Assignment 8 - DEA Analysis

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The Hope Valley Health Care Association owns and operates six nursing homes in adjoining states. An evaluation of their efficiency has been undertaken using two inputs and two outputs. The inputs are staffing labor (measured in average hours per day) and the cost of supplies (in thousands of dollars per day). The outputs are the number of patient-days reimbursed by third party sources and the number of patient-days reimbursed privately.

Let us now calculate the weights to achieve the efficiency values for each nursing home

```
#install.packages("Benchmarking")
library(Benchmarking)
library(tidyverse)
library(dplyr)
```

Loading all the packages. Our problem has 6 Nursing Homes with
2 inputs: staff Hours per day and supplies per day
2 outputs: Reimbursed Patient-days, Privately Paid patient-days.

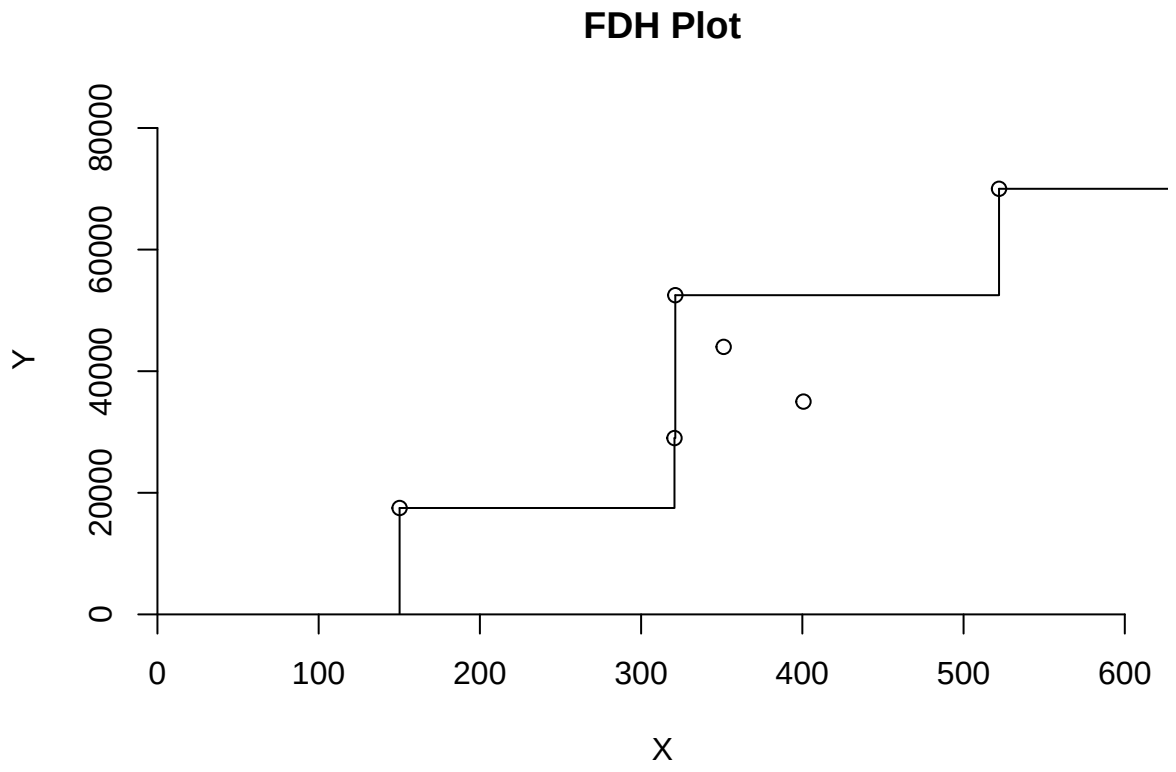
```
input <- matrix(c(150,400,320,520,350,320, 0.2,0.7, 1.2,2.0,1.2,0.7),ncol=2)
output <- matrix(c(14000,14000,42000,28000,19000,14000, 3500,21000, 10500,42000,25000,15000),ncol = 2)
colnames(input) <- c("Staff Hours/day","Supplies/day")
colnames(output) <- c("ReimursedPateients","Privately Paid Patients")
main <- cbind(input, output)
main
```

```
##      Staff Hours/day Supplies/day ReimursedPateients Privately Paid Patients
## [1,]             150          0.2              14000                    3500
## [2,]             400          0.7              14000                   21000
## [3,]             320          1.2              42000                   10500
## [4,]             520          2.0              28000                   42000
## [5,]             350          1.2              19000                   25000
## [6,]             320          0.7              14000                   15000
```

We now run the DEA analysis.

