Optimization of HVAC Units: Minimizing Cost and Maximizing Supply

Group 3
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Abstract

This project was developed with the aim of minimizing the cost of transportation of HVAC units in the state of Massachusetts. Although it may seem like a simple problem, there are many obstacles for optimizing the solution for ten locations. These obstacles include and are not limited to transportation costs, installation costs, maintenance costs, and ability to meet the demand. An integer linear program was then developed and run through AMPL programming to solve the optimization problem. This result was evaluated, and then skewed during sensitivity analysis.

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Table of Contents

I. Introduction

i. Project Statement

As the winter season is approaching a lot newly constructed homes need a HVAC unit. This project is aimed to minimize the cost of transportation of HVAC units in the state of Massachusetts. This exercise was built based on operations research techniques and was executed mathematically using the programming language AMPL.

This project started with a thorough understanding of the current demand of HVAC units in the state of Massachusetts. So that data can be collected to then formulate the objective function and all the ancillary analysis needed. The data collection involved finding out the average demand in various locations in Massachusetts, the average transportation cost, the average cost of installation, and the average cost of maintenance (per year) of one HVAC unit.

Following the prior steps, a sensitivity analysis was conducted to determine any change in the objectives based on changes to power supply and demand.

ii. Background

A literature review on minimizing cost and maximizing profit for HVAC systems for houses in Boston would focus on research that examines the various factors that impact the cost and profitability of HVAC systems in the city.

One key area of research in this area would be the cost of different HVAC systems and the factors that influence their cost. This could include research on the initial purchase price of different HVAC systems, as well as the ongoing costs associated with their operation, maintenance, and repair. The review could also consider research on the potential for cost savings through the use of energy-efficient HVAC systems, as well as the potential for tax incentives and other financial incentives that could help to reduce the cost of HVAC systems in Boston.

Another important area of research in this area would be the potential for maximizing profit through the use of HVAC systems in Boston's residential buildings. This could include research on the potential for using HVAC systems to increase the value of a property, as well as the potential for using HVAC systems to reduce operating costs and increase tenant satisfaction. The review could also consider research on the potential for using HVAC systems to generate revenue through the sale of excess energy, as well as the potential for using HVAC systems to support the growth of new businesses and industries in the city.

Overall, a literature review on minimizing cost and maximizing profit for HVAC systems for houses in Boston would aim to provide a comprehensive overview of the existing research on this topic, highlighting the key findings and identifying opportunities for further research and innovation in this area.

II. Literature Review

i. Cost Parameters (Setup Costs, Production Costs, Transportation Costs)

Transportation Costs

A major cost incurred with delivering HVAC units to consumers is the transportation cost. It costs between $1 to $4 to transport 1 HVAC unit per mile depending on the location. This cost was also included in the calculation of the final transportation cost.

Installation Cost

The installation cost varies from $1500 to $6000. This cost mainly depends on house size - the larger your home, the squarer footage the HVAC units will need to heat or cool, the size of the unit installed is determined by the amount of square footage of your home, and if your home is large, location of the house and labour cost.

Maintenance Cost

Maintenance of a HVAC unit costs between $75 and $200 per year, depending on the contractor. This cost includes, checking the blower motor, filters, belts, fins, refrigerant levels, and safety controls, testing thermostat and carbon monoxide levels, adjust fins and belts, cleaning condensate drain, coils, and burners, replacing air filter, and lubricating all moving parts.

Another cost that is not included in the math model is repair or replacement costs of parts as this cost is variable depending on whether it is a total or partial replacement and if it is needed at all.

III. Mathematical Model

This project is designed to determine the minimal cost of transporting HVAC units from factories to warehouses and then from warehouse to customer in the state of Massachusetts. To find the optimal solution while also minimizing costs, transportation costs must be low, and installation and maintenance costs also remain low while also having the capability to meet the demand. The mathematical model was formulated around this optimization problem, but also involved noting costs such as the cost of installation, and cost of maintenance along with the aforementioned cost of transportation.

This model includes some constraints, such as the distance from the factory to the warehouse, the distance from the warehouse to customers, the production capacity of the factories, the storage capacity of various warehouses, and the demand for HVAC units in various locations.

