Chapter 5

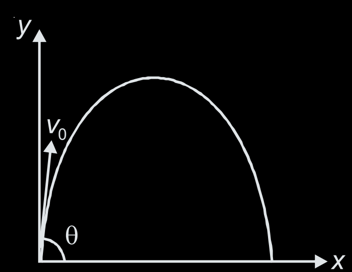
System of Particles and Rotational Motion

1. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of **[AIEEE-2009]**

(1) $\frac{1}{6}\frac{l\omega}{g}$ (2) $\frac{1}{2}\frac{l^2\omega^2}{g}$

(3) $\frac{1}{6}\frac{l^2\omega^2}{g}$ (4) $\frac{1}{3}\frac{l^2\omega^2}{g}$

2. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is **[AIEEE-2010]**



(1) $\frac{1}{2}mg v_0 t^2 \cos\theta\ \hat{i}$ (2) $-mg v_0 t^2 \cos\theta\ \hat{j}$

(3) $mg v_0 t\cos\theta\ \hat{k}$ (4) $-\frac{1}{2}mg v_0 t^2 \cos\theta\ \hat{k}$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively

3. A particle of mass m is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is **[AIEEE-2011]**

(1) $\frac{\sqrt{3}}{16}\frac{mv^3}{g}$ (2) $\frac{\sqrt{3}}{2}\frac{mv^2}{g}$

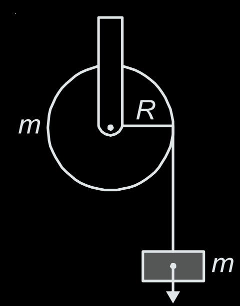
(3) Zero (4) $\frac{mv^3}{\sqrt{2}g}$

4. A ring of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip? **[JEE (Main)-2013]**

(1) $\frac{r\omega_0}{4}$ (2) $\frac{r\omega_0}{3}$

(3) $\frac{r\omega_0}{2}$ (4) $r\omega_0$

5. A mass m is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release? **[JEE (Main)-2014]**



(1) $\frac{2g}{3}$ (2) $\frac{g}{2}$

(3) $\frac{5g}{6}$ (4) g

6. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension **[JEE (Main)-2014]**

(1) Angular momentum is conserved

(2) Angular momentum changes in magnitude but not in direction

(3) Angular momentum changes in direction but not in magnitude

(4) Angular momentum changes both in direction and magnitude

7. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to **[JEE (Main)-2015]**

(1) $\frac{h^2}{4R}$ (2) $\frac{3h}{4}$

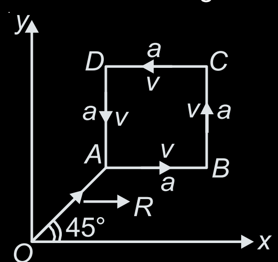
(3) $\frac{5h}{8}$ (4) $\frac{3h^2}{8R}$

8. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is **[JEE (Main)-2015]**

(1) $\frac{MR^2}{32\sqrt{2}\pi}$ (2) $\frac{MR^2}{16\sqrt{2}\pi}$

(3) $\frac{4MR^2}{9\sqrt{3}\pi}$ (4) $\frac{4MR^2}{3\sqrt{3}\pi}$

9. A particle of mass m is moving along the side of a square of side a, with a uniform speed v in the x-y plane as shown in the figure



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin? **[JEE (Main)-2016]**

(1) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D

(2) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C

(3) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A

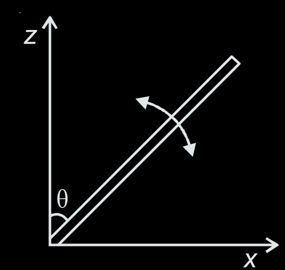
(4) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B

10. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio $\frac{\ell}{R}$ such that the moment of inertia is minimum? **[JEE (Main)-2017]**

(1) $\sqrt{\frac{3}{2}}$ (2) $\frac{\sqrt{3}}{2}$

(3) 1 (4) $\frac{3}{\sqrt{2}}$

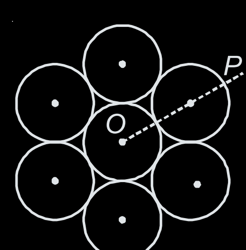
11. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is **[JEE (Main)-2017]**



(1) $\frac{3g}{2\ell}\sin\theta$ (2) $\frac{2g}{3\ell}\sin\theta$

(3) $\frac{3g}{2\ell}\cos\theta$ (4) $\frac{2g}{3\ell}\cos\theta$

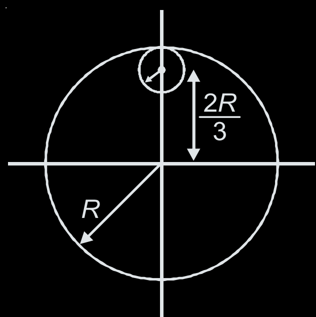
12. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is **[JEE (Main)-2018]**



(1) $\frac{19}{2}MR^2$ (2) $\frac{55}{2}MR^2$

(3) $\frac{73}{2}MR^2$ (4) $\frac{181}{2}MR^2$

13. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is **[JEE (Main)-2018]**

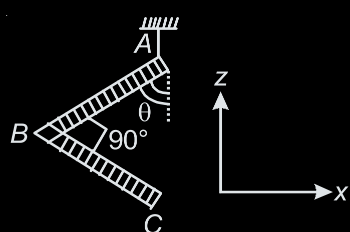


(1) $4MR^2$ (2) $\frac{40}{9}MR^2$

(3) $10MR^2$ (4) $\frac{37}{9}MR^2$

14. An L- shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If $AB = BC$, and the angle made by AB with downward vertical is θ, then

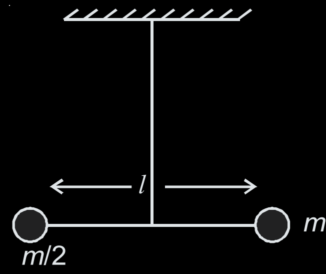
[JEE (Main)-2019]



(1) $\tan\theta = \frac{1}{2}$ (2) $\tan\theta = \frac{2}{\sqrt{3}}$

(3) $\tan\theta = \frac{1}{3}$ (4) $\tan\theta = \frac{1}{2\sqrt{3}}$

15. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be **[JEE (Main)-2019]**

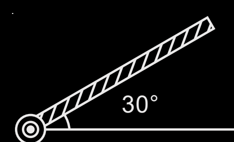


(1) $\frac{k\theta_0^2}{2l}$ (2) $\frac{k\theta_0^2}{l}$

(3) $\frac{2k\theta_0^2}{l}$ (4) $\frac{3k\theta_0^2}{l}$

16. A rod of length 50 cm is pivoted at one end. It is raised such that it makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be ($g = 10\ ms^{-2}$)

[JEE (Main)-2019]



(1) $\frac{\sqrt{20}}{3}$ (2) $\sqrt{30}$

(3) $\sqrt{\frac{30}{2}}$ (4) $\frac{\sqrt{30}}{2}$

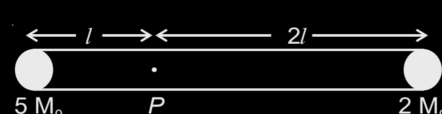
17. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is

[JEE (Main)-2019]

(1) $\frac{F}{2mR}$ (2) $\frac{2F}{3mR}$

(3) $\frac{F}{3mR}$ (4) $\frac{3F}{2mR}$

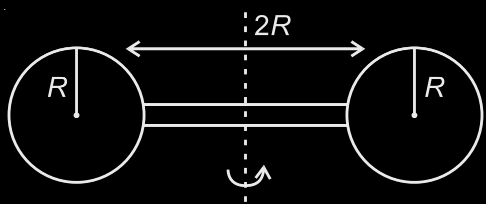
18. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be **[JEE (Main)-2019]**



(1) $\frac{g}{2\ell}$ (2) $\frac{g}{3\ell}$

(3) $\frac{g}{13\ell}$ (4) $\frac{7g}{3\ell}$

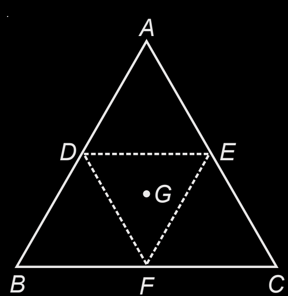
19. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is **[JEE (Main)-2019]**



(1) $\frac{152}{15}MR^2$ (2) $\frac{17}{15}MR^2$

(3) $\frac{209}{15}MR^2$ (4) $\frac{137}{15}MR^2$

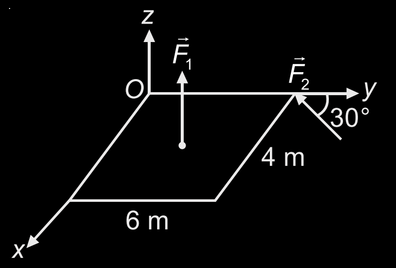
20. An equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then **[JEE (Main)-2019]**



(1) $I=\frac{3}{4}I_0$ (2) $I=\frac{15}{16}I_0$

(3) $I=\frac{I_0}{4}$ (4) $I=\frac{9}{16}I_0$

21. A slab is subjected to two forces $\vec{F_1}$ and $\vec{F_2}$ of same magnitude F as shown in the figure. Force $\vec{F_2}$ is in XY-plane while force F_1 acts along z-axis at the point $(2\vec{i}+3\vec{j})$. The moment of these forces about point O will be **[JEE (Main)-2019]**



(1) $(3\hat{i}-2\hat{j}+3\hat{k})F$ (2) $(3\hat{i}+2\hat{j}-3\hat{k})F$

(3) $(3\hat{i}+2\hat{j}+3\hat{k})F$ (4) $(3\hat{i}-2\hat{j}-3\hat{k})F$

22. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) : **[JEE (Main)-2019]**

(1) 16 rad/s^2 (2) 20 rad/s^2

(3) 12 rad/s^2 (4) 10 rad/s^2

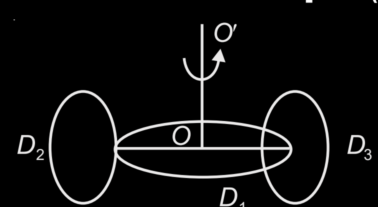
23. The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5 m, the angle between the force and the position vector is (in radians): **[JEE (Main)-2019]**

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

24. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1 as shown in the figure, will be

[JEE (Main)-2019]



(1) $3MR^2$ (2) $\frac{4}{5}MR^2$

(3) MR^2 (4) $\frac{2}{3}MR^2$

25. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is

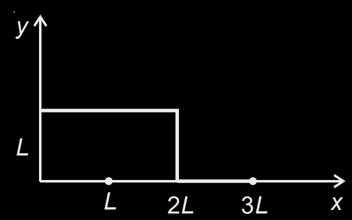
[JEE (Main)-2019]

(1) 14 cm (2) 12 cm

(3) 16 cm (4) 18 cm

26. The position vector of the centre of mass $\vec{r}_{cm}$ of an asymmetric uniform bar of negligible area of cross-section as shown in figure is

[JEE (Main)-2019]



(1) $\vec{r}_{cm} = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$

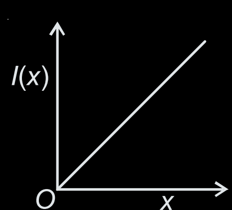
(2) $\vec{r}_{cm} = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$

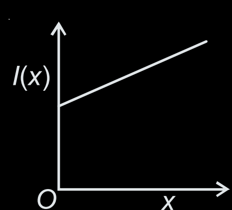
(3) $\vec{r}_{cm} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

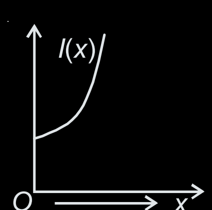
(4) $\vec{r}_{cm} = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$

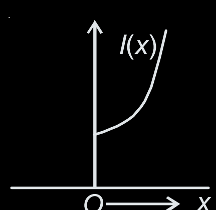
27. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is '$I(x)$'. Which one of the graphs represents the variation of $I(x)$ with x correctly?

[JEE (Main)-2019]

(1) 

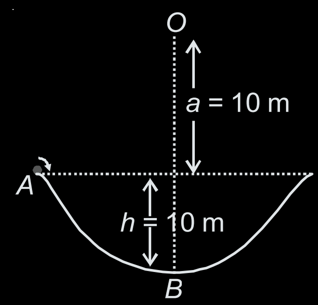
(2) 

(3) 

(4) 

28. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is a height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be

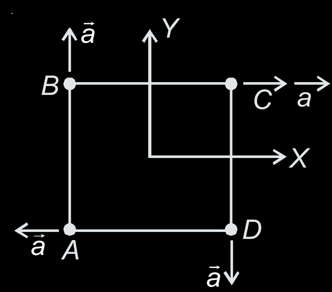
(Take $g = 10$ m/s^2) **[JEE (Main)-2019]**



(1) 2 kg m^2/s (2) 3 kg m^2/s

(3) 8 kg m^2/s (4) 6 kg m^2/s

29. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is **[JEE (Main)-2019]**



(1) Zero (2) $a(\hat{i} + \hat{j})$

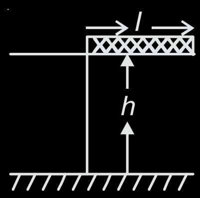
(3) $\frac{a}{5}(\hat{i} + \hat{j})$ (4) $\frac{a}{5}(\hat{i} - \hat{j})$

30. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its center. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = a MR^2$. The value of the coefficient a is **[JEE (Main)-2019]**

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$

(3) $\frac{3}{5}$ (4) $\frac{8}{5}$

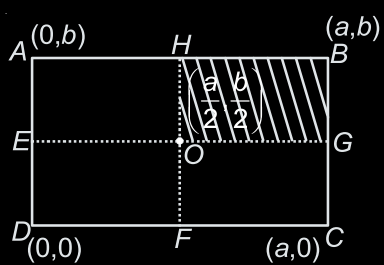
31. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to: **[JEE (Main)-2019]**



(1) 0.3 (2) 0.02

(3) 0.28 (4) 0.5

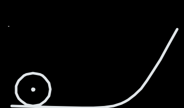
32. A uniform rectangular thin sheet *ABCD* of mass *M* has length a and breadth *b*, as shown in the figure. If the shaded portion *HBGO* is cut-off, the coordinates of the centre of mass of the remaining portion will be: **[JEE (Main)-2019]**



(1) $\left(\frac{2a}{3},\frac{2b}{3}\right)$ (2) $\left(\frac{5a}{12},\frac{5b}{12}\right)$

(3) $\left(\frac{3a}{4},\frac{3b}{4}\right)$ (4) $\left(\frac{5a}{3},\frac{5b}{3}\right)$

33. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by **[JEE (Main)-2019]**



(1) $\frac{4}{5}$ (2) $\frac{2}{\sqrt{5}}$

(3) 1 (4) $\frac{14}{15}$

34. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I, then the angular acceleration of the disc is **[JEE (Main)-2019]**

(1) $\frac{2k}{I}\theta$ (2) $\frac{k}{I}\theta$

(3) $\frac{k}{2I}\theta$ (4) $\frac{k}{4I}\theta$

35. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius *R*, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is **[JEE (Main)-2019]**

(1) 14 : 15 : 20 (2) 10 : 15 : 7

(3) 4 : 3 : 2 (4) 20 : 15 : 14

36. A thin smooth rod of length *L* and mass *M* is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass *m* and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be **[JEE (Main)-2019]**

(1) $\frac{M\omega_0}{M+3m}$ (2) $\frac{M\omega_0}{M+m}$

(3) $\frac{M\omega_0}{M+6m}$ (4) $\frac{M\omega_0}{M+2m}$

37. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of **[JEE (Main)-2019]**

(1) 2.5 s (2) 2 s

(3) 5 s (4) 3 s

38. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$ about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is : **[JEE (Main)-2019]**

(1) $-\frac{I_1\omega_1^2}{12}$ (2) $\frac{3}{8}I_1\omega_1^2$

(3) $\frac{I_1\omega_1^2}{6}$ (4) $-\frac{I_1\omega_1^2}{24}$

39. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is **[JEE (Main)-2019]**

(1) $\frac{MR^2}{3}$ (2) $\frac{MR^2}{6}$

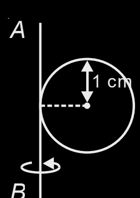
(3) $\frac{MR^2}{2}$ (4) $\frac{2MR^2}{3}$

