Chapter 27

Probability

1. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ... , 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals **[AIEEE-2009]**

   (1) $\frac{1}{7}$ (2) $\frac{5}{14}$

   (3) $\frac{1}{50}$ (4) $\frac{1}{14}$

2. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is **[AIEEE-2010]**

   (1) $\frac{1}{3}$ (2) $\frac{2}{7}$

   (3) $\frac{1}{21}$ (4) $\frac{2}{23}$

3. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}.

   Statement-1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

   Statement-2 : If the four chosen numbers from an AP, then the set of all possible values of common difference is {+1, +2, +3, +4, +5}. **[AIEEE-2010]**

   (1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

   (2) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

   (3) Statement-1 is true, Statement-2 is false

   (4) Statement-1 is false, Statement-2 is true

4. Let A, B, C be pairwise independent events with $P(C) > 0$ and $P(A \cap B \cap C) = 0$. Then $P(A^C \cap B^C|C)$ is equal to **[AIEEE-2011]**

   (1) $P(A^C) - P(B^C)$

   (2) $P(A^C) - P(B)$

   (3) $P(A) - P(B^C)$

   (4) $P(A^C) + P(B^C)$

5. Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is **[AIEEE-2012]**

   (1) $\frac{1}{5}$ (2) $\frac{1}{4}$

   (3) $\frac{2}{5}$ (4) $\frac{3}{8}$

6. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is **[JEE (Main)-2013]**

   (1) $\frac{17}{3^5}$ (2) $\frac{13}{3^5}$

   (3) $\frac{11}{3^5}$ (4) $\frac{10}{3^5}$

7. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are **[JEE (Main)-2014]**

   (1) Independent but not equally likely

   (2) Independent and equally likely

   (3) Mutually exclusive and independent

   (4) Equally likely but not independent

8. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one the boxes contains exactly 3 balls is **[JEE (Main)-2015]**

(1) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (2) $55\left(\frac{2}{3}\right)^{10}$

(3) $220\left(\frac{1}{3}\right)^{12}$ (4) $22\left(\frac{1}{3}\right)^{11}$

9. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true? **[JEE (Main)-2016]**

(1) E_2 and E_3 are independent

(2) E_1 and E_3 are independent

(3) E_1, E_2 and E_3 are independent

(4) E_1 and E_2 are independent

10. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is **[JEE (Main)-2017]**

(1) 6 (2) 4

(3) $\frac{6}{25}$ (4) $\frac{12}{5}$

11. For three events A, B and C, P (Exactly one of A or B occurs) = P(Exactly one of B or C occurs)

= P (Exactly one of C or A occurs) = $\frac{1}{4}$ and

P(All the three events occur simultaneously) = $\frac{1}{16}$.

Then the probability that at least one of the events occurs, is **[JEE (Main)-2017]**

(1) $\frac{7}{16}$ (2) $\frac{7}{64}$

(3) $\frac{3}{16}$ (4) $\frac{7}{32}$

12. If two different numbers are taken from the set {0, 1, 2, 3, ..., 10}; then the probability that their sum as well as absolute difference are both multiple of 4, is **[JEE (Main)-2017]**

(1) $\frac{12}{55}$ (2) $\frac{14}{45}$

(3) $\frac{7}{55}$ (4) $\frac{6}{55}$

13. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is **[JEE (Main)-2018]**

(1) $\frac{3}{10}$ (2) $\frac{2}{5}$

(3) $\frac{1}{5}$ (4) $\frac{3}{4}$

14. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X = 1) + P(X = 2)$ equals **[JEE (Main)-2019]**

(1) $\frac{24}{169}$ (2) $\frac{25}{169}$

(3) $\frac{49}{169}$ (4) $\frac{52}{169}$

15. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is **[JEE (Main)-2019]**

(1) $\frac{26}{49}$ (2) $\frac{21}{49}$

(3) $\frac{32}{49}$ (4) $\frac{27}{49}$

16. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1, 2, 3, ..., 9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is **[JEE (Main)-2019]**

(1) $\frac{13}{36}$ (2) $\frac{15}{72}$

(3) $\frac{19}{36}$ (4) $\frac{19}{72}$

17. If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is **[JEE (Main)-2019]**

(1) 4 (2) 5

(3) 3 (4) 6

18. Two integers are selected at random from the set {1, 2, ..., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is **[JEE (Main)-2019]**

(1) $\frac{3}{5}$ (2) $\frac{7}{10}$

(3) $\frac{1}{2}$ (4) $\frac{2}{5}$

19. A bag contains 30 white ball and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, then $\left(\frac{\text{mean of } X}{\text{Standard deviation of } X}\right)$ is equal to

[JEE (Main)-2019]

(1) $3\sqrt{2}$ (2) $4\sqrt{3}$

(3) $\frac{4\sqrt{3}}{3}$ (4) 4

20. Let S = {1, 2, ..., 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is **[JEE (Main)-2019]**

(1) $\frac{7}{2^{20}}$ (2) $\frac{6}{2^{20}}$

(3) $\frac{4}{2^{20}}$ (4) $\frac{5}{2^{20}}$

21. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to **[JEE (Main)-2019]**

(1) $\frac{150}{6^5}$ (2) $\frac{225}{6^5}$

(3) $\frac{175}{6^5}$ (4) $\frac{200}{6^5}$

22. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither of NCC nor for NSS is

[JEE (Main)-2019]

(1) $\frac{5}{6}$ (2) $\frac{1}{3}$

(3) $\frac{1}{6}$ (4) $\frac{2}{3}$

23. In a game, a man wins Rs. 100 if he gets 5 or 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is **[JEE (Main)-2019]**

(1) $\frac{400}{3}$ gain (2) 0

(3) $\frac{400}{9}$ loss (4) $\frac{400}{3}$ loss

24. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct? **[JEE (Main)-2019]**

(1) $P(A|B) = P(B) - P(A)$

(2) $P(A|B) \leq P(A)$

(3) $P(A|B) \geq P(A)$

(4) $P(A|B) = 1$

25. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is **[JEE (Main)-2019]**

(1) 5 (2) 2

(3) 4 (4) 3

26. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is **[JEE (Main)-2019]**

(1) $\frac{7}{32}$ (2) $\frac{25}{192}$

(3) $\frac{1}{192}$ (4) $\frac{25}{32}$

27. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is **[JEE (Main)-2019]**

(1) $\frac{1}{11}$ (2) $\frac{1}{12}$

(3) $\frac{1}{10}$ (4) $\frac{1}{17}$

28. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is **[JEE (Main)-2019]**

(1) 8 (2) 6

(3) 5 (4) 7

29. Let a random variable X have a binomial distribution with mean 8 and variance 4.

If $P(X \le 2) = \frac{k}{2^{16}}$, then k is equal to **[JEE (Main)-2019]**

(1) 121 (2) 1

(3) 17 (4) 137

30. For an initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is **[JEE (Main)-2019]**

(1) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$ (2) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

(3) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$ (4) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

31. A person throws two fair dice. He wins ₹ 15 for throwing a doublet (same numbers on the two dice), wins ₹ 12 when the throw results in the sum of 9, and loses ₹ 6 for any other outcome on the throw. Then the expected gain/loss (in ₹) of the person is: **[JEE (Main)-2019]**

(1) $\frac{1}{2}$ loss (2) 2 gain

(3) $\frac{1}{2}$ gain (4) $\frac{1}{4}$ loss

32. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for k = 3, 4, 5, otherwise X takes the value –1. Then the expected value of X, is **[JEE (Main)-2020]**

(1) $\frac{3}{16}$ (2) $-\frac{1}{8}$

(3) $-\frac{3}{16}$ (4) $\frac{1}{8}$

33. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to **[JEE (Main)-2020]**

(1) 4 (2) $\frac{17}{4}$

(3) $\frac{17}{8}$ (4) $\frac{17}{2}$

34. Let A and B be two independent events such that $P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{6}$. Then, which of the following is TRUE? **[JEE (Main)-2020]**

(1) $P(A/B)=\frac{2}{3}$

(2) $P(A'/B')=\frac{1}{3}$

(3) $P(A/B')=\frac{1}{3}$

(4) $P(A/(A\cup B))=\frac{1}{4}$

35. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is **[JEE (Main)-2020]**

(1) 0.01 (2) 0.20

(3) 0.02 (4) 0.10

36. In a box, there are 20 cards, out of which 10 are labelled as A and the remaining 10 are labelled as B. Cards are drawn at random, one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is

[JEE (Main)-2020]

(1) $\frac{9}{16}$ (2) $\frac{13}{16}$

(3) $\frac{11}{16}$ (4) $\frac{15}{16}$

37. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is

[JEE (Main)-2020]

(1) $\frac{945}{2^{10}}$ (2) $\frac{965}{2^{11}}$

(3) $\frac{965}{2^{10}}$ (4) $\frac{945}{2^{11}}$

38. A random variable X has the following probability distribution

X :	1	2	3	4	5
$P(X)$:	K^2	$2K$	K	$2K$	$5K^2$

Then $P(X > 2)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{7}{12}$ (2) $\frac{23}{36}$

(3) $\frac{1}{36}$ (4) $\frac{1}{6}$

39. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is **[JEE (Main)-2020]**

(1) $\frac{4}{17}$ (2) $\frac{2}{3}$

(3) $\frac{2}{5}$ (4) $\frac{8}{17}$

40. Let E^C denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$. Then $P(E_2^C \cap E_3^C / E_1)$ is equal to **[JEE (Main)-2020]**

(1) $P\left(E_3^C\right) - P\left(E_2^C\right)$ (2) $P\left(E_2^C\right) + P(E_3)$

(3) $P(E_3) - P\left(E_2^C\right)$ (4) $P\left(E_3^C\right) - P(E_2)$

41. A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is

[JEE (Main)-2020]

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$

(3) $\frac{1}{8}$ (4) $\frac{1}{9}$

42. The probability that a randomly chosen 5-digit number is made from exactly two digits is

[JEE (Main)-2020]

(1) $\frac{150}{10^4}$ (2) $\frac{134}{10^4}$

(3) $\frac{121}{10^4}$ (4) $\frac{135}{10^4}$

43. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is **[JEE (Main)-2020]**

(1) $\frac{5}{31}$ (2) $\frac{31}{61}$

(3) $\frac{5}{6}$ (4) $\frac{30}{61}$

44. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is

[JEE (Main)-2020]

(1) $\frac{5}{101}$ (2) $\frac{10}{99}$

(3) $\frac{5}{33}$ (4) $\frac{15}{101}$

45. The probabilities of three events A, B and C are given by $P(A) = 0.6$, $P(B) = 0.4$ and $P(C) = 0.5$. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \le \alpha \le 0.95$, then β lies in the interval

[JEE (Main)-2020]

(1) [0.25, 0.35] (2) [0.35, 0.36]

(3) [0.36, 0.40] (4) [0.20, 0.25]

46. An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is ________ [JEE (Main)-2020]

47. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ________. [JEE (Main)-2020]

48. Four fair dice are thrown independently 27 times. Then the expected number of times, at least two dice show up a three or a five, is ______. [JEE (Main)-2020]

49. In a bombing attack, there is 50% chance that a bomb will hit the target. Atleast two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is ______. [JEE (Main)-2020]

50. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is : [JEE (Main)-2021]

(1) $\frac{3}{16}$ (2) $\frac{1}{2}$

(3) $\frac{1}{32}$ (4) $\frac{5}{16}$

51. Let B_i (i = 1, 2, 3) be three independent events in a sample space. The probability that only B_1 occurs is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)\,p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to _________. [JEE (Main)-2021]

52. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is : [JEE (Main)-2021]

(1) $\frac{65}{2^7}$ (2) $\frac{135}{2^9}$

(3) $\frac{65}{2^8}$ (4) $\frac{35}{2^7}$

53. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is : [JEE (Main)-2021]

