RV Educational Institutions

RV College of Engineering

Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi | Approved by AICTE, New Delhi

Academic year 2021-2022 (Odd semester 2021)

DEPARTMENT OF MATHEMATICS			
Date	03 January 2021	Time	10 : 00 AM to 11: 30 AM
Test	II	Maximum Marks	50
Course Title	LINEAR ALGEBRA, LAPLACE TRANSFORM AND COMBINATORICS	Course Code	18MA31A
Semester	III	Programs	CSE & ISE

Sl. No.	Questions	M	BT	CO
1.	Verify whether the following sets forms a subspace or not. Justify your answer. a) $P = \{a_0 + a_1x + a_2x^2 + a_3x^3$, set of polynomials of degree 3 for which $a_0 = 0\}$. b) $M = \{M_{2\times2}$, the set of all 2 x 2 matrices such that $\|A\| = 0\}$. c) $S = \{(x, y)$ such that either $x = 0$ or $y = 0\}$ in R^2. d) F = {The set of all polynomials f's such that f (0) = 1}. e) $S = \{(x, y, z): x^2 + y^2 + z^2 \leq 1\}$ in R^3.	10	1	1
2.	a) Determine a value for q such that the following vectors are linearly independent $\{(1, 1, 2, 1), (2, 1, 2, 3), (1, 4, 2, 1), (-1, 3, 5, q)\}$.	4	1	1
	b) Is there a linear transformation $T: R^2 \to R^3$ such that T (1, 1) = (1, 0, 2) and T (2, 3) = (1, -1, 4)? If so compute T (0, 0) and T (8, 11).	6	2	2
3.	Let $\Phi: R^4 \to R^3$ be a linear mapping such that $\Phi(x, y, z, w) = (x + y + 2z + 3w, x + z - w, x + 2y)$. Compute the kernel and image of Φ. What are dim (ker (Φ)) and dim (image (Φ)) relative to the bases $B_1 = \{(1, 1, 1, 2)\ (1, -1, 0, 0)\ (0, 0, 1, 1)\ (0, 1, 0, 0)\}$, $B_2 = \{(1, 2, 3)\ (1, -1, 1)\ (2, 1, 1)\}$.	10	3	3
4.	Let $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & -4 & 8 \\ -1 & 1 & 3 & -5 \\ -1 & 2 & 5 & -6 \\ -1 & -2 & -3 & 1 \end{bmatrix}$ (a) Determine the basis and dimension for row space and column space of A. (b) Determine the basis and the dimension for the set of solutions of Ax = 0. (c) Verify rank – nullity theorem.	10	2	2
5.	Apply the Gram-Schmidt process to construct an orthonormal basis for the subspace $W = \text{span}(x_1, x_2, x_3)$ of R^4, where $x_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, x_2 = \begin{bmatrix} 5 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 2 \\ 3 \\ 4 \\ -1 \end{bmatrix}$	10	3	4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	COS / BT	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Max Marks	14	16	10	10	14	16	20	-	-	-

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DEPARTMENT OF MATHEMATICS

Course: MULTIVARIABLE CALCULUS	IMPROVEMENT TEST	Maximum marks: 10+50=60
Course Code: 21MA11	First semester 2021-2022 Physics Cycle Branch: CS, EC, EE, EI, ET, IS	Time: 10.00am - 12.00pm (120 Minutes) Date: 28-03-2022

Instructions to candidates:

i. Part A must be answered within the first two pages of the Booklet.

ii. Answer all questions.

Q.No	PART A – Quiz	M	BT	CO
1.1	$\beta(1, 0.5)$ = ________.	2	1	1
1.2	The integral $\int_0^\infty e^{-v} v^{3/2} dv$ in terms of Gamma function is ________ and its value is ________.	2	2	1
1.3	The limits for the triple integral to determine the volume of the sphere $x^2+y^2+z^2=1$ is ________	2	2	2
1.4	The value of the integral $\int_1^2\int_0^1\int_0^1 xy\, dzdydx$ is ________.	2	1	1
1.5	The equivalent integral of the integral $\int_0^2\int_0^{\sqrt{4-x^2}} xdydx$ by changing into polar coordinates is ________.	2	1	1

Q.No	PART B – Test	M	BT	CO
1	Evaluate $\iint_R x^2y\, dy\, dx$ where R is the region bounded by the lines $y=x$, $x+y=2$ and y = 0. Represent the region R graphically.	10	2	2
2a	Determine the area enclosed by the curve $r=a(1+\cos\theta)$ and lying above the initial line.	5	2	3
2b	Evaluate $\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$.	5	2	3
3	A plate is in the form of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the first quadrant and of varying thickness $\rho = xy$. Find the coordinates of the centre of gravity of the plane by double integration.	10	3	4
4	Using triple integral evaluate the volume of tetrahedron bounded by the plane $2x+y+2z=2, y=0, x=0$ and $z=0$	10	3	3
5	Obtain the total work done in moving a particle in a force field $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x=t^2+1,\ y=2t^2,\ z=t^3$ from t=1 to 2.	10	3	3

Cos	CO 1	CO 2	CO 3	CO4
Marks	8	12	30	10

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DEPARTMENT OF MATHEMATICS

ODD Semester 2019 – 20

III Semester - Test – 2

Branches: CS & IS

Course: Linear Algebra, Laplace Transforms and Combinatorics (18MA31A)

Date: 09/10/2019 **Marks: 50**

Time: 9:30 AM – 11:00 AM

Sl. No.	Answer All the Question	M	CO	BTL
1	Given $T(2,-4)=(10,-14,14)$, $T(3,2)=(-1,3,5)$. Obtain the transformation $T:\mathbb{R}^2\to\mathbb{R}^3$. Also find the range space, null space, rank, nullity and hence verify the rank-nullity theorem.	10	3	3
2	Suppose A can be factored as $A=QR$. Apply Gram-Schmidt process to the columns of A to obtain Q and hence find R. $A=\begin{bmatrix}1&3&5\\-1&-3&1\\0&2&3\\1&5&2\\1&5&8\end{bmatrix}$	10	4	4
3	If $A=\begin{bmatrix}1&1&3\\1&3&1\\3&1&1\end{bmatrix}$, resolve it as $A=PDP^{-1}$. Give the matrices P,D,P^{-1}.	10	4	4
4(a)	A Givens rotation is a linear transformation from $\mathbb{R}^n$ *to* $\mathbb{R}^n$ used in computer program to create a zero entry in a vector. The standard matrix of a Givens rotation in $\mathbb{R}^2$ has the form $\begin{bmatrix}a&-b\\b&a\end{bmatrix}$, $a^2+b^2=1$. Find a and b such that $\begin{bmatrix}4\\3\end{bmatrix}$ is rotated into $\begin{bmatrix}5\\0\end{bmatrix}$	05	2	3
(b)	If $a\equiv b(mod\ n)$, prove that a and b have the same reminder when divided by n.	05	2	3
5(a)	Examine if the linear congruence $7x\equiv 13(mod\ 24)$ has a unique solution, and hence solve it.	05	3	3
(b)	Determine the reminder when $53^{103}+103^{53}$ is divided by 39.	05	3	3

Course Outcomes: After completing the course, the students will be able to	
CO1:	Understand the fundamental concepts of linear algebra, Laplace and inverse Laplace transforms, number theory and enumeration.
CO2:	Solve the problems of vector spaces, linear transformations, Laplace transform, gcd and generating functions.
CO3:	Apply the acquired knowledge to solve the problems of factorization, transform of special functions and exponential generating functions.
CO4:	Evaluate solution of differential equations using Laplace transform, decomposition of a matrix, public key encryption.

Linear Algebra, Laplace Transforms and Combinatorics
18MA31A - Test 2. Scheme an Solution 09.10.19.

1. $T(2,-4) = (10,-14,14)$, $T(3,2) = (-1,3,5)$ $\begin{vmatrix} 2 & -4 \\ 3 & 2 \end{vmatrix} = 16 \neq 0$

$(x,y) = c_1(2,-4) + c_2(3,2)$ 1

$\Rightarrow (2c_1 + 3c_2 = x)2 \Rightarrow 8c_2 = 2x+y \Rightarrow c_2 = \frac{2x+y}{8}$ or $\frac{4x+2y}{16}$

$-4c_1 + 2c_2 = y$ $\Rightarrow c_1 = \frac{1}{2}\left(x - 3\left(\frac{2x+y}{8}\right)\right) \Rightarrow c_1 = \frac{2x-3y}{16}$ 2

$\therefore (x,y) = \left(\frac{2x-3y}{16}\right)(2,-4) + \left(\frac{4x+2y}{16}\right)(3,2)$ $T(c\alpha) = c\,T(\alpha)$

$\Rightarrow T(x,y) = \left(\frac{2x-3y}{16}\right)T(2,-4) + \left(\frac{4x+2y}{16}\right)T(3,2)$

$\Rightarrow T(x,y) = \left(\frac{2x-3y}{16}\right)(10,-14,14) + \left(\frac{4x+2y}{16}\right)(-1,3,5)$ 2

$\Rightarrow T(x,y) = (x-2y, -x+3y, 3x-2y)$ $\begin{bmatrix} 10 & -14 & 14 \\ -1 & 3 & 5 \end{bmatrix} \sim \begin{bmatrix} 10 & -14 \\ 0 & 16 \end{bmatrix}$

$T(1,0) = (1,-1,3)$, $T(0,1) = (-2,3,-2)$ $\therefore$ Range space $= \{c_1(1,-1,3) + c_2(-2,3,-2)\}$ 1

$\begin{bmatrix} 1 & -1 & 3 \\ -2 & 3 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 4 \end{bmatrix}$ 1 rank $r = 2$

$T(x,y) = (0,0,0) \Rightarrow (x-2y, -x+3y, 3x-2y) = (0,0,0)$ 1

$\Rightarrow x = 2y,\ x = 3y,\ 3x = 2y \Rightarrow x = 0, y = 0$ 1

$\therefore$ Nullspace $= \{(0,0)\}$ nullity $n = 0$

rank + nullity = dimension of domain 1

$2 + 0 = 2$ ✓ hence verified.

2. $x_1 = (1,-1,0,1,1)$, $x_2 = (3,-3,2,5,5)$, $x_3 = (5,1,3,2,8)$

Let $u_1 = x_1 = (1,-1,0,1,1)$ 1

$u_2 = x_2 - \frac{x_2 \cdot u_1}{u_1 \cdot u_1} u_1 = (3,-3,2,5,5) - \frac{(3,-3,2,5,5)\cdot(1,-1,0,1,1)}{(1,-1,0,1,1)\cdot(1,-1,0,1,1)}(1,-1,0,1,1)$

$= (3,-3,2,5,5) - \frac{16}{4}(1,-1,0,1,1) = (-1,1,2,1,1)$ 2

$u_3 = x_3 - \frac{x_3 \cdot u_1}{u_1 \cdot u_1} u_1 - \frac{x_3 \cdot u_2}{u_2 \cdot u_2} u_2 = (5,1,3,2,8) - \frac{(5,1,3,2,8)\cdot(1,-1,0,1,1)}{(1,-1,0,1,1)\cdot(1,-1,0,1,1)}(1,-1,0,1,1)$

$- \frac{(5,1,3,2,8)\cdot(-1,1,2,1,1)}{(-1,1,2,1,1)\cdot(-1,1,2,1,1)}(-1,1,2,1,1)$ 3

$= (5,1,3,2,8) - \frac{14}{4}(1,-1,0,1,1) - \frac{12}{8}(-1,1,2,1,1)$

$= (3,3,0,-3,3)$

$R = Q^T A$

$Q = \begin{bmatrix} 1/2 & -1/2\sqrt{2} & 1/2 \\ -1/2 & 1/2\sqrt{2} & 1/2 \\ 0 & 2/2\sqrt{2} & 0 \\ 1/2 & 1/2\sqrt{2} & -1/2 \\ 1/2 & 1/2\sqrt{2} & 1/2 \end{bmatrix}$

$R = \begin{bmatrix} 1/2 & -1/2 & 0 & 1/2 & 1/2 \\ -1/2\sqrt{2} & 1/2\sqrt{2} & 2/2\sqrt{2} & 1/2\sqrt{2} & 1/2\sqrt{2} \\ 1/2 & 1/2 & 0 & -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 2 \\ 1 & 5 & 8 \end{bmatrix} = \begin{bmatrix} 2 & 8 & \\ 0 & 2\sqrt{2} & 3 \\ 0 & 0 & 6 \end{bmatrix}$

3

3. $A=\begin{bmatrix}1&1&3\\1&3&1\\3&1&1\end{bmatrix}$ $\quad |A-\lambda I|=0 \Rightarrow \lambda^3-(1+3+1)\lambda^2+(2-8+2)\lambda-(2+2-24)=0$

$\Rightarrow \lambda^3-5\lambda^2-4\lambda+20=0$

$$\begin{array}{r|rrrr} 2 & 1 & -5 & -4 & 20 \\ & 0 & 2 & -6 & -20 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

$(\lambda-2)(\lambda^2-3\lambda-10)=0$

$(\lambda-2)(\lambda-5\lambda+2\lambda-10)=0$

$(\lambda-2)(\lambda-5)(\lambda+2)=0 \Rightarrow \lambda=-2,2,5$ 3

$\underline{\lambda=-2}$ $\quad$ $(A+2I)$ $\begin{bmatrix}3&1&3\\1&5&1\\3&1&3\end{bmatrix}$ 1

$\dfrac{x_1}{14}=\dfrac{-x_2}{0}=\dfrac{x_3}{-14}$ $\Rightarrow x_1=\begin{bmatrix}1\\0\\-1\end{bmatrix}$

$\underline{\lambda=2}$ $\quad$ $(A-2I)$ $\begin{bmatrix}-1&1&3\\1&1&1\\3&1&-1\end{bmatrix}$ 1

$\dfrac{x_1}{-2}=\dfrac{-x_2}{-4}=\dfrac{x_4}{-2}$ $\Rightarrow x_2=\begin{bmatrix}-1\\2\\-1\end{bmatrix}$

$\underline{\lambda=5}$ $\quad$ $(A-5I)$ $\begin{bmatrix}-4&1&3\\1&-2&1\\3&1&-4\end{bmatrix}$ 1

$\dfrac{x_1}{7}=\dfrac{-x_2}{-7}=\dfrac{x_3}{7}$ $\Rightarrow x_3=\begin{bmatrix}1\\1\\1\end{bmatrix}$

$P=\begin{bmatrix}1&-1&1\\0&2&1\\-1&-1&1\end{bmatrix}$ 1 $\quad D=\begin{bmatrix}-2&0&0\\0&2&0\\0&0&5\end{bmatrix}$ 1 $\quad P^{-1}=\begin{bmatrix}1/2&0&-1/2\\-1/6&1/3&-1/6\\1/3&1/3&1/3\end{bmatrix}$ 2

a. $\begin{bmatrix}a&-b\\b&a\end{bmatrix}\begin{bmatrix}4\\3\end{bmatrix}=\begin{bmatrix}5\\0\end{bmatrix}$ 1 $\Rightarrow$ $4a-3b=5$, $4b+3a=0$ 1 $\Rightarrow a=-\frac{4}{3}b \Rightarrow 4\times\left(\frac{-4}{3}\right)b-3b=5$

$\Rightarrow -25b=15 \Rightarrow b=-3/5$

$a=4/5$ 3

b. $a\equiv b \pmod n \Rightarrow n\,|\,a-b$ 1 then by division algorithm, when a, b are divided by n, they leave a remainder

$a=nq_1+r_1$, 1 $\quad b=nq_2+r_2$ 1 $\quad \therefore n\,|\,n(q_1-q_2)+(r_1-r_2)$ 2

$\Rightarrow r_1-r_2=0 \Rightarrow r_1=r_2$

5a. $7x\equiv 13 \pmod{24} \Rightarrow 24\,|\,7x-13 \Rightarrow x=\dfrac{24k+13}{7}$ $\quad$ if $k=5$, $x=19$ 4

$\therefore x\equiv 19 \pmod{24}$, $\because \gcd(7,24)=1$, 1

The linear congruence has a unique solution 1

5b. $53^2=2809\equiv 1 \pmod{39}$ 1 $\quad 103^2=10609\equiv 1 \pmod{39}$ 1

~~$(53)^{103}\equiv 53 \pmod{39}$, $(103)^{53}\equiv 103$~~

$(53)^{103}=(53^2)^{50}\cdot 53^3\equiv 1^{50}\cdot 1\cdot 53 \pmod{39}\equiv 14 \pmod{39}$ 1

$(103)^{53}=((103)^2)^{25}\cdot(103)^2\cdot 103 \pmod{39}\equiv 1^{25}\cdot 1\cdot 103 \pmod{39}\equiv 25 \pmod{39}$ 1

$53^{103}+103^{53}\equiv(14+25)\pmod{39}\equiv 0 \pmod{39}$ 1

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RV COLLEGE OF ENGINEERING®

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III Semester B. E. Examinations April-2022

Common CSE / ISE

LINEAR ALGEBRA, LAPLACE TRANSFORMS AND COMBINATORICS

Time: 03 Hours *Maximum Marks: 100*

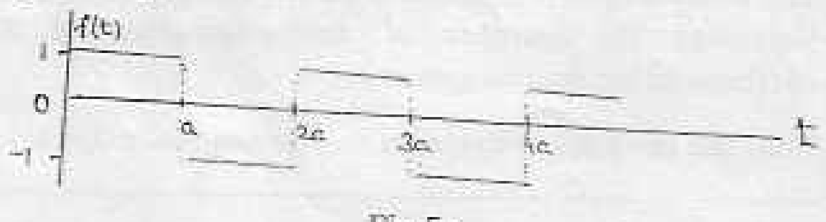
Instructions to candidates:

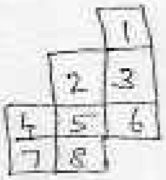
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6.

