Chapter 21

Integrals

1. If the integral

$\int \frac{5\tan x}{\tan x-2}dx = x + a\ln|\sin x - 2\cos x| + k$,

then a is equal to **[AIEEE-2012]**

(1) –2 (2) 1

(3) 2 (4) –1

2. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to **[JEE (Main)-2013]**

(1) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + C$

(2) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$

(3) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$

(4) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right] + C$

3. Let the population of rabbits surviving at a time t be governed by the differential equation

$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t)$ equals

[JEE (Main)-2014]

(1) $600 - 500\, e^{t/2}$ (2) $400 - 300\, e^{-t/2}$

(3) $400 - 300\, e^{t/2}$ (4) $300 - 200\, e^{-t/2}$

4. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals

[JEE (Main)-2015]

(1) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$ (2) $(x^4+1)^{\frac{1}{4}} + c$

(3) $-(x^4+1)^{\frac{1}{4}} + c$ (4) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

5. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}$ is equal to

[JEE (Main)-2016]

(1) $\frac{x^{10}}{2(x^5+x^3+1)^2} + C$

(2) $\frac{x^5}{2(x^5+x^3+1)^2} + C$

(3) $\frac{-x^{10}}{2(x^5+x^3+1)^2} + C$

(4) $\frac{-x^5}{(x^5+x^3+1)^2} + C$

where C is an arbitrary constant.

6. Let $I_n = \int \tan^n x dx, (n > 1)$.

If $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to **[JEE (Main)-2017]**

(1) $\left(\frac{1}{5}, 0\right)$ (2) $\left(\frac{1}{5}, -1\right)$

(3) $\left(-\frac{1}{5}, 0\right)$ (4) $\left(-\frac{1}{5}, 1\right)$

7. The integral

$\int \frac{\sin^2 x\cos^2 x}{(\sin^5 x + \cos^3 x\sin^2 x + \sin^3 x\cos^2 x + \cos^5 x)^2}dx$

is equal to **[JEE (Main)-2018]**

(1) $\frac{1}{3(1+\tan^3 x)} + C$ (2) $\frac{-1}{3(1+\tan^3 x)} + C$

(3) $\frac{1}{1+\cot^3 x} + C$ (4) $\frac{-1}{1+\cot^3 x} + C$

(where C is a constant of integration)

8. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral $\int x\sqrt{\dfrac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}\,dx$ is equal to

(where c is a constant of integration)

[JEE (Main)-2019]

(1) $\frac{1}{2}\log_e\left|\sec(x^2-1)\right|+c$

(2) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(3) $\log_e\left|\cos\left(\frac{x^2-1}{2}\right)\right|+c$

(4) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$

9. If $f(x) = \int \dfrac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx, (x \geq 0)$, and $f(0) = 0$, then the value of $f(1)$ is **[JEE (Main)-2019]**

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $-\frac{1}{2}$ (4) $-\frac{1}{4}$

10. Let $n \geq 2$ be a nutural number and $0 < \theta < \pi/2$.

Then $\int \dfrac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$ is equal to

(where C is a constant of integration)

[JEE (Main)-2019]

(1) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$

(2) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(3) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(4) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

11. If $\int x^5 e^{-4x^3}dx = \frac{1}{48}e^{-4x^3}f(x)+C$, where C is a constant of integration, then $f(x)$ is equal to

[JEE (Main)-2019]

(1) $4x^3 + 1$ (2) $-4x^3 - 1$

(3) $-2x^3 + 1$ (4) $-2x^3 - 1$

12. If $\int \dfrac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function $A(x)$, where C is a constant of integration, then $(A(x))^m$ equals **[JEE (Main)-2019]**

(1) $\frac{-1}{3x^3}$ (2) $\frac{1}{27x^6}$

(3) $\frac{1}{9x^4}$ (4) $\frac{-1}{27x^9}$

13. If $\int \dfrac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1}+C$, where C is a constant of integration, then $f(x)$ is equal to

[JEE (Main)-2019]

(1) $\frac{1}{3}(x+1)$ (2) $\frac{1}{3}(x+4)$

(3) $\frac{2}{3}(x-4)$ (4) $\frac{2}{3}(x+2)$

14. The integral $\int \cos(\log_e x)\,dx$ is equal to (where C is a constant of integration)

[JEE (Main)-2019]

(1) $x\left[\cos(\log_e x) - \sin(\log_e x)\right]+C$

(2) $\frac{x}{2}\left[\cos(\log_e x) + \sin(\log_e x)\right]+C$

(3) $\frac{x}{2}\left[\sin(\log_e x) - \cos(\log_e x)\right]+C$

(4) $x\left[\cos(\log_e x) + \sin(\log_e x)\right]+C$

15. The integral $\int \dfrac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$ is equal to

(where C is a constant of integration)

[JEE (Main)-2019]

(1) $\dfrac{x^4}{6(2x^4+3x^2+1)^3}+C$

(2) $\dfrac{x^{12}}{(2x^4+3x^2+1)^3}+C$

(3) $\dfrac{x^{12}}{6(2x^4+3x^2+1)^3}+C$

(4) $\dfrac{x^4}{(2x^4+3x^2+1)^3}+C$

16. $\displaystyle\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}\,dx$ is equal to

(where c is a constant of integration)

[JEE (Main)-2019]

(1) $2x + \sin x + 2\sin 2x + c$

(2) $x + 2\sin x + 2\sin 2x + c$

(3) $x + 2\sin x + \sin 2x + c$

(4) $2x + \sin x + \sin 2x + c$

17. If $\displaystyle\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}}+C$ where C is a constant of integration, then the function $f(x)$ is equal to: **[JEE (Main)-2019]**

(1) $-\dfrac{1}{2x^3}$ (2) $\dfrac{3}{x^2}$

(3) $-\dfrac{1}{6x^3}$ (4) $-\dfrac{1}{2x^2}$

18. The integral $\int \sec^{2/3}x\,\text{cosec}^{4/3}x\,dx$ is equal to (Here C is a constant of integration)

[JEE (Main)-2019]

(1) $-3\cot^{-1/3}x + C$ (2) $-3\tan^{-1/3}x + C$

(3) $-\dfrac{3}{4}\tan^{-4/3}x + C$ (4) $3\tan^{-1/3}x + C$

19. If $\int e^{\sec x}\left(\sec x\tan x f(x)+\sec x\tan x+\sec^2 x\right)dx$

$= e^{\sec x}f(x)+C$, then a possible choice of $f(x)$ is

[JEE (Main)-2019]

(1) $\sec x-\tan x-\dfrac{1}{2}$ (2) $\sec x+\tan x+\dfrac{1}{2}$

(3) $\sec x+x\tan x-\dfrac{1}{2}$ (4) $x\sec x+\tan x+\dfrac{1}{2}$

20. If $\displaystyle\int \frac{dx}{(x^2-2x+10)^2}$

$= A\left(\tan^{-1}\left(\dfrac{x-1}{3}\right)+\dfrac{f(x)}{x^2-2x+10}\right)+C$

where C is a constant of integration, then

[JEE (Main)-2019]

