Chapter 20

Applications of Derivatives

1. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$ **[AIEEE-2009]**
   (1) $P(-1)$ is not minimum but $P(1)$ is the maximum of P
   (2) $P(-1)$ is minimum but $P(1)$ is not the maximum of P
   (3) Neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P
   (4) $P(-1)$ is the minimum and $P(1)$ is the maximum of P

2. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is **[AIEEE-2009]**
   (1) $\frac{2\sqrt{3}}{8}$ (2) $\frac{3\sqrt{2}}{5}$
   (3) $\frac{\sqrt{3}}{4}$ (4) $\frac{3\sqrt{2}}{8}$

3. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is **[AIEEE-2010]**
   (1) $y = 0$ (2) $y = 1$
   (3) $y = 2$ (4) $y = 3$

4. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by
$$f(x) = \begin{cases} k - 2x, & \text{if } x \le -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$$
   If f has a local minimum at $x = -1$, then a possible value of k is **[AIEEE-2010]**
   (1) 1 (2) 0
   (3) $-\frac{1}{2}$ (4) -1

5. The curve that passes through the point (2, 3) and has the property that the segment of any tangent to it lying between the coordinate axes bisected by the point of contact is given by **[AIEEE-2011]**
   (1) $x^2 + y^2 = 13$ (2) $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$
   (3) $2y - 3x = 0$ (4) $y = \frac{6}{x}$

6. Let f be a function defined by
$$f(x) = \begin{cases} \frac{\tan x}{x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$$
   Statement-1: $x = 0$ is point of minima of f.
   Statement-2: $f'(0) = 0$. **[AIEEE-2011]**
   (1) Statement-1 is true, statement-2 is false.
   (2) Statement-1 is false, statement-2, is true.
   (3) Statement-1 is true, statement-2 is true, statement-2 is a correct explanation for statement-1
   (4) Statement-1 is true, statement-2 is true; statement-2 is ***0*** a correct explanation for statement-1

7. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per unit) at which the radius of the balloon decreases 49 minutes after the leakage began is **[AIEEE-2012]**
   (1) 7/9 (2) 2/9
   (3) 9/2 (4) 9/7

8. Let $a, b \in R$ be such that the function f given be $f(x) = \ln|x| + bx^2 + ax$, $x \ne 0$ has extreme values at $x = -1$ and $x = 2$.
   Statement-1: f has local maximum at $x = -1$ and at $x = 2$.
   Statement-2: $a = \frac{1}{2}$ and $b = \frac{-1}{4}$. **[AIEEE-2012]**

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(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(2) Statement-1 is true, statement-2 is true, statement-2 is **not** a correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is false.

(4) Statement-1 is false, statement-2 is true.

9. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + \beta x^2 + x$ then **[JEE (Main)-2014]**

(1) $\alpha = 2, \beta = -\frac{1}{2}$ (2) $\alpha = 2, \beta = \frac{1}{2}$

(3) $\alpha = -6, \beta = \frac{1}{2}$ (4) $\alpha = -6, \beta = -\frac{1}{2}$

10. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at $(1,1)$ **[JEE (Main)-2015]**

(1) Does not meet the curve again

(2) Meets the curve again in the second quadrant

(3) Meets the curve again in the third quadrant

(4) Meets the curve again in the fourth quadrant

11. Let $f(x)$ be a polynomial of degree four having extreme values at $x = 1$ and $x = 2$. If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$, then $f(2)$ is equal to **[JEE (Main)-2015]**

(1) –8 (2) –4

(3) 0 (4) 4

12. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then **[JEE (Main)-2016]**

(1) $(4-\pi)x = \pi r$ (2) $x = 2r$

(3) $2x = r$ (4) $2x = (\pi + 4)r$

13. The normal to the curve $y(x-2)(x-3) = x + 6$ at the point where the curve intersects the y-axis passes through the point **[JEE (Main)-2017]**

(1) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (2) $\left(\frac{1}{2}, -\frac{1}{3}\right)$

(3) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (4) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

14. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is **[JEE (Main)-2017]**

(1) 10 (2) 25

(3) 30 (4) 12.5

15. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is **[JEE (Main)-2018]**

(1) 6 (2) $\frac{7}{2}$

(3) 4 (4) $\frac{9}{2}$

16. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is **[JEE (Main)-2018]**

(1) 3 (2) –3

(3) $-2\sqrt{2}$ (4) $2\sqrt{2}$

17. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to **[JEE (Main)-2019]**

(1) $\frac{8}{15}$ (2) $\frac{7}{17}$

(3) $\frac{8}{17}$ (4) $\frac{4}{9}$

18. The maximum volume (in cu. m) of the right circular cone having slant height 3 m is **[JEE (Main)-2019]**

(1) $\frac{4}{3}\pi$ (2) $2\sqrt{3}\,\pi$

(3) $3\sqrt{3}\,\pi$ (4) $6\,\pi$

19. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}, (x > 0)$, is **[JEE (Main)-2019]**

(1) $\frac{3}{2}$ (2) $\frac{5}{4}$

(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{\sqrt{5}}{2}$

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20. The tangent to the curve $y = xe^{x^2}$ passing through the point (l, e) also passes through the point **[JEE (Main)-2019]**

(1) $(2, 3e)$ (2) $\left(\frac{4}{3}, 2e\right)$

(3) $(3, 6e)$ (4) $\left(\frac{5}{3}, 2e\right)$

21. A helicopter is flying along the curve given by $y - x^{3/2} = 7$, $(x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is **[JEE (Main)-2019]**

(1) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (2) $\frac{\sqrt{5}}{6}$

(3) $\frac{1}{2}$ (4) $\frac{1}{3}\sqrt{\frac{7}{3}}$

22. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in R: x^2 + 30 \leq 11x\}$ is **[JEE (Main)-2019]**

(1) 122 (2) –122

(3) 222 (4) –222

23. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is **[JEE (Main)-2019]**

(1) $\frac{1}{2}$ (2) $\frac{m+n}{6mn}$

(3) 1 (4) $\frac{1}{4}$

24. Let $f(x) = \frac{x}{\sqrt{a^2+x^2}} - \frac{(d-x)}{\sqrt{b^2+(d-x)^2}}, x \in R$ where a, b and d are non-zero real constants. Then **[JEE (Main)-2019]**

(1) f is an increasing function of x

(2) f is a decreasing function of x

(3) f is neither increasing nor decreasing function of x

(4) f' is not a continuous function of x

25. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point **[JEE (Main)-2019]**

(1) $\left(\frac{1}{4}, \frac{7}{2}\right)$ (2) $\left(\frac{1}{8}, -7\right)$

(3) $\left(\frac{7}{2}, \frac{1}{4}\right)$ (4) $\left(-\frac{1}{8}, 7\right)$

26. If a curve passes through the point (1, –2) and has slope of the tangent at any point (x, y) on it as $\frac{x^2 - 2y}{x}$, then the curve also passes through the point **[JEE (Main)-2019]**

(1) (–1, 2) (2) $(\sqrt{3}, 0)$

(3) (3, 0) (4) $(-\sqrt{2}, 1)$

27. If the function f given by $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0\ (x \neq 1)$ is **[JEE (Main)-2019]**

(1) –7 (2) 5

(3) 6 (4) 7

28. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis, is **[JEE (Main)-2019]**

(1) $x = y\cot\theta - 2\tan\theta$

(2) $y = x\tan\theta + 2\cot\theta$

(3) $x = y\cot\theta + 2\tan\theta$

(4) $y = x\tan\theta - 2\cot\theta$

29. The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is **[JEE (Main)-2019]**

(1) $\frac{11}{4\sqrt{2}}$ (2) $\frac{7}{8}$

(3) 2 (4) $\frac{7}{4\sqrt{2}}$

30. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then **[JEE (Main)-2019]**

(1) $S_1 = \{-2\}; S_2 = \{0, 1\}$

(2) $S_1 = \{-2, 1\}; S_2 = \{0\}$

(3) $S_1 = \{-1\}; S_2 = \{0, 2\}$

(4) $S_1 = \{-2, 0\}; S_2 = \{1\}$

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31. Let $f : [0, 2] \to R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2 - x)$, then ϕ is **[JEE (Main)-2019]**

(1) Decreasing on (0, 2)

(2) Increasing on (0, 2)

(3) Decreasing on (0, 1) and increasing on (1, 2)

(4) Increasing on (0, 1) and decreasing on (1, 2)

32. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is **[JEE (Main)-2019]**

(1) $\frac{2}{3}\sqrt{3}$ (2) $2\sqrt{3}$

(3) $\sqrt{3}$ (4) $\sqrt{6}$

33. If $f(x)$ is a non-zero polynomial of degree four, having local extreme points at $x = -1, 0, 1$; then the set

$S = \{x \in R : f(x) = f(0)\}$

contains exactly **[JEE (Main)-2019]**

(1) Four irrational numbers

(2) Four rational numbers

(3) Two irrational and one rational number

(4) Two irrational and two rational numbers

34. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points $(1, f(1))$ and $(-1, f(-1))$, then S is equal to **[JEE (Main)-2019]**

(1) $\left\{\frac{1}{3}, -1\right\}$ (2) $\left\{-\frac{1}{3}, 1\right\}$

(3) $\left\{-\frac{1}{3}, -1\right\}$ (4) $\left\{\frac{1}{3}, 1\right\}$

35. If the tangent to the curve, $y = x^3 + ax - b$ at the point $(1, -5)$ is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on curve? **[JEE (Main)-2019]**

(1) (–2, 1) (2) (2, –2)

(3) (2, –1) (4) (–2, 2)

36. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic metre per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10 m; is **[JEE (Main)-2019]**

(1) $\frac{2}{\pi}$ (2) $\frac{1}{15\pi}$

(3) $\frac{1}{10\pi}$ (4) $\frac{1}{5\pi}$

37. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall\, x \in R$. Then the set of all $x \in R$, where the function $h(x) = (fog)(x)$ is increasing, is **[JEE (Main)-2019]**

(1) $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$ (2) $[0, \infty)$

(3) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$ (4) $\left[\frac{-1}{2}, 0\right] \cup [1, \infty)$

38. If the tangent to the curve $y = \frac{x}{x^2 - 3}$, $x \in R$, $(x \neq \pm\sqrt{3})$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then **[JEE (Main)-2019]**

(1) $|6\alpha + 2\beta| = 19$ (2) $|2\alpha + 6\beta| = 19$

(3) $|6\alpha + 2\beta| = 9$ (4) $|2\alpha + 6\beta| = 11$

39. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is **[JEE (Main)-2019]**

(1) $\frac{5}{6\pi}$ (2) $\frac{1}{36\pi}$

(3) $\frac{1}{9\pi}$ (4) $\frac{1}{18\pi}$

40. If m is the minimum value of k for which the function $f(x) = x\sqrt{kx - x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when $k = m$, then the ordered pair (m, M) is equal to **[JEE (Main)-2019]**

(1) $(4, 3\sqrt{2})$ (2) $(3, 3\sqrt{3})$

(3) $(5, 3\sqrt{6})$ (4) $(4, 3\sqrt{3})$

41. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is **[JEE (Main)-2019]**

(1) $\frac{120}{13}$ (2) $\frac{13}{2}$

(3) $\frac{13}{5}$ (4) $\frac{60}{13}$

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42. The tangents to the curve $y = (x - 2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point **[JEE (Main)-2019]**

(1) $\left(\frac{5}{2}, 1\right)$ (2) $\left(-\frac{5}{2}, 1\right)$

(3) $\left(\frac{5}{2}, -1\right)$ (4) $\left(-\frac{5}{2}, -1\right)$

43. Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x \to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true?

[JEE (Main)-2020]

(1) f is an odd function

(2) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f

(3) $f(1) - 4f(-1) = 4$

(4) $x = 1$ is a point of maxima and $x = -1$ is a point of minimum of f

44. Let $f(x) = x\cos^{-1}(-\sin|x|)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then which of the following is true? **[JEE (Main)-2020]**

(1) $f'(0) = -\frac{\pi}{2}$

(2) f' is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$

(3) f is not differentiable at $x = 0$

(4) f' is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$

45. Let $f : (1, 3) \to R$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where $[x]$ denotes the greatest integer $\le x$. Then the range of f is

[JEE (Main)-2020]

(1) $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$ (2) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

(3) $\left(\frac{2}{5}, \frac{4}{5}\right]$ (4) $\left(\frac{3}{5}, \frac{4}{5}\right)$

46. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is

[JEE (Main)-2020]

(1) $\frac{1}{36\pi}$ (2) $\frac{1}{18\pi}$

(3) $\frac{1}{54\pi}$ (4) $\frac{5}{6\pi}$

47. Let a function $f : [0, 5] \to R$ be continuous, $f(1) = 3$ and F be defined as

$$F(x) = \int_1^x t^2 g(t)dt, \text{ where } g(t) = \int_1^t f(u)du.$$

Then for the function F, the point $x = 1$ is

[JEE (Main)-2020]

(1) A point of inflection.

(2) Not a critical point.

(3) A point of local minima.

(4) A point of local maxima.

48. If $p(x)$ be a polynomial of degree three that has a local maximum value 8 at $x = 1$ and a local minimum value 4 at $x = 2$; then $p(0)$ is equal to

[JEE (Main)-2020]

(1) 12 (2) 6

(3) –24 (4) –12

49. Let $P(h, k)$ be a point on the curve $y = x^2 + 7x + 2$, nearest to the line, $y = 3x - 3$. Then the equation of the normal to the curve at P is **[JEE (Main)-2020]**

(1) $x - 3y - 11 = 0$ (2) $x - 3y + 22 = 0$

(3) $x + 3y - 62 = 0$ (4) $x + 3y + 26 = 0$

50. If the tangent to the curve $y = x + \sin y$ at a point (a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$, then **[JEE (Main)-2020]**

(1) $b = a$ (2) $b = \frac{\pi}{2} + a$

(3) $|a + b| = 1$ (4) $|b - a| = 1$

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51. The equation of the normal to the curve $y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$ at $x = 0$ is **[JEE (Main)-2020]**

(1) $y = 4x + 2$ (2) $y + 4x = 2$

(3) $x + 4y = 8$ (4) $2y + x = 4$

52. Let $f : (-1, \infty) \longrightarrow R$ be defined by $f(0) = 1$ and $f(x) = \frac{1}{x}\log_e(1+x),\ x \neq 0$. Then the function f **[JEE (Main)-2020]**

(1) Increases in $(-1, \infty)$

(2) Increases in $(-1, 0)$ and decreases in $(0, \infty)$

(3) Decreases in $(-1, 0)$ and increases in $(0, \infty)$

(4) Decreases in $(-1, \infty)$

53. The function, $f(x) = (3x - 7)x^{2/3},\ x \in R$, is increasing for all x lying in : **[JEE (Main)-2020]**

(1) $(-\infty, 0)\cup\left(\frac{14}{15}, \infty\right)$

(2) $(-\infty, 0)\cup\left(\frac{3}{7}, \infty\right)$

(3) $\left(-\infty, -\frac{14}{15}\right)\cup(0, \infty)$

(4) $\left(-\infty, \frac{14}{15}\right)$

54. Suppose $f(x)$ is a polynomial of degree four, having critical points at –1, 0, 1. If $T = \{x \in R \mid f(x) = f(0)\}$, then the sum of squares of all the elements of T is **[JEE (Main)-2020]**

(1) 4 (2) 2

(3) 6 (4) 8

55. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is: **[JEE (Main)-2020]**

(1) 9 (2) 20

(3) 10 (4) 18

56. Let f be a twice differentiable function on (1, 6). If $f(2) = 8$, $f'(2) = 5$, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1, 6)$, then **[JEE (Main)-2020]**

(1) $f(5) + f'(5) \geq 28$ (2) $f'(5) + f''(5) \leq 20$

(3) $f(5) \leq 10$ (4) $f(5) + f'(5) \leq 26$

57. The area (in sq. units) of the largest rectangle $ABCD$ whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the x-axis, is **[JEE (Main)-2020]**

(1) $\frac{4}{3\sqrt{3}}$ (2) $\frac{1}{3\sqrt{3}}$

(3) $\frac{2}{3\sqrt{3}}$ (4) $\frac{4}{3}$

58. The minimum value of $2^{\sin x} + 2^{\cos x}$ is **[JEE (Main)-2020]**

(1) $2^{-1+\sqrt{2}}$ (2) $2^{1-\sqrt{2}}$

(3) $2^{1-\frac{1}{\sqrt{2}}}$ (4) $2^{-1+\frac{1}{\sqrt{2}}}$

59. If the minimum and the maximum values of the function $f : \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \to R$, defined by

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

are m and M respectively, then the ordered pair (m, M) is equal to **[JEE (Main)-2020]**

(1) $(0, 2\sqrt{2})$ (2) (0, 4)

(3) (–4, 4) (4) (–4, 0)

60. Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point (1, 0)? **[JEE (Main)-2020]**

(1) (2, 2) (2) (–2, 4)

(3) (2, 6) (4) (–2, 6)

61. If $x = 1$ is a critical point of the function $f(x) = (3x^2 + ax - 2 - a)e^x$, then **[JEE (Main)-2020]**

(1) $x = 1$ is a local maxima and $x = -\frac{2}{3}$ is a local minima of f

(2) $x = 1$ and $x = -\frac{2}{3}$ are local maxima of f

(3) $x = 1$ and $x = -\frac{2}{3}$ are local minima of f

(4) $x = 1$ is a local minima and $x = -\frac{2}{3}$ is a local maxima of f

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62. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

Then the ordered pair (m, M) is equal to

[JEE (Main)-2020]

(1) $(1, 3)$ (2) $(-3, -1)$

(3) $(-4, -1)$ (4) $(-3, 3)$

63. The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point **[JEE (Main)-2020]**

(1) $2a(t_1 + t_2) + b$

(2) $(t_2 - t_1) / 2$

(3) $a(t_2 - t_1) + b$

(4) $(t_1 + t_2) / 2$

64. The set of all real values of λ for which the function

$f(x) = (1 - \cos^2 x)\cdot(\lambda + \sin x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is

[JEE (Main)-2020]

(1) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (2) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(3) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (4) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

65. If the tangent to the curve, $y = f(x) = x\log_e x$, $(x > 0)$ at a point $(c, f(c))$ is parallel to the line-segment joining the points $(1, 0)$ and (e, e), then c is equal to **[JEE (Main)-2020]**

(1) $\frac{1}{e-1}$ (2) $\frac{e-1}{e}$

(3) $e^{\left(\frac{1}{1-e}\right)}$ (4) $e^{\left(\frac{1}{e-1}\right)}$

66. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10$, $f(1) = -6$, $f(x)$ has a critical point at $x = -1$, and $f'(x)$ has a critical point $x = 1$. Then $f(x)$ has a local minima at x = ________.

