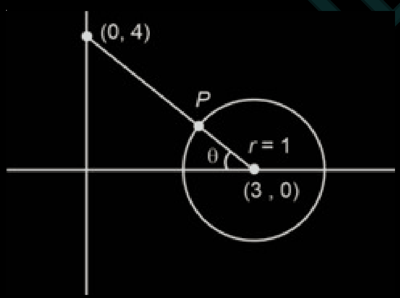
Here $|z - 3| < 1$

$\Rightarrow\ (x - 3)^2 + y^2 < 1$

and $z = (4+3i) + \bar{z}(4-3i) \leq 24$

$\Rightarrow\ 4x - 3y \leq 12$

$\tan\theta = \dfrac{4}{3}$




$\therefore$ Coordinate of $P = (3 - \cos\theta, \sin\theta)$

$$= \left(3 - \frac{3}{5}, \frac{4}{5}\right)$$

$\therefore\ \alpha + i\beta = \dfrac{12}{5} + \dfrac{4}{5}i$

$\therefore\ 25(\alpha + \beta) = 80$

72. Answer (2)

$z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$

Clearly $C \equiv x^2 + y^2 = 25$

Let $z(x, y)$

$$\Rightarrow \left(\frac{y-4}{x-3}\right)\left(\frac{2}{-4}\right) = -1$$

$\Rightarrow\ y = 2x - 2 \equiv L$

$\therefore$ $z$ is intersection of $C$ & $L$

$$\Rightarrow z \equiv \left(\frac{-7}{5}, \frac{-24}{5}\right)$$

$$\therefore\ \operatorname{Arg}(z) = -\pi + \tan^{-1}\left(\frac{24}{7}\right)$$

73. Answer (3)

$$\because\ \frac{z_1}{z_2} = -i \Rightarrow z_1 = -iz_2$$

$$\Rightarrow \arg(z_1) = -\frac{\pi}{2} + \arg(z_2) \quad \ldots\text{(i)}$$

Also $\arg(z_1) - \arg(\bar{z}_2) = \pi$

$\Rightarrow\ \arg(z_1) + \arg(z_2) = \pi \quad \ldots\text{(ii)}$

From (i) and (ii), we get

$$\arg(z_1) = \frac{\pi}{4} \text{ and } \arg(z_2) = \frac{3\pi}{4}$$

74. Answer (2)

$\because\ z^2 + z + 1 = 0 \qquad \Rightarrow\ \omega$ or $\omega^2$

$$\because\ \left|\sum_{n=1}^{15}\left(z^n + (-1)^n \frac{1}{z^n}\right)^2\right|$$

$$= \left|\sum_{n=1}^{15} z^{2n} + \sum_{n=1}^{15} z^{-2n} + 2\cdot\sum_{n=1}^{15}(-1)^n\right|$$

$= |0 + 0 - 2|$

$= 2$

75. Answer (1)

$\bar{z} = iz^2$

Let $z = x + iy$

$x - iy = i(x^2 - y^2 + 2xiy)$

$x - iy = i(x^2 - y^2) - 2xy$

$\therefore\qquad x = -2yx$ or $x^2 - y^2 = -y$

$$x = 0 \quad \text{or} \quad y = -\frac{1}{2}$$

**Case-I**

$x = 0$

$-y^2 = -y$

$y = 0, 1$

**Case-II**

$$\Rightarrow \qquad x^2 - \frac{1}{4} = \frac{1}{2} \quad \Rightarrow x = \pm\frac{\sqrt{3}}{2}$$

$$z = \left\{0,\ i,\ \frac{\sqrt{3}}{2} - \frac{i}{2},\ \frac{-\sqrt{3}}{2} - \frac{i}{2}\right\}$$

$$y = -\frac{1}{2}$$

$$\text{Area of polygon} = \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} & 1 \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 1 \end{vmatrix}$$

