Chapter 2

Motion in a Plane

1. A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is **[AIEEE-2009]**

(1) $7\sqrt{2}$ units (2) 7 units

(3) 8.5 units (4) 10 units

2. A particle is moving with velocity $\vec{v}=K\left(y\hat{i}+x\hat{j}\right)$, where K is a constant. The general equation for its path is **[AIEEE-2010]**

(1) $y^2 = x^2$ + constant (2) $y = x^2$ + constant

(3) $y^2 = x$ + constant (4) xy = constant

3. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the x-axis) **[AIEEE-2010]**

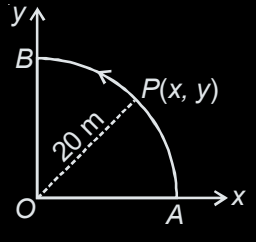
(1) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$

(2) $-\frac{v^2}{R}\cos\theta\hat{i}+\frac{v^2}{R}\sin\theta\hat{j}$

(3) $-\frac{v^2}{R}\sin\theta\hat{i}+\frac{v^2}{R}\cos\theta\hat{j}$

(4) $-\frac{v^2}{R}\cos\theta\hat{i}-\frac{v^2}{R}\sin\theta\hat{j}$

4. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly **[AIEEE-2010]**



(1) 14 m/s^2 (2) 13 m/s^2

(3) 12 m/s^2 (4) 7.2 m/s^2

5. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be **[AIEEE-2012]**

(1) 10 m (2) $10\sqrt{2}$ m

(3) 20 m (4) $20\sqrt{2}$ m

6. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$m/s where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is **[JEE (Main)-2013]**

(1) $y = x - 5x^2$ (2) $y = 2x - 5x^2$

(3) $4y = 2x - 5x^2$ (4) $4y = 2x - 25x^2$

7. A particle is moving with a velocity $\vec{v}=K\left(y\hat{i}+x\hat{j}\right)$, where K is a constant. The general equation for its path is **[JEE (Main)-2019]**

(1) $y^2 = x$ + constant (2) $y = x^2$ + constant

(3) $y^2 = x^2$ + constant (4) xy = constant

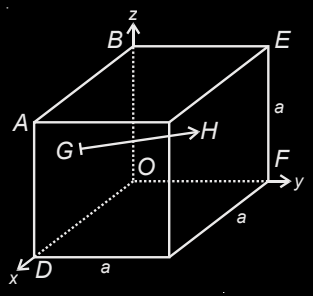
8. The position co-ordinates of a particle moving in a 3-D coordinate system is given by $x = a\cos\omega t$, $y = a\sin\omega t$ and $z = a\omega t$

The speed of the particle is **[JEE (Main)-2019]**

(1) $2a\omega$ (2) $\sqrt{2}a\omega$

(3) $\sqrt{3}a\omega$ (4) $a\omega$

9. In the cube of side 'a' shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be **[JEE (Main)-2019]**



(1) $\frac{1}{2}a(\hat{j}-\hat{i})$ (2) $\frac{1}{2}a(\hat{i}-\hat{k})$

(3) $\frac{1}{2}a(\hat{j}-\hat{k})$ (4) $\frac{1}{2}a(\hat{k}-\hat{i})$

10. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is

[JEE (Main)-2019]

(1) 1 : 4 (2) 1 : 8

(3) 1 : 2 (4) 1 : 16

11. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $\left(\vec{A}+\vec{B}\right)$ is 'n' times the magnitude of $\left(\vec{A}-\vec{B}\right)$. The angle between $\vec{A}$ and $\vec{B}$ is

[JEE (Main)-2019]

(1) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$ (2) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

(3) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$ (4) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$

12. Two forces P and Q, of magnitude $2F$ and $3F$, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is

[JEE (Main)-2019]

(1) 30° (2) 90°

(3) 60° (4) 120°

13. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle? **[JEE (Main)-2019]**

(1) 10 m/s (2) Zero

(3) $10\sqrt{3}$ m/s (4) $10\sqrt{2}$ m/s

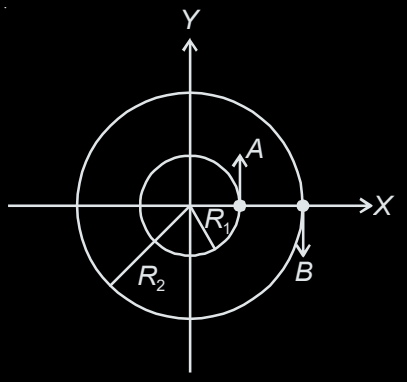
14. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, speed of the plane is

[JEE (Main)-2019]

(1) $\frac{2v}{\sqrt{3}}$ (2) $\frac{\sqrt{3}}{2}v$

(3) $\frac{v}{2}$ (4) v

15. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and direction of motion are shown in the figure **[JEE (Main)-2019]**



The relative velocity $\vec{v}_A - \vec{v}_B$ at $t = \frac{\pi}{2\omega}$ is given by

(1) $\omega(R_2 - R_1)\hat{i}$ (2) $\omega(R_1 - R_2)\hat{i}$

(3) $-\omega(R_1 + R_2)\hat{i}$ (4) $\omega(R_1 + R_2)\hat{i}$

16. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km / hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in **[JEE (Main)-2019]**

(1) 2.2 hrs. (2) 4.2 hrs.

(3) 3.2 hrs. (4) 2.6 hrs.

17. Let $\left|\overrightarrow{A_1}\right| = 3, \left|\overrightarrow{A_2}\right| = 5$ and $\left|\overrightarrow{A_1} + \overrightarrow{A_2}\right| = 5$. The value of $\left(2\overrightarrow{A_1} + 3\overrightarrow{A_2}\right)\cdot\left(3\overrightarrow{A_1} - 2\overrightarrow{A_2}\right)$ is : **[JEE (Main)-2019]**

(1) – 106.5

(2) – 118.5

(3) – 99.5

(4) – 112.5

18. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight? **[JEE (Main)-2019]**

(1) 60° (2) 90°

(3) 150° (4) 120°

19. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$. What is the magnitude of the acceleration at $t = 1$? **[JEE (Main)-2019]**

(1) 50

(2) 100

(3) 40

(4) 25

20. A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$

$y = y_0 + b\sin\omega_2 t$

The torque, acting on the particle about the origin, at $t = 0$ is : **[JEE (Main)-2019]**

(1) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$

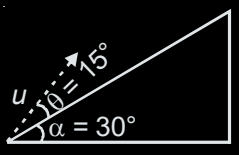
(2) $m(-x_0 b + y_0 a)\omega_1^2\hat{k}$

(3) $+my_0 a\omega_1^2\hat{k}$

(4) Zero

21. A plane is inclined at an angle $\alpha = 30°$ with respect to the horizontal. A particle is projected with a speed $u = 2$ ms^{-1}, from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to :

(Take $g = 10$ ms^{-2}) **[JEE (Main)-2019]**



(1) 18 cm (2) 20 cm

(3) 14 cm (4) 26 cm

22. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product $t_1 t_2$ is : **[JEE (Main)-2019]**

(1) $\frac{R}{2g}$ (2) $\frac{2R}{g}$

(3) $\frac{R}{g}$ (4) $\frac{R}{4g}$

23. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10$ ms^{-2}): **[JEE (Main)-2019]**

(1) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ms^{-1}

(2) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ms^{-1}

(3) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}$ms^{-1}

(4) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}$ms^{-1}

24. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct? **[JEE (Main)-2019]**

(1) $R^2 = 4h_1h_2$ (2) $R^2 = 16h_1h_2$

(3) $R^2 = 2h_1h_2$ (4) $R^2 = h_1h_2$

25. A particle starts from the origin at $t = 0$ with an initial velocity of $3.0\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $\left(6.0\hat{i} + 4.0\hat{j}\right)$m/s^2. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is **[JEE (Main)-2020]**

(1) 60 (2) 32

(3) 40 (4) 50

26. Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}$ ms^{-1}, a particle moves in the x-y plane with a constant acceleration of $\left(10\hat{i} + 4\hat{j}\right)$ ms^{-2}. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0 are, respectively **[JEE (Main)-2020]**

