Chapter 6

Matrices

1. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is **[AIEEE-2010]**

(1) Less than 4 (2) 5

(3) 6 (4) At least 7

2. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define

Tr(A) = sum of diagonal elements of A and $|A|$ = determinant of matrix A.

Statement-1 : Tr(A) = 0.

Statement-2 : $|A|$ = **1.** **[AIEEE-2010]**

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

3. Consider the following relation R on the set of real square matrices of order 3.

$R = \{(A, B)|A = P^{-1} BP$ for some invertible matrix $P\}$.

Statement-1 : R is an equivalence relation.

Statement-2 : For any two invertible 3 × 3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$.

[AIEEE-2011]

(1) Statement-1 is true, statement-2 is false

(2) Statement-1 is false, statement-2 is true

(3) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1

(4) Statement-1 is true, statement-2 is true; statement-2 is ***not*** a correct explanation for statement-1

4. **Statement-1 :** Determinant of a skew-symmetric matrix of order 3 is zero.

Statement-2 : For any matrix A, $\det(A^T) = \det(A)$ and $\det(-A) = -\det(A)$.

Where det(B) denotes the determinant of matrix B. Then **[AIEEE-2011]**

(1) Statement-1 is false and statement-2 is true

(2) Statement-1 is true and statement-2 is false

(3) Both statements are true

(4) Both statements are false

5. If $\omega \neq 1$ is the complex cube root of unity and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to

[AIEEE-2011]

(1) H^2 (2) H

(3) 0 (4) – H

6. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$ if u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to **[AIEEE-2012]**

(1) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

(3) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ (4) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

7. If A is an 3 × 3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals **[JEE (Main)-2014]**

(1) B^{-1} (2) $(B^{-1})'$

(3) $I + B$ (4) I

8. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to **[JEE (Main)-2015]**

(1) (2, –1) (2) (–2, 1)

(3) (2, 1) (4) (–2, –1)

9. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to **[JEE (Main)-2019]**

(1) $\begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

10. If $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$,

then A is **[JEE (Main)-2019]**

(1) Invertible only if $t = \pi$

(2) Invertible for all $t \in$ R.

(3) Invertible only if $t = \frac{\pi}{2}$

(4) Not invertible for any $t \in$ R.

11. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then **[JEE (Main)-2019]**

(1) $-\sqrt{3}$ (2) $\sqrt{3}$

(3) $2\sqrt{3}$ (4) $-2\sqrt{3}$

12. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then $|p|$ is **[JEE (Main)-2019]**

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{1}{\sqrt{6}}$

(3) $\frac{1}{\sqrt{5}}$ (4) $\frac{1}{\sqrt{2}}$

13. Let A and B be two invertible matrices of order 3×3. If $det(ABA^T) = 8$ and $det(AB^{-1}) = 8$, then $det(BA^{-1}B^T)$ is equal to **[JEE (Main)-2019]**

(1) 1 (2) 16

(3) $\frac{1}{16}$ (4) $\frac{1}{4}$

14. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\frac{q_{21}+q_{31}}{q_{32}}$ is equal to **[JEE (Main)-2019]**

(1) 10 (2) 135

(3) 9 (4) 15

15. Let $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, (\alpha \in R)$ such that $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then a value of α is **[JEE (Main)-2019]**

(1) $\frac{\pi}{32}$ (2) $\frac{\pi}{64}$

(3) 0 (4) $\frac{\pi}{16}$

16. Let the numbers 2, b, c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det($A$) $\in$ [2, 16], then c lies in the interval **[JEE (Main)-2019]**

(1) [2, 3) (2) $(2 + 2^{3/4}, 4)$

(3) $[3, 2 + 2^{3/4}]$ (4) [4, 6]

17. If $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\cdot\ldots\cdot\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$,

then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is **[JEE (Main)-2019]**

(1) $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

(3) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$

18. The total number of matrices

$$A=\begin{pmatrix}0 & 2y & 1\\2x & y & -1\\2x & -y & 1\end{pmatrix},\ (x, y \in R, x \neq y) \text{ for}$$

which $A^TA = 3I_3$ is **[JEE (Main)-2019]**

(1) 6 (2) 3

(3) 4 (4) 2

19. If A is a symmetric matrix and B is a skew-symmetric matrix such that $A+B=\begin{bmatrix}2 & 3\\5 & -1\end{bmatrix}$, then AB is equal to **[JEE (Main)-2019]**

(1) $\begin{bmatrix}4 & -2\\1 & -4\end{bmatrix}$ (2) $\begin{bmatrix}4 & -2\\-1 & -4\end{bmatrix}$

(3) $\begin{bmatrix}-4 & -2\\-1 & 4\end{bmatrix}$ (4) $\begin{bmatrix}-4 & 2\\1 & 4\end{bmatrix}$

20. If $B=\begin{bmatrix}5 & 2\alpha & 1\\0 & 2 & 1\\\alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all value of α for which det (A) + 1 = 0, is **[JEE (Main)-2019]**

(1) –1 (2) 2

(3) 0 (4) 1

21. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\1 & \alpha & \alpha^2\\1 & \alpha^2 & \alpha^4\end{bmatrix}$, then the matrix A^{31} is equal to **[JEE (Main)-2020]**

(1) A^2 (2) A

(3) I_3 (4) A^3

22. If $A=\begin{pmatrix}2 & 2\\9 & 4\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$, then $10A^{-1}$ is equal to **[JEE (Main)-2020]**

(1) $6I - A$ (2) $4I - A$

(3) $A - 4I$ (4) $A - 6I$

23. If the matrices $A=\begin{bmatrix}1 & 1 & 2\\1 & 3 & 4\\1 & -1 & 3\end{bmatrix}$, B = adj A and $C = 3A$, then $\frac{|\text{adj } B|}{|C|}$ is equal to

[JEE (Main)-2020]

(1) 16 (2) 2

(3) 72 (4) 8

24. Let A be a 2 × 2 real matrix with entries from {0, 1} and $|A| \neq 0$. Consider the following two statements **[JEE (Main)-2020]**

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then tr(A) = 2,

where I_2 denotes 2 × 2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then

[JEE (Main)-2020]

(1) (P) is true and (Q) is false

(2) Both (P) and (Q) are false

(3) Both (P) and (Q) are true

(4) (P) is false and (Q) is true

25. Let $A = \{X = (x, y, z)^T : PX = 0 \text{ and } x^2 + y^2 + z^2 = 1\}$,

where $P=\begin{bmatrix}1 & 2 & 1\\-2 & 3 & -4\\1 & 9 & -1\end{bmatrix}$, then the set A

[JEE (Main)-2020]

(1) Is an empty set.

(2) Contains more than two elements.

(3) Contains exactly two elements.

(4) Is a singleton.

26. Let A be a 3 × 3 matrix such that

$$\text{adj } A=\begin{bmatrix}2 & -1 & 1\\-1 & 0 & 2\\1 & -2 & -1\end{bmatrix} \text{ and } B = \text{adj(adj } A).$$

If $|A| = \lambda$ and $\left|(B^{-1})^T\right| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to **[JEE (Main)-2020]**

(1) (3, 81) (2) $\left(9, \frac{1}{9}\right)$

(3) $\left(3, \frac{1}{81}\right)$ (4) $\left(9, \frac{1}{81}\right)$

27. If $A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix},\left(\theta=\frac{\pi}{24}\right)$ and $A^5=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, where $i=\sqrt{-1}$, then which one of the following is not true? **[JEE (Main)-2020]**

(1) $a^2-b^2=\frac{1}{2}$ (2) $a^2-c^2=1$

(3) $a^2-d^2=0$ (4) $0\le a^2+b^2\le 1$

28. Let $\theta=\frac{\pi}{5}$ and $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$. If $B=A+A^4$, then det (B) **[JEE (Main)-2020]**

(1) Lies in (2, 3) (2) Is zero.

