Instructions

- Upload (to google classroom) all answers in the same document, but in serial order.
- Name the file in the following format: RollNo.png (or jpeg).
- It is recommended to write your name and roll number in the answer sheet.
- Usual rules of no-plagiarism apply. No consulting each other, no referring to the internet.

1. Given two min-heaps, suggest an algorithm for merging the two heaps into one, and analyze its time complexity. You can assume distinct all elements in both the min-heaps to be distinct. (5 marks)
2. What happens if Kruskal's algorithm runs on a disconnected graph? Does it still output MSTs for individual connected components? Prove or Disprove. (5 marks)
3. Argue that the number of connected components decreases by at most one at every step of Kruskal's algorithm. (5 marks)
4. Suppose the BFS algorithm starting from a vertex u in an unweighted graph reaches the vertex v before it reaches the vertex w. What can you say about the (shortest) distance from u to v and u to w, where distance between two nodes is the number of simple edges on the path connecting them. Justify your answer with a proof. [Hint: Think of the distance from u to the first vertex in the queue] (10 marks)
5. Consider a directed graph $G = (V, E)$ where the nodes represent activities that need to be done to prepare a dish. There is an edge from u to v iff v has to be done after u. Write an algorithm using queues and arrays, to construct a schedule for making the dish: that is, a sequence $\langle u_1, u_2, \ldots, u_n \rangle$ such that for every directed edge (u_i, u_j), u_i appears before u_j in the sequence. (15 marks)