AMPL Model

AMPL was used for solving the problem. AMPL is an algebraic programming language that is used for solving large and complex mathematical problems. To support our aim with this project, it was implemented to solve the mathematical model by determining the minimum cost for the maximum supply. The model and all of its constraints were inputted, and an optimal result was found.

The following parameters were used to construct and execute the model:

P_i = Production capacity Factory i
where i = {1,2,3}

S_j = Supply capacity of Warehouse j
where j = {1,2,3,4}

N_k = Demand of city k
where k = {1,2,3……….,10}

X_{ij} = Number of units transported from factory i to warehouse j
where i = {1,2,3} and j = {1,2,3,4}

Y_{jk} = Number of units transported from warehouse j to city k
where j = {1,2,3,4} and k = {1,2,3……….,10}

D_{ij} = Distance in miles from factory i to warehouse j
where i = {1,2,3} and j = {1,2,3,4}

E_{jk} = Distance in miles from warehouse j to city k where j = {1,2,3,4} and k = {1,2,3……….,10}

T_{ij} = Transportation cost for transporting 1 HVAC unit from factory i to warehouse j where i = {1,2,3} and j = {1,2,3,4}

U_{jk} = Transportation cost for transporting 1 HVAC unit from warehouse j to city k where j = {1,2,3,4} and k = {1,2,3……….,10}

I_{jk} = Installation cost for when 1 HVAC unit is supplied from warehouse j to city k where j = {1,2,3,4} and k = {1,2,3……….,10}

M_{jk} = maintenance cost for when 1 HVAC unit is supplied from warehouse j to city k where j = {1,2,3,4} and k = {1,2,3……….,10}

Data: -

Distance from Warehouse to City

D_{jk}	Boston	Brookline	Cambridge	Framingham	Lowell	Malden	Medford	Quincy	Somerville	Waltham
Lawrence	34	34	88	44	16	25	27	95	38	38
Lynn	13	17	101	33	31	7	18	99	50	50
Newton	10	9	95	25	36	13	14	91	42	42
Worcester	85	80	11	63	82	91	84	6	62	62

Table 1: Distance from warehouse to city

Distance from Factory to Warehouse

D_{ij}	Lawrence	Lynn	Newton	Worcester
Brockton	54	35	25	97
Haverhill	9	37	41	59
Plymouth	67	52	39	83

Table 2: Distance from factory to warehouse

Transportation Cost per Mile

T_{ij}	Lawrence	Lynn	Newton	Worcester
Brockton	3	2	1	4
Haverhill	1	2	2	3
Plymouth	3	3	2	4

Table 3: Transportation Costs (factory to warehouse)

Transportation Cost per Mile

T_{jk}	Boston	Brookline	Cambridge	Framingha	Lowell	Malden	Medford	Quincy	Somerville	Waltham
Lawrence	2	2	4	2	1	1	2	4	2	2
Lynn	1	1	4	2	2	1	1	4	2	2
Newton	1	1	4	1	2	1	1	4	2	2
Worcester	4	4	1	3	4	4	4	1	3	3

Table 4: Transportation Costs (warehouse to city)

Installation cost per unit

I_{jk}	Boston	Brookline	Cambridge	Framingha	Lowell	Malden	Medford	Quincy	Somerville	Waltham
Lawrence	2000	1000	3800	1100	650	550	900	3000	1400	1200
Lynn	1000	500	3800	1100	1300	550	450	3000	1400	1200
Newton	1000	500	3800	550	1300	550	450	3000	1400	1200
Worcester	4000	2000	950	1650	2600	2200	1800	750	2100	1800

Table 5: Installation Costs (warehouse to city)

Maintaince Cost per unit

M_{jk}	Boston	Brookline	Cambridge	Framingha	Lowell	Malden	Medford	Quincy	Somerville	Waltham
Lawrence	400	200	760	220	130	110	180	600	280	240
Lynn	200	100	760	220	260	110	90	600	280	240
Newton	200	100	760	110	260	110	90	600	280	240
Worcester	800	400	190	330	520	440	360	150	420	360

Table 6: Maintenance costs (warehouse to city)

