SOLUTIONS MANUAL

to accompany

Digital Signal Processing: A Computer-Based Approach

Fourth Edition

Sanjit K. Mitra

Prepared by

Chowdary Adsumilli, John Berger, Marco Carli,
Hsin-Han Ho, Rajeev Gandhi, Martin Gawecki, Chin Kaye Koh,
Luca Lucchese, Mylene Queiroz de Farias, and Travis Smith

Chapter 9

9.1 We obtain the solutions by using Eq. (9.3) and Eq. (9.4).

(a) $\delta_p = 1 - 10^{-\alpha_p/20} = 1 - 10^{-0.24/20} = 0.0273,\ \delta_s = 10^{-\alpha_s/20} = 10^{-49/20} = 0.0035.$

(b) $\delta_p = 1 - 10^{-\alpha_p/20} = 1 - 10^{-0.14/20} = 0.016, \delta_s = 10^{-\alpha_s/20} = 10^{-68/20} = 0.000398.$

9.2 We obtain the solutions by using Eqs. (9.3) and (9.4).

(a) $\alpha_p = -20\log_{10}\left(1-\delta_p\right) = -20\log_{10}(1-0.04) = 0.3546\,\text{dB},$

$\alpha_s = -20\log_{10}\left(\delta_s\right) = -20\log_{10}(0.08) = 21.9382\ \text{dB}.$

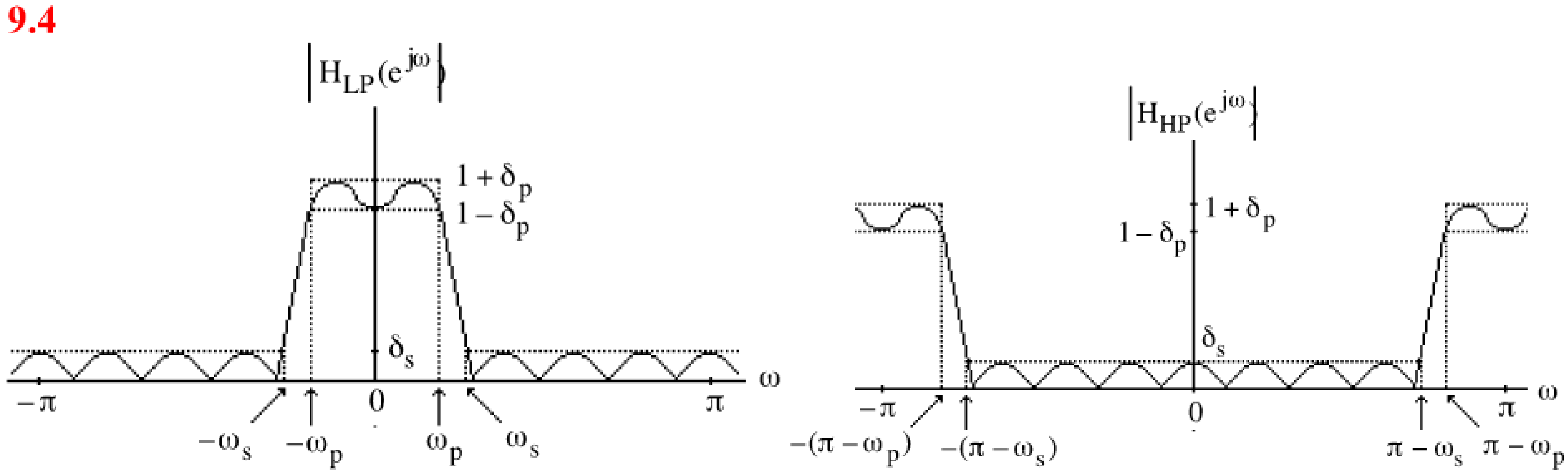
(b) $\alpha_p = -20\log_{10}\left(1-\delta_p\right) = -20\log_{10}(1-0.015) = 0.1313\,\text{dB},$

$\alpha_s = -20\log_{10}\left(\delta_s\right) = -20\log_{10}(0.04) = 27.9588\ \text{dB}.$

9.3 $G(z) = H^2(z)$, or equivalently, $G(e^{j\omega}) = H^2(e^{j\omega})$. $\left|G(e^{j\omega})\right| = \left|H^2(e^{j\omega})\right| = \left|H(e^{j\omega})\right|^2$.

Let δ_p and δ_s denote the passband and stopband ripples of $H(e^{j\omega})$, respectively. Also, let $\delta_{p,2} = 2\delta_p,$ and $\delta_{s,2}$ denote the passband and stopband ripples of $G(e^{j\omega})$, respectively. Then $\delta_{p,2} = 1-(1-\delta_p)^2$, and $\delta_{s,2} = (\delta_s)^2$. For a cascade of M sections, $\delta_{p,M} = 1-(1-\delta_p)^M$, and $\delta_{s,M} = (\delta_s)^M$.

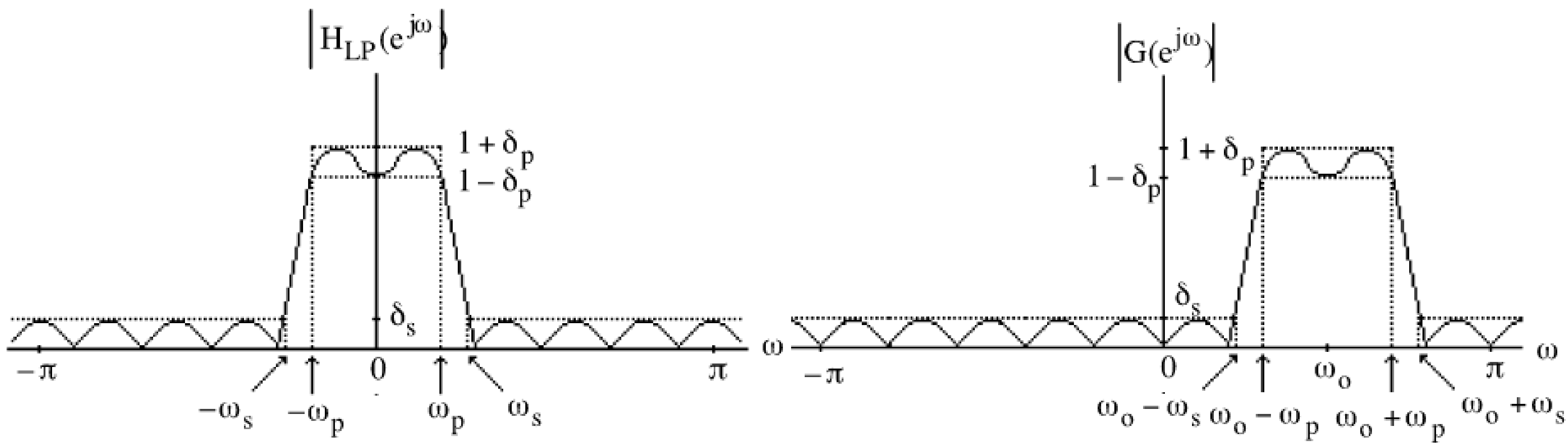
9.4



Therefore, the passband edge and the stopband edge of the highpass filter are given by $\omega_{p,HP} = \pi - \omega_p$, and $\omega_{s,HP} = \pi - \omega_s$, respectively.

9.5 Note that $G(z)$ is a complex bandpass filter with a passband in the range $0 \le \omega \le \pi$. Its passband edges are at $\omega_{p,BP} = \omega_o \pm \omega_p$, and stopband edges at $\omega_{s,BP} = \omega_o \pm \omega_s$. A real coefficient bandpass transfer function can be generated according to $G_{BP}(z) = H_{LP}(e^{j\omega_o}z) + H_{LP}(e^{-j\omega_o}z)$ which will have a passband in the range $0 \le \omega \le \pi$

and another passband in the range $-\pi \le \omega \le 0$. However because of the overlap of the two spectra a simple formula for the bandedges cannot be derived.



9.6 **(a)** $h_p(t) = h_a(t)\cdot p(t)$ where $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$. Thus, $h_p(t) = \sum_{n=-\infty}^{\infty} h_a(nT)\delta(t - nT)$..

We also have, $g[n] = h_a(nT)$. Now, $H_a(s) = \int_{-\infty}^{\infty} h_a(t)e^{-st}dt$ and

$$H_p(s) = \int_{-\infty}^{\infty} h_p(t)e^{-st}dt = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} h_a(nT)\delta(t - nT)e^{-st}dt = \sum_{n=-\infty}^{\infty} h_a(nT)e^{-snT}.$$

Comparing the above expression with $G(z) = \sum_{n=-\infty}^{\infty} g[n]z^{-n} = \sum_{n=-\infty}^{\infty} h(nT)z^{-n}$, we conclude that $G(z) = H_p(s)\Big|_{s=\frac{1}{T}\ln z}$.

We can also show that a Fourier series expansion of $p(t)$ is given by

$p(t) = \dfrac{1}{T}\sum_{k=-\infty}^{\infty} e^{-j(2\pi kt/T)}$. Therefore,

$h_p(t) = \left(\dfrac{1}{T}\sum_{k=-\infty}^{\infty} e^{-j(2\pi kt/T)}\right)h_a(t) = \dfrac{1}{T}\sum_{k=-\infty}^{\infty} h_a(t)e^{-j(2\pi kt/T)}$. Hence,

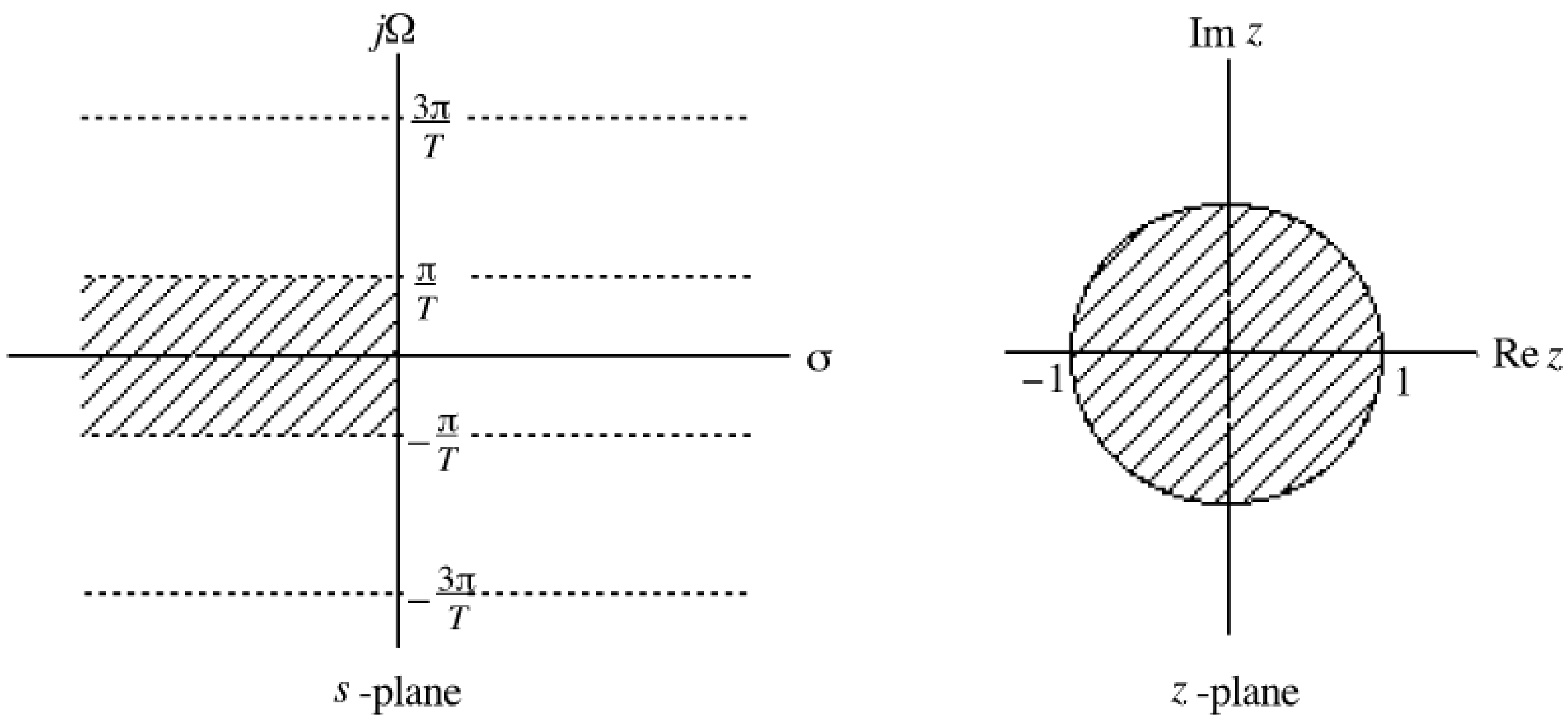
$H_p(s) = \dfrac{1}{T}\sum_{k=-\infty}^{\infty} H_a\left(s + j\dfrac{2\pi kt}{T}\right)$. As a result, we have

$$G(z) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H_a\left(s + j\frac{2\pi kt}{T}\right)\Bigg|_{s=\frac{1}{T}} \ln z. \qquad (7\text{-}1)$$

(b) The transformation from the s-plane to z-plane is given by $z = e^{sT}$. If we express $s = \sigma_o + j\Omega_o$, then we can write $z = re^{j\omega} = e^{\sigma_o T}e^{j\Omega_o T}$. Therefore,

$$|z| = \begin{cases} <1, & \text{for } \sigma_o < 1, \\ =1, & \text{for } \sigma_o = 1, \\ >1, & \text{for } \sigma_o > 1. \end{cases}$$ Or in other words, a point in the left-half s-plane is mapped onto a point inside the unit circle in the z-plane, a point in the right-half s-plane is mapped onto a point outside the unit circle in the z-plane, and a point on the $j\omega$-axis in the s-plane is mapped onto a point on the unit circle in the z-plane. As a result, the mapping has the desirable properties enumerated in Section 9.1.3.

(c) However, all points in the s-plane defined by $s = \sigma_o + j\Omega_o \pm j\frac{2\pi k}{T}$, $k = 0, 1, 2, \ldots,$ a are mapped onto a single point in the z-plane as $z = e^{\sigma_o T} e^{j\left(\Omega_o \pm \frac{2\pi k}{T}\right)T} = e^{\sigma_o T} e^{j\Omega_o T}$. The mapping is illustrated in the figure below



Note that the strip of width $2\pi/T$ in the s-plane for values of s in the range $-\frac{\pi}{T} \le \Omega \le \frac{\pi}{T}$ is mapped into the entire z-plane, and so are the adjacent strips of width $2\pi/T$. The mapping is many-to-one with infinite number of such strips of width $2\pi/T$.

It follows from the above figure and also from Eq. (7-1) that if the frequency response $H_a(j\Omega) = 0$ for $|\Omega| \ge \frac{\pi}{T}$, then $G(e^{j\omega}) = \frac{1}{T} H_a(j\frac{\omega}{T})$ for $|\omega| \le \pi$, and there is no aliasing.

(d) For $z = e^{j\omega} = e^{j\Omega T}$, or equivalently, $\omega = \Omega T$.