(3) Is one (4) Lies in (1, 2)

29. The number of all 3×3 matrices A, with enteries from the set $\{-1, 0, 1\}$ such that the sum of the diagonal elements of AA^T is 3, is ____________.

[JEE (Main)-2020]

30. Let $A=\begin{bmatrix}x & 1\\ 1 & 0\end{bmatrix}, x\in R$ and $A^4=[a_{ij}]$. If $a_{11}=109$, then a_{22} is equal to _________.

[JEE (Main)-2020]

31. Let A and B be 3 × 3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2-B^2A^2)X=O$, where X is a 3 × 1 column matrix of unknown variables and O is a 3 × 1 null matrix, has : **[JEE (Main)-2021]**

(1) exactly two solutions

(2) infinitely many solutions

(3) no solution

(4) a unique solution

32. If for the matrix, $A=\begin{bmatrix}1 & -\alpha\\ \alpha & \beta\end{bmatrix}$, $AA^T=I_2$, then the value of $\alpha^4+\beta^4$ is : **[JEE (Main)-2021]**

(1) 1 (2) 2

(3) 4 (4) 3

[JEE (Main)-2021]

33. Let A be a 3 × 3 matrix with det(A) = 4. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2\to 2R_2+5R_3$ on 2A, then det(B) is equal to : **[JEE (Main)-2021]**

(1) 64 (2) 128

(3) 80 (4) 16

34. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is : **[JEE (Main)-2021]**

(1) 6 (2) 1

(3) 4 (4) 12

35. If the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 3 & 0 & -1\end{bmatrix}$ satisfies the equation $A^{20}+\alpha A^{19}+\beta A=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 1\end{bmatrix}$ for some real numbers α and β, then $\beta-\alpha$ is equal to ______. **[JEE (Main)-2021]**

36. The total number of 3 × 3 matrices A having entries from the set {0, 1, 2, 3} such that the sum of all the diagonal entries of AA^T is 9, is equal to _______. **[JEE (Main)-2021]**

37. Let $A=\begin{bmatrix}a_1\\ a_2\end{bmatrix}$ and $B=\begin{bmatrix}b_1\\ b_2\end{bmatrix}$ be two 2 × 1 matrices with real entries such that A = XB, where $X=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & -1\\ 1 & k\end{bmatrix}$, and $k\in R$. If $a_1^2+a_2^2=\frac{2}{3}(b_1^2+b_2^2)$ and $(k^2+1)\,b_2^2\ne -2b_1b_2$, then the value of k is _____. **[JEE (Main)-2021]**

38. Let $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ and $B=\begin{bmatrix}\alpha\\ \beta\end{bmatrix}\ne\begin{bmatrix}0\\ 0\end{bmatrix}$ such that AB = B and a + d = 2021, then the value of ad – bc is equal to ____. **[JEE (Main)-2021]**

39. Let $A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix}$ and $2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}$. If Tr(A) denotes the sum of all diagonal elements of the matrix A, then Tr(A) – Tr(B) has value equal to:

(1) 1 (2) 3

(3) 0 (4) 2

[JEE (Main)-2021]

40. Define a relation R over a class of n × n real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1}=B$". Then which of the following is true?

[JEE (Main)-2021]

(1) R is reflexive, symmetric but not transitive

(2) R is an equivalence relation

(3) R is symmetric, transitive but not reflexive

(4) R is reflexive, transitive but not symmetric

41. Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in R$ be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to **[JEE (Main)-2021]**

(1) 24 (2) 18

(3) 45 (4) 36

42. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3 × 3. If $B = [b_{ij}]$, then b_{13} is equal to _______. **[JEE (Main)-2021]**

43. Let $A = \{a_{ij}\}$ be a 3 × 3 matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$$ 203117

then $\det(3\,\mathrm{Adj}(2A^{-1}))$ is equal to _____. **[JEE (Main)-2021]**

44. Let $A = [a_{ij}]$ be a real matrix of order 3 × 3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for $i = 1, 2, 3$. Then, the sum of all the entries of the matrix A^3 is equal to **[JEE (Main)-2021]**

(1) 1 (2) 3

(3) 2 (4) 9

45. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3 × 3 matrices B with entries from the set {1, 2, 3, 4, 5} and satisfying AB = BA is _______. **[JEE (Main)-2021]**

46. Let $S = \left\{ n \in N \,\middle|\, \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a, b, c, d \in \mathbb{R} \right\}$, where $i = \sqrt{-1}$. Then the number of 2-digit numbers in the set S is _______. **[JEE (Main)-2021]**

47. If $P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$, then P^{50} is **[JEE (Main)-2021]**

(1) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

48. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in \mathbf{R}$, I is 2 × 2 identity matrix, then $4(\alpha - \beta)$ is **[JEE (Main)-2021]**

(1) $\frac{8}{3}$ (2) 5

(3) 4 (4) 2

49. Let A and B be two 3 × 3 real matrices such that $(A^2 - B^2)$ is invertible matrix. If $A^5 = B^5$ and $A^3B^2 = A^2B^3$, then the value of the determinant of the matrix $A^3 + B^3$ is equal to **[JEE (Main)-2021]**

(1) 1 (2) 2

(3) 4 (4) 0

50. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + \ldots + A^{20}$, then the sum of all the elements of the matrix M is equal to ________. **[JEE (Main)-2021]**

51. Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations

$x + y + z = 5$

$x + 2y + 3z = \mu$

$x + 3y + \lambda z = 1$

is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then **[JEE (Main)-2021]**

(1) $p = \frac{1}{6}$ and $q = \frac{1}{36}$ (2) $p = \frac{5}{6}$ and $q = \frac{5}{36}$

(3) $p = \frac{1}{6}$ and $q = \frac{5}{36}$ (4) $p = \frac{5}{6}$ and $q = \frac{1}{36}$

52. If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of K is : **[JEE (Main)-2021]**

(1) $\frac{1}{2}$ (2) –1

(3) 1 (4) $-\frac{1}{2}$

53. The number of elements is the set

$$\left\{A=\begin{pmatrix}a & b\\ 0 & d\end{pmatrix}: a, b, d \in \{-1, 0, 1\} \text{ and } (I-A)^3 = I - A^3\right\},$$

where I is 2 × 2 identity matrix, is _______.

[JEE (Main)-2021]

54. If $A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and A.adj $A = A\,A^T$, then $5a + b$ is equal to :

[JEE (Main)-2021]

(1) 5
(2) 4
(3) 13
(4) –1

55. If $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to

[JEE (Main)-2021]

(1) $\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$
(2) $\begin{bmatrix}51 & 84\\ 63 & 72\end{bmatrix}$
(3) $\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$
(4) $\begin{bmatrix}72 & -84\\ -63 & 51\end{bmatrix}$

56. Let $A=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}$. If M and N are two matrices given by $M=\sum_{k=1}^{10} A^{2k}$ and $N=\sum_{k=1}^{10} A^{2k-1}$ then MN^2 is :

[JEE (Main)-2022]

(1) a non-identity symmetric matrix
(2) a skew-symmetric matrix
(3) neither symmetric nor skew-symmetric matrix
(4) an identity matrix

57. Let A be a 3 × 3 matrix having entries from the set {–1, 0, 1}. The number of all such matrices A having sum of all the entries equal to 5, is ___________. **[JEE (Main)-2022]**

58. Let $A = \begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}$ and $B = \begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}$. Then the number of elements in the set $\{(n, m) : n, m \in \{1, 2, \ldots, 10\}$ and $nA^n + mB^m = I\}$ is ___________. **[JEE (Main)-2022]**

59. Let $X=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$, $Y = \alpha I + \beta X + \gamma X^2$ and

$$Z = \alpha^2 I - \alpha\beta X + \left(\beta^2 - \alpha\gamma\right)X^2,\ \alpha, \beta, \gamma \in \mathbb{R}.$$

If $Y^{-1}=\begin{bmatrix}\frac{1}{5} & \frac{-2}{5} & \frac{1}{5}\\ 0 & \frac{1}{5} & \frac{-2}{5}\\ 0 & 0 & \frac{1}{5}\end{bmatrix}$, then $(\alpha - \beta + \gamma)^2$ is equal to ___________. **[JEE (Main)-2022]**

60. Let A be a matrix of order 2 × 2, whose entries are from the set {0, 1, 3, 4, 5}. If the sum of all the entries of A is a prime number p, $2 < p < 8$, then the number of such matrices A is ___________. **[JEE (Main)-2022]**

61. Let $A=\begin{pmatrix}1+i & 1\\ -i & 0\end{pmatrix}$ where $i=\sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, \ldots, 100\} : A^n = A\}$ is ________.