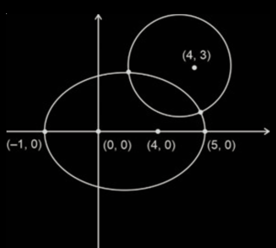
$$= \frac{1}{2}\left|-\sqrt{3} \quad -\frac{\sqrt{3}}{2}\right| = \frac{3\sqrt{3}}{4}$$

76. Answer (3)

$C_1 : |z - (4 + 3i)| = 2$ and $C_2 : |z| + |z - 4| = 6$, $z \in C$

$C_1$: represents a circle with centre (4, 3) and radius 2 and $C_2$ represents a ellipse with focii at (0, 0) and (4, 0) and length of major axis = 6, and length of semi-major axis $2\sqrt{5}$ and (4, 2) lies inside the both $C_1$ and $C_2$ and (4, 3) lies outside the $C_2$

$\therefore$




So, number of intersection points = 2

77. Answer (2)

Given $a = \alpha - i\beta$ and

$4ix + (1 + i)y = 0$ ...(i)

$8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$ .....(ii)

By (i)

$\frac{x}{y} = \frac{-(1+i)}{4i}$ ...(iii)

By (ii)

$\frac{x}{y} = \frac{-\bar{a}}{8\left(\frac{-1}{2} + \frac{\sqrt{3}i}{2}\right)}$ ...(iv)

Now by (iii) and (iv)

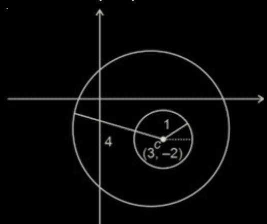
$$\frac{1+i}{4i} = \frac{\bar{a}}{4\left(-1+\sqrt{3}i\right)}$$

$$\Rightarrow \quad \bar{a} = \left(\sqrt{3}-1\right) + \left(\sqrt{3}+1\right)i$$

$$\Rightarrow \quad \alpha + i\beta = \left(\sqrt{3}-1\right) + \left(\sqrt{3}+1\right)i$$

$$\therefore \qquad \frac{\alpha}{\beta} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$$

78. Answer (40)




at line $y = -2$, we have (5, –2) (6, –2) (1, –2) (0, –2) $\Rightarrow$ 4 points

at line $y = -1$, we have (4, –1) (5, –1) (6, –1) (2, –1) (1, –1) (0, –1) $\Rightarrow$ 6 points

at line $y = 0$, we have (0, 0) (1, 0) (2, 0) (3, 0) (4, 0) (5, 0) (6, 0) $\Rightarrow$ 7 points

at line $y = 1$, we have (1, 1), (2, 1), (3, 1), (4, 1), (5, 1) i.e. 5 points

symmetrically

at line $y = -5$, we have 5 points

at line $y = -4$, we have 7 points

at line $y = -3$, we have 6 points

So Total integral points = 2(5 + 7 + 6) + 4

= 40

79. Answer (1)

$x^2 + 2i - 1 = 0$

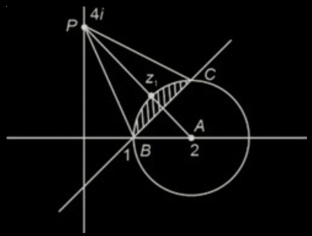
$\alpha^2 = \beta^2 = 1 - 2i$

$\alpha^4 = (1 - 2i)^2 = 1 + (2i)^2 - 4i = -3 - 4i$

$\alpha^8 = (-3 - 4i)^2 = 9 - 16 + 24i = -7 + 24i$

$$\left|\alpha^8 + \beta^8\right| = 2\left|-7 + 24i\right| = 2\sqrt{(-7)^2 + (24)^2} = 50$$

80. Answer (26)




$S$ represents the shaded region shown in the diagram.

Clearly $z_1$ will be the point of intersection of $PA$ and given circle.

$PA$ : $2x + y = 4$ and given circle has equation $(x - 2)^2 + y^2 = 1$.

On solving we get

$$z_1 = \left(2 - \frac{1}{\sqrt{5}}\right) + \frac{2}{\sqrt{5}}i \Rightarrow |z_1|^2 = 5 - \frac{4}{\sqrt{5}}$$

$z_2$ will be either $B$ or $C$.

