Chapter 5

Sequences and Series

1. The sum to infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+.....$ is **[AIEEE-2009]**

(1) 3 (2) 4

(3) 6 (4) 2

2. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference –2, then the time taken by him to count all notes is **[AIEEE-2010]**

(1) 24 minutes (2) 34 minutes

(3) 125 minutes (4) 135 minutes

3. Let a_n be the n^{th} term of an A.P.

If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is **[AIEEE-2011]**

(1) $\beta - \alpha$ (2) $\frac{\alpha-\beta}{200}$

(3) $\alpha - \beta$ (4) $\frac{\alpha-\beta}{100}$

4. **Statement-1:** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) +...+ (361 + 380 + 400) is 8000.

Statement-2: $\sum_{k=1}^{n}(k^3-(k-1)^3)=n^3$ for any natural number n. **[AIEEE-2012]**

(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(2) Statement-1 is true, statement-2 is true, statement-2 is **not** a correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is false.

(4) Statement-1 is false, statement-2 is true.

5. If 100 times the 100^{th} term of an AP with non-zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is **[AIEEE-2012]**

(1) 150 times its 50^{th} term

(2) 150

(3) Zero

(4) –150

6. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is **[JEE (Main)-2013]**

(1) $\frac{7}{81}(179-10^{-20})$ (2) $\frac{7}{9}(99-10^{-20})$

(3) $\frac{7}{81}(179+10^{-20})$ (4) $\frac{7}{9}(99+10^{-20})$

7. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=4$, then the value of $|\alpha - \beta|$ is **[JEE (Main)-2014]**

(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$

(3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

8. If $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + ... + 10(11)^9 = k(10)^9$, then k is equal to **[JEE (Main)-2014]**

(1) 100 (2) 110

(3) $\frac{121}{10}$ (4) $\frac{441}{100}$

9. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is **[JEE (Main)-2014]**

(1) $2-\sqrt{3}$ (2) $2+\sqrt{3}$

(3) $\sqrt{2}+\sqrt{3}$ (4) $3+\sqrt{2}$

10. If m is the A.M. of two distinct real numbers l and n ($l, n > 1$) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals. **[JEE (Main)-2015]**

(1) $4\, l^2mn$ (2) $4\, lm^2n$

(3) $4\, lmn^2$ (4) $4\, l^2m^2n^2$

11. The sum of first 9 terms of the series

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \text{........} \text{ is}$$

[JEE (Main)-2015]

(1) 71 (2) 96

(3) 142 (4) 192

12. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is **[JEE (Main)-2016]**

(1) $\frac{4}{3}$ (2) 1

(3) $\frac{7}{4}$ (4) $\frac{8}{5}$

13. If the sum of the first ten terms of the series

$$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \text{.....},$$

is $\frac{16}{5}m$, then m is equal to **[JEE (Main)-2016]**

(1) 101 (2) 100

(3) 99 (4) 102

14. For any three positive real numbers a, b and c,

$$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c).$$

Then **[JEE (Main)-2017]**

(1) b, c and a are in A.P.

(2) a, b and c are in A.P.

(3) a, b and c are in G.P.

(4) b, c and a are in G.P.

15. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and

$$f(x + y) = f(x) + f(y) + xy,\ \forall\ x, y \in R,$$

then $\sum_{n=1}^{10} f(n)$ is equal to **[JEE (Main)-2017]**

(1) 165 (2) 190

(3) 255 (4) 330

16. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that

$$\sum_{k=0}^{12} a_{4k+1} = 416 \text{ and } a_9 + a_{43} = 66.$$

If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$, then m is equal to **[JEE (Main)-2018]**

(1) 66 (2) 68

(3) 34 (4) 33

17. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots..$

If $B - 2A = 100\lambda$, then λ is equal to

[JEE (Main)-2018]

(1) 232 (2) 248

(3) 464 (4) 496

18. Let $a_1, a_2 \ldots a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to **[JEE (Main)-2019]**

(1) 47 (2) 57

(3) 52 (4) 42

19. If a, b and c be three distinct real numbers in G.P. and $a + b + c = xb$, then x cannot be

[JEE (Main)-2019]

(1) 2 (2) –3

(3) –2 (4) 4

20. The sum of the following series

$$1 + 6 + \frac{9(1^2+2^2+3^2)}{7} + \frac{12(1^2+2^2+3^2+4^2)}{9} + \frac{15(1^2+2^2+\ldots+5^2)}{11} + \ldots$$

up to 15 terms, is **[JEE (Main)-2019]**

(1) 7830 (2) 7820

(3) 7520 (4) 7510

21. Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{2}$ (2) 4

(3) $\frac{7}{13}$ (4) 2

22. Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals **[JEE (Main)-2019]**

(1) 5^3

(2) 5^4

(3) $2(5^2)$

(4) $4(5^2)$

23. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is **[JEE (Main)-2019]**

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$

(3) $\frac{2}{3}$ (4) $\frac{4}{9}$

24. If 19^{th} term of a non-zero A.P. is zero, then its (49^{th} term) : (29^{th} term) is **[JEE (Main)-2019]**

(1) 2 : 1 (2) 1 : 3

(3) 4 : 1 (4) 3 : 1

25. Let $S_n = 1 + q + q^2 + ... + q^n$ and

$$T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$$

where q is a real number and $q \neq 1$.

If ${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + ... + {}^{101}C_{101} \cdot S_{100} = \alpha\, T_{100}$.

[JEE (Main)-2019]

(1) 200 (2) 202

(3) 2^{99} (4) 2^{100}

26. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is **[JEE (Main)-2019]**

(1) 36 (2) 32

(3) 24 (4) 28

27. Let $S_k = \frac{1+2+3+...+k}{k}$. If

$S_1^2 + S_2^2 + ... + S_{10}^2 = \frac{5}{12}A$, then A is equal to

[JEE (Main)-2019]

(1) 303 (2) 156

(3) 301 (4) 283

28. If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + ...$ is equal to 225 k, then k is equal to **[JEE (Main)-2019]**

(1) 108 (2) 27

(3) 9 (4) 54

29. The sum of all natural numbers 'n' such that 100 < n < 200 and H.C.F. (91, n) > 1 is :

[JEE (Main)-2019]

(1) 3303 (2) 3121

(3) 3203 (4) 3221

30. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to **[JEE (Main)-2019]**

(1) $2 - \frac{3}{2^{17}}$ (2) $1 - \frac{11}{2^{20}}$

(3) $2 - \frac{21}{2^{20}}$ (4) $2 - \frac{11}{2^{19}}$

31. Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, ...$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to

[JEE (Main)-2019]

(1) (50, 50 + 46A) (2) (A, 50 + 45A)

(3) (A, 50 + 46A) (4) (50, 50 + 45A)

32. The sum of the series 1 + 2 × 3 + 3 × 5 + 4 × 7 + ... upto 11^{th} term is **[JEE (Main)-2019]**

(1) 916 (2) 946

(3) 945 (4) 915

33. If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11^{th} term is **[JEE (Main)-2019]**

(1) –36 (2) 25

(3) –25 (4) –35

34. If $a_1, a_2, a_3, ..., a_n$ are in A.P. and $a_1 + a_4 + a_7 + ... + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to **[JEE (Main)-2019]**

(1) 98 (2) 38

(3) 64 (4) 76

35. The sum

$$\frac{3 \times 1^3}{1^2} + \frac{5 \times (1^3 + 2^3)}{1^2 + 2^2} + \frac{7 \times (1^3 + 2^3 + 3^3)}{1^2 + 2^2 + 3^2} + ...$$

upto 10^{th} term, is **[JEE (Main)-2019]**

(1) 620 (2) 600

(3) 680 (4) 660

36. The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+...$

$+\frac{1^3+2^3+3^3+...+15^3}{1+2+3+...+15}-\frac{1}{2}(1+2+3+...+15)$

is equal to **[JEE (Main)-2019]**

(1) 1860 (2) 620

(3) 660 (4) 1240

37. Let a, b and c be in G.P. with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If $3a$, $7b$ and $15c$ are the first three terms of an A.P., then the 4^{th} term of this A.P. is **[JEE (Main)-2019]**

(1) $\frac{2}{3}a$ (2) a

(3) $\frac{7}{3}a$ (4) $5a$

38. Let a_1, a_2, a_3, ... be an A.P. with $a_6 = 2$. Then the common difference of this A.P., which maximises the product $a_1\, a_4\, a_5$, is **[JEE (Main)-2019]**

(1) $\frac{2}{3}$ (2) $\frac{8}{5}$

(3) $\frac{3}{2}$ (4) $\frac{6}{5}$

39. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to **[JEE (Main)-2019]**

(1) –260 (2) –380

(3) –320 (4) –410

40. If a_1, a_2, a_3, ... are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is **[JEE (Main)-2019]**

(1) 150 (2) 280

(3) 200 (4) 120

41. The greatest positive integer k, for which $49^k + 1$ is a factor of the sum

$49^{125} + 49^{124} + ... + 49^2 + 49 + 1$, is

[JEE (Main)-2020]

(1) 65 (2) 60

(3) 32 (4) 63

42. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is

[JEE (Main)-2020]

(1) $\frac{21}{2}$ (2) 7

(3) 27 (4) 16

43. Let a_1, a_2, a_3 ... be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to **[JEE (Main)-2020]**

(1) –513 (2) –171

(3) $\frac{511}{3}$ (4) 171

44. If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ...$ is (102)m, then m is equal to

[JEE (Main)-2020]

(1) 5 (2) 20

(3) 25 (4) 10

45. Let $f: R \to R$ be such that for all $x \in R$ $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is **[JEE (Main)-2020]**

(1) 2 (2) 0

(3) 3 (4) 4

46. If the 10^{th} term of an A.P. is $\frac{1}{20}$ and its 20^{th} term is $\frac{1}{10}$, then the sum of its first 200 terms is

[JEE (Main)-2020]

(1) $50\frac{1}{4}$ (2) 50

(3) 100 (4) $100\frac{1}{2}$

47. The product

$2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \cdot ...$ to ∞ is equal to

[JEE (Main)-2020]

(1) $2^{\frac{1}{2}}$ (2) $2^{\frac{1}{4}}$

(3) 2 (4) 1

48. Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to **[JEE (Main)-2020]**

(1) 300 (2) 150

(3) 175 (4) 225

49. If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + ...$ is **[JEE (Main)-2020]**

(1) $\dfrac{x+y+xy}{(1+x)(1+y)}$ (2) $\dfrac{x+y-xy}{(1-x)(1-y)}$

(3) $\dfrac{x+y-xy}{(1+x)(1+y)}$ (4) $\dfrac{x+y+xy}{(1-x)(1-y)}$

50. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in **[JEE (Main)-2020]**

(1) $(-\infty, -9] \cup [3, \infty)$

(2) $[-3, \infty)$

(3) $(-\infty, -3] \cup [9, \infty)$

(4) $(-\infty, 9]$

51. If the sum of first 11 terms of an A.P., $a_1, a_2, a_3, ...$ is 0 ($a_1 \neq 0$), then the sum of the A.P., $a_1, a_3, a_5, ..., a_{23}$ is ka_1, where k is equal to **[JEE (Main)-2020]**

(1) $-\dfrac{121}{10}$ (2) $-\dfrac{72}{5}$

(3) $\dfrac{72}{5}$ (4) $\dfrac{121}{10}$

52. Let S be the sum of the first 9 terms of the series : $\{x + ka\} + \{x^2 + (k + 2)a\} + \{x^3 + (k + 4)a\} + \{x^4 + (k + 6)a\} + ...$ where $a \neq 0$ and $x \neq 1$. If $S = \dfrac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to **[JEE (Main)-2020]**

(1) –3 (2) 1

(3) –5 (4) 3

53. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is **[JEE (Main)-2020]**

(1) $\dfrac{1}{6}$ (2) $\dfrac{1}{4}$

(3) $\dfrac{1}{7}$ (4) $\dfrac{1}{5}$

54. If the sum of the series

$20 + 19\dfrac{3}{5} + 19\dfrac{1}{5} + 18\dfrac{4}{5} + ...$ upto n^{th} term is 488 and the n^{th} term is negative, then **[JEE (Main)-2020]**

(1) $n = 41$ (2) n^{th} term is $-4\dfrac{2}{5}$

(3) $n = 60$ (4) n^{th} term is –4

55. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is **[JEE (Main)-2020]**

(1) 3 : 1 (2) 5 : 3

(3) 9 : 7 (4) 33 : 31

56. If $1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + ...+ (1 - 20^2 \cdot 19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to **[JEE (Main)-2020]**

(1) (10, 103)

(2) (10, 97)

(3) (11, 97)

(4) (11, 103)

57. Let $a_1, a_2, ...a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + ...+ a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to **[JEE (Main)-2020]**

(1) (2490, 249) (2) (2480, 249)

(3) (2490, 248) (4) (2480, 248)

58. If $3^{2 \sin 2\alpha - 1}$, 14 and $3^{4 - 2 \sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P is **[JEE (Main)-2020]**

(1) 65 (2) 78

(3) 81 (4) 66

59. If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + ... + 2 \cdot 3^9 + 3^{10} = S - 2^{11}$, then S is equal to **[JEE (Main)-2020]**

(1) $2 \cdot 3^{11}$ (2) $3^{11} - 2^{12}$

(3) $\dfrac{3^{11}}{2} + 2^{10}$ (4) 3^{11}

60. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is **[JEE (Main)-2020]**

(1) $\dfrac{2}{13}(3^{50} - 1)$ (2) $\dfrac{1}{13}(3^{50} - 1)$

(3) $\dfrac{1}{26}(3^{49} - 1)$ (4) $\dfrac{1}{26}(3^{50} - 1)$

61. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \dots$ is 460, then x is equal to **[JEE (Main)-2020]**

(1) 7^2
(2) e^2
(3) $7^{1/2}$
(4) $7^{46/21}$

62. Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then **[JEE (Main)-2020]**

(1) a, c, p are in G.P.
(2) a, b, c, d are in A.P.
(3) a, c, p are in A.P.
(4) a, b, c, d are in G.P.

63. The common difference of the A.P. $b_1, b_2, \dots, b_m$ is 2 more than the common difference of A.P. $a_1, a_2, \dots, a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to **[JEE (Main)-2020]**

(1) –127
(2) –81
(3) 127
(4) 81

64. The sum $\sum_{k=1}^{20}(1+2+3+\dots+k)$ is ___________. **[JEE (Main)-2020]**

65. The sum, $\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}$ is equal to **[JEE (Main)-2020]**

66. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is ________. **[JEE (Main)-2020]**

67. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots \text{ to } \infty\right)}$ is equal to _________. **[JEE (Main)-2020]**

68. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to _________ . **[JEE (Main)-2020]**

69. Let $A = \{n \in N : n \text{ is a 3-digit number}\}$

$B = \{9k + 2 : k \in N\}$

and $C = \{9k + l : k \in N\}$ for some l $(0 < l < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to _______. **[JEE (Main)-2021]**

70. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1-\sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is _______. **[JEE (Main)-2021]**

71. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is : **[JEE (Main)-2021]**

(1) $\frac{69}{256}$
(2) $-\frac{71}{256}$
(3) $-\frac{69}{256}$
(4) $\frac{71}{256}$

72. The sum of first four terms of a geometric progression (G.P) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is ______. **[JEE (Main)-2021]**

73. If $0 < \theta, \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty}\cos^{2n}\theta,\ y = \sum_{n=0}^{\infty}\sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty}\cos^{2n}\theta \cdot \sin^{2n}\phi$ then : **[JEE (Main)-2021]**

(1) $xyz = 4$
(2) $xy - z = (x + y)z$
(3) $xy + yz + zx = z$
(4) $xy + z = (x + y)z$

74. Let $A_1, A_2, A_3, \dots$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is___________. **[JEE (Main)-2021]**

75. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where a, $x \in R$ and $a > 0$, is equal to : **[JEE (Main)-2021]**

(1) $a + 1$
(2) $2\sqrt{a}$
(3) $a + \frac{1}{a}$
(4) $2a$

76. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of $4^{th}, 6^{th}$ and 8^{th} term is equal to :

[JEE (Main)-2021]

(1) 26 (2) 35

(3) 30 (4) 32

77. The sum of infinite series

$$1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\ldots\ldots \text{is equal to :}$$

[JEE (Main)-2021]

(1) $\frac{13}{4}$ (2) $\frac{9}{4}$

(3) $\frac{11}{4}$ (4) $\frac{15}{4}$

78. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$ is equal to : **JEE (Main)-2021]**

(1) $\frac{41}{8}e+\frac{19}{8}e^{-1}-10$

(2) $\frac{41}{8}e-\frac{19}{8}e^{-1}-10$

(3) $-\frac{41}{8}e+\frac{19}{8}e^{-1}-10$

(4) $\frac{41}{8}e+\frac{19}{8}e^{-1}+10$

79. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence –16, 8, –4, 2, ... satisfy the equation $4x^2 - 9x + 5 = 0$, then p + q is equal to _______.

[JEE (Main)-2021]

80. Consider an arithmetic series and a geometric series having four initial terms from the set {11, 8, 21, 16, 26, 32, 4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to ______. **[JEE (Main)-2021]**

81. Let $S_n(x)=\log_{a^{1/2}}x+\log_{a^{1/3}}x+\log_{a^{1/6}}x+\log_{a^{1/11}}x+\log_{a^{1/18}}x+\log_{a^{1/27}}x+\ldots\ldots$ up to n-terms, where a > 1. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to ________. **[JEE (Main)-2021]**

82. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}, \frac{1}{b}, 6$ be in A.P., where a, b > 0. Then 72(a+b) is equal to_______.