(1) 4 s and 52 m

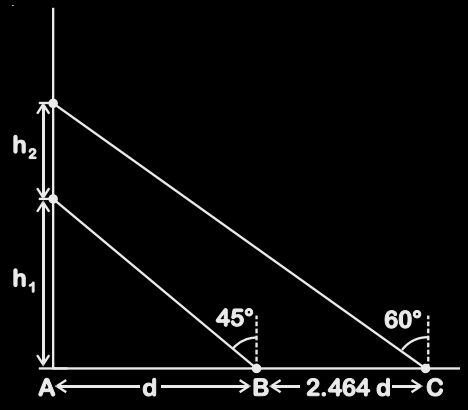
(2) 2 s and 24 m

(3) 5 s and 25 m

(4) 2 s and 18 m

27. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect

to the vertical if the girl moves further by a distance 2.464 d (point *C*). Then the height h_2 is (given tan30° = 0.5774) **[JEE (Main)-2020]**



(1) d (2) 0.732 d

(3) 1.464 d (4) 0.464 d

28. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of **[JEE (Main)-2020]**

(1) 10^{-1} (2) 10^{-2}

(3) 10^{-4} (4) 10^{-3}

29. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45°. The value of β is close to **[JEE (Main)-2020]**

(1) 0.37

(2) 0.41

(3) 0.73

(4) 0.50

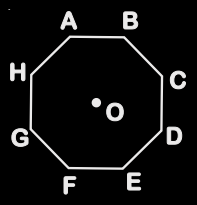
30. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is, _____. **[JEE (Main)-2020]**

31. A particle is moving along the *x*-axis with its coordinate with time '*t*' given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the *y*-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1$ s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is _____. **[JEE (Main)-2020]**

32. In an octagon ABCDEFGH of equal side, what is the sum of

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$,

if $\overrightarrow{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ **[JEE (Main)-2021]**



(1) $-16\hat{i} - 24\hat{j} + 32\hat{k}$

(2) $16\hat{i} + 24\hat{j} + 32\hat{k}$

(3) $16\hat{i} + 24\hat{j} - 32\hat{k}$

(4) $16\hat{i} - 24\hat{j} + 32\hat{k}$

33. The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. Then choose the correct relation for these vectors. **[JEE (Main)-2021]**

(1) $\vec{b} = \vec{a} + 2\vec{c}$

(2) $\vec{b} = 2\vec{a} + \vec{c}$

(3) $\vec{b} = \vec{a} - 2(\vec{a} \cdot \vec{c})\vec{c}$

(4) $\vec{b} = \vec{a} - \vec{c}$

34. The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by **[JEE (Main)-2021]**

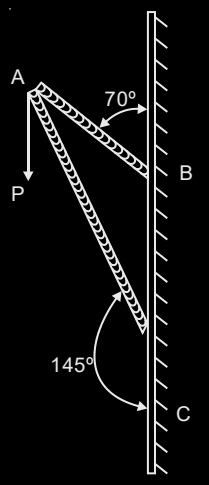
(1) $\tan^{-1}\alpha, \frac{4\alpha^2}{\beta}$

(2) $\tan^{-1}\beta, \frac{\alpha^2}{2\beta}$

(3) $\tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$

(4) $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$

35. Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $\vec{P}$ of magnitude 100 N is applied at point A of the frame. **[JEE (Main)-2021]**



Suppose the force is $\vec{P}$ resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is xN.

The value of x, to the nearest integer, is_____.

[Given : sin(35°) = 0.573,

cos(35°) = 0.819

sin(110°) = 0.939,

cos(110°) = –0.342]

36. A mosquito is moving with a velocity $\vec{v} = 0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2 s? **[JEE (Main)-2021]**

(1) $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$ from y-axis

(2) $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis

(3) $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis

(4) $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis

37. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ where x is _____.

(Round off to the Nearest Integer)

[JEE (Main)-2021]

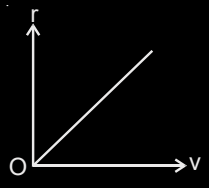
38. A person is swimming with a speed of 10 m/s at an angle of 120° with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is 'x' m/s. The value of 'x' to the nearest integer is _____.

[JEE (Main)-2021]

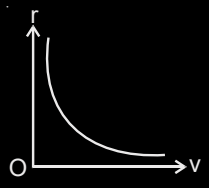
39. A particle of mass m moves in a circular orbit under the central potential field, $U(r) = -\frac{C}{r}$, where C is a positive constant.

The correct radius – velocity graph of the particle's motion is **[JEE (Main)-2021]**

(1)



(2)



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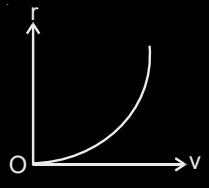

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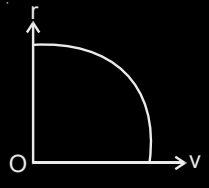
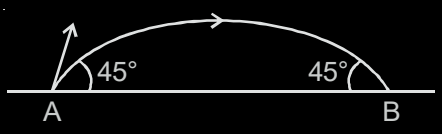


40. The projectile motion of a particle of mass 5 g is shown in the figure. **[JEE (Main)-2021]**



The initial velocity of the particle is $5\sqrt{2}$ ms^{-1} and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is $x \times 10^{-2}$ $kgms^{-1}$.

The value of x, to the nearest integer, is _____.

41. If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A} \times \vec{B} = |\vec{A} \times \vec{B}|$. Then the value of $|\vec{A} - \vec{B}|$ will be **[JEE (Main)-2021]**

(1) $\sqrt{A^2 + B^2}$ (2) $\sqrt{A^2 + B^2 - \sqrt{2}AB}$

(3) $\sqrt{A^2 + B^2 + 2AB}$ (4) $\sqrt{A^2 + B^2 + \sqrt{2}AB}$

42. Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P} + \vec{Q}$ is *n* times the magnitude of $\vec{P} - \vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is **[JEE (Main)-2021]**

(1) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

(2) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$

(3) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

(4) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$

43. A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is _____. **[JEE (Main)-2021]**

44. What will be the projection of vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on vector $\vec{B} = \hat{i} + \hat{j}$? **[JEE (Main)-2021]**

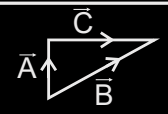
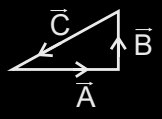
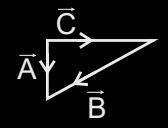
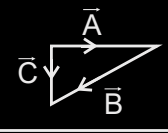
(1) $\sqrt{2}(\hat{i} + \hat{j} + \hat{k})$

(2) $(\hat{i} + \hat{j})$

(3) $\sqrt{2}(\hat{i} + \hat{j})$

(4) $2(\hat{i} + \hat{j} + \hat{k})$

45. Match **List I** with **List II**. **[JEE (Main)-2021]**

List I	List II
(a) $\vec{C} - \vec{A} - \vec{B} = 0$	(i) 
(b) $\vec{A} - \vec{C} - \vec{B} = 0$	(ii) 
(c) $\vec{B} - \vec{A} - \vec{C} = 0$	(iii) 
(d) $\vec{A} + \vec{B} = -\vec{C}$	(iv) 

Choose the correct answer from the options given below:

(1) (a) → (iv), (b) → (i), (c) → (iii), (d) → (ii)

(2) (a) → (i), (b) → (iv), (c) → (ii), (d) → (iii)

(3) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)

(4) (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)

46. Two vectors $\vec{X}$ and $\vec{Y}$ have equal magnitude. The magnitude of $(\vec{X} - \vec{Y})$ is n times the magnitude of $(\vec{X} + \vec{Y})$. The angle between $\vec{X}$ and $\vec{Y}$ is : **[JEE (Main)-2021]**

(1) $\cos^{-1}\left(\frac{n^2+1}{n^2-1}\right)$ (2) $\cos^{-1}\left(\frac{n^2+1}{-n^2-1}\right)$

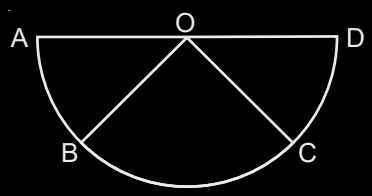
(3) $\cos^{-1}\left(\frac{-n^2-1}{n^2-1}\right)$ (4) $\cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$

47. **Assertion A :** If A, B, C, D are four points on a semi-circular arc with centre at 'O' such that $|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CD}|$, then

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$

Reason R : Polygon law of vector addition yields

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} = 2\overrightarrow{AO}$



In the light of the above statements, choose the **most appropriate** answer from the options given below **[JEE (Main)-2021]**

(1) **A** is not correct but **R** is correct.