9.7 Assume $h_a(t)$ is causal. Now, $h_a(t) = \oint H_a(s) e^{st} ds$. Hence, $g[n] = h_a(nT) = \oint H_a(s) e^{snT} ds$. Therefore,

$$G(z)=\sum_{n=0}^{\infty} g[n]z^{-n}=\sum_{n=0}^{\infty}\oint H_a(s)e^{snT}z^{-n}ds=\oint H_a(s)\sum_{n=0}^{\infty} z^{-n}e^{snT}ds=\oint \frac{H_a(s)}{1-e^{sT}z^{-1}}ds.$$

Hence $G(z)=\sum_{\text{all poles of } H_a(s)} \text{Residues}\left[\frac{H_a(s)}{1-e^{sT}z^{-1}}\right]$.

9.8 $H_a(s)=\dfrac{A}{s+\alpha}$. The transfer function has a pole at $s=-\alpha$. Now

$$G(z)=\underset{\text{at } s=-\alpha}{\text{Residue}}\left[\frac{A}{(s+\alpha)(1-e^{sT}z^{-1})}\right]=\left.\frac{A}{1-e^{sT}z^{-1}}\right|_{s=-\alpha}=\frac{A}{1-e^{-\alpha T}z^{-1}}.$$

9.9 **(a)** $H_a(s)=\dfrac{2(s+2)}{(s+3)(s^2+4s+5)}=\dfrac{-1}{s+3}+\dfrac{0.5-0.5j}{(s+2-j)}+\dfrac{0.5+0.5j}{(s+2+j)}$

$$=\frac{-1}{s+3}+\frac{s+3}{(s+2)^2+1^2}=\frac{-1}{s+3}+\frac{s+2}{(s+2)^2+1^2}+\frac{1}{(s+2)^2+1^2}.$$

Using Eq (9.71), we get

$$G_a(z)=\frac{-1}{1-e^{-3T}z^{-1}}+\frac{1-z^{-1}e^{-2T}\cos(T)}{1-2z^{-1}e^{-2T}\cos(T)+e^{-4T}z^{-2}}+\frac{z^{-1}e^{-2T}\sin(T)}{1-2z^{-1}e^{-2T}\cos(T)+e^{-4T}z^{-2}}.$$

Since $T=0.25$, we get

$$G_a(z)=\frac{-1}{1-0.4724z^{-1}}+\frac{1-0.4376z^{-1}}{1-1.1754z^{-1}+0.3679z^{-2}}..$$

(b) $H_b(s)=\dfrac{2s^2+s-1}{(s+4)(s^2+2s+10)}=\dfrac{1.5}{s+4}+\dfrac{0.25+0.75j}{(s+1-3j)}+\dfrac{0.25-0.75j}{(s+1+3j)}$

$$=\frac{1.5}{s+4}+0.5\frac{s-8}{(s+1)^2+3^2}=\frac{-1}{s+3}+0.5\frac{s+1}{(s+1)^2+3^2}-0.5(3)\frac{3}{(s+1)^2+3^2}.$$

Using Eq (9.71), we get

$$G_b(z)=\frac{1.5}{1-e^{-4T}z^{-1}}+0.5\frac{1-z^{-1}e^{-T}\cos(3T)}{1-2z^{-1}e^{-T}\cos(3T)+e^{-2T}z^{-2}}-1.5\frac{z^{-1}e^{-T}\sin(3T)}{1-2z^{-1}e^{-T}\cos(3T)+e^{-2T}z^{-2}}.$$

Since $T=0.25$, we get

$$G_b(z)=\frac{1.5}{1-0.3679z^{-1}}+\frac{0.5\left(1-2.1624z^{-1}\right)}{1-1.1397z^{-1}+0.6065z^{-2}}..$$

(c) $H_c(s)=\dfrac{-s^2+2s+11}{(s^2+2s+5)(s^2+s+4)}$

$$=\frac{1.5+j}{(s+1-2j)}+\frac{1.5-j}{(s+1+2j)}+\frac{-1.5-0.3\sqrt{15}j}{(s+0.5-0.5\sqrt{15}j)}+\frac{-1.5+0.3\sqrt{15}j}{(s+0.5+0.5\sqrt{15}j)}$$

$$= 3\frac{s-1/3}{(s+1)^2+4} - 3\frac{s-1}{(s+0.5)^2+\left(0.5\sqrt{15}\right)^2}$$

$$= 3\frac{s+1}{(s+1)^2+2^2} + 3(-2/3)\frac{2}{(s+1)^2+2^2} - 3\frac{s+0.5}{(s+0.5)^2+\left(0.5\sqrt{15}\right)^2} - 3\left(-3/\sqrt{15}\right)\frac{0.5\sqrt{15}}{(s+0.5)^2+\left(0.5\sqrt{15}\right)^2}$$

Using Eq (9.71), we get

$$G_c(z) = 3\frac{1-z^{-1}e^{-T}\cos(2T)}{1-2z^{-1}e^{-T}\cos(2T)+e^{-2T}z^{-2}} - 2\frac{z^{-1}e^{-T}\sin(2T)}{1-2z^{-1}e^{-T}\cos(2T)+e^{-2T}z^{-2}}.$$

Since $T = 0.25$, we get

$$G_c(z) = \frac{3\left(1-0.9324z^{-1}\right)}{1-1.3669z^{-1}+0.6065z^{-2}} - \frac{3\left(1-0.4629z^{-1}\right)}{1-1.5622z^{-1}+0.7788z^{-2}}..$$

9.10 **(a)** $G_a(z) = \dfrac{2z}{z-e^{-1.3}} + \dfrac{5z}{z-e^{-2.0}} = \dfrac{A_1 z}{z-e^{-\alpha_1 T}} + \dfrac{A_2 z}{z-e^{-\alpha_2 T}}.$

Since $T = 0.5,\ \alpha_1 = 2.6,\ \alpha_2 = 4,\ A_1 = 2,\ A_2 = 5$, it follows $H_a(s) = \dfrac{2}{s+2.6} + \dfrac{5}{s+4}.$

(b) $G_b(z) = \dfrac{ze^{-1.4}\sin(1.6)}{z^2-2ze^{-1.4}\cos(1.6)+e^{-2.6}} = \dfrac{ze^{-\beta T}\sin(\lambda T)}{z^2-2ze^{-\beta T}\cos(\lambda T)+e^{-2\beta T}}.$

Since $T = 0.5,\ \lambda = 3.2,\ \beta = 2.6$, it follows $H_b(s) = \dfrac{3.2}{(s+2.6)^2+3.2^2}.$

9.11 **(a)** $H_a(s) = G_a(z)\Big|_{z=4\left(\frac{1+s}{1-s}\right)} = \dfrac{4\left(5s^2+18s+9\right)}{75s^2+154s+91}.$

(b) $H_b(s) = G_b(z)\Big|_{z=4\left(\frac{1+s}{1-s}\right)} = \dfrac{105s^3+385s^2+467s+195}{(13s+11)\left(27s^2+46s+23\right)}.$

9.12 For the impulse invariance design:

$$\omega_p = \Omega_p T = 2\pi F_p T = 2\pi\left(0.88\times 10^3\right)\left(0.25\times 10^{-3}\right) = 0.44\pi.$$

For the bilinear transformation method:

$$\omega_p = 2\tan^{-1}\left(\frac{\Omega_p T}{2}\right) = 2\tan^{-1}\left(F_p T\pi\right) = 2\tan^{-1}\left(0.88\times 10^3\cdot 0.25\times 10^{-3}\cdot \pi\right) = 0.385\pi.$$

9.13 For the impulse invariance method: $2\pi F_p = \dfrac{\omega_p}{T} = \dfrac{0.45\pi}{0.4\times 10^{-3}} \Rightarrow F_p = 562.5$ Hz.

For the bilinear transformation method:

$$F_p = \tan\left(\frac{\omega_p}{2}\right)\cdot\frac{1}{\pi T} = \tan\left(\frac{0.45\pi}{2}\right)\cdot\frac{1}{\pi\left(0.4\times 10^{-3}\right)} = 679.7 \text{ Hz}.$$

9.14 The passband and the stopband edges of the analog lowpass filter are assumed to $\Omega_p = 0.25\pi$ and $\Omega_s = 0.55\pi$. The requirements to be satisfied by the analog lowpass filter are thus $20\log_{10}|H_a(j0.25\pi)| \geq -0.5$ dB and $20\log_{10}|H_a(j0.55\pi)| \leq -15$ dB.

From $\alpha_p = 20\log_{10}(\sqrt{1+\varepsilon^2}) = 0.5$ we obtain $\varepsilon^2 = 0.1220184543$. From $\alpha_s = 10\log_{10}(A^2) = 15$ we obtain $A^2 = 31.6227766$. From Eq. (A.6), the inverse discrimination ratio is given by $\dfrac{1}{k_1} = \dfrac{\sqrt{A^2-1}}{\varepsilon} = 15.841979$ and from Eq. (A.5) the inverse transition ratio is given by $\dfrac{1}{k} = \dfrac{\Omega_s}{\Omega_p} = 2.2$. Substituting these values in Eq. (A.9) we obtain

$$N = \frac{\log_{10}(1/k_1)}{\log_{10}(1/k)} = \frac{\log_{10}(15.841979)}{\log_{10}(2.2)} = 3.503885.$$ We choose $N = 4$.

From Eq. (A.7) we have $\left(\dfrac{\Omega_p}{\Omega_c}\right)^{2N} = \varepsilon^2$. Substituting the values of Ω_p, N, and ε^2 we get $\Omega_c = 1.3007568(\Omega_p) = 1.021612$.

Using the statement `[z,p,k] = buttap(4)` we get the poles of the 4-th order Butterworth analog filter with a 3-dB cutoff at 1 rad/s as $p_1 = -0.3827 + j0.9239$, $p_2 = -0.3827 - j0.9239$, $p_3 = -0.9239 + j0.3827$, and $p_4 = -0.9239 - j0.3827$. Therefore,

$$H_{an}(s) = \frac{1}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)} = \frac{1}{(s^2+0.7654\,s+1)(s^2+1.8478\,s+1)}.$$

Next we expand $H_{an}(s)$ in a partial-fraction expansion using the M-file `residue` and arrive at $H_{an}(s) = \dfrac{-0.9238729\,s - 0.7071323}{s^2+0.7654\,s+1} + \dfrac{0.9238729\,s+1.7071323}{s^2+1.8478\,s+1}$. We next denormalize $H_{an}(s)$ to move the 3-dB cutoff frequency to $\Omega_c = 1.021612$ using the M-file `lp2lp` resulting in $H_a(s) = H_{an}\left(\dfrac{s}{1.021612}\right)$

$$= \frac{-0.943847\,s - 0.738039}{s^2+0.781947948\,s+1.0437074244} + \frac{0.943847\,s+1.78174665}{s^2+1.887749436\,s+1.0437074244}$$

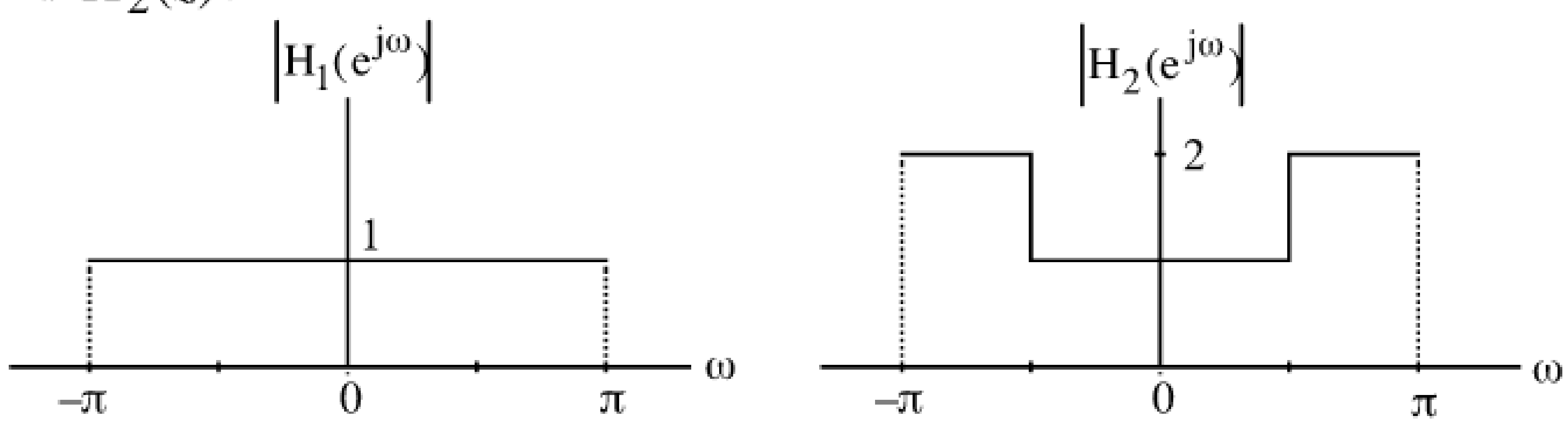
$$= \frac{-0.943847\,s - 0.738039}{(s+0.390974)^2+(0.9438467)^2} + \frac{0.943847\,s+1.78174665}{(s+0.94387471)^2+(0.39090656)^2}.$$

Making use of the M-file `bilinear` we finally arrive at

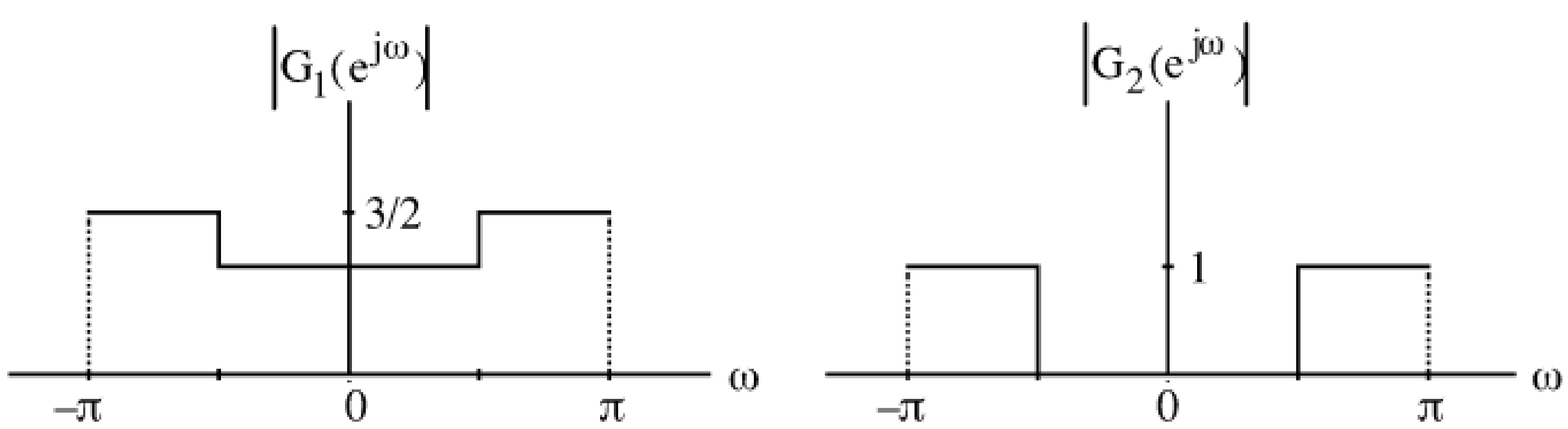
$$G(z) = \frac{-0.943847\,z^2 + 0.68178386\,z}{z^2 - 1.363567724\,z + 0.4575139} + \frac{0.943847\,z^2 - 0.25640047\,z}{z^2 - 0.77823439\,z + 0.1514122}.$$

9.15 The mapping is given by $s = \frac{1}{T}(1 - z^{-1})$ or equivalently, by $z = \frac{1}{1 - sT}$. For $s = \sigma_o + j\Omega_o$, $z = \frac{1}{1 - \sigma_o T - j\Omega_o T}$. Therefore, $|z|^2 = \frac{1}{(1 - \sigma_o T)^2 + (\Omega_o T)^2}$. Hence, $|z| < 1$ for $\sigma_o < 0$. As a result, a stable $H_a(s)$ results in a stable $H(z)$ after the transformation. However, for $\sigma_o = 0, |z|^2 = \frac{1}{1 + (\Omega_o T)^2}$ which is equal to 1 only for $\Omega_o = 0$. Hence, only the point $\Omega_o = 0$ on the $j\Omega$-axis in the s-plane is mapped onto the point $z = 1$ on the unit circle. Consequently, this mapping is not useful for the design of digital filters via analog filter transformation.