[JEE (Main)-2022]

62. Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all i, j = 1, 2, 3. Then, the matrix $A^2 + A^3 + \ldots + A^{10}$ is equal to :

[JEE (Main)-2022]

(1) $\left(\frac{3^{10}-3}{2}\right)A$
(2) $\left(\frac{3^{10}-1}{2}\right)A$
(3) $\left(\frac{3^{10}+1}{2}\right)A$
(4) $\left(\frac{3^{10}+3}{2}\right)A$

63. Let $M=\begin{bmatrix}0 & -\alpha\\ \alpha & 0\end{bmatrix}$, where α is a non-zero real number an $N=\sum_{k=1}^{49} M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is ____ .

[JEE (Main)-2022]

64. Let $A=\begin{pmatrix}2 & -1 & -1\\ 1 & 0 & -1\\ 1 & -1 & 0\end{pmatrix}$ and $B = A - I$. If $\omega=\frac{\sqrt{3}i-1}{2}$, then the number of elements in the set $\{n\in\{1,2,....,100\}: A^n+(\omega B)^n = A+B\}$ is equal to ______.

[JEE (Main)-2022]

65. Let $A=\begin{pmatrix}1 & 2\\ -2 & -5\end{pmatrix}$. Let $\alpha, \beta, \in \mathbb{R}$ be such that $\alpha A^2 + \beta A = 2I$. Then $\alpha + \beta$ is equal to

[JEE (Main)-2022]

(1) –10 (2) –6

(3) 6 (4) 10

66. Let S be the set containing all 3 × 3 matrices with entries from {–1, 0, 1}. The total number of matrices $A \in S$ such that the sum of all the diagonal elements of $A^T A$ is 6 is ________.

[JEE (Main)-2022]

67. Let $A=\begin{pmatrix}4 & -2\\ \alpha & \beta\end{pmatrix}$ If $A^2 + \gamma A + 18I = 0$, then det (A) is equal to ______.

[JEE (Main)-2022]

(1) –18 (2) 18

(3) –50 (4) 50

68. Let $A=\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}$ and $B=\begin{bmatrix}\beta & 1\\ 1 & 0\end{bmatrix}$, $\alpha, \beta \in \boldsymbol{R}$. Let α_1 be the value of α which satisfies $(A+B)^2 = A^2+\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$ and α_2 be the value of α which satisfies $(A + B)^2 = B^2$. Then $|\alpha_1-\alpha_2|$ is equal to ________.

[JEE (Main)-2022]

69. Let A and B be any two 3 × 3 symmetric and skew symmetric matrices respectively. Then Which of the following is **NOT** true?

[JEE (Main)-2022]

(1) $A^4 - B^4$ is a symmetric matrix

(2) $AB - BA$ is a symmetric matrix

(3) $B^5 - A^5$ is a skew-symmetric matrix

(4) $AB + BA$ is a skew-symmetric matrix

70. Which of the following matrices can **NOT** be obtained from the matrix $\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}$ by a single elementary row operation?

[JEE (Main)-2022]

(1) $\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$ (2) $\begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}$

(3) $\begin{bmatrix}-1 & 2\\ -2 & 7\end{bmatrix}$ (4) $\begin{bmatrix}-1 & 2\\ -1 & 3\end{bmatrix}$

71. Let $X=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$ and $A=\begin{bmatrix}-1 & 2 & 3\\ 0 & 1 & 6\\ 0 & 0 & -1\end{bmatrix}$. For $k\in\mathbb{N}$, if $X'A^kX = 33,$ then k is equal to _______.

[JEE (Main)-2022]

72. Let $A=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$ and $B=\begin{bmatrix}9^2 & -10^2 & 11^2\\ 12^2 & 13^2 & -14^2\\ -15^2 & 16^2 & 17^2\end{bmatrix}$, then the value of $A'BA$ is [JEE (Main)-2022]

(1) 1224 (2) 1042

(3) 540 (4) 539

73. Let $S = \left\{\begin{pmatrix}-1 & a\\ 0 & b\end{pmatrix}; a,\ b\in\{1, 2, 3, ...100\}\right\}$ and let $T_n = \{A \in S : A^{n(n+1)} = I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is ____. **[JEE (Main)-2022]**

74. The number of matrices of order 3 × 3, whose entries are either 0 or 1 and the sum of all the entries is a prime number, is _________.

[JEE (Main)-2022]

Matrices

1. Answer (4)

Consider $\begin{pmatrix} 1 & * & * \\ * & 1 & * \\ * & * & 1 \end{pmatrix}$. By placing a1 in any one of the 6 * position and 0 elsewhere. We get 6 nonsingular matrices.

Similarly $\begin{pmatrix} * & * & 1 \\ * & 1 & * \\ 1 & * & * \end{pmatrix}$ gives at least one nonsingular

2. Answer (3)

A satisfies $A^2 - \mathrm{Tr}(A).\ A + (\det A)I = 0$ comparing with $A^2 - I = 0$, it follows Tr $A = 0$, $|A| = -1$.

3. Answer (2)

R is Reflexive

Let ARB

i.e., $A = P^{-1}BP$

$PA = BP$

$PAP^{-1} = B$

$PAP^{-1} \neq P^{-1}AP$

Hence R is not equivalence

$\Rightarrow$ Statement 1 is false

$\Rightarrow$ Statement 2 is true

4. Answer (2)

For skew-symmetric matrix

$A^T = -A$

$\det A^T = \det(-A)$ ($\because \det(-A) = -\det A$ for
$\det A = -\det A$ matrix of odd order)

$2 \det A = 0 \quad \Rightarrow \quad \det A = 0$

Statement 1 is true.

Statement 2 :

For every matrix $\det(A^T) = \det(A)$

But $\det(-A) = -\det A$ is true for matrix of odd order.

$\therefore$ Statement 1 is ture and Statement 2 is false.

