Relations and Functions

2016

1. Show that the binary operation * on $A = \mathbf{R} - \{-1\}$ defined as $a*b = a + b + ab$ for all a, b ϵ A is commutative and associative on A. Also find the identity element of * in A and prove that every element of A is invertible.

2017

1. Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $[-\sqrt{3}, 0]$.

Ans- c= -1

2. Consider $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x + 3}{3x + 4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

OR

Let $A = \mathbb{Q} \times \mathbb{Q}$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Determine, whether $*$ is commutative and associative. Then, with respect to $*$ on A

(i) Find the identity element in A.

(ii) Find the invertible elements of A.

Ans- $f^{-1}(y) = \frac{4y - 3}{4 - 3y}; y \in R - \left\{\frac{4}{3}\right\}$

$f^{-1}(0) = -\frac{3}{4}$

and x = 11/10

OR

i. (1,0)

ii. $\left(\frac{1}{a}, -\frac{b}{a}\right)$, $a \neq 0$ is the inverse of $(a, b) \in A$

2018

1. If $a * b$ denotes the larger of 'a' and 'b' and if $a \circ b = (a * b) + 3$, then write the value of $(5) \circ (10)$, where $*$ and $\circ$ are binary operations.

Ans. $5 \circ 10 = (5 * 10) + 3 = 10 + 3 = 13$ For $5 * 10 = 10$

For Final Answer = 13

2. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

OR

Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \dfrac{x}{x^2 + 1}, \forall x \in \mathbb{R}$ is neither one-one nor onto. Also, if $g : \mathbb{R} \to \mathbb{R}$ is defined as $g(x) = 2x - 1$, find $fog(x)$.

Ans. $\{1, 5, 9\}$ $\{2, 6, 10\}$

or $fog(x) = f(2x - 1) = \dfrac{2x-1}{(2x-1)^2 + 1} = \dfrac{2x-1}{4x^2 - 4x + 2}$

2019

1. Examine whether the operation * defined on R by a * b = ab + 1 is (i) a binary or not. (ii) if a binary operation, is it associative or not ?

Ans. * is a binary operation on R.

'*' operation is not associative

2. Show that the relation R on ℝ defined as R = {(a, b) : a $\leq$ b}, is reflexive, and transitive but not symmetric.

OR

Prove that the function f : N $\rightarrow$ N, defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of f : N $\rightarrow$ S, where S is range of f.

2020

1. If $f(x) = \dfrac{4x+3}{6x-4}$, $x \neq \dfrac{2}{3}$, then show that (fof) $(x) = x$, for all $x \neq \dfrac{2}{3}$. Also, write inverse of f.

OR

Check if the relation R in the set ℝ of real numbers defined as R = {(a, b) : a < b} is (i) symmetric, (ii) transitive

Ans.

$\cdot f^{-1}(x) = \dfrac{4x+3}{6x-4}$

Or

ii) transitive

Inverse Trigonometric Functions

2016

1. Solve the equation for x : $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

OR

If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$, prove that $\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$

Ans- x = 0 or 1/2

2017

1. If $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$, then find the value of x.

Ans- $x = \pm\sqrt{\frac{17}{2}}$

2.

2018

1. Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$.

Ans. $\frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{2}$

2. Prove that :

$$3\sin^{-1}x = \sin^{-1}(3x - 4x^3),\ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

2019

1. Solve for x : $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$.

Ans. $x = \frac{1}{6}$

$x = -1$ is rejected as $\tan^{-1}(-2)$ is negative and $\tan^{-1}(-3)$ is negative but RHS of (i) is positive

2020

1. Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$.

Ans. $-\frac{\pi}{8}$

2. Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}(x) = \frac{\pi}{2}$.

Ans. 0

Matrices

2016

1. Use elementary column operation $C_2 \to C_2 + 2C_1$ in the following matrix equation :

$$\begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

Ans - $\begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}$

2. Write the number of all possible matrices of order 2x2 with each entry 1, 2 or 3.
Ans- No. of possible matrices = 3^4 or 81.

3. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds, they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this question ?

Ans- Rs. 25000

2017

1. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

2. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

Ans- $x = 3,\ y = -2,\ z = -1$

2018

1. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'.

Ans. $a = -2,\ b = 3$

2. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

3. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use it to solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3.$

OR

Using elementary row transformations, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}.$$

Ans.

$$A^{-1} = \frac{1}{|A|} \cdot \mathrm{adj}(A) = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

$x = 1,\ y = 2,\ z = 3$

Or

$$A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

2019

1. Find the value of $x - y$, if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$$

Ans. 0

2. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $(A^2 - 5A)$.

Ans.

$$\mathbf{A^2 - 5A = \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}}$$

3. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Hence solve the following system of equations :

$2x - 3y + 5z = 11,\ \ 3x + 2y - 4z = -5,\ x + y - 2z = -3.$

OR

Obtain the inverse of the following matrix using elementary operations :

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Ans. **x = 1, y = 2 and z = 3 is the solution the given system of equations.**

Or

$$A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$

2020

1. For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ write A^{-1}.

Ans.

$$A^{-1} = \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$$

2. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}.

Using A^{-1}, solve the following system of equations :

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

Ans.

$$A^{-1} = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \qquad x = 1, \; y = 2, \; z = 3$$

Determinants

2016

1. If $x \in \mathrm{N}$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.

2. Using properties of determinants, show that ΔABC is isosceles if :

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C \end{vmatrix} = 0$$

OR

A shopkeeper has 3 varieties of pens 'A', 'B' and 'C'. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of 'A' variety, 3 pens of 'B' variety and 2 pens of 'C' variety for ₹ 60. While Shikha purchased 6 pens of 'A' variety, 2 pens of 'B' variety and 3 pens of 'C' variety for ₹ 70. Using matrix method, find cost of each variety of pen.

Ans- 5, 8, 8

2017

1. If for any 2×2 square matrix A, $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$.

Ans- 8

2. Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

OR

Find matrix A such that

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$

Ans- $\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$

3.

2018

1. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9\,(3xyz + xy + yz + zx)$$

2019

1. Using properties of determinants, prove the following :

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)\,(b+c)\,(c+a).$$

2020

1. If a, b, c are p^{th}, q^{th} and r^{th} terms respectively of a G.P, then prove that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

Continuity and Differentiability

2016

1. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

OR

If $y = 2\cos(\log x) + 3\sin(\log x)$, prove that $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$.

Ans- $\dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx} = x^{\sin x} \cdot \left\{\cos x \cdot \log x + \dfrac{\sin x}{x}\right\} + (\sin x)^{\cos x}\{\cos x \cdot \cot x - \sin x \cdot \log(\sin x)\}$

2. If $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t(1 - \cos 2t)$, find $\dfrac{dy}{dx}$ at $t = \dfrac{\pi}{4}$.

Ans- b/a

2017

1. Determine the value of 'k' for which the following function is continuous at x = 3 :

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x - 3} & , \quad x \neq 3 \\ k & , \quad x = 3 \end{cases}$$

Ans- k= 12

2. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on $\mathbb{R}$.

3. If $x^y + y^x = a^b$, then find $\dfrac{dy}{dx}$.

OR

If $e^y(x + 1) = 1$, then show that $\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$.

Ans- $\frac{dy}{dx} = -\frac{y^x \log y + y \cdot x^{y-1}}{x^y \cdot \log x + x \cdot y^{x-1}}$

2018

1. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x.

Ans. $-\frac{1}{2}$

2. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

OR

If ... $y = a\,(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.

Ans. $\frac{dy}{dx} = \frac{y - 4x^3 - 4xy^2}{4x^2 y + 4y^3 - x}$

Or $\left.\frac{dy}{dx}\right]_{\theta = \frac{\pi}{3}} = \frac{1}{\sqrt{3}}$

2019

1. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$, then find $\frac{d}{dx}(fog)(x)$.

Ans. 1

2. If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

OR

If $x^y - y^x = a^b$, find $\frac{dy}{dx}$.