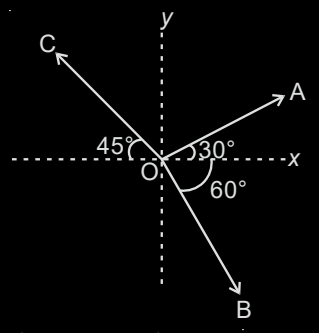
(2) **A** is correct but **R** is not correct.

(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(4) Both **A** and **R** are correct but **R** is not the correct explanation of **A**.

48. The magnitude of vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$ with *x*-axis will be

[JEE (Main)-2021]

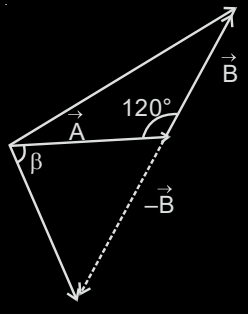


(1) $\tan^{-1}\dfrac{\left(1+\sqrt{3}-\sqrt{2}\right)}{\left(1-\sqrt{3}-\sqrt{2}\right)}$ (2) $\tan^{-1}\dfrac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1+\sqrt{3}-\sqrt{2}\right)}$

(3) $\tan^{-1}\dfrac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1-\sqrt{3}+\sqrt{2}\right)}$ (4) $\tan^{-1}\dfrac{\left(1-\sqrt{3}-\sqrt{2}\right)}{\left(1+\sqrt{3}+\sqrt{2}\right)}$

49. The angle between vector $(\vec{A})$ and $(\vec{A}-\vec{B})$ is:

[JEE (Main)-2021]



(1) $\tan^{-1}\left(\dfrac{-\frac{B}{2}}{A-B\frac{\sqrt{3}}{2}}\right)$ (2) $\tan^{-1}\left(\dfrac{\sqrt{3}B}{2A-B}\right)$

(3) $\tan^{-1}\left(\dfrac{B\cos\theta}{A-B\sin\theta}\right)$ (4) $\tan^{-1}\left(\dfrac{A}{0.7B}\right)$

50. A bomb is dropped by a fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a : **[JEE (Main)-2021]**

(1) Straight line vertically down the plane

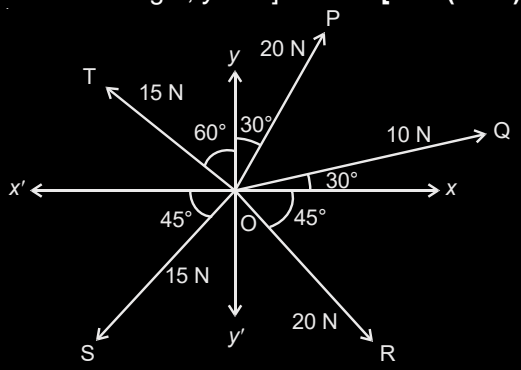
(2) Hyperbola

(3) Parabola in the direction of motion of plane

(4) Parabola in a direction opposite to the motion of plane

51. The resultant of these forces $\overrightarrow{OP}$, $\overrightarrow{OQ}$, $\overrightarrow{OR}$, $\overrightarrow{OS}$ and $\overrightarrow{OT}$ is approximately _____ N.

[Take $\sqrt{3}=1.7$, $\sqrt{2}=1.4$. Given $\hat{i}$ and $\hat{j}$ unit vectors along *x*, *y* axis] **[JEE (Main)-2021]**



(1) $9.25\hat{i}+5\hat{j}$

(2) $3\hat{i}+15\hat{j}$

(3) $-1.5\hat{i}-15.5\hat{j}$

(4) $2.5\hat{i}-14.5\hat{j}$

52. A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion? (Take g = 10 ms^{-2}) **[JEE (Main)-2021]**

(1) h_{max} = 10 m, T = 2.5 s

(2) h_{max} = 15.625 m, T = 1.77 s

(3) h_{max} = 3.54 m, T = 0.125 s

(4) h_{max} = 15.625 m, T = 3.54 s

53. A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped? **[JEE (Main)-2021]**

(1) $\sqrt{\frac{2gh}{v^2}+h^2}$ (2) $\sqrt{2ghv^2+h^2}$

(3) $\sqrt{\frac{2v^2h}{g}+h^2}$ (4) $\sqrt{\frac{2ghv^2+1}{h^2}}$

54. **Statement I:** If three forces $\vec{F}_1,\vec{F}_2$ and $\vec{F}_3$ are represented by three sides of a triangle and $\vec{F}_1+\vec{F}_2=-\vec{F}_3$, then these three forces are concurrent forces and satisfy the condition for equilibrium.

Statement II: A triangle made up of three forces $\vec{F}_1,\vec{F}_2$ and $\vec{F}_3$ as its sides taken in the same order, satisfy the condition for translatory equilibrium.

In the light of the above statements, choose the **most appropriate** answer from the options given below: **[JEE (Main)-2021]**

(1) Statement I is false but statement II is true

(2) Both statement I and statement II are false

(3) Both statement I and statement II are true

(4) Statement I is true but statement II is false

55. **Statement I :** Two forces $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$ where $\vec{P}\perp\vec{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3(P^2+Q^2)}$, when they act at an angle θ_2, the magnitude of their resultant becomes $\sqrt{2(P^2+Q^2)}$. This is possible only when $\theta_1 < \theta_2$.

Statement II : In the situation given above.

$\theta_1 = 60°$ and $\theta_2 = 90°$

In the light of the above statements, choose the **most appropriate** answer from the options given below : **[JEE (Main)-2021]**

(1) Both Statement I and Statement II are true.

(2) Statement I is true but Statement II is false.

(3) Statement I is false but Statement II is true.

(4) Both Statement I and Statement II are false.

56. The ranges and heights for two projectiles projected with the same initial velocity at angles 42° and 48° with the horizontal are R_1, R_2 and H_1, H_2 respectively. Choose the correct option:

[JEE (Main)-2021]

(1) $R_1 > R_2$ and $H_1 = H_2$

(2) $R_1 < R_2$ and $H_1 < H_2$

(3) $R_1 = R_2$ and $H_1 < H_2$

(4) $R_1 = R_2$ and $H_1 = H_2$

57. A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be [use g = 10 m/s^2]

[JEE (Main)-2022]

(1) $\frac{1}{2}\sin^{-1}\left[\frac{5t^2}{4R}\right]$

(2) $\frac{1}{2}\sin^{-1}\left[\frac{4R}{5t^2}\right]$

(3) $\tan^{-1}\left[\frac{4t^2}{5R}\right]$

(4) $\cot^{-1}\left[\frac{R}{20t^2}\right]$

58. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

[JEE (Main)-2022]

Assertion A: Two identical balls A and B thrown with same velocity 'u' at two different angles with horizontal attained the same range R. If A and B reached the maximum height h_1 and h_2 respectively, then $R = 4\sqrt{h_1h_2}$.

Reason R: Product of said heights.

$$h_1h_2 = \left(\frac{u^2\sin^2\theta}{2g}\right)\cdot\left(\frac{u^2\cos^2\theta}{2g}\right)$$

Choose the correct answer :

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(2) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

(3) **A** is true but **R** is false.

(4) **A** is false but **R** is true.

