JADAVPUR UNIVERSITY

INFORMATION TECHNOLOGY

ML ASSIGMENT 5

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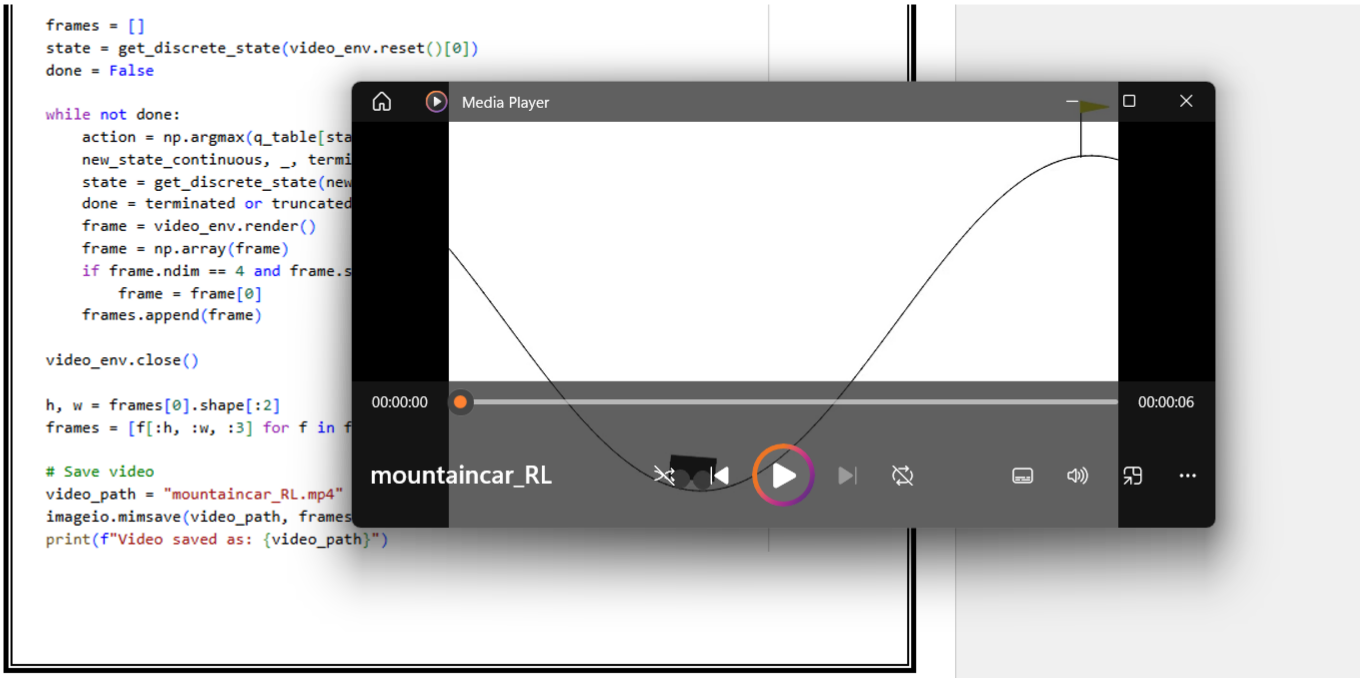
```python
import gymnasium as gym
import numpy as np

env = gym.make('MountainCar-v0')

# Discretize
pos_space = np.linspace(env.observation_space.low[0], env.observation_space.high[0], 20)
vel_space = np.linspace(env.observation_space.low[1], env.observation_space.high[1], 20)

q_table = np.zeros((len(pos_space), len(vel_space), env.action_space.n))

learning_rate = 0.1
discount = 0.95
epochs = 2000
epsilon = 0.5
epsilon_decay = 0.998

def get_discrete_state(state):
    pos, vel = state
    pos_bin = np.digitize(pos, pos_space)
    vel_bin = np.digitize(vel, vel_space)
    return (pos_bin, vel_bin)

# Training loop
for epoch in range(epochs):
    state = get_discrete_state(env.reset()[0])
    terminated = False
    truncated = False

    if epoch % 100 == 0:
        print(f"Epoch: {epoch}")

    while not terminated and not truncated:
        # Epsilon-greedy action selection
        if np.random.random() < epsilon:
            action = env.action_space.sample()
        else:
            action = np.argmax(q_table[state])


        new_state_continuous, reward, terminated, truncated, _ = env.step(action)
        new_state = get_discrete_state(new_state_continuous)

        done = terminated or truncated

        if not done:
            max_future_q = np.max(q_table[new_state])
            current_q = q_table[state + (action,)]

            new_q = current_q + learning_rate * (reward + discount * max_future_q - current_q)
            q_table[state + (action,)] = new_q

        elif terminated:
            q_table[state + (action,)] = 0

        state = new_state

    if epsilon > 0.05:
        epsilon *= epsilon_decay

print("Training finished!")
env.close()
```

```
Epoch: 0
Epoch: 100
Epoch: 200
Epoch: 300
Epoch: 400
Epoch: 500
Epoch: 600
Epoch: 700
Epoch: 800
Epoch: 900
Epoch: 1000
Epoch: 1100
Epoch: 1200
Epoch: 1300
Epoch: 1400
Epoch: 1500
Epoch: 1600
Epoch: 1700
Epoch: 1800
Epoch: 1900
Training finished!
```

```python
import imageio
import numpy as np
import gymnasium as gym

video_env = gym.make('MountainCar-v0', render_mode='rgb_array')

frames = []
state = get_discrete_state(video_env.reset()[0])
done = False

while not done:
    action = np.argmax(q_table[state])
    new_state_continuous, _, terminated, truncated, _ = video_env.step(action)
    state = get_discrete_state(new_state_continuous)
    done = terminated or truncated
    frame = video_env.render()
    frame = np.array(frame)
    if frame.ndim == 4 and frame.shape[0] == 2:
        frame = frame[0]
    frames.append(frame)

video_env.close()

h, w = frames[0].shape[:2]
frames = [f[:h, :w, :3] for f in frames]

# Save video
video_path = "mountaincar_RL.mp4"
imageio.mimsave(video_path, frames, fps=30, macro_block_size=None)
print(f"Video saved as: {video_path}")
```



Mountain Car Using Q-Learning (RL)

In this part of the assignment, the MountainCar-v0 environment is solved using the Q-Learning algorithm. Because the environment provides continuous values for position and velocity, these values are first discretized into 20 bins so that a finite Q-table can be created. A 3-dimensional Q-table is initialized where each entry corresponds to a state–action pair. The agent is trained for 2000 episodes. In each episode, the environment is reset, and the agent repeatedly chooses actions using the epsilon-greedy strategy, which balances exploration and exploitation. After performing each action, the next state and reward are obtained, and the Q-value is updated based on the Q-Learning formula. Over time, epsilon is reduced so that the agent gradually relies more on the learned policy. By the end of training, the Q-table captures an optimal policy that enables the car to gather momentum and reach the goal. The program prints progress during training and displays the message “Training finished!” when learning is complete.

Generating RL Video Output

After training the Q-Learning agent, a second script is used to generate a video demonstrating the learned policy. The Mountain Car environment is recreated in **rgb_array** mode so that individual frames can be captured. Starting from the initial state, the agent uses the learned Q-table to select the optimal action at every step. Each rendered frame is converted into an array and stored in a list. Once the episode ends, all frames are combined and saved as an MP4 video using the imageio library. This video visually shows how the trained Q-Learning agent successfully drives the car up the hill and reaches the goal.

