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# CBSE MATH

## Made Simple

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# Contents

Introduction	iii
<b>1 Intersection of Conics</b>	<b>1</b>
1.1 Chords . . . . .	1
1.2 Curves . . . . .	2
<b>2 Tangent And Normal</b>	<b>3</b>
2.1 Construction . . . . .	6
<b>3 Linear Forms</b>	<b>7</b>
3.1 Equation of a Line . . . . .	7
3.2 Perpendicular . . . . .	11
3.3 Plane . . . . .	12
3.4 Miscellaneous . . . . .	13



# Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.



# Chapter 1

## Intersection of Conics

### 1.1. Chords

1. Using integration, find the area of the region enclosed by the curve  $y = x^2$ , the x-axis and the ordinates  $x = -2$  and  $x = 1$ .

**OR**

2. Using integration, find the area of the region enclosed by line  $y = \sqrt{3}x$  semi-circle  $y = \sqrt{4 - x^2}$  and x-axis in first quadrant.
3. (a) Using integration, find the area of the smaller region enclosed by the curve  $4x^2 + 4y^2 = 9$  and the line  $2x + 2y = 3$ .

**OR**

- (b) If the area of the region bounded by the curve  $y^2 = 4ax$  and the line  $x = 4a$  is  $\frac{256}{3}$  sq. units, then using integration, find the value of  $a$ , where  $a > 0$ .
4. Find the area of the region enclosed by the curves  $y^2 = x$ ,  $x = \frac{1}{4}$ ,  $y = 0$  and  $x = 1$ , using integration.

5. If the area of the region bounded by the line  $y = mx$  and the curve  $x^2 = y$  is  $\frac{32}{3}$  sq. units, then find the positive value of  $m$ , using integration.
6. (a) Find the area bounded by the ellipse  $x^2 + 4y^2 = 16$  and the ordinates  $x = 0$  and  $x = 2$ , using integration.

**OR**

- (b) Find the area of the region  $\{(x, y) : x^2 \leq y \leq x\}$ , using integration.
7. If the area between the curves  $x = y^2$  and  $x = 4$  is divided into two equal parts by the line  $x = a$ , then find the value of  $a$ , using integration.

## 1.2. Curves



## Chapter 2

# Tangent And Normal

1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point **P**.
2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.
3. In Figure 2.1, there are two concentric circles with centre **O**. If  $ARC$  and  $AQB$  are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of  $AC$ , if  $AQ = 5$  cm.

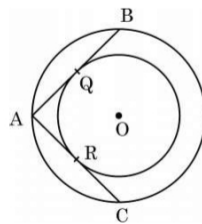


Figure 2.1: Two concentric circles with **O** as centre

4. In Figure 2.2, if a circle touches the side  $QR$  of  $\triangle PQR$  at **S** and

extended sides  $PQ$  and  $PR$  at  $\mathbf{M}$  and  $\mathbf{N}$ , respectively,

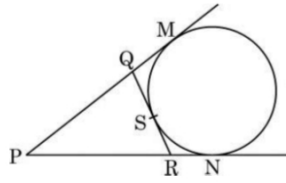


Figure 2.2: Two tangents are drawn from point  $\mathbf{P}$  to the circle

prove that  $PM = \frac{1}{2}(PQ + QR + PR)$

5. In Figure 2.3, a triangle  $ABC$  is drawn to circumscribe a circle of radius 4 cm such that the segments  $BD$  and  $DC$  into which  $BC$  is divided by the point of contact  $\mathbf{D}$  are of lengths 6 cm and 8 cm respectively. If the area of  $\triangle ABC$  is  $84 \text{ cm}^2$ , find the lengths of sides  $AB$  and  $AC$ .