[JEE (Main)-2021]

83. $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+\ldots\ldots+\frac{1}{(201)^2-1}$ is equal to

[JEE (Main)-2021]

(1) $\frac{101}{404}$ (2) $\frac{99}{400}$

(3) $\frac{25}{101}$ (4) $\frac{101}{408}$

84. If α, β, are natural numbers such that $100^{\alpha} - 199\beta$ = (100)(100)+(99)(101)+(98)(102)+.......+(1)(199), then the slope of the line passing through (α, β) and origin is: **[JEE (Main)-2021]**

(1) 550 (2) 530

(3) 540 (4) 510

85. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to :

[JEE (Main)-2021]

(1) 7000 (2) 5000

(3) 3000 (4) 1000

86. If $\sum_{r=1}^{10} r!\left(r^3+6r^2+2r+5\right)=\alpha(11!)$,

then the value of α is equal to _______.

[JEE (Main)-2021]

87. For k $\in$ N, let $\frac{1}{\alpha(\alpha+1)(\alpha+2)\ldots\ldots(\alpha+20)}=\sum_{k=0}^{20}\frac{A_k}{\alpha+k}$,

where $\alpha > 0$. Then the value of $100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$

is equal to_____. **[JEE (Main)-2021]**

88. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1, a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of

$47\sum_{n=1}^{\infty}\frac{a_n}{2^{3n}}$ is equal to ______. **[JEE (Main)-2021]**

89. Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to **[JEE (Main)-2021]**

(1) 1842 (2) 1852

(3) 1862 (4) 1872

90. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is **[JEE (Main)-2021]**

(1) 4 (2) 2

(3) 6 (4) 8

91. If the value of

$$\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+......\text{upto }\infty\right)^{\log_{(0.25)}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+......\text{upto }\infty\right)}$$

is l, then l^2 is equal to ________. **[JEE (Main)-2021]**

92. If $[x]$ be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to **[JEE (Main)-2021]**

(1) 2 (2) –2

(3) 0 (4) 4

93. If $\log_3 2$, $\log_3(2^x - 5)$, $\log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to ________. **[JEE (Main)-2021]**

94. If $\tan\left(\frac{\pi}{9}\right)$, x, $\tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right)$, y, $\tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to **[JEE (Main)-2021]**

(1) 0 (2) 1

(3) 3 (4) 4

95. The sum of the series

$$\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{2^2}{x^4+1}+...+\frac{2^{100}}{x^{2^{100}}+1}$$ when $x = 2$ is **[JEE (Main)-2021]**

(1) $1+\frac{2^{101}}{4^{2^{101}}-1}$ (2) $1-\frac{2^{100}}{4^{2^{100}}-1}$

(3) $1+\frac{2^{100}}{4^{2^{101}}-1}$ (4) $1-\frac{2^{101}}{4^{2^{101}}-1}$

96. If the sum of an infinite GP a, ar, ar^2, ar^3, ... is 15 and the sum of the squares of its each term is 150, then the sum of ar^2, ar^4, ar^6, ... is : **[JEE (Main)-2021]**

(1) $\frac{5}{2}$ (2) $\frac{9}{2}$

(3) $\frac{25}{2}$ (4) $\frac{1}{2}$

97. If $^1P_1 + 2 \cdot {}^2P_2 + 3 \cdot {}^3P_3 + ... + 15 \cdot {}^{15}P_{15} = {}^qP_r - s$, $0 \le s \le 1$, then $^{q+s}C_{r-s}$ is equal to ________. **[JEE (Main)-2021]**

98. $\lim_{x\to 2}\left(\sum_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$ is equal to : **[JEE (Main)-2021]**

(1) $\frac{7}{36}$ (2) $\frac{1}{5}$

(3) $\frac{5}{24}$ (4) $\frac{9}{44}$

99. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is ______. **[JEE (Main)-2021]**

100. Let a_1, a_2, ..., a_{10} be an AP with common difference –3 and b_1, b_2, ..., b_{10} be a GP with common ratio 2. Let $c_k = a_k + b_k$, k = 1, 2, ..., 10. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to ________. **[JEE (Main)-2021]**

101. Let $\frac{\sin A}{\sin B}=\frac{\sin(A-C)}{\sin(C-B)}$, where A, B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then **[JEE (Main)-2021]**

(1) a^2, b^2, c^2 are in A.P. (2) $b^2 - a^2 = a^2 + c^2$

(3) b^2, c^2, a^2 are in A.P. (4) c^2, a^2, b^2 are in A.P.

102. If $0 < x < 1$, then $\frac{3}{2}x^2+\frac{5}{3}x^3+\frac{7}{4}x^4+...$, is equal to **[JEE (Main)-2021]**

(1) $x\left(\frac{1+x}{1-x}\right)+\log_e(1-x)$

(2) $x\left(\frac{1-x}{1+x}\right)+\log_e(1-x)$

(3) $\frac{1+x}{1-x}+\log_e(1-x)$

(4) $\frac{1-x}{1+x}+\log_e(1-x)$

103. If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + ...$, then the value of e^{1+y} at $x = \frac{1}{2}$ is

[JEE (Main)-2021]

(1) 2e (2) $\frac{1}{2}e^2$

(3) $2e^2$ (4) $\frac{1}{2}\sqrt{e}$

104. Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to **[JEE (Main)-2021]**

(1) $7 - 7\sqrt{3}$ (2) $7 + \sqrt{3}$

(3) $7 - \sqrt{3}$ (4) $7 + 3\sqrt{3}$

105. The sum of 10 terms of the series

$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + ...$ is

[JEE (Main)-2021]

(1) $\frac{143}{144}$ (2) $\frac{99}{100}$

(3) $\frac{120}{121}$ (4) 1

106. The mean of 10 numbers

7 × 8, 10 × 10, 13 × 12, 16 × 14, ... is _____.

[JEE (Main)-2021]

107. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\frac{a_1 + a_2 + \ldots + a_{10}}{a_1 + a_2 + \ldots + a_p} = \frac{100}{p^2}, p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to

[JEE (Main)-2021]

(1) $\frac{19}{21}$ (2) $\frac{100}{121}$

(3) $\frac{21}{19}$ (4) $\frac{121}{100}$

108. If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \ldots$, then $160\ S$ is equal to ________. **JEE (Main)-2021]**

109. Let $S_n = 1\cdot(n-1) + 2\cdot(n-2) + 3\cdot(n-3) + \ldots + (n-1)\cdot 1, n \geq 4.$

The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal to

[JEE (Main)-2021]

(1) $\frac{e-1}{3}$ (2) $\frac{e}{3}$

(3) $\frac{e}{6}$ (4) $\frac{e-2}{6}$

110. Let $a_1, a_2, \ldots, a_{21}$ be an AP such that $\sum_{n=1}^{20}\frac{1}{a_n a_n + 1} = \frac{4}{9}$. If the sum of this AP is 189, then $a_6 a_{16}$ is equal to : **JEE (Main)-2021]**

(1) 36 (2) 57

(3) 72 (4) 48

111. If $\{a_i\}_{i=1}^{n}$, where n is an even integer, is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n} a_i = 192, \sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to :

(1) 48 (2) 96

(3) 92 (4) 104

[JEE (Main)-2022]

112. The sum of all the elements of the set $\{\alpha \in \{1, 2, \ldots, 100\} : \text{HCF}(\alpha, 24) = 1\}$ is

[JEE (Main)-2022]

113. If $\frac{1}{2\cdot 3^{10}} + \frac{1}{2^2\cdot 3^9} + \ldots + \frac{1}{2^{10}\cdot 3} = \frac{K}{2^{10}\cdot 3^{10}}$, then the remainder when K is divided by 6 is :

(1) 1 (2) 2

(3) 3 (4) 5

[JEE (Main)-2022]

114. For a natural number n, let $\alpha_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8}$ is ______________.

[JEE (Main)-2022]

115. The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, \ldots$ is equal to ______________.

[JEE (Main)-2022]

116. The sum $1 + 2 \times 3 + 3 \times 3^2 + \ldots + 10 \times 3^9$ is equal to

(1) $\frac{2 \cdot 3^{12} + 10}{4}$ (2) $\frac{19 \cdot 3^{10} + 1}{4}$

(3) $5 \cdot 3^{10} - 2$ (4) $\frac{9 \cdot 3^{10} + 1}{2}$

[JEE (Main)-2022]

117. Let $A = \sum_{i=1}^{10} \sum_{j=1}^{10} \min\{i, j\}$ and $B = \sum_{i=1}^{10} \sum_{j=1}^{10} \max\{i, j\}$. Then $A + B$ is equal to ____________.

[JEE (Main)-2022]

118. If $A = \sum_{n=1}^{\infty} \frac{1}{\left(3+(-1)^n\right)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{\left(3+(-1)^n\right)^n}$, then $\frac{A}{B}$ is equal to:

(1) $\frac{11}{9}$ (2) 1

(3) $-\frac{11}{9}$ (4) $-\frac{11}{3}$

[JEE (Main)-2022]

119. If a_1 (> 0), a_2, a_3, a_4, a_5 are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to ________. **[JEE (Main)-2022]**

120. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$, $abc \neq 0$, then :

(1) x, y, z are in A.P.

(2) x, y, z are in G.P.

(3) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

(4) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

[JEE (Main)-2022]

121. If the sum of the first ten terms of the series

$\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \ldots$ is $\frac{m}{n}$, where m and n are co-prime numbers, then $m + n$ is equal to __________.

[JEE (Main)-2022]

122. Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \cdots$ Then $4S$ is equal to

(1) $\left(\frac{7}{3}\right)^2$ (2) $\frac{7^3}{3^2}$

(3) $\left(\frac{7}{3}\right)^3$ (4) $\frac{7^2}{3^3}$

[JEE (Main)-2022]

123. If a_1, a_2, a_3 ..... and b_1, b_2, b_3 ..... are A.P., and $a_1 = 2$, $a_{10} = 3$, $a_1 b_1 = 1 = a_{10} b_{10}$, then $a_4 b_4$ is equal to

(1) $\frac{35}{27}$ (2) 1

(3) $\frac{27}{28}$ (4) $\frac{28}{27}$

[JEE (Main)-2022]

124. Let $A_1, A_2, A_3, \ldots$ be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to

(1) 33 (2) 37

(3) 43 (4) 47

[JEE (Main)-2022]

125. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1 : 7 and $a + n = 33$, then the value of n is:

(1) 21 (2) 22

(3) 23 (4) 24

[JEE (Main)-2022]

126. Let for $n = 1, 2, \ldots, 50$, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of $\frac{1}{26} + \sum_{n=1}^{50}\left(S_n + \frac{2}{n+1} - n - 1\right)$ is equal to ________.

[JEE (Main)-2022]

127. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$.

Then $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to : **[JEE (Main)-2022]**

(1) $\frac{6}{343}$ (2) $\frac{7}{216}$

(3) $\frac{8}{343}$ (4) $\frac{49}{216}$

128. The sum of the infinite series

$$1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+\frac{35}{6^4}+\frac{51}{6^5}+\frac{70}{6^6}+ \ldots\ldots$$ is equal to

(1) $\frac{425}{216}$ (2) $\frac{429}{216}$

(3) $\frac{288}{125}$ (4) $\frac{280}{125}$

[JEE (Main)-2022]

129. Let 3, 6, 9, 12, … upto 78 terms and 5, 9, 13, 17, … upto 59 terms be two series. Then, the sum of terms common to both the series is equal to ______.

[JEE (Main)-2022]

130. Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p},\frac{1}{q},\frac{1}{r},\frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to _________.

[JEE (Main)-2022]

131. Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_a + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to ______.

[JEE (Main)-2022]

132. Consider two G.Ps. 2, 2^2, 2^3, …. and 4, 4^2, 4^3, … of 60 and n terms respectively. If the geometric mean of all the 60 + n terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to

[JEE (Main)-2022]

(1) 560 (2) 1540

(3) 1330 (4) 2600

133. The series of positive multiples of 3 is divided into sets: {3}, {6, 9, 12}, {15, 18, 21, 24, 27},…… Then the sum of the elements in the 11th set is equal to ________.

[JEE (Main)-2022]

134. Suppose a_1, a_2, … a_n, … be an arithmetic progression of natural numbers. If the ration of the sum of first five terms to the sum of first nine terms of the progression is 5 : 17 and 110 < a_{15} < 120, then the sum of the first ten terms of the progression is equal to

[JEE (Main)-2022]

(1) 290 (2) 380

(3) 460 (4) 510

135. Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \leq 800\}$. Then, the value of $\sum_{n\in S} f(n)$ is equal to ________.

[JEE (Main)-2022]

136. Let the sum of an infinite G.P., whose first term is a and the common ratio is r, be 5. Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is 10ar, n^{th} term is a_n and the common difference is $10ar^2$, is equal to

[JEE (Main)-2022]

(1) 21 a_{11} (2) 22 a_{11}

(3) 15 a_{16} (4) 14 a_{16}

137. $$\frac{2^3-1^3}{1\times 7}+\frac{4^3-3^3+2^2-1^3}{2\times 11}+\frac{6^3-5^3+4^3-3^3+2^3-1^3}{3\times 15}+\ldots+\frac{30^3-29^3+28^3-27^3+\ldots+2^3-1^3}{15\times 63}$$ is equal to _______.

[JEE (Main)-2022]

138. Consider the sequence a_1, a_2, a_3, … such that $a_1 = 1$, $a_2 = 2$ and $a_{n+2} = \frac{2}{a_{n+1}} + a_n$ for n = 1, 2, 3, … If

$$\left(\frac{a_1+\frac{1}{a_2}}{a_3}\right)\cdot\left(\frac{a_2+\frac{1}{a_3}}{a_4}\right)\cdot\left(\frac{a_3+\frac{1}{a_4}}{a_5}\right)\cdots\left(\frac{a_{30}+\frac{1}{a_{31}}}{a_{32}}\right)$$

$= 2^{\alpha}({}^{61}C_{31})$, then α is equal to

[JEE (Main)-2022]

(1) –30 (2) –31

(3) –60 (4) –61

139. For $p, q, \in \mathbf{R}$, consider the real valued function $f(x) = (x - p)^2 - q$, $x \in \mathbf{R}$ and $q > 0$, Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $|f(a_i)| = 500$ for all i = 1, 2, 3, 4, then the absolute difference between the roots of $f(x) = 0$ is

[JEE (Main)-2022]

140. Let $x_1, x_2, x_3, \ldots, x_{20}$ be in geometric progression with $x_1 = 3$ and the common ratio $\frac{1}{2}$. A new data is constructed replacing each x_i by $(x_i - i)^2$. If $\overline{x}$ is the mean of new data, then the greatest integer less than or equal to $\overline{x}$ is _________.

[JEE (Main)-2022]

141. If $\frac{6}{3^{12}} + \frac{10}{3^{11}} + \frac{20}{3^{10}} + \frac{40}{3^9} + \ldots + \frac{10240}{3} = 2^n \cdot m$, where m is odd, then $m \cdot n$ is equal to ________.

[JEE (Main)-2022]

142. Let $S = \{1, 2, 3, \ldots, 2022\}$. Then the probability, that a randomly chosen number n from the set S such that HCF $(n, 2022) = 1$, is

[JEE (Main)-2022]

(1) $\frac{128}{1011}$ (2) $\frac{166}{1011}$

(3) $\frac{127}{337}$ (4) $\frac{112}{337}$

143. Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$, in the increasing powers of $\frac{1}{\sqrt[4]{3}}$ be $\sqrt[4]{6} : 1$. If the sixth term from the beginning is $\frac{\alpha}{\sqrt[4]{3}}$, then α is equal to __________.

[JEE (Main)-2022]

144. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1, \ \forall\ n \geq 0$.

Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to

[JEE (Main)-2022]

(1) 483 (2) 528

(3) 575 (4) 624

145. $\sum_{r=1}^{20}(r^2+1)(r!)$ is equal to

[JEE (Main)-2022]

(1) 22! – 21! (2) 22! – 2(21!)

(3) 21! – 2(20!) (4) 21! – 20!

146. If $\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}$, where m and n are co-prime, then $m + n$ is equal to

[JEE (Main)-2022]

147. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to ________.

[JEE (Main)-2022]

148. If $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + \ldots + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$, then the maximum value of a is **[JEE (Main)-2022]**

(1) 198 (2) 202

(3) 212 (4) 218

149. The sum $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$ is equal to

[JEE (Main)-2022]

(1) $\frac{7}{87}$ (2) $\frac{7}{29}$

(3) $\frac{14}{87}$ (4) $\frac{21}{29}$

150. Different A.P.'s are constructed with the first term 100, the last term 199, and integral common differences. The sum of the common differences of all such A.P.'s having at least 3 terms and at most 33 terms is ______.

[JEE (Main)-2022]

151. If $\frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + \frac{1}{4\times5\times6} + \ldots\ldots + \frac{1}{100\times101\times102} = \frac{k}{101}$

then 34 k is equal to ____________.

❑ ❑ ❑

Chapter 5

Sequences and Series

1. Answer (1)

Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$

$$S - 1 = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$$

$$\frac{S-1}{3} = \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \frac{14}{3^5} + ...$$

$$\Rightarrow \frac{2}{3}(S-1) = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...$$

$$\Rightarrow S - 1 = 1 + \frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + ...$$

$$\Rightarrow S = 2 + \frac{\frac{2}{3}}{1 - \frac{1}{3}}$$

$= 2 + 1$

$= 3$

2. Answer (2)

Number of notes person counts in 10 minutes.

