$$f : \Omega \longrightarrow \mathbb{R}$$

Outcomes of a probability space

C ~~as~~ the outcome when we toss a coin

$$\underline{C} = \{\underline{\underline{H, T}}\}$$

not the numerical outcomes

C is $\underline{\text{NOT}}$ a random variable ?

X = # of heads when we toss a coin

$X \in \{0, 1\}$

$\{H, T\}$

$X = 1$ $\quad$ $X = 0$

X is a random variable.

$Y = \#$ which comes up when we roll a die

$S = \{1, 2, 3, 4, 5, 6\}$

$Y = 1$ $\quad$ $Y = 2$

Z = weight of a randomly selected person in the class

$\{P_1, P_2, P_3, \ldots P_n\}$

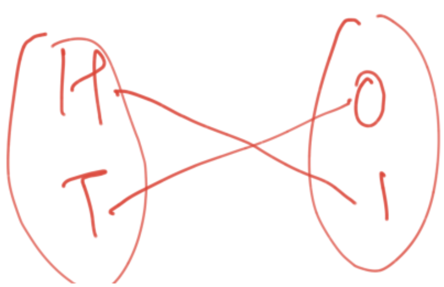
Z

$S = \{H, T\}$ $\quad$ X = # of heads when we toss a coin

$$X(H) \longrightarrow 1$$
$$X(T) \longrightarrow 0$$

$\{H, T\} \longrightarrow \{0, 1\}$ $\quad$ $x \in \{0, 1\}$

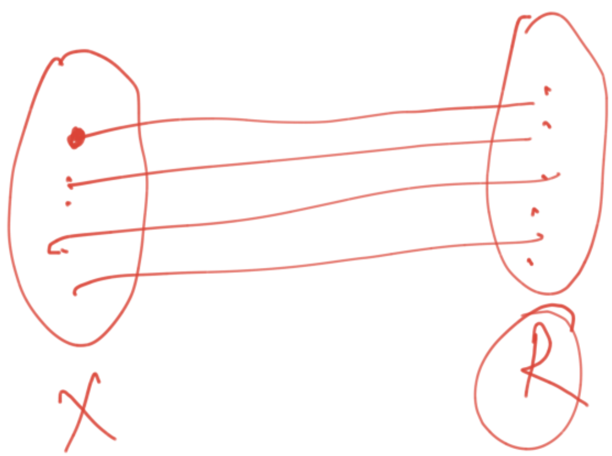
all possible values which X can take.

