

Joint Distributions

DRV

$[0, 1]$

Vector

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$[X_1, X_2]$

↑
2-D random vector

Univariate — Single variable

Bivariate — Two variables

Green one

		$P(X_1=0)$				
		0	1	2	3	$P(X_1=i)$
$i \backslash j$	j					
0	0	$10/220$	$40/220$	$30/220$	$4/220$	$84/220$
1	0	$30/220$	$60/220$	$18/220$	0	$108/220$
2	0	$15/220$	$12/220$	0	0	$27/220$
3	0	$1/220$	0	0	0	$1/220$
$P(X_2=j)$		$56/220$	$112/220$	$48/220$	$4/220$	

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x_1, x_2) dx_1 dx_2 = 1$$

$$f(x, y) = \begin{cases} 2e^{-x}e^{-2y} & 0 < x < \infty, \underline{0 < y < \infty} \\ 0 & \text{otherwise} \end{cases}$$

① $P\{x > 1, y < 1\}$

$$\int_0^1 \int_1^\infty 2e^{-x}e^{-2y} dx dy = \int_0^1 \left(\int_1^\infty 2e^{-x} dx \right) e^{-2y} dy$$

Diagram illustrating the integration region for the probability calculation. The region is defined by $x > 1$ and $y < 1$. The vertical axis is labeled y and the horizontal axis is labeled x . The region is shaded in light blue, bounded by $x=1$ and $y=1$.

$$= 2 \int_0^1 \left[\int_1^{\infty} e^{-x} dx \right] e^{-2y} dy$$

$$= \frac{2}{e} \int_0^1 e^{-2y} dy = \frac{1}{e} \left(1 - \frac{1}{e^2} \right)$$

$$P(X > 1, Y < 1) = P(X > 1 \text{ and } Y < 1)$$

$$\textcircled{2} \quad P(X < a)$$

$$\int_0^{\infty} \int_0^a 2e^{-x} e^{-2y} dx dy$$

$$= 1 - e^{-a}$$

③ $P(X < Y)$

$0 < x < \infty$, $0 < y < \infty$

$x < y$

~~y~~

$x > 0$

$x < y$

$$\int_0^{\infty} \int_0^y f(x, y) dx dy$$

$= \frac{1}{3}$

$$\int_0^{\infty} \left[\int_0^y f(x, y) dx \right] dy = \int_0^{\infty} g(y) dy$$

$\nearrow g(y)$
 \downarrow

$$f(x, y) = \begin{cases} 6x & \underline{\underline{0 < x < y < 1}} \\ 0 & \underline{\underline{\text{otherwise}}} \end{cases} \quad \underline{\underline{0 < x < y < 1}}$$

$$(i) f_1(x) = \int_1 f(x, y) dy \quad x < y < 1$$

$$= \int_x^1 6x dy$$

$$= 6x(1-x)$$

$$P(x < \overset{a'}{a}) = \int_{-\infty}^{\overset{a'}{a}} 6x(1-x) dx$$

$$= \dots$$

$$f_2(y) = \int_0^y f(x, y) dx$$

$$= \int_0^y 6x dx = (3x^2)_0^y \\ = 3y^2.$$

$$P(X_1=1, X_2=2, X_3=0)$$

$$= P(X_1=1) P(X_2=2) P(X_3=0)$$

$$= 0.2 \times 0.1 \times 0.3$$

$$= 0.006$$

$$X \quad f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$Y \quad f(y) = \begin{cases} e^{-y} & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Verify X and Y are independent.

$$f(x, y) = \begin{cases} e^{-(x+y)} & x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases}$$

