E2 - Linear Algebra (MA 4020)

Deadline: Tuesday, October 5, 2021; 3 PM — Maximum Marks 18

Instructions.

1. Write your name and roll number on the answered pages/papers.
2. Scan the document in the pdf file format.
3. Rename the scanned document with your name-E2.
4. Upload the **pdf file** on the Google classroom. (No .jpeg or .jpg file please)

Note. Write answers carefully. Anyone found copying, even for a single question, will be awarded zero marks. Late submission by default will get zero marks. Upload your answers in time, with as many questions as you have done.

1. Let V be a vector space over $\mathbb{R}$. Prove that V is not the union of finitely many proper subspaces.

2. Let $V = \mathbb{R}^n$ be the vector space.

(i) Let $W_1 = \{(x_1, \ldots, x_n) \mid x_1 + \cdots + x_n = 0\}$. Find a basis and dimension of W_1.

(ii) Let $W_2 = \{(x_1, \ldots, x_n) \mid \text{ such that } x_k = 0 \text{ if } k \text{ is even}\}$. Find a basis and dimension of W_2.

3. Let $W \subset \mathbb{R}^4$ be the subspace defined by

$$W = \{x \in R^4 \mid Ax = 0\}$$

where

$$A = \begin{bmatrix} 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 0 \end{bmatrix}$$

Write down a basis for W.

4. Which of the following sets of vectors form a basis for R^3?

(i) $\{(-1, 0, 0), (1, 1, 1), (1, 2, 3)\}$

(ii) $\{(0, 1, 2), (1, 1, 1), (1, 2, 3)\}$

(iii) $\{(-1, 1, 0), (2, 0, 0), (0, 1, 1)\}$

5. Let W_1 be the real vector space of all 5×2 matrices such that the sum of the entries in each row is zero. Let W_2 be the real vector space of all 5×2 matrices such that the sum of the entries in each column is zero. Then the dimension of the subspace $\dim(W_1 \cap W_2)$ is ——. (Justify your claim)

6. Let X and Y are non-empty subsets of the vector space V. We donote by $X + Y$ the subset $\{x + y \mid x \in X, y \in Y\}$. Let W be a vector subspace of V. Answer the following:

(i) What is $w + W$ if $w \in W$?

(ii) What is $W + W$?

(iii) Is it true that $w + W = W$ if and only if $w \in W$?

(iv) Let W_1 and W_2 be vector subspaces of a vector space V. Then $W_1 \cup W_2$ is a subspace if and only if $-------$. Fill in the blanks and prove your claim.