9.16 For no aliasing $T \le \frac{\pi}{\Omega_c}$. Figure below shows the magnitude responses of the digital filters $H_1(z)$ and $H_2(z)$.



(a) The magnitude responses of the digital filters $G_1(z)$ and $G_2(z)$ are shown below:



(b) As can be seen from the above $G_1(z)$ is a multi-passband filter, whereas, $G_2(z)$ is a highpass filter.

9.17 $H_a(s)$ is causal and stable and $|H_a(s)| \le 1, \forall s$, Now, $G(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1-z^{-1}}\right)}$. Thus, $G(z)$ is causal and stable. Now,

$$G(e^{j\omega}) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-e^{j\omega}}{1+e^{j\omega}}\right)} = H_a(s)\Big|_{s=j\frac{2}{T}\tan(\omega/2)} = H_a(j\frac{2}{T}\tan(\omega/2)).$$

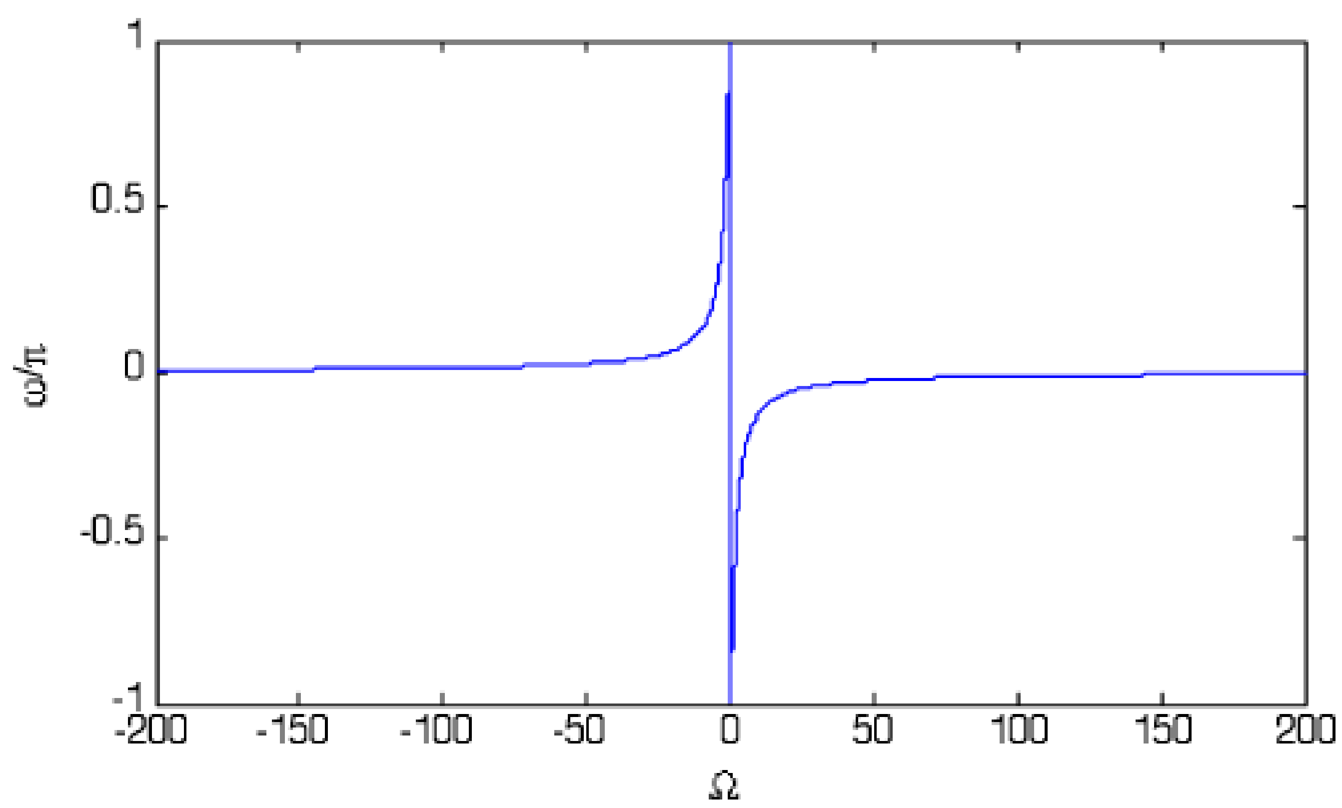
Therefore, $\left|G(e^{j\omega})\right| = \left|H_a(j\frac{2}{T}\tan(\omega/2)\right| \le 1$ for all values of ω. Hence, $G(z)$ is a BR function.

9.18 **(a)** $s = \dfrac{2}{T}\left(\dfrac{1+z^{-1}}{1-z^{-1}}\right) \Rightarrow z^{-1} = \dfrac{s-2/T}{s+2/T}$, or $z = \dfrac{s+2/T}{s-2/T} = \dfrac{(\sigma+2/T)+j\Omega}{(\sigma-2/T)+j\Omega}$.

(b) $|z|^2 = \dfrac{(\sigma+2/T)^2+\Omega^2}{(\sigma-2/T)^2+\Omega^2}$, so $|z|^2\Big|_{\sigma+j\Omega,\sigma=0} = 1$ and $|z|^2\Big|_{s=\sigma+j\Omega,\sigma<0} < 1$. The first proves that a point on the $j\Omega$ axis is mapped to a point on the unit circle, and the second proves that a point in the left-half s-plane is mapped to a point inside the unit circle (stability is preserved). This mapping does indeed have all the desirable properties.

(c) If $s(f_1(z))$ is the bilinear transformation in Eq. (9.14) and $s(f_2(z))$ is the bilinear transformation in Eq. (9.72), then $s(f_2(z)) = s(-f_1(z))$.

(d) $j\Omega = \dfrac{2}{T}\dfrac{1+e^{-j\omega}}{1-e^{-j\omega}} \Rightarrow \omega = -2\cot^{-1}\left(\dfrac{\Omega T}{2}\right)$.



(e) $G(z)$ is a high-pass filter because small Ω (large ω) frequencies passed while large Ω (small ω) frequencies are attenuated.

9.19 $G(z) = H_{LP}(s)\Big|_{s=\frac{2}{T}\frac{1+z^{-1}}{1-z^{-1}}} = \dfrac{1-\alpha}{2}\left(\dfrac{1-z^{-1}}{1+\alpha z^{-1}}\right)$, where α is defined in Eq. (9.25). $G(z)$ is a high-pass filter. $G(z) = G_{LP}(-z)$.G(z)=G_{LP}(−z If β = −α, then

$$G(z) = \frac{1+\beta}{2}\left(\frac{1-z^{-1}}{1-\beta z^{-1}}\right) = G_{HP}(z).$$

9.20 $G(z) = H_{HP}(s)\Big|_{s=\frac{2}{T}\frac{1+z^{-1}}{1-z^{-1}}} = \frac{1+\alpha}{2}\left(\frac{1+z^{-1}}{1+\alpha z^{-1}}\right)$, where α is defined in Eq. (9.25).

G(z) is a low-pass filter. $G(z) = G_{LP}(-z)$. If $\beta = -\alpha$, then

$G(z) = \frac{1-\beta}{2}\left(\frac{1+z^{-1}}{1-\beta z^{-1}}\right) = G_{LP}(z)$..

9.21 Let $y(t) = \frac{d}{dt}x(t)$ be approximated by $y(nT) \approx \frac{x(nT)-x(nT-T)}{T}$. Then

$Y(z) = \frac{1-z^{-1}}{T}X(z)$, which suggests the mapping $s = \frac{1-z^{-1}}{T}$, or

$z = \frac{1}{1-Ts} = \frac{1}{(1-T\sigma)-j(T\Omega)}$. $|z|^2 = \frac{1}{(1-T\sigma)^2+(T\Omega)^2}$, so

$z\big|_{\sigma+j\Omega,\sigma=0} \Rightarrow |z-0.5| = 0.5$ and $|z|^2\Big|_{s=\sigma+j\Omega,\sigma<0} < 1$. This the imaginary axis in the s-domain is mapped to the circle of radius 0.5 centered at $z = 0.5$, but the transformation preserves stability. An analog high-pass filter cannot be mapped to a digital high-pass filter because the poles of the digital filter do not lie in the left-half of the z-plane.

9.22 $G(z) = \frac{1+\alpha}{2}\cdot\frac{1-2\beta z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}$. For $\beta = \cos\omega_o$, the numerator of $G(z)$ becomes

$1-2\cos(\omega_o)z^{-1}+z^{-2} = (1-e^{j\omega_o}z^{-1})(1-e^{-j\omega_o}z^{-1})$ which has roots at $z = e^{\pm j\omega_o}$. The numerator of $G(z^N)$ is then given by $(1-e^{j\omega_o}z^{-N})(1-e^{-j\omega_o}z^{-N})$ whose roots are obtained by solving the equation $z^N = e^{\pm j\omega_o}$, and are given by $z = e^{j(2\pi n \pm \omega_o)/N}$, $0 \le n \le N-1$. Hence $G(z^N)$ has N complex conjugate zero-pairs located on the unit circle at angles of $\frac{2\pi n \pm \omega_o}{N}$ radians, $0 \le n \le N-1$. For $\omega_o = \pi/2$,

there are $2N$ equally spaced zeros on the unit circle starting at $\omega = \pi/2N$.

9.23 Let $H_a(s) = \frac{a_0+a_1s+a_2s^2+\cdots+a_Ns^N}{b_0+b_1s+b_2s^2+\cdots+b_Ns^N}$ denote the analog transfer function and

$G(z) = \frac{q_0+q_1z^{-1}+q_2z^{-2}+\cdots+q_Nz^{-N}}{d_0+d_1z^{-1}+d_2z^{-2}+\cdots+d_Nz^{-N}}$ denote the digital transfer function

obtained after applying the bilinear transformation $s = c\frac{z-1}{z+1}$ where $c = 2/T$. All transfer function coefficients are assumed to be real numbers. The numerator and the denominator coefficients of $H_a(s)$ and $G(z)$ can be represented in vector form as shown below:

$\mathbf{A} = [a_0,\ a_1,\ a_2,\ \ldots,\ a_N]$, $\mathbf{B} = [b_0,\ b_1,\ b_2,\ \ldots,\ b_N]$,

$\mathbf{Q} = [q_0, \ \ q_1, \ \ q_2, \ \ \ldots, \ \ q_N], \ \ \mathbf{D} = [d_0, \ \ d_1, \ \ d_2, \ \ \ldots, \ \ d_N].$

To determine the elements of **Q** and **D** from **A** and **B**, we compare the numerator and denominator coefficients of both transfer functions. It can be shown that for $N = 2$, we have $G(z) = \dfrac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{d_0 + d_1 z^{-1} + d_2 z^{-2}}$

$$= \frac{(a_0 + a_1 c + a_2 c^2) + (2a_0 - 2a_2 c^2) z^{-1} + (a_0 - a_1 c + a_2 c^2) z^{-2}}{(b_0 + b_1 c + b_2 c^2) + (2b_0 - 2b_2 c^2) z^{-1} + (b_0 - b_1 c + b_2 c^2) z^{-2}}.$$

By comparing the coefficients of like powers of z^{-1} we get

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 c \\ a_2 c^2 \end{bmatrix}.$$

A similar relation between the elements of **B** and **D** can be derived. In the general case, these matrix equations can be compactly represented in the form $\mathbf{Q} = \mathbf{P}_N \mathbf{A}_c$ and $\mathbf{D} = \mathbf{P}_N \mathbf{B}_c$ where $\mathbf{P}_N$ is the Pascal matrix of order $N+1$ and $\mathbf{A}_c = \left[a_0, \ \ a_1 c, \ \ a_2 c^2, \ \ \ldots, \ \ a_N c^N\right]$ and $\mathbf{B}_c = \left[b_0, \ \ b_1 c, \ \ b_2 c^2, \ \ \ldots, \ \ b_N c^N\right].$

In the general case, the elements of $\mathbf{P}_N$ are given as follows:

(1) The elements of the first row are all ones,

(2) The elements of the last column are given by

$$p_{i,N+1} = (-1)^{i-1} \frac{n!}{(n-i+1)!(i-1)!}, \ \ 1 \le i \le N+1,$$

(3) The remaining elements are given by

$$p_{i,j} = p_{i-1,j} + p_{i-1,j+1} + p_{i,j+1},$$

where $2 \le i \le N+1, \ \ N \ge j \ge 1$.

