Chapter 25

Vector Algebra

1. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are **[AIEEE-2009]**

(1) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (2) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

(3) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (4) 6, –3, 2

2. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and *p*, *q* are real numbers, then the equality $[3\vec{u}, p\vec{v}, p\vec{w}] - [p\vec{v}, \vec{w}, q\vec{u}] - [2\vec{w}, q\vec{v}, q\vec{u}] = 0$ holds for **[AIEEE-2009]**

(1) Exactly two values of (*p*, *q*)

(2) More than two but not all values of (*p*, *q*)

(3) All values of (*p*, *q*)

(4) Exactly one value of (*p*, *q*)

3. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a}.\vec{b} = 3$ is **[AIEEE-2010]**

(1) $-\hat{i} + \hat{j} - 2\hat{k}$ (2) $2\hat{i} - \hat{j} + 2\hat{k}$

(3) $\hat{i} - \hat{j} - 2\hat{k}$ (4) $\hat{i} + \hat{j} - 2\hat{k}$

4. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$. $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then (λ, μ) = **[AIEEE-2010]**

(1) (–3, 2) (2) (2, –3)

(3) (–2, 3) (4) (3, –2)

5. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a} + 3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + 3\vec{b} + 6\vec{c}$ is **[AIEEE-2011]**

(1) $\vec{0}$ (2) $\vec{a} + \vec{c}$

(3) $\vec{a}$ (4) $\vec{c}$

6. If the vectors $p\hat{i} + \hat{j} + \hat{k}, \hat{i} + q\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + r\hat{k}$ $(p \neq q \neq r \neq 1)$ are coplanar, then the value of $pqr - (p + q + r)$ is **[AIEEE-2011]**

(1) –1 (2) –2

(3) 2 (4) 0

7. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is **[AIEEE-2012]**

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

8. Let *ABCD* be a parallelogram such that $\overrightarrow{AB} = \vec{q}, \overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex *B* to the side *AD*, then $\vec{r}$ is given by **[AIEEE-2012]**

(1) $\vec{r} = -\vec{q} + \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

(2) $\vec{r} = \vec{q} - \left(\frac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

(3) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

(4) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

9. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle *ABC*, then the length of the median through *A* is **[JEE (Main)-2013]**

(1) $\sqrt{18}$ (2) $\sqrt{72}$

(3) $\sqrt{33}$ (4) $\sqrt{45}$

10. The angle between the lines whose direction consines satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is **[JEE (Main)-2014]**

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

11. If $\left[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}\right] = \lambda[\vec{a}\ \vec{b}\ \vec{c}]^2$ then λ is equal to **[JEE (Main)-2014]**

(1) 0 (2) 1

(3) 2 (4) 3

12. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is **[JEE (Main)-2015]**

(1) $\frac{2\sqrt{2}}{3}$ (2) $\frac{-\sqrt{2}}{3}$

(3) $\frac{2}{3}$ (4) $\frac{-2\sqrt{3}}{3}$

13. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is **[JEE (Main)-2016]**

(1) $\frac{\pi}{2}$ (2) $\frac{2\pi}{3}$

(3) $\frac{5\pi}{6}$ (4) $\frac{3\pi}{4}$

14. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a}\times\vec{b})\times\vec{c}| = 3$ and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}$ be 30°. Then $\vec{a}\cdot\vec{c}$ is equal to **[JEE (Main)-2017]**

(1) 2 (2) 5

(3) $\frac{1}{8}$ (4) $\frac{25}{8}$

15. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b} = 24$, then $|\vec{u}|^2$ is equal to **[JEE (Main)-2018]**

(1) 336 (2) 315

(3) 256 (4) 84

16. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$, then $|\vec{c}|^2$ is equal to **[JEE (Main)-2019]**

(1) $\frac{17}{2}$ (2) $\frac{19}{2}$

(3) 9 (4) 8

17. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $\vec{b}$ is equal to **[JEE (Main)-2019]**

(1) $\sqrt{22}$

(2) $\sqrt{32}$

(3) 4

(4) 6

18. Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}, \vec{b} = 4\hat{i} + (3-\lambda_2)\hat{j} + 6\hat{k}$ and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is **[JEE (Main)-2019]**

(1) (1, 3, 1) (2) (1, 5, 1)

(3) $\left(\frac{1}{2}, 4, -2\right)$ (4) $\left(-\frac{1}{2}, 4, 0\right)$

19. Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is **[JEE (Main)-2019]**

(1) – 3 (2) – 4

(3) 3 (4) 4

20. Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is **[JEE (Main)-2019]**

(1) $-14\hat{i} + 5\hat{j}$ (2) $-10\hat{i} - 5\hat{j}$

(3) $-14\hat{i} - 5\hat{j}$ (4) $-10\hat{i} + 5\hat{j}$

21. Let $\sqrt{3}\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ respectively be the position vectors of the points *A*, *B* and *C* with respect to the origin *O*. If the distance of *C* from the bisector of the acute angle between *OA* and *OB* is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is **[JEE (Main)-2019]**

(1) 3 (2) 1

(3) 4 (4) 2

22. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + \mu\hat{j} + \hat{k}$, $\hat{i} + \hat{j} + \mu\hat{k}$ are co-planar, is **[JEE (Main)-2019]**

(1) 2 (2) 1

(3) –1 (4) 0

23. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{1}{2}\vec{b}$, the $|\alpha - \beta|$ is equal to **[JEE (Main)-2019]**

(1) 90° (2) 45°

(3) 30° (4) 60°

24. The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is: **[JEE (Main)-2019]**

(1) $\sqrt{\frac{3}{2}}$ (2) $3\sqrt{6}$

(3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{6}$

25. The vector equation of plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$ is **[JEE (Main)-2019]**

(1) $\vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0$ (2) $\vec{r} \cdot (\hat{i} - \hat{k}) - 2 = 0$

(3) $\vec{r} \times (\hat{i} - \hat{k}) + 2 = 0$ (4) $\vec{r} \times (\hat{i} + \hat{k}) + 2 = 0$

26. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real *x*. Then $\left|\vec{a} \times \vec{b}\right| = r$ is possible if **[JEE (Main)-2019]**

(1) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$ (2) $\sqrt{\frac{3}{2}} < r \le 3\sqrt{\frac{3}{2}}$

(3) $0 < r \le \sqrt{\frac{3}{2}}$ (4) $r \ge 5\sqrt{\frac{3}{2}}$

27. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$ (2) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$

(3) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (4) $3\hat{i} - 9\hat{j} - 5\hat{k}$

28. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is **[JEE (Main)-2019]**

(1) $\frac{5\pi}{12}$ (2) $\frac{2\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{5\pi}{6}$

29. The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i} + 3\hat{j} - 4\hat{k}$ is **[JEE (Main)-2019]**

(1) 7 (2) $4\sqrt{3}$

(3) 6 (4) $2\sqrt{13}$

30. If the volume of parallelopiped (non-zero) formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}, \hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{\sqrt{3}}$ (2) $-\sqrt{3}$

(3) $\sqrt{3}$ (4) $-\frac{1}{\sqrt{3}}$

31. Let $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ has the magnitude 12 then one such vector is **[JEE (Main)-2019]**

(1) $4\left(-2\hat{i}-2\hat{j}+\hat{k}\right)$ (2) $4\left(2\hat{i}+2\hat{j}-\hat{k}\right)$

(3) $4\left(2\hat{i}+2\hat{j}+\hat{k}\right)$ (4) $4\left(2\hat{i}-2\hat{j}-\hat{k}\right)$

32. Let $\alpha \in R$ and the three vectors

$\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and

$\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$. Then the set

$S = \{\alpha : \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar$\}$

[JEE (Main)-2019]

(1) Contains exactly two numbers only one of which is positive

(2) Is singleton

(3) Contains exactly two positive numbers

(4) Is empty

33. A vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ $(\alpha,\beta \in R)$ lies in the plane of the vectors, $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}+4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then

[JEE (Main)-2020]

(1) $\vec{a}\cdot\hat{i}+3=0$ (2) $\vec{a}\cdot\hat{i}+1=0$

(3) $\vec{a}\cdot\hat{k}+2=0$ (4) $\vec{a}\cdot\hat{k}+4=0$

34. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. If $\lambda=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ then the ordered pair, $\left(\lambda, \vec{d}\right)$ is equal to **[JEE (Main)-2020]**

(1) $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$ (2) $\left(-\frac{3}{2}, 3\vec{c}\times\vec{b}\right)$

(3) $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$ (4) $\left(\frac{3}{2}, 3\vec{b}\times\vec{c}\right)$

35. Let the volume of a parallelopiped whose coterminous edges are given by $\vec{u}=\hat{i}+\hat{j}+\lambda\hat{k}, \vec{v}=\hat{i}+\hat{j}+3\hat{k}$ and $\vec{w}=2\hat{i}+\hat{j}+\hat{k}$ be 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then $\cos\theta$ can be **[JEE (Main)-2020]**

(1) $\frac{5}{7}$ (2) $\frac{5}{3\sqrt{3}}$

(3) $\frac{7}{6\sqrt{6}}$ (4) $\frac{7}{6\sqrt{3}}$

36. Let a, b $c \in R$ be such that $a^2 + b^2 + c^2 = 1$. If $a\cos\theta = b\cos\left(\theta+\frac{2\pi}{3}\right) = c\cos\left(\theta+\frac{4\pi}{3}\right)$, where $\theta=\frac{\pi}{9}$, then the angle between the vectors $a\hat{i}+b\hat{j}+c\hat{k}$ and $b\hat{i}+c\hat{j}+a\hat{k}$ is

[JEE (Main)-2020]

(1) 0 (2) $\frac{\pi}{9}$

(3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{2}$

37. Let x_0 be the point of local maxima of

$f(x) = \vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$, where

$\vec{a}=x\hat{i}-2\hat{j}+3\hat{k}, \vec{b}=-2\hat{i}+x\hat{j}-\hat{k}$

and $\vec{c}=7\hat{i}-2\hat{j}+x\hat{k}$. Then the value

of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ at $x = x_0$ is

[JEE (Main)-2020]

(1) 14 (2) –30

(3) –4 (4) –22

38. If the volume of a parallelopiped, whose coterminous edges are given by the vectors $\vec{a}=\hat{i}+\hat{j}+n\hat{k}, \vec{b}=2\hat{i}+4\hat{j}-n\hat{k}$ and $\vec{c}=\hat{i}+n\hat{j}+3\hat{k}$ $(n\geq 0)$, is 158 cu.units, then

[JEE (Main)-2020]

(1) $n = 7$ (2) $\vec{b}\cdot\vec{c}=10$

(3) $n = 9$ (4) $\vec{a}\cdot\vec{c}=17$

39. Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c} \cdot \vec{a} = 0$, then $\vec{c} \cdot \vec{b}$ is equal to

[JEE (Main)-2020]

(1) $-\frac{1}{2}$ (2) $-\frac{3}{2}$

(3) -1 (4) $\frac{1}{2}$

40. If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$, $\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$, and $\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$ $(a \in \mathbb{R})$ are coplanar and $3(\vec{p} \cdot \vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$, then the value of λ is ________.

[JEE (Main)-2020]

41. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 5$, $\vec{b} \cdot \vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b} \times \vec{c}$, then $|\vec{a} \times (\vec{b} \times \vec{c})|$ is equal to ________.

[JEE (Main)-2020]

42. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$. Then $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$ is equal to **[JEE (Main)-2020]**

43. Let the position vectors of points '*A*' and '*B*' be $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$, respectively. A point '*P*' divides the line segment *AB* internally in the ratio $\lambda : 1$ $(\lambda > 0)$. If *O* is the origin and $\overrightarrow{OB} \cdot \overrightarrow{OP} - 3|\overrightarrow{OA} \times \overrightarrow{OP}|^2 = 6$, then λ is equal to ________. **[JEE (Main)-2020]**

44. If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$ is equal to ________.

[JEE (Main)-2020]

45. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}| = 2$, $|\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is .

[JEE (Main)-2020]

46. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$ is ______.

[JEE (Main)-2020]

47. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x} + \vec{y}| = |\vec{x}|$ and $2\vec{x} + \lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is ________ .

[JEE (Main)-2020]

48. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors, if $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $\vec{r} \cdot \vec{a}$ is equal to ________.

[JEE (Main)-2021]

49. A plane passes through the points A(1,2,3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, –1, 1), then the projection of $\overrightarrow{OP}$ on this plane is of length: **[JEE (Main)-2021]**

(1) $\sqrt{\frac{2}{5}}$ (2) $\sqrt{\frac{2}{7}}$

(3) $\sqrt{\frac{2}{3}}$ (4) $\sqrt{\frac{2}{11}}$

50. Let $\vec{a} = \hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - a\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a} \cdot \vec{b}$ is equal to ________.

[JEE (Main)-2021]

51. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \vec{b})))$ is equal to :

[JEE (Main)-2021]

(1) $\vec{a} \times \vec{b}$ (2) $\vec{0}$

(3) $\frac{1}{2}|\vec{a}|^4 \vec{b}$ (4) $|\vec{a}|^4 \vec{b}$

52. If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is : **[JEE (Main)-2021]**

(1) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (2) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

(3) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$ (4) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

53. Let a vector $\alpha\hat{i}+\beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\,\hat{i}+\hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to : **[JEE (Main)-2021]**

(1) $\frac{1}{2}$ (2) $2\sqrt{2}$

(3) 1 (4) $\frac{1}{\sqrt{2}}$

54. Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+5\hat{k}$. If $\vec{r}\times\vec{a}=\vec{b}\times\vec{r}$, $\vec{r}\cdot\left(a\hat{i}+2\hat{j}+\hat{k}\right)=3$ and $\alpha\in R$, then the value of $\alpha+\left|\vec{r}\right|^2$ is equal to : **[JEE (Main)-2021]**

(1) 15 (2) 13

(3) 9 (4) 11

55. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$. If $\vec{c}\cdot\left(\hat{1}+\hat{j}+3\hat{k}\right)=8$ then the value of $\vec{c}\cdot(\vec{a}\times\vec{b})$ is equal to ________.

[JEE (Main)-2021]

56. Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{b}=7\hat{i}+\hat{j}-6\hat{k}$.

If $\vec{r}\times\vec{a}=\vec{r}\times\vec{b}$, $\vec{r}\cdot\left(\hat{i}+2\hat{j}+\hat{k}\right)=-3$, then $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+\hat{k}\right)$ is equal to : **[JEE (Main)-2021]**

(1) 10 (2) 13

(3) 8 (4) 12

57. If $\vec{a}=\alpha\hat{i}+\beta\hat{j}+3\hat{k}$,

$\vec{b}=-\beta\hat{i}-\alpha\hat{j}-\hat{k}$ and

$\vec{c}=\hat{i}-2\hat{j}-\hat{k}$

such that $\vec{a}.\vec{b}=1$ and $\vec{b}.\vec{c}=-3$, then $\frac{1}{3}\left(\left(\vec{a}\times\vec{b}\right).\vec{c}\right)$ is equal to ______.

[JEE (Main)-2021]

58. Let O be the origin. Let $\overrightarrow{OP}=x\hat{i}+y\hat{j}-\hat{k}$ and $\overrightarrow{OQ}=-\hat{i}+2\hat{j}+3x\hat{k}$, $x, y\in R$, $x>0$, be such that $|\overrightarrow{PQ}|=\sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. If $\overrightarrow{OR}=3\hat{i}+z\hat{j}-7\hat{k}$, $z\in R$, is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2+y^2+z^2$ is equal to **[JEE (Main)-2021]**

(1) 7 (2) 9

(3) 2 (4) 1

59. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. If the vector $\vec{x}$ is perpendicular to $\left(3\hat{i}+2\hat{j}-\hat{k}\right)$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $\left|\vec{x}\right|^2$ is equal to ______.

