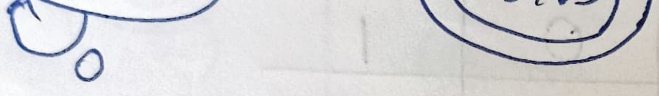


Regular Expression

1) NULL : $\emptyset$: It is a regular expression denoting empty language.

2) ε : It is regular expression indicating the language consisting of empty string.

3) 'a' : It is regular expression corresponding to the language consisting only a's.

4) 'R' : It is RE corresponding to the language L_R and
'S' : It is RE " " L_S then

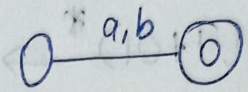
i) R+S is a RE corresponding to the language $L_R \cup L_S$.

ii) R.S is a RE " " $L_R . L_S$

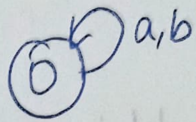
iii) R^* is a RE " " L_R

5) An expression obtained from the above 4 groups is called <u>Regular expression</u> (RE).

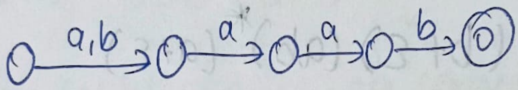
Ex: 1) $(a+b) \rightarrow$ a or b



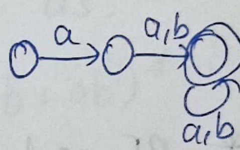
2) $(a+b)^* \rightarrow$ 0 or more a's or b's of any combination



3) $(a+b)^*.aab \rightarrow$ any no of a's / b's i.e ending with aab

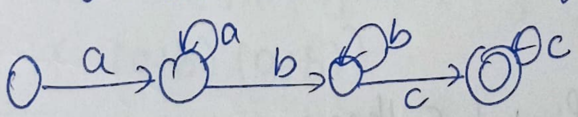


4) $a.(a+b)^* \rightarrow$ string of a's i.e b's but starting with a



5) $(a+b)^*.aa.(a+b)^* \rightarrow$ strings of a's i.e. b's but having substring 'aa'.

6) $a^*.b^*.c^*$ - <u>0</u> or more no of a's <u>followed</u> by 0 or more b's followed by 0 or more c's



7) $a.a^*.b.b^*.c.c^*$ - String start with <u>1</u> or more no of a's followed by <u>1</u> or more b's followed by 1 or more c's.

8) $a^+.b^+.c^+$ same : $aa^* \Leftrightarrow a^+$: (one or more)

a^* (0 or more).

9) $(aa)^* \rightarrow$ even no of a's

10) $(aa)^*.(bb)^*.b \rightarrow$ even no of a's followed by odd no of b's

11) $(0+1)^* \cdot 000$ → strings having 0's and 1's followed by 3 consecutive 0's.

12) $(1+01)^*$ → any no of 1 or 01 means
No consecutive 0's and ending with 1

obtain regular Ex to accept language consisting of strings of a's & b's with alternate a's & b's

ababab... ✓
babababa... ✓

here $(b+\varepsilon)$ means b or nothing.

$RE = (b+\varepsilon)(ab)^*(a+\varepsilon)$

* RE containing atleast one a & atleast 1 b, $\Sigma = \{a,b,c\}$

$RE_1 = (a+b+c)^* a (a+c)^* b (a+b+c)^*$

we don't take b here since b takes to next state, and here any no of a, b, c

This is first a & then b.

So for first b & then a

$RE_2 = (a+b+c)^* b (b+c)^* a (a+b+c)^*$

Total final RE is

$$\left((a+b+c)^* a (a+c)^* b (a+b+c)^*\right) + \left((a+b+c)^* b (b+c)^* a (a+b+c)^*\right)$$

* String of continous a's & b's but with even length

Solution: first element $(a+b)$ a or b
Second element $(a+b)$ a or b
} repeat 2 elements * times => even

$RE = ((a+b).(a+b))^*$

also we can write $RE = (aa+ab+ba+bb)^*$

* string continous a's & b's but with odd length.

$RE = ((a+b).(a+b))^*.(a+b)$

don't write inside Bracket since 6 is even.

* strings of 0's & 1's having consecutive 3 0's.

