Chapter 4

Permutations and Combinations

1. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is **[AIEEE-2009]**
   (1) At least 500 but less than 750
   (2) At least 750 but less than 1000
   (3) At least 1000
   (4) Less than 500

2. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is **[AIEEE-2010]**
   (1) 3 (2) 36
   (3) 66 (4) 108

3. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is **[AIEEE-2012]**
   (1) 629 (2) 630
   (3) 879 (4) 880

4. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is **[JEE (Main)-2013]**
   (1) 7 (2) 5
   (3) 10 (4) 8

5. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is **[JEE (Main)-2015]**
   (1) 216 (2) 192
   (3) 120 (4) 72

6. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is **[JEE (Main)-2016]**
   (1) 59^{th} (2) 52^{nd}
   (3) 58^{th} (4) 46^{th}

7. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is **[JEE (Main)-2017]**
   (1) 468 (2) 469
   (3) 484 (4) 485

8. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is **[JEE (Main)-2018]**
   (1) At least 1000
   (2) Less than 500
   (3) At least 500 but less than 750
   (4) At least 750 but less than 1000

9. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is **[JEE (Main)-2019]**
   (1) 200 (2) 350
   (3) 500 (4) 300

10. The number of natural numbers less than 7,000 which can be formed by using the digits 0, 1, 3, 7, 9 (repetition of digits allowed) is equal to **[JEE (Main)-2019]**
    (1) 374 (2) 375
    (3) 250 (4) 372

11. If $\sum_{r=0}^{25}\left\{{}^{50}C_r \cdot {}^{50-r}C_{25-r}\right\} = K\left({}^{50}C_{25}\right)$, then K is equal to **[JEE (Main)-2019]**
    (1) $2^{25} - 1$ (2) $(25)^2$
    (3) 2^{25} (4) 2^{24}

12. The value of r for which ${}^{20}C_r\, {}^{20}C_0 + {}^{20}C_{r-1}\, {}^{20}C_1 + {}^{20}C_{r-2}\, {}^{20}C_2 + ... + {}^{20}C_0\, {}^{20}C_r$ is maximum, is

[JEE (Main)-2019]

(1) 10 (2) 20

(3) 15 (4) 11

13. Consider three boxes, each containing 10 balls labelled 1, 2, ...,10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is

[JEE (Main)-2019]

(1) 240 (2) 120

(3) 164 (4) 82

14. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is

[JEE (Main)-2019]

(1) 9 (2) 7

(3) 11 (4) 12

15. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be

[JEE (Main)-2019]

(1) 12 (2) 9

(3) 14 (4) 11

16. The sum of the series $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + ... + 62.{}^{20}C_{20}$ is equal to

[JEE (Main)-2019]

(1) 2^{23} (2) 2^{25}

(3) 2^{24} (4) 2^{26}

17. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is **[JEE (Main)-2019]**

(1) 180 (2) 175

(3) 162 (4) 160

18. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is :

[JEE (Main)-2019]

(1) 360 (2) 306

(3) 288 (4) 310

19. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then

[JEE (Main)-2019]

(1) $m = n = 68$ (2) $m + n = 68$

(3) $m = n = 78$ (4) $n = m - 8$

20. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is

[JEE (Main)-2019]

(1) 157 (2) 225

(3) 262 (4) 190

21. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is

[JEE (Main)-2019]

(1) 72 (2) 48

(3) 60 (4) 36

22. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number of beams is

[JEE (Main)-2019]

(1) 210 (2) 180

(3) 170 (4) 190

23. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is **[JEE (Main)-2019]**

(1) $\frac{3}{20}$ (2) $\frac{1}{5}$

(3) $\frac{3}{10}$ (4) $\frac{1}{10}$

24. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is **[JEE (Main)-2019]**

(1) $2^{20} + 1$ (2) 2^{21}

(3) $2^{20} - 1$ (4) 2^{20}

25. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to **[JEE (Main)-2019]**

(1) 24 (2) 27

(3) 25 (4) 28

26. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is **[JEE (Main)-2020]**

(1) $\frac{1}{2}(6!)$ (2) $\frac{5}{2}(6!)$

(3) 5^6 (4) 6!

27. If the number of five digit numbers with distinct digits and 2 at the 10^{th} place is $336k$, then k is equal to **[JEE (Main)-2020]**

(1) 8 (2) 6

(3) 7 (4) 4

28. Let $n > 2$ be an integer. Suppose that there are n Metro stations in a city located along a circular path. Each pair of stations is connected by a straight track only. Further, each pair of nearest stations is connected by blue line, whereas all remaining pairs of stations are connected by red line. If the number of red lines is 99 times the number of blue lines, then the value of n is **[JEE (Main)-2020]**

(1) 199 (2) 201

(3) 101 (4) 200

29. The value of $(2\cdot{}^1P_0 - 3\cdot{}^2P_1 + 4\cdot{}^3P_2 - \ldots$ up to 51^{th} term) + (1! – 2! + 3! – ... up to 51^{th} term) is equal to **[JEE (Main)-2020]**

(1) 1 (2) 1 + (52)!

(3) 1 – 51(51)! (4) 1 + (51)!

30. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is **[JEE (Main)-2020]**

(1) 2250 (2) 3000

(3) 1500 (4) 2255

31. Two families with three members each and one family with four members are to be seated in a row. In how many ways can they be seated so that the same family members are not separated? **[JEE (Main)-2020]**

(1) 2! 3! 4! (2) $(3!)^3\cdot(4!)$

(3) $(3!)^2\cdot(4!)$ (4) $3!\,(4!)^3$

32. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'EXAMINATION' is _________. **[JEE (Main)-2020]**

33. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is _______ **[JEE (Main)-2020]**

34. The total number of 3-digit numbers, whose sum of digits is 10, is _______. **[JEE (Main)-2020]**

35. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is _______. **[JEE (Main)-2020]**

36. The number of words, with or without meaning, that can be formed by taking 4 letters at a time from the letters of the word 'SYLLABUS' such that two letters are distinct and two letters are alike, is _______. **[JEE (Main)-2020]**

37. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is ________ . **[JEE (Main)-2020]**

38. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is : **[JEE (Main)-2021]**

(1) 560 (2) 1050

(3) 1625 (4) 575

39. Let M be any 3 × 3 matrix with entries from the set (0, 1, 2). The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven is _______. **[JEE (Main)-2021]**

40. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is ________. **[JEE (Main)-2021]**

41. The total number of positive integral solutions (x, y, z) such that xyz = 24 is : **[JEE (Main)-2021]**

(1) 36 (2) 30

(3) 45 (4) 24

42. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is ________. **[JEE (Main)-2021]**

43. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is : **[JEE (Main)-2021]**

(1) 77 (2) 42

(3) 82 (4) 35

44. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that y + z = 5 and $y^{-1} + z^{-1} = \frac{5}{6}, y > z$. Then the number of odd divisors of n, including 1, is : **[JEE (Main)-2021]**

