Assignment 1

Suraj - CS20BTECH11050

Download all python codes from

https://github.com/Suraj11050/Assignment1/blob/main/Assignment1.py

and latex-tikz codes from

https://github.com/Suraj11050/Assignment1/blob/main/Assignment1.tex

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PROBLEM 4.11

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X

SOLUTION :

When 2 fair dice are thrown simultaneously we know that each die has 6 possible outcomes and outcome of one dice is independent of the outcome of other dice.
$\therefore$ Total possible outcomes are ${}^6C_1 \times {}^6C_1 = 36$

Let X be a random variable denoting number of sixes in the above case. Then by Binomial Distribution

$$\Pr(X = k) = \binom{n}{k} p^k \ (1-p)^{n-k} \tag{0.0.1}$$

$$k = 0, \ldots, n \tag{0.0.2}$$

Where k = 0,1,2 and n = 2 (Number of dice)
p = Probability of outcome 6 on a dice

$$p = \frac{1}{6}$$

From equation (0.0.1) we obtain the following

$$\Pr(X=0) = \binom{2}{0}\left(\frac{1}{6}\right)^0\left(1-\frac{1}{6}\right)^2 = \frac{25}{36}$$
$$\Pr(X=1) = \binom{2}{1}\left(\frac{1}{6}\right)^1\left(1-\frac{1}{6}\right)^1 = \frac{10}{36}$$
$$\Pr(X=2) = \binom{2}{2}\left(\frac{1}{6}\right)^2\left(1-\frac{1}{6}\right)^0 = \frac{1}{36}$$

The probability distribution table is

X	0	1	2
$\Pr(X)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

$$\begin{aligned}
\mathbb{E}(X=k) &= \sum_{k=0}^{n} k \Pr(k) \\
&= \sum_{k=0}^{n} k \binom{n}{k} p^k \ (1-p)^{n-k} \\
&= n \cdot p \sum_{k=1}^{n-1} \binom{n-1}{k-1} p^{k-1} \ (1-p)^{(n-1)-(k-1)} \\
&= n \cdot p \ (1+(1-p))^{n-1} \\
&= n \cdot p
\end{aligned}$$

$$\mathbb{E}(X=k) = n \cdot p = 2 \times \frac{1}{6} = \frac{1}{3}$$

$$\therefore \mathbb{E}(X) = \frac{1}{3}$$

Hence Expectation value of $X = \frac{1}{3} = 0.333333$
