CBSE MATH

Made Simple

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Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Intersection of Conics

1.1. Chords

1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

OR

2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

3. (a) Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

OR

(b) If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\dfrac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

4. Find the area of the region enclosed by the curves $y^2 = x$, $x = \dfrac{1}{4}$, $y = 0$ and $x = 1$, using integration.

5. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

6. (a) Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

OR

(b) Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration.

7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

1.2. Curves

Chapter 2

Tangent And Normal

1. Draw a circle of radius 2.5 cm. Take a point P outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point P.

2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point P, at a distance of 7 cm from its center O.

3. In Figure 2.1, there are two concentric circles with centre O. If ARC and AQB are tangents to the smaller circle from the point A lying on the larger circle, find the length of AC, if AQ = 5 cm.

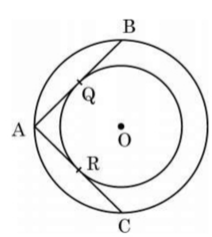

Figure 2.1: Two concentric circles with O as centre

4. In Figure 2.2, if a circle touches the side QR of ΔPQR at S and

extended sides PQ and PR at M and N, respectively,

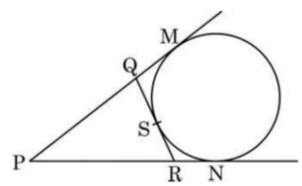

Figure 2.2: Circle touching a triangle with two extended sides as tangents to the circles

prove that $PM = \frac{1}{2}(PQ + QR + PR)$

5. In Figure 2.3, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8 cm respectively. If the area of ΔABC is 84 cm^2, find the lengths of sides AB and AC.

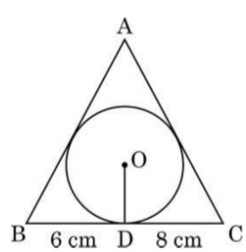

Figure 2.3: Circle with O as center circumscribed in triangle ABC

6. In Figure 2.4, PQ and PR are tangents to the circle centered at O. If $\angle OPR = 45°$, then prove that ORPQ is a square.

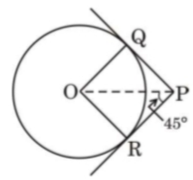

Figure 2.4: Two tangents drawn from point P to a circle whose centre is O

7. In Figure 2.5, O is the centre of a circle of radius 5 cm. PA and BC are tangents to the circle at A and B respectively. If OP is 13 cm, then find the length of tangents PA and BC.

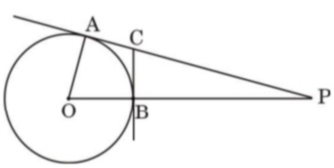

Figure 2.5: Two tangents drawn from point C to a circle whose centre is O

8. In Figure 2.6, AB is diameter of a circle centered at O. BC is tangent to the circle at B. If OP bisects the chord AD and $\angle AOP = 60°$, then find $m\angle C$.

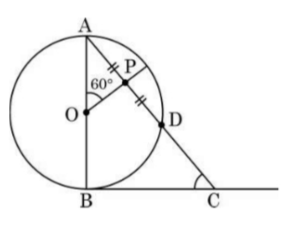

Figure 2.6: Tangent BC is drawn from point C to a circle whose centre is O

9. In Figure 2.7, XAY is a tangent to the circle centered at O. If $\angle ABO =$

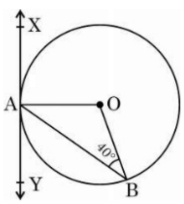

Figure 2.7: The line XAY is tangent to the circle centered at O

60°,then find $m\angle BAY$ and $m\angle AOB$.

10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

11. In Figure 2.8, a triangle ABC with $\angle B = 90°$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point P. Prove that the tangent drawn at point P bisects BC.

