CS1010 DISCRETE STRUCTURES ASSIGNMENT 1

Name: SURAJ TELUGU **Roll NO: CS20BTECH11050**

1.sol Another example of a result that can neither be proved nor disproved by ZFC Axioms

Hilbert's tenth problem

In Number Theory Diophantine Equations are those polynomials with integer coefficients with finite number of unknowns

Hilbert's tenth problem is "Finding an Algorithm (A standard procedure) to determine whether the given Diophantine Equation has integral roots or not"

For example $P(x , y) = 3x^2 + 2y^2$ for his Diophantine Equation we need to find

$$P(k_1 , k_2) = 3(k_1)^2 + 2(k_2)^2 = 0 \text{ such that } k_1 , k_2 \in \mathbb{Z}$$

We know that no solution exist for $P(x , y) = 0$ as always $x^2 \geq 0$

But for problem like $P(x , y , z) = 3x^2 - 2xy - y^2z - 7 = 0$ to find solution we have to verify many cases to find a perfect solution like <u>$x = 1, y = 2, z = -2$</u> and conclude equation has a solution. We cannot prove whether this problem has integral solutions or not using ZFC'S and Axiom of choices. We cannot make any algorithm which verifies the Diophantine equation for all integers and finds integral solution exists or not. It would be like a forever loop and it cannot verify for all values. Therefore, we cannot say whether Hilbert's tenth problem can be solved or no using ZFC's.

<u>Hence, Hilbert's tenth problem in independent of ZFC'S i.e., ZFC'S won't help in solving the problem</u>

Later using some other new axioms from Axiomatic set theory **Yuri Matiyasevich** has concluded Hilbert's tenth problem cannot be solved i.e., No algorithm can be made which efficient solves and decides the possibility of solution for a Diophantine Equation.

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2.sol let $P(n) = (n^2 + n + 2)/2$ we need to prove $\forall n \in N\ P(n)$

Basis Step:

$n = 1$

$P(1) = (1^2 + 1 + 2)/2 = 2$

One Line separates the Plane into 2 regions above the line and below the line.

∴ P(1) True

Inductive Step:

Let us assume P(k) is True (Induction Hypothesis)

$P(k) = (k^2 + k + 2)/2$

k Lines separate the Plane into $(k^2 + k + 2)/2$ regions

We have to prove p(k+1) is True

We know that no two lines are parallel and no three lines are concurrent

Now the new $(k+1)^{th}$ line we introduce intersects with all other k lines and do not pass through intersection point of any other lines as no three lines are concurrent

Thus the $(k+1)^{th}$ line passes through (k+1) regions from the $(k^2 + k + 2)/2$ regions

$(k+1)^{th}$ line divides each of k+1 region into 2 regions

∴ k+1 new regions added

Total regions after $(k+1)^{th}$ line added $= (k^2 + k + 2)/2 + (k+1)$

$$= (k^2 + k + 2 + 2(k+1))/2$$

$$= ((k^2+2k+1) + (k+1) + 2)/2$$

$$= ((k+1)^2 + (k+1) + 2)/2 = P(k+1)$$

∴ p(k+1) True

By Principle of Mathematical Induction $\forall\ n \in N\ P(n)$ is True

∴ n lines separate a plane into $(n\ 2+n+2)/2$ regions if no two lines are parallel and no three pass

through a common point is proved by Mathematical Induction

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3.Sol Consider the following predicates for checking the validity of argument:

1. $P(x)$: x is able to prevent evil
2. $Q(x)$: x is willing to prevent evil
3. $R(x)$: x prevents evil
4. $S(x)$: x is impotent
5. $T(x)$: x is malevolent
6. $U(x)$: x exists

let us assume Superman = s, now for S following arguments are assumed:

1. $(P(s) \wedge Q(s)) \rightarrow R(s)$ (If Superman were able and willing to prevent evil, he would do so)
2. $(\neg P(s)) \rightarrow S(s)$ (If Superman were unable to prevent evil, he would be impotent)
3. $(\neg Q(s)) \rightarrow T(s)$ (If he were unwilling to prevent evil, he would be malevolent)
4. $(\neg R(s))$ (Superman does not prevent evil)
5. $U(s) \rightarrow (\neg S(s) \wedge \neg T(s)$ (If Superman exists, he is neither impotent nor malevolent)

Solving for validity of argument:

Using **Modus Tollens** for 1 and 4: $\neg(P(s) \wedge Q(s)) \equiv (\neg P(s)) \vee (\neg Q(s))$ (argument 6)

$(\neg P(s)) \rightarrow S(s) \equiv P(s) \vee S(s)$ (argument 7) $(\neg Q(s)) \rightarrow T(s) \equiv Q(s) \vee T(s)$ (argument 8)

Using **Resolution** for 6 and 7: $(\neg Q(s)) \vee S(s)$ (argument 9)

Using **Resolution** for 8 and 9: $T(s) \vee S(s)$ (argument 10)

Argument 10 also equivalent to: $T(s) \vee S(s) \equiv \neg(\neg T(s) \vee \neg S(s))$ (argument 11)

Using **Modus Tollens** for 11 and 5: $\therefore\ \neg U(s)$

Hence $(\neg U(s))$ Super man doesn't exist is proved

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4.Sol $(t \rightarrow (r \vee p)) \rightarrow ((\neg r \vee k) \wedge \neg k) \equiv \neg(t \rightarrow (r \vee p)) \vee ((\neg r \vee k) \wedge \neg k)$ **($a \rightarrow b \equiv \neg a \vee b$)**

$\neg(\neg t \vee (r \vee p)) \vee ((\neg r \wedge \neg k) \vee (k \wedge \neg k))$ **($a \rightarrow b \equiv \neg a \vee b$) (Using distributive law)**

$(\neg(\neg t) \wedge (\neg r \wedge \neg p)) \vee ((\neg r \wedge \neg k) \vee F)$ **(Using Negation law)**

$(t \wedge (\neg r \wedge \neg p)) \vee (\neg r \wedge \neg k)$ **(Using Double Negation law) (Using Identity law)**

$(t \wedge (\neg p \wedge \neg r)) \vee (\neg k \wedge \neg r)$

$((t \wedge \neg p) \wedge \neg r) \vee (\neg k \wedge \neg r)$ **(Using Associative law)**

$((t \wedge \neg p) \vee \neg k) \wedge \neg r$ **(Using Distributive law)**

$\therefore\ (t \rightarrow (r \vee p)) \rightarrow ((\neg r \vee k) \wedge \neg k) \equiv ((t \wedge \neg p) \vee \neg k) \wedge \neg r$

Now to prove given Compound Preposition implies $\neg r$ it is enough to prove the proposition

$\underline{(((t \wedge \neg p) \vee \neg k) \wedge \neg r) \rightarrow \neg r}$ is a Tautology

$(((t \wedge \neg p) \vee \neg k) \wedge \neg r) \rightarrow \neg r \equiv \neg(((t \wedge \neg p) \vee \neg k) \wedge \neg r) \vee \neg r$ **($a \rightarrow b \equiv \neg a \vee b$)**

$(((\neg t \vee \neg(\neg p)) \wedge \neg(\neg k)) \vee \neg(\neg r)) \vee \neg r$

$(((\neg t \vee p) \wedge k)) \vee r) \vee \neg r$ **(Using Double negation law)**

$((\neg t \vee p) \wedge k) \vee (r \vee \neg r)$ **(Using Associative law)**

$((\neg t \vee p) \wedge k) \vee T$ **(Using Negation law)**

$((\neg t \vee p) \wedge k) \vee T \equiv T$ **(Using Domination law)**

Hence given $((t \rightarrow (r \vee p)) \rightarrow ((\neg r \vee k) \wedge \neg k)) \rightarrow \neg r$ is a Tautology

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