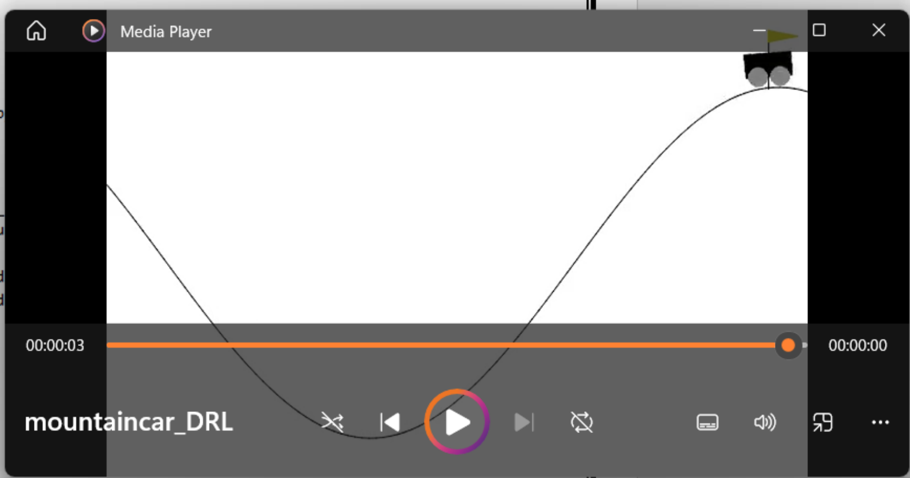
```python
import gymnasium as gym
import torch
import torch.nn as nn
import torch.optim as optim
import numpy as np
import random
from collections import deque

# Q-Network
class QNetwork(nn.Module):
    def __init__(self, state_size, action_size):
        super(QNetwork, self).__init__()
        self.fc1 = nn.Linear(state_size, 64)
        self.fc2 = nn.Linear(64, 64)
        self.fc3 = nn.Linear(64, action_size)

    def forward(self, state):
        x = torch.relu(self.fc1(state))
        x = torch.relu(self.fc2(x))
        return self.fc3(x)

# Replay Buffer
class ReplayBuffer:
    def __init__(self, buffer_size, batch_size):
        self.memory = deque(maxlen=buffer_size)
        self.batch_size = batch_size

    def add(self, state, action, reward, next_state, done): # 'done' here means (terminated or truncated)
        self.memory.append((state, action, reward, next_state, done))

    def sample(self):
        experiences = random.sample(self.memory, self.batch_size)

        states = torch.from_numpy(np.vstack([e[0] for e in experiences if e is not None])).float()
        actions = torch.from_numpy(np.vstack([e[1] for e in experiences if e is not None])).long()
        rewards = torch.from_numpy(np.vstack([e[2] for e in experiences if e is not None])).float()
        next_states = torch.from_numpy(np.vstack([e[3] for e in experiences if e is not None])).float()
        dones = torch.from_numpy(np.vstack([e[4] for e in experiences if e is not None]).astype(np.uint8)).float()

        return (states, actions, rewards, next_states, dones)
```

```python
    def __len__(self):
        return len(self.memory)

device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
print(f"Using device: {device}")

BUFFER_SIZE = 100000
BATCH_SIZE = 64
GAMMA = 0.99
LR = 0.0005
UPDATE_EVERY = 4

env = gym.make('MountainCar-v0')
state_size = env.observation_space.shape[0]
action_size = env.action_space.n

q_network_local = QNetwork(state_size, action_size).to(device)
q_network_target = QNetwork(state_size, action_size).to(device)
optimizer = optim.Adam(q_network_local.parameters(), lr=LR)
memory = ReplayBuffer(BUFFER_SIZE, BATCH_SIZE)

q_network_target.load_state_dict(q_network_local.state_dict())

def learn(experiences, gamma):
    states, actions, rewards, next_states, dones = experiences

    states = states.to(device)
    actions = actions.to(device)
    rewards = rewards.to(device)
    next_states = next_states.to(device)
    dones = dones.to(device)

    best_actions = q_network_local(next_states).detach().argmax(1).unsqueeze(1)
    q_targets_next = q_network_target(next_states).detach().gather(1, best_actions)
    q_targets = rewards + (gamma * q_targets_next * (1 - dones))
    q_expected = q_network_local(states).gather(1, actions)

    loss = nn.MSELoss()(q_expected, q_targets)

    optimizer.zero_grad()
    loss.backward()
    torch.nn.utils.clip_grad_norm_(q_network_local.parameters(), 1)
    optimizer.step()

    # Soft update
    for target_param, local_param in zip(q_network_target.parameters(), q_network_local.parameters()):
        target_param.data.copy_(0.01 * local_param.data + (1.0 - 0.01) * target_param.data)

# Training Loop
episodes = 1200
max_t = 1000
epsilon = 1.0
epsilon_decay = 0.997
epsilon_min = 0.01

scores = []
scores_window = deque(maxlen=100)

for i_episode in range(1, episodes + 1):
    state = env.reset()[0]
    score = 0

    for t in range(max_t):
        state_tensor = torch.from_numpy(state).float().unsqueeze(0).to(device)
        q_network_local.eval()
        with torch.no_grad():
            action_values = q_network_local(state_tensor)
        q_network_local.train()
```

```python
        if random.random() > epsilon:
            action = np.argmax(action_values.cpu().data.numpy())
        else:
            action = random.choice(np.arange(action_size))

        next_state, reward, terminated, truncated, _ = env.step(action)

        # Reward shaping
        position, velocity = next_state
        reward = reward + 0.5 * abs(velocity) + (position + 0.5)
        done = terminated or truncated

        memory.add(state, action, reward, next_state, done)

        if len(memory) > BATCH_SIZE and t % UPDATE_EVERY == 0:
            experiences = memory.sample()
            learn(experiences, GAMMA)

        state = next_state
        score += reward
        if done:
            break

    scores_window.append(score)
    scores.append(score)

    epsilon = max(epsilon_min, epsilon * epsilon_decay)

    if i_episode % 100 == 0:
        print(f'\rEpisode {i_episode}\tAverage Score: {np.mean(scores_window):.2f}')

print('Training finished!')
env.close()
```

```
Using device: cuda
Episode 100     Average Score: -201.15
Episode 200     Average Score: -194.18
Episode 300     Average Score: -189.23
Episode 400     Average Score: -188.20
Episode 500     Average Score: -186.36
Episode 600     Average Score: -180.50
Episode 700     Average Score: -179.71
Episode 800     Average Score: -178.11
Episode 900     Average Score: -175.31
Episode 1000    Average Score: -173.89
Episode 1100    Average Score: -152.00
Episode 1200    Average Score: -131.25
Training finished!
```

```python
frames = []
state = render_env.reset()[0]
done = False

while not done:
    state_tensor = torch.from_nump

    q_network_local.eval()
    with torch.no_grad():
        action_values = q_network_
    action = np.argmax(action_valu

    next_state, reward, terminated
    done = terminated or truncated

    frame = render_env.render()
    frames.append(frame)

    state = next_state

render_env.close()

# Save the video
video_path = "mountaincar_DRL.mp4"
imageio.mimsave(video_path, frames, fps=30, macro_block_size=None)
print(f"Video saved as: {video_path}")
```




```
Video saved as: mountaincar_DRL.mp4
```

```python
import imageio

render_env = gym.make('MountainCar-v0', render_mode='rgb_array')

frames = []
state = render_env.reset()[0]
done = False

while not done:
    state_tensor = torch.from_numpy(state).float().unsqueeze(0).to(device)

    q_network_local.eval()
    with torch.no_grad():
        action_values = q_network_local(state_tensor)
    action = np.argmax(action_values.cpu().data.numpy())

    next_state, reward, terminated, truncated, _ = render_env.step(action)
    done = terminated or truncated

    frame = render_env.render()
    frames.append(frame)

    state = next_state

render_env.close()

# Save the video
video_path = "mountaincar_DRL.mp4"
imageio.mimsave(video_path, frames, fps=30, macro_block_size=None)
print(f"Video saved as: {video_path}")
```

```
Video saved as: mountaincar_DRL.mp4
```

Mountain Car Using Deep Q-Network (DQN)

In this part of the assignment, the MountainCar-v0 problem is solved using a Deep Reinforcement Learning approach based on the Deep Q-Network (DQN). Unlike the Q-Learning implementation that uses a discretized state space, DQN works directly with continuous states by using a neural network to approximate Q-values. The Q-network consists of three fully connected layers, where the input layer receives the two-dimensional state (position and velocity), and the output layer predicts Q-values for the three possible actions.

To stabilize training, a replay buffer is used to store past experiences in the form of (state, action, reward, next state, done). Mini-batches sampled from this buffer are used during training, helping the network learn from uncorrelated data. A target network is also maintained alongside the main network; its weights are periodically updated to provide stable Q-target calculations during learning. The training process runs for 1200 episodes. In each timestep, the agent selects an action using the epsilon-greedy policy, and the neural network estimates the Q-values. The reward is slightly shaped to encourage the agent to build momentum and climb the hill more efficiently.

