Academic year 2023-2024 (Even Sem)

DEPARTMENT OF

COMPUTER SCIENCE & ENGINEERING

Date	June 2024	Maximum Marks	50
Course Code	CS241AT	Duration	90 Min
Sem-IV	**Test-1**	Staff: HKK/ASP/SMS/SGR/MNV	

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS

(Common to CSE, ISE & AIML)

		Marks	BT	CO
1a.	Determine if the expansion of $\left(x^2 - \frac{2}{x}\right)^{18}$ will contain a term containing x^{10}.	5 0	4	2
1b.	If a person places 6 letters into 6 addressed envelopes, what is the probability that exactly two of them are placed correctly.	5 5	3	2
2a.	Find the number of non negative integer solutions of ~~i.~~ $x_1 + x_2 + x_3 + x_4 + x_5 = 40$ ii. $x_1 + x_2 + x_3 + x_4 + x_5 \leq 40$ ~~iii.~~ $x_1 + x_2 + x_3 + x_4 + x_5 = 40$ with $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$, $x_4 \geq 4$, $x_5 \geq 5$ iv. $x_1 + x_2 + x_3 + x_4 + x_5 = 40$ with $x_1 < 20$	6 3-4	3	2
2b.	Simplify using the laws of logic: $\neg[\neg\{(p \vee q) \wedge r\} \vee \neg q]$	4 4	3	1
3a.	Write the recurrence relation to solve the Tower of Hanoi problem. Also solve that recurrence relation using the generating function.	6 3	4	4
3b.	Draw the circuit diagram to represent the following statement: $[p \vee (p \wedge q) \vee (p \wedge q \wedge \neg r)] \wedge [p \wedge r \wedge t) \vee t]$	4 0	2	3
4a.	If a person invests ₹ 25,000 at at 9% annual interest, find the amount he will get at the end of 5 years if • interest compounded half yearly • interest compounded monthly Write the recurrence relation and solve.	6 6	3	4
4b.	Determine the truth values of p, q, r, s, t when $[p \wedge (q \wedge r)] \rightarrow (s \veebar t)$ is false.	4 2	2	1
5a.	Show the validity of the argument: $(\neg p \wedge \neg q) \rightarrow (r \wedge s)$ $r \rightarrow t$ $\neg t$ ----------------------- $\therefore p$	6 3	3	3
5b.	Find the number of ways in which 5 people A, B, C, D, and E can be seated at a round table, such that • C and D always sit together • C and D never sit together	4 4	1	1

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	CO5	L1	L2	L3	L4	L5
	Max Marks	12	16	10	12	-	4	8	27	11	-

Academic year 2023-2024 (Even Sem)

DEPARTMENT OF

COMPUTER SCIENCE & ENGINEERING

Date	July 2024	Maximum Marks	60
Course Code	CS241AT	Duration	120 Min
Sem-IV	Test-II	Staff: HKK/ASP/SMS/SGR/MNV	

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS
(Common to CSE, ISE & AIML)

	PART-A	Marks	BT	CO
1.1	Let A={1, 2, 3, 4}. How many relations on A which are antisymmetric? How many relations on A which are neither reflexive nor irreflexive?	2	1	2
1.2	Let R be the relation on the set A={1, 2, 3, 4, 5} containing the ordered pairs R={(1, 3), (2, 4), (3, 1), (3, 5), (4, 3), (5, 1), (5, 2), (5, 4)}. Find R^4.	2	2	1
1.3	For the POSET (A, |) where A={2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72} find the upper bounds, lower bounds, LUB and GLB of {2, 6, 9, 18}.	2	2	2
1.4	How many ways can one distribute 4 distinct objects among 3 identical containers?	1	1	
1.5	If A={1, 2, 3, 4, 5} and there are 6720 injective functions f: A→B, what is |B|?	1	1	
1.6	Let P(x, y) denote the sentence: x divides y. What are the truth values of $\forall x \exists y P(x, y)$, $\forall x \forall y P(x, y)$, where the domain of x, y is the set {1, 2, 4, 6, 12}?	1	1	
1.7	Express the negation of the below statement so that all negation symbols immediately precede predicates. $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$	1	1	
	PART-B			
2a.	For the following statement state the *converse, inverse,* and *contrapositive*. Also determine the truth value for the given statement, as well as the truth value for its *converse, inverse*, and *contrapositive*. "For all real numbers x, if $x^2+4x-21>0$, then $x>3$ or $x<-7$.	05		3

