CS 2443: Quiz 1

Department of Computer Science, IIT Hyderabad

3-Feb-2022

- Total marks: 10.
- Read the questions carefully and answer only to the questions.
- Submit the answers and mark them towards the questions in gradescope.
- Maintain academic honesty.

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1. Give the best possible asymptotic upper bounds for the following recurrence relations. Prove your answer. The base cases are $T(1) = 1$ for all the relations.

   (a) $T(n) = 5T(\frac{n}{2}) + n$ (1.5)

   (b) $T(n) = 2T(\frac{n}{5}) + T(\frac{n}{6}) + n$ (1.5)

   (c) $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}$ (1.5)

2. Consider the procedure PARTITION taught in the class.

```
PARTITION(A[1..n], p):
  swap A[p] ↔ A[n]
  ℓ ← 0                    ⟨⟨#items < pivot⟩⟩
  for i ← 1 to n − 1
      if A[i] < A[n]
          ℓ ← ℓ + 1
          swap A[ℓ] ↔ A[i]
  swap A[n] ↔ A[ℓ + 1]
  return ℓ + 1
```

   Supoose we call PARTITION with input $[2, 4, 3, 1, 6, 10, 9]$ and 5. Show the changes to the input array after each iteration of the **for** loop. (2)

3. What change the function XYZ will make to the input array $A[1 \dots n]$ of n integers? Write the loop-invariant for the **while** loop which will help us to prove the correctness of XYZ. (3.5)

Algorithm 1: XYZ($A[1 \dots n]$)

```
1  i ← 1;
2  j ← n;
3  while i < j do
4      if A[i] < 0 then
5          i ← i + 1;
6      else if A[j] ≥ 0 then
7          j ← j − 1;
8      else
9          swap A[i] ↔ A[j];
10         i ← i + 1;
11         j ← j − 1;
12     end
13 end
```