Chapter 11

Circle

1. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for **[AIEEE-2009]**
   (1) All except one value of p
   (2) All except two values of p
   (3) Exactly one value of p
   (4) All values of p

2. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **[AIEEE-2010]**
   (1) $-85 < m < -35$ (2) $-35 < m < 15$
   (3) $15 < m < 65$ (4) $35 < m < 85$

3. The equation of the circle passing through the points (1, 0) and (0, 1) and having the smallest radius is **[AIEEE-2011]**
   (1) $x^2 + y^2 + 2x + 2y - 7 = 0$
   (2) $x^2 + y^2 + x + y - 2 = 0$
   (3) $x^2 + y^2 - 2x - 2y + 1 = 0$
   (4) $x^2 + y^2 - x - y = 0$

4. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is **[AIEEE-2012]**
   (1) 3/5 (2) 6/5
   (3) 5/3 (4) 10/3

5. The circle passing through (1, – 2) and touching the axis of x at (3, 0) also passes through the point **[JEE (Main)-2013]**
   (1) (–5, 2) (2) (2, –5)
   (3) (5, –2) (4) (–2, 5)

6. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at (0, 3) is **[JEE (Main)-2013]**
   (1) $x^2 + y^2 - 6y - 7 = 0$
   (2) $x^2 + y^2 - 6y + 7 = 0$
   (3) $x^2 + y^2 - 6y - 5 = 0$
   (4) $x^2 + y^2 - 6y + 5 = 0$

7. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to **[JEE (Main)-2014]**
   (1) $\frac{1}{2}$ (2) $\frac{1}{4}$
   (3) $\frac{\sqrt{3}}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

8. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is **[JEE (Main)-2015]**
   (1) 1 (2) 2
   (3) 3 (4) 4

9. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on **[JEE (Main)-2016]**
   (1) An ellipse which is not a circle
   (2) A hyperbola
   (3) A parabola
   (4) A circle

10. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is **[JEE (Main)-2016]**
    (1) $5\sqrt{3}$
    (2) 5
    (3) 10
    (4) $5\sqrt{2}$

11. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is **[JEE (Main)-2017]**
    (1) $2(\sqrt{2}-1)$ (2) $4(\sqrt{2}-1)$
    (3) $4(\sqrt{2}+1)$ (4) $2(\sqrt{2}+1)$

12. Let the orthocentre and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is **[JEE (Main)-2018]**

(1) $\sqrt{10}$ (2) $2\sqrt{10}$

(3) $3\sqrt{\dfrac{5}{2}}$ (4) $\dfrac{3\sqrt{5}}{2}$

13. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is **[JEE (Main)-2018]**

(1) 195 (2) 185

(3) 85 (4) 95

14. Three circles of radii a, b, c $(a < b < c)$ touch each other externally. If they have x-axis as a common tangent, then **[JEE (Main)-2019]**

(1) a, b, c are in A.P.

(2) $\dfrac{1}{\sqrt{a}} = \dfrac{1}{\sqrt{b}} + \dfrac{1}{\sqrt{c}}$

(3) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

(4) $\dfrac{1}{\sqrt{b}} = \dfrac{1}{\sqrt{a}} + \dfrac{1}{\sqrt{c}}$

15. If the circles $x^2 + y^2 - 16x - 20y + 164 = r^2$ and $(x - 4)^2 + (y - 7)^2 = 36$ intersect at two distinct points, then **[JEE (Main)-2019]**

(1) $1 < r < 11$

(2) $r > 11$

(3) $r = 11$

(4) $0 < r < 1$

16. If a circle C passing through the point (4, 0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is

[JEE (Main)-2019]

(1) 5 (2) $2\sqrt{5}$

(3) $\sqrt{57}$ (4) 4

17. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units

[JEE (Main)-2019]

(1) 13 (2) 25

(3) – 25 (4) 20

18. Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is

[JEE (Main)-2019]

(1) 1 (2) $\sqrt{2}$

(3) $2\sqrt{2}$ (4) 2

19. A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the coordinate axes. Then the distance of the vertex of this square which is nearest to the origin is

[JEE (Main)-2019]

(1) 6 (2) $\sqrt{41}$

(3) 13 (4) $\sqrt{137}$

20. A circle cuts a chord of length $4a$ on the x-axis and passes through a point on the y-axis, distant $2b$ from the origin. Then the locus of the centre of this circle, is **[JEE (Main)-2019]**

(1) A hyperbola

(2) A parabola

(3) An ellipse

(4) A straight line

21. If a variable line, $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval

[JEE (Main)-2019]

(1) (2, 17) (2) (12, 21)

(3) (13, 23) (4) (23, 31)

22. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is **[JEE (Main)-2019]**

(1) $(x^2 + y^2)^2 = 4Rx^2y^2$

(2) $(x^2 + y^2)^2 = 4R^2x^2y^2$

(3) $(x^2 + y^2)^3 = 4R^2x^2y^2$

(4) $(x^2 + y^2)(x + y) = R^2xy$

23. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is **[JEE (Main)-2019]**

(1) 105 (2) 160

(3) 320 (4) 210

24. The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is **[JEE (Main)-2019]**

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{4}{\sqrt{3}}$

(3) $\frac{1}{3}$ (4) $\frac{1}{\sqrt{3}}$

25. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is

[JEE (Main)-2019]

(1) $x^2 + y^2 - 16x^2y^2 = 0$

(2) $x^2 + y^2 - 2x^2y^2 = 0$

(3) $x^2 + y^2 - 4x^2y^2 = 0$

(4) $x^2 + y^2 - 2xy = 0$

26. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point **[JEE (Main)-2019]**

(1) (–6, 4) (2) (–4, 6)

(3) (4, –2) (4) (6, –2)

27. The line $x = y$ touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is **[JEE (Main)-2019]**

(1) $3\sqrt{2}$ (2) 2

(3) $2\sqrt{2}$ (4) 3

28. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2 + y^2) + 2Kx + 3y - 1 = 0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q, for

[JEE (Main)-2019]

(1) Exactly one value of K

(2) Infinitely many values of K

(3) Exactly two values of K

(4) No value of K

29. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is

[JEE (Main)-2019]

(1) $y = \sqrt{1+2x}, x \geq 0$ (2) $x = \sqrt{1+4y}, y \geq 0$

(3) $x = \sqrt{1+2y}, y \geq 0$ (4) $y = \sqrt{1+4x}, x \geq 0$

30. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point **[JEE (Main)-2019]**

(1) (2, 3) (2) (1, 5)

(3) (3, 5) (4) (3, 10)

31. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to

[JEE (Main)-2020]

(1) $\frac{64}{5}$ (2) $\frac{52}{5}$

(3) $\frac{56}{5}$ (4) $\frac{32}{5}$

32. If a line, $y = mx + c$ is a tangent to the circle, $(x - 3)^2 + y^2 = 1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2 + y^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; then

[JEE (Main)-2020]

(1) $c^2 + 6c + 7 = 0$ (2) $c^2 - 7c + 6 = 0$

(3) $c^2 + 7c + 6 = 0$ (4) $c^2 - 6c + 7 = 0$

33. A circle touches the y-axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle?

[JEE (Main)-2020]

(1) $3x - 4y - 24 = 0$ (2) $4x - 3y + 17 = 0$

(3) $4x + 3y - 8 = 0$ (4) $3x + 4y - 6 = 0$

34. The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point **[JEE (Main)-2020]**

(1) (–3, 6) (2) (–1, 3)

(3) (–3, 1) (4) (1, –3)

35. If the length of the chord of the circle, $x^2 + y^2 = r^2$ $(r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to **[JEE (Main)-2020]**

(1) $\frac{9}{5}$ (2) $\frac{24}{5}$

(3) $\frac{12}{5}$ (4) 12

36. The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is ______. **[JEE (Main)-2020]**

37. The diameter of the circle, whose centre lies on the line $x + y = 2$ in the first quadrant and which touches both the lines $x = 3$ and $y = 2$, is ______.

[JEE (Main)-2020]

38. Let PQ be a diameter of the circle $x^2 + y^2 = 9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x + y = 2$ respectively, then the maximum value of $\alpha\beta$ is ______.

[JEE (Main)-2020]

39. Let C_1 and C_2 be the centres of the circles $x^2 + y^2 - 2x - 2y - 2 = 0$ and $x^2 + y^2 - 6x - 6y + 14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is

[JEE (Main)-2019]

(1) 4 (2) 9

(3) 6 (4) 8

40. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (–5, 0). If the locus of the point P is a circle of radius r, then $[4r^2]$ is equal to _________ (where $[\cdot]$ represents g.i.f).

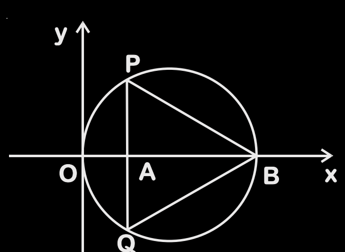
[JEE (Main)-2021]

41. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x - 2)^2 + (y - 3)^2 = 25$ at the point (5, 7) is A, then 24A is equal to _________.

[JEE (Main)-2021]

42. In the circle given below, let OA = 1 unit, OB = 13 unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is :

[JEE (Main)-2021]



(1) $24\sqrt{2}$ (2) $24\sqrt{3}$

(3) $26\sqrt{3}$ (4) $26\sqrt{2}$

43. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, –3) and $(4, -2\sqrt{2})$, and given that $a - 2\sqrt{2}\,b = 3$, then $(a^2 + b^2 + ab)$ is equal to ________. **[JEE (Main)-2021]**

44. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to : **[JEE (Main)-2021]**

(1) $\sqrt{11}$ (2) $\sqrt{7}$

(3) $\sqrt{6}$ (4) $\sqrt{10}$

45. Choose the incorrect statement about the two circles whose equations are given below :

[JEE (Main)-2021]

$x^2 + y^2 - 10x - 10y + 41 = 0$ and

$x^2 + y^2 - 16x - 10y + 80 = 0$

(1) Distance between two centres is the average of radii of both the circles

(2) Circles have two intersection points

(3) Both circles pass through the centre of the each other

(4) Both circles′ centres lie inside region of one another

46. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles′ equations **[JEE (Main)-2021]**

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 24x - 10y + 160 = 0$ is _________.

47. Let the tangent to the circle $x^2 + y^2 = 25$ at the point R(3,4) meet x-axis and y-axis at points P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to :

[JEE (Main)-2021]

(1) $\frac{529}{64}$ (2) $\frac{585}{66}$

(3) $\frac{625}{72}$ (4) $\frac{125}{72}$

48. For the four circles M, N, O and P, following four equations are given :

Circle M : $x^2 + y^2 = 1$

Circle N : $x^2 + y^2 - 2x = 0$

Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P : $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a : **[JEE (Main)-2021]**

(1) Rectangle (2) Parallelogram

(3) Square (4) Rhombus

49. Choose the correct statement about two circles whose equations are given below:

[JEE (Main)-2021]

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 22x - 10y + 137 = 0$

(1) circles have only one meeting point

(2) circles have two meeting points

(3) circles have no meeting point

(4) circles have same centre

50. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x - 2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points:

[JEE (Main)-2021]

(1) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$ (2) $\left(0, \pm\sqrt{3}\right)$

(3) $\left(2, \pm\frac{3}{2}\right)$ (4) $(1, \pm 2)$

51. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point (–4, 1) and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then a + b is equal to

[JEE (Main)-2021]

(1) 3 (2) 7

(3) 11 (4) 5

52. Let the circle S : $36x^2 + 36y^2 - 108x + 120y + c = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then : **[JEE (Main)-2021]**

(1) $\frac{25}{9} < c < \frac{13}{3}$ (2) $81 < c < 156$

(3) $100 < c < 156$ (4) $100 < c < 165$

53. Let

$A = \{(x, y) \in \mathbf{R} \times \mathbf{R} \mid 2x^2 + 2y^2 - 2x - 2y = 1\}$,

$B = \{(x, y) \in \mathbf{R} \times \mathbf{R} \mid 4x^2 + 4y^2 - 16y + 7 = 0\}$ and

$C = \{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x^2 + y^2 - 4x - 2y + 5 \le r^2\}$.