40. The time dependence of the position of a particle of mass $m = 2$ is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time $t = 2$ is **[JEE (Main)-2019]**

(1) $-34\left(\hat{k}-\hat{i}\right)$ (2) $48\left(\hat{i}+\hat{j}\right)$

(3) $36\,\hat{k}$ (4) $-48\,\hat{k}$

41. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is closed to **[JEE (Main)-2019]**



(1) 4.0×10^{-6} Nm (2) 7.9×10^{-6} Nm

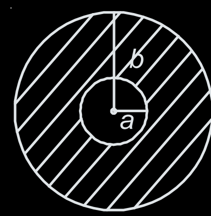
(3) 2.0×10^{-5} Nm (4) 1.6×10^{-5} Nm

42. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by **[JEE (Main)-2019]**

(1) 140 (2) 185

(3) 285 (4) 65

43. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is **[JEE (Main)-2019]**



(1) $\frac{a+b}{2}$ (2) $\sqrt{\frac{a^2+b^2+ab}{2}}$

(3) $\frac{a+b}{3}$ (4) $\sqrt{\frac{a^2+b^2+ab}{3}}$

44. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is **[JEE (Main)-2019]**

(1) 0.20 ms^{-1} (2) 0.14 ms^{-1}

(3) 0.47 ms^{-1} (4) 0.28 ms^{-1}

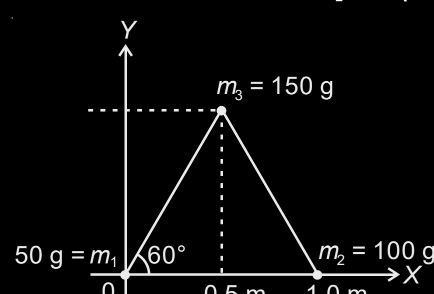
45. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance l (l<<L), is close to **[JEE (Main)-2019]**

(1) Mgl (2) $Mgl\left(1+\theta_0^2\right)$

(3) $Mgl\left(1+\frac{\theta_0^2}{2}\right)$ (4) $Mgl\left(1-\theta_0^2\right)$

46. Three particles of masses, 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be

[JEE (Main)-2019]



(1) $\left(\frac{\sqrt{3}}{8}\text{ m}, \frac{7}{12}\text{ m}\right)$ (2) $\left(\frac{\sqrt{3}}{4}\text{ m}, \frac{5}{12}\text{ m}\right)$

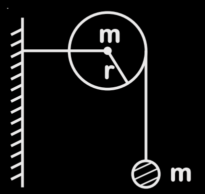
(3) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{8}\text{ m}\right)$ (4) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{4}\text{ m}\right)$

47. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is **[JEE (Main)-2020]**

(1) $\sqrt{\frac{7}{48}}\,l$ (2) $\frac{1}{8}l$

(3) $\frac{1}{4}l$ (4) $\sqrt{\frac{3}{8}}\,l$

48.



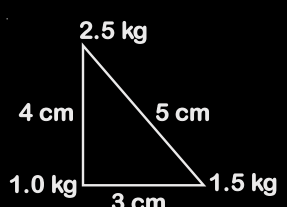
As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be

[JEE (Main)-2020]

(1) $r\sqrt{\frac{3}{2gh}}$ (2) $r\sqrt{\frac{3}{4gh}}$

(3) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (4) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$

49. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of system is at a point **[JEE (Main)-2020]**



(1) 1.5 cm right and 1.2 cm above 1 kg mass

(2) 2.0 cm right and 0.9 cm above 1 kg mass

(3) 0.9 cm right and 2.0 cm above 1 kg mass

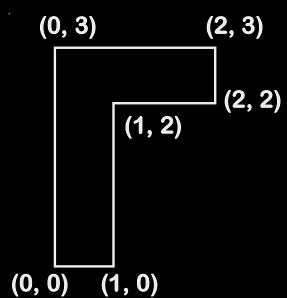
(4) 0.6 cm right and 2.0 cm above 1 kg mass

50. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is **[JEE (Main)-2020]**

(1) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$ (2) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

(3) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (4) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$

51. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4 kg. (The coordinates of the same are shown in figure) are **[JEE (Main)-2020]**



(1) (0.75 m, 1.75 m) (2) (1.25 m, 1.50 m)

(3) (1 m, 1.75 m) (4) (0.75 m, 0.75 m)

52. Consider a uniform rod of mass M = 4 m and length l pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is

[JEE (Main)-2020]

(1) $\frac{4}{7}\frac{v}{l}$ (2) $\frac{3}{7}\frac{v}{l}$

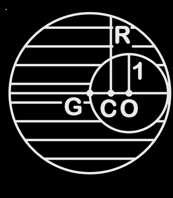
(3) $\frac{3}{7\sqrt{2}}\frac{v}{l}$ (4) $\frac{3\sqrt{2}}{7}\frac{v}{l}$

53. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is **[JEE (Main)-2020]**

(1) 6.25×10^{-4} J (2) 1.13×10^{-3} J

(3) 8.75×10^{-4} J (4) 8.75×10^{-3} J

54. As shown in figure when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e., on the surface of the cavity. R can be determined by the equation **[JEE (Main)-2020]**



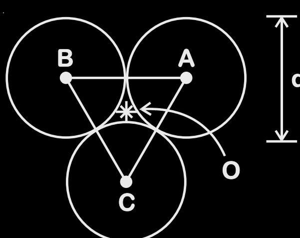
(1) $(R^2 + R + 1)(2 - R) = 1$

(2) $(R^2 - R + 1)(2 - R) = 1$

(3) $(R^2 - R - 1)(2 - R) = 1$

(4) $(R^2 + R - 1)(2 - R) = 1$

55.



Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0/I_A of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is

[JEE (Main)-2020]

(1) $\frac{23}{13}$ (2) $\frac{15}{13}$

(3) $\frac{13}{15}$ (4) $\frac{13}{23}$

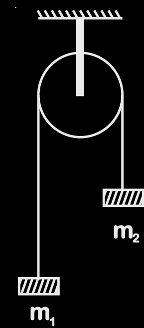
56. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at **[JEE (Main)-2020]**

(1) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$ (2) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$

(3) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$ (4) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

57. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig.), A massless string is warapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is

[JEE (Main)-2020]



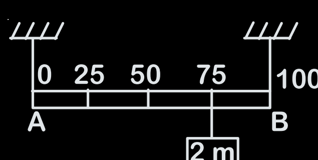
(1) $\left[\frac{(m_1 - m_2)}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}} gh$

(2) $\left[\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}}$

(3) $\left[\frac{m_1 + m_2}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}} gh$

(4) $\left[\frac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}}$

58.

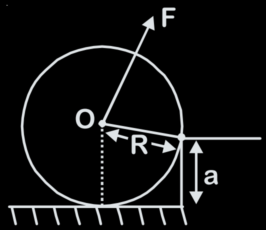


Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is **[JEE (Main)-2020]**

(1) 0.5 mg (2) 2 mg

(3) 0.75 mg (4) 1 mg

59. A uniform cylinder of mass M and radius R is to be pulled over a step of height a ($a < R$) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is **[JEE (Main)-2020]**



(1) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$ (2) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

(3) $Mg\sqrt{1-\frac{a^2}{R^2}}$ (4) $Mg\frac{a}{R}$

60. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is **[JEE (Main)-2020]**

(1) $\frac{20}{3}$ J (2) $\frac{5}{3}$ J

(3) $\frac{10}{3}$ J (4) $\frac{2}{3}$ J

61. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material, the ratio L/R for it to have minimum possible I is

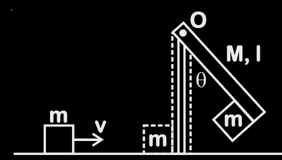
[JEE (Main)-2020]

(1) $\sqrt{\frac{2}{3}}$ (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) $\sqrt{\frac{3}{2}}$

62. A block of mass m = 1 kg slides with velocity v = 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass M = 2 kg and length l = 1 m, the value of θ is approximately (take g = 10 m/s^2)

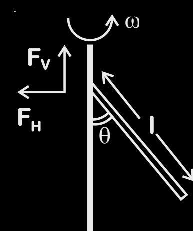
[JEE (Main)-2020]



(1) 49° (2) 55°

(3) 63° (4) 69°

63. A uniform rod of length 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2\sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that

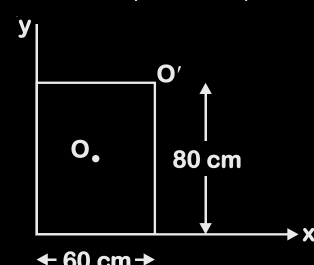


[JEE (Main)-2020]

(1) $\cos\theta = \frac{g}{l\omega^2}$ (2) $\cos\theta = \frac{2g}{3l\omega^2}$

(3) $\cos\theta = \frac{g}{2l\omega^2}$ (4) $\cos\theta = \frac{3g}{2l\omega^2}$

64. For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is



[JEE (Main)-2020]

(1) 1/2 (2) 1/4

(3) 1/8 (4) 2/3

65. A circular coil has moment of inertia 0.8 kg m^2 around any diameter and is carrying current to produce a magnetic moment of 20 Am^2. The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4 T is applied along the

vertical, it starts rotating, around its horizontal diameter. The angular speed the coil acquires after rotating by 60° will be **[JEE (Main)-2020]**

(1) 10 rad s^{-1} (2) 13.16 rad s^{-1}

(3) 20 rad s^{-1} (4) 20π rad s^{-1}

66. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is

[JEE (Main)-2020]

(1) 2 (2) 4

(3) $2\sqrt{2}$ (4) $\sqrt{2}$

67. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia $3I$ initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is **[JEE (Main)-2020]**

(1) $\frac{5}{6}$ (2) $\frac{1}{4}$

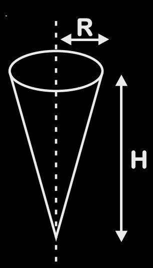
(3) 0 (4) $\frac{3}{4}$

68. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where $v(t)$ is its instantaneous velocity. The instantaneous acceleration of the satellite is **[JEE (Main)-2020]**

(1) $-bv^3(t)$ (2) $-\frac{2bv^3}{M(t)}$

(3) $-\frac{bv^3}{M(t)}$ (4) $-\frac{bv^3}{2M(t)}$

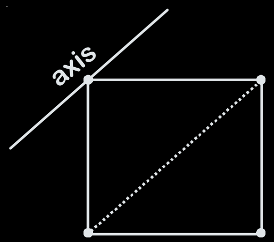
69. Shown in the figure is a hollow icecream cone (it open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is **[JEE (Main)-2020]**



(1) $\frac{M(R^2 + H^2)}{4}$ (2) $\frac{MR^2}{2}$

(3) $\frac{MR^2}{3}$ (4) $\frac{MH^2}{3}$

70. Four point masses, each of mass m, are fixed at the corners of a square of side ℓ. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is **[JEE (Main)-2020]**



(1) $3\ m\ell^2\omega$ (2) $4\ m\ell^2\omega$

(3) $m\ell^2\omega$ (4) $2\ m\ell^2\omega$

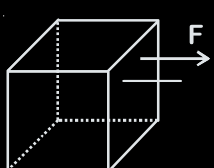
71. The linear mass density of a thin rod AB of length L varies from A to B as

$\lambda(x) = \lambda_0\left(1 + \frac{x}{L}\right)$, where x is the distance from A.

If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is **[JEE (Main)-2020]**

(1) $\frac{2}{5}ML^2$ (2) $\frac{5}{12}ML^2$

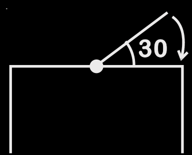
(3) $\frac{3}{7}ML^2$ (4) $\frac{7}{18}ML^2$

72. 

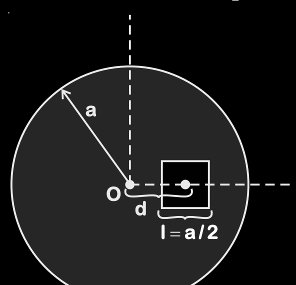
Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for box not to topple before moving is _________.

[JEE (Main)-2020]

73. One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}\ \mathrm{s}^{-1}$, where n is an integer. The value of n is ______ **[JEE (Main)-2020]**

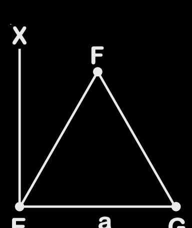


74. A square shaped hole of side $l = \dfrac{a}{2}$ is carved out at a distance $d = \dfrac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\dfrac{a}{X}$, value of X (to the nearest integer) is _________. **[JEE (Main)-2020]**

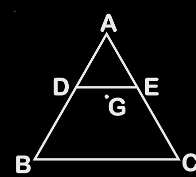


75. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre _________. **[JEE (Main)-2020]**

76. An massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\dfrac{N}{20}ma^2$ where N is an integer. The value of N is _________. **[JEE (Main)-2020]**



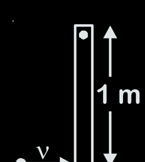
77. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\dfrac{NI_0}{16}$ where N is an integer. Value of N is __________. **[JEE (Main)-2020]**



78. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\dfrac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is _____. **[JEE (Main)-2020]**

79. A force $\vec{F} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ N acts at a point $\left(4\hat{i} + 3\hat{j} - \hat{k}\right)$ m. Then the magnitude of torque about the point $\left(\hat{i} + 2\hat{j} + \hat{k}\right)$ m will be $\sqrt{x}$ N-m. The value of x is ____ . **[JEE (Main)-2020]**

80. A thin rod of mass 0.9 kg and length 1 m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of move 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be ____ . **[JEE (Main)-2020]**



81. The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then value of x is _________. **[JEE (Main)-2020]**

82. Moment of inertia (M. I.) of four bodies, having same mass and radius, are reported as;

I_1 = M.I. of thin circular ring about its diameter,

I_2 = M.I. of circular disc about an axis perpendicular to disc and going through the centre,

I_3 = M.I. of solid cylinder about its axis and

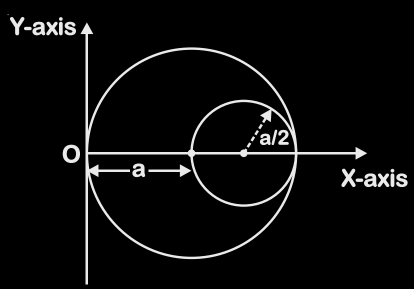
I_4 = M.I. of solid sphere about its diameter.

Then : **[JEE (Main)-2021]**

(1) $I_1 + I_3 < I_2 + I_4$ (2) $I_1 = I_2 = I_3 > I_4$

(3) $I_1 + I_2 = I_3 + \frac{5}{2} I_4$ (4) $I_1 = I_2 = I_3 < I_4$

83. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be **[JEE (Main)-2021]**

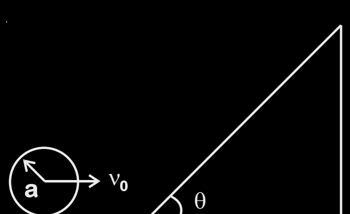


(1) $\frac{1}{6}a$ (2) $\frac{2}{3}a$

(3) $\frac{10}{11}a$ (4) $\frac{5}{6}a$

84. A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is ______ × 10^{-1} kg m^2. **[JEE (Main)-2021]**

85. A sphere of radius 'a' and mass 'm' rolls along a horizontal plane with constant speed v_0. It encounters an inclined plane at angle α and climbs upward. Assuming that it rolls without slipping, how far up the sphere will travel? **[JEE (Main)-2021]**



(1) $\frac{v_0^2}{5g\sin\theta}$ (2) $\frac{v_0^2}{2g\sin\theta}$

(3) $\frac{7v_0^2}{10g\sin\theta}$ (4) $\frac{2}{5}\frac{v_0^2}{g\sin\theta}$

86. If $\vec{P}\times\vec{Q} = \vec{Q}\times\vec{P}$, the angle between $\vec{P}$ and $\vec{Q}$ is θ ($0° < \theta < 360°$). The value of 'θ' will be ______°.