PART-A

1	1.1	Is the set of vectors $\{(1,2,1),(2,1,0),(1,-1,2)\}$ linearly independent or not?	02
	1.2	Write the induced matrix in the following transformations: i. Projection of xz −plane in R^3 ii. Counter clockwise rotation through an angle θ about the positive y −axis in R^3.	02
	1.3	What multiple of $a_1 = \begin{pmatrix}1\\1\end{pmatrix}$ should be subtracted from $a_2 = \begin{pmatrix}4\\0\end{pmatrix}$ to make the result orthogonal to a_1?	02
	1.4	If $\begin{bmatrix}-4.5\\-4\\1\end{bmatrix}$ is an Eigen vector of $\begin{bmatrix}8 & -4 & 2\\4 & 0 & 2\\0 & -2 & -4\end{bmatrix}$, then the Eigen value corresponding to the Eigen vector is ________.	02
	1.5	Evaluate $\int_0^\infty e^{-3t}\cos^2 t\,dt$ using Laplace transforms.	02
	1.6	Find $L^{-1}\left[\frac{1}{\sqrt{2s+3}}\right]$	02
	1.7	The total number of positive divisors of 1412 are ________.	02
	1.8	The Euler's totient function ϕ for the integer 219 is ________.	02
	1.9	Calculate the number of dearrangements of d_4. Hence write corresponding dearrangements.	02
	1.10	Find the generating function for the sequence 2,4,8,16,32,...	02

PART-B

2	a	Determine the basis and dimension for the row space, column space and null space of the matrix $A=\begin{bmatrix}1 & 2 & 3 & -1\\ 2 & -1 & -4 & 8\\ -1 & 1 & 3 & -5\\ -1 & 2 & 5 & -6\\ -1 & -2 & -3 & 1\end{bmatrix}$	08
	b	Examine whether following sets forms a subspace or not? i. $M_{22}=\{\begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}/\, a,b$ are integers $\}$ on the set of all 2×2 matrices. ii. $S=\{(a,b,c)/\ a+b+c=0\ and\ a,b,c\in R\}$ as the set of all real numbers.	08
3	a	Find a third column so that the matrix $Q=\begin{bmatrix}1/\sqrt{3} & 1/\sqrt{14} & -\\ 1/\sqrt{3} & 2/\sqrt{14} & -\\ 1/\sqrt{3} & -3/\sqrt{141} & -\end{bmatrix}$ is orthogonal. Verify that the rows automatically become orthogonal at the same time.	08
	b	Diagonalize the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$.	08
		OR	
4	a	Obtain the QR factorization for the matrix $A=\begin{bmatrix}1 & 1 & -1\\ 2 & 3 & 2\\ 2 & 1 & 3\end{bmatrix}$ using Gram Schmidt process.	08
	b	Obtain the singular value decomposition of the matrix $\begin{bmatrix}1 & -1\\ -2 & 2\\ 2 & -2\end{bmatrix}$	08
5	a	The periodic function $f(t)$ is shown in the Fig 5a below. Write a mathematical expression for $f(t)$ and hence show that $L[f(t)]=\frac{1}{s}\tanh\left(\frac{as}{2}\right)$  Fig 5a	08
	b	Obtain the Laplace transforms of the following functions. i. $e^{-t}\int_0^t \frac{e^{2t}\sin 3t}{t}dt$ ii. $\left(\sqrt{t}+\frac{1}{\sqrt{t}}\right)^3$	08

		OR	
6	a	Using convolution theorem, evaluate $\left(\frac{s^2}{(s^2+16)(s^2+a)}\right)$.	08
	b	Solve by using Laplace transforms $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=3te^{-t}$ given that $x=4,\ \frac{dx}{dt}=2$ when $t=0$.	08
7	a	Find the gcd(12378, 3054) using the Euclidean algorithm and also find the integers x & y to satisfy $12378x+3054y=d$.	08
	b	Given the public key $(e,n)=(7,51)$, encrypt plain text $L\ I\ V$, where the alphabets $A,B,C,\dots X,Y$, are assigned the numbers 3,4,5, ... 26,27,28. Give the cipher text and also find the private key d.	08
8	a	Using expansion formula, find the rook polynomial for the board shown in Fig 8a.  Fig 8a	08
	b	How many integers between 1 and 300 are i. Divisible by at least one of 5,6,8 ? ii. Divisible by none of 5,6,8 ?	08

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Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

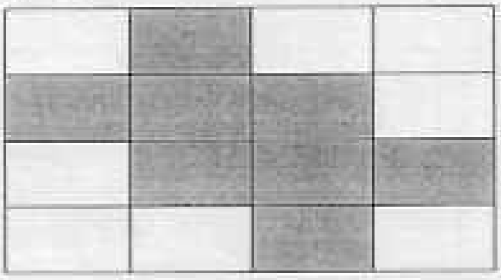
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6.

PART-A

1	1.1	The value of k such that the vectors $(1,0,3),(k,2,5),(2,1,4)$ are linearly dependent is ____.	02
	1.2	If the columns of an 8 by 4 matrix are linearly independent, then the dimension of its null space and left null space are __ and __ respectively.	02
	1.3	Let $W = Span\{v_1, v_2\}$, where $v_1 = (2,4)$ and $v_2 = (3,-1)$. Construct an orthogonal basis $\{u_1, u_2\}$ for W.	02
	1.4	Given $\lambda_1 = 5$, $\lambda_2 = 1$ and corresponding eigenvectors are $\begin{bmatrix}3\\1\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}$ then the matrix A is______	02
	1.5	The Laplace transform of input $y(t)$, if the system is given by $y(t) - \int_0^t y(t)dt - 5\cos(t) = 0$ is _______.	02
	1.6	If $L^{-1}[F(s)] = t\,e^{-t} + 1$, then $L^{-1}\left[F\left(\frac{s}{3}\right)\right] =$ ____.	02
	1.7	The last digit of 17^{37} is ______.	02
	1.8	The number of positive divisors of the integer 1568 are_____.	02
	1.9	There are six letters to six different people to be placed in six different addressed envelopes. Find the number of ways of doing this so that at least one letter gets to the right person?	02
	1.10	Define exponential generating function with an example.	02

PART-B

2	a	Let $V = \{(x, x/2) : x \in R\}$ with standard operations. Is it a vector space? Justify your answer.	06
	b	Apply elementary row operations to the following matrix to reduce it to echelon form and hence obtain the bases and dimension for its Row space, and Null space. $\begin{bmatrix} 1 & -1 & 1 & 1 \\ 4 & -4 & 3 & 6 \\ 2 & -2 & 1 & 3 \end{bmatrix}$	06
	c	The position vector (2, 1) in R^2 is first rotated through an angle of 30^o clockwise and then stretched by a factor of 2. Give the rotation matrix and the stretching matrix for this situation.	04
3	a	The columns of the following matrix A form a basis for the column space of A. Applying suitable process to the columns of A, construct an orthogonal basis for the column space of A. $$A = \begin{bmatrix} 2 & 3 & 5 \\ -1 & -1 & -3 \\ 1 & -1 & 2 \end{bmatrix}$$	08
	b	Decompose the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ into $A = PDP^{-1}$.	08
		OR	
4	a	Obtain the QR factorization for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$	08
	b	Obtain the singular value decomposition of the matrix $\begin{bmatrix} 3 & -1 \\ 1 & -2 \\ 2 & 1 \end{bmatrix}$	08
5	a	i. Evaluate $\int_0^\infty e^{-t}\frac{\sin^2 t}{t}\,dt$ using Laplace transform. ii. Obtain the frequency domain function for $$f(t) = t^{-\frac{1}{2}} + \cos\left(\frac{t}{2}\right) + e^{-\frac{1}{4}t}\sin(3t)$$	08
	b	Show that the Laplace transform of the periodic $f(t)$ is $\frac{p}{s}\,tanh\left(\frac{s}{2}\right)$, where $f(t) = \begin{cases} p & if\ 0 \le t < 1 \\ -p & if\ 1 \le t < 2 \end{cases}$, $f(t+2) = f(t)$.	08
		OR	
6	a	Using convolution theorem, evaluate inverse Laplace transform $\left(\frac{1}{(s-2)(s^2+4s+4)}\right)$.	08
	b	Solve $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}sin(t)$ with $x(0) = 0$ and $\frac{dx}{dt} = 1\ at\ t = 0$ using Laplace transform.	08

7	a	By using the Euclidean algorithm, find the greatest common divisor d of 1389 and 2567 and then find integers x and y to satisfy $1389x + 2567y = d$.	08
	b	Given the public key $(e, n) = (5,95)$, encrypt plain text T J H, where the alphabets $A, B, C, \ldots X, Y$, are assigned the numbers $5, 6, 7, \cdots, 29, 30$. Determine the cipher text and also the private key d.	08
8	a	How many integers between 1 and 300 (inclusive) are divisible by 5 but by neither 3 nor7?	05
	b	Determine the rook polynomial for the following shaded chessboard. 	06
	c	Find the co-efficient of x^{27} in the expansion of the function $(x^4 + x^5 + x^6 + \cdots)^5$	05

Scheme and solution

Linear Algebra, Laplace Transforms and Combinatorics(18MA31A)

PART-A

1.1	$\begin{vmatrix} 1 & 0 & 3 \\ k & 2 & 5 \\ 2 & 1 & 4 \end{vmatrix} = 0,\ k = 3$	2
1.2	0, 4	2
1.3	$v_1 = u_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix},\ v_2 = u_2 - \frac{u_2 v_1}{v_1 v_1} v_1 = \begin{bmatrix} 14/5 \\ -7/5 \end{bmatrix}$	2
1.4	$A = PDP^{-1} = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{4} \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$	2
1.5	$Y(s) - \frac{Y(s)}{s} - \frac{5s}{s^2+1} = 0. \implies Y(s) = \frac{5s^2}{(s^2+1)(s-1)}$	2
1.6	$L^{-1}\left[F\left(\frac{s}{2}\right)\right] = 2f(2t) = 2(2te^{-2t} + 1)$	2
1.7	$17 \equiv x(mod 10) \implies x = 7$	2
1.8	$1568 = 2^5 7^2,\ n(1568) = (1+\alpha_1)(1+\alpha_2) = (1+5)(1+2) = 18$	2
1.9	$6! - d_6 = 720 - 265 = 455$	2
1.10	Given a sequence $\langle a_r \rangle$, suppose there exists a function $E(x)$ such that the expansion of $E(x)$ in a series of powers of x is given by $E(x) = a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \ldots = \sum_{r=0}^{\infty} a_r \frac{x^r}{r!}$ Example:$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots = \sum_{r=0}^{\infty} (-1)^r \frac{x^r}{r!}$ The function e^{-x} is the exponential generating function for the sequence $1, -1, 1, -1, \ldots$	2
	PART - B	
2(a)	V is a vector space. Addition: (i)$\forall x, y \in \mathbb{R}$ we have $(x, \frac{x}{2}) + (y, \frac{y}{2}) = (x + y, \frac{x+y}{2})$, V is closed under addition. (ii)$\forall x, y, z \in \mathbb{R}$ we have $\left((x, \frac{x}{2}) + (y, \frac{y}{2})\right) + (z, \frac{z}{2}) = (x, \frac{x}{2}) + \left((y, \frac{y}{2}) + (z, \frac{z}{2})\right)$ $= (x + y + z, \frac{x+y+z}{2})$, V is associative under addition. (iii) Identity $(0, 0)$. (iv)Inverse element $(-x, -\frac{x}{2})$	1 1 1

	Scalar Multiplication: (v)For a scalar c we have $c(x,\frac{x}{2})=(cx,\frac{cx}{2})$. (vi)$c\left((x,\frac{x}{2})+(y,\frac{y}{2})\right)=\left(c(x,\frac{x}{2})+c(y,\frac{y}{2})\right)$ (vii)$(c+d)(x,\frac{x}{2})=c(x,\frac{x}{2})+d(x,\frac{x}{2})$ (viii)$(1,\frac{2}{2})=(1,1)$, $(1,1)((x,\frac{x}{2})=(x,\frac{x}{2})$	1 1 1
2(b)	$A=\begin{bmatrix}1 & -1 & 1 & 1\\ 4 & -4 & 3 & 6\\ 2 & -2 & 1 & 3\end{bmatrix}$ $R_2:R_2-4R_1,\ R_3:R_3-2R_1 \implies A=\begin{bmatrix}1 & -1 & 1 & 1\\ 0 & 0 & -1 & 2\\ 0 & 0 & -1 & 1\end{bmatrix}$ $R_3:R_3-R_2 \implies A=\begin{bmatrix}1 & -1 & 1 & 1\\ 0 & 0 & -1 & 2\\ 0 & 0 & 0 & -1\end{bmatrix}$ Basis of Row space $Row(A)=\{(1,-1,1,1),(4,-4,3,6),(2,-2,1,3)\}$. Dimension of Row space= 3. $AX=0 \implies \begin{bmatrix}1 & -1 & 1 & 1\\ 0 & 0 & -1 & 2\\ 0 & 0 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\\ 0\end{bmatrix}$ $\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}=\begin{bmatrix}x_2\\ x_2\\ 0\\ 0\end{bmatrix}=x_2\begin{bmatrix}1\\ 1\\ 0\\ 0\end{bmatrix}$ Basis of $Null(A)=\{(1,1,0,0)\}$, Dimension of $Null(A)=1$.	 2 1 2 1
2(c)	$Q_{30}=\begin{bmatrix}cos30 & sin30\\ -sin30 & cos30\end{bmatrix}=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ \frac{-1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$ (Rotational matrix) $A=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$ (is the steching matrix)	2 2
3(a)	$u_1=(2,-1,1), u_2=(3,-1,-1), u_3=(5,-3,2)$ $v_1=u_1=(2,-1,1)$ $v_2=u_2-\frac{u_2v_1}{v_1v_1}v_1=(3,-1,-1)-\frac{(3,-1,-1)(2,-1,1)}{(2,-1,1)(2,-1,1)}(2,-1,1)=(3,-1,-1)-\frac{6}{6}(2,-1,1)$ $v_2=(1,0,-2)$ $v_3=u_3-\frac{u_3v_1}{v_1v_1}v_1-\frac{u_3v_2}{v_2v_2}v_2=(5,-3,2)-\frac{(5,-3,2)(2,-1,1)}{(2,-1,1)(2,-1,1)}(2,-1,1)-\frac{(5,-3,2)(1,0,-2)}{(1,0,-2)(1,0,-2)}(1,0,-2)$	1 1 2 1

	$v_3 = (5,-3,2) - \frac{10+3+2}{4+1+1}(2,-1,1) - \frac{5+0-4}{1+4}(1,0,-2)$	
	$v_3 = (5,-3,2) - (5,-\frac{5}{2},\frac{5}{2}) - (\frac{1}{5} - 0 - \frac{2}{5}) = (\frac{-1}{5},\frac{-1}{2},\frac{-1}{10})$	3
3(b)	$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix} \Longrightarrow \lvert A-\lambda I\rvert = (1-\lambda)(2-\lambda)(3-\lambda) \Longrightarrow \lambda = 1,2,3$	2
	(i) for $\lambda = 1$	
	$\lvert A-I\rvert = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$	1
	(ii) for $\lambda = 2$	
	$\lvert A-I\rvert = \begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$	1
	(iii) for $\lambda = 3$	
	$\lvert A-I\rvert = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$	1
	$P = \begin{bmatrix} -1 & -2 & 1 \\ 1 & 1 & -1 \\ 0 & 2 & -2 \end{bmatrix}, P^{-1} = \begin{bmatrix} 0 & 1 & -\frac{1}{2} \\ -1 & -1 & 0 \\ -1 & -1 & -\frac{1}{2} \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$	1+1+1
4(a)	$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \Longrightarrow u_1 = (1,0,1), u_2 = (2,1,0), u_3 = (0,1,1)$	
	$v_1 = u_1 = (1,0,1)$	1
	$v_2 = u_2 - \frac{u_2 v_1}{v_1 v_1} v_1 = (2,1,0) - \frac{(2,1,0)(1,0,1)}{(1,0,1)(1,0,1)}(1,0,1) = (2,1,0) - \frac{2}{2}(1,0,1) = (1,1,-1)$	2
	$v_3 = u_3 - \frac{u_3 v_1}{v_1 v_1} v_1 - \frac{u_3 v_2}{v_2 v_2} v_2 = (0,1,1) - \frac{(0,1,1)(1,0,1)}{(1,0,1)(1,0,1)}(1,0,1) - \frac{(0,1,1)(1,1,-1)}{(1,1,-1)(1,1,-1)}(1,1,-1)$	2
	$v_3 = (0,1,1) - \frac{1}{2}(1,0,1) - 0 = (-1/2, 1, 1/2) = (-1,2,1)$	1
	$Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{bmatrix}$	1
	$R = Q^T A = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & \sqrt{2} & \frac{1}{\sqrt{2}} \\ 0 & \frac{3}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{3}{\sqrt{6}} \end{bmatrix}$	1

