CS:1010 DISCRETE STRUCTURES

PRACTICE QUESTIONS
LECTURE 8

Instructions

- Try these questions before class. Do not submit!

(1) Prove or disprove that if $a \mid bc$, where a, b, and c are positive integers and $a \neq 0$, then $a \mid b$ or $a \mid c$.

(2) What are the quotient and remainder when: a) -1 is divided by 3 and b) 3 is divided by 5.

(3) What time does a 12-hour clock read
a) 80 hours after it reads $11:00$ and
b) 40 hours before it reads $12:00$?

(4) Suppose that a and b are integers, $a \equiv 4(mod 13)$, and $b \equiv 9(mod 13)$.Find the integer c with $0 \leq c \leq 12$ such that
a)$c \equiv a + b(mod\ 13)$
b) $c \equiv a^3 - b^3(mod\ 13)$.

(5) Show that if a, b, c, and m are integers such that $m \geq 2$,$c > 0$, and $a \equiv b(mod\ m)$, then $ac \equiv bc(mod\ mc)$.

(6) Show using mathematical induction that if a, b and m are integers such that $m \geq 2$,and $a \equiv b(\text{mod } m)$,then $a^n \equiv b^n \ (\text{mod } m)$ for all $n \in \mathbb{N}$.

(7) Convert $(ABCDEF)_{16}$ from hexadecimal to its binary expansion.

(8) Show that the hexadecimal expansion of a positive integer can be obtained from its binary expansion by grouping together blocks of four binary digits, adding initial zeros if necessary, and translating each block of four binary digits into a single hexadecimal digit.

(9) Prove that for every positive integer n, there are n consecutive composite integers.

(10) Show that if a^{m+1} is composite if a and m are integers greater than 1 and m is odd.

(11) Show that if $2^m + 1$ is an odd prime, then $m = 2^n$ for some nonnegative integer n.

(12) Find the last digit of 7^{100}.

(13) In year N, the 300th day of the year is a Tuesday. In year $N + 1$, the 200th day is also a Tuesday. On what day of the week did the 100th day of the year $N - 1$ occur?

(14) P.T. $2^n + 6 \cdot 9^n$ is always divisible by 7 for any positive integer n.

January 16, 2021; Dept of CSE, IIT Hyderabad