

# Assignment 2

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and latex-tikz codes from

<https://github.com/Suraj11050/Assignments-AI1103/blob/main/Assignment%202/Assignment2.tex>

## 1 GATE PROBLEM 78

The joint probability density function of two random variables X and Y is given as

$$f(x, y) = \begin{cases} \frac{6}{5}(x + y^2) & 0 \leq x \leq 1 \quad 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

- |                                    |                                    |
|------------------------------------|------------------------------------|
| a) $\frac{2}{5}$ and $\frac{3}{5}$ | b) $\frac{3}{5}$ and $\frac{3}{5}$ |
| c) $\frac{3}{5}$ and $\frac{6}{5}$ | d) $\frac{4}{5}$ and $\frac{6}{5}$ |

## 2 SOLUTION

For a continuous joint probability distribution  $E(X)$  and  $E(Y)$  are obtained using the following equations (2.0.1) and (2.0.2)

$$E(X) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x \cdot f(x, y) \, dx \, dy \quad (2.0.1)$$

$$E(Y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} y \cdot f(x, y) \, dx \, dy \quad (2.0.2)$$

Using equation (2.0.1)  $E(X)$  is calculated as

$$\begin{aligned} E(X) &= \int_0^1 \int_0^1 x \frac{6}{5} (x + y^2) \, dx \, dy + 0 \\ &= \int_0^1 \frac{6}{5} \left( \int_0^1 x^2 \, dx \right) + \frac{6}{5} y^2 \left( \int_0^1 x \, dx \right) \, dy \\ &= \int_0^1 \frac{6}{5} \left( \frac{1}{3} \right) + \frac{6}{5} y^2 \left( \frac{1}{2} \right) \, dy \\ &= \frac{2}{5} \int_0^1 dy + \frac{3}{5} \int_0^1 y^2 \, dy \\ &= \frac{2}{5} + \frac{3}{5} \left( \frac{1}{3} \right) \\ E(X) &= \frac{3}{5} \end{aligned}$$

Using equation (2.0.2)  $E(Y)$  is calculated as

$$\begin{aligned} E(Y) &= \int_0^1 \int_0^1 y \frac{6}{5} (x + y^2) \, dx \, dy + 0 \\ &= \int_0^1 \frac{6}{5} x \left( \int_0^1 y \, dy \right) + \frac{6}{5} \left( \int_0^1 y^3 \, dy \right) \, dx \\ &= \int_0^1 \frac{6}{5} x \left( \frac{1}{2} \right) + \frac{6}{5} \left( \frac{1}{4} \right) \, dx \\ &= \frac{3}{5} \int_0^1 x \, dx + \frac{3}{10} \int_0^1 dx \\ &= \frac{3}{5} \left( \frac{1}{2} \right) + \frac{3}{10} \\ E(Y) &= \frac{3}{5} \end{aligned}$$

$$\therefore E(X) = \frac{3}{5} \text{ and } E(Y) = \frac{3}{5}$$

Hence the answer is **option b**