

UNIT-IV

Deterministic push down Automata

Left Recursion

if occurs on the left hand side we will have a variable

Ex:-

$$A \rightarrow A\alpha$$

$$A \rightarrow A\alpha_1 / A\alpha_2 / A\alpha_3 / \dots / A\alpha_n /$$

$$B_1 / B_2 / B_3 / \dots / B_m$$

then replace it with new variable A'

$$A \rightarrow B_1 A' / B_2 A' / B_3 A' / \dots / B_m A'$$

$$A' \rightarrow \alpha_1 A' / \alpha_2 A' / \dots / \alpha_n A' / \epsilon$$

- a) Eliminate the left recursion from the following grammar

$$E \rightarrow E + T / T$$

$$T \rightarrow T * F / F$$

$$F \rightarrow (E) / id$$

→

given	substitution	without left recursion
$E \rightarrow E + T / T$ $A = E$ $\alpha_1 = +T$	$A' = E$ $B_1 = T$ $\alpha_1 = +T$	$E \rightarrow TE'$ $A \rightarrow B_1 A'$ $E' \rightarrow +TE' / \epsilon$ $A' \rightarrow \alpha_1 A' / \epsilon$
$T \rightarrow T * F / F$ $A = T$ $B_2 = F$ $\alpha_2 = *F$	$A = T$ $B_2 = F$ $\alpha_2 = *F$	$A \rightarrow B_2 A'$ $T \rightarrow FT'$ $A' \rightarrow \alpha_2 A' / \epsilon$ $T' \rightarrow *FT' / \epsilon$

Third production is not in left recursive
so we will write it as it is.

$$F \rightarrow (E)/id$$

Grammar $G = (V, T, P, S)$

$$V = \{E, E', T, T', F\}$$

$$T \rightarrow \{+, *, id, (,)\}$$

$$P = \{ E \rightarrow TE' \}$$

$$E' \rightarrow TE' / \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' / \epsilon$$

$$F \rightarrow (E)/id$$

}

here start symbol S is E

a) $S \rightarrow Ab/a$

$$A \rightarrow Ab/Sa$$

$\rightarrow S \rightarrow Ab$

$S \rightarrow Sab$ so left recursion

\rightarrow
 $A \rightarrow Ab/Sa$

$$S \rightarrow Ab$$

$$A \rightarrow Ab/(Ab/a)a$$

$$S \rightarrow Sab$$

$$A \rightarrow Ab/Ab/a/a$$

The non terminal S even though there is no immediate left recursion since $S \rightarrow Ab$ so $S \rightarrow Sab$ when A is replaced by Sa ($\because A \rightarrow Sa$). So in

order to eliminate the left recursion, substitute S in the A production can eliminate indirect left recursion.

The productions we get are

$$S \rightarrow Sab$$

$$A \rightarrow Ab/Aba/aa$$

given	substitution	without left recursion
$A \rightarrow Ab/Aba/aa$ $\alpha_1 = a$ $\alpha_2 = ba$ $B_1 = aa$	$A = A$ $\alpha_1 = b$ $\alpha_2 = ba$ $B_1 = aa$	$A \rightarrow B_1 A'$ $A \rightarrow aaA'$ $A' \rightarrow bA'/baA'/\epsilon$

Q1) $A \rightarrow ABd/Aa/a$
 $B \rightarrow Be/b$

→

given	substitution	without left recursion
$A \rightarrow ABd/Aa/a$ $\alpha_1 = B$ $\alpha_2 = a$	$A \rightarrow A$ $\alpha_1 \rightarrow Bd$ $\alpha_2 \rightarrow a$ $B_1 \rightarrow a$	$A \rightarrow B_1 A'$ $A \rightarrow aA'$ $A' \rightarrow \alpha_1 A'/\alpha_2 A'/\epsilon$ $A' \rightarrow BdA'/aA'/\epsilon$
$B \rightarrow Be/b$ $\alpha_1 = B$ $B_1 = b$	$A = B$ $\alpha_1 = e$ $B_1 = b$	$B \rightarrow bB'$ $A \rightarrow B_1 A'$ $B' \rightarrow eB'/\epsilon$ $A' \rightarrow \alpha_1 A'/\epsilon$

grammar $G = (V, T, P, S)$

$$V = \{A, A', B, B'\}$$

$$T = \{a, d, e, b\}$$

$$P = d \quad A \rightarrow aA'$$

$$A' \rightarrow BdA' / aA' / \epsilon$$

$$B \rightarrow bB'$$

$$B' \rightarrow \epsilon B' / \epsilon$$

}

here the start symbol S is A

$$i) S \rightarrow (L) / a$$

$$L \rightarrow L, S / S$$

$$ii) E \rightarrow E + E / E * E / a$$

$$iii) A \rightarrow AA\alpha / \beta$$

i) here we need not substitute S because substituting S will not lead to left recursion

so $S \rightarrow (L) / a$ will be as it is. i.e.

