Unit-I

Simplification of boolean functions

1) Solve the given boolean expression and implement using NAND gates.

$$Y = ABC\bar{D} + AB\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}\bar{C}\bar{D}$$

$$Y = AB\bar{D}(C+\bar{C}) + AB\bar{C}D + A\bar{B}\bar{C}\bar{D}$$

$$Y = ABC(\bar{D}+\bar{C}D) + A\bar{B}\bar{C}\bar{D}$$

$$Y = AB(\bar{D}+\bar{C}) + A\bar{B}\bar{C}\bar{D}$$

$$Y = AB\bar{C} + AB\bar{D} + A\bar{B}\bar{C}\bar{D}$$

$$Y = AB\bar{C} + A\bar{D}(B+\bar{B}\bar{C})$$

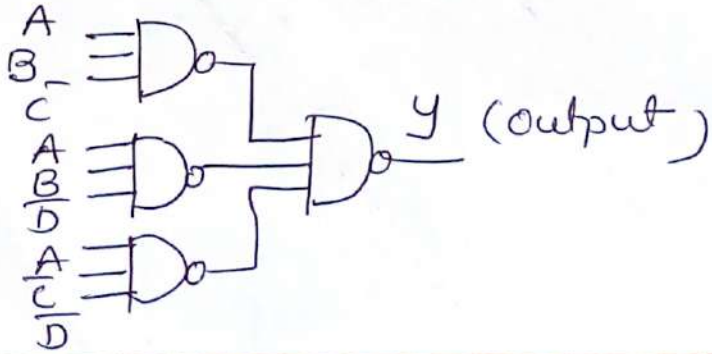
$$\therefore Y = AB\bar{C} + AB\bar{D} + A\bar{C}\bar{D}$$

(Note $C+\bar{C} = 1$ & $B+\bar{B}C = B+C$ as per boolean algebra)

for implementing boolean function using NAND gate take double complement of expression and solve one complement using Demorgan's Theorem.

$$\overline{\overline{Y}} = \overline{[AB\bar{C} + AB\bar{D} + A\bar{C}\bar{D}]}$$

$$\therefore Y = \overline{\left[\overline{(AB\bar{C})} \cdot \overline{(AB\bar{D})} \cdot \overline{(A\bar{C}\bar{D})}\right]}$$



* Sum of Products expression :
(SOP form)

The boolean expression/functions are expressed in two form.

1) SOP 2) POS.

product term: Product of one or more variable A, $A\bar{B}$, ABC ... etc.

minterm : It is fundamental product term for which output is logic '1' for given inputs. It is denoted by m.

for exa. $m_0 = \bar{A}\bar{B}\bar{C}$, $m_7 = ABC$

If the variable value is '0' then, it is complemented otherwise it remains same.

SOP expression ⇒

$f(A,B,C) = \sum m(0,1,6,7)$

It is 3 variable boolean function, so $2^3 = 8$ input combinations 0 to 7.

The output is 1 for minterms given in the function.

$f = m_0 + m_1 + m_6 + m_7$ → SOP form

It is also called as standard form or canonical form.

$f = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$

$f = \bar{A}\bar{B} + AB$ – Simplified SOP

* Product of Sum (POS) expression

Sum term : Sum of one or more variable The variable may be in normal form or complemented form.

for exa. A, $A+\bar{B}$, $A+B+C$, .. etc.

Maxterm : It is fundamental sumterm for which output is '0' for given inputs. exa. M_6 is max term $(6 \rightarrow 110)$. $M_6 = \bar{A}+\bar{B}+C$

POS : In this expression function is expressed as product of sum terms.