(1) 12 (2) 6x

(3) 11 (4) 6

45. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is ______. **[JEE (Main)-2021]**

46. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to: **[JEE (Main)-2021]**

(1) 1890 (2) 717

(3) 795 (4) 1173

47. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to **[JEE (Main)-2021]**

(1) 5 (2) 6

(3) 2 (4) 4

48. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to : **[JEE (Main)-2021]**

(1) 240 (2) 364

(3) 360 (4) 333

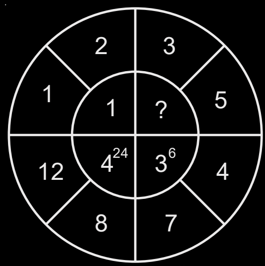
49. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is : **[JEE (Main)-2021]**

(1) 122234 (2) 122664

(3) 22264 (4) 26664

50. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is ______. **[JEE (Main)-2021]**

51. The missing value in the following figure is ______. **[JEE (Main)-2021]**



52. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is ________. **[JEE (Main)-2021]**

53. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to _______. **[JEE (Main)-2021]**

54. Let $M = \left\{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d, \in \{\pm 3, \pm 2, \pm 1, 0\}\right\}$.

Define $f : M \to Z$, as $f(A) = \det(A)$, for all $A \in M$, where Z is set of all integers. Then the number of $A \in M$ such that $f(A) = 15$ is equal to _______. **[JEE (Main)-2021]**

55. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of r is equal to **[JEE (Main)-2021]**

(1) 4 (2) 3

(3) 2 (4) 1

56. Let S = {1, 2, 3, 4, 5, 6, 7}. Then the number of possible functions $f : S \to S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to _______. **[JEE (Main)-2021]**

57. The point P (a, b) undergoes the following three transformations successively :

(a) Reflection about the line $y = x$.

(b) Translation through 2 units along the positive direction of x-axis.

(c) Rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

It the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of 2a + b is equal to

[JEE (Main)-2021]

(1) 7 (2) 9

(3) 5 (4) 13

58. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is______ **[JEE (Main)-2021]**

59. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is ________. **[JEE (Main)-2021]**

60. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is _______. **[JEE (Main)-2021]**

61. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is ______.

[JEE (Main)-2021]

62. Let P_1, P_2 ...., P_{15} be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \neq 15$, is

[JEE (Main)-2021]

(1) 455 (2) 12

(3) 419 (4) 443

63. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is _____.

[JEE (Main)-2021]

64. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is **[JEE (Main)-2021]**

(1) 219 (2) 256

(3) 275 (4) 510

65. Let n be a non-negative integer. Then the number of divisors of the form "4n + 1" of the number $(10)^{10} \cdot (11)^{11} \cdot (13)^{13}$ is equal to _______.

[JEE (Main)-2021]

67. In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct. There are 3 marks for each correct answer, –2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is _____.

[JEE (Main)-2022]

68. The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is _____.

[JEE (Main)-2022]

69. The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is ________.

[JEE (Main)-2022]

70. The total number of three-digit numbers, with one digit repeated exactly two times, is ______.

[JEE (Main)-2022]

71. There are ten boys B_1, B_2, ..., B_{10} and five girls G_1, G_2, ..., G_5 in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is ________.

[JEE (Main)-2022]

72. The total number of 3-digit numbers, whose greatest common divisor with 36 is 2, is _______-.

[JEE (Main)-2022]

73. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is _________.

[JEE (Main)-2022]

74. The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is

(1) 36 (2) 48

(3) 60 (4) 72

[JEE (Main)-2022]

75. The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to:

(1) 205 (2) 615

(3) 510 (4) 430

[JEE (Main)-2022]

76. Let $b_1b_2b_3b_4$ be a 4-element permutation with $b_i \in \{1, 2, 3, \ldots, 100\}$ for $1 \le i \le 4$ and b_i ‘“ b_j for i ‘“ j, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations $b_1b_2b_3b_4$ is equal to _______.

[JEE (Main)-2022]

77. The total number of four digit numbers such that each of first three digits is divisible by the last digit, is equal to _______.

[JEE (Main)-2022]

78. The total number of functions,

$f : \{1, 2, 3, 4\} \to \{1, 2, 3, 4, 5, 6\}$

such that $f(1) + f(2) = f(3)$, is equal to

[JEE (Main)-2022]

(1) 60 (2) 90

(3) 108 (4) 126

79. The letters of the work 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is ______.

[JEE (Main)-2022]

80. The number of 5-digit natural numbers, such that the product of their digits is 36, is _________.

[JEE (Main)-2022]

81. Let S be the set of all passwords which are six to eight characters long, where each character is either an alphabet from $\{A, B, C, D, E\}$ or a number from $\{1, 2, 3, 4, 5\}$ with the repetition of characters allowed. If the number of passwords in S whose at least one character is a number from $\{1, 2, 3, 4, 5\}$ is $\alpha \times 5^6$, then α is equal to _______.

[JEE (Main)-2022]

82. A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then $b + 3g$ is equal to ______.

[JEE (Main)-2022]

83. The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is _______. **[JEE (Main)-2022]**

84. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1, 2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is ______________. **[JEE (Main)-2022]**

85. Let $S = \{4, 6, 9\}$ and $T = \{9, 10, 11, \ldots, 1000\}$. If $A = \{a_1 + a_2 + \ldots + a_k : k \in \mathbf{N}, a_1, a_2, a_3, \ldots, a_k \in S\}$, then the sum of all the elements in the set $T - A$ is equal to ____________. [JEE (Main)-2022]

❑ ❑ ❑

Chapter 4

Permutations and Combinations

1. Answer (3)

   The number of ways in which 4 novels can be selected = ${}^6C_4$ = 15

   The number of ways in which 1 dictionary can be selected = ${}^3C_1$ = 3

   4 novels can be arranged in 4! ways.

   $\therefore$ The total number of ways = 15 × 4! × 3 = 15 × 24 × 3 = 1080.

