# Chapter 18

# Limits

1. Let $f : R \to R$ be a positive increasing function with $\lim\limits_{x\to\infty} \dfrac{f(3x)}{f(x)} = 1$. Then $\lim\limits_{x\to\infty} \dfrac{f(2x)}{f(x)} =$ **[AIEEE-2010]**

(1) 1 (2) $\dfrac{2}{3}$

(3) $\dfrac{3}{2}$ (4) 3

2. Let $f : R \to [0, \infty)$ be such that $\lim\limits_{x\to 5} f(x)$ exists and $\lim\limits_{x\to 5} \dfrac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0$. Then $\lim\limits_{x\to 5} f(x)$ equals

**[AIEEE-2011]**

(1) 2 (2) 3

(3) 0 (4) 1

3. $\lim\limits_{x\to 0} \dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

**[JEE (Main)-2013]**

(1) $-\dfrac{1}{4}$ (2) $\dfrac{1}{2}$

(3) 1 (4) 2

4. $\lim\limits_{x\to 0} \dfrac{\sin(\pi\cos^2 x)}{x^2}$ is equal to **[JEE (Main)-2014]**

(1) $-\pi$ (2) $\pi$

(3) $\dfrac{\pi}{2}$ (4) 1

5. $\lim\limits_{x\to 0} \dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

**[JEE (Main)-2015]**

(1) 4 (2) 3

(3) 2 (4) $\dfrac{1}{2}$

6. Let $p = \lim\limits_{x\to 0+} \left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log $p$ is equal to

**[JEE (Main)-2016]**

(1) 1 (2) $\dfrac{1}{2}$

(3) $\dfrac{1}{4}$ (4) 2

7. $\lim\limits_{n\to\infty} \left(\dfrac{(n+1)(n+2)\ldots 3n}{n^{2n}}\right)^{1/n}$ is equal to

**[JEE (Main)-2016]**

(1) $\dfrac{27}{e^2}$ (2) $\dfrac{9}{e^2}$

(3) 3 log3 – 2 (4) $\dfrac{18}{e^4}$

8. $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\cot x - \cos x}{(\pi - 2\pi)^3}$ equals **[JEE (Main)-2017]**

(1) $\dfrac{1}{16}$ (2) $\dfrac{1}{8}$

(3) $\dfrac{1}{4}$ (4) $\dfrac{1}{24}$

9. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to $t$. Then **[JEE (Main)-2018]**

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots\ldots+\left[\frac{15}{x}\right]\right)$$

(1) Is equal to 0

(2) Is equal to 15

(3) Is equal to 120

(4) Does not exist (in $R$)

10. $\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$ **[JEE (Main)-2019]**

(1) Exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

(2) Does not exist

(3) Exists and equals $\frac{1}{4\sqrt{2}}$

(4) Exists and equals $\frac{1}{2\sqrt{2}}$

11. For each $x \in R$, let $[x]$ be the greatest integer less than or equal to $x$. Then $\lim_{x\to 0^-}\frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to **[JEE (Main)-2019]**

(1) –sin1 (2) 1

(3) sin1 (4) 0

12. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to $t$. Then,

$$\lim_{x\to 1+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{x}{2}[1-x]\right)}{|1-x|[1-x]}$$

**[JEE (Main)-2019]**

(1) Equals 0 (2) Equals 1

(3) Equals –1 (4) Does not exist

13. Let $[x]$ denote the greatest integer less than or equal to $x$. Then

$$\lim_{x\to 0}\frac{\tan(\pi\sin^2 x)+(|x|-\sin(x[x]))^2}{x^2}$$

**[JEE (Main)-2019]**

(1) Equals 0 (2) Equals $\pi + 1$

(3) Equals $\pi$ (4) Does not exist

14. $\lim_{x\to 0}\frac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to

**[JEE (Main)-2019]**

(1) 2 (2) 4

(3) 1 (4) 0

15. $\lim_{x\to\frac{\pi}{4}}\frac{\cot^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ is **[JEE (Main)-2019]**

(1) $8\sqrt{2}$ (2) 4

(3) $4\sqrt{2}$ (4) 8

16. $\lim_{x\to 1}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ is equal to **[JEE (Main)-2019]**

(1) $\sqrt{\frac{2}{\pi}}$ (2) $\sqrt{\frac{\pi}{2}}$

(3) $\sqrt{\pi}$ (4) $\frac{1}{\sqrt{2\pi}}$

17. $\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals **[JEE (Main)-2019]**

(1) $\sqrt{2}$ (2) $2\sqrt{2}$

(3) 4 (4) $4\sqrt{2}$

18. Let $f : R \to R$ be a differentiable function satisfying $f'(3) + f'(2) = 0$. Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to **[JEE (Main)-2019]**

(1) $e$ (2) 1

(3) $e^2$ (4) $e^{-1}$

19. If $f(x) = [x] - \left[\frac{x}{4}\right]$, $x \in R$, where $[x]$ denotes the greatest integer function, then **[JEE (Main)-2019]**

(1) $\lim_{x\to 4+}f(x)$ exists but $\lim_{x\to 4-}f(x)$ does not exist

(2) $f$ is continuous at $x = 4$

(3) $\lim_{x\to 4-}f(x)$ exists but $\lim_{x\to 4+}f(x)$ does not exist

(4) Both $\lim_{x\to 4-}f(x)$ and $\lim_{x\to 4+}f(x)$ exist but are not equal

20. If $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then $k$ is

**[JEE (Main)-2019]**

(1) $\frac{4}{3}$ (2) $\frac{3}{2}$

(3) $\frac{8}{3}$ (4) $\frac{3}{8}$

21. If $\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5$, then $a + b$ is equal to

**[JEE (Main)-2019]**

(1) 5 (2) –4

(3) 1 (4) –7

22. $\lim\limits_{x\to 0}\dfrac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is **[JEE (Main)-2019]**

(1) 3 (2) 6

(3) 1 (4) 2

23. Let $f(x) = 5 - |x - 2|$ and $g(x) = |x + 1|$, $x \in R$. If f(x) attains maximum value at $\alpha$ and $g(x)$ attains minimum value at $\beta$, then

$\lim\limits_{x\to -\alpha\beta}\dfrac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to

**[JEE (Main)-2019]**

(1) 1/2 (2) –1/2

(3) –3/2 (4) 3/2

24. $\lim\limits_{x\to 0}\left(\dfrac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to **[JEE (Main)-2020]**

(1) e (2) $\dfrac{1}{e}$

(3) $\dfrac{1}{e^2}$ (4) $e^2$

25. $\lim\limits_{x\to 0}\left(\tan\left(\dfrac{\pi}{4}+x\right)\right)^{\frac{1}{x}}$ is equal to