5. Answer (2)

$$H^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$$

$$H^2 = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$$

$$H^3 = \begin{bmatrix} \omega^3 & 0 \\ 0 & \omega^3 \end{bmatrix}$$

Similarly $H^{70} = \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$

$= H$

$\because \omega^{70} = \omega$

ω is complex cube root of unity

6. Answer (3)

7. Answer (4)

$BB' = (A^{-1}.A')(A(A^{-1})')$

$= A^{-1}.A.A'.(A^{-1})^1$ {as $AA' = A'A$}

$= I(A^{-1}A)'$

$= I.I = I^2 = I$

8. Answer (4)

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$a + 4 + 2b = 0$

$2a + 2 - 2b = 0$

$a + 1 - b = 0$

$2a - 2b = -2$

$a + 2b = -4$

$3a = -6$

$a = -2$

$-2 + 1 - b = 0$

$b = -1$

$a = -2$

$(-2, -1)$

$$\text{adj}(A) = \begin{bmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & +\cos\theta \end{bmatrix}^T$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = B$$

$$B^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$B^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$

$$\Rightarrow A^{-50} = B^{50} = \begin{bmatrix} \cos(50\theta) & \sin(50\theta) \\ -\sin(50\theta) & \cos(50\theta) \end{bmatrix}$$

$$(A^{-50})_{\theta = \frac{\pi}{12}} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

10. Answer (2)

det(A) = $|A|$

$$= \begin{vmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{vmatrix}$$

$$= e^t \cdot e^{-t} \cdot e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$$= e^{-t} \begin{vmatrix} 0 & 2\cos t + \sin t & 2\sin t - \cos t \\ 0 & -\cos t - 3\sin t & -\sin t + 3\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 + R_3 \end{matrix}$$

$$= e^{-t} \begin{vmatrix} 0 & -5\sin t & 5\cos t \\ 0 & -\cos t - 3\sin t & -\sin t + 3\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix} R_1 \to R_1 + 2R_2$$

$$= 5e^{-t} \neq 0, \forall t \in R$$

$\therefore$ A is invertible

$$\begin{vmatrix} 1 & b & 2 \end{vmatrix}$$

$$= 2(2b^2 + 2 - b^2) - b(2b - b) + 1(b^2 - b^2 - 1)$$

$$= 2b^2 + 4 - b^2 - 1 = b^2 + 3$$

$$\frac{|A|}{b} = b + \frac{3}{b}$$

$$\because \frac{b + \frac{3}{b}}{2} \geq \left(b \cdot \frac{3}{b}\right)^{\frac{1}{2}}$$

$$\frac{|A|}{b} \geq 2\sqrt{3}$$

Minimum value of $\frac{|A|}{b}$ is $2\sqrt{3}$.

Option (3) is correct.

12. Answer (4)

$$A = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}$$

$$\therefore \quad A \cdot A^T = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix} \times \begin{bmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r \end{bmatrix}$$

$$= \begin{bmatrix} 4q^2 + r^2 & 2q^2 - r^2 & -2q^2 + r^2 \\ 2q^2 - r^2 & p^2 + q^2 + r^2 & p^2 - q^2 - r^2 \\ -2q^2 + r^2 & p^2 - q^2 - r^2 & p^2 + q^2 + r^2 \end{bmatrix}$$

$\because$ $AA^T = I$

$\therefore$ $4q^2 + r^2 = p^2 + q^2 + r^2 = 1$

and $2q^2 - r^2 = 0 = p^2 - q^2 - r^2$

$\therefore$ $p^2 = 3q^2$ and $r^2 = 2q^2$

$\therefore$ $p^2 = \frac{1}{2}, q^2 = \frac{1}{6}$ and $r^2 = \frac{1}{3}$

$\therefore$ $|p| = \frac{1}{\sqrt{2}}$.

x y

$\because \quad |ABA^T| = 8 \quad \Rightarrow \quad |A|\,|B|\,|A^T| = 8$

$\Rightarrow \quad x \cdot y \cdot x = 8 \quad \Rightarrow \quad x^2 y = 8 \qquad \ldots$(i)

$\because \quad |AB^{-1}| = 8 \Rightarrow |A|\,|B^{-1}| = 8 \Rightarrow x \cdot \dfrac{1}{y} = 8 \qquad \ldots$(ii)

From (i) & (ii)

$$x = 4,\ y = \frac{1}{2}$$

$$\Rightarrow \quad |BA^{-1}B^T| = |B|\,|A^{-1}|\,|B^T| = y.\frac{1}{x}.y = \frac{y^2}{x} = \frac{1}{16}$$

Option (3) is correct.

14. Answer (1)

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$$

$$Q = I_3 + P^5 = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21} + q_{31}}{q_{32}} = \frac{15 + 135}{15} = 10$$

15. Answer (2)

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

Similarly $A^8 = A^4.A^4 \begin{bmatrix} \\ \sin 8\alpha & \cos 8\alpha \end{bmatrix}$

and so on $A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

So $\sin 32\alpha = 1$ and $\cos 32\alpha = 0$

$\Rightarrow \quad 32\alpha = 2n\pi + \dfrac{\pi}{2} \Rightarrow \alpha = \dfrac{n\pi}{16} + \dfrac{\pi}{64}$ where $n \in Z$

put $n = 0$, $\alpha = \dfrac{\pi}{64}$

16. Answer (4)

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$$

$c_2 \rightarrow c_2 - c_1,\ c_3 \rightarrow c_3 - c_1$

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & (b-2)(b+2) & (c-2)(c+2) \end{vmatrix}$$

$$= (b-2)(c-2)(c-b)$$

2, b, c are in A.P. $\Rightarrow$ $(b - 2) = (c - b) = d$, $c - 2 = 2d$ $\Rightarrow |A| = d.2d.d = 2d^3$

$\because |A| \in [2, 16] \Rightarrow 1 \le d^3 \le 8 \Rightarrow 1 \le d \le 2$

$4 \le 2d + 2 \le 6 \ \Rightarrow 4 \le c \le 6$

17. Answer (2)

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\cdots\begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1+2+3+\ldots.+(n-1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{(n-1)n}{2} = 78$$

$\Rightarrow \quad n = 13$

Now, inverse of $\begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2x & -y & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\Rightarrow$ $8x^2 = 3$, $6y^2 = 3$

$$x = \pm\sqrt{\frac{3}{8}},\ y = \pm\sqrt{\frac{1}{2}}$$

Total combinations of $(x, y) = 2 \times 2 = 4$

19. Answer (2)

$$\text{Let } A = \begin{bmatrix} a & c \\ c & b \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & d \\ -d & 0 \end{bmatrix}$$

$$\Rightarrow \quad A + B = \begin{bmatrix} a & c+d \\ c-d & b \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$$

$\Rightarrow$ $a = 2$, $b = -1$, $c - d = 5$, $c + d = 3$

$\Rightarrow$ $a = 2$, $b = -1$, $c = 4$, $d = -1$

$$\Rightarrow \quad AB = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$$

20. Answer (4)

As $B = A^{-1}$

$$|B| = \frac{1}{|A|}$$

$$\text{Now, } |B| = \begin{vmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{vmatrix} = 2\alpha^2 - 2\alpha - 25$$

Given, $|A| + 1 = 0$

$$\frac{1}{2\alpha^2 - 2\alpha - 25} + 1 = 0$$

$$\Rightarrow \quad \frac{2\alpha^2 - 2\alpha - 24}{2\alpha^2 - 2\alpha - 25} = 0$$

$\alpha = 4, -3$

Sum of values = 1

$$A^2 = \frac{1}{3}\begin{bmatrix} \cdots \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}\begin{bmatrix} \cdots \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

$\Rightarrow$ $A^{31} = A^3$

22. Answer (4)

$$A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}$$

$\because$ Every square matrix satisfying characteristic equation

$\Rightarrow$ $|A - \lambda I| = 0$

$$\because \quad \begin{bmatrix} 2-\lambda & 2 \\ 9 & 4-\lambda \end{bmatrix} = 0$$

$\Rightarrow$ $(\lambda - 4)(\lambda - 2) = 18$

$\Rightarrow$ $\lambda^2 - 6\lambda - 10 = 0$

$\therefore$ $A^2 - 6A - 10I = 0$

(By Pre multiplication of A^{-1} both sides)

$$\Rightarrow \quad \boxed{A - 6I = 10A^{-1}}$$

23. Answer (4)

$$\frac{|\text{adj } B|}{|C|} = \frac{|\text{adj}(\text{adj } A)|}{|3A|} = \frac{|A|^{2^2}}{3^3|A|} = \left(\frac{|A|}{3}\right)^3$$

$$\therefore \quad |A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix} = 1(13) - 1(-1) + 2(-4) = 6$$

$$\text{Hence, } \frac{|\text{adj } B|}{|C|} = \left(\frac{6}{3}\right)^3 = 8$$

$\Rightarrow \quad |A| = ad - bc$

$\therefore \quad ad = 0$ or 1 and $bc = 0$ or 1

so possible values of $|A|$ are 1, 0 or –1

(P) If $A \neq I_2$ then $|A|$ is either 0 or –1

(Q) If $|A| = 1$ then $ad = 1$ and $bc = 0$

$\Rightarrow \quad a = d = 1 \Rightarrow \mathrm{Tr}(A) = 2$

25. Answer (3)

$\because$ det(P) = 0

So the system has infinitely many solutions.

