ELE 215.3 Network Theory (3-1-2)

Evaluation:

	Theory	Practical	Total
Sessional	30	20	50
Final	50	-	50
Total	80	20	100

Course Objectives:

The purpose of the course is to provide the knowledge of network equations and the behavior of network. Moreover, it provides in-depth knowledge to develop one-port and two port networks with given network functions.

Course Contents:

1. **Review of Network Analysis** **(2 hrs)**
   1.4 Mesh and Nodal analysis

2. **Circuit Differential Equations (Formulation and Solutions)** **(5 hrs)**
   2.10 The differential operator
   2.11 Operational impedance
   2.12 Formulation of circuit differential equations
   2.13 Complete response (transient and steady state) of first order differential equations with or without initial conditions

3. **Circuit Dynamics** **(7 hrs)**
   3.7 First order RL and RC circuits
   3.8 Complete response of RL and RC circuit to sinusoidal input
   3.9 RLC circuit
   3.10 Step response of RLC circuit
   3.11 Response of RLC circuit to sinusoidal inputs
   3.12 Damping factors and Damping Coefficients.

4. **Review of Laplace Transform** **(5 hrs)**
   4.7 Definition and properties
   4.8 Laplace transform of common forcing functions
   4.9 Initial and final value theorem
   4.10 Inverse Laplace transform
   4.11 Partial fraction expansion
   4.12 Step response of RL, RC and RLC circuit

4.13 Sinusoidal response of RL, RC and RLC circuits
4.14 Exponential response of RL, RC and RLC circuits

5. **Transfer Functions** **(4 hrs)**
   5.7 Transfer functions of network system
   5.8 Poles and Zeros
   5.9 Time domain behavior from pole-zero locations
   5.10 S Routh'- Hurwitz's stability Criteria

6. **Fourier Series and Transform** **(4 hrs)**
   6.8 Evaluation of Fourier coefficients for periodic non-sinusoidal waveform
   6.9 Fourier Transform
   6.10 Application of Fourier transforms for non-periodic waveforms

7. **Frequency Response of Network** **(7 hrs)**
   7.10 Magnitude and phase responses
   7.11 Bode plots and its applications
   7.12 Concept of ideal and non-ideal low pass, high pass, band pass, and band reject filters

8. **One-port Passive Network** **(7 hrs)**
   8.12 Properties of one-port passive network
   8.13 Positive Real Function
   8.14 Properties of RL, RC and LC network
   8.15 Synthesis of RL,RC and LC networks using Foster's and Cauer's method
   8.16 Properties of RLC one-port network

9. **Two-port Passive Network** **(7 hrs)**
   9.8 Properties of two-port network
   9.9 Reciprocity and symmetry
   9.10 Short circuit and Open circuit parameters
   9.11 transmission parameters
   9.12 Hybrid parameter
   9.13 Relation and transformations between sets of parameters
   9.14 Equivalent T and π section representation

Laboratory:

1. Transient and steady state responses of first order Passive network
   1.1 Measurement of step, impulse and ramp response of RC and RL circuit using oscilloscope
   1.2 Measurement of sinusoidal response of RC and RL circuit using oscilloscope

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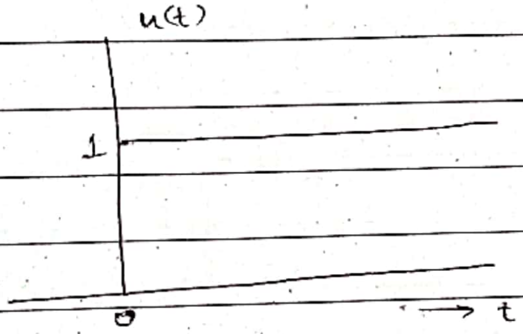
Input ⟶ Network ⟶ Output

(forcing function or driving source) (Response)

* Some Basic Forcing Functions.

1. Unit step function



$u(t) = 1, \quad t \geq 0$

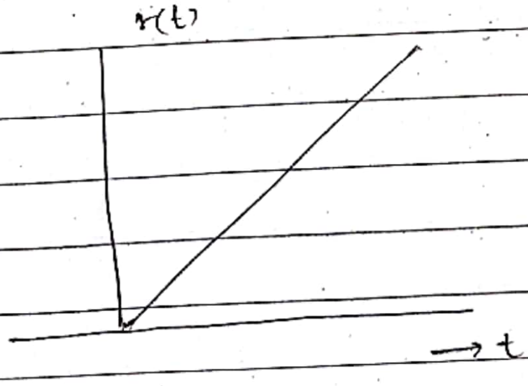
$\quad\;\; = 0, \quad t < 0$

Step function

Can have any value for positive half side.

eg. 10 V, 50 V etc.

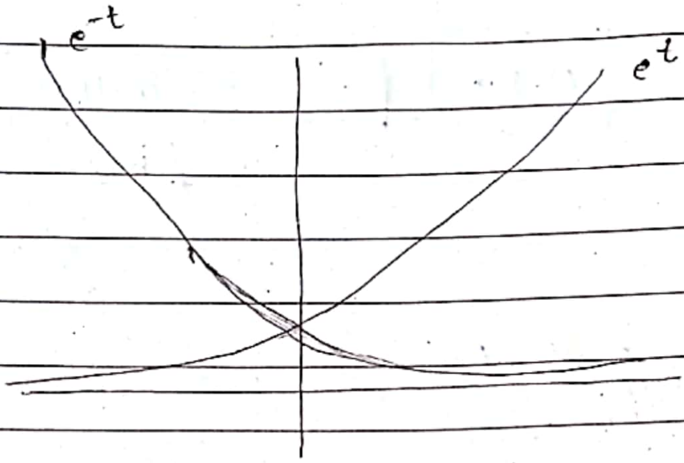
2. Ramp function



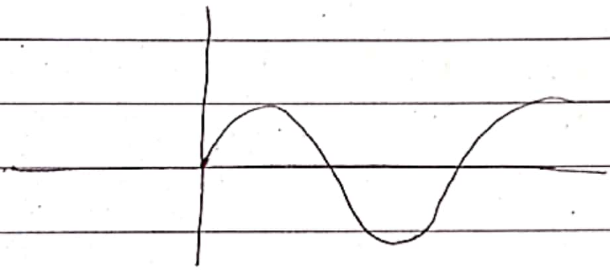
$r(t) = t \quad t \geq 0$

$\quad\;\; = 0 \quad t < 0$

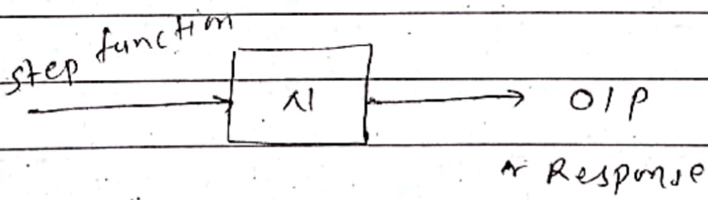
3. Exponential function



4. Sinusoidal function



$v = V_m \sin(\omega t)$, $v = V_m \cos(\omega t)$



(step function response)

* Passive elements and their relations

1) Resistor (R)

Symbol - R 

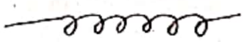
Unit - ohm (Ω)

$$V(t) = i(t)\,R$$

$$\Rightarrow R = \frac{v(t)}{i(t)} \qquad i(t) = \frac{v(t)}{R}$$

The resistor doesn't store any current or voltage just after switching.

2.) Inductor (L)

Symbol - L 

Unit - Henry (H)

mH (milihenry) = 10^{-3} H

$$v \propto \frac{d\,i(t)}{dt}$$

KVL

$$v = L\frac{d\,i(t)}{dt}$$; voltage across inductor.

Inductance.

$$d\,i(t) = \frac{v}{L}\,dt$$

Integrating,

$$i(t) = \frac{1}{L}\int v(t)\,dt$$; current through inductor.

KCL

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Inductor stores the current for a very small time just after switching:

$$i(0^-) = i(0^+)$$

3.7 Capacitor (C)

Symbol → C ⊣⊢

Unit → Farad (F)

μF (micro Farad) → 10^{-6} F.

$q(t) = C\,v(t)$; charge

$$v(t) = \frac{q(t)}{C}$$

$$i(t) = \frac{dq(t)}{dt}$$

$$dq(t) = i(t)\,dt$$

$$q(t) = \int i(t)\,dt$$

$v(t) = \frac{1}{C}\int i(t)\,dt$; voltage across capacitor.

Differentiating.

$$\frac{dv(t)}{dt} = \frac{1}{C}\,i(t)$$

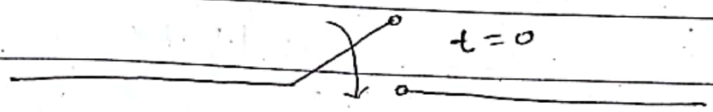
$i(t) = C\frac{dv(t)}{dt}$; current through capacitor.

Capacitor stores the charge or voltage just after switching for very few seconds.

$$V(0^-) = V(0^+) \quad \text{or,} \quad q(0^-) = q(0^+)$$

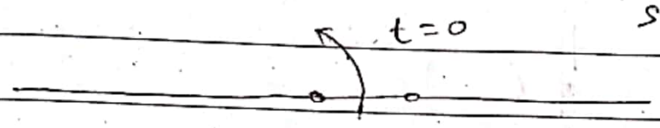
* Switching Function

(1)



(ON)
switch is closed at $t=0$ time.
↑
Action or switch time

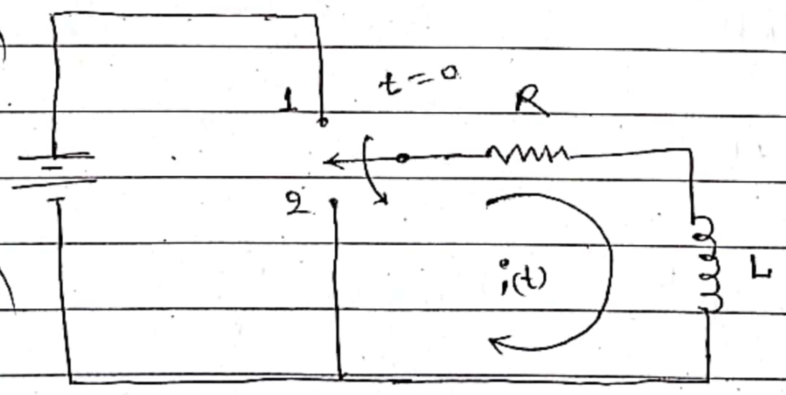
(2)



(OFF)
switch is opened at $t=0$ time

$t=0^-$	$t=0$	$t=0^+$
(open) Initial time or position	↑ switch action.	(closed) Final time or final position.

* Step Function Response of R-L Series Circuit



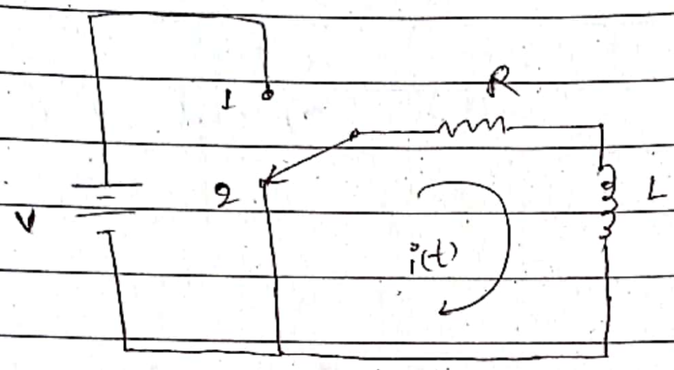
The switch is moved from position 1 to 2 at $t=0$

Find the current response.

Response [illegible] → Final position.

Sol^n

At $t=0$, the final circuit is



Applying mesh analysis.

$$-R\,i(t) - L\frac{d\,i(t)}{dt} = 0$$

$$R\,i(t) + L\frac{d\,i(t)}{dt} = 0$$

$$L\frac{d\,i(t)}{dt} + R\,i(t) = 0$$; First order homogeneous differential equation.

$$\frac{d\,i(t)}{dt} + \frac{R}{L}i(t) = 0$$

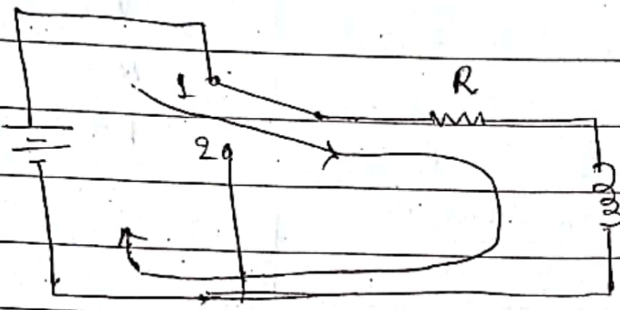
$$\frac{d\,i(t)}{dt} = -\frac{R}{L}i(t)$$

$$\frac{d\,i(t)}{i(t)} = -\frac{R}{L}dt$$

$$i(t) = K e^{-\frac{R}{L}t}$$

General solution.

To find K, we use initial position ckT.



$i(0^-) = \frac{V}{R}$ (due to no reactance)

put in general solution,

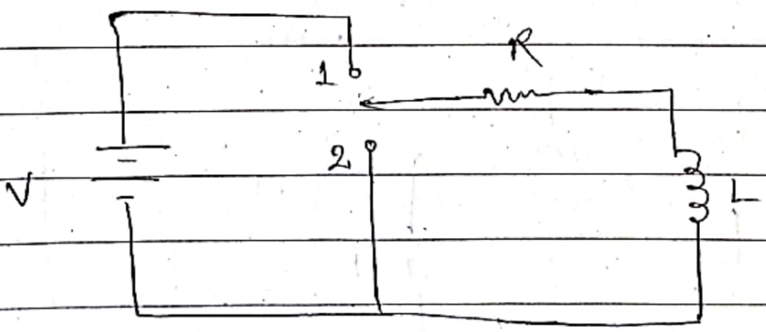
$$\frac{V}{R} = K \cdot e^{-\frac{R}{L} * 0}$$

$$\Rightarrow K = \frac{V}{R}$$

Now, $i(t) = \frac{V}{R} e^{-\frac{R}{L}t}$

particular solution.

H/W



$V = 20V$

$R = 2\Omega$

$L = 1H$

switch position 1 to 2

switching ~~position~~ moved from 1 to 2 at $t = 0$. Find the current Response. →

Also find the voltage across inductor.

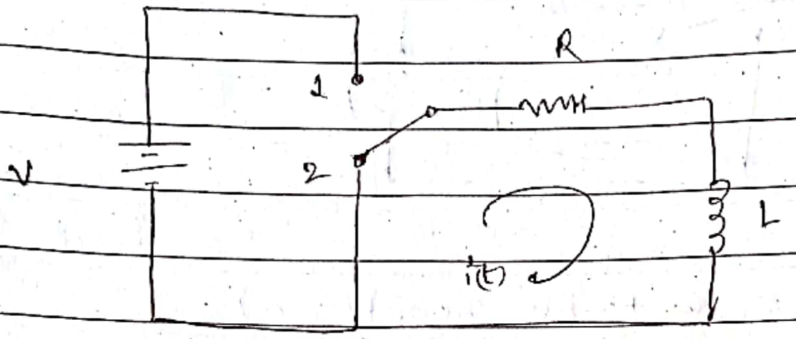
Sol^n

At $t=0$, ckt of final position is.

$V = 20V$

$R = 2\Omega$

$L = 1H$



Now applying mesh analysis, we have

$$-2\,i(t) - 1\,\frac{d\,i(t)}{dt} = 0$$

or, $$2\,i(t) + \frac{d}{dt}\,i(t) = 0$$

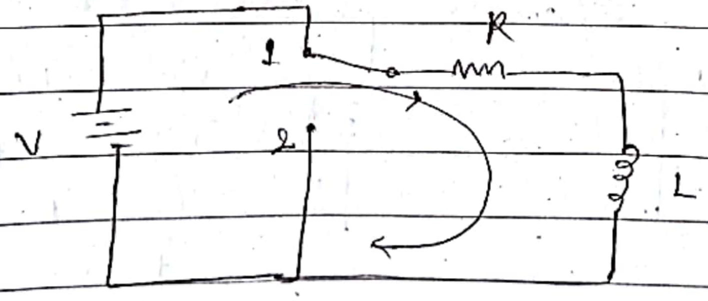
or, $$\frac{d\,i(t)}{dt} = -2\,i(t)$$

or, $$\frac{d\,i(t)}{i(t)} = -2\,dt$$

$$\Rightarrow \quad i(t) = k\,e^{-2t} \qquad \text{——— (i)}$$

To find k, we use initial position of ckt

i.e.



