CS:1010 DISCRETE STRUCTURES

ASSIGNMENT 2

Instructions

- Answer all the questions.
- Please submit a PDF file preferably compiled using a LaTeX editor.
- Max marks: 20, Due date: Feb 22, 2021 end of day

(1) (Lecture 11)

(a) S.T. the recurrence relation

$$f(n)a_n = g(n)a_{n-1} + h(n),$$

for $n \geq 1$ and with $a_0 = C$ can be reduced to a recurrence relation of the form

$$b_n = b_{n-1} + Q(n)h(n),$$

where $b_n = g(n+1)Q(n+1)a_n$ with

$$Q(n) = \frac{(f(1)f(2)\cdots f(n-1))}{(g(1)g(2)\cdots g(n))}.$$

Note that for $Q(1)$ we consider $(f(1)f(2)\cdots f(n-1)) = 1$.

(b) Use previous part to solve the original recurrence relation to obtain

$$a_n = \frac{C + \sum_{i=1}^{n} Q(i)h(i)}{g(n+1)Q(n+1)}.$$

$(1 + 2\ \textit{marks})$

(2) (Lecture 11)

(a) Use the previous exercise to solve the recurrence relation,

$$(n+1)a_n = (n+3)a_{n-1} + n,$$

for $n \geq 1$, with $a_0 = 1$.

(b) There are many computer algebra software (CAS) systems that help you solve recurrences (and other algebraic problems) using inbuilt libraries/functions. Sympy, Sagemath, Macaulay2 are some of the free software available. Mathematica, Maple are some of the paid ones. In fact, for recurrences Maple is the recommended software.
Consider this free online Sympy console: `https://live.sympy.org/`. What are the commands one should type in here to obtain the solution

of the following linear recurrence with *polynomial coefficients* of degree 1. Also, give the solution that Sympy returns.

$$(n+1)g(n) = g(n-1), g(0) = 1.$$

Sympy offers Google Summer of Code (GSoC) projects where you can contribute to this open source project of mathematical software. Check out this page for more details. ***https://github.com/sympy/sympy/wiki/GSoC-Ideas***

($3 + 4$ *marks*)

(3) (Lecture 11)
Prove the theorem relating to the solution of a recurrence relation of degree 2 whose corresponding characteristic equation has only one root. Specifically, prove:

Theorem 0.1. *Let* $c_1, c_2 \in \mathbb{R}$ *with* $c_2 \neq 0$. *Suppose that* $r^2 - c_1 r - c_2 = 0$ *has only one root* r_0. *A sequence* $\{a_n\}$ *is a solution of the recurrence relation* $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ *iff* $a_n = \alpha_1 r_0^n + \alpha_2 n r_0^n$ *for* $n = 0, 1, 2, \ldots,$ *where* α_1 *and* α_2 *are constants.*

(5 *marks*)

(4) (Lecture 12)
Assume that f is an increasing function satisfying the recurrence relation $f(n) = af(n/b) + cn^d$, where $a \geq 1, b$ is an integer greater than 1, and c, d are positive real numbers.
(a) S.T. if $a = b^d$ and n is a power of b, then $f(n) = f(1)n^d + cn^d \; log_b \; n$.
(b) S.T. if $a \neq b^d$ and n is a power of b, then $f(n) = c_1 n^d + c_2 n^{log_b \; a}$, where $c_1 = b^d c/(b^d - a)$ and $c_2 = f(1) + b^d c/(a - b^d)$.
(5 *marks*)