2b.	Test the validity of the following argument: *Some rational numbers are powers of 7.* *All integers are rational numbers.* ――――――――――――― *Some integers are power of 7.*	05	4	3
3a.	Let A={1, 2, 3, 4} and R={(1, 2), (2, 3), (3, 4), (2, 1)}. Write the matrix for R. Find the R^{∞} by computing matrices for R^2, R^3,....	04	2	2
3b.	Suppose A is a set, R is an equivalence relation on A, and a and b are elements of A. Prove the following. i. If aRb, then [a]=[b]. ii. [a]=[b] or [a]$\cap$[b]=ϕ. iii. Distinct equivalence classes of R form a partition of A.	06	4	1
4a.	Define POSET. Show that the set A={1, 2, 3, 6, 12, 15, 24, 36, 48} under the divisibility (|) operation forms a POSET. Draw the Hasse diagram for (A, |)	05	2	
4b.	Let U={1, 2, 3, 4, 5, 6, 7}, with A=P(U) (power set of U), and R be the subset relation on A. For B={{1}, {2}, {2, 3}}$\subseteq$A, determine each of the following. a) The number of upper bounds of B that contains 4 elements. b) The number of upper bounds that exists for B. c) The lub of B d) The number of lower bounds that exists for B. e) The glb of B.	05	3	
5a.	i. Let $f(x)=x^3$ and g(x)=x-1 for all real numbers x. Find g o f and f o g. Verify whether g o f equals f o g. ii. Let f, g: R→R, where $g(x)=1-x+x^2$ and f(x)=ax+b. If $(g \circ f)(x)=9x^2-9x+3$, determine a and b.	04	3	
5b.	If f: A→B and g: B→C are invertible functions, then g o f: A→C is an invertible function and $(g \circ f)^{-1}=f^{-1} \circ g^{-1}$. Prove this.	06	4	
6a.	Let f: A→B, g: B→C and h: C→D. Prove the following. i. If f and g are one-to-one, then g o f is one-to-one. ii. If f and g are onto, then g o f is onto. iii. (h o g) o f=h o (g o f). DMSC-II	06	4	
6b.	Let A={1, 2, 3, 4, 5} and B={6, 7, 8, 9, 10, 11, 12}. i. How many functions f: A→B are there? ii. How many functions are one-to-one? iii. How many functions f: A→B are such that $f^{-1}(\{6, 7, 8\})=\{1, 2\}$?	04	3	

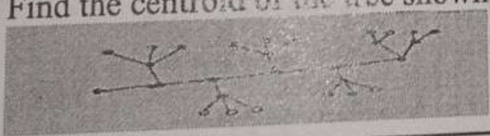
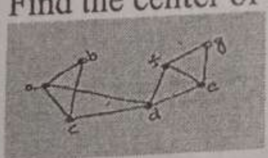
BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

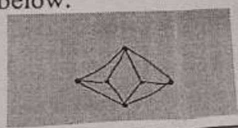
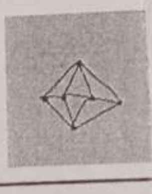
Marks Distribution	Particulars	CO1	CO2	CO3	CO4	CO5	L1	L2	L3	L4	L5
	Max Marks	12	21	17			6	13	18	23	

Course Code	CS241AT	Duration	120 Min
Sem-IV	Improvement Test	Staff: HKK/ASP/SMS/SGR/MNV	

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS

(Common to CSE, ISE & AIML)