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Here, $i(0^+) = \frac{V}{R} = \frac{20}{2} = 10$ Amp.

since, $i(0^-) = i(t)$ at $t = 0$.

from ①

$$10 = K e^{-2 \times 0}$$

$$\Rightarrow K = 10$$

So, $i(t) = 10 e^{-2t}$ —— ⊛

Now, voltage across inductor.

$$v(t) = L \frac{d\, i(t)}{dt}$$

$$= 1 \cdot \frac{d}{dt} 10 e^{-2t}$$

$$= 10 \frac{d}{dt} e^{-2t}$$

$$= 10 \cdot e^{-2t} * -2$$

$$v(t) = -20 e^{-2t}$$

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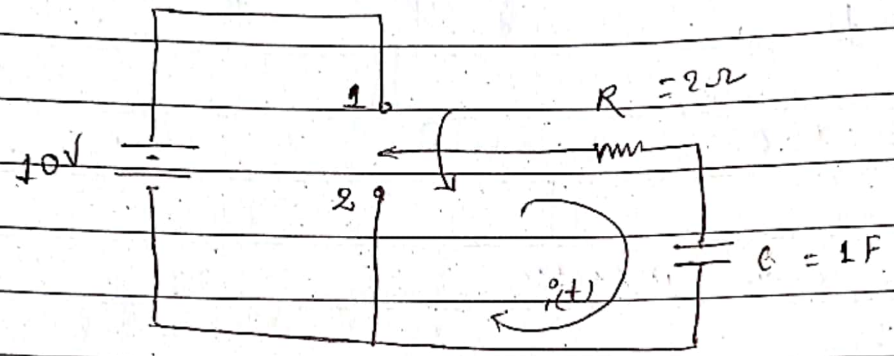
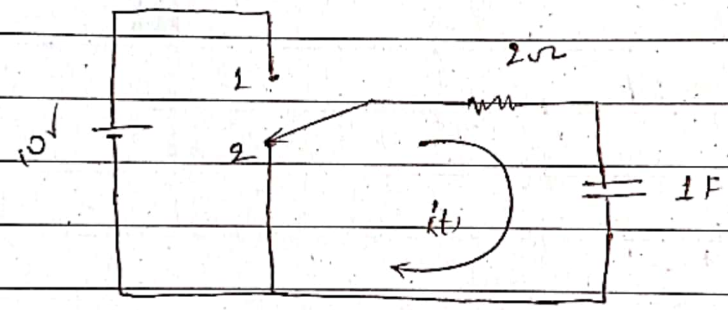
* Step function Response of RC series circuit.

switch is moved from position 1 to 2 at $t=0$.
Find the current response at 5 second.



sol^n

At $t=0$, final ckt



Applying KVL

$$-R\,i(t) - \frac{1}{C}\int i(t)\,dt = 0$$

or $$R\,i(t) + \frac{1}{C}\int i(t)\,dt = 0$$

or $$2\,i(t) + \frac{1}{1}\int i(t)\,dt = 0 \quad \text{--- (1)}$$

Differentiating both sides w.r.to t

$$2\cdot\frac{d\,i(t)}{dt} + i(t) = 0$$

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$$\frac{d\,i(t)}{dt} + \frac{1}{2}\,i(t) = 0$$

First order differential eqn (Homogeneous)

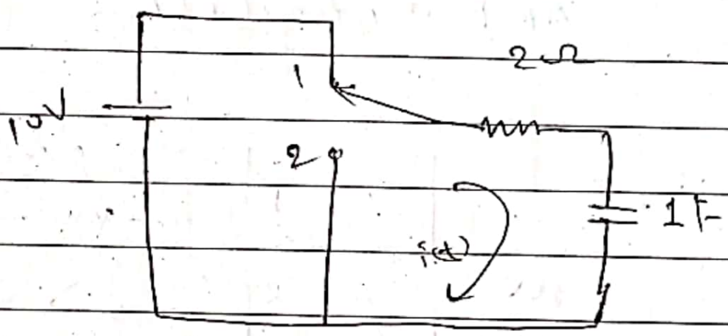
$$\frac{d\,i(t)}{dt} = -\frac{1}{2}\,i(t)$$

$$\frac{d\,i(t)}{i(t)} = -\frac{1}{2}\,dt$$

integrating.

$$i(t) = K\,e^{-1/2\,t}$$

Initially



capacitor stors 10V voltage as initial voltage and capacitor doesnot change its initial value instantly just after switching.

from eqn ①

$$2\,i(0^-) + 10 = 0$$

$$i(0^-) = -5 \text{ Amp.}$$

At 5 second

$$i(t) = -5\,e^{-1/2 * 5} \text{ Amp.}$$

=

putting in general solution

$$-5 = K\,e^{-1/2 * 0}$$

$$K = -5$$

$$\Rightarrow \quad i(t) = -5\,e^{-1/2\,t}$$

which is particular solution.

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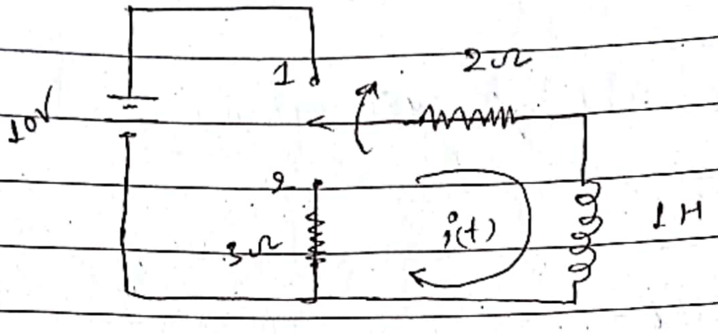
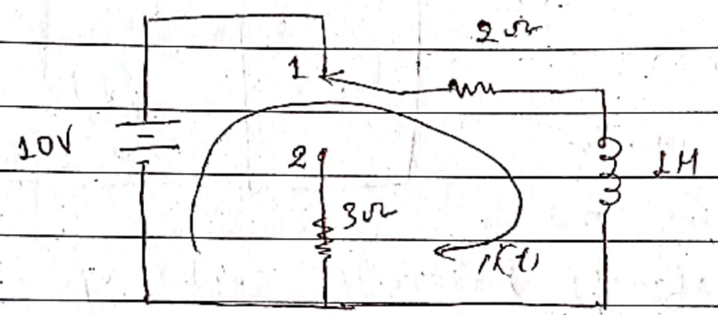


Switch is moved from position 2 to 1 at $t=0$.

Find the current response.

Sol^n

At $t=0$, final ckt.



Applying mesh analysis, or KVL

$$-2i(t) - 1 \cdot \frac{d\,i(t)}{dt} + 10 = 0$$

or $\quad 1\dfrac{d\,i(t)}{dt} + 2\,i(t) = 10$

1st order ~~homogeneous~~ diff. eq^n (non-homogeneous)

Here,

$P = 2, \quad Q = 10$

$$\frac{di(t)}{dt} + P\, i(t) = Q$$

$$i(t) = e^{-Pt} \int Q . e^{Pt} dt + k e^{-Pt}$$

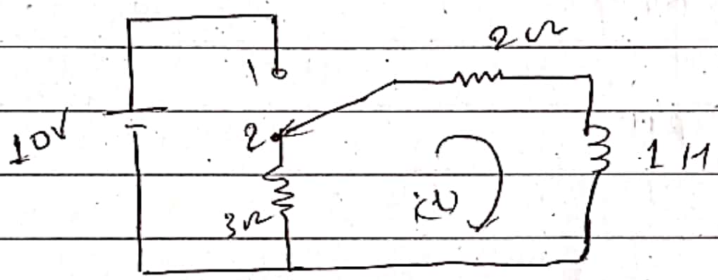
Soln:

$$i(t) = e^{-2t} \int 10\, e^{2t} dt + k e^{-2t}$$

$$i(t) = 10\, e^{-2t} . \frac{e^{2t}}{2} + k e^{-2t}$$

$$i(t) = 5 + k e^{-2t}$$ —— General Solution.

Initial ckt.



position 2 मा हुँदा कुनै पनि voltage source सँग connect हुँदैन त्यसकारण current zero $i(0^-) = 0$ हुन्छ |

i.e. $i(0^+) = 0$

from general solution,

$$0 = 5 + k e^{-2 \times 0}$$

$$0 = 5 + k \times 1$$

$$\Rightarrow k = -5$$

Required solution is

$$i(t) = 5 - 5e^{-2t}$$ which is particular Soln.

↑ steady state solution ↑ Transient state solution.

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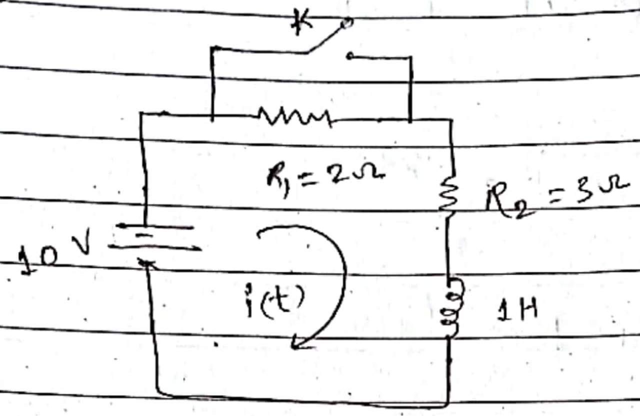
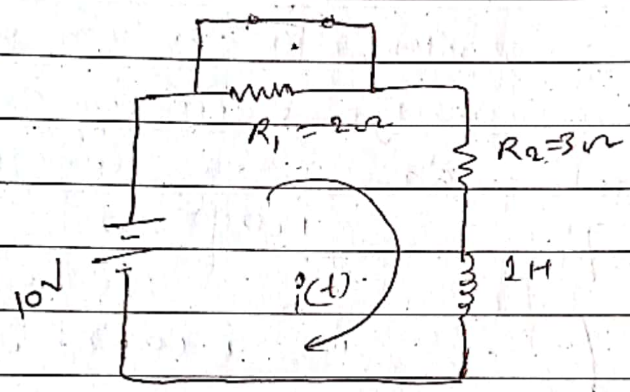


(a) switch is closed at t=0 find the current response

(b) switch is opened at t=0 find the current response

(a) Final ckt.



Applying mesh analysis.

$$-3\ i(t) - 1\ \frac{d\ i(t)}{dt} + 10 = 0$$

$$1\ \frac{d\ i(t)}{dt} + 3\ i(t) = 10$$

which is non-homogeneous 1st order diff eq^n

so on.

$P = 3$

$Q = 10$

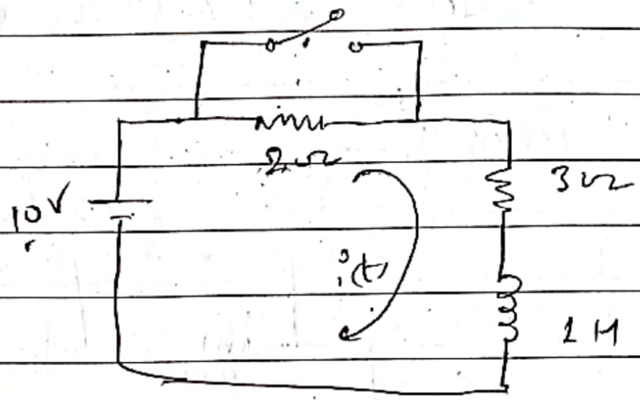
$t=0$ t^{0}

Then $i(t) = e^{-3t}\int 10 \cdot e^{3t}\,dt + k e^{-3t}$

$i(t) = 10 e^{-3t} \cdot \frac{e^{3t}}{3} + k e^{-3t}$

$i(t) = \frac{10}{3} + k e^{-3t}$ —— (1)

Initially



$i(0^-) = \frac{10}{3+2}$

$= 2$

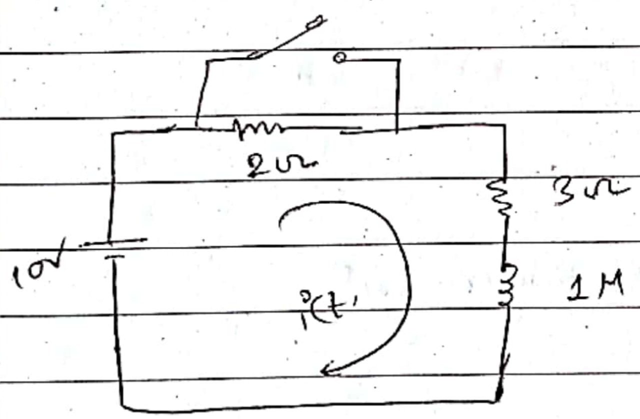
from (1)

$i(0) = 2 = \frac{10}{3} + k e^{-3 \times 0}$ ~~which is~~

$K = -\frac{4}{3}$ from (1)

$\Rightarrow i(t) = \frac{10}{3} - \frac{4}{3} e^{-3t}$ Req. ans.

(b) Final ckt.



Applying KVL or mesh analysis.

$-2\,i(t) - 3\,i(t) - 1 \cdot \frac{d\,i(t)}{d(t)} + 10 = 0$

$5\,i(t) + 1 \cdot \frac{d\,i(t)}{dt} = 10$

$\Rightarrow \frac{d\,i(t)}{dt} + 5\,i(t) = 10$

when $P = 5$ and $Q = 10$

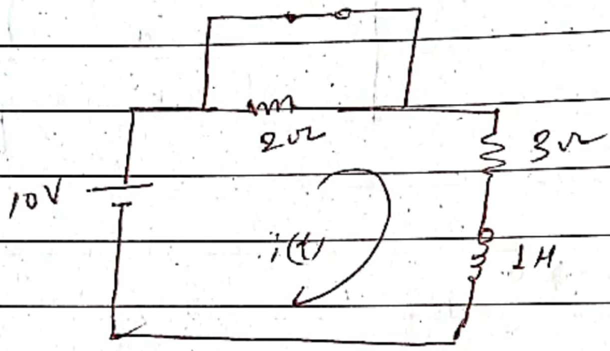
then

$$i(t) = e^{-5t}\int 10\, e^{5t}\, dt + k e^{-5t}$$

$$i(t) = 10\, e^{-5t} \cdot \frac{e^{5t}}{5} + k e^{-5t}$$

$$i(t) = 2 + k e^{-5t} \quad ① \text{ gen. sol}^n$$

initially, $i(0^-) = \frac{10}{3}$



and from ① at $t = 0^-$

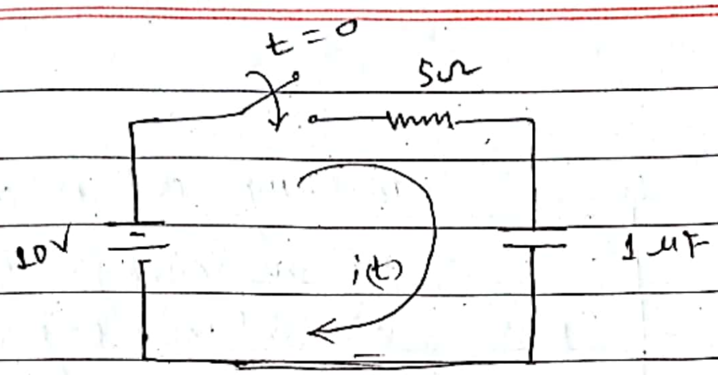
$$\frac{10}{3} = 2 + k e^{-5\times 0}$$

$$\frac{4}{3} = k$$

Then,

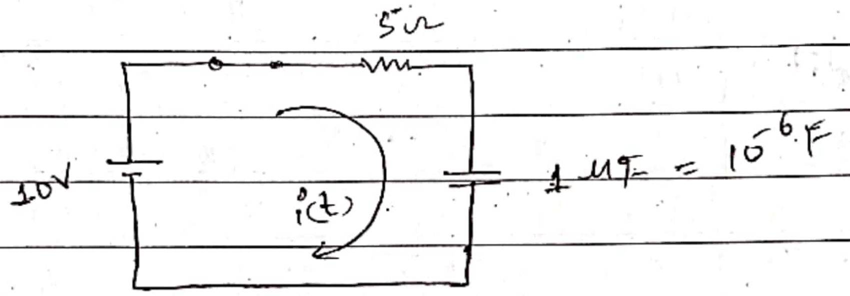
$$i(t) = 2 + \frac{4}{3} e^{-5t}$$ particular sol^n.

*



Switch is closed at $t = 0$. Find the current response.

Sol^n Final ckt.