(1) $e^2$ (2) 1

(3) $e$ (4) 2

26. Let $[t]$ denote the greatest integer $\le t$. If for some $\lambda \in R - \{0, 1\}$, $\lim\limits_{x\to 0}\left|\dfrac{1-x+|x|}{\lambda-x+[x]}\right| = L$, then $L$ is equal to **[JEE (Main)-2020]**

(1) 2 (2) $\dfrac{1}{2}$

(3) 0 (4) 1

27. $\lim\limits_{x\to a}\dfrac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}$ $(a \ne 0)$ is equal to

**[JEE (Main)-2020]**

(1) $\left(\dfrac{2}{9}\right)\left(\dfrac{2}{3}\right)^{\frac{1}{3}}$ (2) $\left(\dfrac{2}{3}\right)\left(\dfrac{2}{9}\right)^{\frac{1}{3}}$

(3) $\left(\dfrac{2}{3}\right)^{\frac{4}{3}}$ (4) $\left(\dfrac{2}{9}\right)^{\frac{4}{3}}$

28. Let $f : (0, \infty) \to (0, \infty)$ be a differentiable function such that $f(1) = e$ and

$\lim\limits_{t\to x}\dfrac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$

If $f(x) = 1$, then $x$ is equal to **[JEE (Main)-2020]**

(1) $2e$ (2) $e$

(3) $\dfrac{1}{2e}$ (4) $\dfrac{1}{e}$

29. If $\alpha$ is the positive root of the equation, $p(x) = x^2 - x - 2 = 0$, then $\lim\limits_{x\to\alpha^+}\dfrac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to

**[JEE (Main)-2020]**

(1) $\dfrac{1}{\sqrt{2}}$ (2) $\dfrac{1}{2}$

(3) $\dfrac{3}{\sqrt{2}}$ (4) $\dfrac{3}{2}$

30. $\lim\limits_{x\to 0}\dfrac{x\left(e^{(\sqrt{1+x^2+x^4}-1)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$ **[JEE (Main)-2020]**

(1) Is equal to 0 (2) Is equal to $\sqrt{e}$

(3) Is equal to 1 (4) Does not exist

31. $\lim\limits_{x\to 1}\left(\dfrac{\int_0^{(x-1)^2} t\cos(t^2)dt}{(x-1)\sin(x-1)}\right)$ **[JEE (Main)-2020]**

(1) Does not exist (2) Is equal to 0

(3) Is equal to 1 (4) Is equal to $\dfrac{1}{2}$

32. $\lim\limits_{x\to 2}\dfrac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}}$ is equal to ________.

**[JEE (Main)-2020]**

33. If $\lim\limits_{x\to 1}\dfrac{x+x^2+x^3+...+x^n-n}{x-1}=820$, $(n \in N)$ then the value of $n$ is equal to ________

**[JEE (Main)-2020]**

34. If $\lim\limits_{x\to 0}\left\{\dfrac{1}{x^8}\left(1-\cos\dfrac{x^2}{2}-\cos\dfrac{x^2}{4}+\cos\dfrac{x^2}{2}\cos\dfrac{x^2}{4}\right)\right\}=2^{-k}$, then the value of $k$ is ________.

**[JEE (Main)-2020]**

35. If $\lim\limits_{x\to 0}\dfrac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$ exists and is equal to b, then the value of a –2b is ________.

**[JEE (Main)-2021]**

36. The value of $\lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$ is equal to **[JEE (Main)-2021]**

(1) 4 (2) –4

(3) –1 (4) 0

37. If α, β are the distinct roots of $x^2 + bx + c = 0$, then $\lim_{x\to\beta}\frac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$ is equal to : **[JEE (Main)-2021]**

(1) $b^2 - 4c$ (2) $b^2 + 4c$

(3) $2(b^2 + 4c)$ (4) $2(b^2 - 4c)$

38. If $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair $(a, b)$ is **[JEE (Main)-2021]**

(1) $\left(1, \frac{1}{2}\right)$ (2) $\left(-1, -\frac{1}{2}\right)$

(3) $\left(-1, \frac{1}{2}\right)$ (4) $\left(1, -\frac{1}{2}\right)$

39. $\lim_{x\to 0}\frac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$ is equal to : **[JEE (Main)-2021]**

(1) $4\pi$ (2) $\pi^2$

(3) $4\pi^2$ (4) $2\pi^2$

40. Let $a$ be an integer such that $\lim_{x\to 7}\frac{18-[1-x]}{[x-3a]}$ exists, where $[t]$ is greatest integer $\le t$. Then $a$ is equal to : **[JEE (Main)-2022]**

(1) –6 (2) –2

(3) 2 (4) 6

41. Let $[t]$ denote the greatest integer $\le t$ and $\{t\}$ denote the fractional part of $t$. The integral value of α for which the left hand limit of the function

$f(x) = [1 + x] + \frac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$ at $x = 0$ is

equal to $\alpha-\frac{4}{3}$, is __________.

**[JEE (Main)-2022]**

42. If $\lim_{x\to 1}\frac{\sin(3x^2-4x+1)-x^2+1}{2x^3-7x^2+ax+b}=-2$, then the value of $(a - b)$ is equal to _______.

**[JEE (Main)-2022]**

43. The value of $\lim_{x\to 1}\frac{\left(x^2-1\right)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to

**[JEE (Main)-2022]**

(1) $\frac{\pi^2}{6}$ (2) $\frac{\pi^2}{3}$

(3) $\frac{\pi^2}{2}$ (4) $\pi^2$

44. $\lim_{x\to 0}\left(\frac{(x+2\cos x)^3+2(x+2\cos x)^2+3\sin(x+2\cos x)}{(x+2)^3+2(x+2)^2+3\sin(x+2)}\right)^{\frac{100}{x}}$ is equal to _________.