Then the minimum value of |r| such that $A \cup B \subseteq C$ is equal to : **[JEE (Main)-2021]**

(1) $\frac{2+\sqrt{10}}{2}$ (2) $\frac{3+2\sqrt{5}}{2}$

(3) $1+\sqrt{5}$ (4) $\frac{3+\sqrt{10}}{2}$

54. Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the ling segments CP and CQ, then the set {P, Q} is equal to: **[JEE (Main)-2021]**

(1) $\left\{\left(2+2\sqrt{2}, 3+\sqrt{5}\right), \left(2-2\sqrt{2}, 3-\sqrt{5}\right)\right\}$

(2) {(4, 0), (0, 6)}

(3) {(–1, 5), (5, 1)}

(4) $\left\{\left(2+2\sqrt{2}, 3-\sqrt{5}\right), \left(2-2\sqrt{2}, 3+\sqrt{5}\right)\right\}$

55. Two tangents are drawn from the point P(–1, 1) to the circle $x^2 + y^2 - 2x - 6y + 6 = 0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to **[JEE (Main)-2021]**

(1) 2 (2) $(3\sqrt{2} + 2)$

(3) 4 (4) $3(\sqrt{2} - 1)$

56. Consider a circle C which touches the *y*-axis at (0, 6) and cuts off an intercept $6\sqrt{5}$ on the *x*-axis. Then the radius of the circle C is equal to

[JEE (Main)-2021]

(1) $\sqrt{53}$ (2) 9

(3) 8 (4) $\sqrt{82}$

57. The locus of a point, which moves such that the sum of squares of its distances from the points (0, 0), (1, 0), (0, 1), (1, 1) is 18 units, is a circle of diameter d. Then d^2 is equal to

[JEE (Main)-2021]

58. A circle C touches the line $x = 2y$ at the point (2, 1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is

(1) $\sqrt{285}$ (2) 15

(3) $4\sqrt{15}$ (4) $7\sqrt{5}$

[JEE (Main)-2021]

59. Let the equation $x^2 + y^2 + px + (1 - p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2$ and q is an integer$\}$ is ______.

[JEE (Main)-2021]

60. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x - 1)^2 + (y - 1)^2 = 1$ and $(x - 9)^2 + (y - 1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is_____.

[JEE (Main)-2021]

61. Let B be the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Let the tangents at two points P and Q on the circle intersect at the point $A(3, 1)$. Then $8\cdot\left(\dfrac{\text{area }\Delta APQ}{\text{area }\Delta BPQ}\right)$ is equal to _______.

[JEE (Main)-2021]

62. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is ________. **[JEE (Main)-2021]**

63. Let a circle $C : (x - h)^2 + (y - k)^2 = r^2$, $k > 0$, touch the x-axis at (1, 0). If the line $x + y = 0$ intersects the circle C at P and Q such that the length of the chord PQ is 2, then the value of $h + k + r$ is equal to ____.

[JEE (Main)-2022]

64. Let a circle C touch the lines $L_1 : 4x - 3y + K_1 = 0$ and $L_2 : 4x - 3y + K_2 = 0$, $K_1, K_2 \in \mathbf{R}$. If a line passing through the centre of the circle C intersects L_1 at (–1, 2) and L_2 at (3, –6), then the equation of the circle C is : [JEE (Main)-2022]

(1) $(x - 1)^2 + (y - 2)^2 = 4$ (2) $(x + 1)^2 + (y - 2)^2 = 4$

(3) $(x - 1)^2 + (y + 2)^2 = 16$ (4) $(x - 1)^2 + (y - 2)^2 =$

65. Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to _________. [JEE (Main)-2022]

66. A circle touches both the y-axis and the line $x + y = 0$. Then the locus of its center is [JEE (Main)-2022]

(1) $y = \sqrt{2}x$ (2) $x = \sqrt{2}y$

(3) $y^2 - x^2 = 2xy$ (4) $x^2 - y^2 = 2xy$

67. Let C be a circle passing through the points $A(2, -1)$ and $B(3, 4)$. The line segment AB is not a diameter of C. If r is the radius of C and its centre lies on the circle $(x-5)^2 + (y-1)^2 = \dfrac{13}{2}$, then r^2 is equal to : [JEE (Main)-2022]

(1) 32 (2) $\dfrac{65}{2}$

(3) $\dfrac{61}{2}$ (4) 30

68. A rectangle R with end points of one of its sides as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is $2x - y + 4 = 0$, then the area of R is ________. [JEE (Main)-2022]

69. The set of values of k, for which the circle $C : 4x^2 + 4y^2 - 12x + 8y + k = 0$ lies inside the fourth quadrant and the point $\left(1, -\dfrac{1}{3}\right)$ lies on or inside the circle C, is [JEE (Main)-2022]

(1) An empty set (2) $\left(6, \dfrac{65}{9}\right]$

(3) $\left[\dfrac{80}{9}, 10\right)$ (4) $\left(9, \dfrac{92}{9}\right]$

70. Let a circle C of radius 5 lie below the x-axis. The line $L_1 : 4x + 3y + 2 = 0$ passes through the centre P of the circle C and intersects the line $L_2 : 3x - 4y - 11 = 0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x - 12y + 51 = 0$ is _____. [JEE (Main)-2022]

71. If the tangents drawn at the points $O(0, 0)$ and $P\left(1+\sqrt{5}, 2\right)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to [JEE (Main)-2022]

(1) $\frac{3+\sqrt{5}}{2}$ (2) $\frac{4+2\sqrt{5}}{2}$

(3) $\frac{5+3\sqrt{5}}{2}$ (4) $\frac{7+3\sqrt{5}}{2}$

72. Let the lines $y+2x=\sqrt{11}+7\sqrt{7}$ and $2y+x=2\sqrt{11}+6\sqrt{7}$ be normal to a circle $C : (x - h)^2 + (y - k)^2 = r^2$. If the line $\sqrt{11}y-3x=\frac{5\sqrt{77}}{3}+11$ is tangent to the circle C, then the value of $(5h - 8k)^2 + 5r^2$ is equal to _______.

[JEE (Main)-2022]

73. If one of the diameters of the circle $x^2+y^2-2\sqrt{2}x-6\sqrt{2}y+14 = 0$ is a chord of the circle $\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$, then the value of r^2 is equal to ________. [JEE (Main)-2022]

74. Let the tangent to the circle $C_1 : x^2 + y^2 = 2$ at the point $M(-1, 1)$ intersect the circle $C_2 : (x - 3)^2 + (y - 2)^2 = 5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to

[JEE (Main)-2022]

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) $\frac{1}{6}$ (4) $\frac{5}{3}$

75. Let the locus of the centre (α, β), $\beta > 0$, of the circle which touches the circle $x^2 + (y - 1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line $y = 4$ is :

[JEE (Main)-2022]

(1) $\frac{32\sqrt{2}}{3}$ (2) $\frac{40\sqrt{2}}{3}$

(3) $\frac{64}{3}$ (4) $\frac{32}{3}$

76. A point P moves so that the sum of squares of its distances from the points $(1, 2)$ and $(-2, 1)$ is 14. Let $f(x, y) = 0$ be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the points C, D. Then the area of the quadrilateral $ACBD$ is equal to [JEE (Main)-2022]

(1) $\frac{9}{2}$ (2) $\frac{3\sqrt{17}}{2}$

(3) $\frac{3\sqrt{17}}{4}$ (4) 9

77. If the circle $x^2 + y^2 - 2gx + 6y - 19c = 0$, $g, c \in \mathbb{R}$ passes through the point $(6, 1)$ and its centre lies on the line $x - 2cy = 8$, then the length of intercept made by the circle on x-axis is

[JEE (Main)-2022]

(1) $\sqrt{11}$ (2) 4

(3) 3 (4) $2\sqrt{23}$

78. A circle C_1 passes through the origin O and has diameter 4 on the positive x-axis. The line $y = 2x$ gives a chord OA of circle C_1. Let C_2 be the circle with OA as a diameter. If the tangent to C_2 at the point A meets the x-axis at P and y-axis at Q, then $QA : AP$ is equal to [JEE (Main)-2022]

(1) 1 : 4 (2) 1 : 5

(3) 2 : 5 (4) 1 : 3

79. For $t \in (0, 2\pi)$, if ABC is an equilateral triangle with vertices $A(\sin t, -\cos t)$, $B(\cos t, \sin t)$ and $C(a, b)$ such that its orthocentre lies on a circle with centre $\left(1, \frac{1}{3}\right)$, then $(a^2 - b^2)$ is equal to

[JEE (Main)-2022]

(1) $\frac{8}{3}$ (2) 8

(3) $\frac{77}{9}$ (4) $\frac{80}{9}$

80. Let C be the centre of the circle $x^2+y^2-x+2y=\frac{11}{4}$ and P be a point on the circle. A line passes through the point C, makes an angle of $\frac{\pi}{4}$ with the line CP and intersects the circle at the Q and R. Then the area of the triangle PQR (in unit^2) is : [JEE (Main)-2022]

(1) 2
(2) $2\sqrt{2}$
(3) $8\sin\left(\frac{\pi}{8}\right)$
(4) $8\cos\left(\frac{\pi}{8}\right)$

81. Let the tangents at two points A and B on the circle $x^2+y^2-4x+3=0$ meet at origin $O(0, 0)$. Then the area of the triangle OAB is

[JEE (Main)-2022]

(1) $\frac{3\sqrt{3}}{2}$
(2) $\frac{3\sqrt{3}}{4}$
(3) $\frac{3}{2\sqrt{3}}$
(4) $\frac{3}{4\sqrt{3}}$

82. Let AB be a chord of length 12 of the circle $(x-2)^2+(y+1)^2=\frac{169}{4}$. If tangents drawn to the circle at points A and B intersect at the point P, then five times the distance of point P from chord AB is equal to _______. [JEE (Main)-2022]

83. If the circles $x^2+y^2+6x+8y+16=0$ and $x^2+y^2+2\left(3-\sqrt{3}\right)x+2\left(4-\sqrt{6}\right)y=k+6\sqrt{3}+8\sqrt{6}$, $k>0$, touch internally at the point $P(\alpha, \beta)$, then $\left(\alpha+\sqrt{3}\right)^2+\left(\beta+\sqrt{6}\right)^2$ is equal to ______.

[JEE (Main)-2022]

84. Let the abscissae of the two points P and Q on a circle be the roots of $x^2-4x-6=0$ and the ordinates of P and Q be the roots of $y^2+2y-7=0$. If PQ is a diameter of the circle $x^2+y^2+2ax+2by+c=0$, then the value of $(a+b-c)$ is

(1) 12
(2) 13
(3) 14
(4) 16

[JEE (Main)-2022]

85. Let the mirror image of a circle $c_1 : x^2+y^2-2x-6y+\alpha=0$ in line $y=x+1$ be $c_2 : 5x^2+5y^2+10gx+10fy+38=0$. If r is the radius of circle c_2, then $\alpha+6r^2$ is equal to _________. **[JEE (Main)-2022]**

❑ ❑ ❑

Chapter 11

Circle

1. Answer (1)

$x^2 + y^2 + 3x + 7y + 2p - 5 + \lambda(x^2 + y^2 + 2x + 2y - p^2) = 0$, $\lambda \neq -1$ passes through point of intersection of given circles.