(1) $\frac{1}{27}$ (2) $\frac{3}{8}$

(3) $\frac{3}{4}$ (4) $\frac{1}{8}$

54. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is : [JEE (Main)-2021]

(1) $\frac{1}{54}$ (2) $\frac{1}{36}$

(3) $\frac{5}{216}$ (4) $\frac{1}{72}$

55. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is : [JEE (Main)-2021]

(1) $\frac{2}{9}$ (2) $\frac{97}{297}$

(3) $\frac{122}{297}$ (4) $\frac{1}{5}$

56. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is : [JEE (Main)-2021]

(1) $\frac{7}{45}$ (2) $\frac{28}{45}$

(3) $\frac{14}{45}$ (4) $\frac{8}{45}$

57. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is : [JEE (Main)-2021]

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$

(3) $\frac{15}{2^{8}}$ (4) $\frac{15}{2^{14}}$

58. A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is : **[JEE (Main)-2021]**

(1) $\frac{1}{7}$ (2) $\frac{6}{7}$

(3) $\frac{4}{7}$ (4) $\frac{3}{7}$

59. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is : **[JEE (Main)-2021]**

(1) $\frac{52}{867}$ (2) $\frac{22}{425}$

(3) $\frac{3}{4}$ (4) $\frac{39}{50}$

60. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of even A is equal to: **[JEE (Main)-2021]**

(1) $\frac{9}{56}$ (2) $\frac{11}{27}$

(3) $\frac{3}{7}$ (4) $\frac{4}{9}$

61. Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is : **[JEE (Main)-2021]**

(1) $\frac{17}{36}$ (2) $\frac{1}{2}$

(3) $\frac{4}{9}$ (4) $\frac{5}{12}$

62. Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to ________. **[JEE (Main)-2021]**

63. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to : **[JEE (Main)-2021]**

(1) $\frac{1}{18}$ (2) $\frac{1}{3}$

(3) $\frac{1}{6}$ (4) $\frac{1}{9}$

64. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to : **[JEE (Main)-2021]**

(1) $\frac{80}{243}$ (2) $\frac{32}{625}$

(3) $\frac{40}{243}$ (4) $\frac{128}{625}$

65. The probability of selecting integers a $\in$ [–5, 30] such that $x^2 + 2(a + 4)x - 5a + 64 > 0$, for all $x \in \mathbf{R}$ is **[JEE (Main)-2021]**

(1) $\frac{1}{4}$ (2) $\frac{1}{6}$

(3) $\frac{7}{36}$ (4) $\frac{2}{9}$

66. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is **[JEE (Main)-2021]**

(1) $\frac{1}{66}$ (2) $\frac{1}{11}$

(3) $\frac{1}{9}$ (4) $\frac{2}{11}$

67. Let A, B and C be three events such that the probability that exactly one of A and B occurs is (1 – k), the probability that exactly one of B and C occurs is (1 – 2k), the probability that exactly one of C and A occurs is (1 – k) and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is **[JEE (Main)-2021]**

(1) Greater than $\frac{1}{2}$

(2) Exactly equal to $\frac{1}{2}$

(3) Greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

(4) Greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

68. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2×2 matrices. The probability that such formed matrices have all different entries and are non-singular, is **[JEE (Main)-2021]**

(1) $\frac{43}{162}$ (2) $\frac{45}{162}$

(3) $\frac{22}{81}$ (4) $\frac{23}{81}$

69. Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability that B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set **[JEE (Main)-2021]**

(1) $\{x \in \mathbf{R} : |x - 3| < 1\}$ (2) $\{x \in \mathbf{R} : |x - 1| < 1\}$

(3) $\{x \in \mathbf{R} : |x - 5| \le 1\}$ (4) $\{x \in \mathbf{R} : |x - 2| \le 1\}$

70. Let X be a random variable such that the probability function of a distribution is given by $P(X=0)=\frac{1}{2}$, $P(X=j)=\frac{1}{3^j}(j=1, 2, 3, ..., \infty)$. Then the mean of the distribution and P(X is positive and even) respectively are **[JEE (Main)-2021]**

(1) $\frac{3}{4}$ and $\frac{1}{9}$ (2) $\frac{3}{4}$ and $\frac{1}{16}$

(3) $\frac{3}{4}$ and $\frac{1}{8}$ (4) $\frac{3}{8}$ and $\frac{1}{8}$

71. A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is ________. **[JEE (Main)-2021]**

72. The probability that a randomly selected 2-digit number belongs to the set $\{n \in N: (2^n - 2)$ is a multiple of 3$\}$ is equal to **[JEE (Main)-2021]**

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) $\frac{1}{6}$

73. A student appeared in an examination consisting of 8 true - false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least 'n' correct answers is less than $\frac{1}{2}$, is **[JEE (Main)-2021]**

(1) 4 (2) 3

(3) 5 (4) 6

74. Let A and B be independent events such that P(A) = p, P(B) = 2p. The largest value of p for which P (exactly one of A, B occurs) = $\frac{5}{9}$, is **[JEE (Main)-2021]**

(1) $\frac{5}{12}$ (2) $\frac{2}{9}$

(3) $\frac{4}{9}$ (4) $\frac{1}{3}$

75. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(X \ge 5 \mid X > 2)$ is : **[JEE (Main)-2021]**

(1) $\frac{5}{6}$ (2) $\frac{125}{216}$

(3) $\frac{11}{36}$ (4) $\frac{25}{36}$

76. When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6}-x$ and its opposite face occurs with probability $\frac{1}{6}+x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to 7 in any die. If $0<x<\frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is **[JEE (Main)-2021]**

(1) $\frac{1}{12}$ (2) $\frac{1}{8}$

(3) $\frac{1}{16}$ (4) $\frac{1}{9}$

77. let n be an odd natural number such that the variance of 1, 2, 3, 4, ..., n is 14. Then n is equal to ______. **[JEE (Main)-2021]**

78. Each of the person A and B independently tosses three fair coins. The probability that both of them get the same number of heads is : **[JEE (Main)-2021]**

(1) $\frac{1}{8}$ (2) 1

(3) $\frac{5}{8}$ (4) $\frac{5}{16}$

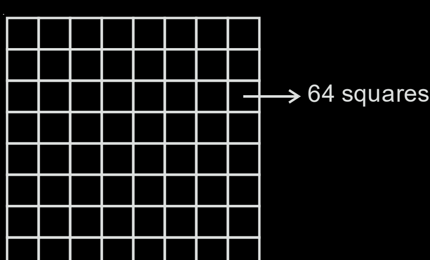
79. The probability distribution of random variable X is given by :

X	1	2	3	4	5
$P(X)$	K	$2K$	$2K$	$3K$	K

Let $p = P(1 < X < 4 \mid X < 3)$. If $5p = \lambda K$, then λ is equal to ______. **[JEE (Main)-2021]**

80. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to ______. **[JEE (Main)-2021]**

81. Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is : **[JEE (Main)-2021]**



(1) $\frac{2}{7}$ (2) $\frac{1}{7}$

(3) $\frac{1}{18}$ (4) $\frac{1}{9}$

82. Let X be a random variable with distribution.

x	-2	-1	3	4	6
$P(X = x)$	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean X is 2.3 and variance of X is σ^2, then $100\sigma^2$ is equal to **[JEE (Main)-2021]**

83. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than **[JEE (Main)-2021]**

(1) $\frac{1}{\log_{10} 4 + \log_{10} 3}$ (2) $\frac{9}{\log_{10} 4 - \log_{10} 3}$

(3) $\frac{4}{\log_{10} 4 - \log_{10} 3}$ (4) $\frac{1}{\log_{10} 4 - \log_{10} 3}$

84. Bag A contains 2 white, 1 black and 3 red balls and bas B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random, are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to ______. **[JEE (Main)-2022]**

(1) 13 (2) 6

(3) 4 (4) 3

85. If a random variable X follows the Binomial distribution $B(33, p)$ such that $3P(X = 0) = P(X = 1)$, then the value of $\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$ is equal to: **[JEE (Main)-2022]**

(1) 1320 (2) 1088

(3) $\frac{120}{1331}$ (4) $\frac{1088}{1089}$

86. A random variable x has the following probability distribution :

X	0	1	2	3	4
$P(X)$	k	$2k$	$4k$	$6k$	$8k$

The value of $P(1 < x < 4 \mid x \le 2)$ is equal to **[JEE (Main)-2022]**

(1) $\frac{4}{7}$ (2) $\frac{2}{3}$

(3) $\frac{3}{7}$ (4) $\frac{4}{5}$

87. In an examination, there are 10 true-false type questions. Out of 10, a student can guess the answer of 4 questions correctly with probability $\frac{3}{4}$ and the remaining 6 questions correctly with probability $\frac{1}{4}$. If the probability that the student guesses the answers of exactly 8 questions correctly out of 10 is $\frac{27k}{4^{10}}$, then k is equal to

[JEE (Main)-2022]

88. Let E_1 and E_2 be two events such that the conditional probabilities $P(E_1 | E_2) = \frac{1}{2}$, $P(E_2 | E_1) = \frac{3}{4}$ and $P(E_1 \cap E_2) = \frac{1}{8}$. Then : **[JEE (Main)-2022]**

(1) $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

(2) $P(E_1' \cap E_2') = P(E_1') \cdot P(E_2)$

(3) $P(E_1 \cap E_2') = P(E_1) \cdot P(E_2)$

(4) $P(E_1' \cap E_2) = P(E_1) \cdot P(E_2)$

89. A biased die is marked with numbers 2, 4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is : **[JEE (Main)-2022]**

(1) $\frac{7}{2^{11}}$ (2) $\frac{7}{2^{12}}$

(3) $\frac{3}{2^{10}}$ (4) $\frac{13}{2^{12}}$

90. Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting atmost two heads is: **[JEE (Main)-2022]**

(1) $\frac{275}{6^5}$ (2) $\frac{36}{5^4}$

(3) $\frac{181}{5^5}$ (4) $\frac{46}{6^4}$

91. If the probability that a randomly chosen 6-digit number formed by using digits 1 and 8 only is a multiple of 21 is p, then 96 p is equal to ________ .