**[JEE (Main)-2022]**

45. $\lim_{x\to\frac{\pi}{4}}\frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$ is equal to

(1) 14 (2) 7

(3) $14\sqrt{2}$ (4) $7\sqrt{2}$

**[JEE (Main)-2022]**

46. $\lim_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+.....+\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$

is equal to

(1) $\frac{1}{2}$ (2) 1

(3) 2 (4) –2

**[JEE (Main)-2022]**

47. $\lim_{x\to\frac{\pi}{2}}\tan^2 x\left(\left(2\sin^2 x+3\sin x+4\right)^{\frac{1}{2}}-\left(\sin^2 x+6\sin x+2\right)^{\frac{1}{2}}\right)$ is equal to

**[JEE (Main)-2022]**

(1) $\frac{1}{12}$ (2) $-\frac{1}{18}$

(3) $-\frac{1}{12}$ (4) $\frac{1}{6}$

48. $\lim\limits_{x\to\frac{1}{\sqrt{2}}}\dfrac{\sin\left(\cos^{-1}x\right)-x}{1-\tan\left(\cos^{-1}x\right)}$ is equal to :

**[JEE (Main)-2022]**

(1) $\sqrt{2}$

(2) $-\sqrt{2}$

(3) $\dfrac{1}{\sqrt{2}}$

(4) $-\dfrac{1}{\sqrt{2}}$

49. $\lim\limits_{x\to 0}\dfrac{\cos(\sin x)-\cos x}{x^4}$ is equal to:

**[JEE (Main)-2022]**

(1) $\dfrac{1}{3}$

(2) $\dfrac{1}{4}$

(3) $\dfrac{1}{6}$

(4) $\dfrac{1}{12}$

50. If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$, then $8(\alpha+\beta)$ is equal to

**[JEE (Main)-2022]**

(1) 4

(2) –8

(3) –4

(4) 8

51. If $\lim\limits_{x\to 0}\dfrac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}=\dfrac{2}{3}$, where $\alpha, \beta, \gamma \in$ **R**, then which of the following is **NOT** correct?

**[JEE (Main)-2022]**

(1) $\alpha^2 + \beta^2 + \gamma^2 = 6$

(2) $\alpha\beta + \beta\gamma + \gamma\alpha + 1 = 0$

(3) $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 0$

(4) $\alpha^2 - \beta^2 + \gamma^2 = 4$

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# Chapter 18

# Limits

1. Answer (1)

We have,

$f : R \to R$

$$\lim_{x\to\infty}\frac{f(3x)}{f(x)} = 1$$

$$\frac{f(2x)}{f(x)} = \frac{f(2x)}{f\left(\frac{2}{3}x\right)}\cdot\frac{f\left(\frac{2}{3}x\right)}{f(x)}$$

$$= \frac{f(2x)}{f\left(\frac{2}{3}x\right)}\cdot\frac{1}{\frac{f(x)}{f\left(\frac{x}{3}\right)}}\cdot\frac{f\left(\frac{x}{3}\right)}{f\left(\frac{2x}{3}\right)}$$

Taking limit $x \to \infty$ and $\lim_{x\to\infty}\frac{f(2x)}{f(x)} = l$

We find that,

$$l = 1\times\frac{1}{1}\times\frac{1}{l}$$

$\Rightarrow l^2 = 1 \Rightarrow l = 1.$

2. Answer (2)

$f : R \to (0, \infty)$

$$\lim_{x\to5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}$$

For existance of limit above form must be $\left(\frac{0}{0}\right)$

This is possible if and only if

$$\lim_{x\to5} f(x) = 3$$

Applying L' Hospital rule.

$$\lim_{x\to5}\frac{2.f(x).f'(x)}{\frac{1}{2}\frac{1}{\sqrt{|x-5|}}\cdot\frac{|x-5|}{x-5}} \quad \because\left(\frac{d}{dx}|x| = \frac{|x|}{x}\right)$$

$$\lim_{x\to5}\frac{4f(x).f'(x)\sqrt{|x-5|}}{\frac{|x-5|}{x-5}}$$

Now R.H.L = $\lim_{x\to5^+}\frac{4f(x).f'(x)}{1}\sqrt{x-5} = 0$

L.H.L = $\lim_{x\to5^-}\frac{4f(x)f'(x)\sqrt{5-x}}{-1} = 0$

3. Answer (4)

$$\lim_{x\to0}\frac{2\sin^2 x\cdot(3+\cos x)}{4x^2\cdot\frac{\tan 4x}{4x}}$$

$$= \frac{2\times4}{4} = 2$$

4. Answer (2)

$$\lim_{x\to0}\frac{\sin(\pi\cos^2 x)}{x^2}$$

$$= \lim_{x\to0}\frac{\sin(\pi(1-\sin^2 x)}{x^2}$$

$$= \lim_{x\to0}\sin\frac{(\pi-\pi\sin^2 x)}{x^2}$$

$$= \lim_{x\to0}\sin\frac{(\pi\sin^2 x)}{x^2} \qquad [\because \sin(\pi-\theta) = \sin\theta]$$

$$= \lim_{x\to0}\sin\frac{(\pi\sin^2 x)}{(\pi\sin^2 x)}\times\frac{\pi\sin^2 x}{x^2}$$

$$= \lim_{x\to0} 1\times\pi\left(\frac{\sin x}{x}\right)^2 = \pi$$

5. Answer (3)

$$\lim_{x\to\infty}\frac{2\sin^2 x\cdot(3+\cos x)}{x^2\frac{\tan 4x}{4x}\times4x}\times\frac{x^2}{x} = 2$$

6. Answer (2)

$$p = \lim_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$$

$$= e^{\lim_{x\to 0} \frac{1}{2x} \tan^2 \sqrt{x}}$$

$$= e^{\lim_{x\to 0} \frac{1}{2}\left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)^2} = e^{\frac{1}{2}}$$

$$\log p = \frac{1}{2}$$

7. Answer (1)

$$p = \lim_{n\to\infty}\left[\frac{(n+1)(n+2)(n+3)....(n+2n)}{n.\,n.\,.....n}\right]^{\frac{1}{n}}$$

$$\log p = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{2n} \log\left(\frac{n+r}{n}\right)$$

$$= \int_0^2 \log(1+x)dx$$

$$= \left[\log(1+x)dx\right]_0^2 - \int_0^2 \frac{1.x}{1+x}dx$$

$$= 2\log 3 - \left[\int_0^2 \left(1 - \frac{1}{1+x}\right)dx\right]$$

$$= 2\log 3 - \left[x - \log(1+x)\right]_0^2$$

$= 2\log3 - (2 - \log3]$

$\log p \; = 3\log 3 - 2$

$$p = e^{3\log 3 - 2} = \frac{e^{\log 27}}{e^2} = \frac{27}{e^2}$$

8. Answer (1)

$$\lim_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$$

Put, $\frac{\pi}{2} - x = t$

$$\lim_{t\to 0}\frac{\tan t - \sin t}{8t^3}$$

$$= \lim_{t\to 0}\frac{\sin t \cdot 2\sin^2 \frac{t}{2}}{8t^3}$$

$$= \frac{1}{16}.$$

9. Answer (3)

As $\frac{1}{x} - 1 < \left[\frac{1}{x}\right] \le \frac{1}{x}$

$$\frac{2}{x} - 1 < \left[\frac{2}{x}\right] \le \frac{2}{x}$$

$$\sum_{r=1}^{15}\left(\frac{r}{x} - 1\right) < \sum_{r=1}^{15}\left(\frac{r}{x}\right) \le \sum_{r=1}^{15}\frac{r}{x}$$

$$120 < \lim_{x\to 0^+} x\left(\sum_{r=1}^{15}\left[\frac{r}{x}\right]\right) \le 120$$

$$\Rightarrow \lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ...... + \left[\frac{15}{x}\right]\right) = 120$$

