SAMPLING DISTRIBUTION OF MEAN (σ UNKNOWN)

t-Distribution

t-distribution is used when sample size is < 30 and the population standard deviation is unknown.

t-statistic is defined as

$$\boxed{t = \left(\frac{\bar{x} - \mu}{s}\right)\sqrt{n}}$$

where Sample standard deviation $s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$

n is Sample size

$\bar{x}$ is sample mean

μ is population mean

Suppose we obtain a frequency distribution of t by computing the value of t for each of the samples of size n drawn from a normal or a nearly normal population. The sampling distribution so obtained is called the "Student's" t- distribution.

Problems

1. A random sample of size 16 has 53 as mean. The sum of squares of the deviation from mean is 135. Compute t-statistic by taking the population mean as 56.

$n=16, \ \bar{X}=53, \ \sum(X-\bar{X})^2=135, \ \mu=56$

$$s=\sqrt{\frac{\sum(X-\bar{X})^2}{n-1}}=\sqrt{\frac{135}{15}}=3$$

t - statistics $t=\left(\frac{\bar{x}-\mu}{s}\right)\sqrt{n}=\left(\frac{53-56}{3}\right)\sqrt{16}$

$$=-4$$

(2) A sample of 20 items has mean 42 units and standard deviation 5 units. Find the t - score if the population mean is 45 units.

$\bar{X}=42$

$n=20$

$s=5$

$\mu=45$

$$t=\left(\frac{\bar{x}-\mu}{s}\right)\sqrt{n}=\left(\frac{42-45}{5}\right)\sqrt{20}$$

$$=-2.6832$$

(3) The 9 items of a sample have the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Find the t-statistics if the population mean is 47.5

$n=9$

$\mu=47.5$

$$\bar{X}=\frac{1}{9}\left[45+47+50+52+48+47+49+53+51\right]$$

$$=\frac{1}{9}(442)=49.111$$

$$\sum(X-\bar{X})^2=(45-49.111)^2+2(47-49.111)^2+(50-49.111)^2$$
$$+(52-49.111)^2+(48-49.111)^2+(49-49.111)^2+(53-49.111)^2$$
$$+(51-49.111)^2$$
$$=16.90+8.9126+0.790321+8.3463+1.2343+0.0123$$
$$+15.124+3.568$$

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{54.8878}{8}} = \sqrt{6.8609} = 2.62$$

$$t = \left(\frac{\bar{x}-\mu}{s}\right)\sqrt{n} = \left(\frac{49.111 - 47.5}{2.62}\right)\sqrt{9}$$

$$= 1.84465$$

4. Ten individuals are chosen at random from a normal population of students and their marks found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 70. Find the statistics t score if the mean marks of the population of students is 66.

$n = 10$

$\mu = 66$

$$\bar{x} = \frac{1}{10}[63+63+66+67+68+69+70+70+71+70] = \frac{677}{10} = 67.7$$

$$\sum (x-\bar{x})^2 = 2(63-67.7)^2 + (66-67.7)^2 + (67-67.7)^2 + (68-67.7)^2 + (69-67.7)^2 + 3(70-67.7)^2 + (71-67.7)^2$$

$$= 44.18 + 2.89 + 0.49 + 0.09 + 1.69 + 15.87 + 10.89$$

$$= 75.61$$

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{75.61}{9}} = 2.9$$

$$t = \left(\frac{\bar{x}-\mu}{s}\right)\sqrt{n} = \left(\frac{67.7-66}{2.9}\right)\sqrt{10} = 1.8537$$

5. A filling machine is expected to fill 5 kg of powder into bags. A sample of 10 bags gave the following weights 4.7, 4.9, 5, 5.1, 5.4, 5.2, 4.6, 5.1, 4.6 & 4.7. Find the t-score.

6. The following values gives the lengths of 12 samples of egyptian cotton taken from a consignment: 48, 46, 49, 46, 52, 45, 43, 47, 47, 46, 45, 50. Find the t-score if the mean length of the consignment is 46.

