Pracical 7(a): Trapezoidal rule

Trapezoidal rule formula:

∫f(x)dx over [a,b] is

h/2[f(a)+f(b)], h=b-a.
Composite trapezoidal rule formula for n equally spaced subintervals:

∫f(x)dx over [a,b] is

h/2[f(a)+2f(x_1)+2f(x_2)+.....+2f(x_(n-1))+f(b)],

where h=b-a/n, x_0=a, x_1= a+h, x_2=a+2h (x_1+h),.......,x_(n-1)=a+(n-1)h, x_n=b.

Q1. Approximate the integral of f(x) =1/(1+x^2) on the interval [0,1] using the trapezoidal rule.

```
(%i6) kill(all)$;
      f(x):=1/(1+x^2);
      a=a:0;
      b=b:1;
      h=h:(b-a);
      print("Integral of ",f(x)," from 0 to 1 =",
      (float(h/2·(f(a)+f(b)))))$
      wxplot2d(f(x),[x,a,b]);
```

(%o1) $f(x):=\frac{1}{1+x^2}$

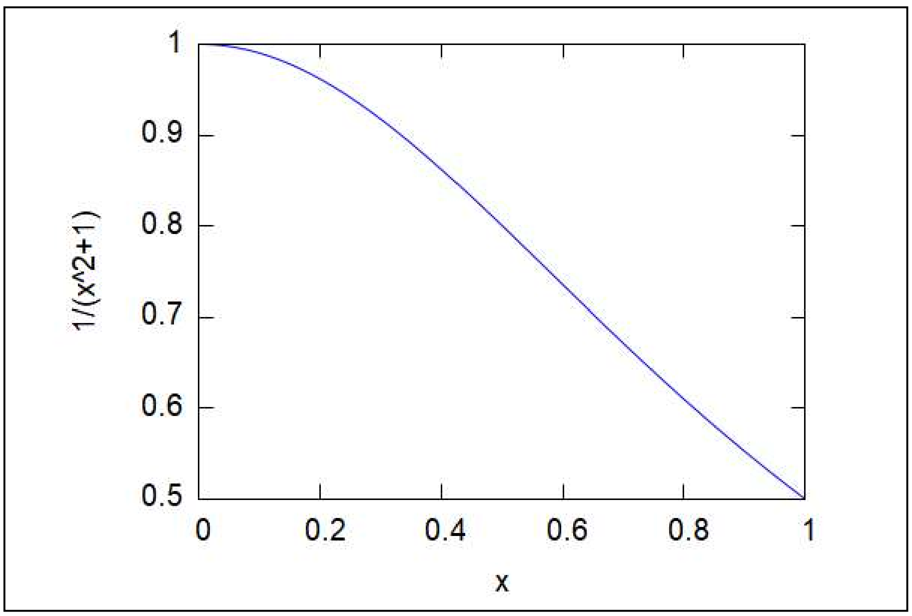
(%o2) $a=0$

(%o3) $b=1$

(%o4) $h=1$

Integral of $\frac{1}{x^2+1}$ from 0 to 1 = 0.75

(%t6)



(%o6)

Q2. Approximate the integral of f(x) =1/(1+x^2)
on the interval [0,1] using
the composite trapezoidal rule with n=6.

```
kill(all)$;
f(x):=1/(1+x^2);
a=a:0;
b=b:1;
n=n:6;
h=h:(b-a)/n;/* defining the step size */
for i:0 thru n do
(
    x[i]:a+((i)·h),/*x[0]=a, x[1]=a+h, x[2]=a+2h.....*/
    y[i]:float(f(x[i])),
    print('x[i]=x[i],"       ",'y[i]=y[i])
)$
sum=sum:0$
for i:1 thru n-1 do /*leaving x_0=a, x_n=b*/
(
    sum:float(sum + (2·y[i]))/*i=1, sum=0+2y[1], i=2,
    sum=2y[1]+2y[2],
     i=3, sum=2y[1]+2y[2]+2y[3],.......,i=n-1,
    sum=2y[1]+2y[2]+2y[3]+...2y[n-1] */
)$
print("Integral of ",f(x)," from 0 to 1 =",
    ( float((h/2·( y[0] + sum + y[n])))))$
wxplot2d(f(x),[x,a,b]);
```

(%o1) $f(x):=\frac{1}{1+x^2}$

(%o2) $a=0$

(%o3) $b=1$

(%o4) $n=6$

(%o5) $h=\frac{1}{6}$

$x_0=0$ $y_0=1.0$

$x_1=\frac{1}{6}$ $y_1=0.972972972972973$

$x_2=\frac{1}{3}$ $y_2=0.9$

$x_3=\frac{1}{2}$ $y_3=0.8$

$x_4=\frac{2}{3}$ $y_4=0.6923076923076923$

$x_5=\frac{5}{6}$ $y_5=0.5901639344262295$

$x_6=1$ $y_6=0.5$

Integral of $\frac{1}{x^2+1}$ from 0 to 1 = 0.7842407666178157