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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU)

III Semester B. E. Examinations Nov/Dec-19

Common to CS / IS

DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A, which is compulsory. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

PART-A

1	1.1	Find the number of 5 digit positive integers such that each of them every digit is greater than the digit to the right.	01
	1.2	Determine the coefficient of x^{12} in the expansion of $x^3(1-2x)^{10}$.	02
	1.3	What is the contrapositive of the conditional statement: "The home team misses whenever it is drizzling"?	01
	1.4	Let S be a set of n elements. What is the number of ordered pair in the largest and the smallest equivalence relations on S?	02
	1.5	For the equivalence relation $R=\{(1,1),(1,2),(2,1),(2,2),(3,4),(4,3),(3,3),(4,4)\}$ defined on the set $A=\{1,2,3,4\}$, determine the partition induced.	02
	1.6	If the function f is defined by $f(x)=x^2+1$ on a set $A=\{-2,-1,0,1,2\}$, find the range of f.	02
	1.7	Find the recurrence relation with initial condition for the sequence and find the general term of the sequence. $2,10,50,250\ldots$	02
	1.8	If $T(x)$ denotes x is a trigonometric function, $P(x)$ denotes x is a periodic function and $C(x)$ denotes x is a continuous function then write the given statement in symbolic form. "It is not the case that some trigonometric functions are not periodic".	01
	1.9	Show that $(Z_6,+)$ is an abelian group	02
	1.10	Obtain a DFA to accept string of $a's$ & $b's$ having exactly one a.	02
	1.11	Check the validity of the following statement: If I drive to work, then I will be tired. I do not drive to work — — — — — — — — — — — — — — — — — $\therefore$ I will not arrive tired	01
	1.12	Define the extended transition function for $\varepsilon-NFA$	01
	1.13	The word $c=1010110$ is transmitted through a binary symmetric channel. If $e=0101101$ is the error pattern, find the word $'r'$ received.	01

PART-B

2	a	Find and solve a recurrence relation for the number of binary sequence of length $n \geq 1$ that have no consecutive $0's$	04
	b	Find the number of integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where $x_1 \geq 3, x_2 \geq 2, x_3 \geq 4, x_4 \geq 6, x_5 \geq 0$.	04
	c	By Mathematical Induction, prove that $n! \geq 2^{n-1}$ for all integers $n \geq 1$.	04
	d	A string of length n is a sequence of form $x_1x_2x_3 \dots x_n$, where each x_i is a digit. The sum $x_1 + x_2 + x_3 \dots + x_n$ is called the weight of the string. If each x_i can be one of 0,1 or 2, Find the number of string of length $n = 10$. Of these, find the number of strings whose weight is an even number.	04
3	a	Examine whether the compound proposition $[(p \vee q) \rightarrow r] \leftrightarrow [\sim r \rightarrow \sim(p \vee q)]$ is a tautology.	04
	b	Verify whether the following argument is valid or not: $\forall x[p(x) \vee q(x)]$ $\forall x[(\neg p(x) \wedge q(x)) \rightarrow r(x)]$ ――――――――――――― $\therefore [(\neg r(x) \rightarrow p(x)]$	06
	c	Simplify the following compound proposition using laws of logic: i) $(p \vee q) \wedge [\sim\{(\sim p) \vee q\}]$ ii) $\sim[\sim\{(p \vee q) \wedge r\} \vee \sim q]$	06
		OR	
4	a	Prove the following logical equivalence without using truth tables: i) $[(p \vee q) \wedge (p \vee \sim q)] \vee q \leftrightarrow p \vee q$ ii) $(p \rightarrow q) \wedge [\sim q \wedge (r \vee \sim q)] \leftrightarrow \sim(q \vee p)$	06
	b	Test the validity of the following argument: $(\sim p \vee q) \rightarrow r, r \rightarrow (s \vee t), \sim s \wedge \sim u, \sim u \rightarrow \sim t \;/\!-p$	04
	c	Find the following: i) Negations of the statement $[\exists x, [p(x) \vee q(x)]$ ii) Negations of the statement For all integers n, if n is not divisible by 2, then n is odd. iii) Proposition in symbolic form Every integer is either even or odd	06
5	a	Let $f: R \rightarrow R$ be defined by $f(x) = \begin{cases} 3x - 1, & x > 0 \\ -3x + 1 & x \leq 0 \end{cases}$ What are $f^1([-5,5])$ and $f^1([-6,5])$?	04
	b	For the poset (A, R) represented for the following Hasse diagram shown in fig 5b, find: i) $GLb\{b, c\}$, ii) $GLB\{b, w\}$, iii) $LUB\{c, e\}$ and iv) $LUB\{a, v\}$.	