9.24 (a) $H(z) = \frac{1}{2}[1 + A_4(z)] = \dfrac{N(z)}{D(z)}$. We can write $A_4(z) = \dfrac{z^{-2} D_1(z^{-1})}{D_1(z)} \cdot \dfrac{z^{-2} D_2(z^{-1})}{D_2(z)}$, where

$D_1(z) = 1 - \beta_1(1 + \alpha_1) z^{-1} + \alpha_1 z^{-2}$ and $D_2(z) = 1 - \beta_2(1 + \alpha_2) z^{-1} + \alpha_2 z^{-2}$. Therefore,

$N(z) = \frac{1}{2}\left[D_1(z) D_2(z) + z^{-4} D_1(z^{-1}) D_2(z^{-1})\right]$. Now,

$z^{-4} N(z^{-1}) = \frac{1}{2}\left[z^{-4} D_1(z^{-1}) D_2(z^{-1}) + D_1(z) D_2(z)\right] = N(z)$. Hence, $N(z)$ is a symmetric polynomial. It follows then

$$P(z) = \frac{1}{2}[\left(\alpha_1 - \beta_1(1+\alpha_1) z^{-1} + z^{-2}\right)\left(\alpha_2 - \beta_2(1+\alpha_2) z^{-1} + z^{-2}\right)$$
$$+ \left(1 - \beta_1(1+\alpha_1) z^{-1} + \alpha_1 z^{-2}\right)\left(1 - \beta_2(1+\alpha_2) z^{-1} + z\alpha_2{}^{-2}\right)]$$

$$= \frac{1+\alpha_1\alpha_2}{2}\Big[1 - \frac{(1+\alpha_1)(1+\alpha_2)(\beta_1+\beta_2)}{1+\alpha_1\alpha_2} z^{-1} + \frac{2[\alpha_1+\alpha_2+\beta_1\beta_2(1+\alpha_1)(1+\alpha_2)]}{1+\alpha_1\alpha_2} z^{-2}$$
$$- \frac{(1+\alpha_1)(1+\alpha_2)(\beta_1+\beta_2)}{1+\alpha_1\alpha_2} z^{-3} + z^{-4}\Big] = a(1+b_1 z^{-1} + b_2 z^{-2} + b_1 z^{-3} + z^{-3}),$$

where
$$b_1 = -\frac{(1+\alpha_1)(1+\alpha_2)(\beta_1+\beta_2)}{1+\alpha_1\alpha_2}, \qquad \text{(7-a)}$$
$$b_2 = \frac{2[\alpha_1+\alpha_2+\beta_1\beta_2(1+\alpha_1)(1+\alpha_2)]}{1+\alpha_1\alpha_2}, \qquad \text{(7-b)}$$

(b) $a = \dfrac{1+\alpha_1\alpha_2}{2}$. (7-c)

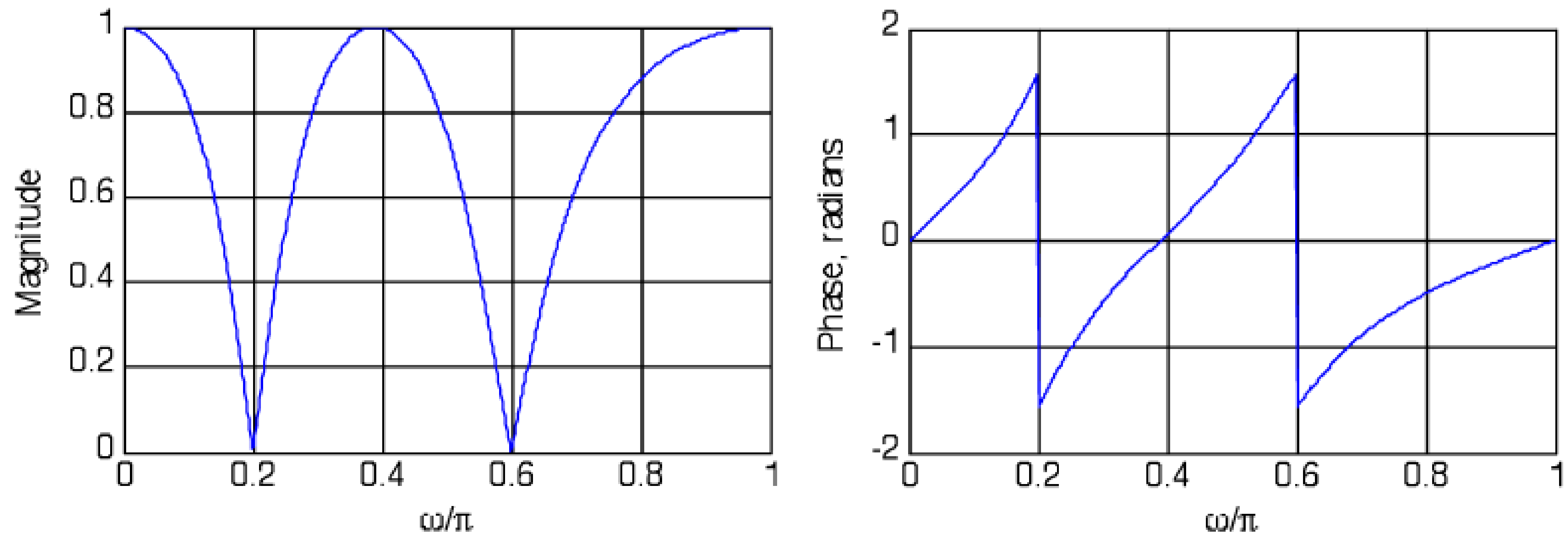
(c) for $z = e^{j\omega}$, we can write $N(e^{j\omega}) = a(1+b_1 e^{-j\omega} + b_2 e^{-j2\omega} + b_1 e^{-j3\omega} + e^{-j4\omega})$ $= ae^{-j2\omega}(b_2 + 2b_1\cos\omega + 2\cos 2\omega)$. Now, $N(e^{j\omega}) = 0$ for $i = 1,\ 2$. For $i = 1$, we get
$$b_2 + 2b_1\cos\omega_1 + 2\cos 2\omega_1 = 0, \qquad \text{(7-d)}$$

for $i = 2$, we get $b_2 + 2b_1\cos\omega_2 + 2\cos 2\omega_2 = 0$, (7-e)

Solving Eqs. (7-d) and (7-e) we get $b_1 = -2(\cos\omega_1 + \cos\omega_2)$, (7-f)
and $b_2 = 2(2\cos\omega_1\cos\omega_2 + 1)$. (7-g)

From Eqs. (7-a) and (7-f) we have $\dfrac{(1+\alpha_1)(1+\alpha_2)(\beta_1+\beta_1)}{1+\alpha_1\alpha_2} = 2(\cos\omega_1 + \cos\omega_2)$, (7-h)
and from Eqs. (7-b) and (7-g) we have
$$\frac{2[\alpha_1+\alpha_2+\beta_1\beta_2(1+\alpha_1)(1+\alpha_2)]}{1+\alpha_1\alpha_2} = 2(2\cos\omega_1\cos\omega_2 + 1). \qquad \text{(7-i)}$$
Substituting $\alpha_1 = \dfrac{1-\tan(B_1/2)}{1+\tan(B_1/2)}$ and $\alpha_2 = \dfrac{1-\tan(B_2/2)}{1+\tan(B_2/2)}$, and after rearrangement we get
$$\beta_1 + \beta_2 = (\cos\omega_1 + \cos\omega_2)[1+\tan(B_1/2)\tan(B_2/2)] \stackrel{\Delta}{=} \theta_1, \qquad \text{(7-j)}$$
and $\beta_1\beta_2 = [1+\tan(B_1/2)\tan(B_2/2)]\cos\omega_1\cos\omega_2 \stackrel{\Delta}{=} \theta_2$. (7-k) The above two nonlinear equations can be solved yielding $\beta_1 = \dfrac{\theta_1 \pm \sqrt{\theta_1^2 - 4\theta_2}}{2}$ and $\beta_2 = \dfrac{\theta_2}{\theta_1}$.

(d) For the double notch filter with the following specifications: $\omega_1 = 0.2\pi$, $\omega_2 = 0.6\pi$, $B_1 = 0.2\pi$, and $B_2 = 0.25\pi$ we get the following values for the parameters of the notch filter transfer function:
$\alpha_1 = 0.5095$, $\alpha_2 = 0.4142$, $\theta_1 = 0.5673$, $\theta_2 = -0.1491$, $\beta_1 = 0.7628$, and $\beta_2 = -0.1955$. $H(z) = \frac{1}{2}\left[1 + A_4(z)\right]$:



9.25 A zero (pole) of $H_{LP}(z)$ is given by the factor $(z - z_k)$. After applying the lowpass-to-lowpass transformation, this factor becomes $\dfrac{\hat{z} - \alpha}{1 - \alpha\hat{z}} - z_k$, and hence the new location of the zero (pole) is given by the roots of the equation

$\hat{z} - \alpha - z_k + \alpha z_k \hat{z} = (1 + \alpha z_k)\hat{z} - (\alpha + z_k) = 0$ or $\hat{z}_k = \dfrac{\alpha + z_k}{1 + \alpha z_k}$. For $z_k = -1$,

$\hat{z}_k = \dfrac{a - 1}{1 - a} = -1.$

9.26 The lowpass-to-bandpass transformation is given by $z \rightarrow \dfrac{-b + a\hat{z} - \hat{z}^2}{1 - a\hat{z} + b\hat{z}^2}$ where $a = \dfrac{2\alpha\beta}{\beta + 1}$ and $b = \dfrac{\beta - 1}{\beta + 1}$. A zero (pole) of $H_{LP}(z)$ is given by the factor $(z - z_k)$. After applying the lowpass-to-bandpass transformation, this factor becomes $\dfrac{-b + a\hat{z} - \hat{z}^2}{1 - a\hat{z} + b\hat{z}^2} - z_k$, and hence, the new location of the zero (pole) of the bandpass transfer function is given by the roots of the equation $(1 + bz_k)z^2 - a(1 + z_k)z + (b + z_k) = 0$, or $z^2 - \dfrac{a(1 + z_k)}{1 + bz_k}z + \dfrac{b + z_k}{1 + bz_k} = 0$, whose solution is given by $\hat{z}_k = \dfrac{a(1 + z_k)}{2(1 + bz_k)} \pm \sqrt{\left[\dfrac{a(1 + z_k)}{2(1 + bz_k)}\right]^2 - \left(\dfrac{b + z_k}{1 + bz_k}\right)}$. For $z_k = -1$, $\hat{z}_k = \pm 1$.

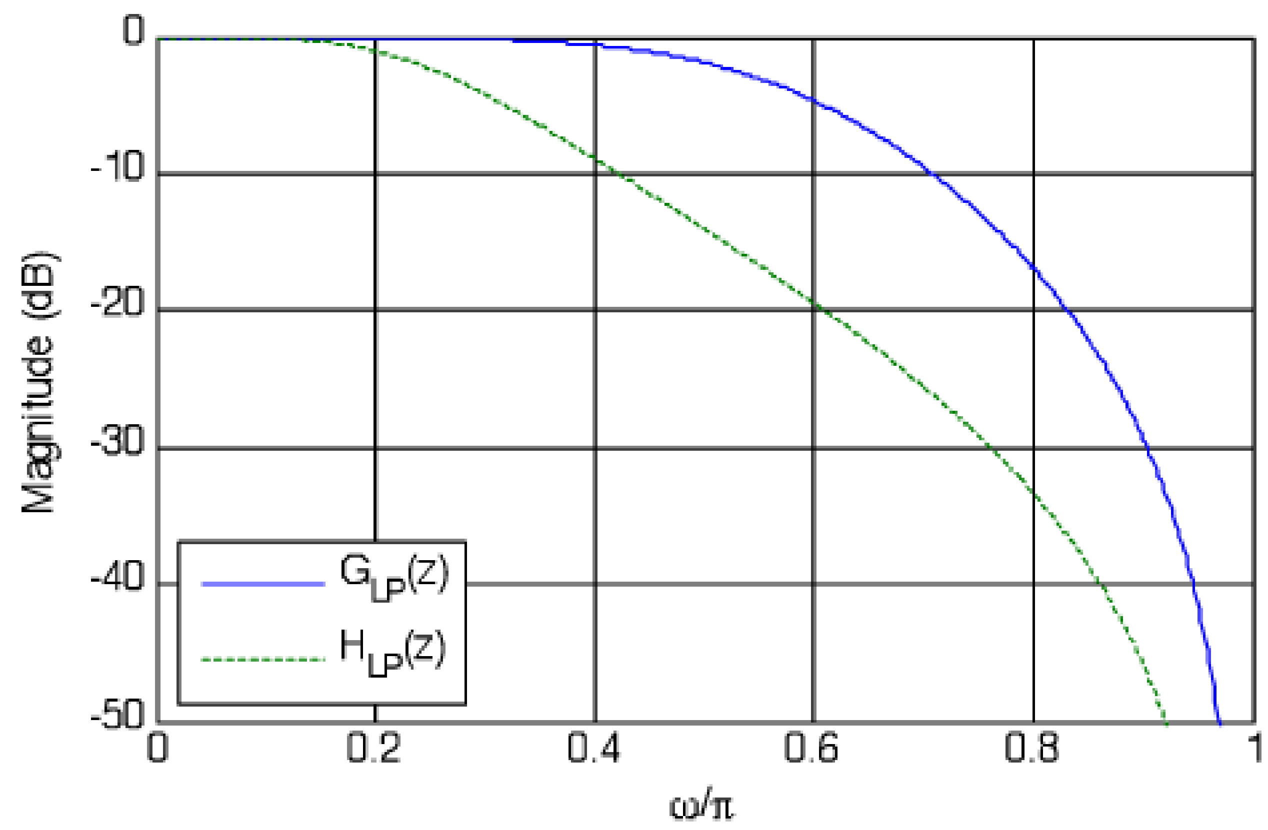
9.27 $G_{LP}(z) = \dfrac{0.3404\left(1 + z^{-1}\right)^2}{1 + 0.1842z^{-1} + 0.1776z^{-2}}$, with $\omega_c = 0.55\pi$.

$H_{LP}(z)$ for $\hat{\omega}_c = 0.27\pi$

$$\alpha = \frac{\tan\left(\frac{0.55\pi}{2}\right) - \tan\left(\frac{0.27\pi}{2}\right)}{\tan\left(\frac{0.55\pi}{2}\right) + \tan\left(\frac{0.27\pi}{2}\right)} = \frac{\sin\left(\frac{0.55\pi - 0.27\pi}{2}\right)}{\sin\left(\frac{0.55\pi + 0.27\pi}{2}\right)} = 0.4434$$

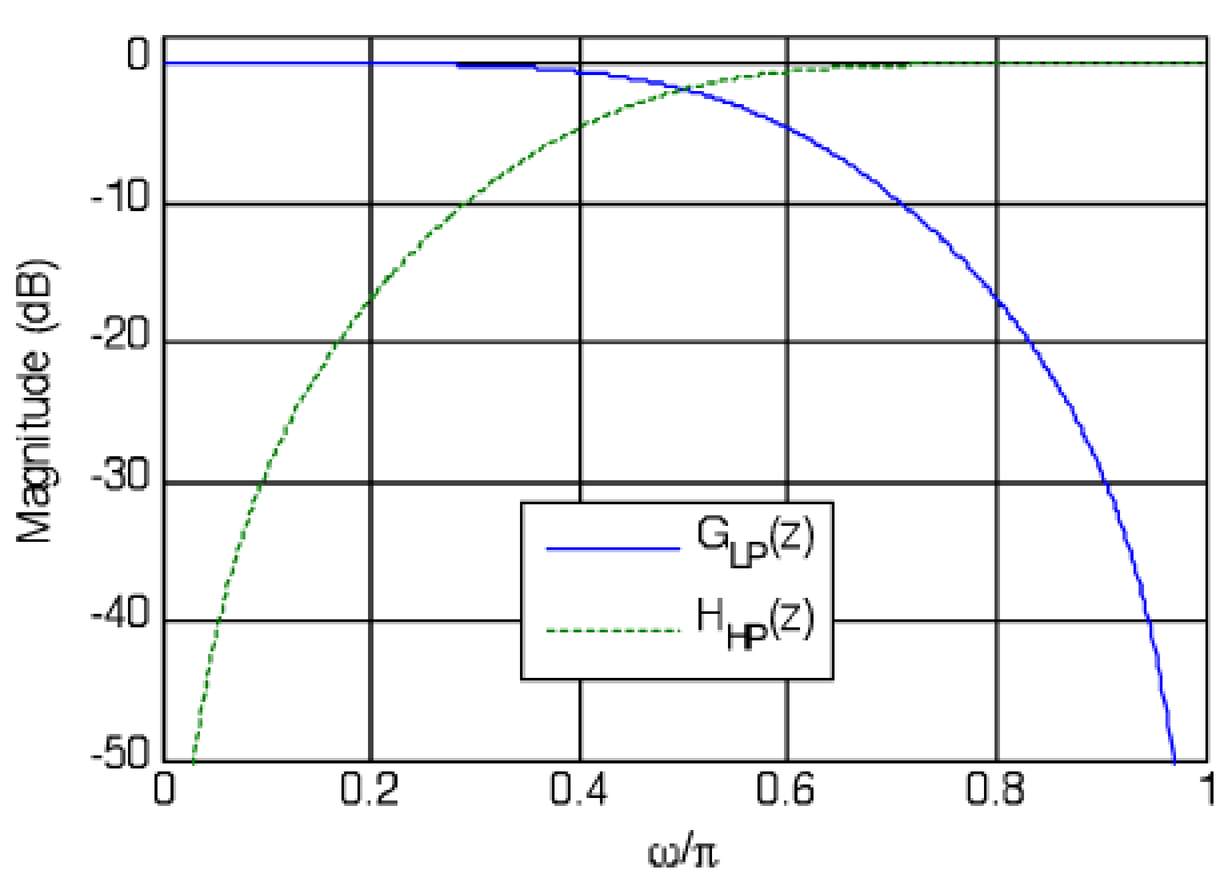
$$H_{LP}(z) = G_{LP}(z)\Big|_{z^{-1} = \frac{\hat{z}^{-1} - \alpha}{1 - \alpha\hat{z}^{-1}}} = \frac{0.3404\left(1 + \frac{\hat{z}^{-1} - \alpha}{1 - \alpha\hat{z}^{-1}}\right)^2}{1 + 0.1842\left(\frac{\hat{z}^{-1} - \alpha}{1 - \alpha\hat{z}^{-1}}\right) + 0.1776\left(\frac{\hat{z}^{-1} - \alpha}{1 - \alpha\hat{z}^{-1}}\right)^2}$$

$$= \frac{0.1055 + 0.2109\hat{z}^{-1} + 0.1055\hat{z}^{-2}}{0.9532 - 0.8239\hat{z}^{-1} + 0.2925\hat{z}^{-2}}.$$