4(b) $A = \begin{bmatrix} 3 & -1 \\ 1 & -2 \\ 2 & 1 \end{bmatrix}$

$$AA^T = \begin{bmatrix} 10 & 5 & 5 \\ 5 & 5 & 0 \\ 5 & 0 & 5 \end{bmatrix} \implies \left|AA^T - \lambda I\right| = \lambda^3 - 20\lambda^2 + 75\lambda = 0 \implies \lambda = 0, 5, 15$$ 1

for $\lambda = 0$

$$\left|AA^T - I\right| = \begin{bmatrix} 10 & 5 & 5 \\ 5 & 5 & 0 \\ 5 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \implies u_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$ $\frac{1}{2}$

for $\lambda = 5$

$$\left|AA^T - 5I\right| = \begin{bmatrix} 5 & 5 & 5 \\ 5 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \implies u_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$ $\frac{1}{2}$

for $\lambda = 15$

$$\left|AA^T - 15I\right| = \begin{bmatrix} -5 & 5 & 5 \\ 5 & -10 & 0 \\ 5 & 0 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \implies u_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$ 1

$$A^T A = \begin{bmatrix} 14 & -3 \\ -3 & 6 \end{bmatrix} \implies \left|A^T A - \lambda I\right| = \lambda^2 - 20\lambda + 75 = 0 \implies \lambda = 5, 15$$

for $\lambda = 5$

$$\left|A^T A - 5I\right| = \begin{bmatrix} 9 & -3 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \implies v_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$ $\frac{1}{2}$

for $\lambda = 15$

$$\left|A^T A - 15I\right| = \begin{bmatrix} -1 & -3 \\ -3 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \implies v_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$ $\frac{1}{2}$

$$U = \begin{bmatrix} -1/\sqrt{3} & 0 & 2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix} \quad V = \begin{bmatrix} 1/\sqrt{10} & -3/\sqrt{10} \\ 3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sqrt{15} & 0 \\ 0 & \sqrt{5} \\ 0 & 0 \end{bmatrix}$$ 1+1+1

5(a)	(i) $sin^2t = \frac{1-cos2t}{2} \implies L(sin^2t) = \frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2+4}\right)$	1
	$L(\frac{sin^2t}{t}) = \frac{1}{2}\int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+4}\right) ds = \frac{1}{2}\left[logs - \frac{1}{2}log(s^2+4)\right]_s^\infty = \frac{1}{2}\left[log\frac{s}{(s^2+4)^{\frac{1}{2}}}\right]_s^\infty$	1
	$L(\frac{sin^2t}{t}) = \frac{1}{2}\left[log(0) - log\frac{s}{(s^2+4)^{\frac{1}{2}}}\right] = \frac{1}{2}\left[log\left(\frac{\sqrt{s^2+4}}{s}\right)\right]$	
	$\int_0^\infty e^{-st}\frac{sin^2t}{t}dt = \frac{1}{2}\left[log\left(\frac{\sqrt{s^2+4}}{s}\right)\right]$	2
	put $s = 1 \implies \int_0^\infty e^{-t}\frac{sin^2t}{t}dt = \frac{1}{2}\left[log\left(\frac{\sqrt{1+4}}{1}\right)\right] = \frac{1}{2}log(\sqrt{5})$	1
	(ii) $L(f(t)) = L(t^{-1/2}) + L(cos(t/2)) + L(e^{-t/4}sin3t) = \frac{\Gamma(1/2)}{s^{1/2}} + \frac{s}{s^2+\frac{1}{4}} + \frac{3}{(s+\frac{1}{4})^2+9}$	1+1+1
5(b)	$f(t) = \begin{cases} p & 0 \le t < 1 \\ -p & 1 \le t < 2 \end{cases} \quad f(t+2) = f(t) \implies T = 2$	1
	$L(f(t)) = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt = \frac{1}{1-e^{-2s}}\left[\int_0^1 e^{-st}(p)dt + \int_1^2 e^{-st}(-p)dt\right]$	1+1
	$= \frac{p}{s}\left(\frac{1}{1-e^{-2s}}\right)\left[1 + e^{-2s} - 2e^{-s}\right] = \frac{p}{s}\left(\frac{1}{1-e^{-2s}}\right)\left[1 - e^{-s}\right]^2$	1+1
	$= \frac{p}{s}\frac{1}{(1-e^{-s})(1+e^{-s})}\left[1 - e^{-s}\right]^2 = \frac{p}{s}\frac{(1-e^{-s})}{(1+e^{-s})}$	1+1
	$= \frac{p}{s}\frac{(e^{s/2}-e^{-s/2})}{(e^{s/2}+e^{-s/2})} = \frac{p}{s}tanh\left(\frac{s}{2}\right)$	1
6(a)	$L^{-1}\left[\frac{1}{(s-2)(s^2+4s+4)}\right] = L^{-1}\left[\frac{1}{(s-2)(s+2)^2}\right]$	
	$F(s) = \frac{1}{s-2} \implies f(t) = e^{2t}$	1
	$G(s) = \frac{1}{(s+2)^2} \implies g(t) = te^{-2t}$	2
	$L^{-1}\left[F(s)G(s)\right] = \int_0^t f(t-u)g(u)du = \int_0^t e^{2(t-u)}e^{-2u}udu = \int_0^t e^{(2t-2u-2u)}udu$	2
	$= e^{2t}\int_0^t e^{-4u}udu = e^{2t}\left[\frac{-ue^{-4u}}{4} + \frac{e^{-4u}}{-16}\right]_0^t = e^{2t}\left[\frac{-4te^{-4t}-e^{-4t}+1}{16}\right]$	1+1+1
6(b)	$L\left[\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x(t) = e^{-t}sint\right]$	
	$\{s^2L\left[x(t)\right] - sx(0) - x'(0)\} + 2\{sL\left[x(t)\right] - x(0)\} + 5L\left[x(t)\right] = \frac{1}{(s+1)^2+1}$	2
	$L\left[x(t)\right](s^2+2s+5) - 1 = \frac{1}{(s+1)^2+1} \implies L\left[x(t)\right] = \frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$	
	$L\left[x(t)\right] = \frac{(s+1)^2+2}{[(s+1)^2+1][((s+1)^2)+4]} \implies x(t) = e^{-t}L^{-1}\left[\frac{s^2+2}{(s^2+1)(s^2+4)}\right]$	1
	$\left[\frac{s^2+2}{(s^2+1)(s^2+4)}\right] = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4} \implies A = 0, B = 1/3, C = 0, D = 2/3$	1+1+1
	$x(t) = e^{-t}L^{-1}\left[\frac{s^2+2}{(s^2+1)(s^2+4)}\right] = e^{-t}L^{-1}\left[\frac{1/3}{(s^2+1)} + \frac{2/3}{(s^2+4)}\right] = e^{-t}\left[\frac{1}{3}sint + \frac{1}{3}sin2t\right]$	1+1
	$x(t) = \frac{e^{-t}}{3}\left[sint + sin2t\right]$	
7(a)	$GCD(1389, 2567) = 1$	
	$1178 = 2567 - 1389, 211 = 1389 - 1178, 123 = 1178 - 211(5), 88 = 211 - 123$	2
	$35 = 123 - 88, 18 = 88 - 35(2), 17 = 35 - 18, 1 = 18 - 17$	2
	$d = 1389x + 2567y \implies 1 = 1389(146) + 2567(-79) \implies x = 146, y = -79$	2+2

7(b)	$e = 5, n = 95 = 5 \times 19 \implies p = 5, q = 19$	1
	$J = 14, H = 12, T = 24$	
	$c_T \equiv 24^5 (mod 95) \equiv 9 (mod 95)$	1
	$c_J \equiv 14^5 (mod 95) \equiv 29 (mod 95)$	1
	$c_H \equiv 12^5 (mod 95) \equiv 27 (mod 95)$	1
	$9, 29, 27 \implies E, Y, W$	1
	$\phi(n) = (p-1)(q-1) = (5-1)(19-1) = 72$ and $GCD(5, 72) = 1$	1
	$de \equiv 1 (mod \phi(n)) \implies d5 \equiv 1 (mod 72) \equiv 1 (mod 72) \implies d = 29$	2
8(a)	$A = \{x : x\|5\}$, $B = \{x : 3\|5\}$ and $C = \{x : x\|7\}$	
	$\|A\| = \lfloor \frac{300}{5} \rfloor = 60$, $\|B\| = \lfloor \frac{300}{3} \rfloor = 100$, $\|C\| = \lfloor \frac{300}{7} \rfloor = 42$,	1
	$\|A \cap B\| = \lfloor \frac{300}{15} \rfloor = 20$, $\|A \cap C\| = \lfloor \frac{300}{35} \rfloor = 8$ and $\|A \cap B \cap C\| = \lfloor \frac{300}{105} \rfloor = 2$	2
	$\|A \cup B \cup C\| - \|B \cup C\|$	
	$= \|A\| - \|A \cap B\| - \|A - C\| + \|A \cap B \cap C\|$	
	$\|A \cup B \cup C\| - \|B \cup C\| = 60 - 20 - 8 + 2 = 34$	2
8(b)	$r_1 = n = 8, r_2 = 16, r_3 = 8, r_4 = 1$	1+2+1+1
	$r(c, x) = 1 + 8x + 16x^2 + 8x^3 + x^4$	1
8(c)	$(x^4 + x^5 + x^6 + ...)^5 = x^{20} (1 + x + x^2 + ...)^5 = x^{20} (1 - x)^5$	1+1
	$= x^{20} \sum_{r=0}^{\infty} \begin{pmatrix} 4+r \\ r \end{pmatrix} x^r$	1
	coefficient of x^{27} is $\begin{pmatrix} 11 \\ 7 \end{pmatrix} = \frac{11}{7!4!} = 330$	2

For alternative answers appropriate marks can be given.

(BOE Chairman)

DEPARTMENT OF MATHEMATICS
RV College of Engineering®, Bengaluru – 59
FIRST TEST - SCHEME OF VALUATION

Semester: III (CSE & ISE) **Date:** 03.09.2019

Course: Linear Algebra, Laplace Transforms and Combinatorics **Course Code:** 18MA31A

Q. No.	Answer	Marks
1	$V = \{a + b\sqrt{2} \,/\, a, b, \in Q\}$ Let $x = a_1 + b_1\sqrt{2}$, $y = a_2 + b_2\sqrt{2}$; $a_1, b_1, a_2, b_2 \in Q$. → 01 M $x + y = (a_1 + a_2) + (b_1 + b_2)\sqrt{2}$ $ax = (aa_1) + (ab_1)\sqrt{2}$ **i. Axioms under vector addition** → 01 M **V_1. Closure.** Let $x, y \in V$ Then $x + y = (a_1 + b_1\sqrt{2}) + (a_2 + b_2\sqrt{2}) = (a_1 + a_2) + (b_1 + b_2)\sqrt{2} \in V$ $[\because a_1, b_1, a_2, b_2 \in Q \Rightarrow a_1 + a_2, b_1 + b_2 \in Q]$ **V_2. Associativity.** Let $x, y, z \in V$ → 01 M Then $x + (y + z) = (a_1 + b_1\sqrt{2}) + [(a_2 + b_2\sqrt{2}) + (a_3 + b_3\sqrt{2})]$ $= ((a_1 + a_2) + a_3) + ((b_1 + b_2) + b_3)\sqrt{2}) = (x + y) + z \in V$ $[\because$ Associative law holds in $Q]$ **V_3. Existence of Identity.** Let $x \in V$, $x = a + b\sqrt{2} \,/\, a, b, \in Q$ → 01 M Then $O = 0 + 0\sqrt{2} \in V \Rightarrow 0 + x = a + b\sqrt{2} = x$ $\therefore O = 0 + 0\sqrt{2}$ is the additive identity in V. **V_4. Existence of Inverse.** Let $x \in V$, $x = a + b\sqrt{2} \,/\, a, b, \in Q$ → 01 M Then $-x = (-a) + (-b)\sqrt{2} \in Q$ $[\because a, b, \in Q \Rightarrow -a, -b, \in Q]$ $x + (-x) = O \Rightarrow -x = (-a) + (-b)\sqrt{2}$ is additive inverse of $x = a + b\sqrt{2}$ in V. **V_5. Commutativity.** Let $x, y \in V$ → 01 M Then $x + y = (a_1 + b_1\sqrt{2}) + (a_2 + b_2\sqrt{2}) = (a_1 + a_2) + (b_1 + b_2)\sqrt{2} = (a_2 + b_2\sqrt{2}) + (a_1 + b_1\sqrt{2}) = y + x$ $\therefore$ Addition is commutative in V. **ii. Axioms under Scalar Multiplication** **V_6.** Let $\alpha \in Q$ and $x \in V$ → 01 M Then $\alpha x = \alpha(a + b\sqrt{2}) = (\alpha a) + (\alpha b)\sqrt{2} \in V$ $[\because \alpha, a \in Q \Rightarrow \alpha a \in Q]$ V is closed under scalar multiplication. **V_7.** Let $\alpha, \beta \in Q$ and $x \in V$ → 01 M Then $(\alpha + \beta)x = (\alpha + \beta)(a + b\sqrt{2}) = (\alpha + \beta)a + (\alpha + \beta)b\sqrt{2}$ $= (\alpha a + \beta a) + (\alpha b + \beta b)\sqrt{2} = \alpha(a + b\sqrt{2}) + \beta(a + b\sqrt{2}) = \alpha x + \beta x$. **$V_8$.** Let $\alpha, \beta \in Q$ and $x \in V$ → 01 M Then $(\alpha\beta)x = (\alpha\beta)(a + b\sqrt{2}) = [(\alpha\beta)a] + [(\alpha\beta)b\sqrt{2}]$ $= \alpha[\beta(a + b\sqrt{2})] = \alpha(\beta x)$. **$V_9$.** Let 1 be the unity element of Q and $x \in V$ → 01 M $1 \cdot x = 1 \cdot (a + b\sqrt{2}) = (1 \cdot a) + (1 \cdot b)\sqrt{2} = a + b\sqrt{2} = x$. V is a vector space over Q.	10
2a	Showing the given 2 x 2 matrices as a subspace of M_{2x2} Let two vectors be $x_1 = \begin{bmatrix} a_1 & b_1 \\ 0 & d_1 \end{bmatrix}$, $x_2 = \begin{bmatrix} a_2 & b_2 \\ 0 & d_2 \end{bmatrix}$ → 01 M $c_1x_1 + c_2x_2 = c_1\begin{bmatrix} a_1 & b_1 \\ 0 & d_1 \end{bmatrix} + c_2\begin{bmatrix} a_2 & b_2 \\ 0 & d_2 \end{bmatrix}$ → 03 M The set H is a subspace of M_{2x2}. The zero matrix is in H, the sum of two upper triangular matrices is upper triangular and any scalar multiple of an upper triangular matrix is again upper triangular.	05

	showing x_1 and x_2 satisfies vector addition and scalar multiplication	→ 01 M	
2b	Showing the vectors linearly independent		05
	$\begin{vmatrix} 2 & -1 & 0 \\ 1 & 2 & 1 \\ 0 & 2 & -1 \end{vmatrix} = -1 \neq 0.$	→ 02 M	
	Expressing the given vectors as linear combination of (3, 2, 1)	→ 02 M	
	$(3, 2, 1) = c_1(2, -1, 0) + c_2(1, 2, 1) + c_3(0, 2, -1)$		
	$c_1 = 8/9$, $c_2 = 11/9$ & $c_3 = 2/9$	→ 01 M	
3	Reduce the given matrix to Echelon form		10
	$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	→ 02 M	
	Null space of A: Set of all solutions of $Ax = 0$.	→ 02 M	
	$\{(-2, 1, 0, 0), (2, 0, -1, 1)\}$ forms the basis for N(A) and $\dim\{N(A)\} = 2$	→ 01 M	
	Column space C(A): The set of all possible linear combination of its column vectors		
	$\{(1, 2, 3), (3, 8, 7)\}$ forms the basis for C(A) and $\dim\{C(A)\} = 2$	→ 01 M	
	Row Space $C(A^T)$: The set of all possible linear combination of its Row vectors		
	$\{(1, 2, 3, 5), (2, 4, 8, 12)\}$ forms the basis for $C(A^T)$ and $\dim\{C(A)\} = 2$	→ 01 M	
	Left null space of A: Set of all solutions of $A^T y = 0$.	→ 02 M	
	$\{(-5, 1, 1)\}$ forms the basis for $N(A^T)$ and $\dim\{N(A^T)\} = 1$	→ 01M	
4a	The number of all positive divisors $8128 = 2^6 \times 127$	→ 01 M	05
	$T(a) = (1 + a_1)(1 + a_2) \cdots (1 + a_n)$	→ 01 M	
	The sum of all positive divisors $S(a) = \left(\frac{p_1^{a_1+1} - 1}{p_1 - 1}\right)\left(\frac{p_1^{a_2+1} - 1}{p_2 - 1}\right) \cdots \left(\frac{p_1^{a_n+1} - 1}{p_n - 1}\right)$	→ 01 M	
	Number of divisors of $8128 = 14$	→ 01 M	
	Sum of divisors of $8128 = 16256$	→ 01 M	
4b	Suppose $p \nmid a$, then $\gcd(p, a) = 1$	→ 01 M	05
	Therefore there exist integers x and y such that $ax + py = 1$		
	$\therefore abx + bpy = b$ .... (1)	→ 01 M	
	Now $p \mid ab \Rightarrow ab = kp$ where $k \in \mathbb{Z}$		
	Substituting for ab in equation (1), we get		
	$kpx + bpy = b$	→ 02 M	
	$\therefore pk_1 = b$ where $k_1 = kx + by \in \mathbb{Z}$		
	Hence $p \mid b$.	→ 01 M	
5	By division algorithm, we get		10
	$1312 = 4001 - 2689$	→ 01 M	
	$65 = 2689 - 2(1312)$		
	$12 = 1312 - 20(65)$	→ 01 M	
	$5 = 65 - 5(12)$		
	$2 = 12 - 2(5)$	→ 01 M	
	$1 = 5 - 2(2)$		
	Since the non-zero remainder is 1, we get $\gcd(2689, 4001) = 1$	→ 01 M	
	$1 = 5 - 2(2)$	→ 01 M	
	$1 = 5(5) - 12(2)$	→ 01 M	
	$1 = 65(5) - 12(27)$	→ 01 M	
	$1 = 65(545) - 1312(27)$	→ 01 M	
	$1 = 2689(545) - 1312(1117)$		
	$1 = 2689x + 4001y$ where $x = 1662$ and $y = -1117$	→ 01 M	
	Disproving x and y are not unique ($x = 5663$ and $y = -3806$)	→ 01 M	