(1) $A=\dfrac{1}{81}$ and $f(x)=3(x-1)$

(2) $A=\dfrac{1}{54}$ and $f(x)=3(x-1)$

(3) $A=\dfrac{1}{27}$ and $f(x)=9(x-1)$

(4) $A=\dfrac{1}{54}$ and $f(x)=9(x-1)^2$

21. If $\int x^5 e^{-x^2}dx = g(x)e^{-x^2}+c$, where c is a constant of integration, then $g(-1)$ is equal to

[JEE (Main)-2019]

(1) -1 (2) $-\dfrac{1}{2}$

(3) 1 (4) $-\dfrac{5}{2}$

22. The integral $\displaystyle\int \frac{2x^3-1}{x^4+x}\,dx$ is equal to

(Here C is a constant of integration)

[JEE (Main)-2019]

(1) $\log_e\left|\dfrac{x^3+1}{x}\right|+C$ (2) $\dfrac{1}{2}\log_e\dfrac{(x^3+1)^2}{|x^3|}+C$

(3) $\dfrac{1}{2}\log_e\dfrac{|x^3+1|}{x^2}+C$ (4) $\log_e\dfrac{|x^3+1|}{x^2}+C$

23. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral $\displaystyle\int \frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx =$ $A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$, where C is a constant of integration, then the functions $A(x)$ and $B(x)$ are respectively **[JEE (Main)-2019]**

(1) $x-\alpha$ and $\log_e|\cos(x-\alpha)|$

(2) $x+\alpha$ and $\log_e|\sin(x-\alpha)|$

(3) $x-\alpha$ and $\log_e|\sin(x-\alpha)|$

(4) $x+\alpha$ and $\log_e|\sin(x+\alpha)|$

24. If $\int \frac{\cos x dx}{\sin^3 x(1+\sin^6 x)^{2/3}} = f(x)(1+\sin^6 x)^{1/\lambda} + c$ where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{9}{8}$ (2) $-\frac{9}{8}$

(3) -2 (4) 2

25. The integral $\int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to

(where C is a constant of integration)

[JEE (Main)-2020]

(1) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{3/7} + C$ (2) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-13/7} + C$

(3) $\left(\frac{x-3}{x+4}\right)^{1/7} + C$ (4) $-\left(\frac{x-3}{x+4}\right)^{-1/7} + C$

26. If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} =$ $\lambda \tan\theta + 2\log_e|f(\theta)| + C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to **[JEE (Main)-2020]**

(1) $(1, 1 - \tan\theta)$ (2) $(-1, 1 + \tan\theta)$

(3) $(-1, 1 - \tan\theta)$ (4) $(1, 1 + \tan\theta)$

27. If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}\left(\sqrt{x}\right) + B(x) + C,$ where C is a constant of integration, then the ordered pair $(A(x), B(x))$ can be

[JEE (Main)-2020]

(1) $\left(x+1, -\sqrt{x}\right)$ (2) $\left(x+1, \sqrt{x}\right)$

(3) $\left(x-1, -\sqrt{x}\right)$ (4) $\left(x-1, \sqrt{x}\right)$

28. The integral $\int\left(\frac{x}{x\sin x + \cos x}\right)^2 dx$ is equal to (where C is a constant of integration)

[JEE (Main)-2020]

(1) $\tan x - \frac{x\sec x}{x\sin x + \cos x} + C$

(2) $\sec x - \frac{x\tan x}{x\sin x + \cos x} + C$

(3) $\tan x + \frac{x\sec x}{x\sin x + \cos x} + C$

(4) $\sec x + \frac{x\tan x}{x\sin x + \cos x} + C$

29. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2}dx \; (x \ge 0)$. Then $f(3) - f(1)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$ (2) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$

(3) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

30. If $\int\left(e^{2x} + 2e^x - e^{-x} - 1\right)e^{(e^x + e^{-x})}dx$

$= g(x)e^{(e^x+e^{-x})} + c$, where c is a constant of integration, then $g(0)$ is equal to

[JEE (Main)-2020]

(1) e^2 (2) 1

(3) 2 (4) e

31. If $\int \frac{\cos\theta}{5 + 7\sin\theta - 2\cos^2\theta}d\theta = A\log_e|B(\theta)| + C,$ where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be **[JEE (Main)-2020]**

(1) $\frac{2\sin\theta + 1}{5(\sin\theta + 3)}$ (2) $\frac{2\sin\theta + 1}{\sin\theta + 3}$

(3) $\frac{5(2\sin\theta + 1)}{\sin\theta + 3}$ (4) $\frac{5(\sin\theta + 3)}{2\sin\theta + 1}$

32. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}}dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c,$ where c is a constant of integration, then the ordered pair (a, b) is equal to :

[JEE (Main)-2021]

(1) $(-1, 3)$ (2) $(3, 1)$

(3) $(1, -3)$ (4) $(1, 3)$

33. The value of the integral

$$\int \frac{\sin\theta.\sin 2\theta\left(\sin^6\theta + \sin^4\theta + \sin^2\theta\right)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta}d\theta \text{ is :}$$

(where c is a constant of integration)

[JEE (Main)-2021]

(1) $\frac{1}{18}\left[9 - 2\cos^6\theta - 3\cos^4\theta - 6\cos^2\theta\right]^{\frac{3}{2}} + c$

(2) $\frac{1}{18}\left[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta\right]^{\frac{3}{2}} + c$

(3) $\frac{1}{18}\left[9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta\right]^{\frac{3}{2}} + c$

(4) $\frac{1}{18}\left[11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta\right]^{\frac{3}{2}} + c$

34. The integral $\int \frac{e^{3\log_e 2x}+5e^{2\log_e 2x}}{e^{4\log_e x}+5e^{3\log_e x}-7e^{2\log_e x}}dx$, $x > 0$, is equal to : **[JEE (Main)-2021]**

(1) $4\log_e\left|x^2+5x-7\right|+c$

(2) $\log_e\left|x^2+5x-7\right|+c$

(3) $\frac{1}{4}\log_e\left|x^2+5x-7\right|+c$

(4) $\log_e\sqrt{x^2+5x-7}+c$

35. For real numbers α, β, γ and δ, if

$$\int \frac{(x^2-1)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$=\alpha\log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)+\beta\tan^{-1}\left(\frac{\gamma(x^2-1)}{x}\right)+\delta\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

Where C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to ________.

[JEE (Main)-2021]

36. The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}dx$ is equal to :

(where c is a constant of integration)

[JEE (Main)-2021]

(1) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$

(2) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

(3) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$

(4) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$

37. If $f(x)=\int \frac{5x^8+7x^6}{(x^2+1+2x^7)^2}dx, (x \ge 0), f(0)=0$ and $f(1)=\frac{1}{K}$, then the value of K is ________.

[JEE (Main)-2021]

38. If $\int \frac{dx}{(x^2+x+1)^2}=a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+b\left(\frac{2x+1}{x^2+x+1}\right)+C$, $x > 0$ where C is the constant of integration, then the value of $9\left(\sqrt{3}a+b\right)$ is equal to ______.

