USN

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU)
III Semester B.E. Fast-track Examinations Jan/Feb-2023
Common to CS/IS

Computer Science and Engineering

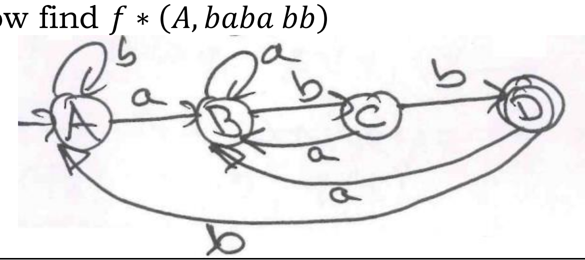
DISCRETE MATHEMATICAL STRUCTURES

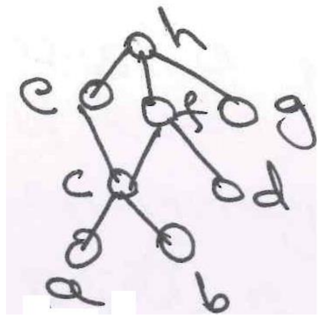
Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

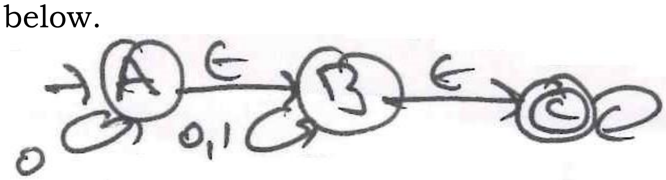
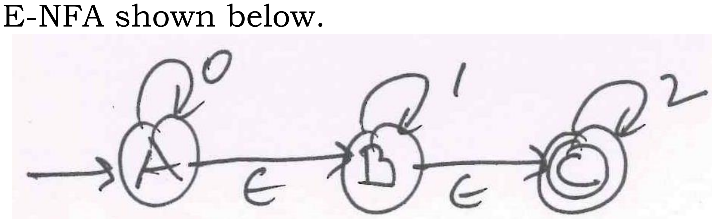
PART-A

1	1.1	Find the number of 5 digit positive integers such that each of them every digit is greater than the digit to the right.	01
	1.2	Determine the number of integer solutions for $x_1 + x_2 + x_3 + x_4 = 18$, where $x_i \leq 7, i = 1,2,3,4$.	01
	1.3	What is the negation of the following statement? $$\exists_x \forall_y \left[\left(P(x,y) V q(x,y)\right) \rightarrow r(x,y)\right]$$	01
	1.4	If T(x) denotes x is a trigonometric function, P(x) denotes x is a periodic function and C(x) denotes x is continuous function then, write the below statement in a symbolic form. “It is not the case that some trigonometric functions are not periodic”	01
	1.5	For the equivalence relations R={(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)} defined on the set A={1,2,3,4}. Determine the partitioned induced by R.	01
	1.6	Consider the functions f and g defined by $f(x) = x^2$ and $g(x) = x^4 - 1$, $\forall_x, x \in R$. Find gof and f^2.	02
	1.7	IF the function f is defined by $f(x) = x^2 + 1$ on a set A={-2, -1, 0, 1,2}. Find the range of f.	01
	1.8	Give DFA accepting the language over $\Sigma_1 = \{0,1\}$, the set of all strings that either begin or end with 01 or both.	02
	1.9	Define the extended transition function for $\epsilon - NFA$.	02
	1.10	In the DFA below find $f * (A, baba\ bb)$ 	02

1.11	Find the maximal and minimal elements in the POSET whose Hasse diagram is as shown below. Also find the greatest element and the least element. 	02
1.12	Sow that $(Z_6, +)$ is a abelian group.	02
1.13	Find the recurrence relation with initial condition for the sequence 2, 10, 50, 250,….. Find the general solutions also.	01
1.14	The word C=1010110 is transmitted through a binary symmetric channel. If e=010101 is the error pattern, find the word 'r' received.	01