$= 10 \times 150 = 1500$

Since, a_{10}, a_{11}, a_{12}, ... are in A.P. with common difference = –2

$\Rightarrow$ Let n be the time taken to count remaining 3000 notes, then

$$\frac{n}{2}[2 \times 148 + (n-1) \times -2] = 3000$$

$\Rightarrow n^2 - 149n + 3000 = 0$

$\Rightarrow (n - 24)(n - 125) = 0$

$\Rightarrow n = 24, 125$

Time taken by the person to count all notes

$= 10 + 24 = 34$ minutes

3. Answer (4)

$a_2 + a_4 + a_6 + ... + a_{200} = \alpha$ ...(i)

$a_1 + a_3 + a_5 + ... + a_{199} = \beta$ ...(ii)

$a_2 - a_1 = a_3 - a_2 = ... = d$ common difference.

subtract (i) & (ii)

$100\,d = \alpha - \beta$

$$d = \frac{\alpha - \beta}{100}$$

4. Answer (1)

5. Answer (3)

6. Answer (3)

S = 0.7 + 0.77 + 0.777 + ... upto 20 terms

$$= \frac{7}{9}[.9 + .99 + .999 + ...]$$

$$= \frac{7}{9}\left[\begin{array}{r}(1-0.1)+(1-0.01)+(1-0.001)\\ +\text{...upto 20 terms}\end{array}\right]$$

$$= \frac{7}{9}\left[20 - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + ... + \frac{1}{10^{20}}\right)\right]$$

$$= \frac{7}{9}\left(20 - \frac{\frac{1}{10}\left(1 - \frac{1}{10^{20}}\right)}{\left(1 - \frac{1}{10}\right)}\right)$$

$$= \frac{7}{9}\left[20 - \frac{1}{9}\left(1 - \frac{1}{10^{20}}\right)\right]$$

$$= \frac{7}{81}\left[179 + \frac{1}{10^{20}}\right]$$

$$= \frac{7}{81}\left[179 + 10^{-20}\right]$$

7. Answer (2)

$\because$ p, q, r are in AP

$2q = p + r$ ...(i)

Also $\frac{1}{\alpha}+\frac{1}{\beta}=4$

$\Rightarrow \frac{\alpha+\beta}{\alpha\beta}=4$

$= \frac{-\frac{q}{p}}{\frac{r}{p}} = 4 \Rightarrow q = -4r$ ...(ii)

From (i)

$2(-4r) = p + r$

$p = -9r$

$q = -4r$

$r = r$

Now $|\alpha-\beta| = \sqrt{(\alpha+\beta)^2-4\alpha\beta}$

$= \sqrt{\left(\frac{-q}{p}\right)^2 - \frac{4r}{p}}$

$= \frac{\sqrt{q^2-4pr}}{|p|}$

$= \frac{\sqrt{16r^2+36r^2}}{|-9r|}$

$= \frac{2\sqrt{13}}{9}$

8. Answer (1)

$10^9 + 2\cdot(11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$

$x = 10^9 + 2\cdot(11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9$

$\frac{11}{10}x = 11\cdot10^8 + 2\cdot(11)^2\cdot(10)^7 + \ldots + 9(11)^9 + 11^{10}$

$x\left(1-\frac{11}{10}\right) = 10^9 + 11(10)^8 + 11^2\times(10)^7 + \ldots + 11^9 - 11^{10}$

$\Rightarrow -\frac{x}{10} = 10^9\left(\frac{\left(\frac{11}{10}\right)^{10}-1}{\frac{11}{10}-1}\right) - 11^{10}$

$\Rightarrow -\frac{x}{10} = (11^{10}-10^{10}) - 11^{10} = -10^{10}$

$\Rightarrow x = 10^{11} = k\cdot10^9$

$\Rightarrow k = 100$

9. Answer (2)

$a, ar, ar^2 \rightarrow$ G.P.

$a, 2ar, ar^2 \rightarrow$ A.P.

$2 \times 2ar = a + ar^2$

$4r = 1 + r^2$

$\Rightarrow r^2 - 4r + 1 = 0$

$r = \frac{4\pm\sqrt{16-4}}{2} = 2\pm\sqrt{3}$

$\boxed{r = 2+\sqrt{3}}$

$r = 2-\sqrt{3}$ is rejected

$\because$ $(r > 1)$

G.P. is increasing.

10. Answer (2)

$\frac{l+n}{2} = m$

$l + n = 2m$ ...(i)

$G_1 = l\left(\frac{n}{l}\right)^{\frac{1}{4}}$

$G_2 = l\left(\frac{n}{l}\right)^{\frac{2}{4}}$

$G_3 = l\left(\frac{n}{l}\right)^{\frac{3}{4}}$

Now $G_1^4 + 2G_2^4 + G_3^3$

$l^4\cdot\frac{n}{l} + 2\cdot(l^2)\left(\frac{n}{l}\right)^2 + l^4\left(\frac{n}{l}\right)^3$

$= nl^3 + 2n^2l^2 + n^3l$

$= 2n^2l^2 + nl(n^2 + l^2)$

$= 2n^2l^2 + nl((n + l)^2 - 2nl)$

$= nl(n + l)^2$

$= nl\cdot(2m)^2$

$= 4\,nlm^2$

11. Answer (2)

$$t_n = \frac{\left[\frac{n(n+1)}{2}\right]^2}{n^2}$$

$$= \frac{(n+1)^2}{4}$$

$$= \frac{1}{4}\left[n^2 + 2n + 1\right]$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{2(n)(n+1)}{2} + 1\right]$$

$$= \frac{1}{4}\left[\frac{9\times10\times19}{6} + 9\times10 + 9\right]$$

= 96

12. Answer (1)

$a + d$, $a + 4d$, $a + 8d$ are in G.P.

$(a + 4d)^2 = (a + d)(a + 8d)$

$\Rightarrow a^2 + 8ad + 16d^2 = a^2 + 9ad + 8d^2$

$\Rightarrow 8d^2 = ad \Rightarrow \frac{a}{d} = 8$

$\therefore$ Common ratio $= \frac{a+4d}{a+d} = \frac{8+4}{8+1} = \frac{4}{3}$

13. Answer (1)

$$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + ... = \frac{16}{5}m$$

$$\Rightarrow \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right) + ...10 \text{ tens} = \frac{16}{6}m$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 [2^2 + 3^2 + 4^2 + 5^2 ...... 10 \text{ terms}] = \frac{16}{5}m$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 [2^2 + 3^2 + 4^2 .... + 11^2] = \frac{16}{5}m$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 [1^2 + 2^2 + 3^2 .... + 11^2 - 1^2] = \frac{16}{5}m$$

$$\Rightarrow \left(\frac{4}{5}\right)^2 \left[\frac{11.12.23}{6} - 1\right] = \frac{16}{5}m \text{ (given)}$$

$$\Rightarrow \frac{16}{25}[22.23 - 1] = \frac{16}{5}m$$

$$\Rightarrow \frac{1}{5}(505) = m$$

$\Rightarrow m = 101$

14. Answer (1)

$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$

$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - 45ab - 15bc - 75ac = 0$

$\Rightarrow (15a - 3b)^2 + (3b - 5c)^2 + (15a - 5c)^2 = 0$

It is possible when

$15a - 3b = 0$ and $3b - 5c = 0$ and $15a - 5c = 0$

$15a = 3b = 5c$

$\frac{a}{1} = \frac{b}{5} = \frac{c}{3}$

$\therefore$ b, c, a are in A.P.

15. Answer (4)

As, $f(x + y) = f(x) + f(y) + xy$

Given, $f(1) = 3$

Putting, $x = y = 1 \Rightarrow f(2) = 2f(1) + 1 = 7$

Similarly, $x = 1$, $y = 2 \Rightarrow f(3) = f(1) + f(2) + 2 = 12$

Now, $\sum_{n=1}^{10} f(n) = f(1) + f(2) + f(3) + ... + f(10)$

$= 3 + 7 + 12 + 18 + ... = S$ (let)

Now, $S_n = 3 + 7 + 12 + 18 + ... + t_n$

Again, $S_n = 3 + 7 + 12 + ... + t_{n-1} + t_n$

We get, $t_n = 3 + 4 + 5 + ...n$ terms

$$= \frac{n(n+5)}{2}$$

i.e., $S_n = \sum_{n=1}^{n} t_n = \frac{1}{2}\left\{\sum n^2 + 5\sum n\right\}$

$$= \frac{n(n+1)(n+8)}{6}$$

So, $S_{10} = \frac{10\times11\times18}{6} = 330$

16. Answer (3)

Let $a_1 = a$ and common difference $= d$

Given, $a_1 + a_5 + a_9 + ... + a_{49} = 416$

$\Rightarrow a + 24d = 32$ ...(i)

Also, $a_9 + a_{43} = 66 \Rightarrow a + 25d = 33$ ...(ii)

Solving (i) & (ii),

We get $d = 1$, $a = 8$

Now, $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$

$\Rightarrow 8^2 + 9^2 + ... + 24^2 = 140m$

$$\Rightarrow \frac{24\times25\times49}{6} - \frac{7\times8\times15}{6} = 140m$$

$\Rightarrow \boxed{m = 34}$

17. Answer (2)

$$A = 1^2 + 2.2^2 + 3^2 + ... + 2.20^2$$

$$= (1^2+2^2+3^2+....+20^2)+4(1^2+2^2+3^2+....+10^2)$$

$$= \frac{20\times21\times41}{6} + \frac{4\times10\times11\times21}{6}$$

= 2870 + 1540 = 4410

$$B = 1^2 + 2.2^2 + 3^2 + ... + 2.40^2$$

$$= (1^2+2^2+3^2+....+40^2)+4(1^2+2^2+3^2+....+20^2)$$

$$= \frac{40\times41\times81}{6} + \frac{4\times20\times21\times41}{6}$$

= 22140 + 11480 = 33620

$\Rightarrow$ $B - 2A = 33620 - 8820 = 24800$

$\Rightarrow$ $100\lambda = 24800$

$\lambda = 248$

18. Answer (3)

$$S = \frac{30}{2}[2a_1 + 29d]$$

$$T = \frac{15}{2}[2a_1 + 14(2d)]$$

According to Question $S - 2T = 75$

$\Rightarrow$ $30a_1 + 29.15d - 30a_1 - 30.14d = 75$

$\Rightarrow$ $d = 5$

Also, $a_5 = 27 \Rightarrow a_1 + 4d = 27 \Rightarrow a_1 = 7, d = 5$

So $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

19. Answer (1)

$\because$ a, b, c, are in G.P.

$$\boxed{b^2 = ac}$$

Now $a + b + c = xb$

$$\Rightarrow a + c = (x-1)b$$ Now square it

$$\Rightarrow a^2 + c^2 + 2ac = (x-1)^2 b^2$$

$$\Rightarrow a^2 + c^2 = (x-1)^2 ac - 2ac$$

$$\Rightarrow a^2 + c^2 = ac[(x-1)^2 - 2]$$

$$a^2 + c^2 = ac[x^2 - 2x - 1]$$

$\because$ $a^2 + c^2$ are positive

and $b^2 = ac$ which is also positive

so $x^2 - 2x - 1$ would be positive

but for $x = 2$, $x^2 - 2x - 1$ is negative

so x cannot take 2.

20. Answer (2)

$$S = 1 + 6 + \frac{9(1^2+2^2+3^2)}{7} +$$

$$\frac{12(1^2+2^2+3^2+4^2)}{9} + \frac{15(1^2+2^2+3^2+4^2+5^2)}{11} + ...$$

$$S = \frac{3\cdot(1)^2}{3} + \frac{6\cdot(1^2+2^2)}{5} + \frac{9\cdot(1^2+2^2+3^2)}{7} +$$

$$\frac{12\cdot(1^2+2^2+3^2+4^2)}{9} + ...$$

n^{th} term of the series

$$= t_n = \frac{3n\cdot(1^2+2^2+...+n^2)}{(2n+1)}$$

$$t_n = \frac{3n\cdot n(n+1)(2n+1)}{6(2n+1)} = \frac{n^3+n^2}{2}$$

$$\therefore \quad S_n = \Sigma t_n = \frac{1}{2}\left\{\left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6}\right\}$$

$$= \frac{n(n+1)}{4}\left(\frac{n(n+1)}{2} + \frac{2n+1}{3}\right)$$

$$\therefore \quad S_{15} = \frac{15\times16}{4}\left\{\frac{15\cdot16}{2} + \frac{31}{3}\right\}$$

$$= 60\times120 + 60\times\frac{31}{3}$$

= 7200 + 620

= 7820

21. Answer (2)

Let first term and common difference be A and D respectively.

$\therefore$ $a = A + 6D$, $b = A + 10D$

and $c = A + 12D$

$\because$ a, b, c are in G.P.

$\therefore$ $b^2 = a.c.$

$\therefore$ $(A + 10D)^2 = (A + 6D)(A + 12D)$

$\therefore$ $14D + A = 0$

$\therefore$ $A = -14D$

$\therefore$ $a = -8D$, $b = -4D$ and $c = -2D$

$$\therefore \quad \frac{a}{c} = \frac{-8D}{-2D} = 4$$

22. Answer (2)

Let $a_1 = a$, $a_2 = ar$, $a_3 = ar^2$ ... $a_{10} = ar^9$

where r = common ratio of given G.P.

As $\dfrac{a_3}{a_1} = 25$

$\Rightarrow \dfrac{ar^2}{a} = 25$

$\Rightarrow r = \pm 5$

Now, $\dfrac{a_9}{a_5} = \dfrac{ar^8}{ar^4} = r^4 = (\pm 5)^4 = 5^4$

23. Answer (3)

Let any series

$a, ar, ar^2, \ldots \infty$

So $\dfrac{a}{1-r} = 3$ ...(i)

Now sum of cubes of its terms is $\dfrac{27}{19}$

So $a^3, a^3r^3, \ldots,$

$\dfrac{a^3}{1-r^3} = \dfrac{27}{19}$

$\Rightarrow \dfrac{a}{1-r} \times \dfrac{a^2}{(1+r^2+r)} = \dfrac{27}{19}$

$\Rightarrow \dfrac{9(1+r^2-2r)\times 3}{1+r^2+r} = \dfrac{27}{19}$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow (3r-2)(2r-3) = 0$

$\Rightarrow r = \dfrac{2}{3}, \dfrac{3}{2}$

As $|r| < 1$

So $r = \dfrac{2}{3}$

24. Answer (4)

Let first term and common difference of AP be a and d respectively.

$\therefore$ $t_{19} = a + 18d = 0$

$\therefore$ $a = -18d$ ...(i)

$\therefore$ $\dfrac{t_{49}}{t_{29}} = \dfrac{a+48d}{a+28d}$

$= \dfrac{-18d+48d}{-18d+28d} = \dfrac{30d}{10d} = 3$

$t_{49} : t_{29} = 3 : 1$

25. Answer (4)

$$S_n = \left(\frac{1-q^{n+1}}{1-q}\right),\ T_n = \frac{1-\left(\dfrac{q+1}{2}\right)^{n+1}}{1-\left(\dfrac{q+1}{2}\right)}$$

$$\Rightarrow T_{100} = \frac{1-\left(\dfrac{q+1}{2}\right)^{101}}{1-\left(\dfrac{q+1}{2}\right)}$$

$$S_n = \frac{1}{1-q} - \frac{q^{n+1}}{1-q} \qquad T_{100} = \frac{2^{101}-(q+1)^{101}}{2^{100}(1-q)}$$

$\Rightarrow$ ${}^{101}C_1 + {}^{101}C_2\, S_1 + {}^{101}C_3\, S_2 + \ldots + {}^{101}C_{101}\, S_{100}$

$= \left(\dfrac{1}{1-q}\right)\left({}^{101}C_2 + \ldots + {}^{101}C_{101}\right) - \dfrac{1}{1-q}\left({}^{101}C_2\, q^2 + {}^{101}C_3\, q^3 + \ldots + {}^{101}C_{101}\, q^{101}\right) + 101$

$= \dfrac{1}{1-q}\left(2^{101}-1-101\right) - \left(\dfrac{1}{1-q}\right)\left((1+q)^{101} - 1 - {}^{101}C_1\, q\right) + 101$

$= \dfrac{1}{1-q}\left[2^{101} - 102 - (1+q)^{101} + 1 + 101q\right] + 101$

$= \dfrac{1}{1-q}\left[2^{101} - 101 + 101q - (1+q)^{101}\right] + 101$

$= \left(\dfrac{1}{1-q}\right)\left[2^{101} - (1+q)^{101}\right]$

$= 2^{100}\, T_{100}$

26. Answer (4)

Let three terms be $\dfrac{a}{r}, a, ar$

$a^3 = 512$

$a = 8$

$\dfrac{8}{r} + 4, 12, 8r$ form an A.P

$24 = \dfrac{8}{r} + 8r + 4$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow (2r-1)(r-2) = 0$

$r = \dfrac{1}{2}$ or 2

Sum of three terms $= \dfrac{8}{2} + 8 + 16$

$= 28$

27. Answer (1)

$$S_k = \frac{k(k+1)}{2k} = \frac{k+1}{2}$$

$$\Rightarrow \frac{5}{12}A = \frac{1}{4}[2^2 + 3^3 + ... + 11^2]$$

$$= \frac{1}{4}\left[\frac{11\times 12\times 23}{6} - 1\right] = \frac{1}{4}[505]$$

$$A = \frac{505}{4}\times\frac{12}{5} = 303$$

28. Answer (2)

$$S = \left(\frac{3}{4}\right)^3 + \left(\frac{3}{2}\right)^3 + \left(\frac{9}{4}\right)^3 + (3)^3 + ...$$

$$S = \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + ...$$

$$T_r = \left(\frac{3r}{4}\right)^3$$

$$225K = \sum_{r=1}^{15} T_r = \left(\frac{3}{4}\right)^3 \sum_{r=1}^{15} r^3$$

$$225K = \frac{27}{64}\times\left(\frac{15\times 16}{2}\right)^2$$

$K = 27$

29. Answer (2)

$\because$ $91 = 13 \times 7$

So the required numbers are either divisible by 7 or 13

Sum of such numbers = Sum of no. divisible by 7 + sum of the no. divisible by 13 – Sum of the numbers divisible by 91

