Chapter 23

Area Under Curve

1. The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent to the parabola at the point (2, 3) and the x-axis is **[AIEEE-2009]**

(1) 6 (2) 9

(3) 12 (4) 3

2. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is **[AIEEE-2010]**

(1) $4\sqrt{2}-2$ (2) $4\sqrt{2}+2$

(3) $4\sqrt{2}-1$ (4) $4\sqrt{2}+1$

3. The area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ is **[AIEEE-2011]**

(1) $\frac{8}{3}$ (2) 0

(3) $\frac{32}{3}$ (4) $\frac{16}{3}$

4. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line $y = 2$ is **[AIEEE-2012]**

(1) $\frac{10\sqrt{2}}{3}$ (2) $\frac{20\sqrt{2}}{3}$

(3) $10\sqrt{2}$ (4) $20\sqrt{2}$

5. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is **[JEE (Main)-2013]**

(1) 9 (2) 36

(3) 18 (4) $\frac{27}{4}$

6. The area of the region described by $A = \{(x, y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is **[JEE (Main)-2014]**

(1) $\frac{\pi}{2}-\frac{2}{3}$ (2) $\frac{\pi}{2}+\frac{2}{3}$

(3) $\frac{\pi}{2}+\frac{4}{3}$ (4) $\frac{\pi}{2}-\frac{4}{3}$

7. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is **[JEE (Main)-2015]**

(1) $\frac{7}{32}$ (2) $\frac{5}{64}$

(3) $\frac{15}{64}$ (4) $\frac{9}{32}$

8. The area (in sq. units) of the region $\left\{(x,y) : y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\right\}$ is **[JEE (Main)-2016]**

(1) $\pi - \frac{8}{3}$ (2) $\pi - \frac{4\sqrt{2}}{3}$

(3) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (4) $\pi - \frac{4}{3}$

9. The area (in sq. units) of the region $\{(x, y) : x \ge 0, x + y \le 3, x^2 \le 4y \text{ and } y \le 1 + \sqrt{x}\}$ is **[JEE (Main)-2017]**

(1) $\frac{3}{2}$ (2) $\frac{7}{3}$

(3) $\frac{5}{2}$ (4) $\frac{59}{12}$

10. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and α, β $(\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$, is **[JEE (Main)-2018]**

(1) $\frac{1}{2}(\sqrt{3}-1)$ (2) $\frac{1}{2}(\sqrt{3}+1)$

(3) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$ (4) $\frac{1}{2}(\sqrt{2}-1)$

11. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is **[JEE (Main)-2019]**

(1) $\frac{32}{3}$ (2) $\frac{8}{3}$

(3) $\frac{56}{3}$ (4) $\frac{14}{3}$

12. The area of the region $A = \{(x, y) : 0 \le y \le x|x| + 1 \text{ and } -1 \le x \le 1\}$ in sq. units, is **[JEE (Main)-2019]**

(1) 2 (2) $\frac{4}{3}$

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

13. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, $(k > 0)$, is 1 square unit. Then k is **[JEE (Main)-2019]**

(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$

(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{2}{\sqrt{3}}$

14. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is **[JEE (Main)-2019]**

(1) $\frac{7}{8}$ (2) $\frac{5}{4}$

(3) $\frac{9}{8}$ (4) $\frac{3}{4}$

15. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is **[JEE (Main)-2019]**

(1) $\frac{187}{24}$ (2) $\frac{8}{3}$

(3) $\frac{14}{3}$ (4) $\frac{37}{24}$

16. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is **[JEE (Main)-2019]**

(1) $\frac{15}{2}$ (2) $\frac{21}{2}$

(3) $\frac{15}{4}$ (4) $\frac{17}{4}$

17. The area (in sq. units) of the region $A = \{(x, y) \in R \times R | 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3x\}$ is: **[JEE (Main)-2019]**

(1) $\frac{26}{3}$ (2) $\frac{59}{6}$

(3) 8 (4) $\frac{53}{6}$

18. Let $S(\alpha) = \{(x, y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals **[JEE (Main)-2019]**

(1) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (2) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$

(3) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (4) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

19. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is **[JEE (Main)-2019]**

(1) $\frac{31}{6}$ (2) $\frac{10}{3}$

(3) $\frac{9}{2}$ (4) $\frac{13}{6}$

20. The area (in sq. units) of the region $A = \left\{(x, y) : \frac{y^2}{2} \le x \le y + 4\right\}$ is **[JEE (Main)-2019]**

(1) 18 (2) 16

(3) $\frac{53}{3}$ (4) 30

21. The region represented by $|x - y| \le 2$ and $|x + y| \le 2$ is bounded by a **[JEE (Main)-2019]**

(1) Square of side length $2\sqrt{2}$ units

(2) Square of area 16 sq. units

(3) Rhombus of side length 2 units

(4) Rhombus of area $8\sqrt{2}$ sq. units

22. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is : **[JEE (Main)-2019]**

(1) $\frac{3}{2} - \frac{1}{\log_e 2}$ (2) $\frac{1}{2}$

(3) $\log_e 2 + \frac{3}{2}$ (4) $\frac{3}{2}$

23. If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x,\ x + y \le 1,\ x \ge 0,\ y \ge 0\}$ is $a\sqrt{2}+b$, then $a - b$ is equal to **[JEE (Main)-2019]**