SCHEME AND SOLUTION JAN -2022

COURSE CODE: 18MA31 **COURSE:** Discrete & Integral Transforms UG

Question No	PART- A	Marks
1 1.1	$\frac{3\sqrt{\pi}}{(s+4)^{5/2}}$ 1.2 $L\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt$	01 01
1.3	$L\{t\cos t\}\big\|_{s=2} = \frac{s^2-1}{(s^2+1)^2}\Big\|_{s=2} = \frac{3}{25}$ 1+1	02
1.4	$\frac{1}{3} e^{-2t}\sin 3t$	01
1.5	$\frac{1}{\pi}\sin(\pi(t-3))\,U(t-3)$	01
1.6	$\int_0^t \frac{\sin\omega t}{\omega}\,dt = \frac{1-\cos\omega t}{\omega^2}$ 1+1	02
1.7	$\frac{1}{2}\{f(a^-)+f(a^+)\}$	01
1.8	$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi x}{\ell}}$	01
1.9	$\cancel{a_0 = \frac{1}{\pi}\int_0^{\pi} x\sin x\,dx = 2}$ 1+1	02
1.10	$F\{e^{-x^2/2}\cos 2x\} = \frac{1}{2}\left[e^{-\frac{(\alpha+2)^2}{2}} + e^{-\frac{(\alpha-2)^2}{2}}\right]$	01
1.11	$\{f(x)\} = \int_{-\infty}^{\infty} e^{-i\alpha x} F(\alpha)\,d\alpha$	01
1.12	$F_c\{f(x)\} = \int_0^{\infty} f(x)\cos\alpha x\,dx = \frac{1}{1-\alpha^2}$ 1+1	02
1.13	$Z\{n^2 e^{an}\} = \frac{e^a z^2 + (e^a)^2 z}{(z-e^a)^3}$	01
1.14	$\cancel{Z^{-1}\left\{\frac{1}{z-a}\right\} = a^{n-1}}$	01
1.15	$u_0 = 0,\quad u_1 = 0,\quad u_2 = 2$ ~~where~~ $\cancel{u_1 = \lim_{z\to\infty} z[U(z)-u_0]}$, $u_2 = \lim_{z\to\infty} z^2\left[U(z)-u_0-\frac{u_1}{z}\right]$ 1+1	02
	PART-B	
2(a).	(i) $L\{\sin 4t\} = \frac{4}{s^2+16}$ —1, $L\{e^{-t}\sin 4t\} = \frac{4}{(s+1)^2+16}$ —1	

Question No		Marks
	$L\{te^{-t}\sin 4t\} = \frac{8(s+1)}{(s^2+2s+17)^2}$ —1	04
	$L\{\int_0^t te^{-t}\sin 4t\,dt\} = \frac{8}{s}\frac{s+1}{(s^2+2s+17)^2}$ —1	
	(ii) $L\{\frac{\sin^2 2t}{\cancel{t}}\} = L\{\frac{1-\cos 4t}{2}\} = \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+16}\right]$	+
	$L\{\frac{\sin^2 2t}{2}\} = \frac{1}{2}\int_s^\infty \left[\frac{1}{s} - \frac{s}{s^2+16}\right] ds$ —2	04
	$= \frac{1}{4}\ln\left(\frac{s^2+16}{s^2}\right)$ —2	
2(b)	$L\{f(t)\} = \frac{1}{1-e^{-4s}}\int_0^4 f(t)e^{-st}dt$ —2	
	$T=4$ $= \frac{1}{1-e^{-4s}}\left[\int_0^2 3t\, e^{-st}dt + \int_2^4 6e^{-st}dt\right]$	
	$= \cancel{\frac{-6e^{-2s}}{s}} - \frac{3e^{-2s}}{s^2} + \frac{3}{s^2} - \frac{6}{s}e^{-4s} + \cancel{\frac{6e^{-2s}}{s}}\Big]$ —4	
	$= \frac{3}{s^2} - \frac{3e^{-2s}}{s^2} - \frac{6}{s}e^{-4s}$ —2	08
3(a)(i)	$L\{t f(t)\} = -\frac{d}{ds}F(s)$. —1	
	$\frac{d}{ds}\{\cot^{-1}s/2\} = \frac{2}{s^2+4}$ —1	
	$\Rightarrow t f(t) = L^{-1}\{\frac{2}{s^2+4}\} = \sin 2t$ —2	4
	$f(t) = \frac{\sin 2t}{t}$	
	(ii) $L^{-1}\left[\frac{3s+7}{s^2-2s-3}\right] = L^{-1}\left[\frac{3(s-1)+10}{(s-1)^2-4}\right]$ —2	
	$= L^{-1}\left[\frac{3(s-1)}{(s-1)^2-4}\right] + L^{-1}\left[\frac{10}{(s-1)^2-4}\right]$ —1	4
	$= 3e^{t}\cosh 2t + 5e^{t}\sinh 2t$ —1	

Question No		Marks
3(b)	Taking Laplace transform on both sides of Eqn. $L\{I''\}+4L\{I'\}+3L\{I(t)\} = L\{e^{-t}\}$ — 1 $s^2I(s)-s\,i(0)-i'(0)+4(sI(s)-i(0))+3I(s)=\frac{1}{s+1}$ $I(s)=\frac{1}{(s^2+4s+3)(s+1)}+\frac{5+s}{s^2+4s+3}$ $i(0)=1$ $i'(0)=1$ — 2 By Partial Fraction Method $\left[\frac{1}{(s+1)^2(s+3)}\right]=\frac{A}{s+1}+\frac{B}{(s+1)^2}+\frac{C}{s+3}=\frac{-1/4}{s+1}+\frac{1/2}{(s+1)^2}+\frac{1/4}{s+3}$ $L^{-1}\left[\frac{1}{(s+1)^2(s+3)}\right]=-\frac{1}{4}e^{-t}+\frac{1}{2}e^{-t}t^2+\frac{1}{4}e^{-3t}$ — 2 $\frac{5+s}{(s+1)(s+3)}=\frac{2}{s+1}+\frac{(-1)}{s+3}$ $L^{-1}\left[\frac{5+s}{(s+1)(s+3)}\right]=2e^{-t}-e^{-3t}$ — 2 $\therefore I(t)=\frac{7}{4}e^{-t}+\frac{1}{2}e^{-t}t^2-\frac{3}{4}e^{-3t}$ — 1	08
	OR	
4(a)	$L^{-1}\left[\frac{s}{s^2+a^2}\right]=\cos at$ $L^{-1}\left[\frac{s}{s^2+b^2}\right]=\cos bt$ — 1+1 $L^{-1}[F(s)\,G(s)]=\int_0^t f(u)\,g(t-u)\,du$ — 1 $=\int_0^t \cos au\,\cos b(t-u)\,du$ $=\frac{1}{2}\int_0^t \left[\cos((a-b)u+bt)+\cos[(a+b)u-bt]\right]du$ $=\frac{1}{2}\left[\frac{\sin at-\sin bt}{a-b}+\frac{\sin at+\sin bt}{a+b}\right]$ — 4 $=\frac{a\sin at-b\sin bt}{a^2-b^2}$ — 1	08
4(b)	Taking Laplace transform on both sides $L\{y'''\}+4L\{y''\}+5L\{y'\}+2L\{y(t)\}=10$ — 1 ~~Yes~~ Apply IC's $y(0)=0$, $y'(0)=0$, $y''(0)=3$	

$$Y(s)\left[s^3+4s^2+5s+2\right]=3 \quad — 2$$

$$Y(s)=\frac{3}{(s+1)^2(s+2)}$$

By Partial Fraction Method

$$L^{-1}[Y(s)]=L^{-1}\left[\frac{3}{(s+1)^2(s+2)}\right] \quad \Bigg\} \; 4$$

Consider $\frac{3}{(s+1)^2(s+2)}=\frac{A}{s+1}+\frac{B}{(s+1)^2}+\frac{C}{s+2}=\frac{-3}{s+1}+\frac{3}{(s+1)^2}+\frac{3}{s+2}$

$$L^{-1}\left[\frac{3}{(s+1)^2(s+2)}\right]=3\left(-e^{-t}+e^{-t}t+e^{-2t}\right) \quad — 1$$

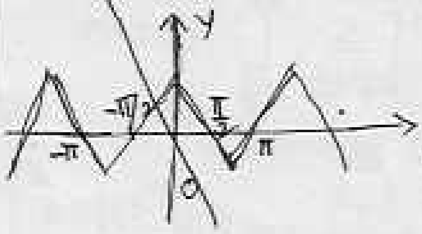
5(a) since $f(x)$ is even function

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx \quad — 1$$

$$a_0=\frac{2}{\pi}\int_0^{\pi}f(x)\,dx=\frac{2}{\pi}\int_0^{\pi}\left(1-\frac{2x}{\pi}\right)dx=0 \quad — 2$$

$$a_n=\frac{2}{\pi}\int_0^{\pi}\left(1-\frac{2x}{\pi}\right)\cos nx\,dx=\frac{+4}{\pi^2n^2}\left(1-(-1)^n\right)$$

$$\therefore f(x)=\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[1-(-1)^n\right] \quad — 3$$



— 2

5(b) Half range cosine series is $f(x)=\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{l}$ — 1

$$b_n=\frac{2}{l}\int_0^{l}f(x)\sin\frac{n\pi x}{l}\,dx \quad — 1$$

$$=\frac{2}{l}\int_0^{l/2}kx\sin\frac{n\pi x}{l}\,dx+\int_{l/2}^{l}k(l-x)\sin\frac{n\pi x}{l}\,dx$$

$$=\frac{2}{l}\Bigg[kx\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right)-(k)\frac{-\sin\frac{n\pi x}{l}}{(n\pi/l)^2}\Bigg]_0^{l/2}$$

$$+k(l-x)\left(\frac{-\cos\frac{n\pi x}{l}}{n\pi/l}\right)-(-k)\frac{-\sin\frac{n\pi x}{l}}{(n\pi/l)^2}\Bigg]_{l/2}^{l}$$

$$= \frac{2}{l}\left[\cancel{-\frac{kl^2}{2n\pi}\cos\frac{n\pi}{2}} + \frac{kl^2}{n^2\pi^2}\sin\frac{n\pi}{2} + \cancel{\frac{kl^2}{2n\pi}\cos\frac{n\pi}{2}} + \frac{l^2k}{n^2\pi^2}\sin\frac{n\pi}{2}\right]$$

$$= \frac{4kl}{n^2\pi^2}\sin\frac{n\pi}{2}$$ 3+3.

$$\therefore f(x) = \frac{4kl}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{l}$$ 08

6(a) $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$ — 1

$$c_n = \frac{1}{2\pi}\int_0^{2\pi}\cos ax\, e^{-inx}\,dx$$ — 1

$$= \frac{1}{2\pi}\left[\frac{e^{-inx}}{a^2-n^2}\left[-in\cos ax + a\sin ax\right]\right]_0^{2\pi}$$ — 2

$$c_n = \frac{1}{2\pi(a^2-n^2)}\left[in(1-\cos 2a\pi) + a\sin 2a\pi\right]$$

$$\therefore f(x) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\frac{e^{inx}}{a^2-n^2}\left[in(1-\cos(2a\pi)) + a\sin 2a\pi)\right]$$ — 1

$$a_n = c_n + c_{-n} = \frac{a\sin 2a\pi}{\pi(a^2-n^2)}$$ — 1

$$b_n = i(c_n - c_{-n}) = \frac{-n(1-\cos 2a\pi)}{\pi(a^2-n^2}$$ — 1

$$a_0 = \frac{1}{\pi a}\sin 2\pi a$$ — 1

$$\therefore f(x) = \frac{1}{2\pi a}\sin 2\pi a + \frac{a\sin 2a\pi}{\pi}\sum_{n=1}^{\infty}\frac{1}{a^2-n^2}\cos nx + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{-n(1-\cos 2a\pi)}{a^2-n^2}\sin nx.$$ 08

6(b). Half range cosine series is in $(0,1)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x$$ — 1

where $l = 1$

$a_0 = \frac{2}{1}\int_0^1 (x-1)^2 dx = \frac{2}{3}$ — 2

$a_n = 2\int_0^1 (x-1)^2 \cos n\pi x \, dx = \frac{4}{n^2\pi^2}$ — 2

$f(x) = \frac{2/3}{2} + \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{1}{n^2}\cos n\pi x$ — 1

put $x=1$ in above series

$0 = \frac{1}{3} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos n\pi$

$\Rightarrow -\frac{1}{3} = \frac{4}{\pi^2}\left(\frac{-1}{1^2} + 0 - \frac{1}{3^2} + 0 - \frac{1}{5^2} + \cdots\right)$

$\frac{\pi^2}{12} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ — 2 8

7(a). $F\{f(x)\} = \int_{-\infty}^{\infty} f(x)e^{i\alpha x}dx$ — 1

$= \int_{-a}^{a}(a^2-x^2)e^{i\alpha x}dx$ — 1

$= \frac{+4a\cos a\alpha}{\alpha^2} + \frac{4}{\alpha^3}\sin\alpha a$

$F(\alpha) = -\frac{4}{\alpha^3}(a\alpha\cos a\alpha - \sin\alpha a)$ — 2

$F^{-1}\{F(\alpha)\} = \int_{-\infty}^{\infty} F(\alpha)e^{-i\alpha x}d\alpha$ — 1

$f(x) = -\int_{-\infty}^{\infty}\frac{4}{\alpha^3}(a\alpha\cos a\alpha - \sin a\alpha)e^{-i\alpha x}d\alpha$

$= -\int_{-\infty}^{\infty}\frac{4}{\alpha^3}(a\alpha\cos a\alpha - \sin a\alpha)(\cos\alpha x - i\sin\alpha x)d\alpha$.