[JEE (Main)-2020]

67. If the tangent to the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point $(1, 2)$ intersect at the same point on the x-axis, then the value of c is ________ .

[JEE (Main)-2020]

68. If the lines $x + y = a$ and $x - y = b$ touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to ________. **[JEE (Main)-2020]**

69. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If $AD = 8$ m, $BC = 11$ m and $AB = 10$ m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is ________.

[JEE (Main)-2020]

70. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is: **[JEE (Main)-2021]**

(1) $-2t^3$ (2) $2t^3$

(3) 0 (4) $-t^3$

71. The function

$$f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x:$$

[JEE (Main)-2021]

(1) Decreases in $\left[\frac{1}{2}, \infty\right)$

(2) Decreases in $\left(-\infty, \frac{1}{2}\right]$

(3) Increases in $\left(-\infty, \frac{1}{2}\right]$

(4) Increases in $\left[\frac{1}{2}, \infty\right)$

72. Let $f: R \to R$ be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let $A = \{x \in R : f \text{ is increasing}\}$. Then A is equal to: **[JEE (Main)-2021]**

(1) $(-5, \infty)$ (2) $(-\infty, -5) \cup (4, \infty)$

(3) $(-5, -4) \cup (4, \infty)$ (4) $(-\infty, -5) \cup (-4, \infty)$

73. If the curve $y = ax^2 + bx + c$, $x \in R$, passes through the point $(1, 2)$ and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are : **[JEE (Main)-2021]**

(1) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$ (2) $a = 1, b = 0, c = 1$

(3) $a = 1, b = 1, c = 0$ (4) $a = -1, b = 1, c = 1$

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74. For which of the following curves, the line $x+\sqrt{3y}=2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2},\frac{1}{2}\right)$? **[JEE (Main)-2021]**

(1) $x^2 + 9y^2 = 9$
(2) $y^2 = \frac{1}{6\sqrt{3}}x$
(3) $2x^2 - 18y^2 = 9$
(4) $x^2 + y^2 = 7$

75. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = –1, and x = 1. If $\lim\limits_{x\to 0}\frac{f(x)}{x^3}=1$, then $5\cdot f(2)$ is equal to _________. **[JEE (Main)-2021]**

76. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to _______. **[JEE (Main)-2021]**

77. The local maximum of slope of the curve $y=\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point : **[JEE (Main)-2021]**

(1) (2, 2)
(2) (0, 0)
(3) $\left(3,\frac{21}{2}\right)$
(4) (2, 9)

78. The triangle of maximum area that can be inscribed in a given circle of radius ‘r’ is : **[JEE (Main)-2021]**

(1) An isosceles triangle with base equal to 2r.
(2) An equilateral triangle of height $\frac{2r}{3}$.
(3) A right angle triangle having two of its sides of length 2r and r.
(4) An equilateral triangle having each of its side of length $\sqrt{3}$ r.

79. The range of $a\in\mathbf{R}$ for which the function $f(x) = (4a - 3)(x + \log_e 5) + 2(a - 7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right)$, $x \neq 2n\pi$, $n\in N$ has critical points, is : **[JEE (Main)-2021]**

(1) (–3, 1)
(2) $[1, \infty]$
(3) $(-\infty, -1)$
(4) $\left[-\frac{4}{3}, 2\right]$

80. Let f be a real valued function, defined on R – {–1, 1} and given by

$$f(x)=3\log_e\left|\frac{x-1}{x+1}\right|-\frac{2}{x-1}.$$

Then in which of the following intervals, function f(x) is increasing? **[JEE (Main)-2021]**

(1) $(-\infty, \frac{1}{2}] - \{-1\}$
(2) $(-1, \frac{1}{2}]$
(3) $(-\infty, \infty) - \{-1, 1\}$
(4) $(-\infty, -1)\cup\left([\frac{1}{2}, \infty) - \{1\}\right)$

81. Consider the function f : R → R defined by

$$f(x)=\begin{cases}\left(2-\sin\left(\frac{1}{x}\right)\right)|x|, & x\neq 0\\ 0, & x=0\end{cases}$$

Then f is : **[JEE (Main)-2021]**

(1) Monotonic on $(0, \infty)$ only
(2) Monotonic on $(-\infty, 0)\cup(0, \infty)$
(3) Monotonic on $(-\infty, 0)$ only
(4) Not monotonic on $(-\infty, 0)$ and $(0, \infty)$

82. Let f : [–1, 1] → R be defined as $f(x) = ax^2 + bx + c$ for all $x \in [-1, 1]$, where a, b, c ∈ R such that f(–1) = 2, f′(–1) = 1 and for x ∈ (–1, 1) the maximum value of f″(x) is $\frac{1}{2}$. If $f(x) \le \alpha$, $x \in [-1, 1]$, then the least value of α is equal to _________. **[JEE (Main)-2021]**

83. Let ‘a’ be a real number such that the function $f(x) = ax^2 + 6x - 15$, $x\in\mathbb{R}$ is increasing in $\left(-\infty,\frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4},\infty\right)$. Then the function $g(x) = ax^2 - 6x + 15$, $x\in\mathbb{R}$ has a : **[JEE (Main)-2021]**

(1) Local maximum at $x=\frac{3}{4}$
(2) Local maximum at $x=-\frac{3}{4}$
(3) Local minimum at $x=-\frac{3}{4}$
(4) Local minimum at $x=\frac{3}{4}$

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84. Let A = $[a_{ij}]$ be a 3 × 3 matrix, where

$$a_{ij} = \begin{cases} 1 & , \text{ if } i = j \\ -x & , \text{ if } |i-j| = 1 \\ 2x+1 & , \text{ otherwise.} \end{cases}$$

Let a function $f : R \to R$ be defined as $f(x) = \det(A)$. Then the sum of maximum and minimum values of f on R is equal to : **[JEE (Main)-2021]**

(1) $-\frac{20}{27}$ (2) $\frac{88}{27}$

(3) $\frac{20}{27}$ (4) $-\frac{88}{27}$

85. The sum of all the local minimum values of the twice differentiable function $f : R \to R$ defined by

$f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$ is

[JEE (Main)-2021]

(1) –22 (2) 5

(3) –27 (4) 0

86. If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right)$ is (α, β), then $2(\alpha + \beta)$ is equal to _____. **[JEE (Main)-2021]**

87. Let f : **R** → **R** be defined as

$$f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x, & x > 0 \\ 3xe^x & , x \le 0 \end{cases}$$

Then f is increasing function in the interval.

[JEE (Main)-2021]

(1) (–3, –1) (2) (0, 2)

(3) $\left(-1, \frac{3}{2}\right)$ (4) $\left(-\frac{1}{2}, 2\right)$

88. Let $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3$, $x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$. Then, f is **[JEE (Main)-2021]**

(1) Decreasing in $\left(-\frac{\pi}{6}, 0\right)$

(2) Decreasing in $\left(0, \frac{\pi}{2}\right)$

(3) Increasing in $\left(-\frac{\pi}{6}, 0\right)$

(4) Increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$

89. A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi} + 1\right)k$ is equal to _______. **[JEE (Main)-2021]**

90. The local maximum value of the function

$f(x) = \left(\frac{2}{x}\right)^{x^2}$, $x > 0$, is **[JEE (Main)-2021]**

(1) $\left(2\sqrt{e}\right)^{\frac{1}{e}}$ (2) $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$

(3) 1 (4) $(e)^{\frac{2}{e}}$

91. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is **[JEE (Main)-2021]**

(1) $\frac{5}{2+\sqrt{3}}$ (2) $\frac{5}{3+\sqrt{3}}$

(3) $\frac{10}{3+2\sqrt{3}}$ (4) $\frac{10}{2+3\sqrt{3}}$

92. A box open from top is made from a rectangular sheet of dimension $a \times b$ by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to **[JEE (Main)-2021]**

(1) $\frac{a+b-\sqrt{a^2+b^2+ab}}{6}$

(2) $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

(3) $\frac{a+b-\sqrt{a^2+b^2-ab}}{12}$

(4) $\frac{a+b+\sqrt{a^2+b^2-ab}}{6}$

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93. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^x - 6 = 0$ is **[JEE (Main)-2021]**

(1) 1 (2) 2

(3) 4 (4) 0

94. If 'R' is the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on [1, 2] and 'S' is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1, 2], then the value of $|R - S|$ is ____. **[JEE (Main)-2021]**

95. An angle of intersection of the curves, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $x^2 + y^2 = ab$, a > b is

[JEE (Main)-2021]

(1) $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$ (2) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$

(3) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$ (4) $\tan^{-1}\left(2\sqrt{ab}\right)$

96. Let $f(x)$ be a cubic polynomial with $f(1) = -10$, $f(-1) = 6$, and has a local minima at $x = 1$, and $f'(x)$ has a local minima at $x = -1$. Then $f(3)$ is equal to ________. **[JEE (Main)-2021]**

97. The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(2) = f(4) = 0$. Consider two statements.

(S1) There exists $x_1, x_2 \in (2, 4)$, $x_1 < x_2$, such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

(S2) There exists $x_3, x_4 \in (2, 4)$, $x_3 < x_4$, such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3}f(x_4)$. **[JEE (Main)-2021]**

(1) Both (S1) and (S2) are true

(2) Both (S1) and (S2) are false

(3) (S1) is false and (S2) is true

(4) (S1) is true and (S2) is false

98. The number of distinct real roots of equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is ______.

[JEE (Main)-2021]

99. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds, it becomes 7 units, then its radius after 9 seconds is :

(1) 9 (2) 10

(3) 11 (4) 12

[JEE (Main)-2022]

100. For the function

$f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5$, $x > 1$, which one of the following is NOT correct?

(1) f is increasing in (1, 2) and decreasing in $(2, \infty)$

(2) $f(x) = -1$ has exactly two solutions

(3) $f'(e) - f''(2) < 0$

(4) $f(x) = 0$ has a root in the interval $(e, e + 1)$

[JEE (Main)-2022]

101. If the tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve :

(1) $x^2 + \frac{y^2}{81} = 2$

(2) $\frac{y^2}{9} - x^2 = 8$

(3) $y = 4x^2 + 5$

(4) $\frac{x}{3} - y^2 = 2$

[JEE (Main)-2022]

102. The sum of absolute maximum and absolute minimum values of the function $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval [0, 1] is :

(1) $3 + \frac{\sin(1)\cos^2\left(\frac{1}{2}\right)}{2}$

(2) $3 + \frac{1}{2}(1 + 2\cos(1))\sin(1)$

(3) $5 + \frac{1}{2}(\sin(1) + \sin(2))$

(4) $2 + \sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

[JEE (Main)-2022]

103. Let $x, y > 0$. If $x^3y^2 = 2^{15}$, then the least value of $3x + 2y$ is

(1) 30 (2) 32

(3) 36 (4) 40

[JEE (Main)-2022]

104. The number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is

(1) 5 (2) 7

(3) 1 (4) 3

[JEE (Main)-2022]

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105. Let λ^* be the largest value of λ for which the function $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$ is increasing for all $x \in \mathbb{R}$. Then $f_{\lambda}^*(1) + f_{\lambda}^*(-1)$ is equal to :

(1) 36
(2) 48
(3) 64
(4) 72

[JEE (Main)-2022]

106. Water is being filled at the rate of 1 cm³/sec in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm²/sec) at which the wet conical surface area of the vessel increase, is

(1) 5
(2) $\frac{\sqrt{21}}{5}$
(3) $\frac{\sqrt{26}}{5}$
(4) $\frac{\sqrt{26}}{10}$

[JEE (Main)-2022]

107. If the angle made by the tangent at the point (x_0, y_0) on the curve $x = 12(t + \sin t \cos t)$, $y = 12(1+\sin t)^2, 0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to:

(1) $6\left(3+2\sqrt{2}\right)$
(2) $3\left(7+4\sqrt{3}\right)$
(3) 27
(4) 48

[JEE (Main)-2022]

108. Let $f(x) = \left|(x-1)\left(x^2-2x-3\right)\right| + x - 3, x \in R$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then $m + M$ is equal to

[JEE (Main)-2022]

109. The sum of the absolute minimum and the absolute maximum values of the function $f(x) = |3x - x^2 + 2| - x$ in the interval [–1, 2] is :

(1) $\frac{\sqrt{17}+3}{2}$
(2) $\frac{\sqrt{17}+5}{2}$
(3) 5
(4) $\frac{9-\sqrt{17}}{2}$

[JEE (Main)-2022]

110. Let S be the set of all the natural numbers, for which the line $\frac{x}{a}+\frac{y}{b}=2$ is a tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a, b), $ab \neq 0$. Then :

(1) $S = \phi$
(2) $n(S) = 1$
(3) $S = \{2k : k \in \mathbf{N}\}$
(4) $S = \mathbf{N}$

[JEE (Main)-2022]

111. Consider a cuboid of sides $2x$, $4x$ and $5x$ and a closed hemisphere of radius r. If the sum of their surface areas is a constant k, then the ratio $x : r$, for which the sum of their volumes is maximum, is

(1) 2 : 5(2)
19 : 45
(3) 3 : 8
(4) 19 : 15

[JEE (Main)-2022]

112. The number of distinct real roots of $x^4 - 4x + 1 = 0$ is :

(1) 4
(2) 2
(3) 1
(4) 0

[JEE (Main)-2022]

113. The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 2x^2$. If for $x = k$, the area of the triangle is maximum, then $3k^2$ is equal to :

(1) 5
(2) 8
(3) 10
(4) 12

[JEE (Main)-2022]

114. The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to ______.

(1) 0
(2) 1
(3) 3
(4) 5

[JEE (Main)-2022]

115. A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is

(1) $\frac{22}{9+4\sqrt{3}}$
(2) $\frac{66}{9+4\sqrt{3}}$
(3) $\frac{22}{4+9\sqrt{3}}$
(4) $\frac{66}{4+9\sqrt{3}}$

[JEE (Main)-2022]

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116. Let $f: R \to R$ be a function defined by

$f(x) = (x-3)^{n_1}(x-5)^{n_2}$, $n_1, n_2 \in N$. Then, which of the following is NOT true?

(1) For $n_1 = 3$, $n_2 = 4$, there exists $\alpha \in (3, 5)$ where f attains local maxima.

(2) For $n_1 = 4$, $n_2 = 3$, there exists $\alpha \in (3, 5)$ where f attains local minima.