(1) $-\frac{2}{3}$ (2) 6

(3) $\frac{10}{3}$ (4) $\frac{8}{3}$

24. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to **[JEE (Main)-2019]**

(1) 48 (2) 24

(3) $4\sqrt{3}$ (4) $2\sqrt{6}$

25. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is **[JEE (Main)-2020]**

(1) $\frac{1}{6}(24\pi - 1)$ (2) $\frac{1}{6}(12\pi - 1)$

(3) $\frac{1}{3}(12\pi - 1)$ (4) $\frac{1}{3}(6\pi - 1)$

26. The area (in sq. units) of the region

$\{(x, y) \in R^2 \mid 4x^2 \le y \le 8x + 12\}$ is

[JEE (Main)-2020]

(1) $\frac{128}{3}$ (2) $\frac{125}{3}$

(3) $\frac{127}{3}$ (4) $\frac{124}{3}$

27. For $a > 0$, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b(0 < b < a)$ intersect the chord OP and the x-axis at points Q and R, respectively. If the line $x = b$ bisects the area bounded by the curves, C_1 and C_2, and the area of $\Delta OQR = \frac{1}{2}$, then 'a' satisfies the equation **[JEE (Main)-2020]**

(1) $x^6 + 6x^3 - 4 = 0$ (2) $x^6 - 12x^3 - 4 = 0$

(3) $x^6 - 6x^3 + 4 = 0$ (4) $x^6 - 12x^3 + 4 = 0$

28. The area (in sq. units) of the region $\{(x, y) \in R^2 : x^2 \le y \le 3 - 2x\}$, is

[JEE (Main)-2020]

(1) $\frac{31}{3}$ (2) $\frac{29}{3}$

(3) $\frac{34}{3}$ (4) $\frac{32}{3}$

29. Given : $f(x) = \begin{cases} x & , \ 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , \ x = \frac{1}{2} \\ 1 - x & , \ \frac{1}{2} < x \le 1 \end{cases}$

and $g(x) = \left(x - \frac{1}{2}\right)^2, x \in R$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is **[JEE (Main)-2020]**

(1) $\frac{\sqrt{3}}{4} - \frac{1}{3}$ (2) $\frac{1}{3} + \frac{\sqrt{3}}{4}$

(3) $\frac{1}{2} - \frac{\sqrt{3}}{4}$ (4) $\frac{1}{2} + \frac{\sqrt{3}}{4}$

30. Area (in sq. units) of the region outside $\frac{|x|}{2} + \frac{|y|}{3} = 1$ and inside the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is

[JEE (Main)-2020]

(1) $3(4 - \pi)$ (2) $6(4 - \pi)$

(3) $6(\pi - 2)$ (4) $3(\pi - 2)$

31. Consider a region $R = \{(x, y) \in R^2 : x^2 \le y \le 2x\}$. if a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true?

[JEE (Main)-2020]

(1) $3\alpha^2 - 8\alpha + 8 = 0$ (2) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$

(3) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$ (4) $\alpha^3 - 6\alpha^2 + 16 = 0$

32. The area (in sq. units) of the region $\left\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2\right\}$ is

[JEE (Main)-2020]

(1) $\frac{79}{16}$ (2) $\frac{23}{6}$

(3) $\frac{79}{24}$ (4) $\frac{23}{16}$

33. The area (in sq. units) of the region $A = \{(x, y) : (x - 1)[x] \le y \le 2\sqrt{x},\ 0 \le x \le 2\}$, where [t] denotes the greatest integer function, is

[JEE (Main)-2020]

(1) $\frac{8}{3}\sqrt{2}-1$ (2) $\frac{4}{3}\sqrt{2}+1$

(3) $\frac{8}{3}\sqrt{2}-\frac{1}{2}$ (4) $\frac{4}{3}\sqrt{2}-\frac{1}{2}$

34. The area (in sq. units) of the region $A = \{(x, y) : |x| + |y| \le 1,\ 2y^2 \ge |x|\}$ is **[JEE (Main)-2020]**

(1) $\frac{1}{6}$ (2) $\frac{7}{6}$

(3) $\frac{5}{6}$ (4) $\frac{1}{3}$

35. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to

[JEE (Main)-2020]

(1) $\frac{7}{2}$ (2) $\frac{4}{3}$

(3) $\frac{8}{3}$ (4) $\frac{16}{3}$

36. The area (in sq. units) of the region enclosed between the parabola $y^2 = 2x$ and the line $x + y = 4$ is _______. **[JEE (Main)-2022]**

37. The area of the region $\left\{(x,y);|x-1| \le y \le \sqrt{5-x^2}\right\}$ is equal to

[JEE (Main)-2022]

(1) $\frac{5}{2}\sin^{-1}\left(\frac{3}{5}\right)-\frac{1}{2}$ (2) $\frac{5\pi}{4}-\frac{3}{2}$

(3) $\frac{3\pi}{4}+\frac{3}{2}$ (4) $\frac{5\pi}{4}-\frac{1}{2}$

38. The area bounded by the curves $y = |x^2 - 1|$ and $y = 1$ is

[26-07-2022 Evening]

(1) $\frac{2}{3}\left(\sqrt{2}+1\right)$ (2) $\frac{4}{3}\left(\sqrt{2}-1\right)$

(3) $2\left(\sqrt{2}-1\right)$ (4) $\frac{8}{3}\left(\sqrt{2}-1\right)$

39. Let the area enclosed by the x-axis, and the tangent and normal drawn to the curve $4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ at the point $(-2, 3)$ be A. Then $8A$ is equal to _______. **[JEE (Main)-2022]**

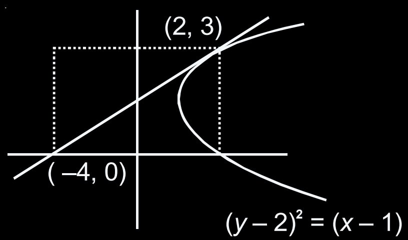
❑ ❑ ❑

Chapter 23

Area Under Curve

1. Answer (2)

The equation of tangent at (2, 3) to the given parabola is $x = 2y - 4$



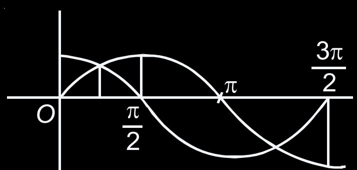
Required area = $\int_0^3 \{(y-2)^2 + 1 - 2y + 4\}dy$

$$= \left[\frac{(y-2)^3}{3} - y^2 + 5y\right]_0^3$$

$$= \frac{1}{3} - 9 + 15 + \frac{8}{3}$$

= 9 sq. units.

