$$P(X=x \mid Y=1) = \frac{P(x,y)}{P_Y(y)} \qquad \frac{P(0,1)}{0.5}$$

$$P(Y=1) = P(X=0, Y=1) + P(X=1, Y=1)$$
$$= 0.2 + 0.3$$
$$= 0.5$$

X	0	1
$P(X=x \mid Y=1)$	$2/5$	$3/5$

$$f(x,y) = \begin{cases} \frac{12}{5} \times x(2-x-y) & 0<x<1, 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)}$$

$$f_Y(y) = \int_0^1 f(x,y)\,dx$$

$$= \int_0^1 \frac{12}{5} \times x(2-x-y)\,dx = \frac{2}{5}(4-3y)$$

$$f_{X|Y}(x|y) = \frac{\frac{\cancel{12}^{6}}{\cancel{5}} x(2-x-y)}{\frac{\cancel{2}}{\cancel{5}}(4-3y)}$$

$$= \frac{6x(2-x-y)}{4-3y}$$

$$f_{Y|X}(y|x) = \frac{f(x,y)}{\int_A f(x,y)\,dy} \qquad f_X(x)$$

$$x \in A$$

$$E[X] = \sum_{x \in A} x \, P(X = x)$$

$$E[X \mid Y = y] = \sum x \, P_{X|y}(x)$$

$$= \sum x \times p(x, y) / P_Y(y)$$

$$f(x,y) = \begin{cases} x^2 + \dfrac{xy}{3}, & 0 \le x \le 1,\ 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$E(Y|x) = \int_0^2 y \times f_{Y|x}(y|x)\, dy$$

$$f_{Y|x}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{f(x,y)}{\int_0^2 f(x,y)\, dy}$$

$$f_X(x) = \int_0^2 \left(x^2 + \frac{xy}{3}\right) dy$$

$$= 2x^2 + \frac{x}{6} \times 4 = \cancel{2\left[x^2\right.}$$

$$= 2x\left[\frac{1}{3} + x\right] = \frac{2x}{3}\left[1 + 3x\right]$$

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{x^2 + \frac{xy}{3}}{\frac{2x}{3}(1+3x)} = \frac{\cancel{x}\left(x + \frac{y}{3}\right)}{\frac{2\cancel{x}}{3}(1+3x)}$$

$$= \frac{\cancel{3}}{2}\left(\frac{3x+y}{1+3x}\right)$$

$$E[Y|x] = \int_0^2 y \times \frac{1}{2}\left(\frac{3x+y}{1+3x}\right) dy$$

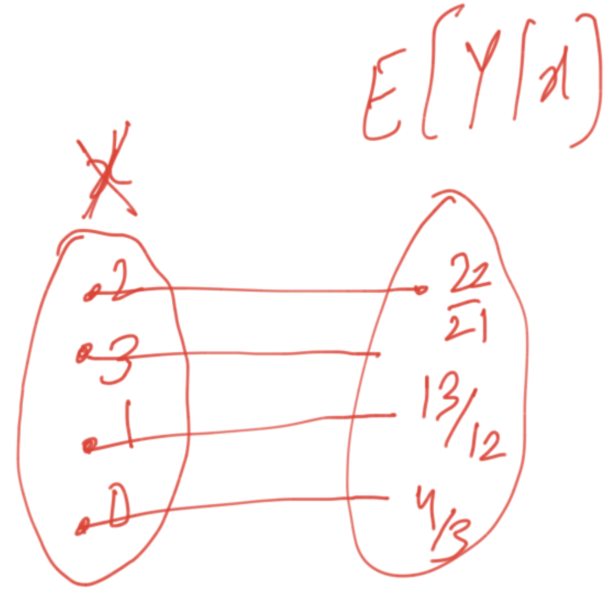
$$= \frac{1}{2(3x+1)} \int_0^2 \left(3xy + y^2\right) dy$$

$$= \frac{1}{2(3x+1)} \times \left[\frac{3}{\cancel{2}} x \times \cancel{4}\,2 + \frac{1}{3} \times 8\right]$$

$$= \frac{1}{2(3x+1)} \times \left(6x + \frac{8}{3}\right) = \frac{\cancel{2}\left(3x + \frac{4}{3}\right)}{2(3x+1)}$$