During training, gradients are clipped to avoid instability, and soft updates are applied to slowly adjust the target network. The average score improves steadily as the agent learns, which is printed every 100 episodes. After training, a separate script records a video of the trained DQN agent by loading the environment in rendering mode and letting the neural network choose the best action at each step. The sequence of frames is saved as an MP4 file, demonstrating that the DQN-based agent successfully learns to reach the goal.

```python
import gym
import gym_toytext
import numpy as np
import warnings

warnings.filterwarnings('ignore')

env = gym.make("Roulette-v0")

num_actions = env.action_space.n
q_table = np.zeros(num_actions)

learning_rate = 0.1
discount = 0.9
epochs = 50_000
epsilon = 1.0
epsilon_decay = 0.9999
epsilon_min = 0.01

for epoch in range(epochs):
    env.reset()

    # Epsilon-greedy action selection
    if np.random.rand() < epsilon:
        action = env.action_space.sample()
    else:
        action = np.argmax(q_table)

    new_state, reward, done, info = env.step(action)

    # Update Q-table
    current_q = q_table[action]
    new_q = current_q + learning_rate * (reward - current_q)
    q_table[action] = new_q

    # Decay epsilon
    epsilon = max(epsilon_min, epsilon * epsilon_decay)

env.close()

print("\n Training finished!")
print("Final Q-values (expected reward for each bet):")
print(q_table.round(4))

best_action = np.argmax(q_table)
print(f"\nBest bet to make: Action {best_action} (Expected reward: {q_table[best_action]:.2f})")
```

```
Gym has been unmaintained since 2022 and does not support NumPy 2.0 amongst other critical functionality.
Please upgrade to Gymnasium, the maintained drop-in replacement of Gym, or contact the authors of your software and request
See the migration guide at https://gymnasium.farama.org/introduction/migration_guide/ for additional information.
/usr/local/lib/python3.12/dist-packages/jupyter_client/session.py:203: DeprecationWarning: datetime.datetime.utcnow() is de
  return datetime.utcnow().replace(tzinfo=utc)

 Training finished!
Final Q-values (expected reward for each bet):
[-0.4947 -0.1888 -0.2211 -0.0306 -0.2207 -0.1152 -0.047  -0.3718 -0.2157
 -0.0865 -0.1771 -0.2512 -0.4696 -0.2177 -0.0663 -0.4659 -0.0707 -0.1681
 -0.0308 -0.1838 -0.1838 -0.288  -0.3199 -0.1801 -0.0613 -0.0674 -0.299
 -0.1828 -0.1979 -0.1443 -0.1643 -0.0048 -0.079  -0.2237 -0.0553 -0.2899
 -0.1817  0.    ]

Best bet to make: Action 37 (Expected reward: 0.00)
```

```python
import gym
import gym_toytext
import numpy as np
import torch
import torch.nn as nn
import torch.optim as optim
from collections import deque
import random

env = gym.make("Roulette-v0")
state_size = 1
action_size = env.action_space.n

# Q-network
class QNetwork(nn.Module):
    def __init__(self, state_size, action_size):
        super(QNetwork, self).__init__()
        self.fc1 = nn.Linear(state_size, 64)
        self.fc2 = nn.Linear(64, 64)
        self.fc3 = nn.Linear(64, action_size)

    def forward(self, x):
        x = torch.relu(self.fc1(x))
        x = torch.relu(self.fc2(x))
        return self.fc3(x)

device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
q_network = QNetwork(state_size, action_size).to(device)
optimizer = optim.Adam(q_network.parameters(), lr=0.001)
criterion = nn.MSELoss()

# Replay buffer
memory = deque(maxlen=5000)
batch_size = 64

episodes = 50000
gamma = 0.9
epsilon = 1.0
epsilon_min = 0.01
epsilon_decay = 0.9999

# Training loop
for ep in range(episodes):
    env.reset()
    state = np.array([0.0], dtype=np.float32)
    done = False

    # Epsilon-greedy
    if np.random.rand() < epsilon:
        action = env.action_space.sample()
    else:
        with torch.no_grad():
            state_tensor = torch.tensor(state).unsqueeze(0).to(device)
            action = torch.argmax(q_network(state_tensor)).item()

    # Take action
    next_state, reward, done, info = env.step(action)
    next_state = np.array([0.0], dtype=np.float32)

    # Store in replay memory
    memory.append((state, action, reward, next_state, done))

    if len(memory) >= batch_size:
        batch = random.sample(memory, batch_size)
        states_b, actions_b, rewards_b, next_states_b, dones_b = zip(*batch)

        states_b = torch.tensor(np.array(states_b), dtype=torch.float32).to(device)
        next_states_b = torch.tensor(np.array(next_states_b), dtype=torch.float32).to(device)
        actions_b = torch.tensor(np.array(actions_b)).unsqueeze(1).to(device)
        rewards_b = torch.tensor(np.array(rewards_b), dtype=torch.float32).unsqueeze(1).to(device)
        dones_b = torch.tensor(np.array(dones_b), dtype=torch.float32).unsqueeze(1).to(device)

        # Compute Q targets
        q_values = q_network(states_b).gather(1, actions_b)
        with torch.no_grad():
            q_next = q_network(next_states_b).max(1)[0].unsqueeze(1)
            q_target = rewards_b + gamma * q_next * (1 - dones_b)

        # Update network
        loss = criterion(q_values, q_target)
        optimizer.zero_grad()
        loss.backward()
        optimizer.step()

    # Decay epsilon
    epsilon = max(epsilon_min, epsilon * epsilon_decay)

env.close()

with torch.no_grad():
    state_tensor = torch.tensor([[0.0]], dtype=torch.float32).to(device)
    final_qs = q_network(state_tensor).cpu().numpy()[0]

print("\n DRL Training finished!")
print("Final Q-values (expected reward for each bet):")
print(final_qs.round(4))
best_action = np.argmax(final_qs)
print(f"\n Best bet to make: Action {best_action} (Expected reward: {final_qs[best_action]:.2f})")
```

```
 DRL Training finished!
Final Q-values (expected reward for each bet):
[-8.144e-01  1.708e-01  2.663e-01  1.542e-01 -4.518e-01  1.337e-01
 -5.167e-01  1.031e-01  1.762e-01 -8.500e-02  1.651e-01 -1.265e-01
  1.485e-01  9.420e-02 -3.996e-01  9.420e-02 -3.400e-03 -1.677e-01
  1.065e-01 -8.030e-02  2.597e-01 -2.960e-02  1.849e-01  2.012e-01
  4.660e-02 -1.529e-01 -1.030e-01  1.788e-01  6.010e-02  2.172e-01
  2.079e-01 -3.580e-01  6.500e-03 -1.354e-01 -3.280e-02  1.827e-01
  2.074e-01 -1.000e-04]

 Best bet to make: Action 2 (Expected reward: 0.27)
```

```python
import numpy as np

# 1. Define the graph structure using an adjacency matrix
# Example: A 5-node graph
# Nodes are indexed from 0 to N-1
# float('inf') represents no direct connection
# Diagonal elements are 0 (cost from a node to itself)

graph_matrix = np.array([
    [0,   1,   5,   float('inf'), float('inf')],  # Node 0
    [float('inf'), 0,   2,   8,   float('inf')],  # Node 1
    [float('inf'), float('inf'), 0,   1,   3],   # Node 2
    [float('inf'), float('inf'), float('inf'), 0,   4],   # Node 3
    [float('inf'), float('inf'), float('inf'), float('inf'), 0]    # Node 4
])

# 2. Specify the start and end nodes
start_node = 0
end_node = 4

# 3. Define reward function parameters
# Negative reward for each step equal to the edge weight
# Positive reward for reaching the goal node
# Large negative reward for invalid moves

goal_reward = 100
invalid_move_penalty = -100

print("Graph Adjacency Matrix:")
print(graph_matrix)
print(f"\nStart Node: {start_node}")
print(f"End Node: {end_node}")
print(f"Goal Reward: {goal_reward}")
print(f"Invalid Move Penalty: {invalid_move_penalty}")
```

```
Graph Adjacency Matrix:
[[ 0.  1.  5. inf inf]
 [inf  0.  2.  8. inf]
 [inf inf  0.  1.  3.]
 [inf inf inf  0.  4.]
 [inf inf inf inf  0.]]

Start Node: 0
End Node: 4
Goal Reward: 100
Invalid Move Penalty: -100
```

```python
import random

# 1. Determine the number of states (nodes) and actions
num_states = graph_matrix.shape[0]
num_actions = graph_matrix.shape[1]

print(f"Number of states (nodes): {num_states}")
print(f"Number of actions per state: {num_actions}")

# 2. Initialize the Q-table
# Initialize with zeros. For shortest path, negative Q-values are expected for costs.
q_table = np.zeros((num_states, num_actions))
# Alternatively, could initialize with small random numbers or a very large negative number if we want to ensure exploration initially.
# q_table = np.random.uniform(low=-1.0, high=1.0, size=(num_states, num_actions))

print(f"Q-table initialized with shape: {q_table.shape}")

# 3. Create a function for epsilon-greedy action selection
def choose_action(current_state, epsilon, graph_matrix, q_table):
    valid_actions = np.where(graph_matrix[current_state, :] != float('inf'))[0]
    if len(valid_actions) == 0:
        return -1 # Indicate no valid actions

    if random.uniform(0, 1) < epsilon: # Explore
        action = random.choice(valid_actions)
    else: # Exploit
        # Select action with max Q-value among valid actions
        q_values_for_state = q_table[current_state, :]
        # Filter for valid actions only
        valid_q_values = q_values_for_state[valid_actions]
        max_q_value = np.max(valid_q_values)

        # Handle multiple actions with the same max Q-value
        best_valid_actions = valid_actions[np.where(valid_q_values == max_q_value)[0]]
        action = random.choice(best_valid_actions)
    return action

# 4. Create a function to calculate the reward
def get_reward(current_state, action, end_node, graph_matrix, goal_reward, invalid_move_penalty):
    if action == end_node:
        return goal_reward

    cost = graph_matrix[current_state, action]
    if cost == float('inf'):
        return invalid_move_penalty
    else:
        return -cost # Negative reward for step cost

print("Epsilon-greedy action selection function 'choose_action' created.")
print("Reward calculation function 'get_reward' created.")

# 5. Define the Q-learning update rule parameters
learning_rate = 0.8 # Alpha (α) - how much new information overrides old information
discount_factor = 0.95 # Gamma (γ) - importance of future rewards

print(f"Q-learning learning rate (alpha): {learning_rate}")
print(f"Q-learning discount factor (gamma): {discount_factor}")
```

```
Number of states (nodes): 5
Number of actions per state: 5
Q-table initialized with shape: (5, 5)
Epsilon-greedy action selection function 'choose_action' created.
Reward calculation function 'get_reward' created.
Q-learning learning rate (alpha): 0.8
Q-learning discount factor (gamma): 0.95
```

Roulette Using Deep Q-Network (DQN)

In this section, the Roulette-v0 environment is solved using a Deep Q-Network (DQN) instead of a simple Q-table. Since the Roulette environment has only one state, the agent relies entirely on learning the expected reward associated with each betting action. A neural network with three fully connected layers is used to approximate the Q-values for all available actions. The model takes the fixed state as input and outputs a Q-value for each bet.

