Span of a set of vectors or Linear span

- Collection of all linear combinations of vectors in a given set.

ie $S = \{v_1, v_2 \cdots v_n\}$

$$\left.\begin{array}{c}\text{Span}(S) \\ \text{or} \\ L[S]\end{array}\right\} = \text{all l.c of } (v_1, v_2 \cdots v_n).$$

problem ** Is the vector $(4,5,1)$ belongs to $\text{Span}\{v_1, v_2, v_3\}$ where $v_1 = (3,5,-4)$, $v_2 = (2,1,-5)$, $v_3 = (-2,1,3)$.

Sol: Given vector w belongs to span $\{v_1, v_2, v_3\}$ only if System of equations

$x v_1 + y v_2 + z v_3 = w$ are in

Consistent. either Unique or Infinite sol.

solving

$$x\begin{bmatrix} 3 \\ 5 \\ -4 \end{bmatrix} + y\begin{bmatrix} 2 \\ 1 \\ -5 \end{bmatrix} + z\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$$

ie $\left.\begin{aligned} 3x+2y-2z &= 4 \\ 5x+y+z &= 5 \\ -4x-5y+3z &= 1 \end{aligned}\right\}$ solving $x=2$, $y=-3$, $z=-2$

$\therefore$ System is Consistent with Unique sol

$\therefore$ $w = (4,5,1) \in \text{span}\{v_1, v_2, v_3\}$

problem Let $P_1(t) = t^2+2t+1$, $P_2(t) = 2t^2+5t+4$, $P_3(t) = t^2+3t+6$. and $P(t) = 3t^2+5t-5$. verify whether $P(t)$ belongs to $\text{Span}\{P_1(t), P_2(t), P_3(t)\}$

Sol:

If the non-homogeneous system

$$xP_1 + yP_2 + zP_3 = P$$

have solution, either unique or Infinite.

Then we Conclude that $P(t) \in \text{span}\{P_1, P_2, P_3\}$

$\therefore$ $x(t^2+2t+1) + y(2t^2+5t+4) + z(t^2+3t+6) = 3t^2+5t-5$

$\Rightarrow$ Comparing t^2 on both Sides

$$x+2y+z = 3 \longrightarrow (1)$$

Comparing t^1 on both sides,

$$2x + 5y + 3z = 5 \longrightarrow (2)$$

Comparing constant terms on both sides,

$$x + 4y + 6z = -5 \longrightarrow (3)$$

Solving (1), (2), (3),

$$\left.\begin{array}{l} x = 3 \\ y = 1 \\ z = -2 \end{array}\right\} \text{Unique solution exist}$$

$\therefore$ $p(t) \in \text{Span}\{P_1, P_2, P_3\}$.