CBSE MATH

Made Simple

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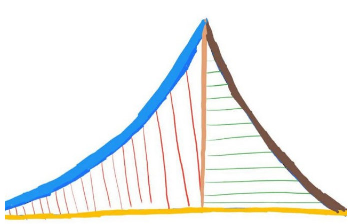


Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Intersection of Conics

1.1. Chords

1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

OR

2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

3. (a) Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

OR

(b) If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\dfrac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

4. Find the area of the region enclosed by the curves $y^2 = x$, $x = \dfrac{1}{4}$, $y = 0$ and $x = 1$, using integration.

5. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

6. (a) Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

OR

(b) Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration.

7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

1.2. Curves

Chapter 2

Tangent And Normal

1. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.

2.1. Construction

Chapter 3

Linear Forms

3.1. Equation of a Line

1. Solve the equations $x + 2y = 6$ and $2x - 5y = 12$ graphically.

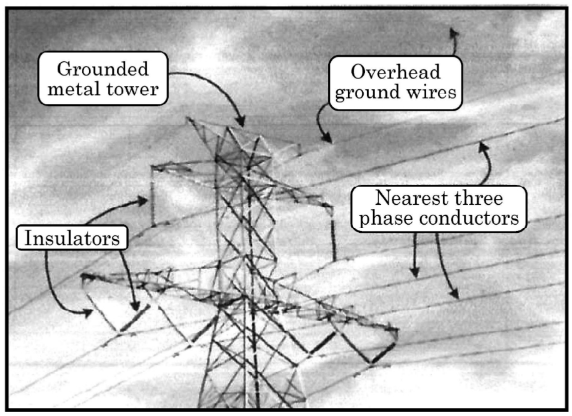
2. Solve the following equations for x and y using cross-multiplication method:

   $(ax - by) + (a + 4b) = 0$

   $(bx + ay) + (b - 4a) = 0$

3. Find the co-ordinates of the point where the line $\dfrac{x-3}{-1} = \dfrac{y+4}{1} = \dfrac{z+5}{6}$ crosses the plane passing through the points $\left(\dfrac{7}{2}, 0, 0\right)$, $(0, 7, 0)$, $(0, 0, 7)$.

4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.



Two such wires lie along the following lines:

$l_1 : \dfrac{x+1}{3} = \dfrac{y-3}{-2} = \dfrac{z+2}{-1}$

$l_2 : \dfrac{x}{-1} = \dfrac{y-7}{3} = \dfrac{z+7}{-2}$

Based on the given information, answer the following questions:

- Are the lines l_1 and l_2 coplanar? Justify your answer.
- Find the point of intersection of lines l_1 and l_2.

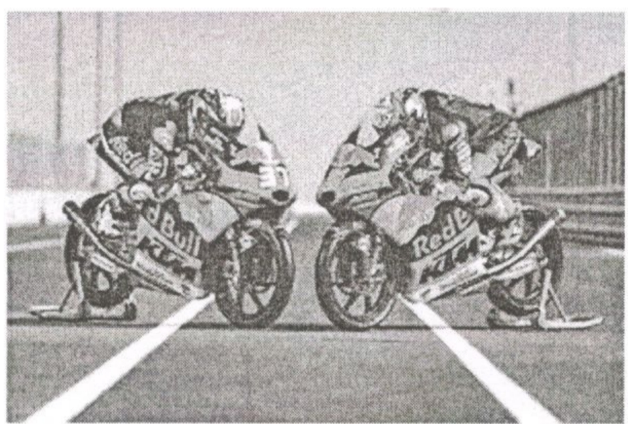
5. Write the cartesian equation of the line PQ passing through points P$(2, 2, 1)$ and Q$(5, 1, -2)$. Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is -2.

6. Find the distance between the lines $x = \dfrac{y-1}{2} = \dfrac{z-2}{3}$ and $x+1 = \dfrac{y+2}{2} = \dfrac{z-1}{3}$.

7. Find the shortest distance between the following lines:

$\vec{\mathbf{r}} = 3\hat{i}+5\hat{j}+7\hat{k}+\lambda(\hat{i}-2\hat{j}+\hat{k})$ and $\vec{\mathbf{r}} = (-\hat{i}-\hat{j}-\hat{k})+\mu(7\hat{i}-6\hat{j}=\hat{k})$.

8. Two motorcycles A and B are running at a speed more than the allowed speed on the roads represented by the lines $\vec{\mathbf{r}} = \lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{\mathbf{r}} = (3\hat{i}+3\hat{j})+\mu(2\hat{i}+\hat{j}+\hat{k})$ respectively.



Based on the following information, answer the following questions:

- Find the shortest distance between the given lines.
- Find a point at which the motorcycles may collide.

9. Find the shortest distance between the lines

$\vec{\mathbf{r}} = (\lambda+1)\hat{i}+(\lambda+4)\hat{j}-(\lambda-3)\hat{k}$, and

$\vec{\mathbf{r}} = (3-\mu)\hat{i}+(2\mu+2)\hat{j}+(\mu+6)\hat{k}$

10. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$\frac{x-1}{2} = \frac{y+1}{3} = z$, $\frac{x+1}{5} = \frac{y-2}{1}$, $z = 2$

3.2. Perpendicular

1. If the distance of the point $(1,1,1)$ from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ.

2. Find the distance of the point $(2,3,4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$.

3. Find the distance of the point $P(4,3,2)$ from the plane determined by the points $A(-1,6,-5), B(-5,-2,3)$ and $C(2,4,-5)$.

4. The distance of the line $\overrightarrow{\mathbf{r}} = (\hat{\imath} - \hat{\jmath}) + \lambda(\hat{\imath} + 5\hat{\jmath} + \hat{k})$ from the plane $\overrightarrow{\mathbf{r}} \cdot (\hat{\imath} - \hat{\jmath} + 4\hat{k}) = 5$ is

   - $\sqrt{2}$
   - $\frac{1}{\sqrt{2}}$
   - $\frac{1}{3\sqrt{2}}$
   - $\frac{-2}{3\sqrt{2}}$

5. Find a unit vector perpendicular to each of the vectors $(\overrightarrow{a} + \overrightarrow{b})$ and $(\overrightarrow{a} - \overrightarrow{b})$ where $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

3.3. Plane

1. Find the equation of the plane passing through the points $(2, 1, 0)$, $(3, -2, -2)$ and $(1, 1, 7)$. Also, obtain its distance from the origin.

2. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane.

3. Find the cartesian and the vector equation of a plane which passes through the point $(3, 2, 0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

4. The distance between the planes $4x - 4y + 2z + 5 = 0$ and $2x - 2y + z + 6 = 0$ is

   - $\frac{1}{6}$
   - $\frac{7}{6}$
   - $\frac{11}{6}$
   - $\frac{16}{6}$

5. Find the equation of the plane through the line of intersection of the

planes $\overrightarrow{\mathbf{r}} \cdot (\hat{\imath} + 3\hat{\jmath}) + 6 = 0$ and $\overrightarrow{\mathbf{r}} \cdot (3\hat{\imath} - \hat{\jmath} - 4\hat{k}) = 0$, which is at a unit distance from the origin.

3.4. Miscellaneous

1. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line
$\overrightarrow{\mathbf{r}} = 4\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + \lambda(3\hat{\imath} - +4\hat{\jmath} + 2\hat{k})$ and the plane $\overrightarrow{\mathbf{r}} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 10$.

2. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

3. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.

4. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 for the following data:

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40
2007	36