Applying mesh analysis:-

$$-5\, i(t) - \frac{1}{10^{-6}} \int i(t)\, dt + 10 = 0$$

or,
$$5\, i(t) + 10^6 \int i(t)\, dt = 10 \quad \text{——— (1)}$$

Differentiating both sides we get.

$$5 \cdot \frac{d\, i(t)}{dt} + 10^6\, i(t) = 0$$

$$\frac{d\, i(t)}{dt} + \frac{10^6}{5}\, i(t) = 0$$

$$\frac{d\, i(t)}{dt} = -\frac{10^6}{5}\, i(t)$$

integrating.

$$i(t) = K e^{-\frac{10^6}{5} t} \quad \text{——— (2)}$$

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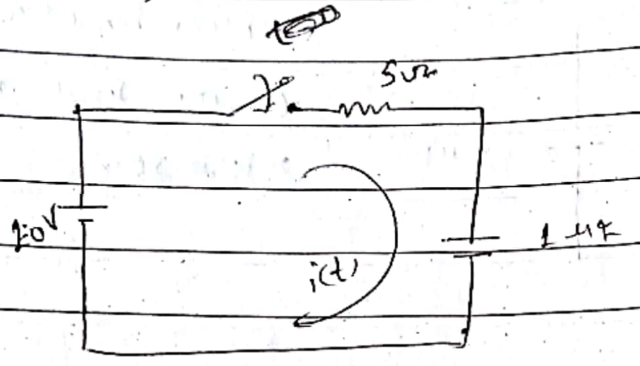
Initially,



Initially ckt is open so no charge is stored in capacitor i.e. $V(0^-) = 0$

for eq ①

$5\,i(t) + \cancel{\ } = 10$

$i(t) = \frac{10}{5}$

$i(t) = 2$ Amp.

for ② $2 = k e^{0}$

At $t = 0^-$ $k = 2$.

then for ② Req^n sol^n is..

$\Rightarrow i(t) = 2 e^{-\frac{10^6}{5} t}$

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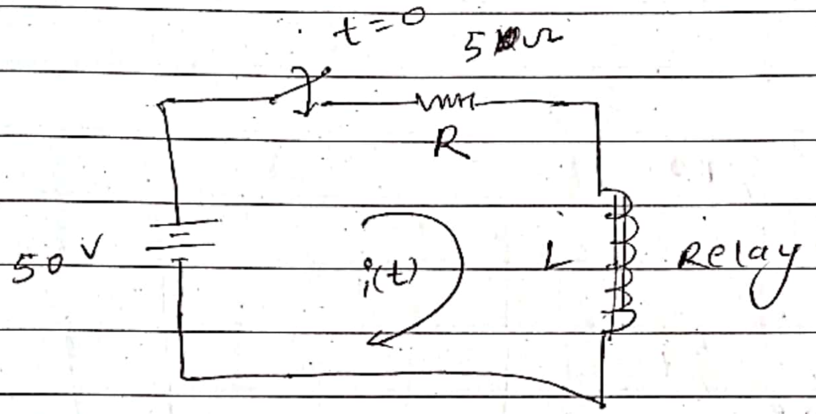
In the given circuit, the relay is adjusted to operate with a current of 7 mA. Switch is closed at $t=0$. It is found the relay operates at $t = 0.2$ sec. Find (i) inductance L of the relay coil
(ii) equation of the current $i(t)$



Sol^n At $t=0$, switch is closed. Applying KVL

$$5\, i(t) + L \frac{d\, i(t)}{dt} = 50$$

or, $$L \frac{d i(t)}{dt} + 5 i(t) = 50$$

or, $$\frac{d i(t)}{dt} + \frac{5}{L} i(t) = \frac{50}{L}$$

$$p = \frac{5}{L}, \quad q = \frac{50}{L}$$

$$i(t) = e^{-pt} \int q. e^{pt} dt + k e^{-pt}$$

or $$i(t) = e^{-\frac{5}{L}t} \int \frac{50}{L} . e^{\frac{5}{L}t} dt + k . e^{-\frac{5}{L}t}$$

$\Rightarrow i(t) = \frac{50}{L} \cdot e^{-\frac{5}{L}t} \cdot \frac{e^{\frac{5}{L}t}}{\frac{5}{L}} + k\, e^{-\frac{5}{L}t}$

$i(t) = 10 + k\, e^{-\frac{5}{L}t}$

general solution:

initially $i(0^-) = 0$

put in general sol^n

$0 = 10 + k\, e^{-\frac{5}{L} \times 0}$

$k = -10$

$\Rightarrow i(t) = 10 - 10\, e^{-\frac{5}{L}t}$

At $t = 0.2$ sec. $i(t) = 7$ mA

$7 \times 10^{-3} = 10 - 10\, e^{-\frac{5}{L} \times 0.2}$

Eq^n of current

$i(t) = 10 - 10\, e^{-\frac{5}{L}t}$

$\Rightarrow 7 \times 10^{-3} = 10 - 10\, e^{-1/L}$

$\Rightarrow i(t) = 10 - 10\, e^{-3.5 \times 10^{-3} t}$

$\Rightarrow e^{-\frac{1}{L}} = 0.9993$

$\Rightarrow -\frac{1}{L} = -7 \times 10^{-4}$

$L = 1428.07$ H

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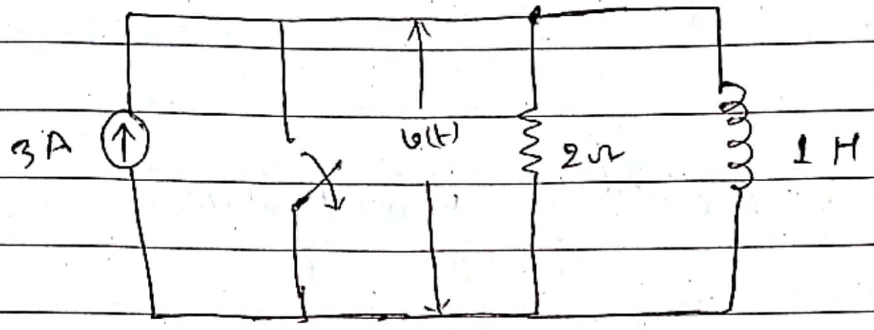
Parallel circuit

$v(t) \Leftarrow$ Node voltage

KCL apply गर्ने
$\downarrow$
parallel & voltage [illegible]

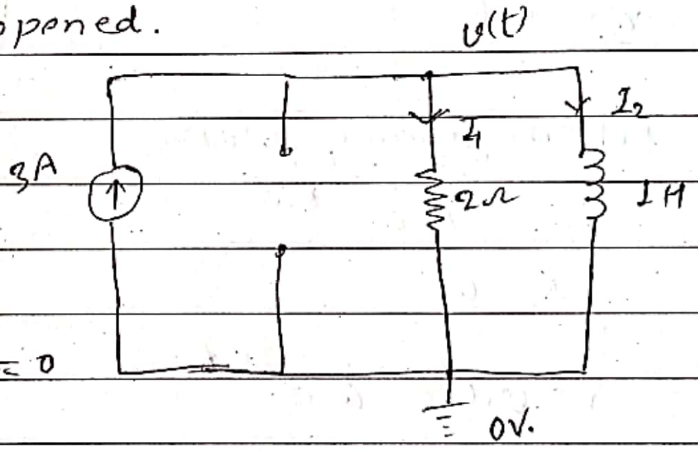
(1)



Switch is opened at $t = 0$.

Find the voltage response $v(t)$. Also find current through inductor.

Sol^n At $t = 0$, switch is opened.



Applying KCL,

$$3 - I_1 - I_2 = 0$$

or $$3 - \frac{v(t) - 0}{2} - \frac{1}{1}\int v(t)\,dt = 0$$

or, $$\frac{v(t)}{2} + \frac{1}{1}\int v(t)\,dt = 3 \quad \text{——— (1)}$$

Differentiating above equation

$$\frac{1}{2}\frac{d\,v(t)}{dt} + \frac{1}{1}v(t) = 0$$

$$\frac{d\,v(t)}{dt} + 2\,v(t) = 0$$

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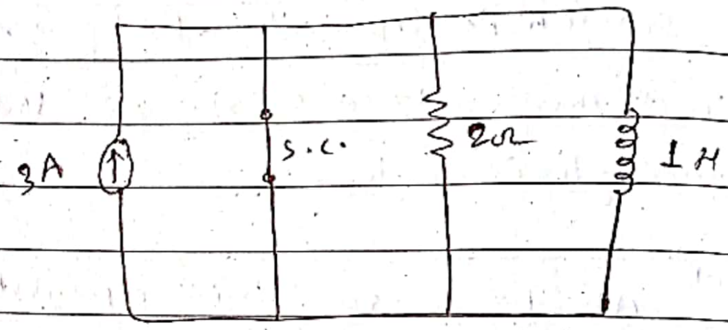
or, $\frac{d\,v(t)}{dt} = -2\,v(t)$ ②

or $\frac{d\,v(t)}{v(t)} = -2dt$

Integrating

$v(t) = Ke^{-2t}$ — general solution

Initially, $t = 0^-$



From ①

$\frac{v(0^-)}{2} + 0 = 3$

$v(0^-) = 6$

put in general solution,

$6 = Ke^{-2*0}$

$K = 6$

$\Rightarrow v(t) = 6e^{-2t}$

current through inductor $i_2(t) = \frac{1}{L}\int v(t)\,dt$

$= \frac{1}{1}\int 6e^{-2t}\,dt$

$= \frac{6e^{-2t}}{-2} + C$

$i_2(t) = -3e^{-2t} + C$

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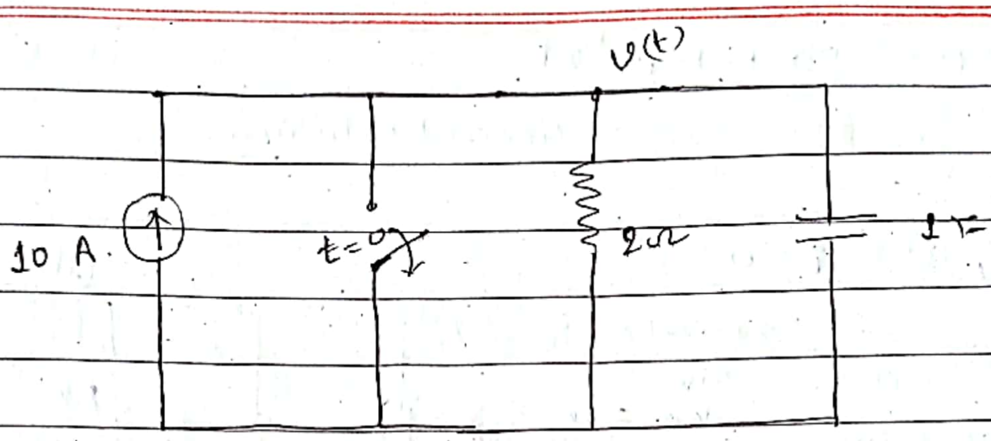
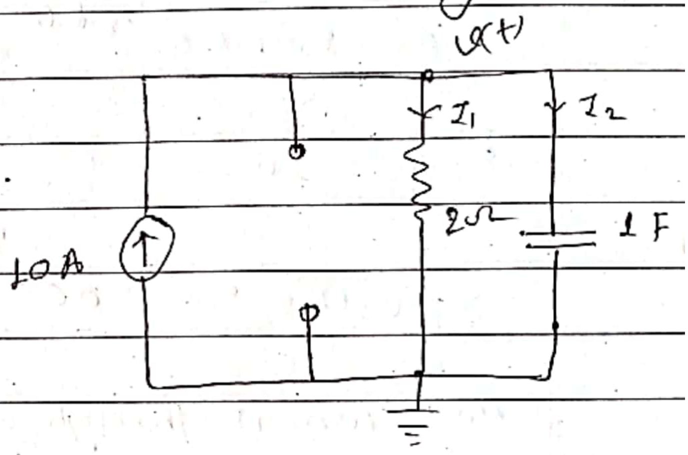
(2)



switch is opened at $t = 0$.

Find the voltage response $v(t)$ and current through capacitor.

sol



⇒ At $t = 0$, switch is opened.

Applying KCL,

$$10 = I_1 + I_2$$

or $$10 = \frac{v(t)}{2} + C\frac{d\,v(t)}{dt}$$

or, $$\frac{d\,v(t)}{dt} + \frac{v(t)}{2} = 10$$

(If initially voltage दिखा रहा तो initial cond^n का use करें)

$$p = \frac{1}{2}, \quad q = 10$$

Then $$v(t) = e^{-pt}\int q \cdot e^{pt}\,dt + k e^{-pt}$$

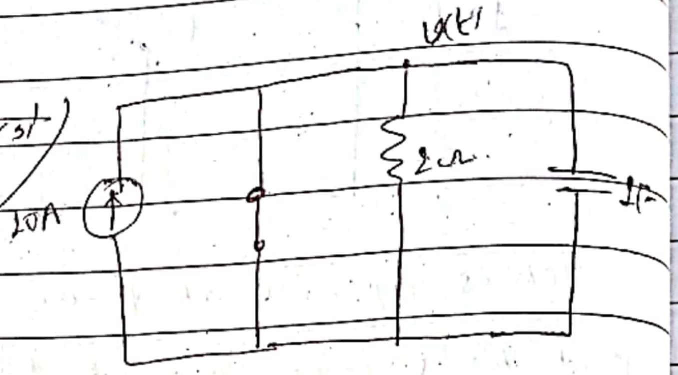
$$v(t) = e^{-\frac{1}{2}t}\int 10 \cdot e^{\frac{1}{2}t}\,dt + k e^{-\frac{1}{2}t}$$

$$= e^{-\frac{1}{2}t} \cdot \frac{10 \cdot e^{\frac{1}{2}t}}{\frac{1}{2}} + k e^{-\frac{1}{2}t}$$

$\Rightarrow \; v(t) = 20 + K e^{-\frac{1}{2}t}$

General solution.

Initially at $t = 0^-$



$v(0^-) = 0 \text{ v}.$

$v(0^+) = 0 \text{ v}.$

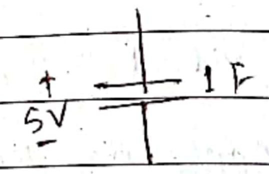
(Initial 5v दिया गया v(0⁻) = 5V लेंगे.)

Then from general solution,

$0 = 20 + K e^{-\frac{1}{2} \times 0}$

same.

