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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU)

V Semester B. E. Fast Track Examinations Oct-2020

Computer Science and Engineering

PROBABILITY, STATISTICS AND QUEUING THEORY (ELECTIVE)

Time: 03 Hours ***Maximum Marks: 100***

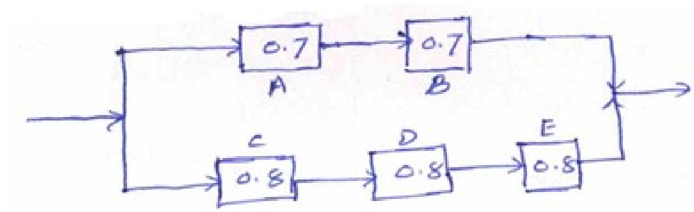
Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6
3. Use of statistical table permitted.

PART-A

1	1.1	The probability that regularly scheduled flight departs on time is 0.83.The probability that it arrives on time is 0.82 and the probability that it arrives and departs on time is 0.78. Find the probability that a plane: a) Arrives on time given that it departed on time and b) Departs on time given that it arrived on time.	02
	1.2	Two defective tubes get mixed up with two good ones. What is the probability that the second defective tube is obtained in the third test?	02
	1.3	In certain experiments, the error X made in determining the solubility of a substance is a random variable having the uniform density function in the interval $(-0.023, 0.023)$. What is the probability that such an error will be between 0.01 and 0.0145.	01
	1.4	A random sample of 10 observations is taken from a normal population having the variance 42.5. Find approximately the probability of obtaining a sample standard deviation S between 3.14 and 8.94.	02
	1.5	Suppose that people immigrate into a territory at a Poisson rate 5 per day. i) What is the expected time until the 12^{th} immigrant arrives? What is the probability that the elapsed time between 14^{th} and 15^{th} arrival exceeds 4 days?	02
	1.6	A student study habits are as follows: If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 40% sure not to study next night as well. In the long run, how often does he study?	02
	1.7	Discuss two methods of Description of a random process.	02
	1.8	Write four relations among $E(N), E(W), W(N_S)$ and $W(N_Q)$.	02
	1.9	Let $\{X(t)\}$ be a Poisson process. Then: a) What is $P(X(t) = x)$? and b) Is $\{X(t)\}$ a covariance stationary?	02
	1.10	Discuss Birth-Death processes.	02
	1.11	Discuss about the extended Fibonacci generator.	01

PART-B

2	a	Two random variables X and Y have joint density function given by $f(x,y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2, 0 \le y \le 1$. Find: i) $P(X > 1/Y < 0.5)$ and ii) $P(X + Y \le 1)$	07
	b	An electronic system consists of five components as illustrated in the following figure. The probability of working of each component is also shown in figure. Find the probability that i) Entire system works and ii) Component A does not work given that the system works. (Assume that the components fail independently) 	06
	c	Find the moment generating function of a binomial distribution.	03
3	a	The following are the weights, in decagrams of 10 packages of grass seed distributed by a certain company: 46.4, 46.1, 45.8, 47.0, 46.1, 45.9, 45.8, 46.9, 45.2 and 46.0. Find a 95% confidence interval for the variance of all such packages of grass seed distributed by this company, assuming a normal population.	07
	b	A random sample of 100 teachers in a large metropolitan area revealed a mean weekly salary of \$487 with a standard deviation of \$48. With what degree of confidence can we assert that the average weekly salary of all teachers in the metropolitan area is between \$472 and \$502?	06
	c	Find Chertoff's bounds for exponential distribution.	03
		OR	
4	a	To test a paint manufacturer's claim that the average drying time of his new "fast-drying" paint is $\mu = 20$ minutes, a 'random sample' of 36 boards is painted with his new paint and his claim is rejected if the mean drying time $\bar{X} > 20.5$ minutes. Find: i) The probability of type I error; ii) The probability of type II error, when $\mu = 21$ minutes (Assume that $\sigma = 2.4$ minutes)	07
	b	State and prove the following: i) Schwarz's inequality and ii) Cauchy-Schwarz's inequality.	06
	c	If immigrants to area A arrive at a Poisson manner at the rate of 10 per week, and if each immigrant is of Bangladesh decent with probability 7/12, then what is the probability that 1 Bangladesh decent will immigrate to area A during the 6 weeks' time? What is the probability that no non-Bangladesh decent will immigrate to area *A* during the month February?	03

5	a	A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 5 or 6 appears. Find i) the probability that he takes a car on the third day and ii) the probability that he catches a train in the long run.	07
	b	Define the following: i) Strongly stationary process, ii) Counting process and iii) Ergodicity process	06
	c	If $\{X(t)\}$ is a WSS with autocorrelation $R(\tau) = Ae^{-\alpha\|\tau\|}$, then determine the second order moment of $\{X(8) - X(5)\}$.	03
		OR	
6	a	Is the random process $X(t) = Asin(\omega t + \theta)$, where A and ω are constants and θ is uniformly distributed random variable in $(0,2\pi)$ a WSS?	08
	b	Is the Markov chain with transition matrix: $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 0 \end{bmatrix}$ Irreducible or regular?	05
	c	Define the following: i) Cross-correlation of two processes, ii) Cross-covariance of two processes and iii) Cross-correlation co-efficient of two processes.	03
7	a	On a network gateway, measurements show that the packets arrive at a mean rate of 125 packets per second(pps) and the gateway takes about 2 milliseconds to forward them. Using an $M/M/1$ model, analyze the gateway. What is the probability of buffer overflow if the gateway had only 13 buffers?	08
	b	Students arrive at the university computer in a poisson manner at an average of 10 per hour .Each student spends an average of 20 minutes at the terminal, and the time can be assumed to be exponentially distributed. The centre currently has 5 terminals. Find: i) An average number of students in the center, iii) Mean response time and mean waiting time.	08
8	a	What are the desired properties of the generator function? Name four types of random number generators.	05
	b	1000 random numbers were generated using the generator $x_n = 125x_{n-1} + mod 2^{12}$ with seed $x_0 = 1$(Mixed LCG).The numbers so obtained were categorized in a histogram using 10 sets at intervals of 0.1 between 0 and 1. At $\alpha = 0.05$ level, can we say that numbers are independent identical uniform distribution $w(0,1)$?	06
	c	Find the period of the generator $x_n = 5x_{n-1} + mod 2^5$ with seed: i) $x_0 = 1$, ii) $x_0 = 2$ and iii) $x_0 = 7$	05

Table for question 8 b:

$Cell$	1	2	3	4	5	6	7	8	9	10
$Observed\ values$	100	96	98	85	105	93	97	125	107	94