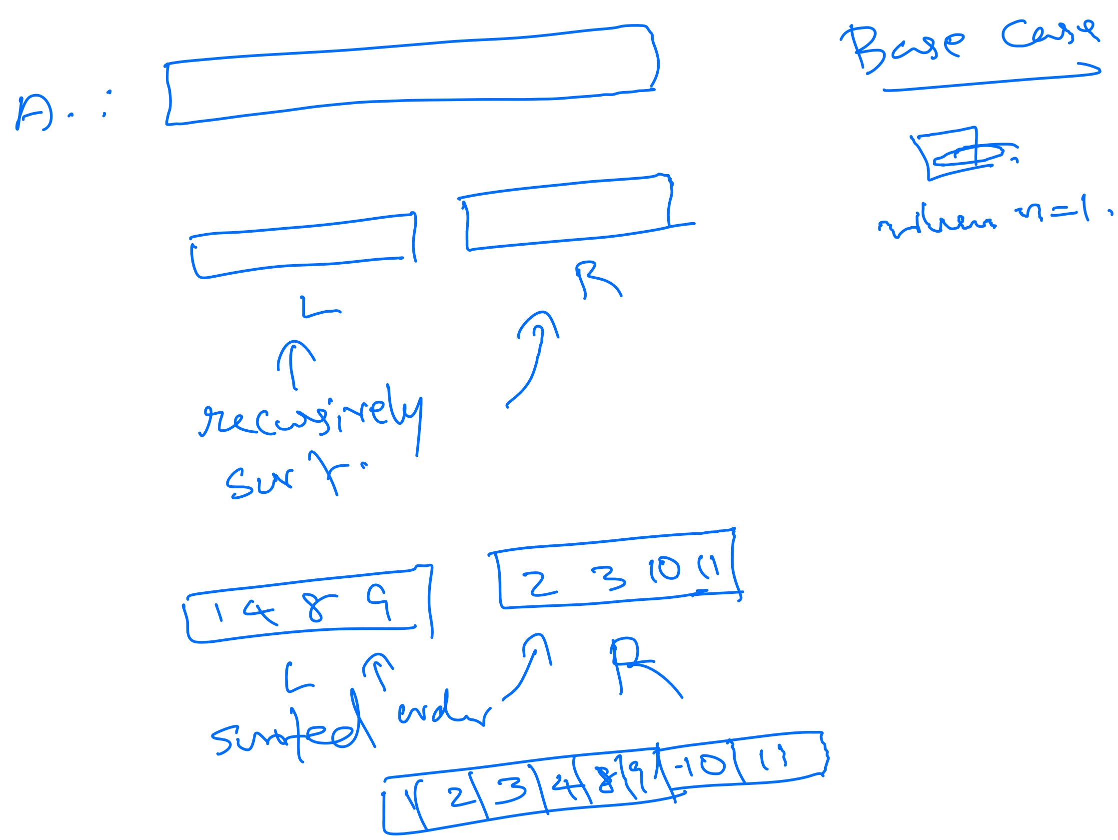
Sorting

I/P: n- integers A[1 ... n]

O/P: Permutation A[i], ... A[n] in non-decreasing order.

1, 2, 1, 3, 4 → 1, 1, 2, 3, 4

Merge Sort.



Merge

I/P: two sorted (sub) arrays

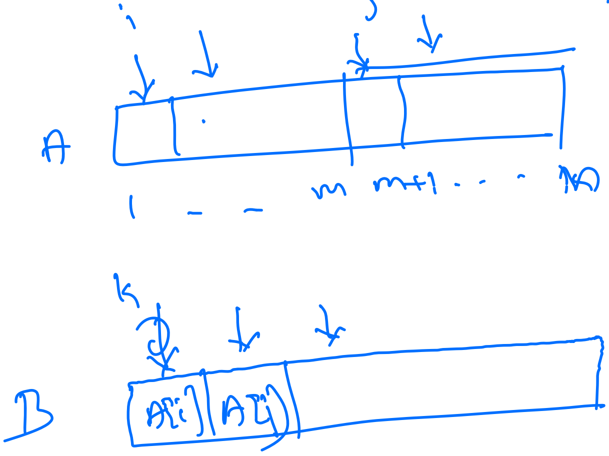
O/P: Sorted array.

```
MergeSort(A[1..n]):
  if n > 1
      m ← ⌊n/2⌋
      MergeSort(A[1..m])        ⟨⟨Recurse!⟩⟩
      MergeSort(A[m+1..n])      ⟨⟨Recurse!⟩⟩
      Merge(A[1..n], m)
```

to m+1

```
Merge(A[1..n], m):
  i ← 1; j ← m+1
  for k ← 1 to n
      if j > n
          B[k] ← A[i]; i ← i+1
      else if i > m
          B[k] ← A[j]; j ← j+1
      else if A[i] < A[j]
          B[k] ← A[i]; i ← i+1
      else
          B[k] ← A[j]; j ← j+1
  for k ← 1 to n
      A[k] ← B[k]
```

Correctness - proof by induction.



Correctness for MERGE.

Assumption: ① A[1...m] is in non-decreasing order. ② A[m+1...n] is in non-decreasing order.

We want to prove: At the end of MERGE, A[1...n] is in non-decreasing order, containing the elements originally present in A[1...n].

Loop-invariant:

At each iteration of

At the end of ... the "first for" loop $\underline{B[1...k]}$ is in non-decreasing order of the elements $\underline{A[1...i-1]}$ union $\underline{A[m+1...j-1]}$. Both $A[i]$ and $A[j]$ are $\geq$ all the elements in $B[1...k]$.

Proof: by induction on k.

$\underline{\text{Base Case}}$: $\underline{k=1}$

At the end of first iteration $B[1]$ contain the smallest ~~among~~ $A[1]$ and $A[m]$. If $A[1] < A[m]$ then $A[2]$ and $A[m]$ are $\geq B[1]$.

$\underline{\text{Induction step}}$

At the end of $k-1^{st}$ iteration $B[1...k-1]$ is in ~~so~~ non-decreasing order containing ~~el~~ ~~the~~ $\underline{A[1...i-1]}$ union $A[m+1...j-1]$.

Suppose $\underline{A[i] < A[j]}$.

$\ast$ $\boxed{B[k] \leftarrow A[i] \text{ before incrementing } i}$

$B[1 \ldots k-1]$ are non-decreasing order.

$A[i-1] \geq$ the elements in $B[1 \ldots k-1]$

$\Rightarrow B[1 \ldots k]$ is in non-decreasing order.

$B[1 \ldots k]$ contain the elements of $A[1 \ldots i-1] \cup A[m+1 \ldots j-1]$.

$$T(n) \leq 2T(n/2) + 10n$$

$$T(1) = 1$$

$$T(n) = O(n \log n)$$

$$T(n) = \Omega(n \log n)$$

$$T(n) \geq 2T(n/2) + n$$

$$T(1) = 1.$$

$$T(n) = \Theta(n \log n).$$