[JEE (Main)-2021]

60. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then a value of p is equal to

[JEE (Main)-2021]

(1) –1 (2) 1

(3) $\frac{4}{5}$ (4) $-\frac{5}{4}$

61. In a triangle ABC, if $\left|\overrightarrow{BC}\right|=8$, $\left|\overrightarrow{CA}\right|=7$, $\left|\overrightarrow{AB}\right|=10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to : **[JEE (Main)-2021]**

(1) $\frac{115}{16}$ (2) $\frac{25}{4}$

(3) $\frac{127}{20}$ (4) $\frac{85}{14}$

62. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $\left|\vec{a}\right|=\left|\vec{b}\right|$. If $\left|\vec{a}\times\vec{b}\right|=\left|\vec{a}\right|$, then the angle between the vectors $\left(\vec{a}+\vec{b}+\left(\vec{a}\times\vec{b}\right)\right)$ and $\vec{a}$ is equal to : **[JEE (Main)-2021]**

(1) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (2) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$ (4) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

63. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|$, $|\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $\left(\vec{a}\times\vec{b}\right)$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|$ is **[JEE (Main)-2021]**

(1) 4 (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) 3

64. Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a}+\vec{b}+\vec{c}$. Then $36\cos^2 2\theta$ is equal to ______. **[JEE (Main)-2021]**

65. In a triangle ABC, if $|\overrightarrow{BC}| = 3, |\overrightarrow{CA}| = 5$ and $|\overrightarrow{BA}| = 7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to : **[JEE (Main)-2021]**

(1) $\frac{13}{2}$ (2) $\frac{19}{2}$

(3) $\frac{15}{2}$ (4) $\frac{11}{2}$

66. For p > 0, a vector $\vec{v}_2 = 2\hat{i} + (p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1 = \sqrt{3}p\hat{i} + \hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta = \frac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of α is equal to ______. **[JEE (Main)-2021]**

67. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, and $|\vec{a}| = \sqrt{10}$. Then a possible value of $[\vec{a}\,\vec{b}\,\vec{c}] + [\vec{a}\,\vec{b}\,\vec{d}] + [\vec{a}\,\vec{c}\,\vec{d}]$ is equal to **[JEE (Main)-2021]**

(1) –42 (2) –40

(3) –38 (4) –29

68. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{a}\times\vec{b} = \vec{c}, \vec{b}\times\vec{c} = \vec{a}$ and $|\vec{a}| = 2$. Then which one of the following is **not** true? **[JEE (Main)-2021]**

(1) $\vec{a}\times\left((\vec{b}+\vec{c})\times(\vec{b}-\vec{c})\right) = \vec{0}$

(2) Projection of $\vec{a}$ on $(\vec{b}\times\vec{c})$ is 2

(3) $[\vec{a}\,\vec{b}\,\vec{c}] + [\vec{c}\,\vec{a}\,\vec{b}] = 8$

(4) $|3\vec{a} + \vec{b} - 2\vec{c}|^2 = 51$

69. Let the vectors

$(2+a+b)\hat{i} + (a+2b+c)\hat{j} - (b+c)\hat{k}$,

$(1+b)\hat{i} + 2b\hat{j} - b\hat{k}$ and

$(2+b)\hat{i} + 2b\hat{j} + (1-b)\hat{k}$, $a, b, c \in \mathbf{R}$

be co-planar. Then which of the following is true? **[JEE (Main)-2021]**

(1) 3c = a + b (2) 2b = a + c

(3) 2a = b + c (4) a = b + 2c

70. Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. If a vector $\vec{r} = (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p}+\vec{q})$ and $(\vec{p}-\vec{q})$, and $|\vec{r}| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to ______. **[JEE (Main)-2021]**

71. Let a, b and c be distinct positive numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are co-planar, then c is equal to **[JEE (Main)-2021]**

(1) $\frac{2}{\frac{1}{a}+\frac{1}{b}}$ (2) $\sqrt{ab}$

(3) $\frac{a+b}{2}$ (4) $\frac{1}{a}+\frac{1}{b}$

72. If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a}\times\vec{b}| = 8$, then $|\vec{a}\cdot\vec{b}|$ is equal to **[JEE (Main)-2021]**

(1) 3 (2) 6

(3) 4 (4) 5

73. If $(\vec{a}+3\vec{b})$ is perpendicular to $(7\vec{a}-5\vec{b})$ and $(\vec{a}-4\vec{b})$ is perpendicular to $(7\vec{a}-2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is ________. **[JEE (Main)-2021]**

74. Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Then the vector product $(\vec{a}+\vec{b})\times\left(\left(\vec{a}\times\left((\vec{a}-\vec{b})\times\vec{b}\right)\right)\times\vec{b}\right)$ is equal to **[JEE (Main)-2021]**

(1) $7(34\hat{i} - 5\hat{j} + 3\hat{k})$ (2) $7(30\hat{i} - 5\hat{j} + 7\hat{k})$

(3) $5(30\hat{i} - 5\hat{j} + 7\hat{k})$ (4) $5(34\hat{i} - 5\hat{j} + 3\hat{k})$

75. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}$ and $\vec{c}=\hat{j}-\hat{k}$ be three vectors such that $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}\cdot\vec{b}=1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a}\times\vec{c}$ is l, then the value of $3l^2$ is equal to ________.

[JEE (Main)-2021]

76. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}=\vec{b}\times(\vec{b}\times\vec{c})$. If magnitudes of the vectors $\vec{a},\vec{b}$ and $\vec{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta\left(0<\theta<\frac{\pi}{2}\right)$, then the value of 1 + tanθ is equal to

[JEE (Main)-2021]

(1) 2
(2) $\frac{\sqrt{3}+1}{\sqrt{3}}$
(3) 1
(4) $\sqrt{3}+1$

77. Let $\vec{a}=\hat{i}-\alpha\hat{j}+\beta\hat{k}, \vec{b}=3\hat{i}+\beta\hat{j}-\alpha\hat{k}$ and $\vec{c}=-\alpha\hat{i}-2\hat{j}+\hat{k}$, where α and β are integers. If $\vec{a}\cdot\vec{b}=-1$ and $\vec{b}\cdot\vec{c}=10$, $(\vec{a}\times\vec{b})\cdot\vec{c}$ is equal to ________.

[JEE (Main)-2021]

78. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$. if $\vec{c}$ is a vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$, then $\vec{a}\cdot(\vec{b}\times\vec{c})$ is equal to

[JEE (Main)-2021]

(1) 6
(2) –2
(3) –6
(4) 2

79. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to ________.

[JEE (Main)-2021]

80. Let $\vec{a}=\hat{i}+5\hat{j}+\alpha\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}+\beta\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{j}-3\hat{k}$ be three vectors such that, $|\vec{b}\times\vec{c}|=5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is ______.

[JEE (Main)-2021]

81. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a}+3\vec{b}|=|3\vec{a}+3\vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

[JEE (Main)-2021]

(1) 8
(2) 4
(3) 5
(4) 6

82. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies

$$\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\}+\vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\}+\vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\}=\vec{0},$$

then $\vec{r}$ is equal to **[JEE (Main)-2021]**

(1) $\frac{1}{3}(\vec{a}+\vec{b}+\vec{c})$
(2) $\frac{1}{2}(\vec{a}+\vec{b}+2\vec{c})$
(3) $\frac{1}{2}(\vec{a}+\vec{b}+\vec{c})$
(4) $\frac{1}{3}(2\vec{a}+\vec{b}-\vec{c})$

83. Let $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$. Let a vector $\vec{v}$ be in the plane containing $\vec{a}$ and $\vec{b}$. If $\vec{v}$ is perpendicular to the vector $3\hat{i}+2\hat{j}-\hat{k}$ and its projection on $\vec{a}$ is 19 units, then $|2\vec{v}|^2$ is equal to ________.

[JEE (Main)-2021]

84. Let $\hat{a}$, $\hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b}=\vec{c}+2(\vec{c}\times\hat{a})$, then $|6\vec{c}|^2$ is equal to:

[JEE (Main)-2022]

(1) $6(3-\sqrt{3})$
(2) $3+\sqrt{3}$
(3) $6(3+\sqrt{3})$
(4) $6(\sqrt{3}+1)$

85. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $|(\hat{a}+\hat{b})+2(\hat{a}\times\hat{b})|=2$. If $\theta\in(0,\pi)$ is the angle between $\hat{a}$ and $\hat{b}$, then among the statements:

(S1) : $2|\hat{a}\times\hat{b}|=|\hat{a}-\hat{b}|$

(S2) : The projection of $\hat{a}$ on $(\hat{a}+\hat{b})$ is $\frac{1}{2}$

[JEE (Main)-2022]

(1) Only (S1) is true
(2) Only (S2) is true
(3) Both (S1) and (S2) are true
(4) Both (S1) and (S2) are false

86. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $a_i > 0, i = 1, 2, 3$ be a vector which makes equal angles with the coordinate axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i} + 4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90°. If $\vec{a}, \vec{b}$ and x-axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is equal to: **[JEE (Main)-2022]**

(1) $\sqrt{7}$ (2) $\sqrt{2}$

(3) 2 (4) 7

87. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 4, |\vec{b}| = 3$ and $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$. Then $\left|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})\right|^2 + 4(\vec{a} \cdot \vec{b})^2$ is equal to ______.

[JEE (Main)-2022]

88. Let $\vec{b} = \hat{i} + \hat{j} + \lambda\hat{k}, \lambda \in \mathbb{R}$. If $\vec{a}$ is a vector such that $\vec{a} \times \vec{b} = 13\hat{i} - \hat{j} - 4\hat{k}$ and $\vec{a} \cdot \vec{b} + 21 = 0$, then $(\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) + (\vec{b} + \vec{a}) \cdot (\hat{i} - \hat{k})$ is equal to

[JEE (Main)-2022]

89. If $\vec{a} \cdot \vec{b} = 1, \vec{b} \cdot \vec{c} = 2$ and $\vec{c} \cdot \vec{a} = 3$, then the value of $\left[\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{b} \times \vec{a})\right]$ is :

[JEE (Main)-2022]

(1) 0 (2) $-6\vec{a} \cdot (\vec{b} \times \vec{c})$

(3) $12\vec{c} \cdot (\vec{a} \times \vec{b})$ (4) $-12\vec{b} \cdot (\vec{c} \times \vec{a})$

90. Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ be three given vectors. Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{2}{\sqrt{3}}$. If $\vec{v} \cdot \hat{j} = 7$, then $\vec{v} \cdot (\hat{i} + \hat{k})$ is equal to :

[JEE (Main)-2022]

(1) 6 (2) 7

(3) 8 (4) 9

91. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, \ldots, 10\}$ is : **[JEE (Main)-2022]**

(1) 0 (2) 1

(3) 2 (4) 3

92. Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute, $|\vec{a}| = 1$, and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$. If $\vec{c} = 2\sqrt{2}(\vec{a} \times \vec{b}) - 2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is **[JEE (Main)-2022]**

(1) $\frac{\pi}{4}$ (2) $-\frac{\pi}{4}$

(3) $\frac{5\pi}{6}$ (4) $\frac{3\pi}{4}$

93. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a} \cdot \vec{c} = 5, \vec{b} \perp \vec{c}$, then $122(c_1 + c_2 + c_3)$ is equal to ________. **[JEE (Main)-2022]**

94. Let, $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$, where $\alpha \in \mathbf{R}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2 + 4)}$, then the value of $2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$ is equal to : **[JEE (Main)-2022]**

(1) 10 (2) 7

(3) 9 (4) 14

95. Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is : **[JEE (Main)-2022]**

(1) $\frac{1}{3}$ (2) 1

(3) $\frac{5}{3}$ (4) $\frac{7}{3}$

96. Let $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in \mathbf{R}$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ is equal to : **[JEE (Main)-2022]**

(1) 3 (2) 4

(3) 5 (4) 6

97. Let A, B, C be three points whose position vectors respectively are

$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in \mathbb{R}$

$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$

If α is the smallest positive integer for which $\vec{a}, \vec{b}, \vec{c}$ are non collinear, then the length of the median, in ΔABC, through A is: **[JEE (Main)-2022]**

(1) $\frac{\sqrt{82}}{2}$ (2) $\frac{\sqrt{62}}{2}$

(3) $\frac{\sqrt{69}}{2}$ (4) $\frac{\sqrt{66}}{2}$

98. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} + (\vec{b} \times \vec{c}) = \vec{0}$ and $\vec{b} \cdot \vec{c} = 5$. Then the value of $3(\vec{c} \cdot \vec{a})$ is equal to ________.

[JEE (Main)-2022]

99. Let ABC be a triangle such that $\overline{BC} = \vec{a}, \overline{CA} = \vec{b}, \overline{AB} = \vec{c}, |\vec{a}| = 6\sqrt{2}, |\vec{b}| = 2\sqrt{3}$ and $\vec{b} \cdot \vec{c} = 12$. Consider the statements :

$(S1) : \left|(\vec{a} \times \vec{b}) + (\vec{c} \times \vec{b})\right| - |\vec{c}| = 6(2\sqrt{2} - 1)$

$(S2) : \angle ACB = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Then **[JEE (Main)-2022]**

(1) Both (S1) and (S2) are true

(2) Only (S1) is true

(3) Only (S2) is true

(4) Both (S1) and (S2) are false

100. Let $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}, \alpha > 0$. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30, then α is equal to **[JEE (Main)-2022]**

(1) $\frac{15}{2}$ (2) 8

(3) $\frac{13}{2}$ (4) 7

101. If the maximum value of a, for which the function $f_a(x) = \tan^{-1}2x - 3ax + 7$ is non-decreasing in $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to

[JEE (Main)-2022]

(1) $8 - \frac{9\pi}{4(9 + \pi^2)}$ (2) $8 - \frac{4\pi}{9(4 + \pi^2)}$

(3) $8\left(\frac{1 + \pi^2}{9 + \pi^2}\right)$ (4) $8 - \frac{\pi}{4}$

102. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{i} + 12\hat{k}$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to

(1) 2 (2) $\frac{39}{5}$

(3) 9 (4) $\frac{46}{5}$

103. Let $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$. If $((\vec{a} \times \vec{b}) \times \hat{i}) \cdot \hat{k} = \frac{23}{2}$, then $|\vec{b} \times 2\hat{j}|$ is equal to

[JEE (Main)-2022]

(1) 4 (2) 5

(3) $\sqrt{21}$ (4) $\sqrt{17}$

104. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors such that $\vec{a} \times \vec{b} = 4\vec{c}, \vec{b} \times \vec{c} = 9\vec{a}$ and $\vec{c} \times \vec{a} = \alpha\vec{b}, \alpha > 0$. If $|\vec{a}| + |\vec{b}| + |\vec{c}| = \frac{1}{36}$, then α is equal to ______.