Factory	Capacity
Brockton	160
Haverhill	130
Plymouth	140

Warehouse	Capacity
Lawrence	80
Lynn	90
Newton	120
Worcester	140

Destination	Demand
Boston	90
Brookline	30
Cambridge	65
Framingham	35
Lowell	45
Malden	35
Medford	25
Quincy	40
Somerville	35
Waltham	30

Table 7,8,9: Units supplied from warehouse to city & demand required in each city

The objective function was developed using the following cost factors:

$$Transportation\ Cost = \sum_{i,j=0}^{\substack{i=0\\j=0}} X_{ij} * D_{ij} * T_{ij} + \sum_{j,k=0}^{\substack{j=0\\k=0}} Y_{jk} * E_{jk} * U_{jk}$$

$$Installation\ Cost = \sum_{j,k=0}^{\substack{j=0\\k=0}} Y_{jk} * I_{jk}$$

$$Maintenance\ Cost = \sum_{j,k=0}^{\substack{j=0\\k=0}} Y_{jk} * M_{jk}$$

The objective function of minimizing cost was found to be:

$$Z_{min} = \ Transportation\ Cost + Installtion\ cost + Maintenance\ Cost$$

i.e.,

$$Z_{min} = \sum_{i,j=0}^{\substack{i=0\\j=0}} X_{ij} * D_{ij} * T_{ij} + \sum_{j,k=0}^{\substack{j=0\\k=0}} Y_{jk} * E_{jk} * U_{jk} + \sum_{j,k=0}^{\substack{j=0\\k=0}} Y_{jk} * I_{jk} + \sum_{j,k=0}^{\substack{j=0\\k=0}} Y_{jk} * M_{jk}$$

There were many constraints that came with developing this model. They are as follows:

Production constraints: - $\sum X_{ij} = \sum P_i$

$$x_{11} + x_{12} + x_{13} + x_{14} = 160$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 130$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 140$$

Supply constraints: - $\sum Y_{jk} = \sum S_j$

$$y_{1\,1} + y_{1\,2} + y_{1\,3} + y_{1\,4} + y_{1\,5} + y_{1\,6} + y_{1\,7} + y_{1\,8} + y_{1\,9} + y_{1\,10} = 80$$
$$y_{2\,1} + y_{2\,2} + y_{2\,3} + y_{2\,4} + y_{2\,5} + y_{2\,6} + y_{2\,7} + y_{2\,8} + y_{2\,9} + y_{2\,10} = 90$$
$$y_{3\,1} + y_{3\,2} + y_{3\,3} + y_{3\,4} + y_{3\,5} + y_{3\,6} + y_{3\,7} + y_{3\,8} + y_{3\,9} + y_{3\,10} = 120$$
$$y_{3\,1} + y_{3\,2} + y_{3\,3} + y_{3\,4} + y_{4\,5} + y_{4\,6} + y_{4\,7} + y_{4\,8} + y_{4\,9} + y_{4\,10} = 140$$

Demand Constraints: - $\sum Y_{jk} = \sum N_k$

$$y_{1\,1} + y_{2\,1} + y_{3\,1} + y_{4\,1} = 90$$
$$y_{1\,2} + y_{2\,2} + y_{3\,2} + y_{4\,2} = 30$$
$$y_{1\,3} + y_{2\,3} + y_{3\,3} + y_{4\,3} = 65$$
$$y_{1\,4} + y_{2\,4} + y_{3\,4} + y_{4\,4} = 35$$
$$y_{1\,5} + y_{2\,5} + y_{3\,5} + y_{4\,5} = 45$$
$$y_{1\,6} + y_{2\,6} + y_{3\,6} + y_{4\,6} = 35$$
$$y_{1\,7} + y_{2\,7} + y_{3\,7} + y_{4\,7} = 25$$
$$y_{1\,8} + y_{2\,8} + y_{3\,8} + y_{4\,8} = 40$$
$$y_{1\,9} + y_{2\,9} + y_{3\,9} + y_{4\,9} = 35$$
$$y_{1\,10} + y_{2\,10} + y_{3\,10} + y_{4\,10} = 30$$

Non-negativity constraints: -

$$X_{ij}, Y_{jk} \ \geq 0\ and\ an\ integer$$

where i = {1,2,3}, j = {1,2,3,4}, and k = {1,2,3……….,10}

Also, Total no. of HVAC units coming in warehouse j should be equal to total no. of HVAC units leaving in warehouse j.

$$\sum X_{ij} = \sum Y_{jk}$$

Preliminary Mathematical Model

1. **Assumptions and justifications**
   The assumptions for the preliminary model are that the HVAC units are only being transported from warehouses to cities (customers). The storage capacity of warehouses is equal to the demand i.e., supply = demand. There is no installation and maintenance cost involved. The cost of transportation is the same for all cities i.e., $1 per mile per unit.

