CS 1010 DISCRETE STRUCTURES

QUIZ 2
LECTURE 5-6

(1) Which of the following sets are uncountable?
  (a) Union of a countable and an uncountable set
  (b) The set of all possible strings made from the English alphabet.
  (c) The set of all infinite binary sequences.(An infinite binary sequence is a unending sequence of 0's and 1's.)
  (d) Set of all real numbers containing only finite number of 1's in their decimal representation.
  Ans-: a,c,d

(2) Suppose $P(n)$ is a propositional function. Which of the following statements are true.
  (a) If $P(1)$ is true and $P(k) \implies P(k+1)$ is true for all positive integers k, then $P(n)$ is true for all positive integers n.
  (b) If $P(1)$ is true and $P(k) \implies P(k+2)$ is true for all positive integers k, then $P(n)$ is true for all positive integers n.
  (c) If $P(1)$ is true and $P(k) \implies P(2k)$ is true for all positive integers k, then $P(n)$ is true for all positive integers n.
  (d) If both $P(1)$ and $P(2)$ are true and $P(k) \wedge P(k+1) \implies P(k+2)$ is true for all positive integers k, then $P(n)$ is true for all positive integers n.
  Ans -: a,d

(3) Which of the following statements are true?
  (a) The union of a countable collection of countable sets is countable.
  (b) If S is an infinite set then $\aleph_0 \leq |S|$.
  (c) If A is an uncountable set and B is a countable set then $A \setminus B$ will always be uncountable.
  (d) Subset of a uncountable set is always uncountable.
  Ans-:a,b,c

(4) Find closed formula for the sequence defined by the recurrence relation $a_n = a_{n-1} - 2$ with $a_1 = 0$ given
  (a) $n - 1$
  (b) $-2(n-1)$
  (c) $-n^2 + 1$
  (d) $-n + 1$
  Answer: b) $-2(n-1)$

(5) The sequence $0, 1, 2, 3, \ldots$ can be represented as a formula which starts from a_1 as
(a) $a_n = \lfloor n/2 + 1 \rfloor + \lceil n/2 - 1 \rceil$
(b) $a_n = \lfloor n/2 \rfloor + \lceil n/2 \rceil - 1$
(c) $a_n = \lceil n/2 \rceil + \lceil n/2 - 1 \rceil$
(d) $a_n = 2 \times \lfloor n/2 \rfloor$
Answer: b

(6) If A is a subset of B and B is a subset of C, then the cardinaliity of $(A \cap B) \cup C$ is equal to
(a) Cardinality of C
(b) Cardinality of B
(c) Cardinality of A
(d) Cannot say
Answer a

(7) Consider the following statements and determine if we can have proofs using mathematical induction for them.
(a) 7 divides $15^n + 6$
(b) Every nonempty finite set of real numbers has a minimum element.
(c) For every $a \in \mathbb{Z}$, 3 divides $(a^3 - a)$

(a) True, True, True
(b) True, False, True
(c) False, False, False
(d) True, False, False
Answer: (a)

(8) Find closed formula for the sequence defined by the recurrence relation $a_{n+1} = 100a_n$ with $a_0 = 10$ given.
(a) $a_n = 100^n$
(b) $a_n = 100^{n-1}$
(c) 10^{2n+1}
(d) 10^{2n-1}
Answer: c

(9) Consider the following induction proof. $P(n)$ is the proposition: For all $n \in \mathbb{N}$,

$$2 + 3 + 4 + \ldots + n = \frac{n(n+1)}{2}.$$

Basis step: $P(0)$ is true since both LHS and RHS is equal to 0 as the sum with no terms is zero.

Inductive step: Assume $P(k)$ is true and we evaluate $P(k+1)$.

$$\begin{aligned} 2+3+\ldots+n+(n+1) &= [2+3+4+\cdots+n]+(n+1) \\ &= \frac{n(n+1)}{2}+(n+1) \\ &= \frac{(n+1)(n+2)}{2} \end{aligned}$$

Now that we have proved basis and inductive step we can conclude that $P(n)$ is true for all $n \in \mathbb{N}$.

(a) Basis step $P(0)$ we considered is not the right one
(b) In the Inductive step we did not consider the correct $P(k)$
(c) In the Inductive step we have made a mistake while evaluating $P(k+1)$
(d) There is nothing wrong with the proof

Answer: (a)

(10) Determine the truth values of the following statements regarding Hilbert's fully occupied Grand Hotel.

(a) The hotel expands to a second building which has countably infinite number of rooms. The current guests can fully occupy both the buildings.
(b) A countably infinite number of ships arrive each carrying a countably infinite number of coaches each carrying a countably infinite number of guests. These guests can be accommodated in Hilbert's Grand Hotel.
(c) A countably infinite number of coaches with countably infinite members in each coach arrive at the Grand Hotel to stay for infinitely many days. The guests have to change their rooms every day such that each guest does not occupy the same room again. This wish of the guests can be taken care of at the Grand Hotel.

(a) True, True, False
(b) True, False, False
(c) True, True, True
(d) False, True, False

Answer: c