**CS:1010 DISCRETE STRUCTURES**

PRACTICE QUESTIONS
LECTURE 12

## Instructions

- Try these questions before class. Do not submit!

(1) Suppose that the function $f$ satisfies the recurrence relation

$$f(n) = 2f(\sqrt{n}) + log\ n,$$

whenever $n$ is a perfect square greater than 1 and $f(2) = 1$.
(a) Find $f(16)$.
(b) Find a big-$\mathcal{O}$ estimate for $f(n)$. [Hint: Make the substitution $m = log\ n$.]

(2) Find a closed form for the generating function for each of these sequences. A closed form means an algebraic expression not involving a summation over a range of values or the use of ellipses. For each sequence, use the most obvious choice of a sequence that follows the pattern of the initial terms listed.
(a) $0, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, \ldots$
(b) $0, 0, 0, 1, 1, 1, 1, 1, \ldots$
(c) $0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, \ldots$
(d) $2, 4, 8, 16, 32, 64, 128, \ldots$
(e) $\binom{7}{0}, \binom{7}{1}, \binom{7}{2}, \ldots, \binom{7}{7}, 0, 0, 0, 0, 0, \ldots$
(f) $2, -2, 2, -2, 2, -2, 2, -2, \ldots$
(g) $1, 1, 0, 1, 1, 1, 1, 1, 1, \ldots$
(h) $0, 0, 0, 1, 2, 3, 4, \ldots$

(3) Use generating functions to find the number of ways to choose a dozen bagels from three varietiesegg, salty, and plainif at least two bagels of each kind but no more than three salty bagels are chosen.

(4) Use generating functions to solve the recurrence relation $a_k = 4a_{k-1} - 4a_{k-2} + k^2$ with initial conditions $a_0 = 2$ and $a_1 = 5$.

(5) Use generating functuns to prove Pascal's identity: $C(n, r) = C(n-1, r) + C(n-1, r-1)$ when $n$ and $r$ are positive integers with $r < n$. Hint : use the identity,

$$(1 + x)^n = (1 + x)^{n-1} + x(1 + x)^{n-1}.$$

FEBRUARY 12, 2021; DEPT OF CSE, IIT HYDERABAD