$RE = (0+1)^* 000 (0+1)^*$

* strings of a's & b's having no consecutive a's.

No consecutive a's & ending with b $= (b+ab)^*$

Since we can have ending with b also

$\therefore RE = (b+ab)^* (a+b)$

* strings of a's & b's of length 2

$RE = (aa+ab+ba+bb)$

also $RE = ((a+b).(a+b))$

* Strings of length 10 $\boxed{=10}$

$RE = (a+b)^{10}$

* strings less than or equal to 10 $\boxed{\leq 10}$

$(a+b+\varepsilon)^{10}$

meaning for each iteration, either a or b or nothing

meaning a or b's less than or equal to 10.

1) even length of string of a's & b's

$$((a+b).(a+b))^*$$

2) odd "

$$((a+b).(a+b))^*.(a+b)$$

3) $L = \{ w \mid w \in (0,1)$ with atleast 3 consecutive 0's $\}$

$$(0+1)^* 000 (0+1)^*$$

4) Starting with a & ending with b

$$a(a+b)^* b$$

5) second last symbol is a $\quad \Sigma = \{a, b\}$

$$(a+b)^* . a . (a+b)$$

6) first & last symbol same

$$(a(a+b)^* a) + (b(a+b)^* b)$$

7) " " different.

$$(a(a+b)^* b) + (b(a+b)^* a)$$

8) length is even or multiple of 3

$$((a+b)(a+b))^* + ((a+b)(a+b)(a+b))^*$$

9) every block of 4 symbols has atleast 2 a's [a's & b's]

$$(aaxx + xxaa + axax + xaxa + axxa + xaax)^*$$

where $x = (a+b)$.

10) $L = \{ a^n b^m \mid n \geq 0, m \geq 0 \}$

$$a^* b^*$$

11) $L = \{a^{2n} b^{2m} \mid n \geq 0, m \geq 0\}$

$(aa)^* (bb)^*$

12) Contain not more that 3 a's

$b^* (a+\varepsilon) b^* (a+\varepsilon) b^* (a+\varepsilon) b^*$

13) $L = \{a^{2n} b^{2m} \mid n > 0, m > 0\}$

$(aa)^+ (bb)^+$

14) Contains not more than 3 a's

15) $L = \{a^n b^m : m+n \text{ is even}\}$

$(aa)^* (bb)^* + (aa)^* a (bb)^* b$

16) $L = \{ w \bmod 3 = 0 \mid w \in (a,b)^* \}$

$((a+b)(a+b)(a+b))^*$

17) $L = \{ w \mid \text{No of a's} \bmod 3 = 0, w \in (a,b) \}$

$(b^* a b^* a b^* a b^*)^* + b^*$

18) $L = \{ a^n b^m ; n \geq 3, m \leq 3 \}$

$aaa a^* (b+\varepsilon)^3$

19) not ending with 01

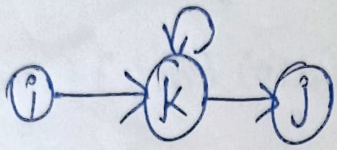
means ending with 00, 10, 11.

$(0+1)^* (00 + 11 + 10)$

20) Ending with ab or ba

$(a+b)^* (ab + ba)$

R.E using Kleen's theorem.

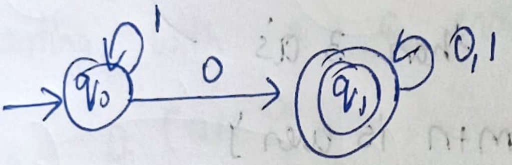


$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} \left(R_{kk}^{(k-1)}\right)^* R_{kj}^{(k-1)}$$

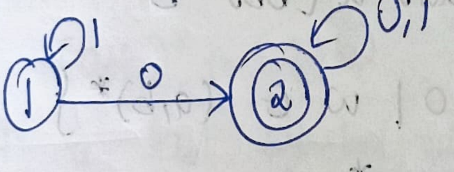
K is state with self loop

k is the no of states (in power)

Ex:



convert to numbers



$k=2,\ i=1,\ j=2$
also $K=2$

$$R_{12}^{(2)} = R_{12}^{(1)} + R_{12}^{(1)} \left(R_{22}^{(1)}\right)^* R_{22}^{(1)}$$

start from $K=0$ i.e find all the transitions from all stats but with RE.