[JEE (Main)-2022]

92. Five numbers, x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3,….., 18 and are arranged in the increasing order ($x_1 < x_2 < x_3 < x_4 < x_5$). The probability that $x_2 = 7$ and $x_4 = 11$ is:

[JEE (Main)-2022]

(1) $\frac{1}{136}$ (2) $\frac{1}{72}$

(3) $\frac{1}{68}$ (4) $\frac{1}{34}$

93. Let X be a random variable having binomial distribution $B(7, p)$. If $P(X = 3) = 5P(X = 4)$, then the sum of the mean and the variance of X is:

[JEE (Main)-2022]

(1) $\frac{105}{16}$ (2) $\frac{7}{16}$

(3) $\frac{77}{36}$ (4) $\frac{49}{16}$

94. If a point $A(x, y)$ lies in the region bounded by the y-axis, straight lines $2y + x = 6$ and $5x - 6y = 30$, then the probability that $y < 1$ is **[JEE (Main)-2022]**

(1) $\frac{1}{6}$ (2) $\frac{5}{6}$

(3) $\frac{2}{3}$ (4) $\frac{6}{7}$

95. Let $S = \{E_1, E_2, \ldots\ldots\ldots\ldots, E_8\}$ be a sample space of a random experiment such that $P(E_n) = \frac{n}{36}$ for every n = 1, 2, ………, 8. Then the number of elements in the set $\left\{A \subseteq S : P(A) \geq \frac{4}{5}\right\}$ is ________ . **[JEE (Main)-2022]**

96. The probability, that in a randomly selected 3-digit number at least two digits are odd, is

[JEE (Main)-2022]

(1) $\frac{19}{36}$ (2) $\frac{15}{36}$

(3) $\frac{13}{36}$ (4) $\frac{23}{36}$

97. The probability that a randomly chosen one-one function from the set $\{a, b, c, d\}$ to the set $\{1, 2, 3, 4, 5\}$ satisfies $f(a) + 2f(b) - f(c) = f(d)$ is :

[JEE (Main)-2022]

(1) $\frac{1}{24}$ (2) $\frac{1}{40}$

(3) $\frac{1}{30}$ (4) $\frac{1}{20}$

98. The probability that a randomly chosen 2×2 matrix with all the entries from the set of first 10 primes, is singular, is equal to : **[JEE (Main)-2022]**

(1) $\frac{133}{10^4}$ (2) $\frac{18}{10^3}$

(3) $\frac{19}{10^3}$ (4) $\frac{271}{10^4}$

99. The probability that a relation R from $\{x, y\}$ to $\{x, y\}$ is both symmetric and transitive, is equal to

[JEE (Main)-2022]

(1) $\frac{5}{16}$ (2) $\frac{9}{16}$

(3) $\frac{11}{16}$ (4) $\frac{13}{16}$

100. If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is :

[JEE (Main)-2022]

(1) $\frac{33}{2^{32}}$ (2) $\frac{33}{2^{29}}$

(3) $\frac{33}{2^{28}}$ (4) $\frac{33}{2^{27}}$

101. If the numbers appeared on the two throws of a fair six faced die are α and β, then the probability that $x^2 + \alpha x + \beta > 0$, for all $x \in R$, is :**[JEE (Main)-2022]**

(1) $\frac{17}{36}$ (2) $\frac{4}{9}$

(3) $\frac{1}{2}$ (4) $\frac{19}{36}$

102. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X = 1) = \frac{4}{243}$, then $P(X = 4 \text{ or } 5)$ is equal to : **[JEE (Main)-2022]**

(1) $\frac{5}{9}$ (2) $\frac{64}{81}$

(3) $\frac{16}{27}$ (4) $\frac{145}{243}$

103. Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1) = \frac{2+3p}{6}$, $P(E_2) = \frac{2-p}{8}$ and $P(E_3) = \frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(p_1 + p_2)$ is equal to :

[JEE (Main)-2022]

(1) $\frac{2}{3}$ (2) $\frac{5}{3}$

(3) $\frac{5}{4}$ (4) 1

104. Let S be the sample space of all five digit numbers. It p is the probability that a randomly selected number from S, is multiple of 7 but not divisible by 5, then $9p$ is equal to **[JEE (Main)-2022]**

(1) 1.0146 (2) 1.2085

(3) 1.0285 (4) 1.1521

105. Let X have a binomial distribution $B(n, p)$ such that the sum and the product of the mean and variance of X are 24 and 128 respectively. If $P(X > n - 3) = \frac{k}{2^n}$, then k is equal to : **[JEE (Main)-2022]**

(1) 528 (2) 529

(3) 629 (4) 630

106. A six faced die is biased such that

$3 \times P$(a prime number) $= 6 \times P$(a composite number) $= 2 \times P(1)$.

Let X be a random variable that counts the number of times one gets a perfect square on some throws of this die. If the die is thrown twice, then the mean of X is : **[JEE (Main)-2022]**

(1) $\frac{3}{11}$

(2) $\frac{5}{11}$

(3) $\frac{7}{11}$

(4) $\frac{8}{11}$

107. Let A and B be two events such that $P(B \setminus A) \frac{2}{5}, P(A \setminus B) = \frac{1}{7}$ and $P(A \cap B) = \frac{1}{9}$. Consider

$(S1)\ P(A' \cup B) = \frac{5}{6}$,

$(S2)\ P(A' \cap B') = \frac{1}{18}$. Then **[JEE (Main)-2022]**

(1) Both ($S1$) and ($S2$) are true

(2) Both ($S1$) and ($S2$) are false

(3) Only ($S1$) is true

(4) Only ($S2$) is true

108. A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then $100\,\sigma^2$ is equal to _____. **[JEE (Main)-2022]**

109. Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is : **[JEE (Main)-2022]**

(1) $\frac{4}{9}$ (2) $\frac{5}{18}$

(3) $\frac{1}{6}$ (4) $\frac{3}{10}$

110. The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. Then the number of trials in the binomial distribution is ________. **[JEE (Main)-2022]**

111. Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then, $54\,P(X \le 2)$ is equal to **[JEE (Main)-2022]**

(1) $\frac{73}{27}$ (2) $\frac{146}{27}$

(3) $\frac{146}{81}$ (4) $\frac{126}{81}$

112. If A and B are two events such that $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$, then $P(A|B') + P(B|A')$ is equal to **[JEE (Main)-2022]**

(1) $\frac{3}{4}$ (2) $\frac{5}{8}$

(3) $\frac{5}{4}$ (4) $\frac{7}{8}$

❑ ❑ ❑

Chapter 27

Probability

1. Answer (4)

Restricting sample space as S = {00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40}.

$\therefore$ P(sum of digits is 8) = $\frac{1}{14}$.

2. Answer (2)

Total number of cases = ${}^9C_3$ = 84

Favourable cases = ${}^3C_1.{}^4C_1.{}^2C_1$ = 24

$$p=\frac{24}{84}=\frac{2}{7}$$

3. Answer (3)

Statement-2 is false.

The outcomes 2, 8, 14, 20 is an AP with common difference 6.

4. Answer (2)

$$P\left(\frac{A^c\cap B^c}{C}\right)=\frac{P(A^c\cap B^c\cap C)}{P(C)}$$

$$=\frac{P(C)-P(C\cap A)-P(C\cap B)+P(A\cap B\cap C)}{P(C)}$$

Let A, B, C be pairwise independent events

$$=\frac{P(C)-P(C).P(A)-P(C).P(B)-0}{P(C)}$$

$= 1 - P(A) - P(B)$ $\quad(\because P(C)\neq 0)$

$= P(A^c) - P(B)$

5. Answer (1)

6. Answer (3)

Required probability $={}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)+{}^5C_5\left(\frac{1}{3}\right)^5$

$$=5\times\frac{1}{81}\times\frac{2}{3}+\frac{1}{3^5}$$

$$=\frac{10}{3^5}+\frac{1}{3^5}=\frac{11}{3^5}$$

7. Answer (1)

$$P(\overline{A\cup B})=\frac{1}{6}\Rightarrow P(A\cup B)=1-\frac{1}{6}=\frac{5}{6}$$

$$P(\bar{A})=\frac{1}{4}\Rightarrow P(A)=1-\frac{1}{4}=\frac{3}{4}$$

$\because\quad P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}$$

$$P(B)=\frac{1}{3}$$

$\because$ $P(A)\neq P(B)$ so they are not equally likely.

Also $P(A)\times P(B)=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}$

$= P(A\cap B)$

$\because\quad P(A\cap B)=P(A)\cdot P(B)$ so A & B are independent.

8. Answer (1)

Question is wrong but the best suitable option is (1).

Required probability = ${}^{12}C_3\frac{2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{3}\right)^{11}$

9. Answer (3)

$$P(E_1)=\frac{1}{6}$$

$$P(E_2)=\frac{1}{6}$$

$P(E_1\cap E_2)$ = P(A shows 4 and B shows 2)

$$=\frac{1}{36}=P(E_1).P(E_2)$$

So E_1, E_2 are independent

Also as $E_1\cap E_2\cap E_3=\phi$

So $P(E_1\cap E_2\cap E_3)\neq P(E_1).P(E_2).P(E_3)$

So E_1, E_2, E_3 are not independent.

10. Answer (4)

$n = 10$

p(Probability of drawing a green ball) = $\dfrac{15}{25}$

$$\therefore \quad p = \frac{3}{5},\ q = \frac{2}{5}$$

var(X) = $n.p.q$

$$= 10 \cdot \frac{6}{25} = \frac{12}{5}$$

11. Answer (1)

$$P(A) + P(B) - P(A \cap B) = \frac{1}{4}$$

$$P(B) + P(C) - P(B \cap C) = \frac{1}{4}$$

$$P(C) + P(A) - P(A \cap C) = \frac{1}{4}$$

$$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) = \frac{3}{8}$$

$$\because \quad P(A \cap B \cap C) = \frac{1}{16}$$

$$\therefore \quad P(A \cup B \cup C) = \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$$

12. Answer (4)

Total number of ways = ${}^{11}C_2$

= 55

Favourable ways are

(0, 4), (0, 8), (4, 8), (2, 6), (2, 10), (6, 10)

Probability = $\dfrac{6}{55}$

13. Answer (2)

E_1 : Event that first ball drawn is red.

E_2 : Event that first ball drawn is black.

E : Event that second ball drawn is red.

$$P(E) = P(E_1).P\left(\frac{E}{E_1}\right) + P(E_2).P\left(\frac{E}{E_2}\right)$$

$$= \frac{4}{10} \times \frac{6}{12} + \frac{6}{10} \times \frac{4}{12}$$

$$= \frac{2}{5}$$

14. Answer (2)

X = number of aces drawn

$$\therefore \quad P(X = 1) + P(X = 2)$$

$$= \left\{\frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52}\right\} + \left\{\frac{4}{52} \times \frac{4}{52}\right\}$$

$$= \frac{24}{169} + \frac{1}{169} = \frac{25}{169}$$

15. Answer (3)

Let drawing a green ball is G and a red ball is R

$\therefore$ The probability that second drawn ball is red

$$= P(G) \cdot P\left(\frac{R}{G}\right) + P(R)P\left(\frac{R}{R}\right)$$

$$= \frac{2}{7} \times \frac{6}{7} + \frac{5}{7} \times \frac{4}{7}$$

$$= \frac{12 + 20}{49} = \frac{32}{49}$$

16. Answer (4)

P (7 or 8 is the sum of two dice) = $\dfrac{11}{36}$

P (7 or 8 is the number of card) = $\dfrac{2}{9}$

Required probability = $\dfrac{1}{2} \times \dfrac{11}{36} + \dfrac{1}{2} \times \dfrac{2}{9}$

$$= \frac{1}{2}\left(\frac{11 + 8}{36}\right) = \frac{19}{72}$$

17. Answer (2)

Let the number of independent shots required to hit the target at least once is n

$$\Rightarrow \quad 1 - \left(\frac{2}{3}\right)^n > \frac{5}{6}$$

$$\left(\frac{2}{3}\right)^n < \frac{1}{6}$$

Least value of n is 5

Option (2) is correct.

18. Answer (4)

Probability that sum of selected two numbers is even

$$= P(E_1) = \frac{{}^6C_2 + {}^5C_2}{{}^{11}C_2}$$

Probability that sum is even and selected numbers are also even $P(E_2) = \dfrac{{}^5C_2}{{}^{11}C_2}$

$$\therefore \quad P\left(\frac{E_2}{E_1}\right) = \frac{{}^5C_2}{{}^6C_2 + {}^5C_2} = \frac{10}{15 + 10} = \frac{2}{5}.$$

19. Answer (2)

$$p=\frac{30}{40}=\frac{3}{4},\quad q=\frac{10}{40}=\frac{1}{4}$$

n = 16

$$\frac{\text{Mean of } X}{\text{standard deviation of } X}=\frac{np}{\sqrt{npq}}=\frac{\sqrt{np}}{\sqrt{q}}$$

$$=\sqrt{\frac{16\times\frac{3}{4}}{\frac{1}{4}}}$$

$$=\sqrt{48}=4\sqrt{3}$$

20. Answer (4)

Total number of subset = 2^{20}

Now sum of all number from 1 to 20 $=\frac{20\times 21}{2}=210$

Now we have to find the sets which has sum 7.