(3) For $n_1 = 3$, $n_2 = 5$, there exists $\alpha \in (3, 5)$ where f attains local maxima.

(4) For $n_1 = 4$, $n_2 = 6$, there exists $\alpha \in (3, 5)$ where f attains local maxima.

[JEE (Main)-2022]

117. Let f and g be twice differentiable even functions on $(-2, 2)$ such that $f\left(\frac{1}{4}\right) = 0$, $f\left(\frac{1}{2}\right) = 0$, $f(1) = 1$ and $g\left(\frac{3}{4}\right) = 0$, $g(1) = 2$ Then, the minimum number of solutions of $f(x)g''(x) + f'(x)g'(x) = 0$ in $(-2, 2)$ is equal to_____.

[JEE (Main)-2022]

118. If the absolute maximum value of the function $f(x) = (x^2 - 2x + 7)\ e^{\left(4x^3 - 12x^2 - 180x + 31\right)}$ in the interval $[-3, 0]$ is $f(\alpha)$, then

(1) $\alpha = 0$ (2) $\alpha = -3$

(3) $\alpha \in (-1, 0)$ (4) $\alpha \in (-3, -1]$

[JEE (Main)-2022]

119. The curve $y(x) = ax^3 + bx^2 + cx + 5$ touches the x-axis at the point $P(-2, 0)$ and cuts the y-axis at the point Q, where y' is equal to 3. Then the local maximum value of $y(x)$ is

(1) $\frac{27}{4}$ (2) $\frac{29}{4}$

(3) $\frac{37}{4}$ (4) $\frac{9}{2}$

[JEE (Main)-2022]

120. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \le 1 \\ -2x + \log_2\left(b^2 - 4\right), & x > 1 \end{cases}$.

Then the set of all values of b, for which $f(x)$ has maximum value at $x = 1$, is

(1) $(-6, -2)$ (2) $(2, 6)$

(3) $[-6, -2) \cup (2, 6]$ (4) $\left[-\sqrt{6}, -2\right) \cup \left(2, \sqrt{6}\right]$

[JEE (Main)-2022]

121. The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is __________.

[JEE (Main)-2022]

122. Let the function $f(x) = 2x^2 - \log_e x$, $x > 0$, be decreasing in $(0, a)$ and increasing in $(a, 4)$. A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point $(8a, 8a - 1)$ but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ then $\alpha + \beta$ is equal to ________.

[JEE (Main)-2022]

123. Let M and N be the number of points on the curve $y^5 - 9xy + 2x = 0$, where the tangents to the curve are parallel to x-axis and y-axis, respectively. Then the value of $M + N$ equals ________.

[JEE (Main)-2022]

124. A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semi-vertical angle is $\tan^{-1}\frac{3}{4}$ Water is poured in it at a constant rate of 6 cubic meter per hour. The rate (in square meter per hour), at which the wet curved surface area of the tank is increasing, when the depth of water in the tank is 4 meters, is

[JEE (Main)-2022]

125. If the minimum value of $f(x) = \frac{5x^2}{2} + \frac{\alpha}{x^5}, x > 0$, is 14, then the value of α is equal to

(1) 32 (2) 64

(3) 128 (4) 256

[JEE (Main)-2022]

126. Let $f : [0, 1] \to \mathbf{R}$ be a twice differentiable function in $(0, 1)$ such that $f(0) = 3$ and $f(1) = 5$. If the line $y = 2x + 3$ intersects the graph of f at only two distinct points in $(0, 1)$ then the least number of points $x \in (0, 1)$ at which $f''(x) = 0$, is ____________.

[JEE (Main)-2022]

127. The function $f(x) = xe^{x(1-x)}, x \in \mathbb{R}$, is

(1) Increasing in $\left(-\frac{1}{2}, 1\right)$

(2) Decreasing in $\left(\frac{1}{2}, 2\right)$

(3) Increasing in $\left(-1, -\frac{1}{2}\right)$

(4) Decreasing in $\left(-\frac{1}{2}, \frac{1}{2}\right)$

[JEE (Main)-2022]

128. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a, b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point $(2, -1)$, then $|2a + 9b|$ is equal to ________.

[JEE (Main)-2022]

129. The sum of the maximum and minimum values of the function $f(x) = |5x - 7| + [x^2 + 2x]$ in the interval $\left[\frac{5}{4}, 2\right]$, where $[t]$ is the greatest integer $\leq t$, is _____.

[JEE (Main)-2022]

130. Let $f(x) = 3^{(x^2-2)^3+4}, x \in \mathbb{R}$. Then which of the following statements are true?

P : $x = 0$ is a point of local minima of f

$Q : x = \sqrt{2}$ is a point of inflection of f

R : f' is increasing for $x > \sqrt{2}$

[JEE (Main)-2022]

(1) Only P and Q (2) Only P and R

(3) Only Q and R (4) All P, Q and R

131. Let $f : R \to R$ and $g : R \to R$ be two functions defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x) = \dfrac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right) > f\left(g\left(\alpha - \frac{5}{3}\right)\right)$ holds?

[JEE (Main)-2022]

(1) $(2, 3)$ (2) $(-2, -1)$

(3) $(1, 2)$ (4) $(-1, 1)$

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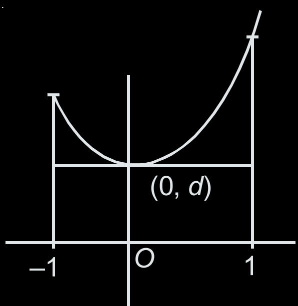
Applications of Derivatives

1. Answer (1)



We have $P(x) = x^4 + ax^3 + bx^2 + cx + d$

$P'(x) = 4x^3 + 3ax^2 + 2bx + c$

$P'(0) = 0 \quad \Rightarrow \quad c = 0$

Also $P'(x) = 0$ only at $x = 0$

$P'(x)$ is a cubic polynomial changing its sign from (–)ve to (+)ve and passing through O.

$\therefore \quad P'(x) < 0 \ \forall \ x < 0$

$P'(x) > 0 \ \forall \ x > 0$

Hence the graph of $P(x)$ is upward concave, where $P'(x) = 0$

Now $P(-1) < P(1)$

$\Rightarrow$ $P(-1)$ cannot be minimum in $[-1, 1]$ as minima in this interval is at $x = 0$.

Hence in $[-1, 1]$ maxima is at $x = 1$

Hence $P(-1)$ is not minimum but $P(1)$ is the maximum of P.

2. Answer (4)

Let there be a point $P(t^2, t)$ on $x = y^2$

Its distance from $x - y + 1 = 0$ is

$$\left|\frac{t^2 - t + 1}{\sqrt{2}}\right|$$

Min $(t^2 - t + 1)$ is $\frac{3}{4}$

Shortest distance $= \left|\frac{3}{4\sqrt{2}}\right| = \frac{3\sqrt{2}}{8}$

3. Answer (4)

We have,

$$y = x + \frac{4}{x^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = 1 - \frac{8}{x^3}$$

The tangent is parallel to x-axis, hence

$$\frac{dy}{dx} = 0$$

$\Rightarrow \quad x^3 = 8$

$\Rightarrow \quad x = 2$

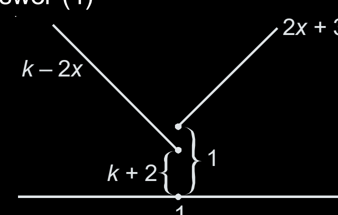
and $\ y = 3$

The equation of the tangent to the given curve at (2, 3) is

$$y - 3 = \left(\frac{dy}{dx}\right)_{(2,\,3)}(x - 2) = 0$$

$\Rightarrow \quad y = 3$

4. Answer (4)



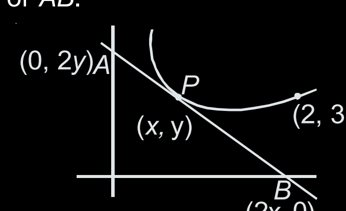
$k + 2 \le 1$

$\therefore \quad k \le -1$

5. Answer (4)

P is point of contact

P is mid point of AB.



$$\frac{dy}{dx} = \frac{2y}{-2x}$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

$$\frac{dy}{y} = \frac{-dy}{x}$$

$\ln y = -\ln x + c$ ...(i)

(i) passes through (2, 3)

$\ln 3 = -\ln 2 + c$

$c = \ln 6$

$\therefore$ Equation of curve is $xy = 6$

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$\lim_{x \to 0} f(x) = 1$

$f(0 + h) \geq f(0)$

and $f(0 - h) \geq f(0)$

$\therefore$ $x = 0$ is point of local minima

$\Rightarrow$ $f(x)$ has local minima at $x = 0$

also, $f'(x) = 0$ at $x = 0$

but statement 2 is not correct explanation of statement 1

7. Answer (2)

8. Answer (1)

9. Answer (1)

$f(x) = \alpha \log|x| + \beta x^2 + x$

$f'(x) = \frac{\alpha}{x} + 2\beta x + 1 = 0$ at $x = -1, 2$

$-\alpha - 2\beta + 1 = 0 \Rightarrow \alpha + 2\beta = 1$ ...(i)

$\frac{\alpha}{2} + 4\beta + 1 = 0 \Rightarrow \alpha + 8\beta = -2$ ...(ii)

$6\beta = -3 \Rightarrow \beta = -\frac{1}{2}$

$\therefore \alpha = 2$

10. Answer (4)

Curve is $x^2 + 2xy - 3y^2 = 0$

Differentiate wr.t. x, $2x + 2\left[x\frac{dy}{dx} + y\right] - 6y \cdot \frac{dy}{dx} = 0$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = 1$

So equation of normal at (1, 1) is

$y - 1 = -1(x - 1)$

$\Rightarrow$ $y = 2 - x$

Solving it with the curve, we get

$x^2 + 2x(2 - x) - 3(2 - x)^2 = 0$

$\Rightarrow$ $-4x^2 + 16x - 12 = 0$

$\Rightarrow$ $x^2 - 4x + 3 = 0$

$\Rightarrow$ $x = 1, 3$

So points of intersections are (1, 1) & (3, –1) *i.e.* normal cuts the curve again in fourth quadrant.

Using $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$

$\Rightarrow \lim_{x \to 0} \frac{f(x)}{x^2} = 2$

$\Rightarrow \lim_{x \to 0} \frac{a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4}{x^2} = 2$

So, $a_0 = 0$, $a_1 = 0$, $a_2 = 2$

i.e., $f(x) = 2x^2 + a_3x^3 + a_4x^4$

Now, $f'(x) = 4x + 3a_3x^2 + 4a_4x^3$

$= x[4 + 3a_3x + 4a_4x^2]$

Given, $f'(1) = 0$ and $f'(2) = 0$

$\Rightarrow$ $3a_3 + 4a_4 + 4 = 0$ ...(i)

and $6a_3 + 16a_4 + 4 = 0$ ...(ii)

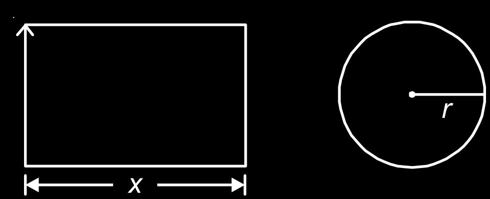
Solving, $a_4 = \frac{1}{2}$, $a_3 = -2$

i.e., $f(x) = 2x^2 - 2x^3 + \frac{1}{2}x^4$

i.e., $f(2) = 0$

12. Answer (2)

Length of wire = 2



Given $4x + 2\pi r = 2$

$\Rightarrow$ $2x + \pi r = 1$ ...(i)

$A = x^2 + \pi r^2 = \left(\frac{1-\pi r}{2}\right)^2 + \pi r^2$

$\Rightarrow \frac{dA}{dr} = 2\left(\frac{1-\pi r}{2}\right)\left(-\frac{\pi}{2}\right) + 2\pi r$

For max and min $\Rightarrow \frac{dA}{dr} = 0$

$\pi(1 - \pi r) = 4\pi r$

$1 = 4r + \pi r$ ...(ii)

from (i) and (ii)

$2x + \pi r = 4r + \pi r$

$x = 2r$

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At y-axis, $x = 0$, $y = 1$

Now, on differentiation.

$$\frac{dy}{dx}(x-2)(x-3)+y(2x-5)=1$$

$$\frac{dy}{dx}(6)+1(-5)=1$$

$$\frac{dy}{dx}=\frac{6}{6}=1$$

Now slope of normal = –1

Equation of normal $y - 1 = -1(x - 0)$

$y + x - 1 = 0$ ... (i)

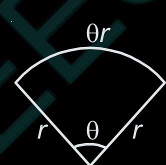
Line (i) passes through $\left(\frac{1}{2},\frac{1}{2}\right)$

14. Answer (2)

$2r+\theta r=20$ ... (i)

A = area = $\frac{\theta}{2\pi}\times\pi r^2=\frac{\theta r^2}{2}$ ... (ii)

$$A=\frac{r^2}{2}\left(\frac{20-2r}{r}\right)$$



$$A=\left(\frac{20r-2r^2}{2}\right)=10r-r^2$$

A to be maximum

$$\frac{dA}{dr}=10-2r=0\Rightarrow r=5$$

$$\frac{d^2A}{dr^2}=-2<0$$

Hence for $r = 5$, A is maximum

Now, $10 + \theta\cdot 5 = 20 \Rightarrow \theta = 2$ (radian)

Area = $\frac{2}{2\pi}\times\pi(5)^2=25$ sq m

also $9x^2+by^2=16;$ slope of tangent at (x_1, y_1) is

$$m_2=\frac{-9x_1}{by_1}$$

As $m_1m_2=-1$

$$\Rightarrow \frac{-27x_1}{by_1^2}=-1$$

$$\Rightarrow b=\frac{9}{2}\left(\text{as } y_1^2=6x_1\right)$$

16. Answer (4)

$$h(x)=\frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}}=\left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}$$

$$x-\frac{1}{x}>0,\ \left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}\in(2\sqrt{2},\ \infty]$$

$$x-\frac{1}{x}<0,\ \left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}\in(-\infty,-2\sqrt{2}]$$

Local minimum is $2\sqrt{2}$

17. Answer (1)

$y = 10 - x^2$ ...(1)

$y = 2 + x^2$ ...(2)

For intersection point of (1) and (2)

(1) + (2)

$2y = 12 \Rightarrow y = 6$

from (1) $x = \pm 2$

differentiate with respect to x equation (1)

$$\frac{dy}{dx}=-2x\Rightarrow\left(\frac{dy}{dx}\right)_{(2,6)}=-4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)}=4$$

differentiate with respect to x equation (2)

$$\frac{dy}{dx}=2x\Rightarrow\left(\frac{dy}{dx}\right)_{(2,6)}=4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)}=-4$$

$$\Rightarrow \tan\theta=\left(\frac{-4-4}{1+(-4)\times 4}\right)=\frac{-8}{15}\Rightarrow|\tan\theta|=\frac{8}{15}$$

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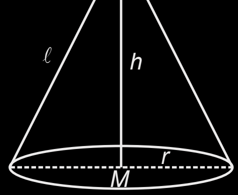


$h^2 + r^2 = \ell^2 = 9$ ...(1)

$V = \frac{1}{3}\pi r^2 h$ ...(2)

From (1) and (2),

$$V = \frac{1}{3}\pi(9 - h^2)h$$

$$V = \frac{1}{3}\pi(9h - h^3)$$

For maxima/minima,

$$\frac{dV}{dh} = 0 \Rightarrow \frac{1}{3}\pi(9 - 3h^2) = 0$$

$\Rightarrow \quad h = \pm\sqrt{3} \Rightarrow h = \sqrt{3} \quad (\because\ h > 0)$

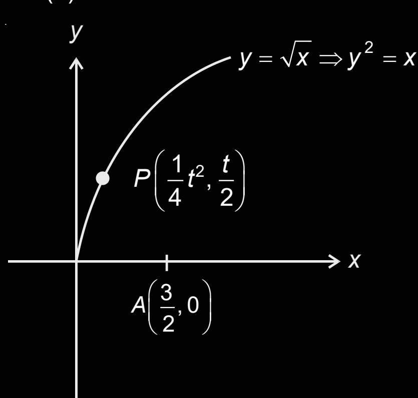
$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(-6h)$$

$$\left(\frac{d^2V}{dh^2}\right)_{\text{at } h=\sqrt{3}} < 0$$

$\Rightarrow$ at $h = \sqrt{3}$, volume is maximum

$\Rightarrow \quad V_{\text{max.}} = \frac{1}{3}\pi(9-3)\sqrt{3} = 2\sqrt{3}\pi$

19. Answer (4)



Normal at P to $y^2 = x$ is

$\Rightarrow \frac{-3}{2}t + \frac{t}{2} + \frac{t^3}{4} = 0 \quad \Rightarrow \quad -4t + t^3 = 0$

$\Rightarrow t(t^2 - 4) = 0 \quad \Rightarrow \quad t = -2, 0, 2 \quad \because t \ge 0 \Rightarrow t = 0, 2$

if $t = 0$, $P(0, 0) \Rightarrow AP = \frac{3}{2}$

if $t = 2$, $P(1, 1) \Rightarrow AP = \frac{\sqrt{5}}{2}$

$\Rightarrow$ Shortest distance $\left(\frac{3}{2}, 0\right)$ and $y = \sqrt{x}$ is $\frac{\sqrt{5}}{2}$

20. Answer (2)

$y = xe^{x^2} \Rightarrow \frac{dy}{dx} = e^{x^2}.1 + x.e^{x^2}.2x$

(1,e) lies on the curve $y = xe^{x^2}$

$\Rightarrow$ equation of tangent at (1, e) is

$$y - e = \left(e^{x^2}(1 + 2x^2)\right)_{x=1}(x - 1)$$

$$y - e = 3e\,(x - 1)$$

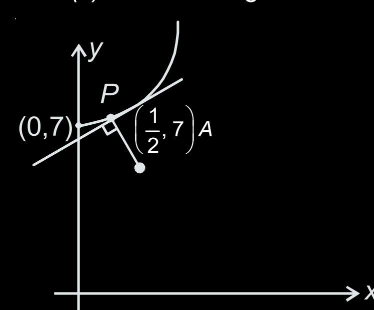
$3ex - y = 2e$ passes through the point $\left(\frac{4}{3}, 2e\right)$.