[JEE (Main)-2021]

39. If $\int \frac{2e^x+3e^{-x}}{4e^x+7e^{-x}}dx=\frac{1}{14}(ux + v\log_e(4e^x + 7e^{-x})) +$ C, where C is a constant of integration, then $u + v$ is equal to ______.

[JEE (Main)-2021]

40. The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}}+C$ (2) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}}+C$

(3) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}}+C$ (4) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$

[JEE (Main)-2021]

41. If $\int \frac{\sin x}{\sin^3 x+\cos^3 x}dx=\alpha\log_e|1+\tan x|+\beta\log_e|1-\tan x+\tan^2 x|+\gamma\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$, when C is constant of integration, then the value of $18(\alpha + \beta + \gamma^2)$ is ________.

[JEE (Main)-2021]

42. If $\cos x\frac{dy}{dx} - y\sin x = 6x$, $\left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right)=0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

[JEE (Main)-2021]

(1) $-\frac{\pi^2}{2}$ (2) $-\frac{\pi^2}{4\sqrt{3}}$

(3) $\frac{\pi^2}{2\sqrt{3}}$ (4) $-\frac{\pi^2}{2\sqrt{3}}$

43. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to

[JEE (Main)-2021]

(1) $(x+1)e^{x+\frac{1}{x}}+c$ (2) $-xe^{x+\frac{1}{x}}+c$

(3) $(x-1)e^{x+\frac{1}{x}}+c$ (4) $xe^{x+\frac{1}{x}}+c$

44. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x)+c,\ g(1)=0,$ then $g\left(\frac{1}{2}\right)$ is equal to

[JEE (Main)-2022]

(1) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)+\frac{\pi}{3}$ (2) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)+\frac{\pi}{3}$

(3) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)-\frac{\pi}{3}$ (4) $\frac{1}{2}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)-\frac{\pi}{6}$

45. If $\int \frac{(x^2+1)e^x}{(x+1)^2}dx = f(x)e^x + C,$ where C is a constant, then $\frac{d^3 f}{dx^3}$ at $x = 1$ is equal to :

[JEE (Main)-2022]

(1) $-\frac{3}{4}$

(2) $\frac{3}{4}$

(3) $-\frac{3}{2}$

(4) $\frac{3}{2}$

46. For $I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x}dx,$ if $I\left(\frac{\pi}{4}\right) = 2^{1011},$ then

[JEE (Main)-2022]

(1) $3^{1010} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

(2) $3^{1010} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

(3) $3^{1011} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

(4) $3^{1011} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

47. Let g : (0, ¥) ® R be a differentiable function such that $\int\left(\frac{x(\cos x - \sin x)}{e^x+1} + \frac{g(x)\left(e^x+1-xe^x\right)}{\left(e^x+1\right)^2}\right)dx$ $= \frac{xg(x)}{e^x+1} + c,$ for all $x > 0$, where c is an arbitrary constant. Then :

(1) g is decreasing in $\left(0, \frac{\pi}{4}\right)$

(2) g' is increasing in $\left(0, \frac{\pi}{4}\right)$

(3) $g + g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$

(4) $g - g'$ is increasing in $\left(0, \frac{\pi}{2}\right)$

[JEE (Main)-2022]

48. The integral $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x - \sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to

[JEE (Main)-2022]

(1) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right| + C$

(2) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{x}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right| + C$

(3) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right| + C$

(4) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right| + C$

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Integrals

1. Answer (3)

2. Answer (3)

$$\int x^5 \cdot f(x^3)dx$$

$$= \frac{1}{3}\int x^3 \cdot f(x^3) \cdot 3x^2 dx$$

$$= \frac{1}{3} \cdot x^3 \int f(x^3) \cdot 3x^2 dx - \frac{1}{3}\int \left(3x^2 \cdot \int f(x^3) \cdot 3x^2 dx\right)dx$$

$$= \frac{1}{3} \cdot x^3 \ \psi\,(x^3) - \int x^2 \cdot \psi(x^3)dx + C$$

3. Answer (3)

$$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$$

$$\int \frac{d(p(t))}{\left(\frac{1}{2}p(t) - 200\right)} = \int dt$$

$$2\log\left(\frac{p(t)}{2} - 200\right) = t + cx$$

$$\frac{p(t)}{2} - 200 = e^{\frac{t}{2}}k$$

Using given condition $p(t) = 400 - 300\, e^{t/2}$

4. Answer (4)

$$I = \int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$$

Let $1 + \frac{1}{x^4} = t \Rightarrow \frac{-4}{x^5}dx = dt$

So, $I = \frac{-1}{4}\int \frac{dt}{t^{3/4}} = \frac{-1}{4}\int t^{-3/4}dt$

$$= \frac{-1}{4}\left(\frac{t^{1/4}}{1/4}\right) + c$$

$$= -\left(1+\frac{1}{x^4}\right)^{1/4} + c$$

where c is an arbitrary constant.

So, option (4) is right answer.

5. Answer (1)

$$\int \frac{2x^{12} + 5x^9}{\left(x^2 + x^3 + 1\right)} dx$$

$$= \int \frac{\left(\frac{2}{x^3} + \frac{5}{x^6}\right)dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

$$1 + \frac{1}{x^2} + \frac{1}{x^5} = t$$

$$\left(-\frac{2}{x^3} - \frac{5}{x^6}\right)dx = \mathrm{dt}$$

$$= \int \frac{-dt}{t^3} = \frac{1}{2t^2} + C$$

$$= \frac{1}{2\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3} + C = \frac{x^{10}}{2\left(x^5 + x^3 + 1\right)^2} + C$$

6. Answer (1)

$$I_n = \int \tan^n x dx,\ n > 1$$

$$I_4 + I_6 = \int (\tan^4 x + \tan^6 x)dx$$

$$= \int \tan^4 x \sec^2 x dx$$

Let $\tan x = t$

$\sec^2 x\ dx = dt$

$$= \int t^4 dt = \frac{t^5}{5} + C = \frac{1}{5}\tan^5 x + C$$

$$a = \frac{1}{5},\ b = 0$$

7. Answer (2)

$$I = \int \frac{\sin^2 x.\cos^2 x\ dx}{\left\{(\sin^2 x + \cos^2 x)\ (\sin^3 x + \cos^3 x)\right\}^2}$$

Dividing the numerator and denominator by $\cos^6 x$

Let, $\tan^3 x = z$

$\Rightarrow \quad 3\tan^2 x.\sec^2 x dx = dz$

$$I = \frac{1}{3}\int \frac{dz}{z^2} = \frac{-1}{3z} + C$$

$$= \frac{-1}{3(1+\tan^3 x)} + C$$

8. Answer (2)

$$I = \int x\sqrt{\frac{2\sin(x^2-1) - 2\sin(x^2-1)\cos(x^2-1)}{2\sin(x^2-1) + 2\sin(x^2-1)\cos(x^2-1)}}\,dx$$

$$I = \int x\sqrt{\frac{1-\cos(x^2-1)}{1+\cos(x^2-1)}}\,dx$$

$$I = \int x\left|\tan\left(\frac{x^2-1}{2}\right)\right| dx, \quad \text{Now let } \frac{x^2-1}{2} = t$$

$$\frac{2x}{2}dx = dt$$

$$I = \int |\tan(t)|\, dt$$

$$I = \ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c = \frac{1}{2}\ln\left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$$