DEA Analysis with FDH

```
e1 <- dea(input,output,RTS = "fdh")             # provide the input and output

effdh<- eff(e1)                                 # put efficiency in a dataset

p1<- peers(e1)                        # identify the peers

colnames(p1)[1] <- "FDH_peer1"

l1<- lambda(e1)                                 # identify the relative weights given to the peers

colnames(l1)[1] <- "FDH_L1"
colnames(l1)[2] <- "FDH_L2"
colnames(l1)[3] <- "FDH_L3"
colnames(l1)[4] <- "FDH_L4"
colnames(l1)[5] <- "FDH_L5"
colnames(l1)[6] <- "FDH_L6"

dea.plot(input,output,RTS="fdh",ORIENTATION="in-out",txt=rownames(input),main="FDH Plot")  # plot the r
```




```
table1<- cbind(main,p1,l1,effdh)              #Summarised table
table1
```

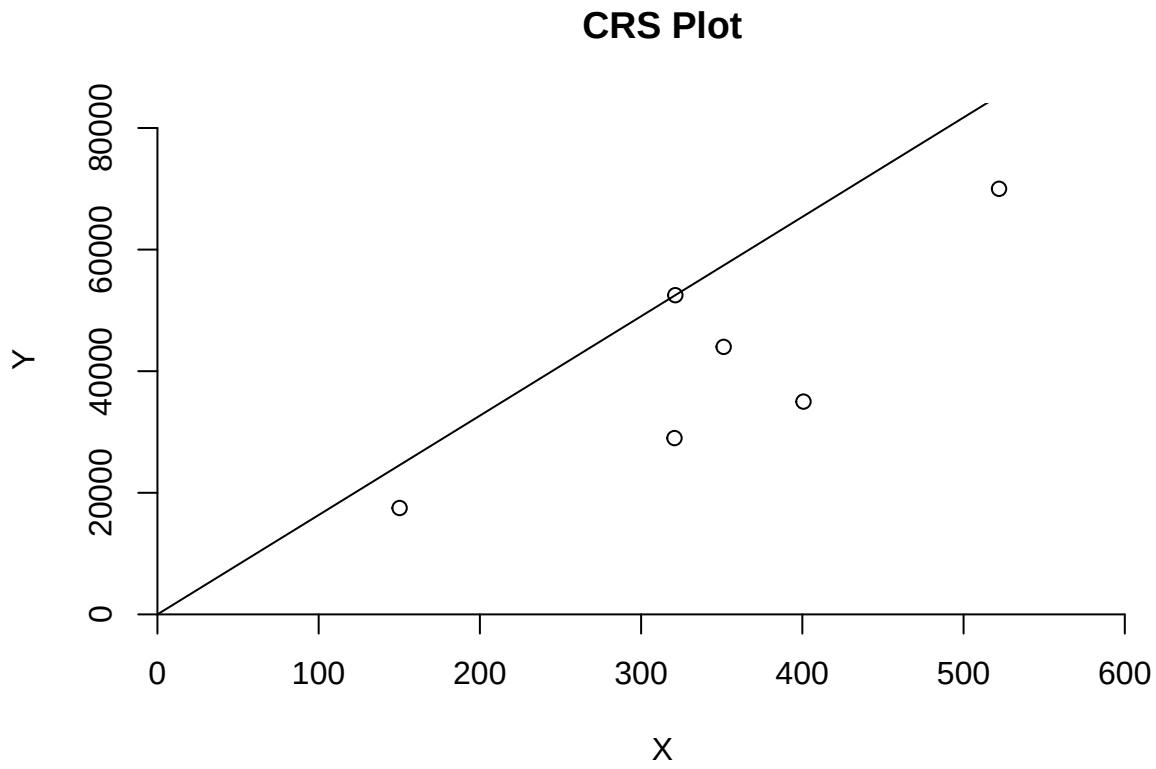
```
##      Staff Hours/day Supplies/day ReimursedPateients Privately Paid Patients
```

```
## [1,]             150          0.2             14000                3500
## [2,]             400          0.7             14000               21000
## [3,]             320          1.2             42000               10500
## [4,]             520          2.0             28000               42000
## [5,]             350          1.2             19000               25000
## [6,]             320          0.7             14000               15000
##      FDH_peer1 FDH_L1 FDH_L2 FDH_L3 FDH_L4 FDH_L5 FDH_L6 effdh
## [1,]         1      1      0      0      0      0      0     1
## [2,]         2      0      1      0      0      0      0     1
## [3,]         3      0      0      1      0      0      0     1
## [4,]         4      0      0      0      1      0      0     1
## [5,]         5      0      0      0      0      1      0     1
## [6,]         6      0      0      0      0      0      1     1
```

This Model suggests that all the Nursing Branches are efficient.

DEA Analysis with CRS

```
e2 <- dea(input,output,RTS = "crs")            # provide the input and output

efcrs<- eff(e2)

p2<- peers(e2)                    # identify the peers

colnames(p2)[1] <- "CRS_peer1"
colnames(p2)[2] <- "CRS_peer2"
colnames(p2)[3] <- "CRS_peer3"

l2<- lambda(e2)                              # identify the relative weights given to the peers

colnames(l2)[1] <- "CRS_L1"
colnames(l2)[2] <- "CRS_L2"
colnames(l2)[3] <- "CRS_L3"
colnames(l2)[4] <- "CRS_L4"

dea.plot(input,output,RTS="crs",ORIENTATION="in-out",main="CRS Plot")         # plot the results
```




```
table2<- cbind(main,p2,l2,efcrs)            #Summarize the result
table2
```

```
##      Staff Hours/day Supplies/day ReimursedPateients Privately Paid Patients
```

```
## [1,]             150          0.2              14000                    3500
## [2,]             400          0.7              14000                   21000
## [3,]             320          1.2              42000                   10500
## [4,]             520          2.0              28000                   42000
## [5,]             350          1.2              19000                   25000
## [6,]             320          0.7              14000                   15000
##      CRS_peer1 CRS_peer2 CRS_peer3    CRS_L1     CRS_L2 CRS_L3    CRS_L4
## [1,]         1        NA        NA 1.0000000 0.00000000      0 0.0000000
## [2,]         2        NA        NA 0.0000000 1.00000000      0 0.0000000
## [3,]         3        NA        NA 0.0000000 0.00000000      1 0.0000000
## [4,]         4        NA        NA 0.0000000 0.00000000      0 1.0000000
## [5,]         1         2         4 0.2000000 0.08048142      0 0.5383307
## [6,]         1         2         4 0.3428571 0.39499264      0 0.1310751
##          efcrs
## [1,] 1.0000000
## [2,] 1.0000000
## [3,] 1.0000000
## [4,] 1.0000000
## [5,] 0.9774987
## [6,] 0.8674521
```

The result indicates that Nursing branches 1,2,3 and 4 are efficient.

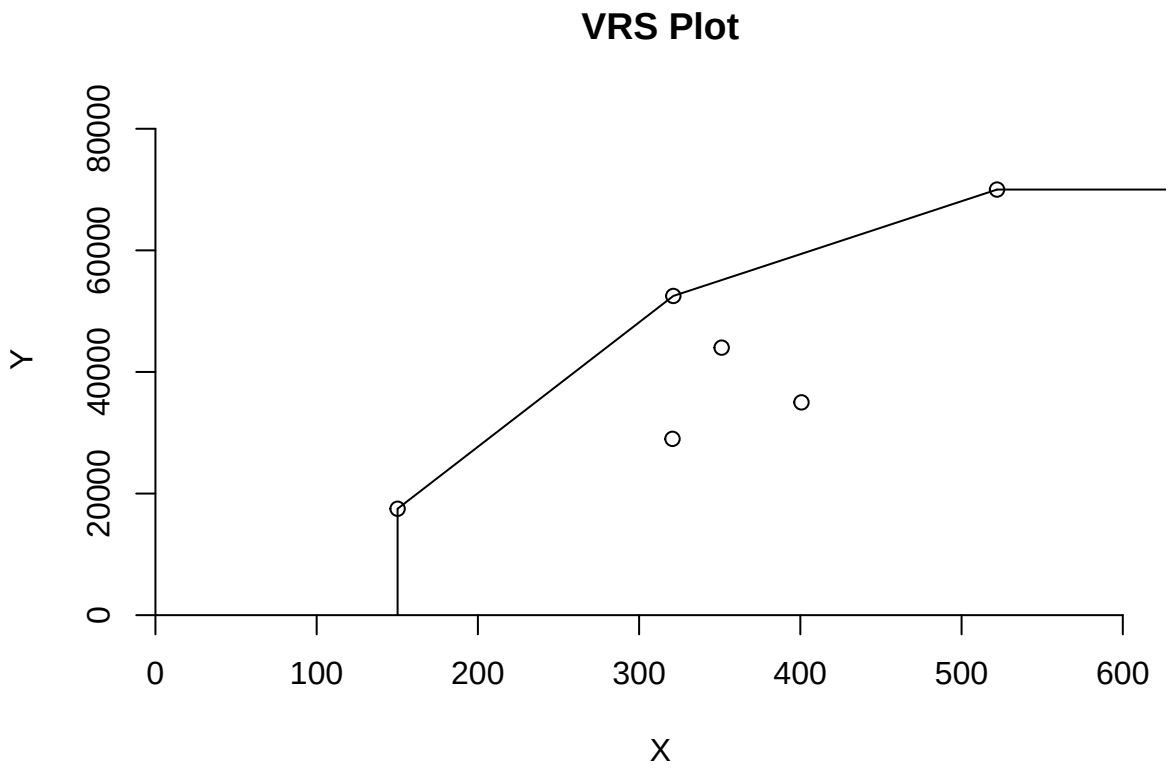
Efficiency of Nursing Branch 5 is 97.74% and of Branch 6 is 86.74%.

Branch 5 has 1, 2, and 4 as it's peers with relative weight of 0.2, 0.08, and 0.53.

Branch 6 has 1, 2, and 4 as it's peers with relative weight of 0.34, 0.39, and 0.13.

DEA Analysis with VRS

```
e3 <- dea(input,output,RTS = "vrs")            # provide the input and output

efvrs<- eff(e3)

p3<- peers(e3)                      # identify the peers

colnames(p3)[1] <- "VRS_peer1"
colnames(p3)[2] <- "VRS_peer2"
colnames(p3)[3] <- "VRS_peer3"

l3<- lambda(e3)                               # identify the relative weights given to the peers

colnames(l3)[1] <- "VRS_L1"
colnames(l3)[2] <- "VRS_L2"
colnames(l3)[3] <- "VRS_L3"
colnames(l3)[4] <- "VRS_L4"
colnames(l3)[5] <- "VRS_L5"

dea.plot(input,output,RTS="vrs",ORIENTATION="in-out",txt=rownames(input), main="VRS Plot")     # plot t
```




```
table3<- cbind(main,p3,l3,efvrs)
table3
```

```
##      Staff Hours/day Supplies/day ReimursedPateients Privately Paid Patients
## [1,]             150          0.2              14000                    3500
## [2,]             400          0.7              14000                   21000
## [3,]             320          1.2              42000                   10500
## [4,]             520          2.0              28000                   42000
## [5,]             350          1.2              19000                   25000
## [6,]             320          0.7              14000                   15000
##      VRS_peer1 VRS_peer2 VRS_peer3    VRS_L1    VRS_L2 VRS_L3 VRS_L4    VRS_L5
## [1,]         1        NA        NA 1.0000000 0.0000000      0      0 0.0000000
## [2,]         2        NA        NA 0.0000000 1.0000000      0      0 0.0000000
## [3,]         3        NA        NA 0.0000000 0.0000000      1      0 0.0000000
## [4,]         4        NA        NA 0.0000000 0.0000000      0      1 0.0000000
## [5,]         5        NA        NA 0.0000000 0.0000000      0      0 1.0000000
## [6,]         1         2         5 0.4014399 0.3422606      0      0 0.2562995
##          efvrs
## [1,] 1.0000000
## [2,] 1.0000000
## [3,] 1.0000000
## [4,] 1.0000000
## [5,] 1.0000000
## [6,] 0.8963283
```