$\because \quad PB = \sqrt{17}$ and $PC = \sqrt{13}$ hence $z_2 = 1$

So $5\left(|z_1|^2 + |z_2|^2\right) = 30 - 4\sqrt{5}$

Clearly $\alpha = 30$ and $\beta = -4 \Rightarrow \alpha + \beta = 26$

81. Answer (1)

$1 + x^2 + x^4 = 0$

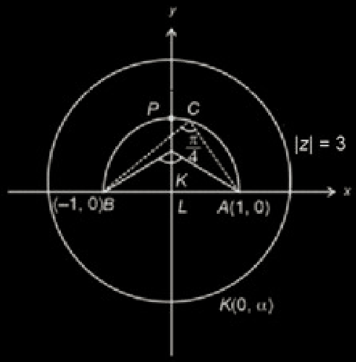
Root is $\omega$(cube root of unity)

$\omega^{1011} + \omega^{2022} - \omega^{3033}$

$= (\omega^3)^{337} + (\omega^3)^{674} - (\omega^3)^{1011}$

$= 1 + 1 - 1 = 1$

82. Answer (3)




$|z| = 3$

$\arg(z - 1) - \arg(z + 1) = \frac{\pi}{4}$

$\angle AKL = \angle ACB = \frac{\pi}{4}$

$\Rightarrow \quad LK = AL = \alpha = 1$

$K(0, 1)$

radius $= \sqrt{2}$

$PL = PK + KL = \sqrt{2} + 1$

$P(0, 1 + \sqrt{2})$

Number of intersection = 0

83. Answer (2)

$x^4 + x^3 + x^2 + x + 1 = 0$ OR $\frac{x^5 - 1}{x - 1} = 0 \ (x \neq 1)$

So roots are $e^{i2\pi/5}, e^{i4\pi/5}, e^{i6\pi/5}, e^{i8\pi/5}$

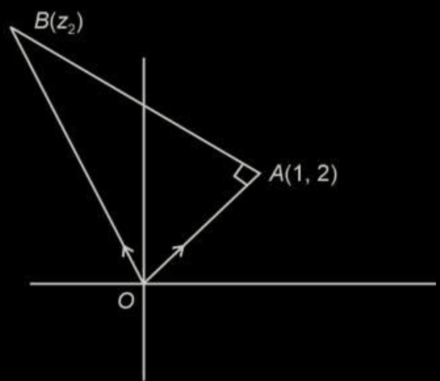
*i.e.* $\alpha$, $\beta$, $\gamma$ and $\delta$