9. Answer (2)

$$f(x) = \int \frac{5x^8 + 7x^6}{\left(x^2+1+2x^7\right)^2}dx,\ x \ge 0$$

$$= \int \frac{5x^8 + 7x^6}{x^{14}\left(x^{-5}+x^{-7}+2\right)^2}dx$$

$$= \int \frac{5x^{-6} + 7x^{-8}}{\left(2+x^{-7}+x^{-5}\right)^2}dx$$

Let $2 + x^{-7} + x^{-5} = t$

$(-7x^{-8} - 5x^{-6})dx = dt$

$$f(x) = \int \frac{-dt}{t^2} = \int -t^{-2}dt = t^{-1} + c$$

$$f(x) = \frac{1}{2+x^{-7}+x^{-5}} + c,\ f(0) = 0 \Rightarrow c = 0$$

$$\therefore \quad f(1) = \frac{1}{4}$$

$$I = \int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}\,d\theta$$

Let $\sin\theta = t$

$\cos\theta d\theta = dt$

$$I = \int \frac{(t^n - t)^{\frac{1}{n}}}{t^{n+1}}\,dt$$

$$= \int \frac{\left(1 - \frac{1}{t^{n-1}}\right)^{\frac{1}{n}}}{t^n}\,dt = \int t^{-n}(1-t^{1-n})^{\frac{1}{n}}\,dt$$

Let $1 - t^{1-n} = u$

$-(1-n)t^{-n}dt = du$

$$t^{-n}dt = \frac{du}{n-1}$$

$$I = \int u^{\frac{1}{n}} \cdot \frac{du}{n-1} = \frac{1}{n-1}\cdot\frac{u^{\frac{1}{n}+1}}{\frac{1}{n}+1}$$

$$= \frac{n}{n^2-1}u^{\frac{n+1}{n}} + C$$

$$= \frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$$

11. Answer (2)

$$I = \int x^5\, e^{-4x^3}\,dx$$

Put $-4x^3 = t$

$\Rightarrow \quad -12x^2\,dx = dt$

$$\Rightarrow \quad x^2 dx = -\frac{dt}{12}$$

$$\Rightarrow \quad I = \int \frac{1}{48}te^t\,dt = \frac{1}{48}[t\,e^t - e^t] + C$$

$$I = \frac{1}{48}e^{-4x^3}(-4x^3 - 1) + C$$

$\Rightarrow \quad f(x) = -4x^3 - 1$

$$A(x)\left(\sqrt{1-x^2}\right)^m + C = \int \frac{\sqrt{1-x^2}}{x^4}\, dx$$

$$= \int \frac{\sqrt{\frac{1}{x^2}-1}}{x^3}\, dx$$

Let $\frac{1}{x^2} - 1 = t^2$

$$\Rightarrow \quad -\frac{2}{x^3} = \frac{2t\, dt}{dx}$$

$$\frac{dx}{x^3} = -\frac{2t}{2}\, dt$$

$$A(x)\left(\sqrt{1-x^2}\right)^m + C = \int (-t^2)\, dt = -\frac{t^3}{3} + C$$

$$= -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}} + C$$

$$= -\frac{1}{3}\cdot\frac{1}{x^3}\cdot(1-x^2)^{\frac{3}{2}} + C$$

$$= \frac{-1}{3x^3}\left(\sqrt{1-x^2}\right)^3 + C$$

$$\therefore \quad A(x) = -\frac{1}{3x^3}$$

$$\therefore \quad (A(x))^3 = \frac{-1}{27x^9}$$

13. Answer (2)

Let $I = \int \frac{x+1}{\sqrt{2x-1}}\, dx$

Let $\sqrt{2x-1} = t$

$\therefore \quad 2x - 1 = t^2$

$\Rightarrow \quad dx = t\,dt$

$$I\int \frac{(t^2+3)}{2}\, dt = \frac{t^3}{6} + \frac{3t}{2} + C$$

$$= \frac{(2x-1)^{\frac{3}{2}}}{6} + \frac{3}{2}(2x-1)^{\frac{1}{2}} + C$$

$$= \sqrt{2x-1}\left(\frac{x+4}{3}\right) + C$$

$$= f(x).\sqrt{2x-1} + C$$

where $f(x) = \frac{x+4}{3}$

$$I = \cos(\ln x).x - \int \frac{-\sin(\ln x)}{x}.x\, dx$$

$$= x\cos(\ln x) + \int \sin(\ln x)\, dx$$

$$= x\cos(\ln x) + \sin(\ln x).x - \int \frac{\cos(\ln x)}{x}.x\, dx$$

$$2I = x(\cos(\ln x) + \sin(\ln x)) + C$$

$$I = \frac{x}{2}\left[\cos(\ln x) + \sin(\ln x)\right] + C$$

15. Answer (3)

$$I = \int \frac{3x^{13} + 2x^{11}}{x^{16}\left(2 + \frac{3}{x^2} + \frac{1}{x^4}\right)^4}\, dx$$

$$I = \int \frac{\frac{3}{x^3} + \frac{2}{x^5}}{\left(2 + \frac{3}{x^2} + \frac{1}{x^4}\right)^4}\, dx$$

Let $2 + \frac{3}{x^2} + \frac{1}{x^4} = t$, $-2\left(\frac{3}{x^3} + \frac{2}{x^5}\right)dx = dt$

$$I = \int \frac{-\frac{dt}{2}}{t^4} = -\frac{1}{2}\,\frac{t^{-4+1}}{-4+1} + C$$

$$I = \frac{-1}{2} \times \frac{1}{(-3)}\,\frac{1}{\left(2 + \frac{3}{x^2} + \frac{1}{x^4}\right)^3} + C$$

$$I = \frac{1}{6}\,\frac{x^{12}}{\left(2x^4 + 3x^2 + 1\right)^3} + C$$

16. Answer (3)

$$\int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)}\, dx = \int \frac{2\cos\frac{x}{2}.\sin\frac{5x}{2}}{2\cos\frac{x}{2}.\sin\frac{x}{2}}\, dx$$

$$= \int \frac{\sin 3x + \sin 2x}{\sin x}\, dx$$

Now use $\sin 2x = 2\sin x\cos x$ and $\sin 3x = 3\sin x - 4\sin^3 x$

$$= \int (1 + 2\cos x + 2\cos 2x)\, dx$$

$$= x + 2\sin x + \sin 2x + c$$

$$I=\int\frac{dx}{x^3(1+x^6)^{2/3}}=\int\frac{dx}{x^7(1+x^{-6})^{2/3}}$$

Put $1+x^{-6}=t^3$

$\Rightarrow\ -6x^{-7}dx=3t^2\,dt$

$$\Rightarrow\ \frac{dx}{x^7}=-\frac{1}{2}t^2dt$$

$$\Rightarrow\ I=\int-\frac{1}{2}\frac{t^2dt}{t^2}$$

$$=-\frac{1}{2}t+C$$

$$=-\frac{1}{2}(1+x^{-6})^{\frac{1}{3}}+C$$

$$=-\frac{1}{2}\frac{(1+x^6)^{\frac{1}{3}}}{x^2}+C$$

$$=-\frac{1}{2x^3}x(1+x^6)^{\frac{1}{3}}+C$$

$$\Rightarrow\ f(x)=-\frac{1}{2x^3}$$

18. Answer (2)

$$I=\int\sec^{\frac{2}{3}}x\cdot\operatorname{cosec}^{\frac{4}{3}}x\,dx$$

$$I=\int\frac{\sec^2x\,dx}{\tan^{\frac{4}{3}}x}$$ Put $\tan x=t$

$\Rightarrow \sec^2x\,dx=dt$

$$\Rightarrow\ I=\frac{t^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+C\quad\Rightarrow I=-3(\tan x)^{\frac{-1}{3}}+C$$