Ans. or

$$\frac{dy}{dx} = \frac{y^x \cdot \log y - x^{y-1} \cdot y}{x^y \cdot \log x - y^{x-1} \cdot x}$$

3. If $x = \cos t + \log \tan\left(\frac{t}{2}\right)$, $y = \sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

Ans. $2\sqrt{2}$

2020

1. If the function f defined as

$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & , x \neq 3 \\ k & , x = 3 \end{cases}$$

is continuous at $x = 3$, find the value of k.

Ans. $k = 6$

2. If $x = a\cos\theta$; $y = b\sin\theta$, then find $\dfrac{d^2y}{dx^2}$.

OR

Find the differential of $\sin^2 x$ w.r.t. $e^{\cos x}$.

Ans. $\dfrac{d^2y}{dx^2} = \dfrac{b}{a}\operatorname{cosec}^2\theta\left(\dfrac{-1}{a\sin\theta}\right) = -\dfrac{b}{a^2}\operatorname{cosec}^3\theta$

Or

$-2\cos x\, e^{-\cos x}$

3. If $y = (\log x)^x + x^{\log x}$, then find $\dfrac{dy}{dx}$.

Ans. $\dfrac{dy}{dx} = (\log x)^x\left[\dfrac{1}{\log x} + \log(\log x)\right] + x^{\log x}.\dfrac{2\log x}{x}$

Application of Derivatives

2016

1. The equation of tangent at (2, 3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b.

Ans- a= 2 and b= -7

2. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}\,r$.

OR

If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

2017

1. The volume of a cube is increasing at the rate of 9 cm^3/s. How fast is its surface area increasing when the length of an edge is 10 cm ?

Ans- $3.6\ cm^2/s$

2. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

2018

1. The total cost C(x) associated with the production of x units of an item is given by $C(x) = 0{\cdot}005x^3 - 0{\cdot}02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

Ans. 30.015

2019

1. Find the equations of the tangent and the normal, to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1), where $x_1 = 2$ and $y_1 > 0$.

OR

Find the intervals in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is (a) strictly increasing, (b) strictly decreasing.

Ans. Equation of tangent: $32x + 27y = 145$

Equation of Normal: $27x - 32y = -42$

Or

$f(x)$ is strictly increasing on $(-3, 2) \cup (4, \infty)$

$f(x)$ is strictly decreasing on $(-\infty, -3) \cup (2, 4)$

2. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in this question ?

Ans. $\frac{x}{2}$

3. Find the equation of tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$. Also, write the equation of normal to the curve at the point of contact.

Ans. $48x - 24y = 23$

$48x + 96y = 113$

4. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find the maximum volume of cone.

2020

1. If $f(x) = x^4 - 10$, then find the approximate value of $f(2.1)$.

OR

Find the slope of the tangent to the curve $y = 2\sin^2(3x)$ at $x = \frac{\pi}{6}$.

Ans. 9.2
Or
0

2. Find the minimum value of $(ax + by)$, where $xy = c^2$.
Ans. $2\sqrt{ab}.c$

Integrals

2016

1. Find : $\int \frac{x^2}{x^4+x^2-2}\,dx$

Ans- $\frac{1}{6}\log\left|\frac{x-1}{x+1}\right| + \frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}} + C$

2. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x}\,dx$

OR

Evaluate : $\int_0^{\frac{3}{2}} |x\cos \pi x|\,dx$

Ans- $\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$ **OR** $\frac{5}{2\pi} - \frac{1}{\pi^2}$

3. Find : $\int (3x+1)\sqrt{4-3x-2x^2}\,dx$

Ans- $-\frac{1}{2}(4-3x-2x^2)^{\frac{3}{2}} + C_1 - \frac{5\sqrt{2}}{4}\left[\frac{1}{2}\left(x+\frac{3}{4}\right)\sqrt{\left(\frac{41}{16}\right)-\left(x+\frac{3}{4}\right)^2} + \frac{1}{2}\left(\frac{41}{16}\right)\sin^-1\left(\frac{x+\frac{3}{4}}{\frac{\sqrt{41}}{4}}\right) + C_2\right]$