SAMPLING DISTRIBUTION OF VARIANCE

CHI-SQUARE DISTRIBUTION

When a coin is tossed 200 times, the theoretical considerations lead us to expect 100 heads and 100 tails. But in practice, these results are rarely achieved. The quantity χ^2 describes the magnitude of discrepancy between theory and observations. If $\chi = 0$, the observed & expected frequencies completely coincide. The greater the discrepancy between the observed and expected frequencies, the greater is the value of χ^2. Thus χ^2 affords a measure of the correspondence between theory and observation.

If $f_1, f_2, f_3 \cdots f_n$ is a set of observed (experimental) frequencies and $e_1, e_2, e_3 \cdots e_n$ is the corresponding set of expected (theoretical or hypothetical) frequencies, then the statistics χ^2 is defined as

$$\chi^2 = \frac{(f_1 - e_1)^2}{e_1} + \frac{(f_2 - e_2)^2}{e_2} + \cdots + \frac{(f_n - e_n)^2}{e_n}$$

$$\boxed{\chi^2 = \sum_{k=1}^{n} \frac{(f_k - e_k)^2}{e_k}}$$

where $\sum_{K=1}^{n} f_K = \sum_{K=1}^{n} e_K = N$ [Total Frequency]

and degrees of freedom (d.f.) $= n-1$

Note :- (i) If $\chi^2 = 0$, the observed and theoretical frequencies agree exactly

(ii) If $\chi^2 > 0$ they do not agree exactly.

PROBLEMS

1. The following table gives the number of road accidents that occured in a large city during the various days of a week. Calculate χ^2 for the above data.

Day	Sun	Mon	Tue	Wed	Thus	Fri	Sat	Total
No of accidents	14	16	08	12	11	9	14	84

$N = 84$, no of days $= 7$

$$\therefore \; e_i = \frac{84}{7} = 12$$

$$\chi^2 = \frac{(14-12)^2}{12} + \frac{(16-12)^2}{12} + \frac{(8-12)^2}{12} + \frac{(12-12)^2}{12} + \frac{(11-12)^2}{12} + \frac{(9-12)^2}{12} + \frac{(14-12)^2}{12} = 4.17$$

2. In 200 tosses of a coin, 118 heads and 82 tails were observed. Find the χ^2.

observed frequency of heads & tails

$$f_1 = 118 \ , \ f_2 = 82$$

$$N = \sum f_i = 118 + 82 = 200$$

Expected frequency of heads & tails on 200 trials are $e_1 = 200 \times \frac{1}{2} = 100$, $e_2 = 200 \times \frac{1}{2} = 100$.

$$\chi^2 = \frac{(f_1 - e_1)^2}{e_1} + \frac{(f_2 - e_2)^2}{e_2}$$

$$= \frac{(118-100)^2}{100} + \frac{(82-100)^2}{100} = 6.48$$

(3) A survey of 240 families with 3 children each revealed the distribution shown in the following table. ~~Is the~~ Find χ^2.

No of children	3B 0G	2B 1G	1B 2G	0B 3G
No of families	37	101	84	18

Sol^n :- Let p = pbty of male births
q = Pbty of female births

$$P = \frac{1}{2} \quad q = \frac{1}{2}$$

Among 3 children, the pbty that x childrens are boys is given by $3c_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{3-x}$

$$P(3B) = 3c_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{3-3} = 1/8$$

$$P(2B) = 3c_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 = 3/8$$

$$P(1B) = 3c_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{3-1} = 3/8$$

$$P(0B) = 3c_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{3-0} = 1/8$$

Among 240 families, the expected no of families with 3B is $e_1 = 240 \times \frac{1}{8} = 30$

2B is $e_2 = 240 \times 3/8 = 90$

1B is $e_3 = 240 \times 3/8 = 90$

0B is $e_4 = 240 \times 1/8 = 30$

From the table $f_1 = 37$, $f_2 = 101$, $f_3 = 84$, $f_4 = 18$

$$\chi^2 = \frac{(37-30)^2}{30} + \frac{(101-90)^2}{90} + \frac{(84-90)^2}{90} + \frac{(18-90)^2}{30}$$

$$= 8.1773$$