

R. V. COLLEGE OF ENGINEERING
Autonomous Institution affiliated to VTU
IV Semester B. E. Fast Track Examinations July-15
Common to ISE / CSE
THEORY OF COMPUTATIONS

Time: 03 Hours**Maximum Marks: 100****Instructions to candidates:**

1. Answer all questions from Part A. Part A questions should be answered in the first three pages of the answer book only.
2. Answer FIVE full questions from Part B.

PART A

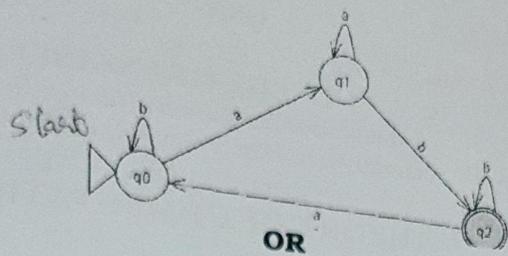
1	1.1	Define a useless variable.	01
	1.2	Define Σ^* .	01
	1.3	Find the string of minimum length in $\{0,1\}^* NOT$ in the language $(0^* + 1^*)(0^* + 1^*)(0^* + 1^*)$.	01
	1.4	In which computation model, Σ is subset of Γ ?	01
	1.5	Show that the language of palindromes on $\Sigma = \{a, b\}$ is context – free.	02
	1.6	Define Post's Correspondence Problem.	02
	1.7	"Every NPDA has an equivalent DPDA". Comment	02
	1.8	Define a Multi-tape Turing machine.	02
	1.9	Identify context – free languages from the following set of languages i) $L = \{a^n \mid n \geq 0\}$ ii) $L = \{a^n b^n \mid n \geq 0\}$ iii) $L = \{a^n b^n c^n \mid n \geq 0\}$	02
	1.10	Show that the following grammar is ambiguous: $S \rightarrow AB \mid CD$ $A \rightarrow aAb \mid \epsilon$ $B \rightarrow cB \mid c$ $D \rightarrow bDc \mid \epsilon$ $C \rightarrow aA \mid a$	02
	1.11	Define recursively enumerable and recursive languages.	02
	1.12	Does the following instance of PCP have a solution? If yes, write the solution.	02

	A	B
I	Wi	Xi
1	10	101
2	011	11
3	101	011

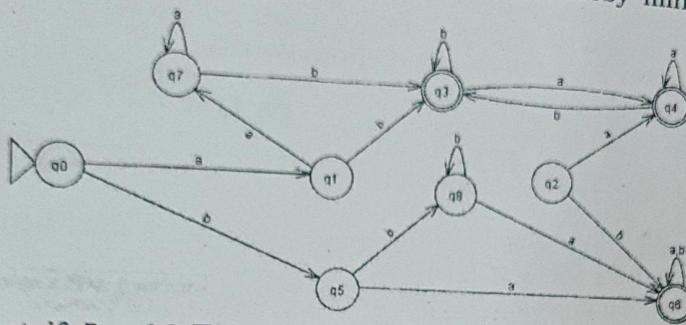
PART B

2 a	Define a DFA. Design a DFA to accept $L = \{w \text{ has starting symbol different from the ending symbol}\}$. Trace the DFA on strings 'abba', 'baaab', 'aabb' and 'babaa'.	08
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b Define an ϵ -NFA. Construct a minimal DFA equivalent to the given ϵ -NFA.

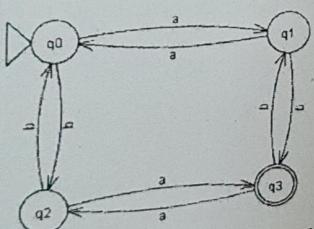


3 a Explain distinguishable and indistinguishable pair of states. Identify all the indistinguishable pairs of states and thereby minimize the following DFA:



b Show that if $D = (Q, \Sigma, \delta, q_0, F)$ is a DFA constructed from the NFA, $N = (Q_N, \Sigma_N, \delta_N, q_0, F_N)$ using subset construction algorithm, then $L(D) = L(N)$.

4 a Explain the rules to convert a given regular expression to an NFA.
Using the same, convert the regular expression $(ab)^* + b^*$ to an NFA.
Obtain a regular expression from the DFA given below. Use state elimination method.



c Show that if L_1 is regular, then so is its complement.

OR

5 a State and prove pumping lemma for regular languages. Using the same show that $L = \{a^n \mid n \text{ is prime}\}$ is not regular.
Write regular expressions for the following languages:

- the set of all odd length strings having different first and last symbol;
- strings whose second and second last symbol are same;
- strings where every 'a' is followed by at least one 'b'.