$R_{11}^{(0)} = 1+\varepsilon$, $R_{12}^{(0)} = 0$, $R_{21}^{(0)} = \emptyset$, $R_{22} = 0+1+\varepsilon$

NOTE: if loop then or $(+\varepsilon)$ is must.

for $K=1$, find all transitions

but $R_{11}^{(1)k}$

$\rightarrow R_{11}^{(1)k} = R_{11}^{(0)} + R_{11}^{(0)} \left(R_{11}^{(0)}\right)^* R_{11}^{(0)}$ substitute,

$= (1+\varepsilon) + (1+\varepsilon)(1+\varepsilon)^* (1+\varepsilon)$

$= (1+\varepsilon) + (1+\varepsilon)(1)^* (1+\varepsilon)$

$= (1+\varepsilon) + 1^*$

$= \underline{1^*}$

$\rightarrow R_{12}^{(1)k} = R_{12}^{(0)} + R_{11}^{(0)} (R_{11}^{(0)})^* R_{12}^{(0)}$ $\boxed{k=1}$

$i\ j$

$= 0 + (1+\varepsilon) \cdot (1+\varepsilon)^* 0$

$= 0 + (1+\varepsilon) 1^* 0$

$= 0 + 1^* 0$

$= \underline{1^* 0.}$

1^* means 0 or more time

$\rightarrow R_{21}^{(1)k} = R_{21}^{(0)} + R_{21}^{(0)} (R_{11}^{(0)})^* (R_{11})$

$i\ j$

$= \emptyset + \underline{\emptyset (1+\varepsilon)^* (1+\varepsilon)}$

$= \emptyset + \emptyset$

$= \emptyset$

$\rightarrow R_{22}^{(1)k} = R_{22}^{(0)} + R_{21}^{(0)} (R_{11}^{(0)})^* (R_{12}^{(0)})$

$= (0+1+\varepsilon) + \underline{\emptyset} (1+\varepsilon)^* (0)$

$= (0+1+\varepsilon) + \emptyset$

$= 0+1+\varepsilon$

Finally $R_{12}^{(2)} = R_{12}^{(1)} + R_{12}^{(1)} ((R_{22}^{(1)}))^* R_{22}^{(1)}$ $\boxed{k=2}$

$R_{12}^{(2)} = 1^* 0 + 1^* 0 (0+1+\varepsilon)^* (0+1+\varepsilon)$

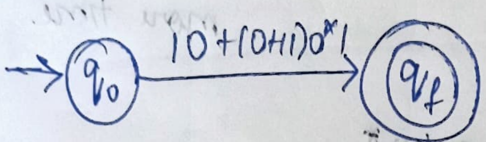
$= 1^* 0 + 1^* 0 (0+1)^*$

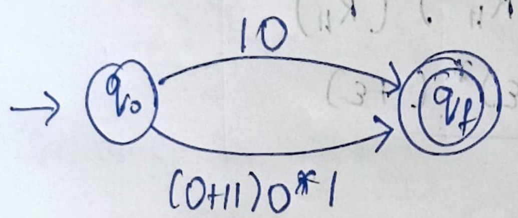
$= \underline{1^* 0 (0+1)^*}$

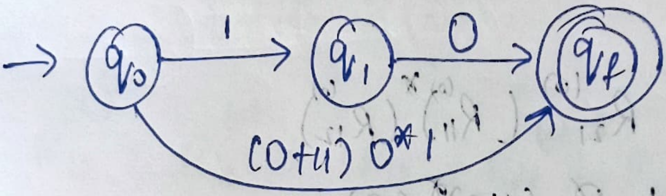
RE to DFA

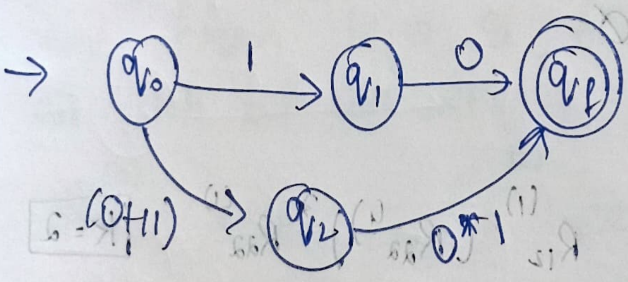
- Design a transition diagram for given RE using NFA or ε-NFA.
- Then convert ε-NFA or NFA to DFA:

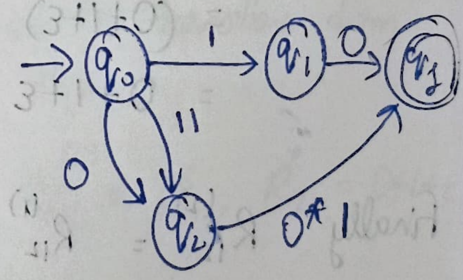
Q1) $10 + (0+11)0^*1$

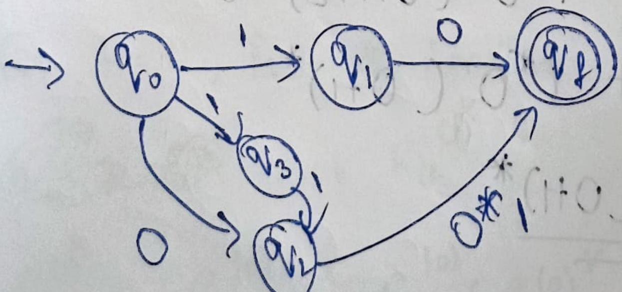


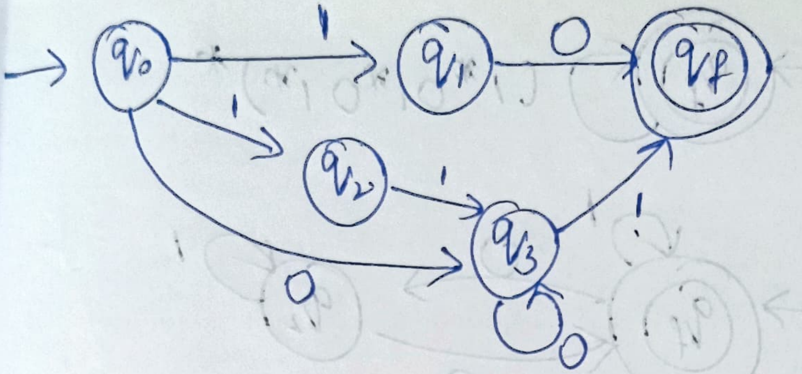








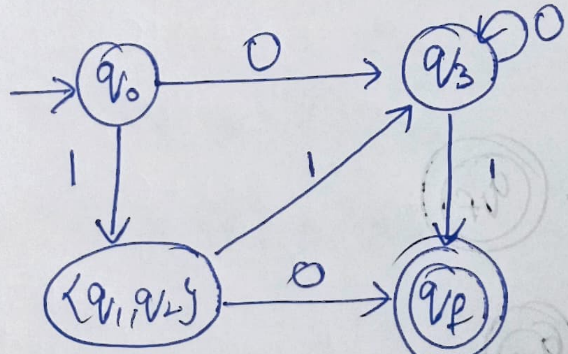




using lazy evaluation, draw the transition table & Then draw DFA

	0	1
$\rightarrow \{q_0\}$	$\{q_3\}$	$\{q_1, q_2\}$
$\{q_3\}$	$\{q_3\}$	$\{q_f\}$
$\{q_1, q_2\}$	$\{q_f\}$	$\{q_3\}$
$\{q_f\}$	$\emptyset$	$\emptyset$

