(1) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$

Soln:. $D^2y - 3Dy + 2y = e^{5x}$ $\qquad D = \frac{d}{dy}$

$(D^2 - 3D + 2)y = e^{5x}$

Auxiliary eqn is $m^2 - 3m + 2 = 0$

$m = 1, 2$

Roots are real and distinct

$\therefore C.F = c_1e^x + c_2e^{2x}$

$P.I = \frac{1}{D^2-3D+2} e^{5x}$ $\qquad$ [Case 1:. e^{ax} Replace $D \to a$]

Here Replace D by 5

$= \frac{1}{5^2 - 3(5) + 2} e^{5x}$

$= \frac{e^{5x}}{12}$

$y = C.F + P.I$

$= c_1e^x + c_2e^{2x} + \frac{e^{5x}}{12}$

(2) Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x$

Soln:. $(D^3 - 3D^2 + 4D - 2)y = e^x$

A.Eqn $m^3 - 3m^2 + 4m - 2 = 0$

Roots are $m = 1, \; 1 \pm i$

$\therefore C.F = c_1e^x + e^x(c_2\cos x + c_3\sin x)$

$$P.I = \frac{1}{D^3-3D^2+4D-2} e^x$$

Replace $D \to 1$

we get zero in the denominator, when we substitute D as 1.

(Differentiate the denominator ~~w.r.to~~ D)

$$\therefore \quad P.I = x \cdot \frac{1}{3D^2-6D+4} e^x$$

Replace $D \to 1$

$$= x e^x$$

$$\therefore \quad y = C.F + P.I$$

③ $$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = e^x$$

Soln:- $(D^3 - D^2 - D + 1)y = e^x$

A.Eqn $\quad m^3 - m^2 - m + 1 = 0$

$$m^2(m-1) - 1(m-1) = 0$$

$$(m^2-1)(m-1) = 0$$

$\therefore \quad m = -1, 1, 1$ (Roots are real and 1 is repeated)

$$\therefore \quad C.F = c_1 e^{-x} + (c_2 + c_3 x)e^x$$

$$P.I = \frac{1}{D^3 - D^2 - D + 1} e^x$$

$D \to 1$

We get Zero in the Denominator

(Differentiate the denominator)

$$\therefore \quad P.I = x \cdot \frac{1}{3D^2-2D-1} e^x$$

$D \to 1$

Again we get Zero in the Denominator

(Differentiating the Denominator again)

$$\therefore \quad P.I = x^2 \frac{1}{6D-2} e^x$$

$D \to 1$

$$= \frac{x^2 e^x}{4}.$$

$$\therefore \quad y = C.F + P.I$$

④ $y'' - 4y = \cosh(2x-1) + 3^x$

use Definition of $\cosh x$

Soln: $(D^2-4)y = \cosh(2x-1) + 3^x$

$$\Rightarrow (D^2-4)y = \frac{e^{2x-1} + e^{-(2x-1)}}{2} + e^{(\log 3)x}$$

A.Eqn $m^2 - 4 = 0$

$m = \pm 2$

$$C.F = c_1 e^{2x} + c_2 e^{-2x}$$

$$P.I = \frac{1}{D^2-4}\left[\frac{e^{2x-1} + e^{-2x+1}}{2}\right] + \frac{1}{D^2-4} e^{(\log 3)x}$$

$$= \frac{1}{2}\left[\frac{1}{D^2-4} e^{2x-1} + \frac{1}{D^2-4} e^{-2x+1}\right] + \frac{1}{D^2-4} e^{(\log 3)x}$$

$D \to 2$ $\quad$ $D \to -2$ $\quad$ $D \to \log 3$

Denominator becomes zero $\quad$ Denominator becomes zero

$$= \frac{1}{2}\left[x \cdot \frac{1}{2D} e^{2x-1} + x \frac{1}{2D} e^{-2x+1}\right] + \frac{1}{(\log 3)^2 - 4} 3^x$$