2017

1. Find :

$$\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x}\,dx$$

Ans- $-\log|\sin 2x| + c$ OR $\log|\sec x| - \log|\sin x| + c.$

2. **Find :**

$$\int \frac{dx}{5 - 8x - x^2}$$

Ans- $\frac{1}{2\sqrt{21}} \log \left| \frac{\sqrt{21} + (x+4)}{\sqrt{21} - (x+4)} \right| + c$

3. **Find :**

$$\int \frac{\cos \theta}{(4 + \sin^2 \theta)(5 - 4\cos^2 \theta)} \, d\theta$$

Ans- $-\frac{1}{30} \tan^{-1}\left(\frac{\sin\theta}{2}\right) + \frac{2}{15} \tan^{-1}(2\sin\theta) + c$

4. **Evaluate :**

$$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} \, dx$$

OR

Evaluate :

$$\int_1^4 \{|x-1| + |x-2| + |x-4|\} dx$$

Ans- $\frac{\pi(\pi - 2)}{2}$ **OR** 23/2

2018

1. **Evaluate :**

$$\tan x + C \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} \, dx$$

Ans.

2.

Find :

$$\int \frac{2 \cos x}{(1 - \sin x)(1 + \sin^2 x)} dx$$

Ans. $-\log(1-\sin x) + \frac{1}{2}\log(1+\sin^2 x) + \tan^{-1}(\sin x) + C$

3. **Evaluate :**

$$\int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9 \sin 2x} dx$$

OR

Evaluate

$$\int_1^3 (x^2 + 3x + e^x)\, dx,$$

as the limit of the sum.

Ans. $-\frac{1}{30}\log\frac{1}{4}$ or $\frac{1}{15}\log 2$

Or

$$\frac{62}{3} + e^3 - e$$

2019

1. Find : $\int \sqrt{1 - \sin 2x}\, dx,\ \frac{\pi}{4} < x < \frac{\pi}{2}$

OR

Find : $\int \sin^{-1}(2x)\, dx.$

Ans. $-\cos x - \sin x + C$

Or

$$x \sin^{-1}(2x) + \frac{1}{2}\sqrt{1-4x^2} + C$$

2. Find : $\int \frac{\tan^2 x \sec^2 x}{1 - \tan^6 x}\, dx.$

Ans. $I = \frac{1}{6} \ln\left|\frac{1+\tan^3 x}{1-\tan^3 x}\right| + C$

3. Find : $\int \frac{3x+5}{x^2+3x-18}\, dx.$

Ans. $\frac{3}{2}\ln\left|x^2+3x-18\right| + \frac{1}{18}\log\left|\frac{x-3}{x+6}\right| + c$

4. Prove that $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$, hence evaluate $\int_0^\pi \frac{x \sin x}{1+\cos^2 x}\, dx.$

Ans. $\frac{\pi^2}{4}$

2020

1. Find the value of $\int_1^4 |x-5|\, dx$.

Ans. 15/2

2. Find $\int \frac{x}{x^2+3x+2}\, dx$.

Ans. $-\log|x+1| + 2\log|x+2| + C$

3. Evaluate $\int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2}\right] e^{2x}\, dx$.

Ans. $\frac{e^4}{\ } - \frac{e^2}{\ }$

4. Find the value of $\int_0^1 x(1-x)^n\, dx$.

Ans. $\frac{1}{(n+1)(n+2)}$

5. Evaluate the following integral as the limit of sums $\int_1^4 (x^2 - x)\, dx$.

Ans. 27/2

2021

1. (a) If $\frac{d}{dx}[F(x)] = \frac{\sec^4 x}{\operatorname{cosec}^4 x}$ and $F\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$, then find $F(x)$.

OR

(b) Find : $\int \frac{\log x - 3}{(\log x)^4}\, dx$.

Ans-

$$F(x) = \frac{\tan^3 x}{3} - \tan x + x + \frac{2}{3}$$

OR

$$\frac{x}{(\log x)^3} + C$$

2. (a) Find : $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$.