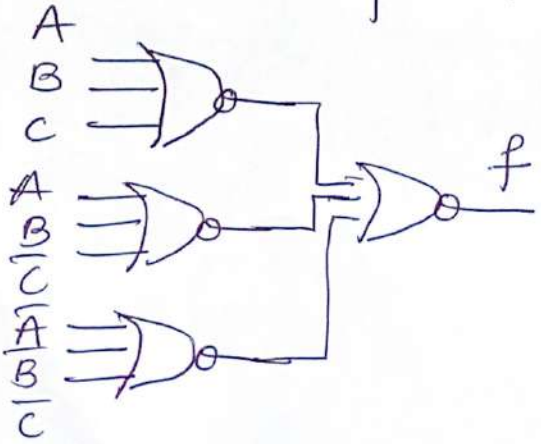
$f(A,B,C) = \Pi M(0,1,7)$

$f = M_0 \cdot M_1 \cdot M_7 \rightarrow$ Standard POS

$f = (A+B+C)\cdot(A+B+\bar{C})\cdot(\bar{A}+\bar{B}+\bar{C})$

POS expressions are implemented by using NOR gates and SOP are implemented by using NAND gates.

NOR implementation



* K map Simplification for SOP expression :

Simplify given 4 variable boolean function in SOP form and realise using NAND gates.

$f(A,B,C,D) = \Sigma m(0,1,2,4,5,8,12) + d(6,14)$

(d → don't care term)

4 variable k-map →

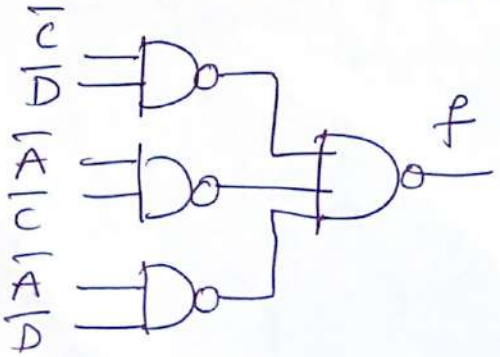
AB \ CD	00	01	11	10
$\bar{A}\bar{B}$ - 00	1	1	0	1
$\bar{A}B$ - 01	1	1	0	X
AB - 11	1	0	0	X
$A\bar{B}$ - 10	1	0	0	0

- Group of 4 ones is quad and it eliminates two variable from product term.
- Group of 2 ones is pair eliminates one variable from product term.

In above map there are three quads one with folding the map. Each group represent a product term

$f = \bar{C}\bar{D} + \bar{A}\bar{C} + \bar{A}\bar{D}$

NAND Realisation



* Complements of variables are taken through inverter.

* K-map simplification for POS expression:

Solve the given boolean function using K-map method in POS form and realise using NOR gates.

$f(A,B,C,D) = \Pi M(0,1,2,3,4,8,12)$

4 variable K map →

AB \ CD	00	01	11	10	
00	0	0	0	0	→ (A+B)
01	0	1	1	1	
11	0	1	1	1	
10	0	1	1	1	

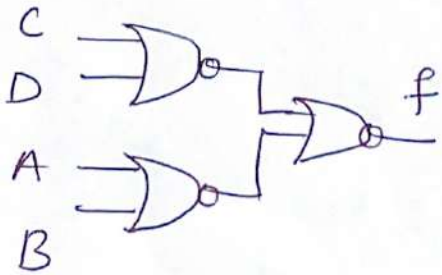
↳ (C+D)

Sum expression

$f = (C+D) \cdot (A+B)$ – POS form

Logic ckt. using NOR gates.

$\therefore \bar{\bar{f}} = \overline{[\overline{(C+D)} + \overline{(A+B)}]}$



Simplification of Boolean function using Q-M (tabulation) method:

Simplify the given boolean function using Q-M method and list the prime implicants.