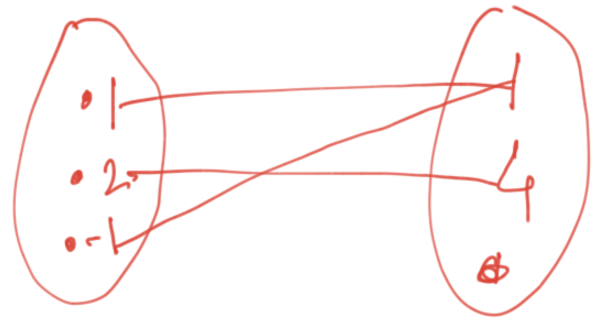


$f : X \longrightarrow R$

Domain Range

$f(x) = x^2$



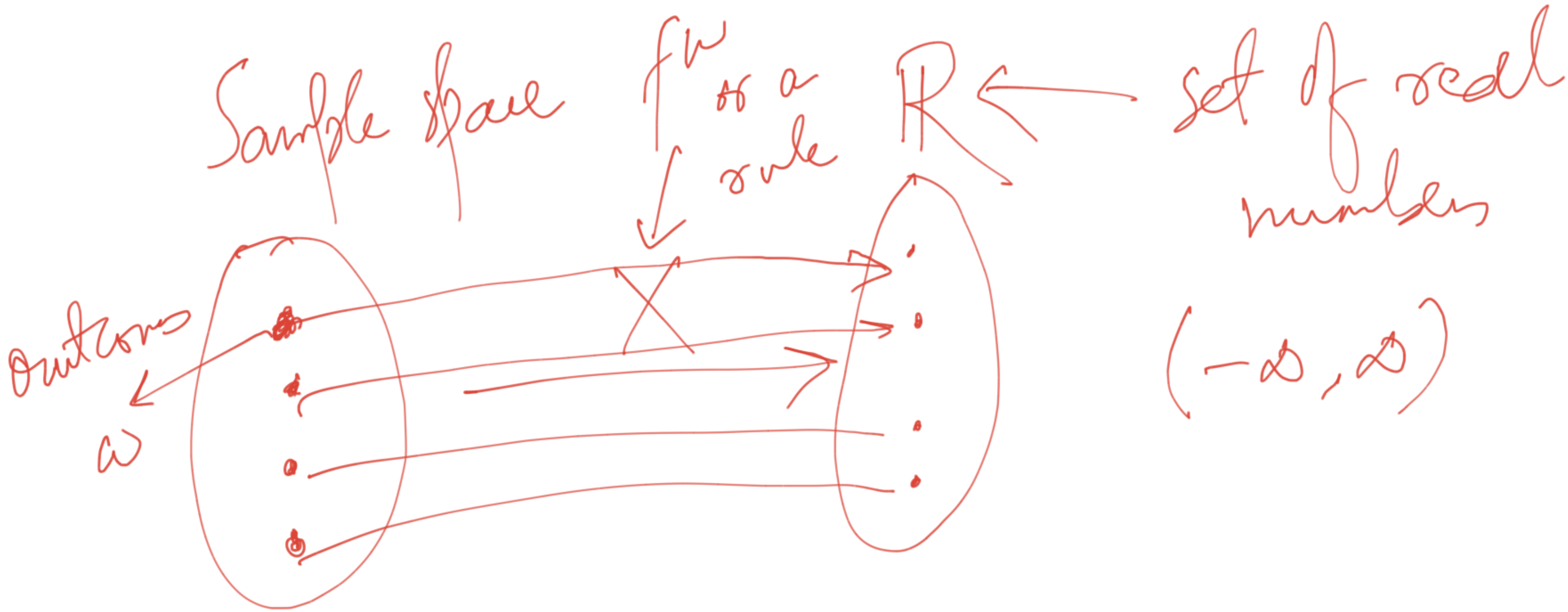


Z

$z \in (0, \infty)$

$[0, \infty)$

0 is included



$$X(\omega) = x$$

Experiment – Tossing a coin

$S = \{H, T\}$ $(0,1]$ $\rightarrow [0,1)$

X = # of heads when we toss a coin

$X(H) = 1$ $x \in \{0,1\}$ all values b/w 0 & 1

$X(T) = 0$

2 values $\rightarrow \{0,1\}$ $(0,1)$ $[0,1]$

All values b/w 0 & 1

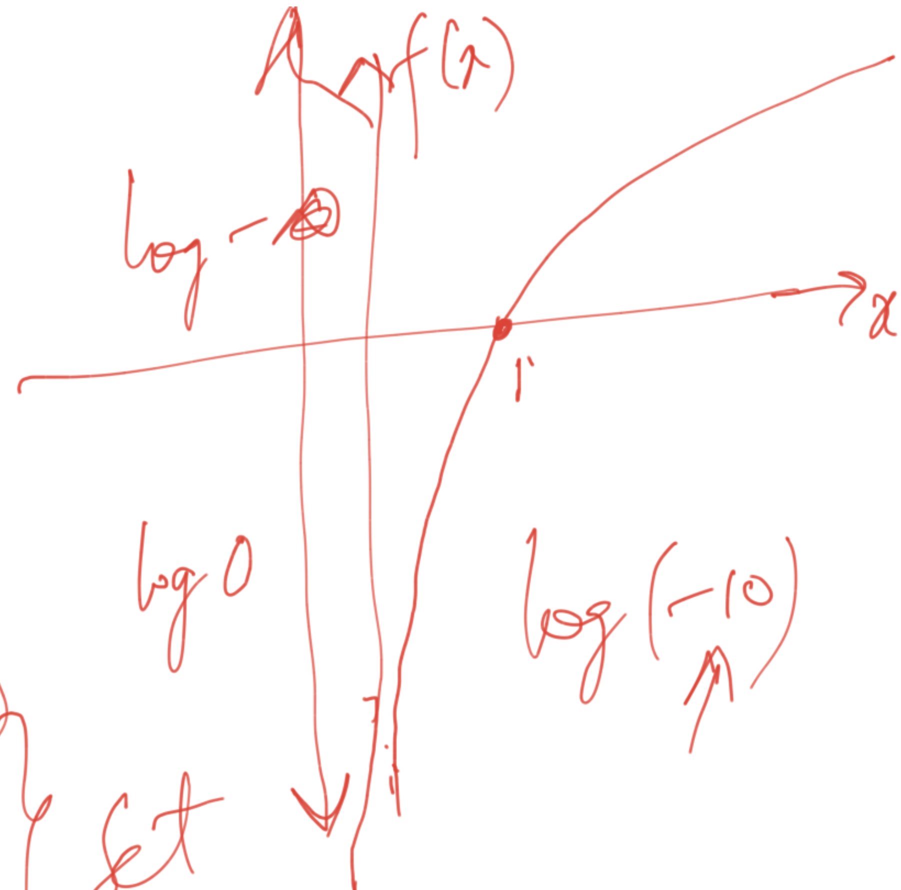
$f(x) = \log x$

Domain of $f(x)$

Range of $f(x)$

Domain $= (0, \infty)$

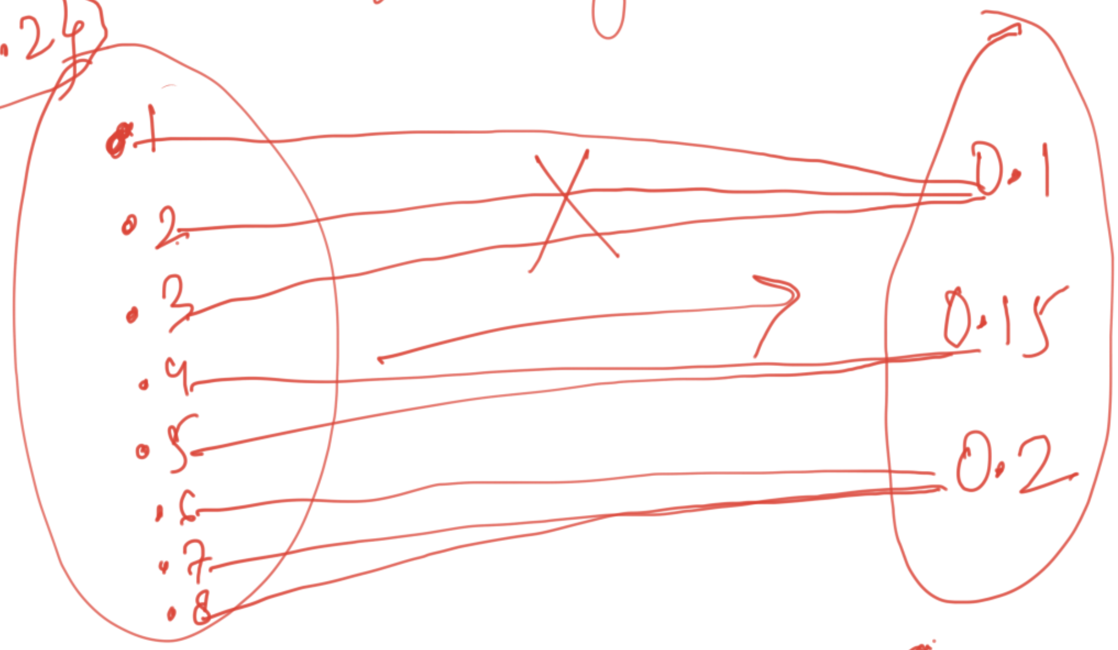
Range $= (-\infty, \infty)$



set convention

X = Weight of a ball selected at random

$x \in \{0.1, 0.15, 0.2\}$



$X(1) = 0.1$

$X(2) = 0.1$

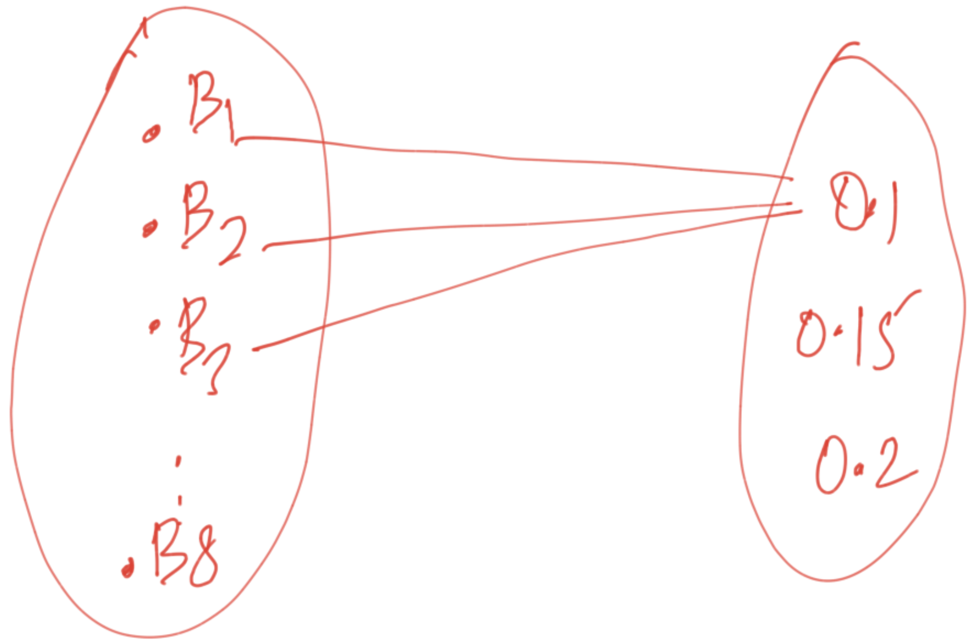
$X(3) = 0.1$

$X(5) = 0.15$

$X(7) = 0.2$

Sample space

set of All possible values which X can take



$$X(B_1) = 0.1$$

$$X(B_8) = 0.2$$

$$X : \Omega \rightarrow R$$

$\{1,2,3,4,5,6\}$ $(0,1)$ $(0, 0.0000000)$

Countably finite

Set of all natural nos

$\{1,2,3,4,5,\ldots\}$

Countably infinite

$S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$X =$ weight of a ball selected at random

$x = 0.1$

$x =$ all possible values which X can take.

$P(X = x) = P(X = 0.1)$ $\quad E = \{1, 2, 3\}$

$= \dfrac{n(E)}{n(S)} = \dfrac{3}{8}$

$P(X = 0.15)$

$= \dfrac{n(E_1)}{n(S)} = \dfrac{2}{8} = 1/4$

X	0.1	0.15	0.2
$f_X(x)$	3/8	1/4	3/8

$$\sum f_X(x) = 1.$$

$$>0$$

$$f_X$$

$S = \{H, T\}$

$X = \#$ of heads

$x \in \{0, 1\}$

$f_X(0) \quad P(X = 0) = 1/2 \qquad f_X(x) > 0$

$f_X(1) \quad P(X = 1) = 1/2$

X	0	20	30
$f_X(x)$	0.58	0.22	0.20

$$\left.\begin{array}{l} 0.58 \\ 0.22 \end{array}\right\} > 0$$

$$\begin{array}{l} 0.20 \\ \hline 1 \end{array}$$

$\rightarrow 1$

$$f_X(x) = P(X = x)$$

Probability distribution fn

$$F_X(x) = P(X \leq x)$$

