
CBSE MATH

Made Simple

G. V. V. Sharma



Copyright ©2023 by G. V. V. Sharma.

<https://creativecommons.org/licenses/by-sa/3.0/>

and

<https://www.gnu.org/licenses/fdl-1.3.en.html>

Contents

Introduction	iii
1 Intersection of Conics	1
1.1 Chords	1
1.2 Curves	2
2 Tangent And Normal	3
2.1 Construction	6
3 Linear Forms	7
3.1 Equation of a Line	7
3.2 Perpendicular	11
3.3 Plane	12
3.4 Miscellaneous	13

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Intersection of Conics

1.1. Chords

1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

OR

2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.
3. (a) Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

OR

- (b) If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a , where $a > 0$.
4. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.

5. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m , using integration.
6. (a) Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

OR

- (b) Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration.
7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a , using integration.

1.2. Curves

Chapter 2

Tangent And Normal

1. Draw a circle of radius 2.5 cm. Take a point P outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point P.
2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point P, at a distance of 7 cm from its center O.
3. In Figure 2.1, there are two concentric circles with centre O. If ARC and AQB are tangents to the smaller circle from the point A lying on the larger circle, find the length of AC, if $AQ = 5$ cm.

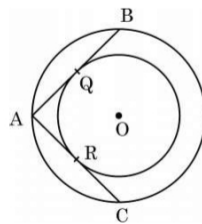


Figure 2.1: Two concentric circles with O as centre

4. In Figure 2.2, if a circle touches the side QR of ΔPQR at S and

extended sides PQ and PR at M and N, respectively,

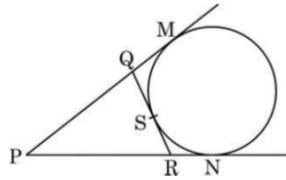


Figure 2.2: Circle touching a triangle with two extended sides as tangents to the circles

prove that $PM = \frac{1}{2}(PQ + QR + PR)$

5. In Figure 2.3, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8 cm respectively. If the area of $\triangle ABC$ is 84 cm^2 , find the lengths of sides AB and AC.

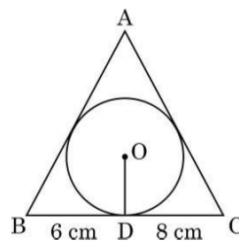


Figure 2.3: Circle with O as center circumscribed in triangle ABC

6. In Figure 2.4, PQ and PR are tangents to the circle centered at O. If $\angle OPR = 45^\circ$, then prove that ORPQ is a square.

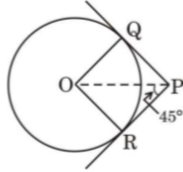


Figure 2.4: Two tangents drawn from point P to a circle whose centre is O

7. In Figure 2.5, O is the centre of a circle of radius 5 cm. PA and BC are tangents to the circle at A and B respectively. If OP is 13 cm, then find the length of tangents PA and BC.

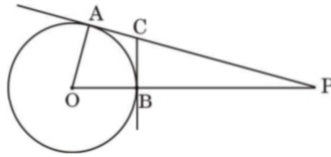


Figure 2.5: Two tangents drawn from point C to a circle whose centre is O

8. In Figure 2.6, AB is diameter of a circle centered at O. BC is tangent to the circle at B. If OP bisects the chord AD and $\angle AOP = 60^\circ$, then find $m\angle C$.

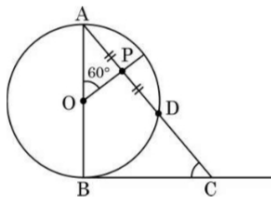


Figure 2.6: Tangent BC is drawn from point C to a circle whose centre is O

9. In Figure 2.7, XAY is a tangent to the circle centered at O. If $\angle ABO = 60^\circ$, then find $m\angle BAY$ and $m\angle AOB$.

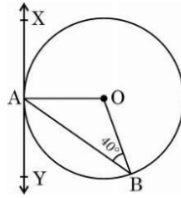


Figure 2.7: The line XAY is tangent to the circle centered at O

10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
11. In Figure 2.8, a triangle ABC with $\angle B = 90^\circ$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point P. Prove that the tangent drawn at point P bisects BC.