[JEE (Main)-2022]

105. Let the vectors $\vec{a}=(1+t)\hat{i}+(1-t)\hat{j}+\hat{k}$, $\vec{b}=(1-t)\hat{i}+(1+t)\hat{j}+2\hat{k}$ and $\vec{c}=t\hat{i}-t\hat{j}+\hat{k}$, $t\in\mathbf{R}$ be such that for α, β, $\gamma\in\mathbf{R}$, $\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}=\vec{0}$ $\Rightarrow \alpha=\beta=\gamma=0$. Then, the set of all values of t is **[JEE (Main)-2022]**

(1) A non-empty finite set

(2) Equal to N

(3) Equal to $\mathbf{R}-\{0\}$

(4) Equal to $\mathbf{R}$

106. Let a vector $\vec{a}$ has magnitude 9. Let a vector $\vec{b}$ be such that for every $(x, y)\in\mathbf{R}\times\mathbf{R}-\{(0, 0)\}$, the vector $(x\vec{a}+y\vec{b})$ is perpendicular to the vector $(6y\vec{a}-18x\vec{b})$. Then the value of $|\vec{a}\times\vec{b}|$ is equal to **[JEE (Main)-2022]**

(1) $9\sqrt{3}$ (2) $27\sqrt{3}$

(3) 9 (4) 81

107. Let S be the set of all $a\in R$ for which the angle between the vectors $\vec{u}=a(\log_e b)\hat{i}-6\hat{j}+3\hat{k}$ and $\vec{v}=(\log_e b)\hat{i}+2\hat{j}+2a(\log_e b)\hat{k}, (b>1)$ is acute. Then S is equal to : **[JEE (Main)-2022]**

(1) $\left(-\infty,-\frac{4}{3}\right)$ (2) ϕ

(3) $\left(-\frac{4}{3},0\right)$ (4) $\left(\frac{12}{7},\infty\right)$

108. Let $\vec{a},\vec{b},\vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})+(\vec{b}\times\vec{c})\cdot(\vec{c}\times\vec{a})+(\vec{c}\times\vec{a})\cdot(\vec{a}\times\vec{b})=168$, then $|\vec{a}|+|\vec{b}|+|\vec{c}|$ is equal to : **[JEE (Main)-2022]**

(1) 10 (2) 14

(3) 16 (4) 18

109. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+2|\vec{b}|^2, \vec{a}\cdot\vec{b}=3$ and $|\vec{a}\times\vec{b}|^2=75$. Then $|\vec{a}|^2$ is equal to ______. **[JEE (Main)-2022]**

110. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{4}$. If θ is the angle between the vectors $(\hat{a}+\hat{b})$ and $(\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b}))$, then the value of $164\cos^2\theta$ is equal to :

(1) $90+27\sqrt{2}$ (2) $45+18\sqrt{2}$

(3) $90+3\sqrt{2}$ (4) $54+90\sqrt{2}$

[JEE (Main)-2022]

111. Let $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ and let $\vec{b}$ be a vector such that $\vec{a}\times\vec{b}=2\hat{i}-\hat{k}$ and $\vec{a}\cdot\vec{b}=3$. Then the projection of $\vec{b}$ on the vector $\vec{a}-\vec{b}$ is :

[JEE (Main)-2022]

(1) $\frac{2}{\sqrt{21}}$ (2) $2\sqrt{\frac{3}{7}}$

(3) $\frac{2}{3}\sqrt{\frac{7}{3}}$ (4) $\frac{2}{3}$

112. Let $\vec{a}=3\hat{i}+\hat{j}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$. Let $\vec{c}$ be a vector satisfying $\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}+\lambda\vec{c}$. If $\vec{b}$ and $\vec{c}$ are non-parallel, then the value of λ is **[JEE (Main)-2022]**

(1) –5 (2) 5

(3) 1 (4) –1

❑ ❑ ❑

Chapter 25

Vector Algebra

1. Answer (2)

Direction ratios are a = 6, b = –3 and c = 2

Then direction cosines are

$$\frac{6}{\sqrt{36+9+4}}, \frac{-3}{\sqrt{36+9+4}}, \frac{2}{\sqrt{36+9+4}}$$

$$= \frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$$

2. Answer (4)

$$[3\vec{u} \ \ p\vec{v} \ \ p\vec{w}] - [p\vec{v} \ \ \vec{w} \ \ q\vec{u}] - [2\vec{w} \ \ q\vec{v} \ \ q\vec{u}]$$

$$= 3p^2[\vec{u}.(\vec{v} \times \vec{w})] - pq[\vec{v}.(\vec{w} \times \vec{u})] - 2q^2[\vec{w}.(\vec{v} \times \vec{u})]$$

$$\Rightarrow (3p^2 - pq + 2q^2)[\vec{u}.(\vec{v} \times \vec{w})] = 0$$

But $\vec{u}.(\vec{v} \times \vec{w}) \neq 0$

$\Rightarrow$ $3p^2 - pq + 2q^2 = 0$

$\Rightarrow$ $p = q = 0$

3. Answer (1)

We have

$$\vec{a} \times \vec{b} + \vec{c} = \vec{0}$$

$$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) + \vec{a} \times \vec{c} = \vec{0}$$

$$\Rightarrow (\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b} + \vec{a} \times \vec{c} = \vec{0}$$

$$\Rightarrow 3\vec{a} - 2\vec{b} + \vec{a} \times \vec{c} = \vec{0}$$

$$\Rightarrow 2\vec{b} = 3\vec{a} + \vec{a} \times \vec{c} \ ; \ \vec{a} \times \vec{c} = -2\hat{i} - \hat{j} - \hat{k}$$

$$= 3\hat{j} - 3\hat{k} - 2\hat{i} - \hat{j} - \hat{k}$$

$$= -2\hat{i} + 2\hat{j} - 4\hat{k}$$

$$\Rightarrow \vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$$

4. Answer (1)

We have

$$\vec{a}.\vec{b} = 2 - 4 + 2 = 0$$

$$\vec{a}.\vec{c} = \lambda - 1 + 2\mu = 0$$

$$\vec{b}.\vec{c} = 2\lambda + 4 + \mu = 0$$

Thus $\lambda = 1 - 2\mu$

and $2 - 4\mu + 4 + \mu = 0$

$\Rightarrow 3\mu = 6, \Rightarrow \mu = 2$

$\lambda = -3$

$(\lambda, \mu) = (-3, 2)$

5. Answer (1)

$$\vec{c} = \mu(\vec{a} + 3\vec{b})$$

$$\vec{b} + 2\vec{c} = \lambda\vec{a}$$

$$\vec{b} + 2\mu(\vec{a} + 3\vec{b}) = \lambda\vec{a}$$

$$(1 + 6\mu)\,\vec{b} + (2\mu - \lambda)\,\vec{a} = 0$$

$6\mu + 1 = 0, 2\mu = \lambda$

$$\mu = -\frac{1}{6}, \ \lambda = -\frac{1}{3}$$

Now, $\vec{c} = -\frac{1}{6}(\vec{a} + 3\vec{b}) = 6\vec{c} + \vec{a} + 3\vec{b} = 0$

6. Answer (2)

Given vectors $p\hat{i} + \hat{j} + \hat{k}, \hat{i} + q\hat{j} + \hat{k}$, $\hat{i} + \hat{j} + r\hat{k}$ to be coplanar

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

$p\,(qr - 1) - 1\,(r - 1) + (1 - q) = 0$

$pqr - p - r + 1 - q + 1 = 0$

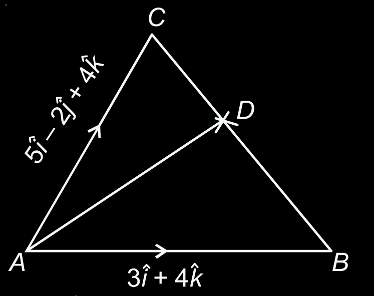
$pqr - p - q - r = -2$

7. Answer (4)

8. Answer (1)

9. Answer (3)

$$\overrightarrow{AD} = \frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$$



$$= \frac{\left(3\hat{i}+4\hat{k}\right)+\left(5\hat{i}-2\hat{j}+4\hat{k}\right)}{2}$$

$$= 4\hat{i}-\hat{j}+4\hat{k}$$

$$\therefore\ |\overrightarrow{AD}| = \sqrt{16+1+16} = \sqrt{33}$$

10. Answer (3)

$l + m + n = 0$

$l^2 = m^2 + n^2$

Now, $(-m-n)^2 = m^2 + n^2$

$\Rightarrow\ mn = 0$

$m = 0$ or $n = 0$

If $m = 0$

then $l = -n$

$l^2 + m^2 + n^2 = 1$

Gives

$$\Rightarrow\ n = \pm\frac{1}{\sqrt{2}}$$

i.e. (l_1, m_1, n_1)

$$= \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

If $n = 0$

then $l = -m$

$l^2 + m^2 + n^2 = 1$

$\Rightarrow\ 2m^2 = 1$

$$\Rightarrow\ m^2 = \frac{1}{2}$$

$$\Rightarrow\ m = \pm\frac{1}{\sqrt{2}}$$

Let $m = \frac{1}{\sqrt{2}}$

$$l = -\frac{1}{\sqrt{2}}$$

$n = 0$

(l_2, m_2, n_2)

$$= \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$\therefore\ \cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

11. Answer (2)

L.H.S.

$$= (\vec{a}\times\vec{b})\cdot[(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})]$$

$$= (\vec{a}\times\vec{b})\cdot[(\vec{b}\times\vec{c}\cdot\vec{a})\vec{c}-(\vec{b}\times\vec{c}\cdot\vec{c})\vec{a}]$$

$$= (\vec{a}\times\vec{b})\cdot[[\vec{b}\,\vec{c}\,\vec{a}]\vec{c}] \qquad [\because\ \vec{b}\times\vec{c}.\vec{c}=0]$$

$$= [\vec{a}\,\vec{b}\,\vec{c}]\cdot(\vec{a}\times\vec{b}\cdot\vec{c}) = [\vec{a}\,\vec{b}\,\vec{c}]^2$$

$$[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = [\vec{a}\,\vec{b}\,\vec{c}]^2$$

So $\lambda = 1$

12. Answer (1)

$$(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$\therefore\ -(\vec{b}\cdot\vec{c}) = \frac{1}{3}|\vec{b}||\vec{c}|$$

$$\therefore\ \cos\theta = -\frac{1}{3}$$

$$\therefore\ \sin\theta = \frac{2\sqrt{2}}{3}$$

13. Answer (3)

$$\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{c} = \frac{\sqrt{3}}{2}\left(\vec{b}+\vec{c}\right) \text{ and}$$

after comparing

$$\vec{a}\cdot\vec{b} = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow\ \cos\theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow\ \theta = \frac{5\pi}{6}$$

14. Answer (1)

$$|(\vec{a}\times\vec{b})\times\vec{c}| = 3 \qquad \vec{a}\times\vec{b} = 2\hat{i}-2\hat{j}+\hat{k}$$

$$\Rightarrow\ |\vec{a}\times\vec{b}||\vec{c}|\sin 30^\circ = 3 \qquad |\vec{a}| = 3 = |\vec{a}\times\vec{b}|$$

$$\Rightarrow\ |\vec{c}| = 2$$

$$|\vec{c}-\vec{a}| = 3$$

$$\Rightarrow\ |\vec{c}|^2+|\vec{a}|^2-2(\vec{a}\cdot\vec{c}) = 9$$

$$\vec{a}\cdot\vec{c} = \frac{9-3-2}{2} = 2$$

15. Answer (1)

Clearly, $\vec{u} = \lambda(\vec{a}\times(\vec{a}\times\vec{b}))$

$\Rightarrow \quad \vec{u} = \lambda((\vec{a}.\vec{b})\vec{a} - |\vec{a}|^2\,\vec{b})$

$\Rightarrow \quad \vec{u} = \lambda(2\vec{a} - 14\vec{b}) = 2\lambda\left\{(2\hat{i}+3\hat{j}-\hat{k}) - 7(\hat{j}+\hat{k})\right\}$

$\Rightarrow \quad \vec{u} = 2\lambda(2\hat{i} - 4\hat{j} - 8\hat{k})$

as, $\vec{u}\cdot\vec{b} = 24$

$\Rightarrow \quad 4\lambda(\hat{i} - 2\hat{j} - 4\hat{k})\cdot(\hat{j}+\hat{k}) = 24$

$\Rightarrow \quad \lambda = -1$

So, $\vec{u} = -4(\hat{i} - 2\hat{j} - 4\hat{k})$

$\Rightarrow \quad |\vec{u}|^2 = 336$

16. Answer (2)

$|\vec{a}\times\vec{c}|^2 = |\vec{a}|^2|\vec{c}|^2 - (\vec{a}\cdot\vec{c})^2$

$\Rightarrow |-\vec{b}|^2 = 2|\vec{c}|^2 - 16$

$\Rightarrow 3 = 2|\vec{c}|^2 - 16$

$\Rightarrow |\vec{c}|^2 = \dfrac{19}{2}$

17. Answer (4)

Projection of $\vec{b}$ on $\vec{a}$ = $\dfrac{\vec{b}.\vec{a}}{|\vec{a}|} = \dfrac{b_1+b_2+2}{4}$

According to question $\dfrac{b_1+b_2+2}{2} = \sqrt{1+1+2} = 2$

$\Rightarrow \quad b_1 + b_2 = 2$ ...(1)

Also $\vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c} = 0$

$\Rightarrow \quad 8 + 5b_1 + b_2 + 2 = 0$ ...(2)

From (1) and (2),

$b_1 = -3,\ b_2 = 5$

$\Rightarrow \quad \vec{b} = -3.\hat{i} + 5\hat{j} + \sqrt{2}\hat{k}$

$|\vec{b}| = \sqrt{9+25+2} = 6$

18. Answer (4)

$\because\ \vec{b} = 2\vec{a}$

$\therefore\ 4\hat{i} + (3-\lambda_2)\hat{j} + 6\hat{k} = 4\hat{i} + 2\lambda_1\hat{j} + 6\hat{k}$

$\therefore\ 3 - \lambda_2 = 2\lambda_1$ ...(1)

$\because\ \vec{a}$ is perpendicular to $\vec{c}$

$\therefore\ 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$

$2 + 2\lambda_1 + \lambda_3 - 1 = 0$

$2\lambda_1 + \lambda_3 + 1 = 0$

$\therefore\ \lambda_3 = -2\lambda_1 - 1$ ...(2)

from equations (1) and (2) one of possible value of

$\lambda_1 = -\dfrac{1}{2}, \lambda_2 = 4$ and $\lambda_3 = 0$

19. Answer (2)

$\vec{\alpha}$ and $\vec{\beta}$ are collinear

$\Rightarrow \quad \vec{\alpha} = t\vec{\beta}$

$(\lambda - 2)\,\vec{a} + \vec{b} = t\left((4\lambda - 2)\,\vec{a} + 3\vec{b}\right)$

$(\lambda - 2 - t(4\lambda - 2))\vec{a} + \vec{b}(1 - 3t) = \vec{0}$

$\vec{a}$ and $\vec{b}$ are non-collinear

$\Rightarrow \lambda - 2 - t(4\lambda - 2) = 0,\ 1 - 3t = 0$

$\Rightarrow \quad t = \dfrac{1}{3}$ and $\lambda - 2 - \dfrac{1}{3}(4\lambda - 2) = 0$

$3\lambda - 6 - 4\lambda + 2 = 0$

$\boxed{\lambda = -4}$

Option (2) is correct

20. Answer (4)

For coplanar vectors,

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & (\lambda^2 - 1) \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - \lambda - 16 + 2(8 - \lambda^2 + 1) + 4(4 - 2\lambda) = 0$

$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

i.e., $(\lambda - 2)(\lambda - 3)(\lambda + 3) = 0$

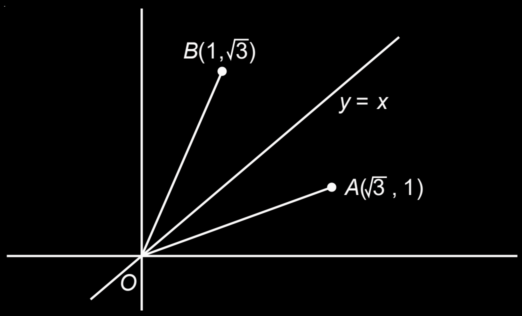
For $\lambda = 2$, $\vec{c} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$$\therefore\quad \vec{a}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{i} + 5\hat{j}$$

For $\lambda = 3$ or -3, $\vec{a}\times\vec{c} = 0$ (Rejected)

21. Answer (2)

By observing point A, B angle bisector of acute angle, OA and OB would be $\boxed{y = x}$



Now, according to question

$$\left|\frac{\beta-(1-\beta)}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$$

$\Rightarrow$ $2\beta = \pm 3 + 1$

$\beta = 2$ or $\beta = -1$

22. Answer (3)

For coplanar vectors,

$$\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$\Rightarrow$ $\mu(\mu^2 - 1) + 1 - \mu + 1 - \mu = 0$

$\Rightarrow$ $(1 - \mu)[2 - \mu(\mu + 1)] = 0$

$\Rightarrow$ $(1 - \mu)\ [\mu^2 + \mu - 2] = 0$

$\Rightarrow$ $\mu = 1, -2$

Sum of all real values $= 1 - 2 = -1$

23. Answer (3)

$\because$ $|\bar{a}| = |\bar{b}| = |\bar{c}| = 1$

Now $\bar{a} \times (\bar{b} \times \bar{c}) = \dfrac{1}{2}\bar{b}$

$$(\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c} = \frac{1}{2}\bar{b}$$

$\therefore$ $\bar{a}\cdot\bar{c} = \dfrac{1}{2}$ and $\bar{a}\cdot\bar{b} = 0$

$|\bar{a}||\bar{c}|\cos\beta = \dfrac{1}{2}$ and $\alpha = 90°$

$\beta = 60°$

$\therefore$ $|\alpha - \beta| = |90° - 60°| = 30°$

24. Answer (1)

Let $\bar{a} = \hat{i} + \hat{j} + \hat{k}$ and $\bar{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ vector perpendicular to $\bar{a}$ and $\bar{b}$ is $\bar{a}\times\bar{b}$

$$\bar{a}\times\bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

Projection of vector $\bar{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ on $\bar{a}\times\bar{b}$ is

$$= \left|\frac{\bar{c}.(\bar{a}\times\bar{b})}{|\bar{a}\times\bar{b}|}\right| = \left|\frac{2-6+1}{\sqrt{6}}\right| = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

