Continuous Distributions

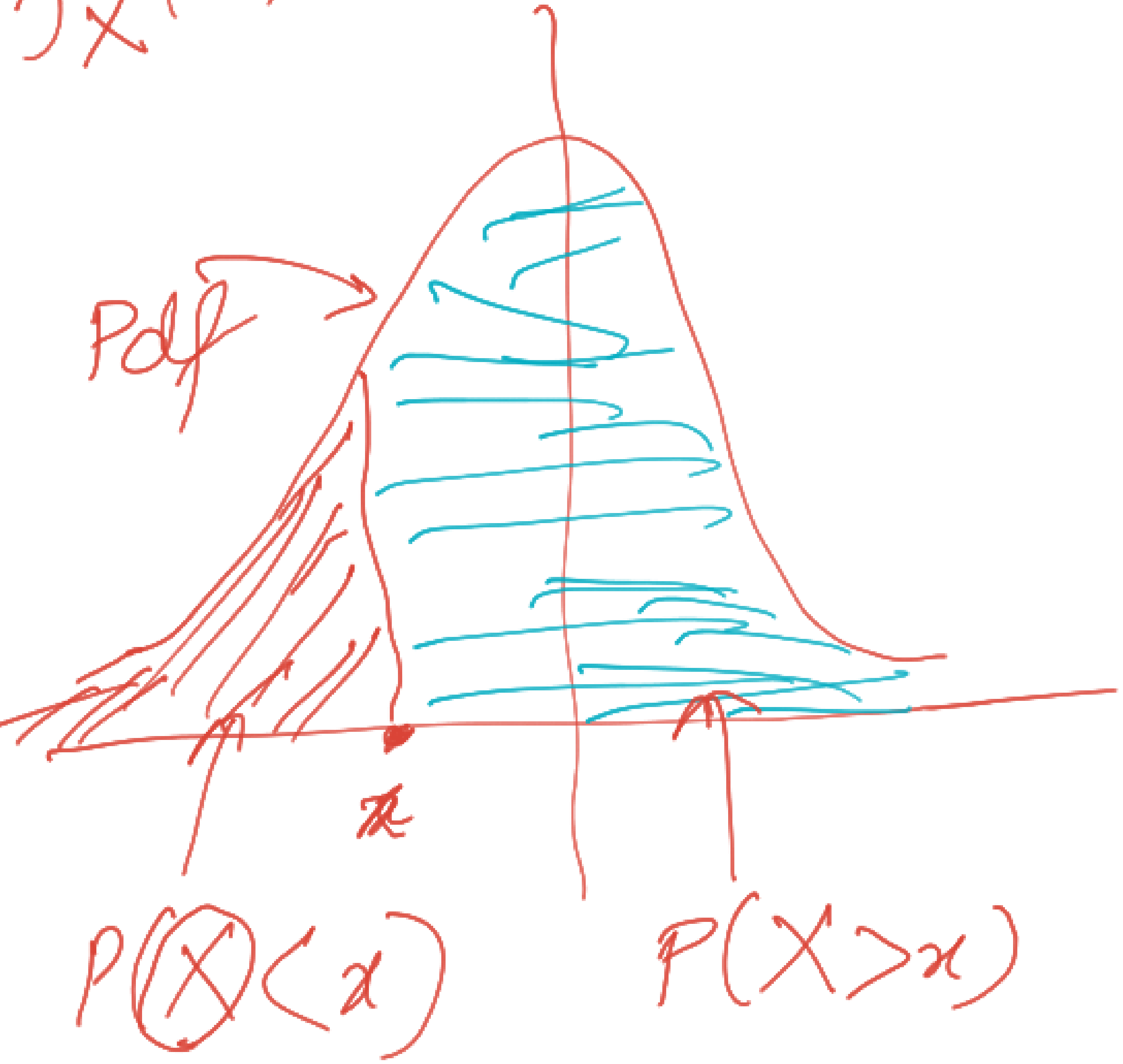
$$P(a<X<b) = \int_a^b f_X(x)\,dx$$

$X \in (a,b)$

$a=b.$

$P(X=a) = 0$

$P(X<x) = 1 - P(X>x)$



Uniform Distribution

$\alpha < X < \beta$ or $X \in (\alpha, \beta)$

$$f_X(x) = \begin{cases} \dfrac{1}{\beta - \alpha}, & \alpha < X < \beta. \\ 0, & \text{elsewhere} \end{cases}$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = \int_{\alpha}^{\beta} \left(\frac{1}{\beta-\alpha}\right) dx = \left(\frac{1}{\beta-\alpha}\right) \int_{\alpha}^{\beta} dx$$

