CS:1010 DISCRETE STRUCTURES

PRACTICE QUESTIONS
LECTURE 2

Instructions

- Try these questions before class. Do not submit!

(1) Determine the truth value of each of these statements if the domain consists of all integers:
 (a) $\forall n(n+1>n)$ True since adding 1 to an integer always makes it greater.
 (b) $\exists n(2n=3n)$ True since it is true for $n=0$.
 (c) $\exists n(n=-n)$ True since $n=0$ this is true
 (d) $\forall n(3n \leq 4n)$ False since since if n is a negative number this is not true.

(2) Let $P(x)$ be the statement $x=x^2$. Domain is $\mathbb{Z}$, the set of integers. What are the truth values?
 (a) $P(0)$
 True
 (b) $P(1)$
 True
 (c) $P(2)$
 False since $2 \neq 2^2 = 4$.
 (d) $\forall x P(x)$
 False since not true for $x=2$

(3) Suppose the domain of $P(x)$ is $\{1,2,3,4\}$ then express $\exists x P(x)$ without a quantifier.
$P(1) \vee P(2) \vee P(3) \vee P(4)$.

(4) Express each of these statements using logical operators, predicates and qauntifiers.
Domain : All propositions. $T(x)$: x is a tautology. $C(x)$: x is a contradiction.
 (a) Some propositions are tautologies.
 $\exists x T(x)$
 (b) The negation of a contradiction is a tautology.
 Implicit is "all/every". Negation of every contradiction is always a tautology.
 That is, if x is a contradiction then $\neg x$ is a tautology.
 $\forall x\ (C(x) \to T(\neg x))$.

(5) What are the truth values of these statements?
 (a) $\exists! x P(x) \rightarrow \exists x P(x)$
 True since if there is a unique x satisfying $P(x)$ then there is an x satisfying $P(x)$.
 (b) $\forall x P(x) \rightarrow \exists! x P(x)$
 Unless the domain has one item in it, this will not be hold.

(6) Let $S(x)$: x is a student, $F(x)$: x is a faculty member and $A(x, y$: x has asked y a question. Domain: all people associated with our school. Use quantifiers to express each of these statements.
 (a) Divya has asked Prof. Gupta a question.
 A(Divya, Prof. Gupta)
 (b) Every student has asked Prof. Gupta a question.
 $\forall x(\ S(x) \rightarrow A(x, Prof.Gupta))$
 (c) Some student has not asked any faculty member a question.
 $\exists x(S(x) \wedge \forall y(F(y) \rightarrow \neg A(x, y))$ (there are other ways of writing it too!)

(7) Express each of the statements using predicated, quantifiers, logical connectives and mathematical operators.
 (a) Every positive real number has exactly two square roots.
 Exactly two objects that meet the same condition: we need two existential quantified variables.
 $\forall x > 0\ \exists a\ \exists b\ (a \neq b \wedge \forall c\ (c^2 = x \leftrightarrow (c = a \vee c = b)))$.
 (b) A negative real number does not have a square root that is a real number.
 $\forall x\ ((x < 0) \rightarrow \neg\exists y(x = y^2))$. (Domain is set of all real numbers.)

(8) Negate the statement such that negation immediately precedes predicates: $\forall x \exists y(P(x, y) \rightarrow Q(x, y))$.

$$\begin{aligned} \neg\forall x \exists y(P(x, y) \rightarrow Q(x, y)) &\equiv \exists x \neg\exists y(P(x, y) \rightarrow Q(x, y)) \\ &\equiv \exists x \forall y \neg(P(x, y) \rightarrow Q(x, y)) \\ &\equiv \exists x \forall y(P(x, y) \wedge \neg Q(x, y)). \end{aligned}$$

(9) Is this argument correct: " Every computer science student takes discrete mathematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science student."

Invalid. Applying universal instantiation it affirmed the conclusion.

(10) Explain the rules of inference used in each step. Each of the 93 students in this class own a laptop. Everyone who owns a laptop can use a PDF viewer.

Therefore, Arun, a student in this class can use a PDF viewer.

$C(x)$: x is in this class. $P(x)$: x owns a laptop. $Q(x)$: x can use a PDF viewer.

(a) $\forall x(C(x) \to P(x))$ – Hypothesis
(b) $C(Arun) \to P(Arun)$ – Universal Instantiation.
(c) $C(Arun)$ – Hypothesis
(d) $P(Arun)$ – Modus Ponens
(e) $\forall x(P(x) \to Q(x))$ – Hypothesis
(f) $P(Arun) \to Q(Arun)$ – Universal instantiation.
(g) $Q(Arun)$ – Modus Ponens

December 3, 2020; Dept of CSE, IIT Hyderabad