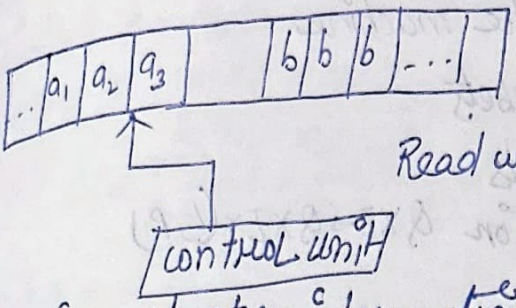
Turing Machines [Module 5]



- Tape is used to store information & is divided into cells.
- each cell can store information of only one symbol.
- The string to be scanned will be stored from the leftmost position on the tape.
- The string to be scanned will end with blank symbols.
- The Tape is assumed to be infinite length from both LHS and RHS.

Read/write head

- Read/write head can scan/read one symbol from where it is pointing to or write into the cell where it is pointing to.

Control Unit:

- The reading/writing from or to the tape is determined by the control unit.
- The different moves of the control head depends on the current symbol scanned and the current state.
- The read/write head can move either left or right.
- The actions performed by the machine are
  - (i) change of state
  - (ii) ~~simple~~ pointing to read/write head can be replaced symbol
  - (iii) Read/write head may move towards left or right.

The delta transition can be indicated as $Q \times \Sigma \rightarrow Q \times \Sigma \times (L/R)$

The Turing machine $M = (Q, \Sigma, \tau, \delta, q_0, B, F)$

where Q → set of states in the machine
Σ → set of input alphabets
τ → set of tape symbols
δ → Transition function $Q \times \tau \to Q \times \tau \times (L, R)$
q_0 → start state
B → special symbol indicating blank character
F → set of final states

Q:) obtain a Turing machine to accept a language
$L = \{0^n 1^n \mid n \geq 1\}$

Ans:) For given, Turing machine, we should have n no of 0's and 1's.
Consider string $w = 00001111$

General procedure:

→ Let q_0 be start state and rewrite
→ Replace leftmost 0 with X and change the state to Y and then move towards right. This is because, after 0 is replaced we have to replace the corresponding 1 so that number of zero match number of 1's.
→ Search for leftmost 1 , and move towards left.
→ when

Consider the situation

X	X	0	0	Y	Y	1	1
		q_0					

where first 2 0's are replaced by X and first 2 1's are replaced by Y.
Read/write head points to the leftmost 0 and machine is in the state

$\delta(q_0, 0) = (q_1, X, R)$

In state q_0, replace 0 by X, change state to q_1 and move the pointer towards right.

X	X	X	0	Y	Y	1	1
			q_1				

$\delta(q_1, 0) = (q_1, 0, R)$
$\delta(q_1, Y) = (q_1, Y, R)$

In state q_1, we have to obtain leftmost 1 and replace it with Y when the pointer is moving towards 1, input symbols encountered are 0 and Y. so replace 0 by 0 and Y by Y, Remain in the same state q_1 and move the pointer towards right.
Transitions are as follows:

$\delta(q_1, 1) = (q_2, Y, L)$

In state q_1, if input symbol scanned is a 1, then replace 1 by Y change the state to q_2 and move the pointer towards left.

$\delta(q_2, Y) = (q_2, Y, L)$
$\delta(q_2, 0) = (q_2, 0, L)$

To obtain the leftmost 0, we need to obtain rightmost X. So in the process, we may encounter Y's and 0's. Replace Y by Y, 0 by 0, remain in state q_2 and move pointer towards left.

$\delta(q_2, X) = (q_0, X, R)$

To get leftmost 0, replace X by X and change state to q_0. Move the pointer towards right.

