04/04/2022

$$c_j = \sum_{0 \le k \le 2n-1} \left[\frac{1}{2n} \cdot R(\omega_{2n}^{k})\right] \cdot \left(\omega_{2n}^{-j}\right)^{k}$$

$$R'(x) = \sum_{0 \le k \le 2n-1} \left[\frac{1}{2n} R(\omega_{2n}^{k})\right] x^{k}$$

$$c_j = R'(\omega_{2n}^{-j}) \qquad 0 \le j \le 2n-1$$

Another way:

$$R''(x) = \sum_{k=0}^{2n-1} R(\omega_{2n}^{k}) x^{k}$$

$$R''(\omega_{2n}^{j}) = \sum_{k=0}^{2n-1} R(\omega_{2n}^{k}) (\omega_{2n}^{j})^{k}$$

$$= \sum_{k=0}^{2n-1} \left(\sum_{i=0}^{2n-1} c_i (\omega_{2n}^{k})^{i}\right) (\omega_{2n}^{j})^{k}$$

$$= \sum_{i=0}^{2n-1} c_i \sum_{k=0}^{2n-1} \left(\omega_{2n}^{k}\right)^{i} \cdot \left(\omega_{2n}^{j}\right)^{k}$$

$$= \sum_{i=0}^{2n-1} c_i \underbrace{\sum_{k=0}^{2n-1} \left(\omega_{2n}^{(i+j)}\right)^{k}}$$

$$\underset{1}{\overset{\parallel}{\omega^0}}, \underbrace{\omega^1, \ldots, \omega^{2n-1}}_{\neq 1}$$

if $i+j=0$ then $\displaystyle\sum_{k=0}^{2n-1} \left(\omega_{2n}^{i+j}\right)^{k}$

$= 2n$

if $i+j \neq 0$ and $< 2n$

then $\displaystyle\sum_{k=0}^{2n-1} \left(\omega_{2n}^{i+j}\right)^{k} = 0$

More generally,

if $i+j \not\equiv 0 \pmod{2n}$ then

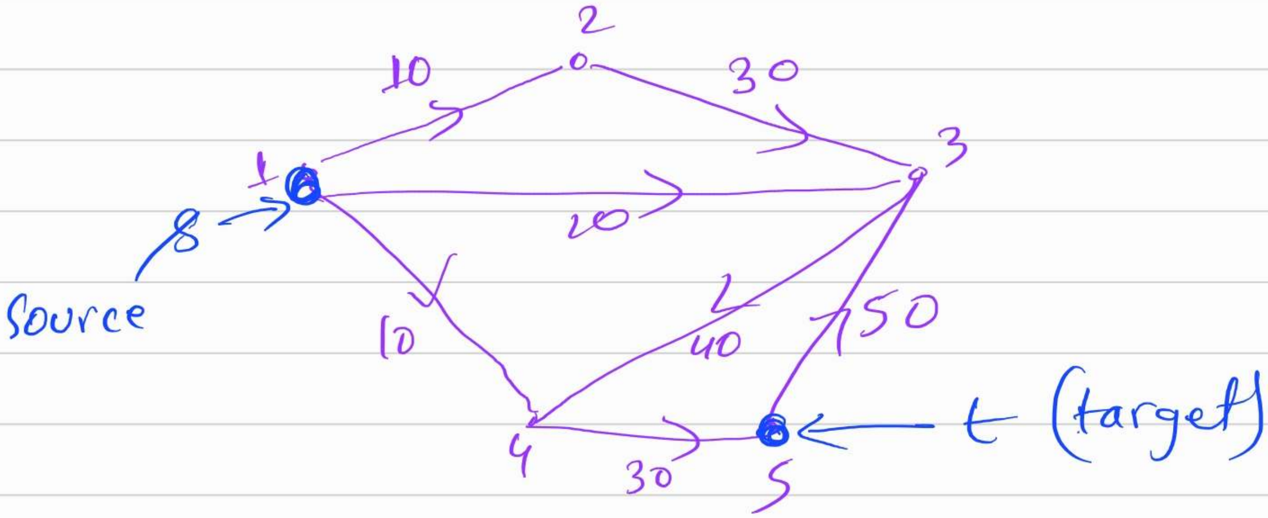
$$\sum_{k=0}^{2n-1} \left(\omega_{2n}^{i+j}\right)^{k} = 0.$$