2. Answer (4)

   Two balls from urn A and two balls from urn B can be selected in ${}^3C_2$ × ${}^9C_2$ ways = 3 × 36 = 108

3. Answer (3)

   No. of ways = (11 × 10 × 8) – 1

4. Answer (2)

   $${}^{n+1}C_3 - {}^nC_3 = 10$$

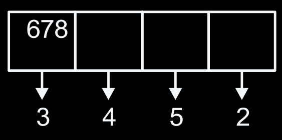
   $$\Rightarrow {}^nC_2 = 10$$

   $\Rightarrow$ n = 5

5. Answer (2)

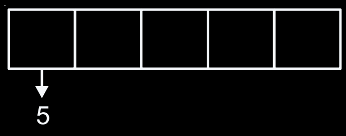
   4 digit numbers

   3, 5, 6, 7, 8



   = 72

   5 digit numbers



   5 × 4 × 3 × 2 × 1 = 120

   Total number of integers = 72 + 120 = 192

6. Answer (3)

   Words starting with $A = \frac{4!}{2!} = 12$

   Words starting with L = 4! = 24

   Words starting with $M = \frac{4!}{2!} = 12$

   Words starting with $SA = \frac{3!}{2!} = 3$

   Words starting with SL = 3! = 6

   Next words is SMALL

   $\therefore$ Rank = 12 + 24 + 12 + 3 + 6 + 1 = 58

7. Answer (4)

X(4 L 3 G)	Y(3 L 4 G)
3 L 0 G	0 L 3 G
2 L 1 G	1 L 2 G
1 L 2 G	2 L 1 G
0 L 3 G	3 L 0 G

   Required number of ways

   $$= {}^4C_3 \cdot {}^4C_3 + \left({}^4C_2 \cdot {}^3C_1\right)^2 + \left({}^4C_1 \cdot {}^3C_2\right)^2 + \left({}^3C_3\right)^2$$

   = 16 + 324 + 144 + 1

   = 485

8. Answer (1)

   Number of ways of selecting 4 novels from 6 novels = ${}^6C_4$

   Number of ways of selecting 1 dictionary from 3 dictionaries = ${}^3C_1$

   Required arrangements = ${}^6C_4$ × ${}^3C_1$ × 4! = 1080

   $\Rightarrow$ Atleast 1000

9. Answer (4)

   Firstly select 2 girls by ${}^5C_2$ ways.

   3 boys can be selected in 3 ways.

   (i) Selection of A and selection of any 2 other boys (except B) in ${}^5C_2$ ways

(ii) Selection of B and selection of any 2 two other boys (except A) in ${}^5C_2$ ways

(iii) Selection of 3 boys (except A and B) in ${}^{15}C_3$ ways

$\Rightarrow$ Number of ways $= {}^5C_2\,({}^5C_2 + {}^5C_2 + {}^5C_3)$

$= 300$

10. Answer (1)

Number of numbers with '1' digit = 4 = 4

Number of numbers with '2' digits = 4 × 5 = 20

Number of numbers with '3' digits = 4 × 5 × 5

= 100

Number of numbers with '4' digits = 2 × 5 × 5 × 5

= 250

Total number of numbers = 4 + 20 + 100 + 250

= 374

11. Answer (3)

$$\sum_{r=0}^{25}\left({}^{50}C_r\cdot{}^{50-r}C_{25-r}\right)=\sum_{r=0}^{25}\left(\frac{\underline{|50}}{\underline{|50-r}\,\underline{|r}}\;\frac{\underline{|50-r}}{\underline{|25}\,\underline{|25-r}}\right)$$

$$=\sum_{r=0}^{25}\left(\frac{\underline{|50}}{\underline{|25}}\times\frac{1}{\underline{|25}}\times\left(\frac{\underline{|25}}{\underline{|25-r}\,\underline{|r}}\right)\right)$$

$$={}^{50}C_{25}\sum_{r=0}^{25}{}^{25}C_r={}^{50}C_{25}(2^{25})$$

$\Rightarrow$ $K = 2^{25}$

$\Rightarrow$ Option (3) is correct.

12. Answer (2)

${}^{20}C_r\;{}^{20}C_0 + {}^{20}C_{r-1}\;{}^{20}C_1 + {}^{20}C_{r-2}\;{}^{20}C_2 + \ldots + {}^{20}C_0\cdot\;{}^{20}C_r$

For maximum value of above expression r should be equal to 20.

as ${}^{20}C_{20}\cdot{}^{20}C_0 + {}^{20}C_{19}\cdot{}^{20}C_1 + \ldots + {}^{20}C_{20}\cdot{}^{20}C_0$

$$=\left({}^{20}C_0\right)^2+\left({}^{20}C_1\right)^2+\cdots+\left({}^{20}C_{20}\right)^2={}^{40}C_{20}.$$

Which is maximum

So $r = 20$

13. Answer (2)

Collecting different labels of balls drawn = 10 × 9 × 8

Now, arrangement is not required so

$$\frac{10\times9\times8}{3!}=120$$

14. Answer (4)

${}^mC_2 \times 2 = {}^mC_1\cdot{}^2C_1\times 2 + 84$

$m(m-1) = 4m + 84$

$m^2 - 5m - 84 = 0$

$m^2 - 12m - 7m - 84 = 0$

$m(m-12) + 7(m-12) = 0$

$m = 12, \quad m = -7$

$\because\; m > 0$

$m = 12$

15. Answer (3)

$$2\,{}^nC_5={}^nC_4+{}^nC_6$$

$$2=\frac{{}^nC_4}{{}^nC_5}+\frac{{}^nC_6}{{}^nC_5}$$

$$2=\frac{5}{n-4}+\frac{n-5}{6}$$

$\Rightarrow$ $12(n-4) = 30 + n^2 - 9n + 20$

$\Rightarrow$ $n^2 - 21n + 98 = 0$

$(n-7)(n-14) = 0$

$(n-7)(n-14) = 0$

$n = 7,\ n = 14$

16. Answer (2)

$2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + \ldots\ldots\ 62.{}^{20}C_{20}$

$$=\sum_{r=0}^{20}(3r+2)\,{}^{20}C_r$$

$$=3\sum_{r=0}^{20}r.\,{}^{20}C_r+2\sum_{r=0}^{20}{}^{20}C_r$$

$$=60\sum_{r=1}^{20}{}^{19}C_{r-1}+2\sum_{r=0}^{20}{}^{20}C_r$$

$= 60 \times 2^{19} + 2 \times 2^{20}$

$= 2^{21}\,[15 + 1] = 2^{25}$

17. Answer (1)

There are total 9 digits; out of which only 3 digits are odd.

$$\_\ \uparrow\ \_\ \uparrow\ \_\ \uparrow\ \_\ \uparrow\ \_$$

Number of ways to arrange odd digits first

$$={}^4C_3\cdot\frac{\underline{|3}}{\underline{|2}}$$

Total number of 9 digit numbers

$$=\left({}^4C_3\cdot\frac{\underline{|3}}{\underline{|2}}\right)\cdot\frac{\underline{|6}}{\underline{|2}\,\underline{|4}}$$

$= 180$

18. Answer (4)

0, 1, 2, 3, 4, 5

$\boxed{5}\boxed{}\boxed{}\boxed{} = 6 \times 6 \times 6 = 216$

6 6 6

$\boxed{4}\boxed{5}\boxed{}\boxed{} = 6 \times 6 = 36$

6 6

$\boxed{4}\boxed{4}\boxed{}\boxed{} = 6 \times 6 = 36$

6 6

$\boxed{4}\boxed{3}\boxed{}\boxed{} = 3 \times 6 = 18$

(5/4/3) 3 6

$\boxed{4}\boxed{3}\boxed{2}\boxed{} = 4$

4

$\Rightarrow$ Required numbers = 216 + 36 + 36 + 18 + 4 = 310

19. Answer (3)

Here, $m = {}^8C_6 \cdot {}^5C_5 + {}^8C_7 \cdot {}^5C_4 + {}^8C_8 \cdot {}^5C_3 = 78$

$n = {}^5C_3 \cdot {}^8C_8 + {}^5C_4 \cdot {}^8C_7 + {}^5C_5 \cdot {}^8C_6 = 78$

So, $m = n = 78$

20. Answer (4)

Balls used in equilateral triangle $= \dfrac{n(n+1)}{2}$

Here, side of equilateral triangle has n-balls.