10. Answer (3)

$$\ell = \lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$$

$$= \lim_{y\to 0} \frac{\left(\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}\right)\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)}$$

$$= \lim_{y\to 0} \frac{1+\sqrt{1+y^4} - 2}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)}$$

$$= \lim_{y\to 0} \frac{\left(\sqrt{1+y^4} - 1\right)\left(\sqrt{1+y^4} + 1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)\left(\sqrt{1+y^4} + 1\right)}$$

$$= \lim_{y\to 0} \frac{1+y^4 - 1}{y^4\left(\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}\right)\left(\sqrt{1+y^4} + 1\right)}$$

$$= \frac{1}{2\sqrt{2}\times 2} = \frac{1}{4\sqrt{2}}$$

11. Answer (1)

$$\lim_{x\to 0^-} \frac{x([x] + |x|)\cdot \sin[x]}{|x|}$$

$$= \lim_{h\to 0} \frac{(0-h)([0-h] + |0-h|)\cdot \sin[0-h]}{|0-h|}$$

$$= \lim_{h\to 0} \frac{(-h)(-1+h)\sin(-1)}{h}$$

$$= \lim_{h\to 0}(1-h)\sin(-1)$$

$= -\sin 1$

12. Answer (1)

$$\lim_{x\to1^+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

$$=\lim_{h\to0}\frac{(1-|1+h|+\sin|1-1-h|\sin\frac{\pi}{2}[1-1-h]}{|1-1-h|[1-1-h]}$$

$$=\lim_{h\to0}\frac{(-h+\sin h)\sin\left(-\frac{\pi}{2}\right)}{h(-1)}$$

= 0

13. Answer (4)

$$\lim_{x\to0^+}\frac{\tan(\pi\sin^2 x)+(x-0)^2}{x^2}$$

$$=\lim_{x\to0^+}\left(\frac{\tan(\pi\sin^2 x)}{x^2}+1\right)$$

$= 1 + \pi$

Also, $\lim_{x\to0^-}\frac{\tan(\pi\sin^2 x)+(-x+\sin x)^2}{x^2}$

$$\lim_{x\to0^-}\frac{\tan(\pi\sin^2 x)+x^2+\sin^2 x-2x\sin x}{x^2}$$

$= \pi + 1 + 1 - 2 = \pi$

As LHL $\neq$ RHL

Limit does not exist

14. Answer (3)

$$\lim_{x\to0}\frac{x\cot 4x}{\sin^2 x\cdot\cot^2 2x}$$

$$=\lim_{x\to0}\frac{x\cdot\tan^2 2x}{\sin^2 x\cdot\tan 4x}$$

$$=\lim_{x\to0}\left(\frac{x}{\sin x}\right)^2\cdot\left(\frac{\tan 2x}{2x}\right)^2\cdot\left(\frac{4x}{\tan 4x}\right)\cdot\frac{4}{2^2}$$

= 1

15. Answer (4)

$$\lim_{x\to\frac{\pi}{4}}\frac{\cot^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{(1+\tan^2 x)(1-\tan x)(1+\tan x)}{\tan^3 x\left(\frac{\cos x-\sin x}{\sqrt{2}}\right)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{(1+\tan^2 x)(1+\tan x)(\cos x-\sin x)}{\frac{\sin^3 x}{\cos^2 x}\left(\frac{\cos x-\sin x}{\sqrt{2}}\right)}$$

$$=\frac{(2)(2)}{\frac{1}{(\sqrt{2})(\sqrt{2})}}=8$$

16. Answer (1)

$$\lim_{x\to1^-}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$$

$$=\lim_{h\to0}f(1-h)$$

$$=\lim_{h\to0}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}(1-h)}}{\sqrt{1-(1-h)}}$$

$$=\lim_{h\to0}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}(1-h)}}{\sqrt{h}}$$

$$=\lim_{h\to0}\frac{-\frac{1}{2\sqrt{2\sin^{-1}(1-h)}}\times2\times\frac{1}{\sqrt{1-(1-h)^2}}(-1)}{\frac{1}{2\sqrt{h}}}$$

$$=\lim_{h\to0}\frac{\frac{1}{\sqrt{2\sin^{-1}(1-h)}}\frac{1}{\sqrt{h(2-h)}}}{\frac{1}{2\sqrt{h}}}$$

$$=2\times\frac{1}{\sqrt{\pi}}\times\frac{1}{\sqrt{2}}=\sqrt{\frac{2}{\pi}}$$

17. Answer (4)

$$\lim_{x\to0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}=\lim_{x\to0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{2\cos^2\frac{x}{2}}}\quad\left[\frac{0}{0}\right]$$

$$=\lim_{x\to0}\frac{\sin^2 x}{\sqrt{2}\left[1-\cos\frac{x}{2}\right]}$$

$$=\lim_{x\to0}\frac{\sin^2 x}{2\sqrt{2}\sin^2\frac{x}{4}}$$

$$=\lim_{x\to0}\frac{\left(\frac{\sin x}{x}\right)^2.16}{2\sqrt{2}\left(\frac{\sin\frac{x}{4}}{\frac{x}{4}}\right)^2}$$

$$=\frac{16}{2\sqrt{2}}=4\sqrt{2}$$

18. Answer (2)

$$I = \lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$$

form : $1^{\infty}$

$\Rightarrow$ $I = e^{I1}$, where

$$I_1 = \lim_{x\to 0}\left(\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}-1\right)\frac{1}{x}\right)$$

$$= \lim_{x\to 0}\left(\left(\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right)\frac{1}{x}\right)$$

form : $\frac{0}{0}$

Using L.H. Rule,

$$I_1 = \lim_{x\to 0}\left(\frac{f'(3+x)+f'(2-x)}{1}\right).\lim_{x\to 0}\left(\frac{1}{1+f(2-x)-f(2)}\right)$$

$= f'(3) + f'(2) = 0$

$\Rightarrow$ $I = e^{I1} = 1$

19. Answer (2)

L.H.L $\displaystyle\lim_{x\to 4^-}[x]-\left[\frac{x}{4}\right]=3-0=3$

$$\left(x<4\Rightarrow[x]=3\ \&\ \frac{x}{4}<1\Rightarrow\left[\frac{x}{4}\right]=0\right)$$

