21CS36

USN

RV COLLEGE OF ENGINEERING

(An Autonomous Institution Affiliated to VTU)

III Semester B. E. Examinations April/May-2023

Common to CS / IS /AIML

DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.

PART-A

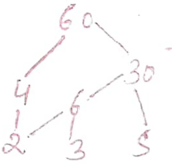
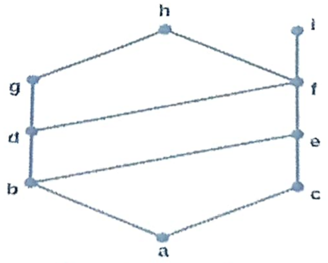
1	1.1	Find the number of ways of travelling from $(2,1)$ to $(5,6)$ considering moves as one unit UP Move or $RIGHT$ move.	02
	1.2	Write the recurrence relation for the integer sequence $3, 15, 75, \ldots$ and solve.	02
	1.3	Negate and simplify the compound statement $(p \wedge q) \rightarrow r$.	02
	1.4	Show that $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$ by writing truth table is tautology.	02
	1.5	Let R be a relation on set A with 3 elements. Find the number of reflexive and antisymmetric relations on set A.	02
	1.6	Let $A = \{1,2,3,4,5,6,7,8\}$ R is an equivalence relation on A which includes the partition $\{1,4,8\} \cup \{3\} \cup \{5,6\} \cup \{2,7\}$. Determine R.	02
	1.7	Write any two applications of Finite Automata.	02
	1.8	Write the DFA to accept all strings of $0's$ and $1's$ which contain at least two $0's$.	02
	1.9	Define homomorphism. Give an example.	02
	1.10	Find the probability of transmitting $c = 10110$ and receiving with 2 bit error with $p = 0.05$ as the probability of incorrect transmission.	02

PART-B

$^{24}P_5 \times 20! \times 2$

2	a	In how many ways can the letters of the English alphabet be arranged so that there are exactly 5 letters between the letters a and b?	04
	b	A student is to answer 7 out of 10 questions. In how many ways can he/she make his selection if i) there are no restrictions? $^{10}C_7$ ii) must answer the first two questions? $^8C_5$ iii) must answer at least four of the first 6 questions? ^{10}C	04
	c	Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 32$, where i) $x_1 \geq 0, 1 \leq i \leq 4$. $^{35}C_{32}$ ii) $x_i > 0, 1 \leq i \leq 4$ $^{31}C_{28}$ iii) $x_1, x_2 \geq 5, x_3, x_4 \geq 7$. $^{11}C_8$ iv) $x_i > -2, 1 \leq i \leq 4$. $^{43}C_{46}$	04
	d	A bank pays 6% (annual) interest in savings, compounding the interest monthly. If Boonie deposits \$1000 on the first day of May, how much will this deposit be worth a year later? Write the recurrence relation.	04

