CS:1010 DISCRETE STRUCTURES

PRACTICE QUESTIONS
LECTURE 11

Instructions

- Try these questions before class. Do not submit!

(1) How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 21,$$

where $x_i, i = 1, 2, 3, 4, 5$ is a nonnegative integer s.t.
(a) $x_1 \geq 1$?
(b) $x_i \geq 2$, for $i = 1, 2, 3, 4, 5$?
(c) $0 \leq x_1 \leq 10$?
(d) $0 \leq x_1 \leq 3$, $1 \leq x_2 < 4$ and $x_3 \geq 15$?

(2) How many solutions are there to the inequality $x_1+x_2+x_3 \leq 11$, where x_1, x_2, and x_3 are nonnegative integers?[Hint: Introduce an auxiliary variable x_4 s.t. $x_1 + x_2 + x_3 + x_4 = 11$.]

(3) How many different bit strings can be transmitted if the string must begin with a 1 bit, must include three additional 1 bits (so that a total of four 1 bits is sent), must include a total of 12 0 bits, and must have at least two 0 bits following each 1 bit?

(4) Solve these recurrences with the initial conditions given.
(a) $a_n = -4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, $a_0 = 0, a_1 = 1$
(b) $a_n = a_{n-2}/4$ for $n \geq 2$, $a_0 = 1, a_1 = 0$

(5) In how many ways can a $2 \times n$ rectangular checkerboard be tiled using 1×2 and 2×2 pieces? *A checkerboard is a board of chequered pattern with alternating dark and light color, typically black and white.*

(6) Find the solution to $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ with $a_0 = 7, a_1 = -4$ and $a_2 = 8$.

(7) Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 2n^2$. Find the solution with initial condition $a_1 = 4$.

(8) Find all solutions of the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$.

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