R.H.L $\displaystyle\lim_{x\to 4^+}[x]-\left[\frac{x}{4}\right]=4-1=3$

$$\left(x>4\Rightarrow[x]=4\ \&\ \frac{x}{4}>1\Rightarrow\left[\frac{x}{4}\right]=1\right)$$

$$f(4)=[4]-\left[\frac{4}{4}\right]=4-1=3$$

LHL = $f(4)$ = RHL

Hence, $f(x)$ is continuous at $x = 4$

20. Answer (3)

If $\displaystyle\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to K}\left(\frac{x^3-k^3}{x^2-k^2}\right)$

L·H·S·

$$\mathop{\mathrm{Lt}}_{x\to 1}\frac{x^4-1}{x-1}=\left(\frac{0}{0}\text{ form}\right)$$

$$\mathop{\mathrm{Lt}}_{x\to 1}\frac{4x^3}{1}=4$$

Now, $\displaystyle\lim_{x\to K}\frac{x^3-k^3}{x^2-k^2}=4$

$$\Rightarrow \lim_{x\to K}\frac{3x^2}{2x}=4$$

$$\Rightarrow \frac{3}{2}k=4$$

$$k=\frac{8}{3}$$

21. Answer (4)

$$\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5$$

As limit is finite, $1 - a + b = 0$

$$\Rightarrow \lim_{x\to 1}\frac{2x-a}{1}=5 \qquad \left(\frac{0}{0}\text{ form}\right)$$

i.e., $2 - a = 5$

or $a = -3$

$\therefore$ $b = -4$

$a + b = -3 - 4 = -7$

22. Answer (4)

$$\lim_{x\to 0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$$

$$=\lim_{x\to 0}\frac{(x+2\sin x)\left[\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right]}{(x^2-\sin^2 x)+(x+2\sin x)}$$

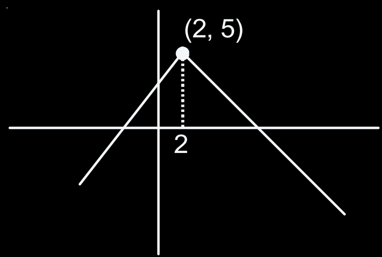
$$=\lim_{x\to 0}\frac{\left[1+2\left(\frac{\sin x}{x}\right)\right]\left[\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right]}{\left(x-\frac{\sin^2 x}{x}\right)+\left(1+2\left(\frac{\sin x}{x}\right)\right)}$$

$$=\frac{3\times 2}{3}=2$$

23. Answer (1)

f($x$) = 5 – |$x$ – 2|

Graph of $y$ = f($x$)

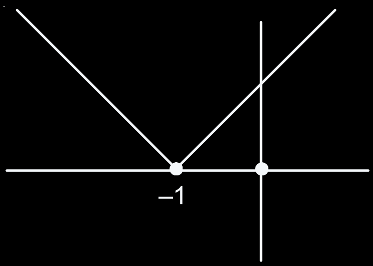



f($x$) is maximum at $x$ = 2

$\alpha = 2$

g($x$) = |$x$ + 1|

Graph of $y = g(x)$




$g(x)$ is minimum at $x = -1$

$\beta = -1$

$$\lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)} = \lim_{x\to 2}\frac{(x-1)(x-3)}{x-4}$$

$$= \frac{1}{2}$$

24. Answer (3)

$$\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$$

$$\Rightarrow \quad e^{\lim_{x\to 0}\left(\frac{3x^2+2-7x^2-2}{7x^2+2}\right)\frac{1}{x^2}}$$

$$\Rightarrow \quad e^{\frac{-4}{2}} = e^{-2}$$

25. Answer (1)

$$\lim_{x\to 0}\left(\frac{1+\tan x}{1-\tan x}\right)^{1/x}$$

$$\Rightarrow \quad e^{\lim_{x\to 0}\left(\frac{1+\tan x-(1-\tan x)}{1-\tan x}\right)\frac{1}{x}}$$

$$\Rightarrow \quad e^{\lim_{x\to 0}\left(\frac{2\tan x}{1-\tan x}\right)\frac{1}{x}} = e^2$$

26. Answer (1)

Here $\lim_{x\to 0}\left|\dfrac{1-x+|x|}{\lambda-x+[x]}\right| = L$

Here L.H.L. $= \lim_{h\to 0}\left|\dfrac{1+h+h}{\lambda+h-1}\right| = \left|\dfrac{1}{\lambda-1}\right|$

R.H.L. $= \lim_{h\to 0}\left|\dfrac{1-h+h}{\lambda+h+0}\right| = \left|\dfrac{1}{\lambda}\right|$

$\because$ Limit exists. Hence L.H.L. = R.H.L.

$\Rightarrow \quad |\lambda - 1| = |\lambda|$

$\Rightarrow \quad \lambda = \dfrac{1}{2}$ and L = 2

27. Answer (2)

The indeterminate form is $\dfrac{0}{0}$

by L′Hospital rule

$$\Rightarrow \quad \lim_{x\to a}\frac{\frac{1}{3}(a+2x)^{-2/3}.2-\frac{1}{3}(3x)^{-2/3}.3}{\frac{1}{3}(3a+x)^{-2/3}-\frac{1}{3}(4x)^{-2/3}.4}$$

Put x = a

$$\Rightarrow \quad \frac{\frac{2}{(3a)^{2/3}}-\frac{3}{(3a)^{2/3}}}{\left(\frac{1}{4a}\right)^{2/3}-\frac{4}{3}\left(\frac{1}{4a}\right)^{2/3}} = \frac{\frac{1}{(3a)^{2/3}}}{\frac{1}{(4a)^{2/3}}}\frac{(-1)}{(-3)}$$

$$\Rightarrow \quad \left(\frac{4}{3}\right)^{2/3}\frac{1}{3}$$

$$\Rightarrow \quad \frac{2}{3}\left(\frac{2}{9}\right)^{1/3}$$

28. Answer (4)

$$\lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x} = 0$$

$$\Rightarrow \quad \lim_{t\to x}\frac{2tf^2(x)-2x^2f(t)\cdot f'(t)}{1} = 0$$

(Using L'Hospital's Rule)

$$\Rightarrow \quad f(x) = xf'(x) \quad \Rightarrow \quad \frac{f'(x)}{f(x)} = \frac{1}{x}$$

