Show $P(x) = PDF$.

Poission Distrubtion.

- This distrubtion is related to probabilities of events which are extremely rare but which have large number of independent oppu. for occurancy.
- This distrubtion is a case of B.D by making 'n' very large & 'P' very small.

The PDF is $\frac{e^{-m} m^x}{x!}$ for $m = np$ (fixed)

NOTE → Mean $= m$, Varience $(V) = m$. Standard dev: $\sqrt{V} = \sqrt{m}$.

(1) If the probability of a bad reaction from a certain injection is 0.001. determine the chance that out of 2000 individuals more than 2 will get bad reation

Soln $P = 0.001$, $n = 2000$. $m = np = 2000 \times 0.001 = 2$

x = no. of bad reactions $P(x) = \frac{e^{-m} x^x}{x!}$

$P(x > 2) = 1 - [P(0) + P(1) + P(2)]$

$= 1 -$

$=$

(2) If a random variable has Probability distribution such that $P(1) = P(2)$. Find

i) mean

(ii) $P(4)$

Soln

$$P(x) = \frac{e^{-m} m^x}{x!}$$

$$P(1) = P(2)$$

$$\frac{e^{-m} m^1}{1!} = \frac{e^{-m} m^2}{2!}$$

$$\boxed{m = 2}$$

(ii) $P(4) = \frac{e^{-m} m^x}{x!} = \frac{e^{-2} 2^4}{4!} = 0.0902.$

(3) If on the average two cars enter certain parking lott per min. What is the probability during any given min 4 or more cars entering the parking lott.

Soln

$m = 2$

$x =$ no. of cars.

$$P(x \geq 4) = 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$= 1 - \left[\frac{e^{-2} 2^0}{0} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!} + \frac{e^{-2} \cdot 2^3}{3!}\right]$$

$$= 1 - [0.135 + 0.27 + 0.2706 + 0.180]$$

$$= 0.144$$

* Continuous Probability Distribution

If for every 'x' belonging to the range of a continuous random variable 'x' we assign a real no. $f(x)$ such that $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)\, dx = 1$. Then $f(x)$ is called Continuous Probability function or Probability Density function.

NOTE :- If (a, b) is subinterval then, the probability that x lies in interval (a, b) is given by.

$$P(a \leq x \leq b) = \int_a^b f(x) \cdot dx$$

* Cummulative Dist

If 'x' is a continuous random variable with PDF $f(x)$ then the function $F(x)$ defined by

$$F(x) = \{ P(X \leq x) = \int_{-\infty}^{x} f(x) \cdot dx.$$

NOTE If 'r' is any Real no :

(i) $P(x \geq r) = \int_r^{\infty} f(x)\, dx$

(ii) $P(x < r) = \int_{-\infty}^{r} f(x) \cdot dx.$

Eg: $P(a \leq x \leq b) = F(b) - F(a)$

$$\int_a^b f(x)\, dx = \int_{-\infty}^{b} f(x)\, dx - \int_{-\infty}^{a} f(x)\, dx.$$

$$= F(b) - F(a).$$

Mode

NOTE :- Mean $(\mu) = \int_{-\infty}^{\infty} x f(x)\, dx$.

Variance $(V) = \int_{-\infty}^{\infty} x^2 f(x) - \mu^2$.

(1) A random variable x has the following density function

$$p(x) = \begin{cases} kx^2 , & -3 \le x \le 3 \\ 0 , & \text{otherwise} \end{cases}$$

Find a) k

b) $P(1 \le x \le 2)$

c) $P(x \le 2)$

d) $P(x > 1)$

Sol^n. WKT $\int_{-\infty}^{\infty} P(x)\, dx = 1$

$$\int_{-\infty}^{3} p(x)\, dx + \int_{-3}^{3} p(x)\, dx + \int_{3}^{\infty} p(x)\, dx = 1$$

$$0 + \int_{-3}^{3} kx^2\, dx + 0 = 1$$

$-\infty$	-3	3	∞
	0	kx^2	0

$$k \int_{-3}^{3} kx^2 = 1$$

$$\boxed{k = \frac{1}{18}}$$

(b) $P(1 \le x \le 2) = \int_{1}^{2} P(x) \cdot dx$

$$= \int_{1}^{2} kx^2\, dx$$

$$k \int_{1}^{2} x^2 \cdot dx$$

$$\frac{7}{54} \,//.$$

(c) $P(x \le 2) = \int_{-\infty}^{2} p(x) \cdot dx$

$= \int_{-3}^{2} p(x) \, dx = \frac{35}{54}$

d) $P(x > 1) = \int_{1}^{\infty} P(x) \, dx$

$= \int_{1}^{3} P(x) \, dx = \frac{13}{27}$

(2) The time 'T' years required to complete a software project has a PDF of the form $f(t) = \begin{cases} kt(1-t), & 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$

Find 'k' and also the probability that the project will be completed in less than 4 months.

