Assignment 2

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and latex-tikz codes from

https://github.com/Suraj11050/Assignments–AI1103/blob/main/Assignment%202/Assignment2.tex

1 GATE PROBLEM 78

The joint probability density function of two random variables X and Y is given as

$$f(x,y) = \begin{cases} \frac{6}{5}\left(x+y^2\right) & 0 \le x \le 1 \;\; 0 \le y \le 1 \\ \\ 0 & \text{elsewhere} \end{cases}$$

$E(X)$ and $E(Y)$ are, Respectively

a) $\frac{2}{5}$ and $\frac{3}{5}$

b) $\frac{3}{5}$ and $\frac{3}{5}$

c) $\frac{3}{5}$ and $\frac{6}{5}$

d) $\frac{4}{5}$ and $\frac{6}{5}$

2 SOLUTION

For a continuous joint probability distribution $E(X)$ and $E(Y)$ are obtained using the following equations (2.0.1) and (2.0.2)

$$E(X) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x \cdot f(x,y)\; dx\, dy \tag{2.0.1}$$

$$E(Y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} y \cdot f(x,y)\; dx\, dy \tag{2.0.2}$$

Using equation (2.0.1) $E(X)$ is calculated as

$$\begin{aligned} E(X) &= \int_0^1\int_0^1 x\,\frac{6}{5}\left(x+y^2\right) dx\,dy \;\; + 0 \\ &= \int_0^1 \frac{6}{5}\left(\int_0^1 x^2\,dx\right) + \frac{6}{5}y^2\left(\int_0^1 x\,dx\right) dy \\ &= \int_0^1 \frac{6}{5}\left(\frac{1}{3}\right) + \frac{6}{5}y^2\left(\frac{1}{2}\right) dy \\ &= \frac{2}{5}\int_0^1 dy + \frac{3}{5}\int_0^1 y^2\,dy \\ &= \frac{2}{5} + \frac{3}{5}\left(\frac{1}{3}\right) \\ E(X) &= \frac{3}{5} \end{aligned}$$

Using equation (2.0.2) $E(Y)$ is calculated as

$$\begin{aligned} E(Y) &= \int_0^1\int_0^1 y\,\frac{6}{5}\left(x+y^2\right) dx\,dy \;\; + 0 \\ &= \int_0^1 \frac{6}{5}x\left(\int_0^1 y\,dy\right) + \frac{6}{5}\left(\int_0^1 y^3\,dy\right) dx \\ &= \int_0^1 \frac{6}{5}x\left(\frac{1}{2}\right) + \frac{6}{5}\left(\frac{1}{4}\right) dx \\ &= \frac{3}{5}\int_0^1 x\,dx \; + \; \frac{3}{10}\int_0^1 dx \\ &= \frac{3}{5}\left(\frac{1}{2}\right) + \frac{3}{10} \\ E(Y) &= \frac{3}{5} \end{aligned}$$

$$\therefore E(X) = \frac{3}{5} \text{ and } E(Y) = \frac{3}{5}$$

Hence the answer is **option b**