9.28 $\alpha = \frac{-\cos\left(\frac{0.55\pi + 0.45\pi}{2}\right)}{\cos\left(\frac{0.55\pi - 0.45\pi}{2}\right)} = 0.$

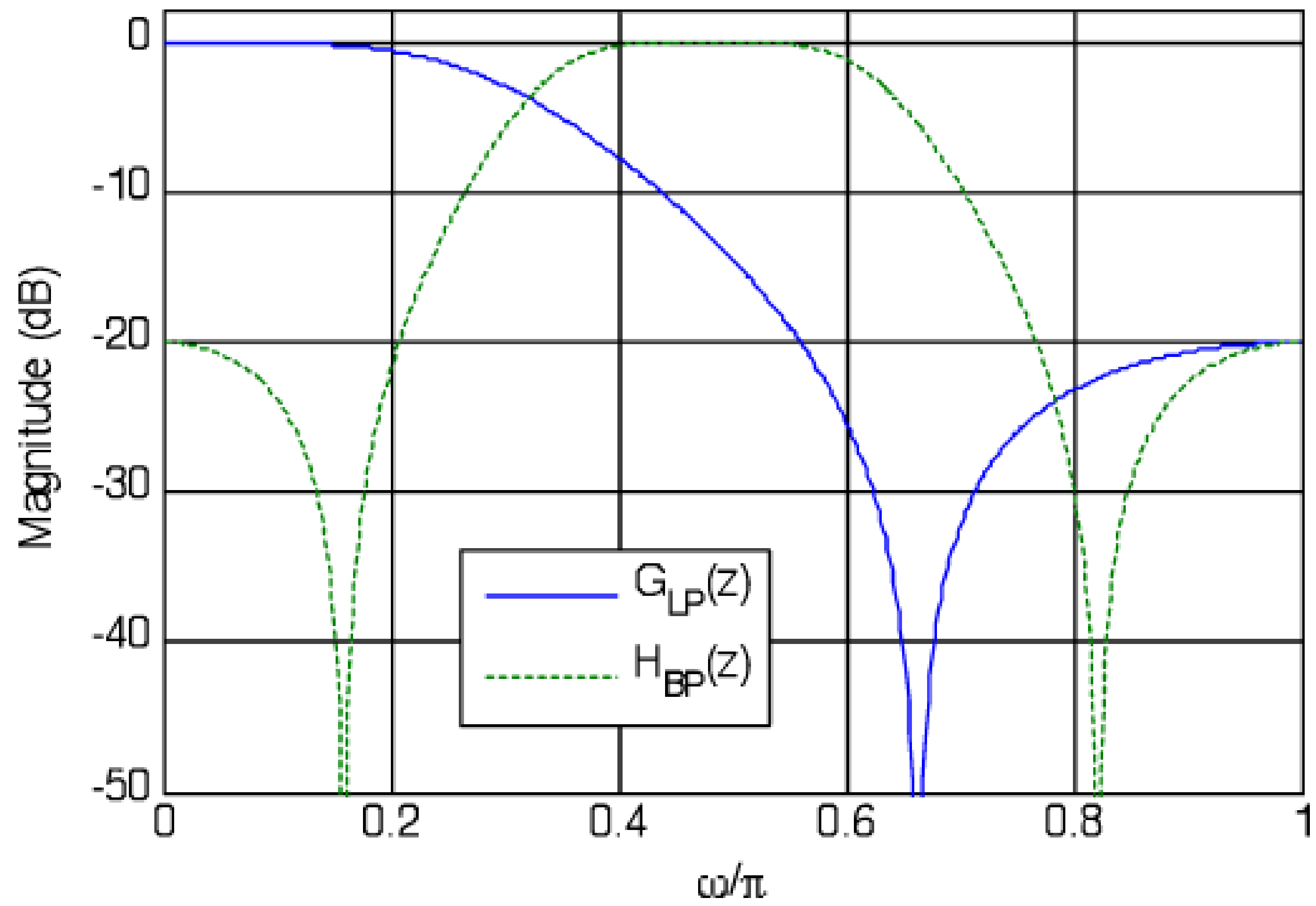
$$H_{HP}(z) = G_{LP}(z)\Big|_{z^{-1} = -\left(\frac{\hat{z}^{-1} + \alpha}{1 + \alpha\hat{z}^{-1}}\right)} = \frac{0.3404 - 0.6808\hat{z}^{-1} + 0.3404\hat{z}^{-2}}{1 - 0.1842\hat{z}^{-1} + 0.1776\hat{z}^{-2}}.$$



9.29 $\omega_c = \hat{\omega}_{c2} - \hat{\omega}_{c1}$, and $\alpha = \cos(\hat{\omega}_c) = 0.0628$

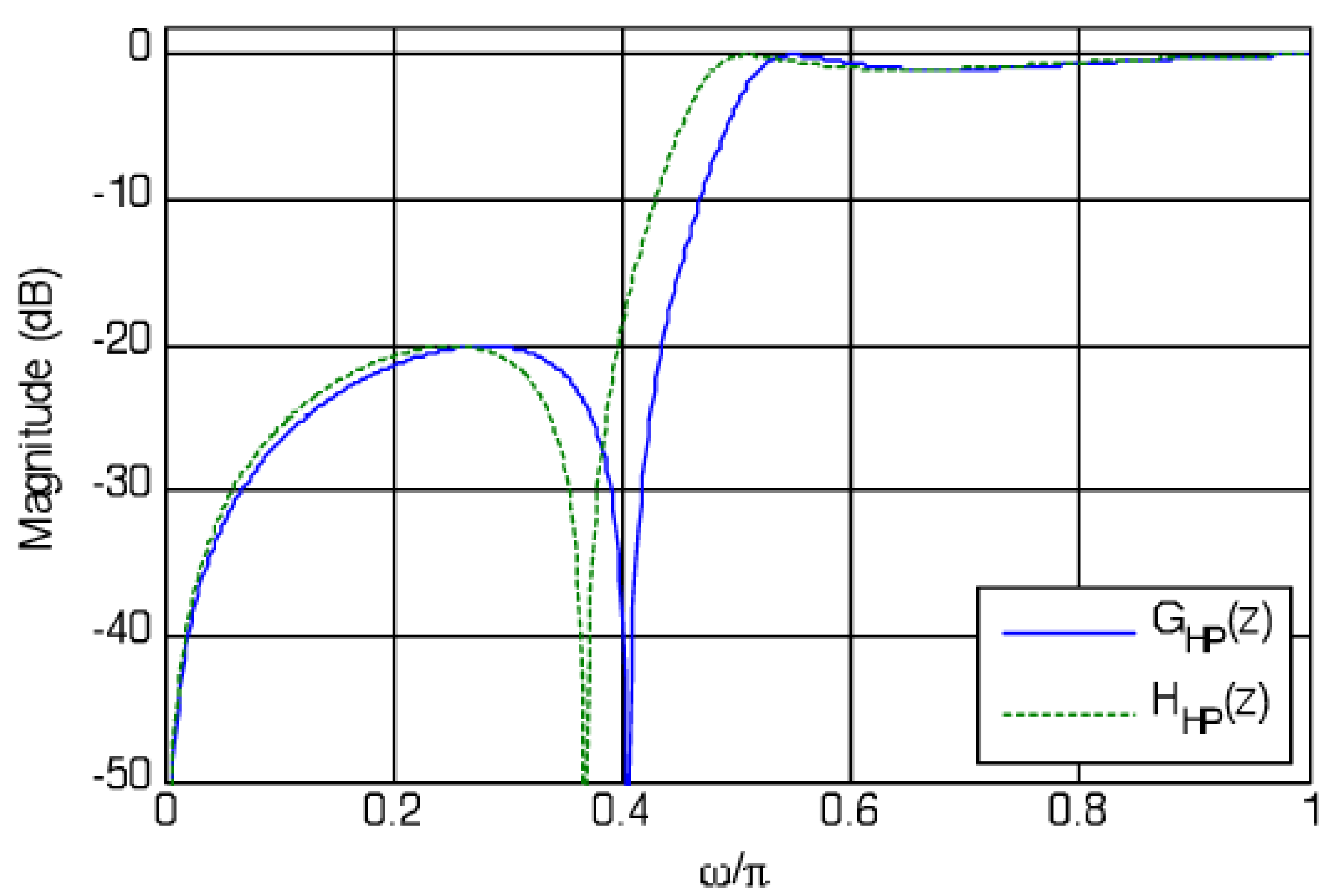
$$z^{-1} = -\hat{z}^{-1}\left(\frac{\hat{z}^{-1}-\alpha}{1-\alpha\hat{z}^{-1}}\right) = \frac{0.0628\hat{z}^{-1}-\hat{z}^{-2}}{1-0.0628\hat{z}^{-1}}.$$

$$H_{BP}(z) = \frac{0.6075-0.0389\hat{z}^{-1}-0.5930\hat{z}^{-2}-0.0389\hat{z}^{-3}+0.6075\hat{z}^{-4}}{3.1250-0.5302\hat{z}^{-1}+2.2169\hat{z}^{-2}-0.2480\hat{z}^{-3}+0.8781\hat{z}^{-4}}.$$



9.30 $\hat{\omega}_p = 0.48\pi$, and $\omega_p = 0.52\pi$. $\lambda = \dfrac{\sin\left(\dfrac{0.52\pi-0.48\pi}{2}\right)}{\sin\left(\dfrac{0.52\pi+0.48\pi}{2}\right)} = 0.0628.$

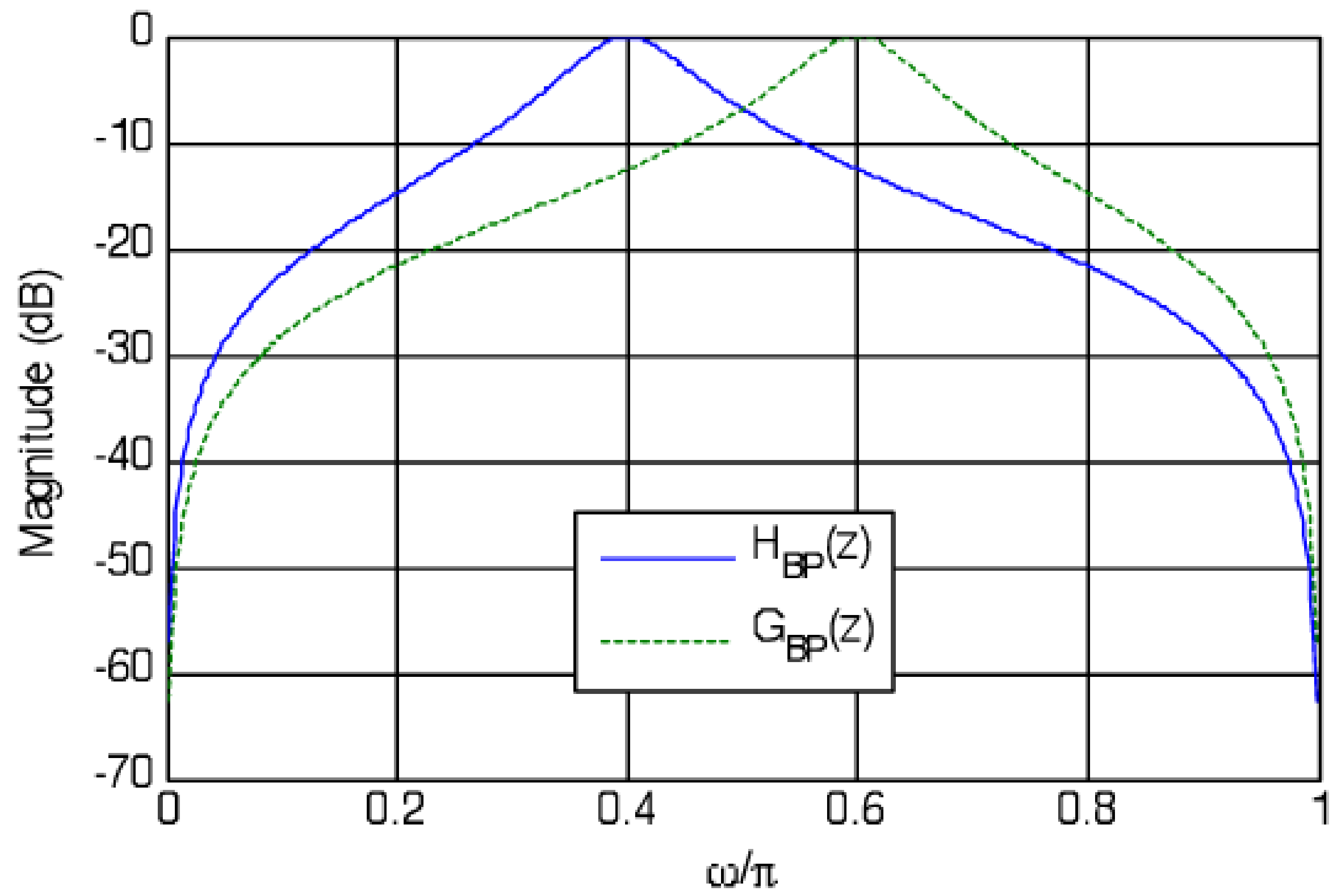
$$H_{HP}(z) = G_{HP}(z)\Big|_{z=\frac{\hat{z}^{-1}-0.0628}{1-0.0628\hat{z}^{-1}}} = \frac{0.3766-0.6803\hat{z}^{-1}+0.6803\hat{z}^{-2}-0.3766\hat{z}^{-3}}{1.3954+0.0705\hat{z}^{-1}+0.9783\hat{z}^{-2}+0.1892\hat{z}^{-3}}.$$