<table>
<tr><th></th><th>PART-A</th><th>Marks</th><th>BT</th><th>C</th></tr>
<tr><td>1.1</td><td>Let G be the set of real numbers not containing -1 and * be the binary operation defined by a*b=a+b+ab. What is the inverse of any number a ∈ G.</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>1.2</td><td>If the binary operation * is associative, then complete the following table.
<table>
<tr><td>*</td><td>a</td><td>b</td><td>c</td><td>d</td></tr>
<tr><td>a</td><td>a</td><td>b</td><td>c</td><td>d</td></tr>
<tr><td>b</td><td>b</td><td>a</td><td>c</td><td>d</td></tr>
<tr><td>c</td><td>c</td><td>d</td><td>c</td><td>d</td></tr>
<tr><td>d</td><td></td><td></td><td>c</td><td>d</td></tr>
</table></td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>1.3</td><td>Let G= (Z_{12}, +) and H= {[0], [4], [8]}. What is the partition of G induced by the subgroup H.</td><td>1</td><td>3</td><td>2</td></tr>
<tr><td>1.4</td><td>A binary symmetric channel has probability p=0.05. What is the probability of sending the code word 110101101 and making at most 2 errors in the transmission?</td><td>1</td><td>2</td><td></td></tr>
<tr><td>1.5</td><td>Let E: $Z_2^3 \rightarrow Z_2^9$ be the encoding function for (9, 3) triple repetition code and D: $Z_2^9 \rightarrow Z_2^3$ is the corresponding decoding function. Find three different received words r for which D(r) =000.</td><td>1</td><td>2</td><td></td></tr>
<tr><td>1.6</td><td>Let G be the Peterson graph. Find $\chi(G)$.</td><td>1</td><td>2</td><td></td></tr>
<tr><td>1.7</td><td>What is the value of $\chi'(G)$ where G is $K_{3,2}$.?</td><td>1</td><td>2</td><td></td></tr>
<tr><td>1.8</td><td>If 5 colors are available, how many proper colorings are possible to color the graph $K_{3,3}$.</td><td>1</td><td>3</td><td></td></tr>
<tr><td>1.9</td><td>Find the centroid of the tree shown below.
</td><td>1</td><td>2</td><td></td></tr>
<tr><td>1.10</td><td>Find the center of the graph shown below.
</td><td>1</td><td>2</td><td></td></tr>
</table>

	PART-B			
~~2a.~~	In a group $(G, *)$, prove that $(a * b)^{-1} = b^{-1} * a^{-1}$ for all $a, b \in G$.	05	3	2
~~2b.~~	Show that $(Z_{12}, +)$ is a cyclic group and find all its generators.	05	4	3
~~3a.~~	Let G be a group and let a be any fixed element of G. Show that the function $f: G \to G$ defined by $f(x) = axa^{-1}$, for $x \in G$, is an isomorphism.	05	3	2
3b.	Let E: W□C be an encoding function with the set of messages $W \subseteq Z_2^m$ and the set of code words $E(W) = C \subseteq Z_2^n$, where $m<n$. For $k \in Z^+$, we can detect transmission errors of weight $\leq k$ iff the minimum distance between code words is at least $k+1$. Prove this.	05	2	3
4a.	Define the encoding function $E: Z_2^3 \to Z_2^6$ by means of the parity check matrix $H = \begin{bmatrix} 1&0&1&1&0&0 \\ 1&1&0&0&1&0 \\ 1&0&1&0&0&1 \end{bmatrix}$ ~~i.~~ Determine all code words. ii. What is the error detection and correction capability? ~~iii.~~ Decode the received words 000011, 111100.	06	2	2
4b.	i. If $x \in Z_2^{10}$, determine $\lvert S(x, 1)\rvert$, $\lvert S(x, 2)\rvert$, $\lvert S(x, 3)\rvert$. ii. For $n, k \in Z^+$ with $1 \leq k \leq n$, if $x \in Z_2^n$, what is $\lvert S(x, k)\rvert$?	04	3	2
5a.	Find $P(G, \lambda)$ for the graph shown below. 	06	3	4
5b.	Give an example of a connected graph that has i. Neither an Euler circuit nor a Hamilton cycle. ii. An Euler circuit but no Hamilton cycle. ~~iii.~~ No Euler circuit but has Hamilton cycle. ~~iv.~~ Both Euler circuit and a Hamilton cycle.	04	1	1
6a.	Prove that in every tree $\lvert V\rvert = \lvert E\rvert + 1$.	05	2	3
~~6b.~~	By applying the decomposition theorem, find the number of spanning trees for the graph shown below.  ~~DAA~~ III DMSC-III	05	3	4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	CO5	L1	L2	L3	L4	L5	L6
	Max Marks	7	26	16	11	-	6	22	27	5	-	