$D \to 2$ $\quad$ $D \to -2$

$$= \frac{1}{2}\left[x \frac{1}{2(2)} e^{2x-1} + x \frac{1}{2(-2)} e^{-2x+1}\right] + \frac{1}{(\log 3)^2 - 4} 3^x$$

$$= \frac{1}{2}\left[\frac{x e^{2x-1}}{4} - \frac{x e^{-2x+1}}{4}\right] + \frac{1}{(\log 3)^2 - 4} 3^x$$

$$= \frac{x}{4}\left[\frac{e^{2x-1} - e^{-(2x-1)}}{2}\right] + \frac{1}{(\log 3)^2 - 4} 3^x$$

$$= \frac{x}{4}\sinh(2x-1) + \frac{1}{(\log 3)^2 - 4} 3^x$$

$y = C.F + P.I$ //

(5) Solve $(D^2+5D+6)y = \sin x$

Soln:- A.Eqn $m^2+5m+6=0$

$m=-2, -3$

$\therefore C.F = c_1 e^{-2x} + c_2 e^{-3x}$

$P.I = \frac{1}{D^2+5D+6} \sin x$ (Case (ii) Replace $D^2 \to -a^2$

$D^2 \to -1$

$= \frac{1}{-1+5D+6} \sin x$

$= \frac{1}{5D+5} \sin x$

[Read the Note given for case ii]

$= \frac{1}{5} \frac{1}{(D+1)} \times \frac{(D-1)}{(D-1)} \sin x$

$\alpha D+\beta$ form

$= \frac{1}{5} \times \frac{1}{D^2-1} (D-1) \sin x$

$D^2 \to -1$

$= \frac{1}{5} \times \frac{1}{(-1-1)} [D(\sin x) - \sin x]$

$D \to \frac{d}{dx}$

$= -\frac{1}{10} [\cos x - \sin x]$

$y = C.F + P.I$

(6) $(D^2+a^2)y = \cos ax$

Soln: $m^2+a^2=0$

$m = \pm ai$

$C.F = e^{0x}[c_1 \cos ax + c_2 \sin ax]$

$P.I = \frac{1}{D^2+a^2} \cos ax$

$D^2 \to -a^2$

we get zero in the Denominator, when we replace D^2 by $-a^2$.

$\therefore \quad P.I = x \frac{1}{2D} \cos ax$

$\frac{1}{D} \rightarrow$ integration

$= \frac{x}{2} \int \cos ax \, dx$

$= \frac{x}{2} \frac{\sin ax}{a}$

$\therefore y = C.F + P.I.$

(7) Solve $\frac{d^2y}{dx^2} + 4y = \sin^2 x$

Soln :- $(D^2+4)y = \frac{1-\cos 2x}{2}$

A. Eqn $\quad m^2+4=0$

$m = \pm 2i$

$\therefore C.F = e^{0x}(c_1 \cos 2x + c_2 \sin 2x)$

$P.I = \frac{1}{D^2+4}\left[\frac{1-\cos 2x}{2}\right]$

$= \frac{1}{2}\left[\frac{1}{D^2+4} e^{0x} - \frac{1}{D^2+4}\cos 2x\right]$

$D \rightarrow 0 \qquad D^2 \rightarrow -2^2$

Denominator becomes zero

$= \frac{1}{2}\left[\frac{1}{4} - x \frac{1}{2D}\cos 2x\right]$

$= \frac{1}{2}\left[\frac{1}{4} - \frac{x}{2}\int \cos 2x \, dx\right]$

$= \frac{1}{8} - \frac{x}{8}\sin 2x$

$y = C.F + P.I$

(8) $(D^2+1)y = \sin x \sin 2x$

Soln: A Eqn $m^2+1=0$

$m = \pm i$

$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$C.F = c_1 \cos x + c_2 \sin x$

$\therefore \sin x \sin 2x$

$= \frac{1}{2}[\cos(-x) - \cos(3x)]$

$$P.I = \frac{1}{D^2+1}[\sin x \sin 2x]$$

$$= \frac{1}{D^2+1}\left[\frac{1}{2}(\cos x - \cos 3x)\right]$$

$$= \frac{1}{2}\left[\frac{1}{D^2+1}\cos x - \frac{1}{D^2+1}\cos 3x\right]$$

$D^2 \to -1$ $\quad$ $D^2 \to -9$

$$= \frac{1}{2}\left[x\frac{1}{2D}\cos x - \frac{1}{-9+1}\cos 3x\right]$$

$$= \frac{1}{2}\left[\frac{x}{2}\sin x + \frac{1}{8}\cos 3x\right]$$

$$= \frac{x\sin x}{4} + \frac{1}{16}\cos 3x.$$

$y = C.F + P.I$

(9) Solve $(D^2+3D+2)y = 1+3x+x^2$

Soln:- A.Eqn $m^2+3m+2=0$

$m = -1, -2$

$C.F = c_1 e^{-x} + c_2 e^{-2x}$

$$P.I = \frac{1}{D^2+3D+2}\, 1+3x+x^2$$

Right hand side is a polynomial in "x". therefore the rules of case (iii).