$\Rightarrow \; K = -20$



$\Rightarrow \; v(t) = 20 - 20 e^{-\frac{1}{2}t}$

(capacitor का voltage दिया है याने initial voltage है।)

Now current through capacitor

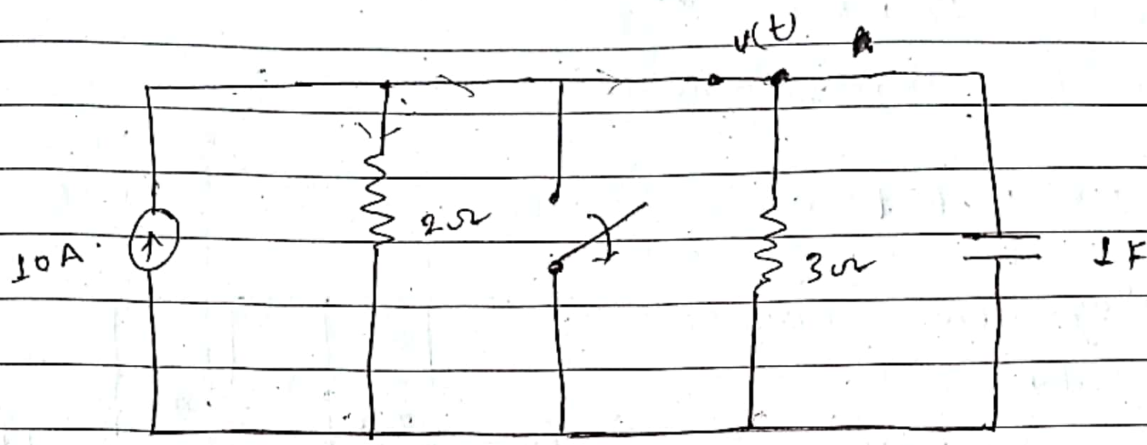
$$i_1(t) = C \frac{d\,v(t)}{dt}$$

$$= 1 \cdot \frac{d}{dt}\left(20 - 20 e^{-\frac{1}{2}t}\right)$$

$$= -20 \times e^{-\frac{1}{2}t} \times -\frac{1}{2}$$

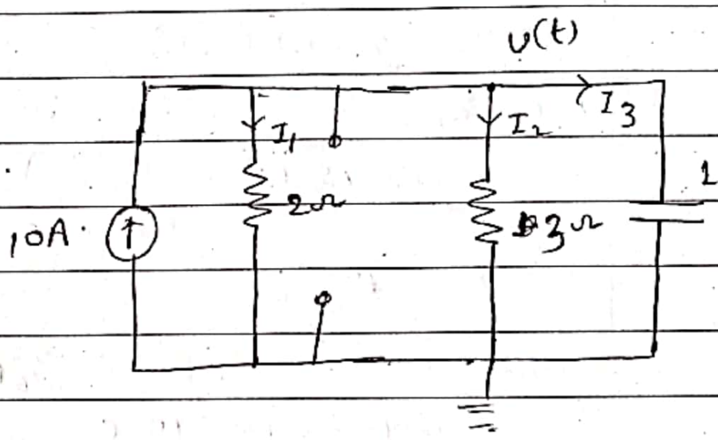
$$= 10 e^{-\frac{1}{2}t}$$

(3)



switch is opened at $t=0$

find $v(t)$ at $t \geq 0$.



sol^n

At $t=0$, switch is opened.

applying KCL

$$10 = I_1 + I_2 + I_3$$

or, $$\frac{v(t)}{2} + \frac{v(t)}{3} + 1 \cdot \frac{d\,v(t)}{dt} = 10$$

or, $$\frac{5\,v(t)}{6} + \frac{d\,v(t)}{dt} = 10$$

or, $$\frac{d\,v(t)}{dt} + \frac{5}{6}v(t) = 10$$

which is non-homogeneous lin diff-eq^n.

$$P = \frac{5}{6}, \quad Q = 10$$

$$v(t) = e^{-Pt}\int Q \cdot e^{Pt}\,dt + K e^{-Pt}$$

$$= e^{-\frac{5}{6}t}\int 10 e^{\frac{5}{6}t}\,dt + K e^{-\frac{5}{6}t}$$

$$= e^{-\frac{5}{6}t} \cdot 10 \cdot e^{\frac{5}{6}t} \cdot \frac{6}{5} + K e^{-\frac{5}{6}t}$$

$v(t) = 12 + k\,e^{-5/6\,t}$

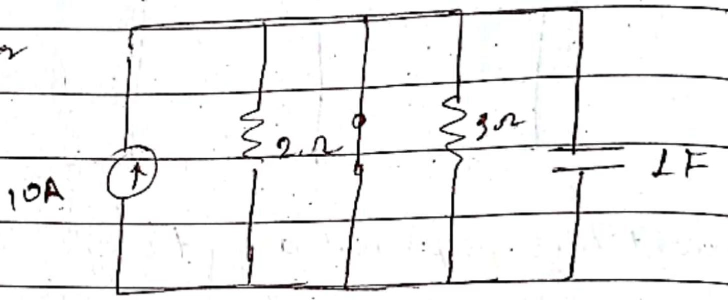
which is general solution.

Initially at $t=0^-$

voltage across capacitor

$= 0V$

So. $V(0^-) = V(0^+) = 0V.$



Then, general sol^n

becomes,

$0 = 12 + k\,e^{-5/6 \cdot 0}$

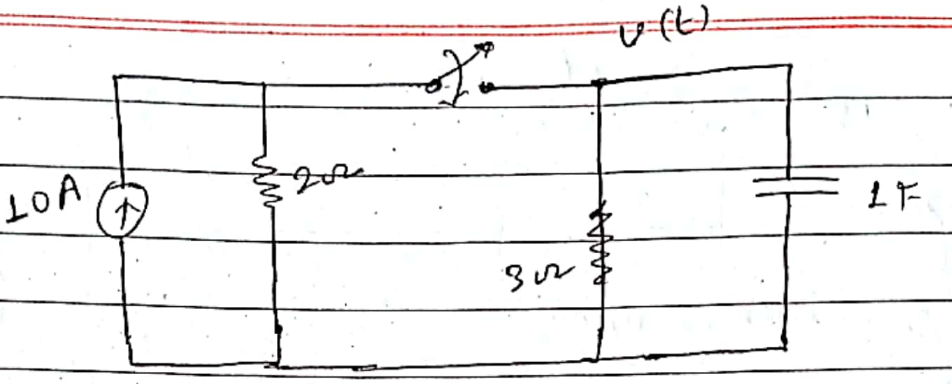
$k = -12$

Then,

$v(t) = 12 - 12\,e^{-5/6\,t}$

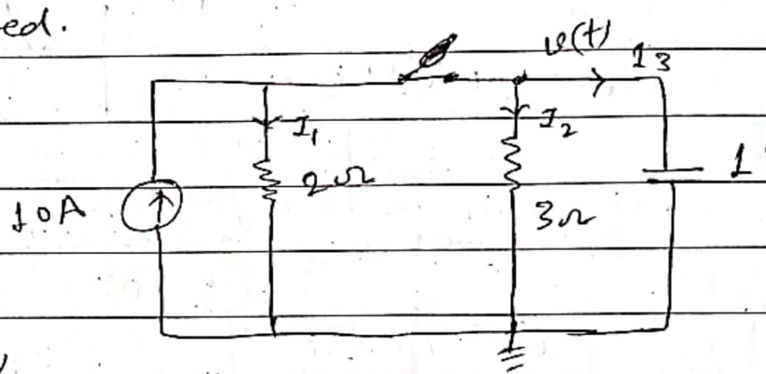
Ans.

H.W



Switch is closed at $t=0$.
Find the voltage response $v(t)$.

At $t=0$ switch is closed.



Applying KCL,

we get,

$$I_1 + I_2 + I_3 = 10$$

or, $$\frac{v(t)}{2} + \frac{v(t)}{3} + 1 \cdot \frac{d\,v(t)}{dt} = 10$$

or, $$\frac{5}{6} v(t) + \frac{d\,v(t)}{dt} = 10$$

or, $$\frac{d\,v(t)}{dt} + \frac{5}{6} v(t) = 10$$

which is non homogeneous diff. eqn.

where $p = \frac{5}{6}$, $q = 10$

$$v(t) = e^{-pt} \int q e^{pt} dt + K e^{-pt}$$

$$= e^{-5/6 t} \int 10 \cdot e^{5/6 t} dt + K e^{-5/6 t}$$

$$= e^{-5/6 t} \cdot \frac{10 \cdot e^{5/6 t}}{5/6} + K e^{-5/6 t}$$

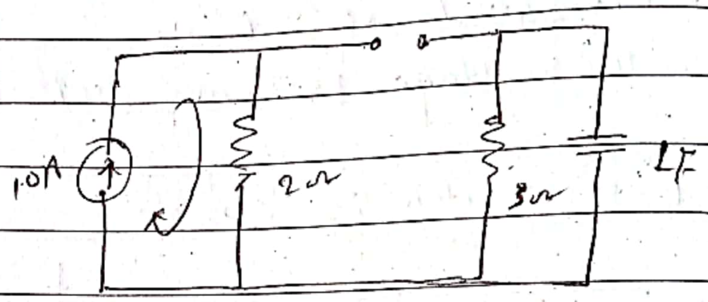
$v(t) = 12 + Ke^{-\frac{5}{6}t}$

general sol^n:

At initially, switch is open,

At $t = 0^-$, voltage across capacitor is zero.

$v(0^-) = v(0^+) = 20$



then from general sol^n

$0 = 12 + Ke^{-\frac{5}{6}\times 0}$

$K = -12$

and, $v(t) = 12 - 12e^{-\frac{5}{6}t}$

At $t = 0^-$, voltage drop across 2Ω ⇒ $2 * 10 = 20V$

$\therefore v(0^-) = v(0^+) = 20V$.

Then $20 = 12 + Ke^{-\frac{5}{6}\times 0}$

$K = 8$

$\therefore v(t) = 12 + 8e^{-\frac{5}{6}t}$

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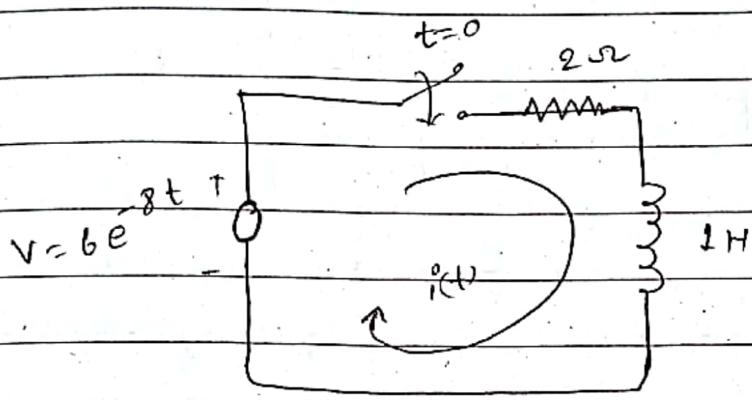
Input as exponential function



Switch is closed at t = 0. Find the current response.

OR,

A resistor of 2Ω and an ~~inductor~~ inductance 1H in series are suddenly connected to a voltage source of $6e^{-8t}$ at t = 0. Find the current response. Assume no initial current through inductor.

Sol^n.

At t = 0, using KVL

$$2\,i(t) + 1\cdot\frac{d\,i(t)}{dt} = 6e^{-8t}$$

or $\dfrac{d\,i(t)}{dt} + 2\,i(t) = 6e^{-8t}$ — non homogeneous.

$p = 2, \quad q = 6e^{-8t}$

$$i(t) = e^{-pt}\int q\cdot e^{pt}\,dt + ke^{-pt}$$

$$= e^{-2t}\int 6e^{-8t}\cdot e^{2t}\,dt + ke^{-2t}$$

$$= e^{-2t}\cdot 6\cdot\frac{e^{-6t}}{-6} + ke^{-2t}$$

$\therefore\ i(t) = -e^{-8t} + k\,e^{-2t}$ — general solution.

Initial. $i(0^-) = 0$

$\therefore\ i(0^+) = i(0^-) = 0$

Then,

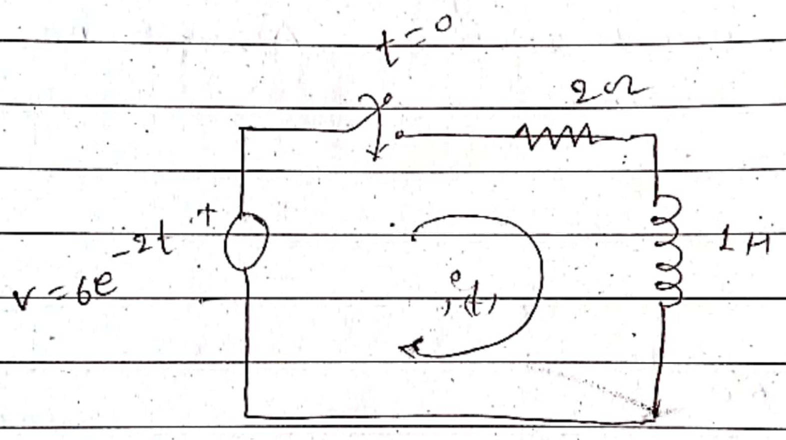
$0 = -e^{-8\times 0} + k\,e^{-2\times 0}$

$0 = -1 + k$

$k = 1.$

$\therefore\ i(t) = -e^{-8t} + e^{-2t}$ Ans.

Q.



Switch is closed at $t=0$

Find current response.

at $t=0$, using KVL

$$2\,i(t) + \frac{d\,i(t)}{dt} = 6e^{-2t}$$

$P = 2,\ Q = 6e^{-2t}$

$$i(t) = e^{-2t}\int 6\cdot \cancel{e^{-2t}}\cdot \cancel{e^{2t}}\, dt + Ke^{-2t}$$

$$i(t) = 6te^{-2t} + Ke^{-2t}$$

Initially, $i(0^-) = 0$

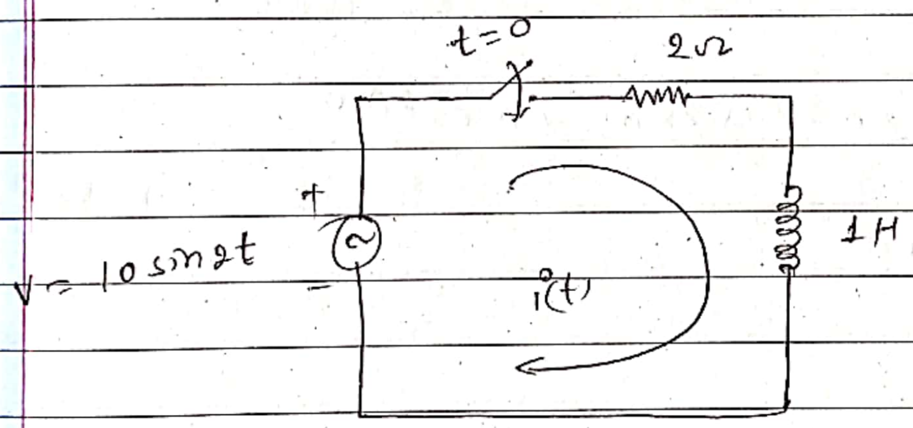
Then,

$$0 = 6\times 0\times e^{-2\times 0} + Ke^{-2\times 0}$$

$$K = 0$$

The $i(t) = 6t\,e^{-2t}$.

Input as sinusoidal Function



Switch is closed at $t=0$, find the current response.

At $t=0$, applying KVL

$$2\,i(t) + 1\cdot\frac{d\,i(t)}{dt} = 10\sin 2t$$

$$\frac{d\,i(t)}{dt} + 2\,i(t) = 10\sin 2t$$

non-homogeneous diff e^n with $P = 2$ and $Q = 10\sin 2t$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

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Then

$$i(t) = e^{-Pt}\int Q e^{Pt}dt + ke^{-Pt}$$

$$= e^{-2t}\int 10\sin 2t\, e^{2t}dt + ke^{-2t}$$

$$= 10e^{-2t}\int e^{2t}\sin 2t\,dt + ke^{-2t}$$

$$= 10e^{-2t}\cdot\frac{e^{2t}}{4+4}(2\sin 2t - 2\cos 2t) + ke^{-2t}$$

$$= \frac{5}{4}(2\sin 2t - 2\cos 2t) + ke^{-2t}$$

$$= \frac{5}{2}(\sin 2t - \cos 2t) + ke^{-2t}.$$

Initially, $i(0^-) = 0$

then,

$$0 = \frac{5}{2}(\sin 2\times 0 - \cos 2\times 0) + ke^{-2\times 0}$$

$$\text{or } 0 = -\frac{5}{2} + ke^{-2\times 0}$$

$$k = \frac{5}{2}.$$

Then, $i(t) = \frac{5}{2}(\sin 2t - \cos 2t) + \frac{5}{2}e^{-2t}.$

$$i(t) = \frac{5}{2}(\sin 2t - \cos 2t + e^{-2t})$$ Amp.

$$\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

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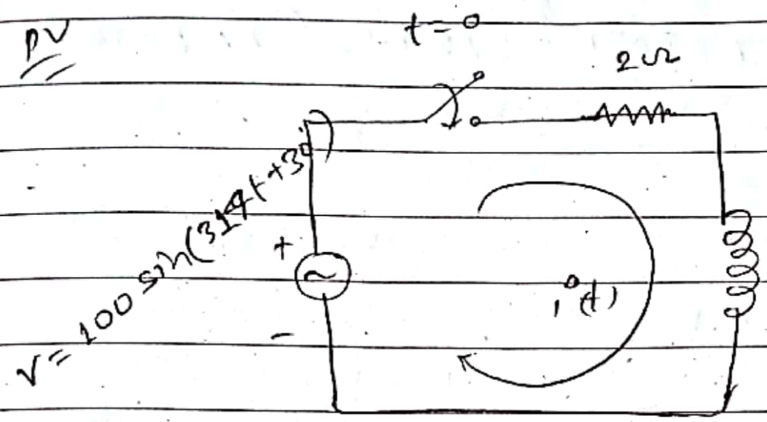
PV



switch is closed at $t=0$, Find the current response.

soln

At $t=0$ second. applying KVL.

$$2\, i(t) + 1\cdot\frac{d\,i(t)}{dt} = 100\sin(314t+30°)$$

which is non homogeneous diff eqn. with

$$P = 2,\quad Q = 100\sin(314t+30°)$$

then,

$$i(t) = e^{-2t}\int 100\sin(314t+30°)\cdot e^{2t}\,dt + k e^{-2t}$$

$$= 100 e^{-2t}\int e^{2t}\sin(314t+30°)\,dt + k e^{-2t}$$

$$= 100\cancel{e^{-2t}}\cdot\frac{\cancel{e^{2t}}}{4+314^2}\left[2\sin(314t+30°) - 314\cos(314t+30°)\right] + k e^{-2t}$$

$$= \frac{100}{98600}\left[2\sin(314t+30°) - 314\cos(314t+30°)\right] + k e^{-2t}$$

$$i(t) = \frac{1}{986}\left\{2\sin(314t+30^\circ) - 314\cos(314t+30^\circ)\right\} + Ke^{-2t}.$$

Initially. $i(0^-) = 0$

Then general eqn becomes,

$$\frac{1}{986}\left\{2\times\sin(0+30^\circ) - 314\cos(0+30^\circ)\right\} + Ke^{-2\times 0} = 0$$

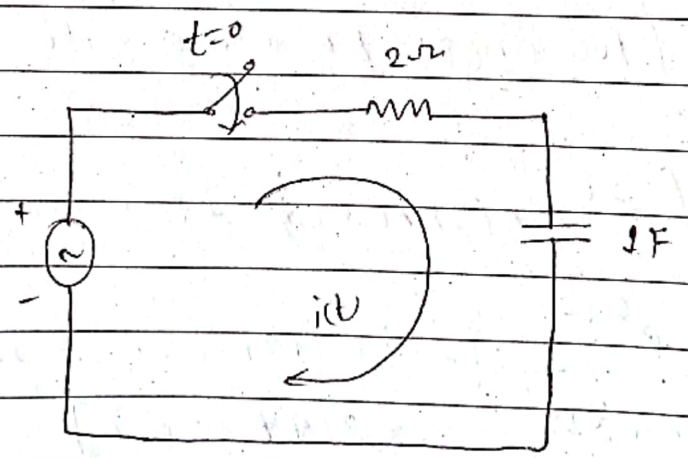
or, $-0.275 + Ke^{0} = 0$

$K = 0.275$

Then,

$$i(t) = \frac{1}{986}\left\{2\sin(314t+30^\circ) - 314\cos(314t+30^\circ)\right\} + 0.275e^{-2t}.$$

Q.



