Chapter 16

Relations

1. Consider the following relations :

   $R = \{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$;

   $S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle| m, n, p \text{ and } q \text{ are integers such that } n, q \neq 0 \text{ and } qm = pn\right\}$. Then **[AIEEE-2010]**

   (1) R is an equivalence relation but S is not an equivalence relation

   (2) Neither R nor S is an equivalence relation

   (3) S is an equivalence relation but R is not an equivalence relation

   (4) R and S both are equivalence relations

2. If $R = \{(x, y) ; x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is **[JEE (Main)-2020]**

   (1) $\{0, 1\}$

   (2) $\{-2, -1, 1, 2\}$

   (3) $\{-1, 0, 1\}$

   (4) $\{-2, -1, 0, 1, 2\}$

3. Let R_1 and R_2 be two relation defined as follows :

   $R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

   $R_2 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$, where Q is the set of all rational numbers. Then

   [JEE (Main)-2020]

   (1) Neither R_1 nor R_2 is transitive.

   (2) R_2 is transitive but R_1 is not transitive.

   (3) R_1 and R_2 are both transitive.

   (4) R_1 is transitive but R_2 is not transitive.

4. Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and

   $g : R - \{1\} \to R$ be defined as $g(x) = \dfrac{x - \frac{1}{2}}{x - 1}$.

   Then the composition function $f(g(x))$ is :

   [JEE (Main)-2021]

   (1) neither one-one nor onto

   (2) onto but not one-one

   (3) both one-one and onto

   (4) one-one but not onto

5. Let $R = \{(P, Q) \mid P$ and Q are at the same distance from the origin$\}$ be a relation, then the equivalence class of $(1, -1)$ is the set :

   [JEE (Main)-2021]

   (1) $S = \{(x, y) \mid x^2 + y^2 = 2\}$

   (2) $S = \{(x, y) \mid x^2 + y^2 = 1\}$

   (3) $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$

   (4) $S = \{(x, y) \mid x^2 + y^2 = 4\}$

6. Let $A = \{2, 3, 4, 5, \ldots, 30\}$ and '$\simeq$' be an equivalence relation on $A \times A$, defined by $(a, b) \simeq (c, d)$, if and only if $ad = bc$. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair $(4, 3)$ is equal to : **[JEE (Main)-2021]**

   (1) 7 (2) 8

   (3) 5 (4) 6

7. Let N be the set of natural numbers and a relation R on N be defined by

   $$R = \left\{(x, y) \in N \times N : x^3 - 3x^2y - xy^2 + 3y^3 = 0\right\}.$$

   Then the relation R is **[JEE (Main)-2021]**

   (1) An equivalence relation

   (2) Reflexive and symmetric, but not transitive

   (3) Reflexive but neither symmetric nor transitive

   (4) Symmetric but neither reflexive nor transitive

8. Let $\mathbb{Z}$ be the set of all integers,

   $$A = \left\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + y^2 \leq 4\right\},$$

   $$B = \left\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 4\right\} \text{ and}$$

   $$C = \left\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + (y^2 - 2)^2 \leq 4\right\}$$

   If the total number of relations from $A \cap B$ to $A \cap C$ is 2^p, then the value of p is

   [JEE (Main)-2021]

   (1) 16 (2) 49

   (3) 25 (4) 9

9. Which of the following is **not** correct for relation R on the set of real numbers?

[JEE (Main)-2021]

(1) $(x, y) \in R \Leftrightarrow |x - y| \le 1$ is reflexive and symmetric.

(2) $(x, y) \in R \Leftrightarrow 0\ |x| - |y| \le 1$ is neither transitive nor symmetric

(3) $(x, y) \in R \Leftrightarrow 0 < |x - y| \le 1$ is symmetric and transitive

(4) $(x, y) \in R \Leftrightarrow |x| - |y| \le 1$ is reflexive but not symmetric

10. Let R_1 and R_2 be relations on the set {1, 2, ..., 50} such that

$R_1 = \{(p, p^n) : p$ is a prime and $n \ge 0$ is an integer$\}$ and
$R_2 = \{(p, p^n) : p$ is a prime and $n = 0$ or $1\}$.

