

Assignment 3

Suraj - CS20BTECH11050

Download all python codes from

<https://github.com/Suraj11050/Assignments-AI1103/blob/main/Assignment%203/Assignment3.py>

Download Latex-tikz codes from

<https://github.com/Suraj11050/Assignments-AI1103/blob/main/Assignment%203/Assignment3.tex>

Using equation (2.0.1) we get the following results

$$\Pr(f = 1, g = 1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$$\Pr(g = 1, h = 1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$$\Pr(f = 1, h = 1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

At least one event among F, G, H should occur is $\Pr(F + G + H)$ from Principle of inclusion and exclusion it is calculated using random variable as

1 GATE 2009 (MA) PROBLEM 16

Let F, G and H be pair wise independent events such that $\Pr(F) = \Pr(G) = \Pr(H) = \frac{1}{3}$ and $\Pr(F \cap G \cap H) = \frac{1}{4}$ Then the probability that at least one event among F, G and H occurs is

- (A) $\frac{11}{12}$ (B) $\frac{7}{12}$ (C) $\frac{5}{12}$ (D) $\frac{3}{4}$

2 SOLUTION

Let f, g, h be three random variables taking values 0 or 1 (Bernoulli random variable) Which represent the occurrence of event F, G, H respectively such that

$$\Pr(f = 0) = \frac{2}{3} \quad \Pr(f = 1) = \frac{1}{3}$$

$$\Pr(g = 0) = \frac{2}{3} \quad \Pr(g = 1) = \frac{1}{3}$$

$$\Pr(h = 0) = \frac{2}{3} \quad \Pr(h = 1) = \frac{1}{3}$$

If two Random variables X_1 and X_2 are independent then

$$\Pr(X_1 X_2) = \Pr(X_1) \times \Pr(X_2) \quad (2.0.1)$$

$$\Pr(f = 1 + g = 1 + h = 1) =$$

$$(\Pr(f = 1) + \Pr(g = 1) + \Pr(h = 1))$$

$$- \Pr(f = 1, g = 1) - \Pr(g = 1, h = 1)$$

$$- \Pr(h = 1, f = 1) + \Pr(f = 1, g = 1, h = 1)$$

$$\Pr(f = 1 + g = 1 + h = 1) = 3\left(\frac{1}{3}\right) - 3\left(\frac{1}{9}\right) + \frac{1}{4}$$

$$\therefore \Pr(f = 1 + g = 1 + h = 1) = \frac{11}{12}$$

Hence Probability that at least one event among F, G, H occurs is $\Pr(F + G + H) = \frac{11}{12}$ and correct answer is **Option (A)**

But we know that

$$(F \cap G \cap H) \subseteq (F \cap G)$$

$$\therefore \Pr(FGH) \leq \Pr(FG)$$

In the given question

$$\Pr(FGH) = \frac{1}{4}$$

$$\Pr(FG) = \frac{1}{9}$$

$$\Pr(FGH) > \Pr(FG)$$

Which is not possible

Some of the probabilities turnout to be negative like

$$\Pr(f = 1, g = 1, h = 0) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36}$$

$$\Pr(f = 1, g = 0, h = 1) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36}$$

$$\Pr(f = 0, g = 1, h = 1) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36}$$

$$\Pr(F^cGH) = -\frac{5}{36}$$

$$\Pr(FG^cH) = -\frac{5}{36}$$

$$\Pr(F^cGH) = -\frac{5}{36}$$

Probability $P \in [0, 1]$ but the data in the question
some of the probabilities turn out to be negative

Similar case with $\Pr(GH)$ and $\Pr(HF)$

Therefore **Question is incorrect**