$L \rightarrow L, S / S$ is a left recursion

so

Given

$$L \rightarrow L, S / S$$

 $\alpha_1 = S$
 $\beta_1 = S$

substitution

$$A = L$$

$$\alpha_1 = S$$

$$\beta_1 = S$$

without left recursion

$$L \rightarrow SL'$$

$$A \rightarrow B.A'$$

$$A' \rightarrow \alpha.A' / \epsilon$$

$$L' \rightarrow S, L' / \epsilon$$

ii) Given

$$E \rightarrow E + E / E * E$$

 $\alpha_1 = +$
 $\alpha_2 = *$
 $\beta_1 = a$

substitution

$$A = E$$

$$\alpha_1 = +E$$

$$\alpha_2 = *E$$

$$\beta_1 = a$$

without left recursion

$$E \rightarrow aE'$$

$$E' \rightarrow +EE' / *EE' / \epsilon$$

Elimination of Useless Symbols

$$S \rightarrow ABCa/bD$$

$$A \rightarrow aA/a$$

$$B \rightarrow bB$$

$$D \rightarrow ab/Ea$$

$$E \rightarrow ac/d$$

→ here we can see that there are few useless symbols which we must eliminate using some standard theorem.

old variables OV	New Variables NV	production
initially no old variables	the variables generating only terminals A, D, E are our new variables these new variables will be our old variables in next step write old variables followed by any new variable	First find only productions generating terminals $\left. \begin{array}{l} A \rightarrow a \\ D \rightarrow ab \\ E \rightarrow d \end{array} \right\} \text{terminal}$
A, D, E look for these variables in productions and select them	A, D, E, S new var	$\begin{array}{l} S \rightarrow aA \\ A \rightarrow aA \\ D \rightarrow E \end{array}$ support use now not listed $D \rightarrow DE$ so we should not list it
A, D, E, S now search only for productions with S	no new variables A, D, E, S	no productions —

Now write Grammar,

$$G_1 = (V_1, T_1, P_1, S)$$

$$V_1 = \{S, A, D, E\} \text{ variables}$$

$$T_1 = \{a, d, b\} \text{ terminals}$$

$P_1 = \{ S \rightarrow aA \}$ productions

$A \rightarrow aA/a$

$D \rightarrow Ea/ab$

$E \rightarrow d$

²
Start symbol is S
Now we need to again filter it

P'	T'	V'
$S \rightarrow aA$	a	S Start symbol
$A \rightarrow aA/a$	a	S, A (new var)
		S, A no new var so stop

Grammar $G_1 = (V', T', P', S)$

$V' = \{ S, A \}$ variables

$T' = \{ a \}$ terminals

$P' = \{ S \rightarrow aA, A \rightarrow aA/a \}$ productions

S is the start symbol

now we have removed all useless productions

2) $S \rightarrow aA/a/bb/cC$

$A \rightarrow aB$

$B \rightarrow a/Aa$

$C \rightarrow cD$

$D \rightarrow ddd$

\rightarrow	OV	NV	productions
	S, B, D		$S \rightarrow a$
			$B \rightarrow a$
			$D \rightarrow ddd$

S, B, D

S, B, D, A

 $S \rightarrow Bb$ $A \rightarrow aB$ $C \rightarrow CDX$ So don't take ^{and} ^{listed}

S, B, D, A

S, B, D, A

 $S \rightarrow aA$ $B \rightarrow Aa$

S, B, D, A

S, B, D, A

—

Let Grammar $G_1 = (V_1, T_1, P_1, S)$ $V_1 = \{S, A, B, D\}$ $T_1 = \{a, b, d\}$ $P_1 = \{S \rightarrow aA/Bb/a$ $A \rightarrow aB$ $B \rightarrow Aa/a$ $D \rightarrow ddd$ $\}$ S is the start symbol.

