Communication Systems

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Chapter 4

Baseband Pulse Transmission

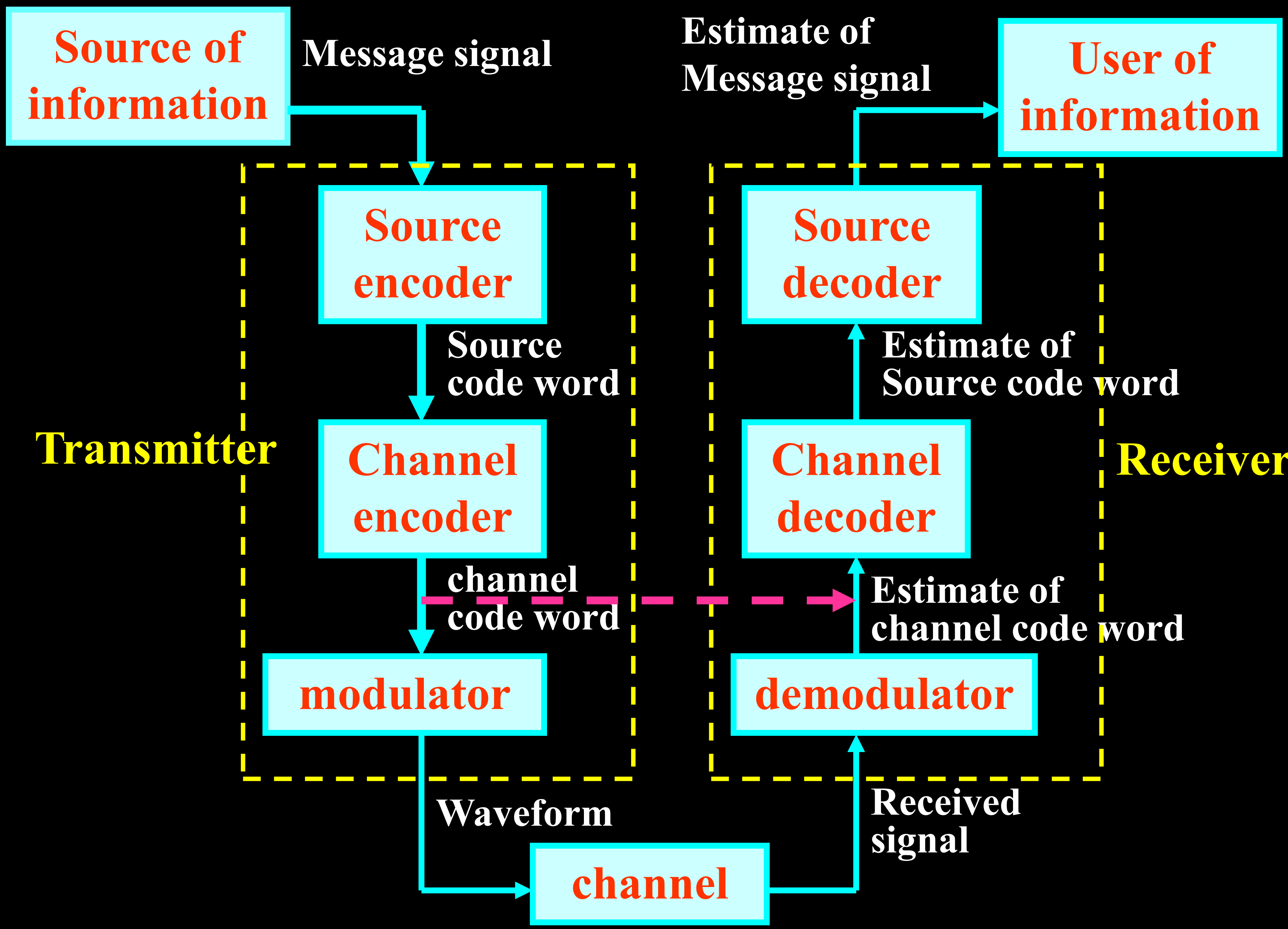
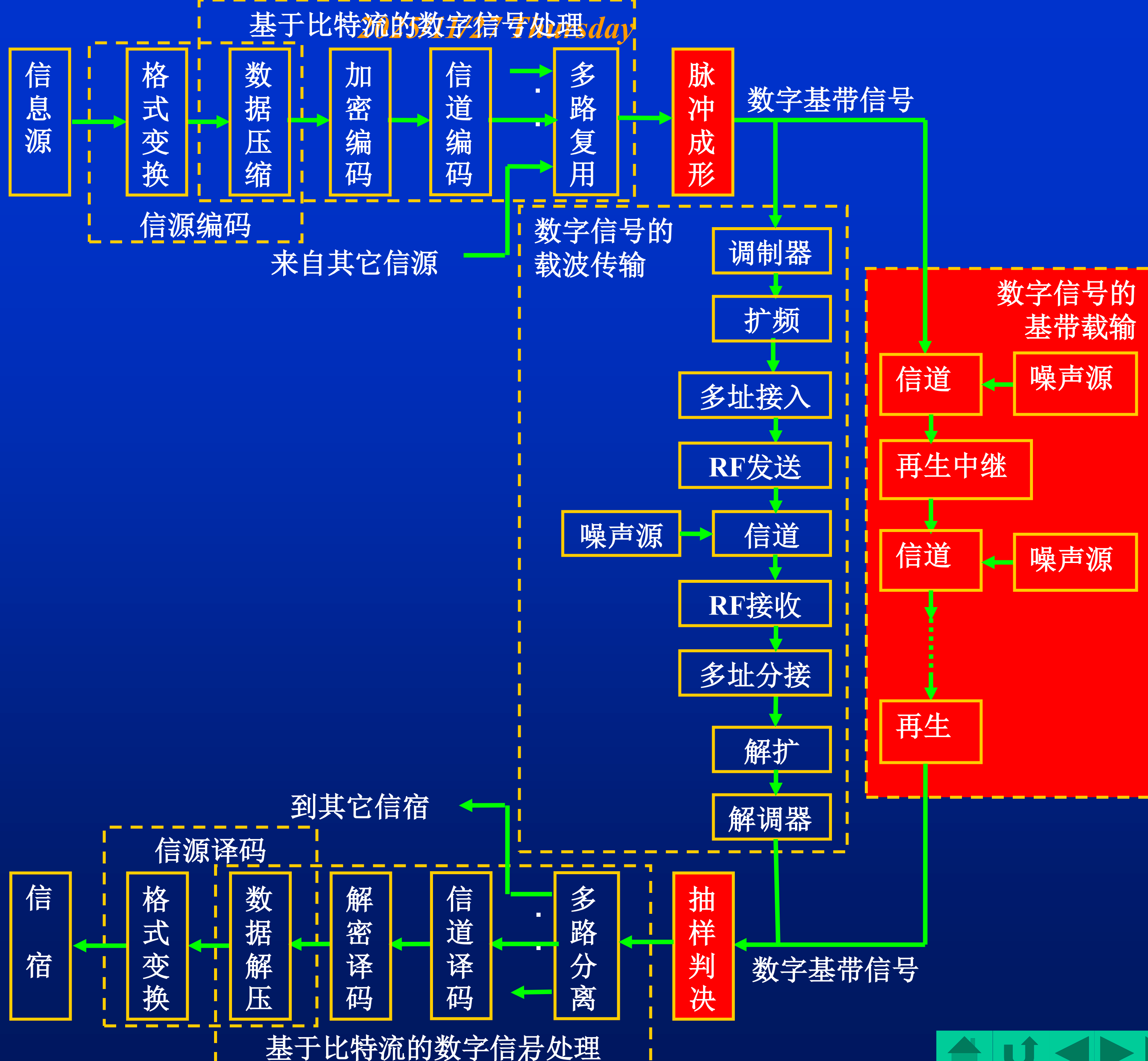


Figure 9 Block diagram of digital communication system.

数字信号的基带传输系统



- **Chapter 3: 信源编码**
  converting an analog signal into digital form.
- **Chapter 10:**
  error-control encoding （信道编码）
- **Chapter 4:**
  transmitting digital data over a baseband channel. 基带传输
- **Chapter 6:**
  transmitting digital data over a band-pass channel. 调制传输，载波传输

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- Matched filter 匹配滤波器
- Calculation of bit error rate BER误比特率计算
- Intersymbol interference ISI 符号间干扰
- Nyquist’s criterion for distortionless baseband transmission 奈奎斯特无失真基带传输准则
- Correlative-level coding or partial-response signaling 相关电平编码，部分响应技术
- Digital subscriber lines 数字用户线
- Equalization of a dispersive baseband channel 弥散基带信道的均衡
- Eye pattern 眼图

4.1 Introduction

- **Review: 通信系统的两大性能指标 ？**

通信的任务是传递信息，传输信息的有效性和可靠性是通信系统最主要的性能指标。

- 有效性（**efficiency**），是指在给定的信道内传输的信息内容的多少，表征通信系统传输信息的数量指标。
- 可靠性（**reliability**），是指接收信息的准确程度，表征通信系统传输信息的质量指标。

有效性和可靠性两者相互矛盾而又相互联系，通常也是可以互换的。

如何度量有效性和可靠性：

	有效性 efficiency	可靠性 reliability
模拟通信系统	有效传输频带	收信机 输出信噪比
数字通信系统	码元传输速率 或信息传输速率	误码率 或 误信率， 误比特率

BER bit error rate

- noise performance in an analog transmission system is measured in terms of SNR.
- The noise performance in a digital transmission system is measured in terms of Bit Error Rate (BER) 错误比特率.

Two main sources of bit errors are:

1 Channel noise 信道噪声

2 Intersymbol Interference (ISI) 符号间干扰

Discussion: Quantization noise 量化噪声会导致误码吗？Why?

Intersymbol Interference ISI
码间干扰，码间串扰

- Digital data have a broad spectrum with significant low-frequency content, thus baseband transmission of digital data requires the use of a low-pass channel. 基带传输需要低通信道。

- However, typically the channel is dispersive 弥散的 in that its frequency response deviates from that of an ideal low-pass filter. The result of data transmission over such a channel is that each received pulse is affected somewhat by adjacent pulses, thereby giving rise to a common form of interference called ISI. 实际信道偏离理想，相邻脉冲之间的波形互相影响，导致误码，称为ISI.

How to eliminate（消除） ISI ？

To correct for ISI, pulse shape should be controlled in the overall system. Thus, much of this chapter is devoted to pulse shaping 脉冲整形 in one form or another.

要纠正ＩＳＩ，必须对整个系统的脉冲波形进行控制，本章的大部分内容就是介绍各种脉冲波形。

Two main sources of bit errors are:

1. **Channel noise 信道噪声**
2. **Intersymbol Interference (ISI) 码间串扰**

Noise and ISI arise in the system simultaneously.

However, in order to understand how they affect the performance of the system, we first consider them separately; then we study their combined effects.

Three Analysis Steps

Step 1: channel noise acting alone, which leads to formulation of the **matched filter** receiver. 只考虑信道噪声的影响

Step 2: intersymbol interference acting alone, which leads to formulation of pulse-shaping transmit filter so as to realize the **Nyquist channel.** 只考虑码间串扰的影响

Step 3: channel noise and ISI acting together, which leads to **optimum linear receiver.** 同时考虑两者的影响

4.2 Matched Filter 匹配滤波器

Premise:

Here, we assume that the major source of system limitation is the channel noise. No ISI.

- Matched Filter is used to detect a pulse signal of known waveform that is immersed in additive white noise.
- The device for the optimum detection of such a pulse involves the use of a linear-time-invariant filter known as a matched filter。

- **Why is it called matched filter ?**
- **Why is it suitable for baseband pulse transmission ?**

Firstly, Two considerations:

- **Sampling and decision 抽样判决**
- **In linear Time Invariant LTI System,**

$x(t) \longrightarrow$ **Impulse response 冲激响应 h(t)** $\longrightarrow y(t) = x(t) * h(t)$

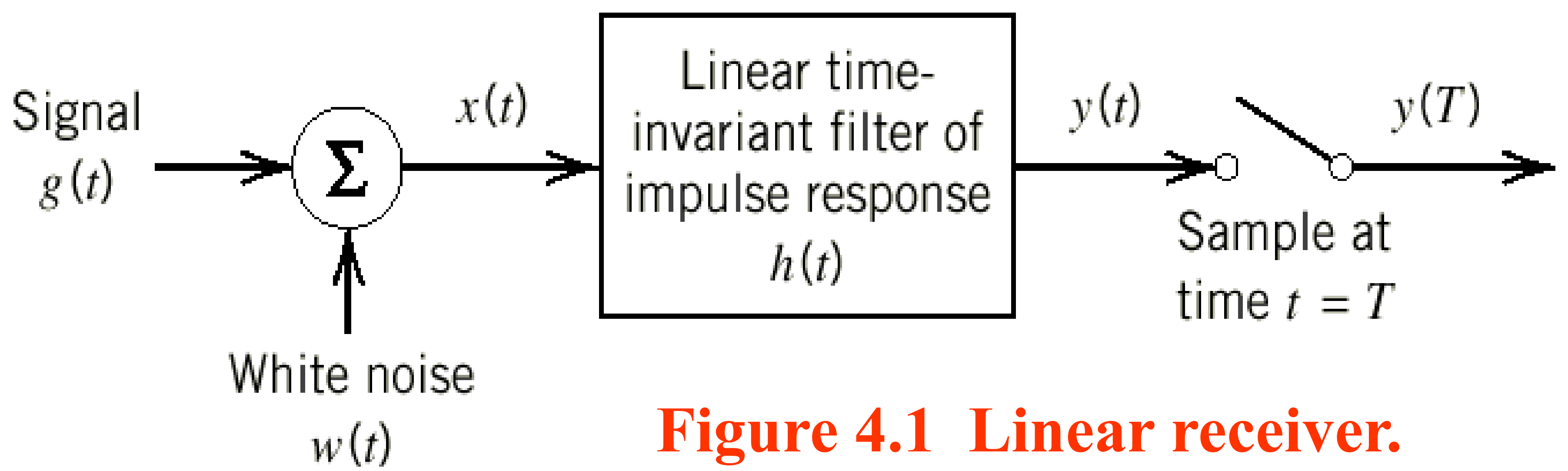


Figure 4.1 Linear receiver.

Impulse response of a matched filter

$$h_{opt}(t) = k\ g(T-t) \qquad 4.16$$

The impulse response of the optimum filter, except for the scaling factor k, is a time-reverse and delayed version of the input signal g(t); that is, it is matched to the input signal. So it is called a matched filter.

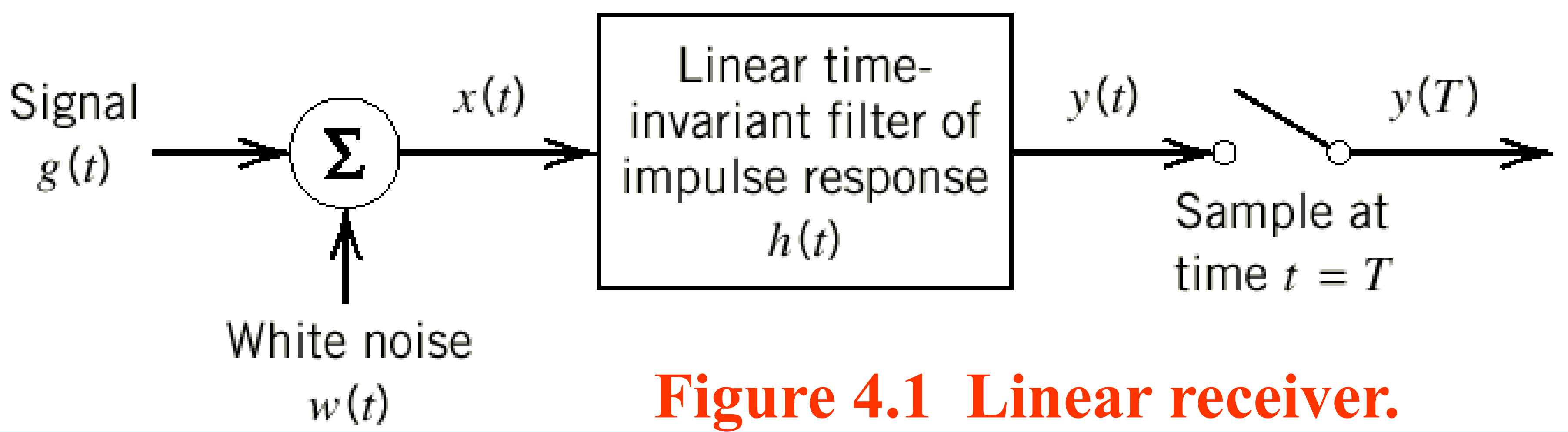


Figure 4.1 Linear receiver.

The peak pulse signal-to-noise 峰值信噪比 has the maximum value:

$$\eta_{\max} = \frac{2E}{N_0} \qquad 4.20$$

E denotes signal energy; N_0 is the power spectral density of white noise.

Example 4.1 Matched filter for rectangular pulse

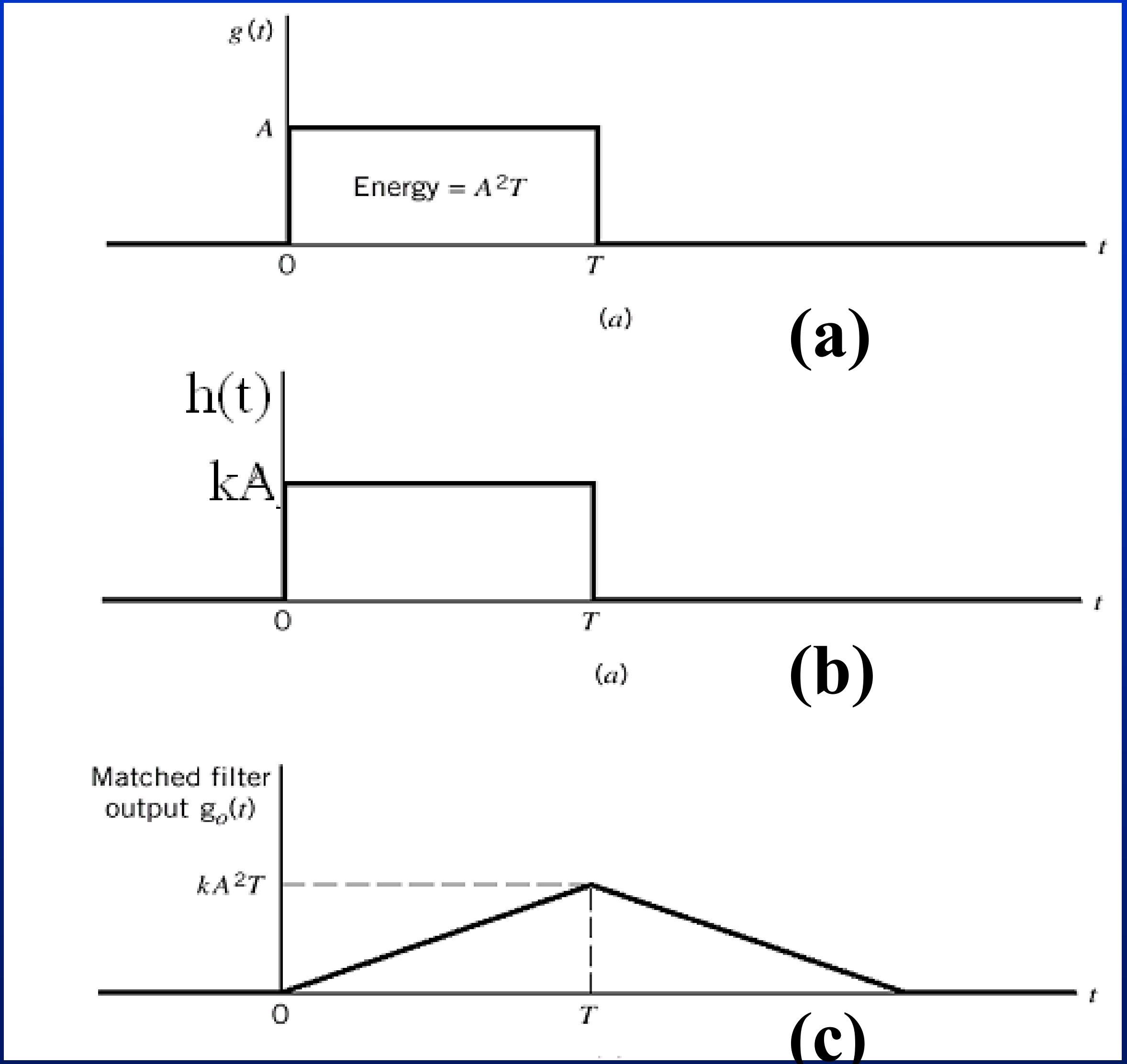


Figure 4.2

(*a*) Rectangular pulse.

