Recurrence Relations $O(n)$ x

(1) $T(n) \leq 4T(n/2) + cn$ $\qquad O(n^2)$.

$T(1) = 1$

(2) $T(n) \leq 3T(n/2) + cn$

$T(1) = 1$.

$S(n) = S(n-1) + n$

$S(0) = 0$

$S(n) = n + n-1 + \dots 1$

$= \frac{n(n+1)}{2} = O(n^2)$

(1) $S(n) = S(n-1) + d(n)$

$S(0) = 0$

$S(n) = \sum_{i=1}^{n} d(i)$

Range Transformation

2 - $R(n) = a\,R(n-1) + d(n)$

$R(0) = 0$

$$S(n) = \frac{R(n)}{a^n}$$

$S(0) = 0$.

$$\frac{R(n)}{a^n} = \frac{R(n-1)}{a^{n-1}} + \frac{d(n)}{a^n}$$

$$\left[\begin{array}{l} S(n) = S(n-1) + \dfrac{d(n)}{a^n} \\ S(0) = 0 \end{array}\right.$$

$$S(n) = \sum_{i=1}^{n} \frac{d(i)}{a^i}$$

$$R(n) = a^n \cdot \sum_{i=1}^{n} \frac{d(i)}{a^i}$$

Example: $a = 2$, $d(n) = n$

$$R(n) = 2^n \cdot \sum_{i=1}^{n} \frac{i}{2^i}$$

$$\underline{\underline{R(n) = \Omega(2^n)}} \qquad \le 2^n \cdot n \cdot 1$$

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots \frac{n}{}$$

$$\le n \cdot \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots \right)$$

Domain Transformation

$$T(n) = T\left(\frac{n}{b}\right) + \underline{\underline{d(n)}}$$

$$T(1) = 1$$

$$\boxed{\text{Example: } T(n) = T(n/2) + 1}$$

Let $n = b^k$ $\qquad k = \log_b n$

$$\begin{cases} T(\underline{b^k}) = T(b^{k-1}) + d(b^k) \\ T(1) = 1 \\ S(k) = T(2^k) \end{cases}$$

$\hookrightarrow \quad S(k) = S(k-1) + d(b^k)$

$S(0) = T(b^0) = T(1) = 1.$

$$T(b^k) = S(k) = \sum_{i=1}^{k} d(b^k)$$

<u>Example</u> $T(n) = T(n/2) + d(b^i)$ $\quad d(n)=1$

$$= \sum_{i=1}^{k=\log n} d(b^i)$$

$$= \sum_{i=1}^{k} 1 = \underline{\underline{\log n}}$$

$$T(n) = T(n/2) + 1.$$

Say $d(n) = 1$ for all n.

$$T(n) = T(n/2) + d(n).$$

$$T(n) = 3T(n/2) + n$$ $n = 2^k$ $k = \log n$

$$T(1) = 1$$

$$T(2^k) = 3T(2^{k-1}) + 2^k$$

$$\begin{cases} R(k) = T(2^k) \\ R(0) = 1 \end{cases}$$

$$R(k) = 3\underline{R(k-1)} + 2^k \text{ —— (a)}$$

$$R(0) = 1$$

$$S(k) = \frac{R(k)}{3^k} \qquad \frac{R(k-1)}{3^{k-1}}$$

$$S(0) = 1$$

$$(a)/3^k \Rightarrow S(k) = S(k-1) + \left(\frac{2}{3}\right)^k$$

k ... i

$$= 1 + \sum_{i=1} \left(\frac{2}{3}\right)$$

$$= 1 + \left[\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \cdots \left(\frac{2}{3}\right)^k \right]_{r<1}$$