[JEE (Main)-2021]

87. Four identical solid spheres each of mass 'm' and radius 'a' are placed with their centres on the four corners of a square of side 'b'. The moment of inertia of the system about one side of square where the axis of rotation is parallel to the plane of the square is : **[JEE (Main)-2021]**

(1) $\frac{4}{5}ma^2$ (2) $\frac{8}{5}ma^2 + mb^2$

(3) $\frac{8}{5}ma^2 + 2mb^2$ (4) $\frac{4}{5}ma^2 + 2mb^2$

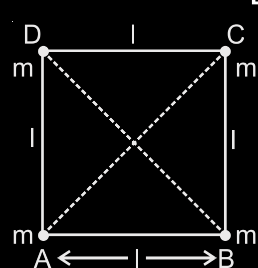
88. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance 'h', the square of angular velocity of wheel will be **[JEE (Main)-2021]**

(1) $2gh$ (2) $\frac{2mgh}{I + 2mr^2}$

(3) $\frac{2mgh}{I + mr^2}$ (4) $\frac{2gh}{I + mr^2}$

89. Four equal masses, m each are placed at the corners of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be:

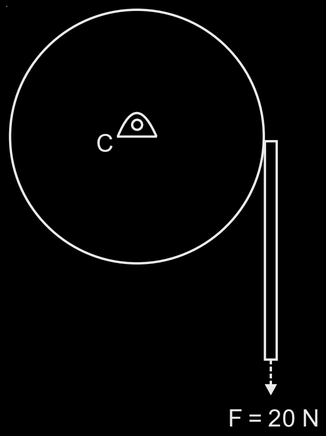
[JEE (Main)-2021]



(1) $3\ ml^2$ (2) $2\ ml^2$

(3) $\sqrt{3}\ ml^2$ (4) ml^2

90. Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force F = 20 N through a massless string wrapped around its periphery as shown in the figure. **[JEE (Main)-2021]**



Suppose the disk makes n number of revolutions to attain an angular speed of 50 rad s^{-1}.

The value of n, to the nearest integer, is ______

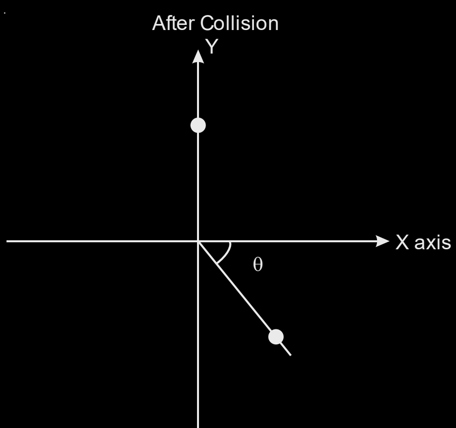
[Given : In one complete revolution, the disk rotates by 6.28 rad]

91. A ball of mass 10 kg moving with a velocity $10\sqrt{3}\ ms^{-1}$ along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle θ (degree) with respect to the X-axis.

The configuration of pieces after collision is shown in the figure.

The value of θ to the nearest integer is______.

[JEE (Main)-2021]

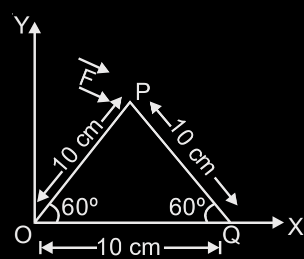


92. A deviation of 2° is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass as 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6 respectively. The refracting angles for crown glass prism will be _______° (in degree) (Round off to the Nearest Integer) **[JEE (Main)-2021]**

93. If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is_______ (Round off to the Nearest Integer)

(M is the mass of earth, R is the radius of earth, G is the gravitational constant) **[JEE (Main)-2021]**

94. A triangular plate is shown. A force $\vec{F} = 4\hat{i} - 3\hat{j}$ is applied at point P. The torque at point P with respect to point 'O' and 'Q' are : **[JEE (Main)-2021]**



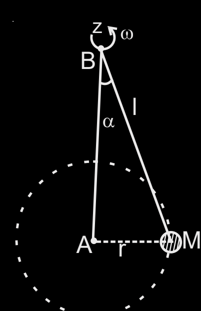
(1) $15 + 20\sqrt{3}, 15 - 20\sqrt{3}$

(2) $-15 - 20\sqrt{3}, 15 - 20\sqrt{3}$

(3) $-15 + 20\sqrt{3}, 15 + 20\sqrt{3}$

(4) $15 - 20\sqrt{3}, 15 + 20\sqrt{3}$

95. A mass M hangs on a massless rod of length l which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive z direction and the angular momentum of M about point B is L_B. The correct statement for this system is : **[JEE (Main)-2021]**



(1) L_A is constant, both in magnitude and direction

(2) L_B is constant in direction with varying magnitude

(3) L_A and L_B are both constant in magnitude and direction

(4) L_B is constant, both in magnitude and direction

96. The following bodies, **[JEE (Main)-2021]**

(1) a ring

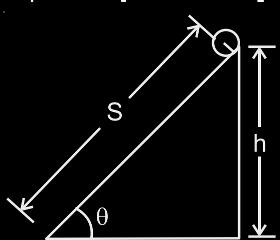
(2) a disc

(3) a solid cylinder

(4) a solid sphere,

of same mass 'm' and radius 'R' are allowed to roll down without slipping simultaneously from the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is ______.

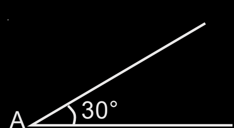
[Mark the body as per their respective numbering given in the question] **[JEE (Main)-2021]**



97. The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 seconds. The number of revolutions by the truck engine during this time is ________. (Assuming the acceleration to be uniform).

[JEE (Main)-2021]

98. A sphere of mass 2 kg and radius 0.5 m is rolling with an initial speed of 1 ms^{-1} goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A? **[JEE (Main)-2021]**

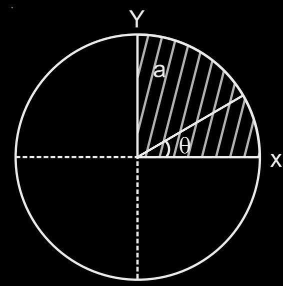


(1) 0.60 s (2) 0.80 s

(3) 0.57 s (4) 0.52 s

99. The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is______.(Round off to the Nearest Integer) [a is an area as shown in the figure]

[JEE (Main)-2021]

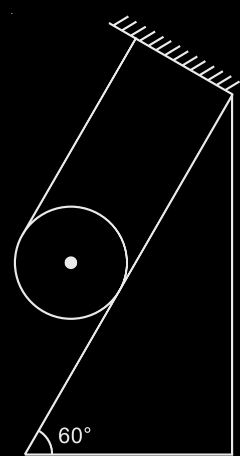


100. A thin circular ring of mass *M* and radius *r* is rotating about its axis with an angular speed ω. Two particles having mass *m* each are now attached at diametrically opposite points. The angular speed of the ring will become: **[JEE (Main)-2021]**

(1) $\omega\frac{M}{M+2m}$ (2) $\omega\frac{M+2m}{M}$

(3) $\omega\frac{M-2m}{M+2m}$ (4) $\omega\frac{M}{M+m}$

101. A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is: **[JEE (Main)-2021]**



[The coefficient of static friction, μ_s, is 0.4]

(1) 0 (2) 5 mg

(3) $\frac{7}{2}$ mg (4) $\frac{mg}{5}$

102. Consider a uniform wire of mass M and length L. It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is: **[JEE (Main)-2021]**

(1) $\frac{2}{5}\frac{ML^2}{\pi^2}$ (2) $\frac{1}{4}\frac{ML^2}{\pi^2}$

(3) $\frac{1}{2}\frac{ML^2}{\pi^2}$ (4) $\frac{ML^2}{\pi^2}$

inclined plane of length 'L'. When it slips down the plane, it takes time 't_1'. When it rolls down the plane, it takes time t_2. The value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$. The value of x will be ________. **[JEE (Main)-2021]**

104. A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits at one end of the rod with a velocity 'u' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio of masses $\left(\frac{m}{M}\right)$ is $\frac{1}{x}$. The value of 'x' will be _____. **[JEE (Main)-2021]**

105. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is: **[JEE (Main)-2021]**

(1) Solid cylinder

(2) Hollow cylinder

(3) Ring

(4) Solid sphere

106. Two bodies, a ring and a solid cylinder of same material are rolling down without slipping an inclined plane. The radii of the bodies are same. The ratio of velocity of the centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is $\frac{\sqrt{x}}{2}$. Then, the value of x is _________. **[JEE (Main)-2021]**

107. Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping. Assume that they start rolling from rest and having identical diameter.

The **correct** statement for this situation is

[JEE (Main)-2021]

(1) All of them will have same velocity.

(2) The ring has greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.

(3) The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

(4) The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

108. A bullet of '4 g' mass is fired from a gun of mass 4 kg. If the bullet moves with the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun and velocity of recoil of gun are: **[JEE (Main)-2021]**

(1) 0.2 kg ms^{-1}, 0.1 ms^{-1}

(2) 0.4 kg ms^{-1}, 0.05 ms^{-1}

(3) 0.2 kg ms^{-1}, 0.05 ms^{-1}

(4) 0.4 kg ms^{-1}, 0.1 ms^{-1}

109. Three particles P, Q and R are moving along the vectors $\vec{A} = \hat{i} + \hat{j}, \vec{B} = \hat{j} + \hat{k}$ and $\vec{C} = -\hat{i} + \hat{j}$ respectively. They strike on a point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is _____. **[JEE (Main)-2021]**

110. The position of the centre of mass of a uniform semi-circular wire of radius 'R' placed in x-y plane with its centre at the origin and the line joining its ends as x-axis is given by $\left(0, \frac{xR}{\pi}\right)$. Then, the value of $|x|$ is ______. **[JEE (Main)-2021]**

111. The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at a speed $\sqrt{x}v_0$. Then the value of x is ______.

[JEE (Main)-2021]

112. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: Moment of inertia of a circular disc of mass 'M' and radius 'R' about X, Y axes (passing through its plane) and Z-axis which is perpendicular to its plane were found to be I_x, I_y & I_z respectively. The respective radii of gyration about all the three axes will be the same.

[JEE (Main)-2021]

Reason R: A rigid body making rotational motion has fixed mass and shape.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.

(2) **A** is correct but **R** is not correct.

(3) **A** is not correct but **R** is correct.

(4) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

113. A particle of mass 'm' is moving in time 't' on a trajectory given by **[JEE (Main)-2021]**

$\vec{r} = 10\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}$

Where α and β are dimensional constants.

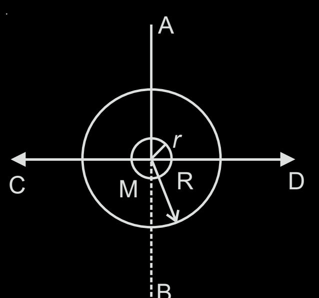
The angular momentum of the particle becomes the same as it was for t = 0 at time t = _________ seconds.

114. A body of mass 2 kg moving with a speed of 4 m/s makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial speed. The speed of the two body centre of mass is $\frac{x}{10}$ m/s. Then the value of x is ______. **[JEE (Main)-2021]**

115. A solid disc of radius 20 cm and mass 10 kg is rotating with an angular velocity of 600 rpm, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc at rest in 10 s is ________ $\pi \times 10^{-1}$ Nm.

[JEE (Main)-2021]

116. The figure shows two solid discs with radius R and r respectively. If mass per unit area is same for both, what is the ratio of MI of bigger disc around axis AB (Which is $\perp$ to the plane of the disc and passing through its centre) to MI of smaller disc around one of its diameters lying on its plane? Given 'M' is the mass of the larger disc. (MI stands for moment of inertia) **[JEE (Main)-2021]**



(1) $2r^4 : R^4$

(2) $R^2 : r^2$

(3) $2R^2 : r^2$

(4) $2R^4 : r^4$

117. **List-I** **List-II**

	List-I	List-II
(a)	MI of the rod (length L, Mass M, about and axis $\perp$ to the rod passing through the midpoint)	(i) 8 $ML^2/3$
(b)	MI of the rod (length L, Mass 2M, about an axis $\perp$ to the rod passing through one of its end)	(ii) $ML^2/3$
(c)	MI of the rod (length 2L, Mass M, about an axis $\perp$ to the rod passing through its midpoint)	(iii) $ML^2/12$
(d)	MI of the rod (length 2L, Mass 2M, about an axis $\perp$ to the rod passing through one of its end)	(iv) 2 $ML^2/3$

Choose the **correct** answer from the options given below : **[JEE (Main)-2021]**

(1) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

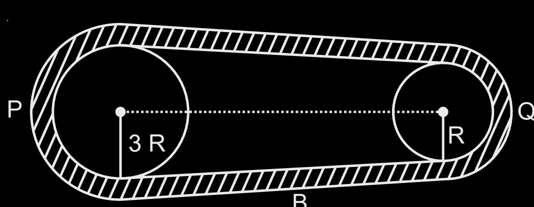
(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

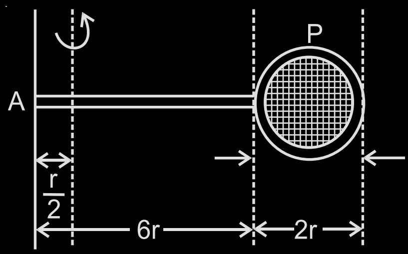
(4) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

118. In the given figure, two wheels P and Q are connected by a belt B. The radius of P is three times as that of Q. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\frac{I_1}{I_2}\right)$ will be x : 1. The value of x will be ________.

[JEE (Main)-2021]

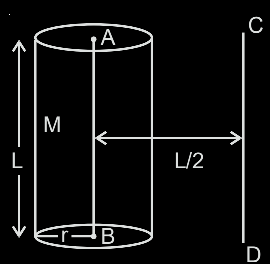


119. Consider a badminton racket with length scales as shown in the figure. **[JEE (Main)-2021]**



If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of ring at, $\frac{r}{2}$ distance from the end A of the handle will be ___ Mr^2.

120. The solid cylinder of length 80 cm and mass M has a radius of 20 cm. Calculate the density of the material used if the moment of inertia of the cylinder about an axis CD parallel to AB as shown in figure is 2.7 kg m^2. **[JEE (Main)-2021]**



(1) 7.5×10^1 kg/m^3

(2) 1.49×10^2 kg/m^3

(3) 7.5×10^2 kg/m^3

(4) 14.9 kg/m^3

121. Moment of interia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of square plate is given by:

[JEE (Main)-2021]

(1) $\frac{2}{3}Ml^2$ (2) $\frac{Ml^2}{6}$

(3) Ml^2 (4) $\frac{Ml^2}{12}$

122. Two discs have moments of intertia I_1 and I_2 about their respective axes perpendicular to the plane and passing through the centre. They are rotating with angular speeds, ω_1 and ω_2 respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by: **[JEE (Main)-2021]**

(1) $\frac{I_1 I_2}{(I_1+I_2)}(\omega_1-\omega_2)^2$ (2) $\frac{(\omega_1-\omega_2)^2}{2(I_1+I_2)}$

(3) $\frac{(I_1-I_2)^2\omega_1\omega_2}{2(I_1+I_2)}$ (4) $\frac{I_1 I_2}{2(I_1+I_2)}(\omega_1-\omega_2)^2$

123. Angular momentum of a single particle moving with constant speed along circular path :

[JEE (Main)-2021]

(1) Is zero

(2) Changes in magnitude but remains same in the direction

(3) Remains same in magnitude and direction

(4) Remains same in magnitude but changes in the direction

124. A system consists of two identical spheres each of mass 1.5 kg and radius 50 cm at the ends of a light rod. The distance between the centres of the two spheres is 5 m. What will be the moment of inertia of the system about an axis perpendicular to the rod passing through its midpoint? **[JEE (Main)-2021]**

(1) 1.875×10^5 kgm^2 (2) 19.05 kgm^2

(3) 18.75 kgm^2 (4) 1.905×10^5 kgm^2

125. A 2 kg steel rod of length 0.6 m is clamped on a table vertically at its lower end and is free to rotate in vertical plane. The upper end is pushed so that the rod falls under gravity. Ignoring the friction due to clamping at its lower end, the speed of the free end of rod when it passes through its lowest position is _____ ms^{-1}. **[JEE (Main)-2021]**

(Take g = 10 ms^{-2})

126. A force $\vec{F} = 4\hat{i}+3\hat{j}+4\hat{k}$ is applied on an intersection point of x = 2 plane and x-axis. The magnitude of torque of this force about a point (2, 3, 4) is ______. (Round off to the Nearest Integer) **[JEE (Main)-2021]**

127. A metre scale is balanced on a knife edge at its centre. When two coins, each of mass 10 g are put one on the top of the other at the 10.0 cm mark the scale is found to be balanced at 40.0 cm mark. The mass of the metre scale is found to be $x \times 10^{-2}$ kg. The value of x is ________. **[JEE (Main)-2022]**

128. If force $\vec{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ acts on a particle having position vector $2\hat{i} + \hat{j} + 2\hat{k}$ then, the torque about the origin will be **[JEE (Main)-2022]**

(1) $3\hat{i} + 4\hat{j} - 2\hat{k}$

(2) $-10\hat{i} + 10\hat{j} + 5\hat{k}$

(3) $10\hat{i} + 5\hat{j} - 10\hat{k}$

(4) $10\hat{i} + \hat{j} - 5\hat{k}$

129. Moment of Inertia (M.I.) of four bodies having same mass 'M' and radius '$2R$' are as follows :

I_1 = M.I. of solid sphere about its diameter

I_2 = M.I. of solid cylinder about its axis

I_3 = M.I. of solid circular disc about its diameter.

