CS:1010 DISCRETE STRUCTURES

PRACTICE QUESTIONS
LECTURE 6

Instructions

- Try these questions before class. Do not submit!

(1) Countable sets are closed under unions and products.

(2) The following sets are countably infinite:

$$\mathbb{Z}^+, \mathbb{Q}^+, \mathbb{Z} \times \mathbb{Z}, \mathbb{Q}.$$

(3) If A is an infinite set and B is countable, then there is a surjection from A to B.

(4) Why cannot we use the same diagonal argument we used for real numbers for integers and conclude that the set of integers are uncountable.

(5) Subsets of countable sets are countable.

(6) S.T.

$$1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1, \forall n \in \mathbb{Z}^+.$$

(7) Using mathematical induction show that $2^n < n!$ for all positive integer $n \geq 4$.

(8) S.T. $H_{2^n} \geq 1 + \frac{n}{2}$, where H_n is the nth Harmonic number.

(9) Prove that

$$H_1 + H_2 + \cdots + H_n = (n+1)H_n.$$

(10) T.S.T if S is a finite set with n elements where n is a nonnegative integer then S has 2^n subsets.

(11) P.T. if $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$ are sets s.t. $A_j \subseteq B_j$ for $j = 1, 2, \ldots, n$, then

$$\cap_{j=1}^{n} A_j \subseteq \cap_{j=1}^{n} B_j$$

(12) P.T. $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer using induction.