$$P.I = \frac{1}{2\left(1+\frac{D^2+3D}{2}\right)} 1+3x+x^2$$

By removing lowest degree term outside to make $1 \pm \phi(D)$ form in denominator.

$$= \frac{1}{2}\left[1+\left(\frac{D^2+3D}{2}\right)\right]^{-1}(1+3x+x^2)$$

$$= \frac{1}{2}\left[1-\left(\frac{D^2+3D}{2}\right)+\left(\frac{D^2+3D}{2}\right)^2 - \cdots\right](1+3x+x^2)$$

$$= \frac{1}{2}\left[1-\frac{D^2}{2}-\frac{3}{2}D+\frac{1}{4}\left[D^4+9D^2+6D^3\right]-\cdots\right](1+3x+x^2)$$

$D(1+3x+x^2) = 3+2x$

$D^2(1+3x+x^2) = 2$

$D^3(1+3x+x^2) = 0$

$$\therefore P.I = \frac{1}{2}\left[1+3x+x^2-\frac{1}{2}(2)-\frac{3}{2}(3+2x)\right.$$
$$\left.+\frac{1}{4}\left[0+9(2)+6(0)\right]-\cdots\right]$$

$$= \frac{1}{2}\left[\cancel{1}+\cancel{3x}+x^2-\cancel{1}-\cancel{\frac{9}{2}}-\cancel{3x}+\cancel{\frac{9}{2}}\right]$$

$$= \frac{1}{2}\left[x^2\right]$$

$$\therefore P.I = \frac{x^2}{2}$$

$$y = C.F + P.I$$

(10) $y'' + 3y' + 2y = 12x^2$

Soln:- $(D^2+3D+2)y = 12x^2$

A.Eqn $m^2+3m+2=0$

$m = -1, -2$

$\therefore$ C.F $= c_1 e^{-x} + c_2 e^{-2x}$

$$P.I = \frac{1}{D^2+3D+2} 12x^2$$

$$= 12 \frac{1}{2\left(1+\left(\frac{D^2+3D}{2}\right)\right)} x^2$$

$$= 6\left[1+\left(\frac{D^2+3D}{2}\right)\right]^{-1} x^2$$

$$= 6\left[1-\left(\frac{D^2+3D}{2}\right)+\left(\frac{D^2+3D}{2}\right)^2 - \cdots\right] x^2$$

$$= 6\left[1-\frac{D^2}{2}-\frac{3}{2}D+\frac{1}{4}(D^4+9D^2+6D^3)-\cdots\right] x^2$$

$D(x^2) = 2x$

$D^2(x^2) = 2$

$D^3(x^2) = 0$

$$= 6\left[x^2-\frac{1}{2}(2)-\frac{3}{2}(2x)+\frac{1}{4}[0+9(2)+6(0)]-\cdots\right]$$

$$= 6\left[x^2-1-3x+\frac{9}{2}\right]$$

$$= 6\left[x^2-3x+\frac{7}{2}\right]$$

$$= 6x^2-18x+21$$

$y = C.F + P.I$

(11) Solve $(D^3+8)y = x^4+2x+1$

Soln: A.Eqn $m^3+8=0$

$m^3+2^3=0$

$(m+2)(m^2-2m+4)=0$

$m=-2,\ 1\pm\sqrt{3}i$

$$C.F = c_1 e^{-2x} + e^{x}[c_2 \cos\sqrt{3}x + c_3 \sin\sqrt{3}x]$$

$$P.I = \frac{1}{D^3+8} x^4+2x+1$$

$$= \frac{1}{8} \frac{1}{(1+\frac{D^3}{8})} x^4+2x+1$$

$$= \frac{1}{8}\left[1+\frac{D^3}{8}\right]^{-1} (x^4+2x+1)$$

$$= \frac{1}{8}\left[1-\left(\frac{D^3}{8}\right)+\left(\frac{D^3}{8}\right)^2 - \cdots\right](x^4+2x+1)$$