I_4 = M.I. of thin circular ring about its diameter

If $2(I_2 + I_3) + I_4 = x \cdot I_1$ then the value of x will be ________. **[JEE (Main)-2022]**

130. A thin circular ring of mass M and radius R is rotating with a constant angular velocity 2 rad s^{-1} in a horizontal plane about an axis vertical to its plane and passing through the center of the ring. If two objects each of mass m be attached gently to the opposite ends of a diameter of ring, the ring will then rotate with an angular velocity (in rad s^{-1}).

[JEE (Main)-2022]

(1) $\dfrac{M}{(M+m)}$

(2) $\dfrac{(M+2m)}{2M}$

(3) $\dfrac{2M}{(M+2m)}$

(4) $\dfrac{2(M+2m)}{M}$

131. Two blocks of masses 10 kg and 30 kg are placed on the same straight line with coordinates (0, 0) cm and (x, 0) cm respectively. The block of 10 kg is moved on the same line through a distance of 6 cm towards the other block. The distance through which the block of 30 kg must be moved to keep the position of centre of mass of the system unchanged is

[JEE (Main)-2022]

(1) 4 cm towards the 10 kg block

(2) 2 cm away from the 10 kg block

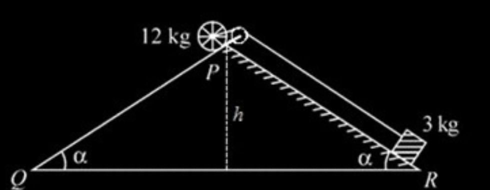
(3) 2 cm towards the 10 kg block

(4) 4 cm away from the 10 kg block

132. A rolling wheel of 12 kg is on an inclined plane at position P and connected to a mass of 3 kg through a string of fixed length and pulley as shown in figure. Consider PR as friction free surface.

The velocity of centre of mass of the wheel when it reaches at the bottom Q of the inclined plane PQ will be $\frac{1}{2}\sqrt{xgh}$ m/s. The value of x is ________.

[JEE (Main)-2022]



133. Match List-I with List-II

[JEE (Main)-2022]

	List-I		List-II
A.	Moment of inertia of solid sphere of radius R about any tangent	I.	$\frac{5}{3}MR^2$
B.	Moment of inertia of hollow sphere of radius (R) about any tangent.	II.	$\frac{7}{5}MR^2$

C. Moment of inertia of circular ring of radius (R) about its diameter. III. $\frac{1}{4}MR^2$

D. Moment of inertia of circular disc of radius (R) about any diameter. IV. $\frac{1}{2}MR^2$

Choose the correct answer from the options given below.

(1) A-II, B-I, C-IV, D-III (2) A-I, B-II, C-IV, D-III

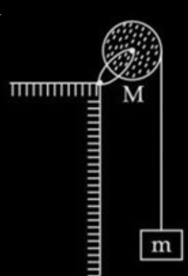
(3) A-II, B-I, C-III, D-IV (4) A-I, B-II, C-III, D-IV

134. The position vector of 1 kg object is $\vec{r} = (3\hat{i} - \hat{j})$ m and its velocity $\vec{v} = (3\hat{j} + \hat{k})\ \text{ms}^{-1}$. The magnitude of its angular momentum is $\sqrt{x}$ Nm where x is

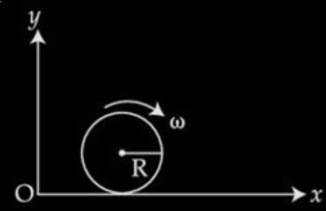
[JEE (Main)-2022]

135. A uniform disc with mass $M = 4$ kg and radius $R = 10$ cm is mounted on a fixed horizontal axle as shown in figure. A block with mass $m = 2$ kg hangs from a massless cord that is wrapped around the rim of the disc. During the fall of the block, the cord does not slip and there is no friction at the axle. The tension in the cord is ______ N.

(Take $g = 10\ \text{ms}^{-2}$) **[JEE (Main)-2022]**



136. A spherical shell of 1 kg mass and radius R is rolling with angular speed ω on horizontal plane (as shown in figure). The magnitude of angular momentum of the shell about the origin O is $\frac{a}{3}R^2\omega$. The value of a will be **[JEE (Main)-2022]**



(1) 2 (2) 3

(3) 5 (4) 4

137. The moment of inertia of a uniform thin rod about a perpendicular axis passing through one end is I_1. The same rod is bent into a ring and its moment of inertia about a diameter is I_2. If $\frac{I_1}{I_2}$ is $\frac{x\pi^2}{3}$, then the value of x will be ______. **[JEE (Main)-2022]**

138. A solid cylinder and a solid sphere, having same mass M and radius R, roll down the same inclined plane from top without slipping. They start from rest. The ratio of velocity of the solid cylinder to that of the solid sphere, with which they reach the ground, will be

[JEE (Main)-2022]

(1) $\sqrt{\frac{5}{3}}$ (2) $\sqrt{\frac{4}{5}}$

(3) $\sqrt{\frac{3}{5}}$ (4) $\sqrt{\frac{14}{15}}$

139. Three identical spheres each of mass M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 3 m each. Taking point of intersection of mutually perpendicular sides as origin, the magnitude of position vector of centre of mass of the system will be $\sqrt{x}$ m. The value of x is ______ **[JEE (Main)-2022]**

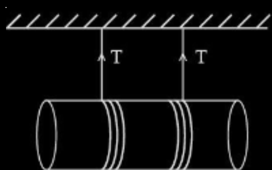
140. A disc of mass 1 kg and radius R is free to rotate about a horizontal axis passing through its centre and perpendicular to the plane of disc. A body of same mass as that of disc of fixed at the highest point of the disc. Now the system is released, when the body comes to the lowest position, it angular speed will be $4\sqrt{\frac{x}{3R}}$ rad s^{-1} where x = ____.

($g = 10\ \text{ms}^{-2}$) **[JEE (Main)-2022]**

141. A pulley of radius 1.5 m is rotated about its axis by a force $F = (12t - 3t^2)$ N applied tangentially (while t is measured in seconds). If moment of inertia of the pulley about its axis of rotation is 4.5 kg m^2, the number of rotations made by the pulley before its direction of motion is reversed, will be $\frac{K}{\pi}$. The value of K is ________. **[JEE (Main)-2022]**

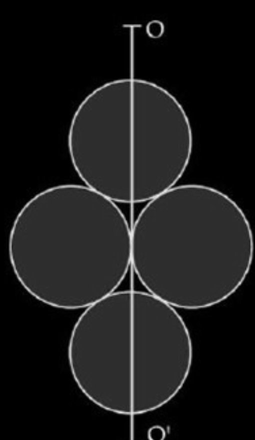
142. A solid cylinder length is suspended symmetrically through two massless strings, as shown in the figure. The distance from the initial rest position, the cylinder should be unbinding the strings to achieve a speed of 4 m/s, is ______ cm. (Take $g = 10$ m/s^2).

[JEE (Main)-2022]



143. Four identical discs each of mass 'M' and diameter 'a' are arranged in a small plane as shown in figure. If the moment of inertia of the system about OO' is $\frac{x}{4}Ma^2$. Then, the value of x will be _______.

[JEE (Main)-2022]



144. The distance of centre of mass from end A of a one dimensional rod (AB) having mass density $\rho = \rho_0\left(1 - \frac{x^2}{L^2}\right)$ kg/m and length L (in meter) is $\frac{3L}{\alpha}$ m. The value of α is ______. (where x is the distance from end A) **[JEE (Main)-2022]**

145. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-3\hat{i} - 2\hat{j} + \hat{k}$ respectively. The magnitude of position vector of centre of mass of this system will be similar to the magnitude of vector :

[JEE (Main)-2022]

(1) $\hat{i} + 2\hat{j} + \hat{k}$

(2) $-3\hat{i} - 2\hat{j} + \hat{k}$

(3) $-2\hat{j} + 2\hat{k}$

(4) $-2\hat{i} - \hat{j} + 2\hat{k}$

146. The torque of a force $5\hat{i} + 3\hat{j} - 7\hat{k}$ about the origin is τ. If the force acts on a particle whose position vector is $2\hat{i} + 2\hat{j} + \hat{k}$, then the value of τ will be

[JEE (Main)-2022]

(1) $11\hat{i} + 19\hat{j} - 4\hat{k}$

(2) $-11\hat{i} + 9\hat{j} - 16\hat{k}$

(3) $-17\hat{i} + 19\hat{j} - 4\hat{k}$

(4) $17\hat{i} + 9\hat{j} + 16\hat{k}$

147. A solid spherical ball is rolling on a frictionless horizontal plane surface about its axis of symmetry. The ratio of rotational kinetic energy of the ball to its total kinetic energy is - **[JEE (Main)-2022]**

(1) $\frac{2}{5}$

(2) $\frac{2}{7}$

(3) $\frac{1}{5}$

(4) $\frac{7}{10}$

148. What percentage of kinetic energy of a moving particle is transferred to a stationary particle when it strikes the stationary particle of 5 times its mass? (Assume the collision to be head-on elastic collision)

[JEE (Main)-2022]

(1) 50.0%

(2) 66.6%

(3) 55.6%

(4) 33.3%

149. A man of 60 kg is running on the road and suddenly jumps into a stationary trolly car of mass 120 kg. Then, the trolly car starts moving with velocity 2 ms^{-1}. The velocity of the running man was ______ ms^{-1}, when he jumps into the car.

[JEE (Main)-2022]

150. A body of mass *M* at rest explodes into three pieces, in the ratio of masses 1 : 1 : 2. Two smaller pieces fly off perpendicular to each other with velocities of 30 ms^{-1} and 40 ms^{-1} respectively. The velocity of the third piece will be **[JEE (Main)-2022]**

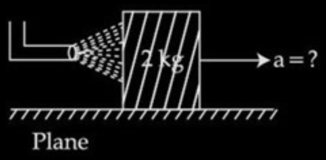
(1) 15 ms^{-1}

(2) 25 ms^{-1}

(3) 35 ms^{-1}

(4) 50 ms^{-1}

151. A block of metal weighing 2 kg is resting on a frictionless plane (as shown in figure). It is struck by a jet releasing water at a rate of 1 kgs^{-1} and at a speed of 10 ms^{-1}. Then, the initial acceleration of the block, in ms^{-2}, will be: **[JEE (Main)-2022]**



(1) 3 (2) 6

(3) 5 (4) 4

152. The radius of gyration of a cylindrical rod about an axis of rotation perpendicular to its length and passing through the center will be__________m.

Given the length of the rod is $10\sqrt{3}$ m.

[JEE (Main)-2022]

❑ ❑ ❑

Chapter 5

System of Particles and Rotational Motion

1. Answer (3)

Loss in kinetic energy = Gain in potential energy

$$\frac{1}{2}I\omega^2 = mgh$$

$$\Rightarrow \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2 = mgh \Rightarrow h = \frac{\ell^2\omega^2}{6g}$$

2. Answer (4)

Angular momentum,

$$\vec{L} = \int \vec{\tau}\, dt$$

$$\vec{L} = -\int mg\ x\ dt\ \hat{k}$$

$$= -\int mg\ v_0 \cos\theta\ t\ dt\ \hat{k}$$

$$= -\frac{mg\ v_0 \cos\theta\ t^2}{2}\hat{k}$$

3. Answer (1)

$L = mu\cos\theta \times h$

$$= mv \times \frac{\sqrt{3}}{2} \times \frac{v^2 \sin^2 30°}{2g}$$

$$= \frac{\sqrt{3}mv^3}{16g}$$

4. Answer (3)

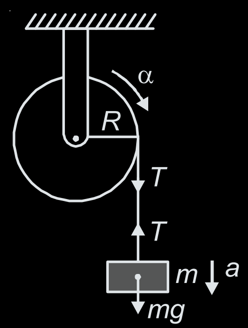
By conservation of angular momentum about a point on ground

$$mr^2\omega_r = mr^2w + mvr$$

$$\Rightarrow mr^2\omega_0 = 2\ mvr$$

$$\Rightarrow v = \frac{r\omega_0}{2}$$

5. Answer (2)



$a = R\alpha$

$mg - T = ma$

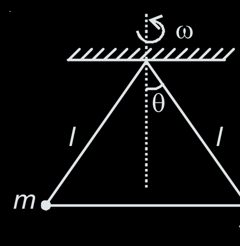
$T \times R = mR^2\alpha$

or $T = ma$

$$\Rightarrow a = \frac{g}{2}$$

6. Answer (3)

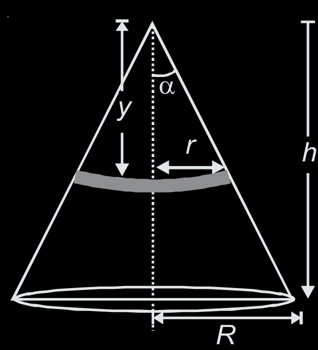
$\tau = mg \times l \sin\theta$. (Direction parallel to plane of rotation of particle)



as τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same.

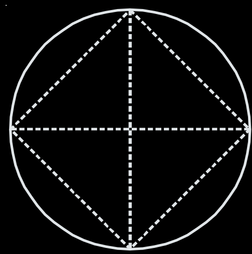
7. Answer (2)

$$dm = \pi r^2.dy.\rho$$



$$y_{CM} = \frac{\int ydm}{\int dm} = \frac{\int_0^h \pi r^2 dy \times \rho \times y}{\frac{1}{3}\pi R^2 h\rho} = \frac{3h}{4}$$

8. Answer (3)



$$d = 2R = a\sqrt{3}$$

$$\Rightarrow \quad a = \frac{2}{\sqrt{3}}R$$

$$\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi$$

$$\Rightarrow \quad M' = \frac{2M}{\sqrt{3}\pi}$$

$$I = \frac{M'a^2}{6} = \frac{2M}{\sqrt{3}\pi} \times \frac{4}{3}R^2 \times \frac{1}{6}$$

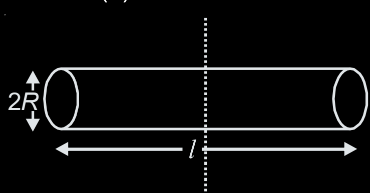
$$I = \frac{4MR^2}{9\sqrt{3}\pi}$$

9. Answer (1, 3)

In option 1, correct $\vec{L}$ should be $mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from C to D.