Integrating w.r.t x, we get

$\Rightarrow$ ln f(x) = ln x + ln C

$\Rightarrow$ f(x) = Cx

$\because$ f(1) = e

$\Rightarrow$ C = e; so f(x) = ex

When f(x) = 1 = ex

$$\Rightarrow \quad x = \frac{1}{e}$$

29. Answer (3)

Eq. $P(x) = x^2 - x - 2 = 0$ $\Rightarrow$ x = 2, –1

$\Rightarrow$ $\alpha = 2$

Now $\lim_{x\to 2^+}\dfrac{\sqrt{1-\cos(x^2-x-2)}}{x-2}$

$$= \lim_{x\to 2^+}\frac{\sqrt{2}\left|\sin\left(\frac{x^2-x-2}{2}\right)\right|}{x-2}$$

$$= \lim_{x\to 2^+} \frac{\sqrt{2}\,\dfrac{\sin(x^2-x-2)}{2}}{\left(\dfrac{x^2-x-2}{2}\right)} \times \frac{(x^2-x-2)}{2(x-2)}$$

$$= \frac{1}{\sqrt{2}} \lim_{x\to 2^+} \left( \frac{\sin\left(\dfrac{x^2-x-2}{2}\right)}{\dfrac{x^2-x-2}{2}} \right) \times \lim_{x\to 2^+} \frac{(x-2)(x+1)}{(x-2)}$$

$$= \frac{1}{\sqrt{2}} \times 1 \times 3 = \frac{3}{\sqrt{2}}$$

30. Answer (3)

$$\lim_{x\to 0} \frac{x\left(e^{\frac{\sqrt{1+x^2+x^4}-1}{x}}-1\right)}{\sqrt{1+x^2+x^4}-1} = \lim_{x\to 0} \frac{e^{\frac{\sqrt{1+x^2+x^4}-1}{x}}-1}{\dfrac{\sqrt{1+x^2+x^4}-1}{x}}$$

$$= \lim_{x\to 0} \frac{e^{\frac{x+x^3}{1+\sqrt{1+x^2+x^4}}}-1}{\dfrac{x+x^3}{1+\sqrt{1+x^2+x^4}}}$$

$$= \lim_{x\to 0} \frac{1+\dfrac{x+x^3}{1+\sqrt{1+x^2+x^4}}+\dfrac{1}{2!}\left(\dfrac{x+x^3}{1+\sqrt{1+x^2+x^4}}\right)^2+\ldots\infty-1}{\dfrac{x+x^3}{1+\sqrt{1+x^2+x^4}}}$$

$= 1$

31. Answer (2)

$$\lim_{x\to 1} \frac{\int_0^{(x-1)^2} t\cos(t^2)\,dt}{(x-1)\sin(x-1)} = \lim_{x\to 1} \frac{\frac{1}{2}\sin(x-1)^4}{(x-1)\sin(x-1)}$$

$$\therefore \quad \text{L.H.L.} = \lim_{h\to 0} \frac{\frac{1}{2}\sin(h^4)}{h\cdot \sin h} = 0$$

$$\text{R.H.L.} = \lim_{h\to 0} \frac{\frac{1}{2}\sin(h^4)}{h\sin h} = 0$$

$\therefore$ L.H.L = R.H.L. = 0

$$\therefore \quad \lim_{x\to 1} \frac{\int_0^{(x-1)^2} t\cos(t^2)\,dt}{(x-1)\sin(x-1)} = 0$$

32. Answer (36)

$$\lim_{x\to 2} \frac{3^x+3^{3-x}-12}{3^{-\frac{x}{2}}-3^{1-x}}$$

Let $3^{x/2} = t$ As $x \to 2$, $t \to 3$

$$\therefore \quad \lim_{t\to 3}\left( \frac{t^2+\dfrac{27}{t^2}-12}{\dfrac{1}{t}-\dfrac{3}{t^2}} \right)$$

$$= \lim_{t\to 3}\left(t^3+3t^2-3t-9\right)$$

$= 36$

33. Answer (40)

$$\lim_{x\to 1} \frac{x+x^2+x^3+\ldots+x^n-n}{x-1} = 820$$

As it is $\left(\frac{0}{0}\right)$ form, Apply L 'Hospital's Rule.

$$\lim_{x\to 1}\left(\frac{1+2x+3x^2+\ldots+nx^{n-1}}{1}\right) = 820$$

$\Rightarrow$ 1 + 2 + 3 + ....... n = 820

$\Rightarrow \dfrac{n(n+1)}{2} = 820$

$\Rightarrow n^2 + n - 1640 = 0$

$\Rightarrow (n-40)(n+41) = 0$

$\therefore$ n = 40

34. Answer (8)

$$\lim_{x\to 0} \frac{1}{x^8}\left\{1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}.\cos\frac{x^2}{4}\right\}$$

$$= \lim_{x\to 0} \frac{\left(1-\cos\dfrac{x^2}{2}\right)}{x^4}\frac{\left(1-\cos\dfrac{x^2}{4}\right)}{x^4}$$

35. Answer (5)

$$L = \lim_{x\to 0} \frac{a-\left(\dfrac{e^{4x}-1}{x}\right)}{a\left(e^{4x}-1\right)}$$

$$L = \lim_{x\to 0} \frac{a-\dfrac{1}{x}\left[\dfrac{4x}{\underline{|1}}+\dfrac{(4x)^2}{\underline{|2}}+\ldots\right]}{a\left[\dfrac{4x}{\underline{|1}}+\dfrac{(4x)^2}{\underline{|2}}+\ldots\right]}$$

Clearly, $a - 4 = 0 \Rightarrow a = 4$

$$L = \frac{-8}{16} = \frac{-1}{2} = b$$

So, $a - 2b = 4 + 1 = 5$

36. Answer (2)

$$\lim_{x\to 0}\frac{x}{\left((1-\sin x)^{\frac{1}{8}}-(1+\sin x)^{\frac{1}{8}}\right)}\times\left(\frac{(1-\sin x)^{\frac{1}{8}}+(1+\sin x)^{\frac{1}{8}}}{(1-\sin x)^{\frac{1}{8}}+(1+\sin x)^{\frac{1}{8}}}\right)$$

$$\Rightarrow \lim_{x\to 0}\frac{x\left((1-\sin x)^{\frac{1}{8}}+(1+\sin x)^{\frac{1}{8}}\right)}{(1-\sin x)^{\frac{1}{4}}-(1+\sin x)^{\frac{1}{4}}}$$

$$\times\frac{(1-\sin x)^{\frac{1}{4}}+(1+\sin x)^{\frac{1}{4}}}{(1-\sin x)^{\frac{1}{4}}+(1+\sin x)^{\frac{1}{4}}}$$

$$\Rightarrow \lim_{x\to 0}\frac{x\,2\cdot 2}{(1-\sin x)^{\frac{1}{2}}-(1+\sin x)^{\frac{1}{2}}}$$

$$\times\frac{(1-\sin x)^{\frac{1}{2}}+(1+\sin x)^{\frac{1}{2}}}{(1-\sin x)^{\frac{1}{2}}+(1+\sin x)^{\frac{1}{2}}}$$

$$\Rightarrow \lim_{x\to 0}\frac{8x}{1-\sin x-1-\sin x}=-4$$

37. Answer (4)

$\alpha$, $\beta$ are roots of

$x^2 + bx + c = 0$

$\therefore\quad x^2 + bx + c = (x-\alpha)(x-\beta)$ ...(i)