From properties of $n^{th}$ root of unity

$$1^{2021} + \alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} = 0$$

$$\Rightarrow \alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} = -1$$

84. Answer (4)



$$\frac{z_2-0}{(1+2i)-0} = \frac{|OB|}{|OA|}e^{\frac{i\pi}{4}}$$

$$\Rightarrow \frac{z_2}{1+2i} = \sqrt{2}e^{\frac{i\pi}{4}}$$

OR $z_2 = (1 + 2i)(1 + i)$

$= -1 + 3i$

$\arg z_2 = \pi - \tan^{-1}3$

$|z_2| = \sqrt{10}$

$z_1 - 2z_2 = (1 + 2i) + 2 - 6i = 3 - 4i$

$$\arg(z_1 - 2z_2) = -\tan^{-1}\frac{4}{3}$$

$|2z_1 - z_2| = |2 + 4i + 1 - 3i| = |3 + i| = \sqrt{10}$

85. Answer (1)

Let $z = x + iy$

$v = x^2 + y^2 + (x-3)^2 + y^2 + x^2 + (y-6)^2$

$= (3x^2 - 6x + 9) + (3y^2 - 12y + 36)$

$= 3(x^2 + y^2 - 2x - 4y + 15)$

$= 3[(x-1)^2 + (y-2)^2 + 10]$

$v_{\min}$ at $z = 1 + 2i = z_0$ and $v_0 = 30$

so $|2(1 + 2i)^2 - (1 - 2i)^3 + 3|^2 + 900$

$= |2(-3 + 4i) - (1 - 8i^3 - 6i(1 - 2i) + 3|^2 + 900$

$= |-6 + 8i - (1 + 8i - 6i - 12) + 3|^2 + 900$

$= |8 + 6i|^2 + 900$

$= 1000$

86. Answer (0)

$\because \; z^2 + \bar{z} = 0$ Let $z = x + iy$

$\therefore \; x^2 - y^2 + 2ixy + x - iy = 0$

$(x^2 - y^2 + x) + i(2xy - y) = 0$

$\therefore \; x^2 + y^2 = 0$ and $(2x - 1)y = 0$

if $x = +\frac{1}{2}$ then $y = \pm\frac{\sqrt{3}}{2}$

And if $y = 0$ then $x = 0, -1$

$\therefore \; z = 0 + 0i, -1 + 0i, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$

$\therefore \; \sum(R_e(z) + m(z)) = 0$

87. Answer (3)

$\because \; \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely imaginary

$$\therefore \; \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha} + \frac{1 + i\sin\alpha}{1 - 2i\sin\alpha} = 0$$

$\Rightarrow \; 1 - 2\sin^2\alpha = 0$

$$\therefore \; \alpha = \frac{5\pi}{4}, \frac{7\pi}{4}$$

and $\frac{1 + i\cos\beta}{1 - 2i\cos\beta}$ is purely real

$$\frac{1 + i\cos\beta}{1 - 2i\cos\beta} - \frac{1 - i\cos\beta}{1 + 2i\cos\beta} = 0$$

$\Rightarrow \; \cos\beta = 0$

$$\therefore \; \beta = \frac{3\pi}{2}$$

$$\therefore \; S = \left\{\left(\frac{5\pi}{4}, \frac{3\pi}{2}\right), \left(\frac{7\pi}{4}, \frac{3\pi}{2}\right)\right\}$$

$Z_{\alpha\beta} = 1 - i$ and $Z_{\alpha\beta} = -1 - i$

$$\therefore \; \sum_{(\alpha,\beta)\in s}\left(iZ_{\alpha\beta} + \frac{1}{i\bar{Z}_{\alpha\beta}}\right) = i(-2i) + \frac{1}{i}\left[\frac{1}{1+i} + \frac{1}{-1+i}\right]$$

$$= 2 + \frac{1}{i}\frac{2i}{-2} = 1$$

88. Answer (3)

$$\because \; \left|z_2 + |z_2 - 1|\right|^2 = \left|z_2 - |z_2 + 1|\right|^2$$

$$\Rightarrow \; \left(z_2 + |z_2 - 1|\right)\left(\bar{z}_2 + |z_2 - 1|\right) = \left(z_2 - |z_2 + 1|\right)\left(\bar{z}_2 - |z_2 + 1|\right)$$

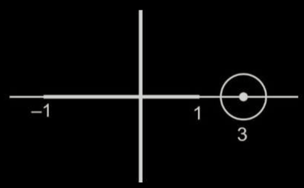
$$\Rightarrow \; z_2\left(|z_2 - 1| + |z_2 + 1|\right) + \bar{z}_2\left(|z_2 - 1| + |z_2 + 1|\right) = |z_2 + 1|^2 - |z_2 - 1|^2$$

$$\Rightarrow \; (z_2 + \bar{z}_2)\left(|z_2 + 1| + |z_2 - 1|\right) = 2(z_2 + \bar{z}_2)$$

$\Rightarrow$ Either $z_2 + \bar{z}_2 = 0$ *or* $|z_2 + 1| + |z_2 - 1| = 2$

So, $z_2$ lies on imaginary axis or on real axis within $[-1, 1]$

Also $|z_1 - 3| = \frac{1}{2} \Rightarrow z_1$ lies on the circle having center 3 and radius $\frac{1}{2}$.