(*b*) h(t) of Matched filter.

(*c*) Matched filter output.

矩形脉冲匹配滤波器的实现—积分-清除电路

For the special case of a rectangular pulse, the matched filter may be implemented using a circuit known as the integrate-and-dump circuit.

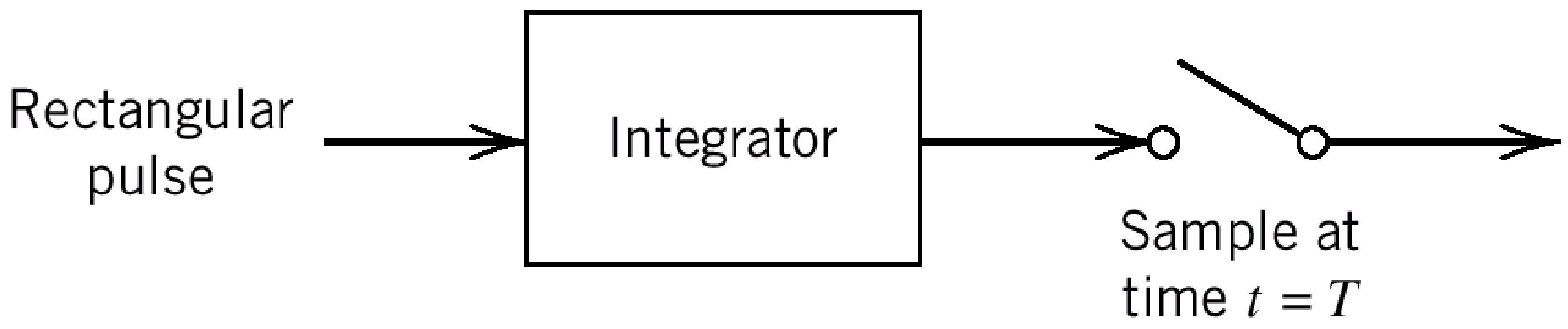
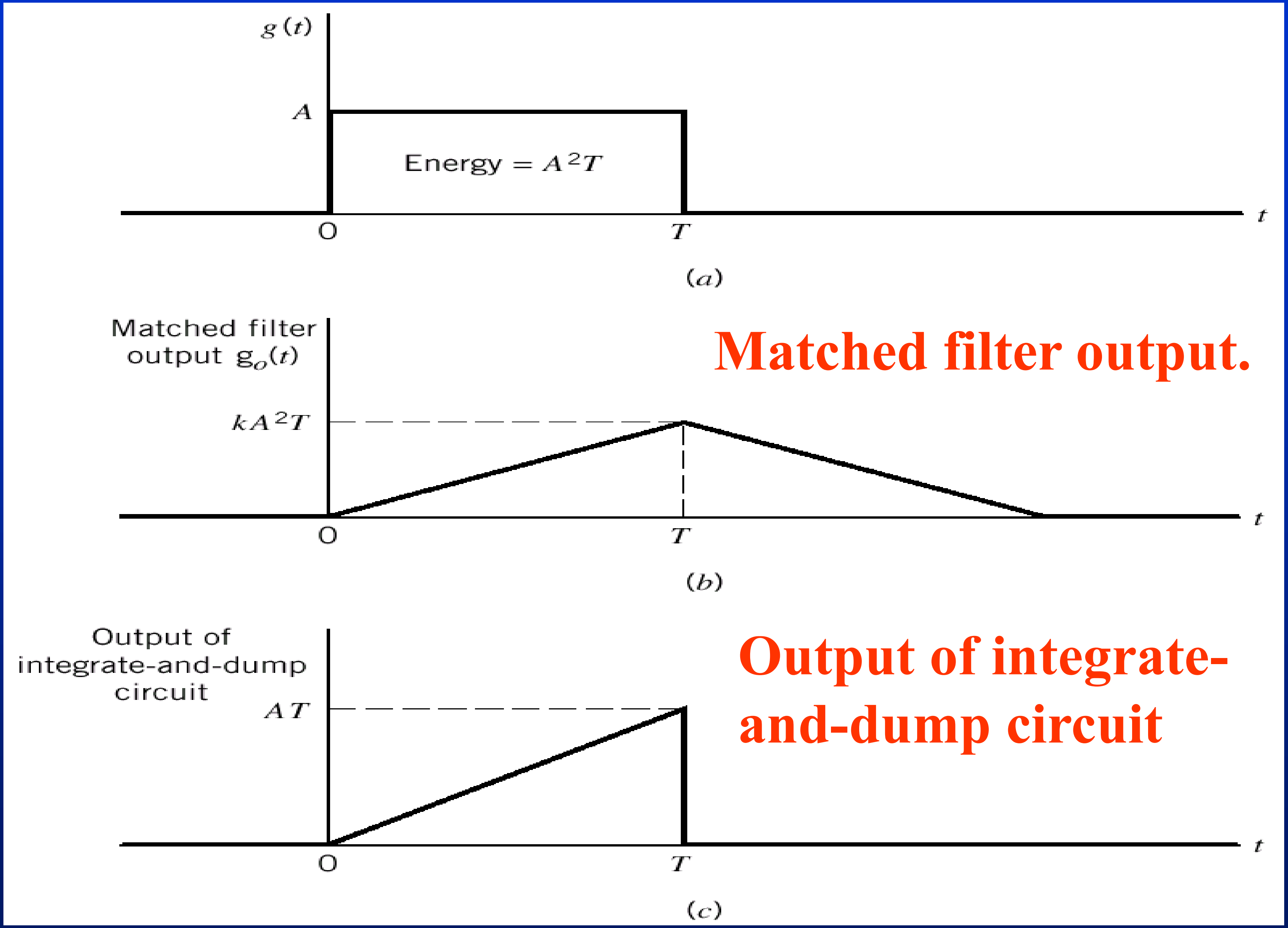


Figure 4.3 Integrate-and-dump circuit
积分-清除电路



Matched filter output.

Output of integrate-and-dump circuit

4.3 Error Rate Due to Noise

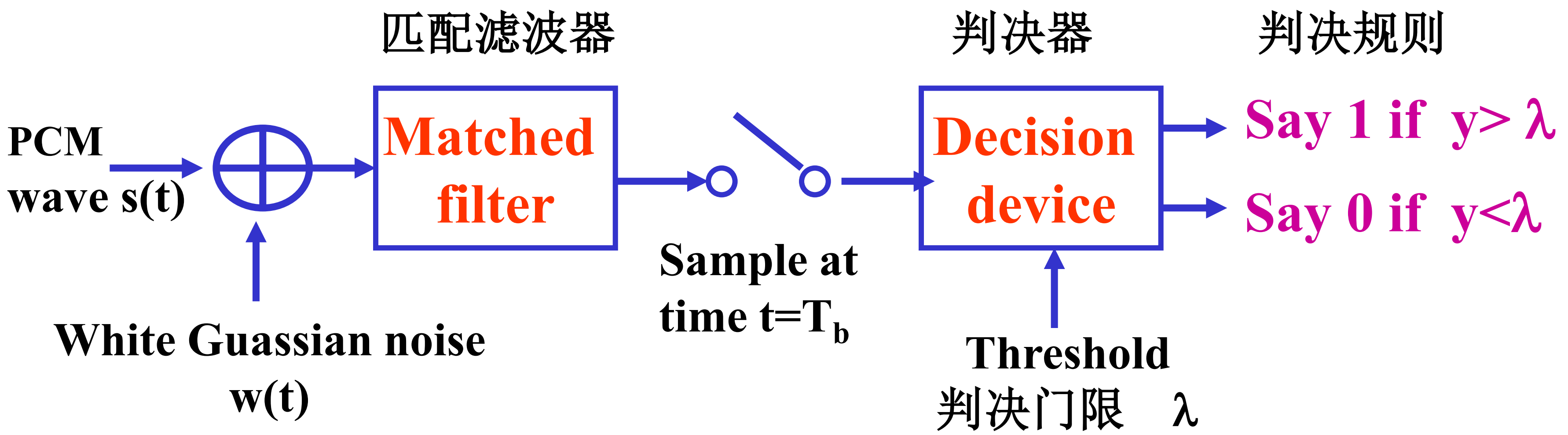
Assumptions:

1. Channel noise w(t) is additive white Gaussian noise.
2. PCM system uses polar nonreturn-to-zero (NRZ) signaling 双极性非归零信号.

Thus, the received signal during a bit duration T_b is

$$x(t)=\begin{cases}+A+w(t), & symbol\ \ 1\ \ was\ \ sent\\ -A+w(t), & symbol\ \ 0\ \ was\ \ sent\end{cases} \qquad (4.21)$$

Figure 4.4 采用双极性ＮＲＺ信号的二进制编码ＰＣＭ波形的基带传输接收机



Two kinds of error:

1. Symbol 1 is chosen when a 0 was actually transmitted----an error of the first kind. 第一类错误 0→1
2. Symbol 0 is chosen when a 1 was actually transmitted----an error of the second kind. 第二类错误 1→0

How to determine the average probability of error?

Suppose that symbol 0 was sent. The received signal is

$$x(t) = -A + w(t), \qquad 0 \le t \le T_b \qquad (4.22)$$

The sampled value at time $t=T_b$ of matched filter output is:

$$let \quad kAT_b = 1$$

$$y = \int_0^{T_b} x(t)dt = -A + \frac{1}{T_b}\int_0^{T_b} w(t)dt \qquad (4.23)$$

It shows that sample value is a random variable.

Since: w(t) is white and Gaussian

Then:

- Y is Gaussian distributed with a mean of –A.
- The variance of the random variable Y is

$$\sigma_Y^2 = E\left[\left(Y+A\right)^2\right] \qquad (4.24)$$

$$= \frac{1}{T_b^2} E\left[\int_0^{T_b}\int_0^{T_b} w(t)w\left(u\right)dtdu\right]$$

$$= \frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b} E\left[w(t)w\left(u\right)\right]dtdu$$

$$= \frac{1}{T_b^2}\int_0^{T_b} R_w\left(t,u\right)du$$

Where $R_W(t)$ is the autocorrelation function of the white noise $w(t)$. Since $w(t)$ is white with a power spectral density $N_0/2$, we have

$$R_w(t,u)=\frac{N_0}{2}\delta(t-u) \qquad (4.25)$$

Hence

$$\sigma_Y^2=\frac{1}{T_b^2}\int_0^{T_b}\int_0^{T_b}\frac{N_0}{2}\delta(t-u)\,dt\,du=\frac{N_0}{2T_b} \qquad (4.26)$$

The conditional probability density function of the random variable Y, given that symbol 0 was sent, 发送0时的条件概率密度函数

$$f_Y(y|0)=\frac{1}{\sqrt{\pi N_0/T_b}}\exp\left(-\frac{(y+A)^2}{N_0/T_b}\right) \qquad (4.27)$$

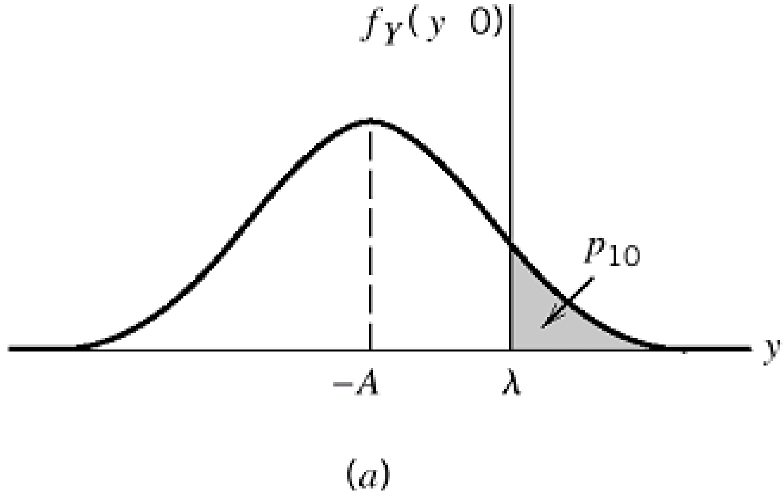


Figure 4.5

Probability density function (a) when 0 is transmitted

P_{10} 发送 0，但错成了 1 的概率.

Conditional probability of error P_{10}

$$P_{10} = P\left(y > \lambda \mid symbol\ \ 0\ \ was\ \ sent\right) \qquad (4.28)$$

$$= \int_{\lambda}^{\infty} f\left(y \mid 0\right) dy = \frac{1}{\sqrt{\pi N_0 / T_b}} \int_{\lambda}^{\infty} \exp\left(-\frac{\left(y+A\right)^2}{N_0 / T_b}\right) dy$$

Complementary error function 互补误差函数

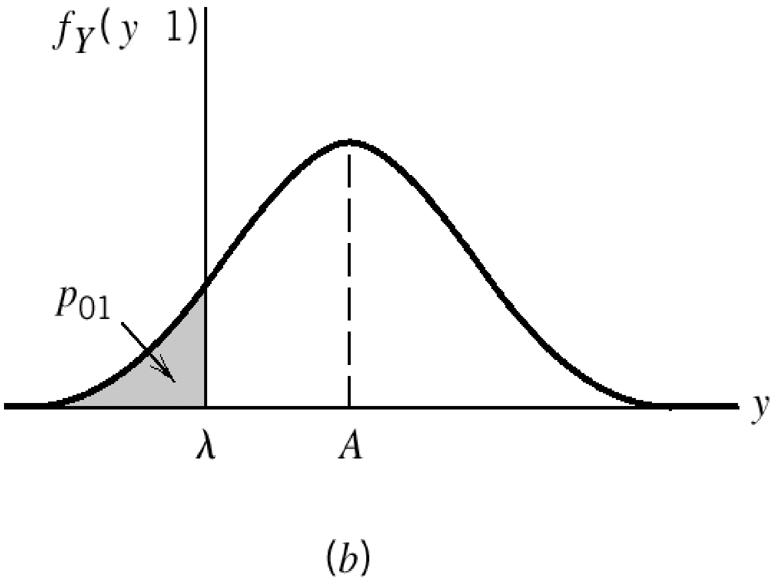
$$erfc(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} \exp(-z^2) dz \qquad 4.29$$

$$p_{10} = \frac{1}{\sqrt{\pi}} \int_{(A+\lambda)/\sqrt{N_0/T_b}}^{\infty} \exp\left(-z^2\right) dz = \frac{1}{2} erfc\left(\frac{A+\lambda}{\sqrt{N_0 / T_b}}\right) \qquad (4.31)$$

Similarly, we can get the conditional probability density function, given that symbol 1 was sent, 发送1时的条件概率密度函数:

$$f_Y(y|1) = \frac{1}{\sqrt{\pi N_0/T_b}} \exp\left(-\frac{(y-A)^2}{N_0/T_b}\right) \qquad (4.32)$$

Figure 4.5
Probability density function (b) when 1 is transmitted



$\mathbf{P_{01}}$ 发送1，但错成了0的概率.

P01: Conditional probability error when 1 is transmitted

$$p_{01} = P\left(y < \lambda \mid symbol\ 1\ was\ sent\right) \qquad (4.33)$$

$$= \int_{-\infty}^{\lambda} f\left(y \mid 1\right) dy = \frac{1}{\sqrt{\pi N_0 / T_b}} \int_{-\infty}^{\lambda} \exp\left(-\frac{\left(y-A\right)^2}{N_0 / T_b}\right) dy$$

Define

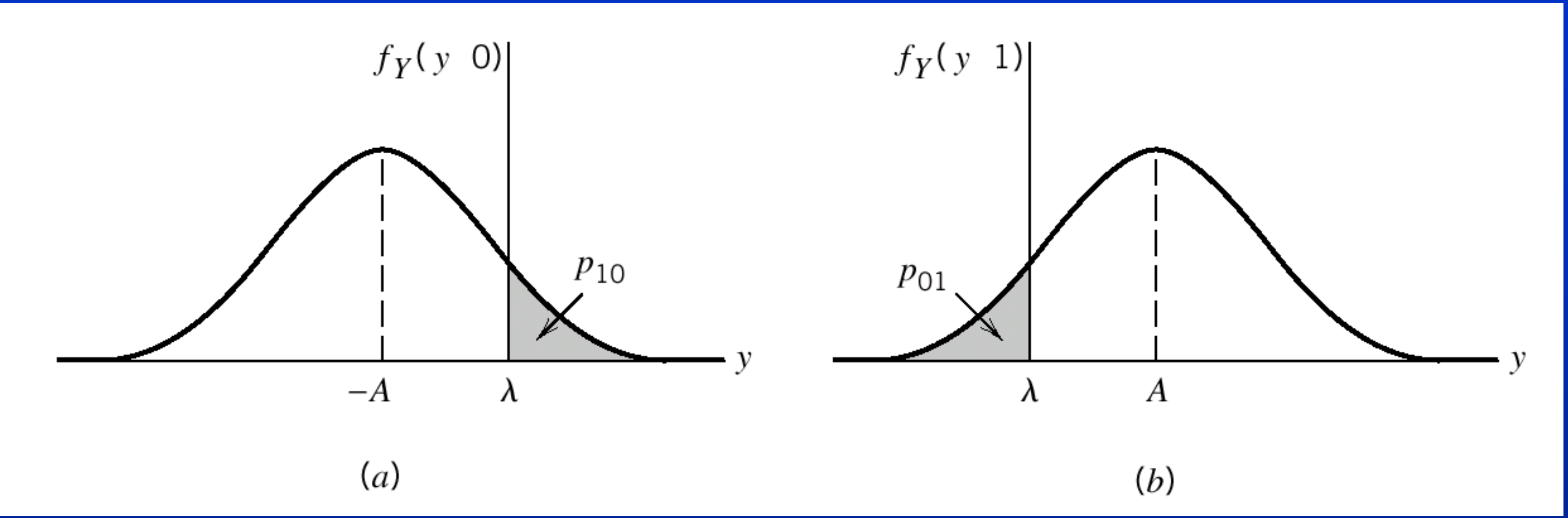
$$z = \frac{A-y}{\sqrt{N_0 / T_b}}$$

Then

$$p_{01} = \frac{1}{\sqrt{\pi}} \int_{(A-\lambda)/\sqrt{N_0/T_b}}^{\infty} \exp\left(-z^2\right) dz \qquad (4.34)$$

$$= \frac{1}{2} erfc\left(\frac{A-\lambda}{\sqrt{N_0 / T_b}}\right)$$

Average probability of symbol error Pe is given by 平均符号差错概率



$$P_e = p_0 p_{10} + p_1 p_{01}$$

$$= \frac{p_0}{2} erfc\left(\frac{A+\lambda}{\sqrt{N_0/T_b}}\right) + \frac{p_1}{2} erfc\left(\frac{A-\lambda}{\sqrt{N_0/T_b}}\right) \quad (4.35)$$

Where, P0 and P1 denote the a priori probabilities of transmitting symbols 0 and 1, respectively.

