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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU)

III Semester B. E. Additional Examinations Dec-2020

Computer Science and Engineering

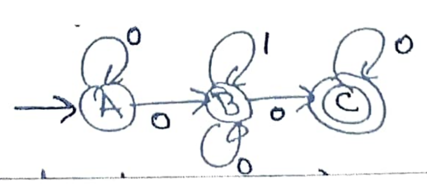
DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours *Maximum Marks: 100*

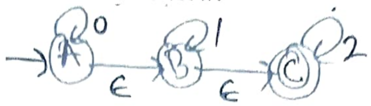
Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

PART-A

1	1.1	How many bit strings of length 8 either start with a 1 bit or end with the two bits 00?	01
	1.2	How many different strings can be made from the letters in $ABRACADABRA$ with all As must be consecutive.	01
	1.3	Give the recursive definition of the sequence $a_n, n = 1,2,3, \ldots..$ if $a_n = n(n+1)$.	01
	1.4	Find $f(5)$ if f is defined recursively by $f(0) = f(1) = 1$ and for $n = 1,2, \ldots \ldots \ldots f(n+1) = f(n)f(n-1)$.	01
	1.5	Let $p(x,y)$ denote the sentence "x divides y". What is the truth value of $\forall x \ni y\, P(x,y)$ where the domain of x, y is the set $\{1,2,4,6,12\}$?	01
	1.6	Express the negation of the statement so that all negation symbols immediately precede predicates. $\forall x \ni y(P(x,y) \wedge \ni zR(x,y,z))$.	01
	1.7	Let $A = \{1,2,3,4\}$. The relation $R\ on\ A$ is given by the matrix M_R. Determine the properties of R. $M_R = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$.	02
	1.8	If $\{\{a,c,e\},\{b,d,f\}\}$ is a partition of the $set\ A = \{a,b,c,d,e,f\}$. Determine the corresponding equivalence relation induced by this partition.	01
	1.9	Find $GLB(\{15,45\})$ and $LUB(\{3,5\})$ in the $POSET(\{3,5,9,15,24,45\}, 1)$.	02
	1.10	Determine the nature of the function $f: R \rightarrow R$, also determine the range of $f(R). f = x^2 + x$.	01
	1.11	For the NFA given below, find the language of the given NFA. 	02

1.12 Find $\delta^*(A, 00122)$ in the $NFA-\epsilon$ below.



02

1.13 If the binary operation $*$ is associative then find X and Y in the following table.

*	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	X	Y	d
d	d	c	c	d

01

1.14 Let $E: 2^3 \to 2^9$ be the encoding function for the (9,3) triple repetition code. If $D: 2_2^9 \to 2_2^3$ is the corresponding decoding function by applying D, decode the received word 111101100. 01

1.15 The encoding function $E: z_2^2 \to z_2^5$ is given by the generator matrix G as below:
$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$. Find the associated parity check matrix H. 02

PART-B

2 a How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$, where $x_i; i = 1,2,\ldots 6$ is a non-negative integer such that.

i) $x_i > 1\ for\ i = 1,2,3,4,5,6$?

ii) $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4, x_5 > 6, x \geq 6$?

iii) $x_1 \leq 5$?

iv) $x_1 < 8\ \&\ x_2 > 8$? 04

b The sequence of Lucas numbers is defined by $l_o = 2, l_1 = 1$ and $l_n = l_{n-1} + l_{n-2}$ for $n = 2,3,4,\ldots\ldots$ prove that $f_n + f_{n+2} = l_{n+1}$ whenever n is a positive integer, where f_i and l_i are the i^{th} Fibonacci and i^{th} Lucas number respectively. 06

c Solve the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$, where $n \geq 0$ and $a_o = 1$ and $a_1 = 4$. 06

3 a State and prove the following rules of inference.

i) Law of syllogism

ii) Rule of destructive dilemma 06

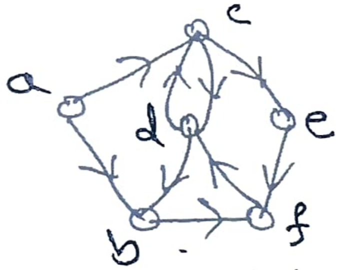
b Let $P(x), q(x)$ and $r(x)$ be the following open statements: $P(x): x^2 - 7x + 10 = 0, q(x): x^2 - 2x - 3 = 0, r(x): x < 0$. Determine the truth or falsity of the following statements where the universe is all integers. If the statement is false provide a counter example or explanation.

