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RV COLLEGE OF ENGINEERING®
(An Autonomous Institution affiliated to VTU)
III Semester B. E. Examinations April/May-2023
Computer Science and Engineering
DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6.

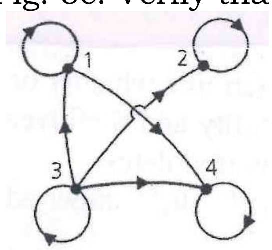
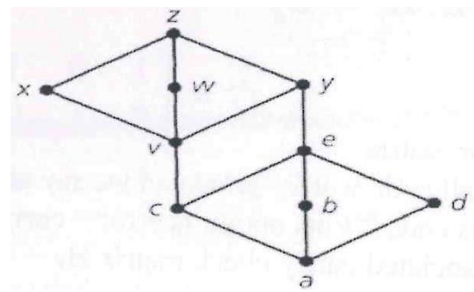
PART-A

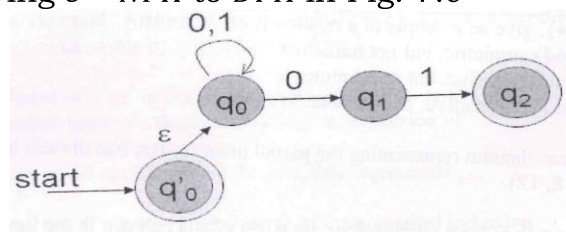
1	1.1	Twelve points are placed on the circumference of a circle and all the chords connecting these points are drawn. What is the largest number of points of intersection for these chords?	01
	1.2	In how many ways can the letters in "$WONDERING$" be arranged with exactly two consecutive vowels.	01
	1.3	How many non-negative integer solutions are there to the pair of equations $x_1 + x_2 + x_3 \ldots + x_4 = 37$ and $x_1 + x_2 + x_3 = 6$?	01
	1.4	A proof that $p \to q$ is true based on the fact that q is true, such proofs are known as ______ proofs.	01
	1.5	The truth value of negation of "$If -1 < 3$ and $3 + 7 = 10$, then $\sin\left(\frac{3\pi}{2}\right) = -1$" is ___________.	01
	1.6	The dual of $p \leftrightarrow q$ is ___________.	01
	1.7	Express this in symbolic form "The product of two negative real numbers is not negative".	01
	1.8	The truth value of inverse of "$If -1 < 3$ and $3 + 7 = 10$, then $\sin\left(\frac{3\pi}{2}\right) = -1$" is ___________.	01
	1.9	If $\lvert A \rvert = 5$, the number of antisymmetric relations on A is ___________.	01
	1.10	A mapping $f: X \to Y$ is one if : ______.	01
	1.11	The number of equivalence relations of the set $(1, 2, 3, 4)$ is ___________.	01
	1.12	The value of $\lvert 1/2. \lvert 5/2 \rvert \rvert$ is ___________.	01
	1.13	The transition function for DFA is given by: ___________.	01
	1.14	Let $L1 = \{a, ab, ba\}$ and $L2 = \{b, aa\}$ then what is the value of $L1L2$?	01
	1.15	For the group (G, o), where $G = \{q \in Q \mid q \neq -1\}$ and binary operation defined as $x\ o\ y = x + y + xy$, the inverse of 5 is given by ___________.	01
	1.16	In the group S_5, let $\alpha = \begin{pmatrix} 1\ 2\ 3\ 4\ 5 \\ 2\ 3\ 1\ 4\ 5 \end{pmatrix}$ $\beta = \begin{pmatrix} 1\ 2\ 3\ 4\ 5 \\ 2\ 1\ 5\ 3\ 4 \end{pmatrix}$ Find $\beta\alpha$ and β^{-1}.	02
	1.17	If $n > 2$, then number of surjectons that can be defined from $\{1, 2, 3, \ldots n\}$ onto $(1, 2)$ is ___________.	01

	1.18	Which of the following relations are functions? i) $N = \{(x, y)/y = x^2, x \in \{-1, 0, 1, 2, 3\}\}$ ii) $P = \{(x, y)/y^2 = x, x \in \{4, 9, 16\}\}$ iii) $Q = \{(x, y)/y = 4x^2 - 14, x \in \{-1, 1, 2, 3\}\}$	02