19. Answer (2)

$$\int e^{\sec x}\left(\sec x\tan x\,f(x)+\left(\sec x\tan x+\sec^2x\right)\right)dx=e^{\sec x}f(x)+C$$

$\because$ We know that

$$\int e^{g(x)}\left((g'(x)f(x))+f'(x)\right)dx=e^{g(x)}\times f(x)+C$$

$\therefore\ \boxed{f(x)=\sec x+\tan x+C}$

20. Answer (2)

$$\int\frac{dx}{(x^2-2x+10)^2}=\int\frac{dx}{((x-1)^2+9)^2}$$

Let $(x-1)^2=9\tan^2\theta$ ...(i)

$$\Rightarrow\ \tan\theta=\frac{x-1}{3}$$

On Differentiating ...(i)

$2(x-1)dx=18\tan\theta\sec^2\theta\,d\theta$

$$\therefore\ I=\int\frac{18\tan\theta\sec^2\theta\,d\theta}{2\times3\tan\theta\times81\sec^4\theta}$$

$$I=\frac{1}{27}\int\cos^2\theta\,d\theta=\frac{1}{27}\times\frac{1}{2}\int(1+\cos2\theta)d\theta$$

$$I=\frac{1}{54}\left\{\theta+\frac{\sin2\theta}{2}\right\}+c$$

$$I=\frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{1}{2}\times\frac{2\left(\frac{x-1}{3}\right)}{1+\left(\frac{x-1}{3}\right)^2}\right]+c$$

$$I=\frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{3(x-1)}{x^2-2x+10}\right]+c$$

So $A=\frac{1}{54}$

$f(x)=3(x-1)$

21. Answer (4)

$$I=\int x^5\cdot e^{-x^2}dx$$

Put $-x^2=t\ \Rightarrow\ -2xdx=dt$

$$I=\int\frac{t^2\cdot e^tdt}{(-2)}=\frac{-1}{2}e^t\left(t^2-2t+2\right)+c$$

$$\therefore\ g(x)=\frac{-1}{2}\left(x^4+2x^2+2\right)$$

$$g(-1)=\frac{-5}{2}$$

$$I=\int\frac{(2x^3-1)dx}{x^4+x}=\int\frac{(2x-x^{-2})dx}{x^2+x^{-1}}$$

Put $x^2 + x^{-1} = t$

$(2x - x^{-2})dx = dt$

$$I=\int\frac{dt}{t}=\ln|t|+c$$

$$=\ln\left|x^2+x^{-1}\right|+c$$

$$=\ln\left|\frac{x^3+1}{x}\right|+c$$

23. Answer (3)

$$\int\frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx$$

$$=\int\frac{\sin(x+\alpha)}{\sin(x-\alpha)}dx$$ let $x - \alpha = t$

$\Rightarrow$ $dx = dt$

$$=\int\frac{\sin(t+2\alpha)}{\sin t}dt$$

= $[\cos2\alpha + \sin2\alpha \cdot \cot t]dt$

= $\cos2\alpha \cdot t + \sin2\alpha \cdot \ln|\sin t| + c$

= $(x - \alpha) \cdot \cos2\alpha + \sin2\alpha \cdot \ln|\sin(x - \alpha)| + c$

24. Answer (3)

Let $\sin x = t$

$\Rightarrow$ $\cos x\, dx = dt$

$$\therefore\quad I=\int\frac{dt}{t^3(1+t^6)^{2/3}}=\int\frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{2/3}}$$

Put $\frac{1}{t^6}+1=k$

$$\Rightarrow\quad\frac{-6}{t^7}dt=dk$$

$$\therefore\quad I=\frac{-1}{6}\int\frac{dk}{k^{2/3}}=\frac{-1}{6}\frac{k^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}+c$$

$$=\frac{-1}{2}k^{1/3}+c$$

$$=\frac{-1}{2}(1+\sin^6 x)^{1/3}.\text{cosec}^2 x+c$$

$\therefore$ $\lambda = 3$ and $f\left(\frac{\pi}{3}\right)=\left(\frac{4}{3}\right)\left(\frac{-1}{2}\right)$ $\Rightarrow \lambda f(\pi/3) = -2$

$$\int\frac{dx}{(x-3)^{\frac{6}{7}}(x+4)^{\frac{8}{7}}}$$

$$=\int\frac{dx}{\left(\frac{x-3}{x+4}\right)^{\frac{6}{7}}\cdot(x+4)^2}$$

Let $\frac{x-3}{x+4}=t=1-\frac{7}{x+4}$

$$=\frac{1}{7}\int\frac{dt}{t^{\frac{6}{7}}}\qquad dt=\frac{7}{(x+4)^2}dx$$

$$=t^{\frac{1}{7}}+C$$

$$=\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}}+C$$

26. Answer (2)

Let $I=\int\frac{\sec^2\theta d\theta}{\sec 2\theta+\tan 2\theta}d\theta$

$$=\int\frac{\sec^2\theta}{\frac{1+\tan^2\theta+2\tan\theta}{1-\tan 2\theta}}d\theta$$

$$=\int\frac{\sec^2\theta(1-\tan\theta)}{1+\tan\theta}d\theta$$

Put $\tan\theta = t$ $\therefore$ $\sec^2\theta d\theta = dt$

$$I=\int\frac{(1-t)dt}{1+t}=\int\left(-1+\frac{2}{1+t}\right)dt=-t+2\log(1+t)+c$$

$\therefore$ $I = -\tan\theta + 2\log_e|1 + \tan\theta| + c$

$\Rightarrow$ $\lambda = -1$, $f(\theta) = 1 + \tan\theta$.