Sol^n $\int_{-\infty}^{\infty} P(x) \, dx = 1$

$\int_{-\infty}^{0} p(t) \cdot dt + \int_{0}^{1} P(t) \, dt + \int_{1}^{\infty} P(t) \, dt = 1.$

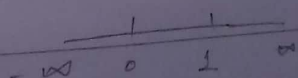
$\int_{0}^{1} kt(1-t) \cdot dt = 1.$

$\int_{0}^{1} kt - kt^2 \cdot dt$

$\left[\frac{kt^2}{2} - \frac{kt^3}{3} \right]_0^1 = \frac{k}{2} - \frac{k}{3} = \frac{k}{6} = 1$

$\boxed{k = 6}$

$P\left(t < \frac{4}{12}\right) = \int_{0}^{4/12} kt(1-t) \, dt$



$6 \int^{4/12} t - t^2 \, dt$

(3) The km run (in 1000km) without any sort of problem w.r.t. certain Vechicle is a random Vehicle having PDF $f(x) = \begin{cases} \frac{1}{40} e^{-x/40} & x \geq 0 \\ 0 & x < 0 \end{cases}$. Find the probability that the Vechicle is having trouble

a) at least 25000 km

b) at most 25000 km

c) b/n 16000 - 32000 km

Sol a) $f(x \geq 25) = \int_{25}^{\infty} f(x)\, dx$

$= \int_{25}^{\infty} \frac{1}{40} e^{-x/40}\, dx.$

$= \frac{1}{40} \left[\frac{e^{-x/40}}{-1/40}\right]_{25}^{\infty} = -\left[0 - e^{-25/40}\right]$

$= 0.5352$

b) $f(x \leq 25) = \int_{-\infty}^{25} f(x)\, dx$

$= \int_{-\infty}^{0} f(x)\, dx + \int_{0}^{25} f(x)\, dx.$

$= 0 + \int_{0}^{25} \frac{1}{40} e^{-x/40}\, dx$

$= 0.4647.$

c) $f(16 < x < 32) = \int_{16}^{32} f(x)\, dx = \int_{16}^{32} \frac{1}{40} e^{-x/40}\, dx$

$= 0.22099$

(4) Find the value of 'k' such that $f(x) = \begin{cases} kxe^{-x} & 0<x<1 \\ 0 & \text{Otherwise} \end{cases}$ is a PDF. Find the mean.

Sol^n WKT $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{1} kxe^{-x}\,dx + 0 = 1$$

$$k(0.2642) = 1$$

$$k = 3.784.$$

Mean $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx$$

$$= 0 + \int_{0}^{1} x \cdot kxe^{-x}\,dx + 0$$

$$k\int_{0}^{1} x^2 e^{-x}\,dx = 0.6078.$$

(i) (ii) $\int_{-\infty}^{\infty} f(x)\,dx$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx$$

$$0 + \int_{0}^{\infty} \lambda e^{-\lambda x}\,dx$$

$$\lambda \left\{ \frac{e^{-\lambda x}}{-\lambda} \right\}_0^{\infty} = -(0-1) = 1$$

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Let 'α' be a constant greater than zero & the PDF

$$f(x) = \begin{cases} \alpha e^{-\alpha/x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

. The distribution with this PDF is known as Exponential distribution.

NOTE → Mean $\mu = 1/\alpha$

Varience $= \frac{1}{\alpha^2}$

1. If 'x' is an exponential variate with mean 3. Find

(i) $P(x>1)$

(ii) $P(x<3)$

Sol^n Mean $= \frac{1}{\alpha} = 3$

$\boxed{\alpha = 1/3}$

(i) $P(x>1) = \int_1^\infty P(x)\,dx$

$= \int_1^\infty \alpha e^{-\alpha x}\,dx$

$= \frac{1}{3}\int_1^\infty e^{-1/3 x}\,dx$

$= \frac{1}{3}\left[\frac{e^{-1/3 x}}{-1/3}\right]_1^\infty = -[0 - e^{-1/3}]$

$= 0.7165.$

(ii) $P(x<3) = \int_{-\infty}^{3} p(x)\,dx = \int_{-\infty}^{0} p(x)\,dx + \int_0^3 p(x)\,dx$

$= 0 + \int_0^3 \alpha e^{-\alpha x}\,dx$

$= \frac{1}{3}\int_0^3 e^{-x/3}\,dx$

$= 0.6321$

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2. Life of a battery is a random Variable which has E.D with mean 200 hrs. Find probability that the life of battery is
(i) less than 100 hrs
(ii) b/n 400 - 600 hrs

