USN

R. V. COLLEGE OF ENGINEERING

Autonomous Institution Affiliated to VTU

III Semester B. E. Examinations, December – 2017

BRANCH: Computer Science & Engineering and Information Science & Engineering

COURSE: FOURIER SERIES, LAPLACE TRANSFORMS AND LINEAR ALGEBRA

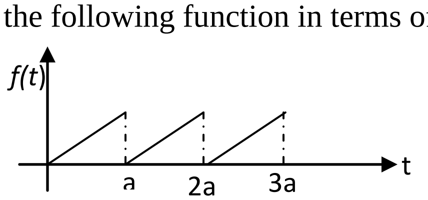
MODEL QUESTION PAPER - II

Time: 03 Hours — ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A question should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6.

PART – A

1	**1.1**	$L\{5^t\} =$ ______________.	**01**
	1.2	If $L\left\{\frac{\sin t}{t}\right\}$ is $\tan^{-1}\left(\frac{1}{s}\right)$, then $L\left\{\frac{\sin 2t}{2t}\right\} =$ ___________.	**01**
	1.3	$L^{-1}\left\{\frac{se^{-\pi s}}{s^2+9}\right\}$ ______________.	**01**
	1.4	$L^{-1}\{F(s)\} = f(t)$, then $L^{-1}\left\{\frac{F(s-a)}{s}\right\} =$ ____________.	**01**
	1.5	The function *sin(nx)* is periodic with period ______________.	**01**
	1.6	The complex form of Fourier series *f(x)* in the interval *(a, a+2l)* is ________.	**01**
	1.7	The subset E of R^2 defined by E = $\{(x, y) \in R^2 \mid 2x + 3y = 4\}$ is not a subspace of R^2. Justify the statement.	**01**
	1.8	State the Rank Nullity theorem for an m x n matrix A.	**01**
	1.9	Let T : $R^2 \to R^2$ be the projection of $\begin{bmatrix} x \\ y \end{bmatrix}$ on to X – axis i. e. T$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$, the Ker (T) = ____________.	**01**
	1.10	In the singular value decomposition $A = U\sum V^T$, C(A) = span of ___________.	**01**
	1.11	Express the following function in terms of unit step function. 	**02**

1.12	Find $L^{-1}\left\{\log\left(1-\frac{1}{s^2}\right)\right\}$.	02
1.13	If $x^2. = \frac{\pi^2}{3} - 4\left(\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2}\cdots\cdots\right)$ in $[-\pi,\pi]$, $0 \le x \le \pi$ then $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\cdots=$ ________.	02
1.14	If A is a 64 x 17 matrix of rank 11, how many independent vectors satisfy Ax = 0? How many independent vectors satisfy $A^T y = 0$.	02
1.15	Verify directly from $c^2 + s^2 = 1$ that reflection matrices satisfy $H^2 = I$.	02

PART - B

2	a	Evaluate $(i) L\left\{\int_0^t te^{-3t}\sin 2t\, dt\right\}$ $\quad (ii)\int_0^\infty e^{-t}\left(\frac{1-\cos t}{t}\right)dt$.	08
	b	Express the following in terms of Heaviside unit step function and also sketch the graph of the function $f(t)=\begin{cases}\cos t & 0<t\le\pi\\ 1 & \pi<t\le 2\pi\\ \sin t & t>2\pi\end{cases}$ and find its Laplace transform.	08
3	a	Find the Inverse Laplace transform of the following $\log\left[\frac{s^2+a^2}{(s-a)(s-b)^2}\right]$.	08
	b	A small oscillations of a certain system with two degrees of freedom are given by $\frac{d^2x}{dt^2}+3x-2y=0,\quad \frac{d^2x}{dt^2}+\frac{d^2y}{dt^2}-3x+5y=0,\quad x=y=0,\quad \frac{dx}{dt}=3,\quad \frac{dy}{dt}=2$ at t = 0. Find the solution.	08
		OR	
4	a	Using convolution theorem, find $L^{-1}\left\{\frac{s}{(2s^2+1)(s^2-4)}\right\}$.	08
	b	Solve $y''-3y'+2y=4t+e^{3t}$, given that $y(0)=1, y'(0)=1$.	08
5	a	A sinusoidal voltage $E\sin\omega t$ is passed through a half-wave rectifier which clips the negative portion of the wave. Find the resulting periodic function $U(t)=\begin{cases}0 & -T/2<t<0\\ E\sin\omega t & 0<t<T/2\end{cases}, T=2\pi/\omega$, in a Fourier series.	08
	b	Find the half- range cosine series for the function $f(x)=\sin\left(\frac{m\pi x}{l}\right)$, where m is a positive integer over the interval (0, l).	08
		OR	

6	**a**	Expand $f(x)=\sqrt{1-\cos x}, 0<x<2\pi$ in a Fourier series, Hence, evaluate $\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots$.	**08**
	b	Obtain the complex form of the Fourier series for $f(x)=e^{x}$ in $-\pi<x<\pi$.	**08**
7	**a**	Find the basis and dimension of the four fundamental subspaces of the matrix $A=\begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$.	**08**
	b	Apply the Gram – Schmidt orthogonalization processes to find an orthonormal basis for the subspace U of R^4 spanned by $v_1 = (1, 1, 1, 1)$, $v_2= (1, -3, -4, -2)$.	**08**
8	**a**	If $T : R^2 \rightarrow R^2$ be defined by $T(x, y) = (2x + 3y, 4x – 5y)$. Find the matrix representation $[T]_B$ of T relative to the basis $B = \{u_1, u_2\}$, where $u_1 = (1, -2)$, $u_2 = (2, -5)$.	**08**
	b	Obtain the singular value decomposition of $A=\begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$.	**08**