CS241AT

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RV COLLEGE OF ENGINEERING
Autonomous Institution affiliated to VTU
IV Semester B.E. Sep - 2024 Examinations
DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

Discrete Mathematical Structures and Combinatorics
Common to CS, IS, CY, CD and AIML
Model Question Paper

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in the first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, and 9 and 10.
3.

PART-A

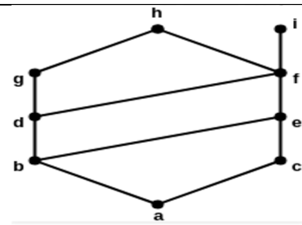
			Marks	BTL
1	1.1	In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together.	2	2
	1.2	Solve the recurrence relation ak = $6a_{k-1} - 9a_{k-2}$ with initial conditions $a_0 = 0$ and $a_1 = 2$. solution to this recurrence relation, provided the constants A and B are chosen correctly?	2	3
	1.3	Simply using Law of Logic ¬[¬[(p v q) ^ r] v ¬q]	2	3
	1.4	Write the converse of the following statements in words: i. if you buy Colgate then your children will brush longer ii. When you serve imported sparkling water, it shows that you had good taste.	2	3
	1.5	Let f : R →R and g: R →R, given by $f(x)=x^2$ and g(x)=x+5 find g∘f.	2	2
	1.6	Let A={1,2,3,4,5,6,7,8} R is an equivalence relation on A which induces the partition {1,4,8} ∪ {3} ∪ {5,6} ∪{2,7}. Determine R.	2	2
	1.7	Identify the identity element of (Z, .) where x . y = x+y+1 for all x, y belongs to Z	2	3
	1.8	If x =1100010 and y=1011101, the Hamming distance h(x, y) = ____	2	3
	1.9	For n≥3, let Gn = (V, E) be the graph obtained from the complete graph Kn upon deletion of one edge, What is χ(Gn)?	2	3
	1.10	If 4 colors are available, in how many ways $K_{2,3}$ can be properly colorable?	2	4

PART-B

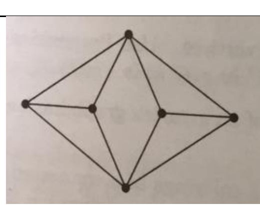
2	a	Find the number of proper divisors of 441000.	4	3
	b	A bank pays a certain % of annual interest on deposits, compounding the interest once in 3 months. If a deposit doubles in 6 years and 6 months, what is the annual % of interest paid by the bank?	4	3

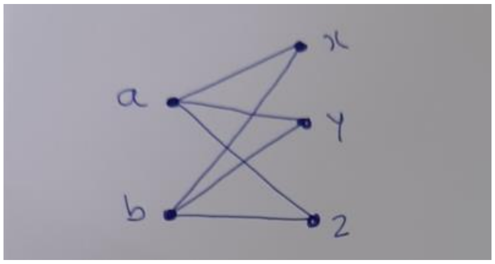
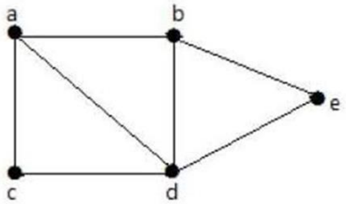
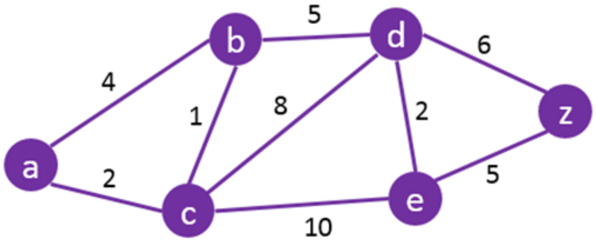
	c	Find a generating function for the recurrence relation $a_{n+2}-3a_{n+1}+2a_n=0$ for $n\geq 0$, $a_0=1$, $a_1=6$. Hence solve it.	4	3
	d	A total of \$10000 is to be distributed to four persons A, B, C, and D in multiples of \$1000. In how many ways can this be done: ● If there is no restriction? ● If every one of these persons should receive at least \$1000?	4	3
3	a	Let p, q, and r be the propositions p : You have the flu. q : You miss the final examination. r : You pass the course. Express each of these propositions as an English sentence. a) $p\rightarrow q$ b) $q\rightarrow \sim r$ c) $(p\rightarrow \sim r)$ v $(q\rightarrow \sim r)$ d) $(p \wedge q)$ v $(\sim' q \wedge r)$	4	4
	b	Prove the logical equivalence using law of logic: $[p\leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r\leftrightarrow p)] \Leftrightarrow [p\wedge q) \wedge (q\rightarrow r) \wedge (r\rightarrow p)]$	6	3
	c	Prove the following logical equivalences: i) $\neg [\forall x, \neg p(x)] \Leftrightarrow \exists x, p(x)$ ii) $\neg [\exists x, \neg p(x)] \Leftrightarrow \forall x, p(x)]$	6	3
		OR		
4	a	Let P(x) and Q(x) are open statements as below P(x): $\lvert x\rvert >3$, Q(x): $x >3$, here the universe consists of all real numbers. For the implication *"If the magnitude of a real number is less than or equal to 3 then the number is less than or equal to 3"*. Give the converse, inverse and contrapositive of the above implication in symbolic form.	4	3
	b	Prove the validity of the following argument $p \rightarrow q$ $\neg r$ v s p v r $\therefore \neg q \rightarrow \neg s$	4	4
	c	Let P(x), Q(x), and R(x) be open statements that are defined for a given universe. Show that the argument is valid. x [P(x)àQ(x)], x [Q(x)àR(x)] ├ x [P(x)àQ(x)]	6	4