$V = 100\sin(314t+30^\circ)$

Switch is closed at $t=0$, Find the current response.

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At $t=0$, circuit is closed then using KVL.

$$2\,i(t) + \frac{1}{1}\cdot\int i(t)\,dt = 100\sin(314t+30°) \quad \text{---(1)}$$

Differentiating w.r.to t.

$$2\frac{d\,i(t)}{dt} + i(t) = 100\cos(314t+30°)\times 314$$

or, $$\frac{d\,i(t)}{dt} + \frac{1}{2}\,i(t) = 31400\cos(314t+30°)$$

Non homogeneous diff. eqn. with

$P = \frac{1}{2}$, $Q = 31400\cos(314t+30°)$

~~Then~~

$$i(t) = e^{-\frac{1}{2}t}\int 31400\cos(314t+30°)\cdot e^{\frac{1}{2}t}\,dt + k e^{-\frac{1}{2}t}$$

$$= 31400\,\cancel{e^{-\frac{1}{2}t}}\cdot\frac{\cancel{e^{\frac{1}{2}t}}}{\frac{1}{4}+314^2}\left[\frac{1}{2}\cos(314t+30°) + 314\sin(314t+30°)\right] + k e^{-\frac{1}{2}t}$$

$$i(t) = \overset{\cancel{0.381}}{0.3185}\left(\frac{1}{2}\cos(314t+30°) + 314\sin(314t+30°)\right) + k e^{-\frac{1}{2}t} \quad \text{---(2)}$$

since, initially ~~ckt is~~ switch is open so voltage across capacitor is 0 V.

also $V(0^-) = V(0^+)$

so from (1)

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$2\ i(t) + 0 = 100 \sin(314 \times 0 + 30^\circ)$

$i(t) = 50 \sin(\cancel{314t} + 30^\circ) = 25$

from (2)

~~$50 \sin(314t + 30^\circ) =$~~

$25 = 0.3185 \left\{ \frac{1}{2} \cos 30^\circ + 314 \sin 30^\circ \right\} + K$

$\Rightarrow K = -78.97$

Again from (2)

$i(t) = 0.3185 \left\{ \frac{1}{2} \cos(314t + 30^\circ) + 314 \sin(314t + 30^\circ) \right\}$

$- 78.97\, e^{-\frac{1}{2} t}$

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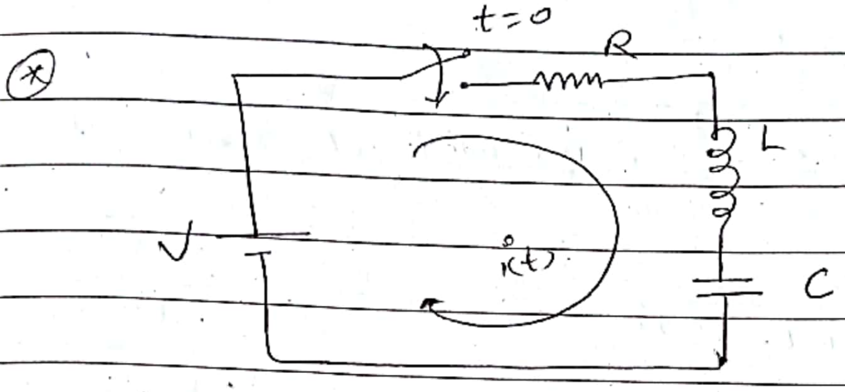
step function response of RLC series circuit;



switch is closed at $t=0$, find the current response.

soln

At $t=0$ switch is closed. applying KVL

$$-Ri(t) - L\frac{di(t)}{dt} - \frac{1}{C}\int i(t)\,dt + V = 0$$

or, $$L\frac{di(t)}{dt} + R\,i(t) + \frac{1}{C}\int i(t)\,dt = V \quad \text{——} \quad (1)$$

Differentiating above eqn.

$$L\frac{d^2 i(t)}{dt^2} + R\frac{d\,i(t)}{dt} + \frac{1}{C}i(t) = 0$$

$$\Rightarrow \frac{d^2 i(t)}{dt^2} + \frac{R}{L}\frac{d\,i(t)}{dt} + \frac{1}{RC}i(t) = 0$$

characteristics eqn,

$$m^2 + \frac{R}{L}m + \frac{1}{RC} = 0$$

Let, m_1 and m_2 be solution or roots of the given eqn.

Then,

$$m_1, m_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

or $m_1, m_2 = \dfrac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$.

Case I : when $\frac{4}{LC} < \frac{R^2}{L^2}$, we will get real and distinct roots.

say m_1 and m_2.

$\Rightarrow i(t) = k_1 e^{m_1 t} + k_2 e^{m_2 t}$.

Case II : when $\frac{4}{LC} = \frac{R^2}{L^2}$, we will get real and equal roots say

$m_1 = m_2 = m$.

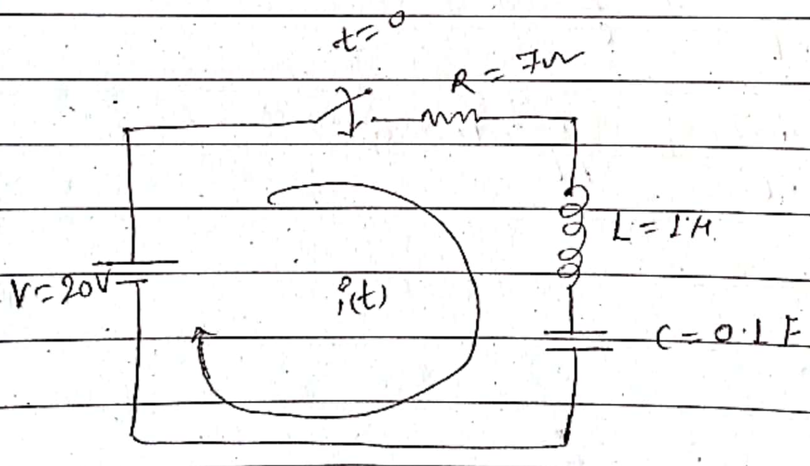
$\Rightarrow i(t) = (k_1 + k_2 t) e^{mt}$

Case III : when $\frac{4}{LC} > \frac{R^2}{L^2}$, we will get imaginary roots

say $m_1 \pm j m_2$.

$\Rightarrow i(t) = e^{m_1 t} (k_1 \cos m_2 t + k_2 \sin m_2 t)$

Q.



Switch is closed at $t = 0$.

Find the complete solution for current.

Sol

At $t=0$, switch is closed.

applying KVL

$$-7\,i(t) - 1\cdot\frac{d\,i(t)}{dt} - \frac{1}{0.1}\int i(t)\,dt + 20 = 0$$

or, $$\frac{d\,i(t)}{dt} + 7\,i(t) + 10\int i(t)\,dt = 20 \quad \text{——— (1)}$$

Diff. w.r. to t.

$$\frac{d^2\,i(t)}{dt^2} + 7\frac{d\,i(t)}{dt} + 10\,i(t) = 0$$

characteristics eqn

$$m^2 + 7m + 10 = 0$$

$$(m+5)(m+2) = 0$$

$$m = -2, -5$$

$$\Rightarrow i(t) = K_1 e^{-2t} + K_2 e^{-5t}$$

It is general equation.

Now, Initial current $i(0^-) = 0$

Then, $$0 = K_1 e^{-2*0} + K_2 \times e^{-5\times 0}$$

$$K_1 + K_2 = 0 \quad \text{——— (2)}$$

Diff. general eqn

$$\frac{d\,i(t)}{dt} = -2K_1 e^{-2t} + 5K_2 e^{-5t}$$

from (1) using initial condition i.e. $V(0^-) = 0$ across capacitor

we get,

$$\frac{d\,i(t)}{dt} + 7\times 0 + 0 = 20$$

$$\frac{d\,i(t)}{dt} = 20$$

Then,

$$20 = -2k_1 e^{2 \times 0} - 5k_2 e^{-5 \times 0}$$

or, $-2k_1 - 5k_2 = 20$ —— ③

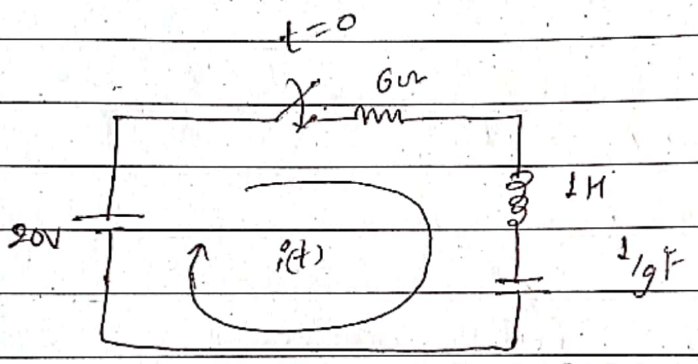
solving ② and ③ we get

$$k_1 = 20/3$$
$$k_2 = -20/3$$

Then general sol^n becomes

$$i(t) = \frac{20}{3} e^{-2t} - \frac{20}{3} e^{-5t}$$ which is complete sol^n

Q.



switch is closed at t=0
Find complete sol^n for current

At t = 0, switch is closed

Applying KVL,

$$-6\,i(t) - 1 \cdot \frac{d\,i(t)}{dt} - 9\int i(t)\,dt + 20 = 0$$

or, $$\frac{d\,i(t)}{dt} + 6\,i(t) + 9\int i(t)\,dt = 20$$ —— ①

Diff ① w.r. to t.

$$\frac{d^2 i(t)}{dt^2} + 6\frac{d\,i(t)}{dt} + 9\,i(t) = 0$$

characteristics eq^n

$$m^2 + 6m + 9 = 0$$

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$(m+3)(m+3)=0$

$m = -3, -3.$

Then the general soln is

$\Rightarrow i(t) = (K_1 + K_2 t) e^{-3t}$

initially, $i(0) = 0$

so, $0 = (K_1 + k_2 \times 0) e^{-3t}$

$K_1 = 0$

$\Rightarrow i(t) = K_2 t e^{-3t}$

Diff. w.r.to t

$$\frac{d i(t)}{dt} = -3e^{-3t} K_2 t + K_2 e^{-3t}$$

Initially from eqn ①

$$\frac{d i(t)}{dt} + 6 \times 0 + 0 = 20$$

Then, $20 = -3e^{-3\times 0} \times K_2 \times 0 + K_2 \times e^{-3\times 0}$

$K_2 = 20$

Hence the complete solution is

$\Rightarrow i(t) = 20 t e^{-3t}.$

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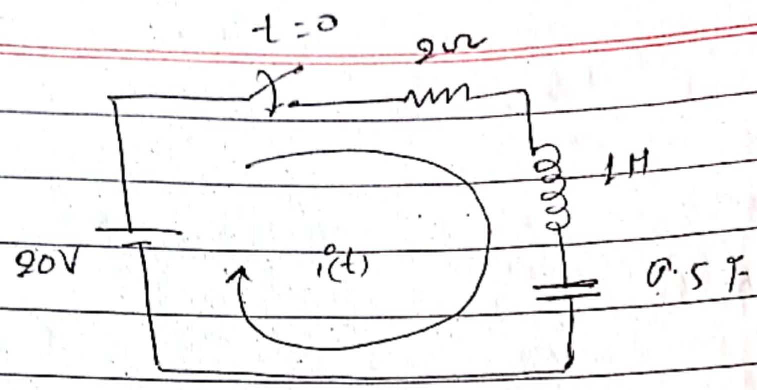
Q.



at t=0 switch is closed. Find current response.

Sol^n

At t=0, switch is closed.

Then applying KVL.

$$1 \cdot \frac{d\, i(t)}{dt} + 2\, i(t) + \frac{1}{0.5}\int i(t)\,dt = 20 \quad \text{—— (1)}$$

Differentiating (1) w.r. to t.

$$\frac{d^2 i(t)}{dt^2} + 2\frac{d\, i(t)}{dt} + 2\, i(t) = 0$$

Its characteristics eq^n

$$m^2 + 2m + 2 = 0$$

$$m_1, m_2 = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$= -1 \pm j1$$

Then general sol^n is

$$\Rightarrow i(t) = e^{-t}\left(k_1 \cos t + k_2 \sin t\right)$$

Initially, $i(0^-) = 0$

and voltage across capacitor $v(0^-) = 0$

then $e^{-0}\left(k_1 \times \cos 0 + k_2 \times \sin 0\right) = 0$

$$k_1 = 0$$

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$\Rightarrow i(t) = e^{-t} \cdot K_2 \sin t$

Diff. w. r. to t.

$$\frac{d\,i(t)}{dt} = -e^{-t} K_2 \sin t + e^{-t} K_2 \cos t$$

from ① initially

$$\frac{d\,i(t)}{dt} + 2\times 0 + 0 = 20$$

Then, $20 = -e^{-0} K_2 \times \sin 0 + e^{-0} K_2 \cos 0$

$K_2 = 20$

Then reqd soln is.

$\Rightarrow i(t) = 20 e^{-t} \sin t$ Ans.

(*) For Non-homogeneous second order equation.

complete solution: $i(t) = i_c(t) + i_p(t)$

where,

$i_c(t)$ = complementary function obtained from the characteristics eqn.

$i_p(t)$ = particular integral obtained from hit trial method by considering the forcing function.

For particular integral.

Forcing funn / Input	Assumption of particular integral
(1) constant r	① constant K
② $r e^{mt}$	② $k e^{mt}$
③ $\cos\omega t$ r $\sin\omega t$	③ $A\cos\omega t + B\sin\omega t$

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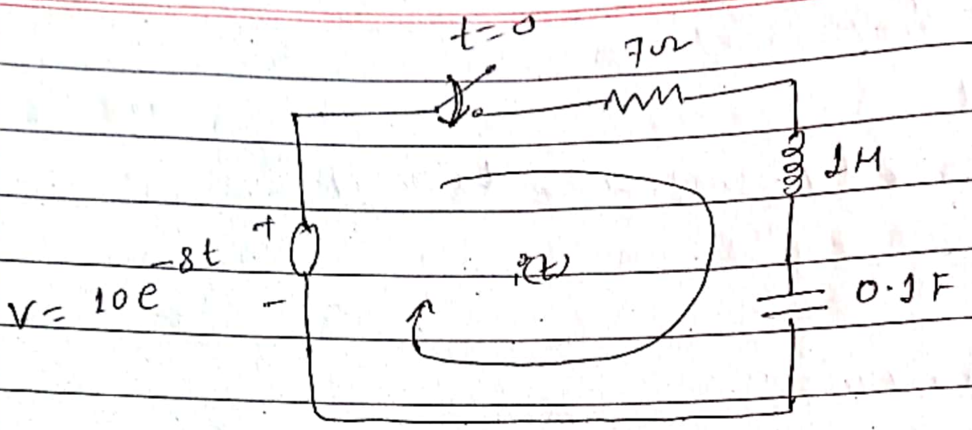
Q.



Find current response.