59. A fighter jet is flying horizontally at a certain altitude with a speed of 200 ms^{-1}. When it passes directly overhead an anti-aircraft gun, a bullet is fired from the gun, at an angle θ with the horizontal, to hit the jet. If the bullet speed is 400 m/s, the value of θ will be ____°. **[JEE (Main)-2022]**

60. A projectile is launched at an angle 'α' with the horizontal with a velocity 20 ms^{-1}. After 10 s, its inclination with horizontal is 'β'. The value of tanβ will be (g = 10 ms^{-2}). **[JEE (Main)-2022]**

(1) $\tan\alpha + 5\sec\alpha$ (2) $\tan\alpha - 5\sec\alpha$

(3) $2\tan\alpha - 5\sec\alpha$ (4) $2\tan\alpha + 5\sec\alpha$

61. A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2}$ kmh^{-1}, the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is **[JEE (Main)-2022]**

(1) 30 kmh^{-1} (2) $\frac{25}{\sqrt{2}}$ kmh^{-1}

(3) $\frac{30}{\sqrt{2}}$ kmh^{-1} (4) 25 kmh^{-1}

62. Motion of a particle in x-y plane is described by a set of following equations $x = 4\sin\left(\frac{\pi}{2} - \omega t\right)$ m and $y = 4\sin(\omega t)$ m. The path of the particle will be **[JEE (Main)-2022]**

(1) Circular (2) Helical

(3) Parabolic (4) Elliptical

63. A ball is spun with angular acceleration $\alpha = 6t^2 - 2t$, where t is in second and α is in $rads^{-2}$. At t = 0, the ball has angular velocity of 10 $rads^{-1}$ and angular position of 4 rad. The most appropriate expression for the angular position of the ball is :**[JEE (Main)-2022]**

(1) $\frac{3}{4}t^4 - t^2 + 10t$

(2) $\frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$

(3) $\frac{2t^4}{3} - \frac{t^3}{6} + 10t + 12$

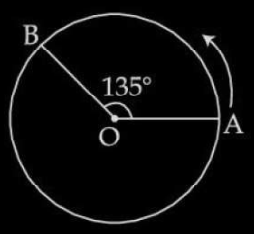
(4) $2t^4 - \frac{t^3}{2} + 5t + 4$

64. A person can throw a ball upto a maximum range of 100 m. How high above the ground he can throw the same ball? **[JEE (Main)-2022]**

(1) 25 m (2) 50 m

(3) 100 m (4) 200 m

65. A person moved from A to B on a circular path as shown in figure. If the distance travelled by him is 60 m, then the magnitude of displacement would be (Given cos135°= –0.7) **[JEE (Main)-2022]**



(1) 42 m (2) 47 m

(3) 19 m (4) 40 m

66. A ball is projected from the ground with a speed 15 ms^{-1} at an angle θ with horizontal so that its range and maximum height are equal, then '$\tan\theta$' will be equal to **[JEE (Main)-2022]**

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$

(3) 2 (4) 4

67. Two projectiles thrown at 30° and 45° with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is : **[JEE (Main)-2022]**

(1) $1:\sqrt{2}$ (2) 2 : 1

(3) $\sqrt{2}:1$ (4) 1 : 2

68. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1 - x)$. The y component vector of the initial velocity is ___ $\hat{j}$. **[JEE (Main)-2022]**

(Take g = 10 m/s²)

69. A ball of mass m is thrown vertically upward. Another ball of mass $2m$ is thrown at an angle θ with the vertical. Both the balls stay in air for the same period of time. The ratio of the heights attained by the two balls respectively is $\frac{1}{x}$. The value of x is _____. **[JEE (Main)-2022]**

70. At time $t = 0$ a particle starts travelling from a height $7\hat{z}$ cm in a plane keeping z coordinate constant. At any instant of time it's position along the $\hat{x}$ and $\hat{y}$ directions are defined at $3t$ and $5t^3$ respectively. At $t = 1$ s acceleration of the particle will be

[JEE (Main)-2022]

(1) $-30\hat{y}$

(2) $30\hat{y}$

(3) $3\hat{x} + 15\hat{y}$

(4) $3\hat{x} + 15\hat{y} + 7\hat{z}$

71. A ball is projected with kinetic energy E, at an angle of 60° to the horizontal. The kinetic energy of this hall at the highest point of its flight will become

[JEE (Main)-2022]

(1) Zero (2) $\frac{E}{2}$

(3) $\frac{E}{4}$ (4) E

72. An object is projected in the air with initial velocity u at an angle θ. The projectile motion is such that the horizontal range R, is maximum. Another object is projected in the air with a horizontal range half of the range of first object. The initial velocity remains same in both the case. The value of the angle of projection, at which the second object is projected, will be _____ degree. **[JEE (Main)-2022]**

73. If $\vec{A} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$ m and $\vec{B} = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ m. The magnitude of component of vector $\vec{A}$ along vector $\vec{B}$ will be _____m

74. A fly wheel is accelerated uniformly from rest and rotates through 5 rad in the first second. The angle rotated by the fly wheel in the next second, will be:

[JEE (Main)-2022]

(1) 7.5 rad

(2) 15 rad

(3) 20 rad

(4) 30 rad

75. A body is projected from the ground at an angle of 45° with the horizontal. Its velocity after 2 s is 20 ms^{-1}. The maximum height reached by the body during its motion is _____ m. (use g = 10 ms^{-2}) **[JEE (Main)-2022]**

76. Two projectiles are thrown with same initial velocity making an angle of 45° and 30° with the horizontal respectively. The ratio of their respective ranges will be **[JEE (Main)-2022]**

(1) $1 : \sqrt{2}$

(2) $\sqrt{2} : 1$

(3) $2 : \sqrt{3}$

(4) $\sqrt{3} : 2$

❑ ❑ ❑

Chapter 2

Motion in a Plane

1. Answer (1)

$\vec{v} = \vec{u} + \vec{a}t$

$= (3\hat{i} + 4\hat{j}) + 10(0.4\hat{i} + 0.3\hat{j})$

$= (3\hat{i} + 4\hat{j}) + (4\hat{i} + 3\hat{j})$

$= 7\hat{i} + 7\hat{j}$

$|\vec{v}| = 7\sqrt{2}$ units

2. Answer (1)

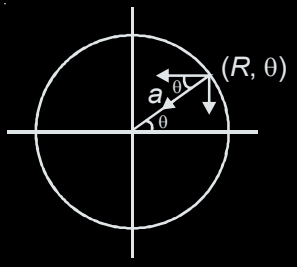
$\frac{dx}{dt} = Ky \; ; \frac{dy}{dt} = Kx$

$\frac{dy}{dx} = \frac{x}{y} \Rightarrow ydy = xdx$

$\Rightarrow \; y^2 = x^2 +$ constant

3. Answer (4)

$\vec{a} = \frac{-v^2}{R} cos\theta \, \hat{i} - \frac{v^2}{R} sin\,\theta \, \hat{j}$



4. Answer (1)

$S = t^3 + 5$

$\frac{dS}{dt} = 3t^2$

$v = 3t^2$

$\frac{dv}{dt} = 6t$

At $t = 2$ s

$v = 12$ m/s

$\Rightarrow \; a_c = \frac{(12)^2}{20} = \frac{144}{20} = 7.2$ m/s

$\frac{dv}{dt} = 12$ m/s^2

$a = \sqrt{a_c^2 + a_t^2} = \sqrt{12^2 + (7.2)^2}$

$\approx$ 14 m/s^2

5. Answer (3)

6. Answer (2)

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

Here $\tan\theta = \frac{v_y}{v_x} = 2$ also $\cos\theta = \frac{1}{\sqrt{5}}$, $u = \sqrt{5}$

$\Rightarrow \; y = 2x - \frac{10x^2}{2 \times 5 \times \frac{1}{5}} = 2x - 5x^2$