PART-B

2	a	Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 40$, where i) $x_i > 0,\ 1 \le i \le 5,$ ii) $x_1,\ x_2 \ge 5,\ x_3, x_4, x_5 \ge 7.$	04
	b	Solve the recurrence relation.	06
	c	Prove the following by mathematical induction $1.3\ +\ 2.4\ +\ 3.5\ +\ \ldots.. + n(n+2) = \frac{(n(n+1)\,(2n+1))}{6}$	06
3	a	Prove the following logical equivalence without using truth table. i) $[(\sim PV\sim q) \rightarrow (P \wedge q \wedge r)] \Leftrightarrow P \wedge q$ ii) $(P \longrightarrow q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow\ \sim(q \vee P)$	06
	b	Show that the following argument is valid $P, P \rightarrow q, s \vee r, r \rightarrow \sim q\ \vdash (s\ \vee t)$	04
	c	For the universe of all integers, let $P(x){:}\,x > 0$, $q(x){:}\,x$ is even, $r(x){:}\,x$ is a perfect square, $s(x){:}\,x$ is divisible by 4, $t(x){:}\,x$ is divisible by 5. For each of the following statements write the equivalent symbolic form: i) At least one integer is even. ii) There exists a positive integer that is even. iii) If x is even, then x is not divisible by 5. iv) No even integer is divisible by 5. v) There exists a even integer divisible 5. vi) If x is even and x is perfect square, then x is divisible by 4.	06
		OR	
4	a	By using truth table find which of the following compound propositions are tautologies: i) $(p \rightarrow q) \rightarrow (q - p)$ ii) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	06
	b	Verify whether the following argument is valid or not $\begin{array}{c} \forall_x[p(x) \vee q(x)] \\ \forall_x[(\sim p(x) \wedge q(x) \rightarrow r(x)] \\ \hline \therefore \forall_x[\sim r(x) \rightarrow p(x)] \end{array}$	10

5	a	For each of the following functions determine whether it is one –to-one and determine its range. i) $f: Z \to Z, f(x) = 2x + 1$ ii) $g: Z \to Z, g(x) = x^3 - 1$	04
	b	Let $A = \{1,2,3,4,5\} \times \{1,2,3,4,5\}$ and define R on A by $(x_1 y_1)\, R(x_2 y_2)$ if $x_1 + y_1 = x_2 + y_2$. i) Verify that R is an equivalence relation on A ii) Determine the equivalence classes $[(1,3), [(2,4)]$ and $[(1,1)]$ iii) Determine the partition of A induced by R.	12
		OR	
6	a	Let R and S are relations and A define by $R = \{(1,2), (1,3), (2,4), (4,4)\}$ and $S = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4)\}$. Find ROS, SOR, R^2, S^2 and write down their matrices.	06
	b	Let $A\{a, b, c, d, e, f\}$ and $B = \{f, g, h, i, j\}$. If a function $f: A \to B$ is defined by $f = \{(a, g), (b, g), (c, h), (d, f), (e, i), (f, i)\}$. If $B_1 = \{g, h\}$, $B_2 = \{h, i, j\}$, then find $f^{-1}(B_1)$ and $f^{-1}(B_2)$	04
	c	Let the set A contains all the positive divisors of 36. Consider the relation R on A such that aRb iff "a divisible b". Show that (A,R) is a POSET. Draw the Hasse diagram for (A,R).	06
7	a	Define DFA and language accepted by DFA.	04
	b	What is E-closure of state? Find the E-closure (A), E-closure ({A,B,C}) in the E-NFA below. 	04
	c	Show that for every E-NFA there exists an equivalent NFA such that the language of these two automata are same. Find the equivalent NFA for the E-NFA shown below. 	08
8	a	Define the binary operation o on Z by $XoY = x + Y + 1.$ Verify that (Z, o) is an abelian group.	05
	b	State and prove Lagrange's Theorem.	05
	c	The encoding function $E: Z_2^3 \to Z_2^6$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ i) Find the code words assigned to 110 & 010. ii) Obtain the associated parity check matrix.	06