$$= \frac{1}{8}\left[x^4+2x+1-\frac{1}{8}[24x]+0\right]$$

$$= \frac{1}{8}[x^4+2x+1-3x]$$

$$= \frac{1}{8}[x^4-x+1]$$

$D(x^4+2x+1) = 4x^3+2$

$D^2(x^4+2x+1) = 12x^2$

$D^3(x^4+2x+1) = 24x$

$$y = C.F + P.I$$

(12) Solve $(D^3-D^2-6D)y = 1+x^2$

Soln:- A.E^n $m^3-m^2-6m=0$

$$m(m^2-m-6)=0$$

$$\therefore m = 0, 3, -2$$

$$C.F = c_1 e^{0x} + c_2 e^{3x} + c_3 e^{-2x}$$

$$P.I = \frac{1}{D^3-D^2-6D} \cdot (1+x^2)$$

*/. Important to note that the lowest degree term in the denominator is $-6D$ //

$$P.I = -\frac{1}{6D}\frac{1}{\left(1-\left(\frac{D^3-D^2}{6D}\right)\right)}(1+x^2)$$

$$= -\frac{1}{6D}\left[1-\left(\frac{D^3-D^2}{6D}\right)\right]^{-1}(1+x^2)$$

$$= -\frac{1}{6D}\left[1-\left(\frac{D^2-D}{6}\right)\right]^{-1}(1+x^2)$$

$$= -\frac{1}{6D}\left[1+\left(\frac{D^2-D}{6}\right)+\left(\frac{D^2-D}{6}\right)^2+\cdots\right](1+x^2)$$

$$= -\frac{1}{6D}\left[1+\frac{D^2}{6}-\frac{D}{6}+\frac{1}{36}\left[D^4+D^2-2D^3\right]+\cdots\right](1+x^2)$$

$$= -\frac{1}{6D}\left[1+x^2+\frac{1}{6}(2)-\frac{1}{6}(2x)+\frac{1}{36}\left[0+2-0\right]\right]$$

$$= -\frac{1}{6D}\left[1+x^2+\frac{1}{3}-\frac{x}{3}+\frac{1}{18}\right]$$

$$= -\frac{1}{108D}\left[18x^2-6x+25\right]$$

$\frac{1}{D} \to \int$ integration

$$= -\frac{1}{108}\left[6x^3-3x^2+25x\right]$$

(13) Solve $(D^2-4D+3)y = e^x\cos 2x$

Soln: A.Eqn $m^2-4m+3=0$

$$m = 1, 3$$

$$C.F = C_1e^x + C_2e^{3x}$$

$$P.I = \frac{1}{D^2-4D+3}e^x\cos 2x$$

The right hand side function is of the form $e^{ax}V$.

$\therefore$ Apply rules of case (iv)

$$P.I = \frac{1}{D^2-4D+3} e^x \cos 2x$$

$D \to D+1$

$$= e^x \frac{1}{(D+1)^2-4(D+1)+3} \cos 2x$$

$$= e^x \frac{1}{D^2+2D+1-4D-4+3} \cos 2x$$

$$= e^x \frac{1}{D^2-2D} \cos 2x$$

Replace $D^2 \to -4$

$$= e^x \frac{1}{-4-2D} \cos 2x$$

$$= -\frac{e^x}{2} \frac{1}{(2+D)} \frac{(D-2)}{(D-2)} \cos 2x$$

$$= -\frac{e^x}{2} \frac{(D-2)\cos 2x}{D^2-4}$$

$D^2 \to -4$

$$= -\frac{e^x}{2} \frac{1}{-4-4} [D(\cos 2x) - 2\cos 2x]$$

$$= \frac{e^x}{16} [-2\sin 2x - 2\cos 2x]$$

$$= -\frac{e^x}{8} [\sin 2x + \cos 2x]$$

$$y = C.F + P.I$$

(14) Solve $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-4y = xe^{-2x}$

Soln:- $(D^2+3D-4)y = e^{-2x}x$

A.Eqn $m^2+3m-4=0$

$m=-4, 1$

$C.F = c_1e^{-4x}+c_2e^{x}$

$$P.I = \frac{1}{D^2+3D-4} e^{-2x}x$$

Replace $D \to D-2$

$$= e^{-2x}\frac{1}{(D-2)^2+3(D-2)-4} x$$

$$= e^{-2x}\frac{1}{D^2-4D+4+3D-6-4} x$$

$$= e^{-2x}\frac{1}{D^2-D-6} x$$

x is a polynomial

Case (iii)