Discussion: Special Case

When symbols 1 and 0 are equiprobable 等概率的, we have

$$p_0 = p_1 = \frac{1}{2}$$

$$\lambda_{opt} = 0$$

$$p_{01} = p_{10}$$

$$P_e = \frac{1}{2} erfc\left(\frac{A}{\sqrt{N_0/T_b}}\right) \qquad (4.38)$$

Now the transmitted signal energy per bit is defined by 每比特发射信号能量

$$E_b = A^2 T_b \qquad (4.39)$$

So

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad (4.40)$$

这说明一个二进制对称信道中的平均符号差错概率仅取决于每比特发射信号的能量与噪声功率谱密度之比Eb/N0.

Brief Review for Last Class

- Main sources of bit error rate (BER)

1. Channel noise 信道噪声
2. Intersymbol Interference (ISI) 码间串扰

Annlysis Steps:

1: channel noise only——matched filter

2: ISI only——Nyquist channel

3: channel noise and ISI together —— optimum linear receiver

Intersymbol Interference ISI
码间干扰，码间串扰

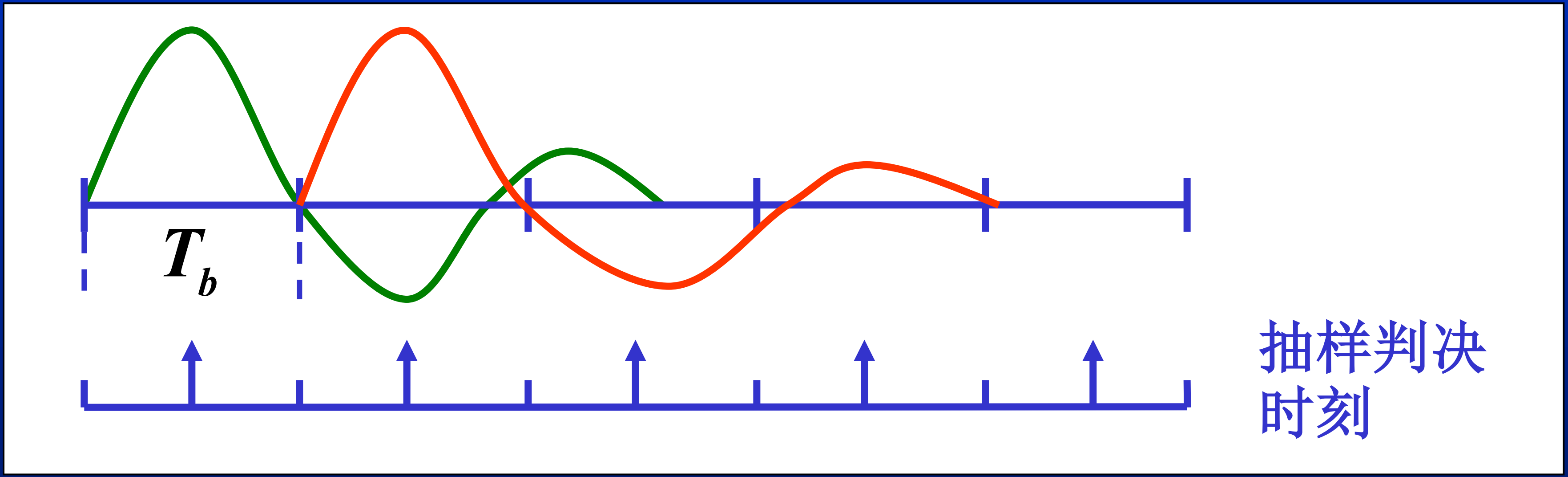
- **Digital data have a broad spectrum with significant low-frequency content, thus baseband transmission of digital data requires the use of a low-pass channel.** 基带传输需要低通信道。

- **However, typically the channel is dispersive** 弥散的 **in that its frequency response deviates from that of an ideal low-pass filter. The result of data transmission over such a channel is that each received pulse is affected somewhat by adjacent pulses, thereby giving rise to a common form of interference called ISI.** 实际信道偏离理想，相邻脉冲之间的波形互相影响，导致误码，称为**ISI.**

4.4 Intersymbol Interference (ISI) 码间干扰

- **Ideally,** baseband transmission of digital data requires an ideal low-pass channel.
- **In practice,** typically the channel is dispersive in that its frequency response deviates from that of an ideal low-pass filter.
- **The result** is that each received pulse is affected by adjacent pulses, thereby giving rise to a form of interference called ISI.

码间干扰： 由于系统传输特性不理想，导致前后码元的波形畸变、展宽，并使前面波形出现很长的拖尾，蔓延到当前码元的抽样时刻上，从而对当前码元的判决造成干扰。



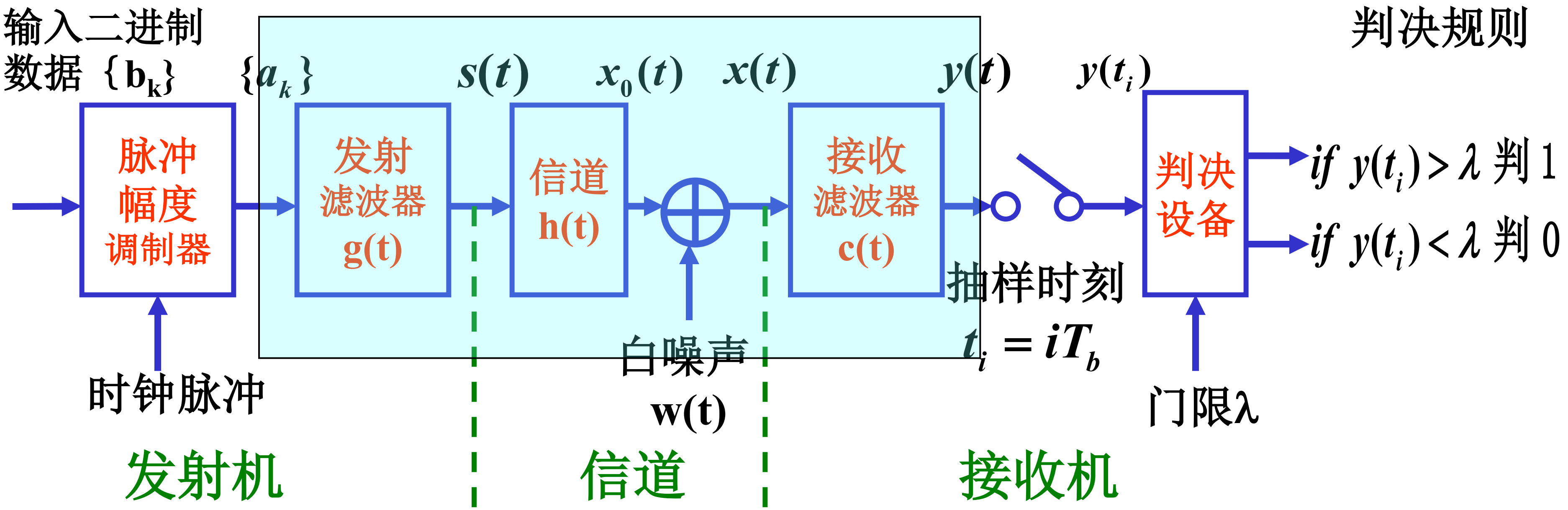
实际抽样判决值不仅有本码元的信号值，还有其它码元在该码元抽样时刻的串扰值以及噪声。

How to eliminate（消除） ISI ？

To correct for ISI, **pulse shape** should be controlled in the overall system. Thus, much of this chapter is devoted to **pulse shaping** 脉冲整形 in one form or another.

要纠正ＩＳＩ，必须对整个系统的脉冲波形进行控制，本章的大部分内容就是介绍各种脉冲波形。

Baseband binary data transmission system



$\{a_k\}$: short pulse sequence, its amplitude is represented in the polar form

$$a_k = \begin{cases} +1 & \text{if symbol } b_k \text{ is } 1 \\ -1 & \text{if symbol } b_k \text{ is } 0 \end{cases} \tag{4.42}$$

transmitted signal:

$$s(t)=\sum_k a_k g(t-kT_b) \qquad (4.43)$$

the receiver filter output :

$$y(t)=\mu\sum_k a_k p(t-kT_b)+n(t) \qquad (4.44)$$

μ is a scaling factor.

Here, *p(t)* satisfies:

$$\mu p(t)=g(t)*h(t)*c(t) \qquad (4.45)$$

FT

$$\mu P(f)=G(f)H(f)C(f) \qquad (4.47)$$

We assume that the pulse p(t) is normalized by setting

$$p(0)=1$$

The receive filter output y(t) is sampled at time $t_i = iT_b$, yielding

$$y(t_i)=\mu\sum_{k=-\infty}^{\infty}a_k p[(i-k)T_b]+n(t_i) \qquad (4.48)$$

$$=\mu a_i+\mu\sum_{\substack{k=-\infty \\ k\neq i}}^{\infty}a_k p[(i-k)T_b]+n(t_i)$$

desired useful value

ISI

Noise sample

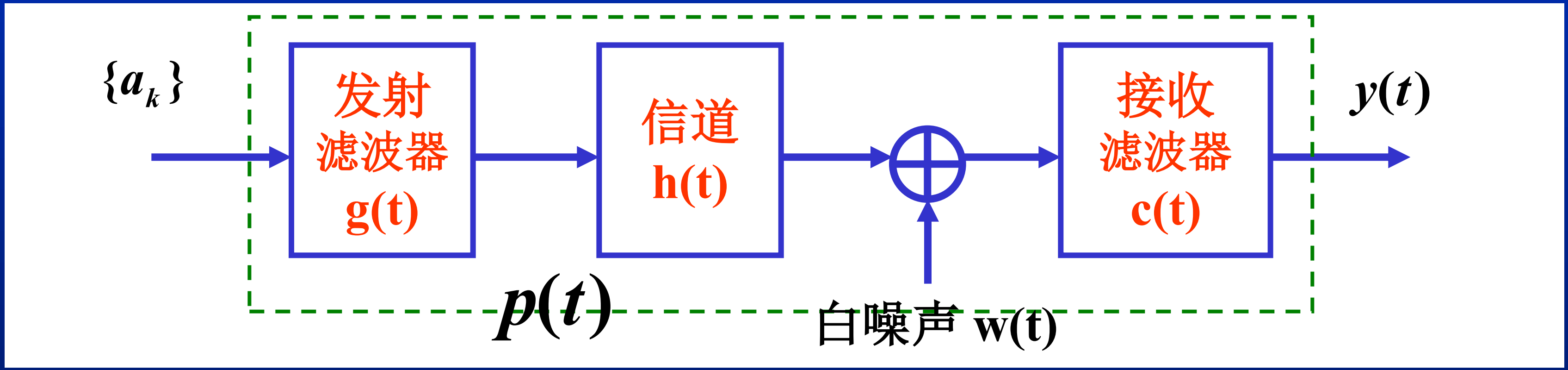
$$y(t_i) = \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p[(i-k)T_b] + n(t_i) \quad (4.48)$$

- **Ideal** case: both ISI and noise are absent, then $y(t_i) = \mu a_i$
- **In practice**, both of them are existed in any communication system and introduce errors in the decision device.

Therefore, in the design of the transmit and receive filters, the objective is to **minimize the effects of noise and ISI.**

How to minimize the effects of noise and ISI ?

- Matched filer is used to minimize noise effect.
- Design the pulse waveform *p(t)* so that ISI can be completely eliminated 完全消除.



$$\mu p(t) = g(t) * h(t) * c(t)$$

$$\mu P(f) = G(f) H(f) C(f)$$

4.5 Nyquist's Criterion for Distortionless Baseband Binary Transmission
无失真基带二进制传输的奈奎斯特准则

$$y(t_i) = \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p[(i-k)T_b] + \cancel{n(t_i)} \qquad (4.48)$$

desired useful value

ISI

Noise sample

In this section, we only consider ISI. That is to say, assume that NO channel noise.

If there is NO noise, then the output is

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k p\left[(i-k)T_b\right] = \underbrace{\mu a_i}_{\text{useful desired value}} + \underbrace{\mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p[(i-k)T_b]}_{\text{ISI}}$$

If

$$p(iT_b - kT_b) = \begin{cases} 1 & , k = i \\ 0 & , k \neq i \end{cases} \qquad (4.49)$$

Then

$$y(t_i) = \mu a_i$$

There is no ISI!

Condition for Zero ISI in time domain

In time domain, the condition for zero ISI can be described as

$$\boxed{p(iT_b - kT_b) = \begin{cases} 1 \quad , k = i \\ 0 \quad , k \neq i \end{cases} \qquad (4.49)}$$

无码间串扰的时域条件

It can be changed into:

$$p(iT_b - kT_b) = p[(i-k)T_b] = p(nT_b) = \begin{cases} 1 \quad , n = 0 \\ 0 \quad , n \neq 0 \end{cases}$$

物理含义是什么呢？怎么来设计这样的系统呢？

Condition for Zero ISI in frequency domain

$$p(t) \xleftrightarrow{FT} P(f)$$

$$\{p(nT_b)\} \xleftrightarrow{FT} \ ? \quad P_\delta(f) = R_b \sum_{n=-\infty}^{\infty} P(f - nR_b), \quad R_b = \frac{1}{T_b}$$

同时，$P_\delta(f)$ 是 $\{p(nT_b)\}$ 的傅立叶变换

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} [p(nT_b)\delta(t - nT_b)]\, e^{-j2\pi ft} dt \qquad (4.51)$$

$$P_\delta(f) = \int_{-\infty}^{\infty} p(0)\delta(t)\, e^{-j2\pi ft} dt = p(0) = 1$$

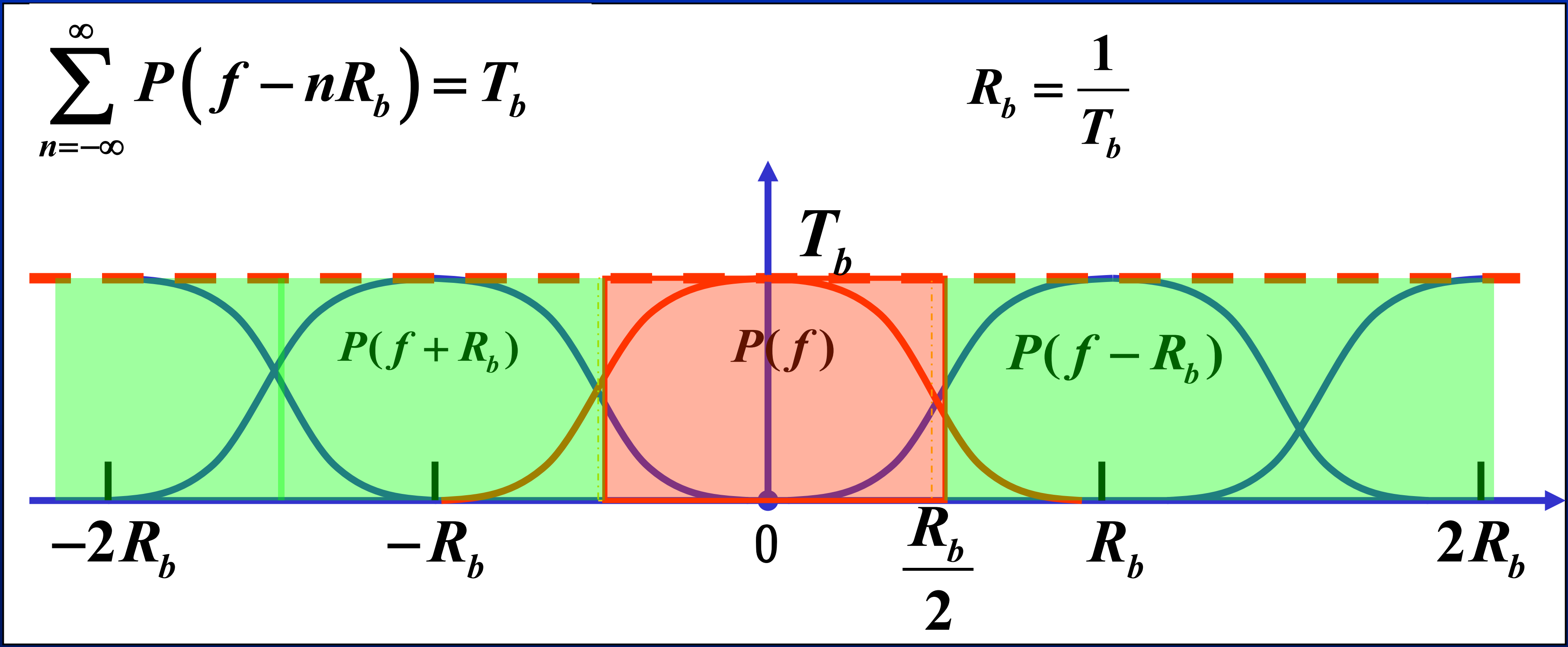
$$\boxed{\sum_{n=-\infty}^{\infty} P\left(f - nR_b\right) = T_b \qquad (4.53)}$$

无码间串扰的频域条件

无失真基带二进制传输的奈奎斯特准则描述为：

如果系统的频率函数满足：$\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b$

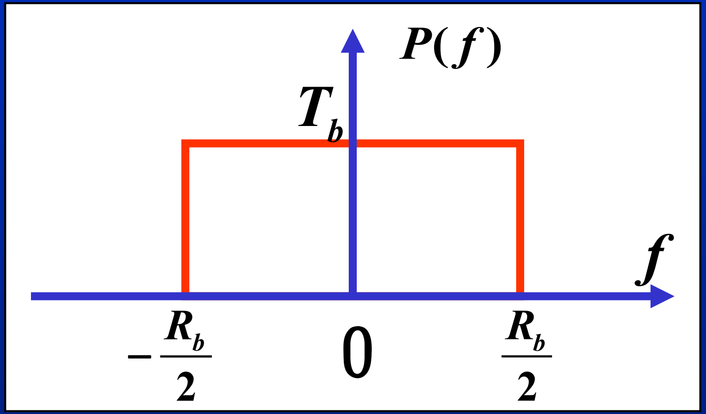
那么就能消除以时间间隔 T_b 进行抽样的ISI。



4.5.1 Ideal Nyquist Channel
理想奈奎斯特信道

The simplest way to satisfy the condition of no ISI is to specify the frequency function *P(f)* to be rectangular 矩形.

$$P(f)=\begin{cases} T_b\ , & |f|<\dfrac{R_b}{2} \\ 0\ , & \text{others} \end{cases}$$