Since it passes through (1, 1), hence

$7 - 2p + \lambda(6 - p^2) = 0$

$\Rightarrow$ $7 - 2p + 6\lambda - \lambda p^2 = 0$

If $\lambda = -1$, then $7 - 2p - 6 + p^2 = 0$

$p^2 - 2p + 1 = 0$

$p = 1$

$\because$ $\lambda \neq -1$ hence $p \neq 1$

$\therefore$ All values of p are possible except $p = 1$

2. Answer (2)

Centre $\equiv (2, 4)$ $\quad r^2 = 4 + 16 + 5 = 25$

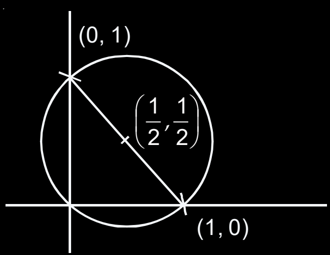
Distance of (2, 4) from $3x - 4y = m$ must be less than radius

$\therefore$ $\dfrac{|6-16-m|}{5} < 5$

$\Rightarrow$ $-25 < 10 + m < 25$

$\therefore$ $-35 < m < 15$

3. Answer (4)



Equation of a circle is

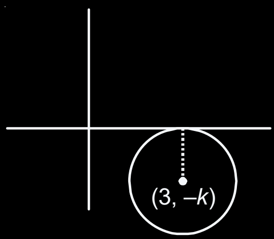
$(x-0)(x-1)+(y-1)(y-0)=0$

$\Rightarrow$ $x^2 + y^2 - x - y = 0$

4. Answer (4)

5. Answer (3)

Let the circle be $(x - 3)^2 + (y + k)^2 = k^2$



It passes through (1, –2)

$4 + (4 + k^2 - 4k) = k^2$

$\Rightarrow$ $k = 2$

$\therefore$ The circle is $(x - 3)^2 + (y + 2)^2 = 4$

Clearly the point (5, –2) lies on it.

6. Answer (1)

$\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

$9 = 16(1 - e^2)$

$e^2 = \dfrac{7}{16}$

$e = \dfrac{\sqrt{7}}{4}$

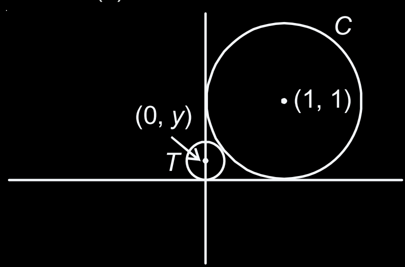
$foci \equiv (\pm\sqrt{7}, 0)$

Equation of required circle is

$(x - 0)^2 + (y - 3)^2 = 7 + 9$

$\Rightarrow$ $x^2 + y^2 - 6y - 7 = 0$

7. Answer (2)



$C \equiv (x-1)^2 + (y-1)^2 = 1$

Radius of $T = |y|$

T touches C externally

$(0 - 1)^2 + (y - 1)^2 = (1 + |y|)^2$

$\Rightarrow\ 1 + y^2 + 1 - 2y = 1 + y^2 + 2|y|$

If $y > 0$,

$y^2 + 2 - 2y = y^2 + 1 + 2y$

$\Rightarrow\ 4y = 1$

$\Rightarrow\ y = \dfrac{1}{4}$

If $y < 0$,

$y^2 + 2 - 2y = y^2 + 1 - 2y$

$\Rightarrow\ 1 = 2$ (Not possible)

$\therefore\ y = \dfrac{1}{4}$

8. Answer (3)

$x^2 + y^2 - 4x - 6y - 12 = 0$

C_1(center) = (2, 3), $r = \sqrt{2^2 + 3^2 + 12} = 5$

$x^2 + y^2 + 6x + 18y + 26 = 0$

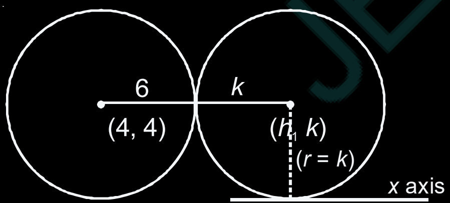
C_2(center) (– 3, –9), $r = \sqrt{9 + 81 - 26}$

$= \sqrt{64} = 8$

$C_1C_2 = 13$, $C_1C_2 = r_1 + r_2$

Number of common tangent is 3.

9. Answer (3)



Radius = $\sqrt{16 + 16 + 4} = 6$

$(6 + k)^2 = (h - 4)^2 + (k - 4)^2$

Replace $h \to x$, $k \to y$

$(y + 6)^2 - (y - 4)^2 = x^2 - 8x + 16$

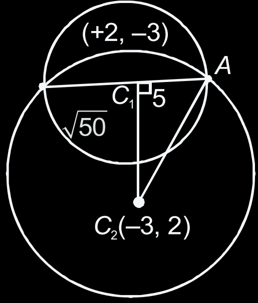
$(2y + 2)\,(10) = x^2 - 8x + 16$

$20y + 20 = x^2 - 8x + 16$

$x^2 - 8x - 20y - 4 = 0$

Centre lies on parabola

10. Answer (1)



Eq. $x^2 + y^2 - 4x + 6y - 12 = 0$

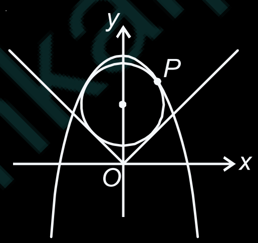
C_1; (2, –3), $r_1 = \sqrt{4 + 9 + 12} = 5$

$C_2 = (-3, 2)$

$C_1C_2 = \sqrt{5^2 + 5^2} = \sqrt{50}$

Then, $C_2A = \sqrt{5^2 + (\sqrt{50})^2} = \sqrt{75} = 5\sqrt{3}$

11. Answer (2)



$x^2 = -(y - 4)$

Let a point on the parabola $P\left(\dfrac{t}{2},\ 4 - \dfrac{t^2}{4}\right)$

Equation of normal at P is

$$y + \frac{t^2}{4} - 4 = \frac{1}{t}\left(x - \frac{t}{2}\right)$$

$$\Rightarrow\ x - ty - \frac{t^3}{4} + \frac{7}{2}t = 0$$

It passes through centre of circle, say (0, k)

$$-tk - \frac{t^3}{4} + \frac{7}{2}t = 0 \qquad \text{...(i)}$$

$t = 0$, $t^2 = 14 - 4k$

Radius = $r = \left|\dfrac{0 - k}{\sqrt{2}}\right|$

(Length of perpendicular from (0, k) to $y = x$)

$\Rightarrow\ r = \dfrac{k}{\sqrt{2}}$

Equation of circle is $x^2+(y-k)^2=\dfrac{k^2}{2}$

It passes through point P

$$\frac{t^2}{4}+\left(4-\frac{t^2}{4}-k\right)^2=\frac{k^2}{2}$$

$$t^4+t^2(8k-28)+8k^2-128k+256=0 \qquad \text{...(ii)}$$

For $t = 0 \Rightarrow \quad k^2-16k+32=0$

$$k=8\pm 4\sqrt{2}$$

$$\therefore \quad r=\frac{k}{\sqrt{2}}=4(\sqrt{2}-1) \quad (\text{discarding } 4(\sqrt{2}+1)) \text{ ...(iii)}$$

For $t=\pm\sqrt{14-4k}$

$$(14-4k)^2+(14-4k)(8k-28)+8k^2-128k+256=0$$

$$2k^2+4k-15=0$$

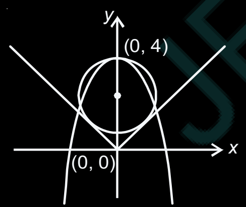
$$k=\frac{-2\pm\sqrt{34}}{2}$$

$$\therefore \quad r=\frac{k}{\sqrt{2}}=\frac{\sqrt{17}-\sqrt{2}}{2} \quad (\text{Ignoring negative value of } r) \quad \text{...(iv)}$$

From (iii) & (iv),

$$r_{\min}=\frac{\sqrt{17}-\sqrt{2}}{2}$$

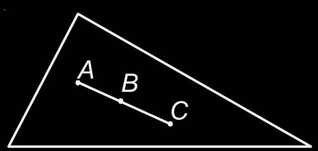
But from options, $r=4(\sqrt{2}-1)$



12. Answer (3)

A (–3, 5)

B (3, 3)



So, $AB=2\sqrt{10}$

Now, as, $AC=\dfrac{3}{2}AB$

So, radius = $\dfrac{3}{4}AB=\dfrac{3}{2}\sqrt{10}=3\sqrt{\dfrac{5}{2}}$

13. Answer (4)

Equation of tangent at (1, 7) to curve $x^2 = y - 6$ is

$$x-1=\frac{1}{2}(y+7)-6$$

$$2x - y + 5 = 0 \qquad \text{...(i)}$$

Centre of circle = (–8, –6)

Radius of circle $=\sqrt{64+36-c}=\sqrt{100-c}$

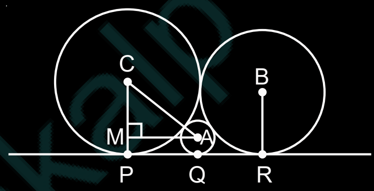
$\because$ Line (i) touches the circle

$$\therefore \quad \left|\frac{2(-8)-(-6)+5}{\sqrt{4+1}}\right|=\sqrt{100-c}$$

$$\sqrt{5}=\sqrt{100-c}$$

$\Rightarrow \quad c = 95$

14. Answer (2)



$AM^2 = AC^2 - MC^2$

$= (a + c)^2 - (a - c)^2 = 4ac$

$\because \quad AM = PQ$

$\Rightarrow PQ=2\sqrt{ac}$

Similarly, $QR=2\sqrt{ba}$ and $PR=2\sqrt{bc}$

$\Rightarrow \quad PR = PQ + QR$

$\Rightarrow 2\sqrt{bc}=2\sqrt{ac}+2\sqrt{ba}$

$$\Rightarrow \frac{1}{\sqrt{a}}=\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}$$

15. Answer (1)

$x^2 + y^2 - 16x - 20y + 164 = r^2$

i.e. $(x - 8)^2 + (y - 10)^2 = r^2$ ...(1)

and $(x - 4)^2 + (y - 7)^2 = 36$ ...(2)

Both the circles intersect each other at two distinct points.

Distance between centres

$$=\sqrt{(8-4)^2+(10-7)^2}=5$$

$\therefore \quad |r - 6| < 5 < |r + 6|$

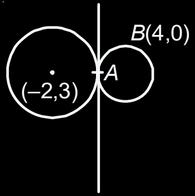
$\therefore \quad$ If $|r - 6| < 5 \Rightarrow r \in (1, 11)$ ...(3)

and $|r + 6| > 5 \Rightarrow r \in (-\infty, -11) \cup (-1, \infty)$ ...(4)

From (3) and (4),

$r \in (1, 11)$

16. Answer (1)



Let $A = (1, -1)$ & $B = (4, 0)$

Equation of tangent at A to the given circle :

$3x - 4y - 7 = 0$ ....(1)

The required circle is tangent to (1) at (1, –1).

