

Matrix Algebra

1] $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 3 & 1 & 3 \end{bmatrix}$

$A^{-1} = ?$

→

$$A^{-1} = \frac{1}{\det(A)} \cdot \text{Adj}(A)^T$$

$$\det(A) = \begin{vmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 3 & 1 & 3 \end{vmatrix}$$

$$\det(A) = 12 + 2 + 3 - (12 + 2 + 3) = 0$$

$$\boxed{A^{-1} = \infty}$$

2] $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ Eigen Values = ?

$$B\vec{v} = \lambda \vec{v}$$

$$(B - \lambda I) \vec{v} = 0$$

$$\begin{vmatrix} 2-\lambda & -\lambda & -\lambda \\ -\lambda & 1-\lambda & -\lambda \\ -\lambda & -\lambda & 5-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(1-\lambda)(5-\lambda) - \lambda^3 - \lambda^3 - [\lambda^2(1-\lambda) + \lambda^2(2-\lambda) + \lambda^2(5-\lambda)]$$

$$\Rightarrow (2-2\lambda-\lambda+\lambda^2)(5-\lambda) - 2\lambda^3 - [\lambda^2 - \lambda^3 + 2\lambda^2 - \lambda^3 + 5\lambda^2 - \lambda^2]$$

$$\Rightarrow [5\lambda^2 - 15\lambda + 10 - \lambda^3 + 3\lambda^2 + 2\lambda - 2\lambda^3 - 7\lambda^2 + 2\lambda^3]$$

$$\Rightarrow -\lambda^3 + 15\lambda^2 - 13\lambda + 10 = 0$$

$$\lambda^3 - 15\lambda^2 + 13\lambda - 10 = 0$$

3) $A = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 3 & 1 & -1 \\ 2 & 4 & 0 & 3 \\ -3 & 1 & 2 & 4 \end{bmatrix}$

$$B = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & -1 \\ 4 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 8 & 0 & 5 \\ 1 & 0 & -4 & 8 \end{bmatrix}$$

$$D = CAB$$

$$= \begin{bmatrix} 9 & 7 \\ 5 & -6 \\ 26 & 6 \\ 16 & -1 \end{bmatrix}$$

CAB is not possible.

2