12CS36 /12IS36

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R. V. COLLEGE OF ENGINEERING

Autonomous Institution affiliated to VTU

III Semester B. E. Examinations Nov/Dec-14

Common to CSE / ISE

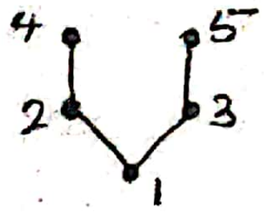
DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B.

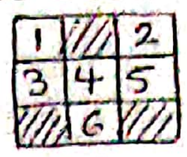
PART-A

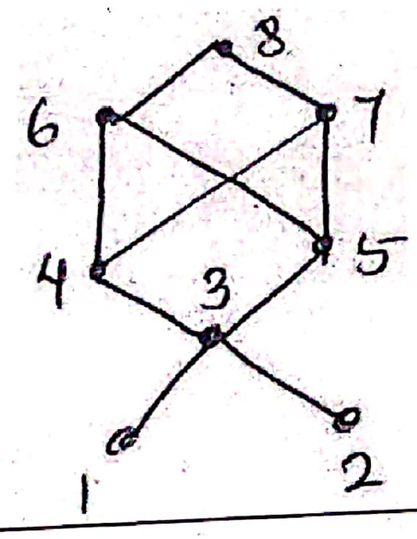
1	1.1	The number of permutations of the letters in the word *COMPUTER* is 8!. What is the number of permutations if only 5 letters are used?	01
	1.2	What is the coefficient of a^5b^2 in the expansion of $(2a-3b)^7$.	02
	1.3	Define power set. If $A = \{0, \phi, \{\phi\}\}$, find $P(A)$.	02
	1.4	Write the duality of $P = \{(A \cup B) \cap \phi\} \cup \{(C \cap D) \cup U\}$.	01
	1.5	Obtain recursive definition of $a_n = n(n+2), n \geq 1$.	02
	1.6	The possible truth values of p, q and r in $p \rightarrow (q \vee r)$ are $p(1,1,1)$, $q = (1,0,0), r = (1,1,0)$. Check and say Yes/No.	02
	1.7	State the law for the negation of a conditional.	01
	1.8	Let $A = \{1,2,3,4\}, R = \{(1,2),(2,3),(3,4),(4,2),(3,3)\}$ is a relation on A. Find indegree and outdegree of all vertices.	01
	1.9	Write the matrix of partial order whose Hasse diagram is given below: 	02
	1.10	If '·' is the operation of Z, defined by $x \cdot y = x + y + 1$ and if Z is an abelian group, find identity element and inverse element.	02
	1.11	Find $S(5,3)$.	02
	1.12	The word $c = 1010110$ is transmitted through a binary symmetric channel. If $e = 0101101$ is the error pattern, find the word 'r' received.	01
	1.13	Find the weights of the following words: 11100, 01101.	01

PART-B

2	a	Find how many possible arrangements are there, for the letters in *MASSASAUGA*. Also find the number of possible arrangements in which all four, *As* are together.	05
	b	In a class of 52 students, 30 are studying C++, 28 are studying Pascal and 13 are studying both. How many in this class are studying at least one of these languages? How many are studying neither of these languages?	04
	c	How many positive integers n can be formed using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000.	07