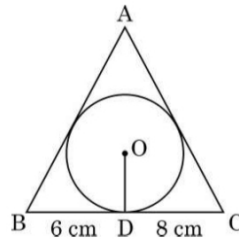


Figure 2.3: Circle with  $\mathbf{O}$  as center circumscribed in triangle  $ABC$

6. In Figure 2.4,  $PQ$  and  $PR$  are tangents to the circle centered at  $\mathbf{O}$ . If  $\angle OPR = 45^\circ$ , then prove that  $ORPQ$  is a square.

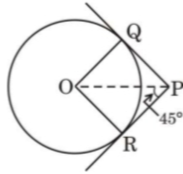


Figure 2.4: Two tangents drawn from point **P** to a circle whose centre is **O**

7. In Figure 2.5, **O** is the centre of a circle of radius 5 cm.  $PA$  and  $BC$  are tangents to the circle at **A** and **B** respectively. If  $OP$  is 13 cm, then find the length of tangents  $PA$  and  $BC$ .

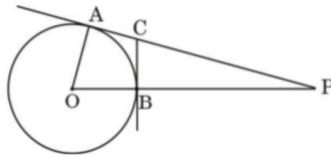


Figure 2.5: Two tangents drawn from point **C** to a circle whose centre is **O**

8. In Figure 2.6,  $AB$  is diameter of a circle centered at **O**.  $BC$  is tangent to the circle at **B. If  $OP$  bisects the chord  $AD$  and  $\angle AOP = 60^\circ$ , then find  $m\angle C$ .**

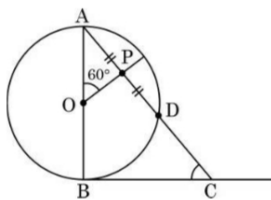


Figure 2.6: Tangent  $BC$  is drawn from point **C** to a circle whose centre is **O**

9. In Figure 2.7,  $XAY$  is a tangent to the circle centered at **O**. If  $\angle ABO =$

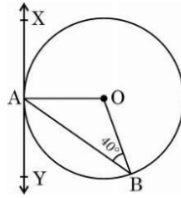


Figure 2.7: The line  $XAY$  is tangent to the circle centered at  $O$

$60^\circ$ , then find  $m\angle BAY$  and  $m\angle AOB$ .

10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
11. In Figure 2.8, a triangle  $ABC$  with  $\angle B = 90^\circ$  is shown. Taking  $AB$  as diameter, a circle has been drawn intersecting  $AC$  at point  $P$ . Prove that the tangent drawn at point  $P$  bisects  $BC$ .

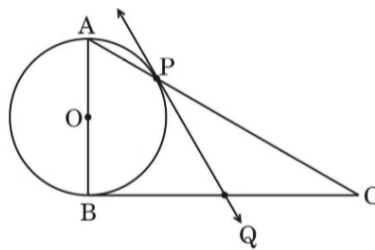


Figure 2.8:  $PQ$  is tangent to the circle centered at  $O$ .  $AB$  is the diameter and  $\angle B = 90^\circ$

## 2.1. Construction

## Chapter 3

# Linear Forms

### 3.1. Equation of a Line

1. Solve the equations  $x + 2y = 6$  and  $2x - 5y = 12$  graphically.
2. Solve the following equations for  $x$  and  $y$  using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \quad (3.1)$$

$$(bx + ay) + (b - 4a) = 0 \quad (3.2)$$

3. Find the co-ordinates of the point where the line  $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$  crosses the plane passing through the points  $\left(\frac{7}{2}, 0, 0\right), (0, 7, 0), (0, 0, 7)$ .

