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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU)

III Semester B. E. Examinations March-2021

Common to CS / IS

DISCRETE MATHEMATICAL STRUCTURES

Maximum Marks: 100

Time: 03 Hours

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

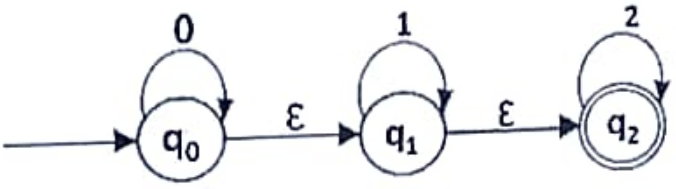
PART-A

1	1.1	Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?	02
	1.2	Determine the coefficient of $x^9\,y^3$ in the expansion of $(2x-3y)^{12}$.	02
	1.3	State inverse and contrapositive of the conditional statement: "If the quadrilateral is a parallelogram, then its diagonal bisects each other".	02
	1.4	Let $A=\{a,b,c\}$ and $B=\{0,1\}$, and $R=\{(a,0),(b,0),(c,1)\}$ be the relation from A to B. Write down the matrix of this relation.	02
	1.5	Evaluate $S(8,7)$, given that $S(7,6)=21$.	02
	1.6	Show that $(Z,X)$is not a group.	02
	1.7	Obtain a DFA to accept string of a's & b's having exactly two a's.	02
	1.8	Check the validity of the following statement: If Sachin hits a century, he gets a free car. Sachin does not get a free car. --- ∴Sachin has not hit a century.	02
	1.9	Define the extended transition function for DFA.	02
	1.10	A binary symmetric channel has probability p=0.05 of incorrect transmission. If the word $c=011011101$ is transmitted, what is the probability that single error occurs.	02

PART-B

2	a	A computer science professor has seven different programming books on a bookshelf. Three of the books deal with C++, the other four with Java. In how many ways can the professor arrange these books on the shelf i) If there are no restrictions? ii) If the languages should alternate? iii) If all the C++ books must be next to each other? iv) If all the C++ books must be next to each other and all the Java books must be next to each other?	06

	b	By Mathematical Induction, prove that 11n-4n is divisible by 7, for $n \geq 1$.	
	c	If $a_0 = 0, a_1 = 1$ and $a_2 = 4, a_3 = 37$ satisfy the recurrence relation $a_{n+2} + ba_{n+1} + ca_n = 0$ for $n \geq 0$. Determine the constants b and c and then solve the relation for a_n.	
3	a	Write down the following proposition in symbolic form and find its negation: "If all triangles are right – angled, then no triangle is equiangular."	06
	b	Let p(x) be the open statement "$x^2 = 2x$" and q(x) be the open statement "$x^3 = 4x$" with the set of all integers as the universe. Write down the truth values of the following quantified statements: i) $\forall x, p(x) \wedge q(x)$ ii) $\exists x, p(x) \wedge q(x)$ iii) $\forall x, p(x) \vee q(x)$	04
	c	Prove the following logical equivalences: $\exists x, [p(x) \wedge q(x)] \Rightarrow \exists x, p(x) \wedge \exists x, q(x)$ Is the converse true?	06 06
		OR	
4	a	Prove the validity of the following argument $p \rightarrow q, \neg r \vee s, p \vee r \therefore \rightarrow q \rightarrow \neg s$	
	b	Establish the validity of the following argument No engineering student of first or second semester studies logic. Anil is an engineering student who studies Logic. Therefore Anil is not in second semester.	06
	c	Write down the following proposition in symbolic form: i) An equilateral triangle has three angles of 60 degree, and conversely. ii) Every rational number is a real number and not every real number is a rational number.	06 04
5	a	Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be any two functions. Then the following are true: i) If f and g are one-to-one, so is gof. ii) If gof is one-to-one, then f is one-to-one.	
	b	Consider the function f and g defined by $f(x) = x^3$ and $g(x) = x^2 + 1$, $\forall x \in R$. Find gof, fog, f^2 and g^2.	06
	c	On the set of all integers, Z, the relation R is defined by $(a, b) \in R$ if and only if $a^2 - b^2$ is an even integer. Show that R is an equivalence relation.	04 06
		OR	
6	a	Let $f: R \rightarrow R$ be defined by $f(x) = \begin{cases} 3x - 5 \text{ for } x > 0 \\ -3x + 1 \text{ for } x \leq 0 \end{cases}$ Determine: i) $f(-5/3)$ and $f(5/3)$ ii) $f^{-1}(-3)$ and $f^{-1}(-6)$ iii) What are $f^{-1}([-5,5])$ and $f^{-1}([-6,5])$?	06

		Let A = {1,2,3,4} and let R be the relation on A defined by x R y if and only if y = 2x. i) Write down R as a set of ordered pairs. ii) Draw the digraph of R. iii) Determine the in-degree and out-degree of the vertices in the digraph.	06
	c	Prove that the set of all positive integers is not totally ordered by the relation of divisibility.	04
7	a	Convert the following ε –NFA to DFA. By first converting it into its equivalent NFA.  	08
	b	Draw a DFA to accept i) Even no. of a's and b's ii) Even no. of a's and odd no of b's iii) Odd no. of a's and even no of b's iv) Odd no. of a's and odd no of b's	08
8	a	The encoding function $E: Z_2^3 \rightarrow Z_2^6$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ i) Determine the code word assigned to 110 and 010. ii) Find the associated parity-check matrix. iii) Use H to decode the received words: 110110, 111101. iv) Show that decoding of 111111 is not possible by using H.	08
	b	State and prove Lagrange's theorem.	04
	c	Let G be a group and H be a subgroup of G. For $a \varepsilon G$, Prove that aH=H if and only if $a \varepsilon H$.	04