Class 12

Chapter 10 - Vector Algebra

The following problem is question 17 from exercise 10.5

1. Let $\mathbf{a}$ and $\mathbf{b}$ be two unit vectors and θ is the angle between them. Then $\mathbf{a}+\mathbf{b}$ is a unit vector if

   a) $\theta = \frac{\pi}{4}$ b) $\theta = \frac{\pi}{3}$ c) $\theta = \frac{\pi}{2}$ d) $\theta = \frac{2\pi}{3}$

Solution:
Given,

$$\|\mathbf{a}\| = \|\mathbf{b}\| = 1 \tag{1}$$

$$\|\mathbf{a}+\mathbf{b}\| = 1 \tag{2}$$

Squaring both sides of (2) , we get

$$\|\mathbf{a}+\mathbf{b}\|^2 = 1^2 \tag{3}$$

$$\implies \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + 2\mathbf{a}^\top\mathbf{b} = 1 \tag{4}$$

Substituting (1) in (4), we get

$$\implies 1 + 1 + 2(\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta) = 1 \tag{5}$$

$$\implies 2 + 2(\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta) = 1 \tag{6}$$

$$\implies 2(\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta) = -1 \tag{7}$$

$$\implies (\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta) = \frac{-1}{2} \tag{8}$$

Subtituting (1) in (8), we get

$$\implies \cos\theta = \frac{-1}{2} \tag{9}$$

$$\implies \theta = \frac{2\pi}{3} \tag{10}$$

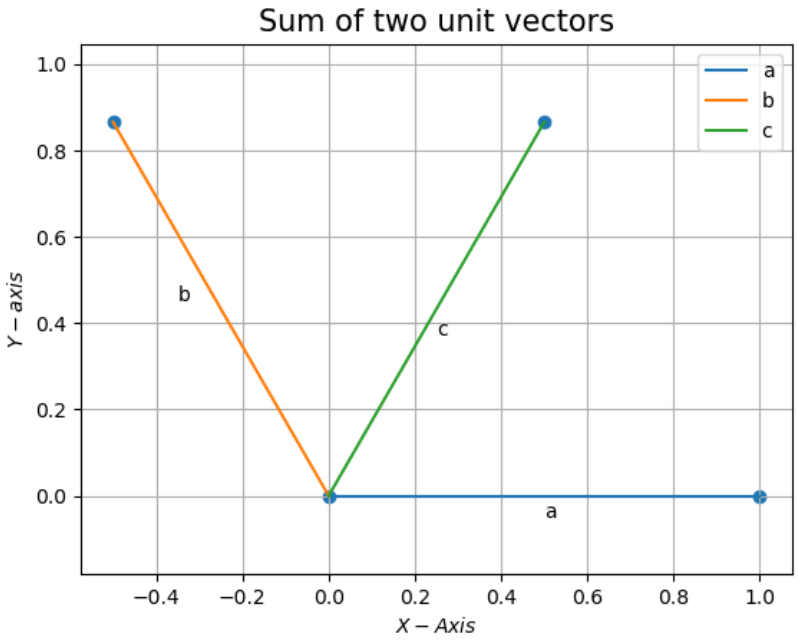

Figure 1: **OA** and **CO** is **a** and **OB** is **b** and **CB** is **a+b**