$\Rightarrow a^2 - x^2 = -\int_{-\infty}^{\infty}\frac{4}{\alpha^3}(a\alpha\cos a\alpha - \sin a\alpha)\cos\alpha x\, d\alpha$ — 2

since $f(x)$ is even function

$a^2 - x^2 = -2\int_0^{\infty}\frac{4}{\alpha^3}(a\alpha\cos a\alpha - \sin a\alpha)\cos\alpha x\, d\alpha$

Take $a=1$, $x=0$

$$1 = 8\int_0^\infty \frac{\sin\alpha - \alpha\cos\alpha}{\alpha^3}\, d\alpha \quad \Big\} - 2$$

$$\Rightarrow \int_0^\infty \frac{\sin\alpha - \alpha\cos\alpha}{\alpha^3}\, d\alpha = \frac{1}{8}. \ / \text{ Alter Ans: } \frac{\pi}{4}.$$ 8

7(b) Since in $(0,\infty) = e^{-|x|} = e^{-x}$. — 1

$$F_s\{f(x)\} = \int_0^\infty f(x)\sin\alpha x\, dx = \left[\frac{e^{-x}}{1+\alpha^2}(-\sin\alpha x - \alpha\cos\alpha x)\right]_0^\infty$$

$$= \frac{\alpha}{1+\alpha^2}. \quad \Big\} - 3.$$

~~By Inverse Fourier transfor.~~

$$\cancel{f(x) = \frac{2}{\pi}\int_0^\infty F_s\{f(x)\}\sin\alpha x\, d\alpha} \quad — 1$$

$$\cancel{e^{-x} = \frac{2}{\pi}\int_0^\infty \frac{\alpha}{1+\alpha^2}\sin\alpha x\, d\alpha.}$$

$$\cancel{\int_0^\infty \frac{\alpha\sin\alpha x}{1+\alpha^2}\, d\alpha = \frac{\pi e^{-x}}{2}}$$ $\Big\} 3$ for $m>0$

$$\cancel{\int_0^\infty \frac{x\sin mx}{1+x^2}\, dx = \frac{\pi e^{-m}}{2}.}$$ 8

8(a). $Z\{\sin(\frac{n\pi}{2}+\frac{\pi}{4})\} = Z\{\sin\frac{n\pi}{2}\cos\frac{\pi}{4} + \cos\frac{n\pi}{2}.\sin\frac{\pi}{4}\}$

(i)
$$= \frac{1}{\sqrt{2}}\left[\frac{z\sin\pi/2}{z^2-2z\cos\frac{\pi}{2}+1} + \frac{z(z-\cos\pi/2)}{z^2-2z\cos\pi/2+1}\right]$$

$$= \frac{1}{\sqrt{2}}\left[\frac{z}{z^2+1} + \frac{z^2}{z^2+1}\right] = \frac{(z+z^2)}{\sqrt{2}(z^2+1)} \quad \Big\} - 2$$

(ii) $Z\{2n + 5\cos\frac{n\pi}{4} + a^n\sinh n\theta - 3a^4\}$

$2Z(n) + 5Z(\cos\frac{n\pi}{4}) + Z(a^n\sinh n\theta)$

$-3a^4 Z\{1\}$. — 1+2

+2+1

08

$$= \frac{2z}{(z-1)^2} + \frac{z(z-\cos\pi/4)}{z^2-2z\cos\pi/4+1} + \frac{a^{-1}z\sinh\theta}{(a^{-1}z)^2-2a^{-1}z\cosh\theta+1}$$

$$-3a^4\frac{z}{z-1}$$ 8

8(b). Taking Z-transform on both sides

$$Z[y_{n+2}] - 5Z[y_{n+1}] + 6Z[y_n] = Z(1)$$ 1

$$z^2[Y(z) - y_0 - y_1 z^{-1}] - 5z[Y(z) - y_0] + 6Y(z) = \frac{z}{z-1}$$

Apply $y_0 = 0$, $y_1 = 1$

$$Y(z)(z^2-5z+6) = \frac{z^2}{z-1}$$ — 2

Consider $$\frac{Y(z)}{z} = \frac{z}{(z-1)(z-2)(z-3)}$$

By Partial Fraction method

$$\frac{z}{(z-1)(z-2)(z-3)} = \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{z-3}$$ — 2

$$\Rightarrow z = A(z-2)(z-3) + B(z-1)(z-3) + C(z-1)(z-2)$$

On taking, $z=2$, $z=3$ & $z=1$

$$\Rightarrow \frac{Y(z)}{z} = \frac{1/2}{z-1} - \frac{2}{z-2} + \frac{3/2}{z-3}$$ — 2

Taking Inverse

$$Z^{-1}[Y(z)] = \frac{1}{2}Z^{-1}\left[\frac{z}{z-1}\right] - 2Z^{-1}\left[\frac{z}{z-2}\right] + \frac{3}{2}Z^{-1}\left[\frac{z}{z-3}\right]$$

$$= \frac{1}{2} - 2\,2^n + \frac{3}{2}(3)^n$$ — 1

08

——— X ———

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RV COLLEGE OF ENGINEERING®

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III Semester B. E. Examinations Nov/Dec-19

Common CSE / ISE

LINEAR ALGEBRA, LAPLACE TRANSFORMS AND COMBINATORICS

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6.

PART-A

1	1.1	In $\mathbb{R}^2$, the vectors $v=\begin{bmatrix}1\\2\end{bmatrix}, w=\begin{bmatrix}1\\-1\end{bmatrix}$ are linearly independent, Justify the statement.	01
	1.2	The nullity of a 3×5 matrix is _______.	01
	1.3	If $A=\begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}$, then the geometric interpretation of the associated linear transformation is _______.	01
	1.4	The orthogonal projection of y onto u is _______ where $y=\begin{bmatrix}-24\\-10\end{bmatrix}$ and $u=\begin{bmatrix}3\\-15\end{bmatrix}$	01
	1.5	The kernel of $\begin{bmatrix}1 & 2\\2 & 4\\3 & 6\end{bmatrix}$ is _______.	01
	1.6	Eigen values of a 4×4 matrix A are given as $2,-3,13$ and 7. Then the value of $\lvert A\rvert$ is _______.	01
	1.7	If $L\{f(t)\}=\frac{e^{-2/s}}{s^2}$, then the $L\{e^t f(2t)\}$ is _______.	01
	1.8	If $L^{-1}\{F(s)\}=f(t)$, then $L^{-1}\left\{\frac{F(s-a)}{s}\right\}=$ _______.	01
	1.9	Find the singular value of the matrix $A=\begin{bmatrix}\sqrt{6} & 1\\0 & \sqrt{6}\end{bmatrix}$.	02
	1.10	Find the Laplace transform of the function $f(t)=\begin{cases}t & ; t\le\pi\\0 & ; t>\pi\end{cases}$	02
	1.11	When the integer n is divided by 8, the remainder is 3. What is the remainder if $6n$ is divided by 8.	02
	1.12	If both 11^2 and 3^3 are factors of the number $a\times4^3\times6^2\times13^{11}$, then the smallest possible value of 'a' is _______.	02
	1.13	The number of partitions of $X=\{a,b,c,d\}$ with a and b in the same block is _______.	02
	1.14	Find the generating function for the sequence $a_n=n$ for $n\ge0$.	02

PART-B

2	a	Determine the dimension and a basis for the four fundamental subspaces for the matrix $A=\begin{bmatrix}1&2&0&1\\0&1&1&0\\1&2&0&1\end{bmatrix}$.	10
	b	Let $T: v \to V$ be given by $T_x: x+v$. Is T a linear map? If not, under what conditions is T a linear map?	03
	c	Let A be an $m \times n$ matrix. Suppose that the null space of A is a plane in $\mathbb{R}^3$ and the range is spanned by a nonzero vector v in $\mathbb{R}^5$. Determine m and n. Also, find the rank and nullity of A.	03
3	a	Apply the Gram-Schmidt process to $a=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $b=\begin{bmatrix}0\\1\\1\end{bmatrix}$, $c=\begin{bmatrix}1\\1\\1\end{bmatrix}$ and write the result in the form $A=QR$.	08
	b	Obtain an invertible matrix P^{-1} and a diagonal matrix D, for the matrix $A=\begin{bmatrix}1&3&3\\-3&-5&-3\\3&3&1\end{bmatrix}$.	08
		OR	
4	a	Find the SVD of the matrix $A=\begin{bmatrix}1&0&1&0\\0&1&0&1\end{bmatrix}$.	10
	b	If $A=\begin{bmatrix}4&3\\1&2\end{bmatrix}$, then find A^{100} by diagonalizing A.	06
5	a	Find the Laplace Transform of $t^6e^{3t}+t^2\cos 2t+\frac{e^{-3t}\sin t}{t}$.	08
	b	Find the Laplace Transform of $f(t)=\begin{cases}t, & 0<t<a\\2a-t, & a<t<2a\end{cases}$ where $f(t+2a)=f(t)$.	08
		OR	
6	a	Find the Laplace Transform of $f(t)=\begin{cases}1 & , 0<t<\frac{a}{2}\\-1, & \frac{a}{2}<t<a\end{cases}$	08
	b	An alternative e.m.f $E\sin(\omega t)$ is applied to an inductance L and a capacitance C in series. The governing equation is given by $L\frac{d^2q}{dt^2}+\frac{q}{c}=E\sin(\omega t)$. Obtain the current in the LC circuit.	08
7	a	Prove that there are infinitely many primes of the form $4n+3$ where n is a positive integer.	04
	b	Find the remainder of 99^{999999} when divided by 23.	04
	c	If the cipher text message produced by the RSA cipher with key $(e,n)=(7,33)$ is 1301 2214 02301 261406, what is the plain text message?	08

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Academic year 2022-2023 (Odd Semester)

DEPARTMENT OF MATHEMATICS				
Date	28^{th} March 2023	Time	09:30 AM to 11:20 PM	
Quiz + Test	II	Maximum Marks	10 + 50	
Course Title	LINEAR ALGEBRA, INTEGRAL TRANSFORMS AND FOURIER SERIES		Course Code	21MA31B
Semester	III	Programs	AS, EC, EE, EI, ET-Lateral Entry	

S. No.	Quiz Questions	M	CO	BT
1	Inverse Laplace transform of $F(s)=\frac{2}{s^2+8}$ is _____.	1	1	1
2	Show that the vectors $(1,2,1)$, $(3,1,5)$ and $(-1,3,-3)$ are linearly independent.	2	2	2
3	Let $A=\begin{bmatrix}1 & 2\\ 3 & 6\end{bmatrix}$ and $v=\begin{bmatrix}-3\\ 1\end{bmatrix}$. Determine if v belongs to left null space of A.	2	1	2
4	Is the subset $W=\{(x_1,x_2,x_3)\ /\ x_1^2+x_2^2+x_3^2\le 1\}$ as subspace of $\mathbb{R}^3$?	2	3	3
5	$L[\sin t\, H(t-\pi)]=$ ______.	2	2	3
6	$L^{-1}\left(\frac{1}{2s-3}\right)$ is ______.	1	1	2

S.No.	Test Questions	M	CO	BT
1	Find the dimension and basis for the four fundamental subspaces of the matrix $A=\begin{bmatrix}1 & 3 & 3 & 2\\ 2 & 6 & 9 & 7\\ -1 & -3 & 3 & 4\end{bmatrix}$	10	1	2
2	Find the range space, null space, rank, nullity and verify rank-nullity theorem for $T:\mathbb{R}^3\to\mathbb{R}^4$, defined by $T(e_1)=(0,1,0,2), T(e_2)=(0,1,1,0), T(e_3)=(0,1,-1,4)$.	10	2	3
3	Show that the set $B=\{u=(1,1,0), v=(1,0,1), w=(0,1,1)\}$ is a basis of the vector space $\mathbb{R}^3$. Express each standard basis vector as a linear combination of u, v and w.	10	3	3
4	Express the following function as a Heaviside step function and hence calculate the Laplace transform $f(t)=\begin{cases}2, & 0\le t<1\\ 3t+1, & 1\le t<2\\ t^2, & 2\le t<3\end{cases}$.	10	2	2
5	Evaluate Inverse Laplace transform of the following functions: (i) $F(s)=\frac{4s+5}{(s+1)^2(s+2)}$ (ii) $F(s)=\frac{s}{s^2-6s+1}$	10	4	4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars		CO1	CO2	CO3	CO4	L1	L2	L3	L4
	Quiz	Max Marks	4	4	2	-	1	5	4	-
	Test	Max Marks	10	20	10	10	-	20	20	10

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RV COLLEGE OF ENGINEERING®

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III Semester B. E. Examinations, December – 2019

BRANCH: Computer Science & Engineering and Information Science & Engineering

COURSE: LINEAR ALGEBRA, LAPLACE TRANSFORMS AND COMBINATORICS

MODEL QUESTION PAPER - I

Time: 03 Hours **Maximum Marks: 100**

Instructions to candidates:

1. Answer all questions from Part A. Part A question should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6.

PART – A

1	1.1	The subset E of R^2 defined by E = {(x, y)∈ R^2 | 2x +3y = 4} is not a subspace of R^2. Justify the statement.	01
	1.2	A is a matrix of order 5 x 6 and dim N(A) = 4. Then dim $C(A^T)$ = _____.	01
	1.3	The product of five reflections and eight rotations is a ___________.	01
	1.4	The eigenvalues of the projection matrix P are ________.	01
	1.5	Let T : $R^2 \to R^2$ be the projection of $\begin{bmatrix} x \\ y \end{bmatrix}$ on to X – axis i. e. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$, the Ker (T) = ___________.	01
	1.6	State the Rank-Nullity theorem for an m x n matrix A.	01
	1.7	Verify directly from $\cos^2\theta + \sin^2\theta = 1$ that reflection matrices satisfy $H^2 = I$.	01
	1.8	In the singular value decomposition $A = U\Sigma V^T$, C(A) = span of __________.	01
	1.9	If $L\left\{\frac{\sin t}{t}\right\}$ is $\tan^{-1}\left(\frac{1}{s}\right)$, then $L\left\{\frac{\sin 2t}{2t}\right\}$ = ___________.	01
	1.10	$L^{-1}\{F(s)\} = f(t)$, then $L^{-1}\left\{\frac{F(s-a)}{s}\right\}$ = __________.	01
	1.11	Find $L^{-1}\left\{\log\left(1 - \frac{1}{s^2}\right)\right\}$.	02
	1.12	Find the sum of positive divisors of the integer 882.	02
	1.13	What is the remainder in the division of 2^{50} by 7.	02
	1.14	Find the rook polynomial for the 2 × 2 board by using expansion formula.	02
	1.15	Find the sequence generated by the function $(3 + x)^3$.	02

PART – B

2	a)	Express $v = t^2 + 4t - 3$ in P(t) as a linear combination of $p_1 = t^2 - 2t + 5$, $p_2 = 2t^2 - 3t$, $p_3 = t + 1$.	04
	b)	Let V be the vector space of function f: R → R. Show that W is a subspace of V where i) W = {f(x) : f(1) = 0} all function whose value at 1 is 0. ii) W = {f(x) : f(3) = f(1)}.	04

	c)	Let $G: R^3 \rightarrow R^3$ be given by $G(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find a basis and dimension of i) Image of G ii) Kernel of G. Verify Rank-Nullity theorem.	08
3	a)	Using Gram-Schmidt orthogonalization process to find an orthonormal basis for the linearly independent set of vectors $\{(1, 2, 1), (-1, 1, 0), (5, -1, 2)\}$ in R^3.	10
	b)	Find the invertible matrix P which diagonalizes the matrix $A = \begin{bmatrix} 5 & 0 & 4 \\ 0 & 5 & 4 \\ 4 & 4 & 9 \end{bmatrix}$.	06
		OR	
4	a)	Diagonalize the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$.	06
	b)	Find the singular value decompositions of the matrix and verify that $A = U \sum V^T$, given $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$.	10
5	a)	Evaluate $(i) L\left\{\int_0^t te^{-3t} \sin 2t \, dt\right\}$ $(ii) \int_0^\infty e^{-t}\left(\frac{1-\cos t}{t}\right) dt$.	08
	b)	The equation of the LRC circuit governing the current i is given by $L\frac{di}{dt} + Ri + \frac{1}{c}\int_0^t i dt = E\delta(t)$ where I = 0, when t = 0. Using Laplace transform method find the current i.	08
		OR	
6	a)	Express the following in terms of Heaviside unit step function and also sketch the graph of the function $f(t) = \begin{cases} t^2 & 0 < t < 2 \\ 4t & 2 < t < 4 \\ 8 & t > 4 \end{cases}$ and find its Laplace transform.	08
	b)	Using convolution theorem, find $L^{-1}\left\{\frac{s}{(2s^2+1)(s^2-4)}\right\}$.	08
7	a)	Prove that there exist infinitely many primes.	04
	b)	If $a \equiv b (\text{mod } m)$ and GCD(a, m) = 1, then prove that GCD(b, m) = 1.	04
	c)	If the cipher text message produced by the RSA cipher with key $(e, n) = (5, 2881)$ is 0504 1874 0347 0515 2088 2356 0736 0468, what is the plain text message.	08
8	a)	Using the exponential generating function, find the number of ways in which 4 of the letters in "ENGINE" be arranged.	08
	b)	In how many ways can 12 Modems be distributed among three networking labs A, B, C so that A gets at least four, B and C gets at least two, but C gets not more than five?	08

R V COLLEGE OF ENGINEERING®
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DEPARTMENT OF MATHEMATICS

ODD SEMESTER 2019-20
III SEMESTER, Test - 1
BRANCHES:CSE, ISE

COURSE : LINEAR ALGEBRA, LAPLACE TRANSFORMS AND COMBINATORICS

DATE: 03.09.2019
COURSE CODE : 18MA31A

TIME : 9.30 AM – 11 AM
MARKS : 50

Q.No.	Answer all the questions	Marks	CO	BTL
1.	Show that the set of all elements of the type $a + b\sqrt{2}$: $a, b \in Q$ form a vector space over the field Q.	10	CO2	L2
2. a)	Determine if the set H of all matrices of the form $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ is a subspace of M_{2x2}, the set of all 2 X 2 Matrices.	05	CO2	L2
b)	If $v_1 = (2, -1, 0)$, $v_2 = (1, 2, 1)$ and $v_3 = (0, 2, -1)$, show that v_1, v_2, v_3 are linearly independent. Express (3, 2, 1) as a linear combination of v_1, v_2, v_3.	05	CO2	L2
3.	Find the dimension and basis for the four fundamental subspaces of the matrix $$A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 4 & 8 & 12 \\ 3 & 6 & 7 & 13 \end{bmatrix}$$	10	CO3	L2
4. a)	Find the number of positive divisors and sum of all positive divisors of 8128.	05	CO1	L2
b)	If p is a prime number and $p\|ab$ where a and b are any integers, then prove that either $p\|a$ or $p\|b$.	05	CO1	L2
5.	By using the Euclidean algorithm, find the greatest common divisor d of 2689 and 4001 and then find integers x and y to satisfy $2689x + 4001y = d$. Also show that x and y are not unique.	10	CO2	L2

COs.

