Assignment 3

Suraj - CS20BTECH11050

Download all python codes from

https://github.com/Suraj11050/Assignments-AI1103/blob/main/Assignment%203/Assignment3.py

Download Latex-tikz codes from

https://github.com/Suraj11050/Assignments-AI1103/blob/main/Assignment%203/Assignment3.tex

1 GATE 2009 (MA) PROBLEM 16

Let F, G and H be pair wise independent events such that $\Pr(F) = \Pr(G) = \Pr(H) = \frac{1}{3}$ and $\Pr(F \cap G \cap H) = \frac{1}{4}$ Then the probability that at least one event among F, G and H occurs is

(A) $\frac{11}{12}$ (B) $\frac{7}{12}$ (C) $\frac{5}{12}$ (D) $\frac{3}{4}$

2 SOLUTION

If two Events A and B are independent then

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B) \tag{2.0.1}$$

F, G, H are pair wise independent events From Equation (2.0.1) we get the following results

$$\Pr(F \cap G) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$
$$\Pr(G \cap H) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$
$$\Pr(H \cap F) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

At least one event among F, G and H occur is $\Pr(F \cup G \cup H)$. From Principal of inclusion and exclusion $\Pr(F \cup G \cup H)$ is calculated as

$$\begin{aligned}\Pr(F \cup G \cup H) &= (\Pr(F) + \Pr(G) + \Pr(H)) \\ &\quad - \Pr(F \cap G) - \Pr(G \cap H) \\ &\quad - \Pr(H \cap F) + \Pr(F \cap G \cap H)\end{aligned}$$

$$\Pr(F \cup G \cup H) = 3 \times \left(\frac{1}{3}\right) - 3 \times \left(\frac{1}{9}\right) + \frac{1}{4}$$
$$\therefore \Pr(F \cup G \cup H) = \frac{11}{12}$$

Hence Probability that at least one event among F, G, H occurs is $\Pr(F \cup G \cup H) = \frac{11}{12}$ and correct answer is **Option (A)**

But we know that

$$(F \cap G \cap H) \subseteq (F \cap G)$$
$$\therefore \Pr(F \cap G \cap H) \leq \Pr(F \cap G)$$

In the given question

$$\Pr(F \cap G \cap H) = \frac{1}{4}$$
$$\Pr(F \cap G) = \frac{1}{9}$$
$$\Pr(F \cap G \cap H) > \Pr(F \cap G)$$

Which is not possible and some of the probabilities turnout to be negative like

$$\Pr(F \cap G \cap H^{c}) = \frac{1}{9} - \frac{1}{4} = -\left(\frac{5}{36}\right)$$
$$\Pr(F \cap G^{c} \cap H) = \frac{1}{9} - \frac{1}{4} = -\left(\frac{5}{36}\right)$$
$$\Pr(F^{c} \cap G \cap H) = \frac{1}{9} - \frac{1}{4} = -\left(\frac{5}{36}\right)$$

Similar case with $\Pr(G \cap H)$ and $\Pr(H \cap F)$
Therefore **Question is incorrect**