$\Rightarrow$ Option (2) is correct.

21. Answer (1)

$f(x) = y = x^{3/2} + 7 \quad \frac{dy}{dx} \Rightarrow \frac{3}{2}\sqrt{x} > 0$

$\Rightarrow$ $f(x)$ is increasing function $\forall\ x > 0$



Let $P\left(x_1, x_1^{3/2} + 7\right)$

$\Rightarrow \quad m_{AP} \cdot m_{\text{at } p} = -1$

$$\Rightarrow \left(\frac{x_1^{3/2}}{x_1 - \frac{1}{2}}\right) \times \frac{3}{2}x_1^{\frac{1}{2}} = -1$$

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$\Rightarrow\ -3x_1^2 = 2x_1 - 1 \Rightarrow\ 3x_1^2 + 2x_1 - 1 = 0$

$\Rightarrow\ 3x_1^2 + 3x_1 - x_1 - 1 = 0$

$\Rightarrow\ 3x_1(x_1+1) - 1(x_1+1) = 0$

$\Rightarrow\ x_1 = \dfrac{1}{3}\quad (\because x_1 > 0) \Rightarrow P\left(\dfrac{1}{3}, 7 + \dfrac{1}{3\sqrt{3}}\right)$

$\Rightarrow\ AP = \sqrt{\dfrac{1}{27} + \dfrac{1}{36}} = \dfrac{1}{6}\sqrt{\dfrac{7}{3}}$

$\Rightarrow$ Option (1) is correct.

22. Answer (1)

$f(x) = 3x(x-3)^2 - 40$

Now $S = \{x \in R : x^2 + 30 \le 11x\}$

So $x^2 - 11x + 30 \le 0$

$x \in [5, 6]$

For given interval, $f(x)$ will have maximum value for $x = 6$

$f(6) = 3 \times 6 \times 3 \times 3 - 40 = 122$

23. Answer (4)

$$E = \frac{x^m y^n}{(1+x^{2m})(1+y^{2n})} = \frac{1}{(x^{-m}+x^m)(y^{-n}+y^n)}$$

$$\frac{x^m + y^{-m}}{2} \ge (x^m . x^{-m})^{\frac{1}{2}} \Rightarrow\ x^m + x^{-m} \ge 2$$

Similarly $y^{-n} + y^n \ge 2$

$\Rightarrow\ (x^m + x^{-m})(y^{-n} + y^n) \ge 4$

$$\Rightarrow\ \frac{1}{(x^m + x^{-m})(y^{-n} + y^n)} \le \frac{1}{4}$$

Option (4) is correct

24. Answer (1)

$$f(x) = \frac{x}{\sqrt{a^2+x^2}} - \frac{(d-x)}{\sqrt{b^2+(d-x)^2}}$$

$$= \frac{x}{\sqrt{a^2+x^2}} + \frac{(x-d)}{\sqrt{b^2+(x-d)^2}}$$

$$f'(x) = \frac{\sqrt{a^2+x^2} - \dfrac{x(2x)}{2\sqrt{a^2+x^2}}}{(a^2+x^2)} + \frac{\sqrt{b^2+(x-d)^2} - \dfrac{(x-d)2(x-d)}{2\sqrt{b^2+(x-d)^2}}}{(b^2+(x-d)^2)}$$

$$\frac{a^2}{(a^2+x^2)^{3/2}} + \frac{b^2}{(b^2+(x-d)^2)^{3/2}} > 0$$

Hence $f(x)$ is increasing function.

25. Answer (2)

$\because$ Tangent is parallel to line $2y = 4x + 1$

Let equation of tangent be $y = 2x + c$ ...(1)

Now line (1) and curve $y = x^2 - 5x + 5$ has only one point of intersection.

$\therefore\ 2x + c = x^2 - 5x + 5$

$x^2 - 7x + (5 - c) = 0$

$\therefore\ D = 49 - 4(5 - c) = 0$

$\therefore\ c = -\dfrac{29}{4}$

$\therefore$ Equation of tangent: $y = 2x - \dfrac{29}{4}$

26. Answer (2)

$$\because\ \frac{dy}{dx} = \frac{x^2 - 2y}{x}$$

$$\frac{dy}{dx} + \frac{2}{x}y = x$$

$$\text{I.F.} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$$

Solution of equation

$$y \cdot x^2 = \int x \cdot x^2 dx$$

$$x^2 y = \frac{x^4}{4} + C$$

This curve passes through point (1, –2)

$\therefore\ C = \dfrac{-9}{4}$

$\therefore$ equation of curve : $y = \dfrac{x^2}{4} - \dfrac{9}{4x^2}$

clearly it passes through $(\sqrt{3}, 0)$

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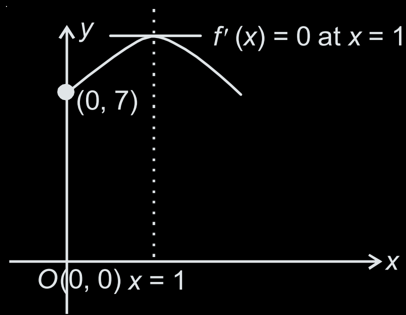


$\Rightarrow$ $f'(x) = 3x^2 - 6(a-2)x + 3a$

$f'(1) = 0$

$\Rightarrow$ $1 - 2a + 4 + a = 0$

$\Rightarrow$ $a = 5$

$$\Rightarrow \frac{f(x)-14}{(x-1)^2} = 0$$

$$\Rightarrow \frac{x^3-9x^2+15x+7-14}{(x-1)^2} = 0$$

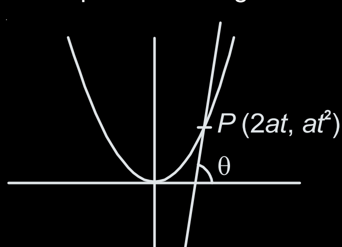
$$\Rightarrow \frac{(x-1)^2(x-7)}{(x-1)^2} = 0$$

$\Rightarrow$ $x = 7$

28. Answer (3)

$x^2 = 8y$

Equation of tangent at P



$tx = y + at^2$

$y = tx - at^2$

$t = \tan\theta$

$y = \tan\theta\, x - 2\tan^2\theta$

$\Rightarrow$ $\cot\theta\, y = x - 2\tan\theta$

$$x = y\cot\theta + 2\tan\theta$$

tangent of parabola having slope 1.

Let equation of tangent of parabola having slope 1 is,

$$y = m(x-2) + \frac{a}{m}$$

where $m = 1$ and $a = \frac{1}{4}$

Equation of tangent $y = x - \frac{7}{4}$

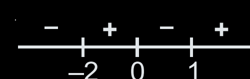
Distance between the line $y = x$ and the tangent

$$= \left|\frac{\frac{7}{4}-0}{\sqrt{1^2+1^2}}\right| = \frac{7}{4\sqrt{2}}$$

30. Answer (2)

$f(x) = 9x^4 + 12x^3 - 36x^2 + 72$

$f'(x) = 36[x^3 + x^2 - 2x] = 36x(x-1)(x+2)$



Whenever derivative changes sign from negative to positive, we get local minima, and whenever derivative changes sign from positive to negative, we get local maxima (while moving left to right on x-axis)

$S_1 = \{-2, 1\}$

$S_2 = \{0\}$

31. Answer (3)

$\phi(x) = f(x) + f(2-x)$

differentiating w.r.t. x

$\phi'(x) = f'(x) - f'(2-x)$

For $\phi(x)$ to be increasing $\phi'(x) > 0$

$\Rightarrow$ $f'(x) > f'(2-x)$

($\because$ $f''(x) > 0$ then $f'(x)$ is an increasing function)

$\Rightarrow$ $x > 2 - x$

$\Rightarrow$ $x > 1$

So $\phi(x)$ is increasing in (1, 2) and decreasing in (0, 1).

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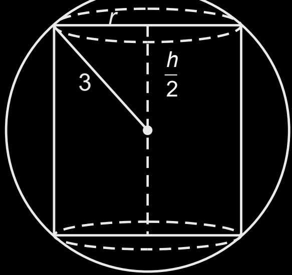


$$\therefore \quad r^2 + \frac{h^2}{4} = 9 \qquad \ldots(i)$$

$\because$ Volume of cylinder

$$V = \pi r^2 h$$

$$V = \pi h\left(9 - \frac{h^2}{4}\right)$$

$$V = 9\pi h - \frac{\pi}{4}h^3$$

$$\therefore \quad \frac{dV}{dh} = 9\pi - \frac{3}{4}\pi h^2$$

For maxima/minima,

$$\frac{dV}{dh} = 0$$

$$\Rightarrow \quad h = \sqrt{12}$$

and $\dfrac{d^2V}{dh^2} = -\dfrac{3}{2}\pi h$

$$\therefore \quad \left(\frac{d^2V}{dh^2}\right)_{h=\sqrt{12}} < 0$$

$\Rightarrow$ Volume is maximum when $h = 2\sqrt{3}$

33. Answer (3)

$f'(x) = A(x + 1) \times (x - 1) = A(x^3 - x)$

$$\Rightarrow \quad f(x) = A\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + C$$

Now, $f(0) = C$

$$\therefore \quad f(x) = f(0) \quad \Rightarrow \quad A\left(\frac{x^4}{4} - \frac{x^2}{2}\right) = 0$$

$$\Rightarrow \quad \frac{x^2}{2}\left(\frac{x^2}{2} - 1\right) = 0$$

$$\Rightarrow \quad x = 0, 0, -\sqrt{2}, \sqrt{2}$$

$$\therefore \quad S = \{0, -\sqrt{2}, \sqrt{2}\}$$

$$\frac{dy}{dx} = 3x^2 - 2x - 2$$

$f(1) = 1 - 1 - 2 = -2, \qquad f(-1) = -1 - 1 + 2 = 0$

According to question,

$$3x^2 - 2x - 2 = \frac{f(1) - f(-1)}{1 - (-1)}$$

$$\Rightarrow \quad 3x^2 - 2x - 2 = \frac{-2 - 0}{2}$$

$$\Rightarrow \quad 3x^2 - 2x - 1 = 0$$

$$\Rightarrow \quad x = \frac{2 \pm 4}{6} = 1, \frac{-1}{3}$$

So, $S = \left\{\dfrac{-1}{3}, 1\right\}$

35. Answer (2)

$f(x) = x^3 + ax - b \quad \Rightarrow \quad f'(x) = 3x^2 + a$

$f(1) = -5 \quad$ and $\quad f'(1) = 3 + a$

$\Rightarrow \quad 1 + a - b = -5$

$\Rightarrow \quad a - b = -6 \qquad \ldots(1)$

Also, slope of tangent $P(1, -5) = -1 = f'(1)$

$\therefore \quad 3 + a = -1$

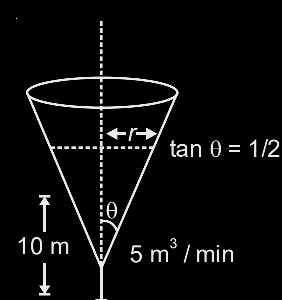
$\Rightarrow \quad a = -4$

$\Rightarrow \quad b = 2$

$\therefore$ Equation of the curve is $f(x) = x^3 - 4x - 2$

$\therefore$ $(2, -2)$ lies on the curve.

36. Answer (4)



Given $\dfrac{dv}{dt} = 5\text{m}^3/\text{min}$

$$V = \frac{1}{3}\pi r^2 h \qquad \ldots(i)$$

(where r is radius and h is height at any time)

Also, $\tan\theta = \dfrac{r}{h} = \dfrac{1}{2} \Rightarrow h = 2r \Rightarrow \dfrac{dh}{dt} = \dfrac{2dr}{dt} \quad \ldots(ii)$

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$$= \frac{1}{3}\left(100\pi\frac{1}{2} + 25\pi\right)\frac{dh}{dt}$$

at $h = 10$, $r = 5$

$$5 = \frac{75\pi}{3}\frac{dh}{dt}$$

$$\Rightarrow \quad \frac{dh}{dt} = \frac{1}{5\pi}\text{m/min}$$

37. Answer (3)

$f(x) = e^x - x$, $g(x) = x^2 - x$

$f(g(x)) = e^{(x^2-x)} - (x^2 - x)$

If $f(g(x))$ is increasing function

$(f(g(x)))' = e^{(x^2-x)} \times (2x-1) - 2x + 1$

$= (2x-1)e^{(x^2-x)} + 1 - 2x$

$= \underset{A}{(2x-1)}\underset{B}{[e^{(x^2-x)} - 1]}$

A & B are either both positive or negative

$$\begin{array}{ccccccc} - & & +ve & & -ve & & + \\ \hline & 0 & & \frac{1}{2} & & 1 & \end{array}$$

for $(f(g(x)))' \geq 0$,

$$x \in \left[0, \frac{1}{2}\right] \cup [1, \infty)$$

38. Answer (1)

$$y = \frac{x}{x^2 - 3}$$

$$\frac{dy}{dx} = \frac{(x^2-3) - x(2x)}{(x^2-3)^2} = \frac{-x^2-3}{(x^2-3)^2}$$

$$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)} = \frac{-\alpha^2-3}{(\alpha^2-3)^2} = -\frac{1}{3}$$

$3(\alpha^2 + 3) = (\alpha^2 - 3)^2$ ...(i)

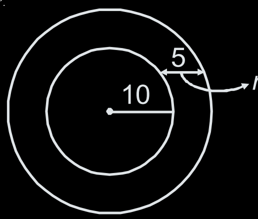
i.e. $\alpha^2 = 9$

Also, $\beta = \frac{\alpha}{\alpha^2-3} \Rightarrow \alpha^2 - 3 = \frac{\alpha}{\beta} \Rightarrow \frac{\alpha}{\beta} = 6$

$$\Rightarrow \quad \alpha = \pm 3,\ \beta = \pm\frac{1}{2}$$

Which satisfies $|6\alpha + 2\beta| = 19$

$$V_{ice} = \frac{4}{3}\pi(10+r)^3 - \frac{4}{3}\pi(10)^3$$



$$\frac{dV}{dt} = \frac{4}{3}\pi\, 3(10+r)^2\frac{dr}{dt}$$

$$= 4\pi(10+r)^2\frac{dr}{dt}$$

At $r = 5, 50 = 4\pi(225)\frac{dr}{dt}$

$$\frac{dr}{dt} = \frac{50}{4\pi(225)}$$

$$= \frac{1}{18\pi}\text{ cm/min}$$

40. Answer (4)

$$f(x) = x\sqrt{kx - x^2} = \sqrt{kx^3 - x^4}$$

$$f'(x) = \frac{(3kx^2 - 4x^3)}{2\sqrt{kx^3 - x^4}} \geq 0 \text{ for } x \in [0, 3]$$