9.31 Eq. (7.79): $H_{BP}(z) = 0.136728736\left(\dfrac{1-z^{-2}}{1-0.53353098z^{-1}+0.726542528z^{-2}}\right).$

$\hat{\omega}_0 = 0.6\pi$, and $\omega_0 = 0.4\pi$. $\lambda = \dfrac{\sin\left(\dfrac{0.4\pi - 0.6\pi}{2}\right)}{\sin\left(\dfrac{0.4\pi + 0.6\pi}{2}\right)} = -0.3090$

$$G_{BP}(z) = H_{BP}(z)\Big|_{z^{-1} = \frac{\hat{z}^{-1}+0.3090}{1+0.3090\hat{z}^{-1}}} = \frac{0.1013 - 0.1013\hat{z}^{-2}}{0.7406 + 0.3951\hat{z}^{-1} + 0.5381\hat{z}^{-2}}.$$



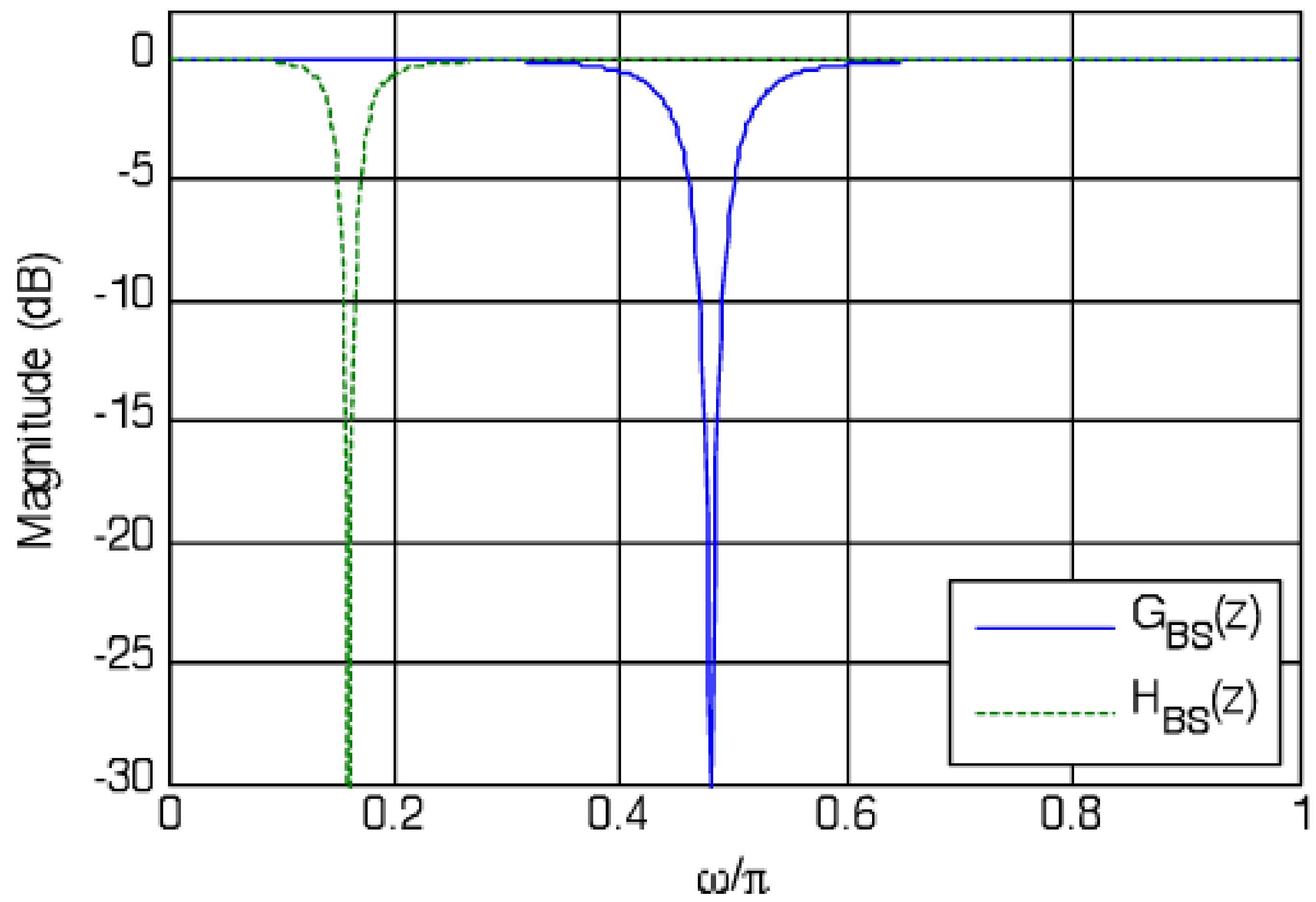
9.32 $\omega_0 = 2\pi\left(\dfrac{120}{500}\right) = 0.48\pi$, $B_\omega = 2\pi\left(\dfrac{5}{400}\right) = 0.06\pi$. $\alpha = \dfrac{1 - \tan\left(\frac{B_\omega}{2}\right)}{1 + \tan\left(\frac{B_\omega}{2}\right)} = 0.8273$

$\beta = \cos(\omega_0) = 0.0628$.

$$G_{BS}(z) = \frac{1}{2}\frac{(1+\alpha) - 2\beta(1+\alpha)z^{-1} + (1+\alpha)z^{-2}}{1 - \beta(1+\alpha)z^{-1} + \alpha z^{-2}} = \frac{0.9137 - 0.1148z^{-1} + 0.9137z^{-2}}{1 - 0.1148z^{-1} + 0.8273z^{-2}}.$$

$\hat{\omega}_0 = \left(\dfrac{40}{500}\right)2\pi = 0.16\pi$. $\alpha = \dfrac{\sin\left(\frac{\omega_0 - \hat{\omega}_0}{2}\right)}{\sin\left(\frac{\omega_0 + \hat{\omega}_0}{2}\right)} = 0.5706$

$$H_{BS}(z) = G_{BS}(z)\Big|_{z^{-1} = \frac{\hat{z}^{-1}-\alpha}{1-\alpha\hat{z}^{-1}}} = \frac{0.3192 - 0.5594\hat{z}^{-1} + 0.3192\hat{z}^{-2}}{0.3337 - 0.5594\hat{z}^{-1} + 0.3046\hat{z}^{-2}}.$$

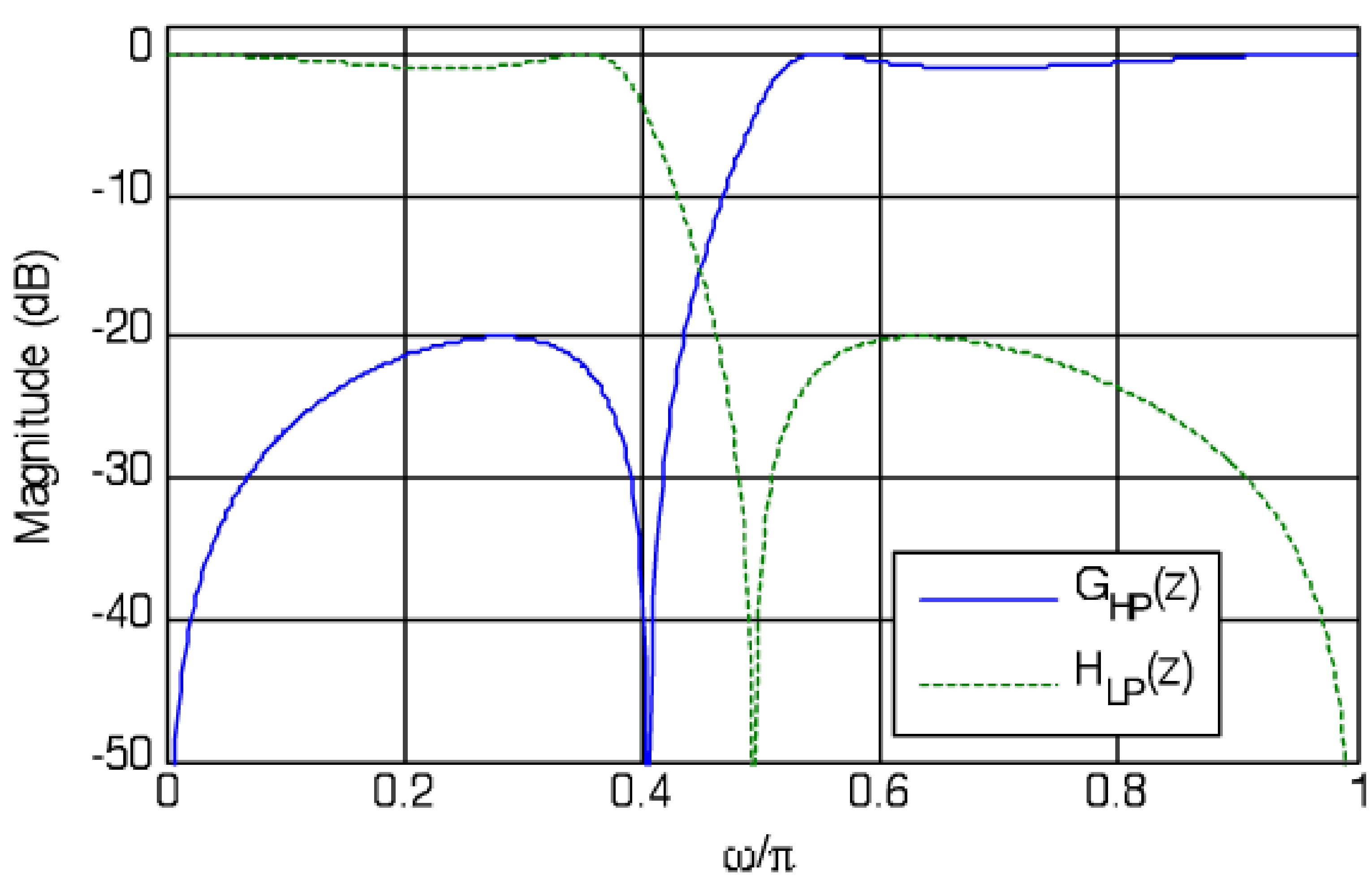


9.33 $\hat{\omega}_p = 0.38\pi$ and $\omega_p = 0.52\pi$.

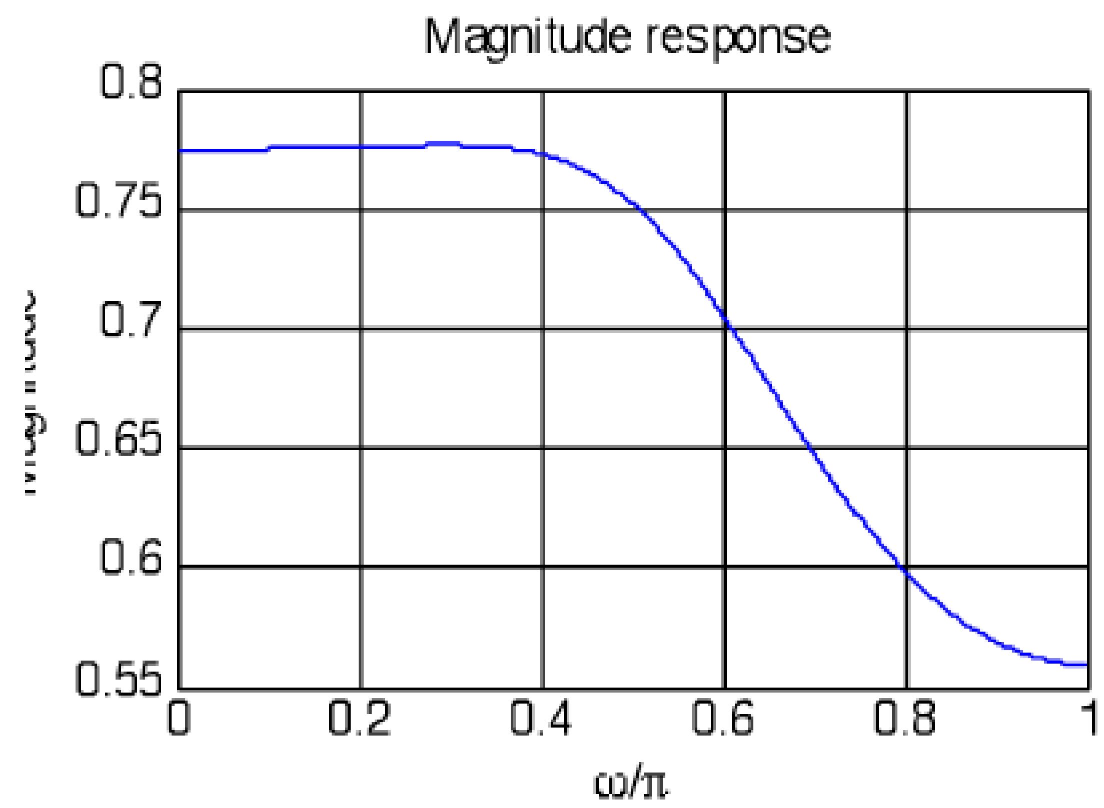
$$G_{HP}(z) = \frac{0.2397\left(1 - 1.5858z^{-1} + 1.5858z^{-2} - z^{-3}\right)}{1 + 0.3272z^{-1} + 0.7459z^{-2} + 0.179z^{-3}}$$

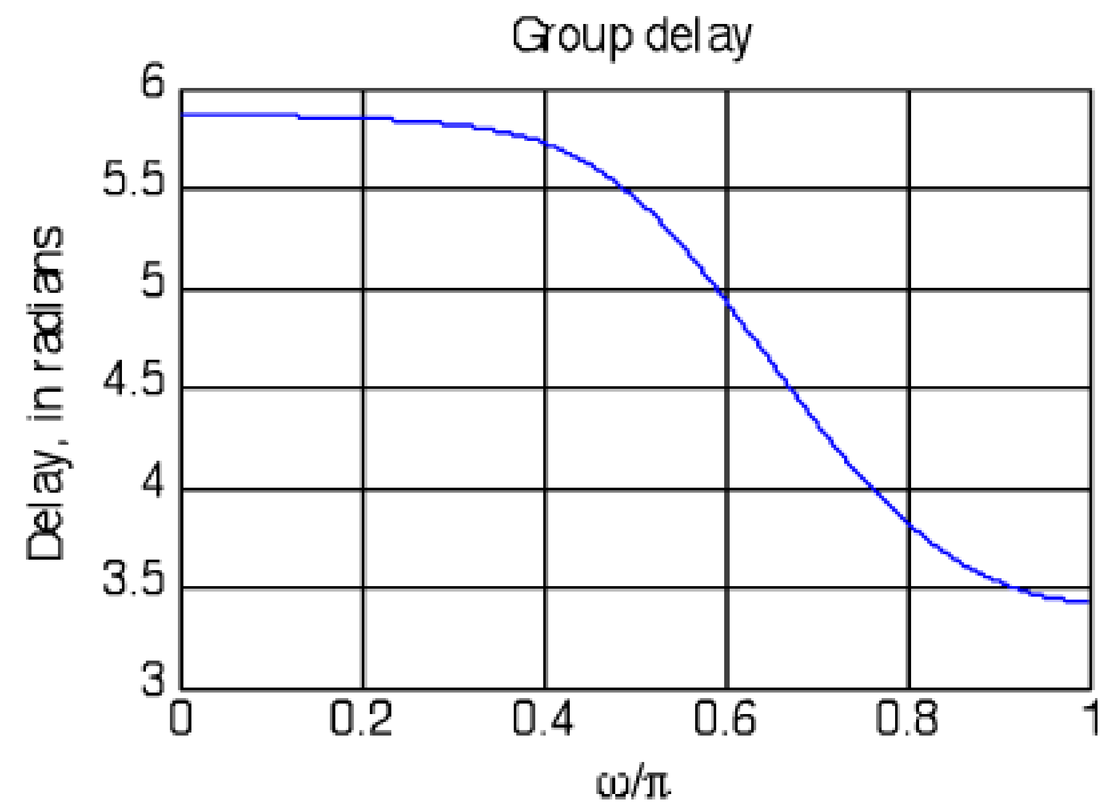
$$\alpha = \frac{-\cos\left(\frac{\omega_p + \hat{\omega}_p}{2}\right)}{\cos\left(\frac{\omega_p - \hat{\omega}_p}{2}\right)} = -0.1603.$$

$$H_{LP}(z) = G_{HP}(z)\Big|_{z^{-1} = \frac{-\hat{z}^{-1} - \alpha}{1 + \alpha\hat{z}^{-1}}} = \frac{-0.1096 - 0.1049\hat{z}^{-1} - 0.1049\hat{z}^{-2} - 0.1096\hat{z}^{-3}}{-0.6269 + 0.6319\hat{z}^{-1} - 0.6160\hat{z}^{-2} + 0.1819\hat{z}^{-3}}$$



9.34 $D = 6.9231$, hence, $N = 6$. $\mathcal{A}(z) = \dfrac{z^{-6}D(z^{-1})}{D(z)}$.