$\therefore$ $(x-1)^2 + (y+1)^2 + \lambda(3x - 4y - 7) = 0$ ....(2)

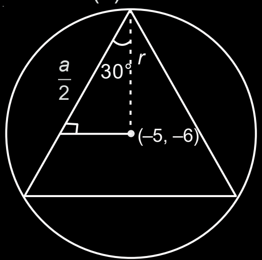
(2) Passes through $B(4, 0)$

$\Rightarrow$ $3^2 + 1^2 + \lambda(12 - 7) = 0 \Rightarrow 5\lambda + 10 = 0 \Rightarrow \lambda = -2$

(2) Becomes $x^2 + y^2 - 8x + 10y + 16 = 0$

radius $= \sqrt{(-4)^2 + (5)^2 - 16} = 5$

17. Answer (2)



Let side of equilateral Δ is a

$\Rightarrow$ $\cos 30° = \dfrac{a}{2r}$

$a = \sqrt{3}r$

$\Delta = 27\sqrt{3} = \dfrac{\sqrt{3}}{4} \cdot a^2 = \dfrac{\sqrt{3}}{4} \times 3r^2$

$r^2 = 36$

$\Rightarrow$ $r = 6$

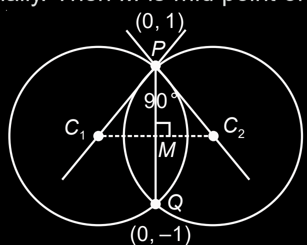
$(-5)^2 + (-6)^2 - c = 36$

$c = 25$

Option (2) is correct.

18. Answer (4)

$\because$ Two circles of equal radii intersect each other orthogonally. Then M is mid point of PQ.



and $PM = C_1M = C_2M$

$PM = \dfrac{1}{2}\sqrt{(0-0)^2 + (1+1)^2} = 1$

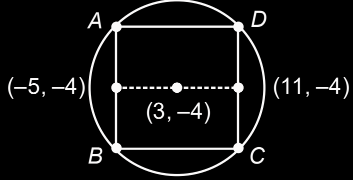
$\therefore$ Distance between centres = 1 + 1 = 2.

19. Answer (2)

$x^2 + y^2 - 6x + 8y - 103 = 0$

$C(3, -4)$, $r = 8\sqrt{2}$

$\Rightarrow$ Length of side of square $= \sqrt{2}r = 16$



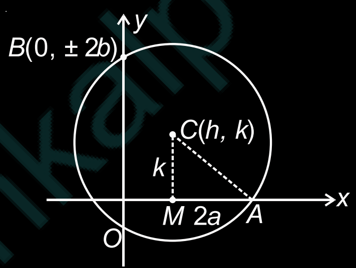
$\Rightarrow$ $A(-5, 4)$, $B(-5, -12)$

$C(11, -12)$, $D(11, 4)$

$\Rightarrow$ Required distance $= OA = \sqrt{41}$

$\Rightarrow$ Option (2) is correct.

20. Answer (2)



Let centre is $C(h, k)$

$CB = CA = r$

$\Rightarrow$ $CB^2 = CA^2$

$(h - 0)^2 + (k \pm 2b)^2 = CM^2 + MA^2$

$h^2 + (k \pm 2b)^2 = k^2 + 4a^2$

$h^2 + k^2 + 4b^2 \pm 4bk = k^2 + 4a^2$

Locus of $C(h, k)$

$x^2 + 4b^2 \pm 4by = 4a^2$

It is a parabola

Option (2) is correct.

21. Answer (2)

Condition 1: (1, 1) and (9, 1) should lie on opposite side of the line $3x + 4y - \lambda = 0$

$(7 - \lambda)(27 + 4 - \lambda) < 0$

$\Rightarrow$ $(\lambda - 7)(\lambda - 31) < 0$

$\lambda \in (7, 31)$ ...(i)

Conditon 2 : Perpendicular distance from centre on line $\geq$ radius of circle.

$\Rightarrow$ $\dfrac{|3 + 4 - \lambda|}{5} \geq 1$

$\Rightarrow$ $|\lambda - 7| \geq 5$

$\lambda \geq 12$ or $\lambda \leq 2$ ...(ii)

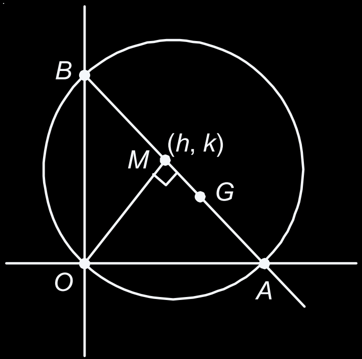
Also $\dfrac{|27+4-\lambda|}{5} \geq 2$

$\lambda \geq 41$ or $\lambda \leq 21$ ...(iii)

Intersection of (i), (ii) and (iii) gives $\lambda \in [12, 21]$

22. Answer (3)

As $\angle AOB = 90°$



$AB \to$ Diameter

$M(h, k)$ is foot of perpendicular

$$M_{AB} = \frac{-h}{k}$$

Equation of AB $(y-k) = \dfrac{-h}{k}(x-h)$

$\Rightarrow$ $hx + ky = h^2 + k^2$

$$A\left(\frac{h^2+k^2}{h}, 0\right)$$

$$B\left(0, \frac{h^2+k^2}{k}\right)$$

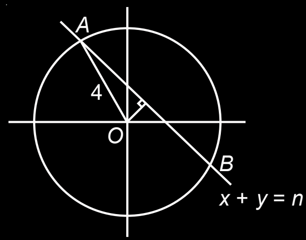
$AB = 2R$

$\Rightarrow$ $AB^2 = 4R^2$

$$\Rightarrow \left(\frac{h^2+k^2}{h}\right)^2 + \left(\frac{h^2+k^2}{k}\right)^2 = 4R^2$$

$\Rightarrow$ Locus is $(x^2 + y^2)^3 = 4R^2 x^2 y^2$

23. Answer (4)



Let the chord $x + y = n$ cuts the circle $x^2 + y^2 = 16$ at A and B length of perpendicular from O on

$$AB = \left|\frac{0+0-n}{\sqrt{1^2+1^2}}\right| = \frac{n}{\sqrt{2}}$$

$$\text{Length of chord } AB = 2\sqrt{4^2 - \left(\frac{n}{\sqrt{2}}\right)^2}$$

$$= 2\sqrt{16 - \frac{n^2}{2}}$$

Here possible values of n are 1, 2, 3, 4, 5.

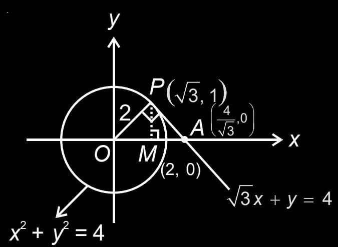
Sum of square of length of chords

$$= \sum_{n=1}^{5} 4\left(16 - \frac{n^2}{2}\right)$$

$$= 64 \times 5 - 2.\frac{5 \times 6 \times 11}{6} = 210$$

24. Answer (1)

Equation of tangent to circle at point $\left(\sqrt{3}, 1\right)$ is $\sqrt{3}x + y = 4$



$\therefore$ Coordinate of $A = \left(\dfrac{4}{\sqrt{3}}, 0\right)$

Area = $\dfrac{1}{2} \times OA \times PM$

$$= \frac{1}{2} \times \frac{4}{\sqrt{3}} \times 1 = \frac{2}{\sqrt{3}} \text{ square units}$$

25. Answer (3)

Let any tangent to circle $x^2 + y^2 = 1$ is

$x\cos\theta + y\sin\theta = 1$

$$\therefore \quad P\left(\frac{1}{\cos\theta}, 0\right); \quad Q\left(0, \frac{1}{\sin\theta}\right)$$

$$\therefore \quad \text{Mid-point of } PQ \text{ let } M\left(\frac{1}{2\cos\theta}, \frac{1}{2\sin\theta}\right) = (h, k)$$

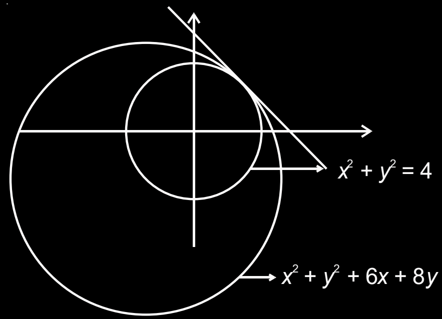
$$\Rightarrow \quad \cos\theta = \frac{1}{2h}; \qquad \sin\theta = \frac{1}{2k}$$

$\therefore$ On squaring and adding,

$$\frac{1}{h^2} + \frac{1}{k^2} = 4 \quad \Rightarrow \quad x^2 + y^2 = 4x^2y^2$$

26. Answer (4)

In given situation $d_{c_1c_2} = |r_1 - r_2|$



Common tangent

$S_1 - S_2 = 0$

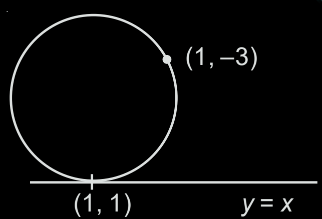
$6x + 8y - 20 = 0 \Rightarrow 3x + 4y - 10 = 0$

Hence, (6, –2) lies on it.

27. Answer (3)

Equation of circle = $(x - 1)^2 + (y - 1)^2 + \lambda(y - x) = 0$

Which passes through (1, –3)



So, $0 + 16 + \lambda(-3 - 1) = 0$

$16 + \lambda(-4) = 0$

$\boxed{\lambda = 4}$

Now equation of circle

$(x - 1)^2 + (y - 1)^2 + 4y - 4x = 0$

$\Rightarrow \quad x^2 + y^2 - 6x + 2y + 2 = 0$

radius = $\sqrt{9 + 1 - 2} = 2\sqrt{2}$

28. Answer (4)

$S_1 \equiv x^2 + y^2 + 5Kx + 2y + K = 0$

$$S_2 \equiv x^2 + y^2 + Kx + \frac{3}{2}y - \frac{1}{2} = 0$$

Equation of common chord is

$S_1 - S_2 = 0$

$$\Rightarrow \quad 4Kx + \frac{y}{2} + K + \frac{1}{2} = 0 \qquad \ldots(1)$$

$$4x + 5y - K = 0 \qquad \ldots(2) \text{ (given)}$$

On comparing (1) and (2),

$$\frac{4K}{4} = \frac{1}{10} = \frac{2K + 1}{-2K}$$

$$\Rightarrow \quad \boxed{K = \frac{1}{10}} \text{ and } -2K = 20K + 10$$

$\Rightarrow$ $22K = -10$

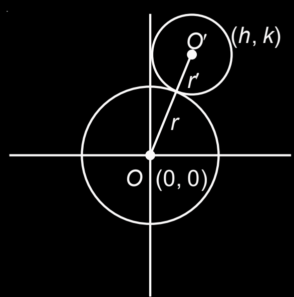
$$\boxed{K = \frac{-5}{11}}$$

$\therefore$ No value of K exists.

29. Answer (1)

Let centre of required circle is (h, k).

$\therefore \quad OO' = r + r'$



$$\Rightarrow \quad \sqrt{h^2 + k^2} = 1 + h$$

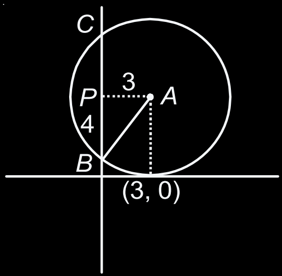
$h^2 + k^2 = 1 + h^2 + 2h$

$k^2 = 1 + 2h$

Locus is $y = \sqrt{1 + 2x}$

30. Answer (4)

Let centre of circle is A and circle cuts the y axis at B and C. Let mid point of chord BC is P.



$$AB = \sqrt{PA^2 + PB^2} = 5 = \text{radius of circle}$$

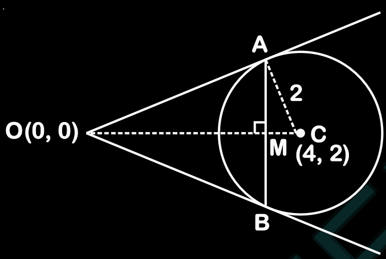
Equation of circle is : $(x - 3)^2 + (y - 5)^2 = 5^2$

Only (3, 10) satisfies this equation.