27. Answer (1)

$$I=\int\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right)dx=\int\tan^{-1}\left(\sqrt{x}\right)dx$$

$$=\int\underset{\text{I}}{\tan^{-1}\sqrt{x}}\cdot\underset{\text{II}}{1}\,dx$$

$$I=x\tan^{-1}\sqrt{x}-\int\frac{x}{1+x}\cdot\frac{1}{2\sqrt{x}}dx$$

$$= x\tan^{-1}\sqrt{x} - \int \frac{t^2}{1+t^2}dt \qquad \begin{bmatrix} \text{Put } x = t^2 \\ dx = 2tdt \end{bmatrix}$$

$$= x\tan^{-1}\sqrt{x} - \int 1dt + \int \frac{1}{1+t^2}dt$$

$$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + c$$

$$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$$

$$\Rightarrow \quad A(x) = (x+1), \;\; B(x) = -\sqrt{x}$$

28. Answer (1)

$$\int \frac{x^2}{(x\sin x + \cos x)^2}dx$$

$$\because \quad \frac{d}{dx}(x\sin x + \cos x) = x\cos x$$

$$= \int \underbrace{\frac{x\cos x}{(x\sin x+\cos x)^2}}_{\text{II}} \cdot \underbrace{\left(\frac{x}{\cos x}\right)}_{\text{I}} dx$$

$$= \frac{x}{\cos x}\left[\frac{-1}{x\sin x + \cos x}\right] - \int \frac{x\sin x + \cos x}{\cos^2 x} \cdot \left[\frac{-1}{x\sin x + \cos x}\right]dx$$

$$= \frac{-x\sec x}{x\sin x + \cos x} + \tan x + C$$

29. Answer (1)

$$\int \frac{\sqrt{x}}{(1+x)^2}dx \; (x > 0)$$

Put x = $\tan^2\theta \Rightarrow 2xdx = 2\tan\theta\sec^2\theta d\theta$

$$I = \int \frac{2\tan^2\theta.\sec^2\theta}{\sec^4\theta}d\theta = \int 2\sin^2\theta d\theta = \int (1-\cos 2\theta)d\theta$$

$$= \theta - \frac{\sin 2\theta}{2} + c$$

$$\Rightarrow \quad f(x) = \theta - \frac{1}{2} \times \frac{2\tan\theta}{1+\tan^2\theta} + c$$

$$f(x) = \theta - \frac{\tan\theta}{1+\tan^2\theta} + c = \tan^{-1}\sqrt{x} - \frac{\sqrt{x}}{1+x} + c$$

Now $f(3) - f(1) = \tan^{-1}\left(\sqrt{3}\right) - \frac{\sqrt{3}}{1+3} - \tan^{-1}(1) + \frac{1}{2}$

$$= \frac{\pi}{3} - \frac{\sqrt{3}}{4} - \frac{\pi}{4} + \frac{1}{2}$$

$$= \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

$$\int (e^{2x} + 2e^{x} - e^{-x} - 1) \cdot e^{(e^x + e^{-x})}dx$$

Let $e^x = t, \; dx = \dfrac{dt}{t}$

$$= \int \left(t^2 + 2t - \frac{1}{t} - 1\right)e^{\left(t+\frac{1}{t}\right)} \cdot \frac{dt}{t}$$

$$= \int \left[\frac{(t^2-1)(t+1)}{t^2} + 1\right]e^{\left(t+\frac{1}{t}\right)}dt$$

$$= \int \underbrace{(t+1)}_{\text{I}} \underbrace{\left(1 - \frac{1}{t^2}\right)e^{\left(t+\frac{1}{t}\right)}dt}_{\text{II}} + \int e^{\left(t+\frac{1}{t}\right)}dt$$

$$= (t+1)\, e^{\left(t+\frac{1}{t}\right)} - \int e^{\left(t+\frac{1}{t}\right)}dt + \int e^{\left(t+\frac{1}{t}\right)}dt$$

$$= (e^x + 1) \cdot e^{(e^x + e^{-x})} + c$$

So, g(x) = 1 + e^x and g(0) = 2

31. Answer (3)

Let sinθ = t

$$\Rightarrow \quad \int \frac{dt}{5 + 7t - 2 + 2t^2}$$

$$\Rightarrow \quad \frac{1}{2}\int \frac{dt}{\left(t + \frac{7}{4}\right)^2 - \left(\frac{5}{4}\right)^2}$$

$$= \frac{1}{5}\ln\left|\frac{t + \frac{1}{2}}{t+3}\right| + c$$

$$= \frac{1}{5}\ln\left|\frac{2t+1}{t+3}\right| + c$$

$$\therefore \quad B(\theta) = \frac{2\sin\theta + 1}{2(\sin\theta + 3)} \text{ and A = } \mathbf{\frac{1}{5}}$$

$$\Rightarrow \quad \frac{B(\theta)}{A} = \frac{5(2\sin\theta + 1)}{(\sin\theta + 3)}$$

$$\int \frac{\cos x - \sin x}{\sqrt{8-\sin 2x}}dx = \int \frac{\cos x - \sin x}{\sqrt{9-(\sin x+\cos x)^2}}dx$$

let sinx + cosx = t

(cosx – sinx)dx = dt

$$= \int \frac{dt}{\sqrt{9-t^2}}$$

$$= \sin^{-1}\left(\frac{t}{3}\right) + c$$

$$= \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c$$

Hence (a, b) = (1, 3)

33. Answer (2)

$$\int \frac{\sin\theta(2\sin\theta)\cos\theta\left(\sin^6\theta+\sin^4\theta+\sin^2\theta\right)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\left(1-2\sin^2\theta\right)}d\theta$$

Put sinθ = t

$\Rightarrow$ cosθ dθ = dt

$$\Rightarrow \int \frac{t^2\left(t^6+t^4+t^2\right)\sqrt{2t^4+3t^2+6}}{t^2}dt$$

$$\int\left(t^5+t^3+t\right)\sqrt{2t^6+3t^4+6t^2}\ dt$$

Put $2t^6 + 3t^4 + 6t^2 = k$

$\Rightarrow$ $12(t^5 + t^3 + t)\ dt = dk$

$$\Rightarrow \frac{1}{12}\int\sqrt{k}\ dk$$

$$\Rightarrow \frac{2k^{\frac{3}{2}}}{12.3}$$

$$\Rightarrow \frac{1}{18}\left(2\sin^6\theta+3\sin^4+6\sin^2\right)^{\frac{3}{2}}+C$$

$$= \frac{1}{18}\left(11-18\cos^2\theta+9\cos^4\theta-2\sin^6\theta\right)^{\frac{3}{2}}+C$$

$$I = \int \frac{e^{3\ln 2x}+5e^{2\ln 2x}}{e^{4\ln x}+5e^{3\ln x}-7e^{2\ln x}}dx$$

$$I = \int \frac{(2x)^3+5(2x)^2}{x^4+5x^3-7x^2}dx = \int\frac{8x+20}{x^2+5x-7}dx$$

Let $x^2 + 5x - 7 = t$

$(2x + 5)dx = dt$

$$I = 4\int\frac{dt}{t} = 4\ln|t|+c$$

$$I = 4\ln\left|x^2+5x-7\right|+c$$

35. Answer (6)

$$I = \int\frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx + \int\frac{1}{x^4+3x^2+1}dx$$

$I = I_1 + I_2$ ...(i)

For I_1, Let $\tan^{-1}\frac{x^2+1}{x} = t$

$$I_1 = \int\frac{1}{t}dt = \ln t = \ln\left|\tan^{-1}\left(\frac{x^2+1}{x}\right)\right| + C_1$$

$$I_2 = \frac{1}{2}\int\frac{(x^2+1)-(x^2-1)}{x^4+3x^2+1}$$

$$= \frac{1}{2}\int\frac{x^2+1}{x^4+3x^2+1} - \frac{1}{2}\int\frac{x^2-1}{x^4+3x^2+1}$$