(1) {7}

(2) {1, 6}

(3) {2, 5}

(4) {3, 4}

(5) {1, 2, 4}

So, there is only 5 sets which has sum 203

So required probability $=\frac{5}{2^{20}}$

21. Answer (3)

To end the experiment in the fifth throw, possibility is 4 × × 4 4, × 4 × 4 4, × × × 4 4 (where × is any number except 4)

$$\text{Probability}=\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)$$

$$\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^2$$

$$=\frac{25+25+125}{6^5}=\frac{175}{6^5}$$

22. Answer (3)

A = Set of students who opted for NCC

B = Set of Students who opted for NSS

$n(A)$ = 40, $n(B)$ = 30, $n(A\cap B)$ = 20

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

= 40 + 30 – 20

= 50

$\therefore$ Required probability $=1-\frac{50}{60}=\frac{1}{6}$

23. Answer (2)

Probability of getting 5 or 6 = $P(E)$ = $\frac{2}{6}=\frac{1}{3}$

Probability of not getting 5 or 6 = $P(E)$ = $1-\frac{1}{3}=\frac{2}{3}$

E will consider as success.

Event	Success in I[st] attempt	Success in II[nd] attempt	Success in 3[rd] attempt	No success in 3 attempt
Probability	$\frac{1}{3}$	$\frac{2}{3}\times\frac{1}{3}$	$\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}$	$\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}$
Gain/loss	100	50	0	–150

His expected gain/loss

$$=\frac{1}{3}\times 100+\frac{2}{9}\times 50+\frac{8}{27}\times(-150)$$

$$=\frac{900+300-1200}{27}=0$$

24. Answer (3)

$\because\quad A\subset B$; so $A\cap B=A$

Now, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$

$\Rightarrow\quad P\left(\frac{A}{B}\right)=\frac{P(A)}{P(B)}$

$\because\quad P(B)\le 1$

So, $P\left(\frac{A}{B}\right)\ge P(A)$

25. Answer (3)

$$p=P(H)=\frac{1}{2},\ q=1-p=\frac{1}{2}$$

$$P(x\ge 1)\ge\frac{9}{10}$$

$$1-P(x=0)\ge\frac{9}{10}$$

$$1-{}^nC_0\left(\frac{1}{2}\right)^n\ge\frac{9}{10}$$

$$\frac{1}{2^n}\le 1-\frac{9}{10}\Rightarrow\frac{1}{2^n}\le\frac{1}{10}$$

$2^n\ge 10\quad\Rightarrow\quad n\ge 4$

$\Rightarrow\ n_{\min}=4$

26. Answer (4)

P(at least one) = 1 – P(none)

$$=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{7}{8}$$

$$=1-\frac{7}{32}$$

$$=\boxed{\frac{25}{32}}$$

27. Answer (1)

A = At least two girls

B = All girls

$$P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}$$

$$=\frac{P(B)}{P(A)}=\frac{\left(\frac{1}{4}\right)^4}{1-{}^4C_0\left(\frac{1}{2}\right)^4-{}^4C_1\left(\frac{1}{2}\right)^4}$$

$$=\frac{1}{16-1-4}=\frac{1}{11}$$

28. Answer (4)

$$1-\left(\frac{1}{2}\right)^n>\frac{99}{100}$$

$$\left(\frac{1}{2}\right)^n<\frac{1}{100}$$

$\therefore\quad n\geq 7$

Minimum value is 7.

29. Answer (4)

$\mu=8,\ \sigma^2=4$

$\Rightarrow\quad \mu = np = 8,\ \sigma^2 = npq = 4,\ p + q = 1$

$\Rightarrow\quad q=\frac{1}{2},\ p=\frac{1}{2},\ n=16$

$$P(X\leq 2)=\frac{k}{2^{16}}$$

$${}^{16}C_0\left(\frac{1}{2}\right)^{16}+{}^{16}C_1\left(\frac{1}{2}\right)^{16}+{}^{16}C_2\left(\frac{1}{2}\right)^{16}=\frac{k}{2^{16}}$$

$\Rightarrow\quad k = (1 + 16 + 120) = 137$

30. Answer (2)

Let p is the probability that candidate can solve a problem.

$p=\frac{4}{5}\ ;\ q=\frac{1}{5}$ $\qquad(\because\ p + q = 1)$

Probability that candidate is able to solve either 50 or 49 problems $= {}^{50}C_{50}p^{50}\cdot q^0 + {}^{50}C_{49}\cdot p^{49}\cdot q^1$

$= p^{49}\,[p + 50q]$

$$=\left(\frac{4}{5}\right)^{49}\cdot\left(\frac{4}{5}+\frac{50}{5}\right)$$

$$=\frac{54}{5}\cdot\left(\frac{4}{5}\right)^{49}$$

31. Answer (1)

Let X be the random variable which denotes the Rs gained by the person.

$$P(X=15)=\frac{6}{36}=\frac{1}{6}\quad\left\{\begin{array}{l}\text{Total cases}=36\\ \text{favourable cases are}\\ (1,1),(2,2),(3,3),(4,4),\\ (5,5),(6,6)\end{array}\right\}$$

$$P(X=12)=\frac{4}{36}=\frac{1}{9}\quad\left\{\begin{array}{l}\text{Favourable cases are}\\ (6,3),(5,4),(4,5),(3,6)\end{array}\right\}$$

$$P(X=-6)=\frac{26}{36}=\frac{13}{18}$$

X	15	12	–6
$P(X)$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{13}{18}$
$X\cdot P(X)$	$\frac{5}{2}$	$\frac{4}{3}$	$\frac{-13}{3}$

$$E(X)=\sum X\cdot P(X)=\frac{5}{2}+\frac{4}{3}-\frac{13}{3}=-\frac{1}{2}$$

32. Answer (4)

k	3	4	5
$P(k)$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

$\therefore$ For k = 1 or 2,

$$\therefore\ P(k)=1-\frac{5}{32}-\frac{2}{32}-\frac{1}{32}=\frac{24}{32}$$

Now expectation

$$=\left(3\times\frac{5}{32}\right)+\left(4\times\frac{2}{32}\right)+\left(5\times\frac{1}{32}\right)+(-1)\left(\frac{24}{32}\right)$$

$$=\frac{4}{32}=\frac{1}{8}$$

33. Answer (3)

Probability that a machine is faulted $=\frac{1}{4}=P$

Probability that a machine is not faulted

$$=1-\frac{1}{4}=\frac{3}{4}=q$$

$\therefore$ Probability that atmost two machine is faulted $= P(X = 0) + P(X = 1) + P(X = 2)$

$$\therefore\ {}^5C_0\left(\frac{1}{4}\right)^0\cdot\left(\frac{3}{4}\right)^5+{}^5C_1\left(\frac{1}{4}\right)^1\cdot\left(\frac{3}{4}\right)^4+{}^5C_2\left(\frac{1}{4}\right)^2\cdot\left(\frac{3}{4}\right)^3$$

$$=\left(\frac{3}{4}\right)^3\cdot k$$

$$\Rightarrow\ \left(\frac{3}{4}\right)^2+5\cdot\frac{1}{4}\cdot\frac{3}{4}+10\cdot\left(\frac{1}{4}\right)^2=K$$

$$\therefore\ k=\frac{10+15+9}{16}=\frac{34}{16}=\frac{17}{8}$$

34. Answer (3)

For option (1) $P(A/B)=\frac{P(A\cap B)}{P(B)}=P(A)=\frac{1}{3}$

Similarly $P(A'/B')=P(A')=\frac{2}{3}$

$$P(A/B')=\frac{P(A)(1-P(B))}{(1-P(B))}=\frac{\frac{1}{3}\cdot\frac{5}{6}}{\frac{5}{6}}=\frac{1}{3}$$

$$P(A/A\cup B)=\frac{P(A\cap(A\cup B))}{P(A\cup B)}$$

$$=\frac{P(A)}{P(A\cup B)}$$

$$=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}-\frac{1}{18}}$$

$$=\frac{6}{6+3-1}=\frac{3}{4}$$

$\because$ Option (3) is correct

35. Answer (4)

Given $P\left(A\cap\overline{B}\right)+P\left(\overline{A}\cap B\right)=\frac{2}{5}$

$$\Rightarrow\ P(A)+P(B)-2P(A\cap B)=\frac{2}{5}$$

$$\Rightarrow\ P(A\cup B)-P(A\cap B)=\frac{2}{5}$$

$$\Rightarrow\ \frac{1}{2}-\frac{2}{5}=P(A\cap B)$$

$$\Rightarrow\ \boxed{P(A\cap B)=\frac{1}{10}}$$

36. Answer (3)

Second A comes before third B, so this process will be finished either in two draws or in three draws or in four draws.

If process is finished in two draws,

$$P\ (AA)=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$$

If process is finished in three draws

$$P\ (ABA\ or\ BAA)=2\left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right)=\frac{1}{4}$$

If process is finished in four draws, P(ABBA or BABA or BBAA) $=3\left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right)=\frac{3}{16}$

Total probability $=\frac{1}{4}+\frac{1}{4}+\frac{3}{16}=\frac{11}{16}$

37. Answer (1)

Total ways of distribution = $4^{10} = 2^{20}$

Favourable ways = $\underbrace{{}^4C_2}_{\text{Selecting two boxes out of four}}\cdot\underbrace{{}^{10}C_5}_{\text{Selecting 5 balls out of 10}}\cdot\underbrace{{}^5C_3\cdot\lfloor 2}_{\text{Distributing 5 balls into two groups of 2 balls and 3 balls}}\cdot\underbrace{2^5}_{\text{Distributing remaining balls into two boxes}}$

Required probability = $\frac{6\cdot 252\cdot 10\cdot 2\cdot 2^5}{2^{20}}$

$$=\frac{3\times 63\times 5}{2^{10}}=\frac{945}{2^{10}}$$

38. Answer (2)

$P(x > 2) = P(x = 3) + P(x = 4) + P(x = 5)$

$= k + 2k + 5k^2$

$= 5k^2 + 3k$

Now $\Sigma p_i = k^2 + 2k + k + 2k + 5k^2$

$= 6k^2 + 5k$

as $\Sigma p_i = 1 \Rightarrow 6k^2 + 5k = 1 \Rightarrow (k + 1)(6k - 1) = 0$

$$\Rightarrow k = \frac{1}{6}$$

$$p(x > 2) = \frac{5}{36} + \frac{3}{6} = \frac{23}{36}$$

39. Answer (4)

Here $P(B_1) = P(B_2) = \frac{1}{2}$ $\{B_1, B_2$ are bags$\}$

and P(Non-prime) $= P(B_1) \times P\left(\frac{NP}{B_1}\right) + P(B_2) \times P\left(\frac{NP}{B_2}\right)$

$$= \frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}$$

$$\text{So, } P\left(\frac{B_1}{NP}\right) = \frac{\frac{1}{2} \times \frac{20}{30}}{\frac{1}{2} \times \frac{20}{30} + \frac{1}{2} \times \frac{15}{20}}$$

$$= \frac{8}{17}$$

40. Answer (4)

Here

$$P\left(\frac{E_2^C \cap E_3^C}{E_1}\right) = \frac{P\left[E_1 \cap \left(E_2^C \cap E_3^C\right)\right]}{P(E_1)}$$

$$= \frac{P(E_1) - P(E_1 \cap E_2) - P(E_2 \cap E_3) + 0}{P(E_1)}$$

$= 1 - P(E_2) - P(E_3)$

$= P\left(E_2^C\right) - P(E_3)$ or $P\left(E_3^C\right) - P(E_2)$

41. Answer (4)

Let A is the event for getting score a multiple of 4

So, n(A) = (1, 3), (3, 1), (2, 2), (2, 6), (6, 2), (3, 5), (5, 3), (4, 4) & (6, 6) = 09

n(A) = 9

& n(B) = (4, 4) = 1

So, $P(E) = \frac{1}{9}$

42. Answer (4)

Sample space $= 9 \times 10^4$

Case-I

Out of exactly two digits selected one is zero then favourable cases $= {}^9C_1(2^4 - 1)$

Case-II

Both selected digits are non-zero then favourable cases $= {}^9C_2(2^5 - 2)$

$$\text{Prob.} = \frac{9(2^4 - 1) + \frac{9.8}{2}(2^5 - 2)}{9 \times 10^4}$$

$$= \frac{15 + 120}{10^4} = \frac{135}{10^4}$$

43. Answer (4)

Probability of sum getting $6 = \frac{5}{36}$ and

Probability of sum getting $7 = \frac{6}{36} = \frac{1}{6}$

Game ends and A wins if A throws 6 in 1st throw or A don't throw 6 in 1st throw, B don't throw 7 in 1st throw and then A throw 6 in his 2nd chance and so on.

i.e $A + \bar{A}\,\bar{B}\,A + \bar{A}\,\bar{B}\,\bar{A}\,\bar{B}\,A + \ldots\ldots\ldots$

$$\Rightarrow \frac{5}{36} + \left(\frac{31}{36}\right)\left(\frac{30}{36}\right)\left(\frac{5}{36}\right) + \ldots\ldots\ldots\infty$$

$$\Rightarrow \frac{5}{36}\left(1 + \frac{155}{216} + \left(\frac{155}{216}\right)^2 + \ldots\ldots\ldots\infty\right)$$

$$\Rightarrow \frac{\frac{5}{36}}{\frac{61}{216}} = \frac{30}{61}$$

44. Answer (3)

For an A.P. of three terms, we must select two even numbers or two odd numbers from given numbers and third number will be fixed automatically.

