

CS 2443: Examination 1

Department of Computer Science, IIT Hyderabad

17-Feb-2022

- Total marks: 15.
- Read the questions carefully and answer only to the questions.
- Submit the answers and mark them towards the questions in gradescope.
- Maintain academic honesty.

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1. Consider the QUICK SORT algorithm discussed in the class where the pivot element is chosen by running the median finding algorithms. What is the number of calls to the median finding algorithm by this version of the QUICK SORT on an input of n numbers. Prove your answer. (2.5)
 2. Give the best possible asymptotic upper bounds for the following recurrence relation and prove your answer using the method of domain and range transformations. (3)

$$\begin{aligned}T(n) &= n^{\frac{1}{3}} \cdot T(n^{\frac{1}{3}}) + 1 \\T(1) &= T(2) = 1\end{aligned}$$

3. Consider the $\mathcal{O}(n)$ time algorithm to find the median of n numbers taught in the class. Suppose we make blocks of size 13, instead of blocks of size 5 in the algorithm. Then, what will be the recurrence relation of the running time? (1.5)
4. Consider the following variant of EDIT DISTANCE. Given two sequence of symbols $A[1 \dots m]$ and $B[1 \dots n]$, where each element in these sequences are from $\{0, 1, 2\}$. We want to convert $A[1 \dots m]$ to $B[1 \dots n]$ using add, deletion and edit operations with minimum cost, where the cost of each operation is as follows. The cost of addition/deletion of a symbol is 0.75 and the cost of an edit from x to y is $0.5 \cdot |x - y|$. (For example, an edit from 0 to 2 costs 1.) Write a recursive formula for this problem that will help you to design a polynomial time dynamic programming algorithm for the problem. (3)
5. You are given an array $A[1, \dots, n]$ such that there exists $1 \leq i \leq j \leq n$ with the following property. $A[r] = 1$ for all $i \leq r \leq j$ and $A[r] = 0$ otherwise. Design an efficient algorithm for finding these indices i and j . What is the lowerbound on the worst-case running time for any comparison based algorithm for solving this problem. (In a comparison based algorithm, the algorithm makes decision by comparing numbers where these numbers are elements in the input array or constants like 0 or 1) (2+3)