~~t=0, for~~

At t=0, switch is closed.

Applying KVL,

$$1 \cdot \frac{d\,i(t)}{dt} + 7\,i(t) + \frac{1}{0.1}\int i(t)\,dt = 10\,e^{-8t} \quad \text{——— (1)}$$

Diff (1) w.r.to t

$$\frac{d^2 i(t)}{dt^2} + 7\frac{d\,i(t)}{dt} + 10\,i(t) = -80\,e^{-8t} \quad \text{——— (2)}$$

Its characteristics eqn

$m^2 + 7m + 10 = 0$

$(m+5)(m+2) = 0$

$m = -2,\ m = -5$

Then its complementry function

$$i_c(t) = K_1 e^{-2t} + K_2 e^{-5t} \quad \text{——— (3)}$$

For particular integral, $Q = -80\,e^{-8t}$

so,

$i_p(t) = K_3\, e^{-8t}$

diff. w.r. to t.

$i_p'(t) = -8K_3\, e^{-8t}$

$i_p''(t) = 64K_3\, e^{-8t}$

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From eqn (2)

$$64 K_3 e^{-8t} + 7\cdot(-8) K_3 e^{-8t} + 10 K_3 e^{-8t} = -80 e^{-8t}$$

comparing equivalent coefficient we get,

$$64 K_3 - 56 K_3 + 10 K_3 = -80$$

$$\Rightarrow K_3 = -\frac{40}{9}$$

$$\Rightarrow i_p(t) = -\frac{40}{9} e^{-8t}$$

Hence the general sol^n

$$i(t) = i_c(t) + i_p(t)$$

$$\Rightarrow i(t) = K_1 e^{-2t} + K_2 e^{-5t} - \frac{40}{9} e^{-8t}$$

Initially, $i(0^-) = 0$. Then

$$K_1 + K_2 = \frac{40}{9} \quad \text{———} \quad (4)$$

Diff general sol^n?

$$\frac{d\,i(t)}{dt} = -2K_1 e^{-2t} - 5K_2 e^{-5t} + \frac{320}{9} e^{-8t}$$

Initially using eq^n (1)

$$\frac{d\,i(t)}{dt} + 7\times 0 + 0 = 10 e^{-8\times 0} = 10$$

So,

$$10 = -2K_1 e^{-2\times 0} - 5K_2 e^{-5\times 0} + \frac{320}{9} e^{-8\times 0}$$

$$-2K_1 - 5K_2 + 320/9 = 10$$

$$-2K_1 - 5K_2 = -\frac{230}{9} \quad \text{———} \quad (5)$$

solving (4) & (5)

$$K_1 = -10/9 \qquad K_2 = 50/9$$

$$\Rightarrow i(t) = -\frac{10}{9} e^{-2t} + \frac{50}{9} e^{-5t} - \frac{40}{9} e^{-8t}$$

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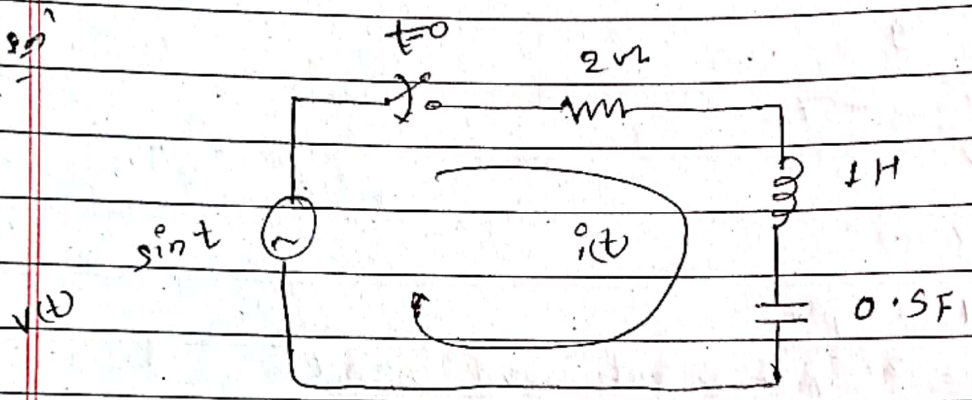
Q. A series RLC circuit with $R = 2\Omega$, $C = 0.5F$ and $L = 1H$ is excited by a voltage of $v(t) = \sin t$ for $t \geq 0$. For the elements values specified, find the current $i(t)$ through the circuit if the switch is closed at $t = 0$.

Sol^n



At $t = 0$, ~~ckt~~ switch is closed.

Using KVL

$$1 \cdot \frac{di(t)}{dt} + \frac{1}{0.5}\int i(t)\,dt + 2\,i(t) = \sin t \quad \text{———} (1)$$

diff.

$$\frac{d^2 i(t)}{dt^2} + \frac{2\,di(t)}{dt} + 2\,i(t) = \cos t \quad \text{———} (2)$$

which is non-homogenes. So

characteristics eq^n is

$$m^2 + 2m + 2 = 0$$

$$m_1, m_2 = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$m_1, m_2 = -1 \pm j1$$

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Then $i_t(t) = e^{-t}(k_1 \cos t + k_2 \sin t)$ —— (3)

For $i_p(t)$.

$i_p(t) = \cos(t)$

$i'_p(t) = -\sin(t)$

$i''_p(t) = -\cos t$

from (2) forcing function [illegible], for $i_p(t)$.

$-\cos t$

For $i_p(t)$

$i_p(t) = k_3 \cos t + k_4 \sin t$

$i'_p(t) = -k_3 \sin t + k_4 \cos t$

$i''_p(t) = -k_3 \cos t - k_4 \sin t$

from (2)

$-k_3 \cos t - k_4 \sin t - 2k_3 \sin t + 2k_4 \cos t + 2k_3 \cos t + 2k_4 \sin t = \cos t$

or $(-k_3 + 2k_4 + 2k_3)\cos t + (-k_4 - 2k_3 + 2k_4)\sin t = \cos t$

equating like terms,

$k_3 + 2k_4 = 1$

$-2k_3 + k_4 = 0$

solving, $k_3 = 1/5$, $k_4 = 2/5$

$i_p(t) = \frac{1}{5}\cos t + \frac{2}{5}\sin t$ —— (4)

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Now general solution is

$i(t) = i_c(t) + i_p(t)$

$i(t) = e^{-t}(k_1 \cos t + k_2 \sin t) + \frac{1}{5}\cos t + \frac{2}{5}\sin t$

Initially, $i(0^-) = 0$ then,

$0 = (k_1 + 0) + \frac{1}{5}$

$k_1 = -1/5$

Diff gen. sn:

$\frac{d\,i(t)}{dt} = -e^{-t}(k_1 \cos t + k_2 \sin t) + e^{-t}(-k_1 \sin t + k_2 \cos t)$

$- \frac{1}{5}\sin t + \frac{2}{5}\cos t$

Initially, from (1)

$\frac{d\,i(t)}{dt} + 0 + 0 = \sin 0$

$\frac{d\,i(t)}{dt} = 0$

then,

$0 = -1.(-1/5) + 1.(k_2) - 0 + \frac{2}{5}$

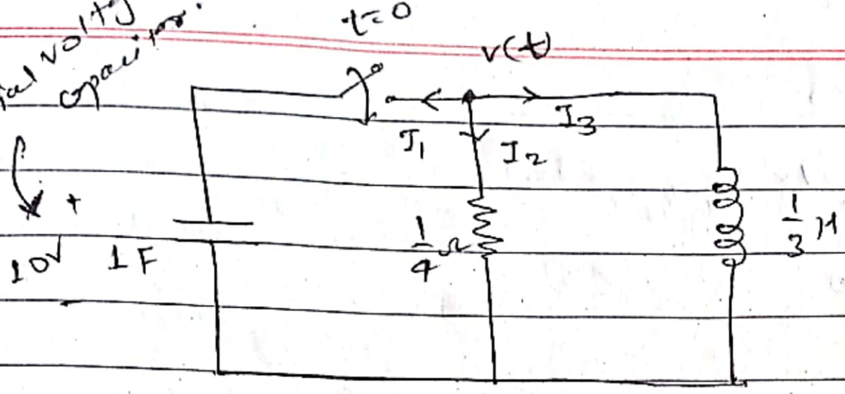
or $0 = \frac{1}{5} + \frac{2}{5} + k_2$

$k_2 = -\frac{3}{5}$

Hence, $i(t) = e^{-t}\left(-\frac{1}{5}\cos t - \frac{3}{5}\sin t\right) + \frac{1}{5}\cos t + \frac{2}{5}\sin t$

PV.

Q. Initial voltage across capacitor.



Switch is closed at $t=0$.
Find the voltage and current through inductor.

Sol^n

At $t=0$, Applying KCL.

$$I_1 + I_2 + I_3 = 0$$

$$C\frac{d\,v(t)}{dt} + \frac{v(t)}{R} + \frac{1}{L}\int v(t)\,dt = 0$$

$$1\cdot\frac{d\,v(t)}{dt} + 4\,v(t) + 3\int v(t)\,dt = 0 \quad \text{——} (1)$$

Diff. w.r. to (t)

$$\frac{d^2 v(t)}{dt^2} + 4\frac{d\,v(t)}{dt} + 3\,v(t) = 0$$

Homogeneous eq^n.

Its characteristics eq^n

$$m^2 + 4m + 3 = 0$$

solving

$$m = -1, -3.$$

$$v(t) = k_1 e^{-1t} + k_2 e^{-3t}$$ — general solution.

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Since $V(0^-) = 10$

so,

$$10 = k_1 e^{-1\times 0} + k_2 e^{-3\times 0}$$

$$k_1 + k_2 = 10 \quad \text{——— (2)}$$

Diff. gen. sol^n:

$$\frac{d\,v(t)}{dt} = -k_1 e^{-t} - 3k_2 e^{-3t}$$

From eq^n (1)

$$1.\frac{d\,v(t)}{dt} + 10\times 7 + 0 = 0$$

$$\frac{d\,v(t)}{dt} = -70$$

$$\text{eq}^n \quad -70 = -k_1 e^{-0} - 3k_2 e^{-3\times 0}$$

$$-k_1 - 3k_2 = -70 \quad \text{——— (3)}$$

solving (2) and (3)

$$k_1 = -5 \qquad k_2 = 15$$

$$\Rightarrow \quad v(t) = -5e^{-t} + 15e^{-3t}$$

current through inductor $\quad i_{ind.} = 3\int(-5e^{-t} + 15e^{-3t})\,dt$

$$i(t)_{ind.} = 15e^{-t} - 135e^{-3t}$$

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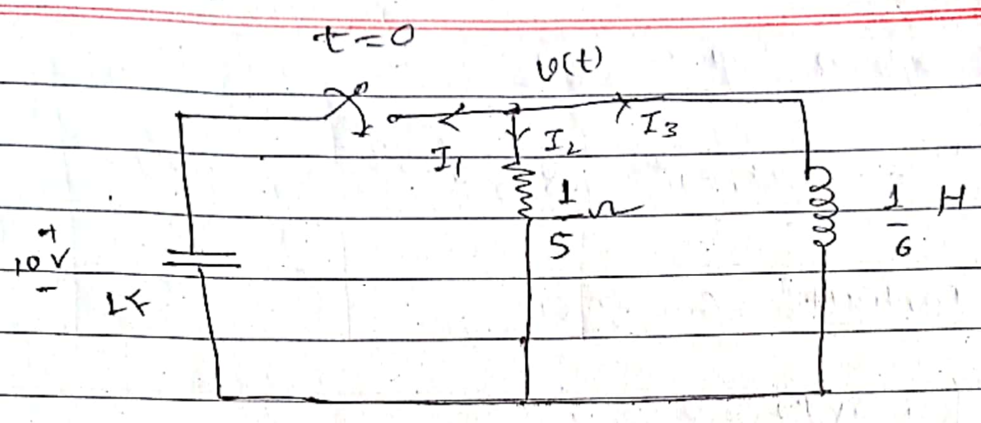

Sol^n At $t=0$, apply KCL.

$I_1 + I_2 + I_3 = 0$

$$1 \cdot \frac{d\,v(t)}{dt} + \frac{v(t)}{1/5} + \frac{1}{1/6}\int v(t)\,dt = 0 \quad \text{——— (1)}$$

diff w.r.to t.

$$\frac{d^2 v(t)}{dt^2} + 5\frac{d\,v(t)}{dt} + 6\,v(t) = 0$$

Its char. eq^n

$m^2 + 5m + 6 = 0$

$m = -2, -3$

$$\therefore \quad v(t) = k_1 e^{-2t} + k_2 e^{-3t} \quad \text{——— (2)}$$

Initially, $v(0^-) = 10V$ Then,

$$10 = k_1 \times e^{-2\times 0} + k_2 \times e^{-3\times 0}$$

$$k_1 + k_2 = 10 \quad \text{——— (3)}$$

Diff eqn ② w.r. to t

$$\frac{d\,v(t)}{dt} = -2k_1 e^{-2t} - 3k_2 e^{-3t} \quad \text{—— ④}$$

now initially from eqn ①

$$\frac{d\,v(t)}{dt} + 5\times 10 + 0 = 0$$

$$\frac{d\,v(t)}{dt} = -50$$

~~then general soln~~

Then from ④

$$-50 = -2k_1 e^{-2\times 0} - 3k_2 e^{-3\times 0}$$

$$-2k_1 - 3k_2 = -50 \quad \text{—— ⑤}$$

solving ③ and ⑤ we get

$k_1 = -20$

$k_2 = 30$

Hence from ②

$$v(t) = -20e^{-2t} + 30e^{-3t}$$

Now current thr inductor, $i_{ind.} = 6\int v(t)\,dt$

$$i(t)_{ind.} = 6\int(-20e^{-2t} + 30e^{-3t})\,dt$$

$$= 6\{10e^{-2t} - 90e^{-3t}\}$$

$$= 270e^{-2t} - 540e^{-3t}$$

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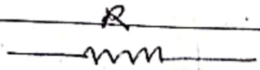
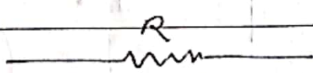
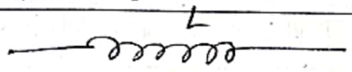
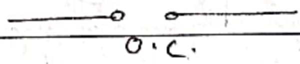
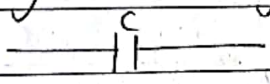
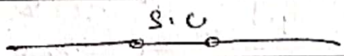
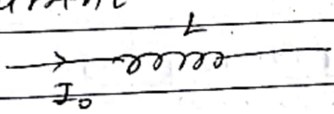
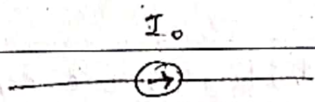
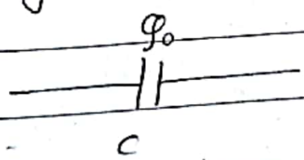
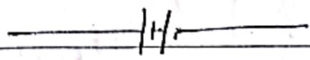
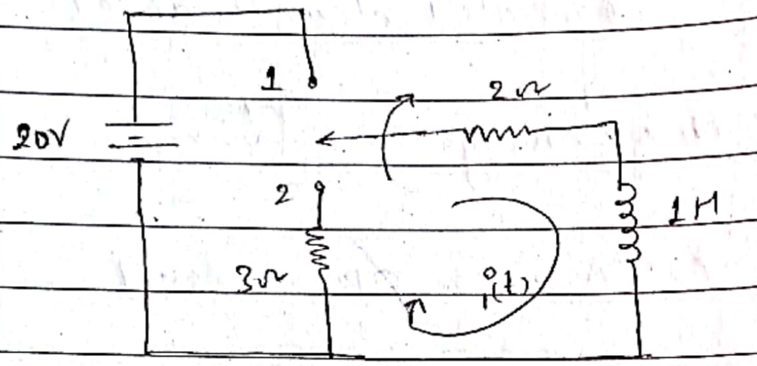
* Calculation of initial values and their derivatives at $t = 0^+$.

