1
Sol Quick sort algorithm using pivot as median
we make calls for median in algorithm

Let number of calls for input n = $T(n)$

After calling median we divide problem into 2 subproblems
we get $T(n) = 2 \cdot T(\frac{n}{2}) + 1$ $T(1) = 0$

Solving the recurrance relation

$T(n) = 2 \cdot T(\frac{n}{2}) + 1$

$n = 2^m$ $m = \log_2 n$ (domain transformation)

$T(2^m) = 2 \cdot T(2^{m-1}) + 1$

$\frac{T(2^m)}{2^m} = \frac{T(2^{m-1})}{2^{m-1}} + \frac{1}{2^m}$ (range transformation)

$s(m) = \frac{T(2^m)}{2^m} \Rightarrow s(m) = s(m-1) + \frac{1}{2^m}$ $s(0) = 0$

$s(m) = \sum_{i=1}^{m} \frac{1}{2^i} = 1 - \frac{1}{2^m}$

$T(2^m) = 2^m - 1$

$\therefore T(n) = n - 1$

$\therefore$ $O(n-1)$ calls are made to find median

$\underline{\underline{\overset{2}{SOI}}}$ $T(n) = n^{1/3} \cdot T(n^{1/3}) + 1$

$\log_3 n = k \Rightarrow n = 3^k \quad n^{1/3} = 3^{k/3}$ (domain transformation)

$T(3^k) = 3^{k/3}\, T(3^{k/3}) + 1$

let $T(3^k) = S(m) \Rightarrow T(3^{k/3}) = S\left(\frac{m}{3}\right)$

$S(m) = \frac{m}{3} \cdot S\left(\frac{m}{3}\right) + 1$

$S(m) = \left(\frac{m}{3} \cdot \frac{m}{9} \cdot \log_3 m\right) S(1) + \left(1 + \frac{m}{3} + \left(\frac{m}{9}\right)^2 \cdots\right)$

$S(m) = \frac{m^{\log_3 m}}{3^{(1+2 \cdots \log_3 m)}} S(1) + \frac{(m/3)^{\log_3 m} - 1}{1 - m/3}$

$S(m) \leq \frac{m^{\log_3 m}}{3^{\frac{\log_3 m(\log_3 m + 1)}{2}}} + O\left(m^{\log_3 m - 1}\right)$

$S(m) \leq \frac{3^{k^2}}{3^{\left(\frac{k^2+k}{2}\right)}} + O\left(3^{k^2 - k}\right)$

$S(m) \leq O\left(3^{k^2-k}\right) + O\left(3^{\frac{k^2-k}{2}}\right)$

$S(m) \leq O\left(\frac{3^{\log_3 n} \cdot 3^{\log_3 n} \cdots \log_3 n \text{ times}}{n}\right)$

$S(m) \leq O\left(\frac{n^{\log_3 n}}{n}\right)$

$\therefore \; T(n) \in O\left(n^{\log_3 n - 1}\right)$

3
Sol Given, Size of each block = 13

no of groups divided = $\lceil n/13 \rceil$

To find the median of each group ⇒ constant time

for $\lceil n/13 \rceil$ groups = $O(n)$

elements smaller than median ≥ 7 $(\lceil 13/2 \rceil)$ $\left(\left\lfloor \frac{1}{2} \left\lceil \frac{n}{13} \right\rceil \right\rfloor - 3\right) \geq \frac{7n}{26} - 21$

elements larger than median $\geq 7 \left(\left\lfloor \frac{1}{2} \left\lceil \frac{n}{13} \right\rceil \right\rfloor - 4\right) \geq \frac{7n}{26} - 28$

Assume rank of median r, k - searching element

If $k < r$ we recurse through all elements except elements larger than median

If $k > r$ we recurse through all elements except elements smaller than median

$$T(n) \leq T\left(\frac{n}{13}\right) + T\left(n - \frac{7n}{26}\right) + O(n)$$

$$\Rightarrow T(n) \leq T\left(\frac{n}{13}\right) + T\left(\frac{19n}{26}\right) + O(n)$$

4
Sol Input: $A[1 \ldots m]$ $B[1 \ldots n]$

Each element from $\{0, 1, 2\}$

We need to edit A and B using addition deletion or insertion with minimum cost

Given, cost of addition/deletion $= 0.75$
cost of edit from x to y $= 0.5 \cdot |x - y|$

We need to modify the orginal EDIT(i,j) recursive function by adding costs as per required operations

$$
EDIT(i,j) = \begin{cases} (0.75)\, i & \text{if } j=0 \text{ (add/delete)} \\ (0.75)\, j & \text{if } i=0 \text{ (add/delete)} \\ \min \begin{cases} EDIT(i-1, j) + 0.75 & \text{(delete)} \\ Edit(i, j-1) + 0.75 & \text{(addition)} \\ EDIT(i-1, j-1) + 0.5\, |A[i] - B[j]| & \text{(edit)} \end{cases} \end{cases}
$$

Since we added constants it does not effect the runtime and Hence run time will be polynomial $(O(mn))$