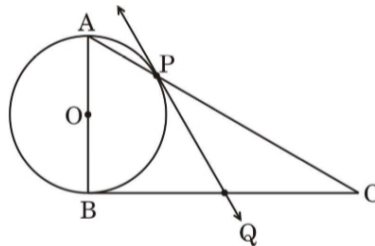


Figure 2.8: PQ is tangent to the circle centered at O. AB is the diameter and $\angle B = 90^\circ$

2.1. Construction

Chapter 3

Linear Forms

3.1. Equation of a Line

1. Solve the equations $x + 2y = 6$ and $2x - 5y = 12$ graphically.
2. Solve the following equations for x and y using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \quad (3.1)$$

$$(bx + ay) + (b - 4a) = 0 \quad (3.2)$$

3. Find the co-ordinates of the point where the line $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ crosses the plane passing through the points $\left(\frac{7}{2}, 0, 0\right), (0, 7, 0), (0, 0, 7)$.

4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

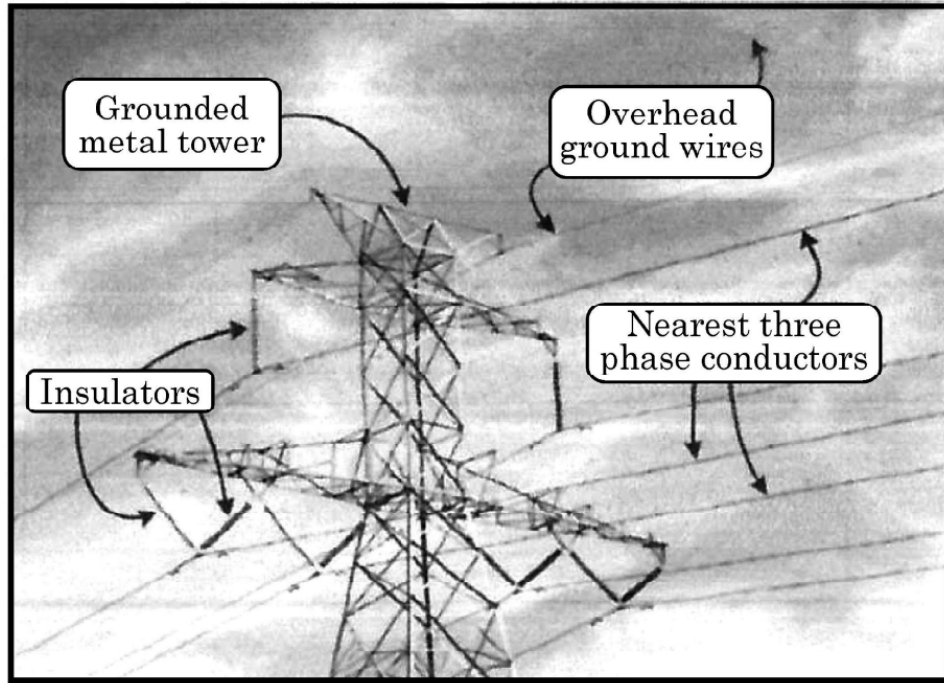


Figure 3.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 3.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \quad (3.3)$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \quad (3.4)$$

Based on the given information, answer the following questions:

- (a) Are the l_1 and l_2 coplanar? Justify your answer.
 - (b) Find the point of intersection of lines l_1 and l_2 .
5. Write the cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, -2). Hence, find the y-coordinate of the point on

the line PQ whose z-coordinate is -2.

6. Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$.

7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k}) \quad (3.5)$$

$$\mathbf{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k}) \quad (3.6)$$

8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 3.2) represented by the following lines