PART-B

2	a	Determine the number of six-digit integers (no leading zeros) in which: i) No digit may be repeated ii) Digits may be repeated Answer i) and ii) with the extra condition that the six-digit integer is: i) Even ii) Divisible by 5 iii) Divisible by 4	04
	b	i) Find the coefficients of x^2yz^2 in the expansion of $[(x/2) + y - 3z]^5$ ii) How many distinct terms are there in the complete expansion of $[(x/2) + y - 3z]^5$ iii) What is the sum of all coefficients in the complete expansion?	03
	c	Prove the following for all $n \geq 1$ by the principle of mathematical induction: $1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = n(2n-1)(2n+1)/3$.	06
	d	For $n \in Z^+$, prove the following by mathematical induction: $5 \mid (n^5 - n)$.	03
3	a	Let p, q and r denote primitive statements. Use the truth table to verify the following logical equivalence: $[p \to (q \vee r)] \Leftrightarrow [\neg r \to (p \to q)]$	03
	b	Establish the validity of the following argument: $p \to (q \to r)$ $p \vee s$ $t \to q$ $\neg s$ $\therefore \neg r \to \neg t$	05
	c	Let $p(x, y)$ denote the open statement "x *divides* y", where the universe for each of the variables x, y comprises of all integers. Determine the truth value of each of the following statements; if a quantified statement is false, provide an explanation or a counter example. i) $p(3, 7)$ ii) $p(3, 27)$ iii) $\forall y\, p(1, y)$ iv) $\forall y\, \exists x\, p(x, y)$ v) $\exists y\, \forall x\, p(x, y)$	05
	d	Give a direct proof of the theorem "If n is an odd integer then n^2 is odd".	03
		OR	
4	a	Use the substitution rules to verify the following: $(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p$	05
	b	Provide the steps and reasons for verifying the following argument: $\forall x\, [p(x) \to (q(x) \wedge r(x))]$ $\forall x [p(x) \wedge s(x)]$ $\therefore \forall x\, [r(x) \wedge s(x)]$	05

	c	Negate the following: $\forall x \, \exists y \left[\left(p(x,y) \wedge q(x,y) \right) \rightarrow r(x,y) \right]$	04
	d	Write the converse and contrapositive of the following: “If Harold passes his $C++$ course and finishes his data structures project then he will graduate at the end of the semester”.	02
5	a	If $A = \{1,2,3,4\}$, give an example of a reaction R on A that is: i) Reflexive and symmetric, but not transitive ii) Reflexive and transitive, but not symmetric iii) Symmetric and transitive, but not reflexive.	03
	b	Draw the Hasse diagram representing the partial ordering $\{(a,b) \mid a \ divides \ b\}$ on $\{1,2,3,4,6,8,12\}$.	04
	c	The directed graph G for a relation R on $set \ A = \{1,2,3,4\}$ shown in Fig. 5c. Verify that (A,R) is a poset.  Fig. 5c  Fig. 5d	04
	d	For $A = \{a,b,c,d,e,v,w,x,y,z\}$, consider the poset (A,R) whose Hasse diagram shown in Fig. 5d. Find: i) $glb \ \{b,c\}$ ii) $lub \ \{c,e\}$ iii) $Is \ there \ a \ least \ element$? iv) $Is \ (A,R) \ a \ lattice$?	05
		OR	
6	a	Let $A = \{1,2,3,4\}$ and $B = \{x,y,z\}$ i) How many functions $f: A \rightarrow B$ are one-to-one? ii) How many functions $f: B \rightarrow A$ are there? iii) How many functions $f: A \rightarrow B$ satisfy $f(1) = x$? iv) How many functions $f: A \rightarrow B$ satisfy $f(1) = f(2) = x$?	04
	b	For each of the following functions, determine whether it is one-to-one an determine its range: i) $f: Z \rightarrow Z, f(x) = 2x + 1$ ii) $f: R \rightarrow R, f(x) = e^x$	04
	c	Prove that: A function $A \rightarrow B$ is invertible if and only if it is one-to-one and onto.	04
	d	Let $f, g: R \rightarrow R$, where $g(x) = 1 - x + x^2$ and $f(x) = ax + b$. If $(g \ o \ f)(x) = 9x^2 - 9x + 3$, determine a, b.	04
7	a	Design a DFA to accept strings of: i) $a's$ and $b's$ ending with abb ii) $0's$ and $1's$ having even number of $0's$ and odd number of $1's$.	06
	b	List any four differences between NFA and DFA.	04

	c	Convert the following $\varepsilon - NFA$ to DFA in Fig. 7.c  Fig. 7.c	06
8	a	Define the binary operation $\boldsymbol{o}$ on Z by $x\ \boldsymbol{o}\ y = x + y + 1$. Verify that $(Z, \boldsymbol{o})$ is an abelian group.	05
	b	For each of the following sets, determine whether or not the set is a group under the standard binary operation. If so, determine its identity and the inverse of each of its elements. If it is not a group, state the condition of the definition it violates: i) $\{-1, 1\}$ under multiplication ii) $\{-1, 0, 1\}$ under addition	03
	c	The encoding function $E: Z_2^2 \rightarrow Z_2^5$ $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ is given by the generator matrix i) Determine all code words. What can we say about the error-detection capability of this code? What about its error-correction capability? ii) Find the associated parity-check matrix H iii) Use H to decode each of the following received words. (a) 1101 (b) 10101 (c) 11101 (d) 00110	08