Required probability = $\frac{{}^6C_2 + {}^5C_2}{{}^{11}C_3} = \frac{25}{165} = \frac{5}{33}$

45. Answer (1)

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

= 1 – 0.8 = 0.2

Now,

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$

$-P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$\alpha = 0.6 + 0.4 + 0.5 - 0.2 - \beta - 0.3 + 0.2$

$\beta = 1.2 - \alpha$

$\therefore \quad \alpha \in [0.85, 0.95]$

then $\beta \in [0.25, 0.35]$

46. Answer (490.00)

Atmost 3 = Total – (All selected balls are red)

$= {}^{12}C_4 - {}^5C_4$

= 495 – 5 = 490

47. Answer (3)

$p = \frac{1}{10}, q = \frac{9}{10}$

P(not hitting target in n trials) = $\left(\frac{9}{10}\right)^n$

P(at least one hit) = $1-\left(\frac{9}{10}\right)^n$

Given, $1-\left(\frac{9}{10}\right)^n > \frac{1}{4}$

$\Rightarrow \quad (0.9)^n < 0.75$

$n_{\text{minimum}} = 3$

48. Answer (11)

Probability of getting at least two 3′s or 5′s in one trial =

$${}^4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2 + {}^4C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right) + {}^4C_4\left(\frac{1}{3}\right)^4$$

$= \frac{33}{3^4} = \frac{11}{27}$

E(x) = np = $27\left(\frac{11}{27}\right) = 11$

49. Answer (11)

Let ‘n’ bombs are required, then

$$1 - {}^nC_1 \cdot \left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1} - {}^nC_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n \ge \frac{99}{100}$$

$\Rightarrow \quad \frac{1}{100} \ge \frac{n+1}{2^n}$

$\Rightarrow \quad 2^n \ge 100(n + 1)$

$\Rightarrow \quad n \ge 11$

50. Answer (2)

Let number of trials be ‘n’

given

$${}^nC_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2} = {}^nC_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{n-3}$$

$\Rightarrow$ n = 5

Probability of getting odd number for odd number of times is

$= (5C_1 + 5C_3 + 5C_5)\frac{1}{2^5}$

$= \frac{2^4}{2^5} = \frac{1}{2}$

51. Answer (06)

Let $P(B_1) = x$, $P(B_2) = y$, $P(B_3) = z$

$\alpha = P(B_1 \cap \bar{B}_2 \cap \bar{B}_3) = P(B_1)P(\bar{B}_2)P(\bar{B}_3)$

$\Rightarrow \quad \alpha = x(1 - y)(1 - z)$ …(i)

Similarly $\beta = (1 - x)y(1 - z)$ …(ii)

$\gamma = (1 - x)(1 - y)z$ …(iii)

$p = (1 - x)(1 - y)(1 - z)$ …(iv)

(i) & (iv) $\Rightarrow \frac{x}{1-x} = \frac{\alpha}{p} \Rightarrow x = \frac{\alpha}{\alpha + p}$

(iii) & (iv) $\Rightarrow \dfrac{z}{1-z} = \dfrac{\gamma}{p} \Rightarrow z = \dfrac{\gamma}{\gamma + p}$

$$\frac{P(B_1)}{P(B_3)} = \frac{x}{z} = \frac{\frac{\alpha}{\alpha+p}}{\frac{\gamma}{\gamma+p}} = \frac{\frac{\gamma+p}{\gamma}}{\frac{\alpha+p}{\alpha}} = \frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}} \qquad \ldots(v)$$

Given that,

$(\alpha - 2\beta)\, p = \alpha\beta \Rightarrow \alpha p = (\alpha + 2p)\beta$ ...(vi)

$(\beta - 3\gamma)\, p = 2\beta\gamma \Rightarrow 3\gamma p = (p - 2\gamma)\beta$ ...(vii)

(vi) & (vii) $\Rightarrow \dfrac{\alpha}{3\gamma} = \dfrac{\alpha + 2p}{p - 2\gamma}$

$\Rightarrow p\alpha - 6p\gamma = 5\gamma\alpha$

$$\frac{p}{\gamma} - \frac{6p}{\alpha} = 5$$

$$\frac{p}{\gamma} + 1 = 6\left(\frac{p}{\alpha} + 1\right) \qquad \ldots(viii)$$

(v) & (viii) $\Rightarrow \dfrac{P(B_1)}{P(B_3)} = 6$

52. Answer (2)

Number of ways of selecting elements common to both A and B = ${}^5C_2$

$\therefore$ Required probability $= \dfrac{{}^5C_2 \cdot 3^3}{4^5} = \dfrac{135}{2^9}$

53. Answer (4)

Given P (when it is intercepted) $= \dfrac{1}{3}$

$\Rightarrow$ P (being not intercepted) $= 1 - \dfrac{1}{3} = \dfrac{2}{3}$ & also when it is not intercepted, probability it hits the target $= \dfrac{3}{4}$

So when such 3 missiles launched then P (all 3 hitting the target)

$$= \left(\frac{2}{3} \times \frac{3}{4}\right) \times \left(\frac{2}{3} \times \frac{3}{4}\right) \times \left(\frac{2}{3} \times \frac{3}{4}\right)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8}$$

54. Answer (3)

For equal roots $b^2 = 4ac$

$a, b, c \in \{1, 2, 3, 4, 5, 6\}$

Favourable case

b = 2	a = c = 1
b = 4	(a, c) = (1, 4), (4, 1) and (2, 2)
b = 6	(a, c) = (3, 3)

Total possible ordered triplets

$(a, b, c) = 6^3 = 216$

Favourable cases = 5

$\therefore$ Required probability $= \dfrac{5}{216}$

55. Answer (2)

Number having exactly one 7 can be

(i) Having 7 at thousand's place $= 9^3 = 729$

(ii) Not 7 at thousand's place $= 3 \times 8 \times 4^2 = 1944$

n(s) = 729 + 1944 = 2673

Favourable cases = having 7 at unit place or having 2 at unit place.

i.e. = $(9 \times 9) + (8 \times 9 \times 2) + (8 \times 9 \times 9) = 873$

Required probability $= \dfrac{873}{2673} = \dfrac{97}{297}$

56. Answer (2)

n(smokers + Non vegetarian) = 160 = $n(A_1)$(Let)

$\Rightarrow P(A_1) = 0.4$

n(smokers + vegetarian) = 100 = $n(A_2)$

similarly $P(A_2) = 0.25$

n(Non-smokers + vegetarian) = 140 = $n(A_3)$ and $P(A_3) = 0.35$

Let event E of getting chest disorder i.e.,

$P(E/A_1) = 0.35$, $P(E/A_2) = 0.2$, $P(E/A_3) = 0.1$

to find $P(A_1/E)$

using Baye's theorem we get

$$P(A_1/E) = \frac{P(E/A_1) \cdot P(A_1)}{P(E/A_1) \cdot P(A_1) + P(E/A_2)P(A_2) + P(E/A_3) \cdot P(A_3)}$$

$$= \frac{0.35 \times 0.4}{(0.35 \times 0.4) + (0.2 \times 0.25) + (0.1 \times 0.35)}$$

$$= \frac{140}{140 + 50 + 35} = \frac{140}{225} = \frac{28}{45}$$

57. Answer (1)

Let n number of tosses

Given,

$$^{n}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^{n}C_9 = \left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$$

$\Rightarrow$ n = 16

$\therefore$ Probability of getting 2 heads

$$= {}^{16}C_2\left(\frac{1}{2}\right)^{16}$$

$$= \frac{15}{2^{13}}$$

58. Answer (4)

For even number, units place should be filled with 4 only.

$$P = \frac{\frac{6!}{2!2!2!}}{\frac{7!}{2!3!2!}} = \frac{6!}{2!}\times\frac{3!}{7!} = \frac{3}{7}$$

59. Answer (4)

Consider the events,

E_1 = missing card is spade

E_2 = missing card is not a spade

A = Two spade cards are drawn

$$P(E_1) = \frac{1}{4} \qquad P(E_2) = \frac{3}{4}$$

$$P\left(\frac{A}{E_1}\right) = \frac{^{12}C_2}{^{51}C_2} \qquad P\left(\frac{A}{E_2}\right) = \frac{^{13}C_2}{^{51}C_2}$$

$$\text{Then } P\left(\frac{E_2}{A}\right) = \frac{P\left(\frac{A}{E_2}\right)\cdot P(E_2)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)} =$$

$$\frac{\frac{3}{4}\cdot\frac{^{13}C_2}{^{51}C_2}}{\frac{1}{4}\cdot\frac{^{12}C_2}{^{51}C_2} + \frac{3}{4}\cdot\frac{^{13}C_2}{^{51}C_2}}$$

$$= \frac{3\cdot{}^{13}C_2}{^{12}C_2 + 3\cdot{}^{13}C_2} = \frac{3.78}{66 + 3.78} = \frac{39}{50}$$

60. Answer (4)

Total number of numbers = 6 × (6 × 5 × 4 × 3 × 2)

= 6 × 6!

Required number of numbers

Case (i) 0 is not included $\rightarrow$ 6!