No. of balls in each side of square is = $(n - 2)$

Given $\dfrac{n(n+1)}{2} + 99 = (n-2)^2$

$\Rightarrow$ $n^2 + n + 198 = 2n^2 - 8n + 8$

$\Rightarrow$ $n^2 - 9n - 190 = 0$

$\Rightarrow$ $n^2 - 19n + 10n - 190 = 0$

$\Rightarrow$ $(n - 19)(n + 10) = 0$

$\Rightarrow$ $n = 19$

Balls used to form triangle

$$= \frac{n(n+1)}{2} = \frac{19 \times 20}{2} = 190$$

21. Answer (3)

$\boxed{a_1}\boxed{a_2}\boxed{a_3}\boxed{a_4}\boxed{a_5}\boxed{a_6}$ digit 0, 1, 2, 5, 7, 9

$(a_1 + a_3 + a_5) - (a_2 + a_4 + a_6) = 11\text{ K}$

so (1, 2, 9) (0, 5, 7)

Now number of ways to arranging them

$= 3! \times 3! + 3! \times 2 \times 2$

$= 6 \times 6 + 6 \times 4$

$= 6 \times 10 = 60$

22. Answer (3)

Required number of beams

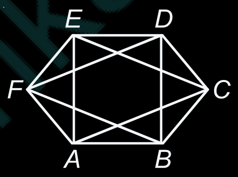
$= {}^{20}C_2 - 20$

$= 190 - 20$

$= 170$

23. Answer (4)

Only two equilateral triangles are possible i.e. ΔAEC and ΔBDF.



Hence, required probability $= \dfrac{2}{{}^6C_3} = \dfrac{1}{10}$

24. Answer (4)

Number of ways of selecting 10 objects

= (10I, 0D) or (9I, 1D) or (8I, 1D) or ... (0I, 10D)

where D signifies distinct object and I indicates identical object

$= 1 + {}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10}$

$= \dfrac{2^{21}}{2} = 2^{20}$

25. Answer (3)

Number of ways of selecting three persons such that there is atleast one boy and atleast one girl in the selected persons

$= {}^{n+5}C_3 - {}^nC_3 - {}^5C_3 = 1750$

$\Rightarrow$ $\dfrac{(n+5)(n+4)(n+3)}{6} - \dfrac{n(n-1)(n-2)}{6} = 1760$

$\Rightarrow$ $n^2 + 3n - 700 = 0$

$\Rightarrow$ $n = -28$ (rejected) or $n = 25$

26. Answer (2)

Exactly 1 digit will repeat which can be selected in ${}^5C_1$ ways

$\therefore$ Total number of ways = ${}^5C_1 . \dfrac{6!}{2!} = \dfrac{5}{2}(6!)$

27. Answer (1)

$\underline{8}$ $\_\_$ $\_\_$ $\underline{1}$ $\_\_$

except '0' and '2' only '2'

There are eight options on first place (except the digits 0 and 2) and only one option at fourth place (digit '2').

Remaining three places can be occupied by any three digits out of 1, 3, 4, 5, 6, 7, 8 and 9.

Number of such numbers = 8 × 8 × 7 × 6 = 336 k

$\Rightarrow k = 8$

28. Answer (2)

Number of two consecutive stations = n

Number of two non-consecutive stations = ${}^nC_2 - n$

Now, According to the question,

$\Rightarrow {}^nC_2 - n = 99n$

$\Rightarrow \dfrac{n(n-1)}{\underline{|2}} - 100n = 0$

$\Rightarrow n - 1 - 200 = 0$

$\Rightarrow \boxed{n = 201}$

29. Answer (2)

$\because (r+1)\cdot {}^rP_{r-1} = (r+1)\cdot\dfrac{\underline{|r}}{\underline{|1}} = \underline{|r+1}$

So $\left(2\cdot {}^1P_0 - 3\cdot {}^2P_1 + \text{......51 terms}\right)$ +

$(\underline{|1} - \underline{|2} + \underline{|3} - \text{........upto 51 terms})$

= $[\underline{|2} - \underline{|3} + \underline{|4} - \text{......} + \underline{|52}] + [\underline{|1} - \underline{|2} + \underline{|3} - \text{.....} + \underline{|51}]$

= $\underline{|52} + \underline{|1} = \underline{|52} + 1$

30. Answer (1)

Each section has 5 questions.

$\therefore$ Total number of selection of 5 questions

= 3 × ${}^5C_1$ × ${}^5C_1$ × ${}^5C_3$ + 3 × ${}^5C_1$ × ${}^5C_2$ × ${}^5C_2$

= 3 × 5 × 5 × 10 + 3 × 5 × 10 × 10

= 750 + 1500

= 2250

31. Answer (2)

No. of arrangement $= (\underline{|3} \times \underline{|3} \times \underline{|4}) \times \underline{|3} = (\underline{|3})^3\, \underline{|4}$

32. Answer (2454)

EXAMINATION has letter distribution as follows

$\overset{8}{2A}, \overset{7}{2N}, \overset{6}{2I}, \overset{5}{E}, \overset{4}{X}, \overset{3}{M}, \overset{2}{T}, \overset{1}{O}$

Case-I, When all letters are different

$\Rightarrow {}^8C_4 \times \underline{|4} = 1680$

Case-II, Two are same and two are different

$\Rightarrow {}^3C_1 \times {}^7C_2 \times \dfrac{\underline{|4}}{\underline{|2}}$

Case-III, Two same of one kind and two same of other kind

$\Rightarrow {}^3C_2 \times \dfrac{\underline{|4}}{\underline{|2} \times \underline{|2}}$

$\therefore$ Total ways = 1680 + 756 + 18 = 2454

33. Answer (309)

EHMORT in alphabetical order

E .... 5!

H .... 5!