2. Answer (1)



Required area

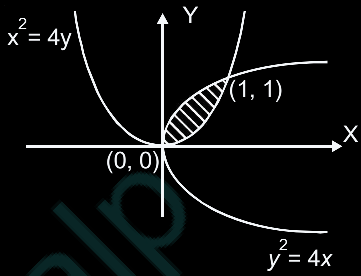
$$= \int_0^{\pi/4}(\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4}(\sin x - \cos x)dx$$

$$+ \int_{5\pi/4}^{3\pi/2}(\cos x - \sin x)dx$$

$= (4\sqrt{2} - 2)$ sq. units

3. Answer (4)

The area loaded by the curves $y^2 = 4x$ and $x^2 = 4y$

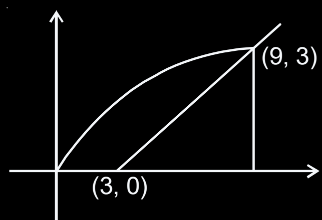


$$A = \int_0^1\left(2\sqrt{x} - \frac{x^2}{4}\right)dx$$

$= \frac{16}{3}$ square units.

4. Answer (1)

5. Answer (1)



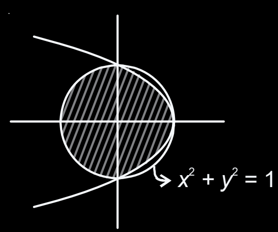
Required area

$$= \int_0^9 \sqrt{x}\,dx - \frac{1}{2}\times 6\times 3$$

$= 18 - 9$

$= 9$

6. Answer (3)



Shaded area

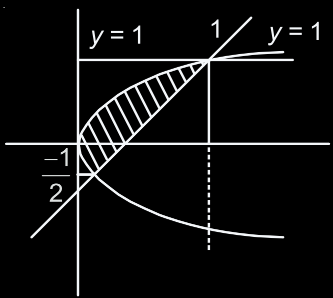
$$= \frac{\pi(1)^2}{2} + 2\int_0^1 \sqrt{(1-x)}\,dx$$

$$= \frac{\pi}{2} + \frac{2(1-x)^{3/2}}{3/2}(-1)\Big|_0^1$$

$$= \frac{\pi}{2} + \frac{4}{3}(0-(-1))$$

$$= \frac{\pi}{2} + \frac{4}{3}$$

7. Answer (4)



After solving $y = 4x - 1$ and $y^2 = 2x$

$$y = 4\cdot\frac{y^2}{2} - 1$$

$2y^2 - y - 1 = 0$

$$y = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} \qquad y = 1, \ \frac{-1}{2}$$

$$A = \int_{-1/2}^{1}\left(\frac{y+1}{4}\right)dy - \int_{-1/2}^{1}\frac{y^2}{2}dy$$

$$= \frac{1}{4}\left[\frac{y^2}{2} + y\right]_{-1/2}^{1} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-1/2}^{1}$$

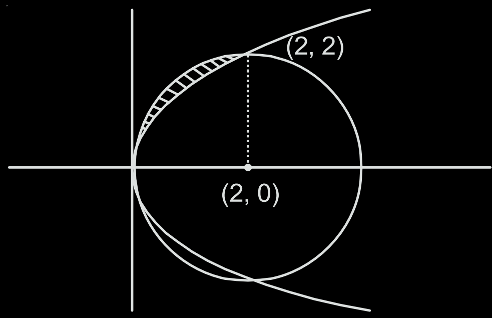
$$= \frac{1}{4}\left[\frac{4+8-1+4}{8}\right] - \frac{1}{2}\left[\frac{8+1}{24}\right]$$

$$= \frac{1}{4}\left[\frac{15}{8}\right] - \frac{9}{48}$$

$$= \frac{15}{32} - \frac{6}{32}$$

$$= \frac{9}{32}$$

8. Answer (1)

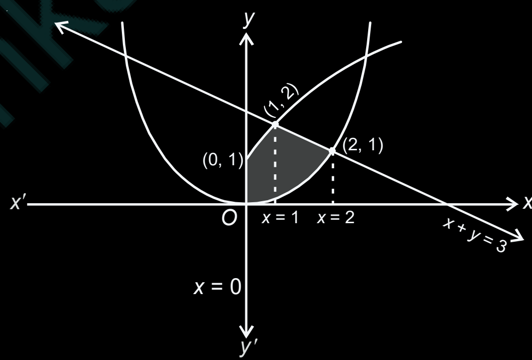


$$\text{Area} = \frac{\pi \cdot 2^2}{4} - \int_0^2 \sqrt{2x}\,dx$$

$$= \pi - \sqrt{2}\cdot\frac{2}{3}x^{\frac{3}{2}}\Big|_0^2$$

$$= \pi - \frac{8}{3}$$

9. Answer (3)



Area of shaded region

$$= \int_0^1\left(\sqrt{x} + 1 - \frac{x^2}{4}\right)dx + \int_1^2\left((3-x) - \frac{x^2}{4}\right)dx$$

$$= \frac{5}{2} \text{ sq. unit}$$

10. Answer (1)

$$18x^2 - 9\pi x + \pi^2 = 0$$

$$(6x - \pi)(3x - \pi) = 0$$

$$\therefore \quad x = \frac{\pi}{6}, \ \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{6}, \ \beta = \frac{\pi}{3}$$

