18CS36

USN

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU)

III Semester B. E. Examinations April - 2022

Common for CSE/ISE

DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours | *Maximum Marks: 100*

Instructions to candidates:

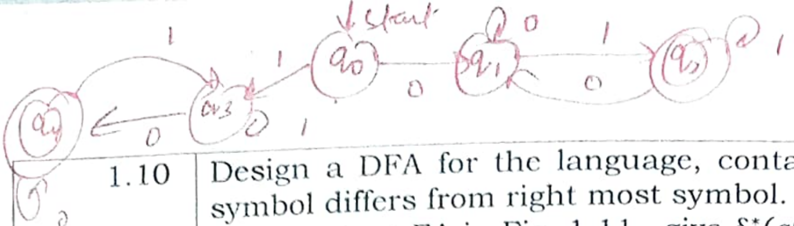
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

PART-A

1	1.1	Find the number of 5-letter words which contain 3 different consonants and two different vowels. $^{21}C_3 \times {}^5C_2 \times 5! = 1596000$	02
	1.2	How many different outcomes are possible by tossing 10 similar coins? $^{11}C_{10} = 11!/10! = 11$	02
	1.3	Find the recurrence relation and the initial condition for the given sequence $a_n = -3a_{n-1}, n \geq 1, a_0 = 6$ $6, -18, 54, -162, \ldots$	02
	1.4	Write down the following proposition in symbolic form, and find its negation. $\{\forall x \in T, p(x)\} \rightarrow \{\forall x \in T, \neg q(x)\}$ "If all triangles are right-angled, then no triangle is equiangular."	02
	1.5	From the information given, determine the truth value required. $p \wedge q$ *is false and* p *is true*; 0 Find the truth value of q.	02
	1.6	Construct the truth table for the compound proposition, $\neg q \wedge ((7r) \rightarrow p)$ 0 1 0 0 1 1 0 0 0 0 0 1 0 0 1 1	01
	1.7	Find the inverse of the function f: A→B $A = \{x \mid x \text{ is real and } x \geq -1\}$ $B = \{y \mid y \text{ is real and } y \geq 0\}, f(a) = a^2 - 1.$	01
	1.8	For the poset (A, R) defined on the set $A = \{2, 3, 5, 6, 7, 11, 12, 35, 385\}$ as represented by the following Hasse diagram in Fig. 1.8 $LUB\{3, 6\}$, $GLB\ \{35, 385\}$ 6 35  Fig. 1.8	02
	1.9	A binary symmetric channel has probability $p = 0.05$ of incorrect transmission. If the word $C = 011011101$ is transmitted, what is the probability that three errors occur, no two of them consecutive?	02

$10 \times (0.05)^3 \times (0.95)^2 + 4\,(0.05)^3 \times (0.95)$

$^5C_3\, p^3 (1-p)^2 + {}^4C_3\, p^3 (1-p)^1$



	1.10	Design a DFA for the language, contains strings in which left most symbol differs from right most symbol. Σ is given $\{0,1\}$.	02
	1.11	For the given FA in Fig. 1.11, give $\delta^*(q0, 10100)$.  Fig. 1.11	02

$\sum_{i=1}^{k+1} H_i = \sum_{i=1}^{k} H_i + H_{k+1}$ **PART-B** $(k+2)H_{k+1} - (k+1)$

2	a	Let $H_1 = 1,\ H_2 = 1 + \frac{1}{2}, H_3 = 1 + \frac{1}{2}, + \frac{1}{3}, \dots, H_n = 1 + \frac{1}{2}, + \frac{1}{3}, \dots, \frac{1}{n}$. Prove that $\sum_{i=1}^{n} H_i = (n+1)H_n - n$ for all positive integers $n \geq 1$. $n \geq 1$	05
	b	Find and solve a recurrence relation for the number of binary sequences of length $n \geq 1$ that has no consecutive 0's.	06
	c	In how many ways can Antony place 24 different books on 4 shelves so that there is at least one book on each shelf? (For any of these arrangements consider the book on each shelf to be placed one next to the other, with the first book at the left of the shelf). $\left({}^{23}C_{20}\right) \cdot 24!$	05
3	a	Test the validity of the given argument. $p \to q,\ q \to s,\ r \to \neg s, \neg p \underline{\vee} r \vdash \neg p$	05
	b	Write down the converse, inverse and contrapositive of each of the following statements for which the set of all real numbers is the universe. i. $\forall x. [(x > 3) \to (x^2 > 9)]$. ii. $\forall x, [\{(x^2 + 4x - 21) > 0\} \to \{(x > 3) \vee (x < -7)\}]$ Also, indicate their truth values. Converse $\forall x [q \to p]$ F; Inverse $x \leq 3 \to x^2 \leq 9$ F; Contr $x^2 \leq 9 \to x \leq 3$ T; T, T, T	06
	c	Prove the given logical equivalence without using truth table. $[\neg p \wedge (\neg q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$	05
		OR	
4	a	Prove the given logical equivalence without using truth table. $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$	05
	b	Establish the validity of the following argument $\forall x, [p(x) \vee q(x)]$ $\forall x, [\{\neg p(x) \wedge q(x)\} \to r(x)]$ ―――――――――― $\therefore \forall x, [\neg r(x) \to p(r)]$	06
	c	State the converse, inverse and contrapositive of the following conditionals: i. If a quadrilateral is a parallelogram, then its diagonals bisect each other. ii. If a real number x^2 is greater than zero, then x is not equal to zero.	05