$\boxed{M}$ **E......................... 4!**

$\boxed{M}$ **H......................... 4!**

$\boxed{M}$ $\boxed{O}$ **E.................... 3!**

$\boxed{M}$ $\boxed{O}$ **H.................... 3!**

$\boxed{M}$ $\boxed{O}$ **R.................... 3!**

$\boxed{M}$ $\boxed{O}$ $\boxed{T}$ **E................ 2!**

$\boxed{M}$ $\boxed{O}$ $\boxed{T}$ $\boxed{H}$ $\boxed{E}$ $\boxed{R}$ **....... 1**

Rank = 2 × 5! + 2 × 4! + 3 × 3! + 2! + 1

= 309

34. Answer (54)

$x + y + z = 10$, $x \geq 1$, $y \geq 0$, $z \geq 0$

Let $x - 1 = x'$

$x' + y + z = 9$, x', y, $z \geq 0$

Number of solutions are ${}^{9+3-1}C_2 = {}^{11}C_2 = 55$

But for $x' = 9 \Rightarrow x = 10$ which is not possible.

$\therefore$ Total required numbers = 55 – 1 = 54

35. Answer (135)

Select any 4 questions in ${}^6C_4$ ways which are correct

Number of ways of answering wrong question = 3

$\therefore$ Required number of ways = ${}^6C_4 \times 3^2 = 135$

36. Answer (240)

LLSSYABU

For two alike and two distinct letters, select any one pair from LL, SS in ${}^2C_1$ ways

Now from rest, select any 2 in ${}^5C_2$ ways and they can be arranged in $\frac{4!}{2!}$ ways

$\therefore$ Required number of ways = ${}^2C_1 \times {}^5C_2 \times \frac{4!}{2!}$

= 240

37. Answer (120)

For vowels not together

1st arrange L,T,T, R in $\frac{4!}{2!}$ ways

Then put both E in 5 gaps formed in ${}^5C_2$ ways

$\therefore$ No. of ways $= \frac{4!}{2!} . {}^5C_2 = 120$

38. Answer (3)

Indians = 6, Foreigners = 8

According to questions

The no. of ways to form the committee are

(2I, 4F) or (3I, 6F) or (4I, 8F)

$\Rightarrow$ ${}^6C_2 \times {}^8C_4 + {}^6C_3 \times {}^8C_6 + {}^6C_4 \times {}^8C_8$

= 15 × 70 + 20 × 28 + 15 × 1

= 1625

39. Answer (540)

Let $\{a_{ij}\}_{3\times3}$

$$T_r\left(M^T \cdot M\right) = \sum_{i=1}^{3} \sum_{j=1}^{3} a_{ij}^2 = 7$$

So there will be two cases.

Case I : Any seven a_{ij}s are 1 and remaining two elements are zero.

Number of such matrices $M = \frac{\lfloor 9}{\lfloor 7 \lfloor 2} = 36$

Case II : Any one elements is 2, any three elements are 1 and remaining elements are 0.

Number of such matrices = $\frac{\lfloor 9}{\lfloor 1 \lfloor 3 \lfloor 5} = 504$

Total number of possible matrices M = 540.

40. Answer (31650)

Number of possible ways when

(i) There is one student in group C

$= {}^{10}C_1 \cdot \left(2^9 - 2\right) = 5100$

(ii) There are two students in group C

$= {}^{10}C_2 \cdot \left(2^8 - 2\right) = 11430$

(iii) There are three students in group C

$= {}^{10}C_3 \cdot \left(2^7 - 2\right) = 15120$

Total number of ways = 31650

41. Answer (2)

Given $xyz = 24 = 2^3 \times 3$

So total number of positive integral solutions (x, y, z)

$= {}^{3+3-1}C_{3-1} \times {}^{1+3-1}C_{3-1}$

$= {}^5C_2 \times {}^3C_2$

= 10 × 3

= 30

42. Answer (32)

The numbers are lying between 100 and 1000 then each number is of three digits.

The possible combination of 3 digits numbers are

1, 2, 3; 1, 2, 4; 1, 2, 5; 1, 3, 4; 1, 3, 5; 1, 4, 5; 2, 3, 4; 2, 3, 5; 2, 4, 5; and 3, 4, 5.

The numbers which are divisible by 3 are 1, 2, 3; 3, 4, 5; 1, 3, 5 and 2, 3, 4.

$\therefore$ Total number of numbers = 4 × 3! = 24

The number divisible by 5 are 1, 2, 5; 2, 3, 5; 1, 4, 5 and 2, 4, 5.

$\therefore$ Number divisible by 5 = 4 × 2! = 8

$\therefore$ Total required number = 24 + 8 = 32

43. Answer (1)

Combination of digits

3, 2, 1, 1, 1, 1, 1 $\rightarrow \frac{7!}{5!} = 42$

2, 2, 2, 1, 1, 1, 1 $\rightarrow \frac{7!}{4!\,3!} = 35$

Total = 42 + 35 = 77

44. Answer (1)

$y + z = 5$ ...(i)

$$\frac{1}{y}+\frac{1}{z}=\frac{y+z}{yz}=\frac{5}{6} \Rightarrow yz=6 \quad \text{...(ii)}$$

Equation with y and z as roots is

$x^2 - 5x + 6 = 0$

$x = 2, 3,$ $\quad y = 3, z = 2\ (y > z)$

$n = 2^x \cdot 3^3 \cdot 5^2$

For odd divisors $x = 1$ only

No. of odd divisors = 1 × 4 × 3 = 12

45. Answer (1000)

Let A denotes a set of number divisible by 3.

B denotes a set of number divisible by 2.

and C denotes a set of number divisible by 9.

Required number of numbers

$= n(A) - n(A \cap B) - n(c) + n(A \cap B \cap C)$

= 3000 – 1500 – 1000 + 500

= 1000

46. Answer (2)

Number of triangles = 5 × 6 × 7 + 6 × 7 × 9 + 7 × 9 × 5 + 9 × 5 × 6

= 210 + 378 + 315 + 270

$\alpha = 1173$

$\beta = 5 \times 6 \times 7 \times 9 = 1890$

$\beta - \alpha = 717$

47. Answer (4)

Total matches of boys can be arranged in 7 × 4 = 28 ways

Total matches of girls can be arranged in n × 6 = 6n ways

Given 28 + 6n = 52

n = 4.

48. Answer (4)

Total number of triangles

$= 14C_3 - {}^3C_3 - {}^5C_3 - {}^6C_3$

= 364 – 31 = 333

49. Answer (4)

Digits to be used 1, 2, 2, 3

Total contribution of 3 →

(3 + 30 + 300 + 3000) = 9999

Similarly total contribution of 1 →

(1 + 10 + 100 + 1000)3 = 3333

And Total contribution of 2 →

(2 + 20 + 200 + 2000)6 = 13332

∴ Sum of number = 26664

50. Answer (300)

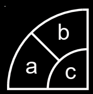
In single digit numbers = 1

In double digit numbers = 10 + 9 = 19

In triple digit numbers = 100 + 90 + 90 = 280

Total = 300 times

51. Answer (04)

In every 

Where $c = |a - b|^{[a]\,[b]}$

Where [ ] is g. i.f.

Hence unknown is $2^2 = 4$

52. Answer (777)

There will be total three cases.