25. Answer (1)

Equation of the plane passing through the line of intersection of $x + y + z = 1$ and $2x + 3y + 4z = 5$ is

$(2x + 3y + 4z - 5) + \lambda\ (x + y + z - 1) = 0$

$(2 + \lambda)x + (3 + \lambda)y + (4 + \lambda)z + (-5 - \lambda) = 0$ ...(i)

(i) is perpendicular to $x - y + z = 0$

$\Rightarrow$ $(2 + \lambda)\ (1) + (3 + \lambda)\ (-1) + (4 + \lambda)\ (1) = 0$

$2 + \lambda - 3 - \lambda + 4 + \lambda = 0$

$\lambda = -3$

$\Rightarrow$ Equation of required plane is

$-x + z - 2 = 0$

$\Rightarrow$ $x - z + 2 = 0$

$\Rightarrow$ $\vec{r}\cdot(\hat{i} - \hat{k}) + 2 = 0$

26. Answer (4)

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix} = (2+x)\hat{i} + (x-3)\hat{j} - 5\hat{k}$$

$$|\vec{a}\times\vec{b}| = r = \sqrt{(2+x)^2 + (x-3)^2 + (-5)^2}$$

$$\Rightarrow r = \sqrt{4 + x^2 + 4x + x^2 + 9 - 6x + 25}$$

$$= \sqrt{2x^2 - 2x + 38} = \sqrt{2\left(x^2 - x + \frac{1}{4}\right) + 38 - \frac{1}{2}}$$

$$= \sqrt{2\left(x - \frac{1}{2}\right)^2 + \frac{75}{2}}$$

$$\Rightarrow r \ge \sqrt{\frac{75}{2}} \Rightarrow r \ge 5\sqrt{\frac{3}{2}}$$

27. Answer (2)

$$\vec{\beta} = \overrightarrow{\beta_1} - \overrightarrow{\beta_2} \qquad \text{...(i)}$$

$$\therefore \quad \overrightarrow{\beta_2} \cdot \vec{\alpha} = 0$$

and Let $\overrightarrow{\beta_1} = \lambda \vec{\alpha}$

$$\vec{\alpha} \cdot \vec{\beta} = \vec{\alpha} \cdot \overrightarrow{\beta_1} - \vec{\alpha} \cdot \overrightarrow{\beta_2}$$

$\Rightarrow \quad 5 = \lambda \alpha^2$

$\Rightarrow \quad 5 = \lambda \times 10$

$$\Rightarrow \quad \lambda = \frac{1}{2}$$

$$\therefore \quad \boxed{\overrightarrow{\beta_1} = \frac{\vec{\alpha}}{2}}$$

Cross product with $\overrightarrow{\beta_1}$ in equation (i),

$$\vec{\beta} \times \overrightarrow{\beta_1} = -\overrightarrow{\beta_2} \times \overrightarrow{\beta_1}$$

$$\Rightarrow \quad \boxed{\vec{\beta} \times \overrightarrow{\beta_1} = \overrightarrow{\beta_1} \times \overrightarrow{\beta_2}} = \frac{(\vec{\beta} \times \vec{\alpha})}{2}$$

$$\Rightarrow \quad \overrightarrow{\beta_1} \times \overrightarrow{\beta_2} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 3 & 1 & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left[-3\hat{i} - \hat{j}(-9) + \hat{k}(5)\right]$$

$$= \frac{1}{2}\left[-3\hat{i} + 9\hat{j} + 5\hat{k}\right]$$

28. Answer (2)

Let $\cos\alpha, \cos\beta, \cos\gamma$ be direction cosines of $\vec{a}$

Hence, by given data

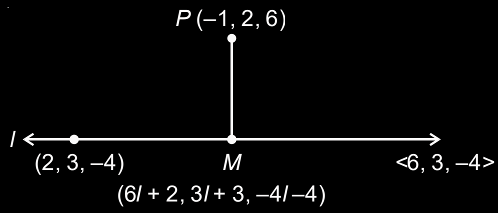
$$\cos\alpha = \cos\frac{\pi}{3}, \ \cos\beta = \cos\frac{\pi}{4} \ \& \ \cos\gamma = \cos\theta$$

$$\therefore \quad \cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \pm\frac{1}{2}, \quad \theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

29. Answer (1)

Equation of l is $\dfrac{x-2}{6} = \dfrac{y-3}{3} = \dfrac{z+4}{-4}$



Let $M\,(6\lambda + 2, 3\lambda + 3, -4\lambda - 4)$

DR's of PM is $< 6\lambda + 3, 3\lambda + 1, -4\lambda - 10 >$

$\Rightarrow \quad (6\lambda + 3)(6) + (3\lambda + 1)(3) + (-4\lambda - 10)(-4) = 0$

$\Rightarrow \quad \lambda = -1$

i.e. $M \equiv (-4, 0, 0)$

$$\therefore \quad PM = \sqrt{9+4+36} = 7$$

30. Answer (1)

Vector are coplanar for $\lambda = \lambda_1$ where $\lambda_1^3 - \lambda_1 + 1 = 0 \Rightarrow$ volume is minimum when $\lambda = \lambda_1$.

$$V = \left\|\begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix}\right\|$$

$= |1(1) + \lambda(\lambda^2) + 1(-\lambda)|$

$= |\lambda^3 - \lambda + 1|$

Let $f(x) = x^3 - x + 1$

$f'(x) = 3x^2 - 1$

For maxima/minima,

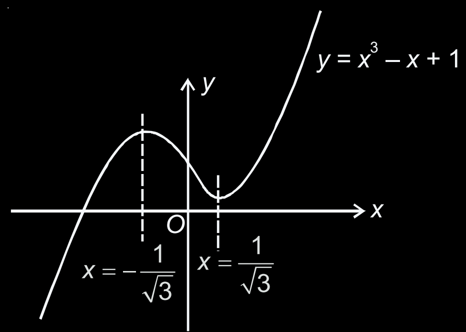
$f'(x) = 0$

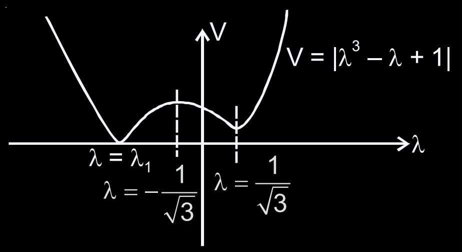
$$x = \pm\frac{1}{\sqrt{3}}$$

$f''(x) = 6x$

$$\because \quad f''\left(\frac{1}{\sqrt{3}}\right) > 0$$

$x = \dfrac{1}{\sqrt{3}}$ is point of local minima





When $\lambda = \lambda_1$, volume of parallelopiped is zero (vectors are coplanar)

31. Answer (4)

Let vector be $\lambda\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right]$

$\vec{a}+\vec{b} = 4\hat{i}+4\hat{j}$

$\vec{a}-\vec{b} = 2\hat{i}+4\hat{k}$

vector $=\lambda\left[\left(4\hat{i}+4\hat{j}\right)\times\left(2\hat{i}+4\hat{k}\right)\right]$

$=\lambda\left[16\hat{i}-16\hat{j}-8\hat{k}\right]$

$=8\lambda\left[2\hat{i}-2\hat{j}-\hat{k}\right]$

$\Rightarrow \quad 12 = 8|\lambda|\sqrt{4+4+1}$

$|\lambda| = \dfrac{1}{2}$

Hence required vector is $\pm 4\left(2\hat{i}-2\hat{j}-\hat{k}\right)$

32. Answer (4)

If $\vec{a},\vec{b},\vec{c}$ are coplanar, then

$\left[\vec{a},\vec{b},\vec{c}\right]=0$

$$\Rightarrow \begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0$$

$\Rightarrow \quad a^2 + 6 = 0$

No value of 'α' exist

Set S is an empty set.

33. Answer (3)

$\vec{a} = \lambda_1(\hat{b}+\hat{c})$

$=\lambda_1\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}+\dfrac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$=\dfrac{\lambda_1}{3\sqrt{2}}\left(4\hat{i}+2\hat{j}+4\hat{k}\right)$

As $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$

$\therefore \quad \lambda_1 = 3\sqrt{2},\ \alpha = 4,\ \beta = 4$

$\vec{a}=4\hat{i}+2\hat{j}+4\hat{k}$ no option is satisfied

Also $\vec{a}=\lambda_2(\hat{b}-\hat{c})$

$=\dfrac{\lambda_2}{3\sqrt{2}}\left((3\hat{i}+3\hat{j})-(\hat{i}-\hat{j}+4\hat{k})\right)=\dfrac{\lambda_2}{3\sqrt{2}}(2\hat{i}+4\hat{j}-4\hat{k})$

$=\dfrac{2\lambda_2}{3\sqrt{2}}\left(\hat{i}+2\hat{j}-2\hat{k}\right)$

$\Rightarrow \quad \alpha = 1,\ \beta = -2$ and $\dfrac{2\lambda_2}{3\sqrt{2}}=1$

$\therefore \quad \vec{a}\cdot\hat{k}+2=0$

34. Answer (3)

$\because \ |\vec{a}|=|\vec{b}|=|\vec{c}|=1$

and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

On squaring both sides

$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{c})=0$

$\therefore \quad \lambda = \vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{c}=-\dfrac{3}{2}$

and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$

$= \vec{a}\times\vec{b}+\vec{b}\times(-\vec{a}-\vec{b})+\left(-\vec{a}-\vec{b}\right)\times\vec{a}$

$= \vec{a}\times\vec{b}-\vec{b}\times\vec{a}-0-0-\vec{b}\times\vec{a}$

$= 3\left(\vec{a}\times\vec{b}\right)$

$\therefore \quad \left(\lambda,\vec{d}\right)=\left(-\dfrac{3}{2},3\left(\vec{a}\times\vec{b}\right)\right)$

35. Answer (4)

$v=\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$$\Rightarrow \begin{vmatrix} 1 & 1 & \lambda \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = \pm 1$$

$\Rightarrow \quad 2 - 1(5) + \lambda(1) = \pm 1$

$\Rightarrow \quad \lambda - 3 = 1$ or $\lambda - 3 = -1$

$\Rightarrow$ $\lambda = 4$ or 2

$\vec{u} = \hat{i} + \hat{j} + 4\hat{k}$ or $\hat{i} + \hat{j} + 2\hat{k}$

$\therefore \quad \cos\theta = \dfrac{2+1+4}{\sqrt{18}\sqrt{6}}$ or $\dfrac{2+1+2}{\sqrt{6}\sqrt{6}}$

$= \dfrac{7}{6\sqrt{3}}$ or $\dfrac{5}{6}$

36. Answer (4)

$\because \quad a\cos\theta = b\cos\left(\theta + \dfrac{2\pi}{3}\right) = c\cos\left(\theta + \dfrac{4\pi}{3}\right) = k$

$$ab + bc + ca = k^2\left[\frac{1}{\cos\theta\cdot\cos\left(\theta+\frac{2\pi}{3}\right)} + \frac{1}{\cos\left(\theta+\frac{2\pi}{3}\right)\cdot\cos\left(\theta+\frac{4\pi}{3}\right)} + \frac{1}{\cos\left(\theta+\frac{4\pi}{3}\right)\cdot\cos\theta}\right]$$

$$= k^2\left[\frac{\cos\theta + \cos\left(\theta+\frac{2\pi}{3}\right) + \cos\left(\theta+\frac{4\pi}{3}\right)}{\cos\theta\cdot\cos\left(\theta+\frac{2\pi}{3}\right)\cdot\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

$= 0$

So, angle between the given vectors will be $\dfrac{\pi}{2}$.

37. Answer (4)

Here $\vec{a}.\left(\vec{b}\times\vec{c}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix} = x^3 - 27x + 26$

$\Rightarrow$ $f(x) = x^3 - 27x + 26$

$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = -3, 3$

$$\begin{array}{ccccc} + & & - & & + \\ & -3 & & 3 & \\ & \text{Max} & & \text{Min} & \end{array}$$

$\Rightarrow$ $x_0 = -3$

Now $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -2x - 2x - 3 - 14 - 2x - x + 7x + 4 + 3x = 3x - 13$

So value at $x = x_0 = 3 \times -3 - 13 = -22$

38. Answer (2)

Volume of parallelopiped = $\left[\vec{a} \ \ \vec{b} \ \ \vec{c}\right]$

$\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 158$

$\Rightarrow$ $(12 + n^2) - 1(6 + n) + n(2n - 4) = 158$

$\Rightarrow$ $3n^2 - 5n - 152 = 0$

$\Rightarrow$ $3n^2 - 24n + 19n - 152 = 0$

$\Rightarrow$ $3n(n - 8) + 19(n - 8) = 0$

$\Rightarrow$ $n = 8$

$\therefore$

$\vec{a} = \hat{i} + \hat{j} + 8\hat{k},\ \vec{b} = 2\hat{i} + 4\hat{j} - 8\hat{k}$ and $\vec{c} = \hat{i} + 8\hat{j} + 3\hat{k}$

$\vec{a}\cdot\vec{c} = 1 + 8 + 24 = 33$

$\vec{b}\cdot\vec{c} = 2 + 32 - 24 = 10$

39. Answer (1)

$\vec{a} = \hat{i} - 2\hat{j} + \hat{k};\ \vec{b} = \hat{i} - \hat{j} + \hat{k}$

$\because \quad \vec{b}\times\vec{c} = \vec{b}\times\vec{a} \ \Rightarrow\ \vec{a}\times(\vec{b}\times\vec{c}) = \vec{a}\times(\vec{b}\times\vec{a})$

$\Rightarrow$ $(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}$

$\because \quad \vec{a}.\vec{c} = 0$

$\Rightarrow$ $\vec{c} = -\dfrac{1}{2}(\hat{i} + \hat{j} + \hat{k})$

$\therefore \quad \boxed{\vec{b}.\vec{c} = \dfrac{-1}{2}}$

40. Answer (1.00)

$\because \quad \vec{P}, \vec{Q}, \vec{R}$ are coplanar, so $\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a+1 & a & a \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -2 & -1 & 1 \end{vmatrix} = 0$

$\Rightarrow$ $3a + 1 = 0$

$\Rightarrow$ $a = -\dfrac{1}{3}$

$\because \quad \vec{P}\cdot\vec{Q} = 3a^2 + 2a = -\dfrac{1}{3}$

and $\vec{R}\times\vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & a & a+1 \\ a & a+1 & a \end{vmatrix}$

$$= (-2a-1)\hat{i} + a\hat{j} + a\hat{k} = -\frac{\hat{i}+\hat{j}+\hat{k}}{3}$$

$$\Rightarrow \quad \left|\vec{R}\times\vec{Q}\right| = \frac{1}{\sqrt{3}}$$

$$\text{Now; } \lambda = \frac{3\left(\vec{P}\cdot\vec{Q}\right)^2}{\left|\vec{R}\times\vec{Q}\right|^2} = \frac{3\times 1}{9\times\frac{1}{3}} = 1$$

41. Answer (30)

$$\vec{b}.\vec{c} = |\vec{b}||\vec{c}|\cos\frac{\pi}{3} = 10$$

$$\Rightarrow \quad 5|\vec{c}|.\frac{1}{2} = 10 \Rightarrow |\vec{c}| = 4$$

$$\text{Now } \left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right| = |\vec{a}|\left|\vec{b}\times\vec{c}\right|\sin\frac{\pi}{2}$$

$$= \sqrt{3}\,|\vec{b}||\vec{c}|.\sin\frac{\pi}{3}$$

$$= \sqrt{3}.5.4\frac{\sqrt{3}}{2}$$

$= 30$

42. Answer (2)

Given, $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Also $\quad |\vec{a}-\vec{b}|^2 + |\vec{a}-\vec{c}|^2 = 8$

$$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 - 2\,\vec{a}\cdot\vec{b} + |\vec{a}|^2 + |\vec{c}|^2 - 2\vec{a}\cdot\vec{c} = 8$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} = -2$$

