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RV COLLEGE OF ENGINEERING®
Autonomous Institution affiliated to VTU
III Semester B. E. Examinations April/May 2024
LINEAR ALGEBRA AND PROBABILITY THEORY
(Common to CS, CD, CY, IS)
(2022 SCHEME)
MODEL QUESTION PAPER

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.

PART – A

1	1.1	The value of k such that the vectors $(1,0,3),(k,2,5),(2,1,4)$ are linearly dependent is _____.	02
	1.2	If the vector $\begin{bmatrix}1\\2\end{bmatrix}$ is rotated through an angle of 30^0counterclockwise, then the resultant vector is _______.	02
	1.3	The orthogonal projection of $(7,6)$ onto $(4,2)$ is ______________.	02
	1.4	Find the matrix A whose eigenvalues are 2 and 5 and corresponding eigenvectors are $\begin{bmatrix}-1\\2\end{bmatrix}$ and $\begin{bmatrix}1\\1\end{bmatrix}$ respectively.	02
	1.5	The probability distribution of a random variable X is given by the following table: $\begin{array}{|c|c|c|c|c|c|c|}\hline x & -2 & -1 & 0 & 1 & 2 & 3 \\ \hline P(x) & 0.1 & k & 0.2 & 2k & 0.3 & k \\ \hline\end{array}$ Find the value of k.	02
	1.6	Find the constant b so that $p(x,y)=\begin{cases} bxe^{-y}, & 0<x<0.5,\ \ y>0 \\ 0, & otherwise\end{cases}$ is a joint probability density function.	02
	1.7	Suppose the life expectancy X (in hours) of a transistor tube is exponential distributed with mean life 180. Then the probability that the tube will last between 36 and 90 hours is ________.	02
	1.8	The weights of 1500 ball bearings are normally distributed with a mean of 635gms and standard deviation of 1.36gms. If 300 random samples of size 36 are drawn from this population, determine the standard deviation of the sampling distribution of means if sampling is done without replacement.	02
	1.9	The mean and variance of a χ^2 distribution with 8 degrees of freedom is ______	02

		and ________ respectively.	
	1.10	The region of rejection for the right tailed test at 0.05 level of significance and that for the left tailed test at 0.01 level of significance are ______ and ______ respectively.	02

PART – B

2	a	Determine the basis for row space, column space and null space of the matrix $\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}$.	6
	b	Show that the set $B = \{(1,1),(1,-1)\}$ is a basis of the vector space $\mathbb{R}^2$.	5
	c	Find the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ such that $T(1,1) = (1,0,2)$ and $T(2,3) = (1,-1,4)$ hence find $T(4,5)$.	5

3	a	Use Gram – Schmidt process to find the orthogonal basis for the column space of the matrix $\begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$.	6
	b	Find the singular value decomposition of the matrix $\begin{bmatrix} 7 & 1 \\ 0 & 0 \\ 5 & 5 \end{bmatrix}$	10
OR			
4	a	Find the QR factorization of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.	8
	b	Suppose the matrix A can be resolved as $A = PDP^{-1}$. Given $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$, find P, D and P^{-1}.	8

5	a	A shipment of 20 similar computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives. Determine (i) Mean and standard deviation of the distribution. (ii) Cumulative distribution	8
	b	Suppose that the error in the reaction temperature X, in ◦C and pressure Y for a controlled laboratory experiment are modelled as continuous random variables with $f(x,y) = \begin{cases} 96x^2y^3, 0 \le x \le 0.5, \quad 0 \le y \le 1 \\ 0, \qquad otherwise \end{cases}$ (i) Show that it is a valid joint probability density function. (ii) Find marginal density functions of X and Y. (iii) Find the conditional probabilities $f(y\|x)$ and $f(x\|y)$. (iv) Covariance of X and Y.	8
OR			
6	a	The weekly demand for a drinking water product, in thousands of liters, from a	8

local chain of efficiency stores is a continuous random variable X with probability function:

$$p(x) = \begin{cases} 0.75(x^2+1), & 0 \le x \le c \\ 0, & \text{otherwise} \end{cases}$$

. For what value of c is $p(x)$ a probability density function. Also

(i) Find $P(X < 0.5)$.

(ii) Evaluate mean and standard deviation of the distribution.

(iii) Find the cumulative distribution function of X.

b The joint probability distribution of two random variables X and Y is given by the following table **8**

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Compute $E(X)$, $E(Y)$, $COV(X, Y)$ and correlation of X and Y.

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7 a From previous tests it is known that the probability of a resistor being out of tolerance is 0.15. From a large batch, a random sample of 20 resistors is chosen so that the probability remains constant. Evaluate using binomial distribution the probabilities that **6**

(i) exactly 7 will be out of tolerance.

(ii) at least 3 will be out of tolerance.

(iii) at most 2 will be out of tolerance

b The sales per day in a shop are distributed exponentially with the average sale amounting to Rs.100 and net profit is 8%. Find the probability that the net profit exceeds Rs.30. **5**

c Suppose it is known that 43% of Americans own an iPhone. If a random sample of 50 Americans were surveyed, what is the probability that the proportion of the sample who owned an iPhone is between 45% and 50%? **5**

OR

8 a A city installs 3000 electric lamps for street lighting. These lamps have a mean burning life of 1000 hours and a standard deviation of 150 hours. What is the probability that a lamp will fail between 950 and 1350 burning hours? **6**

b An automobile manufacturer is concerned about a fault in the braking mechanism of a particular model. The fault can cause a catastrophe at high speed. The distribution of the number of cars per year that will experience the catastrophe is a Poisson random variable with mean 5. **5**

(i) What is the probability that at most 3 cars per year will experience a catastrophe?

(ii) What is the probability that more than 1 car per year will experience a catastrophe?

c The distribution of heights of a certain breed of terrier has a mean of 72 centimeters and a standard deviation of 10 centimeters, whereas the distribution of heights of a certain breed of poodle has a mean of 28 centimeters with a standard deviation of 5 centimeters. Assuming that the sample means can be measured to any degree of accuracy, find the probability that the sample mean for a random sample of heights of 64 terriers exceeds the sample mean for a random sample of heights of 100 poodles by at most 44.2 centimeters. **5**

<table>
<tr><td>9</td><td>a</td><td>The mean and standard deviation of the maximum loads supported by 60 cables are 11.09 tonnes and 0.73 tonnes respectively. Find (a) 95% (b) 99% confidence limits for mean of the maximum loads of all cables produced by the company.</td><td>8</td></tr>
<tr><td></td><td>b</td><td>A die is thrown 264 times and the number appearing on the face(x) follows the following frequency distribution.
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>f</td><td>40</td><td>32</td><td>28</td><td>58</td><td>54</td><td>60</td></tr></table>Calculate the value of χ^2.</td><td>8</td></tr>
<tr><td colspan="4" align="center">OR</td></tr>
<tr><td>10</td><td>a</td><td>The mathematics score at a state university are normally distributed with a mean of 14.3 and a standard deviation of 2.4. The placement coordinator wishes to test whether the mean score on a revised version of the exam has increased from 14.3. She gives the revised exam to 30 entering freshman and mean score is found to be 14.6. State a null hypothesis and alternate hypothesis for testing the revised exam. What are the statistics and P value of the test.</td><td>8</td></tr>
<tr><td></td><td>b</td><td>A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure: $5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4$ (in appropriate units). Can it be concluded that, on the whole, the stimulus will change the blood pressure. Use $t_{005}(11) = 2.201$.</td><td>8</td></tr>
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