$\Rightarrow \quad 3k - 4x \geq 0$

$3k \geq 4x$

$3k \geq 4x$ for $x \in [0, 3]$

Hence $k \geq 4$

i.e., $m = 4$

For $k = 4$,

$$\Rightarrow \quad f(x) = x\sqrt{4x - x^2}$$

For max. value, $f'(x) = 0$

$\Rightarrow \quad x = 3$

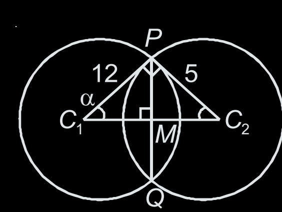
i.e., $y = 3\sqrt{3}$

Hence $M = 3\sqrt{3}$

41. Answer (1)

In ΔPC_1C_2,

$$\tan\alpha = \frac{5}{12}$$



$$\Rightarrow \quad \sin\alpha = \frac{5}{13}$$

In ΔPC_1M,

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$$\Rightarrow \frac{5}{13}=\frac{PM}{12}$$

$$\Rightarrow PM=\frac{60}{13}$$

Length of common chord $(PQ)=\frac{120}{13}$

42. Answer (3)

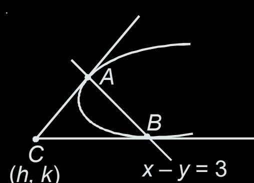
Equation of chord is $T = 0$

$$\Rightarrow \frac{1}{2}(y+k)=(x-2)(h-2)-1$$

$$\Rightarrow \frac{y+k}{2}=xh-2x-2h+3$$

$\Rightarrow (2h-4)x-y-4h+6-k=0$

Given $x-y-3=0$



$$\Rightarrow \frac{2h-4}{1}=\frac{4h-6+k}{3}=1$$

$$h=\frac{5}{2},\ k=-1$$

43. Answer (2)

$\because$ f(x) is a five degree polynomial such that

$$\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4 \text{ then}$$

let $f(x) = ax^5 + bx^4 + cx^3$

$$\lim_{x\to 0}\left(2+\frac{ax^5+bx^4+cx^3}{x^3}\right)=4$$

$\Rightarrow 2 + c = 4 \Rightarrow c = 2.$

Now, $f'(x)=5ax^4+4bx^3+3cx^2$

$$= x^2\left(5ax^2+4bx+3c\right)$$

$\because f'(1)=0 \Rightarrow 5a+4b+6=0$

$$\therefore f(x) = -\frac{6}{5}x^5+2x^3$$

$$f'(x)=-6x^4+6x^2=-6x^2(x+1)(x-1)$$

It is clear that maxima at x = 1 and minima at x = –1.

and $f(1) - 4f(-1) = 4$

44. Answer (2)

$$f(x)=\frac{\pi x}{2}-x\sin^{-1}(\sin-|x|)$$

$$=\frac{\pi x}{2}+x|x|$$

$$\therefore\ f'(x)=\begin{cases}\frac{\pi}{2}+2x & x\ge 0\\ \frac{\pi}{2}-2x & x<0\end{cases}$$

$$\therefore\ f''(x)=\begin{cases}2 & x\ge 0\\ -2 & x<0\end{cases}$$

$\therefore$ $f'(x)$ is decreasing in $x\in\left(-\frac{\pi}{2},0\right)$ and increasing in $x\in\left(0,\frac{\pi}{2}\right)$

45. Answer (2)

$$f(x)=\frac{x[x]}{x^2+1};1<x<3$$

$$\Rightarrow f(x)=\begin{cases}\dfrac{1}{\left(x+\dfrac{1}{x}\right)} & 1<x<2\\[2ex] \dfrac{2}{\left(x+\dfrac{1}{x}\right)} & 2\le x<3\end{cases}$$

$$\Rightarrow f'(x)=\begin{cases}\dfrac{\left(1-x^2\right)}{x^2\left(x+\dfrac{1}{x}\right)^2} & 1<x<2\\[2ex] \dfrac{2\left(1-x^2\right)}{x^2\left(x+\dfrac{1}{x}\right)^2} & 2\le x<3\end{cases}$$

$\therefore$ f(x) is decreasing function.

$\therefore$ Range is $\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{3}{5},\frac{4}{5}\right]$

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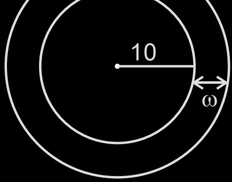

$$V = \frac{4}{3}\pi\left[(10+\omega)^3 - 10^3\right]$$

$$\frac{dv}{dt} = \frac{4}{3}\pi\left[3(10+\omega)^2\frac{d\omega}{dt}\right] = 50$$

when $\omega = 5$

$$\Rightarrow \quad \frac{d\omega}{dt} = \frac{3\times 50}{4\pi\cdot 3\cdot 15^2}$$

$$= \quad \frac{1}{18\pi}$$

47. Answer (3)

$$f(x) = \int_1^x t^2 g(t)dt, \quad g(t) = \int_1^t f(u)du$$

$f'(x) = x^2g(x)$, $g'(t) = f(t)$ and $g(1) = 0$

$\therefore$ $f'(1) = g(1) = 0$

Also, $f''(x) = x^2g'(x) + 2x\,g(x)$

$f''(1) = g'(1) + 2g(1) = 3 > 0$

$\therefore$ Local Minima at $x = 1$

48. Answer (4)

Let $p'(x) = \lambda(x-1)(x-2)$

$$\text{So } \; p(x) = \lambda\left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right] + \mu$$

$$\because \quad p(1) = 8 \Rightarrow \frac{5}{6}\lambda + \mu = 8 \qquad \ldots(1)$$

$$\text{and } \; p(2) = 4 \Rightarrow \frac{2}{3}\lambda + \mu = 4 \qquad \ldots(2)$$

from (1) and (2) ; we get $\lambda = 24$ and $\mu = -12$

Now $p(0) = \mu = -12$

$\therefore \quad \frac{dy}{dx} = 2x + 7$

Slope of tangent parallel to line $y = 3x - 3$

$\therefore$ $2x + 7 = 3$

$\therefore$ $x = -2$

$\therefore$ Point on curve $= (-2, -8)$

Equation of normal at point $(-2, -8)$

$$y + 8 = -\frac{1}{3}(x+2)$$

$x + 3y + 26 = 0$

50. Answer (4)

Curve $y = x + \sin y$ and point is (a, b)

$\Rightarrow$ $b = a + \sin b$ ...(i)

$$\text{Now, } \frac{dy}{dx} = 1 + \cos y\frac{dy}{dx}$$

$$\Rightarrow (1-\cos y)\frac{dy}{dx} = 1$$

$$\Rightarrow \left.\frac{dy}{dx}\right]_{(a,b)} = \frac{1}{1-\cos b} \quad \text{(slope)}$$

$$\text{Now according to question, } \frac{1}{1-\cos b} = \frac{2-\frac{3}{2}}{\frac{1}{2}-0}$$

$$\Rightarrow \quad \cos b = 0 \quad \Rightarrow \quad \boxed{b = \frac{\pi}{2}}$$

Now from (i)

$b = a + \sin b$

$$a = b - \sin b = \frac{\pi}{2} - 1$$

$$\text{So } |b-a| = \left|\frac{\pi}{2} - \frac{\pi}{2} + 1\right| = 1$$

51. Answer (3)

$\therefore$ $y = (1+x)^{2y} + \cos^2(\sin^{-1}x)$ ...(1)

On differentiating both sides w.r.t. x we get

$$\frac{dy}{dx} = 2y(1+x)^{2y-1} + (1+x)^{2y}\ln(1+x).$$

$$2\frac{dy}{dx} - \frac{\sin\left(2\sin^{-1}x\right)}{\sqrt{1-x^2}} \qquad \ldots(2)$$

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$\frac{dy}{dx} = 4 + 0 + 0 = 4$

Slope of normal at x = 0 is $-\frac{1}{4}$

$\therefore$ Equation of normal : $y-2=-\frac{1}{4}(x-0)$

x + 4y = 8

52. Answer (4)

$$f'(x)=\frac{\frac{x}{1+x}-\ln(1+x)}{x^2}$$

$$=\frac{x-(1+x)\ln(1+x)}{(1+x)x^2}<0 \quad \forall x\in(-1,\infty)-\{0\}$$

[as g(x) = x – (1 + x)ln (1 + x) gives g(x) < g(0) for x $\in$ (–1, 0) and g(x) < g(0) for x $\in$ (0, ∞)]

53. Answer (1)

$$f(x)=(3x-7)\cdot x^{2/3}$$

$$f'(x)=3x^{2/3}+(3x-7)\cdot\frac{2}{3}x^{-1/3}$$

$$=\frac{5x-14}{3x^{1/3}}$$

+ × – × +

0 $\frac{14}{15}$

$\therefore$ f′(x) > 0 then $x\in(-\infty,0)\cup\left(\frac{14}{15},\infty\right)$

$\therefore$ f(x) is increasing in $(-\infty,0)\cup\left(\frac{14}{15},\infty\right)$

54. Answer (1)

Critical points = –1, 0, 1.

$\therefore$ f′(x) = a(x – 1)(x + 1)x

$\therefore$ f(x) = $a\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+C$

$\because$ f(x) = f(0)

$$a\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+C=C$$

$$a\frac{x^2}{4}\left(x^2-2\right)=0$$

Sum of square of elements of

$T=0^2+\left(\sqrt{2}\right)^2+\left(-\sqrt{2}\right)^2$

= 4

55. Answer (1)

For cube of side ‘a’

A = $6a^2$ and V = a^3

Given $\frac{dA}{dt}=3.6=12a\frac{da}{dt}$

$$\frac{dV}{dt}=3a^2.\frac{da}{dt}=3a^2\left(\frac{3.6}{12a}\right)$$

at a = 10

$$\frac{dV}{dt}=9$$

56. Answer (1)

By using f′(x) ≥ 1

$\Rightarrow \frac{f(5)-f(2)}{3}\ge 1$

$\Rightarrow$ f(5) ≥ 3 + 8 $\Rightarrow$ f(5) ≥ 11

and also f″(x) ≥ 4

$\Rightarrow \frac{f'(5)-f'(2)}{5-2}\ge 4$

$\Rightarrow$ f′(5) ≥ 17

Hence f(5) + f′(5) ≥ 28

57. Answer (1)

Area of rectangle ABCD

$$A=\left|2t\cdot(t^2-1)\right|$$

$$A=\left|2t^3-2t\right|$$

$$\therefore \frac{dA}{dt}=\left|6t^2-2\right|$$

For maximum area $\frac{dA}{dt}=0\Rightarrow t=\pm\frac{1}{\sqrt{3}}$

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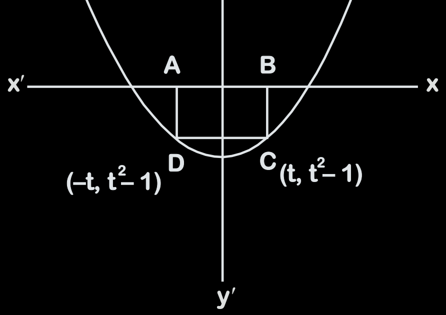


$$\therefore \text{ Maximum area } = \left|\frac{2}{3\sqrt{3}} - \frac{2}{\sqrt{3}}\right| = \frac{4}{3\sqrt{3}}$$

58. Answer (3)

$$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq \left(2^{\sin x + \cos x}\right)^{\frac{1}{2}}$$

$$\Rightarrow \quad 2^{\sin x} + 2^{\cos x} \geq 2.2^{\frac{\sin x + \cos x}{2}}$$

$$\Rightarrow \quad 2^{\sin x} + 2^{\cos x} \geq 2^{1 - \frac{1}{\sqrt{2}}} \text{ (as sinx + cosx} \geq = \sqrt{2}$$

59. Answer (4)

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

$\Rightarrow$ f(θ) = 4cos2θ (after finding determinant)

$\Rightarrow$ f′(θ) = – 8sin2θ < 0 $\forall\theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

So f is decreasing function

So $f(\theta)_{\text{min}} = f\left(\frac{\pi}{2}\right) = 4 \times \cos\pi = -4 = m$

$f(\theta)_{\text{max}} = f\left(\frac{\pi}{4}\right) = 4\cos\frac{\pi}{2} = 0 =$ M

So (m, M) = (–4, 0)

60. Answer (4)

$x^4.e^y + 2\sqrt{y+1} = 3$

Differentiating w.r.t. x, we get

$$\left(4x^3 + x^4.y'\right)e^y + \frac{y'}{\sqrt{1+y}} = 0$$

$\Rightarrow$ $y'_{\text{at }(1,\,0)} = -2$

Equation of tangent;

y – 0 = –2 (x – 1) $\Rightarrow$ 2x + y = 2

Only (–2, 6) lies on it

f′(x) = (3x² + (a + 6) x – 2)eˣ

$\because$ x = 1 is critical point :

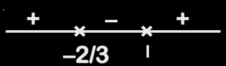
$\therefore$ f′(1) = 0

(3 + a + 6 – 2). e = 0

a = –7

$\therefore$ f′(x) = (3x² – x – 2)eˣ

= (3x + 2) (x – 1)eˣ



$\therefore$ $x = -\frac{2}{3}$ is point of local maxima.

and x = 1 is point of local minima.

62. Answer (2)

Let $f(x) = \begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$

After expanding it we get

f(x) = –2 – sin2x

f′(x) = –2cos2x = 0

$\Rightarrow$ cos2x = 0

$$2x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$

f″(x) = 4sin2x

So $f''\left(\frac{\pi}{4}\right) = 4 > 0$ (minima)

$f''\left(\frac{3\pi}{4}\right) = -4 < 0$ (maxima)

So $m = f_{\text{min}} = f\left(\frac{\pi}{4}\right) = -2-1 = -3$

and $M = f_{\text{max}} = f\left(\frac{3\pi}{4}\right) = -2+1 = -1$

So (m, M) = (–3, –1)

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For time at which it is attained

$$\frac{df(t)}{dt} = \text{Average speed}$$

$2at + b = a(t_1 + t_2) + b$

$$t = \frac{t_1 + t_2}{2}$$

64. Answer (4)

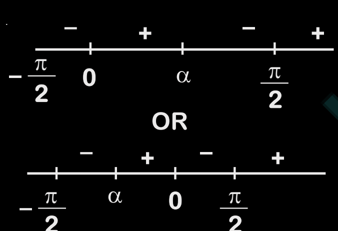
$f(x) = (1 - \cos^2 x)(\lambda + \sin x) = \sin^2 x(\lambda + \sin x)$

$\Rightarrow f(x) = \lambda \sin^2 x + \sin^3 x \quad ...(i)$

$\Rightarrow f'(x) = \sin x \cos x\,[2\lambda + 3\sin x] = 0$

$\Rightarrow x = 0$ and $\sin x = -\frac{2\lambda}{3} \Rightarrow x = \alpha$ (let)

So clearly f(x) will change its sign at x = 0, α because there is exactly one maxima and one minima in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$



Now $\sin x = -\frac{2\lambda}{3}$

$$\Rightarrow -1 \le -\frac{2\lambda}{3} \le 1$$

$$\Rightarrow -\frac{3}{2} \le \lambda \le \frac{3}{2} - \{0\}$$

$\because$ If $\lambda = 0 \Rightarrow f(x) = \sin^3 x$ (from(1))

Which is monotonic. so no maxima/minima

So $\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

$$\Rightarrow \frac{dy}{dx} = 1 + \log_e x$$

$$\Rightarrow \left.\frac{dy}{dx}\right]_{x=c} = 1 + \log_e c \qquad \text{(slope)}$$

$\because$ The tangent is parallel to line joining (1, 0), (e, e)

So, $1 + \log_e c = \frac{e-0}{e-1}$

$$\log_e c = \frac{e}{e-1} - 1$$

$$\Rightarrow \log_e c = \frac{1}{e-1}$$

$$c = e^{\frac{1}{e-1}}$$

66. Answer (03)

Let $f'(x) = a(x+1)(x-3)$

$$\Rightarrow \int_{-1}^{x} f'(x) = \int_{-1}^{x} a(x^2 - 2x - 3)dx$$

$$\Rightarrow f(x) - f(-1) = a\left(\frac{x^3}{3} - x^2 - 3x\right)_{-1}^{x}$$

$$\Rightarrow f(x) = 10 + a\left[\left(\frac{x^3}{3} - x^2 - 3x\right) - \left(-\frac{1}{3} - 1 + 3\right)\right]$$

$\Rightarrow 3f(x) = 30 + a[(x^3 - 3x^2 - 9x) - (-1 - 3 + 9)]$

$\Rightarrow 3f(x) = 30 + a[x^3 - 3x^2 - 9x - 5]$

$\because f(1) = -6$

$\Rightarrow (-)18 = 30 + a[-16] \Rightarrow 16a = 48 \Rightarrow a > 0$

($\because$ a = 3)