$$= \frac{1}{\cancel{\beta-\alpha}} \times \cancel{[\beta-\alpha]} = 1$$

Uniform Distribution

$$E[X] = \int_{\alpha}^{\beta} x f_X(x)\, dx$$

$$= \int_{\alpha}^{\beta} x \times \frac{1}{(\beta-\alpha)}\, dx = \left(\frac{1}{\beta-\alpha}\right)\int_{\alpha}^{\beta} x\, dx$$

$$= \frac{1}{\beta-\alpha} \times \left[\frac{x^2}{2}\right]_{\alpha}^{\beta} = \frac{1}{\beta-\alpha} \times \frac{\left[\beta^2-\alpha^2\right]}{2}$$

$$= \frac{1}{\cancel{(\beta-\alpha)}} \times \frac{\cancel{(\beta-\alpha)}(\beta+\alpha)}{2} = \frac{\beta+\alpha}{2}.$$

Uniform Distribution

$$E[X^2] = \int x^2 \frac{1}{\beta-\alpha}\,dx = \frac{1}{\beta-\alpha} \times \frac{1}{3}\left[\beta^3-\alpha^3\right]$$

$$= \frac{1}{\cancel{\beta-\alpha}} \times \frac{1}{3} \times \left[\beta^2+\alpha^2-\alpha\beta\right]\cancel{(\beta-\alpha)}$$

$$(E[X])^2 = \left(\frac{\alpha+\beta}{2}\right)^2$$

$$Var(X) = \frac{1}{3}\left[\beta^2+\alpha^2-\alpha\beta\right) - \left(\frac{\alpha+\beta}{2}\right)^2$$

$$= \frac{(\beta-\alpha)^2}{12}$$

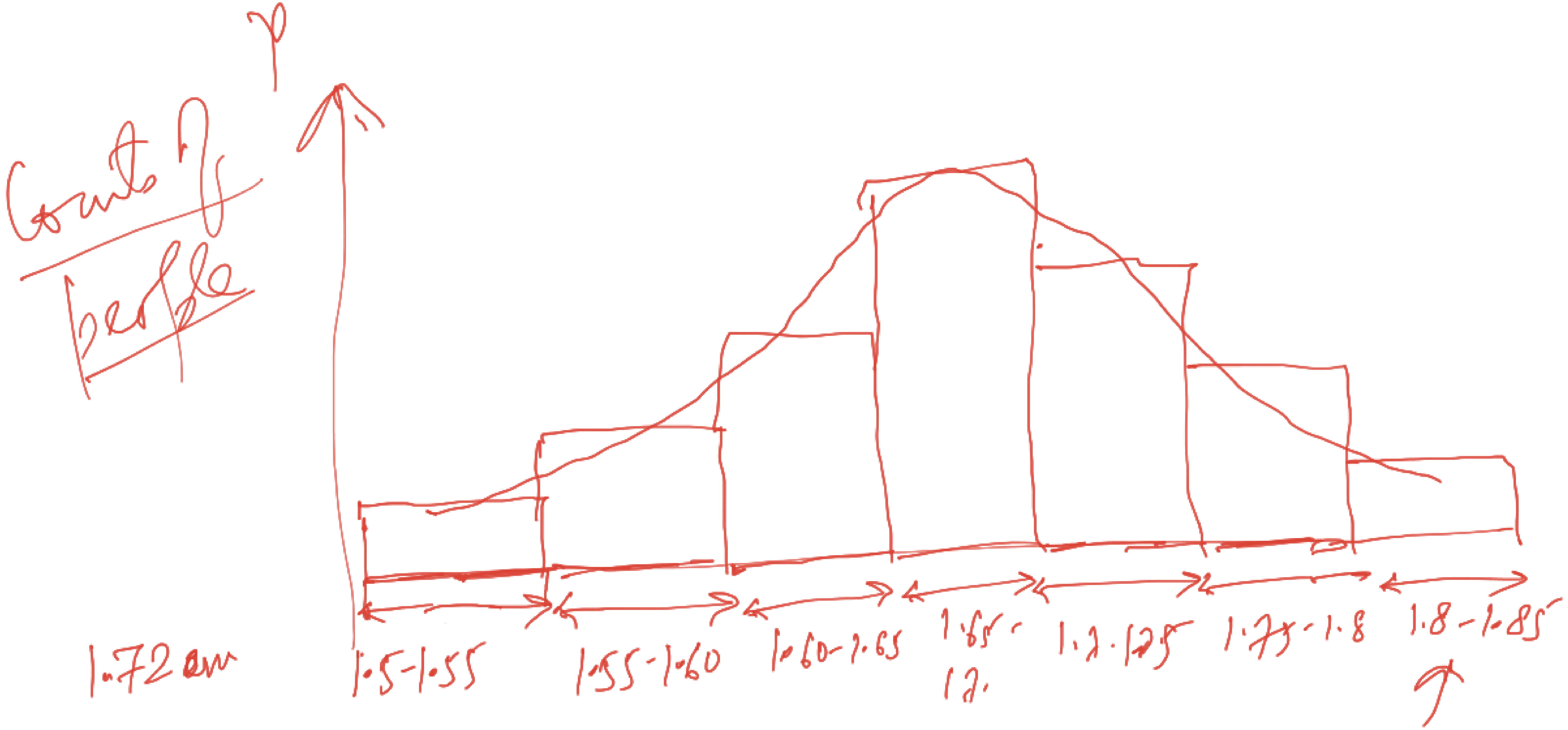
$$f_X(x) = \frac{1}{(\beta-\alpha)^2} \quad x \in [\alpha, \beta]$$

$$\int_{\alpha}^{\beta} \frac{1}{(\beta-\alpha)^2}\, dx = \frac{(\beta-\alpha)}{(\beta-\alpha)^2}$$

$$P(X < x) = \int_{-\infty}^{x} \left(\frac{1}{\beta-\alpha}\right) dx = \int_{\alpha}^{x} \frac{1}{\beta-\alpha}\, dx$$

$$= \left(\frac{1}{\beta-\alpha}\right)\left(x-\alpha\right)$$

Normal Distribution



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \times e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\int f(x)\,dx = 1.$$

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$$z = \frac{x-\mu}{\sigma}$$

$$dz = dx.$$

$$\int \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

$$x \in (-\infty, \infty) \qquad z \in (-\infty, \infty)$$

$$= \int 1$$

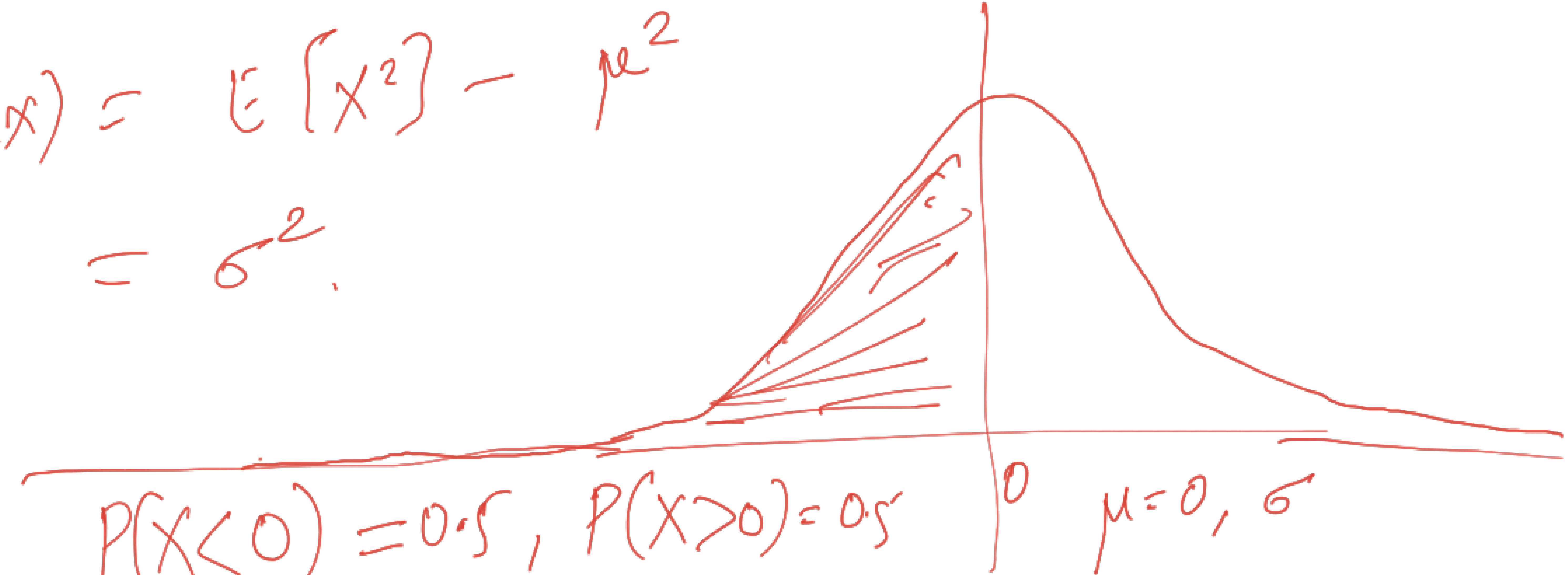
$$E[X] = \int_{-\infty}^{\infty} x \times \frac{1}{\sigma\sqrt{2\pi}} \times e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \mu$$