Although there will be another circle satisfying the same conditions that will lie below the x-axis having equation $(x - 3)^2 + (y - 5)^2 = 5^2$

31. Answer (1)

Equation of chord of contact is



$x \cdot 0 + y \cdot 0 - 4(x + 0) - 2(y + 0) + 16 = 0$

$\therefore\ 2x + y - 8 = 0$

$\therefore$ Length of $\text{CM} = \left[\dfrac{2 \cdot 4 + 2 - 8}{\sqrt{2^2 + 1^2}}\right] = \dfrac{2}{\sqrt{5}}$ units.

$\therefore\ AM = BM = \sqrt{4 - \dfrac{4}{5}} = \sqrt{\dfrac{16}{5}}$

$\therefore$ Length of chord of contact $(AB) = \dfrac{8}{\sqrt{5}}$.

$\therefore$ Square of length of chord of

Contact $= \left(\dfrac{8}{\sqrt{5}}\right)^2 = \dfrac{64}{5}$.

32. Answer (1)

Tangent at $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ on circle $x^2 + y^2 = 1$ is

$\boxed{x + y = \sqrt{2}}$

$\therefore$ Slope of tangent $m = 1$ for circle $(x - 3)^2 + y^2 = 1$

$\because$ Any tangent of circle $(x - 3)^2 + y^2 = 1$ is

$$y = mx - 3m \pm \sqrt{1 + m^2}$$

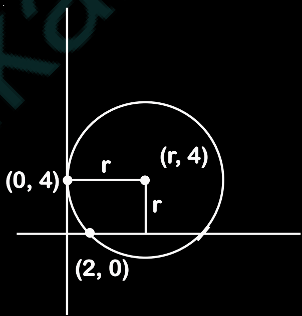
$\therefore\quad c + 3m = \pm\sqrt{1 + m^2} \quad \because m = 1$

$\Rightarrow\quad \boxed{c^2 + 6c + 7 = 0}$

33. Answer (3)

As circle touches the y-axis, let the centre be $(r, 4)$ where r is the radius of the circle

$\because\quad \sqrt{(r - 2)^2 + 4^2} = r^2$



$\Rightarrow\ r = 5$

Now if a line touches the circle then length of perpendicular from (5, 4) to that line must be equals to 5.

Only option (3) is correct

34. Answer (1)

Let circle be $S_1 + \lambda S_2 = 0$

$x^2 + y^2 - 6x + \lambda(x^2 + y^2 - 4y) = 0$

$\Rightarrow\ (1 + \lambda)x^2 + (1 + \lambda)y^2 - 6x - 4\lambda y = 0$

$$\text{Centre} \equiv \left(\frac{3}{1+\lambda}, \frac{2\lambda}{\lambda + 1}\right)$$

Centre lies on $2x - 3y + 12 = 0$ then

$$\frac{6}{\lambda + 1} - \frac{6\lambda}{\lambda + 1} + 12 = 0$$

$\Rightarrow\ \lambda = -3$

$C \equiv -2x^2 - 2y^2 - 6x + 12y = 0$

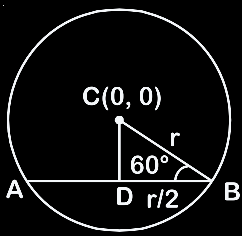
$x^2 + y^2 + 3x - 6y = 0$

It passes through $(-3, 6)$

35. Answer (3)

In right ΔCDB -

$$\sin 60^\circ = \frac{CD}{r}$$



$$\Rightarrow \quad CD = r \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}r}{2}$$

Now equation of AB is

$y - 2x - 3 = 0$

So $\dfrac{\sqrt{3}r}{2} = \dfrac{|0+0-3|}{\sqrt{5}}$

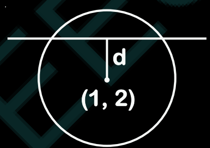
$$\Rightarrow \quad \frac{\sqrt{3}r}{2} = \frac{3}{\sqrt{5}} \Rightarrow r = \frac{2\sqrt{3}}{5} \Rightarrow r^2 = \frac{12}{5}$$

36. Answer (9)

Given circle is $(x-1)^2 + (y-2)^2 = 1$

$\Rightarrow \quad d < r$

(where r is radius of circle)



$$\Rightarrow \quad \left|\frac{3(1)+4(2)-k}{\sqrt{3^2+4^2}}\right| < 1$$

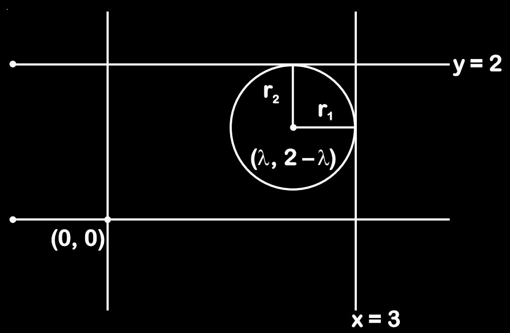
$\Rightarrow \quad |11 - k| < 5$

$\Rightarrow \quad 6 < k < 16$

$\therefore \quad k = 7, 8, ...15$

i.e. 9 values of k

37. Answer (3)



As $\underset{(r_1)}{\text{radius}} = 3 - \lambda$

Also $\underset{(r_2)}{\text{radius}} = 2 - (2 - \lambda)$

$\therefore \quad 3 - \lambda = 2 - (2 - \lambda)$

$$\Rightarrow \quad \lambda = \frac{3}{2}$$

$$r = 3 - \frac{3}{2} = \frac{3}{2}$$

Hence, diameter = 3

38. Answer (7)

Let $P(3\cos\theta, 3\sin\theta)$

$Q(-3\cos\theta, -3\sin\theta)$

$$\alpha = \left|\frac{3\cos\theta + 3\sin\theta - 2}{\sqrt{2}}\right|$$

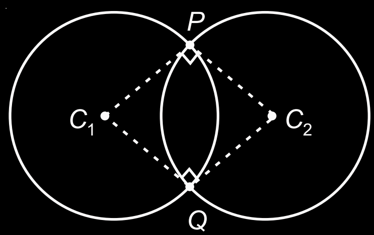
$$\beta = \left|\frac{-3\cos\theta - 3\sin\theta - 2}{\sqrt{2}}\right|$$

$$\alpha\beta = \left|\frac{(3\cos\theta + 3\sin\theta)^2 - 4}{2}\right|$$

$$= \left|\frac{5 + 9\sin 2\theta}{2}\right|$$

$\alpha\beta\,|_{max} = \dfrac{5+9}{2} = 7$ (when $\sin 2\theta = 1$)

39. Answer (1)



$2g_1g_2 + 2f_1f_2 = 2(-1)(-3) + 2(-1)(-3) = 12$

$C_1 + C_2 = 14 - 2 = 12$

As $2g_1g_2 + 2f_1f_2 = C_1 + C_2$

Hence circles intersect orthogonally

$$\therefore \quad \text{Area} = 2\left(\frac{1}{2}(C_1P)(C_2P)\right)$$

$$= 2 \times \frac{1}{2} r_1 r_2 = (2)(2)$$

$= 4$ sq. units

40. Answer (56)

Let P(x, y)

$$\sqrt{(x-5)^2+y^2}=3\sqrt{(x+5)^2+y^2}$$

$\Rightarrow x^2 + 25 - 10x + y^2 = 9(x^2 + y^2 + 10x + 25)$

$\Rightarrow 8x^2 + 8y^2 + 100x + 200 = 0$

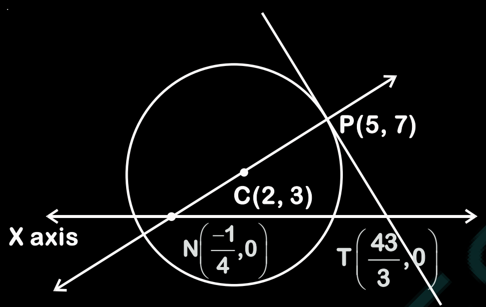
$$\Rightarrow \quad x^2+y^2+\frac{25}{2}x+25=0$$

$$r=\sqrt{\left(\frac{25}{4}\right)^2-25}=5\left(\frac{3}{4}\right)=\frac{15}{4}$$

$\Rightarrow 4r^2 = 56.25 \Rightarrow [4r^2] = 56$

41. Answer (1225)

Equation of normal PN



$$Y-7=\frac{7-3}{5-2}(x-5)$$

$4x-3y+1=0$ ...(i)

$$N\left(\frac{-1}{4},0\right)$$

Equation of Tangent PT

$3x + 4y = 43$ ...(ii)

$$T\left(\frac{43}{3},0\right)$$

$$PT=\frac{43}{3}+\frac{1}{4}=\frac{175}{12}$$

Area of triangle PNT $=\frac{1}{2}\times\frac{175}{12}\times 7=A$

$24A = 1225$

42. Answer (2)

Assume that OB is diameter of the given circle

Using Ptolemy's Theorem,

$OP\cdot QB + OQ\cdot PB = PQ \times OB$

$\Rightarrow 2OP\cdot PB = 13PQ$

Also $PA^2 = OP^2 - 1 = PB^2 - 12^2$

$\Rightarrow PB^2 - OP^2 = 143$

and $OP^2 + PB^2 = 13^2$

then $PB^2 = 156$ and $OP^2 = 13$

So, $PQ=\dfrac{2\sqrt{13}\cdot\sqrt{156}}{13}=4\sqrt{3}$

Area of $\Delta PQB=\frac{1}{2}\cdot 4\sqrt{3}\cdot 12=24\sqrt{3}$

43. Answer (9)

Clearly the curve is a circle with centre (a, b)

Centre lies on the line $x-2\sqrt{2}y=3$ ...(i)

$\because$ Circle passes through A(3, –3) and B(4, $-2\sqrt{2}$)

So centre lies on perpendicular bisector of AB, which is

$x+(3-2\sqrt{2})y=3$ ...(ii)

Clearly x = 3 and y = 0

a = 3 and b = 0

$\Rightarrow a^2 + b^2 + ab = 9$

44. Answer (3)

$2\sqrt{\left(\frac{a}{2}\right)^2-c}=2\sqrt{2}\Rightarrow a^2-4c=8$ ...(i)

$2\sqrt{a^2-c}=2\sqrt{5}\Rightarrow a^2-c=5$ ...(ii)

$\Rightarrow a = -2, c = -1$

Equation of circle

$x^2 + y^2 - 2x - 4y - 1 = 0$

$(x-1)^2+(y-2)^2=\left(\sqrt{6}\right)^2$

$x^2+y^2=\left(\sqrt{6}\right)^2$

m = 2

Tangent $y=2\times+\sqrt{6}\sqrt{1+2^2}$

$y-2=2(x-1)+\sqrt{30}$

$y=2x+\sqrt{30}\Rightarrow 2x-y+\sqrt{30}=0$

Distance from (0, 0) $\dfrac{\sqrt{30}}{\sqrt{5}}=\sqrt{6}$

45. Answer (4)

$S_1 \equiv x^2 + y^2 - 10x - 10y + 41 = 0$

Centre $C_1 \equiv (5, 5)$, radius $r_1 = 3$

$S_2 \equiv x^2 + y^2 - 16x - 10y + 80 = 0$

Centre $C_2 \equiv (8, 5)$, radius $r_2 = 3$

Distance between centres = 3

Hence both circles pass through the centre of each other, have two intersection point and distance between two centres in average of radii of both the circles.