Also $\beta^2 + b\beta + c = 0.$ ...(ii)

$$L=\lim_{x\to\beta}\frac{e^{2\left(x^2+bx+c\right)}-1-2\left(x^2+bx+c\right)}{(x-\beta)^2\times(x-\alpha)^2}\times(x-\alpha)^2$$

$$L=\lim_{x\to\beta}\frac{e^{2\left(x^2+bx+c\right)}-1-2\left(x^2+bx+c\right)}{\left(x^2+bx+c\right)^2}\times\lim_{x\to\beta}(x-\alpha)^2$$

Let $x^2 + bx + c = t$

$x\to\beta \Rightarrow t\to 0$

$$L=\lim_{t\to 0}\frac{e^{2t}-1-2t}{t^2}\times(\beta-\alpha)^2$$

$$L=\lim_{t\to 0}\frac{\left(1+2t+\frac{(2t)^2}{2!}+\frac{(2t)^3}{3!}+\ldots\right)-1-2t}{t^2}\times(\alpha-\beta)^2$$

$L = 2(\alpha-\beta)^2$

$= 2[(\alpha+\beta)^2 - 4\alpha\beta]$

$= 2[(-b)^2 - 4c]$

$L = 2(b^2 - 4c)$

38. Answer (4)

$$L=\lim_{x\to\infty}\sqrt{x^2-x+1}-ax$$

$$=\lim_{x\to\infty}\frac{(x^2-x+1)-(ax)^2}{\sqrt{x^2-x+1}+ax}$$

$$L=\lim_{x\to\infty}\frac{(1-a^2)x^2-x+1}{\sqrt{x^2-x+1}+ax}$$

For limit to exist finitely $1 - a^2 = 0$

$$\Rightarrow L=\lim_{x\to\infty}\frac{-x+1}{\sqrt{x^2-x+1}+ax}=\lim_{x\to\infty}\frac{-1+\frac{1}{x}}{\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a}$$

$$L=\frac{-1}{1+a}=b$$

For $b$ to be finite, $a \neq -1$

$\therefore\quad a = 1,\ b = \dfrac{-1}{2}$

39. Answer (3)

$$\lim_{x\to 0}\frac{\sin^2\left(\pi\cos^4 x\right)}{x^4}$$

$$=\lim_{x\to 0}\frac{\sin^2\left(\pi-\pi\cos^4 x\right)}{x^4}$$

$$=\lim_{x\to 0}\left(\frac{\sin\left(\pi\left(1-\cos^4 x\right)\right)}{x^2}\right)^2$$

$$=\lim_{x\to 0}\left(\frac{\sin\left(\pi\sin^2 x\left(1+\cos^2 x\right)\right)}{x^2}\right)^2$$

$$=\lim_{x\to 0}\left(\frac{\sin\left(\pi\sin^2 x\left(1+\cos^2 x\right)\right)}{\pi\sin^2 x\left(1+\cos^2 x\right)}\times\frac{\pi\sin^2 x\left(1+\cos^2 x\right)}{x^2}\right)^2$$

$$= \lim_{x\to 0}\left(\pi\frac{\sin^2 x}{x^2}\left(1+\cos^2 x\right)\right)^2$$

$= (\pi \times 1 \times (1 + 1))^2$

$= 4\pi^2$

40. Answer (1)

$$\lim_{x\to 7}\frac{18-[1-x]}{[x-3a]} \text{ exist \& } a \in I.$$

$$= \lim_{x\to 7}\frac{17-[-x]}{[x]-3a} \text{ exist}$$

$$\text{RHL} = \lim_{x\to 7^+}\frac{17-[-x]}{[x]-3a} = \frac{25}{7-3a} \qquad \left[a \neq \frac{7}{3}\right]$$

$$\text{LHL} = \lim_{x\to 7^-}\frac{17-[-x]}{[x]-3a} = \frac{24}{6-3a} \qquad [a \neq 2]$$

For limit to exist

LHL = RHL

$$\frac{25}{7-3a} = \frac{24}{6-3a}$$

$$\Rightarrow \frac{25}{7-3a} = \frac{8}{2-a}$$

$$\therefore \boxed{a=-6}$$

41. Answer (3)

$$f(x) = [1+x] + \frac{a^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$$

$$\lim_{x\to 0^-} f(x) = \alpha - \frac{4}{3}$$

$$\Rightarrow \lim_{x\to 0^-} 1+[x]+\frac{\alpha^{x+[x]}+[x]-1}{x+[x]} = \alpha - \frac{4}{3}$$

$$\Rightarrow \lim_{h\to 0^-} 1-1+\frac{\alpha^{-h-1}-1-1}{-h-1} = \alpha - \frac{4}{3}$$

$$\therefore \frac{\alpha^{-1}-2}{-1} = \alpha - \frac{4}{3}$$

$\Rightarrow 3\alpha^2 - 10\alpha + 3 = 0$

$\therefore \alpha = 3$ or $\frac{1}{3}$

$\because \alpha$ is integer, hence $\alpha = 3$

42. Answer (11)

$$\lim_{x\to 1}\frac{\left(\frac{\sin\left(3x^2-4x+1\right)}{3x^2-4x+1}\right)\left(3x^2-4x+1\right)-x^2+1}{2x^3-7x^2+ax+b} = -2$$

$$\Rightarrow \lim_{x\to 1}\frac{3x^2-4x+1-x^2+1}{2x^3-7x^2+ax+b} = -2$$

$$\Rightarrow \lim_{x\to 1}\frac{2(x-1)^2}{2x^3-7x^2+ax+b} = -2$$

So $f(x) = 2x^3 - 7x^2 + ax + b = 0$ has $x = 1$ as repeated root, therefore $f(1) = 0$ and $f'(1) = 0$ gives