Now, $\quad |\vec{a}+2\vec{b}|^2 + |\vec{a}+2\vec{c}|^2$

$$= |\vec{a}|^2 + 4|\vec{b}|^2 + 4\vec{a}\cdot\vec{b} + |\vec{a}|^2 + 4|\vec{c}|^2 + 4\vec{a}\cdot\vec{c}$$

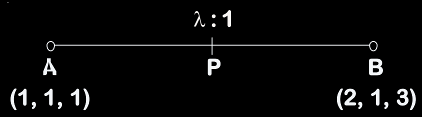
$$= 10 + 4\left(\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}\right)$$

$= 10 + 4\,(-2)$

$= 2$

43. Answer (0.8)

Let position vector of P is $\overrightarrow{OP} = \dfrac{\lambda\vec{b}+\vec{a}}{\lambda+1}$



Given $\overrightarrow{OB}\cdot\overrightarrow{OP} - 3\left|\overrightarrow{OA}\times\overrightarrow{OP}\right|^2 = 6$

$$\Rightarrow \quad \vec{b}\cdot\left(\frac{\lambda\vec{b}+\vec{a}}{\lambda+1}\right) - 3\left|\vec{a}\times\frac{\lambda\vec{b}+\vec{a}}{\lambda+1}\right|^2 = 6$$

$$\Rightarrow \quad \frac{\vec{a}\cdot\vec{b} + \lambda|\vec{b}|^2}{\lambda+1}\ \frac{-3\lambda^2}{(\lambda+1)^2}\,|\vec{a}\times\vec{b}|^2 = 6$$

$$\left(\because\ \vec{a}\times\vec{b} = 2\hat{i} - \hat{j} - \hat{k}\right)$$

$$\Rightarrow \quad \frac{6+14\lambda}{\lambda+1} - \frac{18\lambda^2}{(\lambda+1)^2} = 6$$

$$\Rightarrow \quad 6 + \frac{8\lambda}{\lambda+1} - \frac{18\lambda^2}{(\lambda+1)^2} = 6$$

Let $\dfrac{\lambda}{\lambda+1} = t$

$18t^2 - 8t = 0$

$t = 0,\ \dfrac{4}{9}$

$$\therefore \quad \frac{\lambda}{\lambda+1} = \frac{4}{9}$$

$$\therefore \quad \lambda = \frac{4}{5} = 0.8$$

44. Answer (18)

Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

Now $\hat{i}\times\left(\vec{a}\times\hat{i}\right) = \left(\hat{i}.\hat{i}\right)\vec{a} - \left(\hat{i}.\vec{a}\right)\hat{i}$

$= y\hat{j} + z\hat{k}$

Similarly $\hat{j}\times\left(\vec{a}\times\hat{j}\right) = x\hat{i} + z\hat{k}$

$\hat{k}\times\left(\vec{a}\times\hat{k}\right) = x\hat{i} + y\hat{j}$

Now $\left|y\hat{j}+z\hat{k}\right|^2 + \left|x\hat{i}+z\hat{k}\right|^2 + \left|x\hat{i}+y\hat{j}\right|^2$

$= 2(x^2 + y^2 + z^2) = 2(4 + 1 + 4) = 18$

45. Answer (6)

Projection of $\bar{b}$ on $\bar{a}$ = Projection of $\bar{c}$ on $\bar{a}$

$\Rightarrow \quad \bar{a}\cdot\bar{b} = \bar{a}\cdot\bar{c}$

Given $\bar{b}\cdot\bar{c} = 0$

$\because$

$$|\bar{a}+\bar{b}-\bar{c}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2\bar{a}.\bar{b} - 2\bar{b}\cdot\bar{c} - 2\bar{a}\cdot\bar{c}$$

$$= 4 + 16 + 16$$

$$= 36$$

$\Rightarrow \quad |\bar{a}+\bar{b}-\bar{c}| = 6$

46. Answer (4)

Let angle between $\vec{a}$ and $\vec{b}$ be θ

$$|\vec{a}+\vec{b}| = \sqrt{1+1+2\cos\theta} = 2\left|\cos\frac{\theta}{2}\right|$$

and $|\vec{a}-\vec{b}| = 2\left|\sin\frac{\theta}{2}\right|$

So, $\sqrt{3}|\vec{a}+\vec{b}| + |\vec{a}-\vec{b}| = 2\left[\sqrt{3}\left|\cos\frac{\theta}{2}\right| + \left|\sin\frac{\theta}{2}\right|\right]$

$$\max\{\sqrt{3}|\vec{a}+\vec{b}| + |\vec{a}-\vec{b}|\} = 2\sqrt{(\sqrt{3})^2 + (1)^2}$$

$$= 4$$

47. Answer (01.00)

$|\vec{x}+\vec{y}| = |\vec{x}|$

Squaring both sides we get

$|\vec{x}|^2 + 2\vec{x}.\vec{y} + |\vec{y}|^2 = |\vec{x}|^2$

$\Rightarrow \quad 2\vec{x}.\vec{y} + \vec{y}.\vec{y} = 0 \qquad \ldots(1)$

Also $2\vec{x}+\lambda\vec{y}$ and $\vec{y}$ are perpendicular

$\therefore \quad 2\vec{x}.\vec{y} + \lambda\vec{y}.\vec{y} = 0 \qquad \ldots(2)$

Comparing (1) & (2) $\lambda = 1$

48. Answer (12)

$\because \quad \vec{a}\cdot\vec{b} = -1, \ \ \vec{b}\cdot\vec{c} = 2, \ \ \vec{c}\cdot\vec{a} = 0$

$\vec{r}\times\vec{a} = \vec{c}\times\vec{a} \ \Rightarrow (\vec{r}\times\vec{a})\times\vec{b} = (\vec{c}\times\vec{a})\times\vec{b}$

$\Rightarrow (\vec{r}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{b})\vec{r} = (\vec{b}\cdot\vec{c})\vec{a} - (\vec{a}\cdot\vec{b})\vec{c}$

$\Rightarrow \vec{r} = 2\vec{a} + \vec{c}$

then $\vec{r}\cdot\vec{a} = 2|\vec{a}|^2 + \vec{a}\cdot\vec{c}$

$= 12$

49. Answer (4)

$\overrightarrow{AB} = \hat{i} + \hat{j} - 2\hat{k}$

$\overrightarrow{AC} = \hat{i} + 2\hat{j} - \hat{k}$

Normal to plane $\vec{n} = \overrightarrow{AB}\times\overrightarrow{AC}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

$\overrightarrow{OP} = 2\hat{i} - \hat{j} + \hat{k}$

$$\sin\theta = \frac{\overrightarrow{OP}.\vec{n}}{|\overrightarrow{OP}|\times|\vec{n}|} = \frac{6+1+1}{\sqrt{11}.\sqrt{6}} = \frac{8}{\sqrt{66}}$$

$$\cos\theta = \sqrt{1-\frac{64}{66}} = \frac{1}{\sqrt{33}}$$

Projection = $|\overrightarrow{OP}|\cos\theta = \sqrt{6}\times\frac{1}{\sqrt{33}} = \sqrt{\frac{2}{11}}$

50. Answer (2)

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \alpha & 3 \\ 3 & -\alpha & 1 \end{vmatrix} = 4\alpha\hat{i} + 8\hat{j} - 4\alpha\hat{k} = 4(\alpha\hat{i} + 2\hat{j} - \alpha\hat{k})$$

$\because \quad 8\sqrt{3} = 4\sqrt{2\alpha^2+4} \qquad \Rightarrow \quad \alpha = \pm 2$

$\vec{a}\cdot\vec{b} = 3 - \alpha^2 + 3 = 2$

51. Answer (4)

Let $\hat{c}$ be a unit vector in the direction of $\vec{a}\times\vec{b}$.

$\Rightarrow \quad \hat{a}\times\hat{b} = \hat{c}, \ \hat{b}\times\hat{c} = \hat{a}$ & $\hat{c}\times\hat{a} = \hat{b}$

$\vec{a}\times\vec{b} = |\vec{a}|\times|\vec{b}|\times\hat{c}$

$\vec{a}\times(\vec{a}\times\vec{b}) = -|\vec{a}|^2|\vec{b}|\hat{b}$

$\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b})) = -|\vec{a}|^3|\vec{b}|\hat{c}$

$\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b}))) = |\vec{a}|^4|\vec{b}|\hat{b}$

$$= |\vec{a}|^4\vec{b}$$

52. Answer (1)

$\overrightarrow{a_2} = \lambda\overrightarrow{a_1}$

$\hat{i} + y\hat{j} + z\hat{k} = \lambda(x\hat{i} - \hat{j} + \hat{k})$

$1 = \lambda x, y = -\lambda, z = \lambda$

$$x\hat{i} + y\hat{j} + z\hat{k} = \frac{1}{\lambda}\hat{i} - \lambda\hat{j} + \lambda\hat{k}$$

Unit vector $= \dfrac{\frac{1}{\lambda}i - \lambda\hat{j} + \lambda\hat{k}}{\sqrt{\frac{1}{\lambda^2} + \lambda^2 + \lambda^2}}$

$= \dfrac{\hat{i} - \lambda^2\hat{j} + \lambda^2\hat{k}}{\sqrt{1+2\lambda^4}}$

Let $\lambda^2 = 1$, possible unit vector $= \dfrac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

53. Answer (1)

$\because \quad |\alpha\hat{i} + \beta\hat{j}| = |\sqrt{3}\,\hat{i} + \hat{j}| \Rightarrow \alpha^2 + \beta^2 = 4 \qquad$ ...(i)

Also $\dfrac{\sqrt{3}\alpha + \beta}{2\cdot 2} = \dfrac{1}{\sqrt{2}} \Rightarrow \sqrt{3}\alpha + \beta = 2\sqrt{2} \qquad$ ...(ii)

$\because \quad \alpha, \beta > 0$, then from (i) and (ii)

$\alpha = \dfrac{\sqrt{3}-1}{\sqrt{2}}$ and $\beta = \dfrac{\sqrt{3}+1}{\sqrt{2}}$

Area of required triangle

$= \dfrac{1}{2}\alpha\beta = \dfrac{1}{2}\left(\dfrac{\sqrt{3}-1}{\sqrt{2}}\right)\left(\dfrac{\sqrt{3}+1}{\sqrt{2}}\right)$

$= \dfrac{1}{2}$

54. Answer (1)

$\vec{r} \times \vec{a} = -\vec{r} \times \vec{b}$

$\vec{r} \times (\vec{a} + \vec{b}) = 0$

$\vec{r} = \lambda(\vec{a} + \vec{b}) = \lambda(3\hat{i} - \hat{j} + 2\hat{k})$

$\vec{r}\cdot(\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3 \Rightarrow \alpha\lambda = 1 \quad$ ...(i)

$\vec{r}\cdot(2\hat{i} + 5\hat{j} + \alpha\hat{k}) = -1 \Rightarrow \lambda - 2\alpha\lambda = -1$

$\Rightarrow \quad \lambda = 1$ and $\alpha = 1$ [using (i)]

$\alpha + |\vec{r}|^2 = 1 + (9 + 1 + 4) = 15$

55. Answer (28)

$\vec{a} \times \vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

$C = \lambda(\vec{a} \times \vec{b}) = \lambda(3i - 2j + \hat{k})$

$C\cdot(i + j + 3k) = 8 \Rightarrow \lambda = 2$

$\vec{C} = 2(\vec{a} \times \vec{b})$

$\vec{C}\cdot(\vec{a} \times \vec{b}) = 2(\vec{a} \times \vec{b})\cdot(\vec{a} \times \vec{b})$

$= 2|\vec{a} \times \vec{b}|^2 = 2(9 + 4 + 1) = 28$

56. Answer (4)

$\bar{r} \times \bar{a} = \bar{r} \times \bar{b} \quad \Rightarrow \bar{r} \times (\bar{a} - \bar{b}) = 0$

$\Rightarrow \bar{r} = \lambda(\bar{a} - \bar{b}), \lambda \in R.$

$\Rightarrow \bar{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$

$\because \quad \bar{r}\cdot(\hat{i} + 2\hat{j} + \hat{k}) = -3$

$\Rightarrow \quad \lambda(-5 - 8 + 10) = -3$

$\Rightarrow \quad \lambda = 1$

Hence $\bar{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$

$\bar{r}\cdot(2\hat{i} - 3\hat{j} + \hat{k}) = 12$

57. Answer (2)

$\bar{a}.\bar{b} = 1 \Rightarrow -\alpha\beta - \alpha\beta - 3 = 1$

$\Rightarrow \quad \alpha\beta = -2 \qquad$ ...(i)

$\bar{b}.\bar{c} = -3 \Rightarrow -\beta + 2\alpha + 1 = -3$

$2\alpha - \beta = -4 \qquad$ ...(ii)

Solving (i) & (ii) $\alpha = -1$, $\beta = 2$,

$\dfrac{1}{3}((\bar{a} \times \bar{b})\cdot\bar{c}) = \dfrac{1}{3}\begin{vmatrix} -1 & 2 & 3 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix}$

$= 2$

58. Answer (2)

$\because \quad \overline{OP}\cdot\overline{OQ} = 0 \Rightarrow -x + 2y - 3x = 0 \Rightarrow 2x = y$

and $|\overline{OQ} - \overline{OP}|^2 = 20 \Rightarrow (x + 1)^2 + (y - 2)^2 + (3x + 1)^2 = 20$

$\Rightarrow \quad 14x^2 = 14 \Rightarrow x = 1$

$\because \quad [\overline{OP}\ \overline{OQ}\ \overline{OR}] = 0$

$\Rightarrow \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \\ 3 & z & -7 \end{vmatrix} = 0 \Rightarrow z = -2$

So $x^2 + y^2 + z^2 = 9$

59. Answer (486)

Let $\vec{x} = \lambda(\bar{a} \times \bar{b}) \times \bar{c}$

$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = -\hat{i} + 3\hat{j} + 5\hat{k}$

$(\bar{a} \times \bar{b}) \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & 5 \\ 3 & 2 & -1 \end{vmatrix} = -13\hat{i} + 14\hat{j} - 11\hat{k}$

$\because \quad \frac{\bar{x}\cdot\bar{a}}{|\bar{a}|} = \frac{17\sqrt{6}}{2} \Rightarrow \left|\frac{(-26-14-11)\lambda}{\sqrt{6}}\right| = \frac{17\sqrt{6}}{2}$

$\Rightarrow \quad \lambda = \pm 1$

$|\bar{x}|^2 = 13^2 + 14^2 + 11^2 = 486$

60. Answer (1)

Magnitude of vector remains same hence

$9p^2 + 1 = (p + 1)^2 + 10$

$\Rightarrow \quad 8p^2 - 2p - 10 = 0$

$\Rightarrow \quad 4p^2 - p - 5 = 0$

$\Rightarrow \quad 4p^2 - 5p + 4p - 5 = 0$

$\Rightarrow \quad (p + 1)(4p - 5) = 0 \Rightarrow p = -1 \text{ or } \frac{5}{4}$

61. Answer (4)

Projection of $\overrightarrow{AB}$ on $\overrightarrow{AC} = \frac{(\overrightarrow{AB}).(\overrightarrow{AC})}{|\overrightarrow{AC}|} = p(\text{say})$

$$= \frac{|\overrightarrow{AB}||\overrightarrow{AC}|\cos\theta}{|\overrightarrow{AC}|}$$

where $\cos\theta = \frac{10^2 + 7^2 - 8^2}{2.10.7}$

$\Rightarrow \quad p = \frac{10.85}{2.10.7} = \frac{85}{14}$

62. Answer (4)

Angle required is say θ

$$\Rightarrow \quad \cos\theta = \frac{\vec{a}\cdot\left(\vec{a}+\vec{b}+\left(\vec{a}\times\vec{b}\right)\right)}{|\vec{a}||\vec{a}+\vec{b}+\left(\vec{a}\times\vec{b}\right)|}$$

$$= \frac{|\vec{a}|^2 + 0 + 0}{|\vec{a}||\vec{a}+\vec{b}+\left(\vec{a}\times\vec{b}\right)|}$$

$$= \frac{|\vec{a}|^2}{|\vec{a}|\sqrt{3}|\vec{a}|}$$

(as $\vec{a},\vec{b}$ and $\vec{a} \times \vec{b}$ are mutually perpendicular to each other)

