16CS36

USN | | | | | | | | | | |

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU)

III Semester B. E. Examinations April/May - 2023

Computer Science and Engineering

DISCRETE MATHEMATICS

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

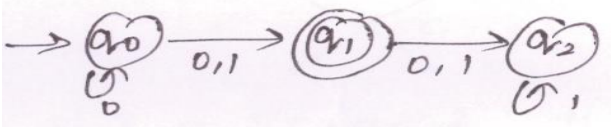
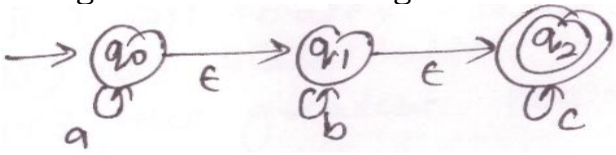
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

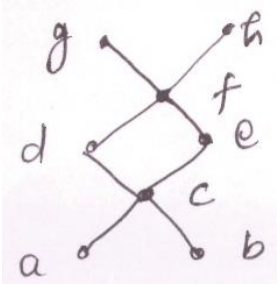
PART-A

1	1.1	How many 3-digit numbers can be formed by using the 6 digits $2,3,4,5,6,8$ if repetitions of digits are allowed?	02
	1.2	Find the value of n so that $2P\ (n,2) + 50\ = P(2n,2)$	02
	1.3	In how many ways can we distribute 7 apples and 6 oranges among 4 children so that each child gets at least 1 apple?	02
	1.4	Find a_{12} if $a_{n+1}^2 = 5a_n^2$, where $a_n > 0$ for $n \geq 0$, given that $a_0 = 2$.	02
	1.5	Indicate how many rows are needed in the truth table for the compound proposition $(p \vee \neg q) \leftrightarrow \{(\neg r \wedge s) \rightarrow t\}$	02
	1.6	Obtain a DFA to accept strings of a's and b's having $4\,a's$ where $\Sigma = \{a, b\}$	02
	1.7	Let $A = \{1,2,3,4,6\}$ and R be the relation on A defined by $(a,b) \in R$ if and only if a is multiple of b. Write down R as a set of ordered pairs.	02
	1.8	Let $A = \{0, \pm 1, \pm 2, 3\}$ consider the function $f : A \rightarrow R$, (where R is the set of all real numbers) defined by $f(x) = x^3 - 2x^2 + 3x + 1,\ \forall x \in A$. Find the range of f.	02
	1.9	Prove that $H = \{0,2,4\}$ is a subgroup of $(Z6, +)$	02
	1.10	A binary symmetric channel has probability $P = 0.05$ of incorrect transmission. If the word $C = 011011101$ is transmitted. What is the probability that single error occurs.	02

PART-B

2	a	Find the number of integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 30$ where $x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x_4 \geq 2, x_5 \geq 0$	05
	b	How many integers between 1 and 300 are i) Divisible by at least one of $5,6,8$? ii) Divisible by none of $5,6,8$?	06
	c	Find the number of permutations of the letters of the word $MASSASAUGA$. In how many of these, all $4A's$ are together? How many of them begin with S?	05

3	a	The number of virus affected files in a system is 1000 (to start with) and this increases 250% every 2 hrs. Use a recurrence relation to determine the number of virus affected files in the system after one day.	05
	b	Prove by mathematical induction, that $1^2 + 3^2 + 5^2 + - - - - - + (2n-1)^2 = \frac{1}{3} n(2n-1)(2n+1)$ for all integers $n \geq 1$.	06
	c	Test the validity of the given arguments $(\neg p \vee q) \rightarrow r, r \rightarrow (s \vee t)$, $(\neg s \wedge \neg u, \neg u \rightarrow \neg t \vdash p$	05
		OR	
4	a	Prove that the following argument is valid. $\forall x, [p(x) \vee q(x)]$ $\exists x, \neg p(x)$ $\forall x, [\neg q(x) \vee r(x)]$ $\forall x, [s(x) \rightarrow \neg r(x)]$ _____ $\therefore \exists x, \neg s(x)$	05
	b	Solve the recurrence relation $a_n = 2(a_{n-1} - a_{n-2})$, for $n \geq 2$ given that $a_0 = 1$ and $a_1 = 2$	06
	c	Prove the following logical equivalence. $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$	05
5	a	Obtain a DFA to accept the language $L = \{awa \mid \epsilon t\ (a+b)*\}$	05
	b	Define the following: i) DFA ii) Language of DFA iii) Extended transition function of DFA.	06
	c	Convert the following NFA shown in Fig 5c to its equivalent DFA:  Fig 5c	05
		OR	
6	a	Convert the following ϵ-NFA shown in Fig 6a to DFA  Fig 6a	06
	b	Define the following: i) NFA ii) Extended transition function of ϵ-NFA.	05
	c	Obtain a DFA to accept strings of a's and b's with at most 2 consecutive $b's$.	05

7	a	Let $A = B = C = R$, and $f: A \longrightarrow B$ and $g: B \longrightarrow C$ be defined by $f(a) = 2a + 1$, $(b) = \frac{1}{3}b$, $\forall$ a $\in$ A, $\forall$ b $\in$ B. Compute gof and show that gof is invertible. What is $(g \circ f) -1$?	05
	b	Let $A = \{1,2,3,4\}$ and $R = \{(1,1),(1,2),(2,3),(3,4)\}$, $S = \{(3,1),(4,4),(2,4),(1,4)\}$ be relations on A. Determine the relations $R \circ S, S \circ R$, R^2 and S^2.	05
	c	Consider the Hasse diagram of a *Poset* (A, R) given in Fig 7c  Fig 7c If $B = \{c, d, e\}$ find: i) All upper bounds of B. ii) All lower bounds of B. iii) The least upper bound of B. iv) The greatest lower bound of B.	06
8	a	Let G be the set of all non-zero real numbers and let $a * b = \frac{1}{2}ab$. Show that $(G, *)$ is an abelian group.	05
	b	An encoding function $\in: Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix. $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ i) Determine all the code words. What can be said about the error-detection capability of this code? What about its error-correction capability? ii) Find the associated parity check matrix H. iii) Use H to decode the received words: 11101,11011	06
	c	State and prove Lagrange's theorem.	05