$$= e^{-2x}\frac{1}{-6(1-(\frac{D^2-D}{6}))} x$$

$$= -\frac{e^{-2x}}{6}\left[1-\left(\frac{D^2-D}{6}\right)\right]^{-1} x$$

$$= -\frac{e^{-2x}}{6}\left[1+\left(\frac{D^2-D}{6}\right)+\left(\frac{D^2-D}{6}\right)^2+\cdots\right] x$$

$$= -\frac{e^{-2x}}{6}\left[x-\frac{1}{6}\right]$$

$D(x)=1$

$D^2(x^2)=0$

$$= -\frac{e^{-2x}}{36}[6x-1].$$ $\therefore y = C.F + P.I$

(15) Solve $\frac{d^4y}{dx^4} = e^x \cos x$

Soln:. $D^4 y = e^x \cos x$

A.Eqn $m^4 = 0$

$m = 0, 0, 0, 0$

$\therefore C.F = (C_1 + C_2 x + C_3 x^2 + C_4 x^3) e^{0x}$

$P.I = \frac{1}{D^4} e^x \cos x$ $\rightarrow$ $D \rightarrow D+1$

$= e^x \frac{1}{(D+1)^4} \cos x$

$= e^x \frac{1}{D^4 + 4D^3 + 6D^2 + 4D + 1} \cos x$

Replace $D^2 \rightarrow -1$

$D^4 \rightarrow 1$

$D^3 \rightarrow D^2 D$

$\rightarrow -1(D)$

$= e^x \frac{1}{1 - 4D - 6 + 4D + 1} \cos x$

$= -\frac{e^x}{4} \cos x$

$y = C.F + P.I$

(16) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \sin x$

Soln:. $(D^2 - 2D + 2)y = e^x \sin x$

A.Eqn $m^2 - 2m + 2 = 0$

$m = 1 \pm i$

$C.F = e^x (C_1 \cos x + C_2 \sin x)$

Case (iv)

$$P.I = \frac{1}{D^2-2D+2} e^x \sin x$$

$D \to D+1$

$$= e^x \frac{1}{(D+1)^2-2(D+1)+2} \sin x$$

$$= e^x \frac{1}{D^2+2D+1-2D-2+2} \sin x$$

$$= e^x \frac{1}{D^2+1} \sin x$$

Denominator becomes zero Replace $D^2 \to -1$

$$\therefore \quad P.I = e^x \, x \frac{1}{2D} \sin x$$

$$= \frac{e^x \cdot x}{2} \int \sin x \, dx$$

Note:- Integrate only $\sin x$ not other terms

$$= -\frac{e^x x \cos x}{2}$$

$$\therefore \quad y = C.F + P.I$$

(17) Solve $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$

Soln:- A.Eqn:- $(m-2)^2 = 0$

$$m = 2, 2$$

$$C.F = (C_1 + C_2 x)e^{2x}$$

$$P.I = \frac{1}{(D-2)^2}[8(e^{2x} + \sin 2x + x^2)]$$

Combination of case (i), case (ii) and case (iii)

$$P.I = 8\left[\frac{1}{(D-2)^2}e^{2x} + \frac{1}{(D-2)^2}\sin 2x + \frac{1}{(D-2)^2}x^2\right]$$

$D \to 2$

Denominator becomes zero
$\therefore$ Differentiate Denominator

$$= 8\left[x\frac{1}{2(D-2)}e^{2x} + \frac{1}{D^2-4D+4}\sin 2x + \frac{1}{(-2)^2}\frac{1}{\left(1-\frac{D}{2}\right)^2}x^2\right]$$

Again Denominator becomes zero

$D^2 \to -4$

$$= 8\left[x^2\frac{1}{2}e^{2x} + \frac{1}{-4-4D+4}\sin 2x + \frac{1}{4}\left[1-\frac{D}{2}\right]^{-2}x^2\right]$$

$$= 8\left[\frac{x^2e^{2x}}{2} - \frac{1}{4}\int \sin 2x\,dx + \frac{1}{4}\left[1+2\left(\frac{D}{2}\right)+3\left(\frac{D}{2}\right)^2+4\left(\frac{D}{2}\right)^3 + \cdots\right]x^2\right]$$

