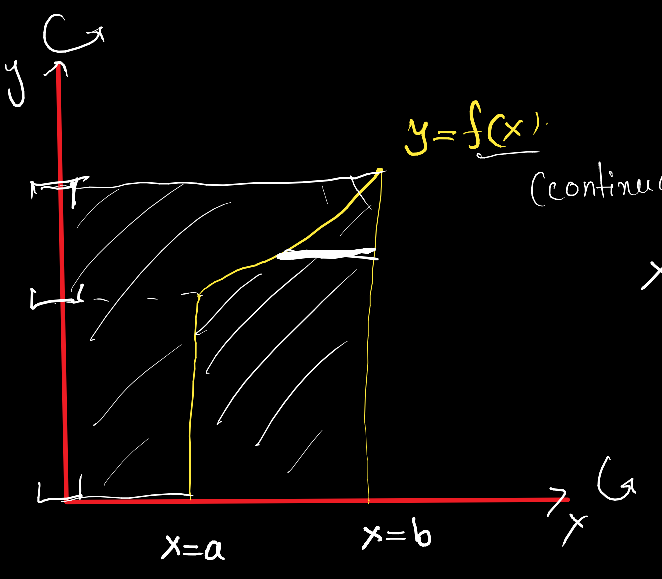
→ Disk method

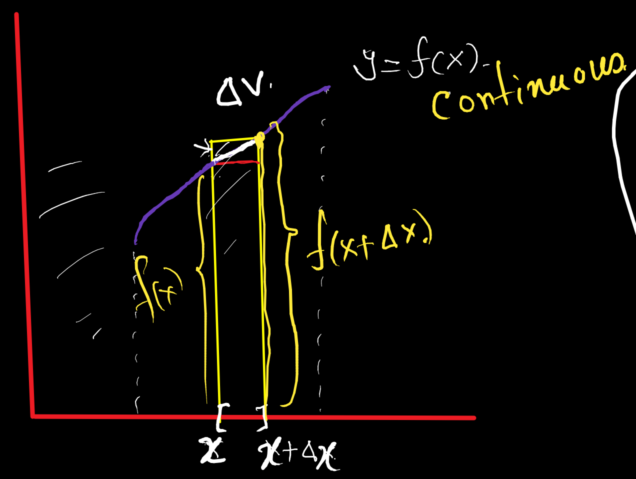
→ $\boxed{\text{Washer method}}$

Rotate around y axis.

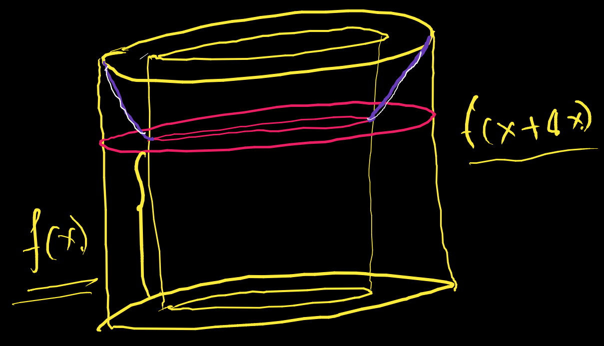


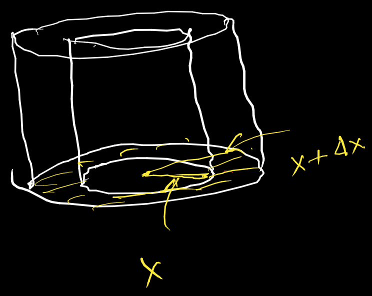
$x = f^{-1}(y)$

$\text{Vol(small hollow cyllinder)} \le \Delta V \le \text{vol. (big hollow cyl)}$



Cyllindrical shells method.





Area of base × height

$\pi((x+\Delta x)^2 - x^2)$.