$\therefore$ Minima occurs at x = 3

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$$\frac{dy}{dx} = e^x$$

$$\left.\frac{dy}{dx}\right|_{x=c} = e^c$$

Tangent is $y - e^c = e^c(x - c)$

Put $y = 0$, $x_1 = c - 1$ ...(i)

For $y^2 = 4x$

$$2y\frac{dy}{dx} = 4 \Rightarrow \left.\frac{-dx}{dy}\right|_{y=2} = -1$$

Normal is $y - 2 = -1(x - 1)$

Put $y = 0$, $x_1 = 3$ ...(ii)

From (i) and (ii) ; $c - 1 = 3$

$\Rightarrow c = 4$

68. Answer (0.50)

$y = (x - 1)(x - 2)$, this curve intersects the x-axis at A(1, 0) and B(2, 0)

$$\therefore \frac{dy}{dx} = 2x - 3\ ;\ \frac{dy}{dx}_{(x=1)} = -1 \text{ and } \frac{dy}{dx}_{(x=2)} = 1$$

Equation of tangent at A(1, 0) ;

$y = -1(x - 1)$

$\Rightarrow x + y = 1$

and equation of tangent at B(2, 0)

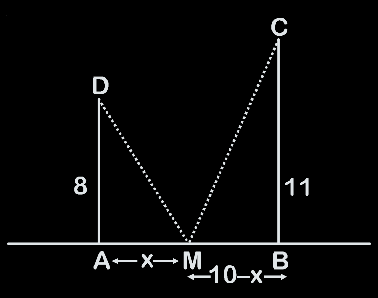
$y = 1(x - 2)$

$\Rightarrow x - y = 2$

So $a = 1$ and $b = 2$

$\Rightarrow \frac{a}{b} = 0.5$

69. Answer (05.00)



$$\frac{d}{dx}$$

$4x = 20 \Rightarrow x = 5$

$$\frac{d^2f(x)}{dx^2} = 2 - 2(-1) > 0$$

$\therefore$ For minimum $x = 5$

70. Answer (1)

Curve is $y = x^3$ ...(1)

So equation of tangent at (t, t^3)

$(y - t^3) = 3t^2(x - t)$ ...(2)

$\because$ It meets again the curve at Q

So solving (1) & (2) we get

$x = -2t \Rightarrow Q = (-2t, -8t^3)$

Now by section formula

$$\text{Ordinate} = \frac{2t^3 - 8t^3}{1+2}$$

$$= \frac{-6t^3}{3}$$

$= -2t^3$

71. Answer (4)

$$\because f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x$$

On differentiating both sides w.r.t. x we get

$$f'(x) = \frac{12x^2 - 6x}{6} - 2\cos x - (2x-1)\sin x + 2\cos x$$

$$f'(x) = 2x^2 - x - (2x-1)\sin x$$

$$f'(x) = (2x-1)(x - \sin x)$$

When $x > \frac{1}{2}$, $2x - 1 > 0$ and $x - \sin x > 0$.

$\therefore f'(x) > 0$ if $x > \frac{1}{2}$

f(x) is increasing in $\left[\frac{1}{2}, \infty\right)$

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$2x^3 - 3x^2 - 36x - 336 \quad , \ x > 4$

Now, $f'(x) = \begin{cases} -55 & , \ x < -5 \\ 6x^2 - 6x - 120 & , \ -5 \le x < 4 \\ 6x^2 - 6x - 36 & , \ x > 4 \end{cases}$

$\Rightarrow \quad f'(x) = \begin{cases} -55 & , \ x < -5 \\ 6(x-5)(x+4) & , \ -5 < x < 4 \\ 6(x-3)(x+2) & , \ x > 4 \end{cases}$

For increasing $f'(x) > 0$

So clearly f(x) is increasing for $x \in (-5, -4) \cup (4, \infty)$

73. Answer (3)

$y = ax^2 + bx + c$ passes through (1, 2)

So $a + b + c = 2$ ...(1)

also (0, 0) satisfies $\Rightarrow$ $c = 0$ ...(2)

also slope of tangent at origin is 1 i.e.

$y' = 2ax + b \quad \Rightarrow \quad b = 1$ and $a = 1$

$\therefore \ a = 1 = b, \ c = 0$

74. Answer (1)

$x + \sqrt{3}y = 2\sqrt{3}$

$m_t = \dfrac{-1}{\sqrt{3}}$ and point of tangency $\left(\dfrac{3\sqrt{3}}{2}, \dfrac{1}{2}\right)$

Option (1) $\quad x^2 + 9y^2 = 9 \quad \Rightarrow 2x + 18yy' = 0$

$\Rightarrow m_t\left(\dfrac{3\sqrt{3}}{2}, \dfrac{1}{2}\right)$

i.e. $y' = \dfrac{-x}{9y} = \dfrac{-3\dfrac{\sqrt{3}}{2}}{9\left(\dfrac{1}{2}\right)} = \dfrac{-1}{\sqrt{3}}$

Option (2) $\quad y' = \dfrac{x}{6\sqrt{3}} \quad \Rightarrow y' = \dfrac{1}{12\sqrt{3}y}$

i.e. $m_t = \dfrac{1}{6\sqrt{3}}$

Option (3) $2x^2 - 18y^2 = 9 \quad \Rightarrow 4x - 36\,yy' = 0$

i.e. $m_t = \dfrac{x}{9y} = \dfrac{3\sqrt{3}}{2.9\dfrac{1}{2}} = \dfrac{1}{\sqrt{3}}$

Option (4) $\quad x^2y^2 = 7 \quad \Rightarrow y' = -\dfrac{x}{y}$

i.e. $m_t = -3\sqrt{3}$

Hence only option (1) is correct

$x \to 0 \quad x^3$

So, $f(x) = x^6 + ax^5 + bx^4 + x^3$

$f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

$\because \ f'(1) = 0 = f'(-1)$

$\Rightarrow \ 5a + 4b = -9$ and $5a - 4b = 3$

$\Rightarrow \ a = -\dfrac{3}{5}$ and $b = -\dfrac{3}{2}$

Then $5.f(2) = 5\left[2^6 - \dfrac{3}{5}.2^5 - \dfrac{3}{2}.2^4 + 2^3\right]$

$= 144$

76. Answer (4)

$C_1 : y^4 = x$ and $C_2 : xy = k$

Point of intersection of C_1 and C_2 is $\left(k^{4/5}, k^{1/5}\right)$

$m_1 = \dfrac{dy_1}{dx} = \dfrac{1}{4y^3} = \dfrac{1}{4k^{3/5}}$

$m_2 = \dfrac{dy_2}{dx} = \dfrac{k}{x^2} = -\dfrac{1}{k^{3/5}}$

$\because m_1 \cdot m_2 = -1 \ \Rightarrow \dfrac{1}{4k^{6/5}} = 1 \Rightarrow 4k^{6/5} = 1$

$\Rightarrow (4k)^6 = 4$

77. Answer (1)

$y = \dfrac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$

Slope = $y' = 2x^3 - 15x^2 + 36x - 19 = g(x)$ say

$g'(x) = 6x^2 - 30x + 36 = 6(x - 2)(x - 3)$

$g'(x) = 0 \qquad \Rightarrow \ x = 2, 3$

Slope g(x) has local maximum at x = 2

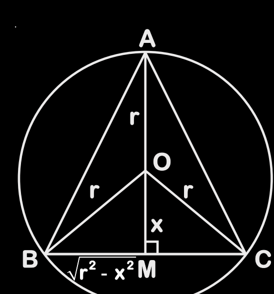
$x = 2 \ \Rightarrow y = 2$

Local maximum at (2, 2)

[Note : Overall maximum (Absolute maximum) value of slope is far greater than that at (2, 2)].

78. Answer (4)

Area of triangle ABC



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$= \frac{1}{2} \times 2\sqrt{r^2 - x^2} \times (r + x)$

$A = (r + x)\sqrt{r^2 - x^2}$

$$\frac{dA}{dx} = \sqrt{r^2 - x^2} - \frac{x}{\sqrt{r^2 - x^2}} \times (r + x) = \frac{r^2 - x^2 - rx - x^2}{\sqrt{r^2 - x^2}}$$

$$= \frac{r^2 - rx - 2x^2}{\sqrt{r^2 - x^2}} = \frac{-(x + r)(2x - r)}{\sqrt{r^2 - x^2}}$$

$\frac{dA}{dx} = 0 \Rightarrow x = \frac{r}{2}$

Sign change of $\frac{dA}{dx}$ at $x = \frac{r}{2} \Rightarrow$ A has maximum

at $x = \frac{r}{2}$ $BC = 2\sqrt{r^2 - x^2} = \sqrt{3}\,r$, $AM = \frac{3}{2}r$

$\Rightarrow AB = AC = \sqrt{3}\,r$

79. Answer (4)

$f(x) = (4a - 3)(x + \ln 5) + (a - 7)\sin x$

$\because$ $f(x)$ is always continuous and differentiable in its domain,

then $f(x)$ has critical points if

$f'(x) = 0$ has solutions

$(4a - 3) + (a - 7)\cos x = 0$

$\Rightarrow \cos x = \frac{3 - 4a}{a - 7}$ has solutions

$\because -1 \le \frac{3 - 4a}{a - 7} \le 1$

$\Rightarrow a \in \left[-\frac{4}{3}, 2\right]$

80. Answer (4)

$$f' = \frac{3(x+1)}{x-1} \times \frac{(x+1)-(x-1)}{(x+1)^2} + \frac{2}{(x-1)^2} > 0$$

$$= \frac{6}{x^2 - 1} + \frac{2}{(x-1)^2} > 0$$

$$= \frac{2(3(x-1)+(x+1))}{(x-1)^2(x+1)} = \frac{4(2x-1)}{(x-1)^2(x+1)} > 0$$

+ − + +

$-\infty$ -1 $\frac{1}{2}$ 1 ∞

$\Rightarrow x \in (-\infty, -1) \cup \left[\frac{1}{2}, \infty\right) - \{1\}$

$$f'(x) = \begin{cases} \cdots \\ -2 + \sin\left(\frac{1}{x}\right) - \frac{1}{x}\cos\left(\frac{1}{x}\right) & \text{if } x < 0 \end{cases}$$

$\because$ $2 - \sin\left(\frac{1}{x}\right) + \frac{1}{x}\cos\left(\frac{1}{x}\right)$ is continuous on either sides of origin.

Also $f'\left(\frac{3}{\pi}\right)$ is +ve and $f'\left(\frac{1}{\pi}\right)$ is –ve, hence $f'(x)$ is changing its sign.

So $f(x)$ is non monotonic in $(0, \infty)$ and $(-\infty, 0)$

82. Answer (5)

$f(x) = ax^2 + bx + c$, $f'(x) = 2ax + b$, $f''(x) = 2a = \frac{1}{2}$

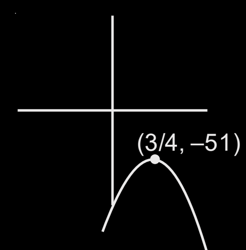
$a = \frac{1}{4}$, $-2a + b = 1 \Rightarrow b = \frac{3}{2}$ and $a - b + c = 2 \Rightarrow c = \frac{13}{4}$

$\because$ $f'(x) > 0 \ \forall \ x \in [-1, 1]$

So, $f(x) \le f(1)$

$\Rightarrow f(x) \le 5$

83. Answer (2)



$\because$ $f(x) = ax^2 + 6x - 15$

$\therefore$ $D = 36 + 60a$

vertex $= \left(-\frac{3}{a}, -\frac{36 + 60a}{a}\right) = \left(-\frac{3}{a}, -\frac{36}{a} - 60\right)$

Here $-\frac{3}{a} = \frac{3}{4}$ $\Rightarrow a = -4$

$\therefore$ $f(x)$ is increasing in $\left(-\infty, \frac{3}{4}\right)$

and decreasing in $\left(\frac{3}{4}, \infty\right)$

Now $g(x) = -4x^2 - 6x + 15$

$\therefore$ $g(x)$ has local maxima

at $x = -\frac{3}{4}$

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$|\; 2x+1 \quad -x \quad 1 \;|$

$$f'(x) = 12x^2 - 8x - 4 = 0 \Rightarrow x = \frac{-1}{3}, 1$$

$$f\left(\frac{-1}{3}\right) + f(1) = \frac{20}{7} - \frac{108}{7} = \frac{-88}{7}$$

85. Answer (3)

$f''(x) = 6x - 6$

$f''(2) = 6,\ f''(1) = 0$

$f(x) = x^3 - 3x^2 - 9x$

$f'(x) = 3x^2 - 6x - 9 = 3(x + 1)(x - 3)$

Local min at $x = 3$

Local min value $= f(3) = -27$

86. Answer (09.00)

Let a point on $y^2 = 6x$ is $P\left(\frac{3}{2}t^2, 3t\right)$

The distance between P and $\left(3, \frac{3}{2}\right)$ is D.

$$\therefore \quad D^2 = \left(\frac{3t^2}{2} - 3\right)^2 + \left(3t - \frac{3}{2}\right)^2$$

$$= 9\left\{\frac{t^4}{4} - t^2 + 1 + t^2 - t + \frac{1}{4}\right\}$$

$$= \frac{9}{4}\left(t^4 - 4t + 5\right)$$

$$\therefore \quad 2D.\frac{dD}{dt} = \frac{9}{4}(4t^3 - 4) = 9(t-1)(t^2 + t + 1)$$

$\therefore$ For t = 1, D^2 will be minimum.

$$\therefore \quad P = \left(\frac{3}{2}, 3\right) = (\alpha, \beta)$$

$\therefore \quad 2(\alpha + \beta) = 9$

87. Answer (3)

$$f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x & , \quad x > 0 \\ 3xe^x & , \quad x \le 0 \end{cases}$$

Here f(x) is differentiable at x = 0

$$\therefore \quad f'(x) = \begin{cases} 4 - (2x-1)^2 & , \quad x > 0 \\ 3e^x(x+1) & , \quad x \le 0 \end{cases}$$

Here $f'(x) > 0$ when $x \in \left(-1, \frac{3}{2}\right)$

$\therefore$ f(x) is increasing in $\left(-1, \frac{3}{2}\right)$

88. Answer (1)

$f(x) = 3\sin^4x + 10\sin^3x + 6\sin^2x - 3$

$f'(x) = 12\sin^3x\cdot\cos x + 30\sin^2x\cos x + 12\sin x\cdot\cos x$

$= 3\sin2x(2\sin^2x + 5\sin x + 2)$

$= 3\sin2x\cdot(\sin x + 2)(2\sin x + 1)$

$\therefore$ Changing points are $-\frac{\pi}{6}, 0$ and $\frac{\pi}{2}$

$$\underset{-\frac{\pi}{6}}{\times} \xrightarrow{\quad - \quad} \underset{0}{\times} \xrightarrow{\quad + \quad} \underset{\frac{\pi}{2}}{\times}$$

$\therefore$ f(x) is decreasing in $\left(-\frac{\pi}{6}, 0\right)$

and increasing in $\left(0, \frac{\pi}{2}\right)$

89. Answer (36)

Let (x) m is used to form circle and $(36 - x)$ m is used to form source. if radius of circle is r

$$2\pi r = x \quad \Rightarrow r = \frac{x}{2\pi}$$

Side of Square $= \frac{36 - x}{4}$

Sum of Area $= \pi\left(\frac{x}{2\pi}\right)^2 + \left(\frac{36 - x}{4}\right)^2 = A$

$$\frac{dA}{dx} = \frac{2x}{4\pi} - \frac{2}{4}\left(\frac{36 - x}{4}\right)$$

To minimize $\frac{dA}{dx} = 0 \Rightarrow x = \frac{36\pi}{4 + \pi} = k$

$$\left(\frac{4}{\pi} + 1\right)\cdot\frac{36\pi}{4 + \pi} = \frac{4 + \pi}{\pi}\cdot\frac{36\pi}{4 + \pi} = 36$$

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$$\ln y = x^2 \ln\left(\frac{2}{x}\right) = x^2(\ln 2 - \ln x)$$