7. Answer (3)

$\frac{dx}{dt} = ky$

$\frac{dy}{dt} = kx$

$\Rightarrow \frac{dy}{dx} = \frac{x}{y}$

$\therefore \; ydy = xdx \Rightarrow ydy - xdx = 0$

$\Rightarrow \; y^2 - x^2 =$ constant

Or, $y^2 = x^2 +$ constant

8. Answer (2)

$v_x = -a\omega \sin \omega t$

$v_y = a\omega \cos \omega t$

$v_z = a\omega$

$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{2}\, a\omega$

9. Answer (1)

P.V. of $G = \frac{a}{2}(\hat{i} + \hat{k})$

P.V. of $H = \frac{a}{2}(\hat{j}+\hat{k})$

$\vec{GH} = \frac{a}{2}(\hat{j}-\hat{i})$

10. Answer (4)

$$\frac{A_1}{A_2} = \frac{\pi R_{1,\max}^2}{\pi R_{2,\max}^2} = \left(\frac{u_1^2}{u_2^2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

11. Answer (2)

Clearly, $\left|\vec{A}+\vec{B}\right|^2 = A^2 + B^2 + 2AB\cos\theta$

and $\left|\vec{A}-\vec{B}\right|^2 = A^2 + B^2 - 2AB\cos\theta$

As, $\left|\vec{A}+\vec{B}\right|^2 = n^2\left|\vec{A}-\vec{B}\right|^2$

$\therefore\ A^2 + B^2 + 2AB\cos\theta = n^2(A^2 + B^2 - 2AB\cos\theta)$

$\therefore\ A^2 + B^2 + 2AB\cos\theta = n^2A^2 + n^2B^2 - 2n^2AB\cos\theta$

$\therefore\ \cos\theta \times 2AB(1+n^2) = A^2(n^2-1) + B^2(n^2-1)$

$$\Rightarrow \cos\theta = \frac{(n^2-1)(A^2+B^2)}{2AB(1+n^2)}$$

$$\Rightarrow \cos\theta = \frac{n^2-1}{n^2+1}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$$

12. Answer (4)

Let θ be the angle between them

$\left|\vec{P}+\vec{Q}\right|^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\left|\vec{P}+\vec{R}\right|^2 = 4F^2 + 9F^2 + 12F^2\cos\theta = F_R{}^2$ ...(i)

Also, $\left|\vec{P}+2\vec{Q}\right|^2 = 4F^2 + 36F^2 + 24F^2\cos\theta = 4F_R{}^2$ ...(ii)

Then (i) and (ii)

$\therefore\ F^2 + 9F^2 + 6F^2\cos\theta = 4F^2 + 9F^2 + 12F^2\cos\theta$

$$\therefore\ -3F^2 = 6F^2\cos\theta \qquad \Rightarrow \cos\theta = \left(-\frac{1}{2}\right)$$

$\Rightarrow\ \theta = 120°$

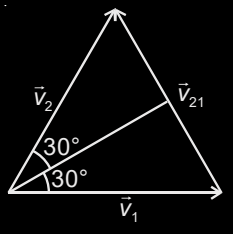
13. Answer (1)

$\vec{v}_1 + \vec{v}_{21} = \vec{v}_2$

$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1$

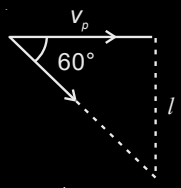
$= |\vec{v}_{21}| = 2v\sin 30°$

$= 2v \times \frac{1}{2} = v$



14. Answer (3)

$$\frac{l\,\text{cosec}\,60°}{v} = \frac{l\cot 60°}{v_p}$$



$\Rightarrow\ v_p = \frac{v}{2}$

15. Answer (1)

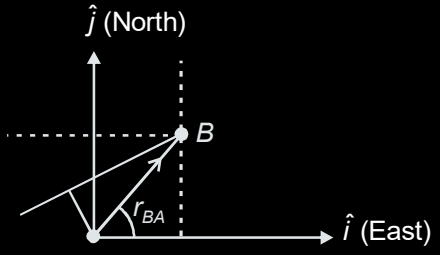
$\omega t = \omega\frac{\pi}{2\omega} = \frac{\pi}{2}$

$\overrightarrow{V_A} = \omega R_1(-\hat{i})$

$\overrightarrow{V_B} = \omega R_2(-\hat{i})$

$\overrightarrow{V_A} - \overrightarrow{V_B} = \omega[R_2 - R_1]\hat{i}$

16. Answer (4)



$\vec{v}_A = 30i + 50j$ km/ hr

$\vec{v}_{BA} = (80\hat{i} + 150\hat{j})$ km

$\vec{v}_B = (-10i)$ km/ hr

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = -10\hat{i} - 30\hat{i} - 50\hat{i} = -40\hat{i} - 50\hat{j}$

Projection of

$$(\vec{r}_{BA})\ on\ \vec{v}_{BA} = \frac{(\vec{r}_{BA})\cdot(\vec{v}_{BA})}{(\vec{v}_{BA})}$$

$$= \frac{(80\hat{i}+150\hat{j})(-40\hat{i}-50\hat{j})}{10\sqrt{41}} = \frac{\sqrt{mk}}{qB}$$

$$\therefore\ t = \frac{10\times 107}{\sqrt{41}\times 10\sqrt{41}} = \frac{107}{41} = 2.6\ \text{Hrs.}$$

17. Answer (2)

$$\left(2\vec{A_1}+3\vec{A_2}\right)\cdot\left(3\vec{A_1}-2\vec{A_2}\right)=6\left|\vec{A_1}\right|^2+5\vec{A_1}\cdot\vec{A_2}-6\left|\vec{A_2}\right|^2$$

$$=(6\times 9)+5\vec{A_1}\cdot\vec{A_2}-(6\times 25) \quad ...(i)$$

As $25=9+25+2\vec{A_1}\cdot\vec{A_2}$

$\Rightarrow \quad 2\vec{A_1}\cdot\vec{A_2}=-9$ ...(ii)

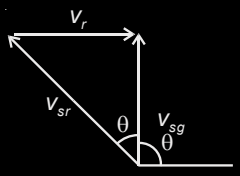
From (i) and (ii),

$$\Rightarrow \quad \left(2\vec{A_1}+3\vec{A_2}\right)\cdot\left(3\vec{A_1}-2\vec{A_2}\right)=54-22.5-150$$

$$=-118.5$$

18. Answer (4)

Draw velocity diagram,



$$\sin\theta=\frac{v_r}{v_{sr}}=\frac{1}{2}$$

$\theta = 30°$

$\phi = 90 + \theta = 120°$

19. Answer (1)

$$\vec{r}=15t^2\hat{i}+4\hat{j}-20t^2\hat{j}$$

$$\frac{d\vec{r}}{dt}=30t\hat{i}-40t\hat{j}$$

$$\frac{d^2\vec{r}}{dt^2}=30\hat{i}-40\hat{j}$$

$$\therefore \quad \left|\frac{d^2\vec{r}}{dt^2}\right|=50 \text{ m/s}^2$$

20. Answer (3)

$x = x_0 + a \cos \omega_1 t$

$y = y_0 + b \sin \omega_2 t$

$\Rightarrow \quad v_x = -a\omega_1 \sin(\omega_1 t), \qquad v_y = b\omega_2 \cos(\omega_2 t)$

$$a_x=-a\omega_1^2\cos(\omega_1 t), \quad a_y=-b\omega_2^2\sin(\omega_2 t)$$

At $t = 0$, $\quad x = x_0 + a, \qquad y = y_0$

$$a_x = -a\omega_1^2, \qquad a_y = 0$$

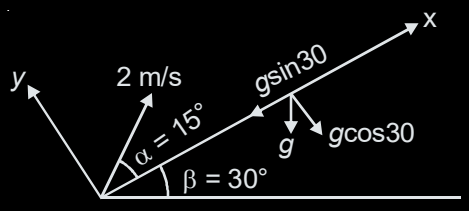
$$\therefore \quad \vec{\tau}=m(-a\omega_1^2)\times y_0(-\hat{k})$$

$$=+my_0a\omega_1^2\,\hat{k}\,.$$

21. Answer (2)

$$\text{Time of flight } (T)=\frac{2u\sin\alpha}{g\cos\beta}$$

$$T=\frac{(2)(2\sin 15)}{g\cos 30}=\frac{4}{10}\frac{\sin 15}{\cos 30}$$



$$\text{Range } (R)=(2\cos 15)T-\frac{1}{2}g\sin 30(T)^2$$

$$=(2\cos 15)\frac{4}{10}\frac{\sin 15}{\cos 30}-\left(\frac{1}{3}\times 10\sin 30\right)\frac{16}{100}\frac{\sin^2 15}{\cos^2 30}$$

$$=\frac{16\sqrt{3}-16}{60}\simeq 0.1952 \text{ m}\simeq 20 \text{ cm}$$

22. Answer (2)

For same horizontal range,

$\theta_1 = \theta$ $\qquad R=\dfrac{u^2\sin 2\theta}{g}$

$\theta_2 = (90 - \theta)$

so $\quad t_1=\dfrac{2u\sin\theta}{g}$ *and* $t_2=\dfrac{2u\cos\theta}{g}$

$$\therefore \quad t_1t_2=\frac{u^2 4\sin\theta\cos\theta}{g^2}$$

$$\Rightarrow \quad t_1t_2=\frac{2u^2\sin 2\theta}{g^2}=\frac{2R}{g}$$

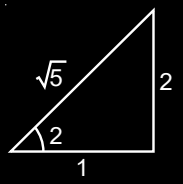
23. Answer (3)

