

**RV COLLEGE OF ENGINEERING®**  
 (An Autonomous Institution Affiliated to VTU)  
 III Semester B. E. Examinations April-2022

**Common CSE / ISE**

**LINEAR ALGEBRA, LAPLACE TRANSFORMS AND  
COMBINATORICS**

*Time: 03 Hours*

*Maximum Marks: 100*

*Instructions to candidates:*

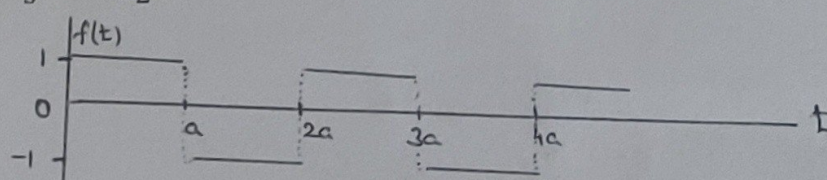
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6.

**PART-A**

1	1.1	Is the set of vectors $\{(1,2,1), (2,1,0), (1,-1,2)\}$ linearly independent or not?	02
	1.2	Write the induced matrix in the following transformations: i. Projection of $xz$ -plane in $R^3$ ii. Counter clockwise rotation through an angle $\theta$ about the positive $y$ -axis in $R^3$ .	02
	1.3	What multiple of $a_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ should be subtracted from $a_2 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ to make the result orthogonal to $a_1$ ?	02
	1.4	If $\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$ is an Eigen vector of $\begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -4 \end{bmatrix}$ , then the Eigen value corresponding to the Eigen vector is _____.	02
	1.5	Evaluate $\int_0^{\infty} e^{-3t} \cos^2 t \, dt$ using Laplace transforms.	02
	1.6	Find $L^{-1} \left[ \frac{1}{\sqrt{2s+3}} \right]$	02
	1.7	The total number of positive divisors of 1412 are _____.	02
	1.8	The Euler's totient function $\phi$ for the integer 219 is _____.	02
	1.9	Calculate the number of dearrangements of $d_4$ . Hence write corresponding dearrangements.	02
	1.10	Find the generating function for the sequence 2,4,8,16,32,...	02

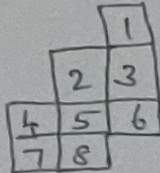


# PART-B

2	a	Determine the basis and dimension for the row space, column space and null space of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & -4 & 8 \\ -1 & 1 & 3 & -5 \\ -1 & 2 & 5 & -6 \\ -1 & -2 & -3 & 1 \end{bmatrix}$	08
	b	Examine whether following sets forms a subspace or not? i. $M_{22} = \{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} / a, b \text{ are integers} \}$ on the set of all $2 \times 2$ matrices. ii. $S = \{ (a, b, c) / a + b + c = 0 \text{ and } a, b, c \in R \}$ as the set of all real numbers.	08
3	a	Find a third column so that the matrix $Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{14} & - \\ 1/\sqrt{3} & 2/\sqrt{14} & - \\ 1/\sqrt{3} & -3/\sqrt{14} & - \end{bmatrix}$ is orthogonal. Verify that the rows automatically become orthogonal at the same time.	08
	b	Diagonalize the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ .	08
OR			
4	a	Obtain the QR factorization for the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ using Gram Schmidt process.	08
	b	Obtain the singular value decomposition of the matrix $\begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$	08
5	a	The periodic function $f(t)$ is shown in the Fig 5a below. Write a mathematical expression for $f(t)$ and hence show that $L[f(t)] = \frac{1}{s} \tanh\left(\frac{as}{2}\right)$ 	08
	b	Obtain the Laplace transforms of the following functions. i. $e^{-t} \int_0^t \frac{e^{2t} \sin 3t}{t} dt$ ii. $\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^3$	08



**OR**

<p>6</p> <p>a</p> <p>b</p>	<p>Using convolution theorem, evaluate <math>\left(\frac{s^2}{(s^2+16)(s^2+a)}\right)</math>.</p> <p>Solve by using Laplace transforms <math>\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 3te^{-t}</math> given that <math>x = 4, \frac{dx}{dt} = 2</math> when <math>t = 0</math>.</p>	<p>08</p> <p>08</p>
<p>7</p> <p>a</p> <p>b</p>	<p>Find the gcd(12378, 3054) using the Euclidean algorithm and also find the integers <math>x</math> &amp; <math>y</math> to satisfy <math>12378x + 3054y = d</math>.</p> <p>Given the public key <math>(e, n) = (7, 51)</math>, encrypt plain text <math>LIV</math>, where the alphabets <math>A, B, C, \dots, X, Y</math>, are assigned the numbers 3, 4, 5, ..., 26, 27, 28.</p> <p>Give the cipher text and also find the private key <math>d</math>.</p>	<p>08</p> <p>08</p>
<p>8</p> <p>a</p> <p>b</p>	<p>Using expansion formula, find the rook polynomial for the board shown in Fig 8a.</p> <div style="text-align: center;">  <p>Fig 8a</p> </div> <p>How many integers between 1 and 300 are</p> <ol style="list-style-type: none"> <li>Divisible by at least one of 5, 6, 8?</li> <li>Divisible by none of 5, 6, 8?</li> </ol>	<p>08</p> <p>08</p>