Divide Nr and Dr by x^2

$$= \frac{1}{2}\int\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+5} - \frac{1}{2}\int\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2+1}$$

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right) - \frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)$$

$$\alpha = 1,\ \beta = \frac{1}{2\sqrt{5}},\ \gamma = \frac{1}{\sqrt{5}},\ \delta = \frac{-1}{2}$$

Required value = $10\left(1+\frac{1}{10}-\frac{1}{2}\right)$

= 6

$4(2x-1)dx = 2tdt$

$$\Rightarrow \int \frac{t}{2}\frac{\cos t}{t}\cdot dt$$

$$\Rightarrow \frac{1}{2}\sin t + c$$

$$\Rightarrow \frac{1}{2}\sin\left(\sqrt{(2x-1)^2+5}\right)+c$$

37. Answer (04)

$$\int \frac{5x^8+7^6}{x^{14}\left(\frac{1}{x^5}+\frac{1}{x^7}+2\right)}dx$$

$$\Rightarrow \int \frac{5x^{-6}+7x^{-8}}{\left(2+\frac{1}{x^5}+\frac{1}{x^7}\right)^2}dx$$

Put $2+\frac{1}{x^5}+\frac{1}{x^7}=t$

$(-5x^{-6}-7x^{-8})dx = dt$

$$\Rightarrow \int \frac{-dt}{t^2}=\frac{1}{t}+c$$

$$f(x)=\frac{x^7}{2x^7+x^2+1}+c$$

$$f(0)=0 \Rightarrow c=0 \Rightarrow f(1)=\frac{1}{4} \Rightarrow k=4$$

38. Answer (15)

$$\int \frac{dx}{\left(\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right)^2} \quad \text{Let } x+\frac{1}{2}=\frac{\sqrt{3}}{2}\tan\theta$$

$$\int \frac{\frac{\sqrt{3}}{2}\cdot\sec^2\theta}{\frac{9}{16}\sec^4\theta}d\theta = \frac{8}{3\sqrt{3}}\int\cos^2\theta d\theta = \frac{4}{3\sqrt{3}}\int 2\cos^2\theta d\theta$$

$$=\frac{4}{\sqrt{3}}\left(\theta+\frac{\sin 2\theta}{2}\right)$$

So $\int \frac{dx}{\left(x^2+x+1\right)^2}$

$$=\frac{4}{3\sqrt{3}}\left(\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{\sqrt{3}}{4}\frac{2x+1}{x^2+x+1}\right)$$

$$a=\frac{4}{3\sqrt{3}},\ b=\frac{1}{3}$$

$9(a+b) = 15$

39. Answer (07)

Write $2e^x + 3e^{-x} = A(4e^x + 7e^{-x}) + B(4e^x - 7e^{-x})$

Comparing both sides

$4A + 4B = 2$ ...(i)

$7A - 7B = 3$ ...(ii)

on solving $A=\frac{13}{28}$ and $B=\frac{1}{28}$

$$I=\int\frac{2e^x+3e^{-x}}{4e^x+7e^{-x}}dx=\int\left(\frac{\frac{13}{28}(4e^x+7e^{-x})+\frac{1}{28}\left(4e^x-7e^{-x}\right)}{4e^x+7e^{-x}}\right)dx$$

$$=\frac{13}{28}x+\frac{1}{18}\ln(4e^x+7e^{-x})+C$$

Comparing LHS and RHS gives $u=\frac{13}{2}$ and $v=\frac{1}{2}$

$\Rightarrow u + v = 7$

40. Answer (4)

$$I=\int\frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$$

$$=\int\frac{dx}{(x+2)^2\left(\frac{x-1}{x+2}\right)^{3/4}}$$

Let $\frac{x-1}{x+2}=t \Rightarrow \frac{3dx}{(x+2)^2}=dt$

$$\therefore \quad I=\int\frac{1}{t^{3/4}}\cdot\frac{1}{3}dt$$

$$=\frac{\ }{3}\left(\frac{\ }{-\frac{3}{4}+1}\right)+C$$

$$=\frac{4}{3}\left(t^{1/4}\right)+C$$

$$\therefore \quad I=\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$$

41. Answer (3)

$$I=\int\frac{\sin x}{\cos^3 x+\sin^3 x}dx=\int\frac{\frac{\sin x}{\cos^3 x}}{\frac{\cos^3 x}{\cos^3 x}+\frac{\sin^3 x}{\cos^3 x}}\cdot dx$$

$$I=\int\frac{\tan x\cdot\sec^2 x}{1+\tan^3 x}dx, \qquad \text{Put } \tan x = t$$

$$\sec^2 x\cdot dx = dt$$

$$I=\int\frac{t}{1+t^3}dt$$

$$\frac{t}{1+t^3}=\frac{A}{1+t}+\frac{Bt+C}{1+t^2-t}$$

$t = A(1 - t + t^2) + (1 + t)(Bt + C)$

By comparing coeff of x, x^2 and constant term,

$$A=-\frac{1}{3},\ B=\frac{1}{3},\ C=\frac{1}{3}$$

$$I=-\frac{1}{3}\int\frac{1}{1+t}dt+\frac{1}{3}\int\frac{t+1}{t^2-t+1}dt$$

$$I=-\frac{1}{3}\ln(1+t)+\frac{1}{6}\left[\int\frac{2t-1}{t^2-t+1}dt+3\int\frac{1}{t^2-t+1}dt\right]$$

$$I=-\frac{1}{3}\ln(1+t)+\frac{1}{6}\left[\log\left(t^2-t+1\right)+3\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)\right]+C$$

$$\ldots\frac{\ }{3}\ldots$$

$$+\frac{1}{6}\cdot\log\left(\tan^2 x-\tan x+1\right)+\frac{1}{\sqrt{3}}\cdot\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)+C$$

$$\alpha=-\frac{1}{3},\ \beta=\frac{1}{6},\ \gamma=\frac{1}{\sqrt{3}}$$

$18(\alpha + \beta + \gamma^2)$

$$18\left(-\frac{1}{3}+\frac{1}{6}+\frac{1}{3}\right)=3$$

42. Answer (4)

$$\cos x dy-(\sin x)y dx=6x\,dx$$

$$\Rightarrow \int d(y\cos x)=\int 6x\,dx$$

$$\Rightarrow y\cos x=3x^2+C$$

$$\text{As } y\left(\frac{\pi}{3}\right)=0\Rightarrow(0)\times\left(\frac{1}{2}\right)=\frac{3\pi^2}{9}+C\Rightarrow C=\frac{-\pi^2}{3}$$

$$\Rightarrow y\cos x=3x^2-\frac{\pi^2}{3}$$

$$\text{For } y\left(\frac{\pi}{6}\right)$$

$$y\frac{\sqrt{3}}{2}=\frac{3\pi^2}{36}-\frac{\pi^2}{3}$$

$$\frac{\sqrt{3}y}{2}=\frac{-3\pi^2}{12}\Rightarrow y=\frac{-\pi^2}{2\sqrt{3}}$$

43. Answer (4)

$$I=\int\left\{e^{\left(x+\frac{1}{x}\right)}+x\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}\right\}dx$$

$$=x.e^{x+\frac{1}{x}}+c$$

$$\text{As }\int(xf'(x)+f(x))dx=xf(x)+c$$

$$\because \int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x)+c$$

$$\int_1^{\frac{1}{2}} \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g\left(\frac{1}{2}\right)-g(1)$$

$$\therefore\ g\left(\frac{1}{2}\right) = \int_1^{\frac{1}{2}} \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx$$