A replay buffer is used to store previous experiences, which helps break correlation between consecutive actions and improves the stability of learning. During training, the agent interacts with the environment for 50,000 episodes. In each episode, the agent selects an action using the epsilon-greedy strategy to balance exploration and exploitation. The experience is stored in memory, and once enough samples are collected, random batches are used to train the neural network. The target Q-values are calculated using the Bellman equation, and the network weights are updated by minimizing the mean squared error loss.

Over time, epsilon is gradually decayed, allowing the agent to depend more on the learned value estimates. At the end of the training process, the neural network provides Q-values for every possible bet, and the action with the highest estimated reward is reported as the optimal betting strategy. This DQN-based approach produces smoother and more stable reward estimates compared to the Q-learning method, demonstrating the benefit of deep learning in reinforcement learning tasks.

```
import torch
import torch.nn as nn
import torch.optim as optim
import random
from collections import deque # For experience replay buffer

# 1. Define the Neural Network for DQN
# Input: current state (one-hot encoded vector of size num_states)
# Output: Q-value for each possible action (size num_actions)

class DQNAgent(nn.Module):
    def __init__(self, state_size, action_size, hidden_size=64):
        super(DQNAgent, self).__init__()
        self.fc1 = nn.Linear(state_size, hidden_size)
        self.relu = nn.ReLU()
        self.fc2 = nn.Linear(hidden_size, action_size)

    def forward(self, state):
        x = self.fc1(state)
        x = self.relu(x)
        q_values = self.fc2(x)
        return q_values

print("DQNAgent neural network class defined.")
```

```
DQNAgent neural network class defined.
```

```
class ReplayBuffer:
    def __init__(self, capacity):
        self.capacity = capacity
        self.buffer = deque(maxlen=capacity)

    def add(self, state, action, reward, next_state, done):
        # state and next_state should be numerical representations, not one-hot encoded yet
        self.buffer.append((state, action, reward, next_state, done))

    def sample(self, batch_size):
        if len(self.buffer) < batch_size:
            return None # Not enough samples to form a batch
        batch = random.sample(self.buffer, batch_size)
        states, actions, rewards, next_states, dones = zip(*batch)
        return (
            torch.tensor(states, dtype=torch.float32),
            torch.tensor(actions, dtype=torch.int64),
            torch.tensor(rewards, dtype=torch.float32),
            torch.tensor(next_states, dtype=torch.float32),
            torch.tensor(dones, dtype=torch.bool)
        )

    def __len__(self):
        return len(self.buffer)

print("ReplayBuffer class defined.")
```

```
ReplayBuffer class defined.
```

```python
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")

# 4. Initialize the main DQN network and a separate target DQN network
policy_net = DQNAgent(num_states, num_actions).to(device)
target_net = DQNAgent(num_states, num_actions).to(device)
target_net.load_state_dict(policy_net.state_dict()) # Copy weights from policy_net to target_net
target_net.eval() # Set target network to evaluation mode

print(f"Policy Network initialized. Device: {device}")
print(f"Target Network initialized. Weights copied from policy_net and set to eval mode. Device: {device}")

# 5. Define the optimizer and the loss function
optimizer = optim.Adam(policy_net.parameters(), lr=0.001)
criterion = nn.MSELoss() # Mean Squared Error Loss

print("Optimizer (Adam) and Loss function (MSELoss) defined.")
```

```
Policy Network initialized. Device: cuda
Target Network initialized. Weights copied from policy_net and set to eval mode. Device: cuda
Optimizer (Adam) and Loss function (MSELoss) defined.
```

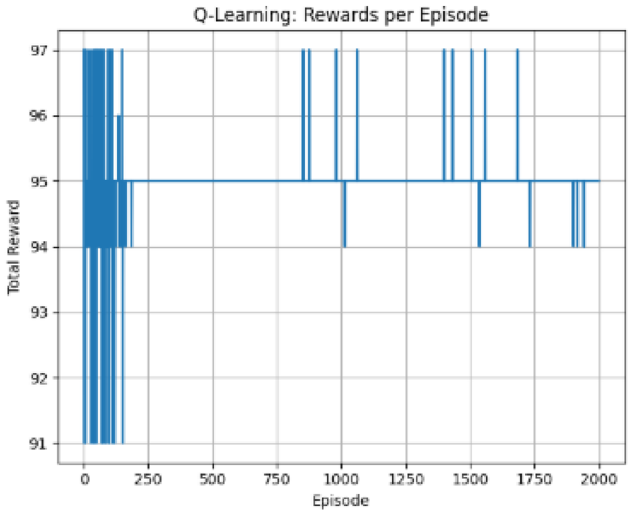
```python
def choose_action_dqn(current_state, epsilon, num_actions, policy_net, device):
    if random.uniform(0, 1) < epsilon: # Explore
        # Choose a random action (node index)
        action = random.randrange(num_actions)
    else: # Exploit
        # Convert current_state to a one-hot encoded tensor and move to device
        state_tensor = torch.zeros(1, num_states).to(device)
        state_tensor[0, current_state] = 1.0

        with torch.no_grad(): # No gradient calculation needed for action selection
            q_values = policy_net(state_tensor)
        action = q_values.argmax(dim=1).item() # Select action with max Q-value
    return action

print("DQN-specific 'choose_action_dqn' function defined for epsilon-greedy action selection.")
```

