Pracical 7(b): Simpson's rule

Simpson's rule formula: It requires three points $x_0=a$, $x_1=x_0+h$, $x_2=b$, $h=(b-a)/2$

∫f(x)dx over [a,b] is

$h/3[f(a)+4f(x_1)+f(b)]$.

Composite Simpson's rule formula for n (n is even) subintervals:

∫f(x)dx over [a,b] is

$h/3[f(a)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+.....+2f(x_{(n-2)})+4f(x_{(n-1)})+f(b)]$,

where $h=b-a/n$, $x_0=a$, $x_1= a+h$, $x_2=a+2h$ (x_1+h),.....
..,$x_{(n-1)}=a+(n-1)h$, $x_n=b$.

Question: Evaluate I = integrate(1/(5+3x), x, 1, 2)
using the Simpson’s rule.

(%i11)
```
kill(all)$
f(x):= 1/(5+3·x);
a = a: 1;
b = b: 2;
h = h: (b-a)/2;     /* Step Size */
print("                         ", "x", "
 ", "y=f(x)")$
for i:0 thru 2 do
(
    x[i]: a+((i)·h),
    y[i]:float(f(x[i])),
    print("                   ",'x[i] = x[i],"
 ",'y[i] =y[i])
)$
I[0]: float((h/3·(y[0] + 4·y[1] + y[2])))$
print("Thus, the approximate value of the integration is:")$
'I[0] = 'integrate(1/(5+3·x), x, 1, 2);/*' sign before I[0]
stops the evaluation of I[0] and prints I[0] as it is.*/
print('I[0] = I[0])$
wxplot2d(f(x),[x,a,b]);
```

(%o1) $f(x):=\frac{1}{5+3\,x}$

(%o2) $a=1$

(%o3) $b=2$

(%o4) $h=\frac{1}{2}$

x $y=f(x)$

$x_0=1$ $y_0=0.125$

$x_1=\frac{3}{2}$ $y_1=$
0.1052631578947368

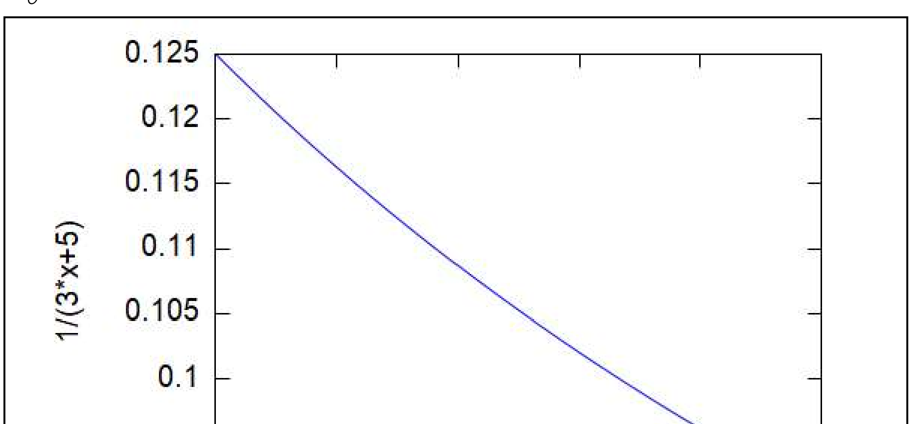
$x_2=2$ $y_2=$
0.09090909090909091

Thus, the approximate value of the integration is:

(%o9) $I_0=\int_1^2 \frac{1}{3\,x+5}$

$I_0=0.1061602870813397$

(%t11)



Question: Evaluate I = integrate(1/(5+3x), x,1, 2) using the composite Simpson’s rule with 8 subintervals.

(%i15)
```
kill(all)$
f(x):= 1/(5+3·x);
a = a: 1;
b = b: 2;
n = n: 8;
h = h: (b-a)/n;
print(" ", "x",
 " ", "y=f(x)")$
for i:0 thru n do
(
 x[i]: a+((i)·h),
 y[i]:float(f(x[i])),
 print(" ",'x[i] = x[i],
 " ",'y[i] =y[i])
)$
sum2=sum2:0$
sum4=sum4:0$
for i:1 thru n-1 do
(
 if (equal(mod(i, 2), 0))
 then(sum2: float(sum2 + y[i]))/*sum2=0+y[2]+y[4]+y[6]...
 .. and sum4=0+y[1]+y[3]+y[5]+.....*/
 else (sum4: float(sum4 + y[i]))
)$
I[0]: float((h/3·(y[0] + 2·sum2 + 4·sum4 + y[n])))$
print("Thus, the approximate value of the integration is:")$
'I[0] = 'integrate(1/(5+3·x), x, 1, 2);
print('I[0] = I[0])$
wxplot2d(f(x),[x,a,b]);
```

(%o1) $f(x):=\frac{1}{5+3x}$

(%o2) $a=1$

(%o3) $b=2$

(%o4) $n=8$

(%o5) $h=\frac{1}{8}$

x $y=f(x)$

$x_0=1$ $y_0=0.125$

$x_1=\frac{9}{8}$ $y_1=0.1194029850746269$

$x_2=\frac{5}{4}$ $y_2=0.1142857142857143$

$x_3=\frac{11}{8}$ $y_3=0.1095890410958904$

$x_4=\frac{3}{2}$ $y_4=0.1052631578947368$

$x_5=\frac{13}{8}$ $y_5=0.1012658227848101$

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