= (105 + 112 + … + 196) +

(104 + 117 + … + 195) – 182

= 2107 + 1196 – 182

= 3121

30. Answer (4)

$$S = \sum_{k=1}^{20} k.\frac{1}{2^k}$$

$$S = \frac{1}{2} + 2.\frac{1}{2^2} + 3.\frac{1}{2^3} + ... + 20.\frac{1}{2^{20}}$$

$$\frac{1}{2}S = \frac{1}{2^2} + 2.\frac{1}{2^3} + ... + 19\frac{1}{2^{20}} + 20\frac{1}{2^{21}}$$

On subtracting,

$$\frac{S}{2} = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^{20}}\right) - 20\frac{1}{2^{21}}$$

$$= \frac{\frac{1}{2}\left(1 - \frac{1}{2^{20}}\right)}{1 - \frac{1}{2}} - 20.\frac{1}{2^{21}} = 1 - \frac{1}{2^{20}} - 10.\frac{1}{2^{20}}$$

$$\frac{S}{2} = 1 - 11.\frac{1}{2^{20}} \Rightarrow S = 2 - 11.\frac{1}{2^{19}} = 2 - \frac{11}{2^{19}}$$

31. Answer (3)

$$\because \quad S_n = \left(50 - \frac{7A}{2}\right) n + n^2 \times \frac{A}{2}$$

$\therefore$ Common difference $= \frac{A}{2}\times 2 = \boxed{A}$

$a_{50} = a_1 + 49 \times d$

$= (50 - 3A) + 49A$

$= 50 + 46A$

So, $(d, a_{50}) = (A, 50 + 46A)$

32. Answer (2)

1 + 2.3 + 3.5 + 4.7 + …

Lets break the sequence as shown

$$1 + \underbrace{(2.3 + 3.5 + 4.7 + ...)}_{S}$$

We find

$$s_{10} = \sum_{n=1}^{10}(n+1)(2n+1)$$

$$= \sum_{n=1}^{10}(2n^2 + 3n + 1)$$

$$= \frac{2n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n(n = 10)$$

$$= \frac{2.10.11.21}{6} + \frac{3.10.11}{2} + 10$$

= 770 + 165 + 10 = 945

Hence, required sum = 1 + 945 = 946

33. Answer (3)

Let terms be $a - d$, a, $a + d$

$\Rightarrow$ $3a = 33 \Rightarrow 11$

Product of terms

$(a - d)\, a\, (a + d) = 11\,(121 - d^2) = 1155$

$\Rightarrow$ $121 - d^2 = 105 \Rightarrow d = \pm 4$

if $d = 4$

$$\left.\begin{matrix} T_1 = 7 \\ T_2 = 11 \\ T_3 = 15 \end{matrix}\right\} \Rightarrow T_{11} = T_1 + 10d = 7 + 10(4) = 47$$

if $d = -4$

$$\left.\begin{matrix} T_1 = 15 \\ T_2 = 11 \\ T_3 = 7 \end{matrix}\right\} \Rightarrow T_{11} = T_1 + 10d = 15 + 10(-4) = -25$$

34. Answer (4)

$3(a_1 + a_{16}) = 114$

$\boxed{a_1 + a_{16} = 38}$

Now $a_1 + a_6 + a_{11} + a_{16} = 2(a_1 + a_{16})$

$= 2 \times 38$

$= 76$

35. Answer (4)

$$T_r = \frac{(2r+1)(1^3 + 2^3 + 3^3 + \ldots + r^3)}{1^2 + 2^2 + 3^2 + \ldots + r^2}$$

$$T_r = (2r+1)\left(\frac{r(r+1)}{2}\right)^2 \times \frac{6}{r(r+1)(2r+1)}$$

$$T_r = \frac{3r(r+1)}{2}$$

Now,

$$S = \sum_{r=1}^{10} T_r = \frac{3}{2}\sum_{r=1}^{10}(r^2 + r)$$

$$= \frac{3}{2}\left\{\frac{10 \times (10+1)(2 \times 10 + 1)}{6} + \frac{10 \times 11}{2}\right\}$$

$$= \frac{3}{2}\left\{\frac{10 \times 11 \times 21}{6} + 5 \times 11\right\} = \frac{3}{2} \times 5 \times 11 \times 8 = 660$$

36. Answer (2)

$$S = 1 + \frac{1^3 + 2^3}{1+2} + \frac{1^3 + 2^3 + 3^3}{1+2+3} + \ldots 15 \text{ terms}$$

$$T_n = \frac{1^3 + 2^3 + \ldots n^3}{1 + 2 + \ldots n} = \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2}} = \frac{n(n+1)}{2}$$

$$S = \frac{1}{2}\left(\sum_{n=1}^{15} n^2 + \sum_{n=1}^{15} n\right) = \frac{1}{2}\left(\frac{15(16)(31)}{6} + \frac{15(16)}{2}\right)$$

$$= 680$$

$$\Rightarrow \quad 680 - \frac{1}{2}\,\frac{15(16)}{2} = 680 - 60 = 620$$

37. Answer (2)

Let $b = ar$, $c = ar^2$

AP : $3a$, $7ar$, $15ar^2$

$14ar = 3a + 15ar^2$

$\Rightarrow$ $15r^2 - 14r + 3 = 0$

$$\Rightarrow \quad r = \frac{1}{3} \text{ or } \frac{3}{5} \text{ (rejected)}$$

Fourth term $= 15ar^2 + 7ar - 3a$

$= a(15r^2 + 7r - 3)$

$$= a\left(\frac{15}{9} + \frac{7}{3} - 3\right)$$

$= a$

38. Answer (1)

$a + 5d = 2$

Let $A = a_1 a_4 a_5 = a(a + 3d)(a + 4d)$

$= a(2 - 2d)(2 - d)$

$A = (2 - 5d)(4 - 6d + 2d^2)$

$$\frac{dA}{dd} = 0$$

$(2 - 5d)(-6 + 4d) + (4 - 6d + 2d^2)(-5) = 0$

$\Rightarrow$ $15d^2 - 34d + 16 = 0$

$$d = \frac{8}{5}, \frac{2}{3}$$

For $d = \frac{2}{3}$, $\frac{d^2A}{dd^2} < 0$

Hence $d = \frac{2}{3}$

39. Answer (3)

$S_4 = 16, \quad S_6 = -48$

$2(2a + 3d) = 16$

$\Rightarrow 2a + 3d = 8$

Also, $3[2a + 5d] = -48$

$\Rightarrow 2a + 5d = -16$

$2d = -24$

$d = -12 \Rightarrow a = 22$

$S_{10} = 5(44 + 9(-12)) = -320$

40. Answer (3)

Let the common difference is 'd'.

$a_1 + a_7 + a_{16} = 40$

$\Rightarrow a_1 + a_1 + 6d + a_1 + 15d = 40$

$\Rightarrow 3a_1 + 21d = 40$

$$\Rightarrow a_1 + 7d = \frac{40}{3}$$

$$S_{15} = \frac{15}{2}[2a_1 + 14d]$$

$= 15(a_1 + 7d)$

$$= 15\left(\frac{40}{3}\right)$$

$= 200$

41. Answer (4)

$1 + 49 + 49^2 + ... + (49)^{125}$

$$\frac{(49)^{126} - 1}{48} = \frac{((49)^{63} - 1)((49)^{63} + 1)}{48}$$

Greatest value of k is 63.

42. Answer (4)

Let 5 terms of A.P. be, $a - 2d$, $a - d$, a, $a + d$ and $a + 2d$

$5a = 25 \quad \Rightarrow \quad a = 5$

Also $(a^2 - 4d^2)(a^2 - d^2)a = 2520$

$\Rightarrow 4d^4 - 125d^2 + 121 = 0$

$\Rightarrow (d^2 - 1)(4d^2 - 121) = 0$

$$d = \pm 1 \text{ or } d = \pm\frac{11}{2}$$

$d = \pm 1$ does not give any term as $\frac{-1}{2}$

Hence rejected

$$\therefore \quad d = \frac{11}{2}$$

$$\text{Greatest term} = 5 + 2\left(\frac{11}{2}\right) = 16$$

43. Answer (2)

Let the G.P. be a, ar, ar^2, ar^3, ... and $a < 0$.

$\because \; a_1 + a_2 = 4 \Rightarrow a(1 + r) = 4$ ...(i)

$a_3 + a_4 = 16 \Rightarrow ar^2(1 + r) = 16$ ...(ii)

$\therefore$ from (i) and (ii), $r = \pm 2$.

if $r = 2$, then $a = \frac{4}{3}$

if $r = -2$, then $a = -4$.

$$\therefore \sum_{i=1}^{9} a_i = \frac{a(r^9 - 1)}{r - 1} = 4\lambda$$

$$= \frac{-4 \cdot ((-2)^9 - 1)}{-2 - 1} = 4\lambda$$

$\therefore \; \lambda = -171$

44. Answer (2)

$S = 3 + 4 + 8 + 9 + 13 + 14 + 18 + ...$ 40 terms.

$= 7 + 17 + 27 + ...$ 20 terms

$$= \frac{20}{2}\{2 \times 7 + (20 - 1).10\}$$

$= 2040$

$(102)m = 2040$

$\therefore \; m = 20$

45. Answer (3)

For A.P.

$2.f(x) = (2^{1-x} + 2^{1+x}) + (3^x + 3^{-x})$

$$\Rightarrow f(x) = 2^x + 2^{-x} + \frac{3^x + 3^{-x}}{2}$$

By AM-GM inequality

$2^x + 2^{-x} \geq 2$ and $3^x + 3^{-x} \geq 2$ at $x = 0$

$\therefore \; f(x) \geq 2 + 1$

$f(x) \geq 3$

46. Answer (4)

$$\therefore \; a + 9d = \frac{1}{20} \qquad \text{...(i)}$$

$$a + 19d = \frac{1}{100} \qquad \text{...(ii)}$$

$$\therefore \text{ By (ii) – (i) } 10d = \frac{1}{10} - \frac{1}{20} = \frac{10}{200} = \frac{1}{20}$$

$$\Rightarrow \boxed{d = \frac{1}{200}} \Rightarrow a = \frac{1}{200} \quad \text{from (i)}$$

$$\therefore \quad S_{200} = 100\left[2\times\frac{1}{200}+\frac{199}{200}\right]$$

$$= 1+\frac{199}{2}=\frac{201}{2}$$

$$= 100\frac{1}{2}$$

47. Answer (1)

$$2^{\frac{1}{4}}\cdot 4^{\frac{1}{16}}\cdot 8^{\frac{1}{48}}\cdot .....$$

$$= 2^{\frac{1}{4}}\cdot 2^{\frac{1}{8}}\cdot 2^{\frac{1}{16}}\cdot .....$$

$$= 2^{\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+.....\right)}$$

$$= 2^{\frac{1/4}{1-1/2}}$$

$$= 2^{\frac{1}{2}} = \sqrt{2}$$

48. Answer (2)

$$\sum_{n=1}^{100} a_{2n+1} = \frac{a_1 r^2\left(r^{200}-1\right)}{r^2-1} = 200 \qquad ...(i) \text{ and}$$

$$\sum_{n=1}^{100} a_{2n} = \frac{a_1 r\left(r^{200}-1\right)}{r^2-1} = 100 \qquad ...(ii)$$

Dividing (i) by (ii) we get $r = 2$ and adding (i) and (ii) we get

$$a_2+a_3+a_4+a_5+...+a_{200}+a_{201}=300$$

$$\Rightarrow \quad a_1r+a_2r+a_3r+...+a_{200}r=300$$

$$\Rightarrow \quad r(a_1+a_2+...+a_{200})=300$$

$$\Rightarrow \quad a_1+a_2+...+a_{200}=150$$

49. Answer (2)

$(x + y) + (x^2 + y^2 + xy) + (x^3 + x^2y + xy^2 + y^3) + ... \infty$

$$= \frac{1}{x-y}\left[\left(x^2-y^2\right)+\left(x^3-y^3\right)+\left(x^4-y^4\right)+...\infty\right]$$

$$= \frac{1}{x-y}\left[\frac{x^2}{1-x}-\frac{y^2}{1-y}\right] = \frac{(x-y)(x+y-xy)}{(x-y)(1-x)(1-y)}$$

$$= \frac{x+y-xy}{(1-x)(1-y)}$$

50. Answer (3)

Let $\frac{a}{r}$, a, ar be terms of G.P.

$$\therefore \quad a\left(\frac{1}{r}+1+r\right)=S \qquad ...(i)$$

and $a^3 = 27$

$$\Rightarrow \quad a = 3 \qquad ...(ii)$$

$$S = 3+3\left(r+\frac{1}{r}\right)$$

As if $f(x)=x+\frac{1}{x}$ then $f(x) \in (-\infty, -2] \cup [2, \infty)$

$\Rightarrow$ $3f(x) \in (-\infty, -6] \cup [6, \infty)$

$\Rightarrow$ $3 + 3f(x) \in (-\infty, -3] \cup [9, \infty)$

$\Rightarrow$ $S \in (-\infty, -3] \cup [9, \infty)$

51. Answer (2)

Let common difference be d.

$\because$ $a_1 + a_2 + a_3 + ... + a_{11} = 0$

$$\therefore \quad \frac{11}{2}\{2a_1+10.d\}=0$$

$\therefore$ $a_1 + 5d = 0$

$$\therefore \quad d=-\frac{a_1}{5} \qquad ...(1)$$

Now $a_1 + a_3 + a_5 + ... + a_{23}$

$= a_1 + (a_1 + 2d) + (a_1 + 4d) + ... + (a_1 + 22d)$

$$= 12a_1 + 2d\ \frac{11\times 12}{2}$$

$$= 12\left(a_1+11.-\frac{a_1}{5}\right)$$

$$= 12\times\left(-\frac{6}{5}\right)a_1$$

$$= -\frac{72}{5}a_1$$

52. Answer (1)

Seires $(x + ka) + (x^2 + (k + 2)a) + ...$ 9 terms

$\Rightarrow$ $S = (x + x^2 + x^3 + ...$ 9 terms$) + a[k + (k + 2) + (k + 4) + ...$ 9 terms$]$

$$\Rightarrow S = \frac{x\left(x^9 - 1\right)}{x - 1} + \frac{9}{2}[2ak + 8 \times (+2a)]$$

$$\Rightarrow S = \frac{x^{10} - x}{x - 1} + \frac{9ka + 72a}{1} = \frac{x^{10} + 45a(x - 1)}{x - 1}$$

(given)

$$\Rightarrow \frac{x^{10} - x + 9a(k + 8)(x - 1)}{x - 1}$$

$$= \frac{x^{10} - x + 45a(x - 1)}{x - 1}$$

$\Rightarrow$ $9a(k + 8) = 45a$

$\Rightarrow$ $k + 8 = 5$

$\Rightarrow$ $\boxed{k = -3}$

53. Answer (1)

Here $a = 3$ & $S_{25} = S^1_{15}$

$\Rightarrow$ $2S_{25} = S_{40}$

$$\Rightarrow 2 \times \frac{25}{2}[6 + 24d] = \frac{40}{2}[6 + 39d]$$

$\Rightarrow$ $25[6 + 24d] = 20[6 + 39d]$

$\Rightarrow$ $150 + 600d = 120 + 780d$

$\Rightarrow$ $180d = 30$

$$\Rightarrow d = \frac{1}{6}$$

54. Answer (4)

$$S_n = 20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + ...$$

$\because$ $S_n = 488$

$$\frac{n}{2}\left\{40 + (n-1)\cdot\left(-\frac{2}{5}\right)\right\} = 488$$

$$n\left(20 - \frac{n-1}{5}\right) = 488$$

$n^2 - 101n + 2440 = 0$

$\therefore$ $(n - 40)(n - 61) = 0$

For negative term $n = 61$

$$\therefore n^{th} \text{ term} = T_{61} = 20 + 60 \cdot \left(-\frac{2}{5}\right) = -4.$$

55. Answer (3)

$\because$ $\alpha, \beta, \gamma, \delta$ are in G.P, so $\alpha\delta = \beta\gamma$

$$\Rightarrow \frac{\alpha}{\beta} = \frac{\gamma}{\delta} \Rightarrow \left|\frac{\alpha - \beta}{\alpha + \beta}\right| = \left|\frac{\gamma - \delta}{\gamma + \delta}\right|$$

$$\Rightarrow \frac{\sqrt{9 - 4p}}{3} = \frac{\sqrt{36 - 4q}}{6}$$

$\Rightarrow$ $36 - 16p = 36 - 4q$

$\Rightarrow$ $q = 4p$

$$\text{So } \frac{2q + p}{2q - p} = \frac{9p}{7p} = \frac{9}{7}$$

56. Answer (4)

$1 + (1 - 2^2.1) + (1 - 4^2.3) + (1 - 6^2.5) + ... (1 - 20^2.19)$

$$S = 1 + \sum_{r=1}^{10}\left[1 - (2r)^2(2r - 1)\right]$$

$$= 1 + \sum_{r=1}^{10}\left(1 - 8r^3 + 4r^2\right) = 1 + 10 - \sum_{r=1}^{10}\left(8r^3 - 4r^2\right)$$

$$= 11 - 8\left(\frac{10 \times 11}{2}\right)^2 + 4 \times \left(\frac{10 \times 11 \times 21}{6}\right)$$

$= 11 - 2 \times (110)^2 + 4 \times 55 \times 7$

$= 11 - 220\ (110 - 7)$

$= 11 - 220 \times 103 = \alpha - 220\ \beta \Rightarrow \alpha = 11$

$\beta = 103$

$\Rightarrow$ $(11, 103)$

57. Answer (3)

$a_1 = 1$ and $a_n = 300$ and $d \in Z$

$300 = 1 + (n - 1)d$

$$\Rightarrow d = \frac{299}{(n-1)} = \frac{23 \times 13}{(n-1)}$$

$\therefore$ $n - 1 = 13$ or 23 (as d is integer)

$\Rightarrow$ $n = 14$ or 24

$\Rightarrow$ $n = 24$ and $d = 13$

$a_{20} = 1 + 19 \cdot 13 = 248$

$$S_{20} = 20\frac{(248 + 1)}{2} = 2490$$

58. Answer (4)

Given $3^{2\sin2\alpha - 1}$, 14, $3^{4 - 2\sin2\alpha}$ are in A.P.