$\frac{20!\,3!}{} = 23 \times 11 \times \dots = 1771$

2.b) $a_n = a_{n-1} + a_{n-2},\ n \geq 3$ — 2M

$a_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$ — 2M

$A = \sqrt{5}+3/2\sqrt{5}$, $B = (\sqrt{5}-3)/2\sqrt{5}$ — 1M

$a_n =$ 1M

$p \to q$

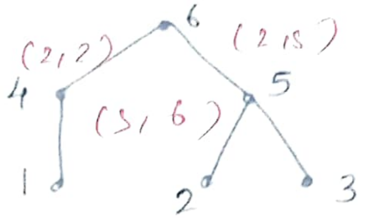
Converse: $q \to p$

Inverse: $\neg p \to \neg q$

contrap: $\neg q \to \neg p$

5	a	Let $F: A \to B$ be a function and C and D be arbitrary nonempty subsets of B. Then prove the following: i. $f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D)$ ii. $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$	06
	b	Let $A = \{1,2,3,4,5,6,7\}$ and R be the equivalence relation on A that induces the partition $A = \{1,2\} \cup \{3\} \cup \{4,5,7\} \cup \{6\}$. Find R. R = {(1,1), (1,2) (2,1) (2,2), (3,3) (4,4) (4,5), (4,7), (5,4) (5,5) (5,7) (7,4) (7,5) (7,7) (6,6)}	05
	c	The Hasse diagram of a partial order R on the set $A = \{1,2,3,4,5,6\}$ is as given in Fig. 5c. Write down R as a subset of $A \times A$. Construct its digraph.	

R = {(1,1) (1,4) (1,6) (2,2) (2,5) (2,6) (3,3) (3,5) (3,6) (4,4) (4,6) (5,5) (5,6) (6,6)} — 2.5 M



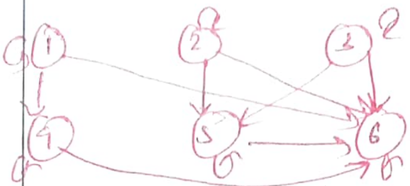


Fig. 5c

05

OR

6	a	For the Poset (A, R) represented by the following Hasse diagram in Fig. 6a, find i. $GLB\ \{B, C\}$ — a ii. $GLB\ (B, W\}$ — a iii. $GLB\ \{e, x\}$ — c iv. $LUB\ \{c, b\}$ — e v. $LUB\ \{d, x\}$ — z	05

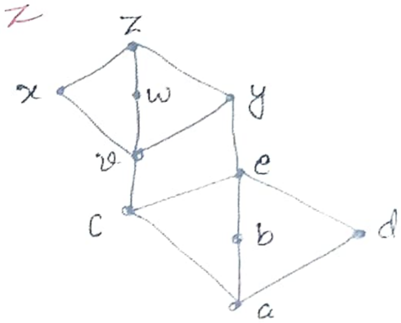
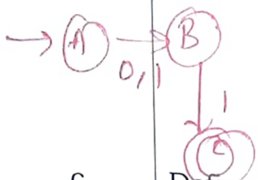
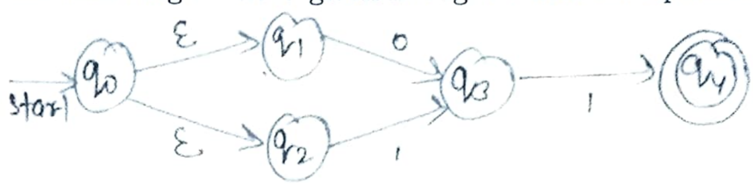


Fig. 6a

	b	On the set Z of all integers, *a relation* R is defined by aRb if and only $a^2 = b^2$. Verify that R is an equivalence relation. Determine the partition induced by this relation. R, S, T } 3 M [a] = {x ∈ Z | x = ±a} [0] = {0} [n] = {n, −n} = [−n] P = {[0], [n]}	05
	c	Let $f: R \to R$ be defined by $f(x) = \begin{cases} x + 7 \ for\ x \le 0 \\ -2x + 5\ for\ 0 \le x < 3 \\ x - 1\ for\ x \ge 3 \end{cases}$ Find $f^{-1}(-10), f^{-1}(0), f^{-1}(4), f^{-1}(6)$. Also determine $f^{-1}([-5,-1]), f^{-1}[-5,0])$. {−17}, {−7, 5/2}, {3, 1/2, 5}, {−1, 7} [−12, −8] [−12, −7] ∪ [5/2, 3]	06
7	a	Design a DFA for the given language $L = \{W: n_a(w) \ge 1, n_b(w) = 2, w(-\{a,b\}^*\}$. Also find $\delta^*(q0, aababab)$.	07

	b	Convert the following $\varepsilon - NFA$ given in Fig. 7b into its equivalent DFA.   Fig. 7b	05
	c	Define language of NFA and extended transition function of t-NFA.	04
8	a	Prove that $(Z_4, +)$ is cyclic group. Find all its generators.	04
	b	Let E: $Z_2^3 \rightarrow Z_2^9$ be the encoding function for the (9, 3) triple repetition code. i. If D: $Z_2^9 \rightarrow Z_2^3$ is the corresponding decoding function. Apply D to decode the following received words: 111101100, 000100011, 010011111, 001110011. ii. Find the three different received words r for which $D(r) =$ 000.	06
	c	For a group G, prove that the function $f: G \rightarrow G$ defined by $f(a) = a^{-1}$ is an isomorphism if and only if G is abelian.	06

i) D (111101100) = 101
000
011
011
3M

ii) 000000000, 100000000, 000000001