

DESIGN AND ANALYSIS OF ALGORITHMS  
ASSIGNMENT - I

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I(a):-

Let  $T(n)$  represent the running time of sorting an array of size  $n$  using the recursive insertion sort:-

- ①- Sort the array  $A[1:n-1]$  recursively, which take  $T(n-1)$  time
- ②- Insert the element  $A[n]$  into the sorted array  $A[1:n-1]$ .  
This step required finding the correct position for the element  $A[n]$  in the sorted subarray and shifting the elements if necessary.

In worst case

to compare  $A[n]$  with half of the element in the sorted subarray which is  $n/2$  comparisons

recurrence Relation

$$T(n) = T(n-1) + n/2$$

This recurrence relation state that the running time of sorting an array of size  $n$  is equal to running time of sorting an array of size  $(n-1)$  plus the time required to insert the element  $A[n]$  into the sorted array  $A[1:n-1]$ , which is  $n/2$  on average.

⑤ what are the minimum and maximum --- height  $h$ :

→ minimum number of element ( $N_{\min}$ ):

In Binary heap, the minimum number of element occurs when the tree is perfectly balanced.

No. of node at height  $h$  given by =  $2^h$

minimum no. of element ( $N_{\min}$ ) =  $2^h$

→ maximum number of element ( $N_{\max}$ ):

The maximum no. element occurred when the heap is complete binary tree up to height  $h-1$ .

max. no. of node =  $2^{(h+1)} - 1$

maximum no. of element =  $2^{(h+1)} - 1$

⑥ Among the given option, insertion sort is stable sorting algorithm, while Quicksort is not.

A stable sorting algorithm maintains the relative order of elements with equal keys. In other word, if elements have same key and appear in particular order in the input, a stable algorithm will ensure that they remain in the same order in the sorted output.

Example:-

we have following array elements with key-value pairs -

[ (5,A), (2,B), (5,C), (1,D), (2,E) ]

using B sorting sort:-

[ (1,D), (2,B), (2,E), (5,A), (5,C) ]

If we use Quicksort on the same input:-

[ (1,D), (2,E), (2,B), (5,A), (5,C) ]

→ the element with equal key, such as  
(2,B) and (2,E) have switched positions  
compared to their original order.

② When determining whether dynamic programming is applicable for an optimization problem, there are two factors considered -

① Overlapping subproblem:

Solving a problem by breaking it down into smaller overlapping subproblems. If the problem can be divided into subproblems that are solved independently and the same problems are encountered multiple times.



Optimal substructure:

If the optimal solution to the problem can be constructed from the optimal solution of its subproblem. If we can recursively breakdown the problem into smaller subproblem and combine their solution to obtain the optimal solution for the original problem.