$\cot x = \cos 2\theta$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{\cos 2\theta}\cdot\frac{\sin\theta}{\cos\theta}(-2\sin 2\theta)\,d\theta$$

$$= -\int_0^{\frac{\pi}{6}} \frac{4\sin^2\theta}{\cos 2\theta}d\theta$$

$$= 2\int_0^{\frac{\pi}{6}} \frac{\left(1-2\sin^2\theta\right)-1}{\cos 2\theta}d\theta$$

$$= 2\int_0^{\frac{\pi}{6}} (1-\sec 2\theta)\,d\theta$$

$$= \frac{\pi}{3} - 2\cdot\frac{1}{2}\left[\ln\left|\sec 2\theta + \tan 2\theta\right|\right]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{3} - \left[\ln\left|2+\sqrt{3}\right| - \ln 1\right]$$

$$= \frac{\pi}{3} + \ln\left(\frac{1}{2+\sqrt{3}}\right)$$

$$= \frac{\pi}{3} + \ln\left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right|$$

45. Answer (2)

$$I = \int \frac{e^x\left(x^2+1\right)}{(x+1)^2}dx = f(x)e^x + c$$

$$= \int \frac{e^x\left(x^2-1+1+1\right)}{(x+1)^2}dx$$

$$\int e^x\left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right]dx$$

$$= e^x\left(\frac{x-1}{x+1}\right)+c$$

$$\therefore\ f(x) = \frac{x-1}{x+1}$$

$$f(x) = 1 - \frac{2}{x+1}$$

$$f'(x) = 2\left(\frac{1}{x+1}\right)^2$$

$$f''(x) = -4\left(\frac{1}{x+1}\right)^3$$

$$f'''(x) = \frac{12}{(x+1)^4}$$

for $x = 1$

$$f'''(1) = \frac{12}{2^4} = \frac{12}{16} = \frac{3}{4}$$

46. Answer (1)

$$I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x}\,dx$$

$$= \int\left(\sec^2 x\cdot\sin^{-2022} x - 2022\sin^{-2022} x\right)dx$$

$$= \sin^{-2022} x\tan x + \int 2022\sin^{-2023} x\cos x\cdot\tan x\ dx$$

$$-\int 2022\sin^{-2022} x\,dx + c$$

$$I(x) = \sin^{-2022} x\ \tan x + c$$

$$\because\ I\left(\frac{\pi}{4}\right) = 2^{1011} \quad\Rightarrow\quad c = 2^{1011} - 2^{1011} = 0$$

$$\therefore\ I\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)^{2022}\sqrt{3},\ I\left(\frac{\pi}{6}\right) = 2^{2022}\frac{1}{\sqrt{3}}$$

So, option (1) : $\frac{3^{1010}2^{2022}}{3^{1011}}\cdot\sqrt{3} - \frac{2^{2022}}{\sqrt{3}} = 0$

$\therefore$ Option (1) is correct

$$\int\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx=\frac{xg(x)}{e^x+1}+c$$

Differentiating on both sides

$$I\,\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x-1)^2}$$

$$=\frac{(e^x+1)\left(g(x)+xg^{\frac{1}{x}}\right)-xg(x)e^x}{(e^x+1)^2}$$

$$=\frac{g(x)[e^x+1-xe^x]}{(e^x+1)^2}+\frac{xg'(x)(e^x+1)}{(e^x+1)^2}$$

$$=\frac{x(\cos x-\sin x)}{e^x+1}=\frac{xg'(x)}{e^x+1}$$

$\Rightarrow\ g'(x)=\cos x-\sin x>0$ in $\left(0,\frac{\pi}{4}\right)$

$g(x)$ is increasing in $\left(0,\frac{\pi}{4}\right)\Rightarrow$ A is wrong

Now, $g''(x)=-\sin x-\cos x<0$ in $\left(0,\frac{\pi}{4}\right)$

$\Rightarrow$ $g(x)$ is increasing in $\left(0,\frac{\pi}{4}\right)\Rightarrow$ *B* is wrong

Let $h(x)=g(x)+g'(x)$

$\Rightarrow h'(x)=g'(x)+g''(x)=-2\sin x<0$ in $x\in\left(0,\frac{\pi}{2}\right)$

$\Rightarrow$ $g+g'$ is decreasing in $\left(0,\frac{\pi}{2}\right)\Rightarrow$ *c* is wrong

Let $J(x)=g(x)-g'(x)$

$J'(x)=g'(x)-g''(x)=2\cos x>0$ in $\left(0,\frac{\pi}{2}\right)$

$\Rightarrow$ $g-g'$ is increasing in $\left(0,\frac{\pi}{2}\right)\Rightarrow$ correct

$$=\int\frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$$

$$=\int\frac{\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)\sqrt{2}\sin\left(\frac{\pi}{4}-x\right)}{\left(\frac{2}{\sqrt{3}}\right)\left(\sin\frac{\pi}{3}+\sin 2x\right)}dx$$

$$=\int\frac{\frac{(\sqrt{3}-1)}{\sqrt{2}}\sin\left(\frac{\pi}{4}-x\right)}{\left(\sin\frac{\pi}{3}+\sin 2x\right)}dx$$

$$=\int\frac{\frac{\sqrt{3}-1}{2\sqrt{2}}\sin\left(\frac{\pi}{4}-x\right)}{\sin\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)}dx$$

$$=\frac{1}{2}\int\frac{2\sin\frac{\pi}{12}\sin\left(\frac{\pi}{4}-x\right)}{\sin\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)}dx$$

$$=\frac{1}{2}\int\frac{\cos\left(\frac{\pi}{6}-x\right)-\cos\left(\frac{\pi}{3}-x\right)}{\sin\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)}dx$$

$$=\frac{1}{2}\left[\int\operatorname{cosec}\left(\frac{\pi}{6}+x\right)dx-\int\sec\left(\frac{\pi}{6}-x\right)dx\right]$$

$$=\frac{1}{2}\left[\ln\left|\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)\right|-\int\operatorname{cosec}\left(\frac{\pi}{3}-x\right)dx\right]$$

$$=\frac{1}{2}\left[\ln\left|\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)\right|-\ln\left|\frac{\pi}{6}+\frac{x}{2}\right|\right]+C$$

$$=\frac{1}{2}\ln\left|\frac{\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)}{\tan\left(\frac{\pi}{6}+\frac{x}{2}\right)}\right|+C$$