Linear Algebra (10GB511)
Assignment – II

Marks: 100 **Last date for submission: 23 – 10 – 2013**

1. (a) The solution to the linear differential equation $\frac{d^2u}{dt^2} = u$ form a vector space, since combination of solutions is still a solution. Find two independent solutions to give a basis for that solution space.
(b) With the initial values u = x and u' = y at t = 0, what combination of basis vectors solves $u''(t) = u$? The transformation from initial values to the solutions is linear. What is its 2 x 2 matrix?

2. On the space P_4 of cubic polynomials, what matrix represents d^2/dt^2 ? Find its null space and column space. What do they mean in terms of polynomials?

3. What is the result of the product of 5 reflections (45^0 line) and 8 rotations (90^0) of the xy-plane?

4. Let T: $P_3 \longrightarrow P_4$ be such that every cubic polynomial is multiplied by (2+3 t). Find the matrix of this transformation. What is its order?

5. Which of these transformations are linear?
(a) $T(v) = v_1 + v_2 + v_3$ (sum of components of v)
(b) $T(v) = \min v_i$ for i = 1, 2, 3
(c) $T(v) = (0, v_1)$

6. The space of 2 x 2 matrices M has the four basis "vectors" as given by
$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
For the linear transformation T: M $\longrightarrow$ M defined by $T(M) = M^T$, find the matrix A with respect to the above basis. Why is $A^2 = I$?

7. Find the 4 x 3 matrix A that represents right shift from R^3 to R^4 : (x_1, x_2, x_3) is transformed to $(0, x_1, x_2, x_3)$. Find also the left shift matrix B from R^4 to R^3, transforming (x_1, x_2, x_3, x_4) to (x_2, x_3, x_4). What are the products AB and BA?

8. Is T^2 is a linear transformation, if T: $R^3 \rightarrow R^3$, is linear? Justify.

9. (a) Which matrix transforms (1, 0) to (1, 1) and (0, 1) to (0, 2)?
(b) Which matrix transforms (2, 5) to (1, 0) and (1, 3) to (0, 1)?
(c) Which matrix transforms (2, 5) to (1, 1) and (1, 3) to (0, 2)?

10. If T: $R^2 \rightarrow R^2$, find T(T(v)) for the following transformations:
(a) T(v) = - v
(b) T(v) = v + (1, 1)
(c) $T(v) = (v_1, 0)$
(d) $T(v) = (v_2, v_1)$

11. Find the range and kernel of the linear transformations:

(a) $T(v_1, v_2) = (v_2, v_1)$
(b) $T(v_1, v_2) = (v_2, v_2)$
(c) $T(v_1, v_2, v_3) = (v_1, v_2)$
(d) $T(v_1, v_2) = (0, 0)$

12. Find the left inverse / right inverse of:
$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ 2 & 5 \\ 4 & 12 \end{pmatrix}$

13. (i) Find the reflection of (-1, 2) about (a) x-axis (b) y-axis (c) the line y = x.
(ii) Find the projection of (2, -5) on (a) x – axis (b) y – axis (c) $\theta = 30^0$ line.
(iii) Find the image of the vector (3, −4) when it is rotated through an angle of (a) $\theta = 30^0$ (b) $\theta = -60^0$ (c) $\theta = 45^0$

14. $A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$ then show that the identity matrix I is not in the range of T. Find $M \neq 0$ such that T(M) = A M is zero.

15. The matrix $M = \begin{pmatrix} r & s \\ t & u \end{pmatrix}$ transforms (1, 0) to (r, t) and (0, 1) to (s, u) respectively.
The matrix $N = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1}$ transforms (a,c) to (1,0) and (b,d) to (0,1). How do you transform (a, c) to (r, t) and (b, d) to (s, u)? What is the matrix of this transformation?

16. Let $\{[1\ 0\ 0]^T, [0\ 1\ 0]^T, [0\ 0\ 1]^T\}$ and $\{[-1\ 2\ 2]^T, [2\ 2\ -1]^T, [2\ -1\ 2]^T\}$ be two bases of R^3. Compute the matrix of transformation from one basis to the other.

17. Which of the transformations satisfy T(u + v) = T(u) + T(v) and which satisfy T(c.u) = c.T(u)?
(a) $T(u) = u_1 + u_2 + u_3$
(b) $T(u) = (u_1, 2u_2, 3u_3)$
(c) $T(u) = \min u_i \quad (1 \le i \le 3)$
(d) $T(u) = (u_1 + 1, u_2 - 1)$

18. A linear transformation $T: R^4 \longrightarrow R^3$ is defined by
$$w_1 = 2x_1 - 3x_2 + x_3 - 5x_4$$
$$w_2 = 4x_1 + x_2 - 2x_3 + x_4$$
$$w_3 = 5x_1 - x_2 + 4x_3$$
Write the matrix of the linear transformation.

19. Which matrix has the effect of rotating every vector through 90° and then projecting the result onto the x -axis? What matrix represents projection onto the x-axis followed by projection onto the y-axis?

20. Suppose $\{v_1, v_2, v_3\}$ are eigenvectors for T. Thus $T(v_i) = v_i$, for i = 1, 2, 3. What is the matrix for T when the input and the output bases are v’s?