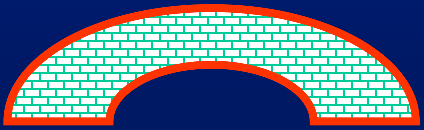
Ideal LPF, 截止频率为:

$$W=\frac{R_b}{2}=\frac{1}{2T_b}$$

R_b: Nyquist rate

W: Nyquist bandwidth

带宽



速率

网络论坛

发帖：100baseT中的100M是指的带宽？还是速率？带宽和速率之间有什么换算关系么？我有点晕了。

回复1

带宽它所指的其实是数据传输率，譬如内存带宽、总线带宽、网络带宽等等，都是以“字节/秒”为单位。数据传输速率指单位时间内传送的二进制位数，单位为bps即位/秒。

回复2

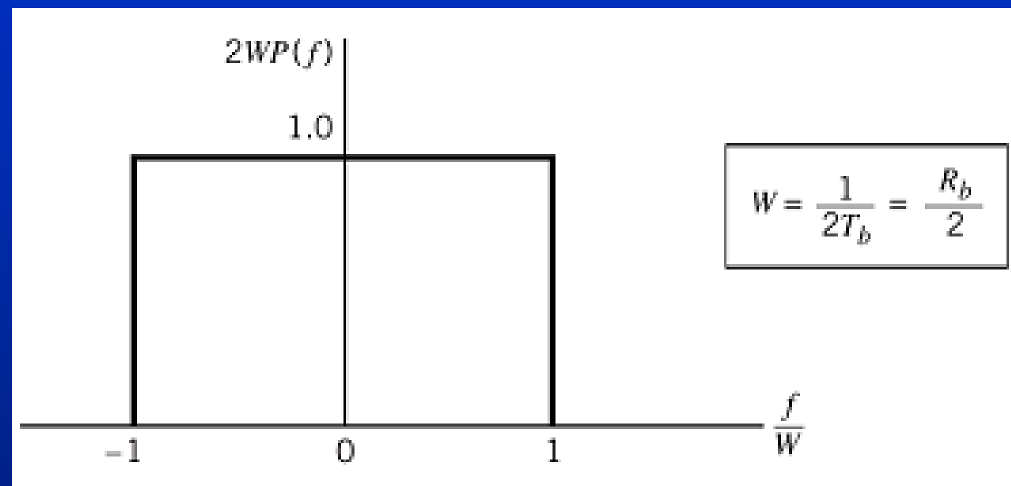
用带宽除以8就是传输速率了吧

回复3

奈奎斯特定理描述了有限带宽、无噪声信道的最大数据传输速率与信道带宽的关系。香农定理则描述了有限带宽、有噪声信道的最大传输速率与信道带宽的关系。 因此，通信信道最大传输速率与信道带宽之间存在着明确的关系，所以人们可以用“带宽”去取代“速率”。

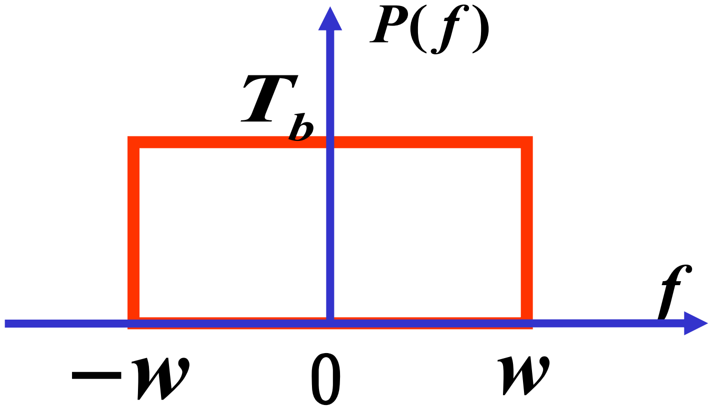
$$P(f)=\begin{cases}\dfrac{1}{2W}, & -W<f<W\\ 0, & |f|>W\end{cases}=\frac{1}{2W}rect\left(\frac{f}{2W}\right)\quad(4.54)$$

IFT



$$p(t)=\frac{\sin(2\pi Wt)}{2\pi Wt}=\sin c(2\pi Wt)\qquad(4.56)$$

理想奈奎斯特信道频率响应与波形形状



$$W = \frac{R_b}{2} = \frac{1}{2T_b}$$

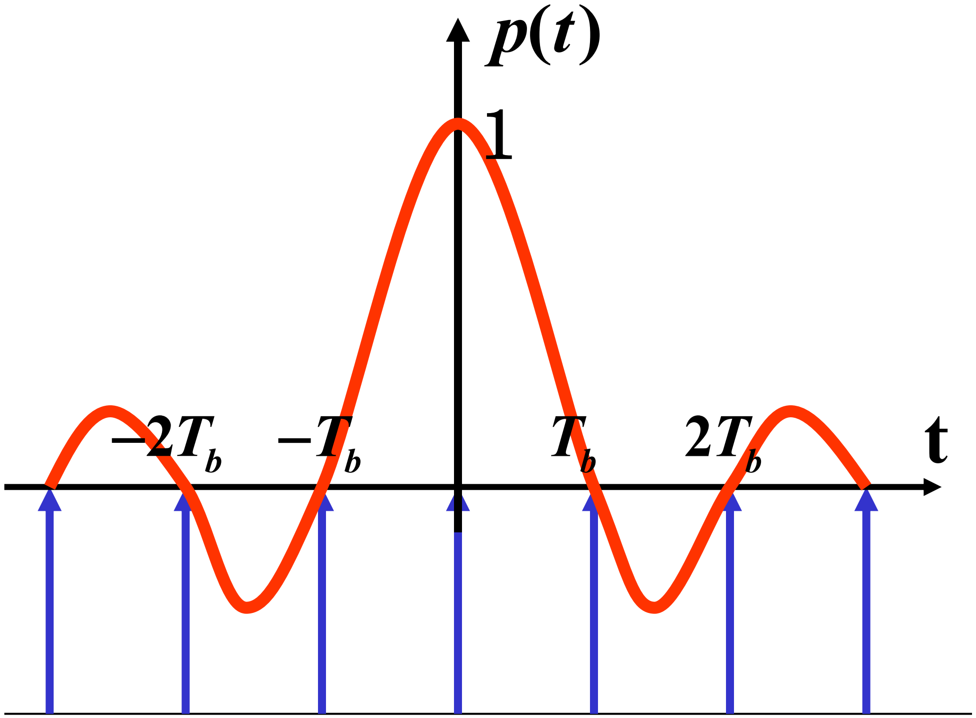
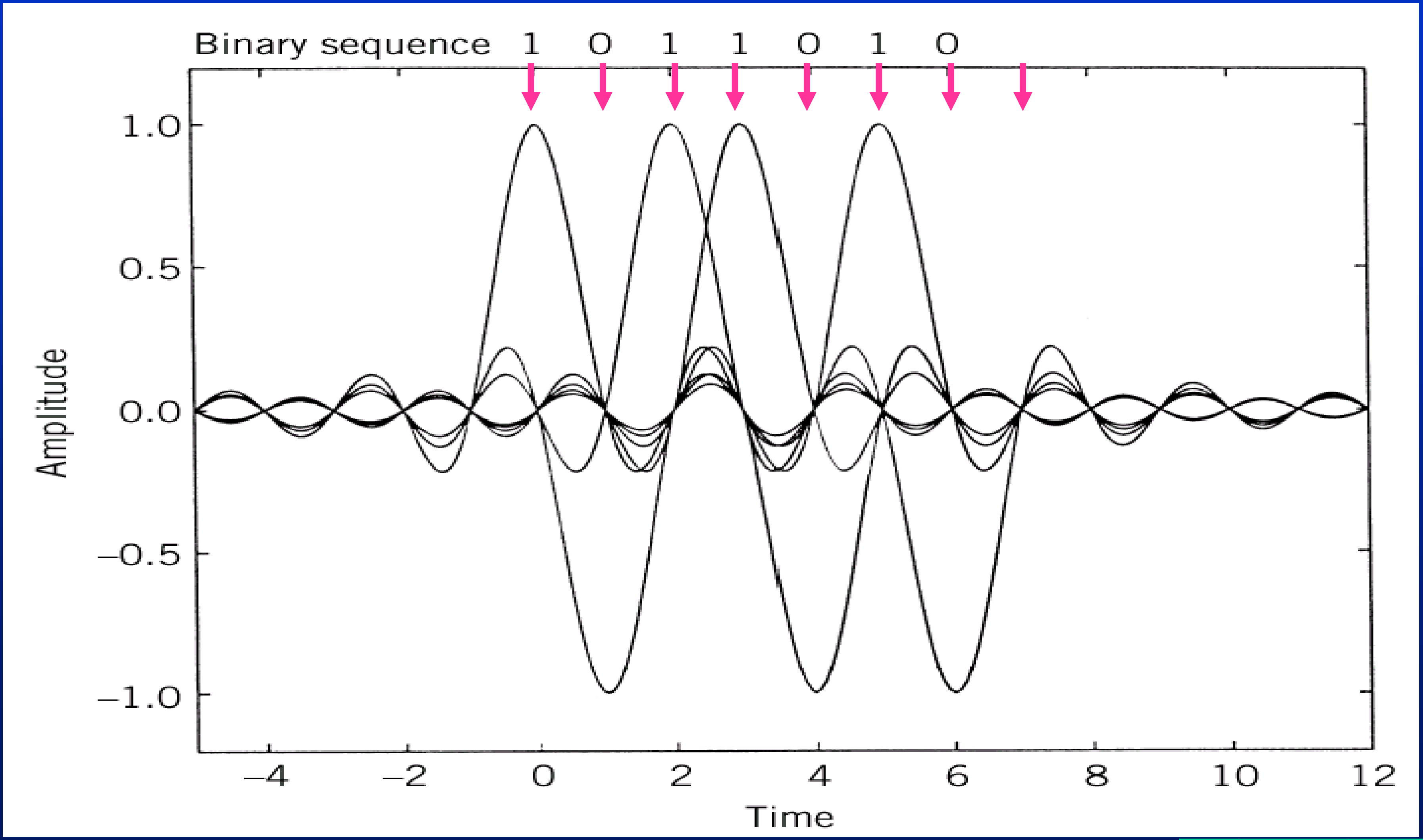


Figure 4.8

(*a*) Ideal magnitude response. (*b*) Ideal basic pulse shape.

Fig 4.9 A series of sinc pulses corresponding to the sequence 1011010



Advantages of Ideal Nyquist Channel

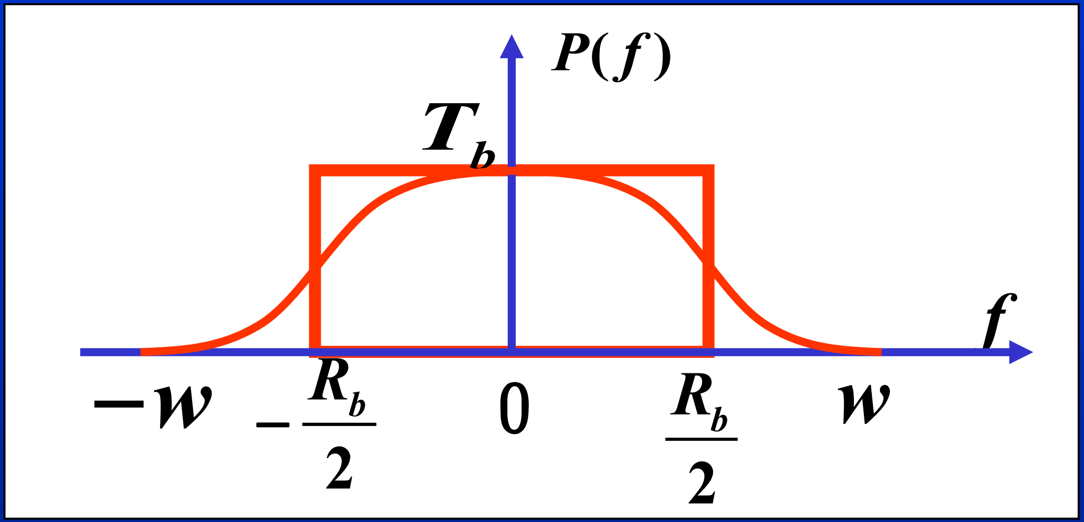
- **No ISI**
- **Minimum bandwidth** $B_T = w = \dfrac{R_b}{2}$

Limitations of Ideal Nyquist Channel

- **It is physically unrealizable because of the abrupt transitions at the band edges.**
- **The function *p(t)* decreases as 1/|t| for large |t|, resulting in a slow rate of decay. There is no margin of error in sampling instants.** 衰减慢，没有错误容限。

How to overcome the limitations of ideal Nyquist channel?

idea ?



Remember: It must satisfy the condition for no ISI.

$$\sum_{n=-\infty}^{\infty} P\left(f - nR_b\right) = T_b$$

Many band-limited functions can satisfy this equation. A particular form that has many desirable features is provided by a raised cosine spectrum.

Raised Cosine Spectrum 升余弦频谱

$$p(f)=\begin{cases}\dfrac{1}{2W}, & 0<|f|<f_1 \\ \dfrac{1}{4W}\left\{1-\sin\left[\dfrac{\pi(|f|-W)}{2W-2f_1}\right]\right\}, & f_1\le|f|\le 2W-f_1 \\ 0, & |f|\ge 2W-f_1\end{cases} \quad (4.60)$$