$f(A,B,C,D) = \sum m(0,1,2,4,5,14,15)$

Step 1: Sort the minterms as per number of 1's in binary representation of decima number (i.e. minterm)

exa. 15 - 1111 (group with 4 one)
12 - 1100 (" " 2 one)

Step 2: pairing the minterms. Each minterm from group 1 is compared with group 2. If two minterm differ in only one position in binary representation, then it is valid pair.

exa. (2, 3) → 0010 & 0011 → The product term with variables ABCD is $\bar{A}\bar{B}C-$ (The group differ in D positio

Step 3: Grouping the minterm pair to next level as Quad (group of 4 one)

If two pair differ in only one position in binary, then it's valid Quad.

exa. (12, 13) & (14, 15). They differ in C position and productterm AB--

Tabulation Method (Q-M)

$f(A,B,C,D) = \Sigma m(0,1,2,4,5,14,15)$

	minterm binary A B C D	1st level of grouping	Binary	*
G-0	0000 - 0 ✓	0, 1 (1) → 000 − ✓		
	0001 - 1 ✓	0, 2 (2) → 00 − 0 →	R	
G-1	0010 - 2 ✓	0, 4 (4) → 0 − 00 ✓		
	0100 - 4 ✓	1, 5 (4) → 0 − 01 ✓		
G-2	0101 - 5 ✓	4, 5 (4) → ~~0 1 0 −~~ 010 − ✓		
G-3	1110 - 14 ✓	14, 15 (1) → 111 − →	Q	
G-4	1111 - 15			

Note: ✓ tick indicates the minterms are carried to next level.
→ indicates the independent group as prime implicant

* 2nd Level of grouping

0, 1, 4, 5 (1, 4) → P

P, Q, R are prime implicants.

$P = \bar{A}\bar{C} \Rightarrow$

$Q = ABC$

$R = \bar{A}\bar{B}\bar{D}$

A	B	C	D
0	0	0	0
0	0	0	1
0	1	0	0
0	1	0	1

$f = P + Q + R$

$f = \bar{A}\bar{C} + ABC + \bar{A}\bar{B}\bar{D}$

[Note: Decimal or Binary notation can be used to solve]

* finding essential prime implicants from prime implicant chart →

In Q-M method (Tabulation method), in some boolean function simplification procedure, extra P.I. are obtained due to overlapping groups or due to don't care terms. The main purpose of simplification is minimization of boolean function. So unwanted P.I. are eliminated using P.I. chart. d → don't care term.

$f(A,B,C,D) = \Sigma m(0,1,4,8,9,13) + d(2,5)$

minterm	1st level of grouping	2nd level of grouping
0 ✓	0, 1 (1) ✓	0, 1, 4, 5 (1, 4) → P
1 ✓	0, 4 (4) ✓	0, 8, 1, 9 (8, 1) → Q
4 ✓	0, 2 (2) → S	1, 5, 9, 13 (4, 8) → R
8 ✓	0, 8 (8) ✓	
2 ✓	1, 5 (4) ✓	
5 ✓	1, 9 (8) ✓	
9	4, 5 (1) ✓	
13 ✓	8, 9 (1) ✓	
	5, 13 (8) ✓	
	9, 13 (4) ✓	

* P, Q, R, S are prime implicants

$f = P + Q + R + S$

$P = \bar{A}\bar{C}$ (0 _ 0 _)

$Q = \bar{B}\bar{C}$ (_ 0 0 _)

$R = \bar{C}D$ (_ _ 0 1)

$S = \bar{A}\bar{B}\bar{D}$ (0 0 _ 0)

Note: Only decimal notation used.

PTO

Prime implicant charts

minterms →

PI	0 ✓	1 ✓	4 ✓	8 ✓	9 ✓	13 ✓
P	×	×	(×)			
Q	×	×		(×)	×	
R		×			×	(×)
S	×					

P, Q, R are essential Prime implicant

Check whether P.I. (essential) cover all the minterms given in the boolean function. Don't consider the don't care terms (d) given in the boolean function. After checking it is observed P, Q, R are essential P.I. covers all minterm given in boolean function. So S is extra P.I.

$$f = P + Q + R$$

$$f = \bar{A}\bar{C} + \bar{B}\bar{C} + \bar{C}D$$

NAND realization