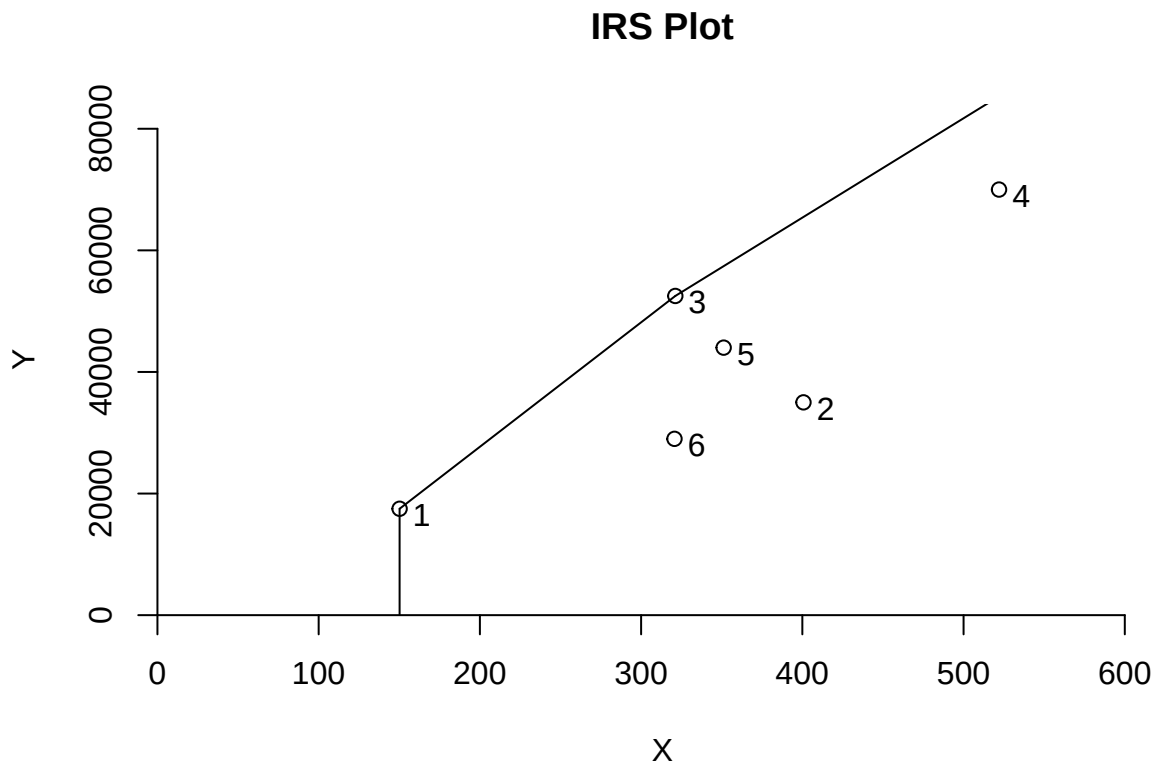
The result indicates that Nursing branches 1,2,3,4 and 5 are efficient.

Efficiency of Nursing Branch 6 is 89.63%.

Branch 6 has 1, 2, and 5 as it's peers with relative weight of 0.4, 0.34, and 0.25.

IRS

```
e4 <- dea(input,output,RTS = "irs")            # provide the input and output

efirs <- eff(e4)

p4<- peers(e4)                      # identify the peers

colnames(p4)[1] <- "IRS_peer1"
colnames(p4)[2] <- "IRS_peer2"
colnames(p4)[3] <- "IRS_peer3"

l4<- lambda(e4)                               # identify the relative weights given to the peers

colnames(l4)[1] <- "IRS_L1"
colnames(l4)[2] <- "IRS_L2"
colnames(l4)[3] <- "IRS_L3"
colnames(l4)[4] <- "IRS_L4"
colnames(l4)[5] <- "IRS_L5"

dea.plot(input,output,RTS="irs",ORIENTATION="in-out",txt=TRUE,main="IRS Plot")    # plot the results
```




```
table4<- cbind(main,p4,l4,efirs)
table4
```

```
##      Staff Hours/day Supplies/day ReimursedPateients Privately Paid Patients
## [1,]             150          0.2              14000                    3500
## [2,]             400          0.7              14000                   21000
## [3,]             320          1.2              42000                   10500
## [4,]             520          2.0              28000                   42000
## [5,]             350          1.2              19000                   25000
## [6,]             320          0.7              14000                   15000
##      IRS_peer1 IRS_peer2 IRS_peer3    IRS_L1    IRS_L2 IRS_L3 IRS_L4    IRS_L5
## [1,]         1        NA        NA 1.0000000 0.0000000      0      0 0.0000000
## [2,]         2        NA        NA 0.0000000 1.0000000      0      0 0.0000000
## [3,]         3        NA        NA 0.0000000 0.0000000      1      0 0.0000000
## [4,]         4        NA        NA 0.0000000 0.0000000      0      1 0.0000000
## [5,]         5        NA        NA 0.0000000 0.0000000      0      0 1.0000000
## [6,]         1         2         5 0.4014399 0.3422606      0      0 0.2562995
##          efirs
## [1,] 1.0000000
## [2,] 1.0000000
## [3,] 1.0000000
## [4,] 1.0000000
## [5,] 1.0000000
## [6,] 0.8963283
```

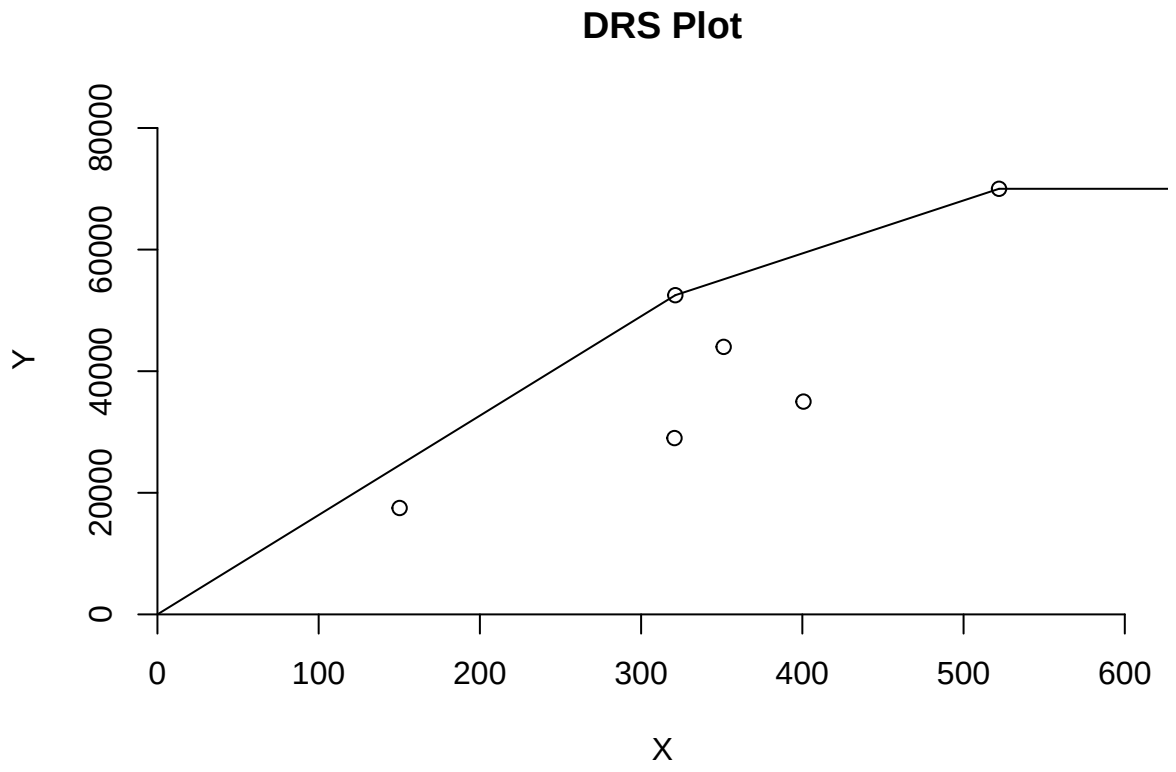
The result indicates that Nursing branches 1,2,3,4 and 5 are efficient.

Efficiency of Nursing Branch 6 is 89.63%.

Branch 6 has 1, 2, and 5 as it's peers with relative weight of 0.4, 0.34, and 0.25.

DRS

```
e5 <- dea(input,output,RTS = "drs")            # provide the input and output

efdrs<- eff(e5)

p5<- peers(e5)                    # identify the peers

colnames(p5)[1] <- "DRS_peer1"
colnames(p5)[2] <- "DRS_peer2"
colnames(p5)[3] <- "DRS_peer3"

l5<- lambda(e5)                             # identify the relative weights given to the peers

colnames(l5)[1] <- "DRS_L1"
colnames(l5)[2] <- "DRS_L2"
colnames(l5)[3] <- "DRS_L3"
colnames(l5)[4] <- "DRS_L4"

dea.plot(input,output,RTS="drs",ORIENTATION="in-out", main="DRS Plot")     # plot the results
```




```
table5<- cbind(main,p5,l5,efdrs)
table5
```

```
##      Staff Hours/day Supplies/day ReimursedPateients Privately Paid Patients
```

```
## [1,]             150          0.2              14000                    3500
## [2,]             400          0.7              14000                   21000
## [3,]             320          1.2              42000                   10500
## [4,]             520          2.0              28000                   42000
## [5,]             350          1.2              19000                   25000
## [6,]             320          0.7              14000                   15000
##      DRS_peer1 DRS_peer2 DRS_peer3    DRS_L1     DRS_L2 DRS_L3    DRS_L4
## [1,]         1        NA        NA 1.0000000 0.00000000      0 0.0000000
## [2,]         2        NA        NA 0.0000000 1.00000000      0 0.0000000
## [3,]         3        NA        NA 0.0000000 0.00000000      1 0.0000000
## [4,]         4        NA        NA 0.0000000 0.00000000      0 1.0000000
## [5,]         1         2         4 0.2000000 0.08048142      0 0.5383307
## [6,]         1         2         4 0.3428571 0.39499264      0 0.1310751
##          efdrs
## [1,] 1.0000000
## [2,] 1.0000000
## [3,] 1.0000000
## [4,] 1.0000000
## [5,] 0.9774987
## [6,] 0.8674521
```

The result indicates that Nursing branches 1,2,3 and 4 are efficient.

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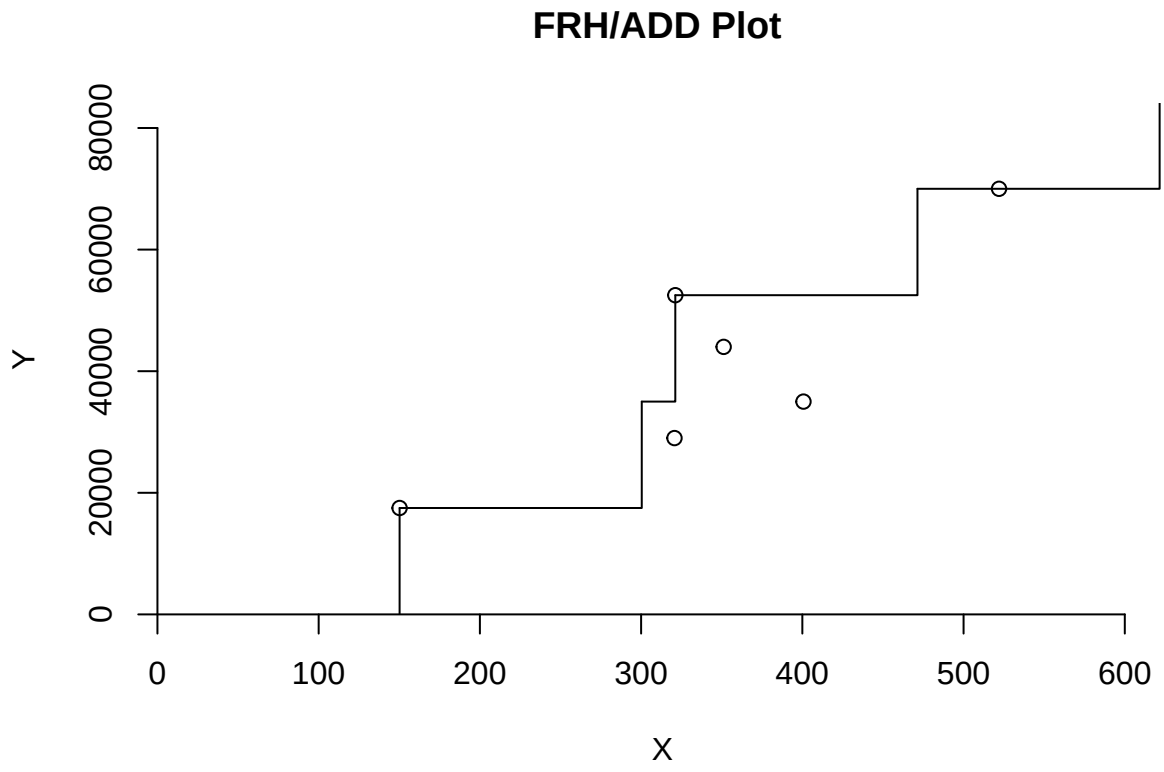
Branch 5 has 1, 2, and 4 as it's peers with relative weight of 0.2, 0.08, and 0.53.

Branch 6 has 1, 2, and 4 as it's peers with relative weight of 0.34, 0.39, and 0.13.

FRH

```
e6 <- dea(input,output,RTS = "add")            # provide the input and output

effrh <- eff(e6)

p6<- peers(e6)                      # identify the peers

colnames(p6)[1] <- "FRH_peer1"

l6<- lambda(e6)                              # identify the relative weights given to the peers

colnames(l6)[1] <- "FRH_L1"
colnames(l6)[2] <- "FRH_L2"
colnames(l6)[3] <- "FRH_L3"
colnames(l6)[4] <- "FRH_L4"
colnames(l6)[5] <- "FRH_L5"
colnames(l6)[6] <- "FRH_L6"

dea.plot(input,output,RTS="add",ORIENTATION="in-out",txt=rownames(input),main="FRH/ADD Plot ")
```




```
table6<- cbind(main,p6,l6,effrh)
table6
```

```
     Staff Hours/day Supplies/day ReimursedPateients Privately Paid Patients
```

```
[1,]             150          0.2              14000                    3500
[2,]             400          0.7              14000                   21000
[3,]             320          1.2              42000                   10500
[4,]             520          2.0              28000                   42000
[5,]             350          1.2              19000                   25000
[6,]             320          0.7              14000                   15000
     FRH_peer1 FRH_L1 FRH_L2 FRH_L3 FRH_L4 FRH_L5 FRH_L6 effrh
[1,]         1      1      0      0      0      0      0     1
[2,]         2      0      1      0      0      0      0     1
[3,]         3      0      0      1      0      0      0     1
[4,]         4      0      0      0      1      0      0     1
[5,]         5      0      0      0      0      1      0     1
[6,]         6      0      0      0      0      0      1     1
```

According to this model, all the Nursing Branches are efficient.

Let's look at the efficiencies of all the models to compare them

```
effic <- cbind(effdh,efcrs,efvrs,efirs,efdrs,effrh)
effic
```

```
##      effdh     efcrs     efvrs     efirs     efdrs effrh
## [1,]     1 1.0000000 1.0000000 1.0000000 1.0000000     1
## [2,]     1 1.0000000 1.0000000 1.0000000 1.0000000     1
## [3,]     1 1.0000000 1.0000000 1.0000000 1.0000000     1
## [4,]     1 1.0000000 1.0000000 1.0000000 1.0000000     1
## [5,]     1 0.9774987 1.0000000 1.0000000 0.9774987     1
## [6,]     1 0.8674521 0.8963283 0.8963283 0.8674521     1
```

So Looking at this we see that our model performs very similarly for:

FDH and FRH CRS and DRS VRS and IRS

In VRS, we can look at what will happen if we rescale a firm. In VRS, there are 2 possibilities: If we increase the input and the output increases - This is Constant Return to Scale. If we decrease the input and the output decreases - Decreasing Return to Scale.

Comparing CRS and VRS CRS - determined by the highest achievable ratio of outputs to inputs in the sample, regardless of size.

VRS - passes through the points where the Nursing Homes have the highest output to input ratios, given their relative size, then runs parallel to the respective axes beyond the extreme points.

Mostly the efficiency scores calculated under variable returns, VRS, will be higher than or equal to those obtained under constant returns,CRS. Which we can observe in our output.

When Efficiency score = 1, VRS and CRS coincide.

IRS, Increasing returns to scale shows a proportional increase in all inputs causes outputs to increase by a greater proportion.

FDH model relaxes the convexity assumption of basic DEA models. The efficiency scores of FDH input-oriented model are always greater than the ones of input-oriented variable returns to scale(VRS) model. We can easily see this in our output also, where we have efficiency score as 1 for all the Nursing Homes but it's not the same with efficiency scores for VRS.

Because of this above relationship: The production possibility set of FDH model is subset of VRS model as well as CRS model.

FRH model permits input and output to enter in only discrete amounts. It's helpful when we want to use mixed integer programming type.