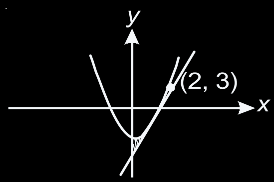
$$y = (gof)(x) = \cos x$$

$$\text{Area} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x\, dx = (\sin x)_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{2}$$

$$= \frac{1}{2}\left(\sqrt{3}-1\right) \text{ sq. units}$$

11. Answer (2)



Tangent at $(2, 3): \dfrac{y+3}{2} = 2x - 1$

$\Rightarrow$ $y + 3 = 4x - 2 \Rightarrow 4x - y - 5 = 0$

$$\text{Area} = \int_0^2 \left[\left(x^2-1\right)-\left(4x-5\right)\right]dx$$

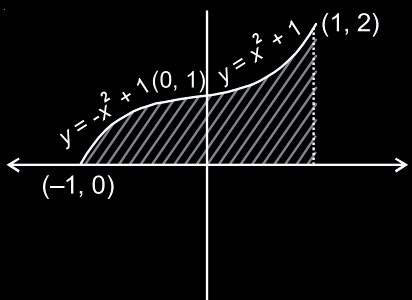
$$= \int_0^2 \left(x^2 - 4x + 4\right)dx$$

$$= \left[\frac{x^3}{3} - 2x^2 + 4x\right]_0^2$$

$$= \frac{8}{3} - 8 + 8 = \frac{8}{3}$$

12. Answer (1)

$A = \{(x, y) : 0 \le y \le x|x| + 1 \text{ and } -1 \le x \le 1\}$



$\therefore$ Area of shaded region

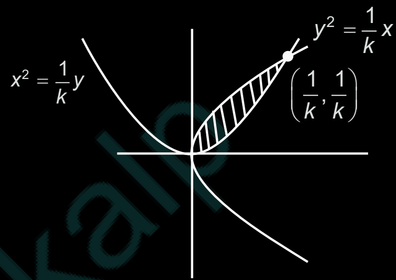
$$= \int_{-1}^{0}\left(-x^2+1\right)dx + \int_0^1 \left(x^2+1\right)dx$$

$$= \left(-\frac{x^3}{3} + x\right)_{-1}^{0} + \left(\frac{x^3}{3} + x\right)_0^1$$

$$= 0 - \left(\frac{1}{3} - 1\right) + \left(\frac{1}{3} + 1\right) - (0+0)$$

$$= \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2 \text{ square units}$$

13. Answer (2)



Area of shaded region = 1.

$$\therefore \int_0^{\frac{1}{k}} \left(\frac{\sqrt{x}}{\sqrt{k}} - kx^2\right)dx = 1$$

$$\left(\frac{1}{\sqrt{k}} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^{\frac{1}{k}} - \left(k \cdot \frac{x^3}{3}\right)_0^{\frac{1}{k}} = 1$$

$$\frac{2}{3\sqrt{k}} \cdot \frac{1}{k^{\frac{3}{2}}} - \frac{k}{3k^3} = 1$$

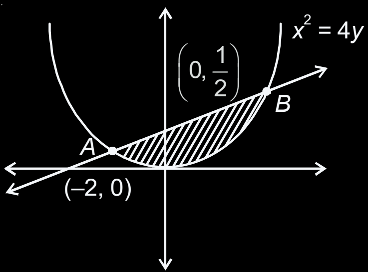
$$\frac{2}{3k^2} - \frac{1}{3k^2} = 1$$

$3k^2 = 1$

$k = \pm\dfrac{1}{\sqrt{3}}$ but $k > 0$

$\therefore k = \dfrac{1}{\sqrt{3}}$

14. Answer (3)



Let points of intersection of the curve and the line be A and B

$x^2 = 4\left(\frac{x+2}{4}\right)$

$x^2 - x - 2 = 0$

x = 2, – 1

Points are (2, 1) and $\left(-1, \frac{1}{4}\right)$

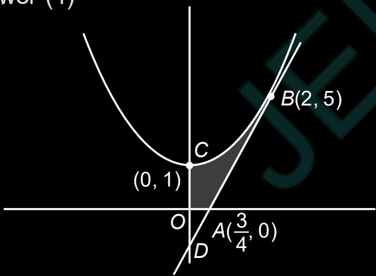
$$\text{Area} = \int_{-1}^{2}\left[\left(\frac{x+2}{4}\right)-\left(\frac{x^2}{4}\right)\right]dx$$

$$= \left[\frac{x^2}{8}+\frac{1}{2}x-\frac{x^3}{12}\right]_{-1}^{2}$$

$$= \left(\frac{1}{2}+1-\frac{2}{3}\right)-\left(\frac{1}{8}-\frac{1}{2}+\frac{1}{12}\right)$$