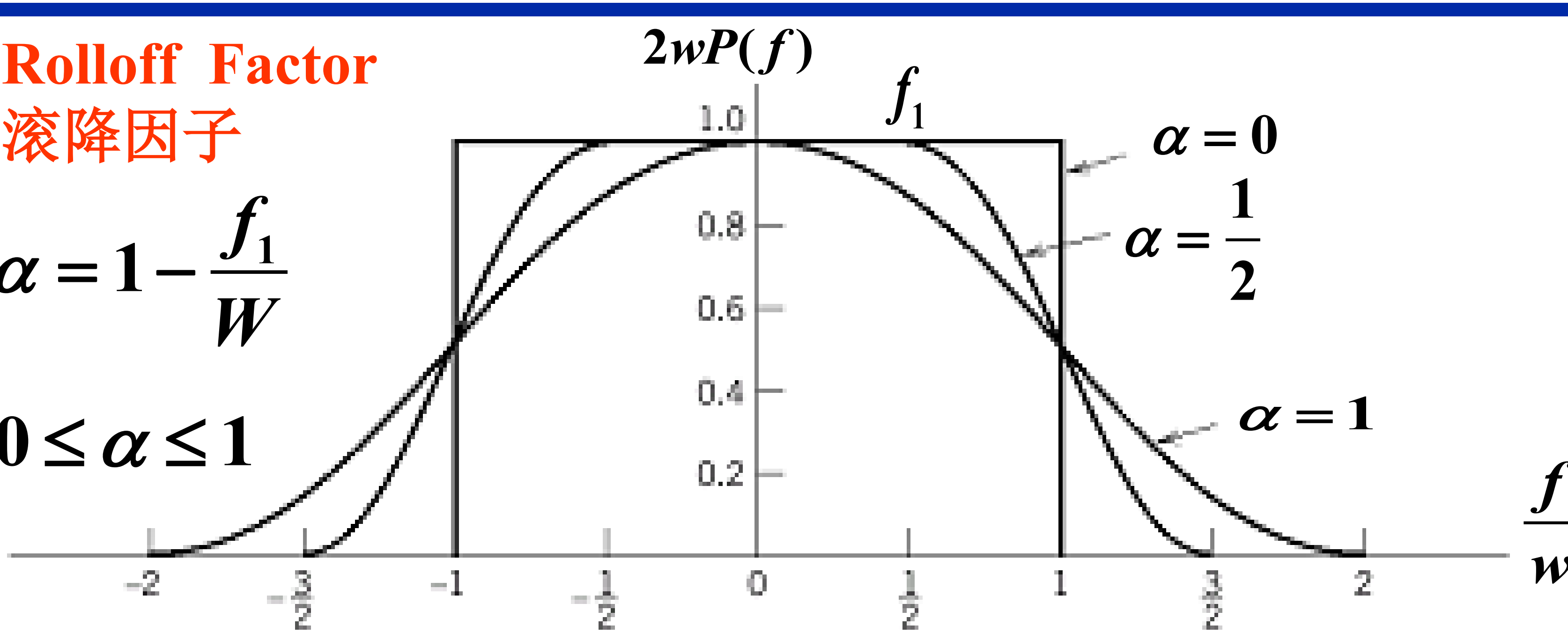
Flat portion 平坦部分

Rolloff portion 滚降部分

Rolloff Factor 滚降因子

$$\alpha=1-\frac{f_1}{W}$$

$$0\le\alpha\le1$$



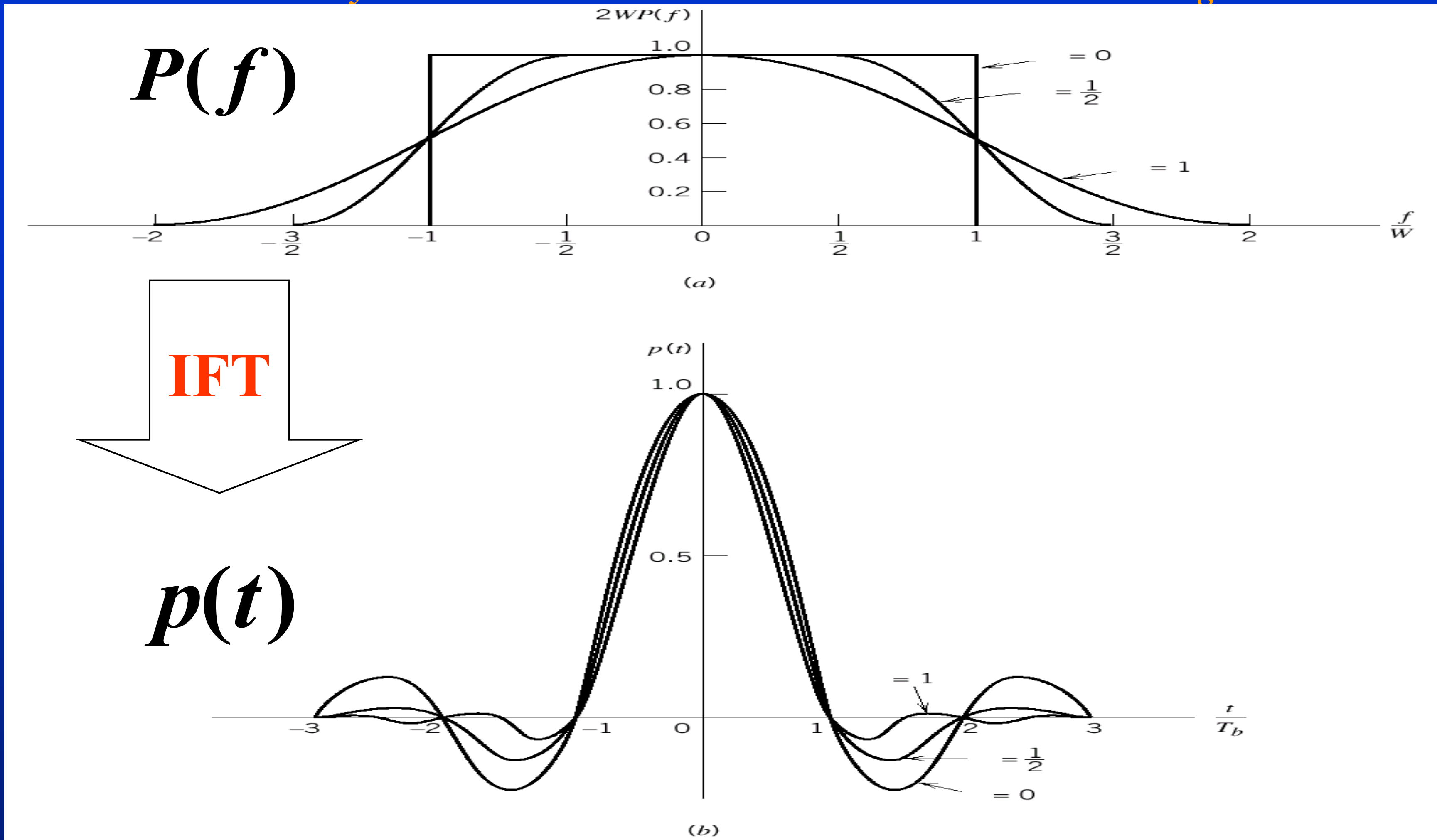
升余弦频谱信道带宽

transmission bandwidth：

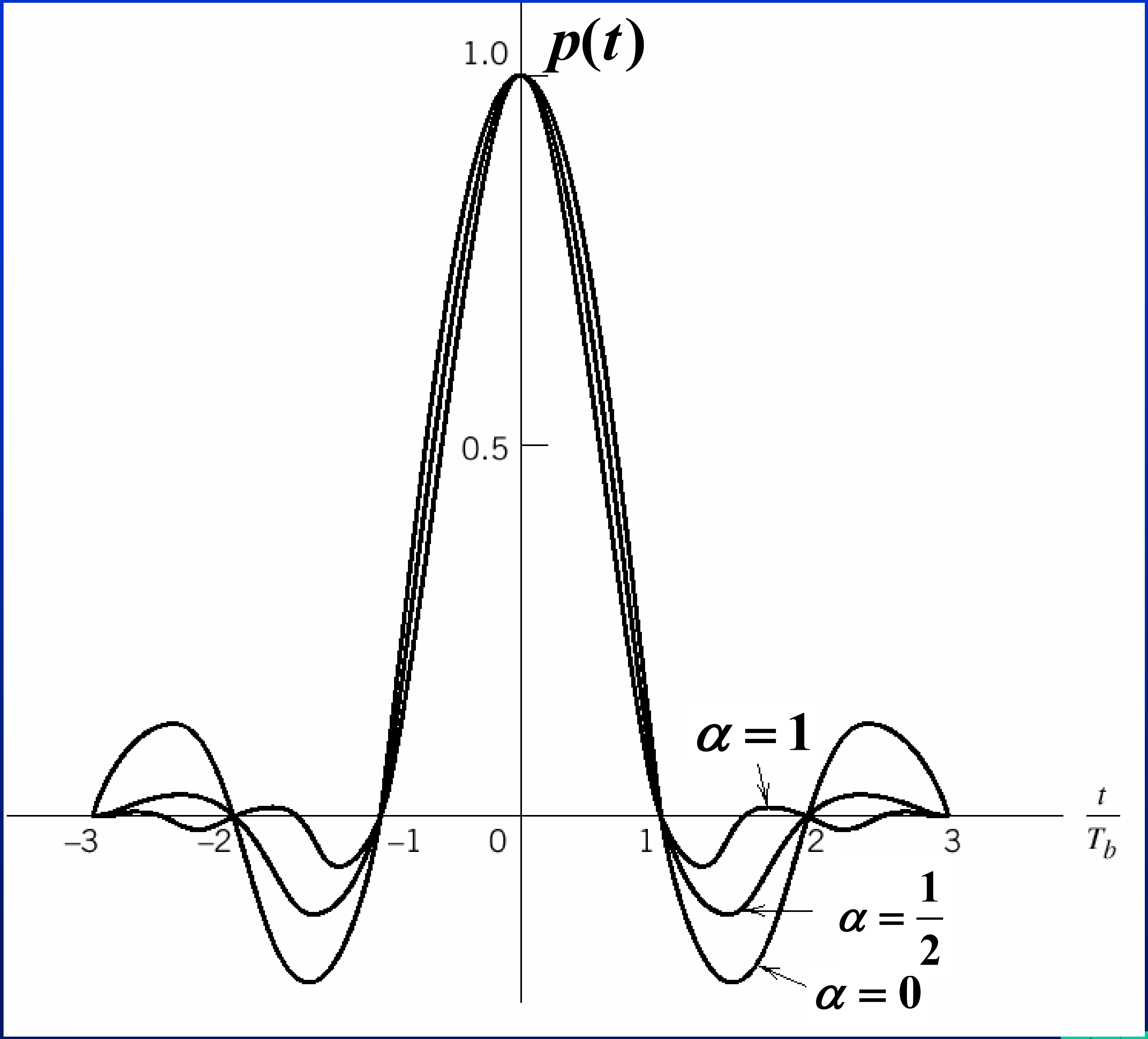
$$B_T = 2W - f_1 = W(1+\alpha)$$

- **When α=0, it corresponds to ideal Nyquist channel, $B_T = w$;**

- **when α=1, it needs maximum bandwidth $B_T = 2w$. In this case, it is called full-cosine rolloff characteristic** 完全升余弦滚降特性.



$$p(t)=\left(\sin c\left(2\pi Wt\right)\right)\left(\frac{\cos\left(2\pi\alpha Wt\right)}{1-16\alpha^{2}W^{2}t^{2}}\right) \qquad (4.62)$$

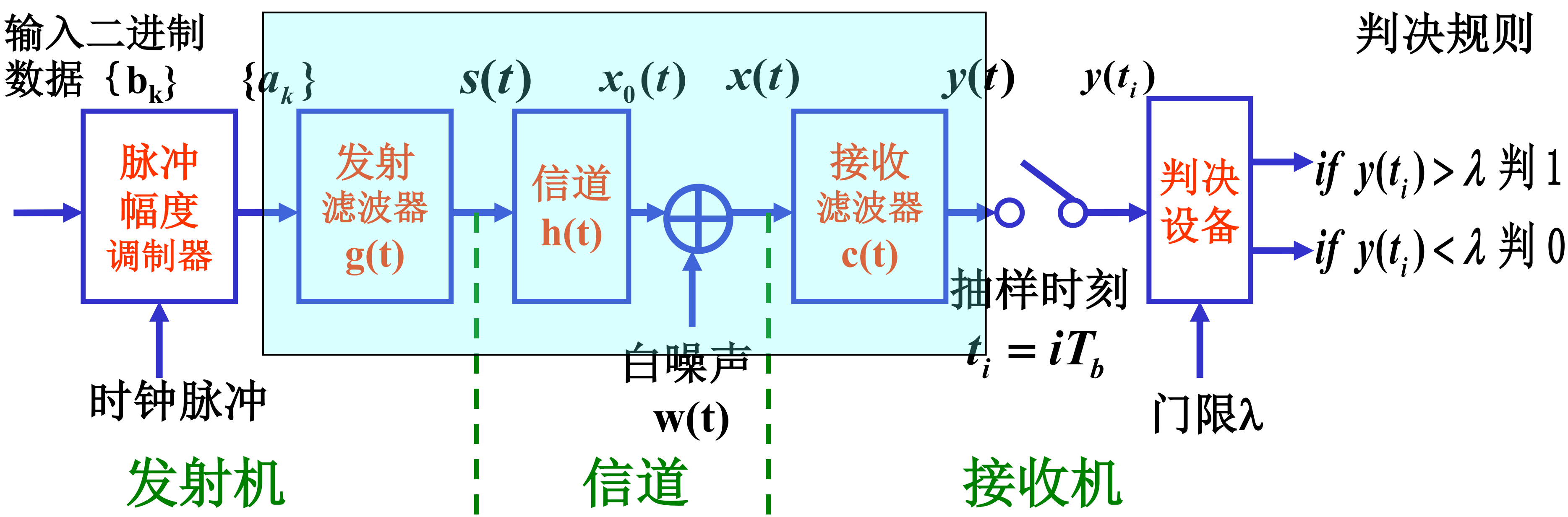

$p(t)$
1.0
0.5
$\alpha = 1$
$\alpha = \frac{1}{2}$
$\alpha = 0$
–3
–2
–1
0
1
2
3
$\frac{t}{T_b}$

Characteristics of Raised Cosine Spectrum

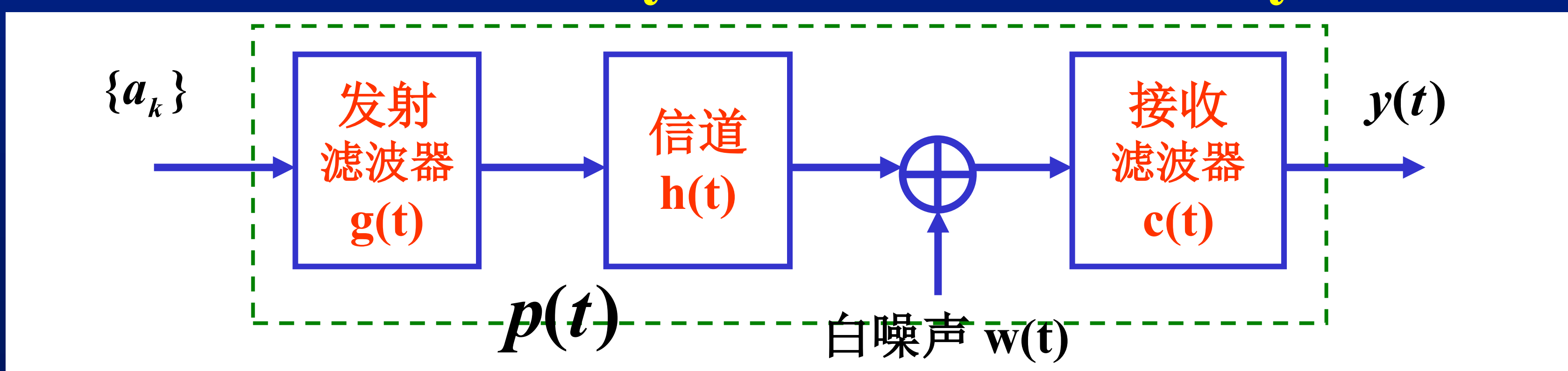
- **No ISI.** 
- **It can be realized.** 
- **It decreases as** $\dfrac{1}{|t|^2}$. 
- **Transmission Bandwidth:**

$$w \le B_T \le 2w$$

Brief Review for Last Class



Baseband binary data transmission system



$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k p\left[(i-k)T_b\right] = \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p[(i-k)T_b]$$

useful desired value **ISI**

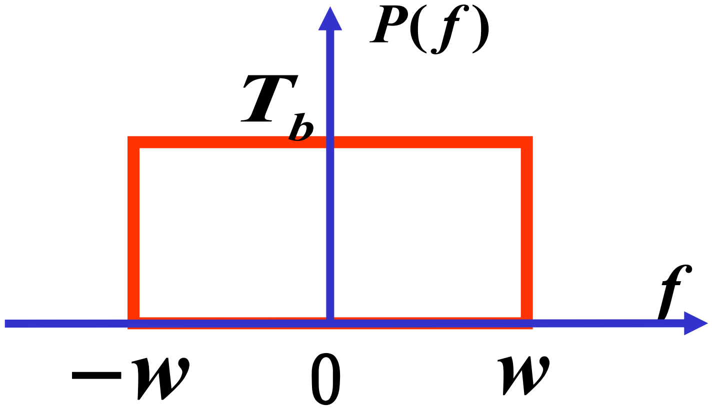
$$\boxed{p(iT_b - kT_b) = \begin{cases} 1 & , k = i \\ 0 & , k \neq i \end{cases} \qquad (4.49)}$$

无码间串扰的时域条件

$$\boxed{\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b \qquad (4.53)}$$

无码间串扰的频域条件

理想奈奎斯特信道频率响应与波形形状



$$W = \frac{R_b}{2} = \frac{1}{2T_b}$$

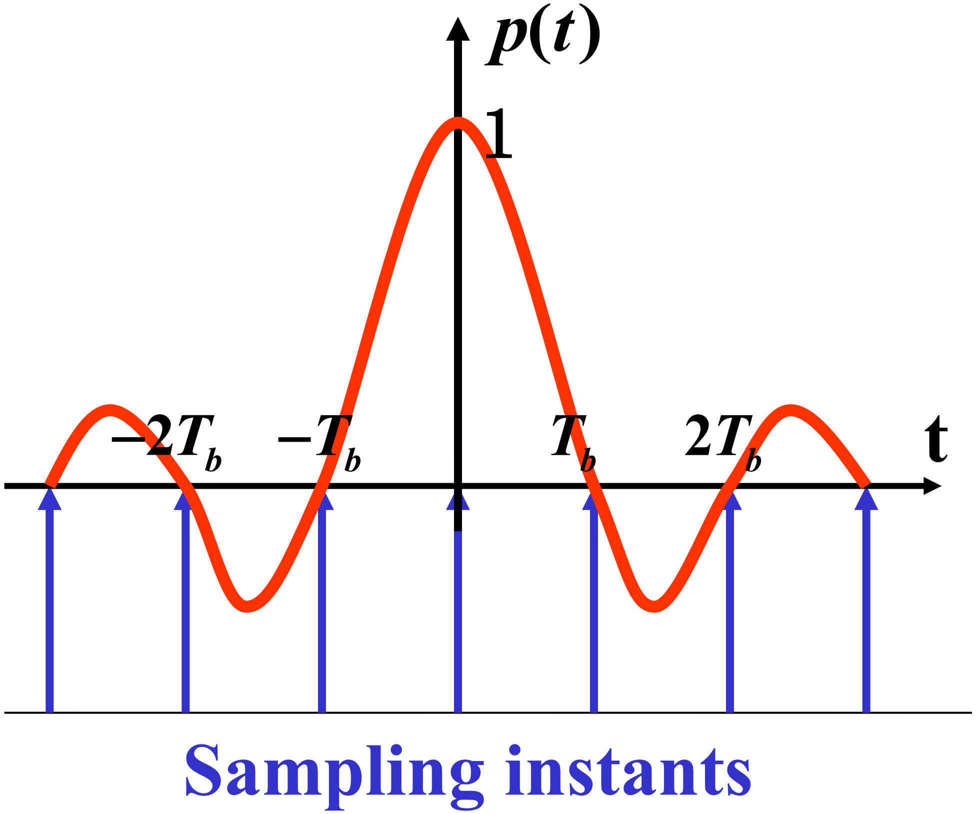


Figure 4.8

(*a*) Ideal magnitude response. (*b*) Ideal basic pulse shape.

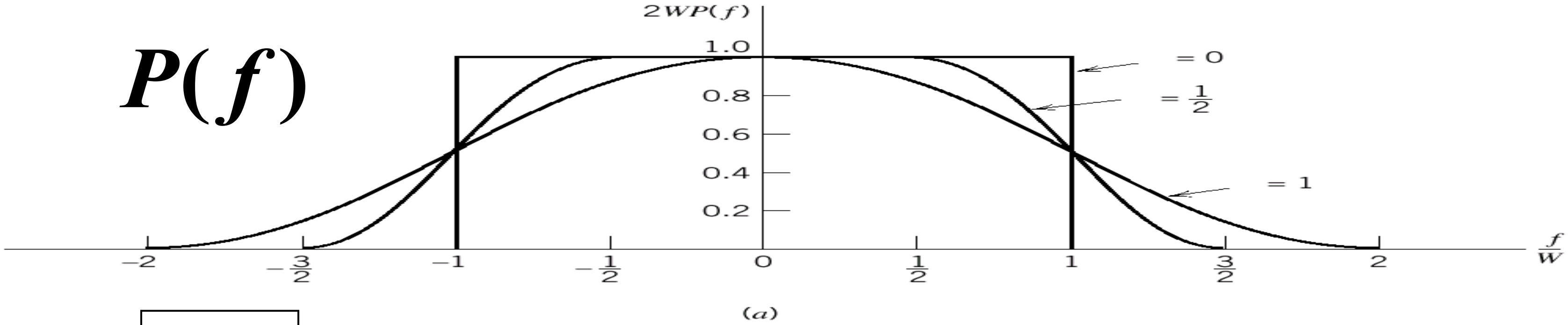
Characteristics of Ideal Nyquist Channel

- **No ISI**
- **Minimum bandwidth**

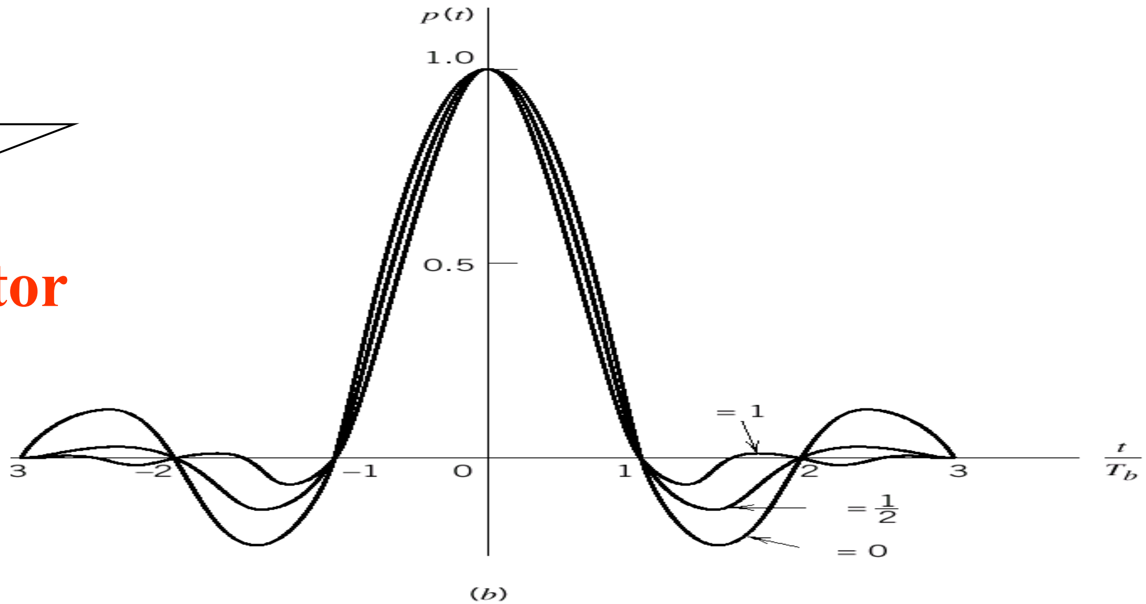
$$B_T = w$$

- **It cannot be realized.**
- **It decreases as** $\dfrac{1}{t}$.

$P(f)$



IFT



Rolloff Factor
滚降因子

$$\alpha = 1 - \frac{f_1}{W}$$

$$0 \le \alpha \le 1$$

$$p(t) = \left(\operatorname{sinc}\left(2\pi W t\right)\right)\left(\frac{\cos\left(2\pi\alpha W t\right)}{1 - 16\alpha^2 W^2 t^2}\right) \qquad (4.62)$$

Characteristics of Raised Cosine Spectrum

- No ISI.
- It can be realized
- It decreases as $\dfrac{1}{|t|^2}$.
- Transmission Bandwidth:

$$w \le B_T \le 2w$$

Example 4.2

Determine the bandwidth requirement of the T1 system.