46. Answer (1)

$S_1 \equiv x^2 + y^2 - 10x - 10y + 41 = 0$

Centre $C_1 \equiv (5, 5)$ radius $r_1 = 3$

$S_2 = x^2 + y^2 - 24x - 10y + 160$

Centre $C_2 \equiv (12, 5)$ radius = 3

Distance between centres > Sum of radii

$\Rightarrow$ Circle are separated,

Required minimum possible distance = 7 – (3 + 3) = 1

47. Answer (3)

T : 3x + 4y = 5

So, $P\left(\frac{5}{3}, 0\right)$ and $Q\left(0, \frac{5}{4}\right)$

Incentre of ΔOPQ is $\left(\frac{25}{12}, \frac{25}{12}\right)$

So, $r^2 = \left(\frac{25}{12}\right)^2 + \left(\frac{25}{12}\right)^2 = 2\left(\frac{625}{144}\right) = \frac{625}{72}$

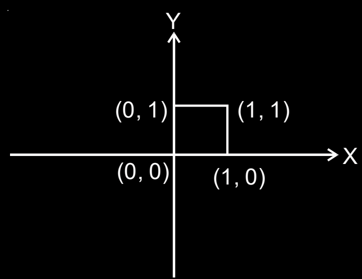
48. Answer (3)

Centre of M = (0, 0)

Centre of N = (1, 0)

Centre of O = (1, 1)

Centre of P = (0, 1)



Clearly these points form a square

(*But every square is also rectangle and parallelogram)

49. Answer (1)

$C_1 \equiv x^2 + y^2 - 10x - 10y + 41 = 0$

$\Rightarrow$ C = (5, 5) R = 3

$C_2 \equiv x^2 + y^2 - 22x - 10y + 137 = 0$

$\Rightarrow$ C = (11, 5) R = 3

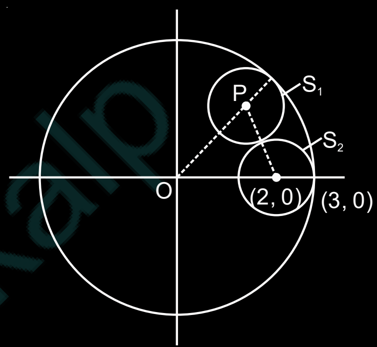
$$d_{c_1c_2} = \sqrt{(11-5)^2 + (5-5)^2} = 6$$

$$d_{c_1c_2} = r_1 + r_2$$

i.e., circles touch each other externally

50. Answer (3)

Let variable centre of required circle (S) be (x_1, y_1) and its radius be r units.



$\because$ S touches S_1 internally.

$\therefore$ OP = 3 – r

$\Rightarrow \sqrt{x_1^2 + y_1^2} = 3 - r$ ...(i)

and S touches S_2 externally.

$\therefore \sqrt{(x_1 - 2)^2 + y_1^2} = 1 + r$ ...(ii)

from eq. (i) and (ii), required locus of centre is

$$\sqrt{x^2 + y^2} + \sqrt{(x-2)^2 + y^2} = 4$$

Clearly point $\left(2, \pm\frac{3}{2}\right)$ lies on the locus.

51. Answer (4)

$C \equiv (x + 1)^2 + (y + 2)^2 = 9$

Distance between (–1, –2) and (–4, 1)

$\sqrt{3^2 + 3^2} = \sqrt{18}$

Maximum radius of required circle $= \sqrt{18} + 3$

Minimum radius of required circle $= \sqrt{18} - 3$

$$\frac{r_1}{r_2} = \frac{3\sqrt{2}+3}{3\sqrt{2}-1} = \frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{\left(\sqrt{2}+1\right)^2}{1} = 3 + 2\sqrt{2}$$

52. Answer (3)

Intersection point of $x - 2y = 4$ and $2x - y = 5$ is $(2, -1)$

$\therefore$ $36(4) + 36(1) - 108(2) + 120(-1) + C < 0$ ...(i)

and $\left(\frac{108}{72}\right)^2 + \left(\frac{-120}{72}\right)^2 - \frac{C}{36} < \frac{3}{2}$
...(ii)

(Neither touches any axis)

$\therefore$ by (i) $C < 156$

and by (ii) $\frac{9}{4} + \frac{25}{9} - \frac{C}{36} < \frac{9}{4}$

$\Rightarrow$ $100 < C$

53. Answer (2)

$A \equiv$ circle of centre $\left(\frac{1}{2}, \frac{1}{2}\right)$ and radius 1

$B \equiv$ circle of centre $(0, 2)$ and radius $\frac{3}{2}$

C is circular disc of centre (2, 1) and radius r

for C to be superset of $A \cup B$

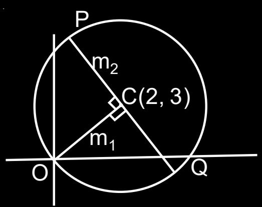
Distance of centre of C from farthest points on A and B both shall be less than radius of C i.e.

$$\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} + 1 \le r \text{ and } \sqrt{2^2 + 1^2} + \frac{3}{2} \le r$$

$$r \ge \frac{3 + 2\sqrt{5}}{2}$$

54. Answer (3)

PQ is a straight line and PQ is a diameter



$$m_1 = \frac{3}{2}$$

$$m_2 = \frac{-2}{3} = \tan\theta$$

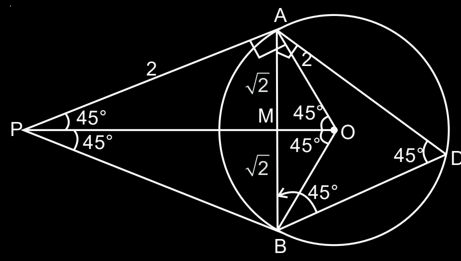
$$\sin\theta = \frac{2}{\sqrt{13}}, \cos\theta = \frac{-3}{\sqrt{13}}$$

$$P(2 + r\cos\theta, 3 + r\sin\theta), r = \sqrt{13}$$

$$Q(2 + r\cos\theta, 3 + r\sin\theta), r = -\sqrt{13}$$

$P \equiv (-1, 5)$, $Q \equiv (5, 1)$

55. Answer (3)



$$PA = \sqrt{1 + 1 + 2 - 6 + 6} = 2$$

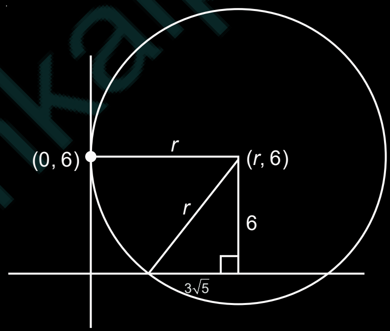
$$MA = PA\sin 45° = \sqrt{2}$$

$$AB = 2\sqrt{2}$$

$$AD = 2\sqrt{2}$$

$$[ABD] = \frac{1}{2} \times \left(2\sqrt{2} \times 2\sqrt{2}\right) = 4$$

56. Answer (2)



$$r^2 = 6^2 + \left(3\sqrt{5}\right)^2 = 81$$

$r = 9$

57. Answer (16)

Let point $P(h, k)$

$A(0, 0)$, $B(1, 0)$, $C(0, 1)$, $d(1, 1)$

$(PA)^2 + (PB)^2 + (PC)^2 + (PD)^2 = 18$

$h^2 + k^2 + (h - 1)^2 + k^2 + h^2 + (k - 1)^2 + (h - 1)^2 + (k - 1)^2 = 18$

$4h^2 + 4k^2 - 4h - 4k = 14$

$h^2 + k^2 - h - k - \frac{7}{2} = 0$

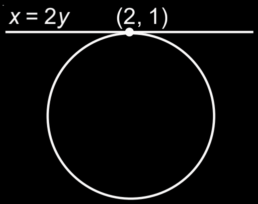
$$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \frac{7}{2}}$$

$$r = \sqrt{\frac{1 + 1 + 14}{4}} = 2$$

$d = 4$

$d^2 = 16$

58. Answer (4)



Equation of C,

$(x-2)^2 + (y-1)^2 + \lambda(x-2y) = 0$

C_1: $x^2 + y^2 + 2y - 5 = 0$ has centre (0, –1)

PQ: $C - C_1 = 0$

$\Rightarrow$ PQ : $x(\lambda - 4) + y(-2\lambda - 4) + 10 = 0$

$\because$ (0, –1) lies on PQ, then $\lambda = -7$

Diameter of $C = 2\sqrt{\dfrac{11^2+12^2}{4} - 5} = \sqrt{245} = 7\sqrt{5}$

59. Answer (61)

$$r^2 = \frac{p^2}{4} + \frac{(1-p)^2}{4} - 5$$

$\Rightarrow 0 < p^2 + (1-p)^2 - 20 \le 100$

$20 < p^2 + (1-p)^2 \le 120$

$$p \in \left(\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2}\right) \cup \left(\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2}\right)$$

$p^2 \in [7, 67]$

Number of integral values = 61

60. Answer (165)

$C_1 \equiv (1, 1)$ and $r_1 = 1$

$C_1 \equiv (9, 1)$ and $r_2 = 2$

$L \equiv 3x + 4y - \alpha - 0$

Distance of line from C_1 should be greater than r_i (i = 1, 2)

$\Rightarrow \left|\dfrac{7-\alpha}{5}\right| > 1$

$\Rightarrow |\alpha - 7| > 5$

$\Rightarrow \alpha \in (-\infty, 2) \cup (12, \infty)$ ...(i)

Also, $\left|\dfrac{27+4-\alpha}{5}\right| > 2 \Rightarrow |\alpha - 31| > 10$

$\Rightarrow \alpha \in (-\infty, 21) \cup (41, \infty)$ ...(ii)

Further C_1 and C_2 should lie on opposite sides.

w.r.t. given lines

$\Rightarrow (3+4-\alpha)\cdot(27+4-\alpha) < 0$

$\Rightarrow (\alpha - 7)(\alpha - 31) < 0$

$\Rightarrow \alpha \in (7, 31)$ ...(iii)

From (i), (ii) and (iii)

$\alpha \in (12, 21)$

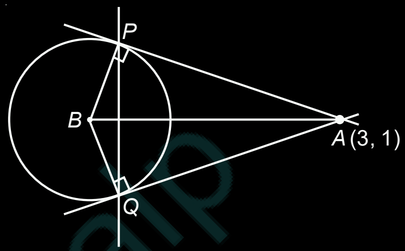
Sum of all the integral values of α.

= 12 + 13 + 14 .... + 21

$= \dfrac{21\times 22}{2} - \dfrac{11\times 12}{2}$

= 165

61. Answer (18)



Let L = length of tangent from A to the circle

& R = radius of circle

$\angle PAB = \angle BPQ = \theta$

$\Rightarrow$ area of ΔPAQ

$= 2.\dfrac{1}{2}\cdot L\sin\theta.\ L\cos\theta = L^2\cdot\sin\theta\cos\theta$

area of $\Delta PBQ = 2\cdot\dfrac{1}{2}\cdot R\sin\theta\cdot R\cos\theta = R^2\cdot\sin\theta.\cos\theta$

Hence $\dfrac{\text{area of } \Delta APQ}{\text{area of } \Delta BPQ} = \dfrac{L^2}{R^2}$

Now, $L = \sqrt{S_1}, = \sqrt{3^2 + 1^2 - 2\times 3 + 4\times 1 + 1} = 3$

& R = 2

$\Rightarrow 8\times\left(\dfrac{\text{area of } \Delta APQ}{\text{area of } \Delta BPQ}\right) = 8\times\left(\dfrac{3}{2}\right)^2 = 18$

62. Answer (3)

Circle $x^2 + y^2 - 2x - 6y + 6 = 0$ has centre O, (1, 3) and radius $r_1 = 2$.