1. Understand the fundamental concepts of linear algebra, properties of Laplace and inverse Laplace transforms, divisibility, properties of prime numbers and principle of inclusion and exclusion.
2. Solve the problems of vector spaces, subspaces, basis and dimension, rank and nullity theorem, orthogonal and orthonormal basis, Laplace transform of different functions, linear transformations, geometrical interpretations and matrix form, greatest common divisor, derangements and generating functions.
3. Apply the acquired knowledge to solve the problems of rank and nullity theorem, Gram-Schmidt process, QR-factorization, transform of periodic functions, convolution theorem modular arithmetic, Euler's theorem and exponential generating functions.
4. Evaluate - solution of differential equations with initial and boundary conditions using Laplace transform, diagonalization of matrix, singular value decomposition, rook polynomials, Turing's code and RSA public key encryption.

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Approved by AICTE, New Delhi

Academic year 2022-2023 (Odd Semester 2022)

DEPARTMENT OF MATHEMATICS			
Date	16th January 2023	Time	11:45AM to 01:45 PM
CIE	I	Maximum Marks	10+50
Course Title	Linear Algebra, Integral Transforms and Fourier Series	Course Code	21MA31B
Semester	III	Programs	AS, EC, EE, EI, ET

Instructions: i) Answer all questions from Part-A and Part-B.
ii) Part-A questions should be answered in first two pages of the answer book only.

Q. No.	PART-A	M	CO	BT
1	If $\mathcal{L}\{f(t)\} = \frac{6s}{(s^2+9)^2}$, then $\mathcal{L}\left\{f\left(\frac{t}{3}\right)\right\} =$ ____.	1	1	L1
2	$\mathcal{L}\left\{\left(\frac{1}{4}\right)^t\right\} =$ ____.	1	1	L1
3	Laplace transform of the exponentially decayed sinusoidal signal $f(t) = e^{-at}\sin(2\pi t)$ is ____.	2	2	L2
4	Obtain $\mathcal{L}\{t\delta(t-5)\}$, where $\delta(t-5)$ represents unit impulse function.	2	1	L1
5	$\mathcal{L}^{-1}\left\{\frac{s\,e^{-\frac{s}{2}}}{s^2-b^2}\right\} =$ ____.	2	3	L2
6	Find inverse Laplace transform of $\frac{1}{s^2+4s+4}$.	2	2	L2

Q. No.	PART-B	M	CO	BT
1	Laplace transform possesses powerful set of properties for analysis of signals and systems. Find Laplace transform of the following time signals: (i) $t^2\sin 3t\cos 3t$ (ii) $\int_0^t \frac{e^{2t}-\cos 4t}{t}\,dt$	10	1	L1
2a	Solve $\int_0^\infty e^{-\frac{t}{2}}\left(\sqrt{t}+\frac{1}{\sqrt{t}}\right)^2 dt$ using Laplace transform technique.	05	2	L2
2b	Show that Laplace transform of the triangular wave of period four given by $f(t) = \begin{cases} t, & 0 \le t < 2 \\ 4-t, & 2 \le t < 4 \end{cases}$ is $\frac{1}{s^2}\tanh(s)$.	05	3	L3
3	Express the following function as a Unit step function and hence determine its Laplace transform $f(t) = \begin{cases} 1, & 0 \le t < \pi \\ 3t, & \pi \le t < 2\pi \\ \sin t, & t \ge 2\pi \end{cases}$. Also draw the graph of the signals (i) $\cos t\, H(t-\frac{\pi}{2})$ and (ii) $\cos(t-\frac{\pi}{2})\,H(t-\frac{\pi}{2})$ for $0 \le t \le 3\pi$.	10	3	L3
4	Transform the s-domain function in to the corresponding time domain $F(s) = \frac{s^2-10s+13}{(s-7)(s^2-5s+6)} + tan^{-1}\left(\frac{4}{s}\right)$.	10	2	L2
5	Convolution is used to express the input and output relationship of Linear Time-Invariant (LTI) systems. Apply convolution theorem to determine the inverse Laplace transform of the function $F(s) = \frac{1}{s^3(s^2+4)}$. Also verify the result.	10	4	L4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars		CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
	Quiz &Test	Max Marks	14	19	17	10	14	21	15	10	-	-

RV COLLEGE OF ENGINEERING
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III Semester B. E. March/April-2022 Examinations
LINEAR ALGEBRA, LAPLACE TRANSFORMS AND COMBINATORICS
(Theory)
SCHEME AND SOLUTION

1.1	$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 4 \\ 1 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & -3 \\ 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & -3 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad |A| = -9 \neq 0 \quad \therefore$ lin. ind. Here only two vectors are independent. Hence the given set of vector does not form a basis for $\mathbb{R}^3$.	1 1
1.2	i) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, ii) $\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \rightarrow \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ only not 'a'	1+1
1.3	Multiple $x^n = \frac{a^T b}{a^T a} = \frac{[1 \;\; 1]\begin{bmatrix} 4 \\ 0 \end{bmatrix}}{[1 \;\; 1]\begin{bmatrix} 1 \\ 1 \end{bmatrix}} = 2$	1+1
1.4	$AX = \lambda X \Rightarrow \begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -4 \end{bmatrix}\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -18 \\ -16 \\ 4 \end{bmatrix} = 4\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$ $\lambda = 4$	1 1
1.5	$L[t\cos 2t] = \frac{s^2-4}{(s^2+4)^2}$ $\int_0^\infty e^{-3t}\, t\cos 2t\, dt = \frac{5}{169}$ $\quad \frac{11}{39}$	1 1
1.6	$\frac{1}{\sqrt{2}} L^{-1}\left[\frac{1}{\sqrt{s+\frac{3}{2}}}\right] = \frac{e^{-\frac{3}{2}t}}{\sqrt{2}} L^{-1}\left[\frac{1}{\sqrt{s}}\right]$ $= \frac{e^{-\frac{3}{2}t}}{\sqrt{2t}\sqrt{\pi}}$	1+1
1.7	$T(a) = (1+a_1)(1+a_2)$ $\quad 2^2\; 3+3$ 6 positive divisors	1+1
1.8	$219 = 3 \times 73$ $\phi(219) = 144$	1 1
1.9	$d_4 = 4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 9$ Derangements	1 1
1.10	$f(x) = a_0 + a_{1x} + a_2x^2 + a_3x^3 + \cdots$ $= 2 + 4x + 8x^2 + 16x^3 + \cdots$ $= 2(1-2x)^{-1}$ $\quad = 2(1 + 2x + (2x)^2 + (2x)^3 + \cdots$ $= 2(1-2x)^{-1}$	1+1

2a

$$A=\begin{bmatrix}1&2&3&-1\\2&-1&-4&8\\-1&1&3&-5\\-1&2&5&-6\\-1&-1&3&1\end{bmatrix} R_2=R_2-2R_1, R_3=R_3+R_1, R_4=R_4+R_1, R_5=R_5+R_1$$

$$\sim\begin{bmatrix}1&2&3&-1\\0&-5&-10&10\\0&3&6&-6\\0&4&8&-7\\0&0&0&0\end{bmatrix} R_2=-\tfrac{1}{5}R_2$$ 2

$$\sim\begin{bmatrix}1&2&3&-1\\0&1&2&-2\\0&3&6&-6\\0&4&8&-7\\0&0&0&0\end{bmatrix} R_3=R_3-3R_2, R_4=R_4-4R_2$$ 1

$$\sim\begin{bmatrix}1&2&3&-1\\0&1&2&-2\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix} R_3\leftrightarrow R_4$$ 1

$$\sim\begin{bmatrix}1&2&3&-1\\0&1&2&-2\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}$$ 1

Basis for $R(A)$ $\{(1,2,3,-1),(0,1,2,-2),(0,0,0,1)\}$ 1

Basis for $C(A)$ $\left\{\begin{bmatrix}1\\2\\-1\\-1\\-1\end{bmatrix},\begin{bmatrix}2\\-1\\1\\2\\2\end{bmatrix},\begin{bmatrix}-1\\8\\-5\\-6\\1\end{bmatrix}\right\}$

Basis for $N(A)$, $Ax=0$ implies $Ux=0$ 1

$$\begin{bmatrix}1&2&3&-1\\0&1&2&-2\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

(1, -2, 1, 0)

$x_1+2x_2+3x_3-x_4=0$
$x_2+2x_3-2x_4=0$
$x_4=0$
Let $x_3=k, x_2=-2k, x_1=2k-3k=-k$

Basis for $N(A)$ $\left\{\begin{bmatrix}-k\\-2k\\k\\0\end{bmatrix}\right\}$ or $\left\{\begin{bmatrix}-1\\-2\\1\\0\end{bmatrix}\right\}$ 1

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RV COLLEGE OF ENGINEERING
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III Semester B.E. April -2023 Examinations
DEPARTMENT OF MATHEMATICS
LINEAR ALGEBRA, INTEGRAL TRANSFORMS AND FOURIER SERIES
(Common to AS, EC, EE, EI, ET)
(2021 SCHEME)
MODEL QUESTION PAPER-1

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, and 9 and 10.

PART-A

1	1.1	Consider the polynomials $p_1(t) = 1 + t^2$, $p_2(t) = 1 - t^2$. Is $\{p_1, p_2\}$ a linearly independent set in P_3? Justify your answer.	2
	1.2	Write which matrix reflects every vector in $\mathbb{R}^2$ about the line $y = x$. Also find the transformation of the vector $(-3, 2)$?	2
	1.3	Show that the set $\{u_1, u_2, u_3\}$ is an orthogonal basis of $\mathbb{R}^3$, where $u_1 = (3, -3, 0)$, $u_2 = (2, 2, -1)$, $u_3 = (1, 1, 4)$	2
	1.4	Choose the second row of $A = \begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix}$ so that A has eigen values 4 and 7.	2
	1.5	The region of convergence for $L[\cosh at] = \frac{s}{s^2 - a^2}$ to hold good is ________.	1
	1.6	The Laplace transform of the signal $(1 - a)^t$, where a is constant is ________.	1
	1.7	If $f(t) = t^{3/2}$ then $L\{f(t)\} =$ ________.	1
	1.8	$L\{(t-2)\,u(t-2)\} =$ ________.	1
	1.9	If $L^{-1}[F(s)] = \sin 2t$, then $L^{-1}\left[\frac{F(s)}{s}\right]$ is ________.	1
	1.10	Find $L^{-1}\left[\frac{5e^{-3s}}{s}\right] =$ ________.	1
	1.11	Inverse Laplace transform of $\left[\frac{1}{(s+a)^{\frac{3}{2}}}\right]$ is ________.	1
	1.12	$L^{-1}(1) =$ ________.	1
	1.13	The Fourier series coefficient a_0 for the signal $x(t) = e^{-2t}$, $0 \le t \le 2$ is________.	1
	1.14	At the point of discontinuity, Fourier series of $f(x) = \begin{matrix} -\pi, -\pi < x < 0 \\ x, \ 0 < x < \pi \end{matrix}$ converges to ________.	1
	1.15	If the Fourier transform of $e^{-\frac{x^2}{2}}$ is $\sqrt{2\pi}\, e^{-\frac{s^2}{2}}$, then the Fourier transform of e^{-2x^2} is________.	1
	1.16	The Fourier cosine transform of x^3 is $\frac{4}{s^4}$, then Fourier sine transform of $x^3 \sin 3x$ is ________.	1

PART-B

UNIT-I

2	a	Show that the set of all vectors of the form $(2s+4t, 2s, 2s-3t, 5t)$ is a subspace of R^4.	4
	b	Obtain the bases for the column space and left null space of the matrix $A=\begin{bmatrix}1 & 2 & -5 & 11 & -3\\ 2 & 4 & -5 & 15 & 2\\ 1 & 2 & 0 & 4 & 5\\ 3 & 6 & -5 & 19 & -2\end{bmatrix}$.	6
	c	Find the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^4$ such that $T(e_1)=(0,1,0,2), T(e_2)=(0,1,1,0), T(e_3)=(0,1,-1,4)$. Also find the rank and nullity of the linear transformation.	6

UNIT-II

3	a	Using the Gram-Schmidt process, orthonormalize the vectors $(3,1,-1,3)$, $(-5,1,5,-7)$, $(1,1,-2,8)$.	8
	b	Find the SVD of the matrix $A=\begin{bmatrix}3 & 2 & 2\\ 2 & 3 & -2\end{bmatrix}$.	8
		OR	
4	a	Compute the QR factorization of the matrix $A=\begin{bmatrix}1 & 2 & 5\\ -1 & 1 & -4\\ -1 & 4 & -3\\ 1 & -4 & 7\\ 1 & 2 & 1\end{bmatrix}$.	8
	b	Obtain the matrix P such that it diagonalises the matrix $A=\begin{bmatrix}7 & -4 & 4\\ -4 & 5 & 0\\ 4 & 0 & 9\end{bmatrix}$. Also find the inverse of P.	8

UNIT-III

5	a	Evaluate $L\left[\frac{2sint\,sin2t}{t}+t\,cos2t\right]$.	6
	b	Determine Laplace transform of the triangular wave given by $f(t)=\begin{cases}\frac{h}{a}t, & 0<t<a\\ \frac{h}{a}(2a-t), & a<t<2a\end{cases}$ with $f(t)=f(t+2a)$.	6
	c	Using Laplace transform show that $\int_0^\infty (t\,e^{-t}sin2t\,dt)=\frac{4}{25}$.	4
		OR	
6	a	Obtain the Laplace transform of $f(t)=cos^3 2t+e^{-3t}(2cos5t-3sin5t)$.	6
	b	Evaluate $L\left\{\int_0^t \frac{e^{-t}sin3t}{t}dt\right\}$.	6
	c	Express $f(t)=\begin{cases}sint, & 0<t\le \pi/2\\ cost, & t>\pi/2\end{cases}$ in terms of the unit step function and hence find its Laplace transform.	4

UNIT-IV			
7	a	Using Convolution theorem, transform the following function in time domain: $F(s) = \left[\frac{s}{(s^2+a^2)\,(s^2+b^2)}\right]$	8
	b	Solve the differential equation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 1 - e^{2t}$ under the conditions $y(0) = 1,\ y'(0) = 0$ using Laplace transform.	8
		OR	
8	a	Determine the inverse Laplace transform of the following: (i) $\frac{s+3}{s^2-4s+13}$ (ii) $\frac{e^{-3s}}{(s^2+1)\,(s^2+9)}$	8
	b	A voltage $E(t)=Ee^{-at}$ is applied at t = 0 to a circuit of inductance L and resistance R satisfying the equation $L\frac{di}{dt} + Ri = E(t)$. Show that the current at any time t is $\frac{E}{R-aL}\left[e^{-at} - e^{-\frac{Rt}{L}}\right]$.	8

UNIT-V			
9	a	Draw the graph of the function $f(t) = \begin{cases} 0, & -2 \le t \le -1 \\ 1+t, & -1 \le t \le 0 \\ 1-t, & 0 \le t \le 1 \\ 0, & 1 \le t \le 2 \end{cases}$ when $f(t+4) = f(t)$. Also express f(t) as trigonometric series.	8
	b	Find the Fourier transform of a parabolic pulse given by $f(t) = \begin{cases} 1-t^2, & \lvert t\rvert < 1 \\ 0, & \lvert t\rvert > 1 \end{cases}$ Hence evaluate the integral $\int_0^\infty \frac{t\cos t - \sin t}{t} \cos\left(\frac{t}{2}\right) dt$.	8
		OR	
10	a	Expand $f(x) = \left(\frac{\pi - x}{2}\right)^2, 0 < x < 2\pi$ in a Fourier series.	8
	b	Determine the Fourier cosine transform of $\frac{1}{1+x^2}$. Hence derive Fourier sine transform of $\frac{x}{1+x^2}$.	8

Signature of Scrutinizer:
Name:

Signature of Chairman
Name:

8	a	A large software development company employs 100 computer programmers. Of them, 45 are proficient in Java, 30 in $C\#$, 20 in Python, 6 in $C\#$ and Java, 1 in Java and Python, 5 in $C\#$ and Python, and just 1 programmer is proficient in all three languages above. Determine the number of computer programmers that are not proficient in any of these three languages.	08
	b	Find the rook polynomial for the following forbidden position problem. You may leave the polynomial in factored form, and you need not go any farther with the problem than finding the rook polynomial. We want to find the number of ways 5 people (A, B, C, D and E) can be assigned 5 tasks (1,2,3,4 and 5) to do if person A cannot do tasks 1 and 2, person B cannot do tasks 2 and 4, person C cannot do tasks 1 and 2, and person D cannot do tasks 3 and 4, and person E cannot do tasks 4 and 5.	08