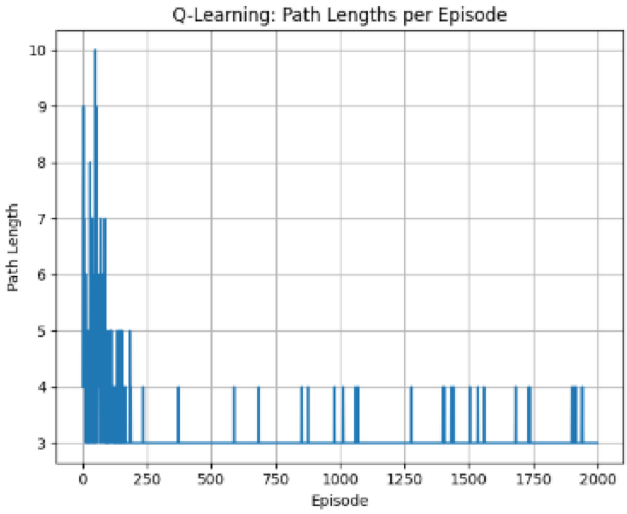
```
DQN-specific 'choose_action_dqn' function defined for epsilon-greedy action selection.
```

```python
import matplotlib.pyplot as plt

# 1. Define training parameters
num_episodes = 2000
max_steps_per_episode = num_states * num_states # A generous upper bound to prevent infinite loops

initial_epsilon = 1.0
min_epsilon = 0.01
epsilon_decay_rate = 0.005 # Rate at which epsilon decays per episode

epsilon = initial_epsilon

print(f"Training Q-learning agent for {num_episodes} episodes.")
print(f"Max steps per episode: {max_steps_per_episode}")
print(f"Epsilon: initial={initial_epsilon}, min={min_epsilon}, decay_rate={epsilon_decay_rate}")

# 2. Initialize lists to store metrics
rewards_per_episode = []
path_lengths_per_episode = []

# 3. Implement the main training loop
for episode in range(num_episodes):
    current_state = start_node
    done = False
    episode_reward = 0
    episode_path = [start_node]

    for step in range(max_steps_per_episode):
        action = choose_action(current_state, epsilon, graph_matrix, q_table)

        if action == -1: # No valid actions from current_state
            episode_reward += invalid_move_penalty # Penalize for being stuck/invalid state
            break

        # Determine next state (action is the next node index)
        next_state = action

        # Calculate reward for taking this action
        reward = get_reward(current_state, action, end_node, graph_matrix, goal_reward, invalid_move_penalty)
        episode_reward += reward
        episode_path.append(next_state)

        # Calculate max Q-value for the next state
        # Ensure we only consider valid actions from the next_state
        valid_actions_next_state = np.where(graph_matrix[next_state, :] != float('inf'))[0]
        if len(valid_actions_next_state) > 0:
            max_future_q = np.max(q_table[next_state, valid_actions_next_state])
        else:
            max_future_q = 0 # No future rewards if stuck or end state

        # Q-learning update rule
        q_table[current_state, action] = q_table[current_state, action] + learning_rate * \
                                         (reward + discount_factor * max_future_q - q_table[current_state, action])
```

```python
        current_state = next_state

        if current_state == end_node:
            done = True
            break

    # After each episode, decay epsilon
    epsilon = max(min_epsilon, epsilon - epsilon_decay_rate)

    # Store metrics
    rewards_per_episode.append(episode_reward)
    path_lengths_per_episode.append(len(episode_path))

    if (episode + 1) % 200 == 0:
        print(f"Episode {episode + 1}/{num_episodes} - Total Reward: {episode_reward:.2f} - Path Length: {len(episode_path)} - Epsilon: {epsilon:.4f}")

print("\nQ-learning training complete.")

# 4. Evaluate the learned Q-table to find the optimal path
print("\nEvaluating learned Q-table for optimal path:")
optimal_path = [start_node]
current_eval_state = start_node
total_path_cost = 0
total_path_reward = 0

for _ in range(max_steps_per_episode):
    # Choose action with max Q-value (greedy policy)
    q_values_current_state = q_table[current_eval_state, :]

    # Filter for valid actions from current_eval_state
    valid_actions_eval = np.where(graph_matrix[current_eval_state, :] != float('inf'))[0]

    if len(valid_actions_eval) == 0:
        print(f"  Stuck at node {current_eval_state} - no valid moves. Path: {optimal_path}")
        break

    # Select action with max Q-value among valid actions
    max_q_value_eval = np.max(q_values_current_state[valid_actions_eval])
    best_eval_actions = valid_actions_eval[np.where(q_values_current_state[valid_actions_eval] == max_q_value_eval)[0]]

    next_eval_node = random.choice(best_eval_actions) # Choose randomly if multiple best actions

    if current_eval_state == end_node:
        break # Reached the goal

    # Accumulate cost and reward
    cost = graph_matrix[current_eval_state, next_eval_node]
    if cost != float('inf'): # Should always be a valid move if chosen correctly
        total_path_cost += cost
        total_path_reward += -cost # Step reward


    optimal_path.append(next_eval_node)
    current_eval_state = next_eval_node

    if current_eval_state == end_node:
        total_path_reward += goal_reward # Add goal reward at the end
        break

print(f"  Optimal path found by Q-learning: {optimal_path}")
print(f"  Total cost of optimal path (sum of edge weights): {total_path_cost:.2f}")
print(f"  Total reward for optimal path (using reward function): {total_path_reward:.2f}")

# Visualize training convergence
plt.figure(figsize=(12, 5))

plt.subplot(1, 2, 1)
plt.plot(rewards_per_episode)
plt.title('Q-Learning: Rewards per Episode')
plt.xlabel('Episode')
plt.ylabel('Total Reward')
plt.grid(True)

plt.subplot(1, 2, 2)
plt.plot(path_lengths_per_episode)
plt.title('Q-Learning: Path Lengths per Episode')
plt.xlabel('Episode')
plt.ylabel('Path Length')
plt.grid(True)

plt.tight_layout()
plt.show()

print("Final Q-table:")
print(q_table)
```

```
Training Q-learning agent for 2000 episodes.
Max steps per episode: 25
Epsilon: initial=1.0, min=0.01, decay_rate=0.005
Episode 200/2000 - Total Reward: 95.00 - Path Length: 3 - Epsilon: 0.0100
Episode 400/2000 - Total Reward: 95.00 - Path Length: 3 - Epsilon: 0.0100
Episode 600/2000 - Total Reward: 95.00 - Path Length: 3 - Epsilon: 0.0100
Episode 800/2000 - Total Reward: 95.00 - Path Length: 3 - Epsilon: 0.0100
Episode 1000/2000 - Total Reward: 95.00 - Path Length: 3 - Epsilon: 0.0100
Episode 1200/2000 - Total Reward: 95.00 - Path Length: 3 - Epsilon: 0.0100
Episode 1400/2000 - Total Reward: 97.00 - Path Length: 4 - Epsilon: 0.0100
Episode 1600/2000 - Total Reward: 95.00 - Path Length: 3 - Epsilon: 0.0100
Episode 1800/2000 - Total Reward: 95.00 - Path Length: 3 - Epsilon: 0.0100
Episode 2000/2000 - Total Reward: 95.00 - Path Length: 3 - Epsilon: 0.0100

Q-learning training complete.

Evaluating learned Q-table for optimal path:
  Optimal path found by Q-learning: [0, 2, 4]
  Total cost of optimal path (sum of edge weights): 8.00
  Total reward for optimal path (using reward function): 92.00
```






```
Final Q-table:
[[ 85.5         87.35        90.          0.          0.        ]
 [  0.         88.34720513  93.         87.          0.        ]
 [  0.          0.          95.         94.        100.        ]
 [  0.          0.           0.         95.        100.        ]
 [  0.          0.           0.          0.          0.        ]]
```

Shortest Path Using Q-Learning (RL)

In this part of the assignment, Q-Learning is applied to find the shortest path in a user-defined graph. The graph is represented using an adjacency matrix, where each entry corresponds to the cost of moving from one node to another. The start node and goal node are specified, and the reward function is designed such that every step gives a negative reward equal to the edge cost, while reaching the goal gives a large positive reward. Invalid moves receive a strong negative penalty, discouraging the agent from selecting non-existent edges.

A Q-table of size *number_of_nodes × number_of_nodes* is initialized, where each row corresponds to a state (current node) and each column represents a possible next node. The agent is trained for 2000 episodes using an epsilon-greedy exploration strategy. During each episode, the agent selects a valid action, receives a reward, updates the Q-value using the Bellman equation, and moves to the next state. Epsilon gradually decays so the agent shifts from exploration to exploitation. At the end of each episode, both cumulative reward and path length are recorded.

After training, the learned Q-table is evaluated using a greedy policy that always selects the action with the highest Q-value among the valid neighbors. The agent successfully identifies the optimal path from the start node to the goal node, and the training plots clearly show convergence in both rewards and path lengths. The final Q-table confirms that the agent has learned the shortest route by favoring actions with lower cost values.

```python
import torch
import torch.nn as nn
import torch.optim as optim
import random
from collections import deque # For experience replay buffer

# DQNAgent, ReplayBuffer, policy_net, target_net, optimizer, criterion, choose_action_dqn, num_states, num_actions, graph_matrix, start_node, end_node, goal_reward, invalid_move_penalty, device are alre

# 1. Define training parameters specific to the DQN agent
dqn_num_episodes = 2000 # Number of training episodes
dqn_max_steps_per_episode = num_states * num_states # A generous upper bound

dqn_initial_epsilon = 1.0 # Initial exploration rate
dqn_min_epsilon = 0.01    # Minimum exploration rate
dqn_epsilon_decay_rate = 0.005 # Rate at which epsilon decays per episode

dqn_batch_size = 64       # Number of transitions to sample from replay buffer
dqn_gamma = 0.95          # Discount factor for future rewards
dqn_learning_rate = 0.001 # Learning rate for the optimizer (already set in optimizer def)

target_update_frequency = 10 # How often to update the target network (in episodes)

print(f"DQN Training Parameters:")
print(f"  Episodes: {dqn_num_episodes}")
print(f"  Max Steps per Episode: {dqn_max_steps_per_episode}")
print(f"  Epsilon: initial={dqn_initial_epsilon}, min={dqn_min_epsilon}, decay_rate={dqn_epsilon_decay_rate}")
print(f"  Batch Size: {dqn_batch_size}")
print(f"  Discount Factor (Gamma): {dqn_gamma}")
print(f"  Learning Rate: {dqn_learning_rate}")
print(f"  Target Network Update Frequency: {target_update_frequency} episodes")

# Initialize Replay Buffer
replay_buffer = ReplayBuffer(capacity=10000)
print(f"Replay Buffer initialized with capacity {replay_buffer.capacity}.")
```

```
DQN Training Parameters:
  Episodes: 2000
  Max Steps per Episode: 25
  Epsilon: initial=1.0, min=0.01, decay_rate=0.005
  Batch Size: 64
  Discount Factor (Gamma): 0.95
  Learning Rate: 0.001
  Target Network Update Frequency: 10 episodes
Replay Buffer initialized with capacity 10000.
```

