



Karnaugh Map



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Abstract—This manual explains Karnaugh maps (K-map) by finding the logic functions for the incrementing decoder.

1 INCREMENTING DECODER

The incrementing decoder takes the numbers 0, 1, ..., 9 in binary as inputs and generates the consecutive number as output. The corresponding truth table is available in Table 0.

Z	Y	X	W	D	C	B	A
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0

TABLE 0

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2 KARNAUGH MAP

Using Boolean logic, output A in Table 0 can be expressed in terms of the inputs W, X, Y, Z as

$$A = W'X'Y'Z' + W'XY'Z' + W'X'YZ' + W'XYZ' + W'X'Y'Z \quad (2.1)$$

1. K-Map for A : The expression in (2.1) can be minimized using the K-map in Fig. 1. In Fig. 1, the *implicants* in boxes 0,2,4,6 result in $W'Z'$. The implicants in boxes 0,8 result in $W'X'Y'$. Thus, after minimization using Fig. 2.2, (2.1) can be expressed as

$$A = W'Z' + W'X'Y' \quad (2.2)$$

Using the fact that

$$\begin{aligned} X + X' &= 1 \\ XX' &= 0, \end{aligned} \quad (2.3)$$

derive (2.2) from (2.1) algebraically.

2. K-Map for B : From Table 0, using boolean logic,

$$B = WX'Y'Z' + W'XY'Z' + WX'YZ' + W'XYZ' \quad (2.4)$$

Show that (2.4) can be reduced to

$$B = WX'Z' + W'XZ' \quad (2.5)$$

using Fig. 2.

3. Derive (2.5) from (2.4) algebraically using (2.3).
4. K-Map for C : From Table 0, using boolean logic,

$$C = WXY'Z' + W'X'YZ' + WX'YZ' + W'XYZ' \quad (2.6)$$

Show that (2.6) can be reduced to

$$C = WXY'Z' + X'YZ' + W'YZ' \quad (2.7)$$

ZY \ XW	00	01	11	10
00			0	0
01			0	
11	0	0	0	0
10			0	0

Fig. 1: K-map for A.

ZY \ XW	00	01	11	10
00	0	0		0
01			0	
11	0	0	0	0
10	0	0	0	0

Fig. 4: K-map for C.

ZY \ XW	00	01	11	10
00	0		0	
01	0		0	
11	0	0	0	0
10	0	0	0	0

Fig. 2: K-map for B.

ZY \ XW	00	01	11	10
00	0	0	0	0
01	0	0		0
11	0	0	0	0
10		0	0	0

Fig. 6: K-map for D.

using Fig. 4.

5. Derive (2.7) from (2.6) algebraically using (2.3).

6. K-Map for D: From Table 0, using boolean logic,

$$D = WXYZ' + W'X'Y'Z \quad (2.8)$$

7. Minimize (2.8) using Fig. 6.

8. Download the code in

```
wget https://raw.githubusercontent.com/gadepall/
  arduino/master/7447/codes/inc_dec/
  inc_dec.ino
```

and modify it using the K-Map equations for A,B,C and D. Execute and verify.

9. Display Decoder: Table 9 is the truth table for the display decoder in Fig. ???. Use K-maps to obtain the minimized expressions for a, b, c, d, e, f, g in terms of A, B, C, D with and without don't care conditions.

D	C	B	A	a	b	c	d	e	f	g	Decimal
0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	1	1	0	0	1	1	1	1	1
0	0	1	0	0	0	1	0	0	1	0	2
0	0	1	1	0	0	0	0	1	1	0	3
0	1	0	0	1	0	0	1	1	0	0	4
0	1	0	1	0	1	0	0	1	0	0	5
0	1	1	0	0	1	0	0	0	0	0	6
0	1	1	1	0	0	0	1	1	1	1	7
1	0	0	0	0	0	0	0	0	0	0	8
1	0	0	1	0	0	0	1	1	0	0	9

TABLE 9: Truth table for display decoder.