- Sampling rate 8000Hz
- 24 independent voice signals
- 8 bits per code word
- 1 bit for synchronization in each frame

$$R_b = 1.544 \text{ Mb/s} \qquad T_b = \frac{1}{R_b} = 0.647 \ \mu s$$

Assuming the use of ideal Nyquist channel, the B_T of T1 system has minimum:

$$B_T = W = \frac{1}{2T_b} = \frac{R_b}{2} = 772 \text{ kHz}$$

Using a full-cosine rolloff characteristic, α=1

$$B_T = W\left(1+\alpha\right) = 2W = \frac{1}{T_b} = 1544 \text{ kHz}$$

Characteristics of Raised Cosine Spectrum

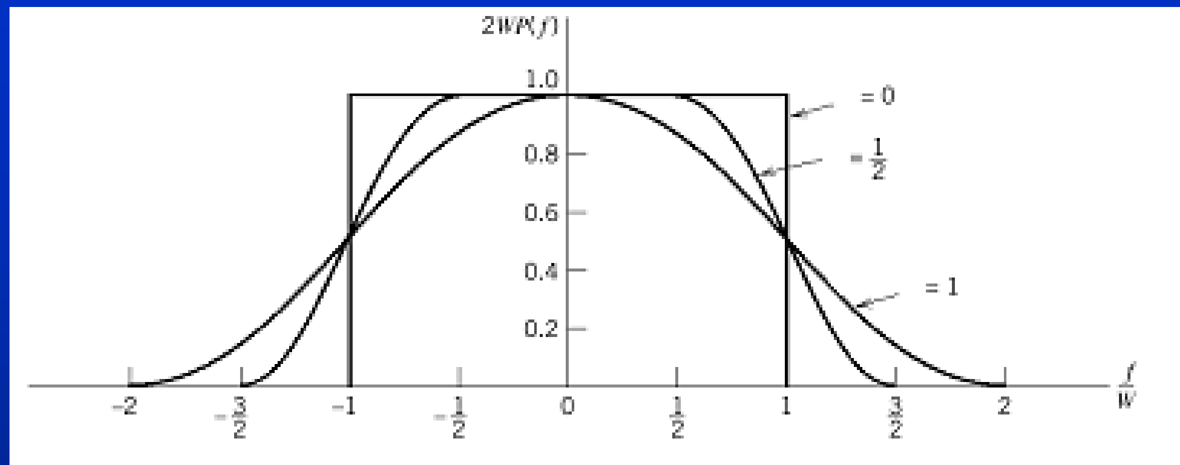
- No ISI. 
- It can be realized 
- It decreases as $\frac{1}{|t|^2}$. 
- Transmission Bandwidth: 

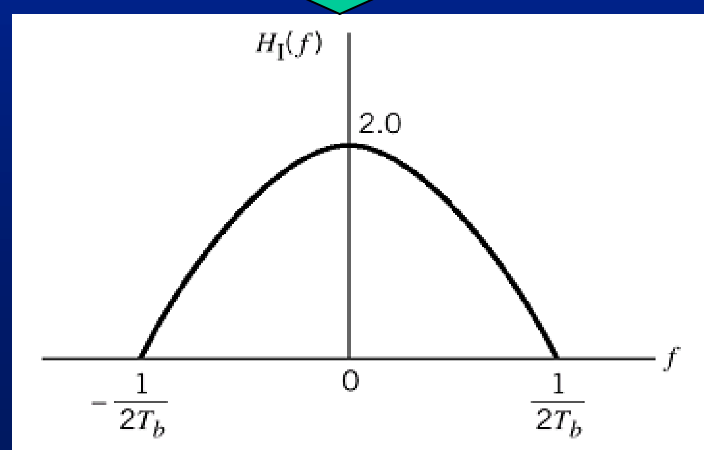
$$w \le B_T \le 2w$$

Can you make it smile?

4.6 Correlative-Level Coding 相关电平编码 Partial-response signaling scheme 部分响应技术

idea

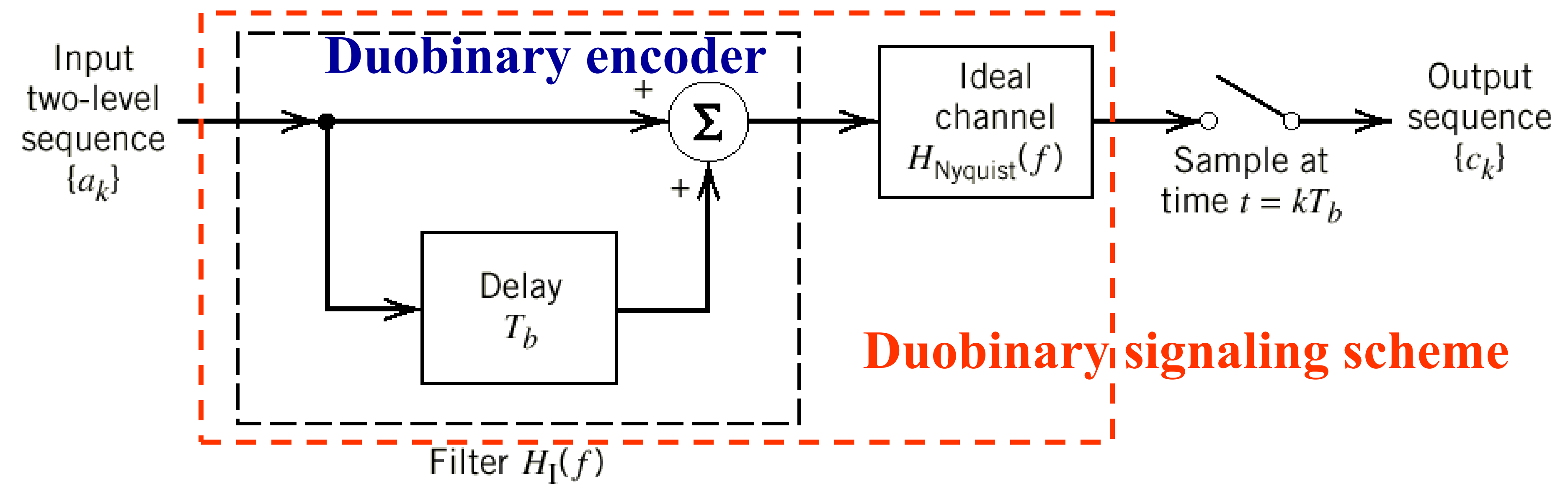




1. **ISI is coming.**
2. **It can be realized.**
3. **It decreases as $1/|t|^2$**
4. **$B_T = w$.**

Fortunately, it introduces ISI in a controlled manner.

Duobinary Signaling 双二进制信号



$$a_k = \begin{cases} +1 & \text{if symbol } b_k \text{ is } 1 \\ -1 & \text{if symbol } b_k \text{ is } 0 \end{cases} \qquad (4.65)$$

$$c_k = a_k + a_{k-1} \qquad (4.66)$$

Why is it called correlative-level coding?

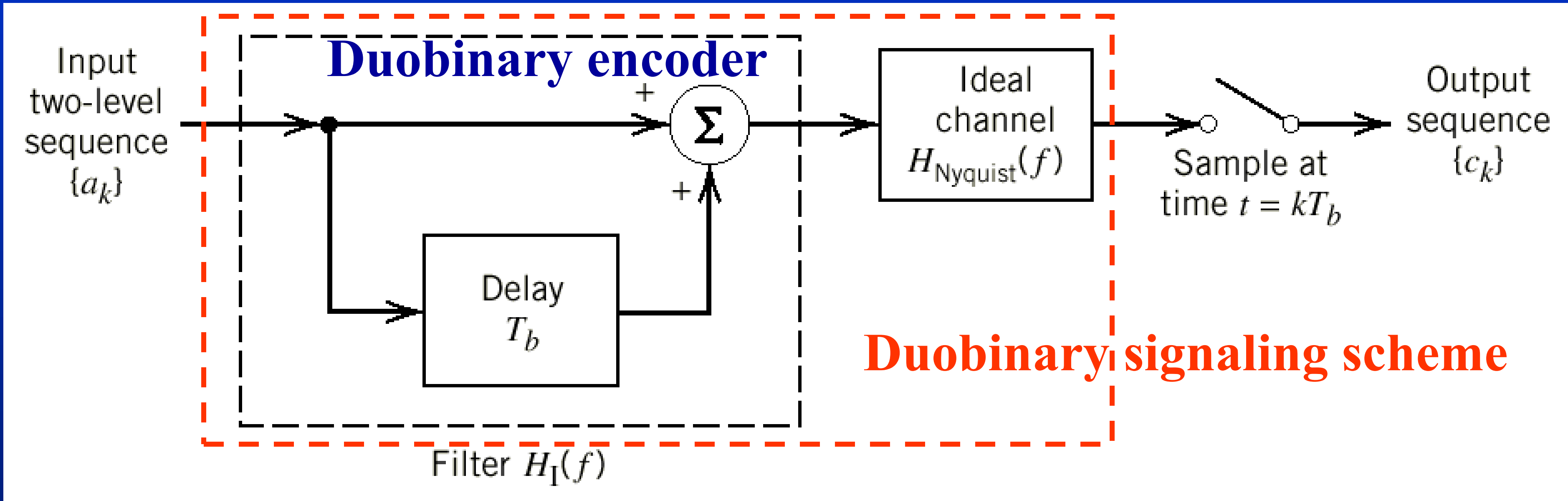
$\{a_k\}$: uncorrelated two-level pulses , +1 and -1

Correlation is introduced.

$\{c_k\}$: correlated three-level pulses, +2, 0 and -2

This correlation between the adjacent pulses may be viewed as introducing ISI. However, this ISI is under the designer's control. So it can be eliminated at the receiver.

Overall frequency response $H_I(f)$



Delay element has frequency response

$$\exp(-j2\pi f T_b)$$

Therefore, $H_I(f)$

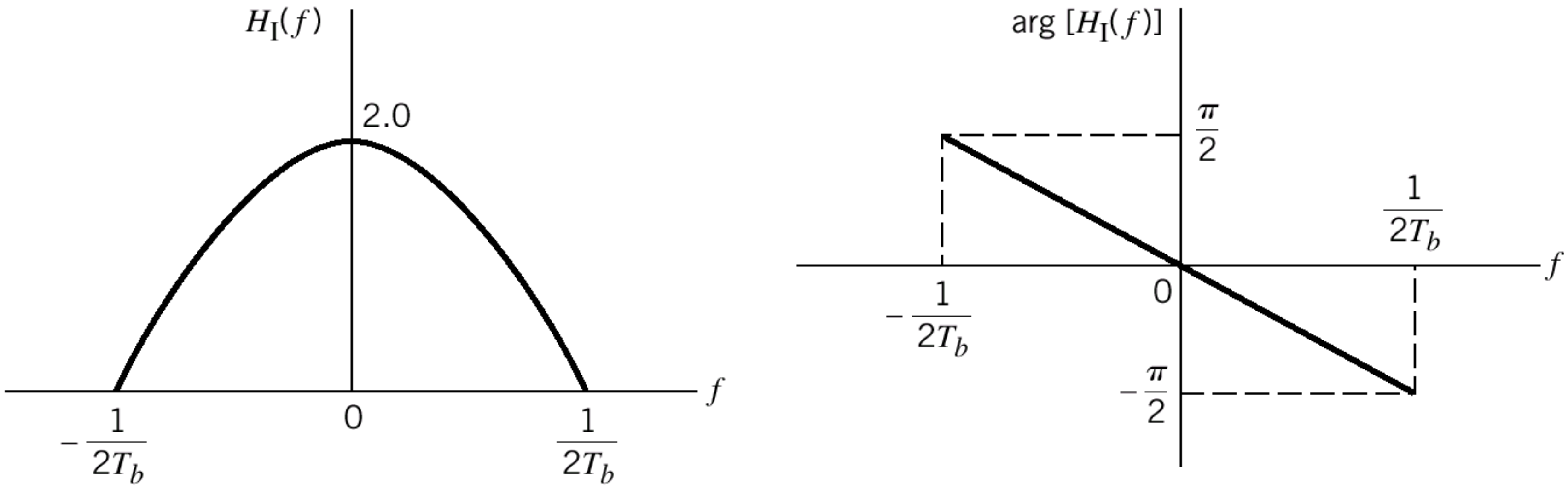
$$\begin{aligned} H_I(f) &= H_{Nyquist}(f)\left[1+\exp(-j2\pi f T_b)\right] \qquad (4.67) \\ &= H_{Nyquist}(f)\left[\exp(j\pi f T_b)+\exp(-j\pi f T_b)\right]\exp(-j\pi f T_b) \\ &= H_{Nyquist}(f)\cos(\pi f T_b)\exp(-j\pi f T_b) \end{aligned}$$

Ideal Nyquist channel

$$H_{Nyquist}(f) = \begin{cases} 1, & |f| \le 1/2T_b \\ 0\ , & otherwise \end{cases} \qquad (4.68)$$

Thus, the overall frequency response of the duobinary signaling schemes has the form of half-cycle cosine function.

$$H_I(f) = \begin{cases} 2\cos(\pi f T_b)\exp(-j\pi f T_b), & |f| \le 1/2T_b \\ 0\ , & otherwise \end{cases} \qquad (4.69)$$



Frequency response of the duobinary conversion filter. (*a*) Magnitude response. (*b*) Phase response.

Impulse response of duobinary conversion filter

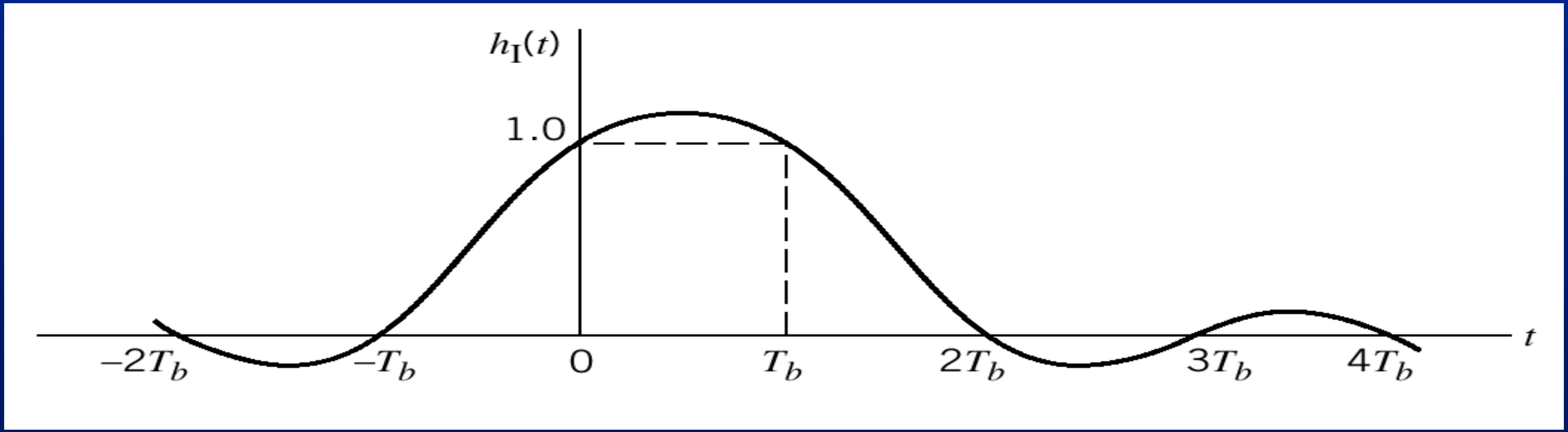
$H_I(f)$ IFT

$$h_1(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[\pi(t-T_b)/T_b]}{\pi(t-T_b)/T_b} \qquad (4.70)$$

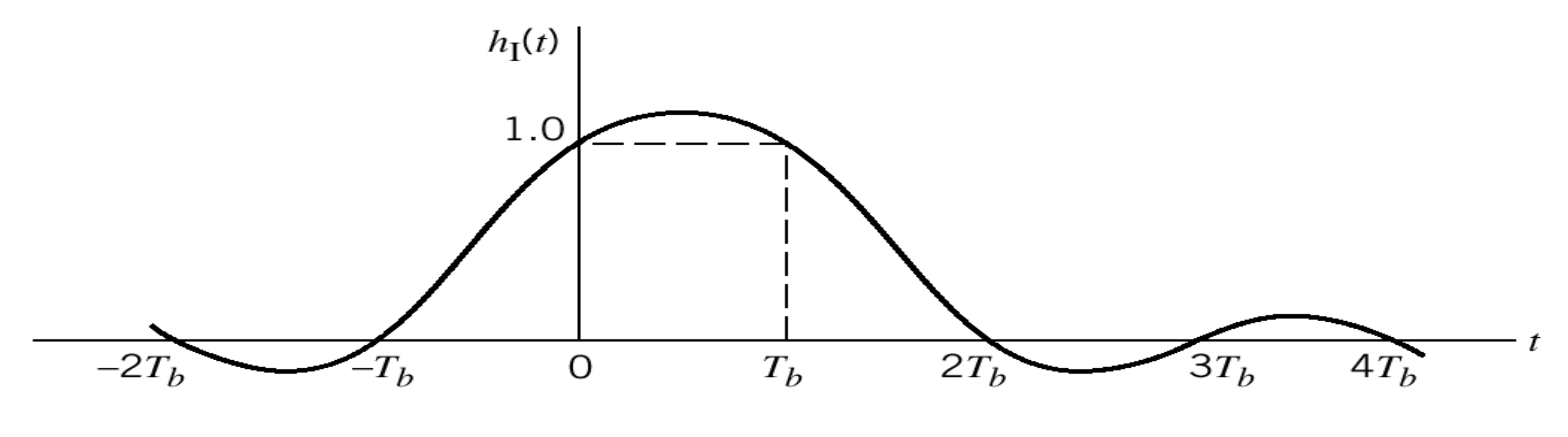
$$= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t-T_b)/T_b}$$

$$= \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(t-T_b)/T_b}$$

It decays as $1/t^2$.



Why is correlative coding also called as partial-response signaling?



The response to an input pulse is spread over more than one signaling interval; in other words, the response in any signaling interval is "partial".

How to get a_k from c_k?

At the transmitter, encoding

$$c_k = a_k + a_{k-1} \qquad (4.66)$$

At the receiver, decoding

$$\hat{a}_k = c_k - \hat{a}_{k-1}$$

Decision feedback 判决反馈：refers to the technique of using a stored estimate of previous symbol.

Example: Duobinary Coding and decoding

$\{b_k\}$		0	0	1	0	1	1	0
$\{a_k\}$	+1	-1	-1	+1	-1	+1	+1	-1
$\{c_k\}$		0	-2	0	0	0	+2	0
接收$\{c_k\}$		0	-2	0	0	0	+2	0
译码$\{a_k\}$	+1	-1	-1	+1	-1	+1	+1	-1
译码$\{b_k\}$		0	0	1	0	1	1	0

Example: Duobinary Coding and decoding with error

$\{b_k\}$		0	0	1	0	1	1	0
$\{a_k\}$	+1	-1	-1	+1	-1	+1	+1	-1
$\{c_k\}$		0	-2	0	0	0	+2	0
接收$\{c_k\}$		0	-2	-2	0	0	+2	0
译码$\{a_k\}$	+1	-1	-1	-1	+1	-1	+3	-3
译码$\{b_k\}$		0	0	0	1	0	1	0

Error propagation happens!

Drawback: error-propagation 误差扩散，错误传播

Once errors are made, they tend to propagate through the output because a decision on the current input a_k depends on the correctness of the decision made on the previous input a_{k-1}.

How to overcome this drawback?