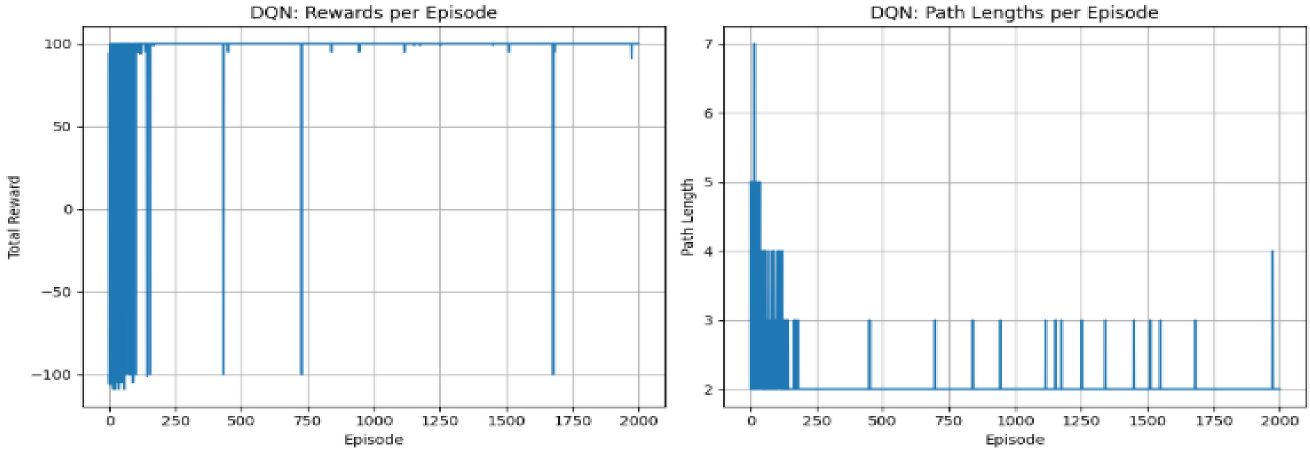
```python
def optimize_model():
    if len(replay_buffer) < dqn_batch_size:
        return # Not enough samples to train

    transitions = replay_buffer.sample(dqn_batch_size)
    if transitions is None:
        return # Not enough samples to form a batch

    # Unpack the batch and convert to tensors on the correct device
    states, actions, rewards, next_states, dones = transitions

    # One-hot encode states and next_states
    states_one_hot = torch.zeros(dqn_batch_size, num_states, device=device)
    states_one_hot.scatter_(1, states.unsqueeze(1).long().to(device), 1.0)

    next_states_one_hot = torch.zeros(dqn_batch_size, num_states, device=device)
    next_states_one_hot.scatter_(1, next_states.unsqueeze(1).long().to(device), 1.0)

    actions = actions.to(device) # Actions are indices, don't one-hot encode here
    rewards = rewards.to(device)
    dones = dones.to(device)

    # Compute Q(s_t, a) - the model predicts Q(s_t), then we select the
    # columns of actions taken.
    current_q_values = policy_net(states_one_hot).gather(1, actions.unsqueeze(1)).squeeze(1)

    # Compute V(s_{t+1}) for all next states.
    # Expected values of actions for next_states are computed based on the target_net
    # This is where the double DQN concept applies (though simple DQN uses policy_net for max(Q') here)
    with torch.no_grad():
        # Filter out states that are 'done'
        next_state_q_values = target_net(next_states_one_hot)
        # For shortest path, valid actions are those with finite cost in graph_matrix
        # We need to ensure we only consider Q-values for valid actions in next_states
        # This is more complex than a simple filter due to batching.
        # For now, let's assume all actions are valid, or simplify this for graph traversal
        # A more robust approach would involve masking invalid actions with -inf before taking max

        # Mask invalid actions for next states (if needed - currently we don't have this in DQN's choose_action)
        # For now, we will simply take the max Q-value from the target network for the next state
        max_next_q_values = next_state_q_values.max(1)[0]
        # Set max_next_q_values to 0 for terminal states
        max_next_q_values[dones] = 0.0

    # Compute the expected Q values: R + gamma * max(Q(s', a'))
    expected_q_values = rewards + dqn_gamma * max_next_q_values

    # Compute loss
    loss = criterion(current_q_values, expected_q_values)

    # Optimize the model
    optimizer.zero_grad()
    loss.backward()
    # Clip gradients to prevent exploding gradients
    for param in policy_net.parameters():
        param.grad.data.clamp_(-1, 1)
    optimizer.step()
```

```python
import matplotlib.pyplot as plt

# 3. Implement the main training loop for the DQN agent
dqn_rewards_per_episode = []
dqn_path_lengths_per_episode = []
dqn_epsilon = dqn_initial_epsilon

print(f"\nStarting DQN training for {dqn_num_episodes} episodes...")

for episode in range(dqn_num_episodes):
    current_state = start_node
    episode_reward = 0
    episode_path = [start_node]
    done = False

    # Convert current_state to one-hot for choose_action_dqn
    current_state_tensor = torch.zeros(1, num_states).to(device)
    current_state_tensor[0, current_state] = 1.0

    for step in range(dqn_max_steps_per_episode):
        # Select action using epsilon-greedy policy with DQN
        action = choose_action_dqn(current_state, dqn_epsilon, num_actions, policy_net, device)

        # Determine next state and reward
        next_state = action # For graph traversal, action is typically the next node
        reward = get_reward(current_state, action, end_node, graph_matrix, goal_reward, invalid_move_penalty)

        episode_reward += reward
        episode_path.append(next_state)

        # Check if the episode ended (reached end_node or invalid move)
        if next_state == end_node or reward == invalid_move_penalty: # If invalid move, it's considered done for this step
            done = True

        # Store the transition in the replay buffer
        # Store states as integers, optimize_model will convert to one-hot
        replay_buffer.add(current_state, action, reward, next_state, done)

        # Move to the next state
        current_state = next_state

        # Perform one optimization step (on the policy network)
        optimize_model()

        if done:
            break

    # Decay epsilon
    dqn_epsilon = max(dqn_min_epsilon, dqn_epsilon - dqn_epsilon_decay_rate)

    # Store metrics
    dqn_rewards_per_episode.append(episode_reward)
    dqn_path_lengths_per_episode.append(len(episode_path))

    # Update the target network periodically
    if (episode + 1) % target_update_frequency == 0:
        target_net.load_state_dict(policy_net.state_dict())

    if (episode + 1) % 200 == 0:
        print(f"Episode {episode + 1}/{dqn_num_episodes} - Total Reward: {episode_reward:.2f} - Path Length: {len(episode_path)} - Epsilon: {dqn_epsilon:.4f}")
```

```python
print("\nDQN training complete.")

# 4. Evaluate the trained DQN agent
print("\nEvaluating learned DQN policy for optimal path:")
optimal_path_dqn = [start_node]
current_eval_state_dqn = start_node
total_path_cost_dqn = 0
total_path_reward_dqn = 0

# Set policy_net to evaluation mode
policy_net.eval()

with torch.no_grad():
    for _ in range(dqn_max_steps_per_episode):
        state_tensor_eval = torch.zeros(1, num_states).to(device)
        state_tensor_eval[0, current_eval_state_dqn] = 1.0

        # Get Q-values from the policy network (greedy action selection)
        q_values_eval = policy_net(state_tensor_eval)
        action_eval = q_values_eval.argmax(dim=1).item() # Select action with max Q-value

        # Check for valid action in graph matrix, similar to Q-learning evaluation
        if graph_matrix[current_eval_state_dqn, action_eval] == float('inf'):
            # If the chosen action is invalid, break and report.
            print(f"  Stuck at node {current_eval_state_dqn} - DQN chose an invalid move. Path: {optimal_path_dqn}")
            total_path_reward_dqn += invalid_move_penalty # Penalize final path if stuck
            break

        if current_eval_state_dqn == end_node:
            total_path_reward_dqn += goal_reward # Add goal reward at the end if it reached
            break

        # Accumulate cost and reward
        cost = graph_matrix[current_eval_state_dqn, action_eval]
        total_path_cost_dqn += cost
        total_path_reward_dqn += -cost # Step reward

        optimal_path_dqn.append(action_eval)
        current_eval_state_dqn = action_eval

        if current_eval_state_dqn == end_node:
            total_path_reward_dqn += goal_reward # Add goal reward at the end
            break

print(f"  Optimal path found by DQN: {optimal_path_dqn}")
print(f"  Total cost of optimal path (sum of edge weights): {total_path_cost_dqn:.2f}")
print(f"  Total reward for optimal path (using reward function): {total_path_reward_dqn:.2f}")

# 5. Visualize DQN training convergence
plt.figure(figsize=(12, 5))

plt.subplot(1, 2, 1)
plt.plot(dqn_rewards_per_episode)
plt.title('DQN: Rewards per Episode')
plt.xlabel('Episode')
plt.ylabel('Total Reward')
plt.grid(True)
```

```
plt.subplot(1, 2, 2)
plt.plot(dqn_path_lengths_per_episode)
plt.title('DQN: Path Lengths per Episode')
plt.xlabel('Episode')
plt.ylabel('Path Length')
plt.grid(True)

plt.tight_layout()
plt.show()

# Set policy_net back to training mode if further training is planned
policy_net.train()
```

```
Starting DQN training for 2000 episodes...
Episode 200/2000 - Total Reward: 100.00 - Path Length: 2 - Epsilon: 0.0100
Episode 400/2000 - Total Reward: 100.00 - Path Length: 2 - Epsilon: 0.0100
Episode 600/2000 - Total Reward: 100.00 - Path Length: 2 - Epsilon: 0.0100
Episode 800/2000 - Total Reward: 100.00 - Path Length: 2 - Epsilon: 0.0100
Episode 1000/2000 - Total Reward: 100.00 - Path Length: 2 - Epsilon: 0.0100
Episode 1200/2000 - Total Reward: 100.00 - Path Length: 2 - Epsilon: 0.0100
Episode 1400/2000 - Total Reward: 100.00 - Path Length: 2 - Epsilon: 0.0100
Episode 1600/2000 - Total Reward: 100.00 - Path Length: 2 - Epsilon: 0.0100
Episode 1800/2000 - Total Reward: 100.00 - Path Length: 2 - Epsilon: 0.0100
Episode 2000/2000 - Total Reward: 100.00 - Path Length: 2 - Epsilon: 0.0100

DQN training complete.

Evaluating learned DQN policy for optimal path:
  Stuck at node 0 - DQN chose an invalid move. Path: [0]
  Optimal path found by DQN: [0]
  Total cost of optimal path (sum of edge weights): 0.00
  Total reward for optimal path (using reward function): -100.00
```




```
DQNAgent(
  (fc1): Linear(in_features=5, out_features=64, bias=True)
  (relu): ReLU()
  (fc2): Linear(in_features=64, out_features=5, bias=True)
)
```