Differentiate both sides

$$\frac{1}{y}\cdot y' = 2x(\ln 2 - \ln x) + x^2\left(\frac{-1}{x}\right)$$

$$= x\left[(2\ln 2 - 1) = 2\ln x\right]$$

$$y' = \left(\frac{2}{x}\right)^{x^2} x\left(\ln\frac{4}{e} - 2\ln x\right)$$

$y' = 0$

$$\Rightarrow \ln\left(\frac{4}{e}\right) = 2\ln x = \ln x^2$$

$$\Rightarrow x^2 = \frac{4}{e}$$

$$\Rightarrow x = \frac{2}{\sqrt{e}}$$

$$\Rightarrow \frac{2}{x} = \sqrt{e}$$

y is maximum at $x = \frac{2}{\sqrt{e}}$ as can be seen from sign change of y' across $x = \frac{2}{\sqrt{e}}$.

$$y_{\max} = y\left(\frac{2}{\sqrt{e}}\right) = \left(\sqrt{e}\right)^{\frac{4}{e}} = e^{\frac{1}{2}\times\frac{4}{e}} = (e)^{\frac{2}{e}}$$

91. Answer (3)

Let side of square be a and that of hexagon be b

Hence, $4a + 6b = 20$ ...(i)

Area of square = a^2 and area of hexagon $= \frac{6\sqrt{3}}{4}b^2$

$$A = a^2 + \frac{6\sqrt{3}}{4}b^2 \qquad ...(ii)$$

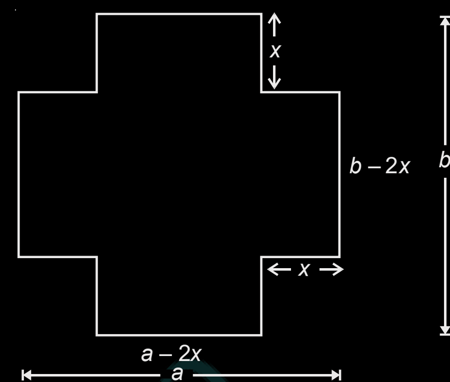
$$\therefore \quad A = \left(\frac{10-3b}{2}\right)^2 + \frac{6\sqrt{3}}{4}b^2$$

$$\frac{dA}{db} = 0$$

$$\Rightarrow 2\left(\frac{10-3b}{2}\right)\left(\frac{-3}{2}\right) + 2\left(\frac{6\sqrt{3}}{4}\right)b = 0$$

(As $\frac{d^2A}{db^2} > 0$ area is minimum for $b = \frac{10}{3+2\sqrt{3}}$)

92. Answer (2)



$V = (a - 2x)(b - 2x)x$

for maximum volume

$$\frac{dV}{dx} = 0$$

$\Rightarrow -2(b - 2x)x + (a - 2x)(-2)x + (a - 2x)(b - 2x) = 0$

$\Rightarrow 12x^2 + x(-2a - 2b - 2b - 2a) + ab = 0$

$\Rightarrow 12x^2 - 4(a + b)x + ab = 0$

$$\Rightarrow x = \frac{4(a+b) \pm \sqrt{16(a+b)^2 - 48ab}}{24}$$

$$= \frac{(a+b) \pm \sqrt{a^2+b^2-ab}}{6}$$

$$\frac{d^2V}{dx^2} = 24x - 4(a+b)$$

$= 4(6x - (a + b)) < 0$

For $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

$\therefore$ Hence $V_{\max}$ at $x = \frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

93. Answer (1)

Let $e^x = t$

and $f(t) = t^4 + 2t^3 - t$ $\qquad (t > 0)$

$f'(t) = 4t^3 + 6t^2 - 1$

$f''(t) = 12t^2 + 12t > 0$ always as $t > 0$

$\therefore$ $f'(t)$ has only 1 root

Also $f'(0) = -1$ and $f'(-1) = 1$

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$f(t) = 6$ will intersects only once

Hence 1 solution

94. Answer (2)

$f'(x) = 2x + a$

For increasing function $2x + a|_{x=1} \geq 0$

$a \geq -2$

For decreasing function $2x + a|_{x=2} \leq 0$

$a \leq -4$

$R = -2,\ S = -4$

$|R - S| = 2$

95. Answer (3)

Let point of intersection be (x_1, y_1)

$$x_1^2 + y_1^2 - ab = 0$$

$$b^2x_1^2 + a^2y_1^2 - a^2b^2 = 0$$

$$x_1^2 = \frac{a^2b}{a+b},\ y_1^2 = \frac{ab^2}{a+b}$$

$$\Rightarrow \quad x_1 = a\sqrt{\frac{b}{a+b}},\ y_1 = b\sqrt{\frac{a}{a+b}} \quad \text{...(i)}$$

Tangent to ellipse $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

Slope $= m_1 = -\frac{b^2x_1}{a^2y_1}$

Tangent to circle $xx_1 + yy_1 = ab$

Slope $= m_2 = -\frac{x_1}{y_1}$

$$\tan\theta = \left|\frac{\frac{-b^2x_1}{a^2y_1} + \frac{x_1}{y_1}}{1 + \frac{b^2x_1}{a^2y_1}\cdot\frac{x_1}{y_1}}\right| = \frac{x_1y_1\left(a^2 - b^2\right)}{a^2b^2}$$

$$= \frac{ab\sqrt{ab}}{a+b}\frac{\left(a^2 - b^2\right)}{a^2b^2} \quad \text{(Using (i))}$$

$$\tan\theta = \frac{a-b}{\sqrt{ab}}$$

$f(x)$ has minima at $x = 1$

and $f'(x)$ has minima at $x = -1$

So, $f''(x) = a(x + 1)$

Integrating both side

$$f'(x) = \frac{a}{2}(x+1)^2 + c$$

$f'(1) = 0 = 2a + c$

$c = -2a$

$$f'(x) = \frac{a}{2}(x+1)^2 - 2a$$

Integrating both side

$$f(x) = \frac{a}{6}(x+1)^3 - 2xa + c'$$

$$f(1) = -10 = \frac{8a}{6} - 2a + c'$$

$f(-1) = 6 = 2a + c'$

$2a + c' = 6$

$4a - 6\,c' = 60$

$a = 6,\ c' = -6$

$f(x) = (x + 1)^3 - 12x - 6$

$f(3) = (4)^3 - 36 - 6$

$f(3) = 22$

97. Answer (1)

$\because$ $f(2) = f(4) = 0 \Rightarrow a = 8$ and $b = 0$

$f(x) = x^3 - 6x^2 + 8x;$

$$f'(x) = 3x^2 - 12x + 8 = 0 \Rightarrow x = 2 + \frac{2}{\sqrt{3}}$$

For statement S1, $x_2 = 2 + \frac{2}{\sqrt{3}}$

$\because$ $f'(2) = -4$ and $f'(x_2) = 0$ hence there exist x_1 such that $x_1 \in (2, x_2)$ and $f'(x_1) = -1$

$\Rightarrow$ Statement S1 is true.

For statement S2; $x_4 = 2 + \frac{2}{\sqrt{3}}$

So $f'(x_3) = \frac{\sqrt{3}}{2} f(x_4) = -\frac{8}{3}$

$f'(2) < f'(x_3) < f'(x_4)$ so statement S2 is also true.

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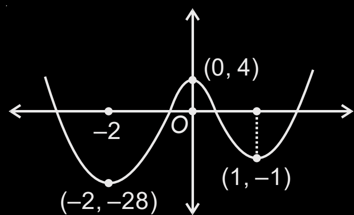
$f(0) = 4$ $f(-2) = -28$

$f(1) = -1$

So, 4 Real Roots



99. Answer (1)

$S = 4\pi r^2$

$$\frac{dS}{dt} = 8\pi r\frac{dr}{dt}$$

$$\frac{dS}{dt} = \text{constant so} \Rightarrow r\frac{dr}{dt} = k \text{ (Let)}$$

$$r\,dr = k\,dt \Rightarrow \frac{r^2}{2} = kt + C$$

at $t = 0$, $r = 3$

$$\frac{9}{2} = C$$

at $t = 5$,

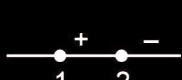
$$\frac{49}{2} = k\cdot 5 + \frac{9}{2} \Rightarrow k = 4$$

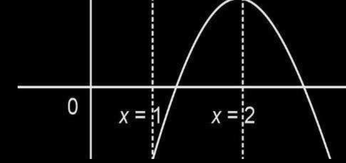
At $t = 9$, $\frac{r^2}{2} = \frac{81}{2}$

So, $r = 9$

100. Answer (3)

$$f'(x) = \frac{4}{x-1} - 4x + 4 = \frac{4\left(2x - x^2\right)}{x-1}$$





So maxima occurs at $x = 2$

$f(2) = 4\times 0 - 2\times 2^2 + 4\times 2 + 5 = 5$

so clearly $f(x) = -1$ has

exactly 2 solutions

$$f''(x) = \frac{4(2-2x)(x-1)}{(x-1)^2} - \left(2x - x^2\right)$$

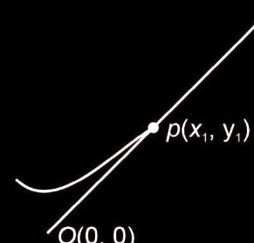
so $f'(e) - f''(2) > 0$

so option c is not correct

101. Answer (4)

$m_{op} - m_{\text{Tangent}}$

$$\frac{y_1}{x_1} = 3x_1^2 + 6x_1$$



$$\Rightarrow \frac{x_1^3 + 3x_1^2 + 5}{x_1} = 3x_1^2 + 6x_1$$

$$\Rightarrow x_1^3 + 3x_1^2 + 5 = 3x_1^3 + 6x_1^2$$

$$\Rightarrow 2x_1^3 + 3x_1^2 - 5 = 0$$

$$\Rightarrow (x_1 - 1)(2x_1^2 + 5x_1 + 5) = 0$$

So, $(x_1, y_1) = (1, 9)$

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$\underline{0 \le x < \frac{1}{2}}$ $\quad f(x) = (1-2x)(x+2) + \frac{\sin 2x}{2}$

$f'(x) = -4x - 3 + \cos 2x < 0$

For $x \ge \frac{1}{2}$: $f'(x) = 4x + 3 + \cos 2x > 0$

So, minima occurs at $x = \frac{1}{2}$

$$f(x)|_{\min} = \left|2\left(\frac{1}{2}\right)^2 + \frac{3}{2} - 2\right| + \sin\left(\frac{1}{2}\right).\cos\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}\sin 1$$

So, maxima is possible at $x = 0$ or $x = 1$

Now checking for $x = 0$ and $x = 1$, we can see it attains its maximum value at $x = 1$

$$f(x)|_{\max} = |2 + 3 - 2| + \frac{\sin 2}{2}$$

$$= 3 + \frac{1}{2}\sin 2$$

Sum of absolute maximum and minimum value

$$= 3 + \frac{1}{2}(\sin 1 + \sin 2)$$

103. Answer (4)

$x, y > 0$ and $x^3y^2 = 2^{15}$

Now, $3x + 2y = (x + x + x) + (y + y)$

So, by A.M $\ge$ G.M inequality

$$\frac{3x + 2y}{5} \ge \sqrt[5]{x^3 \cdot y^2}$$

$$\therefore\ 3x + 2y \ge 5\sqrt[5]{2^{15}}$$

$$\ge 40$$

$\therefore$ Least value of $3x + 4y = 40$

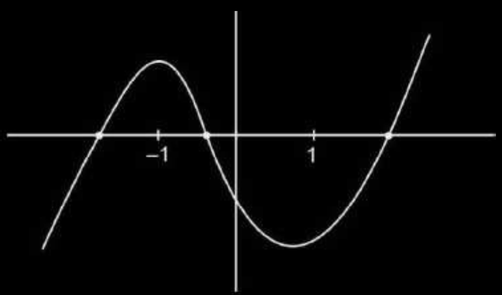
$f'(x) = 7x^6 - 7 = 7(x^6 - 1)$

and $f'(x) = 0 \Rightarrow x = +1$

and $f(-1) = -1 + 7 - 2 = 5 > 0$

$f(1) = 1 - 7 - 2 = -8 < 0$

So, roughly sketch of $f(x)$ will be



So, number of real roots of $f(x) = 0$ and 3

105. Answer (4)

$\because\ f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$

$\therefore\ f_\lambda'(x) = 12(\lambda x^2 - 6\lambda x + 3)$

For $f_\lambda(x)$ increasing : $(6\lambda)^2 - 12\lambda \le 0$

$\therefore\ \lambda \in \left[0, \frac{1}{3}\right]$

$\therefore\ \lambda^* = \frac{1}{3}$

Now, $f_\lambda^*(x) = \frac{4}{3}x^3 - 12x^2 + 36x + 48$

$\therefore\ f_\lambda^*(1) + f_\lambda^*(-1) = 73\frac{1}{2} - 1\frac{1}{2}$

$= 72.$

106. Answer (3)

$\because\ V = \frac{1}{3}\pi r^2 h$ and $\frac{r}{h} = \frac{7}{35} = \frac{1}{5}$

$\Rightarrow V = \frac{1}{75}\pi h^3$

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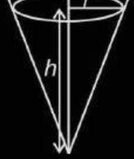


$$\frac{dV}{dt} = \frac{1}{25}\pi h^2 \frac{dh}{dt} = 1$$

$$\Rightarrow \frac{dh}{dt} = \frac{25}{\pi h^2}$$

Now, $S = \pi r l = \pi\left(\frac{h}{5}\right)\sqrt{h^2 + \frac{h^2}{25}} = \frac{\pi}{25}\sqrt{26}h^2$

$$\Rightarrow \frac{dS}{dt} = \frac{2\sqrt{26}\pi h}{25} \cdot \frac{dh}{dt} = \frac{2\sqrt{26}}{h}$$

$$\left.\frac{dS}{dt}\right|_{(h=10)} = \frac{\sqrt{26}}{5}$$

107. Answer (3)

$$\because \frac{dy}{dx} = \frac{24(1+\sin t)\cos t}{12(1+\cos 2t)} = \frac{1+\sin t}{\cos t} = \tan\left(\frac{\pi}{4}+\frac{t}{2}\right)$$

$$\because \left.\frac{dy}{dx}\right|_{(x_0,\, y_0)} = \sqrt{3} = \tan\left(\frac{\pi}{4}+\frac{t}{2}\right)$$

$$\Rightarrow t = \frac{\pi}{6}$$

So, $y_{0\left(\text{at } t=\frac{\pi}{6}\right)} = 12\left(1+\sin\frac{\pi}{6}\right)^2 = 27$

108. Answer (3)

$f(x) = |(x-1)(x+1)(x-3)| + (x-3)$

$$f(x) = \begin{cases} (x-3)(x^2); & 3 \le x \le 4 \\ (x-3)(2-x^2); & 1 \le x < 3 \\ (x-3)(x^2); & 0 < x < 1 \end{cases}$$

$$f'(x) = \begin{cases} 3x^2 - 6x; & 3 < x < 4 \\ -3x^2 + 6x + 2; & 1 < x < 3 \\ 3x^2 - 6x; & 0 < x < 1 \end{cases}$$

$x \in (1, 3)$ $f'(x) = 0$ at one point $\rightarrow$ Maximum

$x \in (3, 4)$ $f'(x) \ne 0$

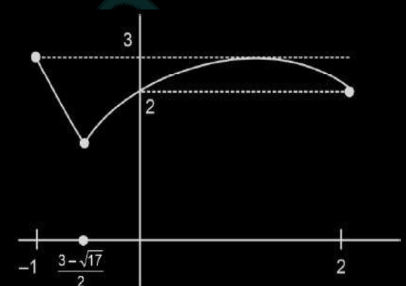
$x \in (0, 1)$ $f'(x) \ne 0$

So, 3 points

109. Answer (1)

$f(x) = |x^2 - 3x - 2| - x \;\; \forall x \in [-1, 2]$

$$\Rightarrow f(x) = \begin{cases} x^2 - 4x - 2 & \text{if } -1 \le x < \frac{3-\sqrt{17}}{2} \\ -x^2 + 2x + 2 & \text{if } \frac{3-\sqrt{17}}{2} \le x \le 2 \end{cases}$$



$f(x)_{\max} = 3$

$$f(x)_{\min} = f\left(\frac{3-\sqrt{17}}{2}\right)$$

$$= \frac{\sqrt{17}-3}{2}$$

110. Answer (4)

$$\Rightarrow \left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$$

$$\Rightarrow \frac{n}{a}\left(\frac{x}{a}\right)^{n-1} + \frac{n}{b}\left(\frac{y}{b}\right)^{n-1}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{b}{a}\left(\frac{bx}{ay}\right)^{n-1}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(a,b)} = -\frac{b}{a}$$

So line always touches the given curve.

