16CS46

USN 1RV17CS051

RV COLLEGE OF ENGINEERING®
(An Autonomous Institution affiliated to VTU)
IV Semester B. E. Examinations April/May-19
Computer Science and Engineering
THEORY OF COMPUTATIONS

Time: 03 Hours ***Maximum Marks: 100***

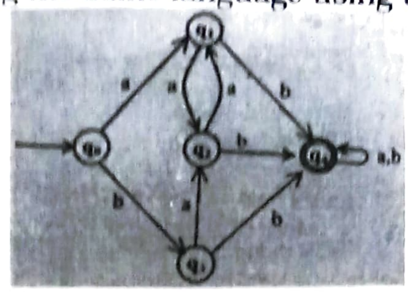
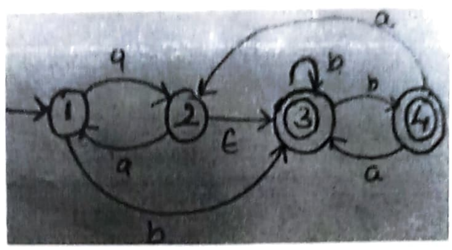
Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B.In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

PART A

<table>
<tr><td>1</td><td>1.1</td><td>What is minimum number of state a deterministic finite automation requires to accept the language $L = \{w|w \in \{0,1\}*$, number of $0s$ and $1s$ in w are divisible by 3 and 5, respectively}.</td><td>02</td></tr>
<tr><td></td><td>1.2</td><td>Identify the shortest string and its length NOT in the language over $\Sigma = \{a, b\}$ of the following regular expression.</td><td>02</td></tr>
<tr><td></td><td>1.3</td><td>What is the equivalent left linear grammar for the following given right linear grammar: $S \rightarrow abA|bB|aba, A \rightarrow b|aB|bA, B \rightarrow aB|aA$</td><td>02</td></tr>
<tr><td></td><td>1.4</td><td>Consider the context free grammars over the alphabet $\{a, b\}$ given below, where S is non – terminal
$X: S = aSa|aSb|\varepsilon$
$Y: S = aaS|bbS|\varepsilon$
What is the length of the shortest string which does not belongs to $L(X)$ but belongs to $L(Y)$.</td><td>02</td></tr>
<tr><td></td><td>1.5</td><td>Consider the CFG
$S \rightarrow XX$
$X \rightarrow XXX/bX/Xb/a$
Construct the parse tree and LMD for the string “$bbaaaab$”.</td><td>02</td></tr>
<tr><td></td><td>1.6</td><td>For the DFA given by transition table below, find its equivalent linear grammar. Here, A is the starting and D is final state.
<table><tr><td></td><td>a</td><td>b</td></tr><tr><td>A</td><td>B</td><td>D</td></tr><tr><td>B</td><td>A</td><td>C</td></tr><tr><td>C</td><td>D</td><td>B</td></tr><tr><td>D</td><td>C</td><td>A</td></tr></table></td><td>02</td></tr>
<tr><td></td><td>1.7</td><td>Write Chomsky hierarchy for formal languages.</td><td>02</td></tr>
<tr><td></td><td>1.8</td><td>Design a turing machine to increment a unary number where $w \in \{0\}^+$.</td><td>02</td></tr>
<tr><td></td><td>1.9</td><td>Mention the Transition function for TM with stay option.</td><td>01</td></tr>
<tr><td></td><td>1.10</td><td>Differentiate recursively enumerable language and recursive languages.</td><td>02</td></tr>
<tr><td></td><td>1.11</td><td>Consider homomorphism h from alphabet $\{0,1,2\}$ to $\{a,b\}$ defined by $h(0) = ab, h(1) = b$ and $h(2) = aa$. Find $h(0210)$ and $h^{-1}(ababb)$.</td><td>01</td></tr>
</table>