Shortest Path Using Deep Q-Network (DQN)

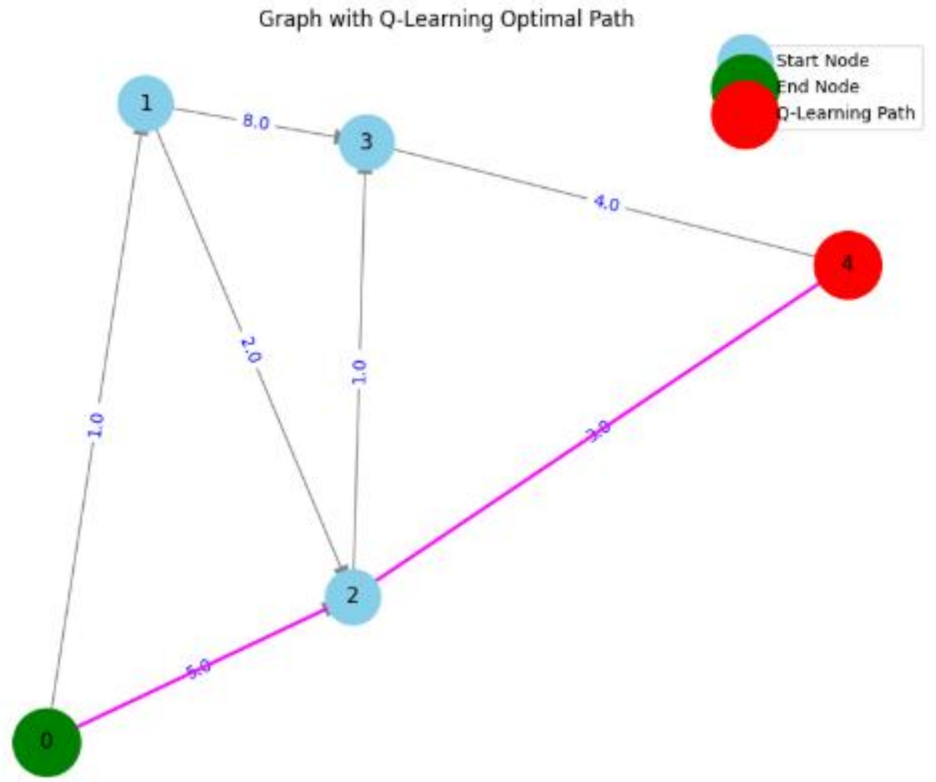
In this part of the assignment, the shortest path problem is solved using a Deep Q-Network (DQN). Unlike the earlier Q-Learning approach that used a Q-table, the DQN method uses a neural network to estimate Q-values for each possible action from a given node. The graph is represented as an adjacency matrix, where valid edges contain numerical costs and invalid moves are marked with infinity. The state is encoded as a one-hot vector, and the neural network outputs Q-values for all possible next nodes.

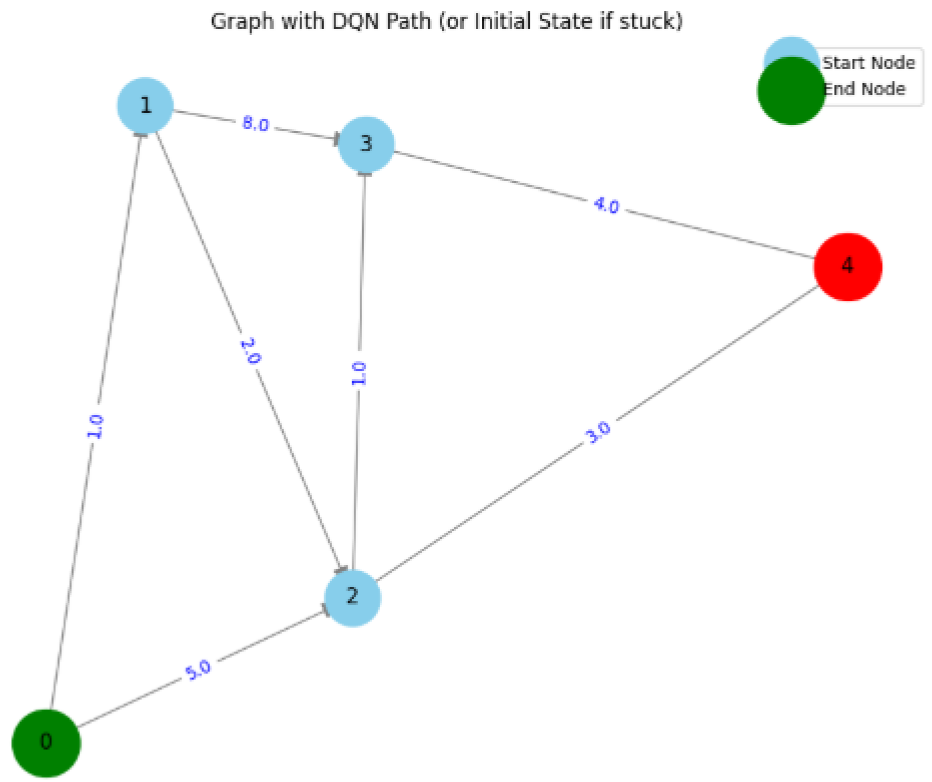
A replay buffer is used to store transitions in the form (state, action, reward, next_state, done). During training, batches of transitions are sampled from this buffer to update the network using the Bellman equation. A separate target network is maintained to stabilize training, and it is periodically updated using the weights of the policy network. The agent is trained for 2000 episodes using epsilon-greedy exploration, and after each step, an optimization function updates the network by minimizing the mean squared error between predicted and target Q-values.

At the end of training, the learned policy is evaluated by starting from the initial node and repeatedly selecting the action with the highest Q-value. The reward and path length for each episode are recorded and plotted to visualize training convergence. Although the DQN agent learns to improve its decisions, it occasionally selects invalid actions due to neural network approximation challenges. The final evaluation reports the best path found by the DQN, along with the corresponding total cost and total reward. The plots clearly show the learning progress in terms of rewards and path lengths as training progresses.

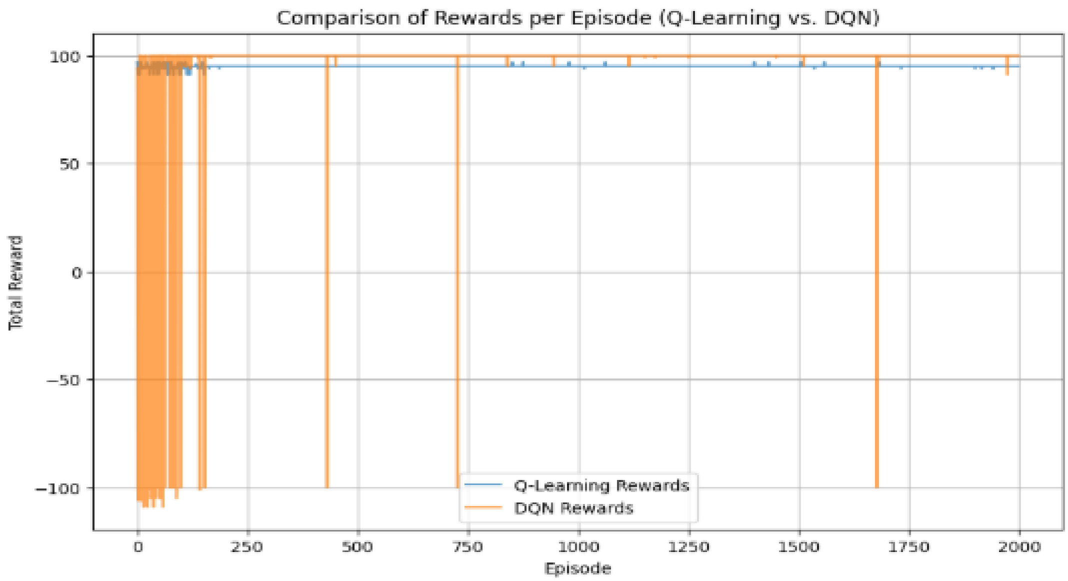
```python
import networkx as nx
import matplotlib.pyplot as plt

# 1. Visualize the input graph and the shortest path found by Q-learning

# Create a directed graph from the adjacency matrix
G = nx.DiGraph()
num_nodes = graph_matrix.shape[0]

for i in range(num_nodes):
    G.add_node(i)

for i in range(num_nodes):
    for j in range(num_nodes):
        weight = graph_matrix[i, j]
        if weight != 0 and weight != float('inf'): # Add edges only if there's a connection
            G.add_edge(i, j, weight=weight)


plt.figure(figsize=(10, 8))
pos = nx.spring_layout(G, seed=42) # For consistent layout

# Draw nodes
nx.draw_networkx_nodes(G, pos, node_color='skyblue', node_size=1000)

# Highlight start and end nodes
nx.draw_networkx_nodes(G, pos, nodelist=[start_node], node_color='green', node_size=1500, label='Start Node')
nx.draw_networkx_nodes(G, pos, nodelist=[end_node], node_color='red', node_size=1500, label='End Node')

# Draw edges
nx.draw_networkx_edges(G, pos, edge_color='gray', arrowsize=20)

# Draw node labels
nx.draw_networkx_labels(G, pos, font_size=12, font_color='black')

# Draw edge labels (weights)
edge_labels = nx.get_edge_attributes(G, 'weight')
nx.draw_networkx_edge_labels(G, pos, edge_labels=edge_labels, font_color='blue')
```