(i) 4 Bowlers + 5 Batsmen + 2 WK

No. of ways = ${}^6C_4 . {}^7C_5 . {}^2C_2 = 315$

(ii) 4 Bowlers + 6 Batsmen + 1 WK

No. of ways = ${}^6C_4 . {}^7C_6 . {}^2C_1 = 210$

(iii) 5 Bowlers + 5 Batsmen + 1WK

No. of ways = ${}^6C_5 . {}^7C_5 . {}^2C_1 = 252$

Total number of ways = 777

53. Answer (96)

$$\underset{\substack{\uparrow \\ 2,4,6,8}}{\underline{4}}\quad \underbrace{\underline{4}\quad \underline{3}\quad \underline{2}\quad \underline{1}}_{\text{No restriction}}$$

Total number of numbers = 4 × 4 × 3 × 2 × 1 = 96

54. Answer (16)

$f(A) = 15 \Rightarrow ad - bc = 15$

(ad, bc) = (9, –6) or (6, –9)

(i) Number of ways to select (a, d) = 2

Number of ways to select (b, c) = 4

(ii) Number of ways to select (a, d) = 4

Number of ways to select (b, c) = 2

Total number of possible matrix A = 2 × 4 + 2 × 4

= 16

55. Answer (3)

$^nP_r = {}^nP_{r+1}$

$\Rightarrow \dfrac{n!}{(n-r)!} = \dfrac{n!}{(n-r-1)!}$

$\therefore$ $(n-r)\cdot(n-r-1)! = (n-r-1)!$

$\therefore$ $(n-r-1)!\,(n-r-1) = 0$

$\therefore$ $n - r - 1 = 0$ ...(i)

$^nC_r = {}^nC_{r-1}$

$\Rightarrow \dfrac{n!}{r!(n-r)!} = \dfrac{n!}{(n-r+1)!\cdot(r-1)!}$

$\Rightarrow$ $r = n - r + 1$

$n - 2r = -1$ ...(ii)

From (i) and (ii) : $r = 2$, $n = 3$

56. Answer (490)

$\because$ $f(m : n) = f(m)\cdot f(n)$

Clearly $f(1) = 1$

and $f(4) = (f(2))^2$

$f(2) = f(4) = 1$
Also $f(3) = f(6)$
$f(5)$, $f(6)$ and $f(7)$ are selected independently in $7 \times 7 \times 7$ ways

$f(2) = 2$ and $f(4) = 4$

$f(3) = 1$, $f(6) = 2$ | $f(3) = 2$, $f(6) = 4$ | $f(3) = 3$, $f(6) = 6$

$f(5)$ and $f(7)$ are selected independently in 7×7 ways

Total number of ways $= 7^3 + 3.7^2 = 490$

57. Answer (2)

Reflection of P(a, b) about line $y = x$ is $P' = (b, a)$. After translation of 2 units the new coordinate in $P'' = (b + 2, a)$

On rotation of $\frac{\pi}{4}$ the new coordinate be (x_1, y_1).

$\therefore \dfrac{(x_1 + iy_1) - 0}{(b + 2 + ai) - 0} = e^{i\frac{\pi}{4}}$

$x_1 + iy_1 = ((b+2) + ai)\left(\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}i\right)$

$= \dfrac{1}{\sqrt{2}}(b + 2 + (b+2)i + ai - a)$

$= \dfrac{1}{\sqrt{2}}((a + b + 2)i + (b - a + 2))$

$\therefore$ $b - a + 2 = -1$, $a + b + 2 = 7$

$\therefore$ $a = 4$, $b = 1$

58. Answer (52)

Three digit even number by 0, 1, 3, 4, 6, 7

When zero is at unit place

Case-I $\overline{\downarrow}\ \overline{\downarrow}\ \overline{\downarrow}$ $\Rightarrow$ $4 \times 5 \times 1 = 20$

4 5 1(0)

When zero is not at unit place

Case-II $\overline{\downarrow}\ \overline{\downarrow}\ \overline{\downarrow}$ $=$ $4 \times 4 \times 2 = 32$

4 4 2(4, 6)

Total three digit even number = 20 + 32 = 52

59. Answer (100)

For divisible by 55 it shall be divisible by 11 and 5 both, for divisibility by 5 unit digit shall be 0 or 5 but as the number is six digit palindrome unit digit is 5.

$\therefore$ Number is of form 5_ _ _ _5

$\therefore$ Now for divisibility by 11 remaining odd places have 10 options each & then even place will have same value as their difference of sum shall be multiple of +1.

$\therefore$ No. of ways $= 10 \times 10 = 100$

60. Answer (576)

Total possible words = 6! = 720

When 4 consonants are together (V, W, L, S)

Total case $\Rightarrow$.

O, *E*, $\boxed{\text{V W L S}}$

Such cases $= 3! \cdot 4! = 144$

Required cases $= 720 - 144 = 576$

61. Answer (5143)

A = set of all four digit integers divisible by 7.

B = set of all four digit integers divisible by 3.

$n(A) = \left[\dfrac{9000}{7}\right] = 1285$

$n(B) = \left[\dfrac{9000}{3}\right] = 3000$

$n(A \cap B) = \left[\dfrac{9000}{21}\right] = 428$

$n(A \cup B) = 3857$

$n\left(\overline{A \cup B}\right) = 9000 - 3857$

$= 5143$

62. Answer (4)

Total number of triangles = ${}^{15}C_3 = 455$

Let $i < j < k$ so $i = 1, 2, 3, 4$ only

When $i = 1$, $i + j + k = 15$ has 5 solutions

$i = 2$, $i + j + k = 15$ has 4 solutions

$i = 3$, $i + j + k = 15$ has 2 solutions

$i = 4$, $i + j + k = 15$ has 1 solution

Required number of triangles = 455 – 12

= 443

63. Answer (77)

First find all possible words and then subtract words from each case that have both R together i.e.,

A………… $\Rightarrow$ $\frac{5!}{2!} - 4!$ = 36

E………… $\Rightarrow$ $\frac{5!}{2!} - 4!$ = 36

FAE……… $\Rightarrow$ $\frac{3!}{2!} - 2$ = 1

FAM……… $\Rightarrow$ $\frac{3!}{2!} - 2$ = 1

FARE…… $\Rightarrow$ 2! = 2

FARMER $\Rightarrow$ 1 = 1

77

$\therefore$ Rank of farmer is 77

64. Answer (1)

$n(A) = 4$, $n(B) = 2$

$n(A \times B) = 8$

Required numbers = ${}^8C_3 + {}^8C_4 + \ldots\ldots + {}^8C_8$

$= 2^8 - ({}^8C_0 + {}^8C_1 + {}^8C_2)$

= 256 – 37

= 219

65. Answer (924)

$N = 2^{10} \cdot 5^{10} \cdot 11^{11} \cdot 13^{13}$

$$\underbrace{(2^0 + 2^1 + \ldots.. + 2^{10})}_{\text{only } 2^0 \text{ is allowed to be selected } (2^0 \text{ is of the type } 4\lambda + 1)} \quad \underbrace{(5^0 + 5^1 + \ldots.. + 5^{10})}_{\text{All terms are of the type } 4\lambda + 1}$$

$$\underbrace{(11^0 + 11^1 + \ldots.. + 11^{11})}_{11^{\text{even}} \text{ are of the type } 4\lambda + 1} \quad \underbrace{(13^0 + 13^1 + \ldots.. + 13^{13})}_{\text{All terms are of the type } 4\lambda + 1}$$

