Assignment 3

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Download all python codes from

https://github.com/Suraj11050/Assignments–AI1103/blob/main/Assignment%203/Assignment3.py

Download Latex-tikz codes from

https://github.com/Suraj11050/Assignments–AI1103/blob/main/Assignment%203/Assignment3.tex

1 GATE 2009 (MA) PROBLEM 16

Let F, G and H be pair wise independent events such that $\Pr(F) = \Pr(G) = \Pr(H) = \frac{1}{3}$ and $\Pr(F \cap G \cap H) = \frac{1}{4}$ Then the probability that at least one event among F, G and H occurs is

(A) $\frac{11}{12}$ (B) $\frac{7}{12}$ (C) $\frac{5}{12}$ (D) $\frac{3}{4}$

2 SOLUTION

If two Events X_1 and X_2 are independent then

$$\Pr(X_1 X_2) = \Pr(X_1) \times \Pr(X_2) \tag{2.0.1}$$

Using equation (2.0.1) we get the following results

$$\Pr(FG) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \tag{2.0.2}$$

$$\Pr(GH) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \tag{2.0.3}$$

$$\Pr(HF) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \tag{2.0.4}$$

At least one event among F, G, H should occur is $\Pr(F + G + H)$ from Principal of inclusion and exclusion it is calculated using random variable as

$$\begin{aligned}\Pr(F + G + H) &= \Pr(F) + \Pr(G) + \Pr(H) \\ &- (\Pr(FG) + \Pr(GH) + \Pr(HF)) + \Pr(FGH)\end{aligned} \tag{2.0.5}$$

Substituting above results from equation (2.0.2), (2.0.3), (2.0.4) in equation (2.0.5)

$$\Pr(F + G + H) = 3\left(\frac{1}{3}\right) - 3\left(\frac{1}{9}\right) + \frac{1}{4}$$

$$\therefore \Pr(F + G + H) = \frac{11}{12}$$

Hence Probability that at least one event among F, G, H occurs is $\Pr(F + G + H) = \frac{11}{12}$ and correct answer is **Option (A)**

But we know that

$$(FG) = (FGH) + (FGH^c) \tag{2.0.6}$$

$$\Pr(FG) = \Pr(FGH) + \Pr(FGH^c) \tag{2.0.7}$$

$$\therefore \Pr(FG) \geq \Pr(FGH) \tag{2.0.8}$$

In the given question

$$\Pr(FGH) = \frac{1}{4} \tag{2.0.9}$$

$$\Pr(FG) = \frac{1}{9} \tag{2.0.10}$$

$$\Pr(FG) < \Pr(FGH) \tag{2.0.11}$$

Which is not possible according to equation (2.0.8).
Similar case with $\Pr(GH)$ and $\Pr(HF)$
Some of the probabilities turnout to be negative like

$$\Pr(F^c GH) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36} \tag{2.0.12}$$

$$\Pr(FG^c H) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36} \tag{2.0.13}$$

$$\Pr(F^c GH) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36} \tag{2.0.14}$$

Probability $P \in [0, 1]$ but because of the data in the question some of the probabilities turn out to be negative. Therefore **Question is incorrect**