$\therefore\ 152x\frac{dx}{dr} + 6\pi r = 0$

$\therefore\ \frac{dx}{dr} = \frac{-6\pi r}{152x}$

Now, $V = 40x^3 + \frac{2}{3}\pi r^3$

$\therefore\ \frac{dv}{dr} = 120x^2 \cdot \frac{dx}{dr} + 2\pi r^2 = 0$

$\Rightarrow\ 120x^2 \cdot \left(\frac{-6\pi}{152}\frac{r}{x}\right) + 2\pi r^2 = 0$

$\Rightarrow\ 120\left(\frac{x}{r}\right) = 2\pi\left(\frac{152}{6\pi}\right)$

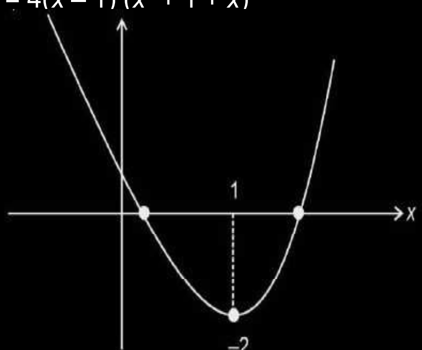
$\Rightarrow\ \left(\frac{x}{r}\right) = \frac{152}{3}\ \frac{1}{120} = \frac{19}{45}$

112. Answer (2)

$f(x) = x^4 - 4x + 1 = 0$

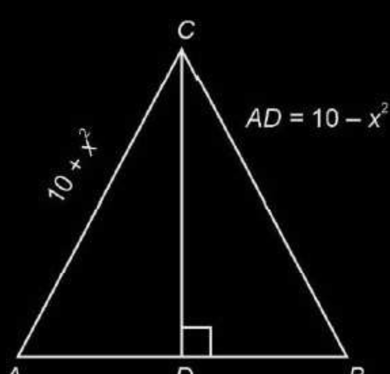
$f'(x) = 4x^3 - 4$

$= 4(x - 1)(x^2 + 1 + x)$



$\Rightarrow$ Two solution

113. Answer (3)



Area $= \frac{1}{2} \times CD \times AB = \frac{1}{2} \times 2\sqrt{10}\,|x|\left(20 - 2x^2\right)$

$A = \sqrt{10}\,|x|\left(10 - x^2\right)$

$\frac{dA}{dx} = \sqrt{10}\frac{|x|}{x}\left(10 - x^2\right) + \sqrt{10}\,|x|\,(-2x) = 0$

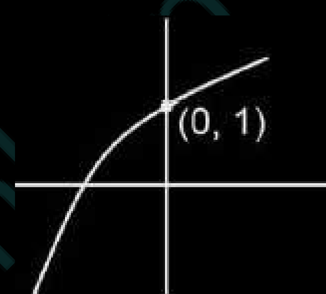
$\Rightarrow\ 10 - x^2 = 2x^2$

$3x^2 = 10$

$x = k$

$\boxed{3k^2 = 10}$

114. Answer (2)



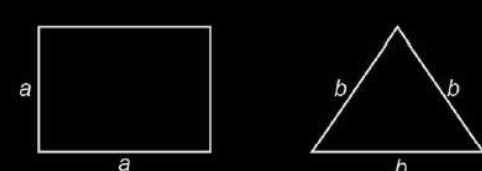
$f'(x) = 7x^6 + 15x^2 + 3 > 0\ \forall x \in R$

$f(x)$ is always increasing

So clearly it intersects

x-axis at only one point

115. Answer (2)



$4a + 3b = 22$

Total area $= A = a^2 + \frac{\sqrt{3}}{4}b^2$

$A = \left(\frac{22 - 3b}{4}\right)^2 + \frac{\sqrt{3}}{4}b^2$

$\frac{dA}{dB} = 2\left(\frac{22 - 3b}{4}\right)\left(\frac{-3}{4}\right) + \frac{\sqrt{3}}{4}.2b = 0$

$4\sqrt{3}b = 66 - 9b$

$$\boxed{b = \frac{66}{9+4\sqrt{3}}}$$

116. Answer (3)

For $n_2 \in$ odd, there will be local minima in (3, 5)
for $n_2 \in$ even, there will be local maxima in (3, 5)

117. Answer (4)

Let $h(x)=f(x)\cdot g'(x)$

As $f(x)$ is even $f\left(\frac{1}{2}\right)=\left(\frac{1}{4}\right)=0$

$$\Rightarrow f\left(-\frac{1}{2}\right)=f\left(-\frac{1}{4}\right)=0$$

and $g(x)$ is even $\Rightarrow g'(x)$ is odd

and $g(1) = 2$ ensures one root of $g'(x)$ is 0.

So, $h(x) = f(x) \,.\, g'(x)$ has minimum five zeroes

$\therefore$ $h'(x) = f'(x)\cdot g'(x) + f(x)\cdot g''(x)= 0,$

has minimum 4 zeroes

118. Answer (2)

Given, $f(x)=\underbrace{\left(x^2-2x+7\right)}_{f_1(x)}\underbrace{e^{\left(4x^3-12x^2-180x+31\right)}}_{f_2(x)}$

$f_1(x) = x^2 - 2x + 7$

$f_1'(x) = 2x - 2$ so $f(x)$ is decreasing in [–3, 0]

and positive also

$$f_2(x) = e^{4x^3-12x^2-180x+31}$$

$$f_2'(x) = e^{4x^3-12x^2-180x+31}.\ 12x^2 - 24x - 180$$

$$= 12(x-5)(x+3)e^{4x^3-12x^2-180x+31}$$

So, $f_2(x)$ is also decreasing and positive in {–3, 0}

$\therefore$ absolute maximum value of $f(x)$ occurs at $x = -3$

$\therefore$ $\boxed{\alpha = -3}$

$\frac{dy}{dx} = 3ax^2 + 2bx + c$ ...(ii)

Touches x-axis at $P(-2, 0)$

$\Rightarrow y|_{x=-2} = 0 \Rightarrow -8a + 4b - 2c + 5 = 0$ ...(iii)

Touches x-axis at $P(-2, 0)$ also implies

$\frac{dy}{dx}\Big|_{x=-2} = 0 \Rightarrow 12a - 4b + c = 0$ ... (iv)

$y = f(x)$ cuts y-axis at (0, 5)

Given, $\frac{dy}{dx}\Big|_{x=0} = c = 3$ ...(v)

From (iii), (iv) and (v)

$$a = -\frac{1}{2}, b = -\frac{3}{4}, c = 3$$

$$\Rightarrow f(x) = \frac{-x^2}{2} - \frac{3}{4}x^2 + 3x + 5$$

$$f'(x) = \frac{-3}{2}x^2 - \frac{3}{2}x + 3$$

$$= \frac{-3}{2}(x+2)(x-1)$$

$f'(x) = 0$ at $x = -2$ and $x = 1$

By first derivative test $x = 1$ in point of local maximum

Hence local maximum value of $f(x)$ is $f(1)$

$i.e., \frac{27}{4}$

120. Answer (3)

$$f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \le 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases}$$

If $f(x)$ has maximum value at $x = 1$ then $f(1+) \le f(1)$

$-2 + \log_2(b^2 - 4) \le 1 - 1 + 10 - 7$

$\log_2(b^2 - 4) \le 5$

$0 < b^2 - 4 \le 32$

(i) $b^2 - 4 > 0 \Rightarrow b \in (-\infty, -2) \cup (2, \infty)$

(ii) $b^2 - 36 \le 0 \Rightarrow b \in [-6, 6]$

Intersection of above two sets

$b \in [-6, -2) \cup (2, 6]$

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$(1-x)+(x-1)=0$

$\Rightarrow$ $(x-1)(x^7-x^5+3x^3-x(x+1)-2x+1)=0$

$\Rightarrow$ $(x-1)(x^7-x^5+3x^3-x^2-3x+1)=0$

$\Rightarrow$ $(x-1)(x^5(x^2-1)+3x(x^2-1)-1(x^2-1))=0$

$\Rightarrow$ $(x-1)(x^2-1)(x^5+3x-1)=0$

$\therefore$ $x=\pm 1$ are roots of above equation and x^5+3x-1 is a monotonic term hence vanishes at exactly one value of x other then 1 or – 1.

$\therefore$ 3 real roots.

122. Answer (45)

$\delta'(x)=\dfrac{4x^2-1}{x}$ so f(x) is decreasing in $\left(0,\dfrac{1}{2}\right)$ and increasing in $\left(\dfrac{1}{2},\infty\right)\Rightarrow a=\dfrac{1}{2}$

Tangent at $y^2=2x \Rightarrow y=mx+\dfrac{1}{2m}$

It is passing through (4, 3)

$$3=4m+\frac{1}{2m}\Rightarrow m=\frac{1}{2}\text{ or }\frac{1}{4}$$

So tangent may be

$$y=\frac{1}{2}x+1 \text{ or } y=\frac{1}{4}x+2$$

But $y=\dfrac{1}{2}x+1$ passes through (–2, 0) so rejected.

<u>Equation of Normal</u>

$$y=-4x-2\left(\frac{1}{2}\right)(-4)-\frac{1}{2}(-4)^3$$

or $y=-4x+4+32$

or $\dfrac{x}{9}+\dfrac{y}{36}=1$

123. Answer (2)

Here equation of curve is

$y^5-9xy+2x=0$ ...(i)

$\therefore$ $\dfrac{dy}{dx}=\dfrac{9y-2}{5y^4-9x}$

When tangents are parallel to x axis then $9y-2=0$

$\therefore$ $M=1$.

For tangent perpendicular to x-axis

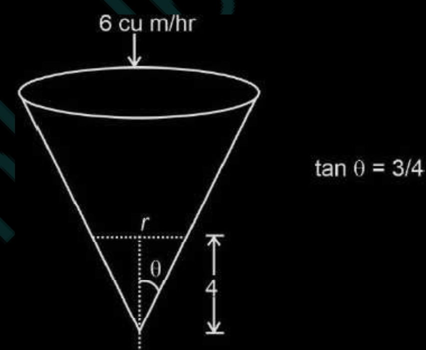
$5y^4-9x=0$ ...(ii)

From equation (i) and equation (ii) we get only one point.

$\therefore$ $N=1$.

$\therefore$ $M+N=2$.

124. Answer (5)



$v=\dfrac{1}{3}\pi r^2 h$ ...(i)

And $\tan\theta=\dfrac{3}{4}=\dfrac{r}{h}$ ...(ii)

i.e. if $h=4$, $r=3$

$$v=\frac{1}{3}\pi r^2\left(\frac{4r}{3}\right)$$

$$\frac{dv}{dt}=\frac{4\pi}{9}3r^2\frac{dr}{dt} \Rightarrow 6=\frac{4\pi}{3}(9)\frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt}=\frac{1}{2\pi}$$

Curved area = $\pi r\sqrt{r^2+h^2}$

$$=\pi r\sqrt{r^2+\frac{16r^2}{9}}$$

$$\frac{dA}{dt} = \frac{10}{3}\pi r\frac{dr}{dt}$$

$$= \frac{10}{3}\pi \cdot 3 \cdot \frac{1}{2\pi}$$

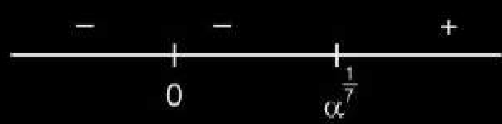
$$= 5$$

125. Answer (3)

$$f(x) = \frac{5x^2}{2} + \frac{\alpha}{x^5} \quad \{x > 0\}$$

$$f'(x) = 5x - \frac{5\alpha}{x^6} = 0$$

$$\Rightarrow x = (\alpha)^{\frac{1}{7}}$$



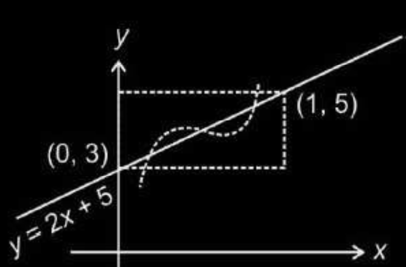
$$f(x)_{\min} = \frac{5(\alpha)^{\frac{2}{7}}}{2} + \frac{\alpha}{\alpha^{\frac{5}{7}}} = 14$$

$$\frac{5}{2}\alpha^{\frac{2}{7}} + \alpha^{\frac{2}{7}} = 14$$

$$\frac{7}{2}\alpha^{\frac{2}{7}} = 14$$

$\alpha = 128$

126. Answer (2)



If a graph cuts y = 2x + 5 in (0, 1) twice then its concavity changes twice

$\therefore$ $f'(x) = 0$ at atleast two points.

$$f'(x) = xe^{x(1-x)} \cdot (1-2x) + e^{x(1-x)}$$

$$= e^{x(1-x)}\left[x - 2x^2 + 1\right]$$

$$= -e^{x(1-x)}\left[2x^2 - x - 1\right]$$

$$= -e^{x(1-x)}(2x+1)(x-1)$$

$\therefore$ $f(x)$ is increasing in $\left(-\frac{1}{2}, 1\right)$ and decreasing in $\left(-\infty, -\frac{1}{2}\right) \cup (1, \infty)$

128. Answer (195)

Slope of tangent to curve $y = 5x^2 + 2x - 25$

$$= m = \left(\frac{dy}{dx}\right)_{\text{at}(2,-1)} = 22$$

$\therefore$ Equation of tangent : $y + 1 = 22(x - 2)$

$\therefore$ $y = 22x - 45$.

Slope of tangent to $y = x^3 - x^2 + x$ at point (a, b)

$= 3a^2 - 2a + 1$

$3a^2 - 2a + 1 = 22$

$3a^2 - 2a - 21 = 0$

$\therefore$ $a = 3$ or $-\frac{7}{3}$

Also $b = a^3 - a^2 + a$

Then $(a, b) = (3, 21)$ or $\left(-\frac{7}{3}, -\frac{151}{9}\right)$

$\left(-\frac{7}{3}, -\frac{151}{9}\right)$ does not satisfy the equation of tangent

$\therefore$ $a = 3$, $b = 21$

$\therefore$ $|2a + 9b| = 195$

129. Answer (15)

$f(x) = |5x - 7| + [x^2 + 2x]$

$= |5x - 7| + [(x + 1)^2] - 1$

Critical points of

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critical points or boundary points

$\therefore \quad f\left(\frac{5}{4}\right)=\frac{3}{4}+4=\frac{19}{4}$

$f\left(\frac{7}{5}\right)=0+4=4$

as both $|5x-7|$ and x^2+2x are increasing in nature after $x=\frac{7}{5}$

$\therefore \quad f(2) = 3 + 8 = 11$

$\therefore \quad f\left(\frac{7}{5}\right)_{\min}=4$ and $f(2)_{\max} = 11$

Sum is 4 + 11 = 15

130. Answer (4)

$$f(x)=3^{\left(x^2-2\right)^3+4},\ x\in R$$

$$f(x)=81.3^{\left(x^2-2\right)^3}$$

$$f'(x)=81.3^{\left(x^2-2\right)^3}\ \ln 2.3\left(x^2-2\right)2x$$

$$=(486\ \ln 2)\left(3^{\left(x^2-2\right)^3}\left(x^2-2\right)x\right)$$

$\Rightarrow \quad x = 0$ is the local minima.

$$f''(x)=(486\ln 2)\left(\begin{array}{c}3^{\left(x^2-2\right)^3}\cdot\left(x^2-2\right)\\ \left(5x^2-2+6x^2\ln 3\left(x^2-2\right)\right)\end{array}\right)$$

$f''\left(\sqrt{2}^{+}\right)>0$

$f''\left(\sqrt{2}^{-}\right)<0$

$\Rightarrow \quad x=\sqrt{2}$ is point of inflection

$f''(x)>0\ \forall\ x>\sqrt{2}$

$\Rightarrow \quad f'(x)$ is increasing for $x>\sqrt{2}$

131. Answer (1)

$$f(x)=\log_e(x^2+1)-e^{-x}+1$$

$$f'(x)=\frac{2x}{x^2+1}+e^{-x}$$

$$=\frac{2}{x+\frac{1}{x}}+e^{-x}>0\quad \forall x\in R$$

$$g(x)=e^{-x}-2e^{x}$$

$$g'(x)=-e^{-x}-2e^{x}<0 \qquad \forall x\in R.$$

$\Rightarrow \quad f(x)$ is increasing and $g(x)$ is decreasing function.

$$f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right)>f\left(g\left(\alpha-\frac{5}{3}\right)\right)$$

$$\Rightarrow \quad \frac{(\alpha-1)^2}{3}<\alpha-\frac{5}{3}$$

$= \alpha^2 - 5\alpha + 6 < 0$

$= (\alpha - 2)(\alpha - 3) < 0$

$= \alpha\in(2, 3)$

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