So $3^{2\sin2\alpha - 1} + 3^{4 - 2\sin2\alpha} = 28$

$$\Rightarrow \frac{3^{2\sin2\alpha}}{3} + \frac{81}{3^{2\sin2\alpha}} = 28$$

$$\Rightarrow \frac{t}{3} + \frac{81}{t} = 28 \quad \{\text{Put } 3^{2\sin2\alpha} = t\}$$

$\Rightarrow t^2 - 84t + 243 = 0 \Rightarrow t = 81, t = 3$

$\Rightarrow$ When $t = 81$, when $t = 3$

$\Rightarrow \sin2\alpha = 2$ (Not possible) $\quad 2\sin2\alpha = 1$

$$\sin2\alpha = \frac{1}{2}$$

$$2\alpha = \frac{\pi}{6}$$

$$\alpha = \frac{\pi}{12}$$

So, I term $a = 3^\circ = 1$, $d = 14 - 1 = 13$

Now, $T_6 = a + 5d = 1 + 65 = 66$

59. Answer (4)

LHS is G.P of common ratio $\frac{3}{2}$

$$\therefore \frac{2^{10}\left(1-\left(\frac{3}{2}\right)^{11}\right)}{\left(1-\frac{3}{2}\right)} = S - 2^{11}$$

$$\Rightarrow 2^{10}\frac{\left(\frac{3^{11}-2^{11}}{2^{11}}\right)}{\frac{1}{2}} = S - 2^{11}$$

$\Rightarrow S = 3^{11}$

60. Answer (4)

Let the first term be 'a' and common ratio be 'r'.

$\because$ $ar(1 + r + r^2) = 3$ ...(i)

and $ar^5(1 + r + r^2) = 243$ ...(ii)

From (i) and (ii),

$r^4 = 81 \Rightarrow r = 3$ and $a = \frac{1}{13}$

$$S_{50} = \frac{a\left(r^{50}-1\right)}{r-1} = \frac{3^{50}-1}{26}$$

61. Answer (1)

$$S = \log_7 x^2 + \log_7 x^3 + \log_7 x^4 + \text{.....20 terms}$$

$$\Rightarrow \log_7(x^2 \cdot x^3 \cdot x^4 \cdot \text{.......} x^{21}) = 460 \text{ given}$$

$$\Rightarrow \log_7 x^{(2+3+4....21)} = 460$$

$$\Rightarrow (2+3+4+....+21)\log_7 x = 460$$

$$\Rightarrow \frac{20}{2}(2+21)\log_7 x = 460$$

$$\log_7 x = \frac{460}{230} = 2$$

$x = 7^2 = 49$

62. Answer (4)

$(a^2p^2 + 2abp + b^2) + (b^2p^2 + 2bcp + c^2) + (c^2p^2 + 2cdp + d^2) = 0$

$\Rightarrow (ap + b)^2 + (bp + c)^2 + (cp + d)^2 = 0$

$\therefore ap + b = bp + c = cp + d = 0$

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \qquad \therefore a, b, c, d$ are in G.P.

63. Answer (2)

Let common difference of series $a_1, a_2, a_3, ..., a_n$ be d.

$\because a_{40} = a_1 + 39d = -159$ ...(i)

and $a_{100} = a_1 + 99d = -399$ ...(ii)

From eqn. (ii) and (i)

$d = -4$ and $a_1 = -3$

Common difference of $b_1, b_2, b_3, ...$ be (-2).

$\because b_{100} = a_{70}$

$\therefore b_1 + 99(-2) = (-3) + 69(-4)$

$\therefore b_1 = 198 - 279$

$\therefore b_1 = -81$

64. Answer (1540.00)

$$\sum_{k=1}^{20} \frac{k(k+1)}{2} \Rightarrow \frac{\sum k^2 + \sum k}{2}$$

$$\Rightarrow \frac{k(k+1)(2k+1)}{12} + \frac{k(k+1)}{4}$$

Put $k = 20$

$$\Rightarrow \frac{20(21)(41)}{12} + \frac{20(21)}{4}$$

$\Rightarrow 1435 + 105 = 1540$

65. Answer (504)

$$\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}=\frac{1}{4}\sum_{n=1}^{7}(n^2+n)(2n+1)$$

$$=\frac{1}{4}\left[\sum_{n=1}^{7}2n^3+3n^2+n\right]$$

$$=\frac{1}{4}\left[2\times(28)^2+\frac{3\times7\times8\times15}{6}+\frac{7\times8}{2}\right]$$

= 7[56 + 15 + 1] = 7 × 72

$$=\boxed{504}$$

66. Answer (14)

The given series are 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51 ... and 2, 9, 16, 23, 30, 37, 44, 51, ...

The series of the common terms is

23, 51, 79, ...

According to question, 23 + (n – 1) 28 ≤ 407

$\Rightarrow$ $28n \le 412$

$$\Rightarrow\ n\le\frac{103}{7}$$

$\therefore$ No. of terms = 14

67. Answer (4)

$$(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\infty\right)}$$

As sum of GP upto infinity $=\dfrac{a}{1-r}$

$$\therefore\ \frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\infty=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{2}$$

$$\therefore\ (0.16)^{\log_{2.5}\left(\frac{1}{2}\right)}$$

$$\text{As }0.16=\frac{16}{100}=\left(\frac{4}{10}\right)^2=\left(\frac{10}{4}\right)^{-2}=(2.5)^{-2}$$

$$\therefore\ (2.5)^{-2\log_{2.5}\left(\frac{1}{2}\right)}=\left(\frac{1}{2}\right)^{-2}=4$$

68. Answer (39)

3, A_1, A_2 ... A_m, 243

As 243 = 3 + (m + 1)d

$$\Rightarrow\ d=\frac{240}{(m+1)}$$

Also 3, G_1, G_2, G_3, 243

$\therefore$ As $G_2^2=243\times3$

$$G_2=\sqrt{243\times3}=27$$

$$\text{Now }A_4=3+4d=3+\frac{960}{(m+1)}$$

As A_4 = G_2

$$27=3+\frac{960}{m+1}$$

$\therefore$ m = 39

69. Answer (5)

Sum of all elements of A ∩ (B ∪ C) is

$$274\times400=\sum_{k=0}^{99}\{(99+9k+l)+(99+9k+2)\}$$

$$\Rightarrow\ 274\times400=200\times100+100l+18.\left(\frac{99\times100}{2}\right)$$

$\Rightarrow$ 274 × 4 = 200 + l + 9 × 99

$\Rightarrow$ l = 5

70. Answer (9)

Let $f(x)=\dfrac{4}{\sin x}+\dfrac{1}{1-\sin x}$ where $\sin x\in(0,1)$

$$\because\ \frac{\frac{2}{\sin x}+\frac{2}{\sin x}+\frac{1}{1-\sin x}}{3}\ge\frac{3}{\frac{\sin x}{2}+\frac{\sin x}{2}+1-\sin x}$$

$$\Rightarrow\ f(x)\ge9$$

So least value of α is 9.

71. Answer (2)

Here, 2b = a + c ...(i)

and centroid of Δ is $\left(\frac{10}{3},\frac{7}{3}\right)$

$$\Rightarrow\ \frac{a+2+a}{3}=\frac{10}{3}\Rightarrow a=4$$

and $\dfrac{c+b+b}{3}=\dfrac{7}{3}\Rightarrow\dfrac{c+(a+c)}{3}=\dfrac{7}{3}\Rightarrow$ 2c + a = 7

$\Rightarrow 2c + 4 = 7$

$\Rightarrow c = \dfrac{3}{2}$

So from (i) $2b = \dfrac{11}{2} \Rightarrow \boxed{b = \dfrac{11}{4}}$

So the Q.E. if $4x^2 + \dfrac{11}{4}x + 1 = 0$

$\Rightarrow$ **$16x^2 + 11x + 4 = 0$** $\Rightarrow \alpha + \beta = \dfrac{-11}{16},\ \alpha\beta = \dfrac{1}{4}$

Now, $\alpha^2 + \beta^2 - \alpha\beta = \alpha^2 + \beta^2 + 2\alpha\beta - 3\alpha\beta$

$$= (\alpha+\beta)^2 - 3\alpha\beta$$

$$= \left(\frac{-11}{16}\right)^2 - \frac{3}{4}$$

$$= \frac{121}{256} - \frac{3}{4} = -\frac{71}{256}$$

72. Answer (3)

Let the G.P be $ar^3, ar, \dfrac{a}{r}, \dfrac{a}{r^3}, \ldots\ldots$

$$\because\ a\left(r^3 + r + \frac{1}{r} + \frac{1}{r^3}\right) = \frac{65}{12} \quad \ldots \text{(i)}$$

$$\frac{1}{a}\left(r^3 + r + \frac{1}{r} + \frac{1}{r^3}\right) = \frac{65}{18} \quad \ldots \text{(ii)}$$

$$\Rightarrow a^2 = \frac{3}{2}$$

Also $a^3r^3 = 1 \Rightarrow r = \dfrac{1}{a}$

and $\alpha = \dfrac{a}{r} = a^2 = \dfrac{3}{2}$

$\Rightarrow 2\alpha = 3$

73. Answer (4)

$$x = \sum_{n=0}^{\infty} \cos^{2n}\theta = 1 + \cos^2\theta + \cos^4\theta + \ldots$$

$$= \frac{1}{1-\cos^2\theta} = \operatorname{cosec}^2\theta$$

$$y = \sum_{n=0}^{\infty} \sin^{2n}\phi = 1 + \sin^2\phi + \sin^4\phi + \ldots$$

$$= \frac{1}{1-\sin^2\phi} = \sec^2\phi$$

$$z = \sum_{n=0}^{\infty} \cos^{2n}\theta \sin^{2n}\phi$$

$$= 1 + \cos^2\theta\sin^2\phi + \left(\cos^2\theta\sin^2\phi\right)^2 + \ldots$$

$$= \frac{1}{1-\cos^2\theta\sin^2\phi}$$

$$\Rightarrow z = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)}$$

$$\Rightarrow 1 - 1 + \frac{1}{x} + \frac{1}{y} - \frac{1}{xy} = \frac{1}{z}$$

$$\Rightarrow \frac{x+y}{xy} = \frac{z+xy}{xyz}$$

$\Rightarrow (x + y)z = xy + z$

74. Answer (9)

$A_1 = 12$, Let side of square 2 be A_2

Given diagonal of A_{n+1} = Side of A_n

$\Rightarrow 2A_2^{\,2} = A_1^{\,2} \Rightarrow A_2 = A_{1/\sqrt{2}}$

(i.e., $A_{n+1} = \dfrac{A_n}{\sqrt{2}}$)

$$\Rightarrow A_2 = \frac{A_1}{\sqrt{2}}, A_3 = \frac{A_2}{\sqrt{2}} = \frac{A_1}{2} \ldots \frac{1}{2}$$

$$A_{n+1} = (\sqrt{2}.\sqrt{2} \ \ldots\ (n-1) \text{ times})^{-1} A_1$$

$$\text{Area} = (A_{n+1})^2 = \frac{A_1^2}{2^{(n-1)}} < 1$$

$144 < 2^{n-1} \Rightarrow n - 1 \geq 8$

$n = 9$

75. Answer (2)

$$f(x) = a^{a^x} + \frac{a}{a^{a^x}}$$

$$\because\ \frac{a^{a^x} + \frac{a}{a^{a^x}}}{2} \geq \sqrt{a^{a^x}.\frac{a}{a^{a^x}}}$$

$$\Rightarrow f(x) \geq 2\sqrt{a}$$

$$f(x)_{min} = 2\sqrt{a}$$

76. Answer (2)

$$a_2 + a_6 = \frac{25}{2}$$

$$a_3 \times a_5 = 25 = a_2 \times a_6 = a_4^2$$

$$a_4^2 = 25 \Rightarrow a_4 = 5$$

a_2 & a_6 are roots of $x^2 - \frac{25}{2}x + 25 = 0$

$x = \frac{5}{2}, 10$

$a_2 = \frac{5}{2}, a_6 = 10$ $\quad$ ($\because$ GP is increasing)

$a_4 = 5$

$a_4 = a_2r^2 \quad \Rightarrow 5 = \frac{5}{2}r^2 \Rightarrow r^2 = 2$

$a_8 = a_6r^2 = 10 \times 2 = 20$

$a_4 + a_6 + a_8 = 5 + 10 + 20 = 35$

77. Answer (1)

$$S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots\ldots\ldots\ldots$$

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \frac{17}{3^5} + \ldots\ldots\ldots\ldots$$

$$- \quad - \quad - \quad - \quad - \quad -$$

$$\frac{2}{3}S = 1 + \frac{1}{3} + \left(\frac{5}{3^2} + \frac{5}{3^3} + \frac{5}{3^4} + \frac{5}{3^5} + \ldots\ldots\right)$$

$$= \frac{4}{3} + \frac{\frac{5}{9}}{1 - \frac{1}{3}} = \frac{5}{3} + \frac{\frac{5}{9}}{\frac{2}{3}}$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{5}{6} = \frac{13}{6}$$

$$S = \frac{13}{4}$$

78. Answer (2)

$$\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{2n^2 + 12n + 20}{(2n+1)!}$$

$$= \frac{1}{2}\sum_{n=1}^{\infty} \frac{n(2n+1) + 11n + 20}{(2n+1)!}$$

$$= \frac{1}{2}\sum_{n=1}^{\infty} \frac{n}{(2n)!} + \frac{1}{2}\sum_{n=1}^{\infty} \frac{11n + \frac{11}{2} + \frac{29}{2}}{(2n+1)!}$$

$$= \frac{1}{4}\sum_{n=1}^{\infty} \frac{2n}{(2n)!} + \frac{1}{2}\cdot\frac{11}{2}\sum_{n=1}^{\infty} \frac{2n+1}{(2n+1)!} + \frac{29}{4}\sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$$

$$= \frac{1}{4}\sum_{n=1}^{\infty} \frac{2n}{(2n-1)!} + \frac{11}{4}\sum_{n=1}^{\infty} \frac{1}{2n!} + \frac{29}{4}\sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$$

$$= \frac{1}{4}\left(\frac{e - e^{-1}}{2}\right) + \frac{11}{4}\left(\frac{e + e^{-1}}{2} - 1\right) + \frac{29}{4}\left(\frac{e - e^{-1}}{2} - 1\right)$$

$$= \frac{15}{2}\left(\frac{e - e^{-1}}{2}\right) + \frac{11}{4}\left(\frac{e + e^{-1}}{2}\right) - 10$$

$$= \frac{41}{8}e - \frac{19}{8}e^{-1} - 10$$

79. Answer (10)

$$T_p = -16\left(-\frac{1}{2}\right)^{p-1} = (-1)^p . 2^{5-p}$$

and $T_q = (-1)^q . 2^{5-q}$

$\because$ A.M. of T_p and T_q is $\frac{5}{4}$ and G.M. is 1

$(-1)^{p+q} 2^{10-p-q} = 1 \Rightarrow p + q = 10$

80. Answer (03)

G.P. from the set will be 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192 .....

and A.P. from the set will be 11, 16, 21, 26 .....

Common terms will be the terms of G.P. having unit digit 1 or 6

i.e. common terms 16, 256, 409

81. Answer (16)

$S_n = (2 + 3 + 6 + 11 + 18 + 27 \ldots\ldots\ldots)\log_a x.$

$T_n = 2, 3, 6, 11, 18, 27, \ldots\ldots\ldots$

$T_n' = 1, 3, 5, 7, 9, \ldots\ldots\ldots$A.P.