IV Semester B. E. Regular Examinations SEP/OCT – 2024

Computer Science and Engineering

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

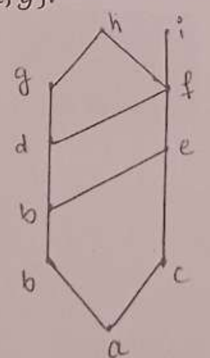
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of statistical tables and formula handbook permitted.

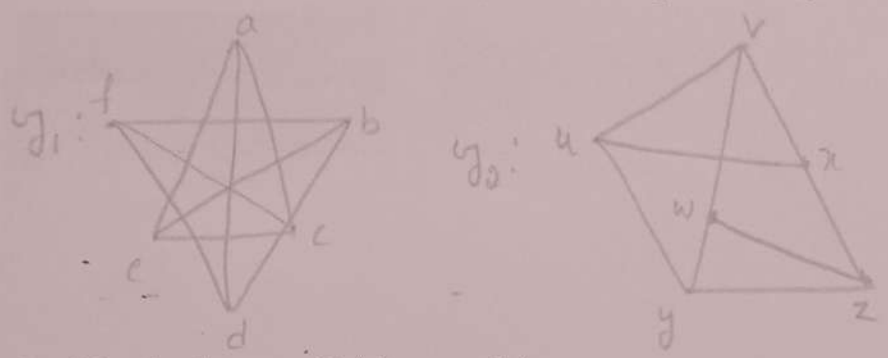
PART-A

			M	BT	CO
1	1.1	Fins the number of ways that the alphabets A, B, C, D, E, F, G are arranged such that A is not first position, B is not in second position, ... G is not in seventh position.	02	2	2
	1.2	Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}, n \geq 2$ subject to the initial conditions $a_1 = 1, a_2 = 3$.	02	2	2
	1.3	Simply using law of logic: $[(p \vee q)\, n (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$	02	1	1
	1.4	Write down the converse, inverse of the following compound proposition. "A person is successful in life if he puts sincere efforts"	02	2	2
	1.5	Let $f: z \to z$ *and* $g: z \to z$, given by $f(x) = x - 1\, g(x) = 2x$ find $f \circ g$ and $g \circ f$.	02	2	2
	1.6	If $R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)\}$ is defined on the set $A = \{1,2,3,4\}$ determine the partition induced.	02	2	2
	1.7	Let $\boldsymbol{G} = \{q \in \theta \mid q \neq -1\}$. Identify the identify element of $\{G, 0\}$ where $xoy = x + y + xy$ for all $x, y \in G$.	02	2	2
	1.8	What is the hamming distance between the codes '11001011' and '10000111'.	02	1	1
	1.9	Let G be a simple graph of order n. If the size of G is 56 and size of $\bar{G}$ is 80, what is n?	02	2	2
	1.10	Given $V = \{(1,2,3,4,5,6)\}$ and $E = \{12, 13, 23, 35, 61, 66\}$ draw undirected and directed graph $G = (V, E)$. Also write down the order and size of G.	02	2	2

PART-B

2	a	Find the number of proper divisors of 38808.	04	2	2
	b	A person inverse some amount at the rate of 11% annual compound interest. Determine the period for principal amount to get doubled.	04	2	3
	c	Find a generating function for the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 0$ for $n \geq 0, a_0 = 5, a_1 = 12$.	04	2	2
	d	In how many ways can 10 identical dimes be distributed among 5 children if i) There are no restrictions ii) Each child gets at least one dime.	04		