$$\mathbf{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) \quad (3.7)$$

$$\mathbf{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k}) \quad (3.8)$$

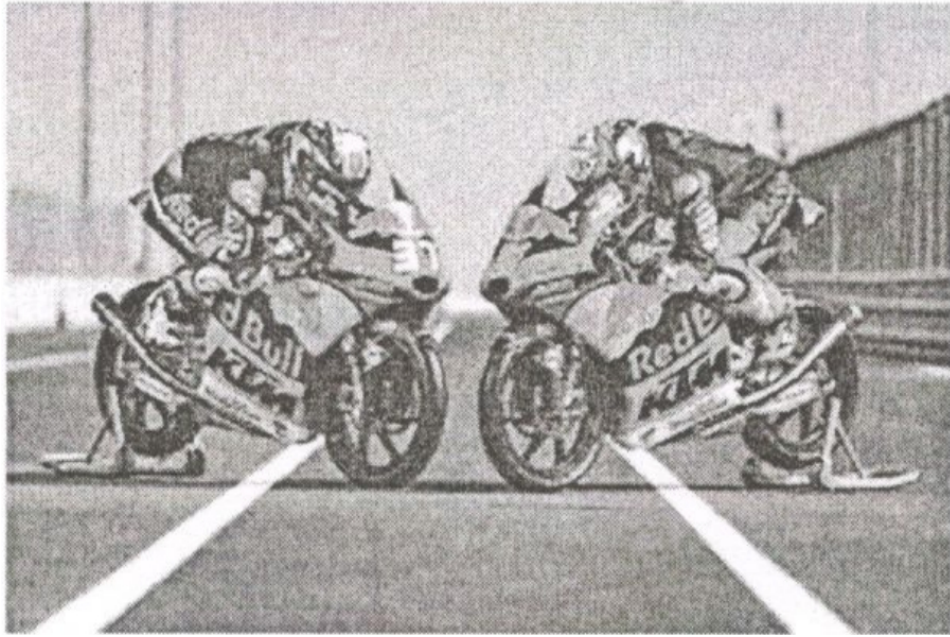


Figure 3.2: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

- (a) Find the shortest distance between the given lines.
- (b) Find a point at which the motorcycles may collide.

9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{i} + (\lambda + 4)\hat{j} - (\lambda - 3)\hat{k} \quad (3.9)$$

$$\mathbf{r} = (3 - \mu)\hat{i} + (2\mu + 2)\hat{j} + (\mu + 6)\hat{k} \quad (3.10)$$

10. Find the shortest distance between the following lines and hence write

whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \quad (3.11)$$

3.2. Perpendicular

1. If the distance of the point $(1, 1, 1)$ from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ .
2. Find the distance of the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$.
3. Find the distance of the point $P(4, 3, 2)$ from the plane determined by the points $A(-1, 6, -5)$, $B(-5, -2, 3)$ and $C(2, 4, -5)$.
4. The distance of the line

$$\mathbf{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + 5\hat{j} + \hat{k}) \quad (3.12)$$

from the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + 4\hat{k}) = 5 \quad (3.13)$$

is

(a) $\sqrt{2}$

(b) $\frac{1}{\sqrt{2}}$

$$(c) \frac{1}{3\sqrt{2}}$$

$$(d) \frac{-2}{3\sqrt{2}}$$

5. Find a unit vector perpendicular to each of the vectors $(\mathbf{a} + \mathbf{b})$ and $(\mathbf{a} - \mathbf{b})$ where

$$\mathbf{a} = \hat{i} + \hat{j} + \hat{k} \quad (3.14)$$

$$\mathbf{b} = \hat{i} + 2\hat{j} + 3\hat{k} \quad (3.15)$$

3.3. Plane

- Find the equation of the plane passing through the points $(2, 1, 0)$, $(3, -2, -2)$ and $(1, 1, 7)$. Also, obtain its distance from the origin.
- The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane.
- Find the cartesian and the vector equation of a plane which passes through the point $(3, 2, 0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.
- The distance between the planes $4x-4y+2z+5=0$ and $2x-2y+z+6=0$ is

$$(a) \frac{1}{6}$$

$$(b) \frac{7}{6}$$

$$(c) \frac{11}{6}$$

(d) $\frac{16}{6}$

5. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{i} + 3\hat{j}) + 6 = 0 \quad (3.16)$$

$$\mathbf{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0 \quad (3.17)$$

which is at a unit distance from the origin.

3.4. Miscellaneous

1. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \quad (3.18)$$

and the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10. \quad (3.19)$$

2. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.
3. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.

4. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 3.1:

Table 3.1: Table showing yearly trend of production of goods in lakh tonnes

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40