$$= \frac{9}{8}$$

15. Answer (4)



Given $x^2 = y - 1$

Equation of tangent at (2, 5) to parabola is

$\boxed{4x - y = 3}$

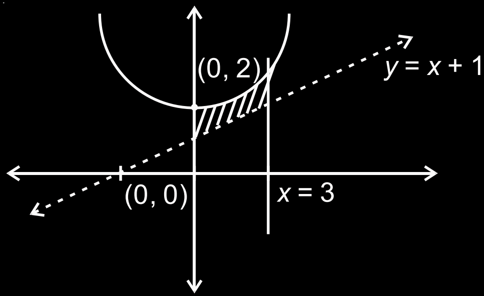
Now required area

$$= \int_{0}^{2}\{(x^2+1)-(4x-3)\}dx - \text{Area of } \Delta AOD$$

$$= \int_{0}^{2}(x^2-4x+4)dx - \frac{1}{2}\times\frac{3}{4}\times 3$$

$$= \left[\frac{(x-2)^3}{3}\right]_0^2 - \frac{9}{8} = \frac{37}{24}$$

16. Answer (1)



$y^2 = 4x$

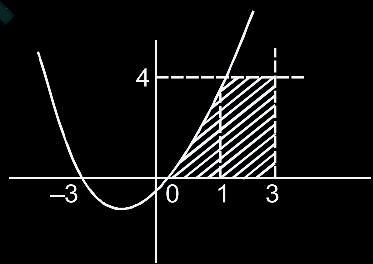
$$\text{Area} = \int_0^3\left[(x^2+2)-(x+1)\right]dx$$

$$= \left[\frac{x^3}{3}-\frac{x^2}{2}+x\right]^3$$

$$= 9 - \frac{9}{2} + 3 = \frac{15}{2}$$

17. Answer (2)

$y \le x^2 + 3x$ represents region below the parabola.



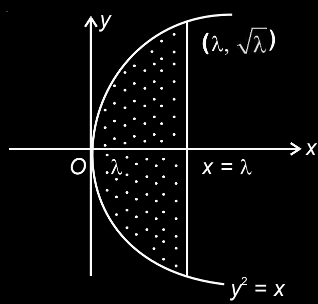
Area of the required region

$$= \int_0^1 (x^2+3x)dx + \int_1^3 4\cdot dx$$

$$= \frac{1}{3}+\frac{3}{2}+8$$

$$= \frac{59}{6}$$

18. Answer (4)

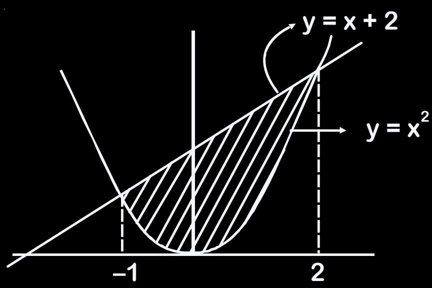


$$A(\lambda) = 2 \times \frac{2}{3}(\lambda \times \sqrt{\lambda}) = \frac{4}{3}\lambda^{3/2}$$

$$\Rightarrow \quad \frac{A(\lambda)}{A(4)} = \frac{2}{5} \qquad \Rightarrow \frac{\lambda^{3/2}}{8} = \frac{2}{5}$$

$$\lambda = \left(\frac{16}{5}\right)^{2/3} = 4.\left(\frac{4}{25}\right)^{1/3}$$

19. Answer (3)



$$\therefore \quad \text{Required area} = \int_{-1}^{2}\left((x+2)-x^2\right)dx$$

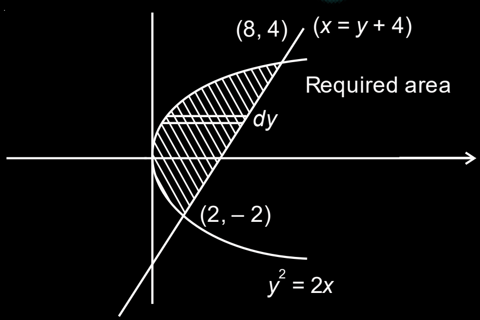
$$= \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)_{-1}^{2}$$

$$= \left(2 + 4 - \frac{8}{3}\right) - \left(+\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= 8 - 3 - \frac{1}{2}$$

$$= 5 - \frac{1}{2} = \frac{9}{2}$$

20. Answer (1)



$$\text{Hence, area} = \int_{-2}^{4} x\,dy$$

$$= \int_{-2}^{4}\left(y + 4 - \frac{y^2}{2}\right)dy$$

$$= \frac{y^2}{2} + 4y - \frac{y^3}{6}\int_{-2}^{4}$$

$$= \left(8 + 16 - \frac{64}{6}\right) - \left(2 - 8 + \frac{8}{6}\right)$$

$$= \left(24 - \frac{32}{3}\right) - \left(-6 + \frac{4}{3}\right)$$

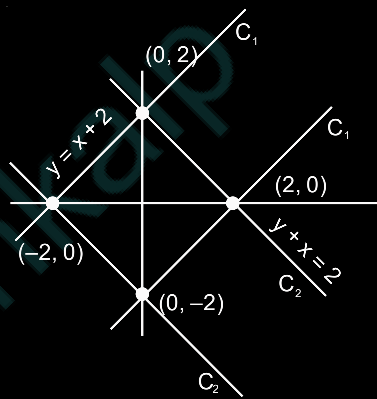
$$= \frac{40}{3} + \frac{14}{3} = \frac{54}{3} = 18$$

