RANDOM VARIABLES

Discrete Random Variables

Let X be a discrete random variable. A function $p(x)$ is a probability mass function of the discrete random variable X if $p(x) \geq 0, \forall x \in X$ and $\sum_x p(x) = 1$.

- Expectation, $E(X) = \sum_x xp(x)$
- If $Y = g(X)$, then $E(Y) = \sum_x g(x)p(x)$
- Variance, $Var(X) = E\left[\left(X - E(X)\right)^2\right] = E(X^2) - [E(X)]^2$
- Standard deviation, $\sigma_X = \sqrt{Var(X)}$
- The cumulative distribution function, $F(t) = P(X \leq t) = \sum_{x \leq t} p(x)$

Continuous Random Variables

Suppose X is a continuous random variable. A function $f(x)$ is called a probability density function of the continuous random variable X if $f(x) \geq 0, \forall x \in X$ and $\int_{-\infty}^{\infty} f(x)dx = 1$.

- Expectation, $E(X) = \int_{-\infty}^{\infty} x\, f(x)dx$
- If $Y = g(X)$, then $E(Y) = \int_{-\infty}^{\infty} g(x)\, f(x)dx$
- Cumulative distribution function, $F(t) = P(X \leq t) = \int_{-\infty}^{t} f(x)dx$
- $P(a \leq X \leq b) = P(a \leq X < b) = P(a < X \leq b) = P(a < X < b) = \int_a^b f(x)dx$

Joint Probability Mass Functions

Suppose X and Y are two discrete random variables. A function $p(x, y)$ is called a joint probability mass function of X and Y if $p(x, y) \geq 0, \forall x \in X, y \in Y$ and $\sum_x \sum_y p(x, y) = 1$.

- Let $Z = g(X, Y)$. Expectation, $E[Z] = \sum_x \sum_y g(x, y)p(x, y)$
- Covariance, $Cov(X, Y) = E(XY) - E(X)E(Y)$
- Correlation of X and Y, $\rho(X, Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$
- If X and Y are independent, then $E(XY) = E(X)E(Y)$