In option 3, correct $\vec{L}$ should be $\frac{-mv}{\sqrt{2}}R\hat{k}$

10. Answer (1)



$$I = \frac{mR^2}{4} + \frac{m\ell^2}{12}$$

$$I = \frac{m}{4}\left[R^2 + \frac{\ell^2}{3}\right] = \frac{m}{4}\left[\frac{v}{\pi\ell} + \frac{\ell^2}{3}\right]$$

$$\frac{dI}{d\ell} = \frac{m}{4}\left[\frac{-v}{\pi\ell^2} + \frac{2\ell}{3}\right] = 0$$

$$\frac{v}{\pi\ell^2} = \frac{2\ell}{3}$$

$$v = \frac{2\pi\ell^3}{3}$$

$$\pi R^2 \ell = \frac{2\pi\ell^3}{3}$$

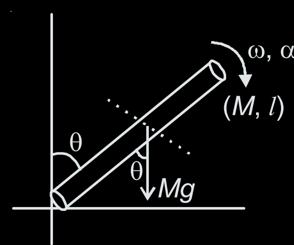
$$\frac{\ell^2}{R^2} = \frac{3}{2}$$

$$\frac{\ell}{R} = \sqrt{\frac{3}{2}}$$

11. Answer (1)

Torque at angle θ

$$\tau = Mg\sin\theta \cdot \frac{\ell}{2}$$



$$\tau = I\alpha$$

$$I\alpha = Mg\sin\theta\frac{\ell}{2} \qquad \therefore\ I = \frac{M\ell^2}{3}$$

$$\frac{M\ell^2}{3} \cdot \alpha = Mg\sin\theta\frac{\ell}{2}$$

$$\frac{\ell\alpha}{3} = g\frac{\sin\theta}{2}$$

$$\alpha = \frac{3g\sin\theta}{2\ell}$$

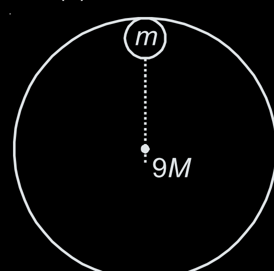
12. Answer (4)

$$I_0 = \frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + M(2R)^2\right)$$

$$I_P = I_0 + 7M(3R)^2$$

$$= \frac{181}{2}MR^2$$

13. Answer (1)



$$m = \frac{(9M)}{9} = M$$

$$I_1 = \frac{(9M) \times R^2}{2}$$

$$I_2 = \frac{M \times \left(\frac{R}{3}\right)^2}{2} + M \times \left(\frac{2R}{3}\right)^2 = \frac{MR^2}{2}$$

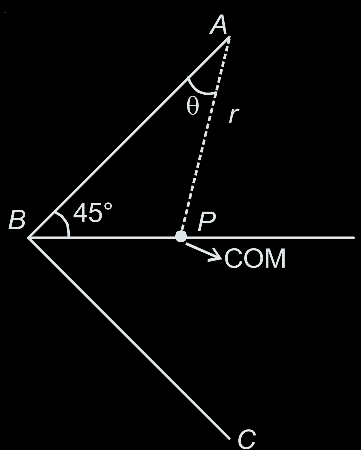
$$\therefore \quad I_{req} = I_1 - I_2$$

$$= \frac{9}{2}MR^2 - \frac{MR^2}{2}$$

$$= 4MR^2$$

14. Answer (3)

Position of COM from B is $\frac{L}{2}\frac{1}{\sqrt{2}} = \frac{L}{2\sqrt{2}}$



Now, $r^2 = L^2 + \frac{L^2}{8} - 2.L.\frac{L}{2\sqrt{2}}\frac{1}{\sqrt{2}}$

$$\Rightarrow \quad r^2 = \frac{9L^2}{8} - \frac{L^2}{2} = \frac{5L^2}{8}$$

$$\therefore \quad r = \sqrt{\frac{5}{8}} \cdot L$$

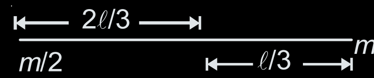
$$\therefore \; r\cos\theta = L - \frac{L}{4} \Rightarrow r\cos\theta = \frac{3L}{4}$$

$$\therefore \quad \cos\theta = \frac{3L\sqrt{8}}{4\sqrt{5} \times L} = \frac{3 \times 2\sqrt{2}}{4\sqrt{5}} = \frac{3}{\sqrt{10}}$$

$$\therefore \quad \tan\theta = \frac{1}{3}$$

15. Answer (2)

$$I = \frac{m\ell^2}{9} + \frac{m}{2} \cdot \frac{4\ell^2}{9}$$



$$\Rightarrow \quad I = \frac{m\ell^2}{9}(1+2) = \frac{m\ell^2}{3}$$

$$\therefore \quad \text{PE at } (\theta_0) = \frac{1}{2}k\theta_0^2$$

Now, $\frac{1}{2}k\theta_0^2 = \frac{1}{2}\frac{m\ell^2}{3} \cdot \omega^2$

$$\Rightarrow \quad \frac{3k{\theta_0}^2}{m\ell^2} = \omega^2$$

$$T = m\omega^2 \frac{\ell}{3} = \frac{m \cdot 3k{\theta_0}^2}{m\ell^2} \cdot \frac{\ell}{3} = \frac{k{\theta_0}^2}{\ell}$$

16. Answer (2)

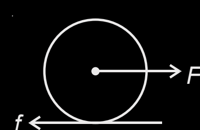
Conservation of mechanical energy

$$\Rightarrow \quad mg\frac{l}{2}\sin 30^\circ = \frac{1}{2}\frac{ml^2}{3} \cdot \omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{3g}{2l} = \frac{30}{1}$$

$$\Rightarrow \quad \omega = \sqrt{30} \text{ rad/s}$$

17. Answer (2)



$$F - f = Ma$$

$$fR = \frac{MR^2}{2} \cdot \frac{a}{R}$$

$$\Rightarrow f = \frac{Ma}{2}$$

$$\Rightarrow F = \frac{3Ma}{2}$$

$$\Rightarrow a = \frac{2F}{3M}$$

$$\alpha = \frac{2F}{3MR}$$

18. Answer (3)

$$\tau = I_p\alpha$$

$$5M_0gl - 4M_0gl = [2M_0(2l)^2 + 5M_0l^2]\,\alpha$$

$$\Rightarrow \quad M_0gl = 13M_0l^2\alpha$$

$$\Rightarrow \quad \alpha = \frac{g}{13l}$$

19. Answer (4)

$I = I_{\text{spheres}} + I_{\text{rod}}$

$$I_{\text{rod}} = \frac{M\times 4R^2}{12} = \frac{MR^2}{3}$$

$$I_{\text{spheres}} = 2\times\left[\frac{2}{5}MR^2 + 4MR^2\right]$$

$$= \frac{44}{5}MR^2$$

$$I = \left[\frac{44}{5}+\frac{1}{3}\right]MR^2$$

$$= \frac{137}{15}MR^2$$

20. Answer (2)

$I_0 = K\cdot ML^2$

$$I_1 = K\cdot\frac{M}{4}\left(\frac{L}{2}\right)^2 = K\cdot\frac{ML^2}{16}$$

$$I_2 = I_0 - I_1$$

$$I_2 = \frac{15}{16}\ \text{KML}^2$$

$$= \frac{15}{16}I_0$$

21. Answer (1)

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$$

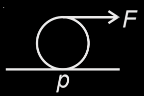
$$\vec{\tau}_1 = \left(2\hat{i}+3\hat{j}\right)\times F\hat{k} = F\left(3\hat{i}-2\hat{j}\right)$$

$$\vec{\tau}_2 = 6\hat{j}\times F\left(-\sin 30^\circ\hat{i} - \cos 30^\circ\hat{j}\right)$$

$$\vec{\tau}_2 = 3F\hat{k}$$

$$\tau = F\left(3\hat{i}-2\hat{j}+3\hat{k}\right)$$

22. Answer (1)



$\tau_p = I_p\alpha$

$F(2R) = 2MR^2\alpha$

$$\alpha = \frac{F}{MR} = \frac{40}{0.5\times 5}$$

$$= 16 \text{ rad/s}^2$$

23. Answer (2)

$\tau = Fr\sin\theta$

$2.5 = 1 \times 5\sin\theta$

$$\Rightarrow \quad \theta = \frac{\pi}{6}$$

24. Answer (1)

$I = I_1 + I_2 + I_3$

$$= \frac{MR^2}{2} + 2\left[\frac{MR^2}{4} + MR^2\right]$$

$$= \frac{MR^2}{2} + 2\times\frac{5MR^2}{4} = 3MR^2$$

25. Answer (3)

$$\frac{M}{2}\left(R_1^2 + R_2^2\right) = MR^2$$

$$R = \sqrt{\frac{R_1^2+R_2^2}{2}}$$

$$= \sqrt{\frac{100+400}{2}}$$

$$= \sqrt{250}$$

≈ 16 cm

26. Answer (3)

Let assume linear mass dinsity is λ

then, $m_1 = 2L\lambda$, and $r_{1\text{cm}} \equiv (L, L)$

$m_2 = L\lambda$, and $r_{2\text{cm}} \equiv \left(2L, \dfrac{L}{2}\right)$

$m_3 = L\lambda$, and $r_{3\text{cm}} \equiv \left(\dfrac{5L}{2}, 0\right)$

$$\therefore \quad X_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3}$$

$\Rightarrow \; X_{cm} = \frac{13}{8}L$

and, $Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{5}{8}L$

27. Answer (3)

$I(x) = I_0 + mx^2$

Hence option (3) is correct.

28. Answer (4)

$$L = mv_0r$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_i^2 + mgh$$

$\Rightarrow \; v_0^2 = 25 + 2 \times 10 \times 10 = 225$

$v_0 = 15$ m/s

Now, $L = 20 \times 10^{-3} \times 15 \times 20 = 6$ kg-m^{2}s^{-1}

29. Answer (4)

$$a_{CM} = \frac{(2m)a\hat{j} + 3m \times a\hat{i} + ma(-\hat{i}) + 4m \times a(-\hat{j})}{2m + 3m + 4m + m}$$

$$= \frac{2a\hat{i} - 2a\hat{j}}{10} = \frac{a}{5}(\hat{i} - \hat{j})$$

30. Answer (4)

$$M = \int_0^R \rho_0 r \times 2\pi r dr = \frac{2\pi\rho_0 R^3}{3}$$

$$I_C = \int_0^R \rho_0 r \times 2\pi r dr \times r^2 = \frac{2\pi\rho_0 R^5}{5}$$

$$\therefore \; I = I_C + MR^2 = 2\pi\rho_0 R^5\left(\frac{1}{3} + \frac{1}{5}\right) = \frac{16\pi\rho_0 R^5}{15}$$

$$= \frac{8}{5}\left[\frac{2}{3}\pi\rho_0 R^3\right]R^2 = \frac{8}{5}MR^2$$

31. Answer (4)

Initial angular acceleration before it slips off

$$mg\frac{l}{2} = I\alpha$$

$$\alpha = \frac{3g}{2l}$$

$\therefore$ Angular speed acquire by the box in time $\tau = 0.01$ s

$$\omega = \alpha t = \frac{3g}{2l} \times 0.01 = \frac{3 \times 10 \times 0.01}{2 \times 0.3} = \frac{1}{2} \text{ rad/sec}$$

$\therefore$ The angle by which it would rotate when hits the ground

$\theta = \omega t'$

Assuming ω = constant and t' = time of fall = $\sqrt{\frac{2H}{g}} = 1$ sec

$$\therefore \; \theta = \frac{1}{2} \text{ radians}$$

32. Answer (2)

X-coordinate of CM of remaining sheet

$$X_{cm} = \frac{MX - mx}{M - m}$$

$$= \frac{(4m) \times \left(\frac{a}{2}\right) - M\left(\frac{3a}{4}\right)}{4m - m} = \frac{5a}{12}$$

Similarly, $y_{cm} = \frac{5b}{12}$

$$\therefore \; CM\left(\frac{5a}{12}, \frac{5b}{12}\right)$$

33. Answer (4)

$$mgh_{sph} = \frac{1}{2}mv^2 + \frac{1}{2} \cdot \frac{2}{5}mR^2 \cdot \left(\frac{v}{R}\right)^2$$

$$= \frac{7}{10}mv^2 \qquad \text{...(i)}$$

$$mgh_{cylinder} = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{v}{R}\right)^2 = \frac{3}{4}mv^2$$

$$\Rightarrow \; \frac{h_{sph}}{h_{cylinder}} = \frac{7 \times 4}{10 \times 3} = \frac{14}{15}$$

34. Answer (1)

$$\tau = \frac{dE}{d\theta}$$

$$2k\theta = I\alpha$$

$$\alpha = \frac{2k\theta}{I}$$

35. Answer (4)

$$mgh = \frac{1}{2} I_p \omega^2$$

For ring $\rightarrow h_1 = \frac{1}{2}\frac{(2mr'^2)}{mg}\frac{v^2}{(r')^2} = \frac{v^2}{g}$

For cylinder $\rightarrow h_2 = \frac{1}{2}\left(\frac{3}{2}mr'^2\right)\frac{v^2}{(r')^2} = \frac{3}{4}\frac{v^2}{g}$

For sphere $\rightarrow h_3 = \frac{1}{2} \times \frac{\frac{7}{5}m(r')^2}{g}\frac{v^2}{(r')^2} = \frac{7}{10}\frac{v^2}{g}$

$h_1 : h_2 : h_3$

$2 : \frac{3}{2} : \frac{14}{10}$

$20 : 15 : 14$

36. Answer (3)

Initial angular momentum

= Final Angular Momentum

$$\frac{ML^2}{12}\omega_0 = \left(\frac{ML^2}{12} + 2\frac{mL^2}{4}\right)\omega$$

$$\Rightarrow \quad \omega = \frac{M\omega_0}{M + 6m}$$

37. Answer (2)

$I = 150;$ $\alpha = 20 \text{ rad/s}^2$

$\therefore \quad \omega = \alpha t$

$$E = \frac{1}{2} I\omega^2 = 1200$$

$$\frac{1}{2} \times 1.5 \times (20t)^2 = 1200 \text{ J} \quad \Rightarrow \quad t = 2 \text{ s}$$

38. Answer (4)

By applying conservation of angular momentum

$(I_1 + I_2)\,\omega_{common} = I_1\omega_1 + I_2\omega_2$

$$\omega_{common} = \frac{I_1\omega_1 + \frac{I_1\omega_1}{4}}{I_1 + \frac{I_1}{2}} = \left(\frac{5}{4} \times \frac{2}{3}\right)\omega_1$$

$$\omega_c = \frac{5\omega_1}{6}$$

$\therefore$ Loss in KE $= \left(\frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2\right) - \frac{1}{2}(I_1 + I_2)\,\omega_c^2$

$\therefore \quad \Delta\text{KE} = -\frac{I_1\omega_1^2}{24}$

39. Answer (4)

Surface mass density $(\sigma) = kr^2$

Mass of disc $M = \int_0^R (kr^2) 2\pi r dr$

$$= 2\pi k\frac{R^4}{4} = \frac{\pi k R^4}{2}$$

$\therefore$ Moment of inertia about the axis of the disc.

$$I = \int dI = \int (dm) r^2 = \int \sigma dA r^2$$

$$= \int (Kr^2)(2\pi r dr) r^2$$

$$= \int_0^R 2\pi k\, r^5 dr = \frac{\pi k R^6}{3} = \frac{2}{3}MR^2$$

40. Answer (4)

$$\vec{r} = 2t\hat{i} - 3t^2\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 6t\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -6\hat{j}$$

$$\vec{F} = m\vec{a} = -12\hat{j}$$

$$\vec{r}\,(at\ t = 2) = 4\hat{i} - 12\hat{j}$$

$$\vec{L} = m(\vec{r} \times \vec{v}) = 2(4\hat{i} - 12\hat{j}) \times (2\hat{i} - 12\hat{j}) = -48\hat{k}$$