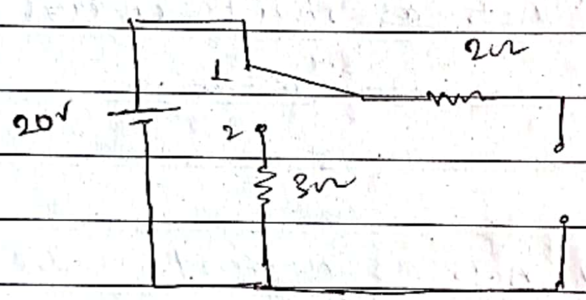
Elements at $t=0$	Equivalent elements at $t=0^+$
(1) Resistor 	(1) No change 
(2) Inductor with no initial current. 	(2). Acts as open circuit 
(3) Capacitor with no initial charge or voltage 	(3) Acts as short circuit 
(4) Inductor with initial current 	(4) Acts as current source 
(5) Capacitor with initial charge or voltage 	(5) Acts as voltage source  $Q_0 = CV_0$ $V_0 = \frac{Q_0}{C}$

Switch is moved from position 2 to position 1 at $t=0$. Find $i(t)$, $\frac{di(t)}{dt}$ and $\frac{d^2i(t)}{dt^2}$ at $t=0^+$



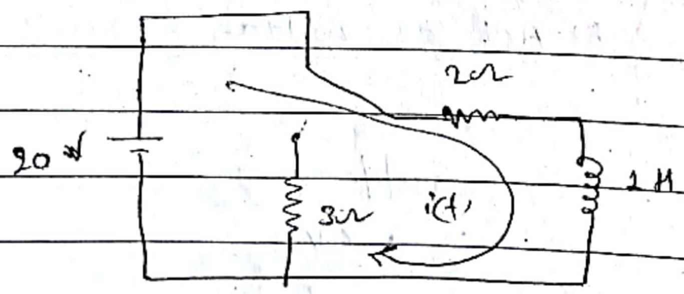
Sol^n

At $t=0^+$



$\Rightarrow i(0^+)=0$

To find derivatives, we consider original ckt at $t=0^+$



Apply KVL.

$$-2\,i(t)-1\cdot\frac{d\,i(t)}{dt}+20=0$$

$$2\,i(t)+\frac{d\,i(t)}{dt}=20 \quad \text{--- (1)}$$

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At $t = 0^+$,

$$2\,i(0^+) + \frac{d\,i(0^+)}{dt} = 20 \quad \text{—— (1)}$$

$$2 \times 0 + \frac{d\,i(0^+)}{dt} = 20$$

$$\Rightarrow \frac{d\,i(0^+)}{dt} = 20 \quad A/s.$$

Diff. eqn (1)

$$2\frac{d\,i(0^+)}{dt} + \frac{d^2 i(0^+)}{dt^2} = 0$$

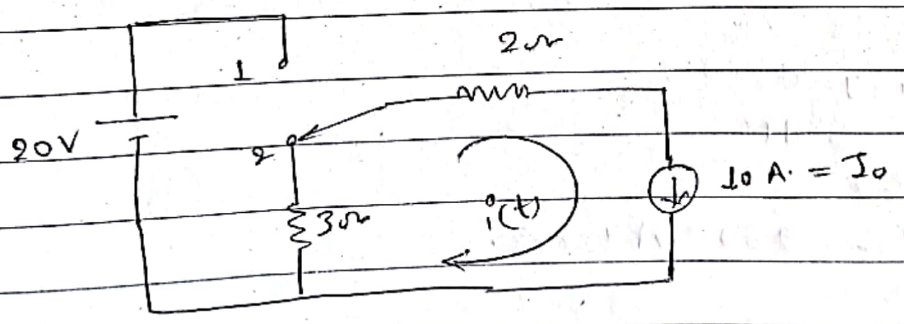
$$2 \times 20 + \frac{d^2 i(0^+)}{dt^2} = 0$$

$$\Rightarrow \frac{d^2 i(0^+)}{dt^2} = -40 \quad A/s^2$$

switch is moved from 1 to 2.

At $t = 0^+$,

$$I_0 = \frac{20V}{2\Omega} = 10A.$$



$$\Rightarrow i(0^+) = 10 \ A.$$

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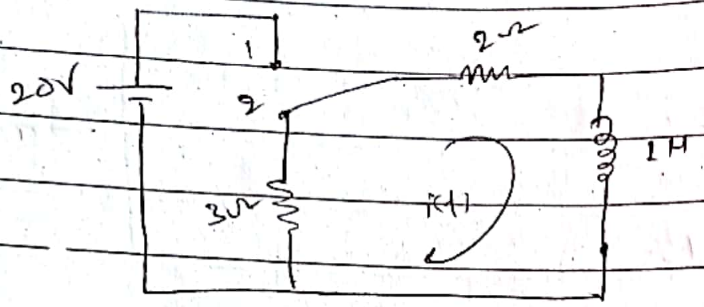
To calculate derivatives.



apply KVL,

$$-5\,i(t) - 1\cdot\frac{d\,i(t)}{dt} = 0$$

At $t = 0+$

$$5\,i(0+) + 1\cdot\frac{d\,i(0+)}{dt} = 0 \quad \text{—— (1)}$$

$$5\times 10 + \frac{d\,i(0+)}{dt} = 0$$

$$\Rightarrow \frac{d\,i(0+)}{dt} = -50 \text{ A/sec}$$

Diff. (1)

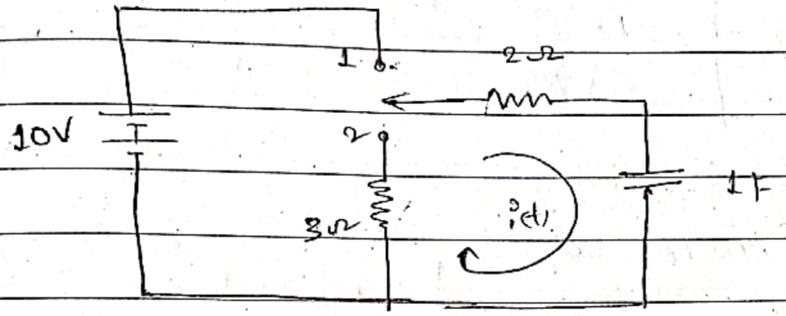
$$5\cdot\frac{d\,i(0+)}{dt} + 1\cdot\frac{d^2\,i(0+)}{dt^2} = 0$$

$$5\times -50 + 1\cdot\frac{d^2(i\,0+)}{dt^2} = 0$$

$$\Rightarrow \frac{d^2(i\,0+)}{dt^2} = 250 \text{ A/sec}^2.$$

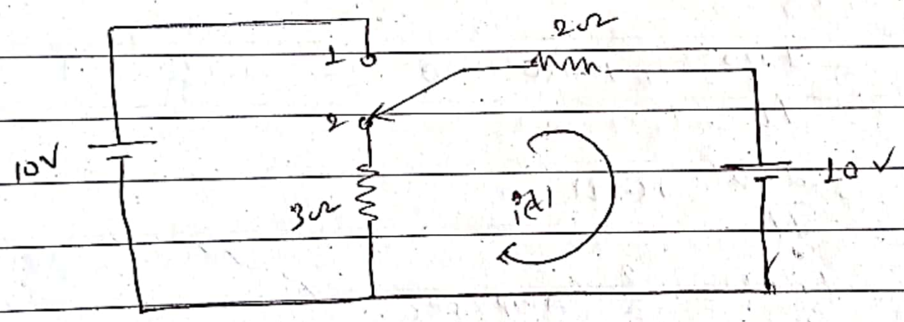
switch is moved from position 1 to 2 at $t=0$.

Find $i(t)$, $\frac{d\,i(t)}{dt}$, and $\frac{d^2 i(t)}{dt^2}$ at $t=0^+$



At $t=0^+$. Equivalent circuit

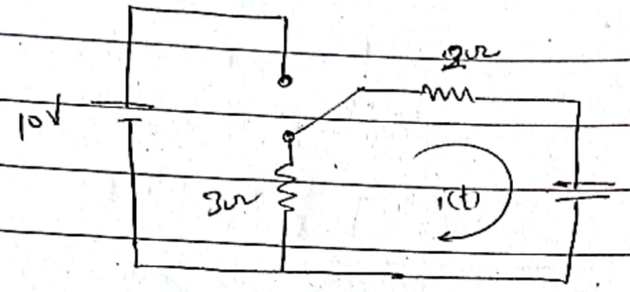
$V_0 = 10V$



using KVL
$$-5\,i(t) - 10 = 0$$
$$5\,i(t) = -10$$

at $t=0^+$
$$5\,i(0^+) = -10$$
$$\Rightarrow i(0^+) = -2\ A.$$

To find derivatives, consider original ckt at $t = 0^+$.



Applying KVL,

$$-5\cdot i(t) - \frac{1}{1}\int i(t)\,dt = 0$$

$$5\,i(t) + \frac{1}{1}\int i(t)\,dt = 0$$

Diff.

$$5\,\frac{d\,i(t)}{dt} + 1\cdot i(t) = 0$$

At $t = 0^+$, $$5\,\frac{d\,i(0^+)}{dt} + 1\cdot i(0^+) = 0 \quad \text{———} \quad (1)$$

$$5\cdot\frac{d\,i(0^+)}{dt} + (-2) = 0$$

$$\Rightarrow \frac{d\,i(0^+)}{dt} = \frac{2}{5} \text{ Amp/sec.}$$

Again Diff (1)

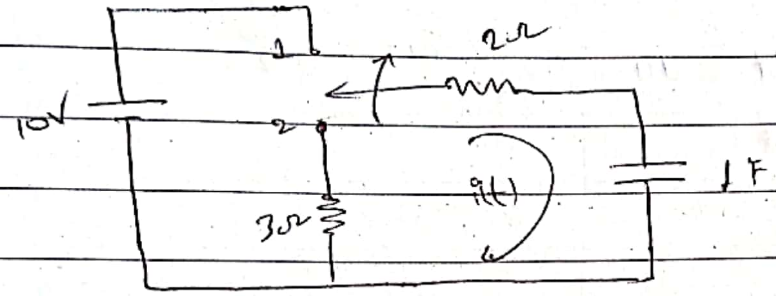
$$5\cdot\frac{d^2\,i(0^+)}{dt^2} + \frac{d\,i(0^+)}{dt} = 0$$

$$5\,\frac{d^2\,i(0^+)}{dt^2} + \frac{2}{5} = 0$$

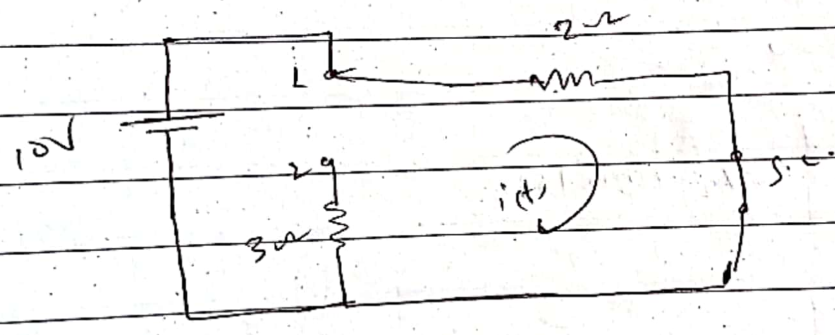
$$\Rightarrow \frac{d^2\,i(0^+)}{dt^2} = -\frac{2}{25} \text{ Amp/sec}^2$$

H.W. From position 2 to 1. at $t=0$.

find $i(t)$, $\frac{d\,i(t)}{dt}$, $\frac{d^2 i(t)}{dt^2}$ at $t=0^+$

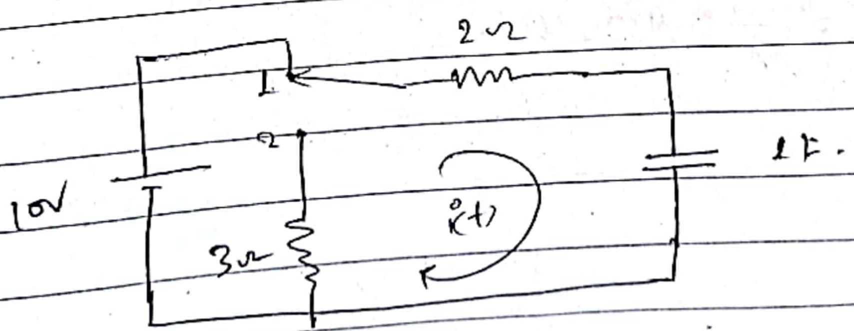


At $t=0^+$, equivalent circuit.



$\Rightarrow i(t) = \frac{10}{2} = 5$ Amp.

To find derivatives,
considering original circuit at $t=0^+$.



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Applying KVL,

$$-2\,i(t) - \frac{1}{1}\int i(t)\,dt + 10 = 0$$

$$2\,i(t) + \frac{1}{1}\cdot\int i(t)\,dt = 10 \quad \text{—— (1)}$$

Diff. (1)

$$2\frac{d\,i(t)}{dt} + i(t) = 0$$

at $t = (0^+)$

$$2\frac{d\,i(0^+)}{dt} + i(0^+) = 0 \quad \text{—— (2)}$$

$$2\frac{d\,i(0^+)}{dt} + 5 = 0$$

$$\Rightarrow \frac{d\,i(0^+)}{dt} = -5/2 \text{ Amp/sec.}$$

Again diff (2)

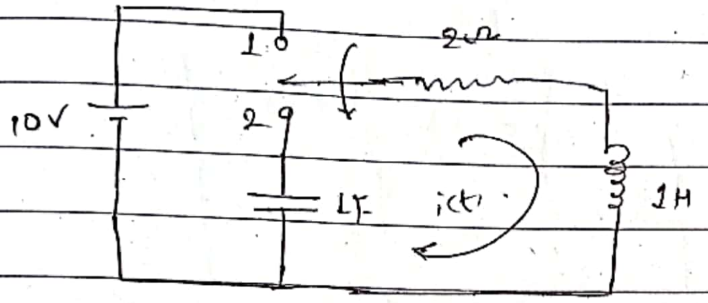
$$2\cdot\frac{d^2 i(0^+)}{dt} + \frac{d\,i(0^+)}{dt} = 0$$

$$2\frac{d^2 i(0^+)}{dt} - \frac{5}{2} = 0$$

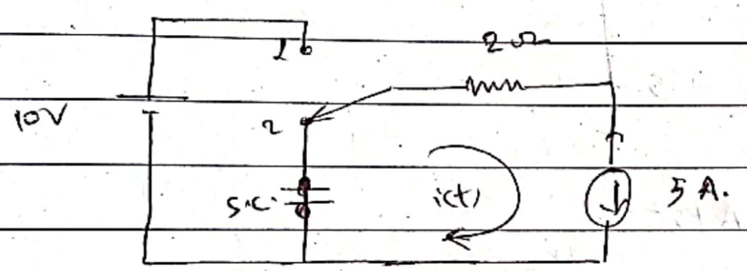
$$\Rightarrow \frac{d^2 i(0^+)}{dt} = \frac{5}{4} \text{ Amp/sec}^2.$$

PV

switch is moved from position 1 to 2 at $t=0$. Find $i(t)$, $\frac{d\,i(t)}{dt}$ and $\frac{d^2 i(t)}{dt^2}$ at $t=0^+$.