Case (ii) 0 is included $\rightarrow$ 5 × 5! × 2

Total 6! + 5 × 5! × 2 = 16 × 5!

$$\text{Probability} = \frac{16\times 5!}{6\times 6!} = \frac{16}{36} = \frac{4}{9}$$

61. Answer (1)

Favourable outcomes are

(1,1), (1,2), (1,3), (1,5), (1,7)

(2,1), (2,2), (2,3), (2,5)

(3,1), (3,2), (3,3), (3,5)

(5,1), (5,2), (5,3)

(7,1)

i.e. total 17 favourable outcomes.

$$\text{Required probability} = \frac{17}{36}.$$

62. Answer (6)

Let $p(E_1) = x$, $p(E_2) = y$ and $p(E_3) = z$

$$\alpha = p(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = p(E_1).p(\bar{E}_2).p(\bar{E}_3)$$

$\Rightarrow$ $\alpha = x(1 - y)(1 - z)$ ...(i)

Similarly

$\beta = (1 - x).y(1 - z)$ ...(ii)

$\gamma = (1 - x)(1 - y).z$ ...(iii)

$p = (1 - x)(1 - y)(1 - z)$ ...(iv)

$$\text{(i) and (iv)} \Rightarrow \frac{x}{1-x} = \frac{x}{p} \Rightarrow x = \frac{\alpha}{\alpha + p}$$

$$\text{(iii) and (iv)} \Rightarrow \frac{z}{1-z} = \frac{\gamma}{p} \Rightarrow z = \frac{\gamma}{\gamma + p}$$

$$\frac{p(E_1)}{p(E_2)} = \frac{x}{z} = \frac{\frac{\alpha}{\alpha+p}}{\frac{\gamma}{\gamma+p}} = \frac{\frac{\gamma+p}{\gamma}}{\frac{\alpha+p}{\alpha}} = \frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}} \quad ...(v)$$

Given that

$(\alpha - 2\beta)p = \alpha\beta \Rightarrow \alpha p = (\alpha + 2p)\beta$ ...(vi)

$(\beta - 3\gamma)p = 2\beta\gamma \Rightarrow 3\gamma p = (p - 2\gamma)\beta$ ...(vii)

(vi) and (vii) $\Rightarrow \dfrac{\alpha}{3\gamma} = \dfrac{\alpha + 2p}{p - 2\gamma}$

$\Rightarrow p\alpha - 6p\gamma = 5\gamma\alpha$

$$\frac{p}{\gamma} - \frac{6p}{\alpha} = 5.$$

$$\frac{p}{\gamma} + 1 = 6\left(\frac{p}{\alpha} + 1\right) \quad \text{...(viii)}$$

(v) and (viii) $\Rightarrow \dfrac{p(E_1)}{p(E_3)} = 6$

63. Answer (4)

'10' is followed by '01' can be if

...1 0 0 1 or ...1 0 0 1

odd position, even position — even position, odd position

$$\Rightarrow \left(\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\right) + \left(\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}\right)$$

$$\Rightarrow \frac{4}{9\cdot 4} = \frac{1}{9}$$

64. Answer (2)

$n = 5$ (given)

also $5c_1\, pq^4 = 0.4096$

and $5c_2\, p^2q^3 = 0.2048$

$$\frac{5q}{10p} = \frac{0.4096}{0.2048}$$

$\Rightarrow q = 4p$

also $p + q = 1$

$\Rightarrow p = \dfrac{1}{5}$ and $q = \dfrac{4}{5}$

p(exactly 3 successes) $= 5c_3 \cdot p^3q^2$

$$= 10\frac{1}{5^3}\cdot\frac{16}{5^2} = \frac{32}{625}$$

65. Answer (4)

For $x^2 + 2(a + 4)x - 5a + 64 > 0$

$\therefore$ $D < 0$

$4(a + 4)^2 - 4(64 - 5a) < 0$

$\Rightarrow a^2 + 8a + 16 - 64 + 5a < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

$\Rightarrow a^2 + 16a - 3a - 48 < 0$

$\Rightarrow (a + 16)(a - 3) < 0$

$a \in (-16, 3)$

in set $[-5, 30]$ total integers 36

favourable integers 8

$$\Pr = \frac{8}{36} = \frac{2}{9}$$

66. Answer (2)

Total number of words formed by letters of word "EXAMINATION" is

$$n(S) = \frac{11!}{2!\cdot 2!\cdot 2!}$$

When 'M' at fourth position then number of words formed $= n(E) = \dfrac{10!}{2!\cdot 2!\cdot 2!}$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{1}{11}$

67. Answer (1)

$P(A) + P(B) - 2P(A \cap B) = 1 - k$ ...(i)

$P(B) + P(C) - 2P(B \cap C) = 1 - 2k$ ...(ii)

$P(C) + P(A) - 2P(C \cap A) = 1 - k$ ...(iii)

(i) + (ii) + (iii)

$$\Rightarrow \sum P(A) - \sum P(A \cap B) = \frac{3 - 4k}{2}$$

$$P(A \cup B \cup C) = \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$$

$$= \frac{3 - 4k}{2} + k^2$$

$$= (k - 1)^2 + \frac{1}{2} > \frac{1}{2}$$

68. Answer (1)

Total matrices = 6^4

Number of matrices with distinct entries

$= 6 \times 5 \times 4 \times 3 = 360$

Number of singular matrices with distinct entries

(i.e. 1, 2, 3, 6 or 2, 3, 4, 6) = 8 + 8 = 16

Favourable cases = 360 – 16 = 344

Required probability $= \dfrac{344}{6^4} = \dfrac{43}{162}$

69. Answer (1)

Number of ways to distribute 9 distinct balls in 4 boxes = n(S) = 4^9

Number of ways of favourable distribution

$$= n(E) = {}^9C_3 \times 3^6 = \frac{10 \times 8 \times 7}{3 \times 2 \times 1} \times 3^6 = 28.3^7$$

$$\therefore \quad \text{Required probability } = \frac{28 \times 3^7}{4^9} = \frac{28}{9} \cdot \left(\frac{3}{4}\right)^9$$

$$\therefore \quad x = \frac{28}{9}$$

$\therefore \quad |x - 3| < 1 \Rightarrow x \in (2, 4)$

and $x \in (3, 4)$

70. Answer (3)

$$\text{Mean} = \sum x_i p_i$$

$$= 0\left(\frac{1}{2}\right) + 1\left(\frac{1}{3}\right) + 2\left(\frac{1}{3^2}\right) + 3\left(\frac{1}{3^3}\right) + \ldots\ldots.$$

$$S = \frac{1}{3} + 2\left(\frac{1}{3^2}\right) + 3\left(\frac{1}{3^3}\right) + \ldots\ldots..\infty$$

$$\frac{S}{3} = \quad \frac{1}{3^2} + \quad 2\left(\frac{1}{3^3}\right) + \ldots\ldots.$$

$$\frac{2S}{3} = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\ldots..\infty$$

$$\frac{2S}{3} = \frac{\frac{1}{3}}{\frac{2}{3}} \Rightarrow \frac{2S}{3} = \frac{1}{2}$$

$$\Rightarrow \quad S = \frac{3}{4}$$

$$\text{Mean} = \frac{3}{4}$$

P(X is positive and even) can be $\dfrac{1}{3^2}, \dfrac{1}{3^4}, \dfrac{1}{3^6}\ldots\ldots.$

P(X is positive and even)

$$= \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \ldots\ldots. = \frac{1}{8}$$

71. Answer (4)

$$\because \quad 1 - \left(\frac{1}{2}\right)^n \geq 0.9$$

$$\Rightarrow \quad \frac{1}{2^n} \leq \frac{1}{10} \Rightarrow 2^n \geq 10$$

$\Rightarrow \quad n \geq 4$

72. Answer (1)

The given set = $\{n \in N : 2^n - 2$ is a multiple of 3$\}$

= {0, 6, 30, 62, 126, ...}

There are only 2, 2 digit numbers out of which only one is divisible by 3

$\therefore$ Required Probability = $\dfrac{1}{2}$.

73. Answer (3)

$$P(x = n) = {}^8C_n\left(\frac{1}{2}\right)^8$$

$$P(x = n+1) = {}^8C_{n+1}\left(\frac{1}{2}\right)^8$$

$$P(x = 8) = {}^8C_8\left(\frac{1}{2}\right)^8$$

$$\left(\frac{1}{2}\right)^8\left({}^8C_n + {}^8C_{n+1} + \ldots.. + {}^8C_8\right) < \frac{1}{2}$$

$$\Rightarrow \quad 2^8 - \left({}^8C_0 + {}^8C_1 + \ldots. + {}^8C_{n-1}\right) < 2^7$$

$\Rightarrow \quad {}^8C_0 + {}^8C_1 + \ldots. + {}^8C_{n-1} > 2^7$

Minimum value of n – 1 = 4

n = 5

74. Answer (1)

P(exactly one of A and B occurs)

$$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$$= P(A) \times P(\bar{B}) + P(\bar{A})P(B)$$

$$= p \times (1-2p) + (1-p)2p$$

$$3p - 4p^2 = \frac{5}{9}$$

$27p - 36p^2 = 5$

$36p^2 - 27p + 5 = 0$

$$p = \frac{27 \pm 3}{27} = \frac{5}{12} \text{ and } \frac{1}{3}$$

$$p_{\max} = \frac{5}{12}$$

75. Answer (4)

$$P(x \geq 5 \mid x > 2) = \frac{P(x \geq 5 \cap x > 2)}{P(x > 2)}$$

$$= \frac{P(x \geq 5)}{P(x > 2)}$$

Now,

$$P(x \geq 5) = \frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{5}{6}\frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{1}{6} + \ldots\infty$$

$$\text{and } P(x > 2) = \frac{5}{6}.\frac{5}{6}.\frac{1}{6} + \left(\frac{5}{6}\right)^3 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \ldots\infty$$

$$\therefore \quad P(x \geq 5) = \left(\frac{5}{6}\right)^4 \frac{1}{6}\left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \ldots\infty\right)$$

$$= \frac{5^4}{6^5}.6 = \left(\frac{5}{6}\right)^4$$

$$\text{and } P(x > 2) = \left(\frac{5}{6}\right)^2 \frac{1}{6}.6 = \left(\frac{5}{6}\right)^2$$

$$\therefore \quad \text{Required probability } = \frac{(5/6)^4}{(5/6)^2} = \frac{25}{36}$$

76. Answer (2)

The required probability

$$= 2\left\{\left(\frac{1}{6} - x\right)\left(\frac{1}{6} + x\right) + \frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6}\right\}$$

$$\because \quad 2\left(\frac{1}{36} - x^2 + \frac{1}{36} + \frac{1}{36}\right) = \frac{13}{96}$$

$$\frac{1}{6} - 2x^2 = \frac{13}{96}$$

$$2x^2 = \frac{1}{6} - \frac{13}{96} = \frac{16-13}{96}$$

$$\therefore \quad x^2 = \frac{1}{64}$$

$$\therefore \quad x = \frac{1}{8}$$

77. Answer (13)

$$\frac{\Sigma x_1^2}{n} - (\bar{x})^2 = 14$$

$$\frac{1^2 + 2^2 + \ldots + n^2}{n} - \left(\frac{\left(\frac{n(n+1)}{2}\right)}{n}\right)^2 = 14$$

$$\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = 14$$

$$n^2 - 1 = 168 \Rightarrow n = 13$$

78. Answer (4)

Let $A_i \rightarrow$ A gets i heads

$B_i \rightarrow$ B gets i heads

$$P = P\left(\sum_{i=0}^{3} (A_i \cap B_i)\right)$$

$$= P\left(\sum_{i=0}^{3} P(A_i)P(B_i)\right)$$

$$= P(A_0)P(B_0) + P(A_1)P(B_1) + P(A_2)P(B_2) + P(A_3)P(B_3)$$

$$= \frac{1}{8} \times \frac{1}{8} + \frac{3}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{3}{8} + \frac{1}{8} \times \frac{1}{8} = \frac{5}{16}$$

79. Answer (30)

$$p = P\left(\frac{1<X<4}{X<3}\right) = \frac{P((1<X<4)\cap(X<3))}{P(X<3)}$$

$$= \frac{P(X=2)}{P(X<3)} = \frac{2K}{K+2K} = \frac{2}{3}$$

Also, $K + 2K + 2K + 3K + K = 1 \Rightarrow K = \frac{1}{9}$

Now, $5p = \lambda K \Rightarrow 5\times\frac{2}{3} = \frac{1}{9}\lambda \Rightarrow \lambda = 30$