All solution lies on the line of intersection of planes

$x + 2y + z = 0, -2x + 3y - 4z = 0$ and $x + 9y - z = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ -2 & 3 & -4 \end{vmatrix} = -11\hat{i} + 2\hat{j} + 7\hat{k}$$

So, $x = -11\lambda$, $y = 2\lambda$, $z = 7\lambda$

$\because \; x^2 + y^2 + z^2 = 1$

$$\Rightarrow \quad \lambda^2 = \frac{1}{174} \Rightarrow \lambda = \pm\frac{1}{\sqrt{174}}$$

Two values of λ gives two triplets of (x, y, z)

26. Answer (3)

Here $|\text{adj } A| = 2(4) + 1(1-2) + 1(2)$

$$|\text{adj } A| = 9$$

and $|\text{adj } A| = |A|^{n-1} = 9$

$$\Rightarrow \quad \lambda^2 = 9 \Rightarrow \lambda = \pm 3 \qquad \{\because |A| = \lambda\}$$

Now

$$|B| = |\text{adj (adj } A)| = |A|^{(n-1)^2} = \lambda^4 = 3^4 = 81$$

Now

$$\mu = |(B^{-1})^T| = |(B^T)^{-1}| = |(B^T)|^{-1} = |B|^{-1} = \frac{1}{|B|} = \frac{1}{81}$$

So $(|\lambda|, \mu) = \left(3, \dfrac{1}{81}\right)$

$$\therefore \quad A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}, n \in N$$

$$\therefore \quad A^5 = \begin{bmatrix} \cos 5\theta & i\sin 5\theta \\ i\sin 5\theta & \cos 5\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\therefore \quad a = \cos 5\theta,\ b = i\sin 5\theta = c,\ d = \cos 5\theta$

$\therefore \quad a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

$a^2 - c^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

$a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 1$

$$a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta = \cos\frac{10\pi}{24}$$

and $0 < \cos\dfrac{5\pi}{12} < 1 \qquad \Rightarrow \quad 0 \leq a^2 + b^2 \leq 1$

$\therefore$ option (1) is not true

28. Answer (4)

$$\because \quad A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore \quad A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in N$$

$$\therefore \quad B = A + A^4 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

$$\therefore \quad B = \begin{bmatrix} \cos\frac{\pi}{5} + \cos\frac{4\pi}{5} & \sin\frac{\pi}{5} + \sin\frac{4\pi}{5} \\ -\sin\frac{\pi}{5} - \sin\frac{4\pi}{5} & \cos\frac{\pi}{5} + \cos\frac{4\pi}{5} \end{bmatrix}$$

$$\det(B) = 2\sin\left(\frac{\pi}{5}\right).\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}$$

$$= \frac{\sqrt{10 - 2\sqrt{5}}}{2} \approx \frac{2.35}{2} \simeq 1.175$$

$\det B \in (1, 2)$

$$AA^{-T} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$\text{tr}(AA^{-T}) = a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2$
$= 3$

$\therefore$ Three among $a, b, c, \ldots i$ should be 1 or -1 rest all 0

$\therefore$ Number of ways $= {}^9C_3 . 2^3 = 672$

30. Answer (10)

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (x^2+1)^2 + x^2 & x(x^2+1)+x \\ x(x^2+1)+x & x^2+1 \end{bmatrix}$$

Given $(x^2 + 1)^2 + x^2 = 109$

Let $x^2 + 1 = t$

$t^2 + t - 1 = 109$

$\Rightarrow (t - 10)(t + 11) = 0$

$\therefore t = 10 = x^2 + 1 = a_{22}$

31. Answer (2)

Let $C = A^2B^2 - B^2A^2$

Then $C^T = (A^2B^2 - B^2A^2)^T$

$= (B^T)^2.(A^T)^2 - (A^T)^2.(B^T)^2$

$= (-B)^2A^2 - A^2.(-B)^2$ $\quad \{\because A^T = A \text{ and } B^T = -B\}$

$= (B)^2A^2 - A^2B^2$

$\therefore C + C^T = 0$

$\therefore$ C is a skew symmetric odd order matrix

$\therefore |C| = |A^2B^2 - B^2A^2| = 0$

$\therefore$ Equation $(A^2B^2 - B^2A^2)\, X = 0$ has infinitely many solutions

32. Answer (1)

$$\mathbf{A} = \begin{bmatrix} \mathbf{1} & -\alpha \\ \alpha & \beta \end{bmatrix}$$

$\because AA^T = I$

So, $I + \alpha^2 = 1 \qquad \Rightarrow \alpha^2 = 0$

and $\alpha^2 + \beta^2 = 1 \qquad \Rightarrow \beta^2 = 1$

then $\alpha^4 + \beta^4 = 1$

34. Answer (3)

Let $\mathbf{A} = \begin{bmatrix} \mathbf{a} & \mathbf{c} \\ \mathbf{c} & \mathbf{b} \end{bmatrix}$

$$\mathbf{A}^2 = \begin{bmatrix} \mathbf{a} & \mathbf{c} \\ \mathbf{c} & \mathbf{b} \end{bmatrix}\begin{bmatrix} \mathbf{a} & \mathbf{c} \\ \mathbf{c} & \mathbf{b} \end{bmatrix} = \begin{bmatrix} \mathbf{a^2+c^2} & \mathbf{ac+bc} \\ \mathbf{ac+bc} & \mathbf{c^2+b^2} \end{bmatrix}$$

$a^2 + b^2 + 2c^2 = 1$ $\qquad$ as $a, b, c \in z$

$c = 0$ and $a, b = \pm 1$

Total 4 matrices are possible

35. Answer (4)

$$\because \mathbf{A^2} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{4} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}, \mathbf{A^4}\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{16} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}, \ldots\ldots$$

$$\text{So, } \mathbf{A^{20}} + \alpha\mathbf{A^{19}} + \beta\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{2^{20}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} + \alpha\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{2^{19}} & \mathbf{0} \\ \mathbf{3} & \mathbf{0} & \mathbf{-1} \end{bmatrix}$$

$$+\beta\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} & \mathbf{0} \\ \mathbf{3} & \mathbf{0} & \mathbf{-1} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{1}+\alpha+\beta & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{2^{20}}+\alpha\mathbf{2^{19}}+\mathbf{2}\beta & \mathbf{0} \\ \mathbf{3}\alpha+\mathbf{3}\beta & \mathbf{0} & \mathbf{1}-\alpha-\beta \end{bmatrix}$$

Clearly $\alpha + \beta = \mathbf{0}$ and $\mathbf{2^{20}} + \alpha\cdot\mathbf{2^{19}} + \mathbf{2}\beta = \mathbf{4}$