3	a	Let p, q and r be the propositions $P: I$ Study; $q: I$ will fail in the examination $r: I$ watch TV in the evening Express each of these proposition as an English sentence (i) $p \rightarrow nq$ (ii) $q \rightarrow r$ (iii) $(p \rightarrow \sim r) \cup (q \rightarrow \sim r)$ (iv) $\sim p \rightarrow (q \cup r)$	04	1	1
	b	Prove that $[(\neg p \vee q) \wedge \{p \wedge (p \wedge q)\}] \Leftrightarrow p \wedge q$ using laws of logic.	06	2	2
	c	Write the following proposition in the symbolic form and find its negation. "If all triangles are right angles then no triangle is equiangular"	06	3	2
		OR			
4	a	For the following statement, state the converse inverse and contrapositive. The universe consist of all integers "If m divides n and n divides p, then m divides p"	04	2	2
	b	Prove the validity of the following argument $p \rightarrow (q \wedge g)$ $r \rightarrow s$ $\underline{\sim(g \wedge s)}$ $\therefore \sim p$	06	2	2
	c	Define open statement and find whether the following variable is valid. No engineering students of 1st or 2nd sem studies logic. Anil is an engineering student who studies logic. $\therefore$ Anil is not in second semester.	06	3	3
5	a	If $f: A \rightarrow B, g: B \rightarrow C, h: C \rightarrow D$ then show that $(h \circ g)\ of = h \circ (g \circ f)$.	04	2	2
	b	If $A = \{1,2,3,4\}$ and R, S are relations on A defined by $R = \{(1,2)(1,3)(2,4)(4,4)\}$ $S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,3)(2,4)\}$. Find $R \circ S,\ S \circ R,\ R^2 and\ S^2$.	05	2	2
	c	Draw the Hasse diagram for all positive integer divisors of 72. Also write R.	07	3	3
		OR			
6	a	Find the lower and upper bounds of the subsets $\{a,b,c\}$; $\{i,h\}$ and $\{a,c,d,f\}$ in the poset with Hasse diagram shown in Fig 6a. Also find the glb and lub of $\{b,d,g\}$.  Fig 6a	08	3	2
	b	Let $f: R \rightarrow R$ be define by $f(x) = \begin{cases} 3x - 5 & if\ x > 0 \\ -3x + 1 & if\ x \leq 0 \end{cases}$. Find $f^{-1}(1)$ and $f^{-1}(3)$.	04	3	3
	c	If R is a relation on $A = \{1,2,3,4\}$ define by xRy if x divides y. Prove that (A, R) is a poset.	04	2	2
7	a	If $\circ$ is an operation on Z define by $x \circ y = x + y + 1$. Prove that $(G.\circ)$ is an Abelian group.	05	3	3
	b	Prove that i) Identity element in a group is unique ii) Inverse of each element in a group is unique	04	2	2

c	The encoding function $E = Z_2^2 \to Z_2^5$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$ i) Find associated parity check matrix H ii) Determine all code words iii) Find decoded word for received msq $[1 \quad 1 \quad 1 \quad 0 \quad 1]$ iv) What is error detection and correction capability.	07	4	4
	OR			
a	If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ Find $\alpha.\beta$ *and* α^{-1}.	04	2	2
b	Define the encoding function $E: Z_2^3 \to Z_2^6$ by means of the parity – check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ Determine all code words.	05	3	3
c	State and prove Lagrange's theorem. Find the right cosets of $H = \{1, -1\}$ in multiplicative group of fourth root of unity.	07	2	3
a	Define Isomorphic and show that G_1 is isomorphic to G_2.  	08	3	3
b	Explain the Konigsberg – Bridge problem.	08	4	4
	OR			
a	If a tree has four vertices of degree 2, one vertex of degree 3, two of degree 4 and one of degree 5, how many pendant vertices does it have?	08	2	2
b	Prove the following for the graph $G = (V, E)$ i) $\sum_{vtV} \deg(V) = 2\lvert E \rvert$ ii) The number of vertices of odd degree must be even.	08	2	2