OR

(b) Evaluate : $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(4+\sin x)}\,dx$.

Ans- $: 2\sqrt{x}-3\sqrt[3]{x}+6x^{1/6}-6\log(x^{1/6}+1)+C$

OR

$: \frac{1}{3}[\log 2-\log 5+\log 4]$

3. Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x}\,dx$.

Ans- $: \pi$

Application of Integrals

2016

1. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of square bounded by $x = 0$, $x = 4$, $y = 4$ and $y = 0$ into three equal parts.

2017

1. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A (4, 1), B (6, 6) and C (8, 4).

OR

Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$.

Ans- Area= 7 sq. units **OR** 27 sq. units

2018

1. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

Ans. 4π

2019

1. Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

OR

Find the area of the region lying about x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$.

Ans. Ans. 7

Or $\frac{32}{3} + 4\pi$

2020

1.

Using integration find the area of the region bounded between the two ci $^2 = 9$ and $(x-3)^2 + y^2 = 9$.

Ans. $\left(6\pi - \frac{9\sqrt{3}}{2}\right)$

2021

1. (a) Using integration, find the area of the region $\{(x, y) : 4x^2 + 9y^2 \leq 36, 2x + 3y \geq 6\}$.

OR

(b) Using integration, find the area of the region bounded by lines $x - y + 1 = 0$, $x = -2$, $x = 3$ and x-axis.

Ans- $\frac{2}{3}\left[\frac{9\pi}{4} - \frac{9}{2}\right]$ *or* $\frac{3\pi}{2} - 3$

OR

17/2

Differential Equations

2016

1. Solve the differential equation :

$$y + x\frac{dy}{dx} = x - y\frac{dy}{dx}$$

Ans- $\frac{1}{2}\log\left|\frac{y^2}{x^2} + \frac{2y}{x} - 1\right| = \log C - \log x$ or, $y^2 + 2xy - x^2 = C^2$

2. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

Ans- $\left(\frac{xy' + yy'}{y' - 1}\right)^2 + \left(\frac{x + y}{y' - 1}\right)^2 = \left(\frac{x + yy'}{y' - 1}\right)^2$

2017

1. Solve the differential equation $(\tan^{-1} x - y)\,dx = (1 + x^2)\,dy$.

Ans- $y = (\tan^{-1} x - 1) + c \cdot e^{-\tan^{-1} x}$

2. Find the particular solution of the differential equation $(x - y)\frac{dy}{dx} = (x + 2y)$, given that $y = 0$ when $x = 1$.

Ans- 0

2018

1. Find the differential equation representing the family of curves $y = a\,e^{bx+5}$, where a and b are arbitrary constants.

Ans. $y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

2. If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$.

3. Find the particular solution of the differential equation $e^x \tan y \, dx + (2 - e^x) \sec^2 y \, dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$.

OR

Find the particular solution of the differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$.

Ans. $\tan y = 2 - e^x$

Or

$$\text{Integrating factor} = e^{\int 2\tan x dx} = \sec^2 x$$

2019

1. Find the order and the degree of the differential equation $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$.

Ans. 2, 1

2. Form the differential equation representing the family of curves $y = e^{2x}(a + bx)$, where 'a' and 'b' are arbitrary constants.

Ans.

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

3. Solve the differential equation : $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$, given that $y = 0$ when $x = 1$.

OR

Solve the differential equation : $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$, subject to the initial condition $y(0) = 0$.

Ans. $y + \sqrt{x^2 + y^2} = x^2$

Or

$$y = \frac{4x^3}{3(1+x^2)}$$

2020

1. Solve the differential equation :

$$x \sin\left(\frac{y}{x}\right)\frac{dy}{dx} + x - y \sin\left(\frac{y}{x}\right) = 0$$

Given that $x = 1$ when $y = \frac{\pi}{2}$.

Ans. $\cos\left(\frac{y}{x}\right) = \log|x|$ is the required solution.