Number of required divisors = 1 × 11 × 6 × 14

= 924

67. Answer (40)

Let student marks x correct answers and y incorrect. So

$3x - 2y = 5$ and $x + y \le 5$ where $x, y \in W$

Only possible solution is $(x, y) = (3, 2)$

Student can mark correct answer by only one choice but for incorrect answer, there are two choices. So total number of ways of scoring 5 marks = ${}^5C_3(1)^3 . (2)^2 = 40$

68. Answer (576)

Sum of all given numbers = 31

I	II	III	IV	V	VI	VII

Difference between odd and even positions must be 0, 11 or 22, but 0 and 22 are not possible.

$\therefore$ Only difference 11 is possible

This is possible only when either 1, 2, 3, 4 is filled in odd position in some order and remaining in other order. Similar arrangements of 2, 3, 5 or 7, 2, 1 or 4, 5, 1 at even positions.

$\therefore$ Total possible arrangements = (4! × 3!) × 4

= 576

69. Answer (63)

For odd number unit place shall be 1, 3, 5, 7 or 9.

$\therefore$ $\underline{x}\ \underline{y}\ \underline{1}$, $\underline{x}\ \underline{y}\ \underline{3}$, $\underline{x}\ \underline{y}$ 5, $x\ y$ 7, $x\ y$ 9 are the type of numbers.

If $x\ y$ 1 then

$x + y = 6, 13, 20 \ldots$ Cases are required

i.e., $6 + 6 + 0 + \ldots = 12$ ways

If $x\ y\ 3$ then

$x + y = 4, 11, 18, \ldots.$ Cases are required

i.e., $4 + 8 + 1 + 0 \ldots = 13$ ways

Similarly for $x\ y\ 5$, we have

$x + y = 2, 9, 16, \ldots$

i.e., $2 + 9 + 3 = 14$ ways

for $x\ y\ 7$ we have

$x + y = 0, 7, 14, \ldots.$

i.e., $0 + 7 + 5 = 12$ ways

And for $x\ y\ 9$ we have

$x + y = 5, 12, 19 \ldots$

i.e., $5 + 7 + 0 \ldots = 12$ ways

$\therefore$ Total 63 ways

70. Answer (243)

C-1 : All digits are non-zero

$${}^{9}C_2 \cdot 2 \cdot \frac{3!}{2} = 216$$

C-2 : One digit is 0

$$0, 0, x \Rightarrow {}^{9}C_1 \cdot 1 = 9$$

$$0, x, x \Rightarrow {}^{9}C_1 \cdot 2 = 18$$

Total = 216 + 27 = 243

71. Answer (1120)

Required number of ways = Total ways of selection – ways in which B_1 and B_2 are present together.

$$= {}^{10}C_3 \cdot {}^{5}C_3 - {}^{8}C_1 \cdot {}^{5}C_3 = 10(120 - 8)$$
$$= 1120$$

72. Answer (150)

$\because$ $x \in [100, 999]$, $x \in N$

Then $\frac{x}{2} \in [50, 499], \frac{x}{2} \in N$

Number whose G.C.D with 18 is 1 in this range have the required condition. There are 6 such number from 18×3 to 18×4. Similarly from 18×4 to 18×5….., 26×18 to 27×18

$\therefore$ Total numbers = 24 × 6 + 6 = 150

The extra numbers are 53, 487, 491, 493, 497 and 499.

73. Answer (56)

First we arrange 5 red cubes in a row and assume x_1, x_2, x_3, x_4, x_5 and x_6 number of blue cubes between them

$$\underbrace{\quad\quad}_{x_1}\square\underbrace{\quad\quad}_{x_2}\square\underbrace{\quad\quad}_{x_3}\square\underbrace{\quad\quad}_{x_4}\square\underbrace{\quad\quad}_{x_5}\square\underbrace{\quad\quad}_{x_6}$$

Here, $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$

and $x_2, x_3, x_4, x_5 \geq 2$

So $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 3$

No. of solutions = ${}^{8}C_5 = 56$

74. Answer (4)

Number should be divisible by 6 and it should be even.

Total sum = 1 + 2 + 3 + 5 + 6 + 7 = 24

So number removed should be of type 3.

C-1 : excluding 3 _ _ _ _ 2 ways = 4! × 2 = 48

C-2 : excluding 6 _ _ _ _ 1 way = 4! = 24

Total cases = 48 + 24 = 72

75. Answer (4)

By multinomial theorem, no. of ways to distribute 30 identical candies among four children C_1, C_2 and C_3, C_4

= Coefficient of x^{30} in $(x^4 + x^5 + \ldots + x^7)(x^2 + x^3 + \ldots + x^6)(1 + x + x^2 \ldots)^2$

$$= \text{Coefficient of } x^{24} \text{ in } \frac{\left(1-x^4\right)}{1-x}\frac{\left(1-x^5\right)}{1-x}\frac{\left(1-x^{31}\right)^2}{(1-x)^2}$$

= Coefficient of x^{24} in $(1 - x^4 - x^5 + x^9)(1 - x)^{-4}$
$= {}^{27}C_{24} - {}^{23}C_{20} - {}^{22}C_{19} + {}^{18}C_{15} = 430$

76. Answer (18915)

There are 98 sets of three consecutive integer and 97 sets of four consecutive integers.

