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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU)

III Semester B. E. Fast Track Examinations July-19

Computer Science and Engineering

DISCRETE MATHEMATICS

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

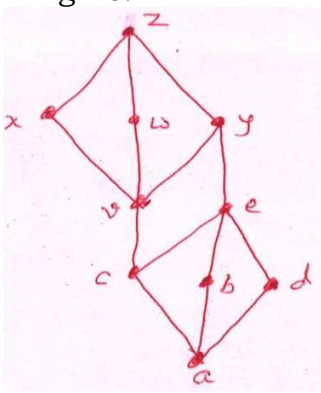
PART A

1	1.1	Determine the co-efficient of $W^3X^2YZ^2$ in $(2W - X + 3Y - 2Z)^8$.	02
	1.2	A six faced die is tossed four times and the numbers shown are recorded in a sequence. How many different sequences are there?	01
	1.3	Define surjective function with an example.	02
	1.4	Find the recurrence relation with initial conditions that uniquely determines each of the following sequence. a) $3, 7, 11, 15, 19, \ldots.$ b) $8, \frac{25}{7}, \frac{72}{49}, \frac{216}{343}, \ldots.$	02
	1.5	Obtain a recursive definition for the sequence $a_n = 2 - (-1)^n$.	02
	1.6	Show that $(W_4, \times)$ is an abelian group where $W_4 = \{1, -1, i, -i\}$.	02
	1.7	Let $g(x) = 2x - 3$, $x \epsilon R$. Is g one-to-one and onto?	02
	1.8	Consider the following open statements with the set of all real numbers as the universe $p(x): \lvert x \rvert > 3, q(x): x > 3$. Find the truth value of the statement $\forall x, [p(x) \rightarrow q(x)]$.	02
	1.9	If $f: A \rightarrow B$ and $\lvert A \rvert = m, \lvert B \rvert = n$, then number of onto functions from A to B is given by formula _____.	01
	1.10	Obtain an NFA to accept strings of 0's and 1's such that left most symbol is different fromthe right most symbol.	02
	1.11	Define DFA.	02

PART B

2	a	In how many ways can 10 identical pencils be distributed among 5 children in the following cases: i) There are no restrictions ii) Each child gets at least one pencil iii) The youngest child gets at least two pencils.	04
	b	i) How many arrangements are there of all letters in $SOCIOLOGICAL$? ii) In how many of the arrangements in part (i) are A and G adjacent? iii) In how many arrangements in part(i) are all vowels adjacent?	06
	c	If A, B and C are sets, prove both analytically and graphically (Venn diagram) $A - (B \cap C) = (A - B) \cup (A - C)$.	06

3	a	Prove that the following argument is valid: $\forall x, [p(x) \vee q(x)]$ $\exists x, \neg p(x)$ $\forall x, [\neg q(x) \vee r(x)]$ $\forall x, [s(x) \rightarrow \neg r(x)]$ _____ $\therefore \exists x, \neg s(x)$	06
	b	Prove the following by mathematical induction. $1.3 + 2.4 + 3.5 + \cdots + n.(n+2) = \frac{n(n+1)(2n+7)}{6}$	06
	c	Define the following: i) Modus tollens. ii) Modus Ponens.	04
		OR	
4	a	Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200, \quad n \geq 0, a_0 = 3000, a_{1=}3300$	06
	b	A bank pays a certain percentage of annual interest on deposits, compounding the interest once in 3 months. If a deposit doubles in 6 years and 6 months, what is the annual percentage of interest paid by the bank?	04
	c	Write the following argument in a symbolic form and validate the argument: If Dominic goes to racetrack, then Helen will be mad. If Ralph plays cards all night, them Carmella will be mad. If either, Helen or Carmella gets mad, then Veronica (their attorney) will be notified. Veronica has not heard from either of these two clients. Consequently, Dominic did not make it to the racetrack and Ralphdid not play cards all night.	06
5	a	Define NFA. Construct anNFA to accept all strings which have second symbol from RHS is 'b' over $\sum = \{a, b\}$. Write the transition table.	05
	b	Convert the following NFA into an equivalentDFA and informally describe the language it accepts. $\rightarrow p$: $\{p,q\}$, $\{p\}$ q: $\{r,s\}$, $\{t\}$ r: $\{p,r\}$, $\{t\}$ $*s$: $\emptyset$, $\emptyset$ $*t$: $\emptyset$, $\emptyset$	06
	c	Obtain DFA to accept the language $L = \{W: \lvert W \rvert mod\ 5 \neq 0\}$ on $\Sigma = \{a\}$	05
		OR	
6	a	Define $\in$-NFA. Construct $\in$-NFA to accept strings over $\sum = \{a, b, c\}$ such that the string contains any number of $a's$ followed by any number of $b's$ followed by any number of $c's$.	05
	b	Give $DFA's$ accepting the following languages over the alphabet $\sum = \{a, b\}$ i) The language of all strings that do not end with ab. ii) The language of all strings in which the number of $a's$ is even.	06
	c	Draw a DFA to accept the language $L = \{W$:every run of $a's$ has length either two or more$\}$	05

7	a	Let $A=\{1,2,3,4\}, B=\{a,b,c\}$ and $C=(w,x,y,z\}$ with $f: A \longrightarrow B$ and $g: B \rightarrow C$ given by $f=\{(1,a),(2,a),(3,b),(4,c)\}, g=\{(a,x),(b,y),(c,z)\}$. Find $g \circ f$.	04
	b	Find: i) $glb\{b,w\}$ ii) $lub\{d,x\}$ iii) lease element and iv) Greatest element. of the Hasse diagram given in fig 7b.	
		 Fig 7b	04
	c	Define an equivalence relation. Find equivalence relation induced by the partition $\{\{1,2\},\{3,4,5\},\{6\}\}$ of the set $\{1,2,3,4,5,6\}$.	04
	d	If $A=\{x \in R: x \neq \frac{1}{2}\}$ and $f: A \rightarrow R$ is defined by $f(x)=\frac{4x}{2x-1}$. Find the range of f and show that f is invertible.	04
8	a	The generator matrix for an encoding function $E: Z_2^3 \rightarrow Z_2^6$ is given by $G=\begin{bmatrix}1 & 0 & 0 & 1 & 1 & 0\\ 0 & 1 & 0 & 0 & 1 & 1\\ 0 & 0 & 1 & 1 & 0 & 1\end{bmatrix}$ i) Find the code words assigned to 110 and 010 ii) Obtain the associated parity check matrix.	06
	b	Define the binary operation '$\circ$' on Z by $x \circ y=x+y+1$. Verify that $(Z,\circ)$ is an abelian group.	05
		Define the following: i) Group homomorphism. ii) Group isomorphism. iii) Subgroup.	05