if $i = 2n - j$ then $i + j \equiv 0 \pmod{2n}$

$$R''(\omega_{2n}^{j}) = c_i \cdot 2n$$

Network flows

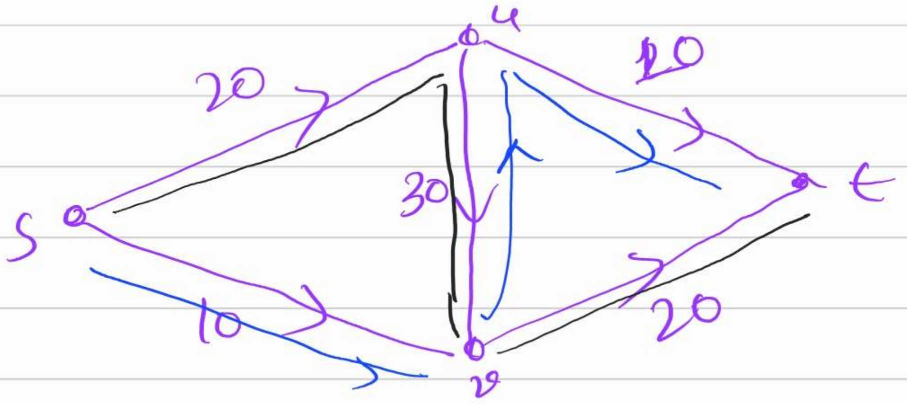
(Max-flow & Min-Cut)



More formally, Given a directed graph two designated vertices s and t.

- every edge, e, has a capacity.

$$c(e) \in \mathbb{Z}^{\geq 0}$$



$$s \xrightarrow{20} u \xrightarrow{30} v \xrightarrow{20} t$$

min over all edges in the path.

Defn:- Residual Graph wrt to a flow f. Let us denote it by G_f.

(1) vertices of G_f are same as vertices of G.

(2) Consider an edge $e = (u, v)$ in G.

$$u \to v$$

s.t. $0 \leq f(e) < C(e)$

then add edge $e = (u, v)$ in G_f

with residual capacity $c(e) - f(e)$

we will call them "Forward" edges.

(3) Consider an edge $e = (u, v)$ in G.

$u \longrightarrow v$

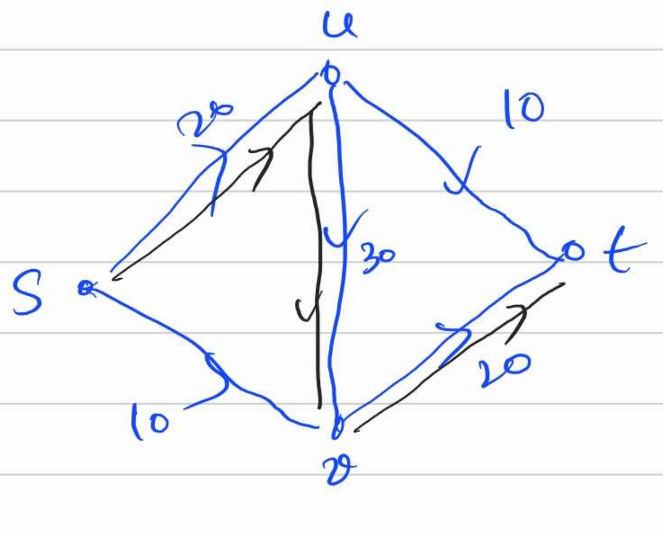
s.t. $0 < f(e) \leq c(e)$

then add edge.

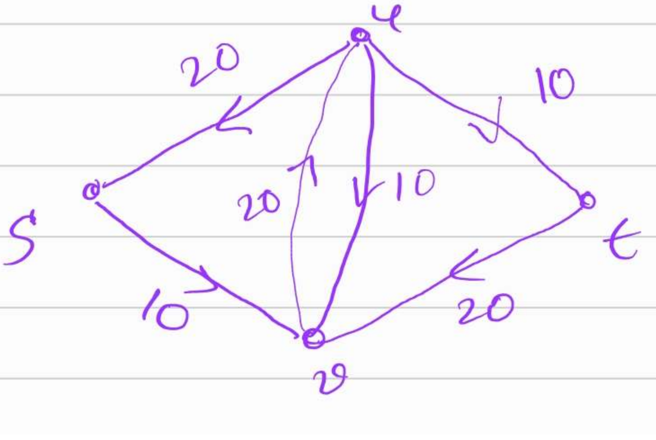
$e' = (v, u)$ in G_f

with residual capacity $f(e)$.

We call such edges "backward" edges.



Residual graph w.r.t f.



Find an $s \rightsquigarrow t$ path P in the residual graph.

$$s \xrightarrow{10} v \xrightarrow{20} u \xrightarrow{10} t$$

$\text{bottleneck}(P, f)$ = min residual capacity on P.

$$= \min_{e \in P} c(e)$$

We now construct a new flow f' s.t. value of f' is more than f.

<u>Augment(P, f)</u>

1) Let P be a $s \rightsquigarrow t$ path in G_f.

2) $b := \text{bottleneck}(P, f)$

3) For every edge e on the path P.

if $e = (u, v)$ is a forward edge

then $f'(e) = f(e) + b$

Else $e = (u, v)$ is a backward edge in G_f.

then (v, u) is an edge in G.

$f'(v \to u) = f(v \to u) - b$

End for

Return (f')