$\cos\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

63. Answer (3)

$\because \quad \vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\Rightarrow \quad |\vec{a}| = \sqrt{4+1+4} = 3$

Now, $|\vec{c} - \vec{a}| = 2\sqrt{2}$

$\Rightarrow \quad (\vec{c} - \vec{a})^2 = 8$

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c}\cdot\vec{a} = 8$

$|\vec{c}|^2 + 9 - 2|\vec{c}| = 8 \qquad (\because \ \vec{a}\cdot\vec{c} = |\vec{c}|)$

$|\vec{c}|^2 - 2|\vec{c}| + 1 = 0$

$\therefore \quad |\vec{c}| = 1$

and angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is θ then

$\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right| = |\vec{a}\times\vec{b}|\cdot|\vec{c}|\cdot\sin\theta$

$\left|2\hat{i} - 2\hat{j} + \hat{k}\right| \cdot 1 \cdot \sin\frac{\pi}{6}$

$= \frac{3}{2}$

64. Answer (4)

$|\bar{a}+\bar{b}+\bar{c}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a})$

$\Rightarrow \quad |\bar{a}+\bar{b}+\bar{c}| = \sqrt{3}|\bar{a}|$

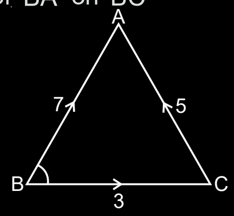
Now $\cos\theta = \frac{\bar{a}.(\bar{a}+\bar{b}+\bar{c})}{|\bar{a}||\bar{a}+\bar{b}+\bar{c}|} = \frac{|\bar{a}|^2}{|\bar{a}|(\sqrt{3}|\bar{a}|)} = \frac{1}{\sqrt{3}}$

So, $36\cos^2 2\theta = 36(2\cos^2\theta - 1)^2$

$$= 36\left(\frac{2}{3} - 1\right)^2 = 4$$

65. Answer (4)

Projection of $\overrightarrow{BA}$ on $\overrightarrow{BC}$



$= \left|\frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BC}|}\right|$

$= \frac{|\overrightarrow{BA}|\cdot|\overrightarrow{BC}|\cos B}{|\overrightarrow{BC}|}$

$= 7\cdot\left(\frac{7^2 + 3^2 - 5^2}{2\times 7\times 3}\right)$

$= \frac{11}{2}$ units

66. Answer (6)

$\because\ \cos\theta = \dfrac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| \cdot |\vec{v}_2|}$ and $|\vec{v}_1| = |\vec{v}_2|$

$\Rightarrow\ \cos\theta = \dfrac{2\sqrt{3}p + p + 1}{|\vec{v}_1|^2}$ and $4 + (p + 1)^2 = 3p^2 + 1$

$\Rightarrow\ p = 2$

$\Rightarrow\ \cos\theta = \dfrac{4\sqrt{3}+3}{13} \Rightarrow \tan\theta = \dfrac{6\sqrt{3}-2}{4\sqrt{3}+3}$

67. Answer (1)

$$[\vec{b}\ \vec{c}\ \vec{d}] = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 3 & 2 & 6 \end{vmatrix} = 2(-6-2) - 1(3) + 1(5)$$

$$= -16 - 3 + 5 = -14$$

Let $\vec{a} = \lambda\vec{b} + \mu\vec{c}$

$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{d}] + [\vec{a}\ \vec{c}\ \vec{d}]$

$= -\mu[\vec{b}\ \vec{c}\ \vec{d}] + \lambda[\vec{b}\ \vec{c}\ \vec{d}]$

$= (\lambda - \mu)[\vec{b}\ \vec{c}\ \vec{d}]$

$\therefore\ \vec{a} = (2\lambda+\mu)\hat{i} + (\lambda-\mu)\hat{j} + (\lambda+\mu)\hat{k}$

$(2\lambda+\mu)^2 + (\lambda-\mu)^2 + (\lambda+\mu)^2 = 10$ (as $|\vec{a}| = \sqrt{10}$)

$\Rightarrow\ 6\lambda^2 + 3\mu^2 + 4\lambda\mu = 10$ ...(i)

& $\hat{a}\cdot\hat{b} = 0 \Rightarrow 3(2\lambda + \mu) + 2(\lambda - \mu) + 6(\lambda + \mu) = 0$

$14\lambda + 7\mu = 0 \Rightarrow \mu = -2\lambda$ ...(ii)

by (i) & (ii), $6\lambda^2 + 12\lambda^2 - 8\lambda^2 = 10$

$\Rightarrow\ \lambda = \pm 1 \Rightarrow \mu = \mp 2$

$(\lambda - \mu) = 3$ or -3

$\therefore$ Required quantity = –42

68. Answer (4)

$\because\ \bar{a}\times\bar{b} = \bar{c} \Rightarrow (\bar{a}\times\bar{b})\cdot\bar{c} = \bar{c}\cdot\bar{c}$

$\therefore\ [\bar{a}\ \bar{b}\ \bar{c}] = |\bar{c}|^2$ ...(i)

and $(\bar{b}\times\bar{c}) = \bar{a} \Rightarrow \bar{a}\cdot(\bar{b}\times\bar{c}) = \bar{a}\cdot\bar{a}$

$\therefore\ [\bar{a}\ \bar{b}\ \bar{c}] = |\bar{a}|^2 = 4$ ...(ii)

$\therefore\ |\bar{a}| = |\bar{c}| = 2$

Option: (1)

$\bar{a}\times((\bar{b}+\bar{c})\times(\bar{b}-\bar{c})) = \bar{a}\times(-\bar{b}\times\bar{c} + \bar{c}\times\bar{b})$

$= 2\bar{a}\times(\bar{c}\times\bar{b}) = 2\bar{a}\times(\bar{a}) = 0$

Option: (2)

Projection of $\bar{a}$ on $(\bar{b}\times\bar{c}) = \dfrac{\bar{a}\cdot(\bar{b}\times\bar{c})}{|\bar{b}\times\bar{c}|} = \dfrac{|\bar{a}|^2}{|\bar{a}|} = 2$

Option: (3)

$[\bar{a}\ \bar{b}\ \bar{c}] + [\bar{c}\ \bar{a}\ \bar{b}] = 2[\bar{a}\ \bar{b}\ \bar{c}] = 8$

Option: (4)

$|3\bar{a} + \bar{b} - 2\bar{c}|^2$

$= 9\bar{a}^2 + \bar{b}^2 + 4\bar{c}^2 + 6\bar{a}\cdot\bar{b} - 4\bar{b}\cdot\bar{c} - 12\bar{a}\cdot\bar{c}$

$= 9.2^2 + 1^2 + 4.2^2 + 0$

= 53

$\because\ \bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{c} = \bar{c}\cdot\bar{a} = 0$

and $[\bar{a}\ \bar{b}\ \bar{c}]^2 = \begin{vmatrix} \bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c} \\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c} \\ \bar{c}\cdot\bar{a} & \bar{c}\cdot\bar{b} & \bar{c}\cdot\bar{c} \end{vmatrix}$

$\therefore\ 16 = |\bar{a}|^2|\bar{b}|^2|\bar{c}|^2$

$\therefore\ |\bar{b}| = 1$

69. Answer (2)

For coplanarity S.T.P. of given vectors shall vanish

i.e. $\begin{vmatrix} 2+a+b & a+2b+c & -(b+c) \\ 1+b & 2b & -b \\ 2+b & 2b & (1-b) \end{vmatrix} = 0$

$R_3 \to R_3 - R_2$ and $R_1 \to R_1 - R_2$

$\begin{vmatrix} 1+a & a+c & -c \\ 1+b & 2b & -b \\ 1 & 0 & 1 \end{vmatrix} = 0$

Expanding

$\therefore$ 1(–b(a + c) + 2bc) + 1(2b(1 + a) – (b + 1)(a + c)) = 0

$\therefore$ a + c = 2b

70. Answer (3)

$$\vec{r} = \sqrt{3}\,\frac{(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})}{|(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})|}$$

$$= \frac{\sqrt{3}\times\begin{vmatrix} i & j & k \\ 3 & 5 & 2 \\ 1 & 1 & 0 \end{vmatrix}}{|(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})|} = \frac{\sqrt{3}(-2\hat{i}+2\hat{j}-2\hat{k})}{2\sqrt{3}}$$

$= -i + j - k$

$|\alpha| = |\beta| = |\gamma| = 1$

71. Answer (2)

$\because \quad a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are co-planar.

$$\therefore \quad \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$$\therefore \quad \begin{vmatrix} a-c & a & c \\ 0 & 0 & 1 \\ c-b & c & b \end{vmatrix} = 0$$

$\therefore \quad (-1)\{ac - c^2 - ca + ab\} = 0$

$\therefore \quad c^2 = ab$

$\therefore \quad c = \sqrt{ab}$

72. Answer (2)

$|\vec{a}| = 2, |\vec{b}| = 5, |\vec{a}\times\vec{b}| = 8$

$$\therefore \quad |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2 \; |\vec{b}|^2 - \left(\vec{a}\cdot\vec{b}\right)^2$$

$$\therefore \quad \left(\vec{a}\cdot\vec{b}\right)^2 = 5^2 \cdot 2^2 - 8^2$$

$= 100 - 64$

$= 36$

$\therefore \quad \vec{a}\cdot\vec{b} = \pm 6$

73. Answer (60)

$\because \; (\vec{a}+3\vec{b})\cdot(7\vec{a}-5\vec{b}) = 0$

$\Rightarrow \quad 7|\vec{a}|^2 - 15|\vec{b}|^2 + 16\vec{a}\cdot\vec{b} = 0 \qquad$ ...(i)

and $(7\vec{a}-2\vec{b})\cdot(\vec{a}-4\vec{b}) = 0$

$\Rightarrow \quad 7|\vec{a}|^2 + 8|\vec{b}|^2 - 30\vec{a}\cdot\vec{b} = 0 \qquad$ ...(ii)

From (i) and (ii)

$|\vec{b}|^2 = 2\vec{a}\cdot\vec{b}$ and $|\vec{a}|^2 = 2\vec{a}\cdot\vec{b}$

$\because \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta \;\Rightarrow\; \vec{a}\cdot\vec{b} = 2\vec{a}\cdot\vec{b}\cos\theta$

$$\Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

74. Answer (1)

$(\vec{a}-\vec{b})\times\vec{b} = \vec{a}\times\vec{b}$

$\therefore \quad$ Given expression is

$$(\vec{a}+\vec{b})\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\times\vec{b}\right)$$

$$\Rightarrow \quad (\vec{a}+\vec{b})\times\left(\vec{a}\times(\vec{b}\cdot\vec{a})\vec{b} - (\vec{b}\cdot\vec{b})\vec{a}\right)$$

$$\Rightarrow \quad \left((\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})\right)\left(\vec{b}\cdot\vec{a}\right)$$

$$\Rightarrow \quad \left(\vec{a}\times(\vec{a}\times\vec{b}) + \vec{b}\times(\vec{a}\times\vec{b})\right)(\vec{b}\cdot\vec{a})$$

$$\Rightarrow \quad \left((\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b} + (\vec{b}\cdot\vec{b})\vec{a} - (\vec{b}\cdot\vec{a})\vec{b}\right)(\vec{b}\cdot\vec{a})$$

Put $\vec{a}\cdot\vec{b} = 7,\; \vec{a}\cdot\vec{a} = 6,\; \vec{b}\cdot\vec{b} = 14$ we get

$$\Rightarrow \quad \left(7\vec{a} - 6\vec{b} + 14\vec{a} - 7\vec{b}\right)7$$

$$\Rightarrow \quad 7\left(21\,\vec{a} - 13\,\vec{b}\right)$$

$$\Rightarrow \quad 7\left(21\hat{i} + 21\hat{j} + 42\hat{k} + 13\,\hat{i} - 26\,\hat{j} - 39\,\hat{k}\right)$$

$$\Rightarrow \quad 7\left(34\,\hat{i} - 5\,\hat{j} + 3\hat{k}\right)$$

75. Answer (2)

$\because \quad \vec{a}\times\vec{b} = \vec{c}$

$$\Rightarrow \quad (\vec{a}\times\vec{b})\cdot\vec{c} = |\vec{c}|^2 = 2 = [\vec{a}\;\vec{b}\;\vec{c}]$$

$$\because \quad l = \left|\frac{\vec{b}\cdot(\vec{a}\times\vec{c})}{|\vec{a}\times\vec{c}|}\right| \quad (\because \vec{a} \text{ and } \vec{c} \text{ are perpendicular})$$

$$\Rightarrow \quad l = \frac{|[\vec{a}\;\vec{b}\;\vec{c}]|}{|\vec{a}||\vec{c}|} = \frac{2}{\sqrt{3}\sqrt{2}} = \sqrt{\frac{2}{3}} \Rightarrow 3l^2 = 2$$

76. Answer (1)

$$\vec{a} = \vec{b}\times\left(\vec{b}\times\vec{c}\right) = \left(\vec{b}\cdot\vec{c}\right)\vec{b} - \left|\vec{b}\right|^2\vec{c}$$

$$= \left(\vec{b}\cdot\vec{c}\right)\vec{b} - \vec{c} \quad \left(\because \left|\vec{b}\right| = 1\right)$$

$$\left|\vec{a}\right|^2 = \left(\vec{b}\cdot\vec{c}\right)^2\left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 - 2\left(\vec{b}\cdot\vec{c}\right)\left(\vec{b}\cdot\vec{c}\right)$$

$$2 = \left|\vec{c}\right|^2 - \left(\vec{b}\cdot\vec{c}\right)^2$$

$2 = 4 - (2\cos\theta)^2$

$(2\cos\theta)^2 = 2$

$$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \tan\theta = 1$$

77. Answer (9)

$\because \quad \vec{a}\cdot\vec{b} = -1 = 3 - 2\alpha\beta \Rightarrow \alpha\beta = 2$

$\vec{b}\cdot\vec{c} = 10 = -3\alpha - 2\beta - \alpha \Rightarrow 2\alpha + \beta = -5$

Clearly $(\alpha,\beta) = (-2, -1)$

$$[\vec{a}\ \ \vec{b}\ \ \vec{c}] = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix} = 9$$

78. Answer (2)

$\vec{a}\cdot(\vec{b}\times\vec{c}) = (\vec{b}\times\vec{c})\cdot\vec{a} = \vec{b}\cdot(\vec{c}\times\vec{a})$

$= \vec{b}\cdot(-\vec{b}) \qquad [\because \vec{a}\times\vec{c} = \vec{b}]$

$= -|\vec{b}|^2$

$= -\left(1^2 + (-1)^2\right) = -2$

79. Answer (5)

$\bar{v}_1 = \hat{i} + 2\hat{j} + \hat{k},\ \bar{v}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k},\ \bar{v}_3 = -\lambda\hat{i} + 2\hat{j} + 3\hat{k}$

$\bar{v}_2 + \bar{v}_3 = (2-\lambda)\hat{i} + 6\hat{j} - 2\hat{k} = \bar{v}_4$

Projection of $\bar{v}_1$ on $\bar{v}_4 = \bar{v}_1\cdot\dfrac{\bar{v}_4}{|\bar{v}_4|}$

$\Rightarrow \dfrac{1\times(2-\lambda) + 2\times 6 + 1\times(-2)}{\sqrt{(2-\lambda)^2 + 6^2 + (-2)^2}} = 1$

$\Rightarrow (12 - \lambda)^2 = (2 - \lambda)^2 + 40$

On solving

$\lambda = 5$

80. Answer (90)

$\vec{b}\times\vec{c} = (-9-2\beta)\hat{i} + (3-\beta)\hat{j} + 5\hat{k}$

$|\vec{b}\times\vec{c}| = 5\sqrt{3} \Rightarrow (9+2\beta)^2 + (3-\beta)^2 + 25 = 75$

$\Rightarrow \beta^2 + 6\beta + 8 = 0$

$\Rightarrow \beta = -2$ or $\beta = -4$

$\vec{a}$ is perpendicular to $\vec{b} \Rightarrow \vec{a}\cdot\vec{b} = 0$

$\Rightarrow \left(\hat{i} + 5\hat{j} + \alpha\hat{k}\right)\cdot\left(\hat{i} + 3\hat{j} + \beta\hat{k}\right) = 0$

