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RV COLLEGE OF ENGINEERING®
(An Autonomous Institution affiliated to VTU)
III Semester B. E. Fast track Examinations Oct-2020
Computer Science and Engineering
DISCRETE MATHEMATICS

*Time: 03 Hours**Maximum Marks: 100**Instructions to candidates:*

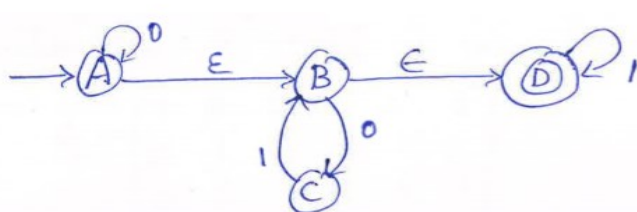
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

PART-A

1	1.1	How many permutations are there for the letters a, b, c, d, e, f, g, h ? How many starts with letter g and end with letter h?	02
	1.2	Find the number of 5 digit positive integer such that in each of them every digit is greater than the digit to the right.	01
	1.3	Determine the co-efficient of x^{12} in the expansion of $x^3(1 - 2x)^{10}$	02
	1.4	Determine the number of integer solutions for $x_1 + x_2 + x_3 + x_4 = 18$, where $x_i \leq 7, i = 1, 2, 3, 4$	02
	1.5	What is the negation of the following statement? $\exists x \forall y[(p(x, y) \vee q(x, y)) \rightarrow r(x, y)]$	02
	1.6	For the partition of the set $A = \{1, 2, 3, 4\}$, given as $P = \{\{1, 2\}, \{3, 4\}\}$. Find the equivalence relation induced by P	01
	1.7	If the function f is defined by $f(x) = x^2 + 1$ on a set $A = \{-2, -1, 0, 1, 2\}$. Find the range of f	02
	1.8	Consider the functions f and g defined by $f(x) = x^2$ and $g(x) = x^4 - 1, \forall x \in R$ Find $g \circ f$ and f^2	02
	1.9	Construct DFA to accept all the strings of a's and b's which is not ending with ab.	02
	1.10	Define the extended transition function δ^* for $\epsilon - NFA$	01
	1.11	Show that $(Z_6, +)$ is an abelian group.	02
	1.12	The word $C = 1010110$ is transmitted through a binary symmetric channel. If $e = 0101101$ is the error pattern, find the receiver word 'r'.	01

PART-B

2	a	if A, B and C are sets, prove both analytically and using Venn diagrams $A - (B \cap C) = (A - B) \cup (A - C)$	06
	b	State and prove the extended addition rule.	06
	c	Find the number of integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where $x_1 \geq 3, x_2 \geq 2, x_3 \geq 4, x_4 \geq 6, x_5 \geq 0$	04
3	a	Prove the following by mathematical induction: $n! \geq 2^{n-1}$ for all integers $n \geq 1$	04

4	b	Find and solve the recurrence relation for the number of binary sequence of length $n \geq 1$ that have consecutive 0's.	04
	c	Solve the recurrence $a_{n+1} - a_n = 3n^2 - n, n \geq 0, a_0 = 3$	08
	OR		
	a	Prove the following logical equivalence without using truth tables i) $[(p \vee q) \wedge (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$ ii) $(p \rightarrow q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow \sim (q \vee p)$	06
	b	Test the validity of the argument $(\sim p \vee q) \rightarrow r, r \rightarrow (s \vee t), \sim s \wedge \sim u, \sim u \rightarrow \sim t \vdash p$	04
	c	Show that the argument below is valid $\forall x[p(x) \vee q(x)]$ $\frac{\forall x[(\sim p(x) \vee q(x)) \rightarrow r(x)]}{\therefore \forall x[\sim r(x) \rightarrow p(x)]}$	06
5	a	Define <i>DFA</i> , Language accepted by <i>DFA</i> . Construct <i>DFA</i> accepting the language over $\Sigma = \{0,1\}$ which has even numbers of 0's and odd number of 1's.	08
	b	Construct <i>NFA</i> to accept all strings which have second symbol from <i>RHS</i> is 'b' over $\Sigma = \{a,b\}$. Find the equivalent <i>DFA</i> for the constructed <i>NFA</i> .	08
OR			
6	a	Define the following terms with respect to ϵ - <i>NFA</i> with suitable examples. i) ϵ -closure(q) ii) Extended transition function δ^*	08
	b	Convert the following ϵ - <i>NFA</i> given below to an equivalent <i>NFA</i>	08
			
7	a	Consider the sets $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$ and the relations $R = \{(a, 1), (b, 1), (c, 2), (c, 3)\}$ and $S = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$ from A to B . Determine $\bar{R}, \bar{S}, R \cup S, R^c$ and $R_0(R \cap S)$.	08
	b	Let $A = \{1, 2, 3, 4\}, B = \{a, b, c\}$ and $C = \{w, x, y, z\}$ with $f: A \rightarrow B$ and $g: B \rightarrow C$ given by $f = \{(1, a), (2, a), (3, b), (4, c)\}, g = \{(a, x), (b, y), (c, z)\}$. Find $g \circ f$.	04
	c	Draw the Harse diagram represented by the positive divisor of 36.	04
8	a	Show that $(U_{14}, ')$ is cyclic and find all its generators.	06
	b	State and prove Lagrange's theorem.	04
	c	The encoding function $E: Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ i) Determine the code words. What is the error detection and correction capability of this code? ii) Find the associated parity check matrix H . iii) Use H to decode the received words 11101, 11011	06

