CS:1010 DISCRETE STRUCTURES

PRACTICE QUESTIONS
LECTURE 4

Instructions

- Try these questions before class. Do not submit!

(1) Show that $\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor$.

(2) Let A, B, C be subsets of $\mathbb{R}^2$ where $A = \{(x, y)|y = 2x + 1\}$, $B = \{(x, y)|y = 3x\}$ and $C = \{(x, y)|x - y = 7\}$. Determine each of the following:
 (a) $A \cap B$
 (b) $B \cap C$
 (c) $\overline{\overline{A} \cup \overline{C}}$
 (d) $\overline{B} \cup \overline{C}$

(3) Let $n, k \in \mathbb{N}$, P.T. $\lceil \frac{n}{k} \rceil = \lfloor \frac{n-1}{k} \rfloor + 1$.

(4) A chemist who has five assistants is engaged in a research project that calls for nine compounds that must be synthesized. In how many ways can the chemist assign these syntheses to the five assistants so that each is working on atleast one synthesis?

(5) Let A_i be the set of all nonempty bit strings (i.e. bit strings of length at least one) of length not exceeding i. Find
 (a) $\bigcup_{i=1}^{n} A_i$
 (b) $\bigcap_{i=1}^{n} A_i$

(6) Prove that if x is a positive real number, then
 (a) $\left\lfloor \sqrt{\lfloor x \rfloor} \right\rfloor = \lfloor \sqrt{x} \rfloor$
 (b) $\left\lceil \sqrt{\lceil x \rceil} \right\rceil = \lceil \sqrt{x} \rceil$

(7) Let $f : A \to A$ be a function. If A is finite then f is one-to-one iff f is onto.

(8) Prove $x - 1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x + 1$.

(9) Prove $\lfloor -x \rfloor = -\lfloor x \rfloor$

(10) Prove $\lceil -x \rceil = -\lceil x \rceil$.

(11) Prove $\lceil x + n \rceil = \lceil x \rceil + n$.

(12) Is $f : \mathbb{Z} \to \mathbb{Z}$ $f(n) = n^3$ one-to-one?

(13) If f and $f \circ g$ are one-to-one does it follow that g is one-to-one? Justify your answer.