Now filter it

P'	T'	V'
—	—	S
$S \rightarrow aA/Bb/a$	a, b	S, A, B
$A \rightarrow aB$	a, b	S, A, B
$B \rightarrow aA/a$	a, b	S, A, B

Let $G_1' = (V', T', P', S)$ $V' = \{S, A, B\}$ $T' = \{a, b\}$

$$P' = \{ S \rightarrow aA/Bb/a$$

$$A \rightarrow aB$$

$$B \rightarrow Aa/a$$

}

S is start symbol

$$3) S \rightarrow ABa/BC$$

$$A \rightarrow ac/Bcc/a$$

$$B \rightarrow bbc$$

$$C \rightarrow CA$$

→

OV

NV

Productions

A, B

$$A \rightarrow ac/a$$

$$B \rightarrow bbc$$

A, B

A, B, S

$$S \rightarrow AB \quad S \rightarrow BC$$

$$A \rightarrow Bcc$$

$$C \rightarrow CA \quad \text{not filter}$$

A, B, S

A, B, S

-

Let Grammar $G' = (V', T', P', S')$

$$V' = \{ A, B, S \}$$

$$T' = \{ a, b, c \}$$

$$P' = \{ S \rightarrow AB$$

$$A \rightarrow (Bcc/ac/a$$

$$B \rightarrow bbc$$

}

Now use filter it

elimination of ϵ -production

$$S \rightarrow ABCa/bD$$

$$A \rightarrow Bc/b$$

$$B \rightarrow b/\epsilon$$

$$C \rightarrow c/\epsilon$$

$$D \rightarrow d$$

old variable	new variable	Production
OV	NV	
-	B, C	$B \rightarrow \epsilon$ ^{production having} $C \rightarrow \epsilon$ ^{ϵ}
B, C	A, B, C	$A \rightarrow Bc$
A, B, C	A, B, C	do not select productions having terminals
		-

$\therefore A, B, C$ are nullable variables

given production	Production without ϵ
$S \rightarrow ABCa$	$ABCa/ABa/Bca/Aca/Aa/Ba/Ca/a$
$S \rightarrow bD$	bD/b
$A \rightarrow Bc/b$	$Bc/Bc/b$
$B \rightarrow b/\epsilon$	b
$C \rightarrow c/\epsilon$	c
$D \rightarrow d$	d

Let $G = (V, T, P, S)$

$V = \{S, A, B, C, D\}$

$T = \{a, b, c, d\}$

$P = \{ABCa/ABa/Bca/ACa/Aa/Ba/Ca/a$
 bD/b

$BC/B/C/b$

b

c

d

$\}$

S is the start stat.

2) $S \rightarrow BAAB$

$A \rightarrow OA2/2AO/\epsilon$

$B \rightarrow AB/1B/\epsilon$

\rightarrow

OV	NV	Production
-	A, B	$A \rightarrow \epsilon$ $B \rightarrow \epsilon$
A, B	A, B, S	$S \rightarrow BAAB$ $B \rightarrow AB$
A, B, S	A, B, S	-

A, B, S are nullable variables

Given Production

$S \rightarrow BAAB$

Production without ϵ

don't eliminate terminals
 $BAAB/BAAB/B/AB/AB/$
 $AB/BAB/BB$

$A \rightarrow OA2/2AO/\epsilon$

$OA2/O2/2AO/2O$

$B \rightarrow AB/1B/\epsilon$

$AB/A/B/1B/1$

Let $G = (V, T, P, S)$

$V = \{A, B, S\}$

$T = \{0, 1, 2\}$

$P = \{BAAB/BAA/B/AAB/AA/A/AB/BAB/BB,$

$0A2/02/2A0/20,$

$AB/A/B/1B/1$

$\}$

$T = \{0, 1, 2\}$

S is the start state

3) $S \rightarrow XYX$

$X \rightarrow 0X/\epsilon$

$Y \rightarrow 1Y/\epsilon$

\rightarrow

OV	NV	Production
-	X, Y	$X \rightarrow \epsilon$
		$Y \rightarrow \epsilon$
X, Y	X, Y, S	$S \rightarrow XYX$
		ϵ
X, Y, S	X, Y, S	

here X, Y, S are nullable variables.

given Production	Production without ϵ
$S \rightarrow XYX$	$XYX/XY/X/YX/XX/Y$
$X \rightarrow 0X$	$0X/0/X$
$Y \rightarrow 1Y$	$1Y/Y/1$

Let $G = (V, T, P, S)$

$V = \{S, X, Y\}$

$T = \{0, 1\}$

$P = \{XX/XY/X/YX/XX/Y,$

$0X/0/X,$

there S is
the start
state.