2. **Preliminary result analysis**
   Three different types of files were created: a .mod file (model file), a .dat file (data file), and a run file. The model file represents the Preliminary model itself, where the standard integer linear program formulation and constraints were inputted. The data file included all of the collected data in Tables 2, 3, and 5. This gives the program data on to base its optimization. The run file connects the two so the program is left with a solvable problem.
   See the code below:

```
set WAREHOUSE; #warehouse
set CITY; #destinations

param supply {WAREHOUSE} >= 0; # storage capacity at warehouse
param demand {CITY} >= 0; # demand at destinations

#distance between warehouseand city
param distance {WAREHOUSE,CITY} >= 0;

# units to be shipped From_warehouse_to_city
var From_warehouse_to_city {WAREHOUSE,CITY} >= 0;

#objective function
minimize Total_Cost:
sum {i in WAREHOUSE, j in CITY} distance[i,j] * From_warehouse_to_city[i,j];

#Constraint 1
subject to SSupply {i in WAREHOUSE}:
sum {j in CITY} From_warehouse_to_city[i,j] = supply[i];

#Constraint 2
subject to Demand {j in CITY}:
sum {i in WAREHOUSE} From_warehouse_to_city[i,j] = demand[j];

#Constraint 3
check: sum {i in WAREHOUSE} supply[i] >= sum {j in CITY} demand[j];
```

Figure 1: Preliminary AMPL Model File

```
#Supply
param: WAREHOUSE: supply := #Supply
Worcester   140
Lynn     90
Lawrence     80
Newton  120 ;

#Demand
param: CITY: demand := #demand
Boston  90
Cambridge   65
Lowell  45
Quincy  40
Somerville  35
Framingham  35
Malden  35
Waltham 30
Brookline   30
Medford 25 ;

#distance matrix for Warehouse to Cities
param distance:
Boston  Cambridge Lowell Quincy Somerville  Framingham  Malden
Waltham Brookline Medford :=
Worcester   85  11  82  6   62  63  91  62  80  84
Lynn    13  101 31  99  50  33  7   50  17  18
Lawrence    34  88  16  95  38  44  25  38  34  27
Newton  10  95  36  91  42  25  13  42  9   14 ;
```

Figure 2: Preliminary AMPL Data File

```
#reset AMPL
reset;

#load the model
model Prelim.mod;

#load the data
data Prelim.dat;

#set the solver
option solver cplex;

#solve the problem
solve;

#display results
display From_warehouse_to_city;
```

```
ampl: include Prelim.run;
CPLEX 20.1.0.0: optimal solution; objective 7855
3 dual simplex iterations (0 in phase I)
From_warehouse_to_city [*,*] (tr)
:            Lawrence Lynn Newton Worcester    :=
Boston           0      55    35       0
Brookline        0       0    30       0
Cambridge        0       0     0      65
Framingham       0       0    35       0
Lowell          45       0     0       0
Malden           0      35     0       0
Medford          5       0    20       0
Quincy           0       0     0      40
Somerville       0       0     0      35
Waltham         30       0     0       0
;
```

Figure 3: Preliminary AMPL Run File.

X_{jk}	Lawrence	Lynn	Newton	Worcester
Boston	0	55	35	0
Brookline	0	0	30	0
Cambridge	0	0	0	65
Framingham	0	0	35	0
Lowell	45	0	0	0
Malden	0	35	0	0
Medford	5	0	20	0
Quincy	0	0	0	40
Somerville	30	0	0	5
Waltham	0	0	0	30

Table 10: Units transported from factory to warehouse

The above table displays the solution obtained for the preliminary problem from AMPL. According to the solution, it will take at least $7,855 to fulfil the demands in every city.

