CS:1010 DISCRETE STRUCTURES

PRACTICE QUESTIONS
LECTURE 1

Instructions

- Try these questions before class. Do not submit!

(1) Let p and q be the propositions
p: It is below freezing.
q:It is snowing.

Write these propositions using p and q and logical connectives.
(a) It is below freezing and snowing.
$p \wedge q$
(b) It is below freezing but not snowing.
$p \wedge \neg q$
(c) If it is below freezing it is also snowing. $p \rightarrow q$
(d) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
$(p \vee q) \wedge (p \rightarrow \neg q)$ or $p \ XOR \ q$.
(e) That it is below freezing is necessary and sufficient for it to be snowing.
$p \leftrightarrow q$

(2) Write each of these statements in the form "if p, then q" in English.
(a) It is necessary to wash the boss's car to get promoted.
(b) Winds from the south imply a spring thaw.
Washing boss's car is necessary - q If you want to get promoted - p.
If you want to get promoted then you need to wash the boss's car.
If the winds are from the south that implies a spring thaw.

(3) How many rows appear in a truth table for each of these compound propositions?
(a) $p \rightarrow \neg p$
(b) $p \vee \neg r) \wedge (q \vee \neg s)$
(c) $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$
A truth table needs 2^n rows if there are n variables. So first one needs 2 rows, second $2^4 = 16$ rows, $2^6 = 64$ rows.

(4) S.T. $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology.

T.S.T if $p \to q$ and $q \to r$ are both true, then $p \to r$ is true. I.e. to show that if p is true then r is true.

Given p and $p \to q$ are both true, we conclude that q is true.
Now with q is true and $q \to r$, we have r is true.

(5) S.T. $(p \to q) \to r$ and $p \to (q \to r)$ is not logically equivalent.
T.S.T these are not logically equivalent, we need only to find one assignment of truth values to p, q and r for which truth values of $(p \to q) \to r$ and $p \to (q \to r)$ differ.

Give F to p, q, r. Then $(p \to q) \to r$ is false and $p \to (q \to r)$ is true.

(6) The dual of a compound proposition that contains only logical operators $\wedge, \vee, \neg$ is the compound proposition obtained by replacing each $\vee$ by $\wedge$, $\wedge$ by $\vee$, T by F and F by T. The dual of s is denoted as s^*.
(a) Find the dual of $p \wedge \neg q \wedge \neg r$.
(b) S.T. $(s^*)^* = s$.
(c) When does $s^* = s$?
(a) $p \vee \neg q \vee \neg r$.
(b) If we apply all the operations one more time every connector returns to what it was originally. So the dual of a dual is the original proposition.
(c) For idempotent, identity and absorption laws it works.
Consider $\hat{s}$: the formula you obtain by replacing every variable in s by its negation. First we can show that $s^* = \neg\hat{s}$.
If s does not contain variables then $\hat{s} = s$ (equal not equivalent) and thus we have $s^* \equiv \neg s$ and so $s^* \neq s$ ever.
But for s that has variables by looking at truth table you can come to the conclusion: for every assignment of truth value in the truth table look for the row where the assignment is the exact opposite value and check if it has the opposite value as the resulting truth value. If yes then for that case you have an example for $s^* = s$.
Note: If $s^* \equiv s$, then you need to have exactly half the rows of the truth table to result in true value.

(7) Determine whether this compound proposition is satisfiable.
$(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$
No truth assignment here. Whatever values you assign here you get $(T \to T) \wedge (T \to F) \wedge (F \to T) \wedge (F \to F)$ in some order.

Some of the queries that came up in class:

(1) Can we think of implication as a transitive relation? A: Yes, it is a transitive relation on propositions.

(2) Unsatisfiablility and Contradiction : Unsatisfiability is a property. Unsatisfiable statements are known as contradictions.
(3) Venn diagrams : I still would strongly advise you all to start looking at proving equivalences using identities than using venn diagrams. It gets harder as we come up with more complicated logical expressions.
(4) Notations: Use anything that is standard but preferably use the ones discussed in class. If you are not using the same ones please specify before using it.

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