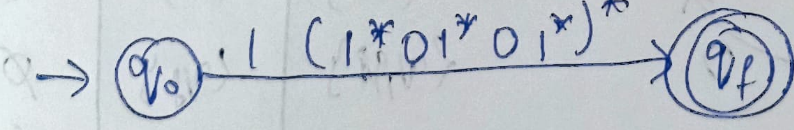
DFA

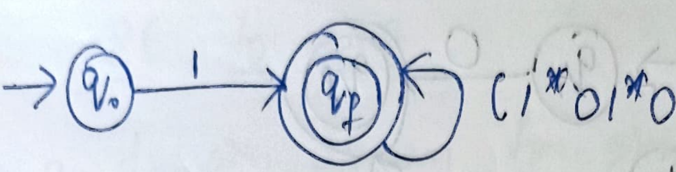


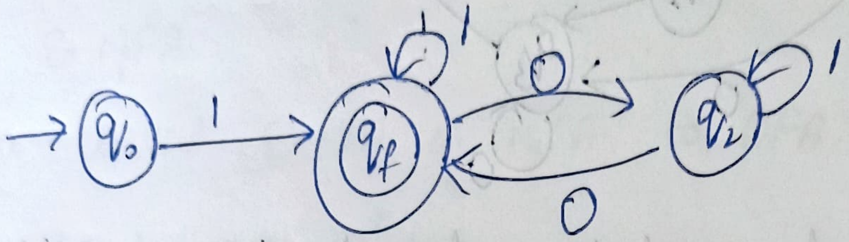
Q2) $1(1^*01^*01^*)^*$

Since after 1, we have → means 0 or more times

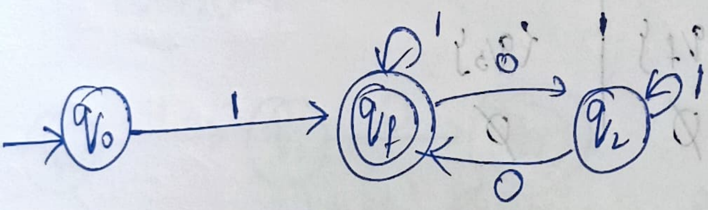
∴ it is loop



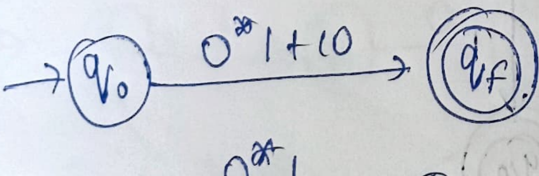


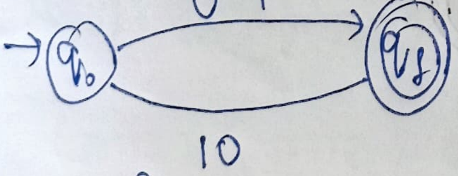


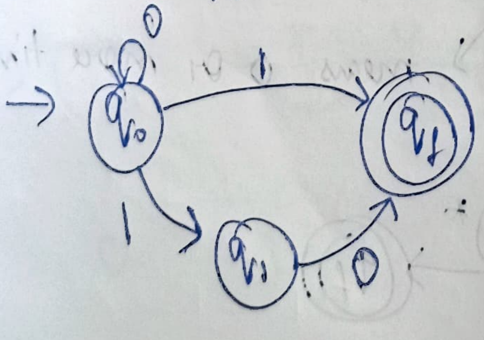
	0	1
→ q_0	$\emptyset$	$\{q_f\}$
* q_f	$\{q_2\}$	$\{q_f\}$
q_2	$\{q_f\}$	$\{q_2\}$