4) $S \rightarrow aAbA$

$S \rightarrow SA$

$A \rightarrow Y/\epsilon$

$Y \rightarrow bY/b$

→

OV

NV

Production

-

A

 $A \rightarrow \epsilon$

A

A

-

here A is the nullable variable

given production

$S \rightarrow aAbA$

Production without

$aAbA/aAb/AbA/ab$

$S \rightarrow SA$

$SA/S/A$

$A \rightarrow Y/\epsilon$

Y

production should not be repeated so Y is not

$Y \rightarrow bY/b$

bY/b

←

Let $G = (C, V, T, P, S)$

$V = \{S, A, Y\}$ $T = \{a, b\}$

$P = \{aAbA/aAb/AbA/ab,$

$SA/S/A,$

$Y,$

$bY/Y/b$

}

here 'S' is the start state

5) $S \rightarrow AB$

$A \rightarrow aAA/\epsilon$

$B \rightarrow bBB/\epsilon$

OV	NV	Production
-	A, B	$A \rightarrow \epsilon$ $B \rightarrow \epsilon$
A, B	A, B, S	$S \rightarrow AB$
A, B, S	A, B, S	-

Here A, B, S are nullable variables.

given Production	Production without ϵ
$S \rightarrow AB$	$AB/A/B$
$A \rightarrow aAA/\epsilon$	$aAA/aA/AA$
$B \rightarrow bBB/\epsilon$	$bBB/bB/BB$

Let $G = (V, T, P, S)$

$V = \{S, A, B\}$ $T = \{a, b\}$

$P = \{$

$AB/A/B,$

$aAA/aA/AA,$

$bBB/bB/BB$

$\}$

$S = \text{start state}$

- 6) $S \rightarrow ACB/cbB/Ba$
 $A \rightarrow da/Bc$
 $B \rightarrow gc/\epsilon$

OV	NV	Production
-	B	$B \rightarrow \epsilon$
B	B	-

here B is the nullable variable

given Production

$S \rightarrow AcB/cbB/Ba$

$A \rightarrow da/Bc$

$B \rightarrow gc/\epsilon$

Production ϵ without

$AcB/Ac/cB/A/c/B/cbB/bB/Ba/a$

$da/d/Bc$

gc/g

Let $G = (V, T, P, S)$

$V = \{S, A, B\}$

$T = \{a, b, c, d\}$

$P = \{AcB/Ac/cB/A/c/B/cbB/bB/Ba/a, da/d/Bc, gc/g\}$

$S = \text{start state}$

Eliminating unit production

1) $S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow c/b$

$C \rightarrow D$

$D \rightarrow E/bC$

$E \rightarrow d/Ab$

→ step 1: Write non-unit production

$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow b$

$D \rightarrow bC$

$E \rightarrow d/Ab$

unit productions are
it tends to a

step 2: Write unit productions

$B \rightarrow C$

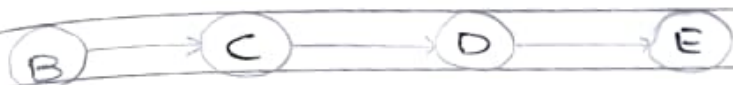
$C \rightarrow D$

single var

single variable
like A, B

$$D \rightarrow E$$

step 3:- Draw dependency graph



step 4:- $D \rightarrow E$ also $E \rightarrow d/Ab$
 $\therefore D \rightarrow d/Ab/bc$

* $C \rightarrow D$ also $D \rightarrow d/Ab/bc$
 $\therefore C \rightarrow d/Ab/bc$

