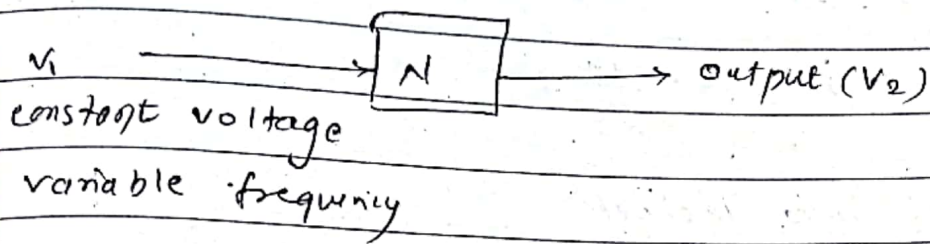


Frequency Response

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$$\text{Gain} = \left(\frac{V_2}{V_1} \right)$$

$$\text{Gain in db} = 20 \log_{10} \left(\frac{V_2}{V_1} \right)$$

plot of magnitude (in db) vs frequency and phase angle
a phase angle in (degree) vs frequency

graph use → semilog graph.

* Bode plot

Plot of asymptotic plot or Bode plot for the following transfer function.

$$G(s) = \frac{(s+10)(s+1)}{s(s+50)}$$

Sol

step-1. convert all the factors in the $\left(1 + \frac{s}{\dots} \right)$ form.

$$\begin{aligned} G(s) &= \frac{10 \left(1 + \frac{s}{10} \right) \times 1 \left(1 + \frac{s}{1} \right)}{s \times 50 \left(1 + \frac{s}{50} \right)} \\ &= \frac{0.2 \left(1 + \frac{s}{10} \right) \left(1 + \frac{s}{1} \right)}{s \left(1 + \frac{s}{50} \right)} \end{aligned}$$

step-2. put $s = j\omega$

(Note: Constant has 0 db/decade or straight line)

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$$G(j\omega) = \frac{0.2 \left(1 + \frac{j\omega}{10}\right) \left(1 + \frac{j\omega}{1}\right)}{j\omega \left(1 + \frac{j\omega}{50}\right)}$$

Step-3 . convert into decibels

$$20 \log_{10} (G(j\omega)) = 20 \log_{10} \left(\frac{0.2 \left(1 + \frac{j\omega}{10}\right) \left(1 + \frac{j\omega}{1}\right)}{j\omega \left(1 + \frac{j\omega}{50}\right)} \right)$$

note.

$$\begin{bmatrix} \log(mn) = \log m + \log n \\ \log\left(\frac{m}{n}\right) = \log m - \log n \end{bmatrix}$$

$$20 \log_{10} [G(j\omega)] = 20 \log_{10} (0.2) + 20 \log_{10} \left(1 + \frac{j\omega}{10}\right) + 20 \log_{10} \left(1 + \frac{j\omega}{1}\right) - 20 \log_{10} (j\omega) - 20 \log_{10} \left(1 + \frac{j\omega}{50}\right)$$

Step-4 Table.

		20db/decade		
S.N.	Component	Cut-off frequency	slope ()	Overall slope
1.	$20 \log_{10} \left(\frac{0.2}{j\omega}\right)$	none	-20 db/decade	-20 db/decade
2.	$20 \log_{10} \left(1 + \frac{j\omega}{1}\right)$	1	+20 db/decade	0 db/decade (straight line)
3.	$20 \log_{10} \left(1 + \frac{j\omega}{10}\right)$	10	+20 db/decade	20 db/decade
4.	$20 \log_{10} \left(1 + \frac{j\omega}{50}\right)$	50	-20 db/decade	0 db/decade (straight line)
starting point $\rightarrow 20 \log_{10} \left(\frac{0.2}{j\omega}\right)$				

Step 4 Table.

