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R. V. COLLEGE OF ENGINEERING

Autonomous Institution affiliated to VTU

V Semester B. E. Examinations Nov/Dec-18

Computer Science and Engineering

PROBABILITY, STATISTICS AND QUEUING THEORY (ELECTIVE)

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B.In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6.
3. Use of statistical table is permitted.

PART-A

1	1.1	Persons in a room are wearing badges marked1 through 10. 3 persons are chosen at random and asked to leave the room simultaneously and their badge numbers are noted. Find the probability that the largest badge number is 4.	02
	1.2	An importer is offered a shipment of machines for $Rs.$ 1,40,000. The probability that he will sell them for $Rs.$ 1,80,000, $Rs.$ 1,70,000and $Rs.$ 1,50,000 are 0.32,0.55 and 0.13. What is his expected profit?	02
	1.3	A discrete random variable x takes the values $-1,0,1$ with the probabilities $\frac{1}{8},\frac{3}{4},\frac{1}{8}$ respectively. Evaluate $P(\lvert X-\mu\rvert \geq 2\sigma)$ and compare it with the upper bound given by Chebyshev's inequality.	02
	1.4	Mention any four properties of a counting process.	02
	1.5	When do you say that a state of a Markov chain is Transient?	02
	1.6	Define the cross co-variance and cross co-relation coefficient of two processes $\{X(t)\}$ and $\{Y(t)\}$.	02
	1.7	Define birth and depth process.	02
	1.8	The local one person barber shop can accommodate a maximum of 6 people at a time (5 waiting and 1 getting hair cut). Customers arrive according to a Poison distribution with a mean 5 per hour. The barber cuts hair at an average rate of 4 per hour, exponentially. What is the average number of customers waiting in the system?	02
	1.9	Define extended Fibonacci generator.	02
	1.10	Define combined generator.	02

PART-B

2	a	Find the $Cov(5X + 3Y, 3X + 4Y)$ if $(E(X) = 3, E(Y) = 2,$ $E(X^2) = 9,$ $E(Y^2) = 10$, and $E(XY) = 7$.	04
	b	Two firms V and W are considered for a road building job which may or may not be awarded based on the bid. The probability that V gets the job is $\frac{3}{4}$ when W does not bid. The probability is $\frac{3}{4}$ that W will bid and if it does then the probability that V gets the job is $\frac{1}{3}$. Find the probability that i) V gets the job. ii) W did not bid if V got the job.	05
	c	If the joint PDF of X and Y is given by $f(x, y) = 24y(1 - x), 0 \leq y \leq x \leq 1$, then find $E(XY)$ and $E(Y/X)$.	07
3	a	The sugar content of the syrup in canned pouches is normally distributed. A random sample of 10 cans yields a sample standard deviation of 4.8 milligrams. Calculate a 98% two sided confidence interval for σ.	04
	b	State and prove Jenson's inequality.	04
	c	If immigrants to area A arrive at a Poison process at the rate of 5 per week, then: i) What is the probability that the elapsed time between the 12^{th} and 13^{th} arrival exceeds 3 weeks? ii) What is the expected time until the 15^{th} immigrants arrives?	04
	d	State and prove Markov's inequality. Hence derive Chebyshev's inequality.	04
		OR	
4	a	State and prove Schwarz's inequality and Cauchy's Schwarz's inequality.	06
	b	If $S_1^2 = 15750, n_1 = 5, S_2^2 = 10920, n_2 = 6.$ Test the null hypothesis $\sigma_1^2 = \sigma_2^2$ against $\sigma_1^2 \neq \sigma_2^2$ at 0.02 level of significance.	04
	c	Suppose we want to test the null hypothesis $\mu = 80$ against alternate hypothesis $\mu = 83$ on the basis of a random sample of size $n = 100$. Assume population standard deviation is 8.4. The null hypothesis is rejected if sample mean $\bar{X} > 82$, otherwise null hypothesis is accepted. What is the probability of type 1 error and find the probability of type 2 error.	06
5	a	Each year, a man trades his car for a new car in 3 brands of popular companies. If he has Alto, he trades it for Zen. If he has Zen, he trades it for Esteem. If he has Esteem , he is just likely to make it for a new Esteem, new Zen or new Alto. In 1996, he bought his first car Esteem . Find the probability that i) He has Esteem in 1998 ii) He has Zen in 1999 iii) He has Esteem in 1999.	06
	b	If $X(t) = P + Qt$ is a random process where P and Q are independent random variables with $E(P) = p, E(Q) = q, Var(P) = \sigma_1^2, Var(Q) = \sigma_2^2.$ Find $E(X(t)), R(t_1, t_2),\ C(t_1, t_2)$. Is the process X(t) stationary?	06

	c	Is the random process $X(t) = A\,sin(\omega t + \theta)$, where A and ω are constants and θ is uniformly distributed random variable in $(0,2\pi)$ wide sense stationary?	04
		OR	
6	a	Events occur according to a Poison process with rate 2 per hour. i) What is the probability that no event occur between $8AM$ to $9AM$? ii) What is the probability that two or more events occur between $6PM$ to $8PM$?	05
	b	Determine which states are transient and which are recurrent of the following Markov chain: $\begin{bmatrix} 0 & 0 & 1/2 & 0 & 1/2 \\ 1/4 & 1/2 & 1/4 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1/2 & 1/2 \end{bmatrix}$	06
	c	A house wife purchases 3 types of cereals $C_{1,}C_2, C_{3,}$ every month. It is known that she never buys the cereal in successive months. If she buys cereal C_1 the first month, then the next month she would buy only cerealC_3. However in case she buys cereal C_2 or $C_{3,}$the first month, then the next month she is twice likely to buy cereal $C_{1,}$ as other brand. Obtain the transition probability matrix and determine how often she would buy each of cereal in the long run.	05
7	a	Car will arrive at a petrol pump, having one petrol unit, in a Poison process at the average rate of 10 cars per hour. The service time is distributed exponentially with a mean of 3 minutes. Find i) Average number of cars in the system. ii) Average waiting time in the queue. iii) Average number of cars in the queue. iv) Response time.	08
	b	A barber shop has two barbers and 3 extra chairs for customers. Assume that each customer arrive in Poison process at the average rate of 2 per hour and each barber services customer according to an exponential distribution with the mean of 20 min. Further if a customer arrives and there are no empty chairs in the shop, he will leave. i) What is the probability that the shop is empty? ii) What is the expected number of customers in the shop? iii) What is the expected number of customers in the queue? iv) What is the response time?	08
8	a	Generate random numbers using seed $X_0 = 1$ in the following linear congruential generator: $X_n = (5)mod\ 32$. At $\alpha = 0.05$ level, can we say that the numbers are uniformly distributed using KS method?[Given $K_{0.95,7} = 1.1537$,$K_{0.95,8} = 1.1586$].	10
	b	Write the desired properties of good generators.	03
	c	Discuss about multiplicative LCG.	03