* $B \rightarrow C$ also $C \rightarrow d/Ab/bc$
 $\therefore B \rightarrow d/Ab/bc/b$

step 5:- The final grammar is

$$S \rightarrow AB$$

$$A \rightarrow a$$

$$B \rightarrow d/Ab/bc/b$$

$$C \rightarrow d/Ab/bc$$

$$D \rightarrow d/Ab/bc$$

$$E \rightarrow d/Ab$$

2) $S \rightarrow AO/B$

$B \rightarrow A/11$ zero so it's not a variable: is a terminal

$A \rightarrow O/12/B$

→ step 1:- non-unit production

$$S \rightarrow AO$$

$$B \rightarrow 11$$

$$A \rightarrow O/12$$

step 2:- unit production

$$S \rightarrow B$$

$$B \rightarrow A$$

$$A \rightarrow A$$

step 3:- dependency graph.



step 4:-

• $B \rightarrow A$

also $A \rightarrow 0/12/B$ and $B \rightarrow 11$

$\Rightarrow A \rightarrow 0/12/11$

$\therefore B \rightarrow 0/12/11$

...

• $S \rightarrow B$ also $B \rightarrow 0/12/11$ and $S \rightarrow AD$

$\therefore S \rightarrow 0/12/11/AD$

...

• $A \rightarrow B$ also $B \rightarrow 0/12/11$

no $\therefore A \rightarrow 0/12/11$

step 5:- The final grammar is

$S \rightarrow 0/12/11/AD$

$B \rightarrow 0/12/11$

$A \rightarrow 0/12/11$

...

3) $S \rightarrow Aa/B/Ca$

$B \rightarrow aB/b$

$C \rightarrow Db/D$

$D \rightarrow E/d$

$E \rightarrow ab$

→ step 1:- Non-unit production

$S \rightarrow Aa/Ca$

$B \rightarrow aB/b$

$C \rightarrow Db$

$E \rightarrow ab$

Step 2:- unit production

$$S \rightarrow B$$

$$C \rightarrow D$$

$$D \rightarrow E$$

Step 3:- Dependency graph

$$(S) \rightarrow (B)$$

$$(C) \rightarrow (D) \rightarrow (E)$$

Step 4:- Since $S \rightarrow B$ also $B \rightarrow aB/b$

$$\therefore S \rightarrow aB/b/Aa/Ca$$

• $D \rightarrow E$ also $E \rightarrow ab$

$$\therefore D \rightarrow ab/d$$

• $C \rightarrow D$ also $D \rightarrow ab/d$

$$\therefore C \rightarrow ab/d/Db$$

Step 5:- The final grammar is

$$S \rightarrow Aa/Ca/aB/b$$

$$B \rightarrow aB/b$$

$$C \rightarrow ab/d/Db$$

$$D \rightarrow ab/d$$

$$E \rightarrow ab$$

4) $S \rightarrow AB$

$$A \rightarrow a$$

$$B \rightarrow C/b$$

$$C \rightarrow D$$

$$D \rightarrow E$$

$$E \rightarrow a$$

→ Step 1:- Non-Unit productions

$$S \rightarrow AB$$

$$A \rightarrow a$$

$$B \rightarrow b$$

$$E \rightarrow a$$

Step 2:- unit productions

$$B \rightarrow C$$

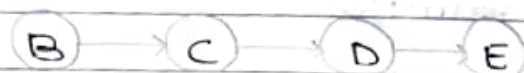
$$C \rightarrow D$$

$$D \rightarrow E$$

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step 3:- Dependency graph



step 4:- * $D \rightarrow E$ also $E \rightarrow a \therefore D \rightarrow a$

* $C \rightarrow D$ also $D \rightarrow a$

So $\therefore C \rightarrow a$

* $B \rightarrow C$ also $C \rightarrow a$

$\therefore B \rightarrow a/b$

step 5:- The final grammar is

$$S \rightarrow AB$$

$$A \rightarrow a$$

$$B \rightarrow a/b$$

$$C \rightarrow a$$

$$D \rightarrow a$$

$$E \rightarrow a$$

s) $S \rightarrow S+T/T$, $T \rightarrow T * F/F$, $F \rightarrow (S)/a$ () are terminals