$T_n = An^2 + Bn + C$

$A + B + C = 2$

$4A + 2B + C = 3$

$9A + 3B + C = 6$

$A = 1, B = -2, C = 3$

$$S_n = \sum\left(n^2 - 2n + 3\right)\log_x$$

$$= \left(\frac{n(n+1)(2n+1)}{6} - 2\frac{n(n+1)}{2} + 3n\right)\log_a x$$

$$S_n(x) = \frac{n}{6}\left[2n^2 - 3n + 13\right]\log_a x$$

$$S_{24}(x) = 1093 \Rightarrow 4 \times 1093 \log_a x = 1093$$

$$\log_a x = \frac{1}{4} \Rightarrow x = a^{\frac{1}{4}} \Rightarrow a = x^4 \quad \ldots(i)$$

$$S_{12}(2x) = 265 \Rightarrow 2(265)\log_a 2x = 265$$

$$\Rightarrow 2x = a^{\frac{1}{2}} \Rightarrow a = 4x^2 \quad \ldots(ii)$$

(i) and (ii) $\Rightarrow$ $x^4 = 4x^2$

$\Rightarrow$ $x^2 = 4$

$a = x^4 = 16$

82. Answer (14)

$$b \times \frac{1}{16} = a^2 \Rightarrow b = 16a^2 \quad ...(i)$$

$$\frac{1}{a} + 6 = \frac{2}{b}$$

$$\frac{6a+1}{a} = \frac{2}{16a^2} \qquad \text{[using (i)]}$$

$48a^2 + 8a - 1 = 0$

$$a = \frac{1}{12} \ (a > 0)$$

$$b = \frac{1}{9}$$

$$a + b = \frac{7}{36}$$

83. Answer (3)

$$T_n = \frac{1}{(2n+1)^2 - 1} = \frac{1}{2n\,(2n+2)} = \frac{1}{4(n\,(n+1))}$$

$$\Rightarrow \quad T_n = \frac{1}{4}\left(\frac{n+1-n}{n(n+1)}\right) = \frac{1}{4}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$S_n = \sum_{n=1}^{100} T_n = \frac{1}{4}\left(1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{100} - \frac{1}{101}\right)$$

$$= \frac{1}{4}\left(1 - \frac{1}{101}\right) = \frac{1}{4} \cdot \frac{100}{101} = \frac{25}{101}$$

84. Answer (1)

$100^{\alpha} - 199\beta = 100^2 + (100 + 1)(100 - 1)$

$\quad + (100 - 2)(100 + 2) + ...(100 - 99)(100 + 99)$

$= 100^2 + 100^2 - 1^2 + 100^2 - 2^2 + ... + 100^2 - 99^2$

$= 100.100^2 - (1^2 + 2^2 + ... + 99^2)$

$$= 100^3 - \frac{99 \cdot 100 \cdot 199}{6}$$

$$= 100^3 - 1650 \cdot 199$$

$\therefore (\alpha, \beta) = (3, 1650)$

$$m = \frac{\beta - 0}{\alpha - 0} = \frac{1650}{3} = 550$$

85. Answer (3)

$$S_1 = \frac{2n}{2}[2a + (2n-1)d]$$

$\left(\begin{array}{l}\text{where } a = T_1 \text{ and d} \\ \text{is common difference}\end{array}\right)$

$$S_2 = \frac{4n}{2}[2a + (4n-1)d]$$

$S_2 - S_1 = 2n[2a + (4n - 1)d] - n[2a + (2n - 1)d] = 1000$

$\qquad = n[2a + d(8n - 2 - 2n + 1)] = 1000$

$\qquad = n[2a + (6n - 1)d] = 1000$

$$S_6 = \frac{6n}{2}[2a + (6n-1)d] = 3(S_2 - S_1) = 3000$$

86. Answer (160)

$$\sum_{r=1}^{10} r!\left(r^3 + 6r^2 + 2r + 5\right)$$

$$= \sum_{r=1}^{10}\left((r+3)! - (r+1)! - 8\left((r+1)! - r!\right)\right)$$

$$= \sum_{r=1}^{10}\left((r+3)! - (r+1)!\right) - 8\sum_{r=1}^{10}\left((r+1)! - r!\right)$$

$= 12! + 13! - 2! - 3! - 8\,(11! - 1!)$

$= 160 \cdot 11!$

Hence $\alpha = 160$

87. Answer (9)

$$\alpha(\alpha+1)(\alpha+2)...(\alpha+20)\sum_{k=0}^{20}\frac{A_k}{\alpha+k} = 1$$

Put $\alpha = -13$, $-A_{13} \cdot \underline{|13}\,\underline{|7} = 1 \Rightarrow A_{13} = \dfrac{1}{\underline{|7}\,\underline{|13}}$

Put $\alpha = -14$, $-A_{14} \cdot \underline{|14}\,\underline{|6} = 1 \Rightarrow A_{14} = \dfrac{-1}{\underline{|14}\,\underline{|6}}$

Put $\alpha = 15$, $-A_{15} \cdot \underline{|15}\,\underline{|5} = 1 \Rightarrow A_{15} = \dfrac{-1}{\underline{|15}\,\underline{|5}}$

$$100\left(\frac{A_{14} + A_{15}}{A_{13}}\right)^2 = \frac{100\left(\dfrac{1}{\underline{|14}\,\underline{|6}} - \dfrac{1}{\underline{|15}\,\underline{|5}}\right)^2}{\left(\dfrac{1}{\underline{|7}\,\underline{|13}}\right)^2}$$

$$= 100\left(\frac{\dfrac{9}{\underline{|15}\,\underline{|6}}}{\dfrac{1}{\underline{|7}\,\underline{|13}}}\right)^2 = 9$$

88. Answer (7)

$a_{n+2} = 2a_{n+1} + a_n$ has its characteristic equation as

$x^2 = 2x + 1 \Rightarrow x = 1 \pm \sqrt{2}$

So $a_n = a(1+\sqrt{2})^{n-1} + b(1-\sqrt{2})^{n-1}$

$\because$ $a_1 = 1 \Rightarrow a + b = 1$

and $a_2 = 1 \Rightarrow (a+b) + \sqrt{2}(a-b) = 1$

$$\Rightarrow a = \frac{1}{2} \text{ and } b = \frac{1}{2}$$

$$\text{So, } a_n = \frac{(1+\sqrt{2})^{n-1} + (1-\sqrt{2})^{n-1}}{2}$$

$$\sum_{n=1}^{\infty} \frac{a_n}{2^{3n}} = \frac{1}{16}\left[\sum_{n=1}^{\infty}\left(\frac{1+\sqrt{2}}{8}\right)^{n-1} + \sum_{n=1}^{\infty}\left(\frac{1-\sqrt{2}}{8}\right)^{n-1}\right]$$

$$= \frac{1}{16}\left[\frac{8}{7-\sqrt{2}} + \frac{8}{7+\sqrt{2}}\right]$$

$$= \frac{7}{47}$$

89. Answer (3)

Let first term of A.P. be a and common difference is d.

$$\therefore \quad S_{10} = \frac{10}{2}\{2a + 9d\} = 530$$

$\therefore$ $2a + 9d = 106$ ...(i)

$$S_5 = \frac{5}{2}\{2a + 4d\} = 140$$

$a + 2d = 28$ ...(ii)

from equation (i) and (ii), a = 8, d = 10

$\therefore$

$$S_{20} - S_6 = \frac{20}{2}\{2\times 8 + 19\times 10\} - \frac{6}{2}\{2\times 8 + 5\times 10\}$$

$= 2060 - 198$

$= 1862$

90. Answer (3)

$$S_{3n} = \frac{3n}{2}\left[2a + (3n-1)d\right]$$

$S_{2n} = n[2a + (2n - 1)d]$

(where a is first term & d is common difference of A.P.)

if $S_{3n} = 3S_{2n}$

$$\Rightarrow \frac{3}{2}\left[2a + (3n-1)d\right] = 3\left[2a + (2n-1)d\right]$$

$\Rightarrow$ $2a + (3n - 1)d = 4a + 2(2n - 1)d$

$\Rightarrow$ $2a = (3n - 1 - 4n + 2)d$

$$\Rightarrow \frac{a}{d} = \frac{1-n}{2} \qquad \text{...(i)}$$

$$\Rightarrow \frac{S_{4n}}{S_{2n}} = \frac{\frac{4n}{2}\left[2a + (4n-1)d\right]}{\frac{2n}{2}\left[2a + (2n-1)d\right]}$$

$$= \frac{2\left[2\left(\frac{1-n}{2}\right) + (4n-1)\right]}{\left[2\left(\frac{1-n}{2}\right) + (2n-1)\right]}$$

$$= \frac{2(1-n+4n-1)}{1-n+2n-1} = \frac{2(3n)}{n} = 6$$

91. Answer (3)

$$S_\infty = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \ldots\infty$$

$$\underline{\quad \frac{1}{3}S_\infty = \quad \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \ldots\infty}$$

$$\frac{2}{3}S_\infty = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \ldots\infty$$

$$\frac{2}{3}S_\infty = \frac{4}{3}\left(1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\infty\right) = \frac{4}{3}\times\frac{1}{1-\frac{1}{3}}$$

$\therefore$ $S_\infty = 3$.

$$\therefore \quad \text{Given expression} = 3^{\log_{\left(\frac{1}{4}\right)}\left(\frac{1}{2}\right)}$$

$$= \sqrt{3} = l$$

$$= l^2 = 3$$

92. Answer (4)

$$\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]=4-5+5-6+6-7+7+....-50+50$$
$$=4$$

93. Answer (03.00)

$\because$ $\log_3 2, \log_3(2^x-5), \log_3\left(2^x-\frac{7}{2}\right)$ are in A.P.

$\therefore$ $2, 2^x-5, 2^x-\frac{7}{2}$ are in G.P.

$(2^x-5)^2=2.\left(2^x-\frac{7}{2}\right)$

$2^{2x}-10.2^x+25=2.2^x-7$

$2^{2x}-12.2^x+32=0$

$(2^x-4)(2^x-8)=0$

$\therefore$ $x=2$ or 3

But $x=2$ is not acceptable

$\therefore$ $x=3$

94. Answer (1)

$$x-2y=\frac{\tan 20°+\tan 70°}{2}-(\tan 20°+\tan 50°)$$

$$\frac{1}{2}(\tan 70°-\tan 20°-2\tan 50°)$$

$$=\frac{1}{2}[(\tan 70°-\tan 50°)-(\tan 20°+\tan 50°)]$$

$$=\frac{1}{2}\left[\frac{\sin 20°}{\cos 70° \cos 50°}-\frac{\sin 70°}{\cos 20° \cos 50°}\right]$$

$$=\frac{1}{2}\left[\frac{1}{\cos 50°}-\frac{1}{\cos 50°}\right]=0$$

95. Answer (4)

$$S=\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{2^2}{x^4+1}+....+\frac{2^{100}}{x^{2^{100}}+1}$$

$$\Rightarrow S-\frac{1}{x-1}=\left(\frac{-1}{x-1}+\frac{1}{x+1}\right)+\frac{2}{x^2+1}+\frac{2^2}{x^4+1}+....+\frac{2^{100}}{x^{2^{100}}+1}$$

$$\Rightarrow S-\frac{1}{x-1}=-\frac{2^{101}}{x^{2^{101}}-1}$$

Put $x=2$

$$\Rightarrow S-1=-\frac{2^{101}}{2^{2^{101}}-1}$$

$$\Rightarrow S=1-\frac{2^{101}}{2^{2^{101}}-1}=1-\frac{2^{101}}{4^{2^{100}}-1}$$

96. Answer (4)

$\because$ $a+ar+ar^2+....\infty=15$

$\therefore$ $\frac{a}{1-r}=15$ ...(i)

and $a^2+a^2r^2+a^2r^4+....\infty=150$

$\therefore$ $\frac{a^2}{1-r^2}=150$ ...(ii)

From (ii)/equation (i)^2 we get : $\frac{1-r}{1+r}=\frac{2}{3}$

$\Rightarrow$ $3-3r=2+2r$

$\therefore$ $r=\frac{1}{5}$

Hence $a=15\times\left(1-\frac{1}{5}\right)=12$

$$\therefore\ ar^2+ar^4+ar^6+....=\frac{ar^2}{1-r^2}=\frac{12\times\frac{1}{25}}{\frac{24}{25}}=\frac{1}{2}$$

97. Answer (136)

$$\sum_{r=1}^{15} r\cdot {}^rP_r=\sum_{r=1}^{15} r\cdot r\,!=\sum_{r=1}^{15}(r+1-1)\cdot r\,!$$

$$=\sum_{r=1}^{15}(r+1)r!-r!=\sum_{r=1}^{15}(r+1)\,!-r\,!$$

$T_1=2!-1!$

$T_2=3!-2!$

$T_3=4!-3!$

$\vdots$

$T_{15}=16!-15!$

Sum $=16!-1={}^{16}P_{16}-1$

$q=16$, $r=16$, $s=1$

${}^{q+s}C_{r-s}={}^{17}C_{15}$

$=136$

98. Answer (4)

$$\lim_{x\to 2}\left(\sum_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$$

$$\Rightarrow \sum_{n=1}^{9}\frac{2}{4n^2+12n+8}$$

$$\Rightarrow \frac{1}{2}\sum_{n=1}^{9}\frac{1}{n^2+3n+2}$$

$$\Rightarrow \frac{1}{2}\sum_{n=1}^{9}\frac{(n+2)-(n+1)}{(n+1)(n+2)}$$

$$\Rightarrow \frac{1}{2}\sum_{n=1}^{9}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{10}-\frac{1}{11}\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{9}{22}\right)=\frac{9}{44}$$

99. Answer (7744)

The required numbers are 209, 220, 231, ..., 495

This A.P. contains 27 terms

Sum of this A.P. $=\frac{27}{2}[209+495]=9504$

Only 231, 319, 341, 418 and 451 are not allowed in this sum.

So required sum = 9504 – 1760 = 7744

100. Answer (2021)

$c_2 = a_2 + b_2 = (a_1 - 3) + 2b_1 = 12 \Rightarrow a_1 = 11$

$c_3 = a_3 + b_3 = (a_1 - 6) + 4b_1 = 13 \Rightarrow b_1 = 2$

$c_k = a_k + b_k = (a_1 - 3(k-1)) + (b_1 \cdot 2^{k-1})$

$= (11 - 3k + 3) + (2^k) = 14 - 3k + 2^k$

$$\sum_{k=1}^{10} c_k = \sum_{k=1}^{10}(2^k - 3k + 14)$$

$$= \sum_{k=1}^{10} 2^k - 3\sum_{k=1}^{10} k + \sum_{k=1}^{10} 14$$

$$= 2(2^{10}-1) - 3\cdot\frac{10\cdot 11}{2} + 140$$

$= 2021$

101. Answer (3)

$$\frac{\sin(B+C)}{\sin(A+C)}=\frac{\sin(A-C)}{\sin(C-B)}$$

$\Rightarrow \sin^2 C - \sin^2 B = \sin^2 A - \sin^2 C$

$= a^2 + b^2 = 2c^2$

102. Answer (1)

$$\because \quad \frac{3}{2}x^2+\frac{5}{3}x^3+\frac{7}{4}x^4+\ldots\infty,\ x\in(0,1)$$

$$=\left(2-\frac{1}{2}\right)x^2+\left(2-\frac{1}{3}\right)x^3+\left(2-\frac{1}{4}\right)x^4+\ldots\infty.$$

$$=2x^2\left(1+x+x^2+\ldots\infty\right)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots\infty\right)$$

$$=2x^2\cdot\frac{1}{1-x}+x-\left(\frac{x}{1}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots\infty\right)$$

$$=x\left(\frac{1+x}{1-x}\right)+\ln(1-x)$$

103. Answer (2)

$$y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\ldots$$

$$v \quad =\left(1-\frac{1}{2}\right)x^2+\left(1-\frac{1}{3}\right)x^3+\left(1-\frac{1}{4}\right)x^4+\ldots$$

$$=\left(x^2+x^3+x^4+\ldots\right)+\left(-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}+\ldots\right)$$

$$=\frac{x^2}{1-x}+x+\left(-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}+\ldots\right)$$

$$y=\frac{x}{1-x}+\ln(1-x)$$

$$y+1=\frac{1}{1-x}+\ln(1-x)$$

$$e^{y+1}=e^{\frac{1}{1-x}+\ln(1-x)}=e^{\frac{1}{1-x}}\times e^{\ln(1-x)}=(1-x)e^{\frac{1}{1-x}}$$

$\therefore$ at $x=\frac{1}{2}$ $y=\frac{1}{2}e^2$

104. Answer (2)

Let number be a, ar, ar^2

$\therefore$ d = 2ar – a = ar^2 – 2ar ...(i)

and $ar^3 = 3r^2 \Rightarrow r=\frac{3}{a}$ ...(ii)

$\Rightarrow \frac{9}{a^2}+1=\frac{12}{a}$ (by (i) and (ii))

Put $\frac{1}{a}=t$

$\Rightarrow 9t^2 - 12t + 1 = 0$

$$t=\frac{2\pm\sqrt{3}}{3}=\frac{1}{a}\Rightarrow\frac{3}{a}=r=2\pm\sqrt{3}$$

As G.P. is increasing $r=2+\sqrt{3}, a=3\left(2-\sqrt{3}\right)$

$d=6-a=6-6+3\sqrt{3}$

$\therefore \quad r^2-d=4+3+4\sqrt{3}-\left(3\sqrt{3}\right)$

$=7+\sqrt{3}$

105. Answer (3)

$$T_n = \frac{2n+1}{n^2(n+1)^2} = \frac{(n+1)^2 - n^2}{(n+1)^2 n^2}$$

$$S_n = \sum_{n=1}^{10} T_n = \sum_{n=1}^{10}\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$= \left(1 - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + ... + \left(\frac{1}{10^2} - \frac{1}{11^2}\right)$$

$$= 1 - \frac{1}{(11)^2} = \frac{120}{121}$$

106. Answer (398)

S = 7× 8 + 10 × 10 + 13 × 12 + ... 10 term

$$S = \sum_{r=1}^{10}(3r+4)(2r+6) = 2\cdot\sum_{r=1}^{10}\left(3r^2 + 13r + 12\right)$$

$$= 2\cdot\left(3\times\frac{10\times 11\times 21}{6} + 13\times\frac{10\times 11}{2} + 12\times 10\right)$$

= 3980

$$\text{Mean} = \frac{3980}{10} = 398$$

107. Answer (3)

$\because$ a_1, a_2, a_3 ... are in A.P.

Let its common difference be d.

$$\therefore \quad \frac{a_1 + a_2 + ... + a_{10}}{a_1 + a_2 + ... a_p} = \frac{100}{p^2}$$

$$\Rightarrow \quad \frac{\frac{10}{2}\{2a_1 + 9d\}}{\frac{p}{2}\{2a_1 + (p-1)d\}} = \frac{100}{p^2}$$

$$\Rightarrow \quad \frac{2a_1 + 9d}{2a_1 + (p-1)d} = \frac{10}{p}$$

$\Rightarrow$ $2pa_1 + 9pd = 20a_1 + 10(p - 1)d$.

= $(2p - 20)\, a_1 = (p - 10)d$.