Clearly $|z_1 - z_2|_{\min} = \frac{3}{2}$

89. Answer (6)

$\because \quad z^2 = \bar{z} \cdot 2^{1-|z|} \qquad ...(1)$

$\Rightarrow \quad |z|^2 = |\bar{z}| \cdot 2^{1-|z|}$

$\Rightarrow \quad |z| = 2^{1-|z|}, \quad \because b \neq 0 \Rightarrow |z| \neq 0$

$\therefore \quad |z| = 1 \qquad ...(2)$

$\because \quad z = a + ib$ then $\sqrt{a^2 + b^2} = 1 \qquad ...(3)$

Now again from equation (1), equation (2), equation (3) we get :

$a^2 - b^2 + i2ab = (a - ib)\, 2^0$

$\therefore \quad a^2 - b^2 = a$ and $2ab = -b$

$\therefore \quad a = -\frac{1}{2}$ and $b = \pm\frac{\sqrt{3}}{2}$

$\therefore \quad z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ or $z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

$$z^n = (z+1)^n \Rightarrow \left(\frac{z+1}{z}\right)^n = 1$$

$$\left(1 + \frac{1}{z}\right)^n = 1$$

$\left(\frac{1+\sqrt{3}i}{2}\right)^n = 1$, then minimum value of $n$ is 6.

90. Answer (4)

$$\left|z - \frac{1}{z}\right| \geq \left||z| - \frac{1}{z}\right|$$

$$\Rightarrow \quad \left||z| - \frac{1}{|z|}\right| \leq 2$$

Let $|z| = r$

$$\left|r - \frac{1}{r}\right| \leq 2$$

$$-2 \leq r - \frac{1}{r} \leq 2$$

$r - \frac{1}{r} \geq -2$ and $r - \frac{1}{r} \leq 2$

$r^2 + 2r - 1 \geq 0$ and $r^2 - 2r - 1 \leq 0$

$r \in \left[-\infty, -1-\sqrt{2}\right] \cup \left[-1+\sqrt{2}, \infty\right]$ and

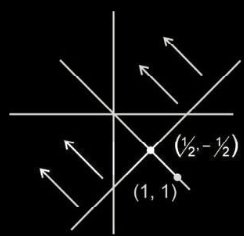
$r \in \left[1-\sqrt{2}, 1+\sqrt{2}\right]$

Taking intersection $r \in \left[\sqrt{2}-1, \sqrt{2}+1\right]$

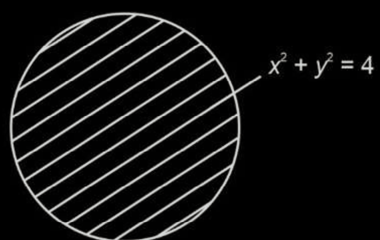
91. Answer (2)

$S : \{z = x + iy : |z - 1 + i| \geq |z|, |z| < 2, |z - i| = |z - 1|\}$

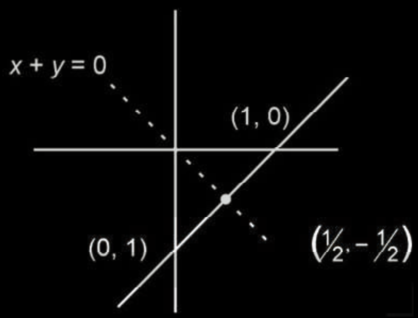
$|z - 1 + i| \geq |z|$

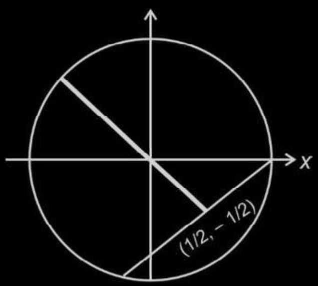



$|z| < 2$



$|z - i| = |z - 1|$




$\because \quad w \in S$ and $w = 2x + iy$

$2x < \dfrac{1}{2} \qquad \therefore \quad x < \dfrac{1}{4}$

$(2x)^2 + (-2x)^2 < 4$

$4x^2 + 4x^2 < 4$

$x^2 < \dfrac{1}{2} \Rightarrow x \in \left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