$y = 2x - 9x^2$

Comparing it with equation of trajectory

$$y=x\tan\theta-\frac{g\,x^2}{24^2\cos^2\theta}$$

$\therefore \quad \tan\theta = 2$



And $9=\dfrac{10\times 5}{2\,v_0^2}$

$$\Rightarrow \quad v_0=\frac{5}{3}\text{ m/s}$$

24. Answer (2)

At complementary angles, ranges are equal.

$$\therefore \quad h_1 = \frac{u^2 \sin^2\theta}{2g},\ h_2 = \frac{u^2 \cos^2\theta}{2g}$$

$$\therefore \quad h_1 \times h_2 = \left(\frac{2u^2 \sin\theta\cos\theta}{g}\right)^2 \times \left(\frac{1}{16}\right)$$

$$\Rightarrow \quad 16h_1h_2 = R^2$$

25. Answer (1)

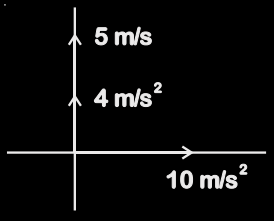
$$\frac{1}{2} \times 4 \times t^2 = 32$$

t = 4 s

$$x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2 = 60$$

26. Answer (4)

At t = t, (20 m, y_0)



$$\therefore \quad t = \sqrt{\frac{20 \times 2}{10}}$$

= 2 s

$$\therefore \quad y_0 = 5 \times 2 + \frac{1}{2} \times 4 \times 2^2$$

= 18 m

27. Answer (1)

$$\frac{h_1}{d} = \tan 45°$$

$$\frac{h_1 + h_2}{d + 2.464d} = \tan 30°$$

$\Rightarrow$ h_2 = d

28. Answer (4)

$a = \omega^2 \times \ell$

$$= \left(\frac{2\pi}{T}\right)^2 \times \ell$$

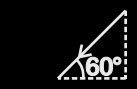
$$= \left(\frac{2\pi}{60}\right)^2 \times 0.1$$

$= 1.1 \times 10^{-3}$ m/s²

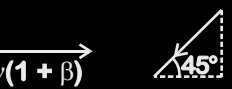
29. Answer (3)

Given

$$\frac{\nu^1}{\nu} = \tan 60 \quad ...(i)$$



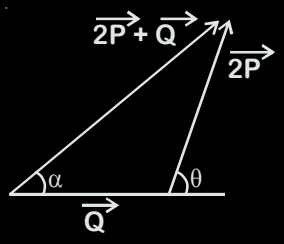
$$\frac{\nu^1}{\nu(1+\beta)} = \tan 45° \quad ...(ii)$$



From (i) and (ii)

β = 0.73

30. Answer (90)



$P^2 = P^2 + Q^2 + 2PQ.\cos\theta$

$$\Rightarrow \quad \cos\theta = -\frac{Q}{2P} ...(i)$$

$$\tan\alpha = \frac{2P\sin\theta}{Q + 2P\cos\theta} = \infty$$

$\Rightarrow$ α = 90°

31. Answer (580)

For particle 'A'	For particle 'B'
$X_A = -3t^2 + 8t + 10$	$Y_B = 5 - 8t^3$
$\vec{V}_A = (8 - 6t)\hat{i}$	$\vec{V}_B = -24t^2\,\hat{j}$
$\vec{a}_A = -6\hat{i}$	$\vec{a}_B = -48t\,\hat{j}$

at t = 1 sec

$$\vec{v}_A = 2\hat{i},\ \vec{v}_B = -24\hat{j}$$

$$\therefore \quad \vec{v}_{B/A} = -\vec{v}_A + \vec{v}_B = -2\hat{i} - 24\hat{j}$$

Speed of B w.r.t. A, $\left|\vec{v}_{B/A}\right| = \sqrt{4 + 576} = \sqrt{580}$

$$\therefore \quad v = 580\ (\text{m/s})^2$$

32. Answer (3)

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$$

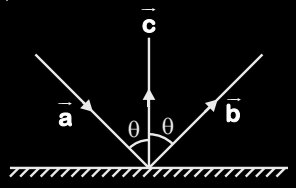
$$= 8\overrightarrow{AO}$$

33. Answer (3)

$\vec{b} - \vec{a} = 2\cos\theta\vec{c}$

$\vec{b} = \vec{a} + 2\cos\theta\vec{c}$

$\vec{b} = \vec{a} - 2(\vec{a}\cdot\vec{c})\vec{c}$



34. Answer (4)

$y = \alpha x - \beta x^2$

$\Rightarrow \quad \tan\theta = \alpha \quad \Rightarrow \quad \theta = \tan^{-1}\alpha$

also, $\frac{dy}{dx} = \alpha - 2\beta x$

$\frac{dy}{dx} = 0 \quad \Rightarrow \quad x = \frac{\alpha}{2\beta}$

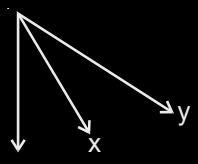
$y = \frac{\alpha^2}{2\beta} - \frac{\alpha^2}{4\beta} = \frac{\alpha^2}{4\beta}$ at $x = \frac{\alpha}{2\beta}$

35. Answer (164)

Let component be x, y

$x\cos35° + y\cos70° = P$

$x\sin35° + y\sin70° = 0$



$y = \frac{-x}{2\cos35°}$

$x\cos35° - \frac{x\cos70°}{2\cos35°} = P$

$x = 2P\cos35°$

36. Answer (1)

$\vec{v} = \left(0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}\right)$ m/s

At t = 2 s

$\vec{v} = \left(2\hat{i} + 6\hat{j} + 9\hat{k}\right)$

$\cos\theta = \frac{\left(v.\hat{j}\right)}{\sqrt{9^2 + 6^2 + 2^2}} = \frac{6}{\sqrt{121}} = \frac{6}{11}$