Precoding 预编码

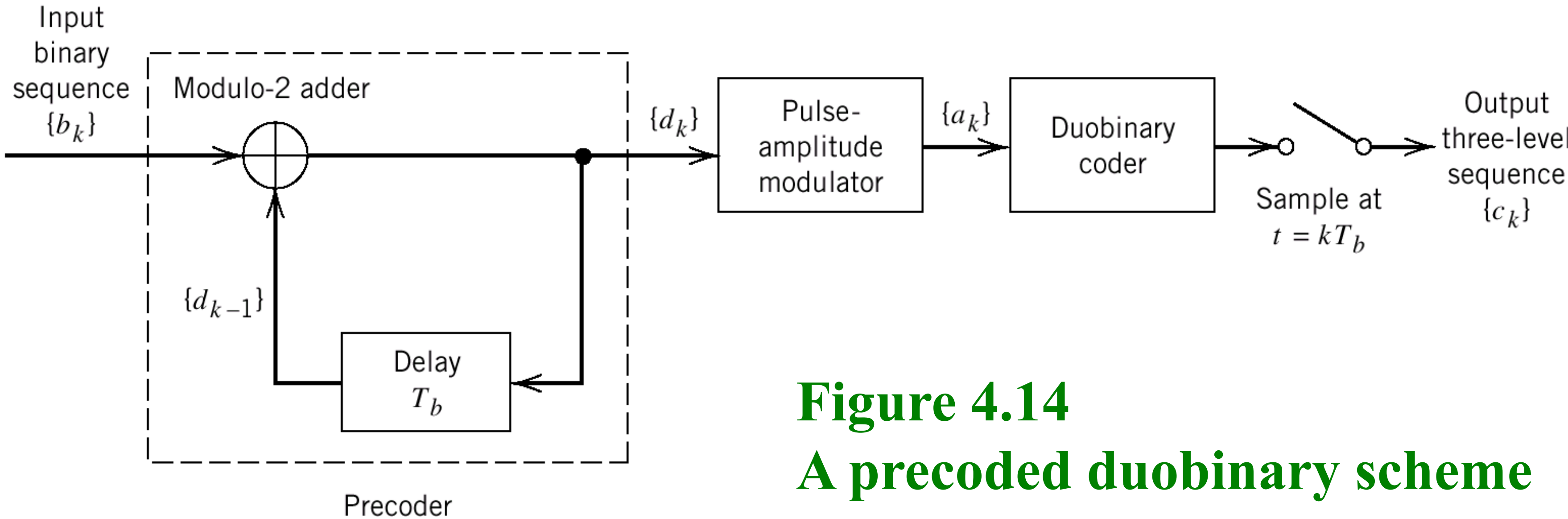


Figure 4.14
A precoded duobinary scheme

Precoding 预编码

$$d_k = b_k \oplus d_{k-1} \qquad (4.72)$$

It is modulo-two addition, Exclusive OR operation.
模2加，异或运算

Duobinary coding

$$c_k = a_k + a_{k-1} \qquad (4.74)$$

$$d_k = b_k \oplus d_{k-1} \qquad (4.72)$$

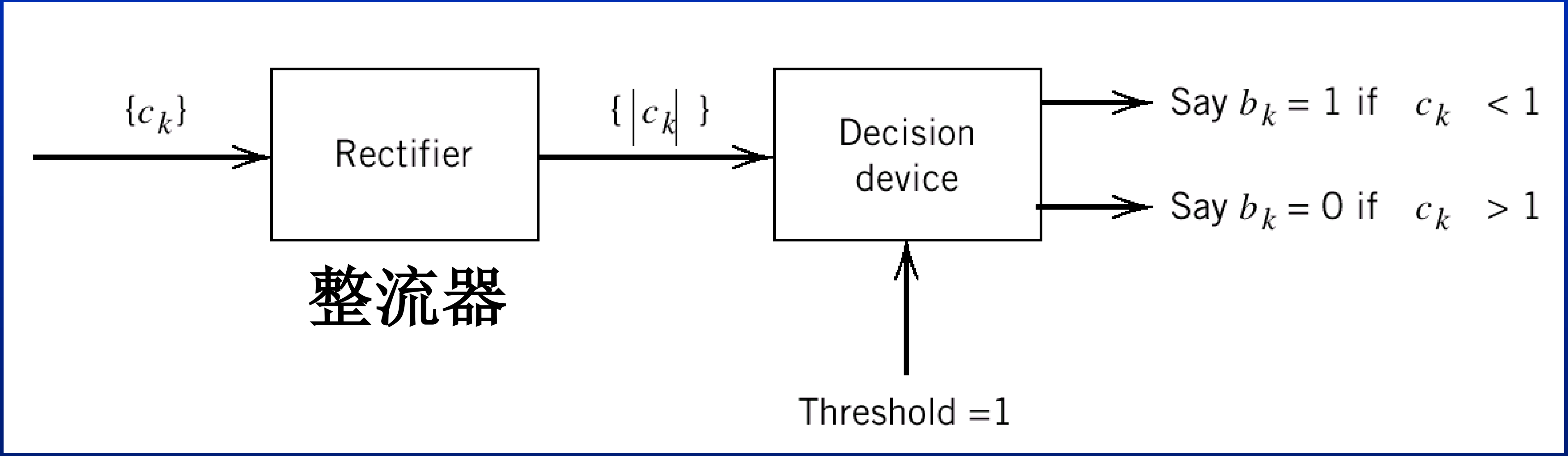
$$c_k = a_k + a_{k-1} \qquad (4.74)$$

$$c_k = \begin{cases} 0 & \text{if data symbol } b_k \text{ is } 1 \\ \pm 2 & \text{if data symbol } b_k \text{ is } 0 \end{cases} \qquad (4.75)$$

Decision rule in the receiver

$$\begin{cases} \text{if } |c_k| < 1, & \text{say symbol } b_k \text{ is } 1 \\ \text{if } |c_k| > 1, & \text{say symbol } b_k \text{ is } 0 \end{cases} \qquad (4.76)$$

Figure 4.15
Detector for recovering original binary sequence from the precoded duobinary coder output.



Example 4.3: Duobinary Coding with Precoding

$\{b_k\}$		0	0	1	0	1	1	0
$\{d_k\}$	1	1	1	0	0	1	0	0
$\{a_k\}$	+1	+1	+1	-1	-1	+1	-1	-1
$\{c_k\}$		+2	+2	0	-2	0	0	-2
接收$\{c_k\}$		+2	+2	0	-2	0	0	-2
译码$\{b_k\}$		0	0	1	0	1	1	0

Example: Duobinary Coding with Precoding with error

$\{b_k\}$		0	0	1	0	1	1	0
$\{d_k\}$	1	1	1	0	0	1	0	0
$\{a_k\}$	+1	+1	+1	-1	-1	+1	-1	-1
$\{c_k\}$		+2	+2	0	-2	0	0	-2
接收$\{c_k\}$		+2	+2	-2	-2	0	0	-2
译码$\{b_k\}$		0	0	0	0	1	1	0

There is no error propagation!

Class N Partial Response

Class I partial response 第一类部分响应

$$h_1(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin[\pi(t-T_b)/T_b]}{\pi(t-T_b)/T_b} \qquad (4.70)$$

Generalization，一般化

$$h(t) = \sum_{n=0}^{N-1} w_n \sin c\left(\frac{t}{T_b} - n\right) \qquad (4.83)$$

Table 4.2 different classes of partial-response signaling schemes

Type of class	N	W0	W1	W2	W3	W4	comments
I	2	1	1				Duobinary coding
II	3	1	2	1			
III	3	2	1	-1			
IV	3	1	0	-1			Modified duobinary coding
V	5	-1	0	2	0	-1	

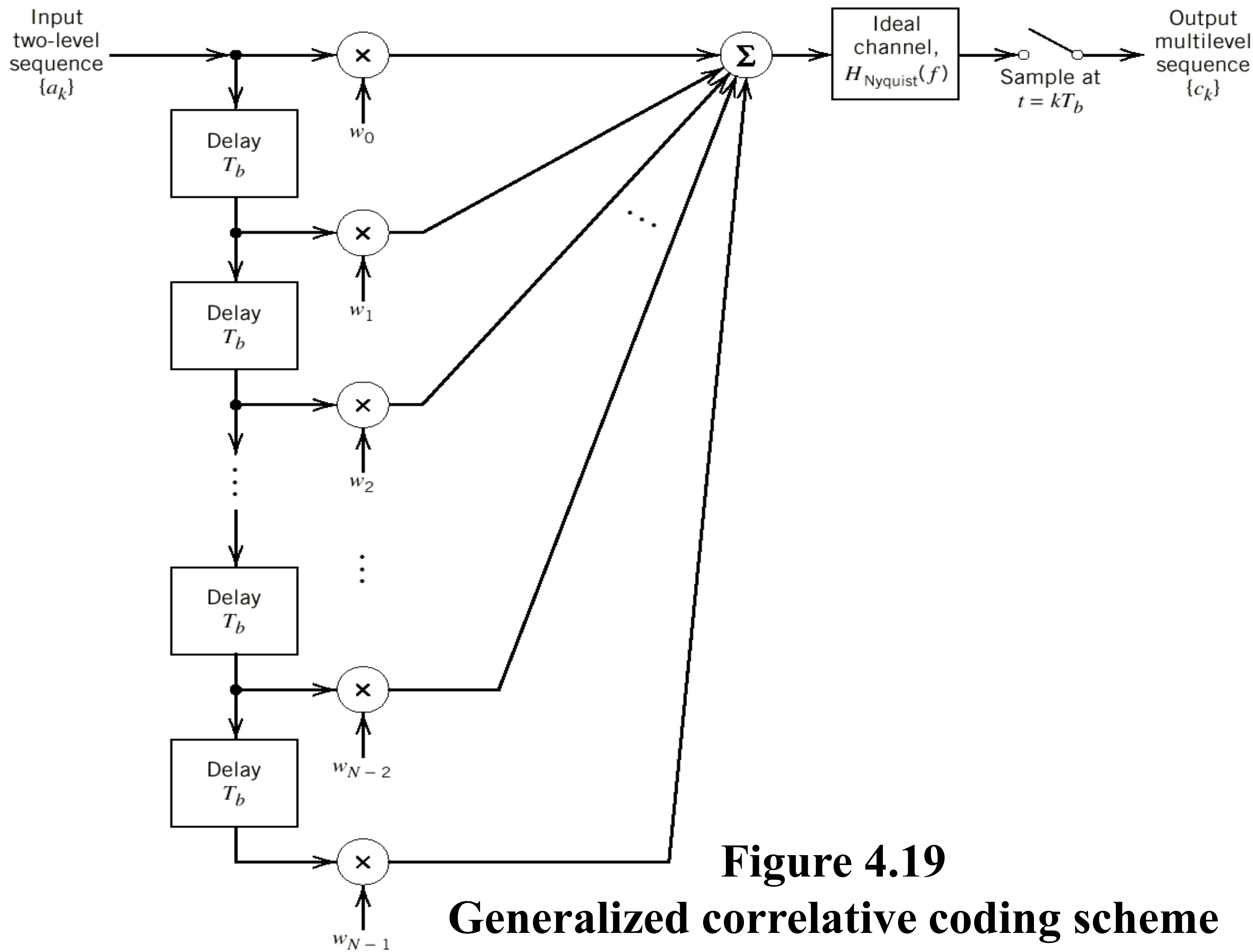
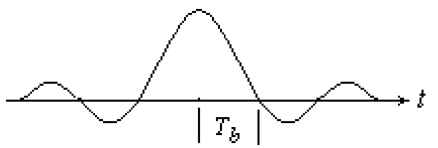
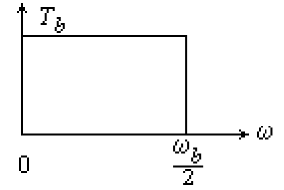
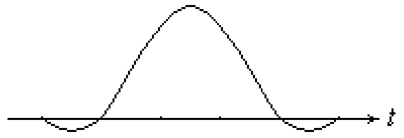
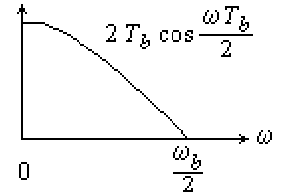
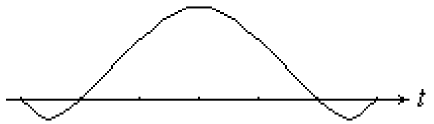
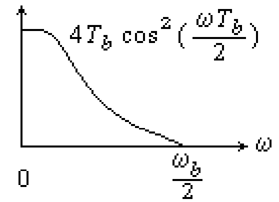
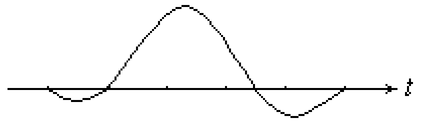
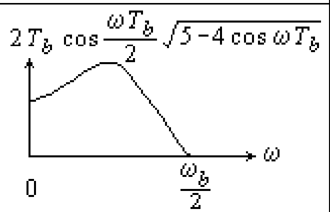
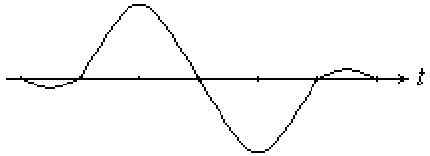
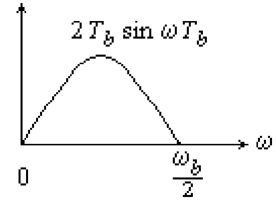
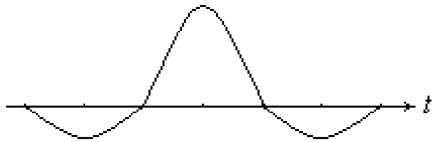
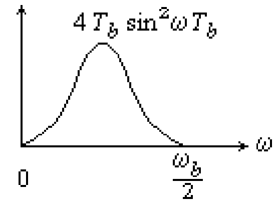


Figure 4.19

Generalized correlative coding scheme

常用五类部分响应波形

表5-3 常见五类部分响应信号

类别	r_0	r_1	r_2	r_3	r_4	$h(t)$	$H(\omega)$	二进制输入时抽样值电平数
0	1							2
I	1	1						3
II	1	2	1					5
III	2	1	-1					5
IV	1	0	-1					3
V	-1	0	2	0	-1			5

Useful characteristics of Partial-response signaling schemes

1. **Binary data transmission over a physical baseband can be accomplished at a rate close to the Nyquist rate, using realizable filters with gradual cutoff characteristics.**
2. **Different spectral shapes can be produced, appropriate for the application at hand.**

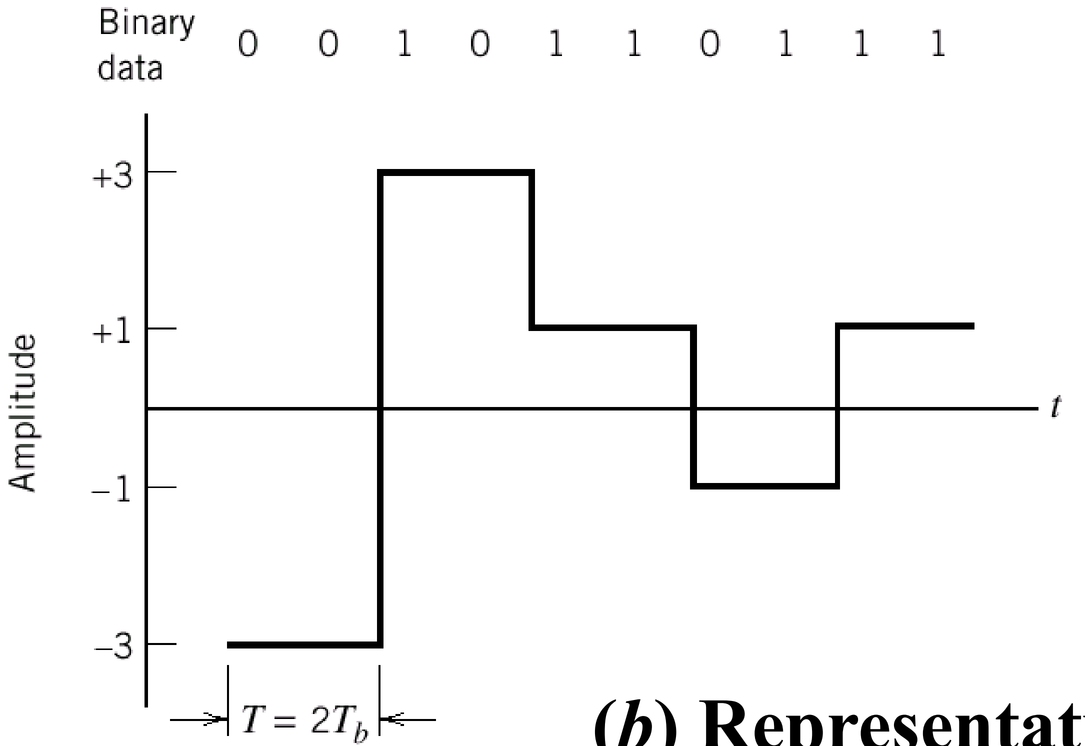
At a price--SNR

A larger signal-to-noise ratio SNR is required because of an increase in the number of signal levels used.

4.7 Baseband M-ary PAM Transmission 基带M进制PAM传输

- In the baseband binary PAM system, the pulse-amplitude modulator produces binary pulses, that is, pulses with one of two possible amplitude levels.
- In a baseband M-ary PAM system, the pulse-amplitude modulator produces one of the M possible amplitude levels.

Quaternary System 四进制系统



(*a*) Waveform.

Dibit	Amplitude
00	–3
01	–1
11	+1
10	+3

格雷码

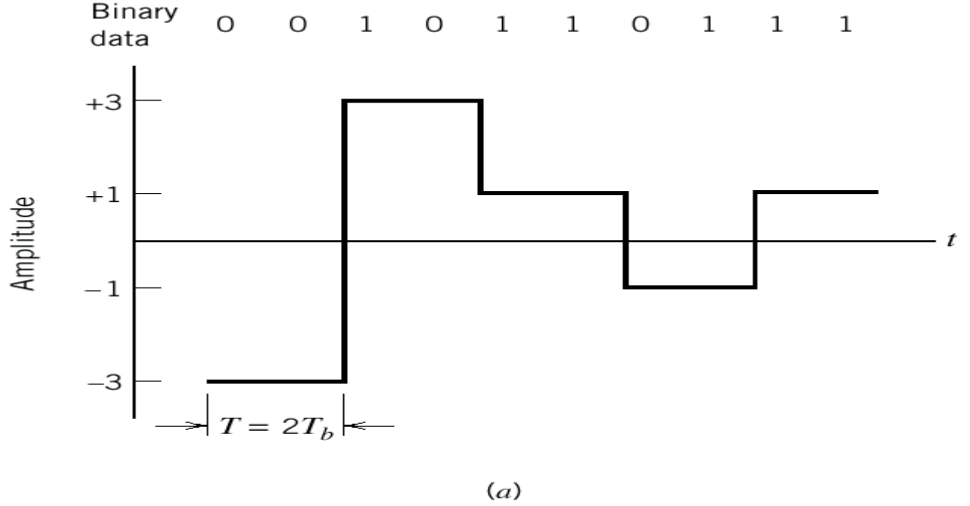
(*b*) Representation of the 4 possible dibits, based on Gray encoding.

Why is Gray encoding used instead of natural encoding 自然编码?

Gray Encoding

	Natural encoding 自然编码	Gray Encoding 格雷编码
Dibit	00 01 10 11	00 01 11 10
Amplitude	-3 -1 +1 +3	-3 -1 +1 +3

Gray encoding means that any dibit in the quaternary alphabet differs from an adjacent dibit in a single bit position.