Cumulative probability distribution

CDF

$$F_X(x) = \begin{cases} 0, & 0 < x < 0.1 \\ 3/8, & 0.1 \le x < 0.15 \\ 5/8 & 0.15 \le x < 0.2 \\ 1; & x \ge 0.2 \end{cases}$$

$$f_W(\omega) = 12\omega^2(1-\omega) \qquad 0 < \omega < 1$$

CRV

$$f_W(\omega) \geq 0$$

$$\int_{-\infty}^{\infty} f_W(\omega)\, d\omega = 1$$

$$-\infty \leq \omega \leq \infty$$

$$0 \quad P(X=a)$$

$$\int_{-\infty}^{\infty} 12\omega^2(1-\omega)\, d\omega = \int_{-\infty}^{0} f_W(\omega)\, d\omega + \int_{0}^{1} f_W(\omega)\, d\omega + \int_{1}^{\infty} f_W(\omega)\, d\omega$$

$$0 \qquad 0$$

$$\int_0^1 12w^2(1-w)\,dw = 12\times\left[\int_0^1 w^2\,dw - \int_0^1 w^3\,dw\right]$$

$$F(0.6) = \int_0^{0.6} f_W(w)\,dw$$

$$= 12\left[\left(\frac{w^3}{3}\right)_0^1 - \left(\frac{w^4}{4}\right)_0^1\right]$$

$$= 12\left[\frac{1}{3} - \frac{1}{4}\right] = \cancel{12}\times\left[\frac{4-3}{\cancel{12}}\right]$$

$$= 1.$$

$$F(0.5) = \int_0^{0.5} f_W(w)\,dw$$