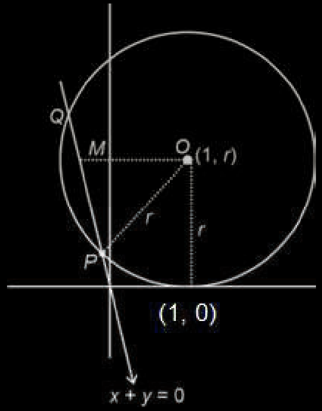
Let centre O_2 (2, 1) of required circle and its radius being R.

So $R^2 = O_1O_2{}^2 = r^2$

$\Rightarrow R^2 = 5 + 4$

$\Rightarrow R = 3$

63. Answer (7)



Here, $OM^2 = OP^2 - PM^2$

$$\left(\frac{|1+r|}{\sqrt{2}}\right)^2 = r^2 - 1$$

$\therefore$ $r^2 - 2r - 3 = 0$

$\therefore$ $r = 3$

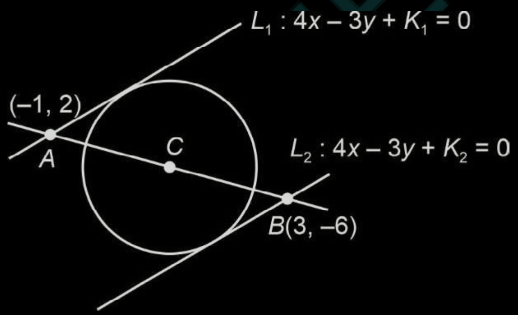
$\therefore$ Equation of circle is

$$(x-1)^2 + (y-3)^2 = 3^2$$

$\therefore$ $h = 1$, $k = 3$, $r = 3$

$\therefore$ $h + k + r = 7$

64. Answer (3)



Co-ordinate of centre

$$C \equiv \left(\frac{3+(-1)}{2}, \frac{-6+2}{2}\right) \equiv (1, -2)$$

L_1 is passing through A

$\Rightarrow$ $-4 - 6 + K_1 = 0$

$\Rightarrow$ $K_1 = 10$

L_2 is passing through B

$\Rightarrow$ $12 + 18 + K_2 = 0$

$\Rightarrow$ $K_2 = -30$

Equation of L_1 : $4x - 3y + 10 = 0$

Equation of L_1 : $4x - 3y - 30 = 0$

$$\text{Diameter of circle} = \left|\frac{10+30}{\sqrt{4^2+(-3)^2}}\right| = 8$$

$\Rightarrow$ Radius = 4

Equation of circle $(x - 1)^2 + (y + 2)^2 = 16$

65. Answer (7)

Let $P(x_1, y_1)$ & $Q(x_2, y_2)$

$\Rightarrow$ $2x^2 - rx + p = 0 \begin{cases} x_1 \\ x_2 \end{cases}$

& $x^2 - sx - q = 0 \begin{cases} y_1 \\ y_2 \end{cases}$

$\therefore$ Equation of circle $\equiv (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$\Rightarrow$ $x^2 - (x_1 + x_2)x + x_1x_2 + y^2 - (y_1 + y_2)y + y_1y_2 = 0$

$\Rightarrow$ $x^2 - \frac{r}{2}x + \frac{p}{2} + y^2 + sy - q = 0$

$\Rightarrow$ $2x^2 + 2y^2 - rx + 2sy + p - 2q = 0$

Compare with $2x^2 + 2y^2 - 11x - 14y - 22 = 0$

We get $r = 11$, $s = 7$, $p - 2q = -22$

$\Rightarrow$ $2r + s + p - 2q = 22 + 7 - 22 = 7$

66. Answer (4)

Let the centre be (h, k)

So, $|h| = \left|\frac{h+k}{\sqrt{2}}\right|$

$\Rightarrow$ $2h^2 = h^2 + k^2 + 2hk$

Locus will be $x^2 - y^2 = 2xy$

67. Answer (2)

Equation of perpendicular bisector of AB is

$$y-\frac{3}{2}=-\frac{1}{5}\left(x-\frac{5}{2}\right)\Rightarrow x+5y=10$$

Solving it with equation of given circle,

$$(x-5)^2+\left(\frac{10-x}{5}-1\right)^2=\frac{13}{2}$$

$$\Rightarrow (x-5)^2\left(1+\frac{1}{25}\right)=\frac{13}{2}$$

$$\Rightarrow x-5=\pm\frac{5}{2}\Rightarrow x=\frac{5}{2}\text{ or }\frac{15}{2}$$

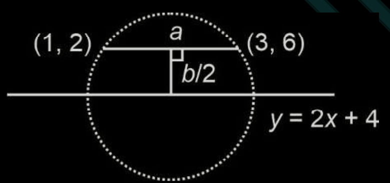
But $x\neq\frac{5}{2}$ because AB is not the diameter.

So, centre will be $\left(\frac{15}{2},\frac{1}{2}\right)$

Now $r^2=\left(\frac{15}{2}-2\right)^2+\left(\frac{1}{2}+1\right)^2$

$$=\frac{65}{2}$$

68. Answer (16)



As slope of line joining (1, 2) and (3, 6) is 2 given diameter is parallel to side

$$\therefore\quad a=\sqrt{(3-1)^2+(6-2)^2}=\sqrt{20}$$

and $b/2=\frac{4}{\sqrt{5}}\Rightarrow b=\frac{8}{\sqrt{5}}$

Area $= ab=2\sqrt{5}\cdot\frac{8}{\sqrt{5}}=16$.

69. Answer (4)

$$C:4x^2+4y^2-12x+8y+k=0$$

$\because\ \left(1,-\frac{1}{3}\right)$ lies on or inside the C

then $4+\frac{4}{9}-12-\frac{8}{3}+k\le 0$

$$\Rightarrow\quad k\le\frac{92}{9}$$

Now, circle lies in 4th quadrant centre $\equiv\left(\frac{3}{2},-1\right)$

$$\therefore\quad r<1\Rightarrow\sqrt{\frac{9}{4}+1-\frac{k}{4}}<1$$

$$\Rightarrow\frac{13}{4}-\frac{k}{4}<1$$

$$\Rightarrow\frac{k}{4}>\frac{9}{4}$$

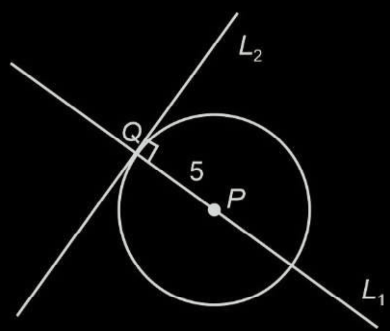
$$\Rightarrow k>9$$

$$\therefore\quad k\in\left(9,\frac{92}{9}\right)$$

70. Answer (11)

$$L_1:4x+3y+2=0$$

$$L_2:3x-4y-11=0$$



Since circle C touches the line L_2 at Q intersection point Q of L_1 and L_2, is (1, –2)

$\because$ P lies of L_1

$\therefore$ $P\left(x, -\frac{1}{3}(2+4x)\right)$

Now, $PQ = 5 \Rightarrow (x-1)^2 + \left(\frac{4x+2}{3} - 2\right)^2 = 25$

$\Rightarrow$ $(x-1)^2\left[1+\frac{16}{9}\right] = 25$

$\Rightarrow$ $(x-1)^2 = 9$

$\Rightarrow$ $x = 4, -2$

$\because$ Circle lies below the x-axis

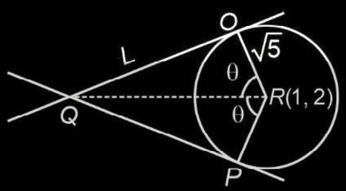
$\therefore$ $y = -6$

$P(4, -6)$

Now distance of P from $5x - 12y + 51 = 0$

$= \left|\frac{20+72+51}{13}\right| = \frac{143}{13} = 11$

71. Answer (3)



$\frac{\sqrt{5}}{L} = \tan\theta$

$\tan 2\theta = 2 \Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = 2$

$\tan\theta = \frac{\sqrt{5}-1}{2}$ (as θ is acute)

$\text{Area} = \frac{1}{2}L^2 \sin 2\theta = \frac{1}{2}.\frac{5}{\tan^2\theta}.2\sin\theta\cos\theta$

$= \frac{5\sin\theta\cos\theta}{\sin^2\theta}.\cos^2\theta$

$= 5\cot\theta.\cos^2\theta$

$= 5.\frac{2}{\sqrt{5}-1}.\frac{1}{1+\left(\frac{\sqrt{5}-1}{2}\right)^2}$

$= \frac{10}{\sqrt{5}-1}.\frac{4}{4+6-2\sqrt{5}}$

$= \frac{40}{2\sqrt{5}(\sqrt{5}-1)^2} = \frac{4\sqrt{5}}{6-2\sqrt{5}}$

$= \frac{4\sqrt{5}(6+2\sqrt{5})}{16}$

$= \frac{\sqrt{5}(3+\sqrt{5})}{2}$

72. Answer (816)

$L_1:\ y + 2x = \sqrt{11} + 7\sqrt{7}$

$L_2:\ 2y + x = 2\sqrt{11} + 6\sqrt{7}$

Point of intersection of these two lines is centre of circle i.e. $\left(\frac{8}{3}\sqrt{7}, \sqrt{11} + \frac{5}{3}\sqrt{7}\right)$

$\perp^r$ from centre to line

$3x - \sqrt{11}y + \left(\frac{5\sqrt{77}}{3} + 11\right) = 0$ is radius of circle

$\Rightarrow$ $r = \left|\frac{8\sqrt{7} - 11 - \frac{5}{3}\sqrt{77} + \frac{5\sqrt{77}}{3} + 11}{\sqrt{20}}\right|$

$= \left|\sqrt[4]{\frac{7}{5}}\right| = \sqrt[4]{\frac{7}{5}}$ units

So $(5h - 8K)^2 + 5r^2$

$= \left(\frac{40}{3}\sqrt{7} - 8\sqrt{11} - \frac{40}{3}\sqrt{7}\right)^2 + 5.16.\frac{7}{5}$

$= 64 \times 11 + 112 = 816.$

73. Answer (10)

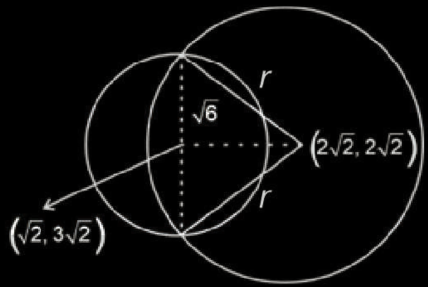
For $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$

Radius = $\sqrt{\left(\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2 - 14} = \sqrt{6}$

$\Rightarrow$ Diameter = $2\sqrt{6}$

If this diameter is chord to

$$\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2 \text{ then}$$



$$\Rightarrow \quad r^2=6+\left(\sqrt{\left(\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2}\right)^2$$