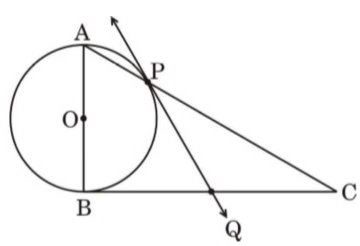

Figure 2.8: PQ is tangent to the circle centered at O. AB is the diameter and $\angle B = 90°$

2.1. Construction

Chapter 3

Linear Forms

3.1. Equation of a Line

1. Solve the equations $x + 2y = 6$ and $2x - 5y = 12$ graphically.

2. Solve the following equations for x and y using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \tag{3.1}$$

$$(bx + ay) + (b - 4a) = 0 \tag{3.2}$$

3. Find the co-ordinates of the point where the line $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ crosses the plane passing through the points $\left(\frac{7}{2}, 0, 0\right)$, $(0, 7, 0)$, $(0, 0, 7)$.

4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

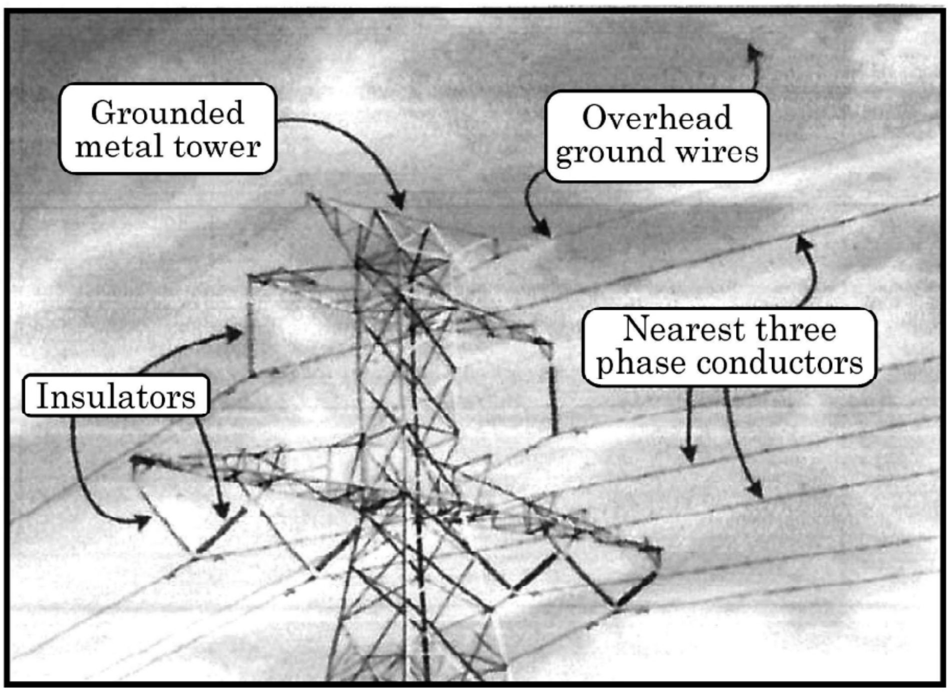

Figure 3.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 3.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \tag{3.3}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \tag{3.4}$$

Based on the given information, answer the following questions:

(a) Are the l_1 and l_2 coplanar? Justify your answer.

(b) Find the point of intersection of lines l_1 and l_2.

5. Write the cartesian equation of the line PQ passing through points P$(2, 2, 1)$ and Q$(5, 1, -2)$. Hence, find the y-coordinate of the point on

the line PQ whose z-coordinate is -2.

6. Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$.

7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + \hat{k}) \tag{3.5}$$

$$\mathbf{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(7\hat{\imath} - 6\hat{\jmath} + \hat{k}) \tag{3.6}$$

8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 3.2) represented by the following lines

$$\mathbf{r} = \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k}) \tag{3.7}$$

$$\mathbf{r} = (3\hat{\imath} + 3\hat{\jmath}) + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k}) \tag{3.8}$$