21. Answer (1)

$C_1 : |y - x| \le 2$

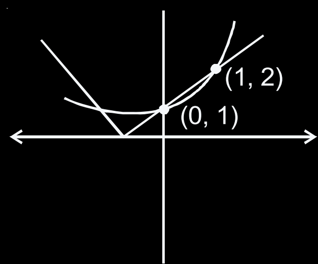
$C_2 : |y + x| \le 2$

Now region is square



Length of side = $\sqrt{2^2 + 2^2} = 2\sqrt{2}$

22. Answer (1)



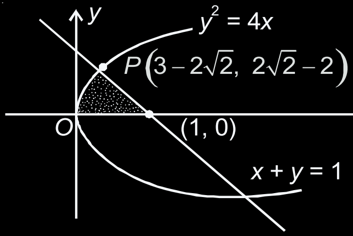
$$\text{Area} = \int_{0}^{1}\left((x+1) - 2^x\right)dx$$

$$= \left[\frac{x^2}{2} + x - \frac{2^x}{\ln 2}\right]_0^1$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{\ln 2}\right) - \left(\frac{-1}{\ln 2}\right)$$

$$= \frac{3}{2} - \frac{1}{\ln 2}$$

23. Answer (2)



$y^2 = 4x$

$x + y = 1$

$y^2 = 4(1 - y)$

$y^2 + 4y - 4 = 0$

$(y + 2)^2 = 8$

$y + 2 = \pm 2\sqrt{2}$

required area

$$= \int_0^{3-2\sqrt{2}} 2\sqrt{x}\,dx + \frac{1}{2}\times\left(2\sqrt{2}-2\right)\times\left(2\sqrt{2-2}\right)$$

$$= \left[2\times\frac{2}{3}x^{3/2}\right]_0^{3-2\sqrt{2}} + \frac{1}{2}\left(8+4-8\sqrt{2}\right)$$

$$= \frac{4}{3}\times\left(3-2\sqrt{2}\right)\sqrt{3-2\sqrt{2}} + 6 - 4\sqrt{2}$$

$$= \frac{4}{3}\left(3-2\sqrt{2}\right)\left(\sqrt{2}-1\right) + 6 - 4\sqrt{2}$$

$$= \frac{4}{3}\left(3\sqrt{2}-3-4+2\sqrt{2}\right) + 6 - 4\sqrt{2}$$

$$= \left(6-\frac{28}{3}\right) + \left(\frac{20}{3}-4\right)\sqrt{2}$$

$$= -\frac{10}{3} + \frac{8}{3}\sqrt{2}$$

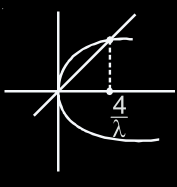
$$\Rightarrow \quad a - b = \frac{10}{3} + \frac{8}{3} = 6$$

24. Answer (2)

$y^2 = 4\lambda x$ and y = λx

On solving ; $(\lambda x)^2 = 4\lambda x$

$x = 0,\ \frac{4}{\lambda}$



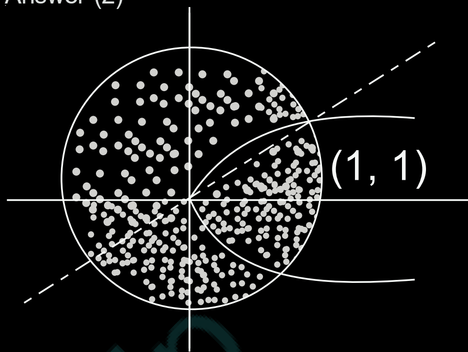
Required area $= \int_0^{\frac{4}{\lambda}} \left(2\sqrt{\lambda x} - \lambda x\right) dx$

$$= \frac{2\sqrt{\lambda}\cdot x^{3/2}}{3/2} - \frac{\lambda x^2}{2}\Bigg|_0^{4/\lambda}$$

$$= \frac{32}{3\lambda} - \frac{8}{\lambda} = \frac{8}{3\lambda} = \frac{1}{9}$$

$\lambda = 24$

25. Answer (2)



Area $= 2\pi - \int_0^1 \left(\sqrt{x} - x\right) dx$

$$= 2\pi - \left[\frac{2x^{\frac{3}{2}}}{3} - \frac{x^2}{2}\right]_0^1$$

$$= 2\pi - \left(\frac{1}{6}\right)$$

$$= \frac{12\pi - 1}{6} \text{ square units}$$

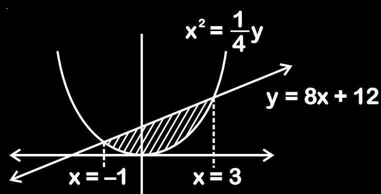
26. Answer (1)

For point of intersections

$4x^2 = 8x + 12$

$x^2 - 2x - 3 = 0$

$\therefore \quad x = -1, 3$



The required area $= \int_{-1}^{3} (8x + 12 - 4x^2)dx$

$$= 4\left(2\cdot\frac{x^2}{2} + 3x - \frac{x^3}{3}\right)_{-1}^{3}$$

$$= 4\left\{(9+9-9) - \left(1-3+\frac{1}{3}\right)\right\}$$

$$= \frac{128}{3} \text{ square units.}$$

27. Answer (4)

Area between $y^2 = ax$ and $x^2 = ay$ is

$$\frac{16\left(\frac{a}{4}\right)\left(\frac{a}{4}\right)}{3} = \frac{a^2}{3}$$

$$\therefore \int_0^b \left(\sqrt{ax} - \frac{x^2}{a}\right)dx = \frac{a^2}{6} \qquad ...(i)$$