Repeating these steps, we will replace all 0 by X and 1's by Y. If there are no more 0's, The instantaneous description as follows:

X	X	X	Y	Y	Y	Y	Y

which means that there are no more 0's. If there are no more 0's, we should see that there are no more 1's.
So, change the state to q_3

$\delta(q_0, Y) = (q_3, Y, R)$

The transition for this can be

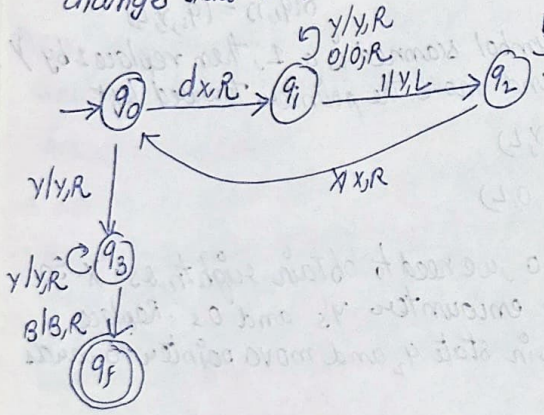
$$\delta(q_3, Y) = (q_3, Y, R)$$

In state q_3, if there are only Y's and no more 1's, replace Y by Y

X	X	X	X	Y	Y	Y	X	B

$$\delta(q_3, B) = (q_f, B, R)$$

The string ends with infinite blank symbols. If we encounter symbol B, it means the end of the string and there exists n number of 0's followed by n number of 1's change state to q_f, replace B by B and move towards right.

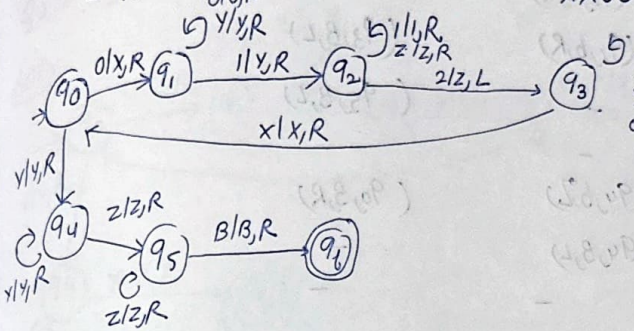


	0	1	X	Y	B
q_0	(q_1, X, R)	–	–	(q_3, Y, R)	–
q_1	$(q_1, 0, R)$	(q_2, Y, R)	–	(q_1, Y, R)	–
q_2	$(q_2, 0, L)$	–	(q_0, X, R)	(q_2, Y, L)	–
q_3	–	–	–	(q_3, Y, R)	(q_f, B, R)
q_f	–	–	–	–	–

Q:) Obtain a Turing machine to accept the language

$L = \{0^n 1^n 2^n \mid n \geq 1\}$

XX00YY11ZZ22

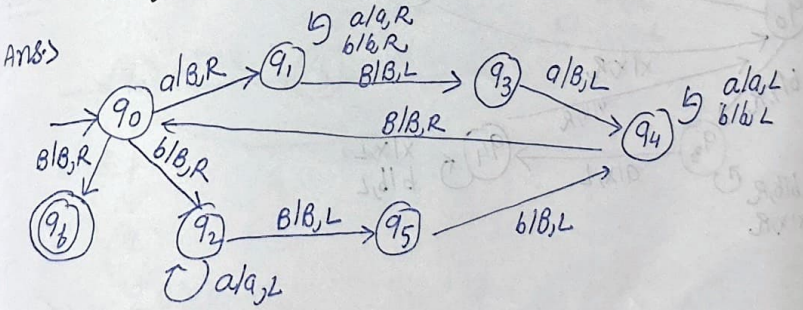


	0	1	X	Y	Z	2	B
q_0	(q_1, X, R)	–	–	(q_4, Y, R)	–	–	–
q_1	$(q_1, 0, R)$	(q_2, Y, R)	–	(q_1, Y, R)	–	–	–
q_2	–	$(q_2, 1, R)$	–	–	(q_2, Z, R)	(q_3, Z, L)	–
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, X, R)	$(q_3, 0, L)$	(q_3, Z, L)	–	–
q_4	–	–	(q_4, Y, R)	–	(q_5, Z, R)	–	–
q_5	–	–	–	–	(q_5, Z, R)	(q_6, B, R)	–
q_6	–	–	–	–	–	–	–