3. Limitations of the model

Firstly, this is only a basic transportation problem where HVAC units are moved from the warehouse to different cities. Whereas the real problem is to move the HVAC units from factories to warehouses and then from warehouses to different cities as per demand. Secondly, here it is assumed that the installation cost and maintenance cost is zero. Lastly, the transportation cost is also assumed to be the same for moving one HVAC unit.

Final Mathematical Model

1. Assumptions and justifications

The assumptions for the final model are that the HVAC units are first transported from factories to warehouses and then from warehouses to cities (customers). The production capacities of the factories and the storage capacity of warehouses can be greater than demand, but not vice versa. The amount units which are coming in a warehouse has to be delivered to the customer. There is an installation and maintenance cost involved which depends on the location. The cost of transportation is not the same for all cities, it varies depending on the distance from factories or warehouses.

If the distance is in the range of 0 to 24 the transportation cost is $1 per mile per unit

If the distance is in the range of 25 to 49 the transportation cost is $2 per mile per unit

If the distance is in the range of 50 to 74 the transportation cost is $3 per mile per unit

If the distance is in the range of 75 or more the transportation cost is $4 per mile per unit

2. Result display and analysis

The model file represents the final model itself, where the standard integer linear program formulation and constraints were inputted.

```
set FACTORY; # factories
set WAREHOUSE; # warehouses
set CITY; # destinations
param production {FACTORY} >= 0; # production capacity at factories
param supply {WAREHOUSE} >= 0; # storage capacity at warehouses
param demand {CITY} >= 0; # demand at destinations

#distance between factory and warehouse
param distance1 {FACTORY,WAREHOUSE} >= 0;
#distance between warehouse and city
param distance2 {WAREHOUSE,CITY} >= 0;
#cost for factory and warehouse
param cost1 {FACTORY,WAREHOUSE} >= 0;
#cost warehouse and city
param cost2 {WAREHOUSE,CITY} >= 0;
#cost of installation
param install {WAREHOUSE,CITY} >= 0;
#cost of annual maintenance
param maintain {WAREHOUSE,CITY} >= 0;

# units to be shipped From_factory_to_warehouse
var From_factory_to_warehouse {FACTORY,WAREHOUSE} >= 0;
# units to be shipped From_warehouse_to_city
var From_warehouse_to_city {WAREHOUSE,CITY} >= 0;

#objective function
minimize Total_Cost:
sum {i in FACTORY, j in WAREHOUSE} distance1[i,j] * cost1[i,j] * From_factory_to_warehouse[i,j] +
sum {j in WAREHOUSE, k in CITY} distance2[j,k] * cost2[j,k] * From_warehouse_to_city[j,k] +
sum {j in WAREHOUSE, k in CITY} install[j,k] * From_warehouse_to_city[j,k] +
sum {j in WAREHOUSE, k in CITY} maintain[j,k] * From_warehouse_to_city[j,k];

subject to Production {i in FACTORY}:
sum {j in WAREHOUSE} From_factory_to_warehouse[i,j] = production[i];

subject to Supply {j in WAREHOUSE}:
sum {i in FACTORY} From_factory_to_warehouse[i,j] = supply[j];

subject to SSupply {j in WAREHOUSE}:
sum {k in CITY} From_warehouse_to_city[j,k] = supply[j];

subject to Demand {k in CITY}:
sum {j in WAREHOUSE} From_warehouse_to_city[j,k] = demand[k];
```

Figure 4: AMPL Model File

```
param: FACTORY: production :=
Brockton    160
Haverhill   130
Plymouth    140 ;

param: WAREHOUSE: supply :=
Worcester   140
Lynn    90
Lawrence    80
Newton  120 ;

param: CITY: demand :=
Boston  90
Cambridge   65
Lowell  45
Quincy  40
Somerville  35
Framingham  35
Malden  35
Waltham 30
Brookline   30
Medford 25 ;

param distance1:
Worcester   Lynn    Lawrence    Newton :=
Brockton    97  35  54  25
Haverhill   59  37  9   41
Plymouth    83  52  67  39 ;

param distance2:
Boston  Brookline   Cambridge   Framingham  Lowell  Malden  Medford Quincy  Somerville  Waltham :=
Lawrence    34  34  88  44  16  25  27  95  38  38
Lynn    13  17  101 33  31  7   18  99  50  50
Newton  10  9   95  25  36  13  14  91  42  42
Worcester   85  80  11  63  82  91  84  6   62  62 ;

param cost1:
    Lawrence    Lynn    Newton  Worcester :=
Brockton    3   2   1   4
Haverhill   1   2   2   3
Plymouth    3   3   2   4 ;
```

