**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans)We have a normal distribution with mean= 45 and std= 8 Let X be the amount of time it takes to complete the repair on a customer's car

To finish in one hour you must have X ≤ 50 we need to find Pr(X > 50)

Pr(X > 50) = 1 -Pr(X ≤ 50)

Z = (X -mean)/std = (X -45)/8

Pr(X ≤ 50) = Pr(Z ≤ (50 -45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans)A)We have a normal distribution with mean = 38 and std = 6. Let X be the number of employees

Probability of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 -Pr(X ≤ 44).

Z = (X -mean)/ std = (X -38)/6

Pr(X ≤ 44) = Pr(Z ≤ (44-38)/6) = Pr(Z ≤ 1)=84.1345%

Probability that the employee will be greater than

age of 44 = 100-84.1345=15.86%

So the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%

Therefore the statement that More employees at the processing center are older than 44 than

between 38 and 44 is TRUE

B) Probability of employees less than age of 30 = Pr(X<30).

Z = (X -mean)/std = (30 -38)/6

Pr(X ≤ 30) = Pr(Z ≤ (30 -38)/6) = Pr(Z ≤ -1.333)=9.12%

So the number of employees with probability 0.912 of them being under age 30 =

0.0912\*400=36.48( or 36 employees)

.

Therefore the statement B of the question is also TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans) As we know that if X1∼N(μ1, σ1^2 ), and X2 ∼N(μ2, σ2^2 ) are two independent random

variables then X1 + X2 ∼N(μ1 + μ2, σ1^2 + σ2^2 ) , and X1− X2 ∼N(μ1 − μ2, σ1^2 + σ2^2 )

Similarly if Z = aX1 + bX2 , where X1 and X2 are as defined above, i.e Z is linear combination of X1 and X2

then Z ∼N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 )

Therefore in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(μ + μ, σ^2 + σ^2 ) ~ N(2 µ, 2σ^2 )

2X1-(X1+X2) = N( 4μ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans) By using the confidence interval method taking X=0.99,mean=100 and std=20

Applying into python code

\* stats.norm.inter(0.99,100,20)

We get 48.5,151.5 which is option D

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans) mean=5+7=12\*45=540 million

Std=sqrt(9+16)\*45=225 million

By using stats.norm.interval(0.95,540,225)

We get rupee range of(99.1,980.9) million

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans ) Here we can use Z distribution formula in different form

Z=X-µ/ σ 🡺 X= µ+z\* σ

Where mean =540,std=225 and 5th % value in z table is -1.645

X=540+(-1.645)\*225=170

The 5th percentile of profit for the company is 170 million rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans) the probability of division1 making a loss is 0.047790

The probability of division2 making a loss is 0.040059

From above results we can see that probability of division1 making loss is greater than division2