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**R. V. COLLEGE OF ENGINEERING**  
**Autonomous Institution affiliated to VTU**  
**III Semester B. E. Examinations Nov/Dec-18**  
**Computer Science and Engineering**  
**DISCRETE MATHEMATICS**

*Time: 03 Hours**Maximum Marks: 100***Instructions to candidates:**

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

**PART-A**

1	1.1	How many permutations are there for the letters $a, c, f, g, i, t, w, x$ ? How many starts with letter $t$ and end with letter $c$ ?	02
	1.2	Consider the following programming segment $\text{for } i := 1 \text{ to } 123 \text{ do}$ $\text{for } j := 1 \text{ to } i \text{ do}$ $\text{print}(i * j)$ How many times the print statement is executed?	02
	1.3	Determine the number of integer solutions for $x_1 + x_2 + x_3 + x_4 = 18$ , where $x_i \leq 7, i = 1, 2, 3, 4$ .	02
	1.4	In the DFA below, find $\delta^*(A, ababb)$	02
			02
	1.5	Define DFA and its language.	02
	1.6	What is the negation of the following statement (simplified form)? $\exists x \forall y [(p(x, y) \vee q(x, y)) \rightarrow r(x, y)]$	02
	1.7	List the properties that must be satisfied by an abelian group $(S, \oplus)$ , where 'S' is a set with a binary operation $\oplus$ defined on S.	02
	1.8	Consider the functions $f$ and $g$ defined by $f(x) = x^2$ and $g(x) = x^4 - 1, \forall x \in R$ . Find $g \circ f$ and $f^2$ .	02
	1.9	What are the maximal and minimal elements in the given Hasse diagram? Also specify the greatest and least element in Fig 1.9.	02
			02
		Fig 1.9	02
1.10		Define Surjective function with an example.	02

## PART-B

2	a	If $A, B$ and $C$ are sets, prove both analytically and graphically (Venn diagram) $A - (B \cap C) = (A - B) \cup (A - C)$ .	06
	b	A gym coach must select 11 seniors to play on a football team. If he can make his selection in 12,376 ways, how many seniors are eligible to play?	04
	c	State and prove the extended addition rule.	06
3	a	Prove the following by mathematical induction: $1.3 + 2.4 + 3.5 + \dots + n(n+2) = \frac{[n(n+1)(2n+1)]}{3}$ .	08
	b	Solve the recurrence $a_{n+2} + 4a_{n+1} + 4a_n = 7, n \geq 0, a_0 = 1, a_1 = 2$ .	08
<b>OR</b>			
4	a	Solve the recurrence $a_{n+1} - a_n = 3n^2 - n, n \geq 0, a_0 = 3$ .	08
	b	Prove that $R \rightarrow S$ is a valid conclusion from the premises: $P \rightarrow (Q \rightarrow S), \neg RVP$ and $Q$	05
	c	Let $m$ and $n$ be integers, prove that $n^2 = m^2$ if and only if $m = n$ or $m = -n$ .	03
5	a	Construct <i>DFA</i> accepting the following language over the alphabet $\{a, b\}$ . $L$ = the set of all strings starting and ending with same symbol.	08
	b	Define the following terms: i) Language of <i>NFA</i> ii) Extended transition function of $\epsilon$ - closure of a state.	08
<b>OR</b>			
6	a	Construct <i>NFA</i> to accept all strings which have second symbol from <i>RHS</i> is 'b' over $\Sigma = \{a, b\}$ . Also convert this <i>NFA</i> into an equivalent <i>DFA</i> .	08
	b	Convert the following $\epsilon$ - <i>NFA</i> to <i>NFA</i>	08
7	a	Draw the Hasse diagram represented by the positive divisors of 36.	03
	b	Consider the sets $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$ and the relations $R = \{(a, 1), (b, 1), (c, 2), (c, 3)\}$ and $S = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$ from $A$ to $B$ . Determine $\bar{R}, \bar{S}, R \cup S, R^c$ and $R \circ (R \cap S)$ .	08
	c	Let $A = \{1, 2, 3, 4\}, B = \{a, b, c\}$ and $C = \{w, x, y, z\}$ with $f: A \rightarrow B$ and $g: B \rightarrow C$ given by $f = \{(1, a), (2, a), (3, b), (4, c)\}, g = \{(a, x), (b, y), (c, z)\}$ . Find $g \circ f$ .	05
8	a	Show that $(U_{14}, \cdot)$ is cyclic and find all its generators.	06
	b	State and prove Lagrange's theorem.	05
	c	The generator matrix for an encoding function $E: Z_2^3 \rightarrow Z_2^6$ is given by $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ <div style="margin-left: 40px;"> i) Find the code words assigned to 110 and 010  ii) Obtain the associated parity check matrix. </div>	05