Volume of the bigger cyllinder $= \pi((x+\Delta x)^2 - x^2)\, f(x+\Delta x)$ ✓

Volume of the smaller cyllinder $= \pi((x+\Delta x)^2 - x^2)\, f(x)$. ✓

$$\pi((x+\Delta x)^2 - x^2) f(x) \le \Delta V \le \pi((x+\Delta x)^2 - x^2)\, f(x+\Delta x).$$

$$\Rightarrow \pi\, \Delta x (2x + \Delta x) f(x) \le \Delta V \le \pi\, \Delta x\, (2x+\Delta x)\, f(x+\Delta x)$$

$\Delta x \ne 0$

> 0

$$\Rightarrow \pi\, (2x+\Delta x)\, f(x) \le \frac{\Delta V}{\Delta x} \le \pi\, (2x+\Delta x)\, f(x+\Delta x)$$

$\lim_{\Delta x \to 0}$ $\quad \Delta x \to 0 \quad$ $\lim_{\Delta x \to 0}$

$$\pi\, 2x\, f(x).$$

Since f is continuous $\lim_{\Delta x \to 0} f(x+\Delta x) = f(x).$

$$\boxed{\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \frac{dV}{dx}.}$$

$$\frac{dV}{dx} = \pi \cdot 2x \cdot f(x)$$

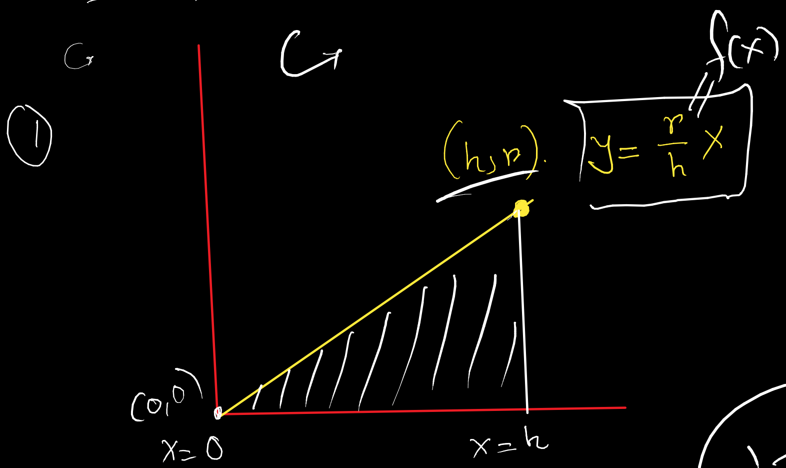
$$V = \int_a^b 2\pi x f(x)\, dx.$$

$$\Rightarrow V = \int_a^b \left(\frac{dV}{dx}\right) dx$$

$$= \int_a^b 2\pi\, x\, f(x)\, dx.$$

Examples:

①



Rotate about y axis:

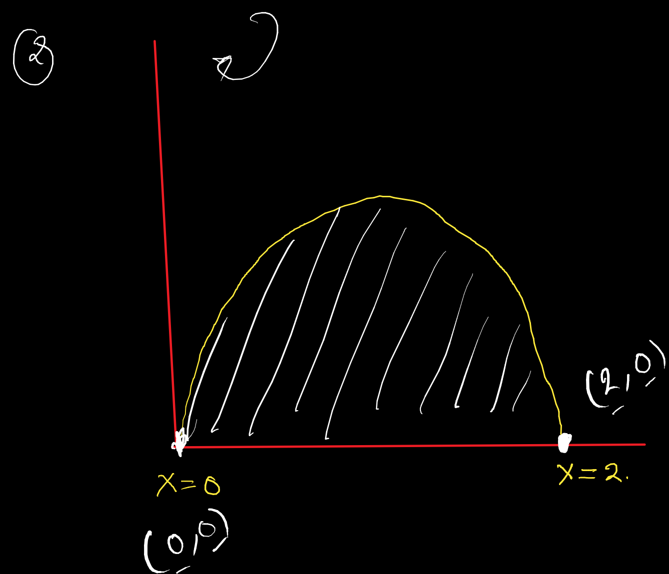
$$\text{Volume} = \int_0^h 2\pi x\, f(x)\, dx$$

$$= \int_0^h 2\pi x \frac{r}{h} x\, dx.$$

$$= \frac{2\pi r h^2}{3}.$$

$\frac{1}{3}\pi r^2 h$

②



Determine the volume of the solid obtained by the region bounded by $y = 2x - x^2$. and x-axis about the y axis.

Ans:

$$\text{Volume} = \int_0^2 2\pi x f(x)\, dx$$

$$= \int_0^2 2\pi x (2x - x^2)\, dx = \frac{8\pi}{3}.$$