$\Rightarrow 1 + 15 + \alpha\beta = 0 \qquad$ ...(i)

$|\vec{a}|^2 = 1 + 25 + \alpha^2 = 26 + \left(\dfrac{-16}{\beta}\right)^2$

from greatest value of $|\vec{a}|^2$ take $\beta = 2$

$\Rightarrow$ greatest value of $|\vec{a}|^2 = 90$

81. Answer (3)

$\left|\dfrac{\vec{a}}{8}\right| = 1 \Rightarrow |\vec{a}| = 8$

$|2\vec{a} + 3\vec{b}|^2 = |3\vec{a} + 3\vec{b}|^2$

$\therefore \quad 4|\vec{a}|^2 + 9|\vec{b}|^2 + 12\vec{a}.\vec{b} = 9|\vec{a}|^2 + |\vec{b}|^2 + 6\vec{a}\cdot\vec{b}$

$8|\vec{b}|^2 + 6|\vec{a}\cdot\vec{b}| - 5|\vec{a}|^2 = 0$

$8|\vec{b}|^2 + 6|\vec{a}|\cdot|\vec{b}|\cos 60 - 5\times 64 = 0$

$8|\vec{b}|^2 + 24|\vec{b}| - 320 = 0$

$|\vec{b}|^2 + 3|\vec{b}| - 40 = 0$

$\left(|\vec{b}| + 8\right)\left(|\vec{b}| - 5\right) = 0$

$|\vec{b}| = 5$, ($|\vec{b}| = -8$ rejected)

82. Answer (2)

$|\vec{a}| = |\vec{b}| = |\vec{c}| = d$ (say)

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

$\vec{a}\times\left((\vec{r}-\vec{b})\times\vec{a}\right) + \vec{b}\times\left((\vec{r}-\vec{c})\times\vec{b}\right) + \vec{c}\times\left((\vec{r}-\vec{a})\times\vec{c}\right) = 0$

$= \sum(a\cdot a)\left(\vec{r}-\vec{b}\right) - \left(\vec{a}\cdot\left(\vec{r}-\vec{b}\right)\right)\vec{a} = 0$

$= \sum d^2\left(\vec{r}-\vec{b}\right) - (\vec{a}-\vec{r})\vec{a} = 0 \quad [\because \ \vec{a}\cdot\vec{b} = \vec{0}]$

$= 3d^2\vec{r} - d^2\left(\vec{a}+\vec{b}+\vec{c}\right) - \left\{(\vec{r}\cdot\vec{a})\vec{a} + (\vec{r}\cdot\vec{b})b + (\vec{r}\cdot\vec{c})\vec{c}\right\} = 0$

$= 3d^2\vec{r} - d^2\left(\vec{a}+\vec{b}+\vec{c}\right) - d^2\vec{r} = 0$

[$\because$ Each term is component of $\vec{r}$]

$2\vec{r} - \left(\vec{a}+\vec{b}+\vec{c}\right) = 0$

$\vec{r} = \dfrac{1}{2}\left(\vec{a}+\vec{b}+\vec{c}\right)$

83. Answer (1494)

Normal of plane containing $\vec{a}$ and $\vec{b}$ is

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 2\\ 1 & 2 & -1\end{vmatrix}=-3\hat{i}+4\hat{j}+5\hat{k}$$

$\vec{v}$ is perpendicular to $(3\hat{i}+2\hat{j}-\hat{k})$ and also $\vec{n}$

$$\vec{v}=\lambda\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & -1\\ -3 & 4 & 5\end{vmatrix}=\lambda[14\hat{i}-12\hat{j}+18\hat{k}]$$

Given

$$\frac{\vec{a}\cdot\vec{v}}{|\vec{a}|}=19\Rightarrow\frac{\lambda((2)(14)+(-12)(-1)+(18)(2))}{3}=19$$

$$\lambda=\frac{3}{4}$$

$$\vec{v}=\frac{3}{4}(14\hat{i}-12\hat{j}+18\hat{k})\Rightarrow 2\vec{v}=3(7\hat{i}-6\hat{j}+9\hat{k})$$

$$|2\vec{v}|^2=1494$$

84. Answer (3)

$\because\quad \hat{b}=\vec{c}+2\left(\vec{c}\times\hat{a}\right)$

$\Rightarrow\quad \hat{b}\cdot\vec{c}=|\vec{c}|^2$ ...(i)

$\therefore\quad \hat{b}-\vec{c}=2\left(\vec{c}\times\vec{a}\right)$

$\Rightarrow\quad |\hat{b}|^2+|\vec{c}|^2-2\,\hat{b}\cdot\vec{c}=4|\vec{c}|^2|\vec{a}|^2\sin^2\frac{\pi}{12}$

$\Rightarrow\quad 1+|\vec{c}|^2-2|\vec{c}|^2=4|\vec{c}|^2\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$

$\Rightarrow\quad 1=|\vec{c}|^2\left(3-\sqrt{3}\right)$

$\Rightarrow\quad 36|\vec{c}|^2=\frac{36}{3-\sqrt{3}}=6\left(3+\sqrt{3}\right)$

85. Answer (3)

$\because\quad \left|\hat{a}+\hat{b}+2(\hat{a}\times\hat{b})\right|=2,\ \theta\in(0,\pi)$

$\Rightarrow\quad |\hat{a}+\hat{b}+2(\hat{a}\times\hat{b})|^2=4.$

$\Rightarrow\quad |\hat{a}|^2+|\hat{b}|^2+4|\hat{a}\times\hat{b}|^2+2\hat{a}\cdot\hat{b}=4.$

$\therefore\quad \cos\theta=\cos2\theta$

$\therefore\quad \theta=\frac{2\pi}{3}$

where θ is angle between $\hat{a}$ and $\hat{b}$.

$\therefore\quad 2|\hat{a}\times\hat{b}|=\sqrt{3}=|\hat{a}-\hat{b}|$

(S1) is correct

And projection of $\hat{a}$ on $(\hat{a}+\hat{b})=\left|\frac{\hat{a}\cdot(\hat{a}+\hat{b})}{|\hat{a}+\hat{b}|}\right|=\frac{1}{2}.$

(S2) is correct.

86. Answer (2)

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1\ \Rightarrow\ \cos^2\alpha=\frac{1}{3}$$

$$\Rightarrow\quad \cos\alpha=\frac{1}{\sqrt{3}}$$

$$\vec{a}=\frac{\lambda}{3}(\hat{i}+\hat{j}+\hat{k}),\ \lambda>0$$

$$\frac{\lambda}{\sqrt{3}}\frac{(\hat{i}+\hat{j}+\hat{k}).(3\hat{i}+4\hat{j})}{\sqrt{3^2+4^2}}=7$$

$$\Rightarrow\quad \frac{\lambda}{\sqrt{3}}(3+4)=7\times5$$

$$\therefore\quad \lambda=5\sqrt{3}$$

$$\vec{a}=5(\hat{i}+\hat{j}+\hat{k})$$

Let $\vec{b}=p\hat{i}+q\hat{j}+r\hat{k}$

$\vec{a}.\vec{b}=0$ and $[\vec{a}\,\vec{b}\,\hat{i}]=0$

$\Rightarrow\quad p+q+r=0$ ...(i)

$$\&\quad \begin{vmatrix}p & q & r\\ 1 & 1 & 1\\ 1 & 0 & 0\end{vmatrix}=0\Rightarrow\begin{array}{l}\boxed{q=r}\\ \boxed{p=-2r}\end{array}$$

$$\vec{b}=-2r\hat{i}+r\hat{j}+r\hat{k}$$

$$\vec{b}=r(-2\hat{i}+\hat{j}+\hat{k})$$

Now $|\vec{a}|=|\vec{b}|$

$$5\sqrt{3}=|r|\sqrt{b}\ \Rightarrow\ |r|=\frac{5}{\sqrt{2}}$$

$\Rightarrow$ Projection of $\vec{b}$ on $3\hat{i}+4\hat{j}=\left|\frac{\vec{b}.(3\hat{i}+4\hat{j})}{\sqrt{3^2+4^2}}\right|$

$$=|r|\frac{(-6+4)}{5}=\left|\frac{-2r}{5}\right|$$

Projection $=\frac{2}{5}\times\frac{5}{\sqrt{2}}=\sqrt{2}$

$\therefore$ 2 is correct

87. Answer (576)

$|(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})|^2+4(\vec{a}\cdot\vec{b})^2$

$\Rightarrow$ $|\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}|^2+4(\vec{a}\cdot\vec{b})^2$

$\Rightarrow$ $|2(\vec{a}\times\vec{b})|^2+4(\vec{a}\cdot\vec{b})^2$

$\Rightarrow$ $4\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}\cdot\vec{b}\right)^2$

$\Rightarrow$ $4|\vec{a}|^2|\vec{b}|^2=4.16.9=576$

88. Answer (14)

Let $\vec{a}=x\hat{i}=y\hat{j}+z\hat{k}$

So, $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ 1 & 1 & \lambda\end{vmatrix}=\hat{i}(\lambda y-z)+\hat{j}(z-\lambda x)+\hat{k}(x-y)$

$\Rightarrow$ $\lambda y-z=13$, $z-\lambda x=-1$, $x-y=-4$

and $x+y+\lambda z=-21$

$\Rightarrow$ clearly, $\lambda=3$, $x=-2$, $y=2$ and $z=-7$

So, $\vec{b}-\vec{a}=3\hat{i}-\hat{j}+10\hat{k}$

and $\vec{b}+\vec{a}=-\hat{i}+3\hat{j}-4\hat{k}$

$\Rightarrow$ $\left(\vec{b}-\vec{a}\right)\cdot\left(\hat{k}-\hat{j}\right)+\left(\vec{b}+\vec{a}\right)\cdot\left(\hat{i}-\hat{k}\right)=11+3=14$

89. Answer (1)

$\because$ $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=3\vec{b}-\vec{c}=\vec{u}$

$\vec{b}\times\left(\vec{c}\times\vec{a}\right)=\vec{c}-2\vec{a}=\vec{v}$

$\vec{c}\times(\vec{b}\times\vec{a})=3\vec{b}-2\vec{a}=\vec{w}$

$\therefore$ $\vec{u}+\vec{v}=\vec{w}$

So vectors $\vec{u},\vec{v}$ and $\vec{w}$ are coplanar, hence their Scalar triple product will be zero.

90. Answer (4)

Let $\vec{v}=\lambda_1\vec{a}+\lambda_2\vec{b}$, where $\lambda_1,\lambda_2\in\mathbb{R}$.

$=(\lambda_1+2\lambda_2)\hat{i}+(\lambda_1-3\lambda_2)\hat{j}+(2\lambda_1+\lambda_2)\hat{k}$

$\because$ Projection of $\vec{v}$ on $\vec{c}$ is $\frac{2}{\sqrt{3}}$.

$\therefore$ $\frac{\lambda_1+2\lambda_2-\lambda_1+3\lambda_2+2\lambda_1+\lambda_2}{\sqrt{3}}=\frac{2}{\sqrt{3}}$

$\therefore$ $\lambda_1+3\lambda_2=1$ ...(i)

and $\vec{v}\cdot\hat{j}=7\Rightarrow\lambda_1-3\lambda_2=7$ ...(ii)

from equation (i) and (ii)

$\lambda_1=4$, $\lambda_2=-1$

$\therefore$ $\vec{v}=2\hat{i}+7\hat{j}+7\hat{k}$

$\therefore$ $\vec{v}\cdot(\hat{i}+\hat{k})=2+7$

$=9$

91. Answer (1)

$\vec{a}=\hat{i}+\hat{j}-\hat{k}$

$\vec{c}=2\hat{i}-3\hat{j}+2\hat{k}$

Now, $\vec{b}\times\vec{c}=\vec{a}$

$\vec{c}\cdot(\vec{b}\times\vec{c})=\vec{c}\cdot\vec{a}$

$\vec{c}\cdot\vec{a}=0$

$\Rightarrow$ $(\hat{i}+\hat{j}-\hat{k})(2\hat{i}-3\hat{j}+2\hat{k})=0$

$=2-3-2=0$

$\Rightarrow$ $-3=0$ (Not possible)

$\Rightarrow$ No possible value of $\vec{b}$ is possible.

92. Answer (4)

$\because$ $\vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$.

$\therefore$ $\frac{1}{2}\left|\vec{a}\times\vec{b}\right|=2\sqrt{2}$

$|\vec{a}||\vec{b}|\sin\theta=4\sqrt{2}$

$\Rightarrow \quad |\vec{b}|\sin\theta = 4\sqrt{2}$ ...(i)

and $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$

$|\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow \quad \boxed{\tan\theta = 1} \quad \therefore \theta = \frac{\pi}{4}$

By (i) $|\vec{b}| = 8$

Now $\vec{c} = 2\sqrt{2}(\vec{a}\times\vec{b}) - 2\vec{b}$

$\Rightarrow \quad \vec{c}\cdot\vec{b} = -2|\vec{b}|^2 = -128$ ...(ii)

and $\vec{c}\cdot\vec{c} = 8|\vec{a}\times\vec{b}|^2 + 4|\vec{b}|^2$

$\Rightarrow \quad |\vec{c}|^2 = 8.32 + 4.64$

$\Rightarrow \quad |\vec{c}| = 16\sqrt{2}$ ...(iii)

From (ii) and (iii)

$|\vec{c}||\vec{b}|\cos\alpha = -128$

$\Rightarrow \quad \cos\alpha = \frac{-1}{\sqrt{2}}$

$\alpha = \frac{3\pi}{4}$

93. Answer (150)

$2C_1 + C_2 + 3C_3 = 5$ ...(i)

$3C_1 + 3C_2 + C_3 = 0$ ...(ii)

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 1 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$= 2(3C_3 - C_2) - 1(3C_3 - C_1) + 3(3C_2 - 3C_1)$

$= 3C_3 + 7C_2 - 8C_1$

$\Rightarrow \quad 8C_1 - 7C_2 - 3C_3 = 0$ ...(iii)

$C_1 = \frac{10}{122}, C_2 = \frac{-85}{122}, C_3 = \frac{225}{122}$

So $122(C_1 + C_2 + C_3) = 150$

94. Answer (4)

$\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$

$$\therefore \quad \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -1 \\ -2 & \alpha & 1 \end{vmatrix} = (2+\alpha)\hat{i} - (\alpha-2)\hat{j} + (\alpha^2+4)\hat{k}$$

Now $|\vec{a}\times\vec{b}| = \sqrt{15(\alpha^2+4)}$

$\Rightarrow \quad (2+\alpha)^2 + (\alpha-2)^2 + (\alpha^2+4)^2 = 15(\alpha^2+4)$

$\Rightarrow \quad \alpha^4 - 5\alpha^2 - 36 = 0$

$\therefore \quad \alpha = \pm 3$

Now, $2|\vec{a}|^2 + (\vec{a}-\vec{b})|\vec{b}|^{-2} = 2.14 - 14 = 14$

95. Answer (3)

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

and $\vec{a}\cdot\left(3\hat{i} - \frac{1}{2}\hat{j} + 2\hat{k}\right) = 0 \Rightarrow 3a_1 + \frac{a_2}{2} + 2a_3 = 0$ ...(i)

and $\vec{a}\times(2\hat{i}+\hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$

$\Rightarrow \quad a_2\hat{i} + (2a_3 - a_1)\hat{j} - 2a_2\hat{k} = 2\hat{i} - 13\hat{j} - 4\hat{k}$

$\therefore \quad a_2 = 2$ ...(ii)

and $a_1 - 2a_3 = 13$ ...(iii)

From eq. (i) and (iii) : $a_1 = 3$ and $a_3 = -5$

$\therefore \quad \vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$

$\therefore$ projection of $\vec{a}$ on $2\hat{i} + 2\hat{j} + \hat{k} = \frac{6+4-5}{3} = \frac{5}{3}$

