Assignment 3

Suraj - CS20BTECH11050

Download all python codes from

https://github.com/Suraj11050/Assignments-AI1103/blob/main/Assignment%203/ Assignment3.py

Download Latex-tikz codes from

https://github.com/Suraj11050/Assignments-AI1103/blob/main/Assignment%203/ Assignment3.tex

1 GATE 2009 (MA) PROBLEM 16

Let F, G and H be pair wise independent events such that $Pr(F) = Pr(G) = Pr(H) = \frac{1}{3}$ and $Pr(F \cap G \cap H) = \frac{1}{4}$ Then the probability that at least one event among F, G and H occurs is

(A)
$$\frac{11}{12}$$
 (B) $\frac{7}{12}$ (C) $\frac{5}{12}$ (D) $\frac{3}{4}$

(B)
$$\frac{7}{12}$$

(C)
$$\frac{5}{12}$$

(D)
$$\frac{3}{4}$$

2 SOLUTION

If two Events A and B are independent then

$$Pr(A \cap B) = Pr(A) \times Pr(B)$$
 (2.0.1)

F, G, H are pair wise independent events From Equation (2.0.1) we get the following results

$$\Pr(F \cap G) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$$\Pr(G \cap H) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$$\Pr(H \cap F) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

At least one event among F, G and H occur is $Pr(F \cup G \cup H)$. From Principal of inclusion and exclusion $Pr(F \cup G \cup H)$ is calculated as

$$Pr(F \cup G \cup H) = (Pr(F) + Pr(G) + Pr(H))$$
$$-Pr(F \cap G) - Pr(G \cap H)$$
$$-Pr(H \cap F) + Pr(F \cap G \cap H)$$

$$\Pr(F \cup G \cup H) = 3 \times \left(\frac{1}{3}\right) - 3 \times \left(\frac{1}{9}\right) + \frac{1}{4}$$

$$\therefore \Pr(F \cup G \cup H) = \frac{11}{12}$$

Hence Probability that at least one event among F, G, H occurs is $Pr(F \cup G \cup H) = \frac{11}{12}$ and correct answer is **Option** (A)

But we know that

$$(F \cap G \cap H) \subseteq (F \cap G)$$
$$\therefore \Pr(F \cap G \cap H) \le \Pr(F \cap G)$$

In the given question

$$\Pr(F \cap G \cap H) = \frac{1}{4}$$

$$\Pr(F \cap G) = \frac{1}{9}$$

$$\Pr(F \cap G \cap H) > \Pr(F \cap G)$$

Which is not possible and some of the probabilities turnout to be negative like

$$\Pr(F \cap G \cap H^{c}) = \frac{1}{9} - \frac{1}{4} = -\left(\frac{5}{36}\right)$$

$$\Pr(F \cap G^{c} \cap H) = \frac{1}{9} - \frac{1}{4} = -\left(\frac{5}{36}\right)$$

$$\Pr(F^{c} \cap G \cap H) = \frac{1}{9} - \frac{1}{4} = -\left(\frac{5}{36}\right)$$

Similar case with $Pr(G \cap H)$ and $Pr(H \cap F)$ Therefore Question is incorrect