80. Answer (28)

$p(E_1) = 0.9$, $p(E_2) = 0.8$

$$p(E) = \frac{p(\overline{E}_1)\cdot p(E_2)}{p(E_1)\cdot p(\overline{E}_2) + p(\overline{E}_1)\cdot p(E_2) + p(\overline{E}_1)\cdot p(\overline{E}_2)}$$

$$= \frac{(.1)(.8)}{(.9)(.2)+(.1)(.8)+(.1)(.2)}$$

$$p = \frac{.08}{.28} = \frac{2}{7}$$

$$98p = 98\times\frac{2}{7} = 28$$

81. Answer (3)

Total ways = ${}^{64}C_2$

Favourable ways = 2(8 × 7) = 112

Required probability $= \frac{112}{32\times 63} = \frac{1}{18}$

82. Answer (781)

$$\frac{1}{5}+\frac{1}{3}+\frac{1}{5}+a+b = 1 \Rightarrow a+b = \frac{4}{15}$$

$$\Sigma P_i X_i = 2\cdot 3 \Rightarrow \frac{1}{5}(-2) - a + 1 + \frac{4}{5} + 6b = \frac{23}{10}$$

$$\left.\begin{aligned} -a+6b &= \frac{9}{10} \\ a+b &= \frac{4}{15} \end{aligned}\right\} \begin{aligned} a &= \frac{1}{10} \\ b &= \frac{1}{6} \end{aligned}$$

Variance $= \Sigma P_i X_i^2 - (\bar{X})^2$

$$= \left(\frac{1}{5}\cdot 4 + a + 3 + \frac{16}{5} + 36b\right) - \left(\frac{23}{10}\right)^2$$

$$= \frac{781}{100}$$

$100\sigma^2 = 781$

83. Answer (4)

$$1 - \left(\frac{3}{4}\right)^n \ge \frac{9}{10}$$

$$\Rightarrow \left(\frac{3}{4}\right)^n \le 1 - \frac{9}{10} = \frac{1}{10}$$

$$\Rightarrow \left(\frac{4}{3}\right)^n \ge 10$$

$\Rightarrow n[\log_4 - \log_3] \ge \log_{10} 10 = 1$

$$\Rightarrow n \ge \frac{1}{\log 4 - \log 3}$$

84. Answer (3)

$$P(1R \text{ and } 1B) = P(A)\cdot P\left(\frac{1R\ 1B}{A}\right) + P(B)\cdot P\left(\frac{1R\ 1B}{B}\right)$$

$$= \frac{1}{2}\cdot\frac{{}^3C_1\cdot{}^1C_1}{{}^6C_2} + \frac{1}{2}\cdot\frac{{}^2C_1\cdot{}^3C_1}{{}^{n+5}C_2}$$

$$P\left(\frac{1R\ 1B}{A}\right) = \frac{\frac{1}{2}\cdot\frac{3}{15}}{\frac{1}{2}\cdot\frac{3}{15} + \frac{1}{2}\cdot\frac{6\cdot 2}{(n+5)(n+4)}} = \frac{6}{11}$$

$$\Rightarrow \frac{\frac{1}{10}}{\frac{1}{10} + \frac{6}{(n+5)(n+4)}} = \frac{6}{11}$$

$$\Rightarrow \frac{11}{10} = \frac{6}{10} + \frac{36}{(n+5)(n+4)}$$

$$\Rightarrow \frac{5}{10\times 36} = \frac{1}{(n+5)(n+4)}$$

$\Rightarrow n^2 + 9n - 52 = 0$

$\Rightarrow n = 4$ is only possible value

85. Answer (1)

$3P(X = 0) = P(X = 1)$

$$3\cdot {}^nC_0 P^0 (1-P)^n = {}^nC_1 P^1 (1-P)^{n-1}$$

$$\frac{3}{n} = \frac{P}{1-P} \Rightarrow \frac{1}{11} = \frac{P}{1-P}$$

$\Rightarrow \quad 1 - P = 11P$

$$\Rightarrow \quad P = \frac{1}{12}$$

$$\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$$

$$\Rightarrow \quad \frac{{}^{33}C_{15} P^{15}(1-P)^{18}}{{}^{33}C_{18} P^{18}(1-P)^{15}} - \frac{{}^{33}C_{16} P^{16}(1-P)^{17}}{{}^{33}C_{17} P^{17}(1-P)^{16}}$$

$$\Rightarrow \quad \left(\frac{1-P}{P}\right)^3 - \left(\frac{1-P}{P}\right)$$

$$\Rightarrow \quad 11^3 - 11 = 1320$$

86. Answer (1)

$\because \quad X$ is a random variable

$\therefore \quad k + 2k + 4k + 6k + 8k = 1$

$$k = \frac{1}{21}$$

Now, $P(1 < X < 4 \mid X \le 2) = \dfrac{4k}{7k} = \dfrac{4}{7}$

87. Answer (479)

Student guesses only two wrong. So there are three possibilities

(i) Student guesses both wrong from 1st section

(ii) Student guesses both wrong from 2nd section

(iii) Student guesses two wrong one from each section

Required probabilities = ${}^4C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^2 \left(\frac{1}{4}\right)^6 +$

$${}^6C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^4 + {}^4C_1 \cdot {}^6C_1 \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^5$$

$$= \frac{1}{4^{10}}\left[6\times 9 + 15\times 9^4 + 24\times 9^2\right]$$

$$= \frac{27}{4^{10}}[2 + 27\times 15 + 72]$$

$$= \frac{27\times 479}{4^{10}}$$

88. Answer (3)

$$P\left(\frac{E_1}{E_2}\right) = \frac{1}{2} \Rightarrow \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1}{2}$$

$$P\left(\frac{E_2}{E_1}\right) = \frac{3}{4} \Rightarrow \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{3}{4}$$

$$P(E_1 \cap E_2) = \frac{1}{8}$$

$$P(E_2) = \frac{1}{4}, P(E_1) = \frac{1}{6}$$

(A) $P(E_1 \cap E_2) = \dfrac{1}{8}$ and $P(E_1).P(E_2) = \dfrac{1}{24}$

$$\Rightarrow \quad P(E_1 \cap E_2) \ne P(E_1).P(E_2)$$

(B) $P\left(E_1' \cap E_2'\right) = 1 - P(E_1 \cup E_2)$

$$= 1 - \left[\frac{1}{4} + \frac{1}{6} - \frac{1}{8}\right] = \frac{17}{24}$$

$$P\left(E_1'\right) = \frac{3}{4} \Rightarrow P\left(E_1'\right)P(E_2) = \frac{3}{24}$$

$$\Rightarrow \quad P\left(E_1' \cap E_2'\right) \ne P\left(E_1'\right).P(E_2)$$

(C) $P\left(E_1 \cap E_2'\right) = P(E_1) - P(E_1 \cap E_2)$

$$= \frac{1}{6} - \frac{1}{8} = \frac{1}{24}$$

$$P(E_1).P(E_2) = \frac{1}{24}$$

$$\Rightarrow \quad P\left(E_1 \cap E_2'\right) = P(E_1).P(E_2)$$

(D) $P\left(E_1' \cap E_2\right) = P(E_2) - P(E_1 \cap E_2)$

$$= \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$$

$$P(E_1)P(E_2) = \frac{1}{24}$$

$$\Rightarrow \quad P\left(E_1' \cap E_2\right) \ne P(E_1).P(E_2)$$

89. Answer (4)

There are only two ways to get sum 48,

which are (32, 8, 8) and (16, 16, 16)

So, required probability

$$= 3\left(\frac{2}{32}\cdot\frac{1}{8}\cdot\frac{1}{8}\right)+\left(\frac{1}{16}\cdot\frac{1}{16}\cdot\frac{1}{16}\right)$$

$$= \frac{3}{2^{10}}+\frac{1}{2^{12}}$$

$$= \frac{13}{2^{12}}$$

90. Answer (4)

Let probability of getting head = p

So, ${}^5C_4 p^4(1-p) = {}^5C_5 p^5$

$\Rightarrow \quad p = 5(1-p) \Rightarrow p = \frac{5}{6}$

Probability of getting atmost two heads =

${}^5C_0(1-p)^5 + {}^5C_1 p(1-p)^4 + {}^5C_2 p^2(1-p)^3$

$= \frac{1+25+250}{6^5}$

$= \frac{276}{6^5} = \frac{46}{6^4}$

91. Answer (33)

Total number of numbers from given

Condition = $n(s) = 2^6$.

Every required number is of the form

$A = 7\cdot(10^{a_1}+10^{a_2}+10^{a_3}+......)+111111$

Here 111111 is always divisible by 21.

$\therefore$ If A is divisible by 21 then

$10^{a_1}+10^{a_2}+10^{a_3}+......$ must be divisible by 3.

For this we have ${}^6C_0 + {}^6C_3 + {}^6C_6$ cases are there

$\therefore \quad n(E) = {}^6C_0 + {}^6C_3 + {}^6C_6 = 22$

$\therefore$ Required probability = $\frac{22}{2^6} = p$

$\therefore \quad \frac{11}{32} = p$

$\therefore \quad 96p = 33$

92. Answer (3)

Total cases = ${}^{18}C_5$

Favourable cases

${}^6C_1$	${}^3C_1$	${}^7C_1$
(Select x_1)	(Select x_3)	(Select x_3)

$$P = \frac{6\cdot 3\cdot 7}{{}^{18}C_5} = \frac{1}{68}$$

93. Answer (3)

Given $P(X=3) = 5P(X=4)$ and $n = 7$

$\Rightarrow \quad {}^7C_3 p^3 q^4 = 5\cdot {}^7C_4 p^4 q^3$

$\Rightarrow \quad q = 5p$ and also $p + q = 1$

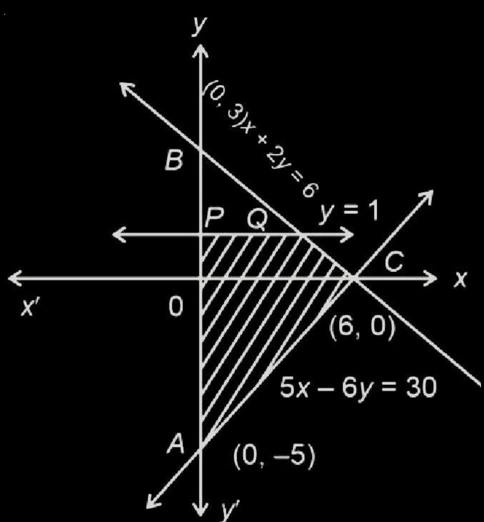
$\Rightarrow \quad p = \frac{1}{6}$ and $q = \frac{5}{6}$

Mean $= \frac{7}{6}$ and variance $= \frac{35}{36}$

Mean + Variance $= \frac{7}{6}+\frac{35}{36} = \frac{77}{36}$

94. Answer (2)

The required probability



$$= \frac{\text{Area of Region } PQCAP}{\text{Area of Region } ABCA}$$

$$= \frac{\frac{1}{2}\times 8\times 6-\frac{1}{2}\times 2\times 4}{\frac{1}{2}\times 8\times 6} = \frac{5}{6}$$

95. Answer (19)

Here $P(E_n) = \dfrac{n}{36}$ for $n = 1, 2, 3, \ldots\ldots\ldots, 8$

Here $P(A)$

$$= \frac{\text{Any possible sum of } (1, 2, 3, \ldots, 8)(= a \text{ say})}{36}$$

$\because \quad \dfrac{a}{36} \geq \dfrac{4}{5}$

$\therefore \quad a \geq 29$

If one of the number from {1, 2, …., 8} is left then total $a \geq 29$ by 3 ways.

Similarly by leaving terms more 2 or 3 we get 16 more combinations.

$\therefore$ Total number of different set A possible is 16 + 3 = 19

96. Answer (1)

Required cases = Total – all digits even – exactly one digit even

Total = 900 ways

All even $\Rightarrow$ 4 5 5 = 100 ways

One digit odd $\Rightarrow$ $\overset{\text{odd}}{5}$ 5 5 = 125 ways

4 $\overset{\text{odd}}{5}$ 5 = 100 ways

4 5 $\overset{\text{odd}}{5}$ = 100 ways

Required probability $= \dfrac{900 - 425}{900} = \dfrac{19}{36}$

97. Answer (4)

Number of one-one function from {a, b, c, d} to set {1, 2, 3, 4, 5} is ${}^5P_4 = 120\ n(s)$.