Then, the number of elements in $R_1 - R_2$ is ______.

[JEE (Main)-2022]

11. Let $R_1 = \{(a, b) \in \mathbf{N} \times \mathbf{N} : |a - b| \le 13\}$ and

$R_2 = \{(a, b) \in \mathbf{N} \times \mathbf{N} : |a - b|$ "“ 13}. Then on **N**:

[JEE (Main)-2022]

(1) Both R_1 and R_2 are equivalence relations

(2) Neither R_1 nor R_2 is an equivalence relation

(3) R_1 is an equivalence relation but R_2 is not

(4) R_2 is an equivalence relation but R_1 is not

12. Let a set $A = A_1 \cup A_2 \cup \dots \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j$, $1 \le i, j \le k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i$ if and only if $x \in A_i$, $1 \le i \le k\}$. Then, R is :

[JEE (Main)-2022]

(1) reflexive, symmetric but not transitive

(2) reflexive, transitive but not symmetric

(3) reflexive but not symmetric and transitive

(4) an equivalence relation

13. Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by $a\ R_1\ b \Leftrightarrow ab \ge 0$ and $a\ R_2\ b \Leftrightarrow a \ge b$. Then,

[JEE (Main)-2022]

(1) R_1 is an equivalence relation but not R_2

(2) R_2 is an equivalence relation but not R_1

(3) Both R_1 and R_2 are equivalence relations

(4) Neither R_1 nor R_2 is an equivalence relation

14. For $\alpha \in \mathbf{N}$, consider a relation R on $\mathbf{N}$ given by $R = \{(x, y) : 3x + \alpha y$ is a multiple of $7\}$. The relation R is an equivalence relation if and only if

[JEE (Main)-2022]

(1) $\alpha = 14$

(2) α is a multiple of 4

(3) 4 is the remainder when α is divided by 10

(4) 4 is the remainder when α is divided by 7

15. Let R be a relation from the set {1, 2, 3, ..., 60} to itself such that $R = \{(a, b) : b = pq$, where $p, q \ge 3$ are prime numbers$\}$. Then, the number of elements in R is : **[JEE (Main)-2022]**

(1) 600 (2) 660

(3) 540 (4) 720

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Chapter 16

Relations

1. Answer (3)

   R is not an equivalence relation because $0\ R\ 1$ but $1 \not{R}\ 0$, S is an equivalence relation.

2. Answer (3)

   Given R = {(x, y) : x, y$\in$Z, $x^2 + 3y^2 \le 8$}

   So R = {(1, 1), (2, 1), (1, –1), (0, 1), (1, 0)}

   So $D_{R^{-1}} = \{-1, 0, 1\}$

3. Answer (1)

   (I) If $(a, b) \in R_1$ and $(b, c) \in R_1$

   $\Rightarrow a^2 + b^2 \in Q$ and $b^2 + c^2 \in Q$

   then $a^2 + 2b^2 + c^2 \in Q$ but we cannot say anything about $a^2 + c^2$, that it is rational or not.

   So R_1 is not transitive

   (II) If $(a, b) \in R_2$ and $(b, c) \in R_2$

   $\Rightarrow a^2 + b^2 \notin Q$ and $b^2 + c^2 \notin Q$

   but we can't say anything about $a^2 + c^2$, that it is rational or irrational.