Q:) Obtain a Turing machine to accept the language

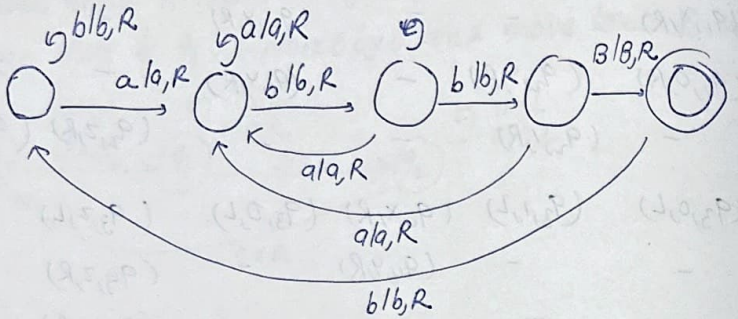
$L = \{ww^R \mid w \in (0+1)^*\}$

Ans:)



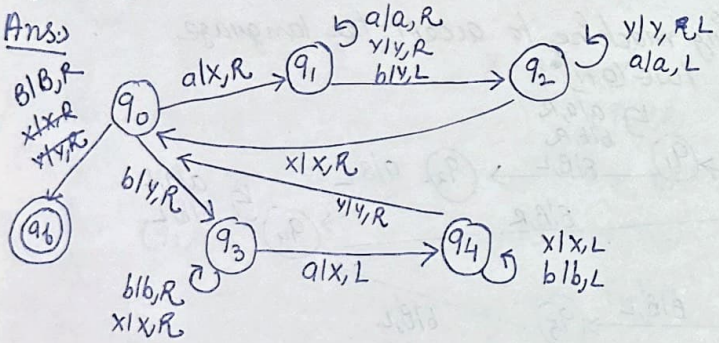
	a	b	B
q_0	(q_1, B, R)	(q_2, B, R)	(q_3, B, R)
q_1	(q_1, a, R)	(q_1, b, R)	(q_3, B, L)
q_2	(q_2, a, L)	–	(q_5, B, L)
q_3	(q_4, B, L)	–	–
q_4	(q_4, a, L)	(q_4, b, L)	(q_0, B, R)
q_5	–	(q_4, B, L)	–
q_6	–	–	–

Q>



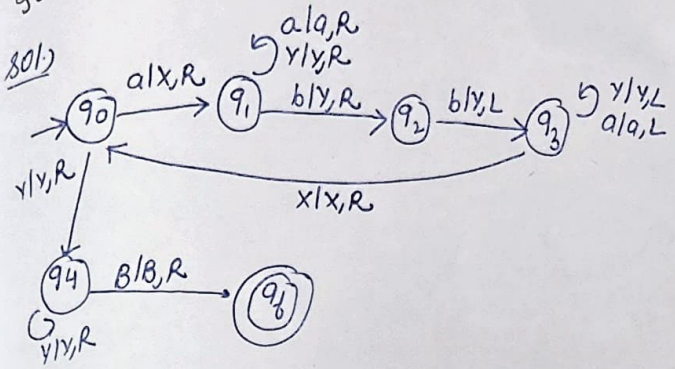
Q> $L = \{ n_a(\omega) = n_b(\omega) \mid \omega \in (a+b)^* \}$

Ans:



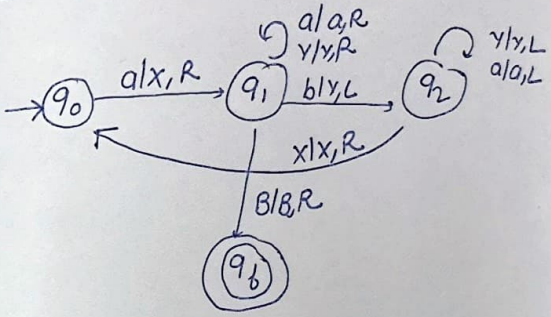
Q> $L = \{ a^n b^{2n},\ n \geq 0 \}$

Sol:



Q> $L = \{ a^m b^n,\ m > n,\ n \geq 1 \}$ aab

Sol:



Q> $L = \{ a^n b^{n+1},\ n > 0 \}$ abb

Sol: