10. PERCENTAGE

IMPORTANT FACTS AND FORMULAE

I. Concept of Percentage : By a certain *percent*, we mean that many hundredths. Thus, x percent means x hundredths, written as $x\%$.

To express x% as a fraction : We have, $x\% = \frac{x}{100}$.

Thus, $20\% = \frac{20}{100} = \frac{1}{5}$; $48\% = \frac{48}{100} = \frac{12}{25}$, etc..

To express $\frac{a}{b}$ as a percent : We have, $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$.

Thus, $\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = 25\%$; $0.6 = \frac{6}{10} = \frac{3}{5} = \left(\frac{3}{5} \times 100\right)\% = 60\%$.

II. If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is

$$\left[\frac{R}{(100 + R)} \times 100\right]\%$$

If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is

$$\left[\frac{R}{(100 - R)} \times 100\right]\%$$

III. Results on Population : Let the population of a town be P now and suppose it increases at the rate of R% per annum, then :

1. Population after n years $= P\left(1 + \frac{R}{100}\right)^n$.

2. Population n years ago $= \frac{P}{\left(1 + \frac{R}{100}\right)^n}$.

IV. Results on Depreciation : Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then :

1. Value of the machine after n years $= P\left(1 - \frac{R}{100}\right)^n$.

2. Value of the machine n years ago $= \frac{P}{\left(1 - \frac{R}{100}\right)^n}$.

V. If A is R% more than B, then B is less than A by

$$\left[\frac{R}{(100 + R)} \times 100\right]\%.$$

If A is R% less than B, then B is more than A by

$$\left[\frac{R}{(100 - R)} \times 100\right]\%.$$

4. SIMPLIFICATION

IMPORTANT CONCEPTS

I. **'BODMAS' Rule :** This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression.
Here, 'B' stands for ***'Bracket'***, 'O' for ***'of'***, 'D' for ***'Division'***, 'M' for ***'Multiplication'***, 'A' for ***'Addition'*** and 'S' for ***'Subtraction'***.
Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order (), {} and [].
After removing the brackets, we must use the following operations strictly in the order :
(*i*) of (*ii*) Division (*iii*) Multiplication (*iv*) Addition (*v*) Subtraction.

II. **Modulus of a Real Number :** Modulus of a real number a is defined as

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a, & \text{if } a < 0. \end{cases}$$

Thus, $|5| = 5$ and $|-5| = -(-5) = 5$.

III. **Virnaculum** (or Bar) : When an expression contains Virnaculum, before applying the 'BODMAS' rule, we simplify the expression under the Virnaculum.

6. AVERAGE

IMPORTANT FACTS AND FORMULAE

1. Average $= \left(\dfrac{\text{Sum of observations}}{\text{Number of observations}} \right)$.
2. Suppose a man covers a certain distance at x kmph and an equal distance at y kmph. Then, the average speed during the whole journey is $\left(\dfrac{2xy}{x+y} \right)$ kmph.

12. RATIO AND PROPORTION

IMPORTANT FACTS AND FORMULAE

1. **RATIO** : The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.
   In the ratio $a : b$, we call a as the **first term** or **antecedent** and b, the **second term** or **consequent**.

   Ex. The ratio $5 : 9$ represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

   Rule : The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.
   Ex. $4 : 5 = 8 : 10 = 12 : 15$ etc. Also, $4 : 6 = 2 : 3$.
2. **PROPORTION** : *The equality of two ratios is called proportion.*
   If $a : b = c : d$, we write, $a : b :: c : d$ and we say that a, b, c, d are in proportion.
   Here a and d are called **extremes**, while b and c are called **mean terms**.
   Product of means = Product of extremes.
   Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.
3. (*i*) **Fourth Proportional** : If $a : b = c : d$, then d is called the fourth proportional to a, b, c.
   (*ii*) **Third Proportional** : If $a : b = b : c$, then c is called the third proportional to a and b.
   (*iii*) **Mean Proportional** : Mean proportional between a and b is $\sqrt{ab}$.
4. (*i*) **COMPARISON OF RATIOS :**
   We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.
   (*ii*) **COMPOUNDED RATIO :**
   The compounded ratio of the ratios $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.
5. (*i*) ***Duplicate ratio*** of $(a : b)$ is $(a^2 : b^2)$.
   (*ii*) ***Sub-duplicate ratio*** of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.
   (*iii*) ***Triplicate ratio*** of $(a : b)$ is $(a^3 : b^3)$.
   (*iv*) **Sub-triplicate ratio** of $(a : b)$ is $\left(a^{\frac{1}{3}} : b^{\frac{1}{3}}\right)$.
   (*v*) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. **(componendo and dividendo)**
6. **VARIATION :**
   (*i*) We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$.
   (*ii*) We say that x is inversely proportional to y, if $xy = k$ for some constant k and we write, $x \propto \frac{1}{y}$.

13. PARTNERSHIP

IMPORTANT FACTS AND FORMULAE

1. **Partnership :** When two or more than two persons run a business jointly, they are called ***partners*** and the deal is known as ***partnership.***
2. **Ratio of Division of Gains :**

   (*i*) *When investments of all the partners are for the same time,* the gain or loss is distributed among the partners in the ratio of their investments.

   Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year :

   (A's share of profit) : (B's share of profit) = $x : y$.

   (*ii*) *When investments are for different time periods,* then equivalent capitals are calculated for a unit of time by taking (capital × number of units of time). Now, gain or loss is divided in the ratio of these capitals.

   Suppose A invests Rs. x for p months and B invests Rs. y for q months, then (A's share of profit) : (B's share of profit) = $xp : yq$.
3. **Working and Sleeping Partners :** A partner who manages the business is known as a ***working partner*** and the one who simply invests the money is a ***sleeping partner.***

11. PROFIT AND LOSS

IMPORTANT FACTS

Cost Price : The price at which an article is purchased, is called its ***cost price***, abbreviated as ***C.P.***

Selling Price : The price at which an article is sold, is called its ***selling price***, abbreviated as ***S.P.***

Profit or Gain : If S.P. is greater than C.P., the seller is said to have a ***profit*** or ***gain***.

Loss : If S.P. is less than C.P., the seller is said to have incurred a ***loss***.

FORMULAE

1. Gain = (S.P.) – (C.P.)
2. Loss = (C.P.) – (S.P.)
3. Loss or gain is always reckoned on C.P.
4. $\text{Gain\%} = \left(\dfrac{\text{Gain} \times 100}{\text{C.P.}}\right)$
5. $\text{Loss\%} = \left(\dfrac{\text{Loss} \times 100}{\text{C.P.}}\right)$
6. $\text{S.P.} = \dfrac{(100 + \text{Gain\%})}{100} \times \text{C.P.}$
7. $\text{S.P.} = \dfrac{(100 - \text{Loss\%})}{100} \times \text{C.P.}$
8. $\text{C.P.} = \dfrac{100}{(100 + \text{Gain\%})} \times \text{S.P.}$
9. $\text{C.P.} = \dfrac{100}{(100 - \text{Loss\%})} \times \text{S.P.}$
10. If an article is sold at a gain of say, 35%, then S.P. = 135% of C.P.
11. If an article is sold at a loss of say, 35%, then S.P. = 65% of C.P.
12. When a person sells two similar items, one at a gain of say, $x\%$, and the other at a loss of $x\%$, then the seller always incurs a loss given by :

$$\text{Loss\%} = \left(\frac{\text{Common Loss and Gain\%}}{10}\right)^2 = \left(\frac{x}{10}\right)^2.$$

13. If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain\%} = \left[\frac{\text{Error}}{(\text{True Value}) - (\text{Error})} \times 100\right]\%.$$

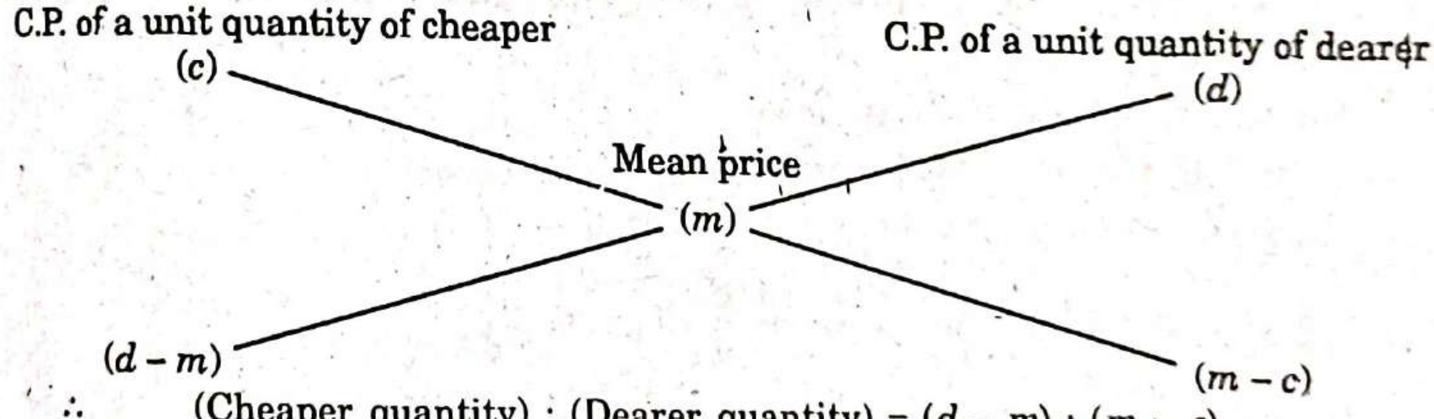
20. ALLIGATION OR MIXTURE

IMPORTANT FACTS AND FORMULAE

1. **Alligation :** It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.
2. **Mean Price :** The cost price of a unit quantity of the mixture is called the mean price.
3. **Rule of Alligation :** If two ingredients are mixed, then

$$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}}\right) = \frac{(\text{C.P. of dearer}) - (\text{Mean price})}{(\text{Mean price}) - (\text{C.P. of cheaper})}.$$

We present as under :



$\therefore$ (Cheaper quantity) : (Dearer quantity) = $(d - m) : (m - c)$.

4. Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid $= \left[x\left(1 - \frac{y}{x}\right)^n\right]$ units.

21. SIMPLE INTEREST

IMPORTANT FACTS AND FORMULAE

1. **Principal :** The money borrowed or lent out for a certain period is called the ***principal*** or the ***sum.***
2. **Interest :** Extra money paid for using other's money is called ***interest.***
3. **Simple Interest (S.I.) :** If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called ***simple interest.***

Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then,

(i) S.I. = $\left(\frac{P \times R \times T}{100}\right)$.

(ii) P = $\left(\frac{100 \times S.I.}{R \times T}\right)$; R = $\left(\frac{100 \times S.I.}{P \times T}\right)$ and T = $\left(\frac{100 \times S.I.}{P \times R}\right)$.

22. COMPOUND INTEREST

Compound Interest : Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say *yearly* or *half-yearly* or *quarterly* to settle the previous account.

In such cases, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.

After a specified period, *the difference between the amount and the money borrowed is called the **Compound Interest*** (*abbreviated as C.I.*) *for that period.*

IMPORTANT FACTS AND FORMULAE

Let Principal = P, Rate = R% per annum, Time = n years.

I. When interest is compound Annually :

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

II. When interest is compounded Half-yearly :

$$\text{Amount} = P\left[1+\frac{(R/2)}{100}\right]^{2n}$$

III. When interest is compounded Quarterly :

$$\text{Amount} = P\left[1+\frac{(R/4)}{100}\right]^{4n}$$

IV. When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{2}{5}R}{100}\right)$$

V. When Rates are different for different years, say R_1%, R_2%, R_3% for 1st, 2nd and 3rd year respectively.

$$\text{Then, Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right).$$

VI. Present worth of Rs. x due n years hence is given by :

$$\text{Present Worth} = \frac{x}{\left(1+\frac{R}{100}\right)^n}.$$

15. TIME AND WORK

IMPORTANT FACTS AND FORMULAE

1. If A can do a piece of work in n days, then A's 1 day's work $= \frac{1}{n}$.
2. If A's 1 day's work $= \frac{1}{n}$, then A can finish the work in n days.
3. If A is thrice as good a workman as B, then :
   Ratio of work done by A and B = 3 : 1.
   Ratio of times taken by A and B to finish a work = 1 : 3.

17. TIME AND DISTANCE

IMPORTANT FACTS AND FORMULAE

1. Speed $= \left(\frac{\text{Distance}}{\text{Time}}\right)$, Time $= \left(\frac{\text{Distance}}{\text{Speed}}\right)$, Distance = (Speed × Time)

2. x km / hr $= \left(x \times \frac{5}{18}\right)$ m / sec

3. x m / sec $= \left(x \times \frac{18}{5}\right)$ km / hr

4. If the ratio of the speeds of A and B is $a : b$, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or $b : a$.

5. Suppose a man covers a certain distance at x km / hr and an equal distance at y km / hr. Then, the average speed during the whole journey is $\left(\frac{2xy}{x+y}\right)$ km / hr

18. PROBLEMS ON TRAINS

IMPORTANT FACTS AND FORMULAE

1. a km / hr $= \left(a \times \frac{5}{18}\right)$ m / s.

2. a m / s $= \left(a \times \frac{18}{5}\right)$ km / hr.

3. Time taken by a train of length l metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover l metres.

4. Time taken by a train of length l metres to pass a stationary object of length b metres is the time taken by the train to cover $(l + b)$ metres.

5. Suppose two trains or two bodies are moving in the same direction at u m / s and v m / s, where $u > v$, then their relatives speed $= (u - v)$ m / s.

6. Suppose two trains or two bodies are moving in opposite directions at u m / s and v m / s, then their relative speed is $= (u + v)$ m / s.

7. If two trains of length a metres and b metres are moving in opposite directions at u m / s and v m / s, then time taken by the trains to cross each other $= \frac{(a + b)}{(u + v)}$ sec.

8. If two trains of length a metres and b metres are moving in the same direction at u m / s and v m / s, then the time taken by the faster train to cross the slower train $= \frac{(a + b)}{(u - v)}$ sec.

9. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then **(A's speed) : (B's speed)** $= (\sqrt{b} : \sqrt{a})$.

19. BOATS AND STREAMS

IMPORTANT FACTS AND FORMULAE

1. In water, the direction along the stream is called ***downstream.*** And, the direction against the stream is called ***upstream.***
2. If the speed of a boat in still water is u km / hr and the speed of the stream is v km / hr, then :

$$\textit{Speed downstream} = (u + v) \text{ km / hr}$$

$$\textit{Speed upstream} = (u - v) \text{ km / hr.}$$

3. If the speed downstream is a km / hr and the speed upstream is b km / hr, then :

$$\textit{Speed in still water} = \frac{1}{2}(a + b) \text{ km / hr}$$

$$\textit{Rate of stream} = \frac{1}{2}(a - b) \text{ km / hr}$$