   So R_2 is not transitive

4. Answer (4)

   Here f : R $\to$ R, f(x) = 2x – 1

   and g : R – {1} $\to$ R g(x) $= \dfrac{x - \frac{1}{2}}{x - 1}$

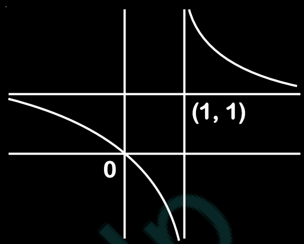
   So, f(g(x)) = 2 g(x) – 1

   $$= 2\left(\frac{x - \frac{1}{2}}{x - 1}\right) - 1$$

   $$= \frac{2x - 1 - x + 1}{x - 1} = \frac{x - 1 + 1}{x - 1}$$

   $$= 1 + \frac{1}{x - 1}$$

   So clearly it is one-one but not onto



5. Answer (1)

   $\therefore$ R = {(P, Q) | P and Q are at the same distance from the origin}.

   Then equivalence class of (1, –1) will contain all such points which lies on circumference of the circle of centre at origin and passing through point (1, –1).

   i.e., radius of circle $= \sqrt{1^2 + 1^2} = \sqrt{2}$

   $\therefore$ Required equivalence class of (S)

   = {(x, y) | $x^2 + y^2 = 2$}.

6. Answer (1)

   Let (4, 3) $\simeq$ (c, d)

   $4d = 3c \Rightarrow \dfrac{c}{4} = \dfrac{d}{3} = k(\text{say})$

   For c, d $\in$ A, k = 1, 2, 3, ..., 7

7. Answer (3)

   $x^2(x - 3y) - y^2(x - 3y) = 0$

   $(x - y)(x + y)(x - 3y) = 0$ ...(i)

   $\therefore$ (i) holds for all (x, x) $\therefore$ R is reflexive

   if (x, y) holds then (y, x) may or may not holds for factors $(x + y)$, $(x - 3y)$ $\therefore$ R is NOT symmetric

   Similarly $(x - 3y)$ factor doesn't hold for transitive

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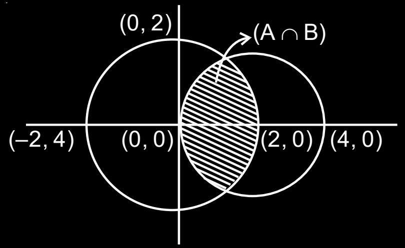
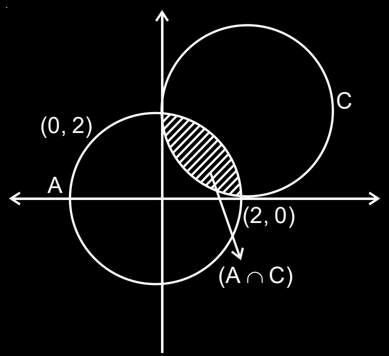
8. Answer (3)

The set A and set B are represented as :



$\therefore$ $A \cap B = \{(0, 0), (1, 0), (2, 0), (1, 1), (1, -1)\}$

The set A and set C are represented as :



$\therefore$ $A \cap C = \{(1, 1), (2, 0), (2, 1), (2, 2), (3, 2)\}$

$\therefore$ Total number relations from $A \cap B$ to $A \cap C = 2^{5\times5}$

$\therefore$ $p = 25$

9. Answer (3)

$(x, y) \in R \Leftrightarrow 0 < |x - y| \le 1.$

R is symmetric because $|x - y| = |y - x|$

But R is not transitive

For example

$x = 0.2,\ y = 0.9,\ z = 1.5$

$0 \le |x - y| = 0.7 \le 1$

$0 \le |y - z| = 0.6 \le 1$

But $|x - z| = 1.3 > 1$

10. Answer (8)

$R_1 - R_2 = \{(2, 2^2), (2, 2^3), (2, 2^4), (2, 2^5), (3, 3^2)$ $(3, 3^3), (5, 5^2), (7, 7^2)\}$

So number of elements = 8

11. Answer (2)

$R_1 = \{(a, b) \in N \times N : |a - b| \le 13\}$ and

$R_2 = \{(a, b) \in N \times N : |a - b| \le 13\}$

In R_1: $\because$ $|2 - 11| = 9 \le 13$

$\therefore$ $(2, 11) \in R_1$ and $(11, 19) \in R_1$ but

$(2, 19) \notin R_1$

$\therefore$ R_1 is not transitive

Hence R_1 is not equivalence

In R_2: $(13, 3) \in R_2$ and $(3, 26) \in R_2$ but

$(13, 26) \notin R_2$ $(\because |13 - 26| = 13)$

$\therefore$ R_2 is not transitive

Hence R_2 is not equivalence.