$P_n = 1.005 P_n \Rightarrow P_n = P_0(1.005)^n$ = \$1061.65

3	a	Simplify using Laws of Logic: $(p \vee q) \wedge \neg(\neg p \wedge q)$.	
	b	Write the symbolic representation of the statement "If Joan goes to Lake George, then Mary will pay for Joan's shopping spree". Also write its negation in words as well as symbolic form. p $p \to q$ negation: $\neg(p \to q)$ $\equiv p \wedge \neg q$ p: Joan goes to Lake George q: Mary will pay for Joan's shopping spree	04
	c	Prove the validity of the following argument: $\neg r(c)$ $\forall t[p(t) \to q(t)]$ $\forall t[q(t) \to r(t)]$ $\therefore \neg p(c)$	04
		OR	08
4	a	Write inverse, converse and contrapositive of the statement: *If you brush with brite, then your teeth will be pearly white.*	
	b	Show that argument is invalid $[(p \wedge \neg q) \wedge (p \to (q \to r))] \to \neg r$.	04
	c	Find the negation of the following: $\forall x \exists y\ [(p(x,y) \wedge q(x,y) \to r(x,y)]$ $\exists x\ \forall y\ [p(x,y) \wedge q(x,y) \wedge \neg r(x,y)]$	04
	d	Write the symbolic representation of the statement if x is odd, then x^2 -1 is even. Also write its negation in words as well as symbolic form.	04
			04
5	a	Let $f: R \to R$ and $g: R \to R$, given by $f(x) = x^2$ and $g(x) = x + 5$ find $g \circ f$ and $f \circ g$. $gof(x) = x^2+5$, $fog(x) = (x+5)^2$	04
	b	Given that the set $A = \{a, b, c\}$ with the relations $R = \{(a,a), (a,c), (b,a), (c,b)\}$ and $S = \{(a,b), (b,c), (c,c)\}$. Find the i) Converse of R $\{(a,a), (c,a), (a,b), (b,c)\}$ ii) Complement of S $\{(a,a), (a,c), (b,a), (b,b), (c,a), ...\}$ iii) The composition $R \circ S$ $\{(a,b), (a,c), (b,b), ...\}$ iv) Show that $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$	05
	c	i) Draw a Hasse diagram for set A with divisibility relation, where $A = \{2, 3, 4, 5, 6, 30, 60\}$. ii) Find the greatest and least elements. 60, ϕ iii) Find LB, UB, LUB and GLB for $\{6, 30\}$. $\{2,3,6\}$, $\{30,60\}$, $\{30\}$, $\{6\}$  2M 4M	07
		OR	
6	a	$\{a\}$ $\{h,i\}$ $\{a\}$ Find minimal, maximal and least elements for the following Hasse diagram in Fig.6a.  Fig.6a $\{e,f,h,i\}$ $\{a\}$ $\{e\}$ $\{a\}$ Also, find the upper bound, lower bound, LUB and GLB for the set $\{b, c\}$.	08
	b	Let the function $f: R \to R$ be defined by $f(x,y) = \{(x,y) \mid y = mx + b$, where $m, b \in R$. Then find f^{-1}. $(x-b)/m$	04
	c	Prove that the composition of binary relations is associative.	04
7	a	Define DFA and design DFA for $L = \{w \mid \Sigma = \{a,b\}, N_a(w)$ *is odd and* $N_b(w)$ *is divisible by* $3\}$.	06

	b	Convert the following NFA shown in Fig. 7b to DFA. Represent equivalent DFA using the Transition table also.  Fig. 7b	06
	c	List any four differences DFA and NFA.	04

OR

8	a	Define language accepted by epsilon NFA and construct an epsilon NFA to accept any number of zeros followed by any number of ones followed by any number of twos. Also construct an equivalent NFA without epsilon.	10
	b	Construct a DFA which accepts all ternary strings divisible by 4.	06
9	a	With $m = 3, W \in Z_2^3: E: Z_2^3 \rightarrow Z_2^9$ and decoding function $D: Z_2^9 \rightarrow Z_2^3$ of triple repetition code, find i) $E(10110111)$ = 101101111101101111101101111 ii) $D(111101100)$ = 101 iii) three different received words r for which $D(r) = 001$ 001 001 001 iv) For each $W \in Z_2^3$, what is $\lvert D^{-1}(W) \rvert$? 2 6	06
	b	Show that a group of complex numbers is an abelian group under multiplication. Is it a cyclic group? Table + abelian + generator {i}	06
	c	State and prove Lagrage's theorem.	04

OR

000000, 100110, 010011, 001101, 110101, 011110, 101000, 111000

10	a	With $E: Z_2^3 \rightarrow Z_2^6$ and $G = \begin{matrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{matrix}$ $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ Find i) All possible code words ii) Error detection and correction capability 2, 1 iii) H matrix iv) Decoded word for received words $r = 110110$ and $r = 000111$. 100, not possible	10
	b	Show $(Z_5, +)$ is a cyclic group.	06

4+2M generators = {1, 2, 3, 4}

p(x): x is odd, q(x): x^2-1 is even. $\boxed{\forall x[p(x) \rightarrow q(x)]}$

Negation: $\neg[\forall x[p(x) \rightarrow q(x)]] \Leftrightarrow \exists x, [\neg(p(x) \rightarrow q(x))]$

$\Leftrightarrow \boxed{\exists x [p(x) \land \neg q(x)]}$

There exist an integer such that x is odd & x^2-1 is odd.

Converse: If you do not brush with white, then your teeth will be pearly white.

If your teeth is pearly white then you brush with white.

If your teeth are not pearly white then you do not brush with white.