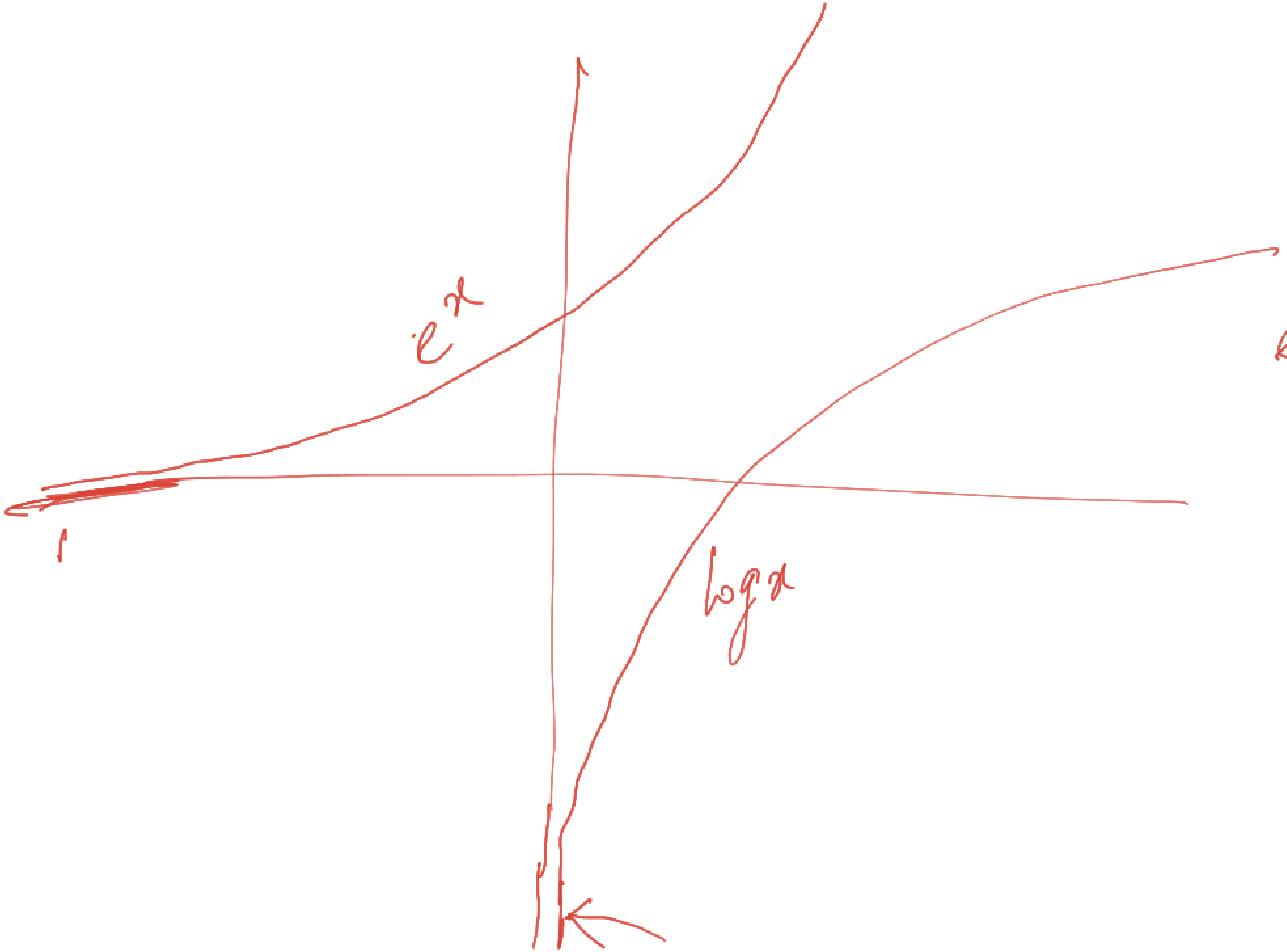
Asymptote

$$Var(X) = E[X^2] - \mu^2$$

$$= \sigma^2.$$



$P(X<0) = 0.5$, $P(X>0) = 0.5$ $\quad \mu = 0, \sigma$



e^x

$e^{-\infty} = 0$

$e^{\infty} \to \infty$

Normal Distribution

$$X \sim \mathcal{N}(\mu, \sigma)$$

Standard Normal

$$X \sim \mathcal{N}(0, 1)$$

(standard scaler) $\rightarrow$ $Z = \dfrac{X - \mu}{\sigma}$

Z-value,

$$\dfrac{X - X_{min}}{X_{max} - X_{min}} \leftarrow \text{Minmaxscaler}$$

Exponential Distribution

X — waiting time, or the time it takes for an event to occur

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x < 0 \end{cases}$$

$$\int_0^\infty \lambda e^{-\lambda x}\,dx = \lambda \int_0^\infty e^{-\lambda x}\,dx$$

$$= \cancel{\lambda} \times \left[-\frac{1}{\cancel{\lambda}} \left[e^{-\lambda x} \right]_0^\infty \right] = 1$$

$$E[X] = \int_0^{\infty} x \times \lambda e^{-\lambda x} dx$$

$$= \lambda \left[\int_0^{\infty} x e^{-\lambda x} dx \right]$$

$$= \frac{1}{\lambda}$$

$$\boxed{\begin{array}{l} \lambda = 5 \text{ persons/hr} \\ \text{Poisson process} \end{array}}$$

$$\frac{1}{\lambda} = \frac{1}{5} hr = \frac{60 m}{5}$$

$$= 12 \text{ min}$$

$\lambda t = 5$ person/hr $\times 0.5$ hr

$= 2.5$ person

$t=0$ $\quad$ $t=t$ $\quad$ $t=T$

X is a Poisson RV

0 events occurring defined by the Poisson process

$$P(T>t) = P(X=0)$$

$$= \frac{\lambda^x e^{-\lambda}}{x!} = e^{-\lambda t}$$

$$\chi^2 = \sum_{i=1}^{K} Z_i^2$$

Equivalent to sum of the squares of K independent standard normal RVs

K = degree of freedom