$$= 8\left[\frac{x^2e^{2x}}{2} - \frac{1}{4}\frac{(-\cos 2x)}{2} + \frac{1}{4}\left[x^2+2x+\frac{3}{4}(2)\right]\right]$$

$$= 8\left[\frac{x^2e^{2x}}{2} + \frac{\cos 2x}{8} + \frac{x^2}{4} + \frac{x}{2} + \frac{3}{8}\right]$$

$$= 4x^2e^{2x} + \cos 2x + 2x^2 + 4x + 3$$

$\therefore\ y = C.F + P.I.$

(18) Solve $(D^2-2D+1)y = xe^x\sin x$

Soln: A. Eqn $m^2-2m+1=0$

$m = 1, 1$

$C.F = (c_1+c_2x)e^x$

$$P.I = \frac{1}{D^2-2D+1} e^x x \sin x$$

case (iv)

$D \to D+1$

$$= e^x \frac{1}{(D+1)^2-2(D+1)+1} x \sin x$$

$$= e^x \frac{1}{D^2+2D+1-2D-2+1} x \sin x$$

$$= e^x \frac{1}{D^2} x \sin x$$

$$= e^x \frac{1}{D} \int x \sin x \, dx$$

Integrating by parts

$$= e^x \frac{1}{D} \left[x(-\cos x) - (1)(-\sin x) \right]$$

$$= e^x \frac{1}{D} \left[-x\cos x + \sin x \right]$$

$$= e^x \int (\sin x - x\cos x) \, dx$$

$$= e^x \left[\int \sin x \, dx - \int x\cos x \, dx \right]$$

↓ Integration by parts

$$= e^x \left[-\cos x - \left[x \sin x - (-\cos x) \right] \right]$$

$$= e^x \left[-\cos x - x \sin x - \cos x \right]$$

$$= -e^x \left[2\cos x + x \sin x \right]$$

$y = C.F + P.I$ //

(19) Solve $(D^2-1)y = x^2\cos x$

Soln:- A.Eqn $m^2-1=0$

$m=\pm 1$

$C.F = c_1e^{x}+c_2e^{-x}$

$P.I = \frac{1}{D^2-1}x^2\cos x$ [we cannot apply any cases here]

$\therefore e^{ix} = \cos x + i\sin x$

Imaginary Part

Real part of e^{ix}

$= \frac{1}{D^2-1}x^2$ (Real part of e^{ix})

$=$ Real part of $\frac{1}{D^2-1}e^{ix}x^2$ → Case iv $e^{ax}v$

$D \to D+i$

$=$ Real part of $e^{ix}\frac{1}{(D+i)^2-1}x^2$

$=$ Real part of $e^{ix}\frac{1}{D^2-1+2iD-1}x^2$

$=$ Real part of $e^{ix}\frac{1}{D^2+2iD-2}x^2$ → case (iii) Polynomial

$=$ Real part of $e^{ix}\frac{1}{-2(1-(\frac{D^2+2iD}{2}))}x^2$

$= -\frac{1}{2}$ R.P of $e^{ix}(1-(\frac{D^2+2iD}{2}))^{-1}x^2$

$= -\frac{1}{2}$ R.P of $e^{ix}\left[1+(\frac{D^2+2iD}{2})+(\frac{D^2+2iD}{2})^2+\cdots\right]x^2$

$$P.I = -\frac{1}{2} \text{ R.P of } e^{ix}\left[1+\frac{D^2}{2}+iD+\frac{1}{4}\left[D^4-4D^2+4iD^3\right]+\cdots\right]x^2$$

$D(x^2)=2x, \quad D^2(x^2)=2$

$$= -\frac{1}{2} \text{ R.P of } e^{ix}\left[x^2+1+i(2x)+\frac{1}{4}\left[0-4(2)+4(0)\right]\right]$$

$$= -\frac{1}{2} \text{ R.P of } e^{ix}\left[x^2+1+i2x-2\right]$$

$$= -\frac{1}{2} \text{ R.P of } (\cos x+i\sin x)\left[(x^2-1)+i2x\right]$$

$$= -\frac{1}{2} \text{ R.P of } \left[\underbrace{(x^2-1)\cos x}_{R.P}+i2x\cos x+i(x^2-1)\sin x \underbrace{-2x\sin x}_{R.P}\right]$$

$$\therefore P.I = -\frac{1}{2}\left[(x^2-1)\cos x-2x\sin x\right]$$

$$y = C.F + P.I$$