Equation of AB is $y = x$

$$\therefore \frac{1}{2}.b.b = \frac{1}{2} \quad \Rightarrow \quad b = 1 \qquad ...(ii)$$

by (i) and (ii)

$$\int_0^1 \left(\sqrt{a}\sqrt{x} - \frac{x^2}{a}\right)dx = \frac{a^2}{6}$$

$$\Rightarrow \left.\frac{\sqrt{a}x^{3/2}}{3/2} - \frac{x^3}{3a}\right|_0^1 = \frac{a^2}{6}$$

$$\Rightarrow \frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$$

$\Rightarrow$ $4a^{3/2} - 2 = a^3$

$\Rightarrow$ $4a^{3/2} = a^3 + 2$

$\Rightarrow$ $16a^3 = a^6 + 4a^3 + 4$

$\Rightarrow$ $a^6 - 12a^3 + 4 = 0$

Hence a satisfy $x^6 - 12x^3 + 4 = 0$

28. Answer (4)

$\because$ $x^2 - y \le 0$ and $2x + y - 3 \le 0$

For Point of intersection we have

$x^2 + 2x - 3 = 0 \quad \Rightarrow x = 1, x = -3$

$\therefore$ P(1, 1) and Q(–3, 9) are point of intersection

$$\therefore \text{ Required area } = \int_{-3}^{1} (3 - 2x - x^2)dx$$

$$= 12 - (x^2)_{-3}^{1} - \frac{1}{3}(x^3)_{-3}^{1}$$

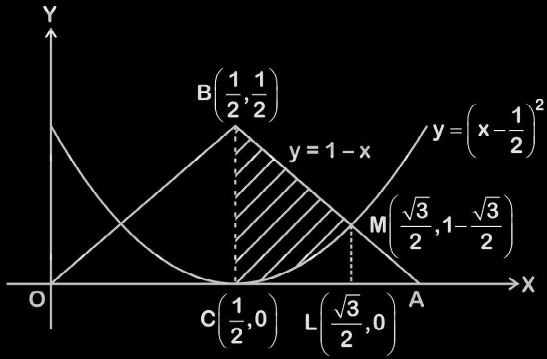
$$= 12 - (1 - 9) - \frac{1}{3}[1 + 27]$$

$$= 20 - \frac{28}{3} = 11 - \frac{1}{3} = \frac{32}{3}$$

29. Answer (1)

Required Area = Area of the Region CMBC

= Area of trapezium CLMBC – Area of the region CLMC



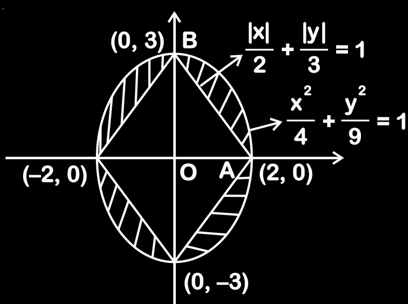
$$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \left[(1-x) - \left(x - \frac{1}{2}\right)^2\right]dx$$

$$= \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \left(\frac{3}{4} - x^2\right)dx$$

$$= \left[\frac{3}{4}x - \frac{x^3}{3}\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{3}}{4} - \frac{1}{3}$$

30. Answer (3)



$\therefore$ Required area = Area of ellipse

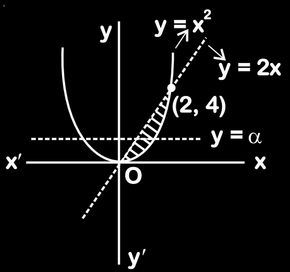
– 4 (Area of triangle OAB)

$$= \pi(2)(3) - 4\left(\frac{1}{2} \times 2 \times 3\right)$$

$= 6\pi - 12 = 6(\pi - 2)$ sq.units

31. Answer (3)

According to given condition



$$\therefore \quad \int_0^{\alpha}\left(\sqrt{y}-\frac{y}{2}\right)dy=\int_{\alpha}^{4}\left(\sqrt{y}-\frac{y}{2}\right)dy$$

$$\left[\frac{y^{3/2}}{\frac{3}{2}}\right]_0^{\alpha}-\left[\frac{y^2}{4}\right]_0^{\alpha}=\left[\frac{y^{3/2}}{\frac{3}{2}}\right]_{\alpha}^{4}-\left[\frac{y^2}{4}\right]_{\alpha}^{4}$$

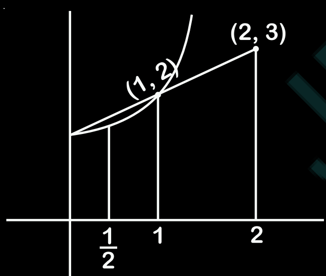
$$\frac{2}{3}\alpha^{3/2}-\frac{\alpha^2}{4}=\frac{2}{3}\left(8-\alpha^{3/2}\right)-4+\frac{\alpha^2}{4}$$

$$\frac{4}{3}\alpha^{3/2}-\frac{\alpha^2}{2}=\frac{4}{3}$$

$$\therefore \quad 8\alpha^{3/2}-3\alpha^2=8$$

$$\therefore \quad 3\alpha^2-8\alpha^{3/2}+8=0$$

32. Answer (3)



Required area = $\int_{\frac{1}{2}}^{1}(x^2+1)dx+\int_{1}^{2}(x+1)dx$

$$=\left[\frac{x^3}{3}+x\right]_{\frac{1}{2}}^{1}+\left.\frac{(x+1)^2}{2}\right|_1^2$$

$$=\left[\frac{4}{3}-\frac{13}{24}\right]+\frac{5}{2}$$

$$=\frac{79}{24}$$

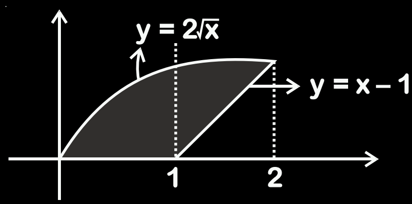
33. Answer (3)

