CS:1010 DISCRETE STRUCTURES

PRACTICE QUESTIONS
LECTURE 13,14,15

Instructions

- Try these questions before class. Do not submit!

(1) Which of these collections of subsets are partitions of the set of integers?
   (a) the set of even integers and the set of odd integers
   (b) the set of positive integers and the set of negative integers
   (c) the set of integers not divisible by 3, the set of even integers and the set of integers that leave a remainder of 3 when divided by 6

(2) For the given set and relations below, determine which define equivalence relations.
   (a) S is the set of all people in the world today, a is related to b if a and b have an ancestor in common.
   (b) S is the set of all people in the world today, a is related to b if a and b have the same father.
   (c) S is the set of real numbers a is related to b if $a = \pm b$.
   (d) S is the set of all straight lines in the plane, a is related to b if a is parallel to b.

(3) If G is a group of even order, prove that it has an element $a \neq e$, where e is the identity element satisfying $a^2 = e$, i.e. a is its own inverse.

(4) Show that the complete graph of n vertices K_n has $n(n-1)/2$ edges.

(5) Show that the number of edges in $K_{m,n}$ is mn.

(6) Show that every regular bipartite graph has a perfect matching.

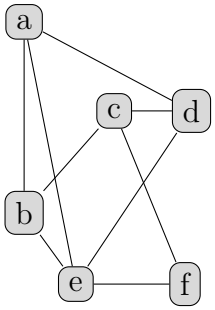
(7) Every simple graph has a bipartite subgraph with at least $|E|/2$ edges.

(8) Prove that for a bipartite graph G on n vertices the number of edges in G is at most $\frac{n^2}{4}$.

(9) Show that for all graphs $\kappa(G) \leq \lambda(G) \leq \min_{v \in V} \deg(v)$.

(10) Show that the existence of a simple circuit of a particular length is a graph invariant.

(11) Count the number of paths between c and d in the graph below of length 2 and 3:



(12) Show that K_n has a Hamilton circuit whenever $n \geq 3$.

(13) If G is a connected planar simple graph then G has a vertex of degree not exceeding 5.

(14) A graph with at least 3 vertices is 2-connected iff every pair of vertices lie in a cycle.

(15) If G_1 and G_2 are two connected subgraphs of G having at least one vertex in common then $G_1 \cup G_2$ is connected.

(16) The complementary graph $\hat{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\hat{G}$ if and only if they are not adjacent in G. If a graph G is not connected, prove that its complement graph is connected.

(17) Show that the property that a graph is bipartite is an isomorphic invariant.

(18) How many distinct Hamiltonian cycles are there in a complete graph $K_n, n \geq 3$?

(19) What is the height of a full and balanced 7-ary tree with 340 leaves?

(20) Consider a simple graph G.
   (a) If G has k connected components and each of these components have $n_1, n_2, \ldots, n_k$ vertices respectively, then the number of edges of G does not exceed $\sum_{i=1}^{k} C(n_i, 2)$.
   (b) Use the previous result to show that a simple graph with n vertices and k connected components has at most $\frac{(n-k)(n-k+1)}{2}$ edges.
   (c) Use the previous result to show that a simple graph with n vertices is connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges.

(21) Ore's theorem : If G is a simple graph with n vertices $n \geq 3$, s.t. $deg(u) + deg(v) \geq n$ for every pair of nonadjacent vertices u and v in G then G has a Hamilton circuit.

(22) Design recursive algorithms for preorder, inorder, postorder traversals.

(23) What does the inorder traversal of a BST give rise to?

(24) Let x and y be two nodes of a binary tree B. Prove that x is an ancestor of y iff x stands before y in the pre-order traversal of B and x stands after y in the post-order traversal of B.