$\therefore$ $2a_1 = d$ $\qquad$ ($\because$ p $\neq$ 10)

$$\therefore \quad \frac{a_{11}}{a_{10}} = \frac{a_1 + 10d}{a_1 9d} = \frac{a_1 20a_1}{a_1 + 18a_1} = \frac{21}{19}$$

108. Answer (305)

$$S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + ...$$

$$\frac{1}{5}S = \frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + ...$$

Subtracting,

$$\Rightarrow \quad \frac{4S}{5} = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + ...$$

$$\Rightarrow \quad \frac{4S}{5} = \frac{7}{5} + M \qquad ...(i)$$

Where, $M = \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + ...$

$$\frac{1}{5}M = \frac{2}{5^3} + \frac{4}{5^4} + \frac{6}{5^5} + ...$$

$$\frac{4M}{5} = \frac{2}{5^2} + 2\left(\frac{1}{5^3} + \frac{1}{5^4} + ...\right)$$

$$\Rightarrow \quad M = \frac{1}{8}$$

Putting in (i) gives $S = \frac{61}{32}$

$\Rightarrow$ $160S = 305$

109. Answer (1)

$$S_n = 1(n-1) + 2(n-2) ... + (n-1)n$$

i.e. $T_k = k(n-k)$

$$S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}\left(kn - k^2\right)$$

$$= \frac{n(n(n+1))}{2} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{2}\left(\frac{3n - (2n+1)}{3}\right) = \frac{n(n^2-1)}{6} = S_n$$

$$\sum_{n=4}^{\infty}\left(\frac{2\cdot S_n}{n!} - \frac{1}{(n-2)!}\right) = \sum_{n=4}^{\infty}\left(\frac{n(n-1)(n+1)}{3n(n-1)(n-2)!} - \frac{1}{(n-2)!}\right)$$

$$= \sum_{n=4}^{\infty}\left(\frac{(n-2)+3}{3(n-2)(n-3)!} - \frac{1}{(n-2)!}\right)$$

$$= \sum_{n=4}^{\infty}\left(\frac{1}{3(n-3)!} + \frac{1}{(n-2)!} - \frac{1}{(n-2)!}\right) = \frac{1}{3}(e-1)$$

110. Answer (3)

Let first term a and common difference d

$$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+....+\frac{1}{a_{20}a_{21}}=\frac{4}{9} \quad ...(i)$$

Also, $a_1 + a_2 + ... + a_{21} = 189$ ...(ii)

by (i)

$$\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+....+\frac{1}{a_{20}}-\frac{1}{a_{21}}=\frac{4d}{9}$$

$$\Rightarrow \frac{1}{a}-\frac{1}{a+20d}=\frac{4d}{9}$$

$$\Rightarrow \frac{20d}{a(a+20d)}=\frac{4d}{9} \Rightarrow 45 = a(a+20d) \quad ...(iii)$$

and

$$21a + 210d = 189 \Rightarrow a + 10d = 9 \quad ...(iv)$$

by (iii) and (iv)

$$d=\frac{3}{5} \text{ and } a = 3$$

$\therefore \quad a_6a_{16} = (3 + 3)(3 + 9) = 72$

111. Answer (2)

$$a_1+a_2+...+a_n=192 \Rightarrow \frac{n}{2}(a_1+a_n)=192 \quad ...(1)$$

$a_2 + a_4 + a_6 + ... + a_n = 120$

$$\Rightarrow \frac{n}{4}(a_1+1+a_n)=120 \quad ...(2)$$

From (2) & (1)

$$\frac{480}{n}-\frac{384}{n}=1 \Rightarrow n=96$$

112. Answer (1633)

The numbers upto 24 which gives g.c.d. with 24 equals to 1 are 1, 5, 7, 11, 13, 17, 19 and 23.

Sum of these numbers = 96

There are four such blocks and a number 97 is there upto 100.

$\therefore$ Complete sum

= 96 + (24 × 8 + 96) + (48 × 8 + 96) + (72 × 8 + 96) + 97

= 1633

113. Answer (4)

$$\frac{1}{2\cdot 3^{10}}+\frac{1}{2^2\cdot 3^9}+...+\frac{1}{2^{10}\cdot 3}=\frac{K}{2^{10}\cdot 3^{10}}$$

$$\Rightarrow \frac{1}{2\cdot 3^{10}}\left[\frac{\left(\frac{3}{2}\right)^{10}-1}{\frac{3}{2}-1}\right]=\frac{K}{2^{10}\cdot 3^{10}}$$

$$=\frac{3^{10}-2^{10}}{2^{10}\cdot 3^{10}}=\frac{K}{2^{10}\cdot 3^{10}} \Rightarrow K=3^{10}-2^{10}$$

Now $K = (1 + 2)^{10} - 2^{10}$

$= {}^{10}C_0 + {}^{10}C_1\, 2 + {}^{10}C_2\, 2^3 + ... + {}^{10}C_{10}\, 2^{10} - 2^{10}$

$= {}^{10}C_0 + {}^{10}C_1\, 2 + 6\lambda + {}^{10}C_9 \times 2^9$

$= 1 + 20 + 5120 + 6\lambda$

$= 5136 + 6\lambda + 5$

$= 6\mu + 5$

$\lambda, \mu \in N$

$\therefore$ remainder = 5

114. Answer (4)

$\alpha_n = 19^n - 12^n$

Let equation of roots 12 & 19 i.e.

$$x^2-31x+228=0$$

$$\Rightarrow (31-x)=\frac{228}{x} \quad \text{(where } x \text{ can be 19 or 12)}$$

$$\therefore \frac{31\alpha_9-\alpha_{10}}{57\alpha_8}=\frac{31\left(19^9-12^9\right)-\left(19^{10}-12^{10}\right)}{57\left(19^8-12^8\right)}$$

$$=\frac{19^9(31-19)-12^9(31-12)}{57\left(19^8-12^8\right)}$$

$$=\frac{228\left(19^8-12^8\right)}{57\left(19^8-12^8\right)}=4.$$

115. Answer (98)

$$S=\frac{1}{3}+\frac{5}{9}+\frac{19}{27}+\frac{65}{81}+...$$

$$=\sum_{r=1}^{100}\left(\frac{3^r-2^r}{3^r}\right)$$

$$= 100-\frac{2}{3}\frac{\left(1-\left(\frac{2}{3}\right)^{100}\right)}{1/3}$$

$$= 98+2\left(\frac{2}{3}\right)^{100}$$

$\therefore [S] = 98$

116. Answer (2)

Let $\begin{array}{l} S = 1.3^{\circ}+2.3^{1}+3.3^{2}+.......+10.3^{9} \\ \underline{3S = \quad\quad 1.3^{1}+2.3^{2}+..................+10.3^{10}} \end{array}$

$-2S = (1.3^{\circ} + 1.3^{1} + 1.3^{2} + ... + 1.3^{9}) - 10.3^{10}$

$$\Rightarrow S=\frac{1}{2}\left[10.3^{10}-\frac{3^{10}-1}{3-1}\right]$$

$$\Rightarrow S=\frac{19.3^{10}+1}{4}$$

117. Answer (1100)

Each element of ordered pair $\{i, j\}$ is either present in A or in B.

So, $A + B$ = Sum of all elements of all ordered pairs $\{i, j\}$ for $1 \le i \le 10$ and $1 \le j \le 10$

$= 20\ (1 + 2 + 3 + ... + 10)$

$= 1100$

118. Answer (3)

$$A=\sum_{n=1}^{\infty}\frac{1}{\left(3+(-1)^{n}\right)^{n}} \text{ and } B=\sum_{n=1}^{\infty}\frac{(-1)^{n}}{\left(3+(-1)^{n}\right)^{n}}$$

$$A=\frac{1}{2}+\frac{1}{4^{2}}+\frac{1}{2^{3}}+\frac{1}{4^{4}}+....$$

$$B=\frac{-1}{2}+\frac{1}{4^{2}}-\frac{1}{2^{3}}+\frac{1}{4^{4}}+....$$

$$A=\frac{\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{16}}{1-\frac{1}{16}},\ B=\frac{-\frac{1}{2}}{1-\frac{1}{4}}+\frac{\frac{1}{16}}{1-\frac{1}{16}}$$

$$A=\frac{11}{15},\ B=\frac{-9}{15}$$

$$\therefore \frac{A}{B}=\frac{-11}{9}$$

119. Answer (40)

Let G.P. be $a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, ...

$\because 3a_2+a_3=2a_4$

$\Rightarrow 3ar+ar^2=2ar^3$

$\Rightarrow 2ar^2-r-3=0$

$\therefore r=-1 \text{ or } \frac{3}{2}$

$\because a_1=a>0$ then $r\neq -1$

Now, $a_2+a_4=2a_3+1$

$ar+ar^3=2ar^2+1$

$$a\left(\frac{3}{2}+\frac{27}{8}-\frac{9}{2}\right)=1$$

$$\therefore a=\frac{8}{3}$$

$$\therefore a_2+a_4+2a_5=a\left(r+r^3+2r^4\right)$$

$$=\frac{8}{3}\left(\frac{3}{2}+\frac{27}{8}+\frac{81}{8}\right)$$

$= 40$

120. Answer (3)

$$x=\sum_{n=0}^{\infty}a^{n}=\frac{1}{1-a};\ y=\sum_{n=0}^{\infty}b^{n}=\frac{1}{1-b};\ z=\sum_{n=0}^{\infty}c^{n}=\frac{1}{1-c}$$

Now,

$a,\ b,\ c \rightarrow$ AP

$1-a,\ 1-b,\ 1-c \rightarrow$ AP

$$\frac{1}{1-a},\ \frac{1}{1-b},\ \frac{1}{1-c} \rightarrow \text{HP}$$

$x, y, z \rightarrow$ HP

$$\therefore \frac{1}{x},\ \frac{1}{y},\ \frac{1}{z} \rightarrow \text{AP}$$

121. Answer (276)

$$T_r = \frac{r}{(2r^2)^2+1}$$

$$= \frac{r}{(2r^2+1)^2-(2r)^2}$$

$$= \frac{1}{4}\frac{4r}{(2r^2+2r+1)(2r^2-2r+1)}$$

$$S_{10} = \frac{1}{4}\sum_{r=1}^{10}\left(\frac{1}{(2r^2-2r+1)}-\frac{1}{(2r^2+2r+1)}\right)$$

$$= \frac{1}{4}\left[1-\frac{1}{5}+\frac{1}{5}-\frac{1}{13}+.....+\frac{1}{181}-\frac{1}{221}\right]$$

$$\Rightarrow S_{10} = \frac{1}{4}\cdot\frac{220}{221}=\frac{55}{221}=\frac{m}{n}$$

$\therefore m + n = 276$

122. Answer (3)

$$S = 2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+.... \quad ...(i)$$

$$\frac{1}{7}S = \frac{2}{7}+\frac{6}{7^2}+\frac{12}{7^3}+\frac{20}{7^4}+.... \quad ...(ii)$$

(i) – (ii)

$$\frac{6}{7}S = 2+\frac{4}{7}+\frac{6}{7^2}+\frac{8}{7^3}+.... \quad ...(iii)$$

$$\frac{6}{7^2}S = \frac{2}{7}+\frac{4}{7^2}+\frac{6}{7^3}+.... \quad ...(iv)$$

(iii) – (iv)

$$\left(\frac{6}{7}\right)^2 S = 2+\frac{2}{7}+\frac{2}{7^2}+\frac{2}{7^3}+.....$$

$$= 2\left[\frac{1}{1-\frac{1}{7}}\right]=2\left(\frac{7}{6}\right)$$

$$\therefore\ 4S = 8\left(\frac{7}{6}\right)^3=\left(\frac{7}{3}\right)^3$$

123. Answer (4)

a_1, a_2, a_3 ... are in A.P. (Let common difference is d_1)

b_1, b_2, b_3 ... are in A.P. (Let common difference is d_2)

and $a_1 = 2$, $a_{10} = 3$, $a_1 b_1 = 1 = a_{10} b_{10}$

$\because\ a_1 b_1 = 1 \quad \therefore \quad b_1 = \frac{1}{2}$

$a_{10} b_{10} = 1 \quad \therefore \quad b_{10} = \frac{1}{3}$

Now, $a_{10} = a_1 + 9d_1 \Rightarrow d_1 = \frac{1}{9}$

$$b_{10} = b_1 + 9d_2 \Rightarrow d_2 = \frac{1}{9}\left[\frac{1}{3}-\frac{1}{2}\right] = -\frac{1}{54}$$

Now, $a_4 = 2+\frac{3}{9}=\frac{7}{3}$

$$b_4 = \frac{1}{2}-\frac{3}{54}=\frac{4}{9}$$

$$\therefore \quad a_4 b_4 = \frac{28}{27}$$

124. Answer (3)

$$\frac{A_4}{r^3}.\frac{A_4}{r}.A_4 r.A_4 r^3 = \frac{1}{1296}$$

$$A_4 = \frac{1}{6}$$

$$A_2 = \frac{7}{36}-\frac{1}{6}=\frac{1}{36}$$

So $A_6 + A_8 + A_{10} = 1 + 6 + 36$

$= 43$

125. Answer (3)

a, A_1, A_2 .... A_n, 100

Let d be the common difference of above A.P. then

$$\frac{a+d}{100-d}=\frac{1}{7}$$

$\Rightarrow\ 7a + 8d = 100$ ...(i)

and $a + n = 33$ ...(ii)

and $100 = a + (n + 1)d$

$$\Rightarrow\ 100 = a+(34-a)\frac{(100-7a)}{8}$$

$\Rightarrow \quad 800 = 8a + 7a^2 - 338a + 3400$

$\Rightarrow \quad 7a^2 - 330a + 2600 = 0$

$\Rightarrow \quad a = 10, \ \frac{260}{7}, \text{ but } a \neq \frac{260}{7}$

$\therefore \quad n = 23$

126. Answer (41651)

$$S_n = \frac{n^2}{1-\frac{1}{(n+1)^2}} = \frac{n(n+1)^2}{n+2} = (n^2+1) - \frac{2}{n+2}$$

$$\text{Now } \frac{1}{26} + \sum_{n=1}^{50}\left(S_n + \frac{2}{n+1} - n - 1\right)$$

$$= \frac{1}{26} + \sum_{n=1}^{50}\left\{(n^2-n) + 2\left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right\}$$

$$= \frac{1}{26} + \frac{50\times 51\times 101}{6} - \frac{50\times 51}{2} + 2\left(\frac{1}{2} - \frac{1}{52}\right)$$

$$= 1 + 25\times 17(101-3)$$

$$= 41651$$

127. Answer (2)

$a_{n+2} = 2a_{n+1} - a_n + 1 \ \& \ a_0 = a_1 = 0$

$a_2 = 2a_1 - a_0 + 1 = 1$

$a_3 = 2a_2 - a_1 + 1 = 3$

$a_4 = 2a_3 - a_2 + 1 = 6$

$a_5 = 2a_4 - a_3 + 1 = 10$

$$\sum_{n=2}^{\infty} \frac{a_n}{7^n} = \frac{a_2}{7^2} + \frac{a_3}{7^3} + \frac{a_4}{7^4} + \dots$$

$$s = \frac{1}{7^2} + \frac{3}{7^3} + \frac{6}{7^4} + \frac{10}{7^5} + \dots$$

$$\frac{1}{7}s = \frac{1}{7^3} + \frac{3}{7^4} + \frac{6}{7^5} + \dots$$

$$\frac{6s}{7} = \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \dots$$

$$\frac{6s}{49} = \frac{1}{7^3} + \frac{2}{7^4} + \dots$$

$$\frac{36s}{49} = \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \dots$$

$$\frac{36s}{49} = \frac{\frac{1}{7^2}}{1-\frac{1}{7}}$$

$$\frac{36s}{49} = \frac{7}{49\times 6}$$

$$\boxed{s = \frac{7}{216}}$$

128. Answer (3)

$$S = 1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \dots.$$

$$\frac{1}{6}S = \frac{1}{6} + \frac{5}{6^2} + \frac{12}{6^3} + \dots.$$

$$\frac{5S}{6} = 1 + \frac{4}{6} + \frac{7}{6^2} + \frac{10}{6^3} + \frac{13}{6^4} + \dots.$$

$$\frac{5S}{36} = \frac{1}{6} + \frac{4}{6^2} + \frac{7}{6^3} + \frac{10}{6^4} + \dots.$$

$$\frac{25S}{36} = 1 + \frac{3}{6} + \frac{3}{6^2} + \frac{3}{6^3} + \frac{3}{6^4} + \dots.$$

$$\frac{25S}{36} = 1 + \frac{\frac{3}{6}}{1-\frac{1}{6}}$$

$$\frac{25S}{36} = \frac{8}{5}$$

$$S = \frac{288}{125}$$

129. Answer (2223)

S_1 : 3, 6, 9, 12,……78-terms

S_2 : 5, 9, 13, 17,…….59-terms

Common terms are 9, 21,……….

T_{78} of S_1 = 3 + (77).3 = 234

T_{59} of S_2 = 5 + (58) 4 = 237

So n^{th} common term ≤ 234

$\Rightarrow \quad 9 + (n-1)\, 12 \leq 234$

$$\Rightarrow \quad n < \frac{225}{12} + 1$$

$$\Rightarrow \quad n < \frac{237}{12} \Rightarrow n = 19$$

S_{19} of common terms

$= \frac{19}{2}\left[2(9)+18.12\right]$

$=$ 19 (9 + 108)

$=$ 117 × 19 = 2223

130. Answer (38)

$\because$ Roots of $2ax^2 - 8ax + 1 = 0$ are $\frac{1}{p}$ and $\frac{1}{r}$

and roots of $6bx^2 + 12bx + 1 = 0$ are $\frac{1}{q}$ and $\frac{1}{s}$.