41. Answer (3)

$\tau = I\alpha$

$\omega = \omega_0 + \alpha t$

$\Rightarrow \quad 25 \times 2\pi = (\alpha)5$

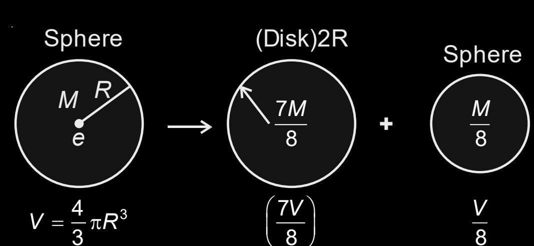
$\alpha = 10\pi$

$$\Rightarrow \quad \tau = \left(\frac{5}{4}mR^2\right)\alpha$$

$$\approx \left(\frac{5}{4}\right)(5 \times 10^{-3})(10^{-4})10\pi$$

$= 2.0 \times 10^{-5}$ Nm

42. Answer (1)



$$I_1 = \left(\frac{7M}{8}\right)(2R)^2\frac{1}{2} = \frac{14}{8}MR^2$$

$$I_2 = \frac{2}{5}\left(\frac{M}{8}\right)r^2$$

$$\begin{bmatrix} \text{We have} \\ \frac{4}{3}\pi r^3\rho = \frac{M}{8} \\ \Rightarrow \quad r = \left(\frac{R}{2}\right) \end{bmatrix}$$

$$\Rightarrow \quad I_2 = \frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R^2}{4}\right) = \frac{MR^2}{80}$$

$$\frac{I_1}{I_2} = \frac{14\times 80}{(8)} = 140$$

43. Answer (4)

$$\Rightarrow \quad \sigma = \frac{\sigma_0}{r} \qquad\qquad \therefore \quad \frac{\sigma_0}{r}2\pi r dr = dm$$

$$\Rightarrow \quad m = \sigma_0 2\pi\ (b - a)$$

$$I = \sigma_0 2\pi\int_a^b r^2 dr = \frac{2\pi\sigma_0}{3}(b^3 - a^3)$$

$$\therefore \quad mk^2 = I \ \Rightarrow\ 2\pi\sigma_0(b-a)k^2 = \frac{2\pi\sigma_0}{3}(b^3 - a^3)$$

$$\Rightarrow \quad k^2 = \frac{1}{3}(b^2 + ab + a^2)$$

$$\therefore \quad k = \sqrt{\frac{1}{3}\frac{(b^3 - a^3)}{b-a}}$$

44. Answer (1)

$50\ V_1 = 20\ V_2$

$V_1 + V_2 = 0.70$

$V_1 = 0.20$

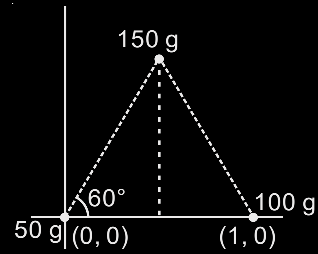
45. Answer (2)

$$W_{\text{man}} = Mg_{\text{eff}}\ l$$

$$g_{\text{eff}} = g(1 + \theta_0^2)$$

$$W_{\text{man}} = Mgl(1 + \theta_0^2)$$

46. Answer (4)



$$X_{CM} = \frac{m\times 0 + 2m\times 1 + 3m\times\left(\frac{1}{2}\right)}{6\ m} = \frac{7}{12}\ \text{m}$$

$$Y_{CM} = \frac{m\times 0 + 2m\times 0 + 3m\times\left(\frac{\sqrt{3}}{2}\right)}{6\ m} = \frac{3\sqrt{3}}{12}\ \text{m}$$

$$= \frac{\sqrt{3}}{4}\ \text{m}$$

47. Answer (1)

$$I = \frac{Ml^2}{12} + M\times\left(\frac{l^2}{16}\right) = \frac{7Ml^2}{48}$$

$$\therefore \ MK^2 = \frac{7Ml^2}{48}$$

$$\Rightarrow K = \sqrt{\frac{7}{48}}\ l$$

48. Answer (3)

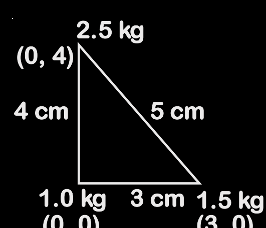
$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}m(\omega r)^2 + \frac{1}{2}\times\frac{mr^2}{2}\times\omega^2$$

$$\Rightarrow \ mgh = \frac{3}{4}m\omega^2 r^2$$

$$\Rightarrow \ \omega = \sqrt{\frac{4gh}{3r^2}} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$$

49. Answer (3)



$$X_{cm} = \frac{1\times 0 + 1.5\times 3 + 2.5\times 0}{1+1.5+2.5} = \frac{1.5\times 3}{5} = 0.9 \text{ cm}$$

$$Y_{cm} = \frac{1\times 0 + 1.5\times 0 + 2.5\times 4}{1+1.5+2.5} = \frac{2.5\times 4}{5} = 2 \text{ cm}$$

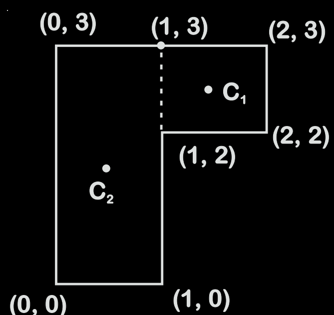
50. Answer (3)

$$I = \int dmr^2 = \int \sigma\, 2\pi r dr \cdot r^2$$

$$\Rightarrow \quad I = 2\pi\int_0^a (A+Br)r^3 dr = 2\pi\left[\frac{Aa^4}{4} + \frac{Ba^5}{5}\right]$$

$$\Rightarrow \quad I = 2\pi a^4\left[\frac{A}{4} + \frac{Ba}{5}\right]$$

51. Answer (1)



For given Lamina

A_1 = 1, C_1 = (1.5, 2.5)

A_2 = 3, C_2 = (0.5, 1.5)

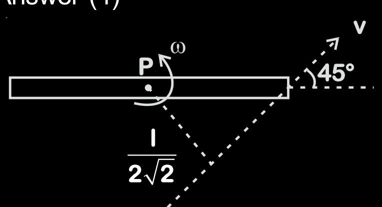
$$\therefore \quad X_{cm} = \frac{1.5+1.5}{4} = 0.75$$

$$Y_{cm} = \frac{2.5+4.5}{4} = 1.75$$

$\therefore$ Coordinate of centre of mass

(0.75, 1.75)

52. Answer (4)



About point P, angular momentum (L) of the system

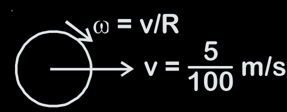
$$L_{\text{initial}} = \frac{mv}{\sqrt{2}} \times \frac{l}{2}$$

$$L_{final} = \left[\frac{(4m)l^2}{12} + \frac{ml^2}{4}\right]\omega$$

As $L_{\text{initial}} = L_{\text{final}}$

$$\omega = \frac{3\sqrt{2}v}{7l}$$

53. Answer (3)



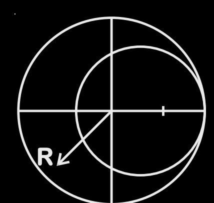
$$\text{KE} = \frac{1}{2}\left(\frac{2}{5}mR^2 + mR^2\right)\left(\frac{v}{R}\right)^2$$

$$\Rightarrow \quad KE = \frac{1}{2}mR^2 \times \frac{7}{5} \times \frac{v^2}{R^2} = \frac{7}{10} \times \frac{1}{2} \times \frac{25}{10^4}$$

$$\text{KE} = \frac{35}{4} \times 10^{-4} \text{ Joule}$$

KE = 8.75×10^{-4} Joule

54. Answer (1)



$$M_0 = \frac{4}{3}\pi R^3 \rho$$

$$M_{cavity} = \frac{4}{3}\pi(1)^3 \rho$$

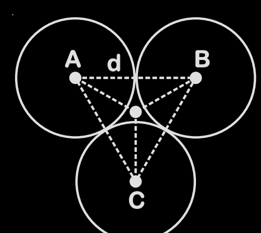
$$M_{(\text{Remaining})} = \frac{4}{3}\pi R^3 \rho - \frac{4}{3}\pi(1)^3 \rho$$

$$\therefore \quad \left(\frac{4}{3}\pi R^3 \rho - \frac{4}{3}\pi(1)^3 \rho\right) \times 2 + \frac{4}{3}\pi(1)^3 \rho = \frac{4}{3}\pi R^3 \rho \cdot R$$

$\Rightarrow \quad R^4 - 2R^3 + 1 = 0 \qquad \because \quad R \neq 1$

$\therefore \quad R^3 - R^2 - R - 1 = 0 \;\Rightarrow\; (R^2 + R + 1)(2 - R) = 1$

55. Answer (4)



$I_O = 3I_1$

$$I_1 = \frac{2}{5}m\left(\frac{d}{2}\right)^2 + m(AO)^2$$

$$AO = \frac{d}{\sqrt{3}}$$

$$\Rightarrow \quad I_O = \frac{13}{10}Md^2$$

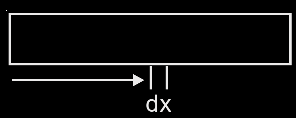
$$I_A = 2\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + Md^2\right] + \frac{2}{5}M\left(\frac{d}{2}\right)^2$$

$$= \frac{23}{10}Md^2$$

$$\frac{I_O}{I_A} = \frac{13}{23}$$

56. Answer (4)

$$X_{cm} = \frac{\int_0^L X\, dM}{M}$$



$$M = aL + \frac{bL}{3}$$

$$\int_0^L X\, dM = \int_0^L\left(aX + \frac{bX^3}{l^2}\right)dx = \left(\frac{aL^2}{2} + \frac{bL^2}{4}\right)$$

$$X_{cm} = \frac{3L}{4}\left(\frac{2a+b}{3a+b}\right)$$

57. Answer (2)

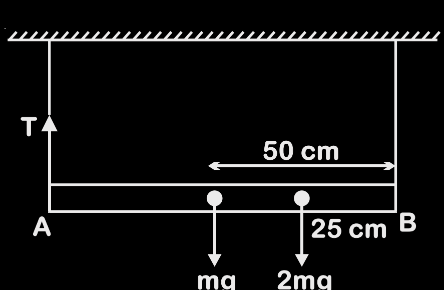
$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\frac{v^2}{r^2} = (m_1 - m_2)gh$$

$$v = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2 + \frac{I}{r^2}}}$$

$$w = \frac{V}{r}$$

58. Answer (4)

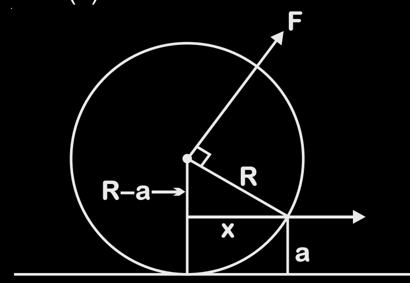


Net torque about B = 0

$\Rightarrow$ T × 100 = mg × 50 + 2mg × 25

$\Rightarrow$ T = mg

59. Answer (1)



To Step up,

F × R $\geq$ Mg × x

$$\Rightarrow F_{min} = \frac{Mg}{R}\times\sqrt{R^2 - (R-a)^2}$$

$$= Mg\sqrt{1 - \left(\frac{R-a}{R}\right)^2}$$

60. Answer (1)

$(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2$

$$\omega = \frac{0.1\times10 + 0.2\times5}{0.1 + 0.2}$$

$$= \frac{20}{3}\text{ rad/s}$$

$$K.E. = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$= \frac{20}{3}\text{J}$$

61. Answer (4)

$$I = M\left[\frac{R^2}{4} + \frac{L^2}{12}\right]$$

$M = \rho\pi R^2 L$

$$\therefore\ I = M\left[\frac{M}{4\rho\pi L} + \frac{L^2}{12}\right]$$

$$\therefore\ \frac{dI}{dL} = 0 \Rightarrow L^3 = \frac{6}{4}\frac{M}{\rho\pi}$$

$$\therefore\ \frac{L^3}{LR^2} = \frac{6}{4}$$

$$\Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

62. Answer (3)

$$mvl = \left(\frac{Ml^2}{3} + ml^2\right)\omega$$

$$\frac{1}{2}\left(\frac{Ml^2}{3} + ml^2\right)\omega^2 = \left[Mg\frac{l}{2} + mgl\right](1-\cos\theta)$$

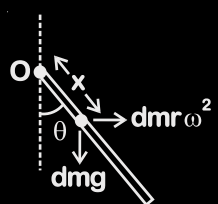
$$\omega = \frac{18}{5}\text{rad/s}$$

$$\frac{1}{2}\left(\frac{5}{3}\right)\times\frac{324}{25} = (10+10)(1-\cos\theta)$$

$$20(1-\cos\theta) = \frac{324}{30}$$

$$\cos\theta = 1-\frac{324}{600} = \frac{276}{600} = 0.46$$

63. Answer (4)



From a rotating frame rod will appear in equilibrium. Net torque about suspension point must be zero.

$$\int_0^L\left(\frac{M}{L}dx\right)(x\sin\theta)\omega^2 x\cos\theta = Mg\frac{L}{2}\sin\theta$$

$$\Rightarrow \cos\theta = \frac{3g}{2l\omega^2}$$

64. Answer (2)

$$I_O = \frac{M}{12}\left((80)^2 + (60)^2\right)$$

$$I_{O'} = I_O + M(50)^2$$

$$\Rightarrow \frac{I_O}{I_{O'}} = \frac{\frac{M}{12}(10000)}{\frac{M}{12}(10000)+(M)(2500)} = \frac{1}{4}$$

65. Answer (2)

$$|U_f - U_i| = \frac{1}{2}I\omega^2$$

$$20\times4\times\frac{\sqrt{3}}{2} - 0 = \frac{1}{2}\times(0.8)\omega^2$$

$\Rightarrow$ w= 13.16 rad/s

66. Answer (1)

$I = MR^2$

$M \propto R^2$

$I \propto R^4$

$$\frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^4$$

67. Answer (4)

$I\omega = 4I\omega'$

$$\Rightarrow \omega' = \frac{\omega}{4}$$

$$(\Delta K)_{Loss} = \frac{1}{2}I\omega^2 - \frac{1}{2}(4I)\left(\frac{\omega}{4}\right)^2$$

$$\Rightarrow \text{Fractional loss} = \frac{(\Delta K)_{Loss}}{K_1} \quad \left(K_1 = \frac{1}{2}I\omega^2\right)$$

$$= \frac{3}{4}$$

68. Answer (3)

$$F = \frac{dP}{dt} = \left(\frac{dM}{dt}\right)v$$

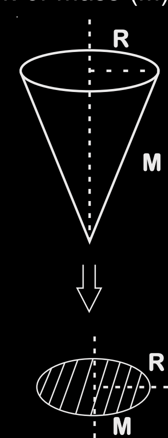
$$\Rightarrow M(t)a = -(bv^2)v$$

$$\Rightarrow a = -\frac{bv^3}{M(t)}$$

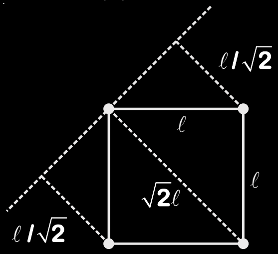
69. Answer (2)

$$I = \frac{MR^2}{2}$$

Moment of inertia of this cone will same as circular disk of mass (M) and radius R.