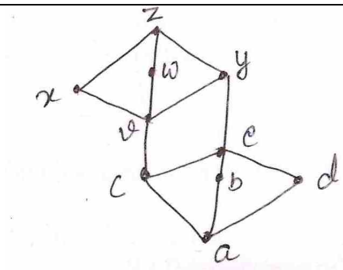


Fig 5b — 04

c — For a fixed integer $n > 1$, prove that the relation "congruent modulo n" is an equivalence relation on the set of all integers Z. — 04

d — A function $f: A \rightarrow B$ is invertible if and only if it is one to one and onto. — 04

OR

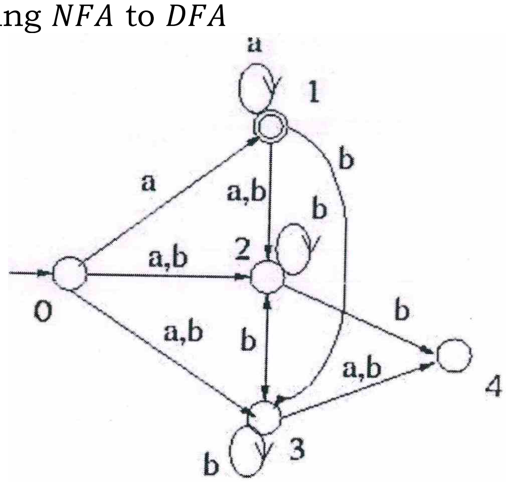
6 a — Let $A = B = C$, and $f: A \rightarrow B$ and $g: B \rightarrow C$ be defined by $f(a) = 2a + 1$ and $g(b) = b/3$, $\forall a \in A, \forall b \in B$. Compute gof and show that gof is invertible. What is $(g \circ f) - 1$? — 04

b — Let $A = \{1,2,3,4,5,6\}$ and $B = \{6,7,8,9,10\}$. If a function $f: A \rightarrow B$ is defined by $f = \{(1,7),(2,7),(3,8),(4,6),(5,9),(6,9)\}$. If $B_1 = \{7,8\}$ and $B_2 = \{8,9,10\}$, then find $f^{-1}(B_1)$ and $f^{-1}(B2)$ — 04

c — Consider the sets $A = \{1,2,3,4\}$ and R, S are a relation on A defined by $R = \{(1,2),(1,3),(2,4),(4,4)\}$ and $S = \{(1,1),(1,2),(1,3),(1,4),(2,3),(2,4)\}$. Find $R \circ S, S \circ R, R^2, S^2$ and write down their matrices — 04

d — On the set Z of all integers, a relation R is defined by aRb if and only if $a^2 = b^2$. Verify that R is an equivalence relation. Determine the partition induced by this relation. — 04

7 a — Convert the following NFA to DFA



— 08

b — Obtain a DFA to accept strings of $0's$ and $1's$ which, when interpreted as a binary number is multiple of 5. — 08

8 a — The encoding function $E: Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix.

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

i) Determine all code words. What can we say about the error-detection capability of this code? What about its error-correcting capacity?

ii) Find the associated parity-check matrix H.

iii) Use H to decode the received words: 11101, 11011. — 06

b	If $f: G \to H$ and $g: H \to K$ are homomorphism, prove that $g \circ f: G \to K$ defined by $(g \circ f)(x) = g\{f(x)\}$ is an homomorphism.	05
c	Prove that a group G is abelien if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$	05