96. Answer (1)

$\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$

$\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$

Projection of $\vec{a}$ on $\vec{c}$ is

$\frac{\vec{a}\cdot\vec{c}}{|\vec{b}|} = \frac{10}{3}$

$$\frac{\alpha+6+2}{\sqrt{1^2+2^2+(-2)^2}}=\frac{\alpha+8}{3}=\frac{10}{3}$$

$\therefore \quad \boxed{\alpha=2}$

$\vec{b}\times\vec{c}=-6\hat{i}+10\hat{j}+7\hat{k}$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -\beta & 4\\ 1 & 2 & -2\end{vmatrix}=(2\beta-8)\hat{i}+10\hat{j}+(6+\beta)\hat{k}=-6\hat{i}+10\hat{j}+7\hat{k}$$

$2\beta-8=-6 \;\;\&\;\; 6+\beta=7$

$\therefore \quad \boxed{\beta=1}$

$\alpha + \beta = 2 + 1 = 3$

97. Answer (1)

$\overline{AB} \,||\, \overline{AC}$ if

$$\frac{1}{2}=\frac{\alpha-4}{-6}=\frac{1}{2}$$

$\Rightarrow \quad \alpha = 1$

$\vec{a},\vec{b},\vec{c}$ are non-collinear for $\alpha = 2$ (smallest positive integer)

Mid point of $BC=M\left(\frac{5}{2},0,\frac{9}{2}\right)$

$$AM=\sqrt{\frac{9}{4}+16+\frac{9}{4}}=\frac{\sqrt{82}}{2}$$

98. Answer (10*)

Data not correct

$\because \quad \vec{a}\cdot\vec{b}=1-2+3=2 \quad \left\{\text{according to } \vec{a} \text{ and } \vec{b}\right\}$

but given that

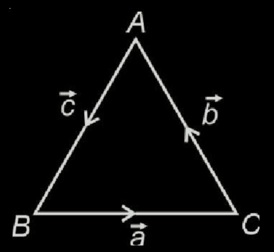
$\vec{a}+\left(\vec{b}\times\vec{c}\right)=\vec{0}$

$\vec{a}=-\left(\vec{b}\times\vec{c}\right)$

$\Rightarrow \quad \vec{a}\perp\vec{b}$ and $\vec{a}\perp\vec{c}$ -

$\therefore \quad \vec{a}\cdot\vec{b}=0$ {Contradicts}

99. Answer (3*)



$\because \quad \vec{a}+\vec{b}+\vec{c}=0 \qquad \ldots$(i)

then $\vec{a}+\vec{c}=-\vec{b}$

then $\left(\vec{a}+\vec{c}\right)\times\vec{b}=-\vec{b}\times\vec{b}$

$\therefore \quad \vec{a}\times\vec{b}+\vec{c}\times\vec{b}=\vec{0} \qquad \ldots$(i)

For $(S1):\left|\vec{a}\times\vec{b}+\vec{c}\times\vec{b}\right|-\left|\vec{c}\right|=6\left(2\sqrt{2}-1\right)$

$\left|\left(\vec{a}+\vec{c}\right)\times\vec{b}\right|-\left|\vec{c}\right|=6\left(2\sqrt{2}-1\right)$

$\left|\vec{c}\right|=6-12\sqrt{2}$ (not possible)

Hence $(S1)$ is not correct

For $(S2)$: from (i) $\vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow \quad \vec{b}\cdot\vec{b}+\vec{c}\cdot\vec{b}=-\vec{a}\cdot\vec{b}$

$\Rightarrow \quad 12+12=-6\sqrt{2}\cdot 2\sqrt{3}\cos(\pi-\angle ACB)$

$\therefore \quad \cos(\angle ACB)=\sqrt{\frac{2}{3}}$

$\therefore \quad \angle ACB=\cos^{-1}\sqrt{\frac{2}{3}}$

$\therefore \quad S(2)$ is correct.

100. Answer (4)

Given : $\vec{a}=(\alpha,1,-1)$ and $\vec{b}=(2,1,-\alpha)$

$$\vec{c}=\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & 1 & -1\\ 2 & 1 & -\alpha\end{vmatrix}$$

$=(-\alpha+1)\hat{i}+\left(\alpha^2-2\right)\hat{j}+(\alpha-2)\hat{k}$

Projection of $\vec{c}$ on $\vec{d}=-\hat{i}+2\hat{j}-2\hat{k}$

$$= \left|\vec{c} \cdot \frac{\vec{d}}{|\vec{d}|}\right| = 30 \text{ \{Given\}}$$

$$\Rightarrow \quad = \left|\frac{\alpha - 1 - 4 + 2\alpha^2 - 2\alpha + 4}{\sqrt{1+4+4}}\right| = 30$$

On solving $\alpha = \frac{-13}{2}$ (Rejected as $\alpha > 0$)

and $\alpha = 7$

101. Answer (*)

$f_a(x) = \tan^{-1} 2x - 3ax + 7$

$\because \quad f_a(x)$ is non-decreasing in $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$

$$\therefore \quad f_a'(x) \geq 0 \quad \Rightarrow \quad \frac{2}{1+4x^2} - 3a \geq 0$$

$$\Rightarrow \quad 3a \leq \frac{2}{1+4x^2}$$

So, $a_{max} = \frac{2}{3}\left(\frac{1}{1 + 4 \times \frac{\pi^2}{36}}\right) = \frac{6}{9+\pi^2} = \bar{a}$

$$\therefore \quad f_{\bar{a}}\left(\frac{\pi}{8}\right) = \tan^{-1}\frac{\pi}{4} - 3\frac{\pi}{8} \cdot \frac{6}{9+\pi^2} + 7$$

102. Answer (4)

$\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$

$\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & \beta \\ 3 & -5 & 4 \end{vmatrix} = -\hat{i} + 9\hat{j} + 12\hat{k}$$

$4 + 5\beta = -1 \Rightarrow \beta = -1$

$-5\alpha - 3 = 12 \Rightarrow \alpha = -3$

$\vec{b} - 2\vec{a} = 3\hat{i} - 5\hat{j} + 4\hat{k} - 2\left(-3\hat{i} + \hat{j} - \hat{k}\right)$

$\vec{b} - 2\vec{a} = 9\hat{i} - 7\hat{j} + 6\hat{k}$

$\vec{b} + \vec{a} = \left(3\hat{i} - 5\hat{j} + 4\hat{k}\right) + \left(-3\hat{i} + \hat{j} - \hat{k}\right)$

$\vec{b} + \vec{a} = -4\hat{j} + 3\hat{k}$

Projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is $= \frac{\left(\vec{b} - 2\vec{a}\right) \cdot \left(\vec{b} + \vec{a}\right)}{\left|\vec{b} + \vec{a}\right|}$

$$= \frac{28+18}{5} = \frac{46}{5}$$

103. Answer (2)

Given, $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$

Also, $\left(\left(\vec{a} \times \vec{b}\right) \times i\right) \cdot \hat{k} = \frac{23}{2}$

$$\Rightarrow \quad \left(\left(\vec{a} \cdot \hat{i}\right)\vec{b} - \left(\vec{b} \cdot \hat{i}\right) \cdot \vec{a}\right) \cdot \hat{k} = \frac{23}{2}$$

$$\Rightarrow \quad \left(2 \cdot \vec{b} - \alpha \cdot \vec{a}\right) \cdot \hat{k} = \frac{23}{2}$$

$$\Rightarrow \quad 2 \cdot 2 - 5\alpha = \frac{23}{2} \Rightarrow \alpha = \frac{-3}{2}$$

Now, $\left|\vec{b} \times 2j\right| = \left|\left(\alpha\hat{i} + \beta\hat{j} + 2\hat{k}\right) \times 2\hat{j}\right|$

$$= \left|2\alpha\hat{k} + 0 - 4\hat{i}\right|$$

$$= \sqrt{4\alpha^2 + 16}$$

$$= \sqrt{4\left(\frac{-3}{2}\right)^2 + 16}$$

$$= 5$$

104. Answer (*)

Given $\vec{a} \times \vec{b} = 4 \cdot \vec{c}$ ...(i)

$\vec{b} \times \vec{c} = 9 \cdot \vec{a}$ ...(ii)

$\vec{c} \times \vec{a} = \alpha \cdot \vec{b}$ ...(iii)

Taking dot products with $\vec{c}, \vec{a}, \vec{b}$ we get

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

Hence (i) $\Rightarrow |\vec{a}| \cdot |\vec{b}| = 4 \cdot |\vec{c}|$ ...(iv)

(ii) $\Rightarrow |\vec{b}| \cdot |\vec{c}| = 9 \cdot |\vec{a}|$ ...(v)

$(iii) \Rightarrow |\vec{c}|\cdot|\vec{a}| = \alpha\cdot|\vec{b}|$ ...(vi)

Multiplying (iv), (v) and (vi)

$\Rightarrow \ |\vec{a}|\cdot|\vec{b}|\cdot|\vec{c}| = 36\alpha$ ...(vii)

Dividing (vii) by (iv) $\Rightarrow |\vec{c}|^2 = 9\alpha \Rightarrow |\vec{c}| = 3\sqrt{\alpha}$

...(viii)

Dividing (vii) by (v) $\Rightarrow |\vec{a}|^2 = 4\alpha \Rightarrow |\vec{a}| = 2\sqrt{\alpha}$

Dividing (viii) by (vi) $\Rightarrow |\vec{b}|^2 = 36 \Rightarrow |\vec{b}| = 6$

Now, as given, $3\sqrt{\alpha} + 2\sqrt{\alpha} + 6 = \frac{1}{36} \Rightarrow \sqrt{\alpha} = \frac{-43}{36}$

105. Answer (3)

Clearily $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar

$$\begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix} \neq 0$$

$\Rightarrow \ (1 + t)(1 + t + 2t) - (1 - t)(1 - t - 2t) + 1(t^2 - t - t - t^2) \neq 0$

$\Rightarrow \ (3t^2 + 4t + 1) - (1 - t)(1 - 3t) - 2t \neq 0$

$\Rightarrow \ (3t^2 + 4t + 1) - (3t^2 - 4t + 1) - 2t \neq 0$

$\Rightarrow \ t \neq 0$

106. Answer (2)

$(x\vec{a} + y\vec{b}).(6y\vec{a} - 18x\vec{b}) = 0$

$\Rightarrow \ \left(6xy|\vec{a}|^2 - 18xy|\vec{b}|^2\right) + \left(6y^2 - 18x^2\right)\vec{a}.\vec{b} = 0$

As given equation is identity

Coefficient of x^2 = coefficient of y^2 = coefficient of $xy = 0$

$\Rightarrow \ |\vec{a}|^2 = 3|\vec{b}|^2 \ \Rightarrow |\vec{b}| = 3\sqrt{3}$

and $\vec{a}.\vec{b} = 0$

$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$= 9.3\sqrt{3}.1 = 27\sqrt{3}$

107. Answer (2)

$\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$

$\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}$

For acute angle $\vec{u}\cdot\vec{v} > 0$

$\Rightarrow \ a(\log_e b)^2 - 12 + 6a(\log_e b) > 0$

$\because \ b > 1$

Let $\log_e b = t \ \Rightarrow \ t > 0$ as $b > 1$

$at^2 + 6at - 12 > 0 \quad \forall\, t > 0$

$\Rightarrow \ a \in \phi$

108. Answer (3)

$|\vec{a}||\vec{b}||\vec{c}| = 14$

$\vec{a}\wedge\vec{b} = \vec{b}\wedge\vec{c} = \vec{c}\wedge\vec{a} = \theta = \frac{2\pi}{3}$

$\vec{a}\cdot\vec{b} = -\frac{1}{2}|\vec{a}||\vec{b}|$

$\vec{b}\cdot\vec{c} = -\frac{1}{2}|\vec{b}||\vec{c}|$

$\vec{c}\cdot\vec{a} = -\frac{1}{2}|\vec{c}||\vec{a}|$

Now,

$(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c}) + (\vec{b}\times\vec{c})\cdot(\vec{c}\times\vec{a}) + (\vec{c}\times\vec{a})\cdot(\vec{a}\times\vec{b})$

$= 168$ ...(i)

$(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c}) - (\vec{a}\cdot\vec{c})|\vec{b}|^2$

$= \frac{1}{4}|\vec{b}|^2|\vec{a}||\vec{c}| + \frac{1}{2}|\vec{a}||\vec{b}|^2|\vec{c}|$

$= \frac{3}{4}|\vec{a}||\vec{b}|^2|\vec{c}|$ ...(ii)

Similarly $(\vec{b}\times\vec{c})\cdot(\vec{c}\times\vec{a}) = \frac{3}{4}|\vec{a}||\vec{b}||\vec{c}|^2$ ...(iii)

$(\vec{c}\times\vec{a})\cdot(\vec{a}\times\vec{b}) = \frac{3}{4}|\vec{a}|^2|\vec{b}||\vec{c}|$ ...(iv)

Substitute (ii), (iii), (iv) in (i)

$$\frac{3}{4}|\vec{a}||\vec{b}||\vec{c}|\left[|\vec{a}|+|\vec{b}|+|\vec{c}|\right]=168$$

$$\frac{3}{4}\times 14\left[|\vec{a}|+|\vec{b}|+|\vec{c}|\right]=168$$

$$|\vec{a}|+|\vec{b}|+|\vec{c}|=16$$

109. Answer (14)

$$\because \quad |\vec{a}+\vec{b}|^2=|\vec{a}|^2+2|\vec{b}|^2$$

or $|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=|\vec{a}|^2+2|\vec{b}|^2$

$$\therefore \quad |\vec{b}|^2=6 \qquad ...(i)$$

Now $|\vec{a}\times\vec{b}|^2=|\vec{a}|^2|\vec{b}|^2-(\vec{a}\cdot\vec{b})^2$

$$75=|\vec{a}|^2\cdot 6-9$$

$$\therefore \quad |\vec{a}|^2=14$$

110. Answer (1)

$\hat{a}\cdot\hat{b}=\frac{1}{\sqrt{2}}$ and $|\vec{a}\times\vec{b}|=\frac{1}{\sqrt{2}}$

$$\frac{(\hat{a}+\hat{b})\cdot(\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b}))}{|\hat{a}+\hat{b}||\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b})|}=\cos\theta$$

$$\Rightarrow \quad \cos\theta=\frac{1+3\hat{a}\hat{b}+2}{|\hat{a}+\hat{b}||\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b})|}$$

$$|\hat{a}+\hat{b}|^2=2+\sqrt{2}$$

$$|\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b})|^2=1+4+4|\hat{a}\times\hat{b}|^2+4\hat{a}\hat{b}$$

$$=5+4\cdot\frac{1}{2}+\frac{4}{\sqrt{2}}=7+2\sqrt{2}$$

So, $\cos^2\theta=\dfrac{\left(3+\frac{3}{\sqrt{2}}\right)^2}{(2+\sqrt{2})(7+2\sqrt{2})}=\dfrac{9\sqrt{2}(5\sqrt{2}+3)}{164}$

$$\Rightarrow \quad 164\cos^2\theta=90+27\sqrt{2}$$

111. Answer (1)

$$\vec{a}=\hat{i}-\hat{j}+2\hat{k}$$

$$\vec{a}\times\vec{b}=2\hat{i}-\hat{k}$$

$$\vec{a}\cdot\vec{b}=3$$

$$|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=|\vec{a}|^2\cdot|\vec{b}|^2$$

$$\Rightarrow 5+9=6|\vec{b}|^2$$

$$\Rightarrow |b|^2=\frac{7}{3}$$

$$|\vec{a}-\vec{b}|=\sqrt{|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}}=\sqrt{\frac{7}{3}}$$

projection of $\vec{b}$ on $\vec{a}-\vec{b}=\dfrac{\vec{b}\cdot(\vec{a}-\vec{b})}{|\vec{a}-\vec{b}|}$

$$=\frac{\vec{b}\cdot\vec{a}-|\vec{b}|^2}{|\vec{a}-\vec{b}|}=\frac{3-\frac{7}{3}}{\sqrt{\frac{7}{3}}}$$

$$=\frac{2}{\sqrt{21}}$$

112. Answer (1)

$\vec{a}=3\hat{i}+\hat{j}$ & $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$

$$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\vec{b}+\lambda\vec{c}$$

If $\vec{b}$ & $\vec{c}$ are non-parallel

then $\vec{a}\cdot\vec{c}=1$ & $\vec{a}\cdot\vec{b}=-\lambda$

but $\vec{a}\cdot\vec{b}=5\Rightarrow\lambda=-5$

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