Coordinate Geometry

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Class 10^{th} Maths - Chapter 7

This is Problem-5 from Exercise 7.3

1. QUESTION: Median of a triangle divides it into two equal triangles of same areas.Verify this result for triangle ABC whose vertices are A(4,-6) B(3,-2) C(5,2)

Solution:

$$\mathbf{A} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \tag{3}$$

$$\tag{4}$$

Let the **AP** be the median from **A** to side **BC**. Hence,

$$\mathbf{P} = \frac{(1)\mathbf{C} + (1)\mathbf{B}}{1+1} \tag{5}$$

$$\mathbf{P} = \frac{(1)\begin{pmatrix} 5 \\ 2 \end{pmatrix} + (1)\begin{pmatrix} 3 \\ -2 \end{pmatrix}}{2} \tag{6}$$

$$\mathbf{P} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{7}$$

$$\tag{8}$$

$$(9)$$

$$Areaoftriangle ABP = \frac{1}{2}\left\|\left(\mathbf{BP}\times\mathbf{BA}\right)\right\| \tag{10}$$

$$= \frac{1}{2}\begin{vmatrix}1 & 1\\ 2 & -4\end{vmatrix} \tag{11}$$

$$= \frac{1}{2}\left\|-4-(2)\right\| \tag{12}$$

$$= \frac{1}{2}(-6) \tag{13}$$

$$= -3 sq.units \tag{14}$$

However the area cannot be negative . Therefore the area of triangle ABP is equal to 3 square units

$$(15)$$

$$Areaoftriangle ACP = \frac{1}{2}\left|\left(\mathbf{CP}\times\mathbf{CA}\right)\right| \tag{16}$$

$$= \frac{1}{2}\begin{vmatrix}-1 & -1\\ -2 & -8\end{vmatrix} \tag{17}$$

$$= \frac{1}{2}\left\|8-2\right\| \tag{18}$$

$$= \frac{1}{2}(6) \tag{19}$$

$$= 3 sq.units \tag{20}$$

The area of both sides is the same. Thus, median AD has divided ABC into two triangles of equal areas.

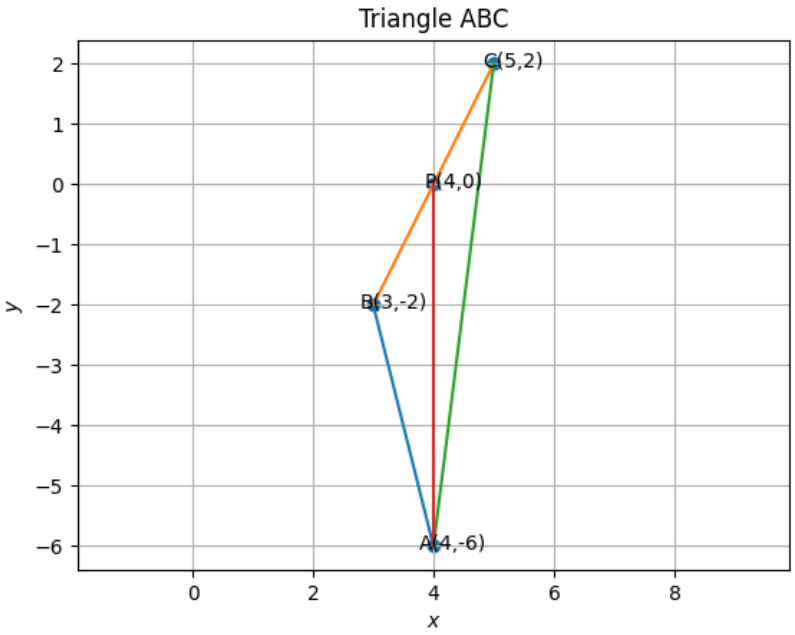

Figure 1: Triangle ABC