Matrix transformation:

Reflection about line $y = mx$.

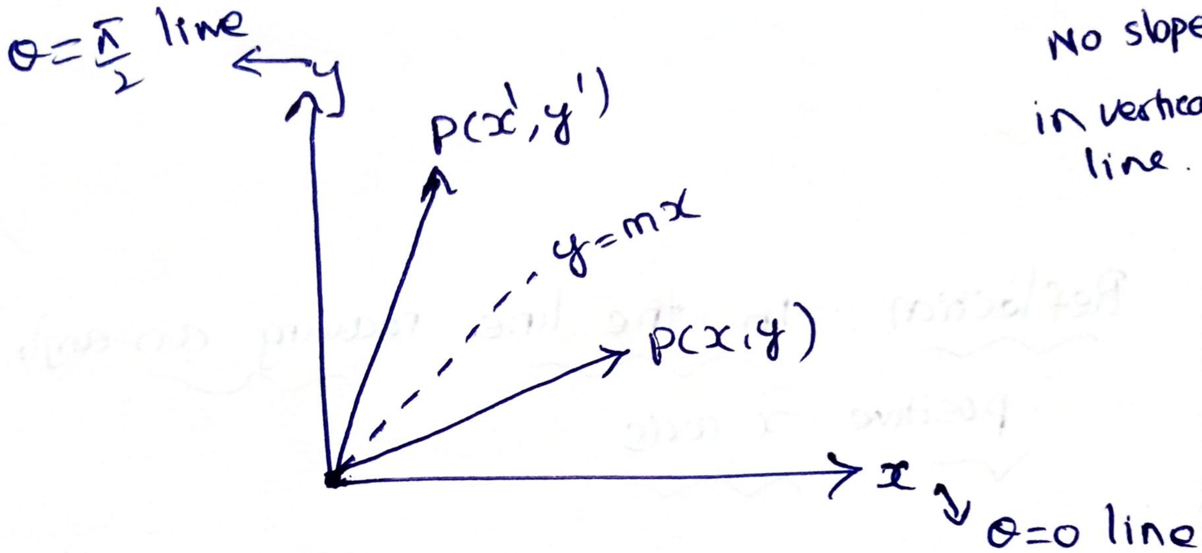
$$\text{Matrix} = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$$

→ NOTE

Not suitable y-axis

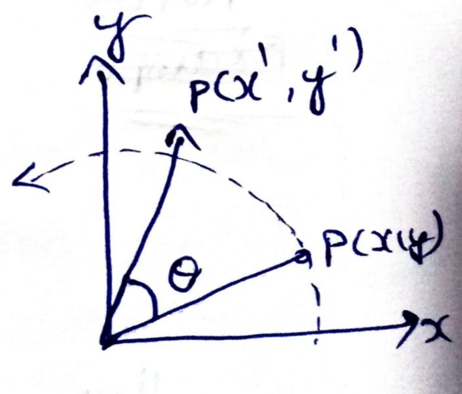
ie $x=0$

No slope in vertical line.



Rotation in Counterclockwise direction with angle θ:

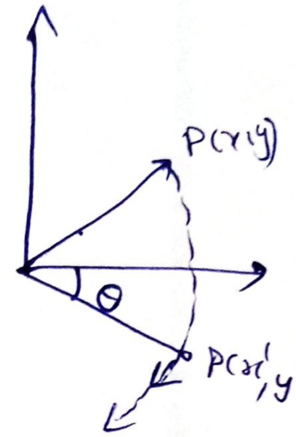
$$\text{Matrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



Rotation in clockwise direction with an angle θ

$$\text{matrix} = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$$

$$\text{Matrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$



Reflection in the line making an angle θ with the positive x-axis

$$\text{Matrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

NOTE:
~~Example~~: Reflection in the y-axis. ie $x=0$

Then $\frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$ not suitable

Using $\theta = \frac{\pi}{2}$, Reflection Matrix $\begin{bmatrix} \cos 2(\frac{\pi}{2}) & \sin 2(\frac{\pi}{2}) \\ \sin 2(\frac{\pi}{2}) & -\cos 2(\frac{\pi}{2}) \end{bmatrix}$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

problem Let $T: R^2 \to R^2$ be rotation through $-\frac{\pi}{2}$ followed by reflection in the y-axis what is the resultant matrix?