2021

1. Find the general solution of the differential equation

$\sec^2 x.\tan y\, dx + \sec^2 y.\tan x\, dy = 0$

Ans- 2

2. Find the particular solution of the differential equation $x\frac{dy}{dx} + x\cos^2\left(\frac{y}{x}\right) = y$; given that when $x = 1$, $y = \frac{\pi}{4}$.

Ans- Particular solution is $\tan\frac{y}{x} = -\log|x| + 1$

Vector Algebra

2016

1. Write the position vector of the point which divides the join of points with position vectors $3\vec{a} - 2\vec{b}$ and $2\vec{a} + 3\vec{b}$ in the ratio 2 : 1.

Ans- $\frac{7}{3}\vec{a} + \frac{4}{3}\vec{b}$

2. Write the number of vectors of unit length perpendicular to both the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$.

Ans- 2

3. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

Ans-
Area of parallelogram $= \frac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right| = \sqrt{404}$ or $2\sqrt{101}$ sq. units

2018

1. Find the magnitude of each of the two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

Ans. $|\vec{a}| = |\vec{b}| = 3$

2. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$.

Ans. $\frac{2\sqrt{6}}{7}$

3. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d} \cdot \vec{a} = 21$.

Ans. $\vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$

2019

1. If a line makes angles 90°, 135°, 45° with the x, y and z axes respectively, find its direction cosines.

OR

Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.

Ans. $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Or

$\vec{r} = (3\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$

2. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

OR

If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, find $[\vec{a}\ \vec{b}\ \vec{c}]$.

Ans. or
-30

3. If $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not.

Ans. Yes.

2020

1. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

OR

Using vectors, find the area of the triangle ABC with vertices A (1, 2, 3), B(2, –1, 4) and C (4, 5, – 1).

Ans. unit vectors are $\frac{(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|} = \frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$ and $\frac{(\vec{a}-\vec{b})}{|\vec{a}-\vec{b}|} = -\frac{1}{\sqrt{69}}\hat{i} - \frac{2}{\sqrt{69}}\hat{j} + \frac{8}{\sqrt{69}}\hat{k}$

Or

$\frac{1}{2}\sqrt{274}$

2021

1. In a parallelogram PQRS, $\overrightarrow{PQ} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{PS} = -\hat{i} - 2\hat{k}$. Find $|\overrightarrow{PR}|$ and $|\overrightarrow{QS}|$.

Ans- 6

2. (a) If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are four non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d}$, then show that $(\vec{a} - 2\vec{d})$ is parallel to $(2\vec{b} - \vec{c})$ where $\vec{a} \neq 2\vec{d}$, $\vec{c} \neq 2\vec{b}$.

OR

(b) The two adjacent sides of a parallelogram are represented by $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

Ans- or

$2\sqrt{101}$

Three Dimensional Geometry

2016

1. Find the vector equation of the plane with intercepts 3, -4 and 2 on x, y and z-axis respectively.

Ans - $\vec{r}\cdot(4\hat{i}-3\hat{j}+6\hat{k}) = 12$

2. Find the coordinates of the point where the line through the points A(3, 4, 1) and B(5, 1, 6) crosses the XZ plane. Also find the angle which this line makes with the XZ plane.

Ans- Co-ordinate of required point $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

$$\theta = \sin^{-1}\left(\frac{3}{\sqrt{38}}\right)$$

3. Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i}+3\hat{j}+4\hat{k}$ to the plane $\vec{r}\,.\left(2\hat{i}+\hat{j}+3\hat{k}\right)-26=0$. Also find image of P in the plane.

Ans- Position vector of foot of perpendicular are $3\hat{i}+\frac{7}{2}\hat{j}+\frac{11}{2}\hat{k}$

Length of perpendicular: √7/2

The coordinates of the image of the point P in the given plane are (4, 4, 7).

2017

1. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2{\cdot}5y + 5z = 20$.

