Joint Distributions

DRV

$[0, 1]$ — Vector

$[X_1, X_2]$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

2-D random vector

Univariate — Single variable

Bivariate — Two variables

Green one

$P(X_1=0)$

$i \backslash j$	0	1	2	3	$P(X_1=i)$
0	$10/220$	$40/220$	$30/220$	$4/220$	$84/220$
1	$30/220$	$60/220$	$18/220$	0	$108/220$
2	$15/220$	$12/220$	0	0	$27/220$
3	$1/220$	0	0	0	$1/220$
$P(X_2=j)$	$56/220$	$112/220$	$48/220$	$4/220$	

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x_1, x_2)\,dx_1\,dx_2 = 1$$

$$f(x,y) = \begin{cases} 2e^{-x}e^{-2y} & 0<x<\infty,\ \underline{0<y<\infty} \\ 0 & \text{otherwise} \end{cases}$$

1. $P\{X>1, Y<1\}$

$$\int_0^1 \int_1^\infty 2e^{-x}e^{-2y}\,dx\,dy = \int_0^1 \left(\int_1^\infty 2e^{-x}\,dx \right) e^{-2y}\,dy$$

y x

$$= 2\int_0^1 \left[\int_1^\infty e^{-x}\,dx\right] e^{-2y}\,dy$$

$$= \frac{2}{e}\int_0^1 e^{-2y}\,dy = \frac{1}{e}\left(1 - \frac{1}{e^2}\right)$$

$P(X>1, Y<1) = P(X>1 \text{ and } Y<1)$

(2) $P(X < a)$

$$\int_0^\infty \int_0^a 2e^{-x} e^{-2y} \, dx \, dy$$

$$= 1 - e^{-a}$$

③ $P(X<Y)$

$0<x<\infty$, $0<y<\infty$

$x<y$

$\cancel{y}$

$$\int_0^\infty \int_0^y f(x,y)\,dx\,dy$$

$x>0$

$x<y$

$$= \frac{1}{3}$$

$g(y)$

$$\int_0 \left[\int_0^y f(x,y)\,dx \right] dy = \int_0^\infty g(y)\,dy$$

$$f(x,y) = \begin{cases} 6x & 0 < x < y < 1 \quad 0 < x < y \\ 0 & \text{otherwise} \quad x < y < 1 \end{cases}$$

(i) $f_1(x) = \int f(x,y)\,dy \qquad x < y < 1$

$$= \int_x^1 6x\,dy \qquad P(x < a) = \int_{-\infty}^{a} 6x(1-x)\,dx$$

$$= 6x(1-x)$$

$$= \qquad\qquad = \ldots$$

$$f_2(y) = \int_0^y f(x,y)\,dx$$

$$= \int_0^y 6x\,dx = \left[3x^2\right]_0^y$$

$$= 3y^2.$$

$$P(X_1=1, X_2=2, X_3=0)$$

$$= P(X_1=1)\, P(X_2=2)\, P(X_3=0)$$

$$= 0.2 \times 0.1 \times 0.3$$

$$= 0.006$$

$X \quad f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$

$Y \quad f(y) = \begin{cases} e^{-y} & y > 0 \\ 0 & \text{otherwise} \end{cases}$

Verify X and Y are independent.

$f(x, y) = \begin{cases} e^{-(x+y)} & x > 0,\ y > 0 \\ 0 & \text{otherwise} \end{cases}$