$\Rightarrow \alpha = \mathbf{-2}$ and $\beta = \mathbf{2}$

36. Answer (766)

Let matrix be $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Then $AA^T = \begin{bmatrix} a^2+b^2+c^2 & - & - \\ - & d^2+e^2+f^2 & - \\ - & - & g^2+h^2+i^2 \end{bmatrix}$

$$= \frac{9!}{6!2!1!} = 252 \text{ cases}$$

Case-III One 2's, five 1's and three zero

$$= \frac{9!}{5!3!} = 504 \text{ cases}$$

Case-IV Nine ones = 1 case

$\therefore$ Total cases = 9 + 252 + 504 + 1 = 766

37. Answer (1)

A = XB

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\sqrt{3}a_1 = b_1 - b_2 \Rightarrow 3a_1^2 = b_1^2 + b_2^2 - 2b_1b_2 \quad ...(i)$$

$$\sqrt{3}a_2 = b_1 + kb_2 \Rightarrow 3a_2^2 = b_1^2 + k^2b_2^2 + 2kb_1b_2 \quad ...(ii)$$

(i) + (ii) $\Rightarrow$

$$3\left(a_1^2 + a_2^2\right) = 2b_1^2 + \left(k^2 + 1\right)b_2^2 + 2(k-1)b_1b_2$$

$$\Rightarrow 2\left(b_1^2 + b_2^2\right) = 2b_1^2 + \left(k^2 + 1\right)b_2^2 + 2(k-1)b_1b_2$$

$$\left(1 - k^2\right)b_2^2 = 2(k-1)b_1b_2$$

$$(k-1)\left[(k+1)b_2^2 + 2b_1b_2\right] = 0$$

$\Rightarrow$ k = 1

38. Answer (10)

$$\because \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$\Rightarrow$ $a\alpha + b\beta = \alpha$ and $c\alpha + d\beta = \beta$

$$\Rightarrow \frac{\alpha}{\beta} = \frac{b}{1-a} = \frac{1-d}{c}$$

$\Rightarrow$ bc = ad – a – d + 1

$\Rightarrow$ ad – bc = a + d – 1

= 2020

$$= \frac{1}{5}\left(\begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{bmatrix}\right)$$

$$= \frac{1}{5}\begin{bmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{bmatrix} \Rightarrow \text{tr}(A) = 1$$

Similarly,

$$B = \frac{1}{5}(2(A+2B) - (2A-B))$$

$$= \frac{1}{5}\left(\begin{bmatrix} 2 & 4 & 0 \\ 12 & -6 & 6 \\ -10 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{5}\begin{bmatrix} 0 & 6 & -5 \\ 10 & -5 & 0 \\ -10 & 5 & 0 \end{bmatrix} \Rightarrow \text{tr}(B) = -1$$

Tr(A) – Tr(B) = 1 – (–1) = 2

40. Answer (2)

For reflexive,

$PAP^{-1} = A$ is true if P = I

For symmetric,

If $PAP^{-1} = B$ then $PBP^{-1} = A$ must be true

$\because$ $PAP^{-1} = B \Rightarrow A = P^{-1}BP$

and $PBP^{-1} = P(PAP^{-1})P^{-1} = P^2A(P^{-1})^2$ is equal to A if P is involutory matrix (i.e. $P^2 = I$)

For transitive,

If $PAP^{-1} = B$ and $PBP^{-1} = C$ then $PAP^{-1} = C$ must be true

$\because$ $C = PBP^{-1} = P^2AP^{-1}$ will be equal to PAP^{-1} if P is idempotent matrix (i.e. $P^2 = P$)

Hence relation R is an equivalence relation.

$$= \frac{1}{2}\begin{bmatrix} 0 & 3-a \\ a-3 & 0 \end{bmatrix}$$

$$\det(Q) = \frac{1}{4}(a-3)^2 = 9 \Rightarrow a-3 = \pm 6$$

$$a = 9, -3$$

$$P = \frac{A + A^T}{2} = \frac{1}{2}\begin{bmatrix} 4 & 3+a \\ a+3 & 0 \end{bmatrix}$$

$$\det(P) = \frac{1}{4} - (a+3)^2$$

$\therefore$ $\det(P) = 0$ or $\frac{-144}{4} = 36$

$\therefore$ Required sum 36

42. Answer (910)

Let $A = I + C$ where $C = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$C^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $C^n = 0$ for $n \ge 3$

So $A^n = (I + C)^n = I + nC + {}^nC_2.C^2$.

So $(b_{13})_n = 0 + 0 + {}^nC_2.1 = {}^nC_2$

Now b_{13} element of $7.A^{20} - 20.A^7 + 2I$

$$= 7({}^{20}C_2) - 20({}^7C_2) + 0$$
$$= 7 \times 190 - 20 \times 21$$
$$= 70[19 - 6] = 910$$

43. Answer (108)

$$\text{adj}(2A^{-1}) = |2A^{-1}|(2A^{-1})^{-1} = \frac{8}{|A|}\cdot\frac{1}{2}A = \frac{4A}{|A|}$$

So, $|3\text{adj}\,(2A^{-1})| = \left|12\frac{A}{|A|}\right| = \left(\frac{12}{|A|}\right)^3 \cdot |A| = \frac{12^3}{|A|^2}$

$\because$ $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \Rightarrow |A| = 4$

Hence, $|3\text{adj}\,(2A^{-1})| = \frac{12^3}{4^2} = 108$

$\because$ $A \cdot B = B$

$\because$ Sum of all entries of A^3 is equal to the only element of $B^T \cdot A^3 \cdot B$

$\because$ $B^T \cdot A^3 \cdot B = B^T \cdot A^2 \cdot (AB) = B^T \cdot A^2 \cdot B = B^T \cdot B$

$= B^T \cdot B = [3]_{1\times 1}$

45. Answer (3125)

Let $B = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \\ \alpha_1 & \alpha_2 & \alpha_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix}$$

$$BA = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha_2 & \alpha_1 & \alpha_3 \\ \beta_2 & \beta_1 & \beta_3 \\ \gamma_2 & \gamma_1 & \gamma_3 \end{bmatrix}$$

$AB = BA \Rightarrow \beta_1 = \alpha_2,\ \beta_2 = \alpha_1,\ \beta_3 = \alpha_3,\ \gamma_1 = \gamma_2$

5 places can be filled independently in $5^5 = 3125$ ways = 3125 matrices

46. Answer (11)

Let $B = \begin{bmatrix} 0 & i \\ 1 & 0 \end{bmatrix}$

$\Rightarrow$ $B^2 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$

$\Rightarrow$ $B^4 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow$ $B^8 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

hence n must be a multiple of 8.

So n = 16, 24, 32, ..., 96

No. of values of n = 11.

$$P^3 = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$$

$$\Rightarrow \quad P^n = \begin{bmatrix} 1 & 0 \\ \frac{n}{2} & 1 \end{bmatrix}$$

48. Answer (3)

$$\because \quad A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$$

$$\text{Then } A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 \\ 1/6 & 1/6 \end{bmatrix}$$

$$\therefore \quad \alpha I + \beta A = \begin{bmatrix} \alpha+\beta & 2\beta \\ -\beta & \alpha+4\beta \end{bmatrix}$$

$$\therefore \quad \beta = -\frac{1}{6} \text{ and } \alpha = \frac{5}{6}$$

$$\therefore \quad 4(\alpha-\beta) = 4\left(\frac{5}{6}+\frac{1}{6}\right) = 4$$

49. Answer (4)

$A^5 = B^5$ ...(i)

$A^3B^2 = A^2B^3$ ...(ii)