Sol^n Mean $= \frac{1}{\alpha} = 200$

$\boxed{\alpha = 1/200}$

(i) $P(x < 100) = \int_{-\infty}^{100} P(x) \cdot dx$

$= \int_{-\infty}^{0} P(x) \cdot dx + \int_{0}^{100} P(x)\, dx$

$0 + \int_{0}^{100} \alpha e^{-\alpha x}\, dx$

$= \int_{0}^{100} \frac{1}{200} e^{-x/200}$

$= 0.3934$

(ii) $P(400 < x < 600) = \int_{400}^{600} \alpha e^{-\alpha x} = \frac{1}{200} \int_{400}^{600} e^{-x/200}$

$= 0.0855$

pQ.

3. The duration of shower in certain town during the period of depression is exponentially distributed with mean 5 min. What is the probability that the duration of the downpore is
(i) 10 min or more
(ii) less than 10 min

Sol^n Mean $= \frac{1}{\alpha} = \alpha = \frac{1}{5}$

(i) $P(x \geq 10) = \int_{10}^{\infty} \alpha e^{-\alpha x}\, dx = \int_{10}^{\infty} \frac{1}{5} e^{-x/5}\, dx$

$= \frac{1}{5} \int_{10}^{\infty} e^{-x/5}\, dx = \frac{1}{5} \left[\frac{e^{-x/5}}{-1/5} \right]_{10}^{\infty} = -\left[0 - e^{-2}\right]$

$= 0.135$

(ii) $P(x<10) = \int_{-\infty}^{10} p(x)\,dx$.

$= \int_{-\infty}^{0} p(x)\,dx + \int_{0}^{10} p(x)\,dx$

$= 0 + \int_{0}^{10} \alpha e^{-\alpha x}\,dx$

$\frac{1}{5}\int_{0}^{10} e^{-x/5}\,dx$

$= \underline{\underline{0.86466}}$

Fitting of P-Distributⁿ

*. Fit the P-D for the following frequency distribution.

x :	0	1	2	3	4
$f(x)$:	46	38	22	9	1

Solⁿ

$m = \frac{\sum f_i x_i}{\sum f_i} = \frac{113}{116} = 0.974$

$N = \sum f_i = 116$

$f(x) = N \times P(x)$

$= N \times \frac{e^{-m} m^x}{x!}$

$P(x=0) = 43.7980 \approx 44$

$P(x=1) = 42.659 \approx 43$ $P(x=2) = 20.775 \approx 21$

$P(x=3) = 6.744 \approx 7$ $P(x=4) = 1.642 \approx 1$.

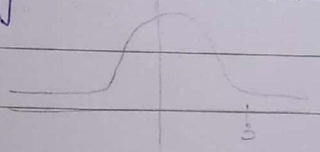
Derivatⁿ of mean & Variance

Normal distrubtion :

The continuous probability dist. having PDF $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$ where μ & σ are the mean & standard deviation of the distrubtion & 'x' assume the value $-\infty$ to ∞.

NOTE 1. The graph of the normal distrubtion is called as Normal curve. It is bell shape & symmetric abt mean (μ). The 2 tales of the curve extends to $-\infty$ & ∞ gradually approching the x-axis without meating it.

2. The area under the normal curve b/n the ordinates represent the probability of the values falling into the given intervals.



$P(t<3) = 0.5 + A(3)$

$P(t>3) = 0.5 - A(3)$

Standard Normal distrubtion :

The normal distrubtion for which the mean is 0 & standard dev. is 1 is called standard Normal distr. & is given by $\frac{1}{\sqrt{2\pi}} e^{-t^2/2} = \phi(t)$.

NOTE For conversion we use $t = \frac{x-\mu}{\sigma}$ (or z)

eg $P(\underline{x}>3) \rightarrow$ Normal.

$P\left(\frac{x-\mu}{\sigma} > \frac{3-\mu}{\sigma}\right)$

1) If x is a normal variate with mean 30 & SD 5 find probability that

a) $26 \le x \le 40$ c) $|x-30| > 5$

b) $x \geq 45$

Soln a) $P(26 \le x \le 40)$

$P\left(\frac{26-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{40-\mu}{\sigma}\right)$

$P\left(\frac{26-30}{5} \le t \le \frac{40-30}{5}\right)$

$P(-0.8 \le t \le 2)$

$A(0.8) + A(2)$

$0.2881 + 0.4772$

$= 0.7653$

b) $P(x \geq 45)$

$P\left(\frac{x-\mu}{\sigma} \geq \frac{45-\mu}{\sigma}\right)$

$P\left(t \geq \frac{45-30}{5}\right)$

$P(t \geq 3)$

$0.5 - \phi(3)$

$0.5 - 0.4987 = 0.0014$ //

c) $P(|x-30| > 5) \rightarrow P(|x-30| \le 5)$

$P(-5 \le x-30 \le 5)$

$P(-5+30 \le x \le 5+30)$

$P(25 \le x \le 35)$

$P\left(\frac{25-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{35-\mu}{\sigma}\right)$