$\therefore \quad x \in \left(-\dfrac{1}{2}, \dfrac{1}{4}\right]$

92. Answer (1)

$z = (2 + 3i)$

$\Rightarrow \quad z^5 = (2+3i)\left((2+3i)^2\right)^2$

$= (2+3i)(-5+12i)^2$

$= (2+3i)(-119-120i)$

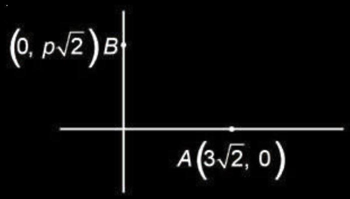
$= -238 - 240i - 357i + 360$

$= 122 - 597i$

$\bar{z}^5 = 122 + 597i$

$z^5 + \bar{z}^5 = 244$

93. Answer (3)




It is sum of distance of $z$ from $\left(3\sqrt{2}, 0\right)$ and $\left(0, p\sqrt{2}\right)$

For minimising, $z$ should lie on $AB$ and $AB = 5\sqrt{2}$

$(AB)^2 = 18 + 2p^2$

$p = \pm 4$

94. Answer (3)

$|z - i| = |z + 5i|$

So, $z$ lies on $\perp^r$ bisector of (0, 1) and (0, –5)

i.e., line $y = -2$

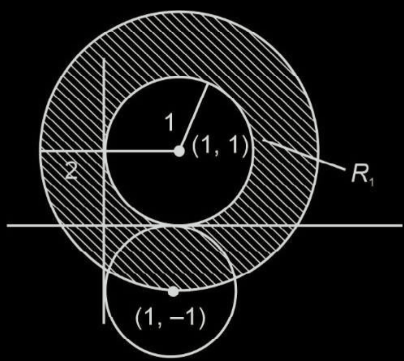
as $|z| = 2$

$\Rightarrow \quad z = -2i$

$x = 0$ and $y = -2$

so, $x + 2y + 4 = 0$

95. Answer (4)

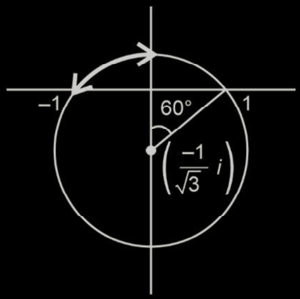



Set $A$ represents region 1 *i.e.* $R_1$ and clearly set $B$ has infinite points in it.

96. Answer (2)

$\left|\dfrac{z+1}{z-1}\right| < 1 \Rightarrow |z+1| < |z-1| \Rightarrow \text{Re}(z) < 0$

and $\arg\left(\dfrac{z-1}{z+1}\right) = \dfrac{2\pi}{3}$ is a part of circle as shown.




97. Answer (2)

Let $z = x + iy$

So $2x = (1 + i)(x^2 - y^2 + 2xyi)$

$\Rightarrow \quad 2x = x^2 - y^2 - 2xy \qquad$ ...(i) and

$\quad x^2 - y^2 + 2xy = 0 \qquad$ ...(ii)

From (i) and (ii) we get

$x = 0 \quad \text{or} \quad y = -\frac{1}{2}$

When $x = 0$ we get $y = 0$

When $y = -\frac{1}{2}$ we get $x^2 - x - \frac{1}{4} = 0$

$\Rightarrow \quad x = \frac{-1 \pm \sqrt{2}}{2}$

So there will be total 3 possible values of $z$, which are $0, \left(\frac{-1+\sqrt{2}}{2}\right) - \frac{1}{2}i$ and $\left(\frac{-1-\sqrt{2}}{2}\right) - \frac{1}{2}i$

Sum of squares of modulus

$$= 0 + \left(\frac{\sqrt{2}-1}{2}\right)^2 + \frac{1}{4} + \left(\frac{\sqrt{2}+1}{2}\right)^2 = +\frac{1}{4}$$

$= 2$

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