If $x\in(0,1)$ we have [x] = 0

$0\le y\le 2\sqrt{x}$

& if $x\in(1,2)$ we have [x] = 1

$(x-1)\le y\le 2\sqrt{x}$



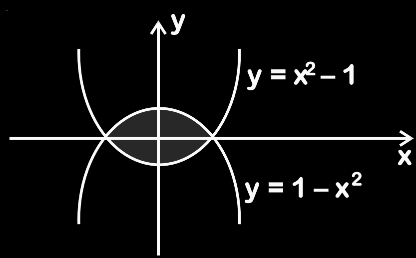
$$\therefore \quad A=\int_0^1 2\sqrt{x}dx+\int_1^2\left(2\sqrt{x}-(x-1)\right)dx$$

$$=\left.\frac{4x^{3/2}}{3}\right|_0^1+\left.\frac{4x^{3/2}}{3}\right|_1^2-\left.\frac{x^2}{2}\right|_1^2+\left.x\right|_1^2$$

$$\frac{4}{3}+\frac{4}{3}\left(2\sqrt{2}-1\right)-\left(2-\frac{1}{2}\right)+1=\frac{8\sqrt{2}}{3}-\frac{1}{2}$$

34. Answer (3)

Required area



$$\text{Area}=2\int_0^1\left((1-x^2)-(x^2-1)\right)dx$$

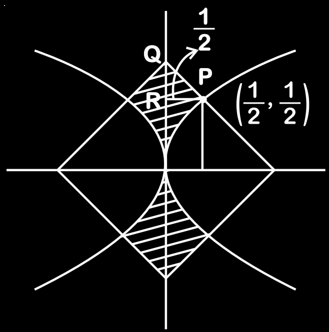
$$=4\int_0^1(1-x^2)dx$$

$$=4\left.\left(x-\frac{x^3}{3}\right)\right|_0^1=4\cdot\frac{2}{3}=\frac{8}{3}$$

35. Answer (3)

Here, $|x| + |y| \le 1,\ 2y^2 \ge |x|$

Here $P=\left(\frac{1}{2},\frac{1}{2}\right)$

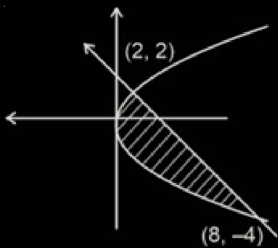


So area $= 4\left[\int_0^{\frac{1}{2}} 2y^2 dy + \frac{1}{2}\text{area }(\Delta PQR)\right]$

$$= 4\left[\frac{2}{3}\left[y^3\right]_0^{\frac{1}{2}} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right]$$

$$= 4\left[\frac{2}{3}\times\frac{1}{8}+\frac{1}{8}\right] = 4\times\frac{5}{24} = \frac{5}{6}$$

36. Answer (18)



The required area $= \int_{-4}^{2}\left(4 - y - \frac{y^2}{2}\right)dy$

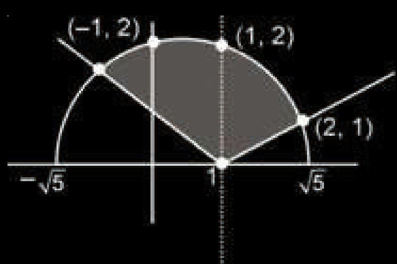
$$= \left[4y - \frac{y^2}{2} - \frac{y^3}{6}\right]_{-4}^{2}$$

= 18 square units

37. Answer (4)

$$A = \int_{-1}^{1}\left(\sqrt{5-x^2} - (1-x)\right)dx$$

$$+\int_{1}^{2}\left(\sqrt{5-x^2} - (x-1)\right)dx$$

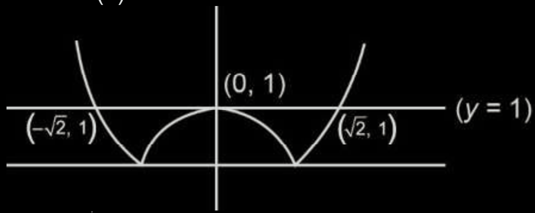


$$A = 2\left(\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right) - 2x\Big|_0^1$$

$$+\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}} - \frac{x^2}{2} + x\Big|_1^2$$

$$= \left(\frac{5\pi}{4} - \frac{1}{2}\right) \text{ sq. units}$$

38. Answer (4)



$$\text{Area} = 2\int_0^{\sqrt{2}} (1 - |x^2 - 1|)\, dx$$

$$2\left[\int_0^1 \left(1 - (1 - x^2)\right)dx + \int_1^{\sqrt{2}}\left(2 - x^2\right)dx\right]$$

$$= 2\left[\left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^3}{3}\right]_1^{\sqrt{2}}\right]$$

$$= 2\left(\frac{4\sqrt{2}-4}{3}\right) = \frac{8}{3}\left(\sqrt{2}-1\right)$$

39. Answer (170)

$4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$

differentiating both sides we get

$$12x^2 - 3y^2 - 6xyy' + 12x - 5y - 5xy' - 16yy' + 9 = 0$$

$$\downarrow (-2, 3)$$

$\Rightarrow$ $48 - 27 + 36y' - 24 - 15 + 10y' - 48y' + 9 = 0$

$\Rightarrow$ $2y' = -9$

$\Rightarrow$ $m_T = \frac{-9}{2}$ & $m_N = \frac{2}{9}$

$T \equiv y - 3 = \frac{-9}{2}(x+2)$ & $N \equiv y - 3 = \frac{2}{9}(x+2)$

$\downarrow y = 0$ $\qquad\qquad$ $\downarrow y = 0$

$x = \frac{-4}{3}$ $\qquad\qquad$ $x = \frac{-31}{2}$

$\therefore$ Area $= \frac{1}{2}\times$ Base × Height

$$A = \frac{1}{2}\times\left(\frac{-4}{3}+\frac{31}{2}\right)(3) = \frac{1}{2}\left(\frac{85}{6}\right)\cdot 3 = \frac{85}{4}$$

= 8A = 170