$A^5 - A^3B^2 = B^5 - A^2B^3$

$A^3(A^2 - B^2) = B^3(B^2 - A^2) = -B^3(A^2 - B^2)$

$A^3(A^2 - B^2) + B^3(A^2 - B^2) = 0$

$(A^3 + B^3)(A^2 - B^2) = 0$

$$\left|(A^3+B^3)(A^2-B^2)\right| = 0$$

$$\left|A^3+B^3\right| \times \left|A^2-B^2\right| = 0$$

$$\Rightarrow \quad \left|A^3+B^3\right| = 0 \quad \left(\because \left|A^2-B^2\right| \neq 0\right)$$

50. Answer (2020)

$$A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

$M = A + A^2 + \dots\dots\dots + A^{20}$

$$\begin{bmatrix} 0 & 0 & \sum 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 20 \end{bmatrix}$$

$$\text{Because } \sum 1 = 20,\ \sum_{n=1}^{20} n = \frac{20\times 21}{2} = 210$$

$$\frac{1}{2}\sum_{n=1}^{20} n(n+1) = \frac{1}{2}\times\frac{20\times 21\times 22}{3} = 1540$$

Sum = 20 + 20 + 20 + 210 + 210 + 1540 = 2020

51. Answer (2)

$$\begin{vmatrix} 1 & 3 & \lambda \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 1 - 3(2) + \lambda(1) = 0 \Rightarrow \lambda = 5$$

For $\lambda \neq 5$ there will be unique solution

$$p = 1 - \frac{1}{6} = \frac{5}{6}$$

For $\lambda = 5$ and $\mu = 3$ there will be infinitely many solutions and for $\lambda = 5$ and $\mu \neq 3$ there will be no solution.

$$q = \frac{1}{6}\cdot\left(1-\frac{1}{6}\right) = \frac{5}{36}$$

52. Answer (1)

$$A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$$

characteristic equation is

$|A - xI| = 0$

$$\begin{vmatrix} -x & 2 \\ K & -1-x \end{vmatrix} = 0$$

$x(x + 1) - 2K = 0$

$x^2 + x - 2K = 0$

A satisfies its characteristic equation

i.e. $A^2 + A - 2KI = 0$

$\Rightarrow A^2 = 2KI - A$ ...(i)

$\Rightarrow A^3 = 2KA - A^2 = 2KA - (2KI - A)$ (using (i))

$\Rightarrow A^3 = (2K + 1) A - 2KI$

$\Rightarrow A^4 = (2K + 1) A^2 - 2KA$

$= (2K + 1) (2KI - A) - 2KA$

$A^4 + 3A = 2I$ ...(iii)

Comparing the coefficients

$4K + 1 = 3$ and $4K^2 + 2K = 2$

$K = \frac{1}{2}$ and $2K^2 + K - 1 = 0$

$(2K - 1)(K + 1) = 0$

$K = \frac{1}{2}, -1$

$\therefore \quad K = \frac{1}{2}$

53. Answer (8)

$(I - A)^3 = I - A^3$

$I - A^3 - 3A(I - A) = I - A^3$

$3A(I - A) = 0 \quad \Rightarrow A(I - A) = 0$

$$\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}\begin{bmatrix} 1-a & -b \\ 0 & 1-d \end{bmatrix} = 0$$

$$\begin{bmatrix} a(1-a) & -b(a-1+d) \\ 0 & d(1-d) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a(1 - a) = 0,\ d(1 - d) = 0,\ b(a - 1 + d) = 0$

$$\begin{array}{l|l|l|l|} a = 0 & a = 1 & a = 0 & a = 1 \\ d = 0 & d = 0 & d = 1 & d = 1 \\ b = 0 & b = -1, 0, 1 & b = -1, 0, 1 & b = 0 \end{array}$$

So Total 8 cases

54. Answer (1)

A - adj $A = |A| = A.A^T$

$\Rightarrow$ adj $A = A^T$

$$\begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$\Rightarrow \quad 5a = 2,\ b = 3$

So, $5a + b = 5$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & -3 \\ -4 & 1-\lambda \end{vmatrix}$$

$= (2 - 2\lambda - \lambda + \lambda^2) - 12$

$f(\lambda) = \lambda^2 - 3\lambda - 10$

$\because \quad A$ satisfies $f(\lambda)$

$\therefore \quad A^2 - 3A - 10I = 0$

$A^2 - 3A = 10I$

$3A^2 - 9A = 30I$

$3A^2 + 12A = 30I + 21A$

$$= \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix} + \begin{bmatrix} 42 & -63 \\ -84 & 21 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\text{adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

56. Answer (1)

$$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$$

$M = A^2 + A^4 + A^6 + \ldots + A^{20}$

$= -4I + 16I - 64I + \ldots$ upto 10 terms

$= -I\,[4 - 16 + 64 \ldots +$ upto 10 terms$]$

$$= -I.4\left[\frac{(-4)^{10} - 1}{-4 - 1}\right] = \frac{4}{5}(2^{20} - 1)I$$

$N = A^1 + A^3 + A^5 + \ldots + A^{19}$

$= A - 4A + 16A + \ldots$ upto 10 terms

$$N^2 = \frac{(2^{20}-1)^2}{2^5}A^2 = \frac{-4}{25}(2^{20}-1)^2 I$$

$$MN^2 = \frac{-16}{125}(2^{20}-1)^3 I = KI \qquad (K \neq \pm 1)$$

$(MN^2)^T = (KI)^T = KI$

57. Answer (414)

Let matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

We need

$a + b + c + d + e + f + g + h + i = 5$

Possible cases	Number of ways
$5 \rightarrow$ 1's, $4 \rightarrow$ zeroes	$\frac{9!}{5!4!} = 126$
$6 \rightarrow$ 1's, $2 \rightarrow$ zeroes, $1 \rightarrow -1$	$\frac{9!}{6!2!} = 252$
$7 \rightarrow$ 1's, $2 \rightarrow -1$'s	$\frac{9!}{7!2!} = 36$

Total ways = 126 + 252 + 36 = 414

58. Answer (1)

$$A^2 = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = A \qquad \Rightarrow A^K = A,\ K \in I$$

$$B^2 = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = B$$

So, $B^K = B,\ K \in I$

$nA^n + mB^m = nA + mB$

$$= \begin{bmatrix} 2n - 2n \\ n - n \end{bmatrix} + \begin{bmatrix} -m & 2m \\ -m & 2m \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, $2n - m = 1, -n + m = 0, 2m - n = 1$

So, $(m, n) = (1, 1)$

$$\therefore X = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\therefore X^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore Y = \alpha I + \beta X + \gamma X^2 \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}$$

$\because \quad Y \cdot Y^{-1} = I$

$$\therefore \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}\begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{\alpha}{5} & \frac{\beta - 2\alpha}{5} & \frac{\alpha - 2\beta + \gamma}{5} \\ 0 & \frac{\alpha}{5} & \frac{\beta - 2\alpha}{5} \\ 0 & 0 & \frac{\alpha}{5} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore \quad \alpha = 5, \beta = 10, \gamma = 15$

$\therefore \quad (\alpha - \beta + \gamma)^2 = 100$

60. Answer (180)

$\because$ Sum of all entries of matrix A must be prime p such that $2 < p < 8$ then sum of entries may be 3, 5 or 7.

If sum is 3 then possible entries are (0, 0, 0, 3), (0, 0, 1, 2) or (0, 1, 1, 1).

$\therefore$ Total number of matrices = 4 + 4 + 12 = 20

If sum of 5 then possible entries are

(0, 0, 0, 5), (0, 0, 1, 4), (0, 0, 2, 3), (0, 1, 1, 3), (0, 1, 2, 2) and (1, 1, 1, 2).