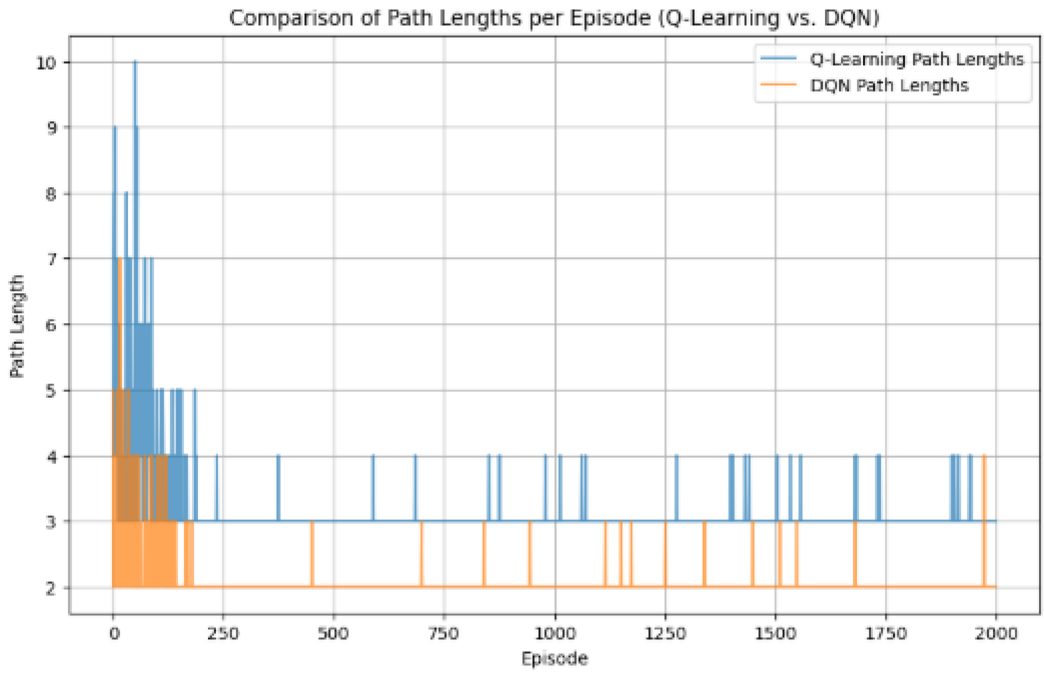
```python
# Highlight the Q-learning optimal path
q_path_edges = []
for i in range(len(optimal_path) - 1):
    q_path_edges.append((optimal_path[i], optimal_path[i+1]))

nx.draw_networkx_edges(G, pos, edgelist=q_path_edges, edge_color='magenta', width=2, label='Q-Learning Path')

plt.title('Graph with Q-Learning Optimal Path')
plt.legend(['Start Node', 'End Node', 'Q-Learning Path'])
plt.axis('off')
plt.show()


# 2. Visualize the graph and the path found by DQN agent
plt.figure(figsize=(10, 8))

# Draw nodes
nx.draw_networkx_nodes(G, pos, node_color='skyblue', node_size=1000)

# Highlight start and end nodes
nx.draw_networkx_nodes(G, pos, nodelist=[start_node], node_color='green', node_size=1500, label='Start Node')
nx.draw_networkx_nodes(G, pos, nodelist=[end_node], node_color='red', node_size=1500, label='End Node')

# Draw edges
nx.draw_networkx_edges(G, pos, edge_color='gray', arrowsize=20)

# Draw node labels
nx.draw_networkx_labels(G, pos, font_size=12, font_color='black')

# Draw edge labels (weights)
nx.draw_networkx_edge_labels(G, pos, edge_labels=edge_labels, font_color='blue')

# Highlight the DQN optimal path (or lack thereof)
dqn_path_edges = []
# Check if DQN found a path with more than one node
if len(optimal_path_dqn) > 1:
    for i in range(len(optimal_path_dqn) - 1):
        dqn_path_edges.append((optimal_path_dqn[i], optimal_path_dqn[i+1]))
    nx.draw_networkx_edges(G, pos, edgelist=dqn_path_edges, edge_color='orange', width=2, label='DQN Path')
elif optimal_path_dqn[0] == start_node and len(optimal_path_dqn) == 1:
    # If DQN got stuck at the start node, indicate this visually if possible
    # For simplicity, we just won't draw a path, and the title will reflect it
    pass # No path to draw

plt.title('Graph with DQN Path (or Initial State if stuck)')
plt.legend(['Start Node', 'End Node', 'DQN Path'] if len(optimal_path_dqn) > 1 else ['Start Node', 'End Node'])
plt.axis('off')
plt.show()
```


Graph with Q-Learning Optimal Path
Start Node
End Node
Q-Learning Path
1
3
4
2
0
8.0
4.0
2.0
1.0
1.0
3.0
5.0


Graph with DQN Path (or Initial State if stuck)
Start Node
End Node
1
3
4
2
0
8.0
4.0
2.0
1.0
1.0
3.0
5.0

```python
import matplotlib.pyplot as plt

# 6. Generate a single plot comparing the 'Rewards per Episode' for both Q-learning and DQN agents.
plt.figure(figsize=(10, 6))
plt.plot(rewards_per_episode, label='Q-Learning Rewards', alpha=0.7)
plt.plot(dqn_rewards_per_episode, label='DQN Rewards', alpha=0.7)
plt.title('Comparison of Rewards per Episode (Q-Learning vs. DQN)')
plt.xlabel('Episode')
plt.ylabel('Total Reward')
plt.legend()
plt.grid(True)
plt.show()

# 7. Generate a single plot comparing the 'Path Lengths per Episode' for both Q-learning and DQN agents.
plt.figure(figsize=(10, 6))
plt.plot(path_lengths_per_episode, label='Q-Learning Path Lengths', alpha=0.7)
plt.plot(dqn_path_lengths_per_episode, label='DQN Path Lengths', alpha=0.7)
plt.title('Comparison of Path Lengths per Episode (Q-Learning vs. DQN)')
plt.xlabel('Episode')
plt.ylabel('Path Length')
plt.legend()
plt.grid(True)
plt.show()
```





Graph Visualization and Comparison of Q-Learning vs. DQN

In this section, NetworkX and Matplotlib are used to visually analyze the shortest-path results obtained from both Q-Learning and DQN. The graph is first created from the adjacency matrix, where each node represents a graph state and each directed edge represents a valid movement along with its associated cost. Start and end nodes are highlighted in green and red respectively, while all other nodes are shown in sky-blue for clarity. Edge labels display the weights, making it easy to interpret the graph structure.

After plotting the base graph, the optimal path discovered by the Q-Learning agent is superimposed in magenta by connecting consecutive nodes in the learned path. A separate plot is then generated for the DQN agent, where the discovered path (if any) is shown in orange. If DQN fails to identify a complete path due to invalid actions, the visualization indicates that the agent remained stuck at the initial node.

Finally, two comparison plots are produced to evaluate the performance of Q-Learning and DQN across episodes. The first plot compares the total rewards, showing how Q-Learning stabilizes quickly while DQN exhibits more variability because of neural-network approximation. The second plot compares the path lengths, where Q-Learning consistently converges to the optimal length, while DQN sometimes produces shorter but invalid or inconsistent paths. These visualizations clearly highlight the difference in stability, learning behavior, and overall reliability of both RL and DRL methods in the shortest-path problem.

This assignment successfully demonstrates the application of both Reinforcement Learning (RL) and Deep Reinforcement Learning (DRL) techniques across multiple environments. Q-Learning was effectively used to solve the Mountain Car, Roulette, and Shortest Path problems by relying on tabular value updates and exploration strategies. The Deep Q-Network (DQN) approach extended these capabilities by using neural networks to approximate Q-values, enabling learning directly from continuous or high-dimensional state spaces. Through performance graphs, video visualizations, and path comparisons, the results clearly show that while Q-Learning is simple and stable for smaller discrete problems, DQN provides greater flexibility and scalability at the cost of higher complexity and occasional instability. Overall, the assignment highlights the strengths, limitations, and practical differences between RL and DRL methods, offering a complete understanding of how both approaches can be applied to solve diverse decision-making tasks.