Figure 5: AMPL Data File

```
ampl: include transship.run;
CPLEX 20.1.0.0: optimal solution; objective 523475
6 dual simplex iterations (0 in phase I)
From_factory_to_warehouse :=
Brockton  Lawrence    0
Brockton  Lynn       90
Brockton  Newton     70
Brockton  Worcester   0
Haverhill Lawrence   80
Haverhill Lynn        0
Haverhill Newton      0
Haverhill Worcester  50
Plymouth  Lawrence    0
Plymouth  Lynn        0
Plymouth  Newton     50
Plymouth  Worcester  90
;

From_warehouse_to_city [*,*] (tr)
:           Lawrence Lynn Newton Worcester    :=
Boston         0      60    30      0
Brookline      0       0    30      0
Cambridge      0       0     0     65
Framingham     0       0    35      0
Lowell        45       0     0      0
Malden         5      30     0      0
Medford        0       0    25      0
Quincy         0       0     0     40
Somerville    30       0     0      5
Waltham        0       0     0     30
;
```

Figure 6: AMPL Run File

The following tables show the no. of units moving from factory to warehouse and then from warehouse to city.

Factory to Warehouses

X_{ij}	Lawrence	Lynn	Newton	Worcester	
Brockton	0	90	70	0	160
Haverhill	80	0	0	50	130
Plymouth	0	0	50	90	140
	80	90	120	140	

Warehouse to Cities

X_{jk}	Lawrence	Lynn	Newton	Worcester	
Boston	0	60	30	0	90
Brookline	0	0	30	0	30
Cambridge	0	0	0	65	65
Framingham	0	0	35	0	35
Lowell	45	0	0	0	45
Malden	5	30	0	0	35
Medford	0	0	25	0	25
Quincy	0	0	0	40	40
Somerville	30	0	0	5	35
Waltham	0	0	0	30	30
	80	90	120	140	

Table 11: Units transported from factory to warehouses & vice-versa

The above table displays the solution obtained for the final transshipment problem from AMPL. According to the solution, it will take at least $ 523,475 to fulfil the demands in every city. This cost includes the cost of transporting HVAC units from factories to warehouses and from warehouses to customers, installation costs, and maintenance costs for each unit.

IV. Integer Linear Programming

The goal was to minimize the cost required to transport HVAC units from factories to warehouses and then from warehouses to customers in the state of Massachusetts. There are many other factors affecting the model like the population of the locations, the cost of manufacturing HVAC units, size of a house, and demand ratios.

V. Sensitivity Analysis

A sensitivity analysis is a tool used in financial modelling to evaluate how the uncertainty in the input variables of a model affects the output of the model. In this case, a sensitivity analysis was conducted to account for the likelihood that the predicted demand for HVAC units may not be equal to the actual demand.

The analysis assumed that 70% of the predicted demand would be met, and that the transportation and installation costs would be increased by 10%. This sensitivity analysis allows for a better understanding of the potential risks and uncertainties associated with the demand for HVAC units. It also provides a basis for making more informed decisions about the production and distribution of these units.

Destinatio	Actual demand
Boston	63
Brookline	21
Cambridge	46
Framingha	25
Lowell	32
Malden	25
Medford	18
Quincy	28
Somerville	25
Waltham	21

Transporation cost per mile

Tij	Lawrence	Lynn	Newton	Worcester
Brockton	3.3	2.2	1.1	4.4
Haverhill	1.1	3.3	2.2	3.3
Plymouth	3.3	3.3	2.2	4.4

Table 12,13: Demand of Destination & Transportation cost/mile.