$\sin\theta = \frac{\sqrt{85}}{11}$

$\tan\theta = \frac{\sqrt{85}}{6}$

37. Answer (4)

$v = \sqrt{\frac{1}{k\rho}}$

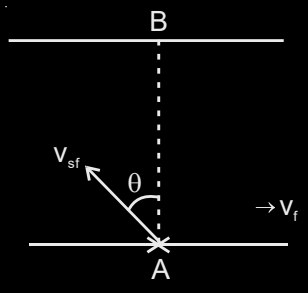
$\frac{3v_1}{4L} = \frac{2v_2}{2L_2}$

$\Rightarrow L_2 = \frac{4}{3}L\left(\frac{v_2}{v_1}\right)$

$= \frac{4}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$

x = 4

38. Answer (5)



$\sin\theta = \frac{v_f}{v_{sf}}$ $\qquad$ $\sin30° = \frac{x}{10}$

x = 5

39. Answer (2)

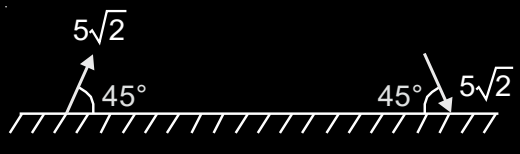
$F_r = -\frac{dU}{dr}$

$= -\frac{d}{dr}\left(-\frac{C}{r}\right) = -\frac{C}{r^2}$

$\Rightarrow \quad \frac{mv^2}{r} = \frac{C}{r^2}$

$\Rightarrow \quad v^2 = \frac{C}{mr}$

40. Answer (5)



$\left|\overrightarrow{\Delta P}\right| = 2 \times (5 \times 10^{-3})\ (5)$

$= 5 \times 10^{-2}$ kg ms^{-1}

41. Answer (2)

$\vec{A}\cdot\vec{B} = |\vec{A}\times\vec{B}|$

$\Rightarrow$ ABcosθ = ABsinθ

$\Rightarrow$ cosθ = sinθ $\Rightarrow$ $\theta = 45°$

$\therefore |\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos\theta}$

$= \sqrt{A^2+B^2-\sqrt{2}AB}$

42. Answer (1)

$|\vec{P}+\vec{Q}|^2 = n^2|\vec{P}-\vec{Q}|^2$

$\Rightarrow$ $P^2 + Q^2 + 2PQ\cos\theta = n^2 (P^2 + Q^2 - 2PQ\cos\theta)$

$\Rightarrow$ $1 + 1 + 2\cos\theta = n^2 (1 + 1 - 2\cos\theta)$

$\Rightarrow$ $1 + \cos\theta = n^2 (1 - \cos\theta)$

$\Rightarrow \dfrac{1+\cos\theta}{1-\cos\theta} = \dfrac{n^2}{1}$

$\Rightarrow \dfrac{1}{\cos\theta} = \dfrac{n^2+1}{n^2-1}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{n^2-1}{n^2+1}\right)$

43. Answer (200)

$4\pi^2\dfrac{(18^2-6^2)10^4}{60^2} = \dfrac{2\times1200}{10}\times\dfrac{\pi 2\times\theta}{60}$

$\theta = \dfrac{\pi\times24\times12\times10^4}{120\times60} = \dfrac{\pi24\times10^3}{60}$

$\theta = \dfrac{(12000)(2\pi)}{60}$

$= (200)2\pi$

44. Answer (2)

Projection $= \dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|}(\hat{B})$

$= \dfrac{(\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}+\hat{j})}{\sqrt{2}}\dfrac{(\hat{i}+\hat{j})}{\sqrt{2}}$

$= \dfrac{2}{\sqrt{2}}\dfrac{(\hat{i}+\hat{j})}{\sqrt{2}}$

$= (\hat{i}+\hat{j})$

45. Answer (4)

(a) $\vec{C} = \vec{A}+\vec{B}$ $\Rightarrow$ (iv)

(b) $\vec{A} = \vec{B}+\vec{C}$ $\Rightarrow$ (iii)

(c) $\vec{B} = \vec{A}+\vec{C}$ $\Rightarrow$ (i)

(d) $-\vec{C} = \vec{A}+\vec{B}$ $\Rightarrow$ (ii)

46. Answer (4)

$|\vec{X}+\vec{Y}|^2 = X^2 + Y^2 + 2XY\cos\theta$

$|\vec{X}-\vec{Y}|^2 = X^2 + Y^2 - 2XY\cos\theta$

$n^2 = \dfrac{2-2\cos\theta}{2+2\cos\theta}$

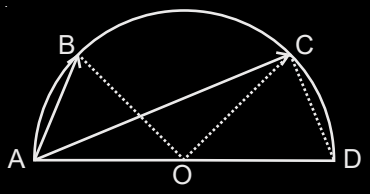
$n^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

$n^2 + n^2\cos\theta = 1 - \cos\theta$

$\cos\theta = -\left(\dfrac{n^2-1}{n^2+1}\right)$

47. Answer (4)

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}$



$= (\overrightarrow{AO}+\overrightarrow{OB})+(\overrightarrow{AO}+\overrightarrow{OC})+2\overrightarrow{AO}$

$= 4\overrightarrow{AO}+\overrightarrow{OB}+\overrightarrow{OC}$

48. Answer (4)

Sum of *x* component,

R_x = acos30° + acos60° + acos45°

$= a\left(\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}+\dfrac{1}{\sqrt{2}}\right)$

$= a\dfrac{\sqrt{3}+1+\sqrt{2}}{2}$

Sum of *y* component,

Ry = a sin30° – a sin60° – a sin45°

$= \dfrac{a}{2}-\dfrac{a\sqrt{3}}{2}+\dfrac{a}{\sqrt{2}} = a\dfrac{1-\sqrt{3}-\sqrt{2}}{2}$

Now, $\tan\theta = \dfrac{R_y}{R_x} = \dfrac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+1+\sqrt{2}}$

$\theta = \tan^{-1}\left(\dfrac{1-\sqrt{3}-\sqrt{2}}{1+\sqrt{3}+\sqrt{2}}\right)$

49. Answer (2)

Angle between $\vec{A}$ and $\vec{B}$ = 60°

So angle between $\vec{A}$ and $-\vec{B}$ = 120°

If angle between $\vec{A}$ and $\vec{A}-\vec{B}$ is θ

then $\tan\theta = \dfrac{|-\vec{B}|\sin\theta}{\vec{A}+|-\vec{B}|\cos\theta}$

$$= \frac{B\sin120°}{A+B\cos120°} = \frac{B\frac{\sqrt{3}}{2}}{A-\frac{B}{2}}$$

$$= \frac{\sqrt{3}\,B}{2A-B}$$

$$\theta = \tan^{-1}\left(\frac{B\sqrt{3}}{2A-B}\right)$$

50. Answer (1)

Relative velocity of bomb w.r.t. observer in plane = 0

Bomb will fall down vertically. So, it will move in straight line w.r.t. observer.

51. Answer (1)

$$\sum F_n = 10\times\frac{\sqrt{3}}{2} + 20\times\frac{1}{2} - 15\times\frac{\sqrt{3}}{2} - 15\times\frac{1}{\sqrt{2}} + 20\times\frac{1}{\sqrt{2}}$$

= 8.5 + 10 – 12.75 – 10.71 + 14.28 = 9.32

$$\sum F_y = 10\times\frac{1}{2} + 20\times\frac{\sqrt{3}}{2} + 15\times\frac{1}{2} - 15\times\frac{1}{\sqrt{2}} - 20\times\frac{1}{\sqrt{2}}$$

= 5 + 17 + 7.5 – 10.71 – 14.28 = 4.54

$$\vec{F}_{net} = 9.32\hat{i} + 4.54\hat{j}$$

52. Answer (2)

$$h_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{25^2\times\frac{1}{2}}{2\times10} = 15.625 \text{ m}$$

$$T = \frac{u\sin\theta}{g} = \frac{25\times\frac{1}{\sqrt{2}}}{10} = 1.77 \text{ s}$$

53. Answer (3)

Time of flight $= \sqrt{\dfrac{2h}{g}}$

Now displacement in horizontal direction $= \sqrt{\dfrac{2h}{g}}\,v$

So, distance of man from helicopter

$$= \sqrt{h^2 + \left(\sqrt{\frac{2h}{g}}\,v\right)^2}$$

$$= \sqrt{h^2 + \frac{2v^2h}{g}}$$

54. Answer (3)

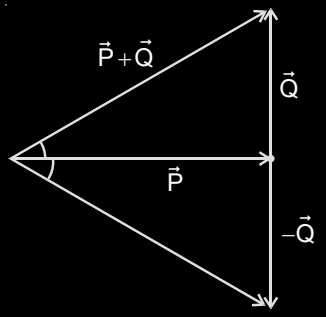
As, $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

So three forces are concurrent and object is in equilibrium.