Dibit	Amplitude
00	−3
01	−1
11	+1
10	+3

格雷码

(b)

T: symbol duration 符号持续时间

T_b: bit duration 比特持续时间

1/T: 符号速率，码元速率，波特率

Signaling rate, symbols per second, or Bauds

$1/T_b$: 比特率 Bit rate

$$T = T_b \log_2^M$$

$$\frac{1}{T_b} = \frac{1}{T} \log_2^M$$

$$\frac{1}{T_b} = \frac{1}{T}\log_2^M$$

$$R_b = \log_2^M R_B$$

M进制基带传输系统中,
信息速率和码元速率的关系.

Therefore, in a given channel bandwidth, we find that by using an M-ary PAM system, we are able to transmit information at a rate that is log_2^M faster than the corresponding binary PAM system.

The cost is increased transmitted power and complexity.

In a binary baseband transmission system, if the signaling rate is 2000 Bauds, then the corresponding information rate is ____bit/s.
The required minimum bandwidth is____ Hz.

In a quaternary baseband transmission system, if the signaling rate is 2000 Bauds, then the corresponding information rate is ____ bit/s.
The required minimum bandwidth is____ Hz.

In a M-ary baseband transmission system, if the signaling rate is 2000 Bauds, then the corresponding information rate is ____bit/s.
The required minimum bandwidth is____ Hz.

4.8 Digital Subscriber Lines DSL

This topic is reserved for group presentation.

4.9 Optimum Linear Receiver

So far, we have treated the following two channel conditions separately:

- **Channel noise acting alone, which led to information of the matched filter.**
- **Intersymbol interference acting alone, which led to formulation of the pulse-shaping transmit filter so as to realize the Nyquist channel.**

However, channel and ISI act together in practical systems.

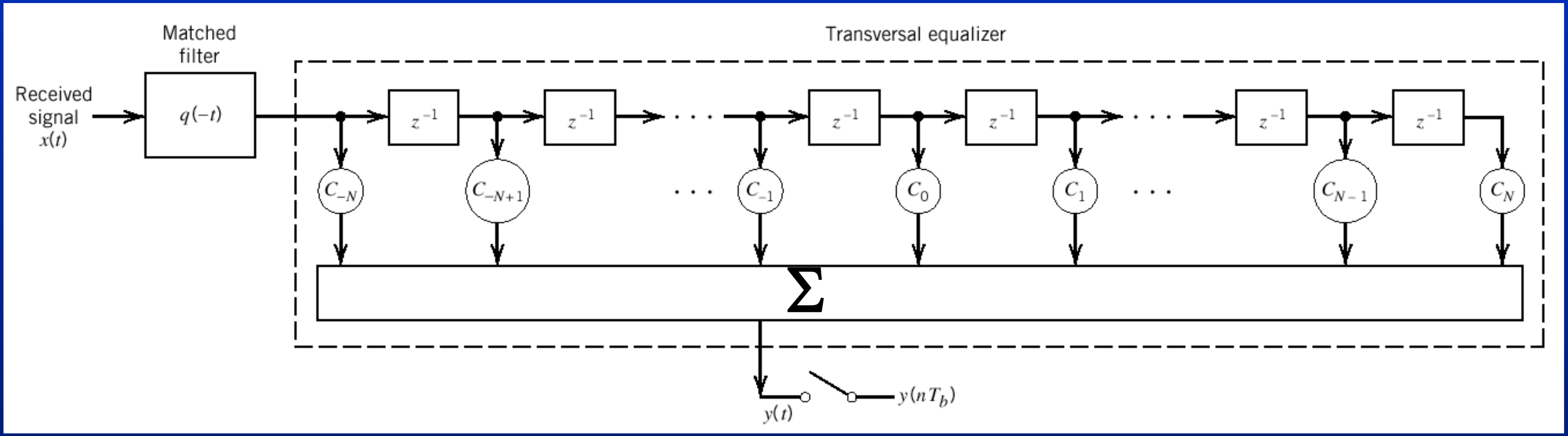
When ISI and noise are both significant, how to design the receiver?

- The resulting optimum linear receiver is called the minimum mean-square error receiver 最小均方误差接收机.
- It consists of the cascade connection of a matched filter and linear transversal (tapped-delay line) equalizer.

linear transversal equalizer 线性横向均衡器

Tapped-delay-line 抽头延迟线

Figure 4.27
Optimum linear receiver consisting of the cascade connection of matched filter and transversal equalizer.



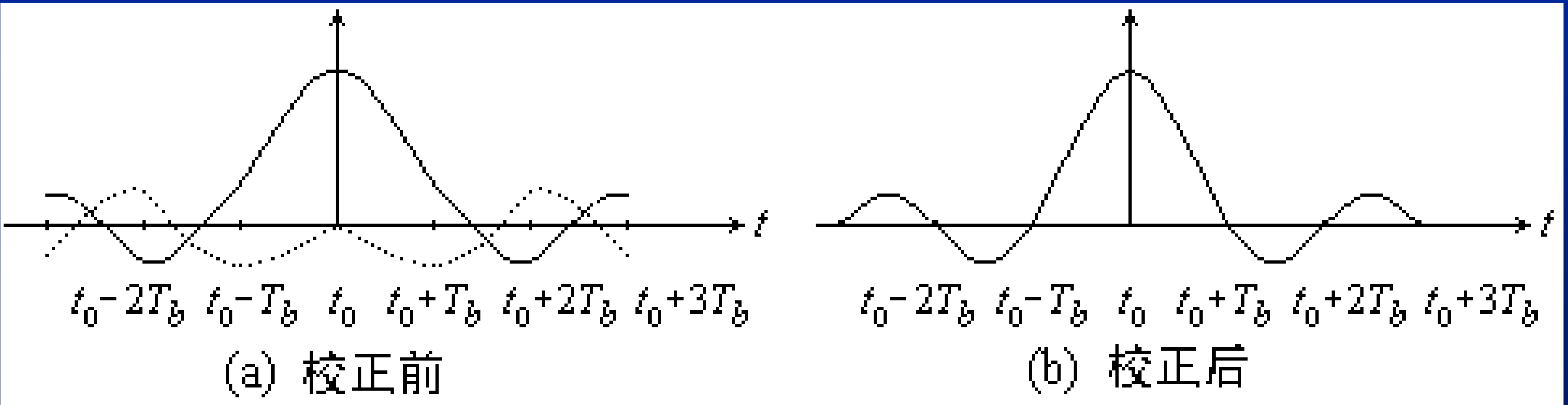
What is the purpose of equalizer?

为什么要研究均衡技术?

各种减少码间干扰的方法，其前提都是要知道信道和某些部件的特性，然后精心设计发送滤波器和接收滤波器，使总的系统特性满足无失真的传输条件。但是实际的信道特性不可能完全知道，而且是经常变化的。尤其是经过交换的线路，每次都不同。此外，也不可能设计出完全满足最佳传输条件的发送与接收滤波器的特性。因此，一个实际的基带传输系统中总是存在不同程度的码间干扰。为此，往往在系统中加入可调滤波器，一般称之为均衡器，用来校正这些失真，使信道达到最小误码率。

时域均衡原理

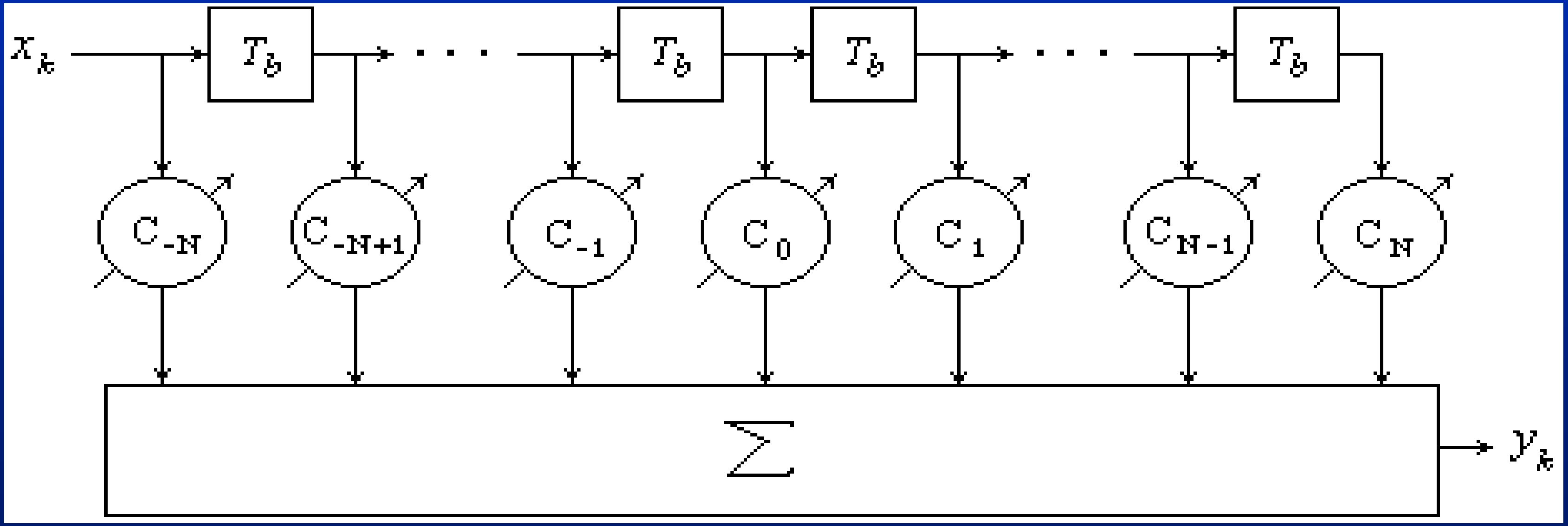
所谓时域均衡是直接利用波形补偿的方法来校正由于基带特性不理想引起的波形畸变，使校正后的波形在抽样时刻的码间串扰尽可能小。



均衡前后波形示意图

横向均衡滤波器

时域均衡最常用的方法是在基带信号接收滤波器之后插入一个横向滤波器，它由抽头延时线加上一些可变增益放大器构成。



4.11 Eye Pattern 眼图

In previous sections of this chapter we have discussed various techniques for dealing with the effects of channel noise and ISI on the performance of a baseband pulse-transmission system.

However, in an operational environment, how to evaluate the combined effect of these impairments on overall system performance?

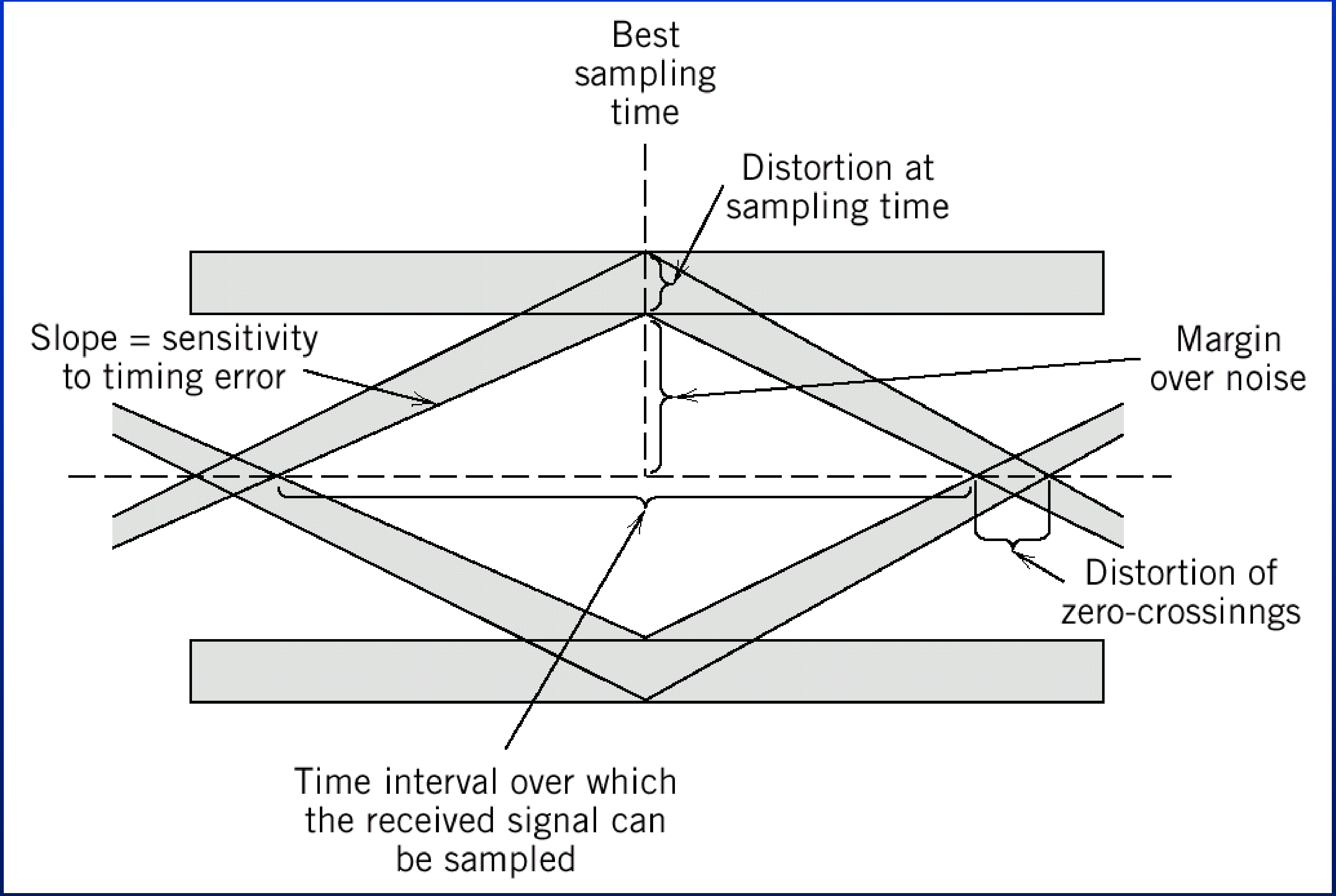
Why is it called Eye Pattern?

- An experimental tool for such an evaluation is eye pattern, which is defined as the **synchronized superposition 同步重叠** of all possible realization within a particular **signaling interval**.
- For binary waves, the result of observation is like the human eye, so it is called Eye Pattern.

Eye Pattern 眼图

在实际的信道中，要使传输系统的特性满足奈奎斯特准则是困难的，因此，总是或多或少存在码间串扰，而噪声又是不可避免的。码间串扰和噪声的干扰可以利用示波器对接收电压进行扫描来观察，因而评价基带传输系统性能的一种定性而方便的方法是观察接收到的基带信号波形。如果将接收波形输入示波器的垂直放大器，把示波器的水平扫描频率调整为与码元速率同步，则可把相隔的各码元波形调整成为一个重叠的图形，这个图形与人眼相象，故常称它为眼图。

Figure 4.33 Interpretation of the eye pattern.



Characteristics of Eye Pattern

1) The **width** of the **eye opening** defines the time interval over which the received signal can be sampled without error from ISI. It is apparent that **the best time for sampling is the instant of time at which the eye is open the widest.**

2) The **sensitivity** of the system to timing error is determined by the **slope 斜率**.

3) The **height** of the eye opening defines the **noise margin 噪声容限** of the system.

What kind of eye do you like?

A beautiful clear big eye!

The number of eyes

Binary system: 1 eye

M-ary system: M-1 eyes why?

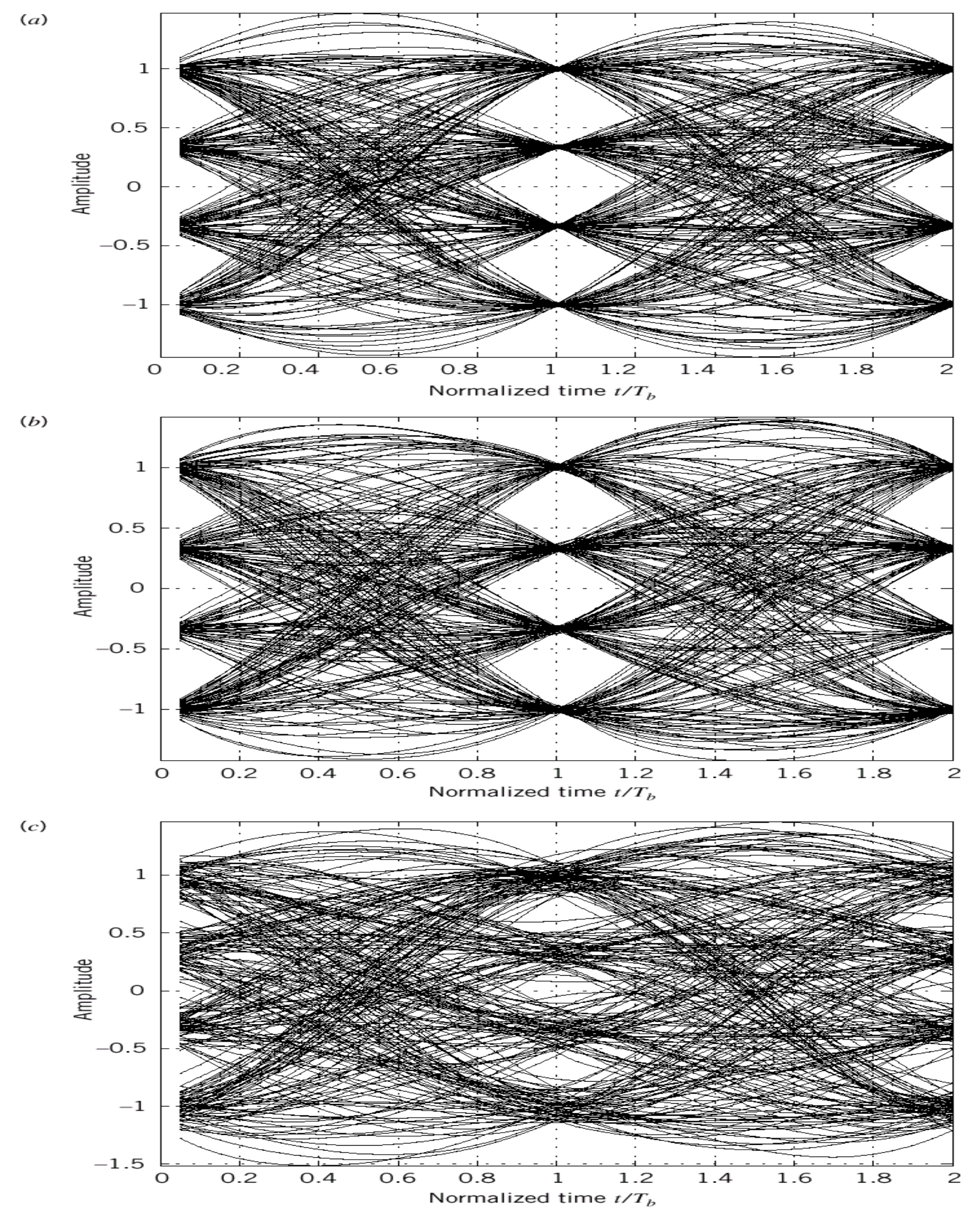


Figure 4.34
(*a*) Eye diagram for quaternary system

(*a*) Noiseless

(*b*) SNR = 20 dB.

(*c*) SNR = 10 dB.

Conclusion

- Matched filter
- Calculation of bit error rate BER
- Intersymbol interference ISI
- Nyquist's criterion for distortionless baseband transmission
- Correlative-level coding or partial-response signaling
- Eye pattern

Homework

4.14 4.16 4.21 4.22