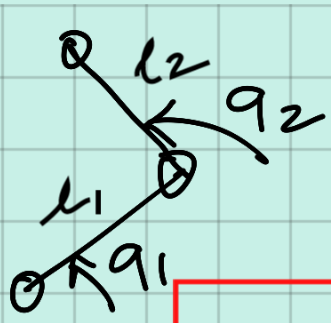


$x_1 = l_1 \cos q_1$

$y_1 = l_1 \sin q_1$

$$\boxed{\begin{aligned} x_2 &= l_1 \cos q_1 + l_2 \cos q_2 \\ y_2 &= l_1 \sin q_1 + l_2 \sin q_2 \end{aligned}}$$

Forward kinematics — 1

Differentiating above equation

$$\dot{x}_2 = -l_1 s q_1 \dot{q}_1 - l_2 s q_2 \dot{q}_2$$

$$\dot{y}_2 = l_1 c q_1 \dot{q}_1 + l_2 c q_2 \dot{q}_2$$

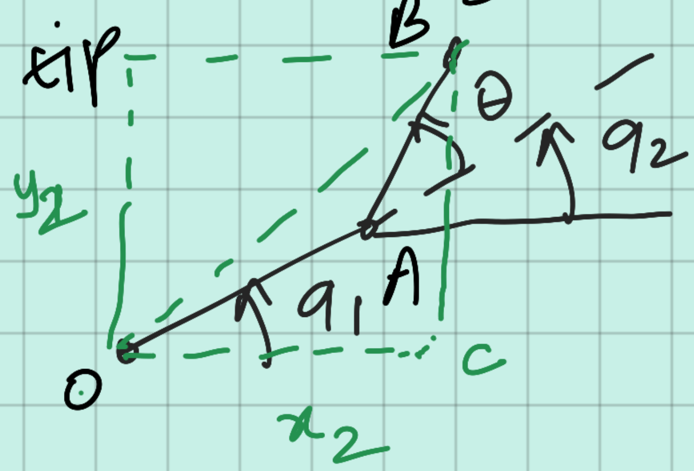
$$\boxed{\begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & -l_2 s q_2 \\ l_1 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}}$$

velocity at tip

Jacobian

vel at joint

— (2)



ΔOAC,

$l = x_2^2 + y_2^2$

Applying cosine law in Δ OAB :-

$$l^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos(\pi - \theta)$$

$$\cos\theta = \frac{-(l_1^2 + l_2^2) + l^2}{-2 l_1 l_2} = D$$

$$\theta = \cos^{-1}\left(\frac{l^2 - (l_1^2 + l_2^2)}{2 l_1 l_2}\right)$$

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \pm\sqrt{1 - D^2}$$

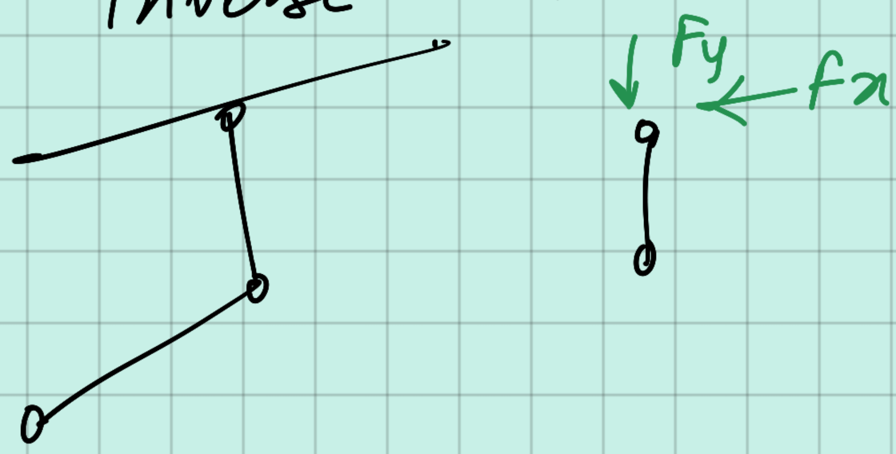
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\pm\sqrt{1 - D^2}}{D}$$

$\pm$ gives elbow up and down configuration

$$\boxed{\theta = \tan^{-1} \frac{\pm\sqrt{1 - D^2}}{D}}$$

$$\boxed{\begin{aligned} q_1 &= \tan^{-1}\left(\frac{y_1}{x_1}\right) \\ q_2 &= q_1 + D \end{aligned}}$$

Inverse kinematics. —(3)



Fy, Fx are horizontal & vertical forces on manipulator

Manipulator is stationary.

$$\tau_1 = F_x\, l_1 S q_1 - F_y\, l_2 S q_1$$

$$\tau_2 = F_x\, l_2 S q_2 - F_y\, l_2 C q_2$$

$$\boxed{\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} l S q_1 & -l_1 C q_1 \\ l_2 S q_2 & -l_2 C q_2 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix}}$$

—(4)

Statics equation

Dynamics equations in
Dynamics pdf
codes in folder.

For spring like motion.

$$\tau = K\Delta q + \text{Dynamics}$$

dynamics other than $\dot{q}$

effect of gravity

(5)

Δq can be found by
inverse kinematics.
from Δx

$\downarrow$

$$\Delta x = x - x_0$$

K - stiffness matrix.