

Class 12

Chapter 10 - Vector Algebra

The following problem is question 17 from exercise 10.5

1. Let \mathbf{a} and \mathbf{b} be two unit vectors and θ is the angle between them. Then $\mathbf{a} + \mathbf{b}$ is a unit vector if

a) $\theta = \frac{\pi}{4}$ b) $\theta = \frac{\pi}{3}$ c) $\theta = \frac{\pi}{2}$ d) $\theta = \frac{2\pi}{3}$

Solution:

Given,

$$\|\mathbf{a}\| = \|\mathbf{b}\| = 1 \quad (1)$$

$$\|\mathbf{a} + \mathbf{b}\| = 1 \quad (2)$$

Squaring both sides of (2), we get

$$\|\mathbf{a} + \mathbf{b}\|^2 = 1^2 \quad (3)$$

$$\Rightarrow \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + 2\mathbf{a}^\top \mathbf{b} = 1 \quad (4)$$

Substituting (1) in (4), we get

$$\Rightarrow 1 + 1 + 2(\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta) = 1 \quad (5)$$

$$\Rightarrow 2 + 2(\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta) = 1 \quad (6)$$

$$\Rightarrow 2(\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta) = -1 \quad (7)$$

$$\Rightarrow (\|\mathbf{a}\|\|\mathbf{b}\|\cos\theta) = \frac{-1}{2} \quad (8)$$

Substituting (1) in (8), we get

$$\Rightarrow \cos\theta = \frac{-1}{2} \quad (9)$$

$$\Rightarrow \theta = \frac{2\pi}{3} \quad (10)$$

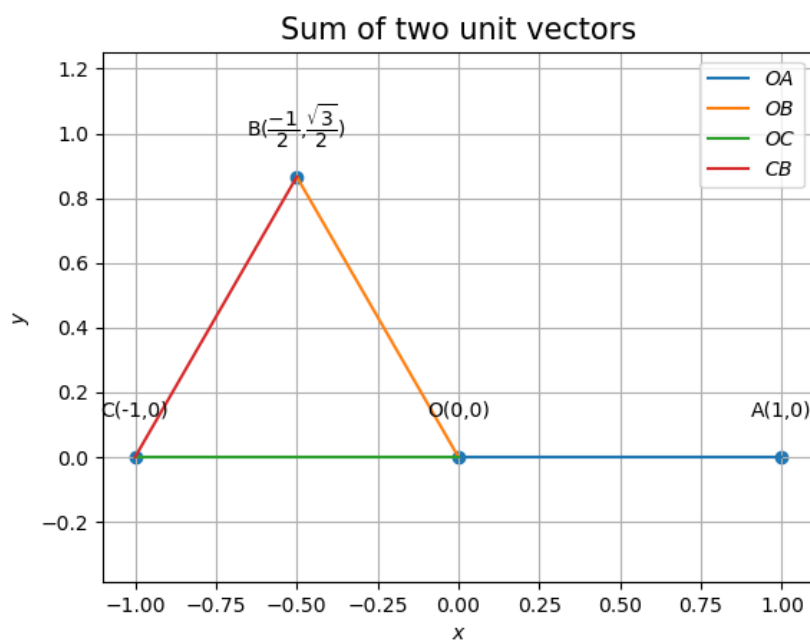


Figure 1: \mathbf{OA} and \mathbf{CO} is \mathbf{a} and \mathbf{OB} is \mathbf{b} and \mathbf{CB} is $\mathbf{a}+\mathbf{b}$