Transportation Cost per Mile

T_{jk}	Boston	Brookline	Cambridge	Framingha	Lowell	Malden	Medford	Quincy	Somerville	Waltham
Lawrence	2.2	2.2	4.4	2.2	1.1	1.1	2.2	4.4	2.2	2.2
Lynn	1.1	1.1	4.4	2.2	2.2	1.1	1.1	4.4	2.2	2.2
Newton	1.1	1.1	4.4	1.1	2.2	1.1	1.1	4.4	2.2	2.2
Worcester	4.4	4.4	1.1	3.3	4.4	4.4	4.4	1.1	3.3	3.3

Table 14: Transportation cost/mile from warehouse to city

Installation cost per unit

I_{jk}	Boston	Brookline	Cambridge	Framingha	Lowell	Malden	Medford	Quincy	Somerville	Waltham
Lawrence	2200	1100	4180	1210	715	605	990	3300	1540	1320
Lynn	1100	550	4180	1210	1430	605	495	3300	1540	1320
Newton	1100	550	4180	605	1430	605	495	3300	1540	1320
Worcester	4400	2200	1045	1815	2860	2420	1980	825	2310	1980

Table 15: Installation costs/unit from warehouse to city

With this, the new values were taken into consideration and a new minimum cost was found using AMPL:

AMPL Model for Sensitivity Analysis

1. The Sensitivity Analysis Model File:

sensi.mod | sensi.dat | sensi.run

```
param supply {WAREHOUSE} >= 0; # storage capacity at warehouses
param demand {CITY} >= 0; # demand at destinations

#distance between factory and warehouse
param distance1 {FACTORY,WAREHOUSE} >= 0;
#distance between warehouse and city
param distance2 {WAREHOUSE,CITY} >= 0;
#cost for factory and warehouse
param cost1 {FACTORY,WAREHOUSE} >= 0;
#cost warehouse and city
param cost2 {WAREHOUSE,CITY} >= 0;
#cost of installation
param install {WAREHOUSE,CITY} >= 0;
#cost of annual maintenance
param maintain {WAREHOUSE,CITY} >= 0;

# units to be shipped From_factory_to_warehouse
var From_factory_to_warehouse {FACTORY,WAREHOUSE} >= 0;
# units to be shipped From_warehouse_to_city
var From_warehouse_to_city {WAREHOUSE,CITY} >= 0;

check: sum {i in FACTORY} production[i] >= sum {k in CITY} demand[k];

#objective function
minimize Total_Cost:
sum {i in FACTORY, j in WAREHOUSE} distance1[i,j] * cost1[i,j] * From_factory_to_warehouse[i,j] +
sum {j in WAREHOUSE, k in CITY} distance2[j,k] * cost2[j,k] * From_warehouse_to_city[j,k] +
sum {j in WAREHOUSE, k in CITY} install[j,k] * From_warehouse_to_city[j,k] +
sum {j in WAREHOUSE, k in CITY} maintain[j,k] * From_warehouse_to_city[j,k];

subject to Production {i in FACTORY}:
sum {j in WAREHOUSE} From_factory_to_warehouse[i,j] = production[i];

subject to Supply {j in WAREHOUSE}:
sum {i in FACTORY} From_factory_to_warehouse[i,j] <= supply[j];

subject to SSupply {j in WAREHOUSE}:
sum {k in CITY} From_warehouse_to_city[j,k] <= supply[j];

subject to Demand {k in CITY}:
sum {j in WAREHOUSE} From_warehouse_to_city[j,k] = demand[k];
```

Figure 7: AMPL Model File (2)

2. The Sensitivity Analysis Data File:

sensi.mod | sensi.dat | sensi.run

```
param: C:\Users\suraj\Desktop\OR Project\sensi.mod
Brockton    160
Haverhill   130
Plymouth    140 ;

param: WAREHOUSE: supply :=
Worcester   140
Lynn    90
Lawrence    80
Newton  120 ;

param: CITY: demand :=
Boston  63
Brookline   21
Cambridge   46
Framingham  25
Lowell  32
Malden  25
Medford 18
Quincy  28
Somerville  25
Waltham 21 ;

param distance1:
Worcester   Lynn    Lawrence    Newton :=
Brockton    97  35  54  25
Haverhill   59  37  9   41
Plymouth    83  52  67  39 ;

param distance2:
Boston  Brookline   Cambridge   Framingham  Lowell  Malden  Medford Quincy  Somerville  Waltham :=
Lawrence    34  34  88  44  16  25  27  95  38  38
Lynn    13  17  101 33  31  7   18  99  50  50
Newton  10  9   95  25  36  13  14  91  42  42
Worcester   85  80  11  63  82  91  84  6   62  62 ;

param cost1:
    Lawrence    Lynn    Newton  Worcester :=
Brockton    3.3 2.2 1.1 4.4
Haverhill   1.1 3.3 2.2 3.3
Plymouth    3.3 3.3 2.2 4.4 ;
```