OR

3	a	Find the Rook polynomial for the board shown below: 	06
	b	Obtain the formula for number of derangements of n objects by using Rook polynomials.	04
	c	Determine the number of positive integers $n, 1 \leq n \leq 2000$, that are not divisible by 2, 3, 5 or 7.	06
4	a	Let $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$. Prove that $a_n \leq 3^n$ for all +ve integers n.	05
	b	The Lucas numbers are defined recursively by $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$. Evaluate L_2 to L_{10}.	03
	c	Solve the following: $a_{n+2} + 3a_{n+1} + 2a_n = 3^n, n \geq 0$, $a_0 = 0$, $a_1 = 1$ using generating function method.	08
		OR	
5	a	In how many ways can a police captain distribute 24 rifle shells to four police officers so that each officer gets at least three shells, but not more than eight?	06
	b	Define Ackermann's numbers and evaluate $A_{1,3}$ and $A_{2,3}$.	05
	c	Solve: $a_{n+2} - 10a_{n+1} + 21a_n = 3n - 2, n \geq 0$.	05
6	a	Prove that for any three prepositions $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology.	05
	b	Find the negation of each of the following quantified statements: i) $\forall x, \forall y, [(x > y) \to ((x - y) > 0)]$ ii) $\forall x, \forall y, [(x < y) \to \exists z, (x < z < y)]$ iii) $\forall x, \forall y, [(\lvert x \rvert = \lvert y \rvert) \to (y = \pm x)]$.	06
	c	Prove the following identities: i) $p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$ ii) $[\neg p \wedge (\neg q \wedge r)] \vee (q \wedge r) \vee (q \wedge r) <=> r$	05
		OR	
7	a	Draw and simplify the switching network for compound preposition: $(q \vee p \vee \neg q) \wedge r$ and show that the given network has four switches and equivalent to a network which contains only one switch.	05
	b	Find whether the following argument is valid: i) If a triangle has two equal sides, then it is isosceles ii) If a triangle is isosceles, then it has two equal angles iii) The triangle ABC does not have two equal angles. $\therefore ABC$ does not have two equal sides.	06
	c	Consider the open statements $p(x), q(x), s(x)$ and $t(x)$, where $p(x): x > 0, q(x) = x$ is even, $s(x) = x$ is divisible by 3, $t(x) = x$ is divisible by 7. Express the following statements in words and find its truth value: i) $\forall x, [r(x) \to p(x)]$ ii) $\exists x, [s(x) \wedge \neg q(x)]$ iii) $\forall x, [\neg r(x)]$ iv) $\forall x, [r(x) \vee t(x)]$	05

8	a	For $A = \{1,2,3,4,5\}$ and $B = \{w,x,y,z\}$, a function $f: A \to B$ is defined by $f = \{(1,w),(2,x),(3,x),(4,y),(5,y)\}$. Find images of the following subsets of A under f. $A_1 = \{1\}$, $A_2 = \{1,2\}$, $A_{13} = \{1,2,3\}$, $A_4 = \{2,3\}$, $A_5 = \{2,3,4,5\}$.	05
	b	Let $f: R \to R$ be defined by: $f(x) = \begin{cases} 3x-5 & for \quad x > 0 \\ -3x+1 & for \quad x \leq 0 \end{cases}$ Find $f(0), f(-1), f(5/3), f^{-1}(-3), f^{-1}(-6)$.	05
	c	Let f, g, h be functions from Z to Z defined by $f(x) = x-1$, $g(x) = 3x$, $h(x) = \begin{cases} 0 & if \quad x\ is\ even \\ 1 & if \quad x\ is\ odd \end{cases}$ Verify that $f \circ (g \circ h) = (f \circ g) \circ h$.	06
		OR	
9	a	Let $A = \{a,b,c,d,e\}, R = \{(a,a),(a,b),(b,c),(c,d),(c,e),(d,e)\}$. Find R^∞. Draw digraph of R^∞, write M_{R^∞}, check whether R is strongly connected or not.	06
	b	Let $A = \{1,2,3,4\}$. Determine the nature of the following relation on : $R = \{(1,2),(1,3),(3,1),(1,1),(3,3),(3,2),(1,4),(4,2),(3,4)\}$. Also check whether R is an equivalence relation or not. Also check whether R is a partial ordered relation or not.	05
	c	Find all the extremal elements in Poset, whose Hasse diagram is shown below. Check whether for $B_1 = \{1,2\}$, where $B_1 \subseteq A$, given $A = \{1,2,3,4,5,6,7,8\}$, the Hasse diagram is a lattice or not. 	05
10	a	For a group G, prove that the function $f = G \to G$ defined by $f(a) = a^{-1}$ is an isomorphism iff a is abelian.	06
	b	If every element of a group G is its own inverse, then show that a is an abelian group.	05
	c	Show that the group $(G,*)$, whose multiplication table is given below, is cyclic. Also find all generators.	05

$*$	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	c	d	e	f	a
c	c	d	e	f	a	b
d	d	e	f	a	b	c
e	e	f	a	b	c	d
f	f	a	b	c	d	e

OR

11	a	The word $c = 1010110$ is sent through a binary symmetric channel. If $p = 0.02$ is the probability of incorrect receipt of a signal, find the probability that c is received as $r = 1011111$. Find the error pattern also.	05
	b	Explain encoding and decoding of a message.	06
	c	Let $E: Z_2^3 \rightarrow Z_2^9$ be the encoding function for the (9,3) triple repetition code. If $D: Z_2^9 \rightarrow Z_2^3$ is the corresponding decoding function, apply D to decode the following received words. 111101100, 000100011, 010011111, 001110011.	05