Ans- 1 unit

2. The x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, – 2) is 4. Find its z-coordinate.

Ans- -1

3. Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

Ans- Area of $\Delta = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{BC}| = \frac{1}{2}\sqrt{210}$

4. Find the value of λ, if four points with position vectors $3\hat{i} + 6\hat{j} + 9\hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$, $2\hat{i} + 3\hat{j} + \hat{k}$ and $4\hat{i} + 6\hat{j} + \lambda\hat{k}$ are coplanar.

Ans- 2

5. Find the coordinates of the point where the line through the points (3, – 4, – 5) and (2, – 3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, – 3) and (0, 4, 3).

OR

A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.

Ans- (1, -2, 7)

2018

1. Find the shortest distance between the lines

$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$.

Ans. $\frac{6\sqrt{5}}{5}$

2. Find the distance of the point (– 1, – 5, – 10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

Ans. 13

2019

1. Find the value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

Ans. $\lambda = 7$, lines do not intersect.

2. Find the vector and Cartesian equations of the plane passing through the points (2, 2 –1), (3, 4, 2) and (7, 0, 6). Also find the vector equation of a plane passing through (4, 3, 1) and parallel to the plane obtained above.

OR

Find the vector equation of the plane that contains the lines $\vec{r} = (\hat{i}+\hat{j}) + \lambda(\hat{i}+2\hat{j}-\hat{k})$ and the point (–1, 3, – 4). Also, find the length of the perpendicular drawn from the point (2, 1, 4) to the plane thus obtained.

Ans. $\vec{r}\cdot(5\hat{i}+2\hat{j}-3\hat{k})=17$ $\quad$ $\vec{r}\cdot(5\hat{i}+2\hat{j}-3\hat{k})=23$

$5x + 2y - 3z = 17$

Or

$\vec{r}\cdot(-\hat{i}+\hat{j}+\hat{k})=0$ $\quad$ $\sqrt{3}$ units

2020

1. Find the vector and cartesian equations of the line which is perpendicular to the lines with equations

$$\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4} \text{ and } \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

and passes through the point (1, 1, 1). Also find the angle between the given lines.

Ans. Equation of line in Cartesian form is: $\frac{x-1}{-4}=\frac{y-1}{4}=\frac{z-1}{-1}$

Vector form of line is $\vec{r}=(\hat{i}+j+k)+\lambda(-4\hat{i}+4j-k)$

$$\theta=\cos^{-1}\left(\frac{24}{\sqrt{21}\sqrt{29}}\right)$$

2021

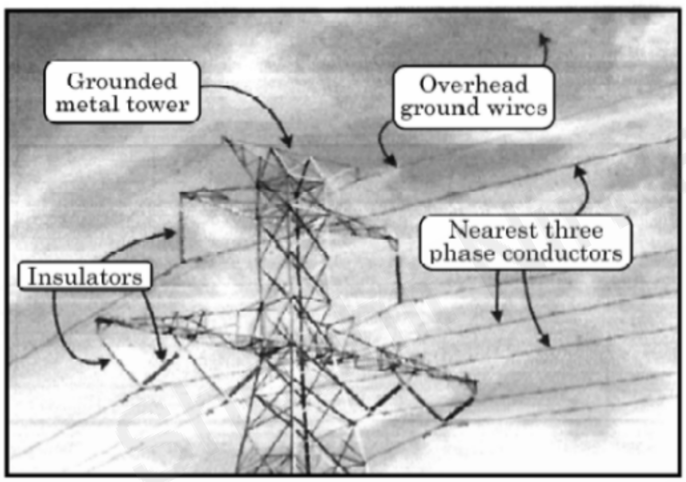
1. Find the values of λ, for which the distance of point $(2, 1, \lambda)$ from plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units.

Ans- $\lambda = \pm 5$

2. Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$ and $\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ and through the point (2, 1, 3).

Ans- Equation is $\vec{r} \cdot (38\hat{i} + 68\hat{j} + 3\hat{k}) = 153$

3. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.



Two such wires lie along the following lines :

$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$

$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2}$

Based on the given information, answer the following questions :

(i) Are the lines l_1 and l_2 coplanar ? Justify your answer.