The required possible set of value

($f(a)$, f(b), $f(c)$, $f(d)$) such that $f(a) + 2f(b) - f(c) = f(d)$ are (5, 3, 2, 1), (5, 1, 2, 3), (4, 1, 3, 5), (3, 1, 4, 5), (5, 4, 3, 2) and (3, 4, 5, 2)

$\therefore \quad n(E) = 6$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{6}{120} = \dfrac{1}{20}$

98. Answer (3)

Let A = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

Let E be the event that matrix of order 2 × 2 is singular

Case-I

All entries are same example $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$= {}^{10}C_1$

Case-II

Matrix with two prime numbers only example $\begin{bmatrix} 3 & 5 \\ 3 & 5 \end{bmatrix}$

$= {}^{10}C_2 \times 2! \times 2!$

$$P(E) = \frac{{}^{10}C_1 + {}^{10}C_2 \times 2! \times 2!}{10^4} = \frac{190}{10^4} = \frac{19}{10^3}$$

99. Answer (1)

Total number of relations = $2^{2^2} = 2^4 = 16$

Relations which are symmetric as well as transitive are

ϕ, {(x, x)}, {(y, y)}, {(x, x), (x, y), (y, y), (y, x)}, {(x, x), (y, y)}

$\therefore$ favourable cases = 5

$\therefore \quad P_r = \dfrac{5}{16}$

100. Answer (3)

If n is number of trails, p is probability of success and q is probability of unsuccess then,

Mean = np and variance = npq.

Here $np + npq = 24$ …(i)

$np.npq = 128$ …(ii)

and $q = 1 - p$ …(iii)

from eq. (i), (ii) and (iii) : $p = q = \dfrac{1}{2}$ and $n = 32$.

$\therefore$ Required probability $= p(X = 1) + p(X = 2)$

$$= {}^{32}C_1 \cdot \left(\frac{1}{2}\right)^{32} + {}^{32}C_2 \cdot \left(\frac{1}{2}\right)^{32}$$

$$= \left(32 + \frac{32 \times 31}{2}\right) \cdot \frac{1}{2^{32}} = \frac{33}{2^{28}}$$

101. Answer (1)

For $x^2 + \alpha x + \beta > 0 \ \forall \ x \in R$ to hold, we should have $\alpha^2 - 4\beta < 0$

If $\alpha = 1$, β can be 1, 2, 3, 4, 5, 6 *i.e.*, 6 choices

If $\alpha = 2$, β can be 2, 3, 4, 5, 6 *i.e.*, 5 choices

If $\alpha = 3$, β can be 3, 4, 5, 6 *i.e.*, 4 choices

If $\alpha = 4$, β can be 5 or 6 *i.e.*, 2 choices

If $\alpha = 6$, No possible value for β *i.e.*, 0 choices

Hence total favourable outcomes

$= 6 + 5 + 4 + 2 + 0 + 0$

$= 17$

Total possible choices for α and $\beta = 6 \times 6 = 36$

Required probability $= \dfrac{17}{36}$

102. Answer (3)

Given, mean $= np = \alpha$.

and variance $= npq = \dfrac{\alpha}{3}$

$\Rightarrow \quad q = \dfrac{1}{3}$ and $p = \dfrac{2}{3}$

$P(X = 1) = n.p^1.q^{n-1} = \dfrac{4}{243}$

$\Rightarrow \quad n.\dfrac{2}{3}.\left(\dfrac{1}{3}\right)^{n-1} = \dfrac{4}{243}$

$\Rightarrow \quad n = 6$

$P(X = 4 \text{ or } 5) = {}^6C_4.\left(\dfrac{2}{3}\right)^4.\left(\dfrac{1}{3}\right)^2 + {}^6C_5.\left(\dfrac{2}{5}\right)^5.\dfrac{1}{3}$

$= \dfrac{16}{27}$

103. Answer (2)

$0 \le \dfrac{2+3P}{6} \le 1 \Rightarrow P \in \left[-\dfrac{2}{3}, \dfrac{4}{3}\right]$

$0 \le \dfrac{2-P}{8} \le 1 \Rightarrow P \in [-6, 2]$

$0 \le \dfrac{1-P}{2} \le 1 \Rightarrow P \in [-1, 1]$

$0 < P(E_1) + P(E_2) + P(E_3) \le 1$

$0 < \dfrac{13}{12} - \dfrac{P}{8} \le 1$

$P \in \left[\dfrac{2}{3}, \dfrac{26}{3}\right]$

Taking intersection of all

$P \in \left[\dfrac{2}{3}, 1\right)$

$P_1 + P_2 = \dfrac{5}{3}$

104. Answer (3)

Among the 5 digit numbers,

First number divisible by 7 is 10003 and last is 99995.

$\Rightarrow$ Number of numbers divisible by 7.

$= \dfrac{99995 - 10003}{7} + 1$

$= 12857$

First number divisible by 35 is 10010 and last is 99995.

$\Rightarrow$ Number of numbers divisible by

$35 = \dfrac{99995 - 10010}{35} + 1$

$= 2572$

Hence number of number divisible by 7 but not by 5

$= 12857 - 2572$

$= 10285$

$9p. = \dfrac{10285}{90000} \times 9$

$= 1.0285$

105. Answer (2)

Mean $= np = 16$

Variance $= npq = 8$

$\Rightarrow \quad q = p = \dfrac{1}{2}$ and $n = 32$

$P(x > n - 3) = p(x = n - 2) + p(x = n - 1) + p(x = n)$

$= \left({}^{32}C_2 + {}^{32}C_1 + {}^{32}C_0\right) \cdot \dfrac{1}{2^n}$

$= \dfrac{529}{2^n}$

106. Answer (4)

Let P(a prime number) = α

P(a composite number) = β

and $P(1) = \gamma$

$\because\ 3\alpha = 6\beta = 2\gamma = k$ (say)

and $3\alpha + 2\beta + \gamma = 1$

$$\Rightarrow\quad k + \frac{k}{3} + \frac{k}{2} = 1 \Rightarrow k = \frac{6}{11}$$

Mean = np where $n = 2$

and p = probability of getting perfect square

$$= P(1) + P(4) = \frac{k}{2} + \frac{k}{6} = \frac{4}{11}$$

So, mean = $2\cdot\left(\frac{4}{11}\right) = \frac{8}{11}$

107. Answer (1)

$$P(A/B) = \frac{1}{7} \Rightarrow \frac{P(A\cap B)}{P(B)} = \frac{1}{7}$$

$$\Rightarrow\quad P(B) = \frac{7}{9}$$

$$P(B/A) = \frac{2}{5} \Rightarrow \frac{P(A\cap B)}{P(A)} = \frac{2}{5}$$

$$P(A) = \frac{5}{2}\cdot\frac{1}{9} = \frac{5}{18}$$

$$S2:\ P(A'\cap B') = \frac{1}{18}$$

$$S1: \text{and } P(A'\cup B) = \frac{1}{9} + \frac{6}{9} + \frac{1}{18} = \frac{5}{6}.$$

108. Answer (56)

X = Number of white ball drawn

$$P(X=0) = \frac{{}^6C_3}{{}^{10}C_3} = \frac{1}{6}$$

$$P(X=1) = \frac{{}^6C_2 \times {}^4C_1}{{}^{10}C_3} = \frac{1}{2},$$

$$P(X=2) = \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3} = \frac{3}{10}$$

and $P(X=3) = \dfrac{{}^6C_0 \times {}^4C_3}{{}^{10}C_3} = \dfrac{1}{30}$

$$\text{Variance} = \sigma^2 = \sum P_i X_i^2 - \left(\sum P_i X_i\right)^2$$

$$\sigma^2 = \frac{1}{2} + \frac{12}{10} + \frac{3}{10} - \left(\frac{1}{2} + \frac{6}{10} + \frac{1}{10}\right)^2$$

$$= \frac{56}{100}$$

$100\sigma^2 = 56.$

109. Answer (2)

Let $E \to$ Ball drawn from Bag II is black.

$E_R \to$ Bag I to Bag II red ball transferred.

$E_B \to$ Bag I to Bag II black ball transferred.

$E_W \to$ Bag I to Bag II white ball transferred.

$$P\left(\frac{E_R}{E}\right) = \frac{P\left(\frac{E}{E_R}\right)\cdot P(E_R)}{P\left(\frac{E}{E_R}\right)P(E_R) + P\left(\frac{E}{E_B}\right)P(E_B) + P\left(\frac{E}{E_W}\right)P(E_W)}$$

Here,

$$P(E_R) = 3/10,\ P(E_B) = 4/10,\ P(E_W) = 3/10$$

and

$$P\left(\frac{E}{E_R}\right) = 5/10,\ P\left(\frac{E}{E_B}\right) = 6/10,\ P\left(\frac{E}{E_W}\right) = 5/10$$

$$\therefore\quad P\left(\frac{E_R}{E}\right) = \frac{15/100}{15/100 + 24/100 + 15/100}$$

$$= \frac{15}{54} = \frac{5}{18}$$

110. Answer (96)

Given $np + npq = 82.5$ ... (1)

and $np\,(npq) = 1350$ ... (2)

$$\therefore\quad x^2 - 82.5x + 1350 = 0 \begin{matrix}\nearrow \text{Mean}\\ \searrow \text{Variance}\end{matrix}$$

$\Rightarrow \quad x^2 - 22.5\,x - 60x + 1350 = 0$

$\Rightarrow \quad x - (x - 22.5) - 60\,(x - 22.5) = 0$

Mean = 60 and Variance = 22.5

$np = 60,\ npq = 22.5$

$$\Rightarrow \quad q = \frac{9}{24} = \frac{3}{8},\ \ p = \frac{5}{8}$$

$$\therefore \quad n\frac{5}{8} = 60 \ \Rightarrow \ n = 96$$

111. Answer (2)

Mean = 4 = μ = np

Variance = $\sigma^2 = np(1-P) = \dfrac{4}{3}$

$$4(1-P) = \frac{4}{3}$$

$$P = \frac{2}{3}$$

$$n \times \frac{2}{3} = 4$$

$n = 6$

$$P(X = k) = {}^nC_k\ P^k(1-P)^{n-k}$$

$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= {}^6C_0\,P^0(1-p)^6 + {}^6C_1\,P^1\,(1-P)^5 + {}^6C_2\,P^2\,(1-P)^4$$

$$= {}^6C_0\left(\frac{1}{3}\right)^6 + {}^6C_1\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^5 + {}^6C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4$$

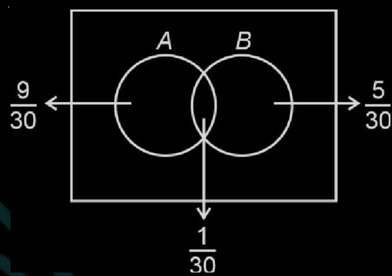
$$= \left(\frac{1}{3}\right)^6 [1+12+60] = \frac{73}{3^6}$$

$$54P\,(X \le 2) = \frac{73}{3^6} \times 54 = \frac{146}{27}$$

112. Answer (2)

$$P(A) = \frac{1}{3}, P(B) = \frac{1}{5} \text{ and } P(A \cup B) = \frac{1}{2}$$

$$\therefore \quad P(A \cap B) = \frac{1}{3} + \frac{1}{5} - \frac{1}{2} = \frac{1}{30}$$



$$\text{Now, } P(A|B') + P(B|A') = \frac{P(A \cap B')}{P(B')} + \frac{P(B \cap A')}{P(A')}$$

$$= \frac{\frac{9}{30}}{\frac{4}{5}} + \frac{\frac{5}{30}}{\frac{2}{3}} = \frac{5}{8}$$

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