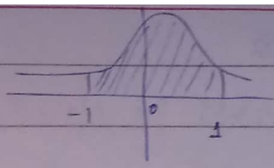
$P\left(\frac{25-30}{5} \le \frac{x-30}{5} \le \frac{35-30}{5}\right)$

$P(-1 \le t \le 1)$

$A(1) + A(1)$

$= 0.3413 + 0.3413$

$= 0.6826$



$= 1 - 0.6826$

$= \underline{0.3174}$

2) The mean height of 500 students is 151 cm & SD is 15cm Assuming that the heights are normally distributed find how many students' height lie b/n 120 & 155cm

$\underline{\text{sol}^n}$

$\sigma = 15\text{ cm}$ mean $(\mu) = 151$

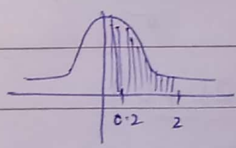
$P(120 < x < 155)$

$$P\left(\frac{120-\mu}{\sigma} < \frac{x-\mu}{\sigma} < \frac{155-\mu}{\sigma}\right)$$

$$P\left(\frac{120-151}{15} < t < \frac{155-151}{15}\right)$$

$P(2.06 < t < 0.26)$

$A(2.1) + A(0.26)$



$0.4821 + 0.0193$

$= 0.5829$

for 500 students 0.5829×500

$= 291.45$ students

3) In a test of 2000 electric bulb it was found that the life of a particular make was normally distributed with an average life of 2040 hrs & SD of 160 hrs estimate the No. of bulbs likely to burn for

1) more than 2150 hrs

2) less than 1950 hrs

3) more than 1920 hrs but less than 2160 hrs.

Date
Page

Sol^n $\mu = 2040$ $\sigma = 60$

1) $P(x > 2150)$

$P\left(\frac{x-\mu}{\sigma} > \frac{2150-\mu}{\sigma}\right)$

$P\left(t > \frac{2150-2040}{60}\right)$

$P(t > 1.83)$

$0.5 - A(1.83)$

$0.5 - 0.4644$

$= 0.0336$

$= 0.0336 \times 2000 = 67$

2) $P(x < 1950)$

$P\left(\frac{x-\mu}{\sigma} < \frac{1950-\mu}{\sigma}\right)$

$P\left(t < \frac{1950-2040}{60}\right)$

$P(t < - \cancel{60}\ 1.5)$

$0.5 - A(1.5)$

$0.5 - 0.4332$

0.0668

$0.0668 \times 2000 = 133.6$

$- 134$ //

4] In a normal dist. 31% of items are under 45 & 8% of the items are over 64. Find the mean & ~~st~~ the standard deviation of dist.

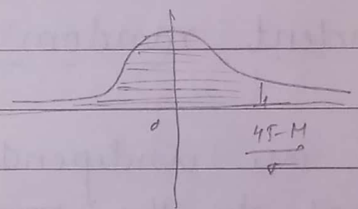
Sol^n $P(x < 45) = 0.31$

$P\left(\frac{x-\mu}{\sigma} < \frac{45-\mu}{\sigma}\right) = 0.31$

$P\left(t < \frac{45-\mu}{\sigma}\right) = 0.31$

$0.5 + A\left(\frac{45-\mu}{\sigma}\right) = 0.31$

$A\left(\frac{45-\mu}{\sigma}\right) = 0.31 - 0.5$



$A\left(\frac{45-\mu}{\sigma}\right) = -0.19$

$\frac{45-\mu}{\sigma} = -0.5$

$45 - \mu = -0.5\sigma$

$\mu - 0.5\sigma = 45 \rightarrow (1)$

$P(x > 64) = 0.08$

$P\left(\frac{x-\mu}{\sigma} > \frac{64-\mu}{\sigma}\right) = 0.08$

$P\left(t > \frac{64-\mu}{\sigma}\right) = 0.08$

$0.5 - A\left(\frac{64-\mu}{\sigma}\right) = 0.08$

$A\left(\frac{64-\mu}{\sigma}\right) = 0.5 - 0.08$

$A\left(\frac{64-\mu}{\sigma}\right) = 0.42$

$\frac{64-\mu}{\sigma} = 1.4$

~~64μ~~

$64 - \mu = 1.4\sigma$

$\mu + 1.4\sigma = 64 \rightarrow (2)$

$\boxed{\mu = 50}$ $\boxed{\sigma = 10}$