$\Rightarrow \quad r^2 = 6 + 4 = 10$

$\Rightarrow \quad r^2 = 10$

74. Answer (3)

Tangent to C_1 at M: $-x + y = 2 \equiv T$

Intersection of T with $C_2 \Rightarrow (x-3)^2 + x^2 = 5$

$\Rightarrow \quad x = 1, 2$

$A(1, 3)$ and $B(2, 4)$

Let $N \equiv (\alpha, \beta)$

Then $-x + y = 2$ shall be chord of contact for

$x^2+y^2-6x-4y+8=0$

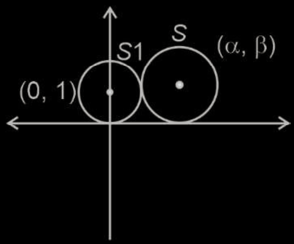
$\therefore \quad \alpha x+\beta y-3x-3\alpha-2y-2\beta+8=0$ is same as $-x + y = 2$

$$\frac{\alpha-3}{-1}=\frac{\beta-2}{1}=\frac{3\alpha-8+2\beta}{2}$$

$$\Rightarrow \quad (\alpha,\beta)\equiv\left(\frac{4}{3},\frac{11}{3}\right)$$

$$\Delta=\frac{1}{2}\begin{vmatrix}1 & 3 & 1\\ 2 & 4 & 1\\ \frac{4}{3} & \frac{11}{3} & 1\end{vmatrix}=\frac{1}{6}\text{ units}$$

75. Answer (3)



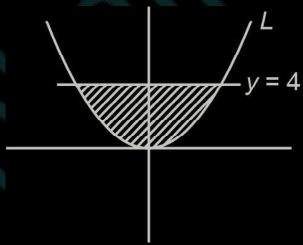
Radius of circle S touching x-axis and centre (α, β) is $|\beta|$. According to given conditions

$\alpha^2 + (\beta - 1)^2 = (|\beta| + 1)^2$

$\alpha^2 + \beta^2 - 2\beta + 1 = \beta^2 + 1 + 2|\beta|$

$\alpha^2 = 4\beta$ as $\beta > 0$

$\therefore$ Required louse is $L : x^2 = 4y$



The area of shaded region $=2\int_0^4 2\sqrt{y}\,dy$

$$=4.\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$

$$=\frac{64}{3}\text{ square units.}$$

76. Answer (2)

Let point $P : (h, k)$

$(h-1)^2 + (k-2)^2 + (h+2)^2 + (k-1)^2 = 14$

$2h^2 + 2k^2 + 2h - 6k - 4 = 0$

Locus of $P : x^2 + y^2 + x - 3y - 2 = 0$

Intersection with x-axis,

$x^2 + x - 2 = 0$

$\Rightarrow \quad x = -2, 1$

Intersection with y-axis,

$y^2 - 3y - 2 = 0$

$$\Rightarrow \quad y = \frac{3 \pm \sqrt{17}}{2}$$

Area of the quadrilateral $ACBD$ is

$$= \frac{1}{2}\left(|x_1| + |x_2|\right)\left(|y_1| + |y_2|\right)$$

$$= \frac{1}{2} \times 3 \times \sqrt{17} = \frac{3\sqrt{17}}{2}$$

77. Answer (4)

Circle : $x^2 + y^2 - 2gx + 6y - 19c = 0$

It passes through $h(6, 1)$

$\Rightarrow \quad 36 + 1 - 12g + 6 - 19c = 0$

$= 12g + 19c = 43 \qquad \ldots(1)$

Line $x - 2cy = 8$ passes though centre

$\Rightarrow \quad g + 6c = 8 \qquad \ldots(2)$

From (1) & (2)

$g = 2,\ c = 1$

$C : x^2 + y^2 - 4x + 6y - 19 = 0$

$$x\text{int} = 2\sqrt{g^2 - C} = 2\sqrt{4 + 19}$$

$$= 2\sqrt{23}$$

78. Answer (1)

Equation of C_1

$x^2 + y^2 - 4x = 0$

Intersection with

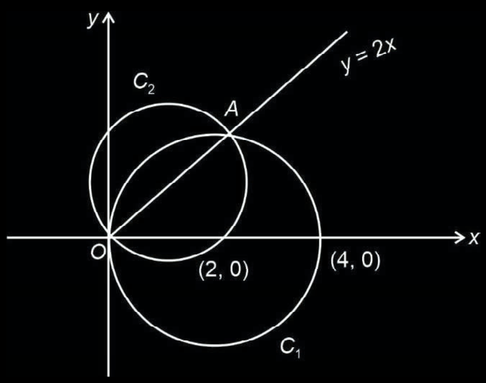
$y = 2x$

$$x^2 + 4x^2 - 4x = 0$$

$$5x^2 - 4x = 0 \quad \Rightarrow \quad x = 0, \frac{4}{5}$$

$$y = 0, \frac{8}{5}$$

$$A : \left(\frac{4}{5}, \frac{8}{5}\right)$$



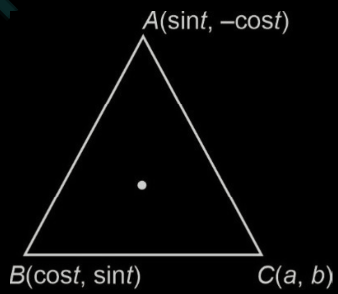
Tangent of C_2 at $A\left(\frac{4}{5}, \frac{8}{5}\right)$

$$x + 2y = 4 \Rightarrow P : (4, 0),\ Q : (0, 2)$$

$$QA : AP = 1 : 4$$

79. Answer (2)

Let $P(h, k)$ be the orthocentre of ΔABC



Then

$$h = \frac{\sin t + \cos t + a}{3}, k = \frac{-\cos t + \sin t + b}{3}$$

(orthocentre coincide with centroid)

$\therefore \quad (3h - a)^2 + (3k - b)^2 = 2$

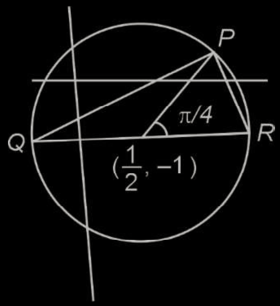
$$\therefore \quad \left(h - \frac{a}{3}\right)^2 + \left(k - \frac{b}{3}\right)^2 = \frac{2}{9}$$

$\because \quad$ orthocentre lies on circle with centre $\left(1, \frac{1}{3}\right)$

$\therefore \quad a = 3,\ b = 1$

$\therefore \quad a^2 - b^2 = 8$

80. Answer (2)



$QR = 2r = 4$

$$P = \left(\frac{1}{2} + 2\cos\frac{\pi}{4}, -1 + 2\sin\frac{\pi}{4}\right)$$

$$= \left(\frac{1}{2} + \sqrt{2}, -1 + \sqrt{2}\right)$$

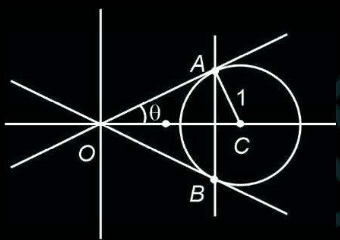
Area of $\Delta PQR = \frac{1}{2} \times 4 \times \sqrt{2}$

$= 2\sqrt{2}$ sq. units

81. Answer (2)

$x^2 + y^2 - 4x + 3 = 0$

$\Rightarrow \quad (x-2)^2 + y^2 = 1$



$$AO = \sqrt{(OC)^2 - (AC)^2}$$

$$= \sqrt{4-1} = \sqrt{3}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

Also, $AO = BO$

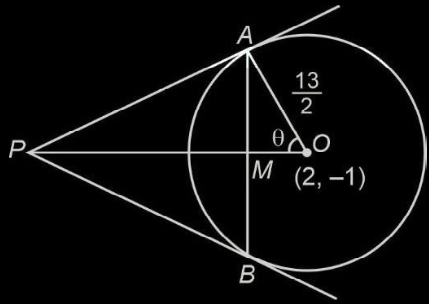
Area of $\Delta OAB = \frac{1}{2} \cdot OA \cdot OB \sin 60^\circ$

$$= \frac{1}{2} \times \sqrt{3} \cdot \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$$

82. Answer (72)

Here $AM = BM = 6$

$$OM = \sqrt{\left(\frac{13}{2}\right)^2 - 6^2} = \frac{5}{2}$$



$$\sin\theta = \frac{12}{13}$$

In ΔPAO:

$$\frac{PO}{OA} = \sec\theta$$

$$PO = \frac{13}{2} \cdot \frac{13}{5} = \frac{169}{10}$$

$$\therefore \quad PM = \frac{169}{10} - \frac{5}{2} = \frac{144}{10} = \frac{72}{5}$$

$\therefore \quad 5\,PM = 72.$

83. Answer (25)

The circle $x^2 + y^2 + 6x + 8y + 16 = 0$ has centre $(-3, -4)$ and radius 3 units.

The circle $x^2 + y^2 + 2\left(3 - \sqrt{3}\right)x + 2\left(4 - \sqrt{6}\right)y =$

$k + 6\sqrt{3} + 8\sqrt{6},\ k > 0$ has centre $\left(\sqrt{3} - 3,\ \sqrt{6} - 4\right)$

and radius $\sqrt{k + 34}$

$\because$ These two circles touch internally hence

$$\sqrt{3 + 6} = \left|\sqrt{k + 34} - 3\right|.$$

Here, $k = 2$ is only possible $\quad (\because k > 0)$

Equation of common tangent to two circles is

$$2\sqrt{3}x + 2\sqrt{6}y + 16 + 6\sqrt{3} + 8\sqrt{6} + k = 0$$

$\because$ $k = 2$ then equation is

$$x+\sqrt{2}y+3+4\sqrt{2}+3\sqrt{3}=0 \qquad \ldots(i)$$

$\because$ (α, β) are foot of perpendicular from $(-3, -4)$

To line (i) then

$$\frac{\alpha+3}{1}=\frac{\beta+4}{\sqrt{2}}=\frac{-\left(-3-4\sqrt{2}+3+4\sqrt{2}+3\sqrt{3}\right)}{1+2}$$

$$\therefore\ \alpha+3=\frac{\beta+4}{\sqrt{2}}=-\sqrt{3}$$

$$\Rightarrow \left(\alpha+\sqrt{3}\right)^2=9 \text{ and } \left(\beta+\sqrt{6}\right)^2=16$$

$$\therefore \left(\alpha+\sqrt{3}\right)^2+\left(\beta+\sqrt{6}\right)^2=25$$

84. Answer (1)

Abscissae of PQ are roots of $x^2 - 4x - 6 = 0$

Ordinates of PQ are roots of $y^2 + 2y - 7 = 0$

and PQ is diameter

$\Rightarrow$ Equation of circle is

$$x^2+y^2-4x+2y-13=0$$

But, given $x^2+y^2+2ax+2by+c=0$

By comparison $a = -2$, $b = 1$, $c = -13$

$\Rightarrow$ $a + b - c = -2 + 1 + 13 = 12$

85. Answer (12)

$c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$

Then centre = (1, 3) and radius $(r)=\sqrt{10-\alpha}$

Image of (1, 3) w.r.t. line $x - y + 1 = 0$ is (2, 2)

$c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$

or $x^2+y^2+2gx+2fy+\frac{38}{5}=0$

Then $(-g, -f) = (2, 2)$

$\therefore\ g = f = -2 \qquad \ldots(i)$

Radius of $c_2 = r = \sqrt{4+4-\frac{38}{5}}=\sqrt{10-\alpha}$

$$\Rightarrow \frac{2}{5}=10-\alpha$$

$$\therefore\ \alpha=\frac{48}{5} \text{ and } r=\sqrt{\frac{2}{5}}$$

$$\therefore\ \alpha+6r^2=\frac{48}{5}+\frac{12}{5}$$

$$= 12$$

❑ ❑ ❑