6 a D
ti
F

7 a b

b

8 a b

b

08

08

9 10

08

03

10

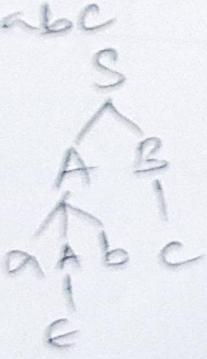
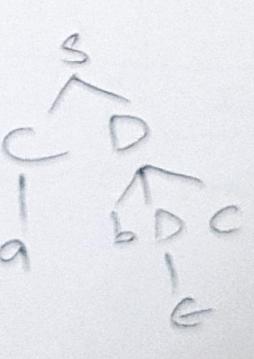
06

6	a	Define a <i>CFG</i> . Obtain <i>CFGs</i> to recognize $L = \{a^i b^j \mid i \leq 2j\}$. Give parse trees for the strings 'ab', 'aaaabbb', and 'abbb'. Prove that if L_1 is regular and L_2 is a <i>CFL</i> , then $L_1 \cap L_2$ is context-free.	08
	b		08
7	a	Define useless productions. Eliminate useless productions, if any, from the following grammar and then represent the grammar in <i>CNF</i> : $S \rightarrow 0A0 \mid 1B1 \mid BB$ $A \rightarrow C \mid G$ $B \rightarrow S \mid A$ $C \rightarrow S \mid \epsilon$ $D \rightarrow aABS \mid \epsilon$ $G \rightarrow CG \mid aG$	10
	b	Is the given grammar ambiguous? If yes, prove it and find an equivalent unambiguous grammar. $S \rightarrow ABA$ $A \rightarrow aA \mid \epsilon$ $B \rightarrow bB \mid \epsilon$.	06
8	a	Design a <i>PDA</i> P such that $L = L(P) = \{a^{2n} b^n \mid w \in \{a, b\}^*\}$. State whether P is deterministic or not. Show the sequence of IDs to accept the string "aab".	10
	b	Write the algorithm to convert a grammar to a <i>PDA</i> . Using the same convert the grammar $S \rightarrow aA \mid bA \mid \epsilon$ $A \rightarrow bS \mid aS$ to a <i>PDA</i> that accepts the same language by empty stack.	06
9	a	Construct an <i>NPDA</i> to accept the language $L = \{aww^r a \mid n \geq 1\}$ and trace the <i>PDA</i> for the string 'aabbaa'.	08
	b	Define instantaneous description of a <i>PDA</i> and the languages accepted by a <i>PDA</i> .	04
	c	Design a <i>PDA</i> that accepts $L = \{a^n b^n \mid n \geq 1\}$ by empty stack .	04
10	a	Design a Turing machine such that $(q_0, BwB) \xrightarrow{*} (q_f, BwBw^r B)$ where q_0 is the initial state, q_f is the final state, B is blank. Trace the Turing machine for the string 'aba'. Show that:	08
	b	i) if L and its complement are recursively enumerable, then L is recursive; ii) If L is recursive, then so is its complement.	08
11	a	Design a Turing machine to recognize palindromes of odd length. Give the sequence of IDs for the strings 'abbba' and 'abba'.	10
	b	Explain Chomsky hierarchy.	06

SCHEME AND SOLUTION

SUBJECT CODE: 121543

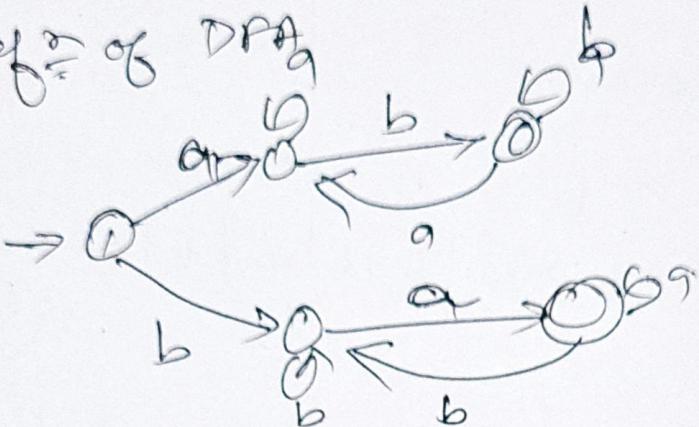
SUBJECT: TOC -

Question No		Marks
1(a)	The Variable which is either not reachable or non-generating	
1.b	$Z^* = Z^0 \cup Z^1 \cup Z^2 \cup \dots$	
1.c	1010 or 0101	
1.d	Turing machine	
1.e	Convert a PDA or CFG $S \Rightarrow aSb \mid bSb \mid a \mid b \mid \epsilon$	
1.f	Df 2:	
1.g	The Statement is false. WW^* has an NPDFA but no DPDA	
1.h	Df 2:	
1.i	$a \times b$ (\because Every regular language is also context free)	
1.j	$w = abc$  	

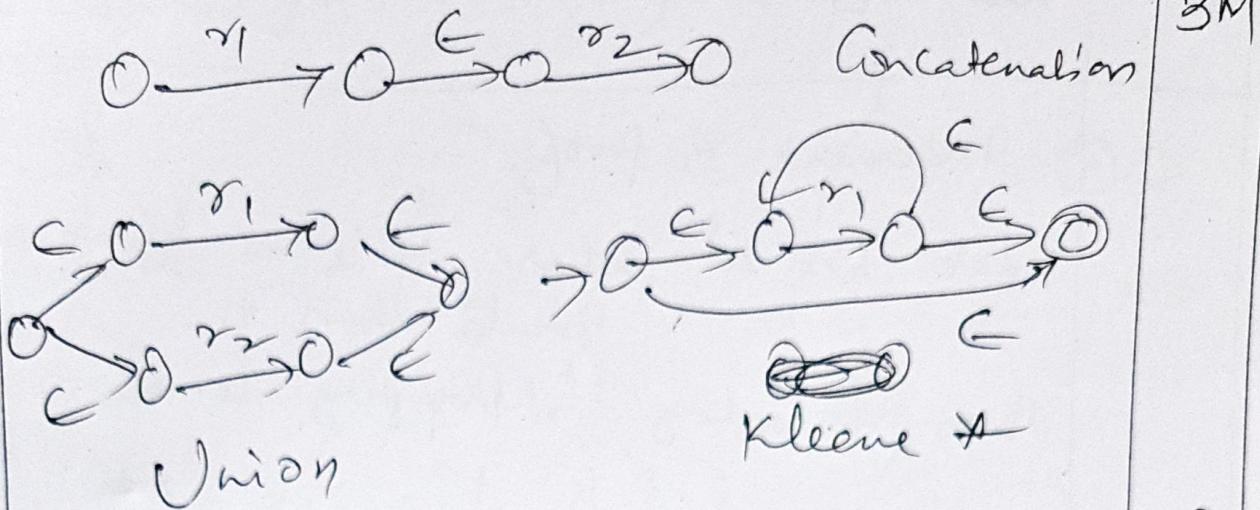
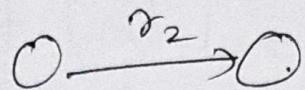
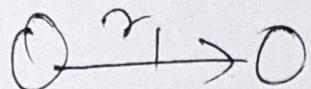
SCHEME AND SOLUTION

SUBJECT CODE: 121SU3

SUBJECT: Toc

Question No		Marks
1.1 (a)	Def ⁿ s	
1.1 (b)	PCP has no solution	
	<u>PART B</u>	
2(a)	Def ⁿ of DFA	2
		4
	Tracing	2
2(b)	Def ⁿ , Conversion to DFA Minimization	2 4 2
3.(a)	Explanation Minimization	2 6
(b)	Proof	8

4.0

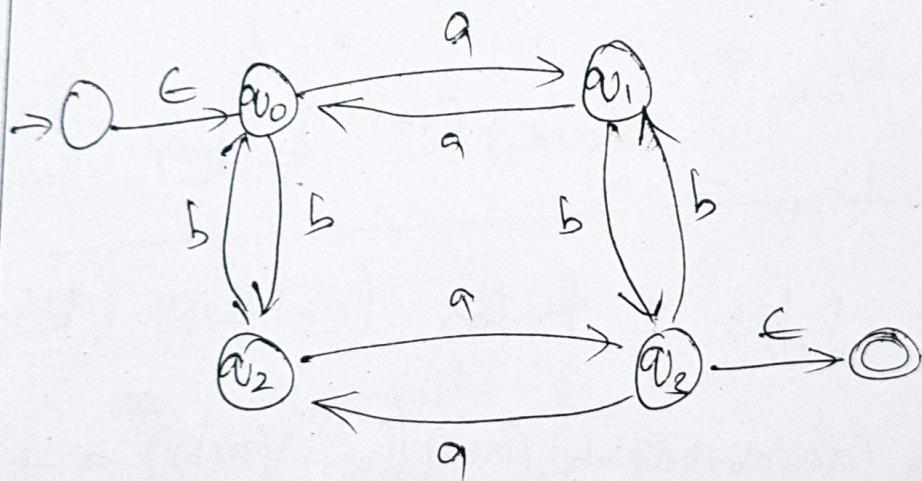


ϵ -NFA for $(ab)^* + b^*$

2

①

Eliminate w_1, w_2, w_3 & w_4 in that order from the DFA $\rightarrow (2 \times 4 = 8)$



(ii)

SCHEME AND SOLUTION

SUBJECT CODE: 121SU3

SUBJECT: T.O.C

Question No	Marks
4.(c) Let $L_1 = L(M)$ for $\tilde{M} = (Q, Z, \delta, q_0, F)$, then $\tilde{L}_1 = L(\tilde{M})$ for $\tilde{M} = (Q, Z, \delta, q_0, Q-F)$	(3)
5.(a) Statement & Proof Let $w = a^p$, where p is a prime no. greater than n . Then, According to Pumping Lemma $w = a^i a^j a^k \mid i+j+k = p$ $\&$ $i \leq j \leq n$	(6)
Pump 'y' $i+k$ times $w_{i+k} = a^i a^{j+(i+k)} a^k$ $w_{i+k} = a^{(i+k)+(i+j)} \notin L$ as p has 2 factors $(i+k) \& (i+j)$ i) $a(a+ab+ba+bb)^*(ab)((ab)(ab))^* a +$ $b((ab)(ab))^*(ab)((ab)(ab))^* b + a+ b$ ii) $(ab)a(ab)^* a(ab) + (ab)b(ab)^* b$ (ab)	(4)

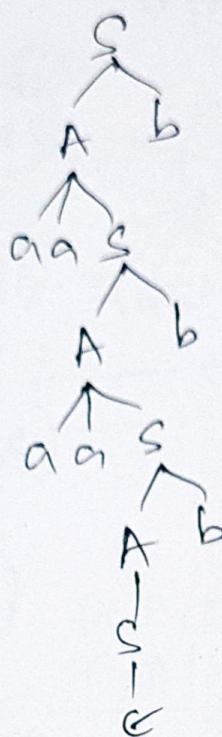
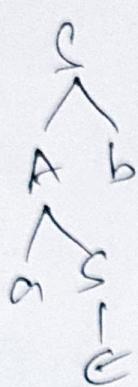
iii) $(b+ab^*)^*$

TOL

6.(a) CFG Def. → 2

$$S \rightarrow Ab \mid \epsilon$$

$$A \rightarrow S \mid as \mid aas$$



→ 1x3

6.(b) Proof → 8 marks.

7.(a) Def. → 2

D is unreachable, G is non-generating

remove them → 2

eliminate null productions → 2

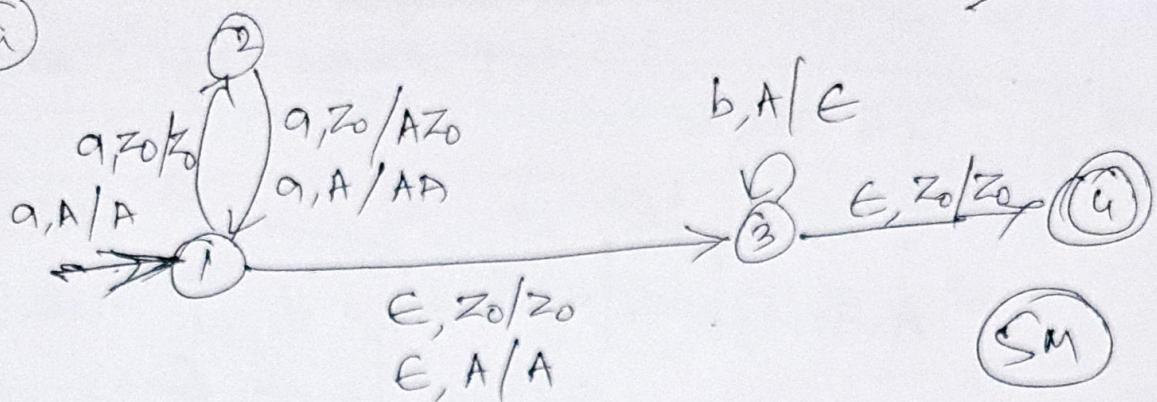
eliminate Unit productions → 2

CNF → 2

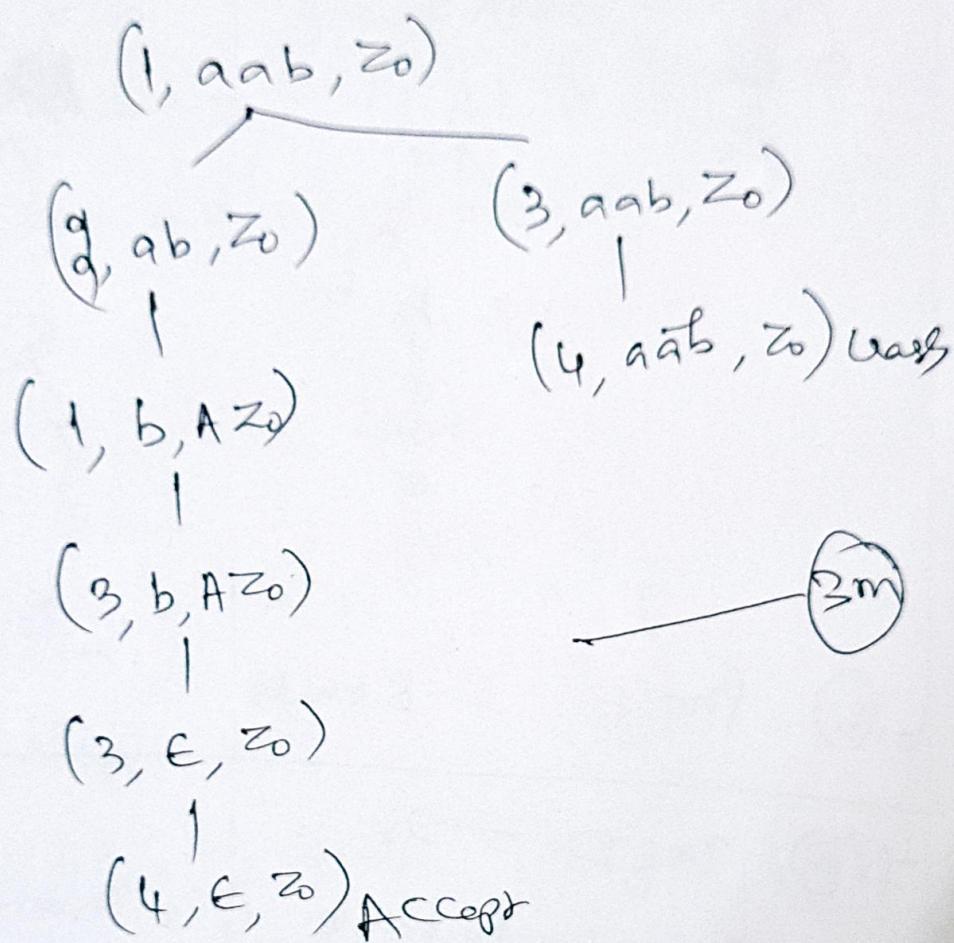
⑤

ToC

8(a)



It is NPDA — 2m

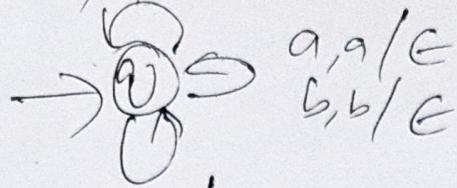


8(b) Algorithm — 2m

6

$\epsilon, S / \epsilon$
 $\epsilon, S / bA$
 $\epsilon, S / aA$

ToC



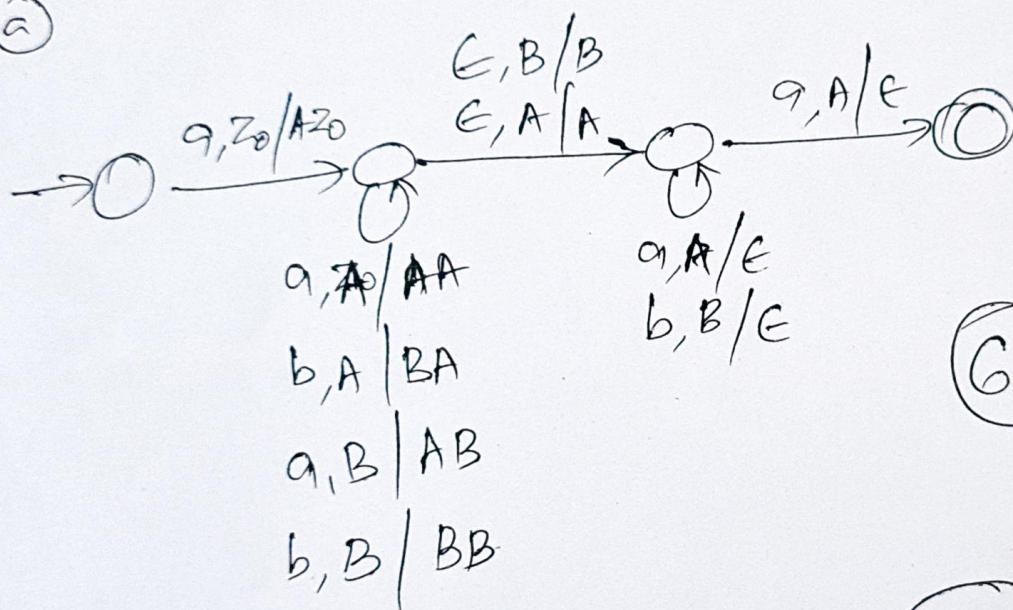
$\epsilon, A / bS$
 $\epsilon, A / aS$



The PDA is $M = (\{a\}, \{a, b\}, \{ab, A, S\}, \delta, a, S)$ where δ is

given above.

9.(a)



(6m)

(2m)

Tracing

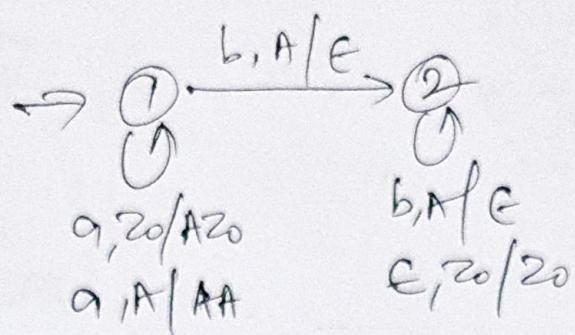
(b) Def \equiv_s

2+2m

(7)

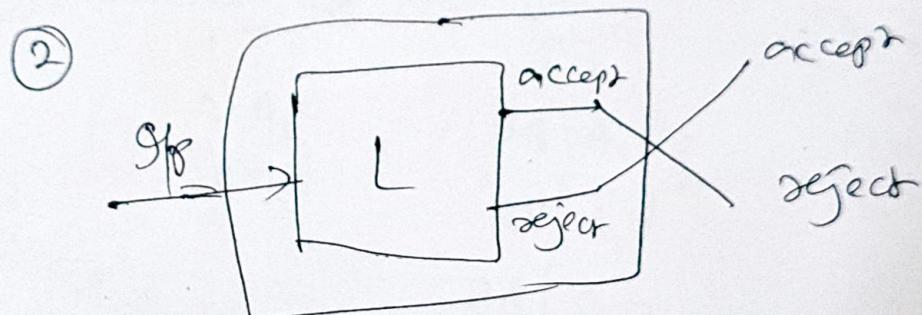
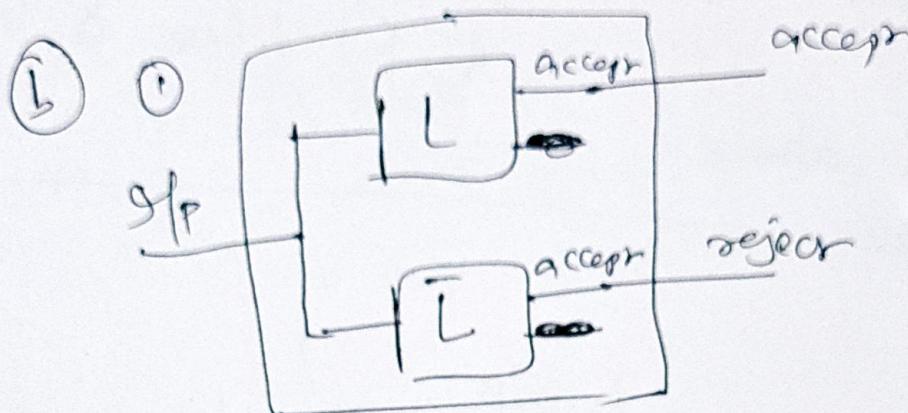
To C

9.①



10. ① Design — 6m

Tracing — 2m



11. ① Design + Tracing

6+2+2

② Explanation

③

④