RV Educational Institutions ®
RV College of Engineering ®
Autonomous Institution Affiliated to Visvesvaraya Technological University, Belagavi
Approved by AICTE, New Delhi

DEPARTMENT OF MATHEMATICS

Course: Linear Algebra and Probability Theory	CIE-I	Maximum marks: 50
Course code: MAT231CT	Third semester 2023-2024 Branch: CS, CD, CY	Time: 10:00AM-11:30AM Date: 08-01-2024

SCHEME AND SOLUTION

Q.No	Solutions	Marks
1.	i) Not a subspace, $\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\4\end{bmatrix} \in S_1$ but $\begin{bmatrix}1\\1\end{bmatrix}+\begin{bmatrix}2\\4\end{bmatrix}=\begin{bmatrix}3\\5\end{bmatrix} \notin S_1$(or any other suitable justification) ii) Subspace. Justification. iii) Subspace. Justification. iv) Not a subspace. Justification. v) Not a subspace. Justification.	1+1 1+1 1+1 1+1 1+1
2.a	$t = c_1u + c_2v + c_3w$ $2 = c_1 + 2c_2 + 2c_3, 5 = 3c_1 - 2c_2 - c_3, -4 = 2c_1 - 5c_2 + 3c_3, 0 = c_1 + 4c_2 + 6c_3$ $\begin{bmatrix}1 & 2 & 2\\3 & -2 & -1\\2 & -5 & 3\\1 & 4 & 6\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}2\\5\\-4\\0\end{bmatrix}$ this system reduces to $\begin{bmatrix}1 & 2 & 2\\0 & 1 & -6\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}2\\-1\\8\\0\end{bmatrix}$ $c_3 = -1\ c_2 = 1$ and $c_3 = 2$ $t = 2u + 1v - 1w$	1+3 1 1
2.b	Suppose $c_1(1 + x - 2x^2) + c_2(2 + 5x - x^2) + c_3(x + x^2) = 0$ $c_1 + 2c_2 = 0, c_1 + 5c_2 + c_3 = 0, -2c_1 - c_2 + c_3 = 0$ $\begin{bmatrix}1 & 2 & 0\\1 & 5 & 1\\-2 & -1 & 1\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$ $\det\begin{bmatrix}1 & 2 & 0\\1 & 5 & 1\\-2 & -1 & 1\end{bmatrix}=0$ Therefore, the above homogeneous system has non-trivial solution. Hence, the given set of polynomials is a linearly dependent set in P_2	1 1 2 1 1
3.a	i) $12k^2 + 9k = 1 \Rightarrow k = 0.0982$ ii) $P(X \geq 5) = 0.214,\ P(X < 3) = 0.2946,\ P(2 < X \leq 5) = 0.51993$ ii) $E[X] = 3.6789$	2 2 2
3.b	$X = \{0, 1, 2, 3\}$ x | 0 | 1 | 2 | 3 $p(x)$ | $\frac{C(5,3)}{C(7,3)}=\frac{10}{35}$ | $\frac{C(5,2)C(2,1)}{C(7,3)}=\frac{20}{35}$ | $\frac{C(5,1)C(2,2)}{C(7,3)}=\frac{5}{35}$ | 0 CDF | 10/35 | 30/35 | 35/35=1 | 1	1 2 1
4.a	i) $P(X < 1.2) = \int_0^1 x dx + \int_1^{1.2}(2 - x)dx = 0.68$ ii) $P(0.5 < X < 1) = \int_{0.5}^1 x\, dx = 0.375$	3 3
4.b	Probability mass function $= \frac{d(F(x))}{dx} = -8e^{-8x}$ $P(X < 12) = 1 - e^{-96}$	2 2
5.a	i) Marginal distribution of X and Y x | 1 | 2 | 3 p(x) | 0.1 | 0.35 | 0.55 y | 1 | 3 | 5 p(y) | 0.2 | 0.5 | 0.3 ii) $P(X > 1, Y \geq 3) = 0.75,\ P(X < 3, Y = 3) = 0.15.$ iii) $Cov(X, Y) = E[XY] - E[X]E[Y] = 7.85 - 1.8 \times 3.2 = 2.$	1+1 2 2

5.b

Let $X = \{x \mid x = a + b < 5, (a,b) \in \Omega\} = \{2,3,4\}$, $Y = \{y \mid y = \max(a,b)\} = \{1,2,3,4\}$

Joint probability distribution $p(x,y)$ — 1

$P(X = 2, Y = 1) = P\big((1,1)\big) = 1/16$

$p(x,y)$		x		
		2	3	4
y	1	1/16	0	0
	2	0	1/8	1/16
	3	0	0	1/8
	4	0	0	0

3