i) $\forall x(p(s) \to \sim r(x))$

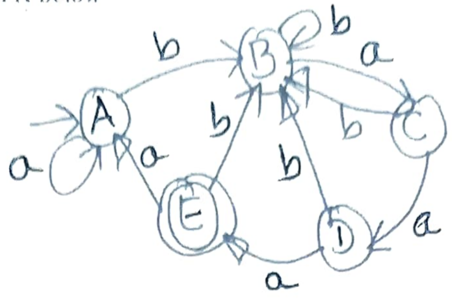
ii) $\exists\, x\,(p(x) \to r(x))$

iii) $\exists\, x(q(x) \to r(x))$

iv) $\forall x\,(q(x) \to r(x))$ 04

	c	Show that the argument $h \to i, (h \wedge i) \to j, \sim k \to (h \vee i), (j \vee \sim j) \to (i \to h) \vdash j \vee k$.	06
		OR	
4	a	Establish the validity of the following argument. $\forall x [p(x) \vee q(x)]$ $\exists x \sim p(x)$ $\forall x [\sim q(x) \vee r(x)]$ $\forall x [s(x) \to \sim r(x)]$ $\therefore \forall x \sim s(x)$	06
	b	Let $p(x), q(x) and\ r(x)$ be the following open statements $p(x): \lvert x \rvert > 3, q(x): x > 3$ and $r(x): x < -3$. Write the converse inverse and contrapositive of the implication $\forall x\ (p(x) \to (r(x) \vee q(x)))$.	04
	c	Express each of the following statements in equivalent predicate logic formula. What is their negation? i) All humming birds are richly colored. ii) No large birds live on honey. iii) Birds that do not live on honey are dull in color iv) Humming birds are small.	06
5	a	Let R be a relation on Z such that aRb iff $ab > 0$ for all $a, b \in Z$. Verify whether R is an equivalence relation.	04
	b	Let R be the relation whose digraph is given below. i) Draw digraph of R^3. ii) Write M_R^6. 	06
	c	Let $f, g, h : z \to z$ defined by $f(x) = x - 1, g(x) = 3x, h(x) = \begin{cases} 0, x\ even \\ 1, x\ odd \end{cases}$. Determine fog , hog, fo(goh) and h^3.	06
		OR	
6	a	Define *POSET*. Draw the Hasse diagram representing the partial ordering $\{(a,b) \mid a\ divides\ b\}$ on the set $\{1,2,3,6,12,24,36,48\}$. Find the lower bounds, upper bounds, GLB, LUB for the sets $\{2,6,12\}$ and $\{12,36,48\}$.	05
	b	Let $f: R \to R$ determine whether f is invertible and if so determine f^{-1} where $f_1 = \{ (x,y) / 2x + 3y = 7 \}$.	05
	c	Let $A = \{1,2,3,4,5,6,7\} and\ B = \{v, w, x, y, z\}$. Determine the number of functions $f: A \to B$ where i) $\lvert f(A) \rvert = 2$ ii) $f(A) = \{w, x, y\}$ iii) $\lvert f(A) \rvert = 4$	06

a Consider the DFA below



i) What is the language of DFA?

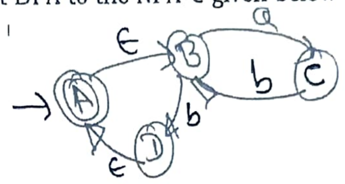
ii) Find $\delta^*(A, ababaaa)$ & $\delta^*(A, ababaaba)$. 04

b Construct NFA which accepts the following language over $\sum = \{0,1\}$.

i) Set of all strings ends with 01 *or* 10

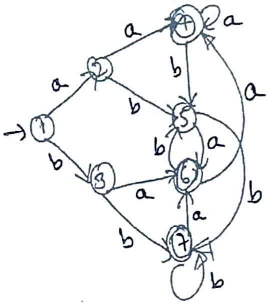
ii) Set of all strings consists of 101 as a substring. 04

c Find the equivalent DFA to the NFA-Є given below



04

d If possible to minimize the states , find the minimum state DFA for the given DFA.



04

8 a Define the binary operation $*$ *on* Z by $x * y = x + y + 1$. Verify that $(z,*)$ is an abelian group. 04

b Find all subgroups of $(z_{18}, +)$. 04

c Prove that if f is an isomorphism from $G_1 to\ G_2$, then f^{-1} is an isomorphism from $G_2 to\ G_1$. 04

d The encoding function $E: Z_2^2 \rightarrow z_2^5$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$. Determine all the code words. Find the associated parity check matrix H. Use H to decode the received words $00111, 00110$. 04