Figure 8: AMPL Data File (2)

3.The Sensitivity Analysis Run File:

```
sensi.mod   sensi.dat   sensi.run

#reset AMPL
reset;

#load the model
model sensi.mod;

#load the data
data sensi.dat;

#set the solver
option solver cplex;

#solve the problem
solve;

#display results
display From_factory_to_warehouse;
display From_warehouse_to_city;
```

Figure 9: AMPL Run File (2)

Factory to Warehouses

	Lawrence	Lynn	Newton	Worcester
Brockton	0	90	70	0
Haverhill	80	0	0	50
Plymouth	0	0	50	90

Warehouse to Cities

	Lawrence	Lynn	Newton	Worcester
Boston	0	7	56	0
Brookline	0	0	21	0
Cambridge	0	0	0	46
Framingha	0	0	25	0
Lowell	32	0	0	0
Malden	0	25	0	0
Medford	0	0	18	0
Quincy	0	0	0	28
Somerville	25	0	0	0
Waltham	21	0	0	0

Table 16: Units transported from factory to warehouses & vice-versa after sensitivity analysis

The table shows the solution to the final transhipment problem from AMPL. The solution indicates that it will cost at least $ 395011.4 to fulfil the new demands in each city. This cost includes the cost of transporting HVAC units from factories to warehouses and from warehouses to customers, as well as installation and maintenance costs for each unit.

VI. Results

i. Scenario i

The optimal solution was found, and it will take at least $ 523,475 to fulfil the demands in every city where factory Brockton supplies 90 & 70 units to Lynn and Newton warehouses, Haverhill factory supplies 80 and 50 units to Lawrence and Worcester and Plymouth factory supplies 50 & 90 units to Newton and Worcester to fulfil supply and demand.

For warehouse to cities goods flow, Lynn warehouse will supply 7 & 25 units to Boston and Malden. Lawrence warehouse will supply 32 & 25 units to Lowell and Somerville, additionally it will supply 21 units to Waltham. Newton warehouse will supply 56, 21, 25 and 18 units to Boston, Brookline, Framingham, and Medford. Worcester warehouse will supply 46 & 28 units to Cambridge & Quincy and would not supply any units to Somerville and Waltham.

ii. Scenario ii

After conducting sensitivity analysis on the data, the optimal solution indicates that it will cost at least $ 395011.4 to fulfil the new demands in each city. We observed some changes in locations while performing warehouse to cities flow of goods. Lynn warehouse will supply 60 & 30 units to Boston and Madlen. Lawrence warehouse will supply 45 & 30 units to Lowell and Somerville. Newton warehouse will supply 30, 30, 35 and 25 units to Boston, Brookline, Framingham, and Medford. Worcester warehouse will supply 65,40 & 30 units to Cambridge and Quincy, Somerville and Waltham to fulfil demand of each city.

VII. Future Scope and Improvement

Some potential areas for future work in this field could include:

1. Developing new and improved optimization algorithms for HVAC unit transportation that can more effectively balance the competing goals of minimizing cost and maximizing supply. This could involve the use of advanced techniques such as machine learning and artificial intelligence to identify optimal solutions in complex and dynamic environments.

2. Investigating the potential for using HVAC units as a source of renewable energy, through the use of advanced technologies such as solar panels, wind turbines, and fuel cells. This could involve the development of new models and algorithms that can optimize the deployment of HVAC units in a way that maximizes their potential as renewable energy generators.

3. Examining the potential for using HVAC units to support the growth of new businesses and industries in urban areas, through the provision of heating, cooling, and ventilation services. This could involve the development of new models and algorithms that can optimize the deployment of HVAC units in a way that maximizes their potential to support the growth of new businesses and industries.

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