$a + b + 5$ and $a = 8$

So, $a - b = 11$

43. Answer (4)

$$\lim_{x\to 1}\frac{\left(x^2-1\right)\sin^2 \pi x}{x^4-2x^3+2x-1}$$

$$= \lim_{x\to 1}\frac{(x+1)(x-1)\sin^2 \pi x}{(x-1)^3(x+1)}$$

Let $x - 1 = t$

$$\lim_{t\to 0}\frac{(2+t)t\sin^2 \pi t}{t^3(t+2)}$$

$$= \lim_{t\to 0} = \frac{\sin^2 \pi t}{\pi^2 t^2}\cdot\pi^2$$

$= \pi^2$

44. Answer (01)

Let $x + 2\cos x = a$

$x + 2 = b$

as $x \to 0$, $a \to 2$ and $b \to 2$

$$\lim_{x\to 0}\left(\frac{a^3+2a^2+3\sin a}{b^3+2b^2+3\sin b}\right)^{\frac{100}{x}}$$

$$= e^{\lim\limits_{x\to 0}\cdot\frac{100}{x}\cdot\frac{(a^3-b^3)+2(a^2-b^2)+3(\sin a-\sin b)}{b^3+2b^2+3\sin b}}$$

$$\because \lim_{x\to 0}\frac{a-b}{x}=\lim_{x\to 0}\frac{2(\cos x-1)}{x}=0$$

$= e^0$

$= 1$

45. Answer (1)

$$\lim_{x\to\frac{\pi}{4}}\frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{-7(\cos x+\sin x)^6(-\sin x+\cos x)}{-2\sqrt{2}\cos 2x}$$ using L–H Rule

$$=\lim_{x\to\frac{\pi}{4}}\frac{56(\cos x-\sin x)}{2\sqrt{2}\cos 2x} \qquad \left(\frac{0}{0}\right)$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{-56(\sin x+\cos x)}{-4\sqrt{2}\sin 2x}$$ using L–H Rule

$$=7\sqrt{2}\cdot\sqrt{2}=14$$

46. Answer (3)

$$\beta=\lim_{x\to 0}\frac{\alpha x-(e^{3x}-1)}{\alpha x(e^{3x}-1)},\ \alpha\in\mathbb{R}$$

$$=\lim_{x\to 0}\frac{\frac{\alpha}{3}-\left(\frac{e^{3x}-1}{3x}\right)}{\alpha x\left(\frac{e^{3x-1}}{3x}\right)}$$

So, $\alpha = 3$ (to make indeterminant form)

$$\beta=\lim_{x\to 0}\frac{1-\left(\frac{e^{3x}-1}{3x}\right)}{3x}=\frac{1-\frac{\left(3x+\frac{9x^2}{2}+.....\right)}{3x}}{3x}$$

$$=\frac{-\left(\frac{9}{2}x^2+\frac{(3x)^3}{31}+....\right)}{9x^2}=\frac{-1}{2}$$

$$\therefore\ \alpha+\beta=3-\frac{1}{2}=\frac{5}{2}$$

47. Answer (1)

$$\lim_{x\to\frac{\pi}{2}}\tan^2 x\left\{\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right\}$$

$$=\lim_{x\to\frac{\pi}{2}}\frac{\tan^2 x(\sin^2 x-3\sin x+2)}{\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2}}$$

$$=\frac{1}{6}\lim_{x\to\frac{\pi}{2}}\frac{(2-\sin x)\sin^2 x}{1+\sin x}$$

$$=\frac{1}{12}$$

48. Answer (4)

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sin(\cos^{-1}x)-x}{1-\tan(\cos^{-1}x)} \qquad \text{let } \cos^{-1}x=\frac{\pi}{4}+\theta$$

$$=\lim_{\theta\to 0}\frac{\sin\left(\frac{\pi}{4}+\theta\right)-\cos\left(\frac{\pi}{4}+\theta\right)}{1-\tan\left(\frac{\pi}{4}+\theta\right)}$$

$$=\lim_{\theta\to 0}\frac{\sqrt{2}\sin\left(\frac{\pi}{4}+\theta-\frac{\pi}{4}\right)}{1-\frac{1+\tan\theta}{1-\tan\theta}}$$

$$=\lim_{\theta\to 0}\frac{\sqrt{2}\sin\theta}{-2\tan\theta}(1-\tan\theta)=-\frac{1}{\sqrt{2}}$$

49. Answer (3)

$$\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}=\lim_{x\to 0}\frac{2\sin(x+\sin x)\cdot\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$$

$$=\lim_{x\to 0}2\cdot\left(\frac{\left(\frac{x+\sin x}{2}\right)\left(\frac{x-\sin x}{2}\right)}{x^4}\right)$$

$$=\lim_{x\to 0}\frac{1}{2}\cdot\left(\frac{\left(x+x-\frac{x^3}{3!}+\frac{x^5}{5!}...\right)\left(x-x+\frac{x^3}{3!}...\right)}{x^4}\right)$$

$$=\lim_{x\to 0}\frac{1}{2}\cdot\left(2-\frac{x^2}{3!}+\frac{x^4}{5!}...\right)\left(\frac{1}{3!}-\frac{x^2}{5!}-1\right)=\frac{1}{6}$$

50. Answer (3)

$$\lim_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$$

$$=\lim_{n\to\infty} n\left[\sqrt{1-\frac{1}{n}-\frac{1}{n^2}}+\alpha+\frac{\beta}{n}\right]=0$$

$\therefore\quad \alpha = -1$

Now,

$$\lim_{n\to\infty} n\left[\left\{1-\left(\frac{1}{n}+\frac{1}{n^2}\right)\right\}^{1/2}+\frac{\beta}{n}-1\right]=0$$

$$=\lim_{n\to\infty}\frac{\left(1-\frac{1}{2}\left(\frac{1}{n}+\frac{1}{n^2}\right)+.....\right)+\frac{\beta}{n}-1}{\frac{1}{n}}=0$$

$$\Rightarrow\quad \beta-\frac{1}{2}=0$$

$$\therefore\quad \beta=\frac{1}{2}$$

Now, $8(\alpha+\beta)=8\left(-\frac{1}{2}\right)=-4$

51. Answer (3)

$$\lim_{x\to 0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}=\frac{2}{3}$$

$\Rightarrow\quad \alpha + \beta = 0$ (to make indeterminant form) ...(i)

Now,

$$\lim_{x\to 0}\frac{\alpha e^x-\beta e^{-x}+\gamma\cos x}{3x^2}=\frac{2}{3}\ \text{(Using L-H Rule)}$$

$\Rightarrow\quad \alpha - \beta + \gamma = 0$ (to make indeterminant form) ...(ii)

Now,

$$\lim_{x\to 0}\frac{\alpha e^x+\beta e^{-x}-\gamma\sin x}{6x}=\frac{2}{3}\ \text{(Using L-H Rule)}$$

$$\Rightarrow\quad \frac{\alpha-\beta-\gamma}{6}=\frac{2}{3}$$

$\Rightarrow\quad \alpha - \beta - \gamma = 4$ ...(iii)

$\Rightarrow\quad \gamma = -2$

and (i) + (ii)

$2\alpha = -\gamma$

$\Rightarrow\quad \alpha = 1$ and $\beta = -1$

and $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 1 - 4 - 2 + 3 = -2$

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