(0, 0, 2, 5), (0, 0, 3, 4), (0, 1, 1, 5), (0, 3, 3, 1), (0, 2, 2, 3), (1, 1, 1, 4), (1, 2, 2, 2), (1, 1, 2, 3) and (0, 1, 2, 4)

Total number of matrices with sum 7 = 104

$\therefore$ Total number of required matrices

= 20 + 56 + 104

= 180

61. Answer (25)

$$\therefore \quad A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}\begin{bmatrix} 1+i & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix}\begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix} = I$$

So $A^5 = A$, $A^9 = A$ and so on.

Clearly n = 1, 5, 9, ..., 97

Number of values of n = 25

62. Answer (1)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 2^0 & 2^1 & 2^2 \\ 2^{-1} & 2^0 & 2^1 \\ 2^{-2} & 2^{-1} & 2^0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 12 \\ \frac{3}{2} & 3 & 6 \\ \frac{3}{4} & \frac{3}{2} & 3 \end{bmatrix} = 3A$$

$A^2 = 3A$

$A^3 = A.A^2 = A(3A) = 3A^2 = 3^2A$

$A^4 = 3^3A$

Now

$A^2 + A^3 + \ldots + A^{10}$

$A[3^1 + 3^2 + 3^3 + \ldots + 3^9]$

$$= \frac{3\left[3^9 - 1\right]}{3-1}A$$

$$= \frac{\left(3^{10} - 3\right)}{2}A$$

$$N = M^2 + M^4 + \ldots + M^{98}$$

$$= \left[-\alpha^2 + \alpha^4 - \alpha^6 + \ldots\right]I$$

$$= \frac{-\alpha^2\left(1-(-\alpha^2)^{49}\right)}{1+\alpha^2} \cdot I$$

$$I - M^2 = (1+\alpha^2)I$$

$$(I - M^2)N = -\alpha^2(\alpha^{98} + 1) = -2$$

$$\therefore \quad \alpha = 1$$

64. Answer (17)

$$\text{Here } A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

We get $A^2 = A$ and similarly for

$$B = A - I = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

We get $B^2 = -B \Rightarrow B^3 = B$

$\therefore$ $A^n + (\omega B)^n = A + (\omega B)^n$ for $n \in \mathrm{N}$

For ω^n to be unity n shall be multiple of 3 and for B^n to be B. n shell be 3, 5, 7, … 99

$\therefore$ n = {3, 9, 15,….. 99}

Number of elements = 17.

65. Answer (4)

$$A^2 = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix}$$

$$\alpha A^2 + \beta A = \begin{bmatrix} -3\alpha & -8\alpha \\ 8\alpha & 21\alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -2\beta & -5\beta \end{bmatrix}$$

$$= \begin{bmatrix} -3\alpha + \beta & -8\alpha + 2\beta \\ 8\alpha - 2\beta & 21\alpha - 5\beta \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

So, $\alpha + \beta = 10$

66. Answer (5376)

Sum of all diagonal elements is equal to sum of square of each element of the matrix.

i.e., $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

then $t_r\ (A \cdot A^T)$

$= a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 + c_1^2 + c_2^2 + c_3^2$

$\because \quad a_i, b_i, c_i \in \{-1, 0, 1\}$ for $i = 1, 2, 3$

$\therefore$ Exactly three of them are zero and rest are 1 or – 1.

Total number of possible matrices ${}^9C_3 \times 2^6$

$= \dfrac{9 \times 8 \times 7}{6} \times 64$

$= 5376$

67. Answer (2)

Characteristic equation of A is given by

$|A - \lambda I| = 0$

$\begin{vmatrix} 4-\lambda & -2 \\ \alpha & \beta-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - (4 + \beta)\lambda + (4\beta + 2\alpha) = 0$

So, $A^2 - (4 + \beta)A + (4\beta + 2\alpha)I = 0$

$|A| = 4\beta + 2\alpha = 18$

68. Answer (2)

$(A + B)^2 = A^2 + B^2 + AB + BA$

$= A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$\therefore \quad B^2 + AB + BA = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \quad \ldots(1)$

$BA = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} \beta+2 & \alpha-\beta \\ 1 & -1 \end{bmatrix}$

$B^2 = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \beta^2+1 & \beta \\ \beta & 1 \end{bmatrix}$

By (1) we get

$\begin{bmatrix} \beta^2+2\beta+2 & \alpha+1 \\ \alpha+3\beta+1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$\therefore \quad \alpha = 1\ \beta = 0 \ \Rightarrow \alpha_1 = 1$

Similarly If $A^2 + AB + BA = 0$ then

$\left(A^2 = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} -1 & -1-\alpha \\ 2+2\alpha & \alpha^2-2 \end{bmatrix}\right)$

$\begin{bmatrix} 2\beta & \alpha-\beta+1-1-\alpha \\ \alpha+2\beta+1+2+2\alpha & \alpha^2-2+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \beta = 0$ and $\alpha = -1 \Rightarrow \alpha_2 = -1$

$\therefore \quad |\alpha_1 - \alpha_2| = |2| = 2.$

69. Answer (3)

(A) $M = A^4 - B^4$

$M^T = (A^4 - B^4)^T = (A^T)^4 - (B^T)^4$

$= A^4 - (-B)^4 = A^4 - B^4 = M$

(B) $M = AB - BA$

$M^T = (AB - BA)^T = (AB)^T - (BA)^T$

$= B^TA^T - A^TB^T$

$= -BA - A(-B)$

$= AB - BA = M$

(C) $M = B^5 - A^5$

$M^T = (B^T)^5 - (A^T)^5 = -(B^5 + A^5) \neq -M$

(D) $M = AB + BA$

$M^T = (AB)^T + (BA)^T$

$= B^TA^T + A^TB^T = -BA - AB = -M$

(2) By $R_1 \leftrightarrow R_2$, $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ is possible

(3) This matrix can't be obtained

(4) By $R_2 \to R_2 + 2R_1$, $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ is possible

71. Answer (10*)

Given $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A^4 = \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \quad A^k = \begin{bmatrix} 1 & 0 & 3k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \quad X'A^kX = [1\,1\,1]\begin{bmatrix} 1 & 0 & 3k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [3k+3]$

$\Rightarrow$ $[3k + 3] = 33$ (here it shall be [33] as matrix can't be equal to a scalar)

i.e. $[3k + 3] = 33$

$3k + 3 = [33] \quad \Rightarrow k = 10$

If k is odd and apply above process, we don't get odd value of k

$\therefore \quad k = 10$

72. Answer (4)

$A'BA = [1 \;\; 1 \;\; 1]\begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}A$

$= \left[9^2 + 12^2 - 15^2 - 10^2 + 13^2 + 16^2 + 11^2 - 14^2 + 17^2\right]$

$= [(9^2 - 10^2) + (11^2 + 12^2) + (13^2 - 14^2) + (16^2 - 15^2) + 17^2]$

$= [-19 + 265 + (-27) + 31 + 289]$

$= [585 - 46] = [539]$

73. Answer (100)

$S = \left\{\begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix} : a, b \in \{1, 2, 3, ..., 100\}\right\}$

$\because \quad A = \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}$ then even powers of

A as $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, if $b = 1$ and $a \in \{1, \ldots, 100\}$

Here, $n(n + 1)$ is always even.

$\therefore \quad T_1, T_2, T_3, \ldots, T_n$ are all I for $b = 1$ and each value of a.

$\therefore \quad \bigcap_{n=1}^{100} T_n = 100$

74. Answer (282)

In a 3 × 3 order matrix there are 9 entries.

These nine entries are zero or one.

The sum of positive prime entries are 2, 3, 5 or 7.

Total possible matrices

$= \frac{9!}{2!\cdot 7!} + \frac{9!}{3!\cdot 6!} + \frac{9!}{5!\cdot 4!} + \frac{9!}{7!\cdot 2!}$

$= 36 + 84 + 126 + 36$

$= 282$

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