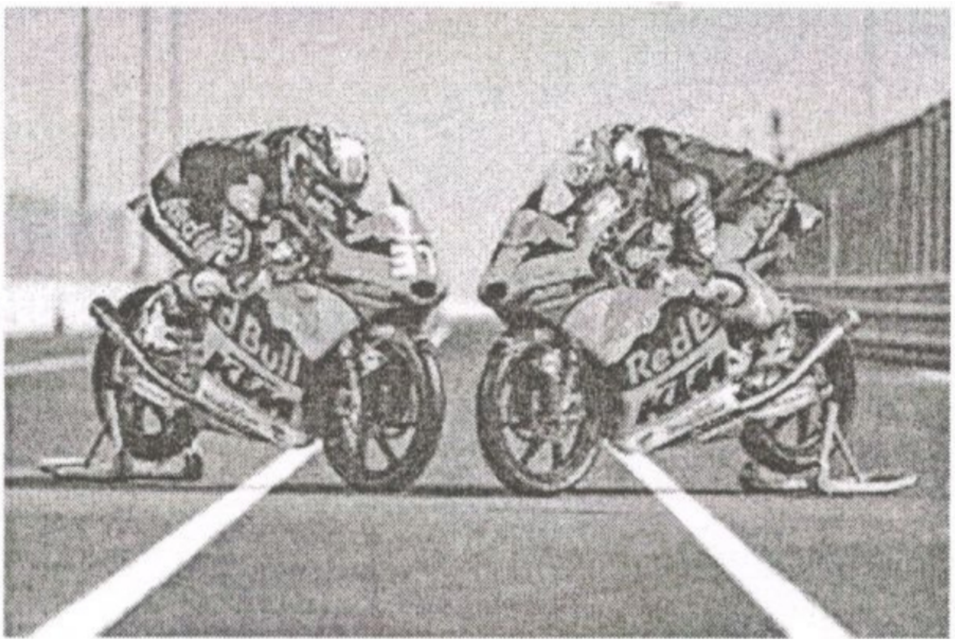

Figure 3.2: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

(a) Find the shortest distance between the given lines.

(b) Find a point at which the motorcycles may collide.

9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{i} + (\lambda + 4)\hat{j} - (\lambda - 3)\hat{k} \tag{3.9}$$

$$\mathbf{r} = (3 - \mu)\hat{i} + (2\mu + 2)\hat{j} + (\mu + 6)\hat{k} \tag{3.10}$$

10. Find the shortest distance between the following lines and hence write

whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \tag{3.11}$$

3.2. Perpendicular

1. If the distance of the point $(1, 1, 1)$ from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ.

2. Find the distance of the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$.

3. Find the distance of the point $P(4, 3, 2)$ from the plane determined by the points $A(-1, 6, -5), B(-5, -2, 3)$ and $C(2, 4, -5)$.

4. The distance of the line

$$\mathbf{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + 5\hat{j} + \hat{k}) \tag{3.12}$$

from the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + 4\hat{k}) = 5 \tag{3.13}$$

is

(a) $\sqrt{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\dfrac{1}{3\sqrt{2}}$

(d) $\dfrac{-2}{3\sqrt{2}}$

5. Find a unit vector perpendicular to each of the vectors $(\mathbf{a}+\mathbf{b})$ and $(\mathbf{a}-\mathbf{b})$ where

$$\mathbf{a} = \hat{\imath} + \hat{\jmath} + \hat{k} \tag{3.14}$$

$$\mathbf{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} \tag{3.15}$$

3.3. Plane

1. Find the equation of the plane passing through the points $(2, 1, 0), (3, -2, -2)$ and $(1, 1, 7)$. Also, obtain its distance from the origin.

2. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane.

3. Find the cartesian and the vector equation of a plane which passes through the point $(3, 2, 0)$ and contains the line $\dfrac{x-3}{1} = \dfrac{y-6}{5} = \dfrac{z-4}{4}$.

4. The distance between the planes $4x-4y+2z+5=0$ and $2x-2y+z+6=0$ is

   (a) $\dfrac{1}{6}$

   (b) $\dfrac{7}{6}$

   (c) $\dfrac{11}{6}$

(d) $\frac{16}{6}$

5. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{\imath} + 3\hat{\jmath}) + 6 = 0 \tag{3.16}$$

$$\mathbf{r} \cdot (3\hat{\imath} - \hat{\jmath} - 4\hat{k}) = 0 \tag{3.17}$$

which is at a unit distance from the origin.

3.4. Miscellaneous

1. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}) \tag{3.18}$$

and the plane

$$\mathbf{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 10. \tag{3.19}$$

2. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

3. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.

4. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 3.1:

Table 3.1: Table showing yearly trend of production of goods in lakh tonnes

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40