Also if $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

then $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ lie along sides of triangle taken in order

55. Answer (1)



$$|\vec{P}+\vec{Q}+\vec{P}-\vec{Q}| = \sqrt{3}\sqrt{P^2+Q^2}$$

$4P^2 = 3(P^2 + Q^2)$

$P^2 = 3Q^2$

$$\theta_1 = 2\tan^{-1}\left(\frac{Q}{P}\right)$$

= 60°

For case-II

$4P^2 = 2(P^2 + Q^2)$

$P = Q$

$$\theta_2 = 2\tan^{-1}\left(\frac{Q}{P}\right) = 90°$$

56. Answer (3)

$$R = \frac{U^2\sin2\theta}{g}$$

$$H = \frac{U^2\sin^2\theta}{2g}$$

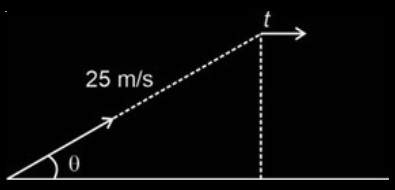
$\theta_1 + \theta_2 = 90$

$\Rightarrow R_1 = R_2$

$\theta_1 < \theta_2$

$\Rightarrow H_1 < H_2$

57. Answer (4)



$$t = \frac{25\sin\theta}{g}$$

and, $R = \frac{(25)^2(2\sin\theta\cos\theta)}{g}$

$$\Rightarrow \quad R = \frac{25\times25\times2}{g}\times\frac{gt}{25}\times\cos\theta$$

$$\Rightarrow \quad R = 50t\cos\theta$$

$$\therefore \quad \tan\theta = \frac{gt}{25}\times\frac{50t}{R}$$

$$= \frac{20t^2}{R}$$

$$\Rightarrow \quad \theta = \cot^{-1}\left(\frac{R}{20t^2}\right)$$

58. Answer (1)

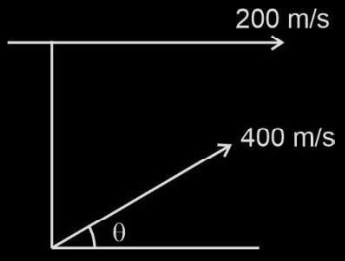
$$h_1 = \frac{u^2\sin^2\theta}{2g}$$

$$h_2 = \frac{u^2\cos^2\theta}{2g}$$

$$\therefore \quad \sqrt{h_1h_2} = \frac{u^2\sin\theta\cos\theta}{2g} = \frac{R}{4}$$

$$\Rightarrow \quad R = 4\sqrt{h_1h_2}$$

59. Answer (60)



To hit the jet

400 cosθ = 200

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60°$$

60. Answer (2)

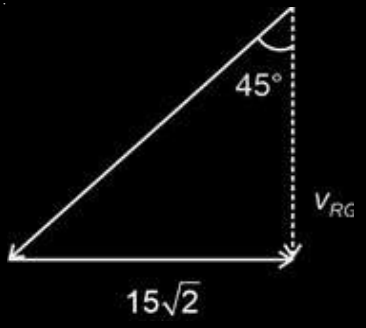
$$v_y = 20\times\sin\alpha - 10\times10$$

$$v_x = 20\cos\alpha$$

$$\therefore \quad \tan\beta = \frac{v_y}{v_x} = \frac{20\sin\alpha - 100}{20\cos\alpha}$$

$$= \tan\alpha - 5\sec\alpha$$

61. Answer (3)



From graph,

$$v_{RG} = 15\sqrt{2}\tan45°$$

$$= 15\sqrt{2}$$

$$= \frac{30}{\sqrt{2}}$$

62. Answer (1)

$$x = 4\sin\left(\frac{\pi}{2} - \omega t\right)$$

$$= 4\cos(\omega t)$$

$$y = 4\sin(\omega t)$$

$\Rightarrow$ $x^2 + y^2 = 4^2$

$\Rightarrow$ The particle is moving in a circular motion with radius of 4 m.

63. Answer (2)

$$\alpha = \frac{d\omega}{dt} = 6t^2 - 2t$$

$$\int_0^{\omega} d\omega = \int_0^t (6t^2 - 2t)dt$$

so $\omega = 2t^3 - t^2 + 10$

and $\frac{d\theta}{dt} = 2t^3 - t^2 + 10$

so $\int_4^{\theta} d\theta = \int_0^t (2t^3 - t^2 + 10)dt$

$\theta = \frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$

64. Answer (2)

$R_{max} = \frac{u^2}{g} = 100 \text{ m}$

So, $H_{max} = \frac{u^2}{2g} = 50 \text{ m}$

65. Answer (2)

Distance travelled = 60 m

$\Rightarrow$ Angle covered = 135°

$\text{Displacement} = 2R\sin\left(\frac{135°}{2}\right)$

$= 2\left(\frac{60}{135} \times \frac{180}{\pi}\right)\left[\frac{1-\cos(135°)}{2}\right]^{1/2}$

$= 2\left(\frac{80}{\pi}\right)(0.85)^{1/2}$

≈ 47 m

66. Answer (4)

$\because \quad \tan\theta = \frac{4H}{R}$

$\Rightarrow \tan\theta = 4 \times 1$

$\Rightarrow \tan\theta = 4$

67. Answer (3)

$\because \quad t_a = \frac{u\sin\theta}{g}$

$\Rightarrow \quad \frac{u_1\sin(30°)}{g} = \frac{u_2\sin(45°)}{g}$

$\Rightarrow \quad \frac{u_1}{u_2} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{\sqrt{2}}{1}$

68. Answer (5)

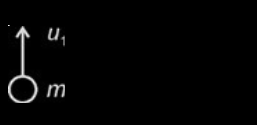
$y = 5x - 5x^2$

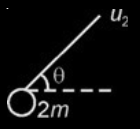
$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{v^2}$

$\tan\theta = 5 = \frac{u_y}{u_x}$

$\Rightarrow u_y = 5$

69. Answer (1)





$T_1 = \frac{2u_1}{g}$ $\qquad$ $T_2 = \frac{2u_2\sin\theta}{g}$

$\therefore \quad u_1 = u_2\sin\theta$

$\frac{H_1}{H_2} = \frac{\frac{u_1^2}{2g}}{u_2^2\frac{\sin^2\theta}{2g}}$

$= \left(\frac{u_1}{u_2\sin\theta}\right)^2$

$= 1$

70. Answer (2)

$x = 3t \Rightarrow a_x = 0$

$y = 5t^3 \Rightarrow a_y = 30t$

$\vec{a}(t = 1) = 30\hat{y}$

71. Answer (3)

$\text{K}\cdot\text{E}\cdot = E = \frac{1}{2}mv^2$

at highest point

$\text{K}\cdot\text{E}' = \frac{1}{2}mv^2\cos^2\theta$

$= \frac{1}{2}mv^2\left(\frac{1}{4}\right)$

$= \frac{E}{4}$

72. Answer (15)

$\theta = 45°$

$R_1 = \frac{R}{2}$

$\frac{u^2 \sin 2\theta_1}{g} = \frac{u^2 \sin(90°)}{2g}$

$\Rightarrow \quad 2\theta_1 = 30°$

$\theta_1 = 15°$

73. Answer (2)

$A\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{B}|} = \frac{2+6-2}{3} = \frac{6}{3} = 2$

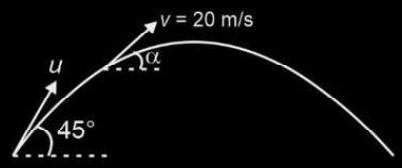
74. Answer (2)

$\theta_1 = \frac{1}{2}\alpha(2\times 1 - 1) = 5 \text{ rad}$

$\Rightarrow \quad \alpha = 10 \text{ rad/sec}^2$

So $\theta_2 = \frac{1}{2} \times \alpha\,(2 \times 2 - 1) = 15$ rad

75. Answer (20)



$\Rightarrow \quad v\cos\alpha = u\cos 45° \qquad \ldots(i)$

& $v\sin\alpha = u\sin 45° - gt \quad \ldots(ii)$

Solve for u we get

$u = 20\sqrt{2}$ m / s

$\Rightarrow \quad H = \frac{u^2 \sin^2 45°}{20} = 20$ m

76. Answer (3)

$R = \frac{u^2 \sin 2\theta}{g}$

$\Rightarrow \quad \frac{R_1}{R_2} = \frac{\sin 2\theta_1}{\sin 2\theta_2} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

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