$$a + ar + ar^2 + \cdots$$

$$\frac{a}{1-r}$$

$$\leq 1 + \boxed{\frac{2/3}{1 - 2/3}}$$

$$\cancel{8} = 1 + \frac{2/3}{1/3} = \underline{\underline{3}}$$

$$S(k) \leq 3.$$

$$R(k) = 3^k \cdot S(k)$$

$$\leq 3 \cdot 3^k.$$

$$\leq 3^{\log_2 n}$$

$$\log n$$

$$= \left(2^{\log_2 3}\right)^{\log_2 n}$$

$$= \left(2^{\log_2 n}\right)^{\log_2 3}$$

$$= n^{\log_2 3}$$

$$= n^{1.584}$$

$$T(n) = O\left(n^{1.58495}\right)$$

$$T(n) = T(n/3) + T(2n/3) + n$$

$$T(1) = 1 \quad \Rightarrow O(n \log n)$$

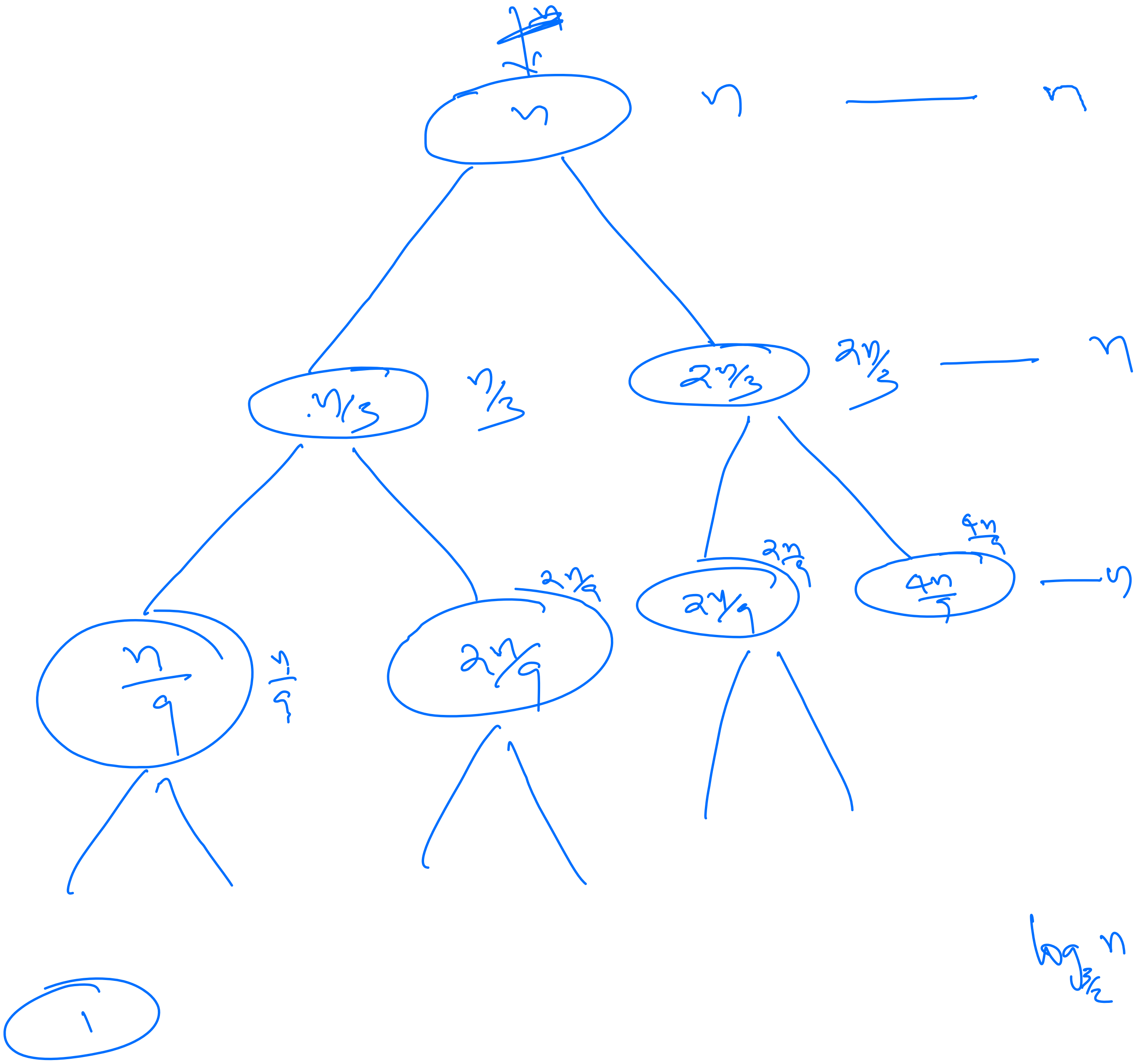
$$T(n) \leq 2T(2n/3) + n$$

$$T(1) = 1$$

Solve

$\cancel{O(n \log n)}$

Recurrence Tree Method



Total running $\leq n \cdot \log_{3/2} n$

~~Ex~~ $\log_3 n = O(\log_2 n)$

$= O(\log_{10} n)$

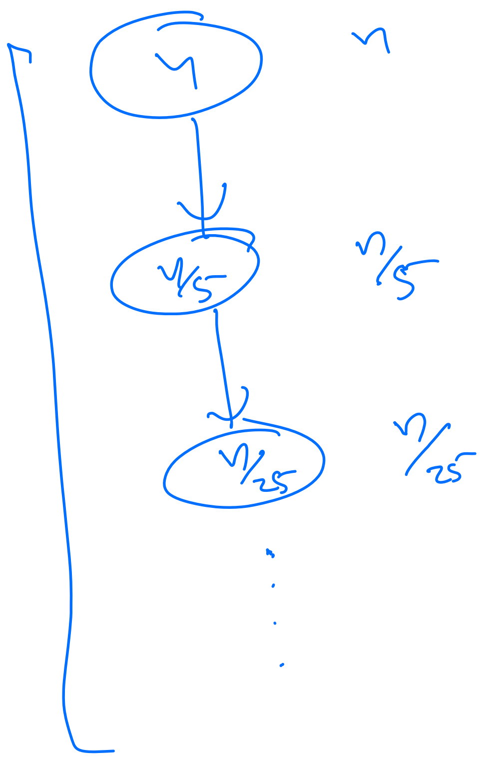
$T(n) = T\left(\frac{n}{5}\right) + n$ Domain

$T(1) = 1$

$T(n) =$ ① $O(n)$ ② $O(n \log n)$ ③ $O(n^2)$

depth

$\log_5 n$



$n + \frac{n}{5} + \frac{n}{25} + \cdots$

$$\le n\left(1+\frac{1}{5}+\frac{1}{25}+\cdots\right)$$

$$\le n\left(\frac{1}{1-1/5}\right) = \underline{\underline{\frac{5}{4}n}}.$$

① Induction.

② Range and Domain Transformation

③ Recurrence Tree method.