70. Answer (1)



$$I = m\left(\frac{\ell^2}{2}\right)\times 2 + m\times(\sqrt{2}\ell)^2 = 3m\ell^2$$

$\therefore$ L = Iω = $3m\ell^2\omega$

71. Answer (4)

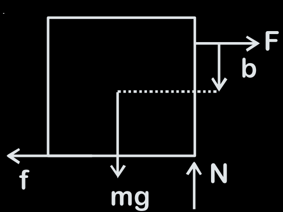
$$M = \int_0^L \lambda dx = \int_0^L \lambda_0\left(1+\frac{x}{L}\right)dx$$

$$= \frac{3}{2}\lambda_0 L$$

$$I = \int_0^L (\lambda dx)x^2 = \frac{7}{12}\lambda_0 L^3$$

$$I = \frac{7}{18}ML^2$$

72. Answer (75)



When the block slides,

F = f = 0.4 mg

For the block not to topple

$$F\left(\frac{a}{2}+b\right) < mg\frac{a}{2}$$

$$\Rightarrow 0.4\, mg\left(\frac{a}{2}+b\right) < mg\frac{a}{2}$$

$\Rightarrow$ 0.2 a + 0.4 b < 0.5 a

$$\Rightarrow \frac{b}{a} < \frac{3}{4}$$

$\Rightarrow$ Maximum possible value of $\frac{100\,b}{a}$ is 75

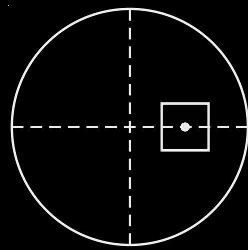
73. Answer (15)

ΔPE = ΔKE

$$\Rightarrow (mg)\frac{l}{2}\sin 30^\circ = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$\Rightarrow$ $\omega = \sqrt{15}$ rad/s

74. Answer (23)



Let the density is σ

Then original mass $m_0 = \pi a^2\sigma$

Remaining mass

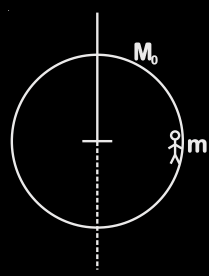
$$m' = \left(\pi a^2 - \frac{a^2}{4}\right)\sigma = a^2\left(\frac{4\pi - 1}{4}\right)\sigma$$

Removed mass $m = \frac{a^2}{4}\cdot\sigma$

$$\therefore\ \pi a^2\sigma(0) = \frac{a^2}{4}\cdot\sigma\times\frac{a}{2} + \frac{\sigma a^2}{4}(4\pi - 1)r$$

$$\Rightarrow r = \frac{-a}{2(4\pi-1)} = \frac{-a}{23.13} \approx \frac{-a}{23}$$

75. Answer (09.00)



M_0 = 200 kg

m = 80 kg

$$I = (I_M + I_m) = \left(\frac{M_0R^2}{2} + mR^2\right)$$

$\therefore$ $I_1W_1 = I_2W_2$

$$\therefore\ W_2 = \left(\frac{M_0R^2}{2} + mR^2\right)\times\frac{5}{\frac{M_0R^2}{2}}$$

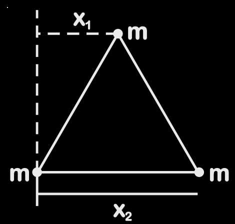
$$= \frac{5R^2}{R^2}\times\frac{(80+100)}{100}$$

W_2 = 9 RPM

76. Answer (25)

$x_1 = \frac{a}{2}$; $x_2 = a$

and $x_3 = 0$



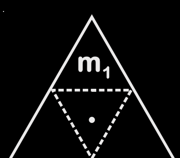
$$I = \frac{ma^2}{4} + ma^2 + 0$$

$$= \frac{5}{4}ma^2$$

$$I = \frac{25}{20}ma^2 = \frac{N}{20}ma^2$$

$\therefore$ N = 25

77. Answer (11)



Let I_0 is about 'G' mass be m, and length be a

$\therefore$ $kma^2 = I_0$ ...(i)

Then MI of $(m_1) = \frac{km}{4}\frac{a^2}{4} = \frac{I_0}{16}$

Let moment of inertia of one triangular part (side) be I_1 about 'G' then

$$3I_1 + \frac{I_0}{16} = I_0 \Rightarrow 3I_1 = I_0 - \frac{I_0}{16} = \frac{15I_0}{16}$$

$$\Rightarrow I_1 = \frac{5I_0}{16}$$

So moment of inertia of remaining portion

$$I'' = I_0 - \frac{5I_0}{16}$$

$$\Rightarrow I'' = \frac{11}{16}I_0$$

78. Answer (20.00)

$I_1\omega_1 = I_2\omega_2$

$$\omega_2 = \frac{\left(\frac{MR^2}{2}\right)\omega_1}{\left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)} = \frac{MR^2}{2}\frac{\omega_1 \times 8}{MR^2 \times 5} = \frac{4\omega_1}{5}$$

$\therefore$

$$\left(\frac{\Delta E}{E} \times 100\right) = P = \left(\frac{\frac{1}{2}\cdot\frac{MR^2}{2}\cdot\omega_1^2 - \frac{1}{2}\frac{5MR^2}{8}\frac{16}{25}\omega_1^2}{\frac{1}{2}\frac{MR^2}{2} \times \omega_1^2}\right) \times 100$$

$$= \frac{\frac{1}{2}\frac{MR^2}{2}\omega_1^2\left(1 - \frac{4}{5}\right)}{\frac{1}{2}\frac{MR^2}{2}\cdot\omega_1^2} \times 100 = 20$$

79. Answer (195.00)

$$\vec{\tau} = \vec{r} \times \vec{f}$$

$$\vec{r} = \left(4\hat{i} + 3\hat{i} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + \hat{k}\right)$$

$$\Rightarrow \vec{r} = \left(3\hat{i} + j - 2\hat{k}\right)$$

$$\vec{\tau} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) \times \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$\vec{\tau} = 6\hat{k} - 9\hat{j} - \hat{k} + 3\hat{i} - 2\hat{i} + 4\hat{i}$$

$$\vec{\tau} = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

$$\therefore \quad (\tau) = \sqrt{195}$$

80. Answer (20.00)

Angular momentum will remain conserved about pivotal point

$$(0.1 \times 80 \times 1) = \left[\left(\frac{1}{10} \times 1\right) + \left(\frac{9}{10} \times \frac{(1)}{3}\right)\right]\omega$$

$$\Rightarrow \omega = \frac{8 \times 10}{4} = 20 \text{ rad/s}$$

81. Answer (03)

$$x = \frac{3R}{8} = 3 \text{ cm}$$

82. Answer (2)

$$I_1 = \frac{MR^2}{2}$$

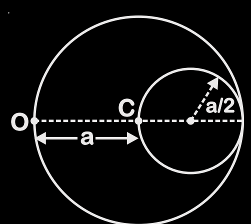
$$I_2 = \frac{MR^2}{2}$$

$$I_3 = \frac{MR^2}{2}$$

$$I_4 = \frac{2}{5}MR^2$$

83. Answer (4)

Let mass of hole be (–m), then mass of complete disc will be 4m.

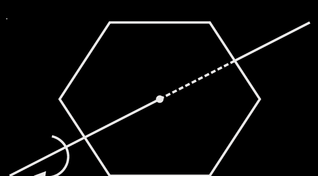


$\therefore$ Centre of mass w.r.t. point O is

$$X_{CM} = \frac{4m \times a + (-m) \times \left(\frac{3}{2}a\right)}{(4m - m)} = \frac{5}{6}a$$

84. Answer (8)

$$m = \frac{M}{6} \text{ and } \ell = \frac{L}{6}$$



$$I = 6\left(\frac{m\ell^2}{12} + \frac{3m}{4}\ell^2\right)$$

$$= 5\ m\ell^2$$

$$= \frac{5}{216}ML^2$$

$$= \frac{5}{216} \times 6 \times (24)^2$$

$$= 0.8$$

85. Answer (3)

$$K_T = \frac{1}{2} \times \frac{7}{5} mv_0^2$$

$$\frac{7}{10}mv_0^2 = mg\ell\sin\theta$$

$$\ell = \frac{7v_0^2}{10g\sin\theta}$$

given answer in official option is $\frac{10v_0^2}{7g\sin\theta}$.

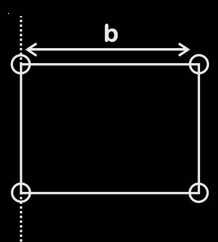
86. Answer (180)

$\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$

$\Rightarrow \quad \vec{P} \times \vec{Q} = 0$

$\Rightarrow \quad \theta = 0°$ or $180°$

87. Answer (3)



$$I = \frac{2}{5}ma^2 \times 4 + 2 \times mb^2$$

$$= \frac{8}{5}ma^2 + 2mb^2$$

88. Answer (3)

$\Delta PE = \Delta KE$

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}m(r\omega)^2$$

$$\Rightarrow \quad \omega^2 = \frac{2mgh}{I + mr^2}$$

89. Answer (1)

$$I_A = m \times \left(\frac{l}{\sqrt{2}}\right)^2 \times 2 + m \times \left(\sqrt{2}\,l\right)^2$$

$$= \frac{ml^2}{2} \times 2 + 2ml^2$$

$= 3ml^2$

90. Answer (20)

$$\alpha = \frac{F.R}{I}$$

$$= \frac{20 \times 0.2}{\frac{1}{2} \times 20 \times 0.2^2}$$

$$= 10 \text{ rad / s}^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$n = \frac{\theta}{2\pi} + \frac{\omega^2}{4\pi\alpha} = \frac{2500}{4 \times 3.14 \times 10} = 19.90$$

91. Answer (30)

Using conservation of linear momentum

$-mv_1 \sin\theta + mv_2 = 0$

$$\sin\theta = \frac{v_2}{v_1} = \frac{10}{20}$$

$\theta = 30°$

92. Answer (12°)

$$\delta = \delta_1 \left(1 - \frac{\omega_1}{\omega_2}\right)$$

$$2 = \delta_1 \left(1 - \frac{.02}{.03}\right)$$

$$\Rightarrow \quad \delta_1 = 6 = A(1.5 - 1) \quad \Rightarrow \quad A = 12°$$

93. Answer (3)

Binding energy of uniform sphere = $\frac{3}{5}\frac{GM^2}{R}$

94. Answer (2)

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{r}_{OP} = \left(5\hat{i} + \frac{5\sqrt{3}}{2}\hat{j}\right)$$

$$\vec{\tau} = 5\left(\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) \times \left(4\hat{i} - 3\hat{j}\right)$$

$$= 5\left(-3\hat{k} - 4\sqrt{3}\hat{k}\right)$$

$$= \left(-15 - 20\sqrt{3}\right)\hat{k}$$

$$\overrightarrow{r_{QP}} = \left(-5\hat{i} + 5\sqrt{3}\hat{j}\right)$$

$$\vec{\tau} = \vec{r}_{QP} \times \vec{f}$$

$$= \left(15 - 20\sqrt{3}\right)\hat{k}$$

95. Answer (1)

Net force on M is towards A, hence torque is zero about A.

$$\Rightarrow \vec{L}_A = \text{constant}$$

96. Answer (4)

The body having maximum acceleration will reach the bottom first.

$$a = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}$$

$\frac{K^2}{R^2}$ is least for solid sphere.

97. Answer (728)

$$\omega = \omega_0 + \alpha t$$

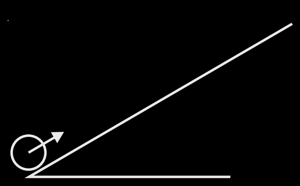
$$\alpha = 2\pi \text{ rad/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$2\pi n = 900 \times \frac{2\pi}{60} \times 26 + \frac{1}{2} \times 2\pi \times (26)^2$$

n = 728

98. Answer (3)



$$a = \frac{mg\sin\theta \times r^2}{\left(\frac{7}{5}mr^2\right)}$$

$$= \frac{5}{7}g\sin\theta$$

$$= \frac{5}{7} \times 10 \times \frac{1}{2} = \frac{25}{7} \text{ m/s}^2$$

$$\therefore \Delta t = 2\times\left(\frac{u}{a}\right)$$

$$= 2\times\frac{1}{\left(\frac{25}{7}\right)}$$

$$\simeq 0.57 \text{ s}$$

99. Answer (4)

$$\left(X_{CM}, Y_{CM}\right) = \left(\frac{4a}{3\pi}, \frac{4a}{3\pi}\right)$$

100. Answer (1)

External torque being absent, net angular momentum should be conserved about the axis of ring

$\Rightarrow$ $I_{ring}\omega = (I_{ring} + I_{particles})\omega_f$

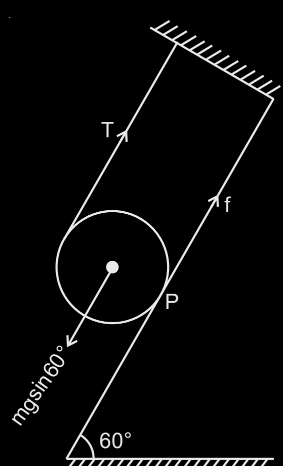
$$\Rightarrow \quad \omega_f = \frac{Mr^2\omega}{Mr^2 + 2mr^2}$$

$$\Rightarrow \quad \omega_f = \frac{M}{M+2m}\omega$$

101. Answer (4)

$T + f = mg\sin 60°$ ...(1)

$\vec{\tau}_p = 0$



$\Rightarrow$ $T \times 2R = mg\sin 60° \times R$

$$\Rightarrow \quad T = \frac{mg\sqrt{3}}{4} \qquad \text{...(2)}$$

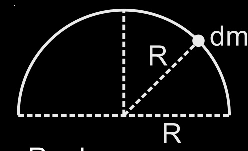
from (1) and (2)

$$f = \frac{mg\sqrt{3}}{2} - \frac{mg\sqrt{3}}{4} = \frac{mg\sqrt{3}}{4}$$

$$f^{max} = (\mu)\frac{mg}{2} = \frac{mg}{5}$$

$$f > f_s^{max} \quad \Rightarrow f = \frac{mg}{5}$$

102. Answer (4)



$$I = \int (dm)r^2$$

$$I = (R^2)\int dm$$

$\pi R = L$

$$= MR^2$$

$$\Rightarrow R = \frac{L}{\pi}$$

$$= \frac{ML^2}{\pi^2}$$

103. Answer (2)

Assuming frictionless

$$t_1 = \sqrt{\frac{2L}{g\sin\theta}}$$

When it rolls down

$$t_2 = \sqrt{\frac{\frac{2L}{g\sin\theta}}{\left(1+\frac{1}{2}\right)}} = \sqrt{\frac{3}{2}\cdot\frac{2L}{g\sin\theta}}$$

$$\frac{t_2}{t_1} = \sqrt{\frac{3}{2}}$$

104. Answer (4)

$mu = Mv$

$$u = v + \frac{\omega L}{2}$$

$$0 = \frac{MvL}{2} - \frac{ML^2}{12}\omega$$

$$\Rightarrow \quad \omega L = 6v$$

or $u = v + 3v$

$u = 4v$

$mu = Mv$

$$\Rightarrow \quad \frac{m}{M} = \frac{1}{4}$$

105. Answer (1)

$$\frac{k_T}{k_R} = \frac{k_T}{\frac{1}{2}k_T} = 2$$

$$\frac{MR^2}{I_{CM}} = 2$$

$$\Rightarrow \quad I_{CM} = \frac{MR^2}{2}$$

106. Answer (3)

$$v = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}$$

$$v_{ring} = \sqrt{gh}$$

$$v_{cylinder} = \sqrt{\frac{4}{3}gh}$$

$$\frac{v_{ring}}{v_{cylinder}} = \sqrt{\frac{3}{4}}$$

107. Answer (3)

$$\frac{K_T}{K_R} = \frac{MR^2}{I_{CM}}$$

I_{CM} is maximum for ring.

$\Rightarrow$ v is least for ring.

108. Answer (3)

m_{Bullet} = 4 g, M_{Gun} = 4 kg

$v_{Bullet} \simeq 50$ m/s

Now $P_B = P_g$

$P_g = m \times v_{Bullet}$

$= \frac{4}{1000} \times 50$

= 0.2 kg m/s

So impulse = 0.2 kg m/s

$$v_G = \frac{0.2}{M_{Gun}} = \frac{0.2}{4} = 0.05 \text{ m/s}$$

109. Answer (3)

$$\hat{n}_1 = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$$

$$\hat{n}_2 = \frac{\vec{A} \times \vec{C}}{|\vec{A} \times \vec{C}|} = \hat{k}$$

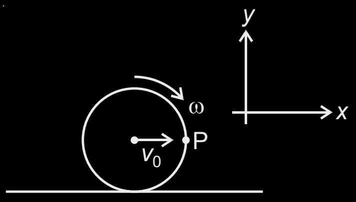
$$\cos\theta = \hat{n}_1 \cdot \hat{n}_2 = \frac{1}{\sqrt{3}}$$

110. Answer (2)

Centre of mass of half ring is located at a distance $\frac{2R}{\pi}$ from centre of the ring on its axis of symmetry so position of centre of mass in the given question will be $\left(0, \frac{2R}{\pi}\right)$

$\Rightarrow \quad |x| = 2$

111. Answer (02)



$$|\omega| = \frac{v_0}{R}$$

$$\vec{v}_P = v_0\hat{i} + \omega R(-\hat{j}) = v_0\hat{i} - v_0\hat{j}$$

$$|\vec{v}_P| = \sqrt{2}v_0$$

x = 02

112. Answer (3)

I_x, I_y, I_z are not equal.

So, radius of gyration can not be equal.