→ step 1:- Non-unit production

$$S \rightarrow S+T$$

$$T \rightarrow T * F$$

$$F \rightarrow (S)/a$$

step 2:- unit production :

$$S \rightarrow T$$

$$T \rightarrow F$$

step 3:- dependency graph



step 4:- * $T \rightarrow F$ also $F \rightarrow (S)/a$

$\therefore T \rightarrow (S)/a / T * F$

• $S \rightarrow T$ also $T \rightarrow (S)/a/T * F$

$\therefore S \rightarrow (S)/a/T * F/S + T$

step 5:- The final grammar is

$S \rightarrow (S)/a/T * F/S + T$

$T \rightarrow (S)/a/T * F$

$F \rightarrow (S)/a$

6) $S \rightarrow A/B/C$

$A \rightarrow aAa/B$

$B \rightarrow bB/bb$

$C \rightarrow aCaa/D$

$D \rightarrow baD/abD/aa$

→ step 1:- non-unit productions

$A \rightarrow aAa$

$B \rightarrow bB/bb$

$C \rightarrow aCaa$

$D \rightarrow baD/abD/aa$

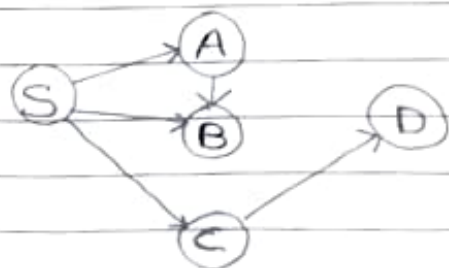
step 2:- unit productions

$S \rightarrow A/B/C$

$A \rightarrow B$

$C \rightarrow D$

step 3:- dependency graph



Step 4:-

* $A \rightarrow B$ also $B \rightarrow bB/bb$

$\therefore A \rightarrow bB/bb/aAa$

* $C \rightarrow D$ also $D \rightarrow baD/abD/aa$

$\therefore C \rightarrow baD/abD/aa/aCaa$

* $S \rightarrow A/B/C$

$\therefore S \rightarrow bB/bb/aAa/baD/abD/aa/aCaa$

Step 5:- The final grammar is

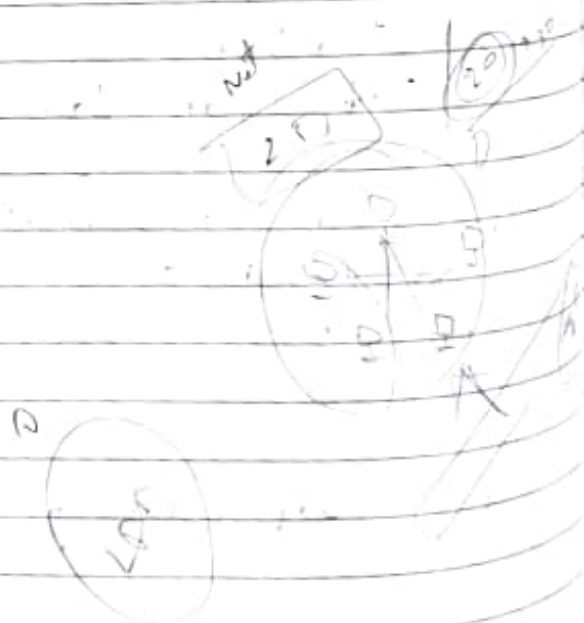
$S \rightarrow bB/bb/aAa/baD/abD/aa/aCaa$

$A \rightarrow aAa/bb/bB$

$B \rightarrow bB/bb$

$C \rightarrow aCaa/baD/abD/aa$

$D \rightarrow baD/abD/aa$



Chomsky Normal form (CNF)

step 1:- Eliminate the start symbol from right hand side create a new production $S_0 \rightarrow S$ where S_0 is the new start symbol

step 2:- Get rid of all ϵ -productions (Eliminate epsilon (ϵ) productions)

step 3:- Eliminate unit productions

step 4:- Eliminate all useless symbols

step 5:- Replace long productions by short ones

Ex:- if we have $A \rightarrow BCD$

then we can write $E \rightarrow CD$ and write $A \rightarrow BE$
new production

Chomsky normal form can be only of this form

$$A \rightarrow BC$$

$$A \rightarrow a$$

step 6:- move terminals to unit production

Ex:- if we have $A \rightarrow bc$ which is not in CNF so we introduce $B \rightarrow b$ and write as $A \rightarrow BC$

Q) $S \rightarrow 0A/1B$

$$A \rightarrow 0AA/1S/1$$

$$B \rightarrow 1BB/0S/0$$

$B \rightarrow B_0 BB / A_0 S / D$ (not in CNF)

$A_0 \rightarrow 0$

$B_0 \rightarrow 1$

again,

GD	AT	RP
$A \rightarrow A_0 AA$	$D_0 \rightarrow AA$	$A \rightarrow A_0 D_0$ $D_0 \rightarrow AA$
$B_0 \rightarrow B_0 BB$	$D_1 \rightarrow BB$	$B \rightarrow B_0 D_1$ $D_1 \rightarrow BB$