(ii) Find the point of intersection of the lines l_1 and l_2.

Ans-

i. Coplanar

ii. (2, 1, -3)

Linear Programming

2016

1. There are two types of fertilisers 'A' and 'B'. 'A' consists of 12% nitrogen and 5% phosphoric acid whereas 'B' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If 'A' costs ₹ 10 per kg and 'B' cost ₹ 8 per kg, then graphically determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost.

Ans- The minimum requirement of fertilizer of type A will be 30 kg and that of type B will be 210 kg.

2017

1. Two tailors, A and B, earn ₹ 300 and ₹ 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

2. Maximise $Z = x + 2y$
   subject to the constraints

$$x + 2y \geq 100$$
$$2x - y \leq 0$$
$$2x + y \leq 200$$
$$x, y \geq 0$$

   Solve the above LPP graphically.

Ans- Max (= 400) at $x = 0$, $y = 200$

2018

1. A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of ₹ 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit ? Formulate the above LPP and solve it graphically and find the maximum profit.

Ans. Max. profit is ₹ 41 at $x = 30$, $y = 20$.

2019

1. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is ₹ 15 and on an item of model B is ₹ 10. How many of items of each model should be made per day in order to maximize daily profit ? Formulate the above LPP and solve it graphically and find the maximum profit.

Ans. **The maximum profit is ₹ 350**

For the maximum profit, 10 items of model A and 20 items of model B should be made.

2020

1. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type A souvenir is ₹ 100 each and for type B souvenir, profit is ₹ 120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit ? Formulate the problem as an LPP and solve it graphically.

Ans. For Maximum profit, No. of souvenirs of Type A = 8
No. of souvenirs of Type B = 20

Probability

2016

1. In a game, a man wins ₹ 5 for getting a number greater than 4 and loses ₹ 1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.

OR

A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white ?

Ans- Rs. 19/9 **OR** 3/5

2. Five bad oranges are accidently mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence find the mean and variance of the distribution.

Ans-

X	0	1	2	3	4
P(x)	$\left(\frac{4}{5}\right)^4 = \frac{256}{625}$	${}^4C_1\left(\frac{4}{3}\right)^3\left(\frac{1}{5}\right) = \frac{256}{625}$	${}^4C_2\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)^2 = \frac{96}{625}$	${}^4C_3\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^3 = \frac{16}{625}$	$\left(\frac{1}{5}\right)^4 = \frac{1}{625}$

Mean= 4/5 and Variance= 16/25

2017

1. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

Ans- No

2. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X.

Ans- Mean = 8 and Variance= 20/3

3. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance ? Is regularity required only in school ? Justify your answer.

Ans- 3/4

2018

1. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Ans. $\frac{1}{9}$

2. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die ?

Ans. $\frac{8}{11}$

3. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and variance of X.

Ans. 4, 1

2019

1.

A die is thrown 6 times. If "getting an odd number" is a "success", what is the probability of (i) 5 successes ? (ii) atmost 5 successes ?

OR

The random variable X has a probability distribution P(X) of the following form, where 'k' is some number.

$$P(X = x) = \begin{cases} k\ , & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k'.

Ans. i) $\frac{3}{32}$ ii) $\frac{63}{64}$

Or

k 1

2. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not.

Ans. **A and B are not independent events.**

3. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A ?

Ans.

$$P\left(\frac{E_1}{K}\right) = \frac{5}{34}$$

2020

1. **Given two independent events A and B such that P(A) = 0.3 and P(B) = 0.6, find P(A′ ∩ B′)**

Ans. 0.28

2. Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples.

OR

In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y.

Ans. 9/10
Or
21/43

2021

1. Let A and B be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A/B) = \frac{3}{4}$. Find the value of P(B/A).

Ans- 3/5

2. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable X denotes the number of red balls. Find the probability distribution of X.

Ans-

X	0	1	2
$P(X)$	$\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}$	$2\left(\frac{2}{5}\times\frac{3}{4}\right)=\frac{6}{10}$	$\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$

3. A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace.

Ans- 1/25