SCHEME AND SOLUTION

COURSE CODE: 18MA31A COURSE: LINEAR ALGEBRA, LAPLACE TRANSFORM AND COMBINATORICS

UG

Q. No		PART - A	Marks
1	1.1	$\alpha\begin{bmatrix}1\\2\end{bmatrix}+\beta\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\Rightarrow \alpha+\beta=0,\ 2\alpha-\beta=0\Rightarrow \alpha=0,\ \beta=0$ is trivial solution. Therefore linearly independent.	01
	1.2	2, 3, 4 or 5 – rank.	01
	1.3	Rotates counter clockwise through 90^0 and doubles the length.	01
	1.4	$\hat{y}=\frac{y\cdot u}{u\cdot u}u=\frac{(-24,10)\cdot(3,-15)}{(3,-15)\cdot(3,-15)}(3,-15)=(1,-5)$	01
	1.5	The line $x_1+2x_2=0$ or Kernel = (2,-1)	01
	1.6	2 x -3 x 13 x 7 = -546	01
	1.7	$L[f(2t)]=\frac{2e^{-\frac{4}{s}}}{s^2}$ then $L[e^t f(2t)]=\frac{2e^{-\frac{4}{(s-1)}}}{(s-1)^2}$	01
	1.8	$L^{-1}\left\{\frac{F(s-a)}{s}\right\}=\int_0^t e^{at}f(t)dt$	01
	1.9	$AA^T=\begin{bmatrix}7 & \sqrt{6}\\ \sqrt{6} & 6\end{bmatrix},\ \lvert AA^T-\lambda I\rvert=0\Rightarrow \lambda^2-13\lambda+36=0\Rightarrow\lambda=9,\ 4.$ Singular values of matrix A are 3, 2.	02
	1.10	$L[f(t)]=\int_0^{\pi} e^{st}t\,dt=\frac{1-\pi s e^{-s\pi}-e^{-s\pi}}{s^2}$	02
	1.11	n = 8k + 3 ⇒ 6n = 6(8k + 3) = 8(6k) + 18 = 8(6k + 2) + 2 ⇒ 2 is the remainder	02
	1.12	The smallest value of a is $\frac{a\times 4^3\times 6^7\times 13^{11}}{11^2\times 3^3}\Rightarrow 11^2\times 3=363$	02
	1.13	Insufficient data, Grace marks to be awarded.	02
	1.14	$\frac{1}{1-x}=1+x+x^2+x^3+\cdots$ *Differentiating* $\frac{1}{(1-x)^2}=1+2x+3x^2+4x^3+\cdots$ Multiply by x $\frac{x}{(1-x)^2}=x+2x^2+3x^3+4x^4+\cdots=\sum_{n=0}^{\infty}nx^n$ So $G(x)=\frac{x}{(1-x)^2}$	01 01

PART - B

2 a)

$$A=\begin{bmatrix}1&2&0&1\\0&1&1&0\\1&2&0&1\end{bmatrix}\xrightarrow{R_3-R_1}U=\begin{bmatrix}1&2&0&1\\0&1&1&0\\0&0&0&0\end{bmatrix}$$ 1

i) Column space of A

Basis for C(A) = {(1, 0, 1), (2, 1, 2)} 1

Dim of C(A) = 2

ii) Row space of A

Basis for R(A) = {(1, 2, 0, 1), (0, 1, 1, 0)}

Dim of R(A) = 2 1

iii) Null space of A

$$U=\begin{bmatrix}1&2&0&1\\0&1&1&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}\Rightarrow\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=x_3\begin{bmatrix}2\\-1\\1\\0\end{bmatrix}+x_4\begin{bmatrix}-1\\0\\0\\1\end{bmatrix}$$ 1

Basis for N(A) = {(2, -1, 1, 0), (-1, 0, 0, 1)} 1

Dim of N(A) = 2. 1

iii) Left Null space of A

$$A^T=\begin{bmatrix}1&0&1\\2&1&2\\0&1&0\\1&0&1\end{bmatrix}\xrightarrow{R_2-2R_1,\ R_4-R_1,\ R_3-R_2}U=\begin{bmatrix}1&0&1\\0&1&0\\0&0&0\\0&0&0\end{bmatrix}$$ 2

$$U=\begin{bmatrix}1&0&1\\0&1&0\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}\Rightarrow\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=y_3\begin{bmatrix}-1\\0\\1\end{bmatrix}$$ 1

Basis for N(A) = {(-1, 0, 1)}

Dim of N(A^T) = 1. 1

b) Let T: v → V given T(x) : x + v.

T a linear map

Here T(x + y) = x + y + v

T(x) + T(y) = (x + v)+(y + v) ⇒ T(x + y) = T(x) + T(y) iff v = 0. 1

T(αx) = αx + v , αT(x) = α(x + v) 1

T(αx) = αT(x) iff v = 0.

Therefore T is a linear transformation iff v = 0. 1

c) For an m x n matrix A, the null space consists of vectors X such that AX = 0. Thus such X must be n – dimensional. Since the null space is a subspace in R^3, then n = 3. 1

The range of A consists of vectors y such that y = AX. As the range is the subspace of R^5, then m = 5. 1

Since a plane is a 2 dimensional subspace, the nullity of A is 2 and the rank is 1. 1

3	a)	$v_1 = a = (0,0,1)$	1
		$v_2 = b - \frac{b \cdot v_1}{v_1 \cdot v_1} v_1 = (0,1,0)$	2
		$v_3 = c - \frac{c \cdot v_1}{v_1 \cdot v_1} v_1 - \frac{c \cdot v_2}{v_2 \cdot v_2} v_2 = (1,0,0)$	2
		$Q = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad R = Q^T A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	1+2
	b)	$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$	
		$\lvert A - \lambda I \rvert = 0 \Rightarrow \lambda^3 + 3\lambda^2 - 4 = 0 \Rightarrow \lambda = 1, -2, -2$	1+1
		for $\lambda = 1$ $X_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, for $\lambda = -2$ $X_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, for $\lambda = 1$ $X_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$	1+1+1
		$P = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix} \Rightarrow D = P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$	2+1
		OR	
4	a)	$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$	
		$A^T A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \Rightarrow \lvert A^T A - \lambda I \rvert = 0 \Rightarrow \lambda^4 - 4\lambda^3 + 4\lambda^2 = 0 \Rightarrow \lambda = 0, 0, 2, 2$	1+2
		Finding eigen vectors	2
		$V = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix},$	1
		$AA^T = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow \lvert AA^T - \lambda I \rvert = 0 \Rightarrow \lambda^2 - 4\lambda + 4 = 0 \Rightarrow \lambda = 2, 2$	1+1
		$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \sum = \begin{bmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \end{bmatrix} \Rightarrow SVD = U \sum V^T$	1+1

	b)	$A=\begin{bmatrix}4&3\\1&2\end{bmatrix}$	1+1
		$\lvert A-\lambda I\rvert=0\Rightarrow\lambda^2-6\lambda+5=0\Rightarrow\lambda=5,1$	
		when $\lambda=5,\ X_1=\begin{bmatrix}3\\1\end{bmatrix}$, when $\lambda=1,\ X_1=\begin{bmatrix}-1\\1\end{bmatrix}\Rightarrow P=\begin{bmatrix}3&-1\\1&1\end{bmatrix}$ & $P^{-1}=\frac{1}{4}\begin{bmatrix}1&1\\-1&3\end{bmatrix}$	2+1
		$D=P^{-1}AP,\quad A=PDP^{-1}\Rightarrow A^{100}=PD^{100}P^{-1}$	
		$\Rightarrow A^{100}=\frac{1}{4}\begin{bmatrix}3&-1\\1&1\end{bmatrix}\begin{bmatrix}5^{100}&0\\0&1^{100}\end{bmatrix}\begin{bmatrix}1&1\\-1&3\end{bmatrix}$	1
5	a)	$L\left\{t^6e^{3t}+t^2\cos 2t+\frac{e^{-3t}\sin t}{t}\right\}=\frac{6!}{(s-3)^7}+(-1)^2\frac{d^2}{ds^2}\left(\frac{s}{s^2+4}\right)+\int_s^\infty\frac{1}{(s+3)^2+1}ds$	1+1+1
		$=\frac{6!}{(s-3)^7}+\frac{d}{ds}\left[\frac{1}{s^2+4}-\frac{2s^2}{(s^2+4)^2}\right]+\tan^{-1}(s+3)\Big\rvert_s^\infty$	0+1+1
		$=\frac{6!}{(s-3)^7}+\frac{8s^3-6s(s^2+4)^2}{(s^2+4)^4}+\frac{\pi}{2}-\tan^{-1}(s+3)$ $\quad\frac{2s^3-24s}{(s^2+4)^3}$	0+2+1
	b)	$L\{f(t)\}=\frac{1}{1-e^{-2as}}\left\{\int_0^a e^{-st}t\,dt+\int_a^{2a}e^{-st}(2a-t)dt\right\}$	1+1
		$=\frac{1}{1-e^{-2as}}\left\{\left[t\frac{e^{-st}}{-s}-\frac{e^{-st}}{s^2}\right]_0^a+\left[(2a-t)\frac{e^{-st}}{-s}-(-1)\frac{e^{-st}}{s^2}\right]_a^{2a}\right\}$ $=\frac{1}{(1-e^{-2as})}\left\{\left[\frac{-ae^{-as}}{s}-\frac{e^{-as}}{s^2}+\frac{1}{s^2}\right]+\left[\frac{e^{-2as}}{s^2}+\frac{ae^{-as}}{s}-\frac{e^{-as}}{s^2}\right]\right\}$	2+2
		$=\frac{1}{1-e^{-2as}}\left[\frac{1-2e^{-as}+e^{-2as}}{s^2}\right]$	2
		OR	
6	a)	$L\{f(t)\}=\left\{\int_0^{\frac{a}{2}}e^{-st}(1)dt+\int_{\frac{a}{2}}^{a}e^{-st}(-1)dt\right\}$	2+2
		$=\left\{\left[\frac{e^{-st}}{-s}\right]_0^{\frac{a}{2}}-\left[\frac{e^{-st}}{-s}\right]_{\frac{a}{2}}^{a}\right\}=\left[\frac{1-2e^{-\frac{as}{2}}+e^{-as}}{s}\right]=\cancel{\frac{1}{s}\tanh\left(\frac{as}{4}\right)}$ $\quad\frac{(1-e^{-as/2})^2}{s}$	2+2
	b)	$L\left\{L\frac{d^2q}{dt^2}+\frac{q}{c}=E\sin(\omega t)\right\}=L\left[s^2Q(s)-sq(0)-q'(0)\right]+\frac{1}{c}Q(s)=E\frac{\omega}{s^2+\omega^2}$	1+1
		$\left(Ls^2+\frac{1}{C}\right)Q(s)=E\frac{\omega}{s^2+\omega^2}+Lsq(0)+Lq'(0)$ $\quad \div L$	1
		$\left(s^2+\frac{1}{LC}\right)Q(s)=E\frac{\omega}{s^2+\omega^2}+\frac{sq(0)+q'(0)}{L}$	1
		$Q(s)=\frac{E}{L}\frac{\omega}{(s^2+\omega^2)(s^2+p^2)}+\frac{sq(0)+q'(0)}{L}$ $\quad\frac{1}{LC}=p^2$	1
		$Q(s)=\frac{E\omega}{L}\frac{1}{(\omega^2-p^2)}\left[\frac{1}{(s^2+p^2)}-\frac{1}{(s^2+\omega^2)}\right]+\frac{sq(0)+q'(0)}{L}$	1+1
		$q(t)=\frac{E\omega}{L}\frac{1}{(\omega^2-p^2)}\left[\frac{\sin pt}{p}-\frac{\sin\omega t}{\omega}\right]+L^{-1}\left[\frac{sq(0)+q'(0)}{L}\right]$ $\quad i(t)=\frac{E\omega}{L(\omega^2-p^2)}\left\{\cos pt-\cos\omega t\right\}$	1

$L\frac{d^2q}{dt^2}\quad q=\int_0^t i\,dt$

$L\frac{di}{dt}+\frac{1}{C}\int_0^t i\,dt=E\sin\omega t$

$I(s)$

$\left(Ls+\frac{1}{Cs}\right)I(s)=\frac{E\omega}{s^2+\omega^2}+Li_0$

$I(s)=\frac{E\omega s}{L(s^2+\frac{1}{LC})(s^2+\omega^2)}+\frac{s\,i_0}{\ldots}$

a) The existence of infinitely many primes proof by contradiction

Suppose there are finitely many primes of the form 4n+3 and they are exactly $\{p_1, p_2, \ldots, p_k\}$. 1

Consider $N = (p_1, p_2, \ldots, p_k)^2+2$. Then $PN \equiv 3 \pmod 4$. But N is odd and not divisible by any p_i. 1+1

It follows that all prime divisoCrs of N are congruent to 1 mod 4, which is $N \equiv 1 \pmod 4$ a contradiction. 1

b) $99 \equiv 7 \pmod{23}$ $99^2 \equiv 3 \pmod{23}$ $99^4 \equiv 9 \pmod{23}$ 1

$99^3 \equiv 7^3 \pmod{23} \equiv 21 \pmod{23} \equiv (-2) \pmod{23}$ 1

$(99^3)^{11} \equiv (-2)^{11} \pmod{23} \equiv -2048 \equiv (-1) \pmod{23}$ $99^{22} \equiv 1 \pmod{23}$ 1

$(99^{33})^{30303} \equiv (-1)^{30303} \pmod{23} \equiv -1 \equiv 22 \pmod{23}$ $99^{11} \equiv -1 \pmod{23}$ 1

Hence the remainder of 99^{999999} when divided by 23 is 22. $99^9 \equiv 18 \pmod{23}$ 1

c) Given e = 7, n = 33, the choice of two primes p and q may be 3 and 11. 1

$\phi = (p-1)(q-1) = (2)(10) = 20$ 1

$ed \equiv 1 \pmod{\phi} \Rightarrow 7d \equiv 1 \pmod{20} \Rightarrow d = 3$ 1

$130^3 \equiv 5 \pmod{33}$

$2214_5^3 \equiv 27 \pmod{33}$

$02301 \equiv 30 \pmod{33}$

$26150^3 \equiv 19 \pmod{33}$

for decryption

$13^3 \pmod{33} = 19$	$1^3 \pmod{33} = 1$	$22^3 \pmod{33} = 22$
$23^3 \pmod{33} = 23$	$26^3 \pmod{33} = 20$	$6^3 \pmod{33} = 18$
$14^3 \pmod{33} = 5$	$1^3 \pmod{33} = 1$	$14^3 \pmod{33} = 5$

4

the decrypted sequence will be

SAVE WATER 1

Since there was a typographical error, marks may be awarded for alternative methods.

8 a) Let U denote the set of all employed computer programmers and let J, C and P denote the set of programmers proficient in Java, C# and Python respectively 1

$|U| = 100$, $|J| = 45$, $|C| = 30$, $|P| = 20$, $|J \cap C| = 6$, $|J \cap P| = 1$, $|C \cap P| = 5$, $|J \cap C \cap P| = 1$. 4

To find the cardinality of the complement of $J \cup C \cup P$.

$|J \cup C \cup P| = 84$ 1

$|\overline{J \cup C \cup P}| = |U| - |J \cup C \cup P| = 100 - 84 = 16$ 1

16 programmers are not proficient in any of the three languages. 1

b) Board

	1	2	3	4	5
A	X	X			
B		X		X	
C	X	X			
D			X	X	
E				X	X

Rewrite the board as

	1	2	3	4	5
A	X	X			
B	X	X			
C		X		X	
D			X	X	
E				X	X

1

X	X				
X	X				
	X		X		
		X	X		
			X	X	

$r_0=1$

$r_1=10$ 1

$r_2=33$ 1

$r_3=42$ 2

$r_4=20$ 1

$r_5=2$ 1

$R(C) = 1 + 10x + 33x^2 + 42x^3 + 20x^4 + 2x^5$ 1

Note: Marks may be awarded for alternative proofs and methods.