9.35 $H(e^{j\omega}) = H_{DIFF}(e^{j\omega})H_{DEL}(e^{j\omega}) = j\omega e^{-j\omega/2}$.

The phase response is

$$\theta(\omega) = \tan^{-1}\left(\frac{H_{im}(e^{j\omega})}{H_{re}(e^{j\omega})}\right) = \tan^{-1}\big(\cot(\omega/2)\big) = \begin{cases} -(\pi+\omega)/2, & \omega < 0, \\ (\pi-\omega)/2, & \omega > 0. \end{cases}$$ which has linear phase. The magnitude response is unchanged since

$$\left|H(e^{j\omega})\right|^2 = \omega^2 = \left|H_{DIFF}(e^{j\omega})\right|^2.$$

M9.1 $F_T = 100$ kHz, $\alpha_p = 0.3$ dB, $F_p = 15$ kHz, $\alpha_s = 45$ dB, and $F_s = 25$ kHz,

$$\omega_p = \frac{2\pi F_p}{F_T} = 0.9425, \quad \omega_s = \frac{2\pi F_s}{F_T} = 1.5708.$$

Let $T = 2$. $\Omega_p = \tan\left(\frac{\omega_p}{2}\right) = 0.5095$, and $\Omega_s = \tan\left(\frac{\omega_s}{2}\right) = 1$. Therefore,

$\frac{1}{k} = \frac{\Omega_s}{\Omega_p} = 1.9626$. Now, $20\log_{10}\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right) = -0.3$. Hence, $\varepsilon^2 = 0.0715$.

From $20\log_{10}\left(\frac{1}{A}\right) = -45$ we obtain $A^2 = 31623$. Therefore,

$k_1 = \frac{\varepsilon}{\sqrt{A^2-1}} = 0.0015$, or $\frac{1}{k_1} = 665.03$. As a result,

$N = \frac{\log_{10}(1/k_1)}{\log_{10}(1/k)} = 9.64 \to 10$. Next, solving $\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = A^2 - 1 = 31622$ we get

$$\Omega_c = \frac{\Omega_s}{31622^{1/20}} = 0.5957\Omega_s = 0.5957$$

Using the M-file `buttap`, we determine the normalized analog Butterworth transfer function of 10th order with a 3-dB cutoff frequency at $\Omega_c = 1$, which is:

$$H_{an}(s) = \frac{1}{s^{10} + 6.4s^9 + 20.4s^8 + 42.8s^7 + 64.9s^6 + 74.2s^5 + 64.9s^4 + 42.8s^3 + 20.4s^2 + 6.4s^1 + 1}.$$

Denormalize $H_{an}(s)$ to move Ω_c to 0.5957:

$$H_a(s) = H_{an}\left(\frac{s}{0.5957}\right)$$

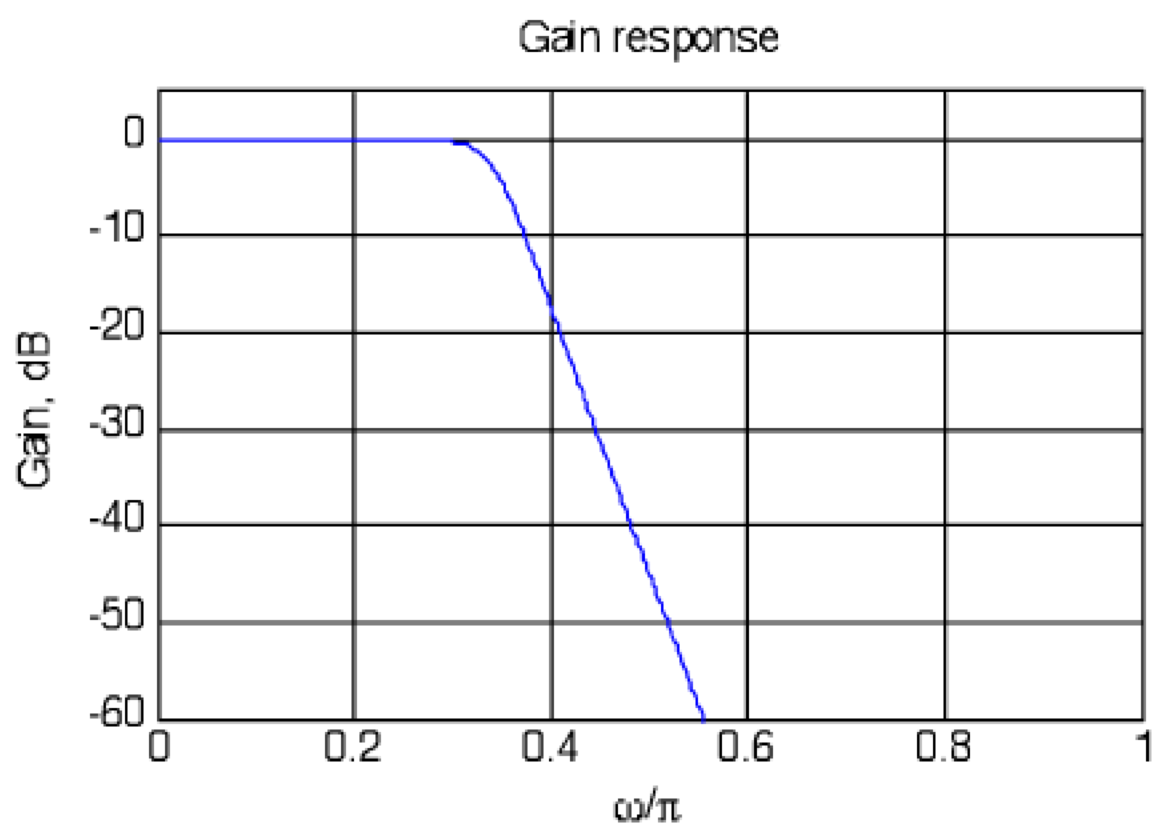
$$= \frac{1}{177.71\left(s^{10} + 3.8s^9 + 7.3s^8 + 9.0s^7 + 8.2s^6 + 5.6s^5 + 32.9s^4 + 1.1s^3 + 0.32s^2 + 0.06s^1 + 0.006\right)}.$$

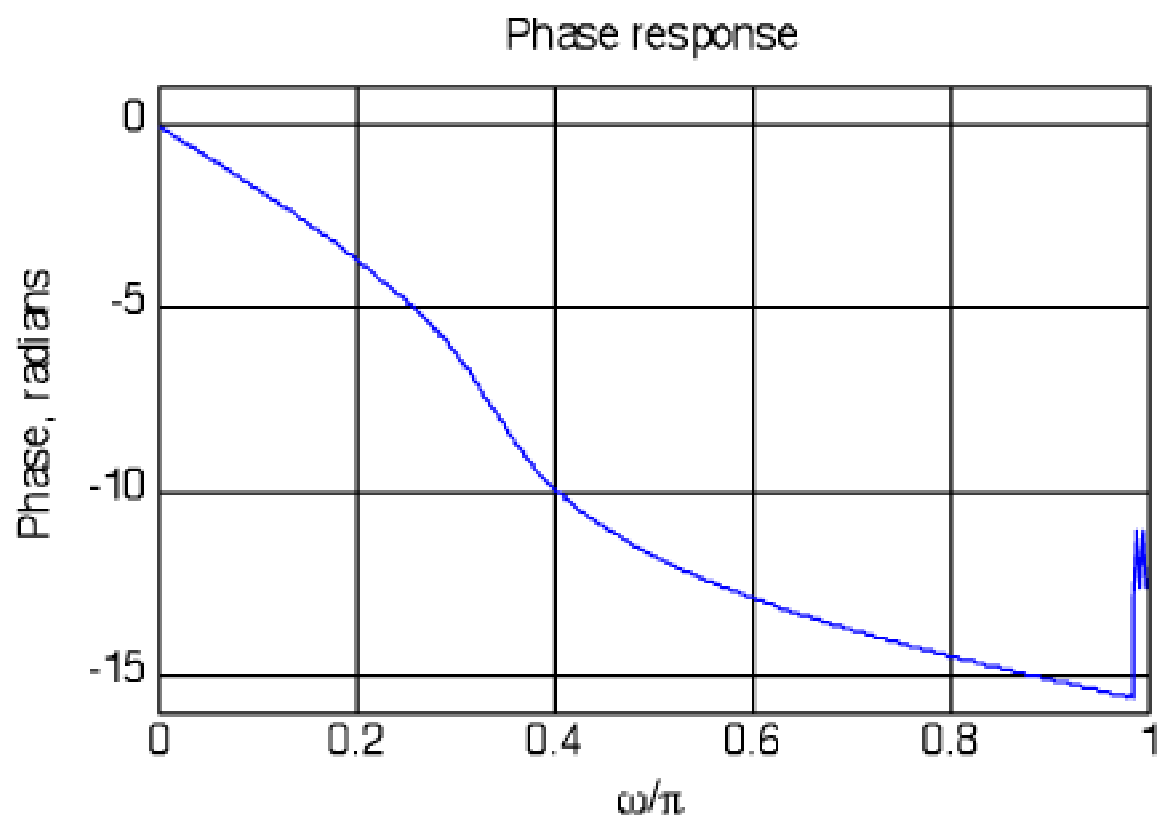
$$G(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{1e-3\left(0.14 + 1.43z^{-1} + 6.45z^{-2} + 17.19z^{-3} + 30.09z^{-4} + 36.11z^{-5} + 30.09z^{-6} + 17.19z^{-7} + 6.45z^{-8} + 1.43z^{-9} + 0.14z^{-10}\right)}{1 - 3.15z^{-1} + 5.54z^{-2} - 6.34z^{-3} + 5.16z^{-4} - 3.05z^{-5} + 1.32z^{-6} - 0.41z^{-7} + 0.09z^{-8} + -0.01z^{-9} + 0.0006z^{-10}}$$

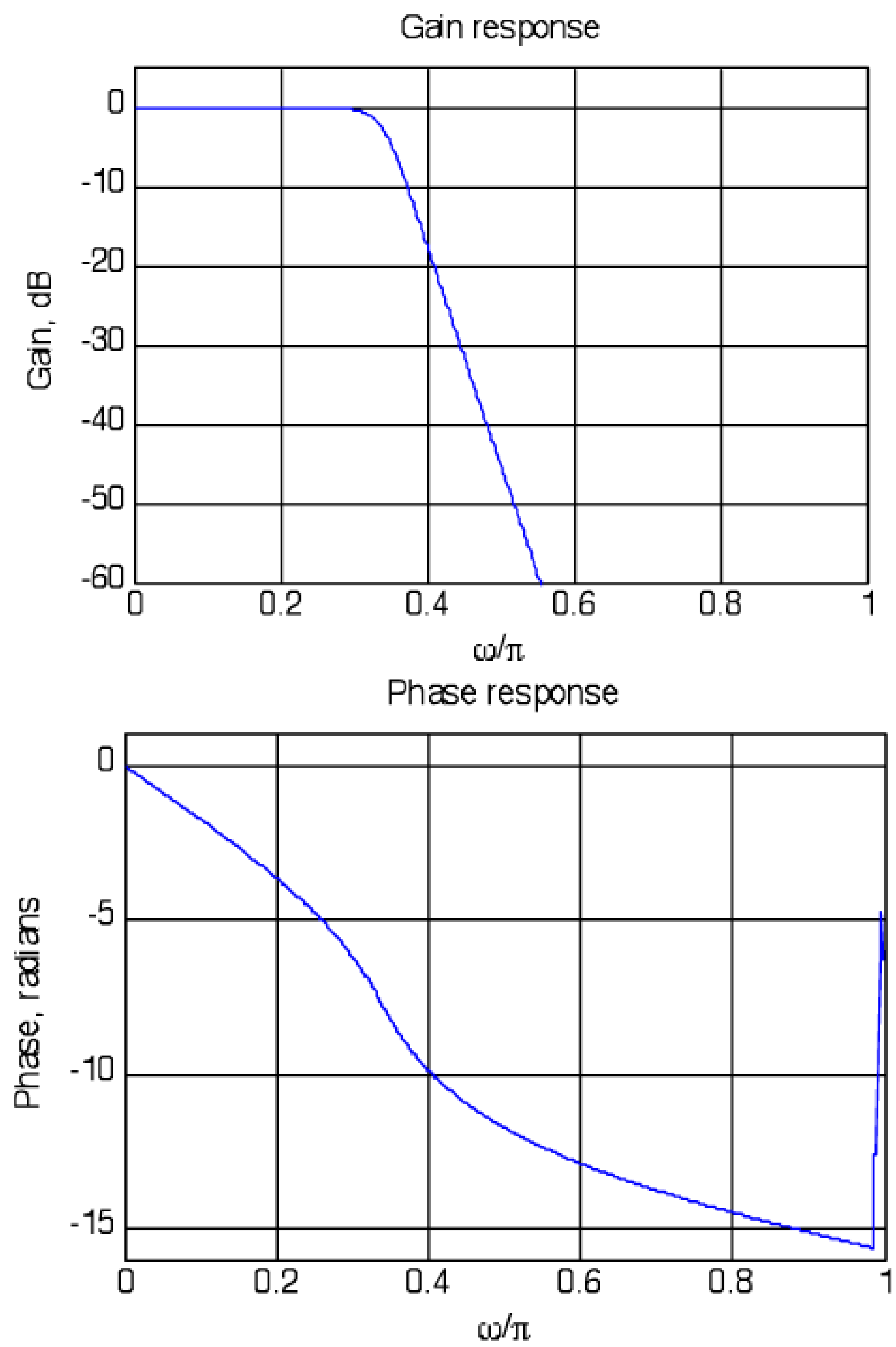
Matlab code is as follows:

```
% Program M9.01
N = 10;
[z, p, k] = buttap(N);
[num, den] = zp2tf(z, p, k);
% s -> s/0.5957
den = den.*(1/0.5957).^[N:-1:0]
% compute z, p, and k
[z, p, k] = tf2zp(num, den);
% perform bilinear transformation with T = 2;
[zd, pd, kd] = bilinear(z, p, k, 1/2);
% get the digital transfer function
[n2, d2] = zp2tf(zd, pd, kd);
% get the frequency response
[h, w] = freqz(n2, d2, 512);
figure; plot(w/pi, 20*log10(abs(h))); grid;
axis([0 1 -60 5]);
xlabel('\omega/\pi'); ylabel('Gain, dB');
title('Gain response');
figure; plot(w/pi, unwrap(angle(h))); grid;
axis([0 1 -16 1]);
xlabel('\omega/\pi'); ylabel('Phase, radians');
title('Phase response');
```