Using principle of inclusion and exclusion, Number of permutations of $b_1b_2b_3b_4$ = Number of permutations when $b_1b_2b_3$ are consecutive + Number of permutations when $b_2b_3b_4$ are consecutive – Number of permutations when $b_1b_2\ b_3b_4$ are consecutive

$= 97 \times 98 + 97 \times 98 - 97 = 97 \times 195 = 18915.$

77. Answer (1086)

If unit digit is 1 then $\rightarrow 9 \times 10 \times 10 = 900$ numbers

If unit digit is 2 then $\rightarrow 4 \times 5 \times 5 = 100$ numbers

If unit digit is 3 then $\rightarrow 3 \times 4 \times 4 = 48$ numbers

If unit digit is 4 then $\rightarrow 2 \times 3 \times 3 = 18$ numbers

If unit digit is 5 then $\rightarrow 1 \times 2 \times 2 = 4$ numbers

If unit digit is 6 then $\rightarrow 1 \times 2 \times 2 = 4$ numbers

For 7, 8, 9 $\rightarrow 4 + 4 + 4 = 12$ Numbers

Total = 1086 Numbers

78. Answer (2)

Case 1: If $f(3) = 3$ then $f(1)$ and $f(2)$ take 1 OR 2

No. of ways = $2\times6 = 12$

Case 2: If $f(3) = 5$ then $f(1)$ and $f(2)$ take 2 OR 3 OR 1 and 4

No. of ways = $2\times6\times2 = 24$

Case 3: If $f(3) = 2$ then $f(1) = f(2) = 1$

No. of ways = 6

Case 4: If $f(3) = 4$ then $f(1) = f(2) = 2$

No. of ways = 6

OR $f(1)$ and $f(2)$ take 1 and 3

No. of ways = 12

Case 5: If $f(3) = 6$ then $f(1) = f(2) = 3 \Rightarrow 6$ ways

OR $f(1)$ and $f(2)$ take 1 and 5 $\Rightarrow$ 12 ways

OR $f(2)$ and $f(1)$ take 2 and 4 $\Rightarrow$ 12 ways

79. Answer (1492)

Arranging letter in alphabetical order A D I K M N N for finding rank of MANKIND making arrangements of dictionary we get

$A \ldots\ldots\ldots\ldots \rightarrow \dfrac{6!}{2!} = 360$

$D \ldots\ldots\ldots\ldots \rightarrow 360$

$I \ldots\ldots\ldots\ldots \rightarrow 360$

$K \ldots\ldots\ldots\ldots \rightarrow 360$

$MAD \ldots\ldots\ldots\ldots \rightarrow \dfrac{4!}{2!} = 12$

$MAI \ldots\ldots\ldots\ldots \rightarrow 12$

$MAK \ldots\ldots\ldots\ldots \rightarrow 12$

$MAND \ldots\ldots\ldots\ldots \rightarrow 3! = 6$

$MANI \ldots\ldots\ldots\ldots \rightarrow 6$

$MANKD \ldots\ldots\ldots\ldots \rightarrow 2$

$MANKID \ldots\ldots\ldots\ldots \rightarrow 1$

$MANKIND \ldots\ldots\ldots\ldots \rightarrow 1$

$\therefore$ Rank of MANKIND = 1440 + 36 + 12 + 2 + 2

= 1492

80. Answer (180)

Factors of 36 = $2^2 \times 3^2 \times 1$

Five-digit combinations can be

(1, 2, 2, 3, 3) (1, 4, 3, 3, 1), (1, 9, 2, 2, 1)

(1, 4, 9, 11) (1, 2, 3, 6, 1) (1, 6, 6, 1, 1)

i.e., total numbers

$$\frac{5!}{2!2!} + \frac{5!}{2!2!} + \frac{5!}{2!2!} + \frac{5!}{3!} + \frac{5!}{2!} + \frac{5!}{3!2!}$$

$= \quad (30 \times 3) + 20 + 60 + 10 = 180.$

81. Answer (7073)

If password is 6 character long, then

Total number of ways having atleast one number = $10^6 - 5^6$

Similarly, if 7 character long = $10^7 - 5^7$

and if 8-character long = $10^8 - 5^8$

Number of password

$= (10^6 + 10^7 + 10^8) - (5^6 + 5^7 + 5^8)$

$= 5^6 (2^6 + 5.2^7 + 25.2^8 - 1 - 5 - 25)$

$= 5^6(64 + 640 + 6400 - 31)$

$= 7073 \times 5^6$

$\therefore \quad \alpha = 7073.$

82. Answer (17)

$^bC_3 \cdot {^gC_2} = 168$

$\Rightarrow \dfrac{b(b-1)(b-2)}{6} \cdot \dfrac{g(g-1)}{2} = 168$

$\Rightarrow b(b-1)(b-2) \quad g(g-1) = 2^5 \cdot 3^2 \cdot 7$

$\Rightarrow b(b-1)(b-2) \quad g(g-1) = 6.\ 7.\ 8.\ 3.\ 2$

$\therefore \quad b = 8$ and $g = 3$

$\therefore \quad b + 3g = 17$

83. Answer (6)

Case-I : When number is 4-digit number $\left(\overline{a\ b\ c\ d}\right)$

here d is fixed as 5

So, (a, b, c) can be (6, 4, 3), (3, 4, 6), (2, 3, 6), (6, 3, 2), (3, 2, 4) or (4, 2, 3)

$\Rightarrow$ 6 numbers

Case-II : No number possible

84. Answer (30)

Number must start by 1 or 2 and for divisibility by 4 last two digits shall be divisible by 4

$\therefore \quad \_\ \_\ \underline{1}\ \underline{2}$! 0 cases

$\underline{2}\ \underset{3}{\uparrow}\ \underline{1}\ \underline{6} \rightarrow$ 3 cases

$\underline{1}\ \underset{3}{\uparrow}\ \underline{2}\ \underline{4} \rightarrow$ 3 cases

$\underline{1}\ \underset{3}{\uparrow}\ \underline{3}\ \underline{2} \rightarrow$ 3 cases

$\underline{2}\ \underset{3}{\uparrow}\ \underline{3}\ \underline{6} \rightarrow$ 6 cases

$\underline{1}\ \underset{3}{\uparrow}\ \underline{5}\ \underline{2} \rightarrow$ 3 cases

$\underline{2}\ \underset{3}{\uparrow}\ \underline{5}\ \underline{6} \rightarrow$ 6 cases

$\underline{2}\ \underset{3}{\uparrow}\ \underline{6}\ \underline{4} \rightarrow$ 6 cases

$\Rightarrow$ Total 30 numbers

85. Answer (11.00)

Here $S = \{4, 6, 9\}$

And $T = \{9, 10, 11, \ldots\ldots, 1000\}$.

We have to find all numbers in the form of

$4x + 6y + 9z$, where $x, y, z \in \{0, 1, 2, \ldots..\}$.

If a and b are coprime number then the least number from which all the number more than or equal to it can be express as $ax + by$ where $x, y \in \{0, 1, 2, \ldots.\}$ is $(a - 1) \cdot (b - 1)$.

Then for $6y + 9z = 3(2y + 3z)$

All the number from $(2 - 1) \cdot (3 - 1) = 2$ and above can be express as $2x + 3z$ (say t).

Now $4x + 6y + 9z = 4x + 3(t + 2)$

$= 4x + 3t + 6$

again by same rule $4x + 3t$, all the number from $(4 - 1)(3 - 1) = 6$ and above can be express from $4x + 3t$.

Then $4x + 6y + 9z$ express all the numbers from 12 and above.

again 9 and 10 can be express in form $4x + 6y + 9z$.

Then set $A = \{9, 10, 12, 13, \ldots., 1000\}$.

Then $T - A = \{11\}$

Only one element 11 is there.

Sum of elements of $T - A = 11$

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