4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

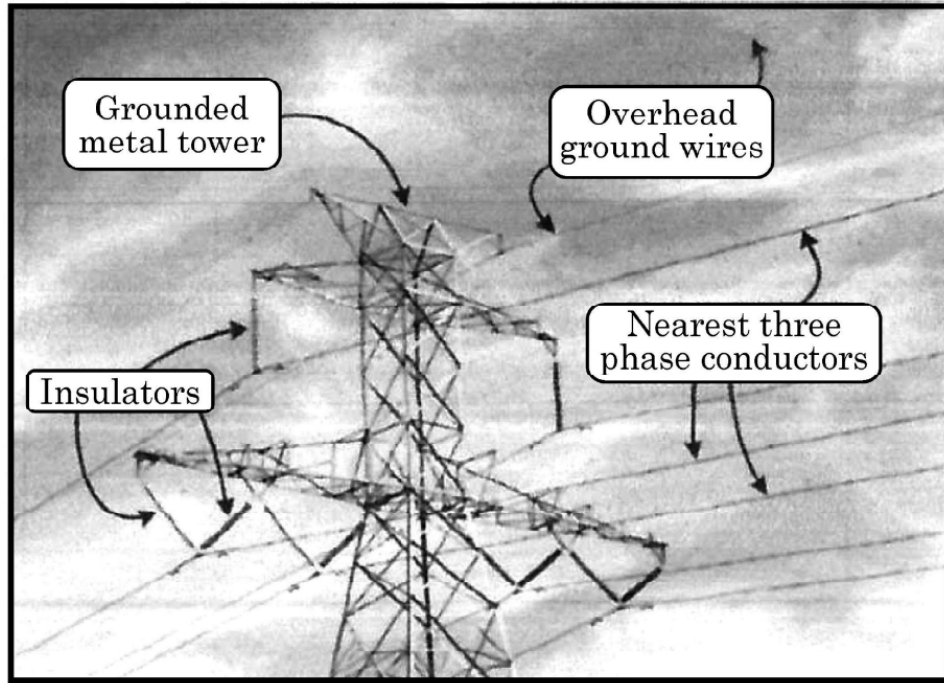


Figure 3.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 3.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \quad (3.3)$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \quad (3.4)$$

Based on the given information, answer the following questions:

- (a) Are the  $l_1$  and  $l_2$  coplanar? Justify your answer.
  - (b) Find the point of intersection of lines  $l_1$  and  $l_2$ .
5. Write the cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, -2). Hence, find the y-coordinate of the point on

the line PQ whose z-coordinate is -2.

6. Find the distance between the lines  $x = \frac{y-1}{2} = \frac{z-2}{3}$  and  $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$ .

7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k}) \quad (3.5)$$

$$\mathbf{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k}) \quad (3.6)$$

8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 3.2) represented by the following lines