PART B

2	a	Define regular expressions. Give regular expression which generates the following languages over the alphabet $\Sigma = \{0,1\}$. i) Strings of a's and b's ending with b and has no substring aa. ii) Strings that do not end with 01.	05
	b	For the DFA shown in fig 2b, use the minimization algorithm to find a minimum DFA recognizing the same language using table filling algorithm.  Fig 2b	06
	c	Obtain a regular expression for the FA shown fig 2c using state elimination method.  Fig 2c	05
3	a	Convert the following grammar into GNF: $S \to AA\|0, A \to SS\|1$.	06
	b	Define a CFG and write a CFG for the language $L = \{a^n b^n c^m d^m \| n \geq 1, m \geq 1\} \cup \{a^n b^m c^m d^n, n \geq 1, m \geq 1\}$	05
	c	Describe the decision algorithms to answer the following questions: i) Given two finite automata M_1 and M_2, are there any strings that are accepted by both? ii) Given a FAM, is it a minimum state FA accepting the language $L(M)$?	05
		OR	
4	a	Convert the following grammar into CNF: $S \to AB\|aB, A \to aab\| \in, B \to bbA$.	04
	b	Using Pumping lemma for regular sets show that $L = \{a^n b^n \| n \geq 0\}$ is not regular.	04
	c	Define ambiguity in grammar. Is the following grammar ambiguous? Justify your answer : $S \to iCtS\|iCtSeS\|a, C \to b$	04
	d	Show that the regular languages are closed under complement operation.	04
5	a	Define push down automata and instantaneous description (ID). Construct a PDA to accept the language $L = \{w \| w \in (a,b)^* and\ n_a(w) = n_b(w)\}$ by a final state and show by ID that the string $abbaa$ is accepted.	07
	b	If L_1 is CFL and L_2 is regular language, then prove that $L_1 \cap L_2$ is a CFL.	04
	c	Convert the given CFG to its equivalent PDA. $S \to aABB\|aAA, A \to aBB\|a, B \to bBB\|A, C \to a$	05
		OR	

6 a State and prove pumping lemma for CFL. Show that $L = \{a^a b^n c^n | n \geq 0\}$ is not CFL. — 06

b Define the language accepted by PDA by final state and by empty stack. — 04

c Find the equivalent CFG for the PDA given below
$\delta(q_0, a, z_0) = \{(q_0, AZ_0)\}, \delta(q_0 a, A) = \{(q_0, AA)\}, \delta(q_0, b, A) = \{(q_1, \varepsilon)\}$,
$\delta(q_1, b, A) = \{(q_1, \varepsilon)\}, \delta(q_1, \varepsilon, Z_0) = \{(q_2, Z_0)\}$. — 06

7 a Obtain the left linear grammar for the DFA given below.

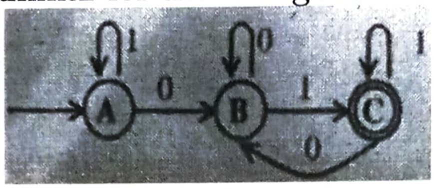


Fig 7a — 04

b Obtain a right linear grammar for the language $L = \{a^n b^n | n \geq 2, m \geq 3\}$. — 04

c Define context sensitive grammar. Give context sensitive grammar to generate the language $L = \{a^n b^n c^n | n \geq 1\}$. Show that the string "$aaabbbccc$" is generated. — 04

d Construct a DFA to accept the language generated by the following grammar: $S \rightarrow aA|\varepsilon, A \rightarrow aA|bB|\varepsilon, B \rightarrow bB|\varepsilon$. — 04

8 a Define post correspondence problem. Solve the PCP given below.

	List A	List B
i	w_i	x_i
1	1	111
2	10111	10
3	10	0

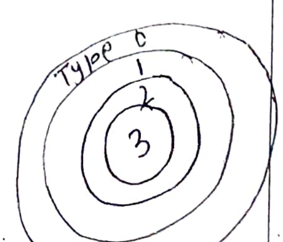
— 04

b If L_1 and L_2 are recursively enumerable languages over Σ, then prove that $L_1 \cap L_2$ and $L_1 \cup L_2$ are recursively enumerable. — 06

c During Turing machine and language of TM. Design a TM to perform $x + y$ where x and y are two positive integers. — 06

UG

COURSE CODE: 18CS46 **COURSE:** Theory of computation

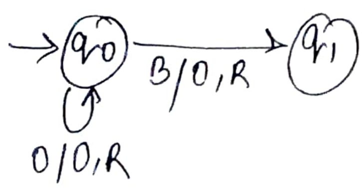
Question No	Part - A	Marks
1.1	Language will contain the strings such as {ε, 000, 11111, 00011111, 00111101 - - - } so strings accepted by the automata have to be of length 0, 3, 5, 8, 11, 13, 14, 16 - - - and so on. i.e $3x+5y$ ($x, y \geq 0$) modulo 3 gives remainder as (0, 1, 2) & modulo 5 gives remainder as (0, 1, 2, 3, 4). Hence 3×5 states in the state i.e there will be 15 states in the automata.	2M
1.2	3 [1M], bab [1M] can not be generated from given language.	2M
1.3	$S \to Ab/aba, A \to ab/Ab/Ba, B \to a/Aa/Ba$	2M
1.4	Here $L(X)$ = set of even palindrome & $L(Y) = (aa+bb)^*$ So string "aabb" or "bbaa" belongs to $L(Y)$ but does not belong to $L(X)$ ∴ shortest string is 4. String - 1M, length 4 - 1M. bb ∉ L(x) & bb ∈ L(Y) length = 2	2M
1.5	$S \to XX$ $\to bXX$ $\to bbXX$ $\to bbXXX$ $\to bbaXXX$ $\to bbaaaX$ $\to bbaaaXb$ $\to bbaaaab$ or $S \to XX$ $\to bXX$ $\to bbXX$ $\to bba\ X$ $\to bbaXX$ $\to bbaaXb$ $\to bbaaab$ LMD - 1M Parse tree - 1M	2M
1.6	$A \to aB/bD, B \to aA/bC$ [1M], $C \to aD/bB, D \to aC/bA/\epsilon$ [1M] or $A \to aB/bD/b, B \to aA/bC, C \to aD/bB/a, D \to aC/bA$	2M
1.7	Type 0 - Unrestricted grammar Type 1 - Context sensitive grammar Type 2 - Context free grammar Type 3 - Regular grammar 	2M

SCHEME AND SOLUTION APR/MAY-19

COURSE CODE: COURSE: UG

Question No | | **Marks**

1.8



I/p : B 000 B
O/p : 000 0 — 2M

1.9 δ is a transition function from $Q \times \Gamma$ to $Q \times \Gamma \times \{L, R, S\}$ indicating the TM may move towards left or right or stay in the same position after updating the symbol on the tape. — 2M

1.10 Difference between recursive enumerable language & recursive language. 1+1 — 2M

1.11 $h(0210) = abaabab$. $h^{-1}(ababb) = 021$ — 01M

Part-B

2.a. Definition of Regular expression — 1M

i) $(ab+b)^* (b+ab)$ — 2M

ii) $((0+1)^* (0+11)) + 1 + \epsilon$ — 2M

2.b.

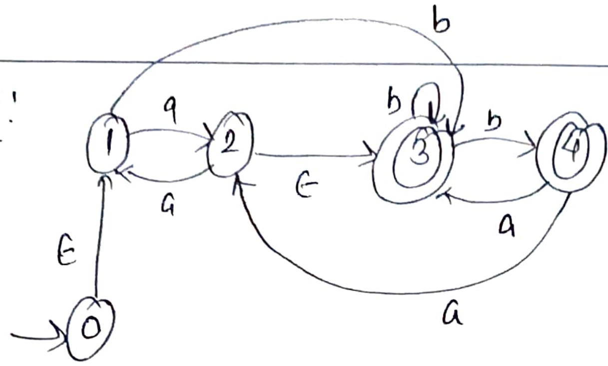
q_1	X			
q_2	X			
q_3	X			
q_4	X	X	X	X
	q_0	q_1	q_2	q_3

pairs (q_1, q_2), (q_1, q_3) and (q_2, q_3) are not at all marked & are considered to be indistinguishable state. — 4M

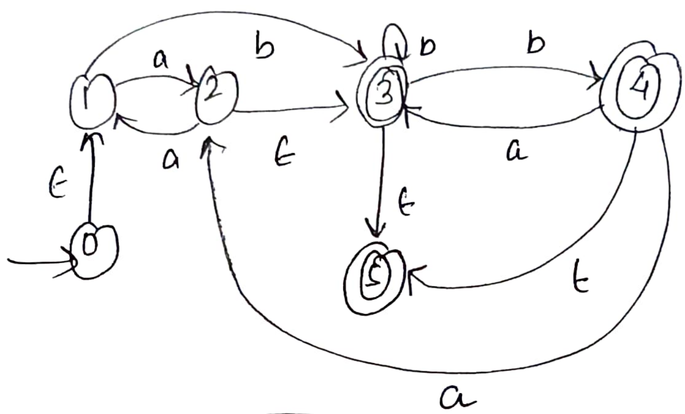
minimized transition table

states	Σ: a	b
$\rightarrow q_0$	(q_1, q_2, q_3)	(q_1, q_2, q_3)
(q_1, q_2, q_3)	(q_1, q_2, q_3)	q_4
$* q_4$	q_4	q_4

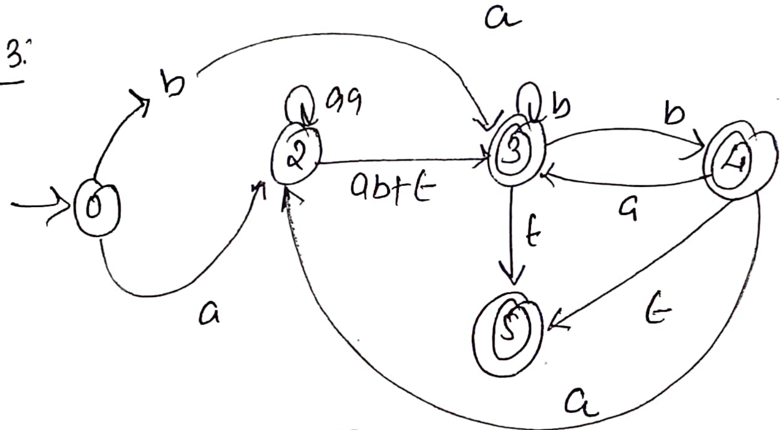
— 2M

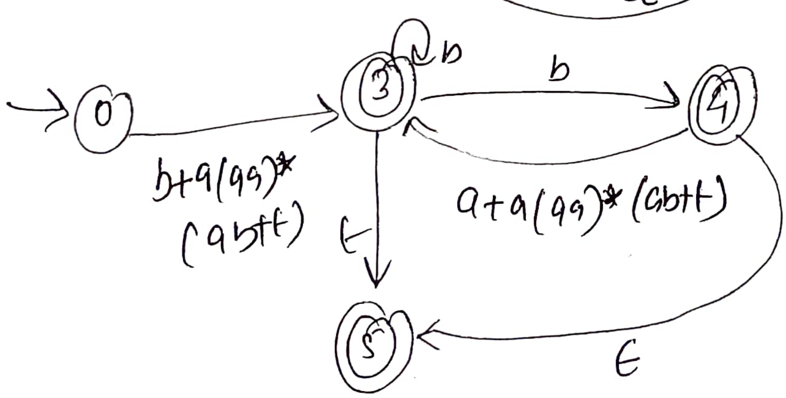
Question No		Marks
2.c.	Step 1: 	

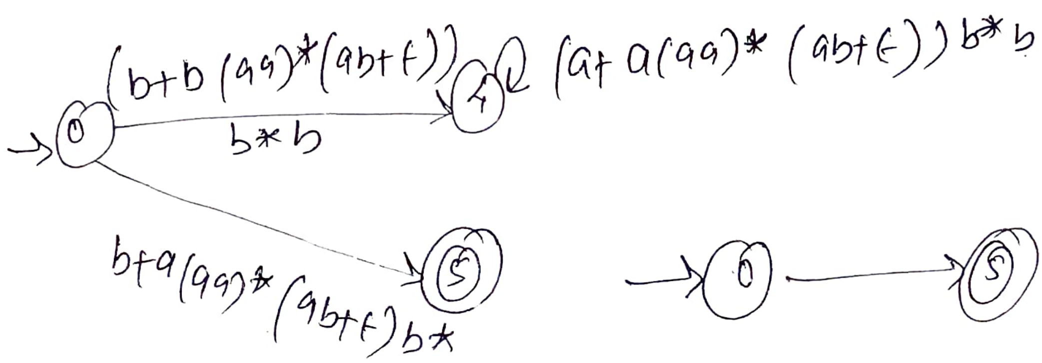
Step 2:



Step 3:





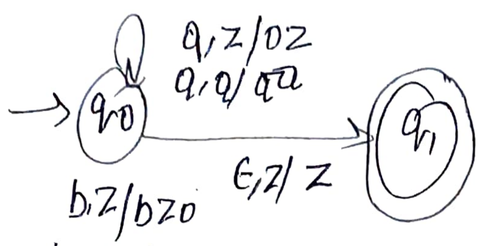


Question No		Marks
	$(b+a((aa)^*(ab+\epsilon))b^+ + ((b+a(aa)^*(ab+\epsilon))b^*b((a+a(aa)^*(ab+\epsilon))b^*b)^*(\epsilon+(a+a(aa)^*(ab+\epsilon))b^*)$	5M
3.a	$S \rightarrow AA / 0,\ A \rightarrow SS / 1$	
	$A_1 \rightarrow A_2A_2 / 0$ $A_2 \rightarrow A_1A_1 / 1$	1M
	Consider A_2 - production. $A_2 \rightarrow A_2A_2A_1 / 0A_1 / 1$	
	$A_2 \rightarrow 0A_1 / 1 / 0A_1Z / 1Z$ $Z \rightarrow A_2A_1 / A_2A_1Z$	2M
	Consider A_1 - production: $A_1 \rightarrow 0A_1A_2 / 1A_2 / 0A_1ZA_2 / 1ZA_2 / 0$	1M
	Consider Z production: $\cancel{Z} \rightarrow 0A_1A_1 / 1A_1 / 0A_1ZA_1 / 1ZA_1$ $Z \rightarrow 0A_1A_1Z / 1A_1Z / 0A_1ZA_1Z / 1ZA_1Z$	2M
3.b.	Definition of CFG	1M
	P_1: $S \rightarrow AB$ $A \rightarrow aAb / ab$ $B \rightarrow cBd / cd$ P_2: $S_2 \rightarrow aDd$ $D \rightarrow aDd / E$ $E \rightarrow bEc / bc$	1+1M
	P: $S \rightarrow S_1 / S_2$ $S_1 \rightarrow AB$ $A \rightarrow aAb / ab$ $B \rightarrow cBd / cd$ $S_2 \rightarrow DE$ $D \rightarrow aDd / E$ $E \rightarrow bEc / bc$	2M

<table>
<tr><th>Question No</th><th></th><th>Marks</th></tr>
<tr><td>3.C.</td><td>i.) Construct a finite automata M recognizing $L(M_1) \cap L(M_2)$</td><td>2.5M</td></tr>
<tr><td></td><td>ii) Apply the minimization algorithm to M and see if the number of state is reduced.</td><td>2.5M</td></tr>
<tr><td>4.a.</td><td>Grammar does not have any ε - production.
$A \to B$ is a Unit production.
$A \to 00A / 1A1$ after removing Unit production.</td><td>-1M</td></tr>
<tr><td>Elimination of ε-product -1
Rest 3m</td><td>$S \to AB1 / 0$
$A \to 00A / 1A1$, $B \to 1A1$
Now the grammar has to be converted into CNF.
$S \to ABA_1 \mid 0$
$A \to A_0A_0A \mid A_1AA_1$
$B \to A_1AA_1$
$A_1 \to 1$
$A_0 \to 0$</td><td>-1M</td></tr>
<tr><td></td><td>$S \to AD_1 \mid 0$, $A \to A_0D_2 \mid A_1D_3$, $B \to A_1D_3$, $A_1 \to 1$
$A_0 \to 0$, $D_1 \to BA_1$, $D_2 \to A_0A$, $D_3 \to AA_1$</td><td>2M</td></tr>
<tr><td>4.b.</td><td>Let $L = \{a^n b^n \mid n \geq 0\}$ is regular
& n be the integer of pumping lemma.
Let $x = 0^n 1^n$, $x = uvw$
$|x| = 2n > n$ Let $uv = 0^K$ where $K \leq n$
Let $V = 0^j$, $j > 0$</td><td></td></tr>
</table>

Question No		Marks
	Consider $uv^m w = (uv)\, v^{m-1}\, w$	
	$= 0^k (01)^{m-1} \cdot 0^{n-k} 1^n$	
	$= 0^{n+k-k} \cdot 1^n \, (01)^{m-1}$	1M
	$= 0^n 1^n (01)^{m-1}$ Here $J(m-1)+n > n$	
	Here $uv^m w \notin L$ $\therefore$ L is not regular.	
4.c	Definition of ambiguity in grammar	1M
	LMD 1: $S \Rightarrow iCtS \Rightarrow ibtS \Rightarrow ibtiCtSeS \Rightarrow ibtibtSeS \Rightarrow ibtibtaeS \Rightarrow ibtibtaea$	
	LMD 2: $S \Rightarrow iCtSeS \Rightarrow ibtSeS \Rightarrow ibtiCtSeS \Rightarrow ibtibtSeS \Rightarrow ibtibtaeS \Rightarrow ibtibtaea$	3M
	$\therefore$ Given grammar is ambiguous.	
4.d	Proof: Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA which accepts the language L. Now let us define the machine $M_1 = (Q, \Sigma, \delta, q_0, Q-F)$. The Non-final state of M are the final states of M_1 & final states of M are the non final states of M_1. So the language which is rejected by M is accepted by M_1. Also a language accepted by a DFA is regular. So the language accepted by M_1 is regular.	4M

Question No		Marks
5.a	Definition of PDA	1M
	Definition of ID	1M



b,Z/bZ0
b,b/bb
a,b/ε
b,a/ε

3M

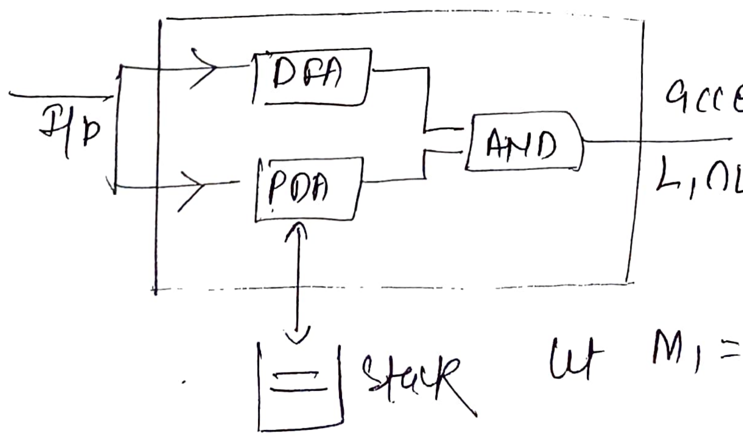
ID: "abbbaa"

$(q_0, abbbaa, Z) \vdash (q_0, bbbaa, aZ)$
$\vdash (q_0, bbaa, Z)$
$\vdash (q_0, baa, bZ_0)$
$\vdash (q_0, aa, bbZ)$
$\vdash (q_0, a, bZ_0)$
$\vdash (q_0, \epsilon, Z)$
$\vdash (q_1, \epsilon, Z)$

2M

Given string abbaa is rejected.

5.b



7M

let $M_1 = (Q_1, \Sigma, \Gamma, \delta_1, q_1, Z, A)$ be the PDA which accepts L_1 and $M_2 = (Q_2, \Sigma, \delta_2, q_2, f_2)$ be the DFA

Question No		Marks
	to accept L_2 Construct M_3 such that	
	$M_3 = (Q_1 \times Q_2, \Sigma_3, \Gamma, \delta, [q_1, q_2], Z, F_1 \times F_2)$	
	$\delta((p,q), a, A) = ((r,s), BC)$ where	
	$\delta_1(p, a, A) = (r, BC)$	
	$\delta_2(q, a) = s$	-3M
	the language accepted by M_3 is	
	$L(M_3) = L(M_1) \cap L(M_2)$	
5.c.	$\delta(q_0, \epsilon, Z) = (q_1, SZ)$	
	$\delta(q_1, \epsilon, S) = \{(q_1, aABB), (q_1, aAA)\}$	
	$\delta(q_1, \epsilon, A) = \{(q_1, aBB), (q_1, a)\}$	
	$\delta(q_1, \epsilon, C) = \{q_1, a)$	5M
	$\delta(q_1, \epsilon, B) = \{(q_1, bBB), (q_1, A)\}$	
	$\delta(q_1, a, a) = \{(q_1, \epsilon)\}$	
	$\delta(q_1, \epsilon, Z) = (q_2, Z)$	
6.a.	Statement of pumping lemma	
	Let L be the context free language & is infinite	
	let Z is long string $Z \in L$ so that $\|Z\| \geq n$	
	where n is some positive integer	
	$Z = uvwxy$	1M
	Such that $\|vwx\| \leq n$, $\|vx\| \geq 1$ then	
	$uv^i w x^i y \in L$ for $i = 0, 1, 2 --$	