S/N.	Component	corner freq.	db/decade	
			slope	overall slope
1.	$20 \log_{10} (2 (\omega)^2)$	none	40 db/decade	40 db/decade
2.	$20 \log_{10} (1 + \frac{j\omega}{4})$	4	-20 db/decade	20 db/decade
3.	$20 \log_{10} (1 + \frac{j\omega}{40})$	40	-20 db/decade	0 db/decade

starting point.

$$20 \log_{10} (2 (\omega)^2)$$

$$= 20 \log_{10} (2 \times 0.1^2)$$

$$= -33.919 \text{ db.}$$

phase angle plot:

$$H(s) = \frac{(s+10)(s+1)}{s(s+50)}$$

step 1, 2, 3 — same as magnitude plot

$$= +20 \log_{10}(0.2) + 20 \log_{10}\left(1 + \frac{j\omega}{10}\right) + 20 \log_{10}\left(1 + \frac{j\omega}{1}\right) - 20 \log_{10}(j\omega) - 20 \log_{10}\left(1 + \frac{j\omega}{50}\right)$$

Ignore constant value in phase

$$- \tan^{-1}\left(\frac{\omega}{0}\right) = -90^\circ$$

then

$$\phi = -90^\circ - \tan^{-1}\left(\frac{\omega}{50}\right) + \tan^{-1}\left(\frac{\omega}{10}\right) + \tan^{-1}\left(\frac{\omega}{1}\right)$$

phase angle ϕ vs ω

ω	0.1	0.2	0.3	0.8	1	2	6	8
ϕ	-83.83	-77.77	-71.92	-97.68	-90.93	-17.57	14.65	22.97

10	20
27.97	38.77

$$Q. \quad H(s) = \frac{300(s+8)}{s(7s+1)(3s^2+11s+12)(s+10)}$$

Magnitude plot

step-1. convert into $\left(1 + \frac{s}{\dots}\right)$

$$= \frac{60 \times 8 \left(1 + \frac{s}{8}\right)}{s}$$

$$\left(1 + \frac{s}{0.5}\right) \times 10 \left(1 + \frac{s}{10}\right) \times \frac{12}{12} \left(1 + \frac{11}{12}s + \frac{3s^2}{12}\right)$$

$$= \frac{s \left(1 + \frac{s}{8}\right)}{\left(1 + \frac{s}{0.5}\right) \left(1 + \frac{s}{10}\right) \left(1 + \frac{11}{12}s + \frac{s^2}{4}\right)}$$

$$\left(1 + \frac{s}{0.5}\right) \left(1 + \frac{s}{10}\right) \left(1 + \frac{11}{12}s + \frac{s^2}{4}\right)$$

step-2. put $s = j\omega$

$$2 \left(1 + \frac{j\omega}{8}\right)$$

$$H(s) =$$

$$\left(1 + \frac{j\omega}{0.5}\right) \left(1 + \frac{j\omega}{10}\right) \left(1 + \frac{11}{12}j\omega + \left(\frac{j\omega}{2}\right)^2\right)$$

step-3. convert into db

$$= 20 \log_{10}(2) + 20 \log_{10} \left(1 + \frac{j\omega}{8}\right) - 20 \log_{10} \left(1 + \frac{j\omega}{0.5}\right) -$$

$$20 \log_{10} \left(1 + \frac{j\omega}{10}\right) - 40 \log_{10} \left(1 + \frac{11}{12}j\omega + \left(\frac{j\omega}{2}\right)^2\right)$$

Table

S.N.	Component	corner freq.	slope	overall slope
1.	$20 \log_{10} (2)$	none	0 db/decade	0 db/decade
2.	$20 \log_{10} \left(1 + \frac{j\omega}{0.5} \right)$	0.5	-20 db/decade	-20 db/decade
3.	$20 \log_{10} \left(1 + \frac{11}{12} j\omega + \left(\frac{j\omega}{2} \right)^2 \right)$	2	-40 db/decade	-60 db/decade
4.	$20 \log_{10} \left(1 + \frac{j\omega}{4} \right)$	4	-20 db/decade	-40 db/decade
5.	$20 \log_{10} \left(1 + \frac{j\omega}{10} \right)$	10	-20 db/decade	-60 db/decade

starting point $\Rightarrow 20 \log_{10} (2) = 6.02$

Phase angle plot:

$$\phi = \tan^{-1} \left(\frac{\omega}{4} \right) - \tan^{-1} \left(\frac{\omega}{0.5} \right) - \tan^{-1} \left(\frac{\omega}{10} \right) - \tan^{-1} \left(\frac{11\omega}{12 - \omega^2} \right)$$

ω	0.1	0.2	0.4	0.8	1	2	4	6	8	10	20
ϕ	-15.69	-30.5	-55.69	-89.09	-100.12	-130.12	-25.12	-10.12	-2.22	-12.59	-43.76