RV Educational Institutions
RV College of Engineering

Autonomous
Institution Affiliated
to Visvesvaraya
Technological
University, Belagavi

Approved by AICTE,
New Delhi

DEPARTMENT OF MATHEMATICS

Course: Linear Algebra and Probability Theory	**CIE-I**	**Maximum marks: 50**
Course code: MAT231CT	**Third semester 2023-2024** **Branch: CS, CD, CY**	**Time: 10:00AM-11:30AM** **Date: 08-01-2024**

Instructions to candidates:

Answer all questions.

Q.No	QUESTIONS	M	BT	CO
1	Let $\mathbb{R}^2, \mathbb{R}^3, P_4$ (set of all polynomials of degree 4 or less with real coefficients) and $M_{2\times 2}$ (set of all 2×2 real matrices) be vector spaces with usual addition and scalar multiplication. Verify whether the following sets forms a subspace or not. Justify your answer. i) $S_1 = \left\{\begin{bmatrix}x\\y\end{bmatrix} \in \mathbb{R}^2 \middle\vert y = x^2\right\}$ of $\mathbb{R}^2$ ii) $S_2 = \left\{\begin{bmatrix}x\\y\end{bmatrix} \in \mathbb{R}^2 \middle\vert 2x = 3y\right\}$ of $\mathbb{R}^2$ iii) $S_3 = \left\{\begin{bmatrix}x\\y\\z\end{bmatrix} \in \mathbb{R}^3 \middle\vert x - 4y + 5z = 0\right\}$ of $\mathbb{R}^3$ iv) $S_4 = \{f(x) \in P_4 \vert f(1) \text{ is an integer}\}$ v) $S_5 = \{A \in M_{2\times 2} \vert det(A) \neq 0\}$.	10	2	2
2.a	Let $u = (1,3,2,1), v = (2,-2,-5,4), w = (2,-1,3,6)$ be vectors in $\mathbb{R}^4$. If possible express $t = (2,5,-4,0)$ as a linear combination of u, v and w.	5	2	2
2.b	Determine whether the set of vectors $S = \{1 + x - 2x^2, 2 + 5x - x^2, x + x^2\}$ in P_2 is linearly independent or linearly dependent.	5	2	2
3.a	Discrete random variable has the probability mass function as follows: $\begin{array}{\|c\|c\|c\|c\|c\|c\|c\|c\|c\|} \hline x & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline p_X(x) & 0 & k & 2k & 2k & 3k & 3k^2 & 2k^2 & 7k^2 + k \\ \hline \end{array}$ i) Determine the value of k ii) Compute the probabilities $P(X \geq 5), P(X < 3)$ and $P(2 < X \leq 5)$ iii) Find expectation of X.	6	1	1
3.b	A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, find the probability and cumulative distribution of X.	4	2	2
4.a	The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable X that has the density function $p(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \leq x < 2, \\ 0, & elsewhere. \end{cases}$ Find the probability that over a period of one year, a family runs their vacuum cleaner i) less than 120 hours; ii) between 50 and 100 hours.	6	2	2

<table>
<tr><td>4.b</td><td>The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ 1 - e^{-8x}, & x \geq 0. \end{cases}$$

Find the probability density function and the probability of waiting less than 12 minutes between successive speeders.</td><td>4</td><td>2</td><td>2</td></tr>
<tr><td>5.a</td><td>Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their probability distribution is given as

<table>
<tr><th colspan="2" rowspan="2">$p(x,y)$</th><th colspan="3">x</th></tr>
<tr><th>1</th><th>2</th><th>3</th></tr>
<tr><td rowspan="3">y</td><td>1</td><td>0.05</td><td>0.05</td><td>0.1</td></tr>
<tr><td>3</td><td>0.05</td><td>0.10</td><td>0.35</td></tr>
<tr><td>5</td><td>0.00</td><td>0.20</td><td>0.10</td></tr>
</table>

i) Evaluate the marginal distributions of X and Y
ii) Find $P(X > 1, Y \geq 3)$, $P(X < 3, Y = 3)$.
iii) Covariance of X and Y.</td><td>6</td><td>3</td><td>3</td></tr>
<tr><td>5.b</td><td>A fair tetrahedral die (four faced die) is rolled twice. Let X denote the sum of two tosses less than 5 and let Y denote the maximum of the two tosses. Obtain the joint probability distribution function of X and Y.</td><td>4</td><td>2</td><td>2</td></tr>
</table>

***