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RV COLLEGE OF ENGINEERING®
(An Autonomous Institution affiliated to VTU)
V Semester B. E. Examinations Nov/Dec-19
Computer Science and Engineering
PROBABILITY, STATISTICS AND QUEUING THEORY (ELECTIVE)

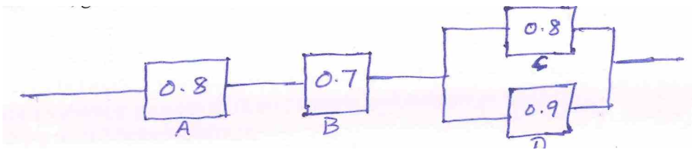
Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:
1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6
3. Statistical Table permitted.

PART-A

1	1.1	If the moment generating function of a random variable X is $M_X(t)$, then what is t he moment generating function of $Y = aX + b$?	01
	1.2	If the customers arrive at a bank according to a Poisson variate with mean rate of 2 per minute, find the probability that no customer arrives in one minute interval.	01
	1.3	Show that an exponential distribution is memoryless.	02
	1.4	A random sample of 10 observations is taken from a normal population having the variance 42.5. Find approximately the probability of obtaining a sample standard deviation S between 3.24 and 8.94.	02
	1.5	Some generators do not repeat an initial part of the sequence. This part is called __________. In this case what is the period of the generator?	01
	1.6	State and prove Markov's inequality.	01
	1.7	A student study habits are as follows: If he studies one night, he is 70% $sure$ not to study the next night. On the other hand, if he does not study one night, he is 40% sure not to study the next night as well. In the long run, how often does he study?	02
	1.8	What are the characteristics of a Queuing system?	01
	1.9	The maximum possible period for a multiplicative LCG with $m = 2^k$ is __________.	01
	1.10	Discuss classification of states of Markov Chain.	02
	1.11	Define the cross-covariance and cross-correlation co-efficient of two processes $\{X(t)\}$ and $\{Y(t)\}$.	02
	1.12	Define birth and death process.	02
	1.13	Find the third moment about mean in terms of moments about 0.	01
	1.14	A contractor has to choose a job between two jobs. The first job promises a profit of $Rs.2{,}40{,}000$ with probability 0.75 and a loss of $Rs.60{,}000$ with probability 0.25. The second job promises a profit of $Rs.3{,}60{,}000$ with probability 0.5 and a loss of $Rs.90{,}000$ with probability 0.5. Which job should the contractor choose to maximize his expected profit?	01

PART-B

2	a	Two random variables X and Y have the joint density function given by $f(x,y) = 24xy, x > 0, y > 0, x + y \leq 1$ and $f(x,y) = 0$ elsewhere. Find conditional mean and variance of Y given X.	06
	b	All the screws in a machine come from the same factory, but it is likely to be from factory A as from factory B. The percentage of defective screws is 5% from A and 1% from B. Two screws are inspected. i) If the first is found to be defective, what is the probability that the second is also defective? ii) If the first is found to be good, what is the probability that the second is also good?	05
	c	An electrical system consists of four components as illustrated in the following fig 2c. The system works if components A and B work and either of the components C or D work. Assume that four components work independently. The probability of working of each component is also shown in the figure. Find the probability that: i) The entire system works and ii) The component C does not work, given that the entire system works.  Fig 2c	05
3	a	State and prove the following: i) Schwartz inequality, ii) Cauchy-Schwartz inequality.	05
	b	On an average, a submarine on patrol sights 6 enemy ships per hour. Assuming that the number of ships sighted in a given length of time is Poisson variate; find the probability of sighting: i) 6 ships in the next half an hour, ii) 4 ships in the next two hours and iii) At least 1 ship in the next 15 minutes.	05
	c	A manufacturer claims that the average mileage of a car is 14kms per liter. In an attempt to show that it differs from the claim, 5 measurements are made and the results are 14.5, 14.2, 14.4, 15.3 and 14.8. Test the hypothesis at 0.05 level of significance, a summing normality.	06
		OR	
4	a	State and prove Chertoff's bounds. Also find Chertoff's bounds for an exponential distribution.	05
	b	i) Show that the sum of two independent Poisson processes is a poisson processes. ii) Is difference of independent Poisson processes a Poisson processes?	05
	c	To test a paint manufacturer's claim that the average drying time of his new "fast drying" paint is $\mu = 20$ minutes, a 'random sample' of 36 boards is painted with his new paint and his claim is rejected if the mean drying time $\overline{X} > 20.5$ minutes. Find the: i) Probability of type I error; ii) The probability of type II error, when $\mu = 21$ minutes. $(assume\ that\ \sigma = 2.4\ minutes)$.	06

5	a	There are two white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each earn and the 2 marbles selected are interchanged. Let the state a_i of the system be the number of red marbles in urn A after I changes. What is the probability that there are 2 red marbles in urn A after 3 steps? In the long run, what is the probability that there are 2 red marbles in urn A?	06
	b	A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again he is to travel by train. Now suppose that on first day of the week, the man tossed a fair dice and drove to work if and only if a 5 or 6 appears. Find i) The probability that he takes a car on the third day and ii) The probability that he catches a train in the long run.	05
	c	Show that the random process $X(t) = A\cos(\omega_o t + \theta)$ is wide sense stationary, if A and ω_0 are constants and θ is a uniformly distributed random variable in $(0,2\pi)$.	05
		OR	
6	a	Find the auto-correlation, the auto co-variance and the correlation coefficient of the Poisson process $\{X(t)\}$.	05
	b	If the customers arrive at a counter in accordance with a mean rate of 2 per minute. Find the probability that the interval between 2 consecutive arrivals is: i) More than one minute, ii) Between 1 minute and 2 minute and iii) 4 minutes and less.	04
	c	Define counting process with an example and state its properties.	04
	d	If the tpm of Markov chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$, find the steady state distribution of the chain.	03
7	a	A car service station has 2 boys where service can be offered simultaneously. Due to space limitation, only 4 cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time is exponentially distributed with $\mu = 8\, cars$ per day per boy. Find the probability distribution table. Also find the average number of cars waiting for service and the average time a car spends in the system.	08
	b	In a railway yard, goods train arrives at a rate of 30 $trains$ per day. Assuming that inter arrival time follows exponential distribution and the service time is exponential with an average of 36 minutes. Further it is known that the line capacity of the yard is 9 trains. With usual notations, find L_S, L_Q, W_S and W_Q	08

8	a	One thousand random numbers were generated using the following generator with a seed $x_0 = 1$: $x_n = (125x_{n-1} + 1)mod2^{12}$. The numbers so obtained were categorized in a histogram using 5 cells at intervals of 0.2, between 0 and 1.At the $\alpha = 0.05$ level, can we say that the numbers are $IID\ U(0,1)$?	06
	b	Determine any four primitive roots of 31.	06
	c	Discuss the following: i) Fibonacci generators, ii) Combined generators.	04

$Cell$	$Observed$
1	196
2	183
3	198
4	222
5	201