Now we get,

$A \rightarrow A_0 D_0$

$D_0 \rightarrow AA$

$B \rightarrow B_0 D_1$

$D_1 \rightarrow BB$

Next write $G = (V, T, P, S)$

$V = \{S_0, S, A, B, A_0, B_0, D_0, D_1\}$

$T = \{0, 1\}$

$P = \{$

all productions obtained above

S_0 is the start state

$S \rightarrow aXbX$

$X \rightarrow aY/bY/\epsilon$

$Y \rightarrow X/c$

no S on RHS, so not needed

There is ϵ production, so we get

$S \rightarrow aXbX / aXb / abX / ab$

$X \rightarrow aY / a / bY / b$

$Y \rightarrow X / c$

There is unit production
 $Y \rightarrow X$ since $X \rightarrow aY/a/bY/b$
 we get

$$Y \rightarrow aY/a/bY/b/a$$

There is no useless symbol
 so now,

Given Production G/P	action taken AT	Resulting Production on RP
$S \rightarrow axbx/axb/abx/ab$	$A_0 \rightarrow a$ $B_0 \rightarrow b$	$S \rightarrow A_0XB_0X/A_0XB_0X/A_0XB_0X/A_0XB_0$ $A_0 \rightarrow a$ $B_0 \rightarrow b$
$X \rightarrow aY/a/bY/b$	$A_0 \rightarrow a$ $B_0 \rightarrow b$	$X \rightarrow A_0Y/a/B_0Y/b$ $A_0 \rightarrow a$ $B_0 \rightarrow b$
$Y \rightarrow aY/a/bY/b/c$	$A_0 \rightarrow a$ $B_0 \rightarrow b$	$Y \rightarrow A_0Y/a/B_0Y/b/c$ $A_0 \rightarrow a$ $B_0 \rightarrow b$

we got,

$$S \rightarrow A_0XB_0X/A_0XB_0/A_0B_0X/A_0B_0 \quad \text{not in CNF}$$

$$X \rightarrow A_0Y/a/B_0Y/b$$

$$Y \rightarrow A_0Y/a/B_0Y/b/c$$

$$A_0 \rightarrow a$$

$$B_0 \rightarrow b$$

These are in CNF

again.

G.P	AT	R.P
$S \rightarrow A_0 X B_0 X / A_0 X B_0 / A_0 B_0 X$	$D_0 \rightarrow B_0 X$ $D_1 \rightarrow X B_0$	$S \rightarrow A_0 X D_0 / A_0 D_0 / A_0 D_0$ $D_0 \rightarrow B_0 X$ $D_1 \rightarrow X B_0$
$S \rightarrow A_0 X D_0$	$D_2 \rightarrow X D_0$	$S \rightarrow A_0 D_0$ $D_2 \rightarrow X D_0$ all are in CNF

finally, we get

$$S \rightarrow A_0 D_2 / A_0 D_1 / A_0 D_0 / A_0 B_0$$

$$X \rightarrow A_0 Y / a / B_0 Y / b$$

$$Y \rightarrow A_0 Y / a / B_0 Y / b / c$$

$$A_0 \rightarrow a$$

$$B_0 \rightarrow b$$

$$D_0 \rightarrow B_0 X$$

$$D_1 \rightarrow X B_0$$

$$D_2 \rightarrow X D_0$$

$$\text{Let } G_1 = (V, T, P, S)$$

$$V = \{S, X, Y, A_0, B_0, D_0, D_1, D_2\}$$

$$T = \{a, b, c\}$$

$$P = \{$$

$\}$

S is the start state

$$3a) S \rightarrow a / aA / B / c$$

$$A \rightarrow aB / E$$

$$B \rightarrow aA / E$$

$$C \rightarrow cCD$$

$$D \rightarrow ddd$$

40) $S \rightarrow ABC / BaB$
 $A \rightarrow aA / BaC / aaa$
 $B \rightarrow bBb / a / D$
 $C \rightarrow CA / AC$
 $D \rightarrow \epsilon$

50) $S \rightarrow 0A0 / 1B1 / BB$
 $A \rightarrow C$
 $B \rightarrow S / A$
 $C \rightarrow S$