$$= \left(\frac{9x+4}{3x+1}\right) \times \frac{1}{3}$$

$$E[Y|x] = \frac{1}{3} \times \frac{[9x+4]}{[3x+1]}$$



$$E[Y|x] = g(x)$$

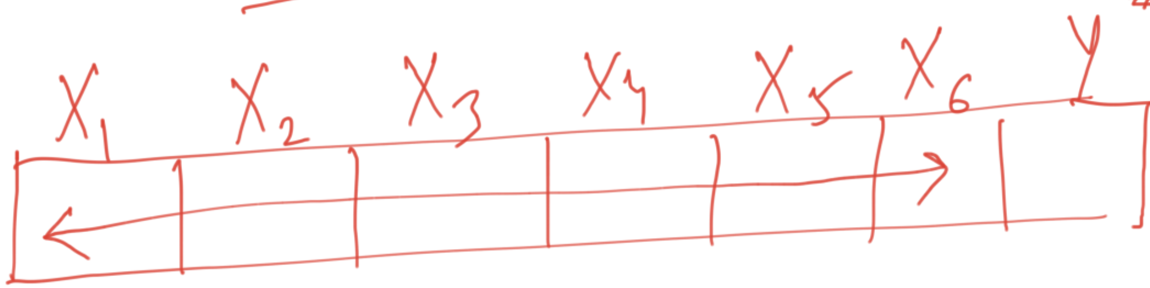
$$g(x) = \frac{1}{3}\,\frac{(9x+4)}{(3x+1)}$$

$$E[Y|X=2] = \frac{1}{3} \times \left(\frac{9 \times 2 + 4}{3 \times 2 + 1}\right) = \frac{22}{21}$$

$$E[X|y] = g'(y)$$

In ML

Dependent variable



1st row

$$X \xrightarrow{f(\theta)} Y$$

y takes continuous

$$f(\ldots) \rightarrow$$

$$E[Y|x] = \theta_0 + \theta_1 x$$

Univariate linear regression

$f(\theta, x)$

Training dataset

X Y

Var1	Var2			

(X, Y)

$\rightarrow 2 + 3x$

$\rightarrow 1 + 6x$

$$Var(X) = E[X^2] - \mu^2$$

$$\mu = E[X]$$

$$Cov(X,Y) = E[(X-\mu_x)(Y-\mu_y)]$$

$$= E[XY - X\mu_y - \mu_x Y + \mu_x \mu_y]$$

$$= E[XY] - \mu_y E[X] - \mu_x E(Y) + \mu_x \mu_y$$

$$= E[XY] - E[Y]E[X] - \cancel{E[X]E[Y]} + \cancel{E[X]E[Y]}$$

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ \end{cases}$$

$$f(y) = \begin{cases} e^{-y} & y > 0 \\ \end{cases}$$

$$E[XY] = E[X]E[Y].$$

$$Cov(X,Y) = E[XY] - E[X]E[Y]$$

$$Cov(X,Y) = Cov(Y,X)$$

$$E[XY] - E[X]E[Y] = E[YX] - E[Y]E[X]$$

$$Cov(X,X) = E[X^2] - (E[X])^2$$

$$= Var(X)$$

$$= a \times Cov(X,Y)$$

$$E[aXY] - E[aX]E[Y]$$

$$= a \times E[XY] - aE[X]E(Y)$$

$$f(x,y) = \begin{cases} 6x & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

$E[X]$

$E[Y]$

$E[Y^2]$

$E[X^2]$

$E[XY]$

$$Cov(X,Y) = E[XY] - E[X]E[Y]$$

$$Var X = E[X^2] - (E[X])^2$$

$$Var Y = E[Y^2] - (E[Y])^2$$

$$f_X(x) = \int f(x,y)\,dy = 6x(1-x)$$

$$f_Y(y) = \int f(x,y)\,dx = 3y^2$$

$$E[X] = \int x \times f_X(x)\,dx = 0.5$$

$$E[X^2] = \int x^2 f_X(x)\,dx = \int_0^1 x^2 \times 6x(1-x)\,dx$$

$$= 6 \times \left(\int_0^1 (x^3 - x^4)\,dx \right.$$

$$= 6 \times \left(\frac{1}{4} - \frac{1}{5} \right) = 6 \times \frac{1}{20} = \frac{3}{10}$$

$$E[Y] = \int_0^1 y \times 3y^2\,dy = \frac{3}{4}$$

$$E[Y^2] = \int_0^1 y^2 \times 3y^2\,dy = \frac{3}{5}$$

$$Var(X) = E[X^2] - (E[X])^2 = 1/20$$

$$Var(Y) = E[Y^2] - (E[Y])^2 = \frac{3}{5} - \frac{9}{16}$$

$$= \frac{48-45}{80} = \frac{3}{80}$$

$$E[XY] = \int_0^1 \int_0^y xy\, 6x \, dx\, dy \qquad 0<x<y$$

$$= 2/5$$

$$\mathrm{Corr}(X,Y) = \rho = \frac{6\, 2/5 - 1/2 \times 3/4}{\sqrt{\frac{1}{20} \times \frac{3}{80}}}$$

$$=$$