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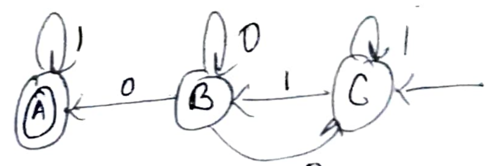
Question No		Marks
	proof of lemma	3M
	To prove $L = \{a^n b^n c^n \mid n \geq 0\}$ is not CFL	2M
6.b	Definition language accepted by PDA	2M
	Definition language accepted by empty stack	2M
6.C	Give the equivalent PDA using	
	i) $\delta(q_i, a, Z) = (q_j, AB)$ introduce the produ	
	ction $(q_i Z q_k) \rightarrow a(q_j A q_l)(q_l B q_k)$	2M
	where q_k & q_l will take all possible value from Q	
	ii) $\delta(q_i, a, Z) = (q_j, \epsilon)$ introduce the production	
	$(q_i Z q_j) \rightarrow a$	
	Equivalent CFG is below	4M
	$[q_0 Z_0 q_2] \rightarrow a[q_0 A q_1][q_1 Z_0 q_2] / \epsilon$	
	$[q_0 Z_0 q_1] \rightarrow a[q_0 A q_1][q_0 Z_0 q_1]$	
	$[q_0 A q_1] \rightarrow a[q_0 A q_1][q_1 A q_1) / b$	
	$[q_0 A q_1] \rightarrow b$	
	$[q_1 Z_0 q_2] \rightarrow \epsilon$	
	$[q_1 Z_0 q_1] \rightarrow \epsilon$	
7.a.	Reverse the DFA 	1M
	RLG $C \rightarrow 1C / 1B$, $B \rightarrow 0B / 0A / 0C$, $A \rightarrow 1A / \epsilon$	2M
	LLG $C \rightarrow C1 / B1$, $B \rightarrow B0 / A0 / C0$, $A \rightarrow A1 / \epsilon$	1M
7.b.	$V = \{S, A, B\}$, $T = \{a, b\}$, $P = \{ S \rightarrow aaA$, $A \rightarrow aA / bbbB$, $B \rightarrow bB / \epsilon \}$	4M
7.C.	Definition	1M
	CSG Grammar	2M

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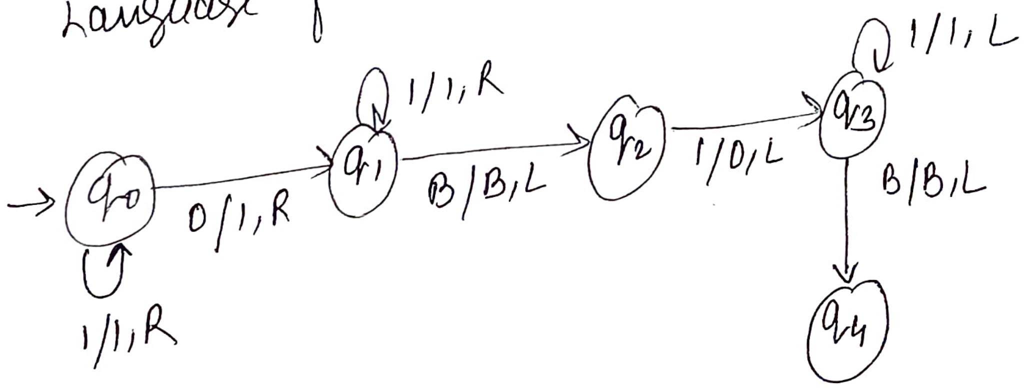
Question No		Marks
	String "aaabbbccc" generation	1M
8.a.	Definition of PCP	1M
	Solution to PCP is 2, 1, 1, 3 $w_2 w_1 w_1 w_3 = x_2 x_4 x_1 x_3$ 10 111 1110 = 10 111 111 0	3M
8.b.	i) Short note on Decidability & Un-decidability	2M
	ii) Counter machine	2M
	iii) Linear bounded Automata	2M
8.c.	Definition of TM	1M
	Language of TM	1M

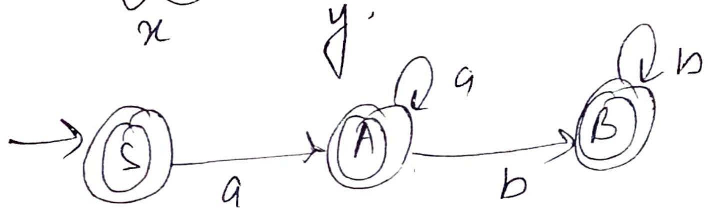


4M

I/p: $\underbrace{1111}_{x}$ 0 $\underbrace{111111}_{y}$ B

O/p: $\underbrace{1111}_{x}$ $\underbrace{111111}_{y}$ 0

7.d.



4M