I/C Shivakumar N.
Dr. N. SHIVAKUMAR.
(BOE) 12.12.19

Co-ordinator.
12/12/19

RV COLLEGE OF ENGINEERING
Autonomous Institution affiliated to VTU
III Semester B. E. March/April-2022 Examinations
LINEAR ALGEBRA, LAPLACE TRANSFORMS AND COMBINATORICS
(Theory)
SCHEME AND SOLUTION

2b i	Let $u = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $v = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \in M_{22}$ and a, b, c, d and $\alpha \in Z$	1
	$u + v = \begin{bmatrix} a+c & 0 \\ 0 & b+d \end{bmatrix} \in M_{22}$ and $\alpha u = \begin{bmatrix} \alpha a & 0 \\ 0 & \alpha b \end{bmatrix} \in M_{22}$	2
	Therefore M_{22} is closed under vector addition and scalar multiplication, and hence M_{22} is a subspace	1
2b ii	$S = \{(a,b,c) \mid a+b+c=0, a,b,c \in \mathbb{R}\}$ Let $u = (a_1, b_1, c_1), v = (a_2, b_2, c_2) \in S, \alpha \in \mathbb{R}$ (i) $u + v = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$ $a_1 + a_2 + b_1 + b_2 + c_1 + c_2 = (a_1 + b_1 + c_1) + (a_2 + b_2 + c_2) = 0$	2
	Hence $u + v \in S$	1
	(ii) $\alpha u = (\alpha a_1, \alpha b_1, \alpha c_1)$ $\alpha +, \alpha b_1 + \alpha c_1 = \alpha(a_1 + b_1 + c_1) = 0$ Hence $\alpha u \subset S$	1
	Therefore S is closed under vector addition and scalar multiplication, and hence S is a subspace of $\mathbb{R}^3$	
3a	Let the third matrix be (x, y, z). Then $\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}y + \frac{1}{\sqrt{3}}z = 0 \Rightarrow x + y + z = 0$ and $\frac{1}{\sqrt{14}}x + \frac{2}{\sqrt{14}}y - \frac{3}{\sqrt{14}}z = 0 \Rightarrow x + 2y - 3z = 0$ Also, $\sqrt{x^2 + y^2 + z^2} = 1 \Rightarrow x^2 + y^2 + z^2 = 1$ Solving (1) and (2) with $z = 1$ we get $x = -5, y = 4$ $\therefore (x, y, z) = (-5, 4, 1)$ Normalizing, the 3rd column is $\left(\frac{-5}{\sqrt{42}}, \frac{4}{\sqrt{42}}, \frac{1}{\sqrt{42}}\right)$ Rows automatically become zero.	1+1+1+2+2+1

3b	Characteristic equation is $\lvert A-\lambda I\rvert = 0$ The eigenvalues are: -3, -3, 5 and the corresponding eigenvectors are $\begin{pmatrix}-2\\1\\0\end{pmatrix}, \begin{pmatrix}3\\0\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\-2\\1\end{pmatrix}$ $\therefore A = SAS^{-1} = \begin{bmatrix}-2&3&-1\\1&0&-2\\0&1&1\end{bmatrix}\begin{bmatrix}-3&0&0\\0&-3&0\\0&0&5\end{bmatrix}\begin{bmatrix}-2&3&-1\\1&0&-2\\0&1&1\end{bmatrix}^{-1}$ $A^{-1} = (SAS^{-1})^{-1} = SA^{-1}S^{-1}$ $A^{-1} = \begin{bmatrix}-2&3&-1\\1&0&-2\\0&1&1\end{bmatrix}\begin{bmatrix}-1/3&0&0\\0&-1/3&0\\0&0&1/5\end{bmatrix}\begin{bmatrix}-2&3&-1\\1&0&-2\\0&1&1\end{bmatrix}^{-1}$	1+2+1+1+1+1+1+1
4a	$q_1 = \begin{pmatrix}1/3\\2/3\\2/3\end{pmatrix};$ (1, 2, 2) (0, 1, −1) (−2, ½, ½) $B = b - (q_1T_b)q_1 = (0, 1, -1) \qquad q_2 = \begin{pmatrix}0\\1/\sqrt{2}\\-1/\sqrt{2}\end{pmatrix}$ $C = C - (q_1T_c)q_1 - (q_2T_c)q_2$ $C = (-2,\ 1/2,\ 1/2)\ ;\ q_3 = (-2\sqrt{2}/3,\ \sqrt{2}/6,\ \sqrt{2}/6)$ $\therefore A = QR = \begin{bmatrix}1/3&0&-2\sqrt{2}/3\\2/3&1/\sqrt{2}&-\sqrt{2}/6\\2/3&-1/\sqrt{2}&\sqrt{2}/6\end{bmatrix}\begin{bmatrix}\overset{3}{0}&3&3\\0&\sqrt{2}&-\sqrt{2}/2\\0&0&3/\sqrt{2}\end{bmatrix}$	1+2+2+1+1+1
4b	$A^TA = \begin{pmatrix}9&-9\\-9&9\end{pmatrix}$ The eigenvalues of A^TA are 18 and 0 with corresponding eigenvectors $v_1 = \begin{bmatrix}1/\sqrt{2}\\-1/\sqrt{2}\end{bmatrix}, v_2 = \begin{bmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix}, \Sigma = \begin{pmatrix}3\sqrt{2}&0\\0&0\\0&0\end{pmatrix}$ Now, $u_1 = \frac{1}{3\sqrt{2}}Av_1 = \begin{pmatrix}1/3\\-2/3\\2/3\end{pmatrix}$ $u_2 = \begin{pmatrix}2/\sqrt{5}\\1/\sqrt{5}\\0\end{pmatrix}; u_3 = \begin{pmatrix}-2/\sqrt{45}\\4/\sqrt{45}\\5/\sqrt{45}\end{pmatrix}$	1+1+1+1+1+2+1

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	$\therefore A = U\Sigma V^T = \begin{bmatrix} 1/3 & 2/\sqrt{5} & -2/\sqrt{45} \\ -2/3 & 1/\sqrt{5} & 4/\sqrt{45} \\ 2/3 & 0 & 5/\sqrt{45} \end{bmatrix} \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$	
5a	$f(t) = \begin{cases} 1 & 0 \le t \le a \\ -1 & a \le t \le 2a \end{cases}$ $L[f(t)] = \frac{1}{1-e^{-2as}}\left(\int_0^a e^{-st}dt - \int_a^{2a} e^{-st}dt\right)$ $= \frac{1}{s(1-e^{-2as})}(e^{-2as} - 2e^{-as} + 1)$ $= \frac{1}{s}\tanh\left(\frac{as}{2}\right)$	2+2+1+1+1+1
5b	i) $L[\sin 3t] = \frac{3}{s^2+9}$ $L\left[\frac{\sin 3t}{t}\right] = \int_s^\infty \frac{3}{s^2+9}ds = \frac{\pi}{2} - tan^{-1}\left(\frac{s}{3}\right)$ $L\left[\frac{e^{2t}\sin 3t}{t}\right] = \frac{\pi}{2} - tan^{-1}\left(\frac{s-2}{3}\right) = cot^{-1}\left(\frac{s-2}{3}\right)$ $L\left[\int_0^t \frac{e^{2t}\sin 3t}{t}\ dt\right] = \frac{1}{s}cot^{-1}\left(\frac{s-2}{3}\right)$ $L\left[e^{-t}\int_0^t \frac{e^{2t}\sin 3t}{t}\ dt\right] = \frac{1}{(s+1)}cot^{-1}\left(\frac{s-1}{3}\right)$	1+1+1+1+1
	ii) $\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^3 = t^{\frac{3}{2}} + 3t^{\frac{1}{2}} + 3t^{\frac{-1}{2}} + t^{\frac{-3}{2}}$ $L\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^3 = L(t^{\frac{3}{2}}) + 3L(t^{\frac{1}{2}}) + 3L(t^{\frac{-1}{2}}) + L(t^{\frac{-3}{2}})$ $= \frac{\sqrt{\pi}}{4}\left(\frac{3}{s^{\frac{5}{2}}} + \frac{6}{s^{\frac{3}{2}}} + \frac{12}{s^{\frac{1}{2}}} + \frac{8}{s^{\frac{-1}{2}}}\right)$	1+1+1

6 a)	$F(s) = \frac{s}{s^2+16},\ G(s) = \frac{s}{s^2+9} \Rightarrow\ f(t) = \cos 4t,\ \ g(t) = \cos 3t$ $g(t) = \cos\sqrt{a}\,t$ $L^{-1}[F(s).G(s)] = \int_0^t \cos 4u \cos(3t - 3u)\,du$ $\int_0^t \cos 4u \cos(\sqrt{a}t - \sqrt{a}u)\,du$ $= \frac{1}{2}\int_0^t [\cos(u + 3t) + \cos(3t - 7u)]\,du$ $\frac{1}{2}\int_0^t \{\cos[(4-\sqrt{a})u + \sqrt{a}t] + \cos[\sqrt{a}t - (4+\sqrt{a})u]\}\,du$ $= \frac{4\sin 4t - 3\sin 3t}{7}$ $\frac{1}{2}\left\{\frac{\sin 4t}{4-\sqrt{a}} - \frac{\sin 4t}{4+\sqrt{a}} - \frac{\sin\sqrt{a}t}{4-\sqrt{a}} + \frac{\sin\sqrt{a}t}{4+\sqrt{a}}\right\}$	2 1 2 (1+1+1)
6 b)	$(s^2 + 2s + 1)L[x(t)]\text{-}4s\text{-}10 = \frac{3}{(s+1)^2}$ $L[x(t)] = \frac{4s+10}{(s+1)^2} + \frac{3}{(s+1)^4}$ $x(t) = L^{-1}\left[\frac{4(s+1)+6}{(s+1)^2}\right] + L^{-1}\left[\frac{3}{(s+1)^4}\right]$ $= e^{-t}\left[4L^{-1}\left[\frac{1}{s}\right] + 6L^{-1}\left[\frac{1}{s^2}\right] + 3L^{-1}\left[\frac{1}{s^4}\right]\right]$ $x(t) = e^{-t}\left[4 + 6t + \frac{t^3}{2}\right]$	2 1 1 (1+1+1) 1
a)	12378 = 4x3054+162 3054 = 18x162+138 162 = 1x738+24 138 = 5x24 +18 24=1x18+6 18=3x6+0 $\therefore gcd(12378,3054) = 6$ 6 = 24 -18= 6x24 – 138 = 6x162 -7x138 =132x12378 + (-535) (3054) x=132, y= -535	(1+1+1 +1) 1 (1 + 1 + 1)
b)	p = 3, q = 17, $\varphi(51) = 32$ $c = m^e \bmod n = 14^7 \bmod 51 = 23 = U$ $= 11^7 \bmod 51 = 20 = R$ $= 24^7 \bmod 51 = 21 = J$ 12 $d_e = 1 \bmod \varphi(n)\ \Rightarrow 7d = 1\ mod(32) = 1.d = 23$	1 2 2 2 1

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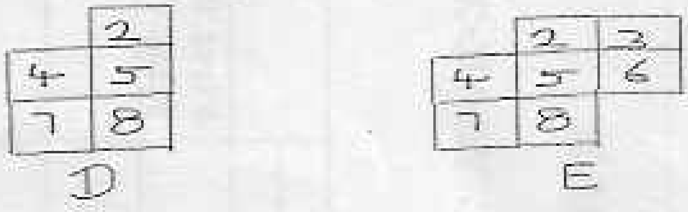
8 a)	Let us mark the top most square 1 	1
	In D, $r_1 = 5, r_2 = 4$ and $r_3 = r_4 = 0$	2
	$\therefore r(D,1) = 1 + 5x + 4x^2$	2
	In E, $r_1 = 7, r_2 = 11, r_3 = 3, r_4 = r_5 = \cdots \ldots \ldots = 0$ $r(E,x) = 1 + 7x + 11x^2 + 3x^3$ By using expansion formula, we get $r(C,x) = x\,r(D,x) + r(E,x)$ $= 1 + 8x + 16x^2 + 7x^3$	(1+1+1)
8 b)	$\lvert A_1 \cup A_2 \cup A_3\rvert = \lvert A_1\rvert + \lvert A_2\rvert + \lvert A_3\rvert - \{\lvert A_1 \cap A_2\rvert + \lvert A_1 \cap A_3\rvert + \lvert A_2 \cap A_3\rvert\} + \lvert A_1 \cap A_2 \cap A_3\rvert$	1
	$= 60 + 50 + 37 - \{10 + 7 + 12\} + 2 = 120$	(1+1+1+1)
	$\lvert \bar{A} \cup \overline{A_2} \cap \overline{A_3}\rvert = \lvert S\rvert - \lvert A_1 \cup A_2 \cup A_3\rvert = 300 - 120 = 180$	(1+1+1)

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(1) $\frac{4}{s+4}$ (2) $\frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt$

(3) $\frac{2}{s}$ (4) $(1-\cancel{\pi})\,u(t-\pi)$

(5) $L^{-1}\left(\frac{1}{as-b}\right)=\frac{1}{a}L^{-1}\left(\frac{1}{s-b/a}\right)=\frac{1}{a}e^{+\frac{b}{a}t}$

(6) $\int_0^t \frac{\sin\omega t}{\omega}\,dt=\frac{1-\cos\omega t}{\omega^2}$

(7) $\frac{1}{2}\left(f(a-t)+f(a+t)\right)$

(8) $f(x)=\sum_{n=-\infty}^{\infty} c_n e^{\frac{in\pi x}{l}}$

(9) $a_0=\frac{2}{\pi}\int_0^\pi \sin x\,dx=\frac{2}{\pi}\left[-\cos x\right]_0^\pi=\frac{2}{\pi}\left[-(-1)+1\right]=\cancel{\frac{4}{\pi}}$

(10) $F\left[e^{-x^2}\cos 3x\right]=\frac{1}{2}\left[e^{-\frac{(\alpha+3)^2}{2}}+e^{-\frac{(\alpha-3)^2}{2}}\right]$

(11) $F^{-1}\left[F(\alpha)\right]=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\alpha)\,e^{-i\alpha x}\,d\alpha=f(x)$

(12) $Z\left(n^2 e^{an}\right)=\frac{e^a z^2+(e^a)^2 z}{(z-e^a)^3}$

(13) $F_c\left[e^{-x}\right]=\int_0^\infty e^{-x}\cos\alpha x\,dx=\frac{as}{s^2+a^2}$ $a,s=1$ $\frac{1}{1+\alpha^2}$

(14) $Z^{-1}\left[\frac{z}{a(z-a)}\right]=\frac{1}{a}Z^{-1}\left(\frac{z}{z-a}\right)=\frac{1}{a}a^n=a^{n-1}$

(15) $4x=2$

3(a) $f(x) = e^{-ax}$ $(-\pi, \pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\,dx = \frac{1}{\pi}\left(\frac{e^{-ax}}{-a}\right)_{-\pi}^{\pi}$$

$$= \frac{2\sinh(a\pi)}{a\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{-ax}}{a^2+n^2}\left(-a\cos nx + n\sin nx\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{-a\pi}}{a^2+n^2}\left(-a\cos n\pi + 0\right) - \frac{e^{a\pi}}{a^2+n^2}\left(-a\cos(n\pi) + 0\right)\right]$$

$$= \frac{1}{\pi}\frac{a(-1)^n}{(a^2+n^2)}\left(e^{a\pi} - e^{-a\pi}\right)$$

$$= \frac{1}{\pi}\frac{2a(-1)^n}{(a^2+n^2)}\sinh a\pi$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\sin nx\,dx = \frac{2n(-1)^n}{\pi(a^2+n^2)}\sinh(a\pi)$$

$$f(x) = \frac{\sinh a\pi}{\pi a} + \sum_{n=1}^{\infty}\frac{2a(-1)^n}{\pi(a^2+n^2)}\sinh a\pi\cos nx + \sum_{n=1}^{\infty}\frac{2n(-1)^n}{\pi(a^2+n^2)}\sinh a\pi \sin nx$$

$$e^{-ax} = \frac{\sinh a\pi}{\pi}\left[\frac{1}{a} + \sum_{n=1}^{\infty}\frac{2a\cos nx}{(a^2+n^2)}(-1)^n + \frac{2n\sin nx\,(-1)^n}{a^2+n^2}\right]$$

$a=1$ $x=0$

$$1 = \frac{\sinh \pi}{\pi}\left(1 + \sum_{n=1}^{\infty} \frac{2(-1)^n}{1+n^2} + 0\right)$$

$$\frac{\pi}{\sinh \pi} = 1 + 2\left(\frac{-1}{1+1} + \frac{1}{1+2^2} + \frac{-1}{1+3^2} + \frac{1}{1+4^2} + \cdots\right)$$

$$= 2\left[\cdots \frac{1}{5} - \frac{1}{10} + \frac{1}{17} - \cdots\right]$$

5(b) $$f(x) = \begin{cases} kx & 0 \le x \le 1 \\ k(2-x) & 1 \le x \le 2 \end{cases}$$

$$a_0 = \frac{2}{2}\cancel{\left[\int_0^1 kx\,dx + \int_1^2 k(2-x)\,dx\right]}$$

$$= k$$

$$a_n = \frac{2}{2}\cancel{\left[\int_0^1 kx\cos\left(\frac{n\pi x}{2}\right)dx + \int_1^2 k(2-x)\cos\left(\frac{n\pi x}{2}\right)dx\right]}$$

$$= \frac{2k}{n^2\pi^2}\left(2\cos n\pi - 1 - (-1)^n\right)$$

$$b_n = \frac{2}{2}\int_0^1 kx\sin\left(\frac{n\pi x}{2}\right)dx + \int_1^2 k(2-x)\sin\left(\frac{n\pi x}{2}\right)dx$$

$$= \frac{8k}{n^2\pi^2}\left[\sin\frac{n\pi}{2}\right]$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right) = \frac{8k}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right)\sin$$

$$= \frac{8k}{\pi^2}\left(\frac{1}{1^2}\sin\frac{\pi x}{2} - \frac{1}{3^2}\sin\frac{3\pi x}{2} + \frac{1}{5^2}\sin\frac{5\pi x}{2} + \cdots\right)$$

7 (b) FST of $e^{-|x|}$

$$F_s(e^{-|x|}) = \int_0^\infty e^{-|x|} \sin \alpha x \, dx$$

$$= \frac{\alpha^2}{1+\alpha^2}$$

let $f(t) = \frac{t}{t^2+1}$ $F(\alpha) = \frac{\pi}{2} e^{-\alpha}$

by Parseval's Identities $\frac{2}{\pi} \int_0^\infty [F_s(\alpha)]^2 d\alpha = \int_0^\infty [f(x)]^2 dx$

$$\int_0^\infty \left(\frac{x}{x^2+1}\right)^2 dx = \frac{2}{\pi} \left(\frac{\pi}{2} e^{-\alpha}\right)^2 d\alpha$$

$$= \frac{\pi}{2} \int_0^\infty e^{-2\alpha} d\alpha$$

$$\boxed{\int_0^\infty \frac{x^2}{(x^2+1)^2} dx = \frac{\pi}{4}}$$