Let $\frac{1}{p},\frac{1}{q},\frac{1}{r},\frac{1}{s}$ as $\alpha - 3\beta$, $\alpha - \beta$, $\alpha + \beta$, $\alpha + 3\beta$

So sum of roots $2\alpha - 2\beta = 4$ and $2\alpha + 2\beta = -2$

Clearly $\alpha = \frac{1}{2}$ and $\beta = -\frac{3}{2}$

Now product of roots, $\frac{1}{p}\cdot\frac{1}{r} = \frac{1}{2a} = -5 \Rightarrow \frac{1}{a} = -10$

and $\frac{1}{q}\cdot\frac{1}{x} = \frac{1}{6b} = -8 \Rightarrow \frac{1}{b} = -48$

So, $\frac{1}{a} - \frac{1}{b} = 38$

131. Answer (27560)

$a_1 = b_1 = 1$

$a_n = a_{n-1} + 2$ (for $n \geq 2$) $\quad b_n = a_n + b_{n-1}$

$a_2 = a_1 + 2 = 1 + 2 = 3 \quad b_2 = a_2 + b_1 = 3 + 1 = 4$

$a_3 = a_2 + 2 = 3 + 2 = 5 \quad b_3 = a_3 + b_2 = 5 + 4 = 9$

$a_4 = a_3 + 2 = 5 + 2 = 7 \quad b_4 = a_4 + b_3 = 7 + 9 = 16$

$a_{15} = a_{14} + 2 = 29 \quad b_{15} = 225$

$$\sum_{n=1}^{15} a_n\, b_n = 1\times1 + 3\times4 + 5\times9 + ...29\times225$$

$$\therefore \quad \sum_{n=1}^{11} a_n\, b_n = \sum_{n=1}^{15}(2n-1)n^2 = \sum_{n=1}^{15}2n^3 - \sum_{n=1}^{15}n^2$$

$$= 2\left[\frac{15\times16}{2}\right]^2 - \left[\frac{15\times16\times31}{6}\right] = 27560.$$

132. Answer (3)

Given G.P's 2, 2^2, 2^3, … 60 terms

4, 4^2, … n terms

Now, $G.M = 2^{\frac{225}{8}}$

$$\left(2.2^2...4.4^2...\right)^{\frac{1}{60+n}} = 2^{\frac{225}{8}}$$

$$\left(2^{\frac{n^2+n+1830}{60+n}}\right) = 2^{\frac{225}{8}}$$

$$\Rightarrow \frac{n^2+n+1830}{60+n} = \frac{225}{8}$$

$\Rightarrow 8n^2 - 217n + 1140 = 0$

$n = \frac{57}{8}$, 20, so $n = 20$

$$\therefore \sum_{k=1}^{20} k(20-k) = 20\times\frac{20\times21}{2} - \frac{20\times21\times41}{6}$$

$$= \frac{20\times21}{2}\left[20 - \frac{41}{3}\right] = 1330$$

133. Answer (6993)

Given series

$$\underbrace{\{3\times1\}}_{\text{1-term}}, \underbrace{\{3\times2, 3\times3, 3\times4\}}_{\text{3-terms}}, \underbrace{\{3\times5, 3\times6, 3\times7, 3\times8, 3\times9\}}_{\text{5-terms}}, ...$$

$\therefore$ 11th set will have 1 + (10)2 = 21 term

Also upto 10th set total 3 × k type terms will be 1 + 3 + 5 +……+19 = 100 – term

$\therefore$ Set 11 = {3 × 101, 3 × 102,……3 × 121}

$\therefore$ Sum of elements = 3 × (101 + 102 +…+121)

$$= \frac{3\times222\times21}{2} = 6993$$

134. Answer (2)

$\because$ $a_1, a_2, \ldots a_n \ldots$ be an A.P of natural numbers and

$$\frac{S_5}{S_9} = \frac{5}{17} \Rightarrow \frac{\frac{5}{2}[2a_1+4d]}{\frac{9}{2}[2a_1+8d]} = \frac{5}{17}$$

$\Rightarrow\ 34a_1 + 68d = 18a_1 + 72d$

$\Rightarrow\ 16a_1 = 4d$

$\therefore\ \boxed{d = 4a_1}$

And $110 < a_{15} < 120$

$\therefore\ 110 < a_1 + 14d < 120 \Rightarrow 110 < 57a_1 < 120$

$\therefore\ a_1 = 2\ (\because a_i \in N)$

$d = 8$

$\therefore\ S_{10} = 5\,[4 + 9 \times 8] = 380$

135. Answer (10620)

$\because\ |f(n)| \le 800$

$\Rightarrow\ -800 \le 2n^2 - n - 1 \le 800$

$\Rightarrow\ 2n^2 - n - 801 \le 0$

$$\therefore\ n \in \left[\frac{-\sqrt{6409}+1}{4}, \frac{\sqrt{6409}+1}{4}\right] \text{ and } n \in z.$$

$\therefore\ n = -19, -18, -17, \ldots\ldots\ldots, 19, 20.$

$$\therefore\ \sum\left(2x^2 - x - 1\right) = 2\sum x^2 - \sum x - \sum 1.$$

$$= 2\cdot 2\cdot\left(1^2 + 2^2 + \ldots + 19^2\right) + 2.20^2 - 20 - 40$$

$$= 10620$$

136. Answer (1)

Let first term of G.P. be a and common ratio is r

Then, $\dfrac{a}{1-r} = 5$ ...(i)

$$a\frac{(r^5-1)}{(r-1)} = \frac{98}{25} \Rightarrow 1 - r^5 = \frac{98}{125}$$

$$\therefore\ r^5 = \frac{27}{125},\ r = \left(\frac{3}{5}\right)^{\frac{3}{5}}$$

$$\therefore\ \text{Then, } S_{21} = \frac{21}{2}\left[2\times 10ar + 20\times 10ar^2\right]$$

$$= 21\left[10ar + 10\cdot 10ar^2\right]$$

$$= 21\,a_{11}$$

137. Answer (120)

$$T_n = \frac{\sum_{k=1}^{n}\left[(2k)^3 - (2k-1)^3\right]}{n(4n+3)}$$

$$= \frac{\sum_{k=1}^{n} 4k^2 + (2k-1)^2 + 2k(2k-1)}{n(4n+3)}$$

$$= \frac{\sum_{k=1}^{n}(12k^2 - 6k + 1)}{n(4n+3)}$$

$$= \frac{2n(2n^2+3n+1) - 3n^2 - 3n + n}{n(4n+3)}$$

$$= \frac{n^2(4n+3)}{n(4n+3)} = n$$

$$\therefore\ T_n = n$$

$$S_n = \sum_{n=1}^{15} T_n = \frac{15\times 16}{2} = 120$$

138. Answer (3)

$$a_{n+2} = \frac{2}{a_{n+1}} + a_n$$

$$\Rightarrow\ a_n\,a_{n+1} + 1 = a_{n+1}\,a_{n+2} - 1$$

$$\Rightarrow\ a_{n+2}\,a_{n+1} - a_n\cdot a_{n+1} = 2$$

For $n = 1$ $a_3a_2 - a_1a_2 = 2$

$n = 2$ $a_4a_3 - a_3a_2 = 2$

$n = 3$ $a_5a_4 - a_4a_3 = 2$

$\vdots$

$n = n$ $a_{n+2}a_{n+1} - a_na_{n+1} = 2$

$a_{n+2}a_{n+1} = 2n + a_1a_2$

Now,

$$\frac{(a_1a_2+1)}{a_2a_3}\cdot\frac{(a_2a_3+1)}{a_3a_4}\cdot\frac{(a_3a_4+1)}{a_4a_5}\cdot\ldots\ldots\cdot\frac{(a_{30}a_{31}+1)}{a_{31}a_{32}}$$

$$= \frac{3}{4}\cdot\frac{5}{6}\cdot\frac{7}{8}\cdot\ldots\ldots\cdot\frac{61}{62}$$

$$= 2^{-60}\left({}^{61}C_{31}\right)$$

139. Answer (50)

$\because$ a_1, a_2, a_3, a_4 are in A.P and its mean is p.

$\therefore$ $a_1 = p - 3d$, $a_2 = p - d$, $a_3 = p + d$ and $a_4 = p + 3d$

Where $d > 0$

$\because$ $|f(a_i)| = 500$

$\Rightarrow$ $|9d^2 - q| = 500$

and $|d^2 - q| = 500$ ...(i)

either $9d^2 - q = d^2 - q$

$\Rightarrow$ $d = 0$ not acceptable

$\therefore$ $9d^2 - q = q - d^2$

$\therefore$ $5d^2 - q = 0$ ...(ii)

Roots of $f(x) = 0$ are $p + \sqrt{q}$ and $p - \sqrt{q}$

$\therefore$ absolute difference between roots $= |2\sqrt{q}|$

$= 50$

140. Answer (142)

$x_1, x_2, x_3, \dots, x_{20}$ are in G.P.

$x_1 = 3,\ r = \frac{1}{2}$

$$\bar{x} = \frac{\sum x_i^2 - 2x_i i + i^2}{20}$$

$$= \frac{1}{20}\left[12\left(1 - \frac{1}{2^{40}}\right) - 6\left(4 - \frac{11}{2^{18}}\right) + 70 \times 41\right]$$

$$\begin{cases} S = 1 + 2\cdot\frac{1}{2} + 3\cdot\frac{1}{2^2} + \dots \\ \frac{S}{2} = \qquad \frac{1}{2} + \frac{2}{2^2} + \dots \end{cases}$$

$$\left.\frac{S}{2} = 2\left(1 - \frac{1}{2^{20}}\right) - \frac{20}{2^{20}} = 4 - \frac{11}{2^{18}}\right\}$$

$$\therefore \quad [\bar{x}] = \left[\frac{2858}{20} - \left(\frac{12}{240} - \frac{66}{2^{18}}\right)\cdot\frac{1}{20}\right] = 142$$

141. Answer (12)

$$\frac{1}{3^{12}} + 5\left(\frac{2^0}{3^{12}} + \frac{2^1}{3^{11}} + \frac{2^2}{3^{10}} + \dots\dots + \frac{2^{11}}{3}\right) = 2^n \cdot m$$

$$\Rightarrow \quad \frac{1}{3^{12}} + 5\left(\frac{1}{3^{12}}\frac{\left((6)^2 - 1\right)}{(6-1)}\right) = 2^n \cdot m$$

$$\Rightarrow \quad \frac{1}{3^{12}} + \frac{5}{5}\left(\frac{1}{3^{12}}\cdot 2^{12}\cdot 3^{12} - \frac{1}{3^{12}}\right) = 2^n \cdot m$$

$$\Rightarrow \quad \frac{1}{3^{12}} + 2^{12} - \frac{1}{3^{12}} = 2^n \cdot m$$

$\Rightarrow$ $2^n \cdot m = 2^{12}$

$\Rightarrow$ $m = 1$ and $n = 12$

$m \cdot n = 12$

142. Answer (4)

$S = \{1, 2, 3, \dots\dots 2022\}$

HCF $(n, 2022) = 1$

$\Rightarrow$ n and 2022 have no common factor

Total elements = 2022

$2022 = 2 \times 3 \times 337$

M : numbers divisible by 2.

$\{2, 4, 6, \dots, 2022\}$ $n(M) = 1011$

N : numbers divisible by 3.

$\{3, 6, 9, \dots, 2022\}$ $n(N) = 674$

L : numbers divisible by 6.

$\{6, 12, 18, \dots, 2022\}$ $n(L) = 337$

$n(M \cup N) = n(M) + n(N) - n(L)$

$= 1011 + 674 - 337$

$= 1348$

0 = Number divisible by 337 but not in $M \cup N$

$\{337, 1685\}$

Number divisible by 2, 3 or 337

$= 1348 + 2 = 1350$

Required probability $= \frac{2022 - 1350}{2022}$

$$= \frac{672}{2022}$$

$$= \frac{112}{337}$$

143. Answer (84)

Fifth term from beginning $= {}^nC_4\left(2^{\frac{1}{4}}\right)^{n-4}\left(3^{\frac{-1}{4}}\right)^4$

Fifth term from end = $(n-5+1)^{\text{th}}$ term from begin

$$= {}^nC_{n-4}\left(2^{\frac{1}{4}}\right)^3\left(3^{\frac{-1}{4}}\right)^{n-4}$$

Given $\dfrac{{}^nC_4\, 2^{\frac{n-4}{4}}\cdot 3^{-1}}{{}^nC_{n-3}\, 2^{\frac{4}{4}}\cdot 3^{-\left(\frac{n-4}{4}\right)}} = 6^{\frac{1}{4}}$

$\Rightarrow\ 6^{\frac{n-8}{4}} = 6^{\frac{1}{4}}$

$\Rightarrow\ \dfrac{n-8}{4} = \dfrac{1}{4} \qquad \Rightarrow n = 9$

$T_6 = T_{5+1} = {}^9C_5\left(2^{\frac{1}{4}}\right)^4\left(3^{\frac{-1}{4}}\right)^5$

$$= \frac{{}^9C_5\cdot 2}{3^{\frac{1}{4}}\cdot 3} = \frac{84}{3^{\frac{1}{4}}} = \frac{\alpha}{3^{\frac{1}{4}}}$$

$\Rightarrow\ \alpha = 84.$

144. Answer (2)

$a_{n+2} = 3a_{n+1} - 2a_n + 1,\ \forall\ n \ge 0 \quad (a_0 = a_1 = 0)$

$(a_{n+2} - a_{n+1}) - 2(a_{n+1} - a_n) - 1 = 0$

Put $n = 0$

$(a_2 - a_1) - 2(a_1 - a_0) - 1 = 0$

$n = 1$

$(a_3 - a_2) - 2(a_2 - a_1) - 1 = 0$

$n = 2$

$(a_4 - a_3) - 2(a_3 - a_2) - 1 = 0$

$\vdots$

$n = n$

$(a_{n+2} - a_{n+1}) - 2(a_{n+1} - a_n) - 1 = 0$

Adding,

$(a_{n+2} - a_1) - 2(a_{n+1} - a_0) - (n+1) = 0$

$\therefore\ a_{n+2} - 2a_{n+1} - (n+1) = 0$

$n \to n-2$

$a_n - 2a_{n-1} - n + 1 = 0$

Now, $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$

$= a_{25}(a_{23} - 2a_{22}) - 2a_{24}(a_{23} - 2a_{22})$

$= (a_{25} - 2a_{24})(a_{23} - 2a_{22})$

$= 24\cdot 22 = 528$

145. Answer (2)

$\sum_{r=1}^{20}\left(r^2 + 1 + 2r - 2r\right)r! = \sum_{r=1}^{20}\left((r+1)^2 - 2r\right)r!$

$= \sum_{r=1}^{20}\left[(r+1)(r+1)! - rr!\right] - \sum_{r=1}(r+1)r! = r!$

$= (2\cdot 2! - 1!) + (3\cdot 3! - 2\cdot 2!) + \ldots + (21.21! - 20.20!)$

$-\left[(2! - 1!) + (3! - 2!) + \ldots + (21! - 20!)\right]$

$= (21\cdot 21! - 1) - (21! - 1)$

$= 20.21! = (22 - 2)21!$

$= 22! - 2(21!)$

146. Answer (166)

$\sum_{k=1}^{10}\dfrac{k}{k^4 + k^2 + 1}$

$= \dfrac{1}{2}\left[\sum_{k=1}^{10}\left(\dfrac{1}{k^2 - k + 1} - \dfrac{1}{k^2 + k + 1}\right)\right]$

$= \dfrac{1}{2}\left[1 - \dfrac{1}{3} + \dfrac{1}{3} - \dfrac{1}{7} + \dfrac{1}{7} - \dfrac{1}{13} + \ldots + \dfrac{1}{91} - \dfrac{1}{111}\right]$

$= \dfrac{1}{2}\left[1 - \dfrac{1}{111}\right] = \dfrac{110}{2.111} = \dfrac{55}{111} = \dfrac{m}{n}$

$\therefore\ m + n = 55 + 111 = 166$

147. Answer (16)

Given

$$S = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \frac{a_4}{2^4} + \ldots\ldots\infty$$

$$\frac{1}{2}S = \frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots\ldots\ldots\infty$$

$$\frac{S}{2} = \frac{a_1}{2} + \frac{(a_2 + a_1)}{2^2} + \frac{(a_3 + a_2)}{2^3} + \ldots\infty$$

$$\Rightarrow \frac{S}{2} = \frac{a_1}{2} + \frac{d}{2}$$

$\Rightarrow$ $a_1 + d = a_2 = 4$ $\Rightarrow 4a_2 = 16$

148. Answer (3)

$$\frac{1}{20}\left(\frac{1}{20-a} - \frac{1}{40-a} + \frac{1}{40-a} - \frac{1}{60-a} + \ldots\ldots + \frac{1}{180-a} - \frac{1}{200-a}\right) = \frac{1}{256}$$

$$\Rightarrow \frac{1}{20}\left(\frac{1}{20-a} - \frac{1}{200-a}\right) = \frac{1}{256}$$

$$\Rightarrow \frac{1}{20}\left(\frac{180}{(20-a)(200-a)}\right) = \frac{1}{256}$$

$\Rightarrow$ $(20 - a)(200 - a) = 9.256$

or $a^2 - 220a + 1696 = 0$

$\Rightarrow$ $a = 212, 8$

149. Answer (2)

$$\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)} = \frac{3}{4}\sum_{n=1}^{21} \frac{1}{4n-1} - \frac{1}{4n+3}$$

$$= \frac{3}{4}\left[\left(\frac{1}{3} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{11}\right) + \ldots\ldots + \left(\frac{1}{83} - \frac{1}{87}\right)\right]$$

$$= \frac{3}{4}\left[\frac{1}{3} - \frac{1}{87}\right] = \frac{3}{4}\ \frac{84}{3.87} = \frac{7}{29}$$

150. Answer (53)

$$d_1 = \frac{199-100}{2} \notin I$$

$$d_2 = \frac{199-100}{3} = 33$$

$$d_3 = \frac{199-100}{4} \notin I$$

$$d_n = \frac{199-100}{i+1} \in I$$

$d_i = 33 + 11, 9$

Sum of CD's = 33 + 11 + 9

= 53

151. Answer (286)

$$S = \frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + \frac{1}{4\times5\times6} + \ldots + \frac{1}{100\times101\times102}$$

$$= \frac{1}{(3-1)\cdot 1}\left[\frac{1}{2\times3} - \frac{1}{101\times102}\right]$$

$$= \frac{1}{2}\left(\frac{1}{6} - \frac{1}{101\times102}\right)$$

$$= \frac{143}{102\times101} = \frac{k}{101}$$

$\therefore$ $34k = 286$

❑ ❑ ❑