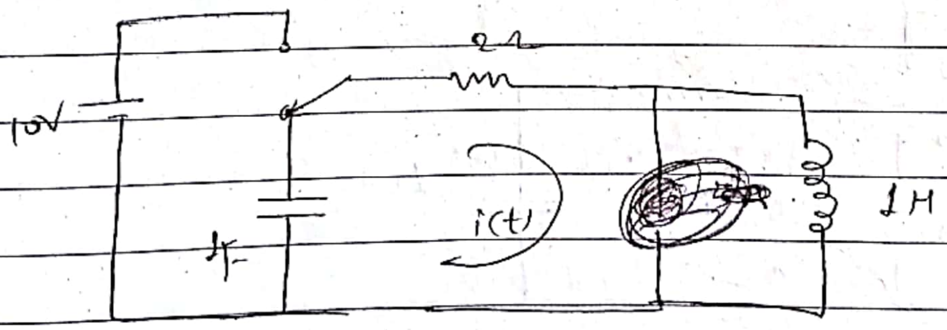


At $t=0^+$, equivalent circuit



$\Rightarrow \quad i(0^+) = 5\,A.$

To find derivatives, considering original ckt



Applying KVL,

$$-2\,i(t) - 1\cdot\frac{d\,i(t)}{dt} - \frac{1}{1}\int i(t)\,dt = 0$$

$$\Rightarrow \quad 2\,i(t) + \frac{d\,i(t)}{dt} + \int i(t)\,dt = 0$$

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At $t = 0^+$

using initial condⁿ

$$2\, i(0^+) + \frac{d\, i(0^+)}{dt} + \int i(0^+)\,dt \overset{0}{=} 0 \quad \text{--- (1)}$$

$$2 \times 5 + \frac{d\, i(0^+)}{dt} = 0$$

$$\Rightarrow \frac{d\, i(0^+)}{dt} = -10 \ A/s$$

Diff. (1)

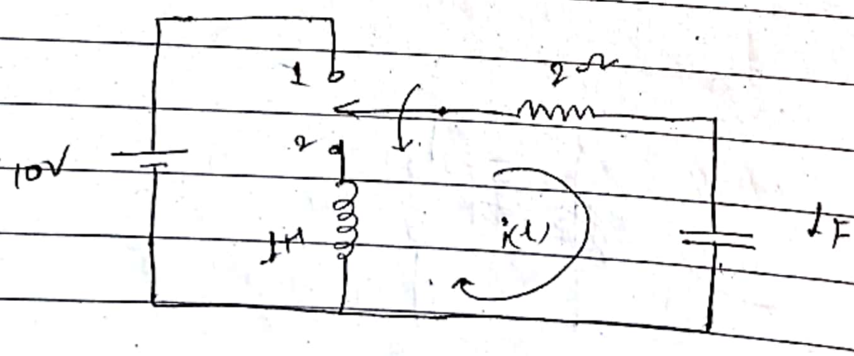
$$2 \frac{d(i(0^+))}{dt} + \frac{d^2 i(0^+)}{dt^2} + i(0^+) = 0$$

$$2 \times -10 + \frac{d^2 i(0^+)}{dt^2} + 5 = 0$$

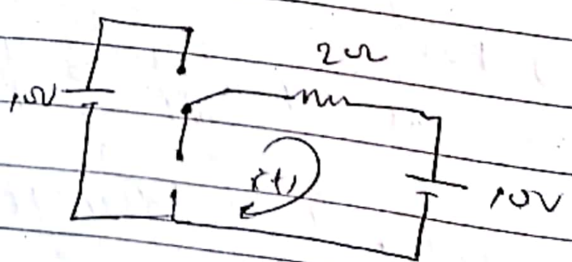
$$\Rightarrow \frac{d^2 i(0^+)}{dt^2} = 15 \ A/s^2$$

Ans

same qⁿ

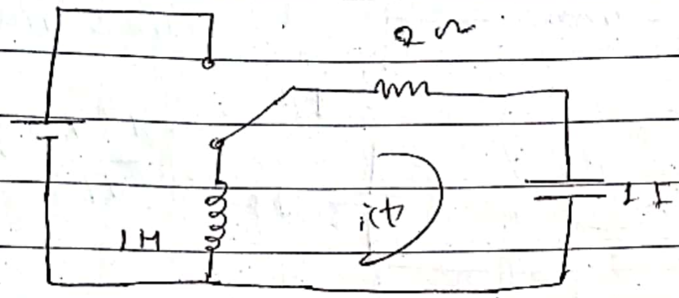


At $t = 0^+$, eq ckt.



$\Rightarrow i(0^+) = 0$ A.

To find derivatives.



applying KVL,

$$2\,i(t) + 1 \cdot \frac{d\,i(t)}{dt} + \frac{1}{1}\int i(t)\,dt = 0$$

at $t = 0^+$

$$2\,i(0^+) + \frac{d\,i(0^+)}{dt} + 1 \cdot \underbrace{\int i(0^+)\,dt}_{10V} = 0 \qquad \text{---- (1)}$$

$$2 \times 0 + \frac{d\,i(0^+)}{dt} + 10 = 0$$

$$\Rightarrow \frac{d\,i(0^+)}{dt} = -10 \quad A/s.$$

diff. (1)

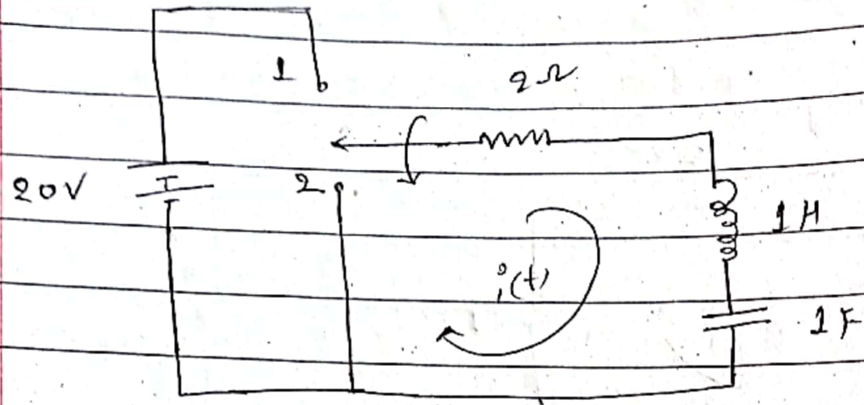
$$2\frac{d\,i(0^+)}{dt} + \frac{d^2\,i(0^+)}{dt^2} + i(0^+) = 0$$

or, $2 \times -10 + \dfrac{d^2\,i(0^+)}{dt^2} + 0 = 0$

$$\Rightarrow \frac{d^2\,i(0^+)}{dt^2} = 20 \quad A/s^2.$$

HW

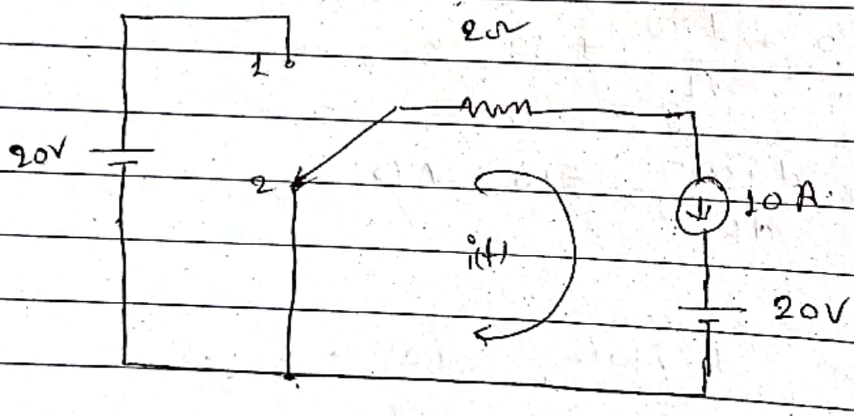
#



switch moved from 1 to 2 at $t=0$. Find $i(0^+)$, $\frac{d\,i(0^+)}{dt}$ and $\frac{d^2 i(0^+)}{dt^2}$.

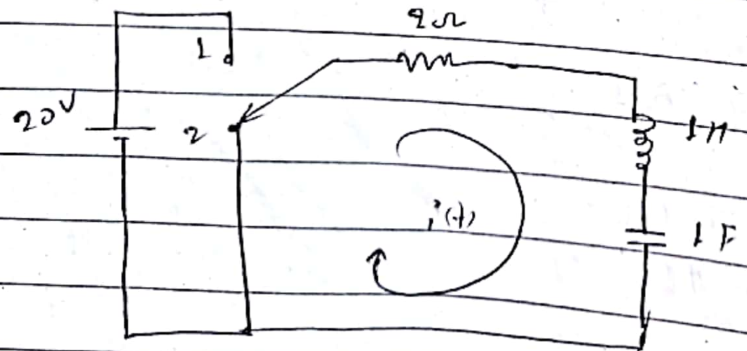
OR, switch was at 1 for long time and the ckt reached to steady sate before switching. Find —.

At $t=0+$, equivalent circuit



$$\Rightarrow\ i(0^+) = 10A - \frac{20}{2}A = 0\ A.$$

To find derivatives, we consider original ckt at $t=0^+$



Applying KVL,

$$2\,i(t) + 1\cdot\frac{d\,i(t)}{dt} + \frac{1}{1}\int i(t)\,dt = 0$$

or, ~~2i(0+)~~

At $t = 0^+$,

$$2\,i(0^+) + 1\cdot\frac{d\,i(0^+)}{dt} + \frac{1}{1}\int i(0^+)\,dt = 0 \quad \text{--- (1)}$$

$$\Rightarrow 2\times 0 + \frac{d\,i(0^+)}{dt} + 20 = 0$$

$$\Rightarrow \frac{d\,i(0^+)}{dt} = -20 \text{ A/sec.}$$

Diff (1) we get,

$$2\frac{d\,i(0^+)}{dt} + \frac{d^2\,i(0^+)}{dt^2} + i(0^+) = 0$$

$$\Rightarrow 2\times -20 + \frac{d^2\,i(0^+)}{dt^2} + 0 = 0$$

$$\Rightarrow -40 + \frac{d^2\,i(0^+)}{dt^2} + 0 = 0$$

$$\Rightarrow \frac{d^2\,i(0^+)}{dt^2} = 40 \text{ A/sec}^2.$$

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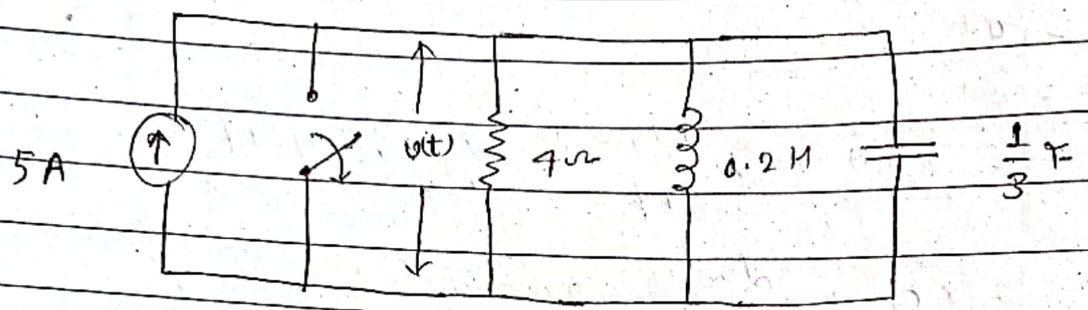
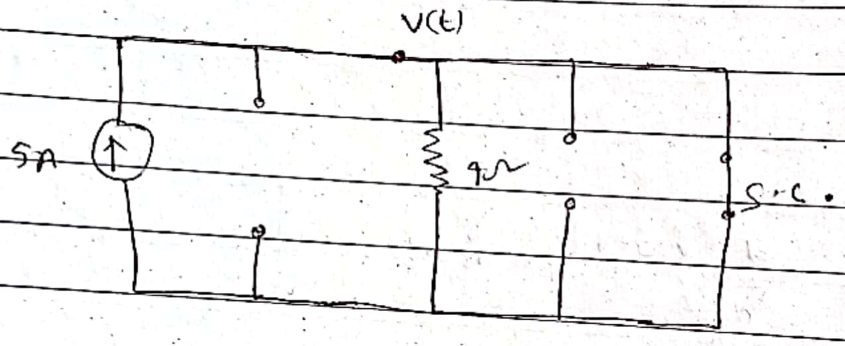
~~For~~ the network shown below, the switch K is opened at $t=0$. Find $v(t)$, $\frac{dv(t)}{dt}$ and $\frac{d^2v(t)}{dt^2}$ at $t=0^+$



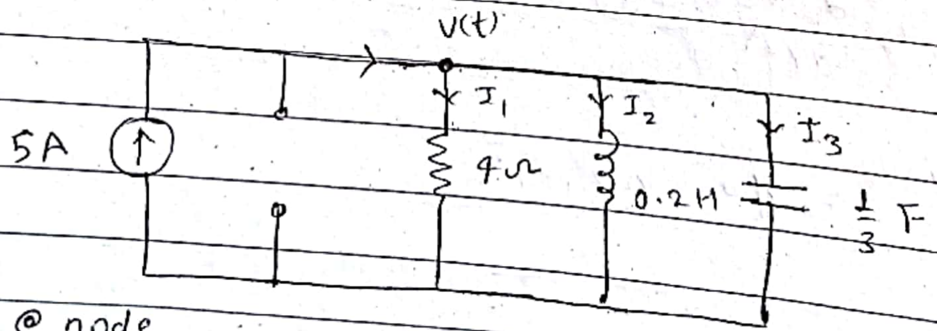
Sol^n

At $t=0^+$, equivalent circuit is



$v(0^+) = 0\ V$ (Due to short circuit)

To find derivatives,



KCL @ node,

$$I_1 + I_2 + I_3 = 5$$

$$\frac{v(t)}{4} + \frac{1}{0.2}\int v(t)\,dt - \frac{1}{3}\frac{dv(t)}{dt} = 5$$

At $t = 0+$,

$$\frac{v(0+)}{4} + \frac{1}{0.2}\int \overset{0}{v(0+)}\,dt + \frac{1}{3}\frac{d\,v(0+)}{dt} = 5 \qquad ①$$

$$0 + 0 + \frac{1}{3}\frac{d\,v(0+)}{dt} = 5$$

$$\Rightarrow \frac{d\,v(0+)}{dt} = 15 \text{ V/s}$$

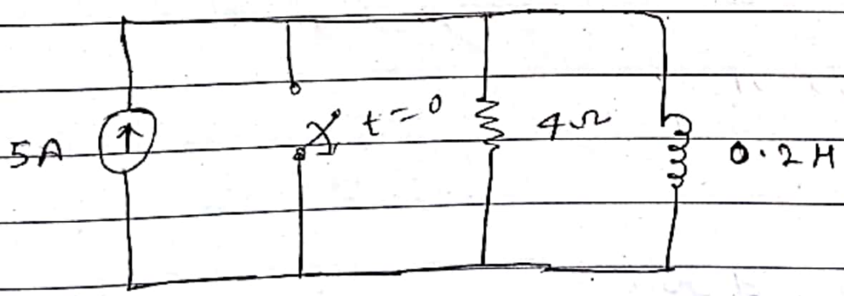
Diff eqn ①

$$\frac{1}{4}\frac{d\,v(0+)}{dt} + \frac{1}{0.2}v(0+) + \frac{1}{3}\frac{d^2\,v(0+)}{dt^2} = 0$$

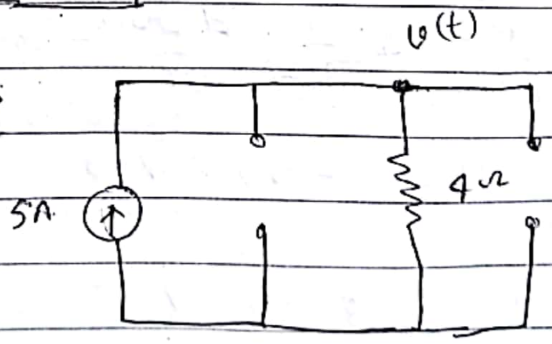
$$\Rightarrow \frac{1}{4}\times 15 + \frac{1}{0.2}\times 0 + \frac{1}{3}\frac{d^2\,v(0+)}{dt^2} = 0$$

$$\Rightarrow \frac{d^2\,v(0+)}{dt^2} = -\frac{45}{4} \text{ V/s}^2$$

Find $v(t)$, $\frac{dv(t)}{dt}$ & $\frac{d^2 v(t)}{dt^2}$ at $t = (0+)$

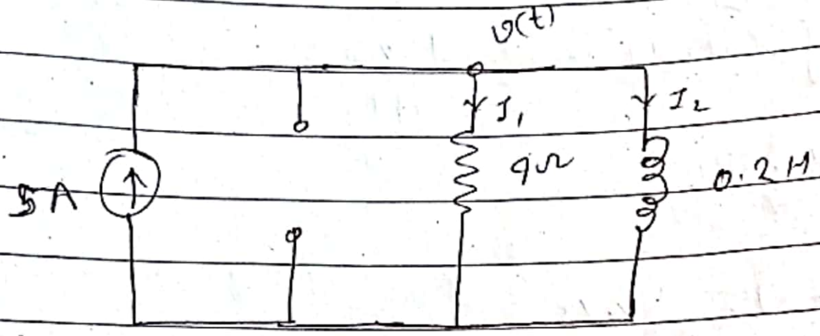


At $t = 0+$, eqn ckt. is



$$v(0+) = 5\times 4 = 20 \text{ V}$$

To find derivatives,



At node, applying KCL.

$$I_1 + I_2 = 5$$

$$\frac{v(t)}{4} + \frac{1}{0.2}\int v(t)\,dt = 5$$

at $t = 0^+$;

$$\frac{v(0^+)}{4} + \frac{1}{0.2}\int v(0^+)\,dt = 5$$

Diff. we get,

$$\frac{1}{4}\frac{d\,v(0^+)}{dt} + \frac{1}{0.2}\,v(0^+) = 0 \quad \text{—— (1)}$$

$$\frac{1}{4}\frac{d\,v(0^+)}{dt} = -5\times 20$$

$$\frac{d\,v(0^+)}{dt} = -400 \text{ V/s.}$$

Diff (1)

$$\frac{1}{4}\frac{d^2 v(0^+)}{dt^2} + \frac{1}{0.2}\frac{d\,v(0^+)}{dt} = 0$$

$$\frac{1}{4}\frac{d^2 v(0^+)}{dt^2} = -5\times -400$$

$$\frac{d^2 v(0^+)}{dt^2} = 8000 \text{ V/s}^2$$