Assignment 3

Suraj - CS20BTECH11050

Download all python codes from

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https://github.com/Suraj11050/Assignments–
    AI1103/blob/main/Assignment%203/
    Assignment3.py
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Download Latex-tikz codes from

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https://github.com/Suraj11050/Assignments–
    AI1103/blob/main/Assignment%203/
    Assignment3.tex
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1 GATE 2009 (MA) PROBLEM 16

Let F, G and H be pair wise independent events such that $\Pr(F) = \Pr(G) = \Pr(H) = \frac{1}{3}$ and $\Pr(F \cap G \cap H) = \frac{1}{4}$ Then the probability that at least one event among F, G and H occurs is

(A) $\frac{11}{12}$ (B) $\frac{7}{12}$ (C) $\frac{5}{12}$ (D) $\frac{3}{4}$

2 SOLUTION

Let f, g, h be three random variables taking values 0 or 1 (Bernoulli random variable) Which represent the occurrence of event F, G, H respectively such that

$$\Pr(f = 0) = \frac{2}{3} \quad \Pr(f = 1) = \frac{1}{3} \tag{2.0.1}$$

$$\Pr(g = 0) = \frac{2}{3} \quad \Pr(g = 1) = \frac{1}{3} \tag{2.0.2}$$

$$\Pr(h = 0) = \frac{2}{3} \quad \Pr(h = 1) = \frac{1}{3} \tag{2.0.3}$$

$$\tag{2.0.4}$$

If two Random variables X_1 and X_2 are independent then

$$\Pr(X_1 X_2) = \Pr(X_1) \times \Pr(X_2) \tag{2.0.5}$$

Using equation (2.0.5) we get the following results

$$\Pr(f = 1, g = 1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \tag{2.0.6}$$

$$\Pr(g = 1, h = 1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \tag{2.0.7}$$

$$\Pr(f = 1, g = 1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \tag{2.0.8}$$

At least one event among F, G, H should occur is $\Pr(F + G + H)$ from Principal of inclusion and exclusion it is calculated using random variable as

$$\begin{aligned} \Pr(f = 1 + g = 1 + h = 1) = \\ (\Pr(f = 1) + \Pr(g = 1) + \Pr(h = 1)) \\ - \Pr(f = 1, g = 1) - \Pr(g = 1, h = 1) \\ - \Pr(h = 1, f = 1) + \Pr(f = 1, g = 1, h = 1) \end{aligned} \tag{2.0.9}$$

$$\Pr(f = 1 + g = 1 + h = 1) = 3\left(\frac{1}{3}\right) - 3\left(\frac{1}{9}\right) + \frac{1}{4}$$

$$\therefore \Pr(f = 1 + g = 1 + h = 1) = \frac{11}{12}$$

Hence Probability that at least one event among F, G, H occurs is $\Pr(F + G + H) = \frac{11}{12}$ and correct answer is **Option (A)**

But we know that

$$(F \cap G \cap H) \subseteq (F \cap G) \tag{2.0.10}$$

$$\therefore \Pr(FGH) \leq \Pr(FG) \tag{2.0.11}$$

In the given question

$$\Pr(FGH) = \frac{1}{4} \tag{2.0.12}$$

$$\Pr(FG) = \frac{1}{9} \tag{2.0.13}$$

$$\Pr(FGH) > \Pr(FG) \tag{2.0.14}$$

Which is not possible

Some of the probabilities turnout to be negative like

$$\Pr(f = 1, g = 1, h = 0) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36}$$
$$\Pr(f = 1, g = 0, h = 1) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36}$$
$$\Pr(f = 0, g = 1, h = 1) = \frac{1}{9} - \frac{1}{4} = -\frac{5}{36}$$

$$\Pr(F^{c}GH) = -\frac{5}{36} \qquad (2.0.15)$$
$$\Pr(FG^{c}H) = -\frac{5}{36} \qquad (2.0.16)$$
$$\Pr(F^{c}GH) = -\frac{5}{36} \qquad (2.0.17)$$

Probability $P \in [0, 1]$ but the data in the question some of the probabilities turn out to be negative Similar case with $\Pr(GH)$ and $\Pr(HF)$
Therefore **Question is incorrect**