Given

Sol: $\theta = -\frac{\pi}{2}$ (clockwise)

$$\text{matrix} = \begin{bmatrix} \cos(-\frac{\pi}{2}) & -\sin(-\frac{\pi}{2}) \\ \sin(-\frac{\pi}{2}) & \cos(-\frac{\pi}{2}) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

Reflection on y-axis

ie y-axis $\Rightarrow$ $x = 0$

$\therefore$ ie slope $m = 0$

from matrix $\Rightarrow \frac{1}{1+m^2} \begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

problem Find the matrix of linear transformation which is obtained by first rotating all vectors through angle $\frac{\pi}{6}$ and the reflecting through x-axis

Sol: T_1: Rotation with $\theta = \pi/6$

$$\text{Rotation matrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

T_2: Reflection about x-axis ie $\theta = 0$ line $y = 0$

$\therefore$ slope $m = 0$

from matrix $\frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$

put $m=0$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(OR)

$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

with $\theta = 0$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\therefore$ ~~Reflection~~ Rotation followed by reflection

T_1 followed by T_2

$$\cancel{T_2(T_1(x))} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ +\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

Reflection followed by a rotation:

ie 1st Reflection Then Rotation.

R_θ – indicates rotation through angle θ.

R_ϕ – indicates reflection with angle ϕ through origin

Then

$$\text{matrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{bmatrix}$$

Rotation Followed by reflection

ie First Rotation Then reflection

$$\begin{bmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Rotation followed by Rotation:

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

problem Find matrix for $T: R^2 \to R^2$ ~~first~~ which rotates vectors through $-\frac{3\pi}{4}$ Then reflects through horizontal axis. (x-axis)

↳ $\cancel{\phi} = 0$ line

sol:

Given $\theta = -\frac{3\pi}{4}$, $\phi = 0$

$$\text{matrix} = \begin{bmatrix} \cos 0 & \sin 0 \\ \sin 0 & \cos 0 \end{bmatrix} \begin{pmatrix} \cos(-\frac{3\pi}{4}) & \sin(-\frac{3\pi}{4}) \\ \sin(-\frac{3\pi}{4}) & \cos(-\frac{3\pi}{4}) \end{pmatrix}$$

Ex (1) Lep $P(3,2)$ after rotation of 90°

New point = — ?

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 3 \end{bmatrix}.$$

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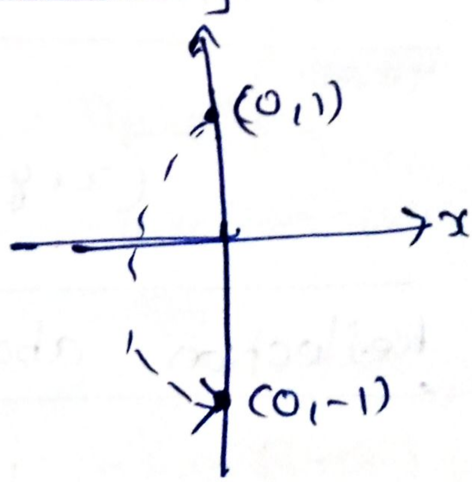
[3] Reflection through x-axis (or) 0°-line



$$\text{matrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

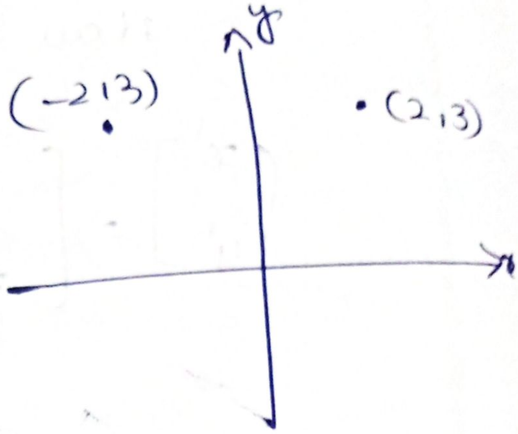
ie $(x, y) \longrightarrow (x, -y)$

$\boxed{y \text{ becomes } -y}$

iii)

Reflection in y-axis (or) 90° - line

(x,y) becomes (-x,y)



matrix = $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Reflection about y = x (or) 45° - line

matrix = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(x,y) becomes (y,x)

Reflection about y = -x (or) 135° line

matrix = $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(x,y) becomes (-y,-x)

Reflection about line y = Kx (or) line with slope = K

matrix = $\dfrac{1}{1+K^2}\begin{bmatrix} 1-K^2 & 2K \\ 2K & K^2-1 \end{bmatrix}$

* Rotation followed by a rotation

Let A_{θ_1} is matrix which rotates vectors through angle θ_1 ie $A_{\theta_1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix}$

A_{θ_2} is matrix which rotates vectors through angle θ_2 ie $A_{\theta_2} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}$

Then if we apply these two rotations in succession Then

$$\text{matrix} = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix}$$