E3 - Linear Algebra (MA 4020)

Deadline: Sunday, October 10, 2021; 10 PM Maximum Marks 8

Instructions.

1. Write your name and roll number on the answered pages/papers.
2. Scan the document in the pdf file format.
3. Rename the scanned document with your name-E3.
4. Upload the **pdf file** on the Google classroom. (No .jpeg or .jpg file please)

Note. Write answers carefully. Anyone found copying, even for a single question, will be awarded zero marks. Late submission by default will get zero marks. Upload your answers in time, with as many questions as you have done.

1. Let $V = \mathcal{P}_3(\mathbb{R})$ be the vector space of all polynomials in a single variable x with real coefficients and of degree less than, or equal to, 3. Assume that $(1, x, x^2, x^3)$ be the ordered basis for V. Consider the linear transformation $T : V \longrightarrow V$ defined by

$$T(p)(x) = xp^{''}(x) + 3xp^{'}(x) + 2p(x) \quad \text{for all } p \in V.$$

Write down the corresponding matrix of T with respect to the standard basis. Moreover find p such that

$$xp^{''}(x) + 3xp^{'}(x) + 2p(x) = 11x^3 + 14x^2 + 7x + 2.$$

2. Let $T : V \longrightarrow W$ be a linear transformation. Assume that $(v_1, v_2, \ldots, v_n)$ is linearly independent set of vectors in V. Is it true that $(T(v_1), \ldots, T(v_n))$ is also linearly independent in W. (Justify your answer)

3. Find the range and kernel of $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ defined by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \longmapsto \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Also compute dim (ker T) and dim (imT).