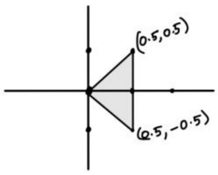
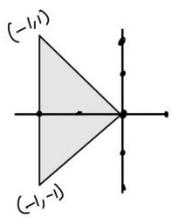
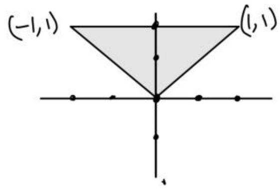
DEPARTMENT OF MATHEMATICS

Course: Linear Algebra and Probability Theory	**CIE-II**	**Maximum marks: 50**
Course code: MA231CT	**Third semester 2023-2024** **Branch: CS, CD, CY**	**Time: 10:00AM-11:30AM** **Date: 20-02-2024**

SCHEME AND SOLUTION

Q.No	Solutions	Marks
1.	Row echelon form of $A = \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	3
	Basis for row space is $\{(-3,6,-1,1,-7)^T, (1,-2,2,3,-1)^T\}$	1
	Basis for column space is $\{(-3,1,2)^T, (-1,2,5)^T\}$	1
	Dimension of row space =Dimension of column space $= 2$	1
	Consider $\begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	
	$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$	2
	Basis of Null space is $\{ (2,1,0,0,0)^T, (1,0,-2,1,0)^T, (-3,0,2,0,1)^T \}$	1
	Dimension of Null space $= 3$	1
2.a	Matrix representations and diagrams:	
	i) $\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ ii) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ iii) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	1+1+1
	  	1+1+1
2.b	Verify: i) $T\big((x_1,y_1,z_1)+(x_2,y_2,z_2)\big) = T\big((x_1,y_1,z_1)\big) + T((x_2,y_2,z_2))$	2
	ii) $T\big(\alpha(x_1,y_1,z_1)\big) = \alpha T((x_1,y_1,z_1))$ for all $\alpha \in \mathbb{R}$, (x_1,y_1,z_1), $(x_2,y_2,z_2) \in \mathbb{R}^3$	2
3.	Suppose $\{v_1,v_2,v_3\}$ is orthogonal basis. Then $v_1 = u_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = u_2 - \frac{(u_2 \cdot v_1)}{v_1 \cdot v_1} v_1 = \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix} - \frac{-24}{12} \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$	1+3
	$v_3 = u_3 - \frac{u_3 \cdot v_1}{v_1 \cdot v_1} v_1 - \frac{u_3 \cdot v_2}{v_2 \cdot v_2} v_2 = \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} - \frac{18}{12} \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix} - \frac{48}{72} \begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} -5/6 \\ -1/6 \\ 9/2 \\ -19/6 \end{bmatrix}$	3

	Projection is $$\vec{p} = \frac{u \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{u \cdot v_2}{v_2 \cdot v_2} v_2 + \frac{u \cdot v_3}{v_3 \cdot v_3} v_3 = \frac{1}{6}\begin{bmatrix}1\\3\\1\\1\end{bmatrix} + \frac{1}{9}\begin{bmatrix}8\\-2\\0\\-2\end{bmatrix} - \frac{7}{93}\begin{bmatrix}-5/6\\-1/6\\9/2\\-19/6\end{bmatrix} = \begin{bmatrix}104/93\\9/31\\-16/93\\17/93\end{bmatrix}$$	3
4.a	i) $\int_{y=0}^{1}\int_{x=0}^{1} c(x^2+y^2)\,dx\,dy = 1 \Rightarrow c = \frac{3}{2}$ ii) Marginal distributions $f_X(x) = \int_{y=0}^{1} c(x^2+y^2)\,dy = \frac{1}{2}(3x^2+1)$ $f_Y(y) = \int_{x=0}^{1} c(x^2+y^2)\,dx = \frac{1}{2}(3y^2+1)$ iii) $P(X+Y<1) = \int_{y=0}^{1}\int_{x=0}^{1-y} c(x^2+y^2)\,dx\,dy = \frac{1}{4}$	2 1 1 2
4.b	Let X denote the number of engine running, it is a binomial r.v. with parameter $p = 0.6$ and n. When $n = 4$, the probability that plane has a successful flight is $P(X \geq 2) = 1 - P(X < 2) = 1 - b(0;4,0.6) - b(1;4,0.6) = 1 - (0.4)^4 - 4(0.6)(0.4)^3 = 0.8208$ When $n = 2$, the probability that plane has a successful flight is $P(X \geq 1) = 1 - P(X < 1) = 1 - b(0;2,0.6) = 1 - (0.4)^2 = 0.84$ Thus 2- engine flight has a higher probability for a successful flight than the 4 – engine flight.	 2 1 1
5.a	Let K denote number of arrival of messages and X denote time interval between two successive messages. Given $\lambda = 10$, number of messages in one hour. i) $P(1 < K < 10) = \sum_{k=2}^{9} \frac{e^{-2\lambda}(2\lambda)^k}{k!} = 1 - 0.004995$ ii) $P(X > 40min \mid X > 30min) = P(X > 10min) = P\left(X > \frac{1}{6}\right) = e^{-\frac{\lambda}{6}} = 0.1888$ iii) $P(X > x) = 0.8 \Rightarrow e^{-\lambda x} = 0.8 \Rightarrow x = 0.0223hr = 1.339min.$	 2 2 2
5.b	$\mu = 6000$ and $\sigma = 100$. Let X denote strength of samples of cement, corresponding standard normal r.v., $Z = \frac{X-\mu}{\sigma}$ i) $P(5800 < X < 5900) = P(-2 < Z < -1) = \Phi(-1) - \Phi(-2) = 0.158655 - 0.02275 = 0.1359$ ii) $P(X > x) = 0.95 \Rightarrow P(Z > z) = 0.95 \Rightarrow \Phi(z) = 1 - 0.95 \Rightarrow z = -1.65$ Thus the strength that is exceeded by 95% of the samples is, $x = -1.65\sigma + \mu = 5835.$	 2 1 1