5	a	Show that composition of function is associative.	4	2
	b	Given the set A={a, b, c} with the relations R={(a,a),(a,c),(b,a),(c,b)} and S={(a,b),(b,c),(c,c)}. Find i) Converse of R ii) Complement of S iii) the composition $R \circ S$ iv) Show that $(R\circ S)^{-1} = S^{-1}\circ R^{-1}$	5	3
	c	i) Draw a Hasse diagram for set A with divisibility relation, where A= {2, 3, 4, 5, 6, 30, 60}. ii) Find the greatest and least elements. iii) Find LB, UB, LUB and GLB for {6, 30}	7	3
		OR		
6	a	Find minimal, maximal, greatest and least elements for the following Hasse diagram	8	3

		 Also find upper bound, lower bound, LUB and GLB for the set {b, c}		
	b	Let the function f: R → R be defined by f(x,y) = {(x,y) | y=mx+b} , where m, b ∈ R. Then, find f^{-1}.	4	4
	c	Prove that the divisibility relation is a partial order relation on a set of integers.	4	2

7	a	Let G = {q ∈Q / q ≠ -1}. Define the binary operation **o** on G by x **o y** = x + y + xy. Prove that (G, **o**) is an abelian group.	5	3
	b	Prove that (i) Identity element in a group is unique (ii) Inverse of each element in a group is unique	4	3
	c	The encoding function $E: \mathbf{Z}_2^2 \rightarrow \mathbf{Z}_2^5$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$. (i) Find the associated parity-check matrix H. (ii) Use H to decode each of the following received words. (a) 1101 (b) 10101 (c) 11101 (d) 00110	7	3
		OR		
8	a	In the group S_5, let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 3 & 4 \end{pmatrix}$. Find $\beta\alpha$, and β^{-1}	4	3
	b	Define the encoding function $E: \mathbf{Z}_2^3 \rightarrow \mathbf{Z}_2^6$ by means of the parity-check matrix, $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ Determine all code words.	5	3
	c	State and prove Lagrange's theorem. Find the right cosets of H = {0, 4} in group G = {Z8, +} ($H = \{e, g^4, g^8\}$ in $C_{12} = \{e, g, g^2, \ldots, g^{11}\}$.)	7	3

9	a		8	3

		Determine the chromatic polynomial for the below graph. Find $\chi(G)$.		
	b	Consider the graph $K_{2,3}$. Let λ denote the number of colors available to properly color the vertices of $K_{2,3}$. i. How many proper colorings of $K_{2,3}$ have vertices a, b colored with the same color? ii. What is the chromatic polynomial for $K_{2,n}$? What is $\chi(K_{2,n})$? 	8	4
		OR		
10	a	Define spanning tree of a connected graph. How many spanning trees are there for the below connected graph. 	8	4
	b	Explain the Prims algorithm to find the minimum cost spanning tree. Find the minimum cost spanning tree for the weighted connected graph below by using the Prims algorithm. 	8	4