12. Answer (4)

$R = \{(x, y) : y \in A_i,$ iff $x \in A_i, 1 \le i \le k\}$

(1) Reflexive

$(a, a) \Rightarrow a \in A_i$ iff $a \in A_i$

(2) Symmetric

$(a, b) \Rightarrow a \in A_i$ iff $b \in A_i$

$(b, a) \in R$ as $b \in A_i$ iff $a \in A_i$

(3) Transitive

$(a, b) \in R$ & $(b, c) \in R$.

$\Rightarrow a \in A_i$ iff $b \in A_i$ & $b \in A_i$ iff $c \in A_i$

$\Rightarrow a \in A_i$ iff $c \in A_i$

$\Rightarrow (a, c) \in R.$

$\Rightarrow$ Relation is equivalence

13. Answer (4)

$a R_1 b \Leftrightarrow ab \ge 0$

So, definitely $(a, a) \in R_1$ as $a^2 \ge 0$

If $(a, b) \in R_1$ $\Rightarrow$ $(b, a) \in R_1$

But if $(a, b) \in R_1$, $(b, c) \in R_1$

$\Rightarrow$ Then (a, c) may or may not belong to R_1

{Consider $a = -5$, $b = 0$, $c = 5$ so (a, b) and $(b, c) \in R_1$ but $ac < 0$}

So, R_1 is not equivalence relation

$a R_2 b \Leftrightarrow a \ge b$

$(a, a) \in R_2 \Rightarrow$ so reflexive relation

If $(a, b) \in R_2$ then (b, a) may or may not belong to R_2

$\Rightarrow$ So not symmetric

Hence it is not equivalence relation

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14. Answer (4)

$R = \{(x, y) : 3x + \alpha y$ is multiple of 7$\}$, Now R to be an equivalence relation

(1) R should be reflexive : $(a, a) \in R\ \forall\ a \in N$

$\therefore\ 3a + a\alpha = 7k$

$\therefore\ (3 + \alpha)\, a = 7k$

$\therefore\ 3 + \alpha = 7k_1 \Rightarrow \alpha = 7k_1 - 3$

$= 7k_1 + 4$

(2) R should be symmetric : $aRb \Leftrightarrow bRa$

$aRb : 3a + (7k - 3)\, b = 7\text{ m}$

$\Rightarrow 3(a - b) + 7kb = 7\text{ m}$

$\Rightarrow 3(b - a) + 7\, ka = 7\text{ m}$

So, $aRb \Rightarrow bRa$

$\therefore\ R$ will be symmetric for $a = 7k_1 - 3$

(3) Transitive : Let $(a, b) \in R$, $(b, c) \in R$

$\Rightarrow 3a + (7k - 3)b = 7k_1$ and

$3b + (7k_2 - 3)\, c = 7k_3$

Adding $3a + 7kb + (7k_2 - 3)\, c = 7\,(k_1 + k_3)$

$3a + (7k_2 - 3)\, c = 7\text{ m}$

$\therefore\ (a, c) \in R$

$\therefore\ R$ is transitive

$\therefore\ \alpha = 7k - 3 = 7k + 4$

15. Answer (2)

b can take its values as 9, 15, 21, 33, 39, 51, 57, 25, 35, 55, 49

b can take these 11 values

and a can take any of 60 values

So, number of elements in R $= 60 \times 11$

$= 660$

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