113. Answer (10)

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$= (10\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}) \times m(20\alpha t\hat{i} + 5\beta\hat{j})$$

$$= 50\alpha\beta mt(10-t)\hat{k}$$

$$\vec{L}_{t=0} = 0$$

$$\vec{L}_t = 0 \quad \Rightarrow \quad t = 10 \text{ sec.}$$

114. Answer (25)

$$m_1\frac{v_0}{4} + m_2v_2 = m_1v_0 \qquad \text{... (1)}$$

$$v_2 - \frac{v_0}{4} = v_0 \qquad \text{... (2)}$$

$$\Rightarrow \quad v_2 = 5 \text{ m/s and } m_2 = \frac{6}{5} \text{ kg}$$

$$v_c = \frac{m_1v_0}{m_1 + m_2} = 2.5 \text{ m/s}$$

115. Answer (4)

$\Delta L = \tau \Delta t$

$$\tau = \frac{\frac{1}{2}mR^2\omega}{10} = \frac{\frac{1}{2}\times 10\times 0.04\times 600\times 2\pi}{10\times 60}$$

$$= \frac{24\pi}{60}$$

$$= 0.4\pi$$

$$= 4 \times \pi \times 10^{-1}$$

116. Answer (4)

$$I_1 = \frac{MR^2}{2}$$

$$I_2 = \frac{mr^2}{4}$$

$$m = (\rho)\pi r^2 = \frac{m}{\pi R^2}\pi r^2 = \frac{Mr^2}{R^2}$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{2R^4}{r^4}$$

117. Answer (1)

(a) $I_1 = \frac{ML^2}{12}$

(b) $I_2 = \frac{(2M)L^2}{3} = \frac{2ML^2}{3}$

(c) $I_3 = \frac{(M)(2L)^2}{12} = \frac{ML^2}{3}$

(d) $I_4 = \frac{(2M)(2L)^2}{3} = \frac{8}{3}ML^2$

118. Answer (9)

$$\frac{1}{2}I_1\omega_1^2 = \frac{1}{2}I_2\omega_2^2$$

$$\frac{1}{2}I_1\left(\frac{v}{R_1}\right)^2 = \frac{1}{2}I_2\left(\frac{v}{R_2}\right)^2$$

$$\frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^2 = 9$$

119. Answer (52)

$$I = \left(\frac{1}{2}Mr^2 + M\left(\frac{13}{2}r\right)^2\right) + \left(\frac{M(6r)^2}{12} + M\left(\frac{5}{2}r\right)^2\right)$$

$$= Mr^2\left(\frac{1}{2} + \frac{169}{4} + 3 + \frac{25}{4}\right) = 52\,Mr^2$$

120. Answer (2)

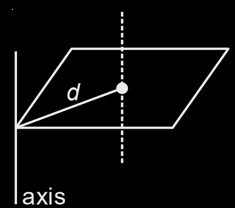
$$I = \frac{Mr^2}{2} + \frac{ML^2}{4}$$

$\Rightarrow$ M = 15 kg

$15 = \pi r^2 L\rho$

$\rho = 1.49 \times 10^2$ kg/m^3

121. Answer (1)



$I_{axis} = I_{cm} + Md^2$

$$= \frac{Ml^2}{6} + M\times\left(\frac{l}{\sqrt{2}}\right)^2$$

$$= \frac{Ml^2}{6} + \frac{Ml^2}{2}$$

$$= \frac{2}{3}Ml^2$$

122. Answer (4)

Using conservation of angular momentum

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

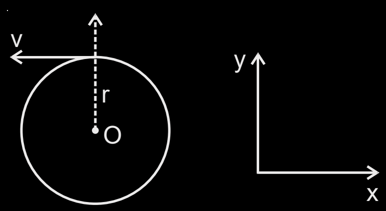
$$\text{Loss in KE.} = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

$$-\frac{1}{2}(I_1 + I_2)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$$

$$= \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2) - \frac{(I_1\omega_1 + I_2\omega_2)^2}{2(I_1 + I_2)}$$

$$= \frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$$

123. Answer (3)

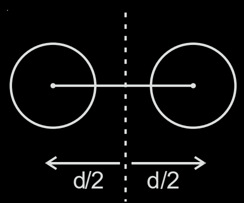


$\vec{L}_0 = \vec{r} \times \vec{p}$

$= mvr(\hat{k})$

It does not change in direction and magnitude.

124. Answer (2)



$$I = 2 \times \left[\frac{2}{5}mR^2 + \frac{md^2}{4}\right]$$

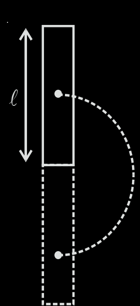
$$I = 2\left[\frac{2}{5} \times 1.5 \times (0.5)^2 + 1.5 \times (2.5)^2\right]$$

$= 1.2 \times (0.5)^2 + 3 \times [2.5]^2$

$I = 18.75 + 0.30$

$= 19.05$

125. Answer (6)



$$\frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2 = mg\ell$$

$$\Rightarrow \omega\ell = \sqrt{6g\ell} = 6 \text{ m/s}$$

126. Answer (20)

$\vec{\tau} = \vec{r} \times \vec{F}$

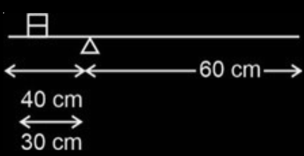
$= \left[(2-2)\hat{i} + (0-3)\hat{j} + (0-4)\hat{k}\right] \times \left(4\hat{i} + 3\hat{j} + 4\hat{k}\right)$

$= \left(-3\hat{j} - 4\hat{k}\right) \times \left(4\hat{i} + 3\hat{j} + 4\hat{k}\right)$

$= -16\hat{j} + 12\hat{k}$

$|\vec{\tau}| = 20$ units

127. Answer (6)



If λ is the mass per unit length of the scale then

$0.02 \times (30) \times 10 + \lambda 40 \times 20 \times 10 = \lambda 60 \times 30 \times 10$

$0.006 = \lambda 10$

Or $100\ \lambda = 0.06$ kg

$= 6 \times 10^{-2}$ kg

$x = 6$

128. Answer (2)

$\vec{\tau} = \vec{r} \times \vec{F}$

$= (2\hat{i} + \hat{j} + 2\hat{k}) \times (3\hat{i} + 4\hat{j} - 2\hat{k})$

$= -10\hat{i} + 10\hat{j} + 5\hat{k}$

129. Answer (5)

$$2\left(\frac{1}{2} + \frac{1}{4}\right) \times M(2R)^2 + \frac{1}{2}M(2R)^2 = x\frac{2}{5}M(2R)^2$$

$$\Rightarrow 1 + \frac{1}{2} + \frac{1}{2} = x \times \frac{2}{5}$$

$\Rightarrow x = 5$

130. Answer (3)

$I_1\omega_1 = I_2\omega_2$

$\Rightarrow MR^2 \times 2 = (MR^2 + 2mR^2)\omega_2$

$$\Rightarrow \omega_2 = \frac{2M}{M + 2m}$$

131. Answer (3)

For COM to remain unchanged,

$m_1x_1 = m_2x_2$

$\Rightarrow$ 10 × 6 = 30 × x_2

$\Rightarrow$ x_2 = 2 cm towards 10 kg block

132. Answer (3)

For rolling wheel

$[12g\sin\alpha - 3g\sin\alpha] \times R = (2 \times 12R^2 + 3R^2) \times \frac{a}{R}$

$\Rightarrow \frac{9g\sin\alpha}{27} = a$

$\Rightarrow a = \frac{g\sin\alpha}{3}$

$\therefore v = \sqrt{2 \times \frac{g\sin\alpha}{3} \times \frac{h}{\sin\alpha}} = \sqrt{\frac{2}{3}gh}$

$= \frac{1}{2} \times \sqrt{\frac{8}{3}gh}$

$\therefore x = \frac{8}{3} = 2.67$

133. Answer (1)

(A) Moment of inertia of solid sphere of radius R about a tangent $= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

$\Rightarrow$ A – (II)

(B) Moment of inertia of hollow sphere of radius R about a tangent $= \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$

$\Rightarrow$ B – (I)

(C) Moment of inertia of circular ring of radius (R) about its diameter $= \frac{(MR^2)}{2}$

$\Rightarrow$ C – (IV)

(D) Moment of inertia of circular ring of radius (R) about any diameter $= \frac{MR^2/2}{2} = \frac{MR^2}{4}$

$\Rightarrow$ D – (III)

134. Answer (91)

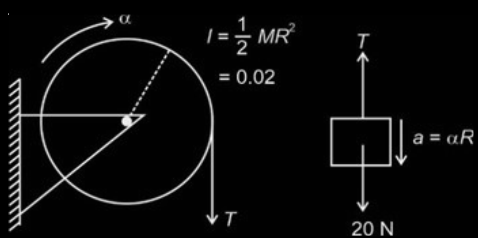
$|\vec{l}| = |\vec{r} \times (m\vec{v})|$

$= |(3\hat{i} - \hat{j}) \times (3\hat{j} + \hat{k})|$

$= |-\hat{i} - 3\hat{j} + 9\hat{k}|$

$= \sqrt{91}$

135. Answer (10)



20 – T = 2a

and 0.1 × T = 0.02 $\alpha = \frac{0.02a}{0.1}$

T = 2a

$\Rightarrow$ a = 5 m/sec^2

So T = 10 N

136. Answer (3)

$\vec{L}_0 = \vec{L}_{\text{of cm}} + \vec{L}_{\text{about cm}}$

$\Rightarrow \frac{a}{3}R^2\omega = mvR + \frac{2}{3}mR^2\omega = \frac{5}{3}mR^2\omega$

$\Rightarrow$ a = 5

137. Answer (8)

$I_1 = \frac{ML^2}{3}$ ....(1)

For ring : $I_2 = \frac{MR^2}{2}$

and $2\pi R = L$

$\Rightarrow I_2 = \frac{M}{2}\left(\frac{L^2}{4\pi^2}\right)$ ....(2)

$\Rightarrow \frac{I_1}{I_2} = \frac{8\pi^2}{3}$

$\Rightarrow$ x = 8

138. Answer (4)

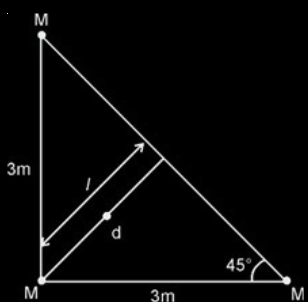
$$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$$

$$v = \sqrt{\frac{2Sg\sin\theta}{1+\frac{K^2}{R^2}}}$$

$$\Rightarrow \frac{v_c}{v_{ss}}\sqrt{\frac{1+\frac{K_{ss}^2}{R^2}}{1+\frac{K_c^2}{R^2}}} = \sqrt{\frac{1+\frac{2}{5}}{1+\frac{1}{2}}}$$

$$\Rightarrow \sqrt{\frac{\frac{7}{5}}{\frac{3}{2}}} = \sqrt{\frac{14}{15}}$$

139. Answer (2)

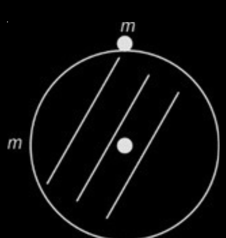


$$d_{cm} = 3\sin 45° = \frac{3}{\sqrt{2}}$$

$$d_{cm} = \frac{2}{3}\times\frac{3}{\sqrt{2}} = \sqrt{2} = \sqrt{x}$$

$x = 2$

140. Answer (5)



Loss in P.E = Gain in K.E.

$$2mg\,R = \frac{1}{2}\left[\frac{1}{2}mR^2 + mR^2\right]w^2$$

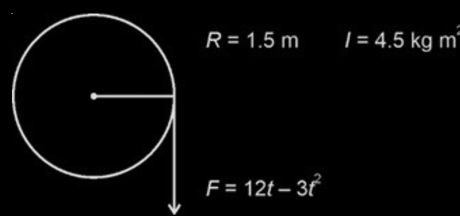
$$2mg\ R = \frac{1}{2}\times\frac{3}{2}mR^2\ w^2$$

$$w^2 = \frac{8g}{3R}$$

$$w = \sqrt{\frac{8g}{3R}} = 4\sqrt{\frac{g}{2\times 3R}}$$

$$\Rightarrow x = \frac{g}{2} = 5$$

141. Answer (18)



$FR = I\alpha$

$$\alpha = \frac{\left(12t - 3t^2\right)\times 1.5}{4.5} = 4t - t^2$$

$$w = \int \alpha dt = 2t^2 - \frac{t^3}{3}$$

$$w = 0$$

$$\Rightarrow t^2\left[2 - \frac{t}{3}\right] = 0$$

$t = 6$ sec

$$\theta = \int_0^6\left[2t^2 - \frac{t^3}{3}\right]dt = \left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6$$

$$= \left[\frac{2}{3}\times 6^3 - \frac{6^4}{12}\right] = 36$$

$$n = \frac{36}{2\pi}$$

$$= \frac{18}{\pi}$$

142. Answer (120)

$$\alpha = \frac{(mg)(r)}{\frac{3}{2}mr^2} = \frac{2g}{3r}$$

$$\Rightarrow \quad a = \frac{2g}{3}$$

$$\Rightarrow \quad v^2 = 2as$$

$$16 = \frac{40}{3} \times s \Rightarrow s = 0.3 \times 4 = 120 \text{ cm}$$

143. Answer (3)

$$I = 2 \times \left(\frac{M\left(\frac{a}{2}\right)^2}{4} \right) + 2 \times \left(\frac{M\left(\frac{a}{2}\right)^2}{4} + M\left(\frac{a}{2}\right)^2 \right)$$

$$= \frac{Ma^2}{8} + \frac{5Ma^2}{8} = \frac{6Ma^2}{8} = \frac{3}{4}Ma^2$$

144. Answer (8)

$$\rho = \rho_0 \left(1 - \frac{x^2}{L^2}\right) \text{kg/m}$$

$$x_{cm} = \frac{A\int_0^L \rho_0 \left(1 - \frac{x^2}{L^2}\right) x\,dx}{A\int_0^L \rho_0 \left(1 - \frac{x^2}{L^2}\right) dx}$$

$$x_{cm} = \frac{\frac{L^2}{2} - \frac{L^2}{4}}{L - \frac{L}{3}} = \frac{\frac{L^2}{4}}{\frac{2L}{3}} = \frac{3L}{8}$$

$$\Rightarrow \quad \alpha = 8$$

145. Answer (1)

$$\bar{r}_{com} = \frac{m_1\bar{r}_1 + m_2\bar{r}_2}{m_1 + m_2}$$

$$= \frac{(1-9)\hat{i} + (2-6)\hat{j} + (1+3)\hat{k}}{4}$$

$$= \frac{-8\hat{i} - 4\hat{j} + 4\hat{k}}{4}$$

$$\bar{r}_{com} = -2\hat{i} - \hat{j} + \hat{k}$$

$$|\bar{r}| = \sqrt{4+1+1} = \sqrt{6}$$

$$\left|\hat{i} + 2\hat{j} + \hat{k}\right| = \sqrt{6}$$

146. Answer (3)

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 5 & 3 & -7 \end{vmatrix}$$

$$= \hat{i}(-14-3) + \hat{j}(5+14) + \hat{k}(6-10)$$

$$= -17\hat{i} + 19\hat{j} - 4\hat{k}$$

147. Answer (2)

$$KE_R = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2} \times \frac{2}{5} \times \omega^2 \times (mR^2)$$

$$KE_{\text{total}} = \frac{1}{2} \times \frac{7}{5} \times mR^2 \times \omega^2$$

$$\therefore \quad \frac{KE_R}{KE_{\text{total}}} = \frac{2}{7}$$

148. Answer (3)

For a head on elastic collision

$$v_2 = \frac{mu_1}{m+5m} + \frac{mu_1}{m+5m}$$

$$= \frac{2u_1}{6} \text{ or } \frac{u_1}{3}$$

Initial kinetic energy of first mass $= \frac{1}{2}mu_1^2$

Final kinetic energy of second mass $= \frac{1}{2} \times 5m\left(\frac{u_1}{3}\right)^2$

$$= \frac{5}{9}\left(\frac{1}{2}mu_1^2\right)$$

$\Rightarrow$ kinetic energy transferred = 55% of initial kinetic energy of first colliding mass

149. Answer (6)

$$v_m = \frac{(120+60)v_T}{60}$$

$$= \frac{180 \times 2}{60} = 6 \text{ m/s}$$

150. Answer (2)

Conserving momentum:

$$m(30\hat{i}) + m(40\hat{j}) + 2m(\vec{v}) = \vec{0}$$

$$\Rightarrow \quad \vec{v} = -15\hat{i} - 20\hat{j}$$

$$\Rightarrow \quad |\vec{v}| = 25 \text{ m/s}$$

151. Answer (3)

$F = \rho v^2 a$

$\Rightarrow$ 10 × 1 = 2 × acceleration

$\Rightarrow$ Acc. = 5 m/s^2

152. Answer (5)

$$I = \frac{ML^2}{12} = MK^2$$

$$K = \frac{L}{\sqrt{12}} = \frac{10\sqrt{3}}{\sqrt{12}} = 5 \text{ m}$$

❑ ❑ ❑