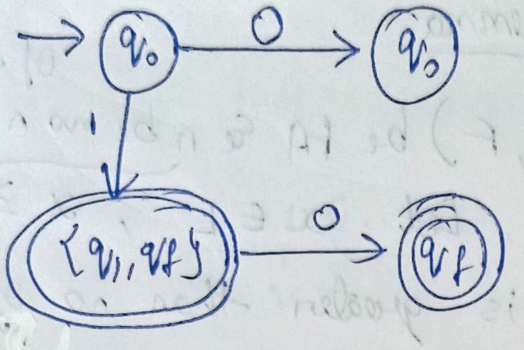
Q34 $0^*1 + 10$



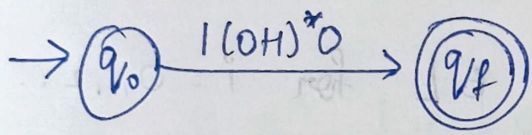


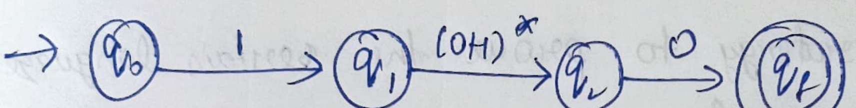


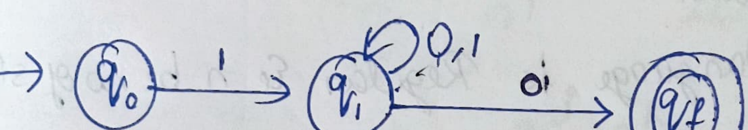
	0	1
$\{q_0\}$	$\{q_0\}$	$\{q_f, q_1\}$
$\{q_1, q_f\}$	$\{q_f\}$	$\emptyset$
q_f	$\emptyset$	$\emptyset$



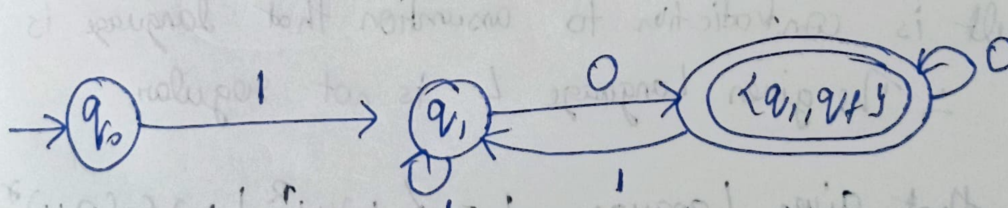
Q44. $1\,(0+1)^*\,0$







	0	1
$\rightarrow q_0$	$\emptyset$	$\{q_1\}$
q_1	$\{q_1, q_f\}$	$\{q_1\}$
$*\{q_1, q_f\}$	$\{q_1, q_f\}$	$\{q_1\}$



Pumping lemma

of states

Let $M = (Q, \Sigma, \delta, q_0, F)$ be FA & n be no n & assume that language is regular, Let $x \in L$, $x \geq n$, length of the string x is greater than no of states in the finite automata. If the string x can be decomposed into $X = uvw$ such that

$|u v| \leq n$

$|v| \geq 1$ Then $uv^iw \in L$ for $i = 0, 1, 2 \ldots$

* The general stratergy to prove the certain language is not regular

Step 1: Assume that language is Regular & n be no of states

Step 2: Select a string X & break it into UVW, such that $x = UVW$ & with the constraints, $X \geq n$ & $|U V| \leq n$ & $|V| \geq 1$

Step 3: Find any i such that UV^iW does not belong to L $uv^iw \notin L$.

According to pumping lemma $UV^iW \in L$ means regular. So result is contradiction to assumtion that language is regular. ∴ The given Language L is not regular.

Q: Show that given Language $L = \{ ww^R \mid w \in (0+1)^* \}$ is not regular.

Step 1: Let L be regular & n be no of states in automata.

$$X = \underbrace{\overbrace{\underbrace{1 \ldots 1}_{n}\underbrace{0 \ldots 0}_{n}}^{w}\overbrace{\underbrace{0 \ldots 0}_{n}\underbrace{1 \ldots 1}_{n}}^{WR}}$$

WR ⟶ reverse of w

$W = 1 \ldots 1 0 \ldots 0$ ('n' number of 1's followed by 'n' number of 0's)

$W^R = 0 \ldots 0 1 \ldots 1$ ('n' number of 0's followed by 'n' number of 1's)

Step 2 : since $x \geq n$ we can split this x into uvw such that $|uv| \leq n$ & $|v| \geq 1$

$$X = \underbrace{1 \ldots}_{u} \underbrace{1}_{v} \underbrace{0 \ldots 00 \ldots 01 \ldots 1}_{w}$$

where $|u| = n-1$, $|v| = 1$

$\therefore$ $|uv| = |u| + |v| = n-1+1 = \underline{\underline{n}}$

— step 3 continued.

Q5 $L = \{ a^{n!} \mid n \geq 0 \}$

Soln we know $n! > x$

$\therefore$ $x = a^{n!}$ consider.

Now breaking $x = a^{j} . a^{k} \; a^{n!-j-k}$

$= a^{j} \, (a^{k})^{i} \; a^{n!-j-k}$

in the form $uv^{i}w$