$$\mathbf{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) \quad (3.7)$$

$$\mathbf{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k}) \quad (3.8)$$

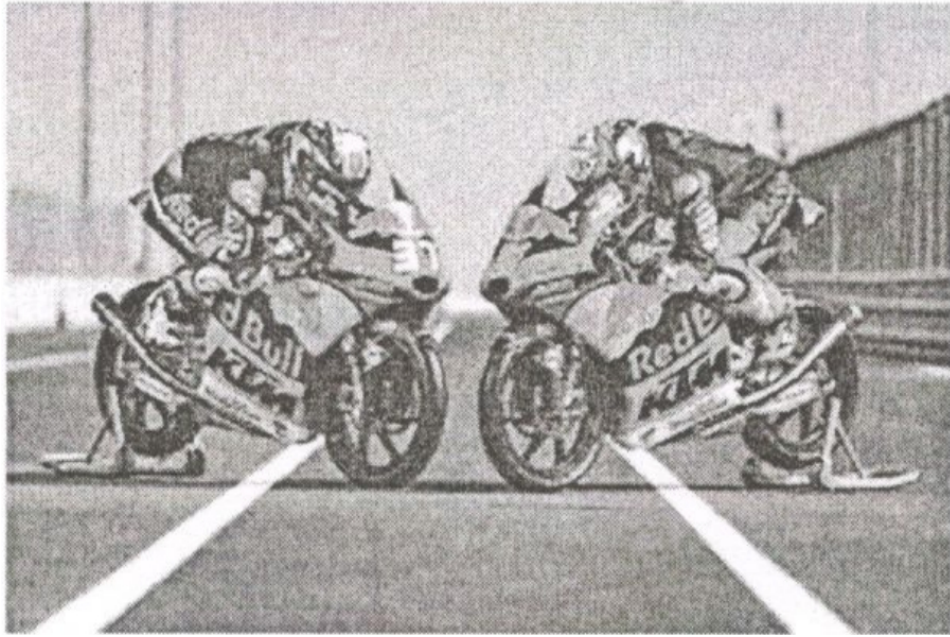


Figure 3.2: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

- (a) Find the shortest distance between the given lines.
- (b) Find a point at which the motorcycles may collide.

9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{i} + (\lambda + 4)\hat{j} - (\lambda - 3)\hat{k} \quad (3.9)$$

$$\mathbf{r} = (3 - \mu)\hat{i} + (2\mu + 2)\hat{j} + (\mu + 6)\hat{k} \quad (3.10)$$

10. Find the shortest distance between the following lines and hence write



whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \quad (3.11)$$

## 3.2. Perpendicular

1. If the distance of the point  $(1, 1, 1)$  from the plane  $x - y + z + \lambda = 0$  is  $\frac{5}{\sqrt{3}}$ , find the value(s) of  $\lambda$ .
2. Find the distance of the point  $(2, 3, 4)$  measured along the line  $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$  from the plane  $3x + 2y + 2z + 5 = 0$ .
3. Find the distance of the point  $P(4, 3, 2)$  from the plane determined by the points  $A(-1, 6, -5)$ ,  $B(-5, -2, 3)$  and  $C(2, 4, -5)$ .
4. The distance of the line

$$\mathbf{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + 5\hat{j} + \hat{k}) \quad (3.12)$$

from the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + 4\hat{k}) = 5 \quad (3.13)$$

is

(a)  $\sqrt{2}$

(b)  $\frac{1}{\sqrt{2}}$

$$(c) \frac{1}{3\sqrt{2}}$$

$$(d) \frac{-2}{3\sqrt{2}}$$

5. Find a unit vector perpendicular to each of the vectors  $(\mathbf{a} + \mathbf{b})$  and  $(\mathbf{a} - \mathbf{b})$  where

$$\mathbf{a} = \hat{i} + \hat{j} + \hat{k} \quad (3.14)$$

$$\mathbf{b} = \hat{i} + 2\hat{j} + 3\hat{k} \quad (3.15)$$

### 3.3. Plane

- Find the equation of the plane passing through the points  $(2, 1, 0)$ ,  $(3, -2, -2)$  and  $(1, 1, 7)$ . Also, obtain its distance from the origin.
- The foot of a perpendicular drawn from the point  $(-2, -1, -3)$  on a plane is  $(1, -3, 3)$ . Find the equation of the plane.
- Find the cartesian and the vector equation of a plane which passes through the point  $(3, 2, 0)$  and contains the line  $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ .
- The distance between the planes  $4x-4y+2z+5=0$  and  $2x-2y+z+6=0$  is

$$(a) \frac{1}{6}$$

$$(b) \frac{7}{6}$$

$$(c) \frac{11}{6}$$

(d)  $\frac{16}{6}$

5. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{i} + 3\hat{j}) + 6 = 0 \quad (3.16)$$

$$\mathbf{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0 \quad (3.17)$$

which is at a unit distance from the origin.

### 3.4. Miscellaneous

1. Find the distance of the point  $(1, -2, 9)$  from the point of intersection of the line

$$\mathbf{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \quad (3.18)$$

and the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10. \quad (3.19)$$

2. Find the area bounded by the curves  $y = |x - 1|$  and  $y = 1$ , using integration.
3. Find the coordinates of the point where the line through  $(4, -3, -4)$  and  $(3, -2, 2)$  crosses the plane  $2x + y + z = 6$ .

4. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 3.1:

Table 3.1: Table showing yearly trend of production of goods in lakh tonnes

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40