M9.2
```
% Problem M9.02
Fp = input('Passband edge frequency in Hz = ');
Fs = input('Stopband edge frequency in Hz = ');
FT = input('Sampling frequency in Hz = ');
Rp = input('Passband ripple in dB = ');
Rs = input('Stopband minimum attenuation in dB = ');
Wp = 2*Fp/FT;
Ws = 2*Fs/FT;
[N, Wn] = buttord(Wp, Ws, Rp, Rs)
[b, a] = butter(N, Wn);
disp('Numerator polynomial');
disp(b)
disp('Denominator polynomial');
disp(a)
[h, w] = freqz(b, a, 512);
figure; plot(w/pi, 20*log10(abs(h))); grid;
axis([0 1 -60 5]);
xlabel('\omega/\pi'); ylabel('Gain, dB');
title('Gain response');
figure; plot(w/pi, unwrap(angle(h))); grid;
axis([0 1 -16 1]);
xlabel('\omega/\pi'); ylabel('Phase, radians');
title('Phase response');
```

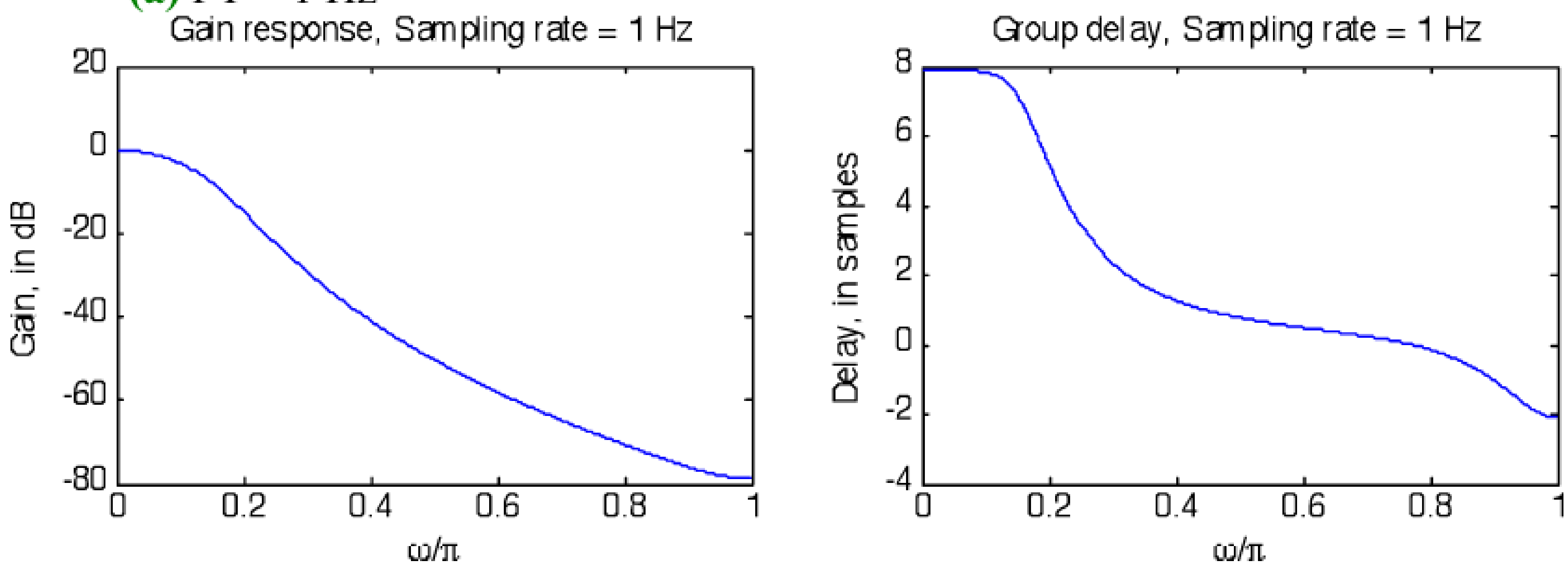


M9.3

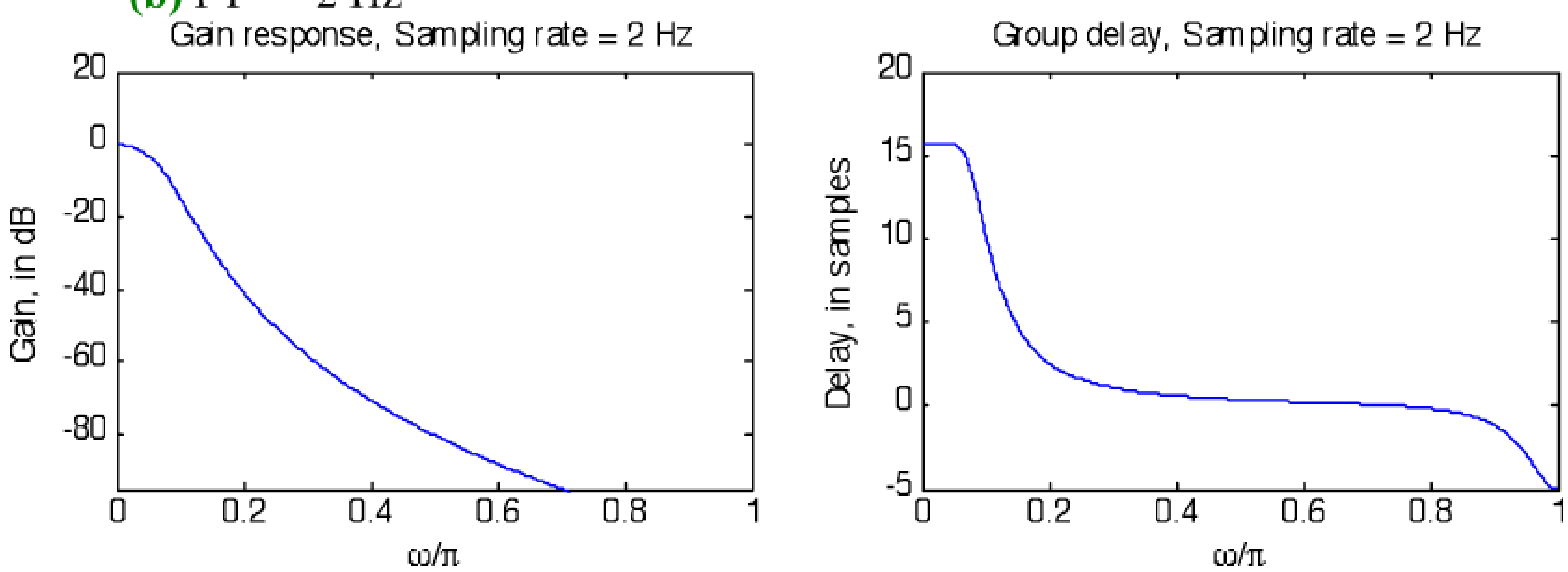
```
% Program #M9.03
close all;
clear;
clc;

% (a.) Ft = 1Hz;
[B, A] = besself(5, 0.5);
[BZ,AZ] = impinvar(B,A,1);
 [h, w] = freqz(BZ, AZ, 512);
[Gd,W] = grpdelay(BZ,AZ,512);

figure(1);
plot(w/pi, 20*log10(abs(h)));
title('Gain response, , Sampling rate = 1 Hz');
xlabel('\omega/\pi'); ylabel('Gain, in dB');

figure(2);
plot(W/pi, (Gd));
title('Group delay, Sampling rate = 1 Hz');
xlabel('\omega/\pi'); ylabel('Delay, in samples');
```

```
% (b.) Ft = 2Hz;
[B, A] = besself(5, 0.5);
[BZ,AZ] = impinvar(B,A,2);
[h, w] = freqz(BZ, AZ, 512);
[Gd,W] = grpdelay(BZ,AZ,512);

figure(3);
plot(w/pi, 20*log10(abs(h)));
title('Gain response, Sampling rate = 2 Hz');
xlabel('\omega/\pi');ylabel('gain response');

figure(4);
plot(W/pi, (Gd));
title('Group delay, Sampling rate = 2 Hz');
xlabel('\omega/\pi'); ylabel('Delay, in samples');
```

(a) FT = 1 Hz



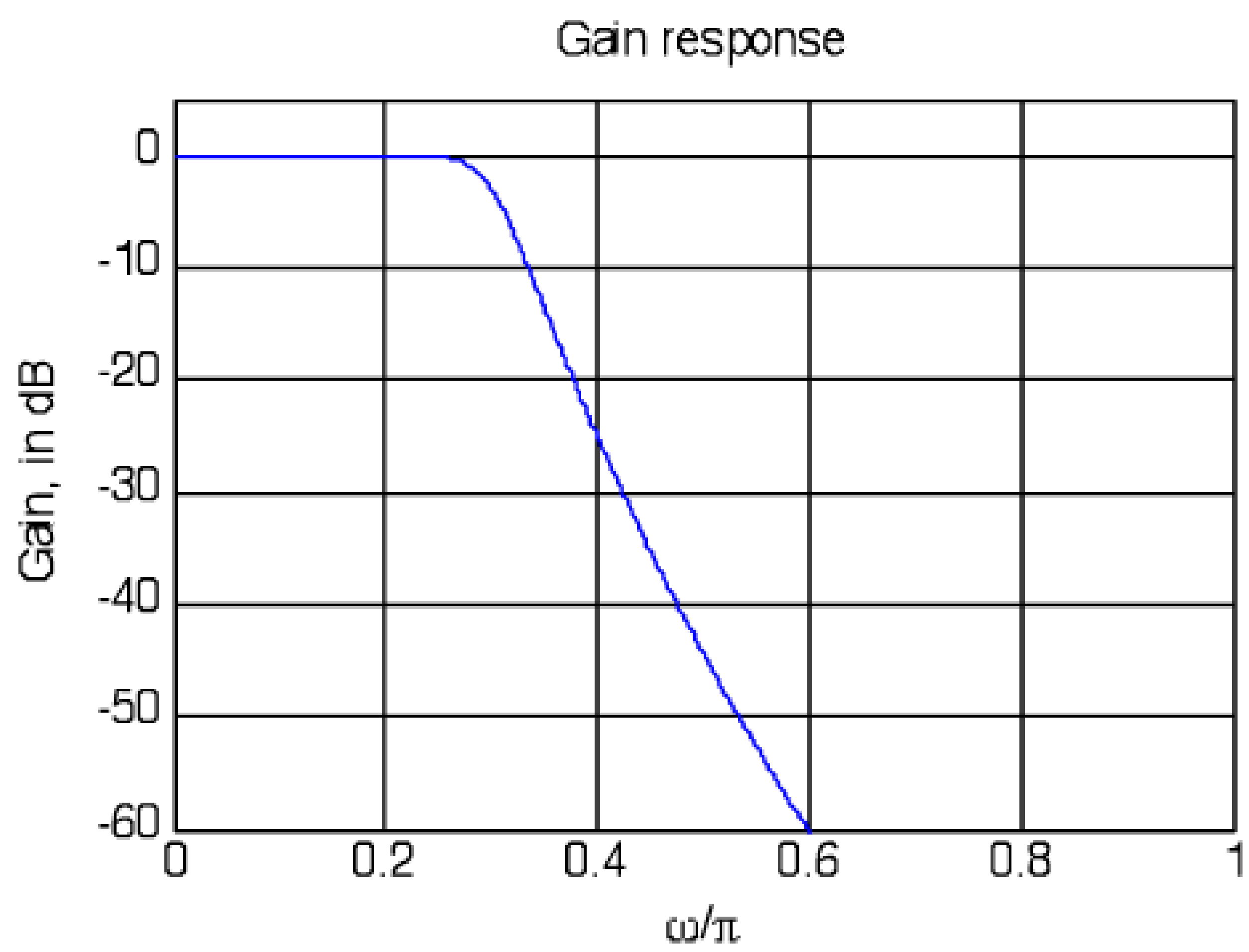
(b) FT = 2 Hz



M9.4

```
% Problem M9.04
[B, A] = butter(10, 0.9425, 's');
```

```
[num, den] = impinvar(B, A, 1);
% get the frequency response
[h, w] = freqz(num, den, 512);
plot(w/pi, 20*log10(abs(h))); grid;
axis([0 1 -60 5]);
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('Gain response');
```



M9.5 $F_T = 100$ kHz, $F_p = 15$ kHz, $F_s = 25$ kHz, $\alpha_p = 0.3$ dB, $\alpha_s = 45$ dB

$\omega_p = \dfrac{2\pi F_p}{F_T} = 0.3\pi$, and $\omega_s = \dfrac{2\pi F_s}{F_T} = 0.5\pi$.

$\Omega_p = \tan\left(\dfrac{\omega_p}{2}\right) = 0.5095$, and $\Omega_s = \tan\left(\dfrac{\omega_s}{2}\right) = 1$.

<u>Impulse invariance method</u>:
Let $T = 1$ and assume no aliasing. In this case, the specifications for $H_a(s)$ are same as that for $G(z)$.

Now, $20\log_{10}\left(\dfrac{1}{\sqrt{1+\varepsilon^2}}\right) = -0.4 \Rightarrow \varepsilon^2 = 0.0715$, and

$20\log_{10}\left(\dfrac{1}{A}\right) = -45 \Rightarrow A^2 = 31623$. Hence,

$\dfrac{1}{k_1} = \sqrt{\dfrac{A^2-1}{\varepsilon^2}} = 665.03$ and $\dfrac{1}{k} = \dfrac{1.5708}{0.9425} = 1.667$.

Order of the Type I Chebyshev filter is $N = \dfrac{\cosh^{-1}(1/k_1)}{\cosh^{-1}(1/k)} = 6.55 \to 7$

<u>Bilinear transformation method</u>:

Let $T = 2$. $\Omega_p = 0.5095$, $\Omega_s = 1$. Here, $\dfrac{1}{k} = \dfrac{\Omega_s}{\Omega_p} = 1.9627$, and

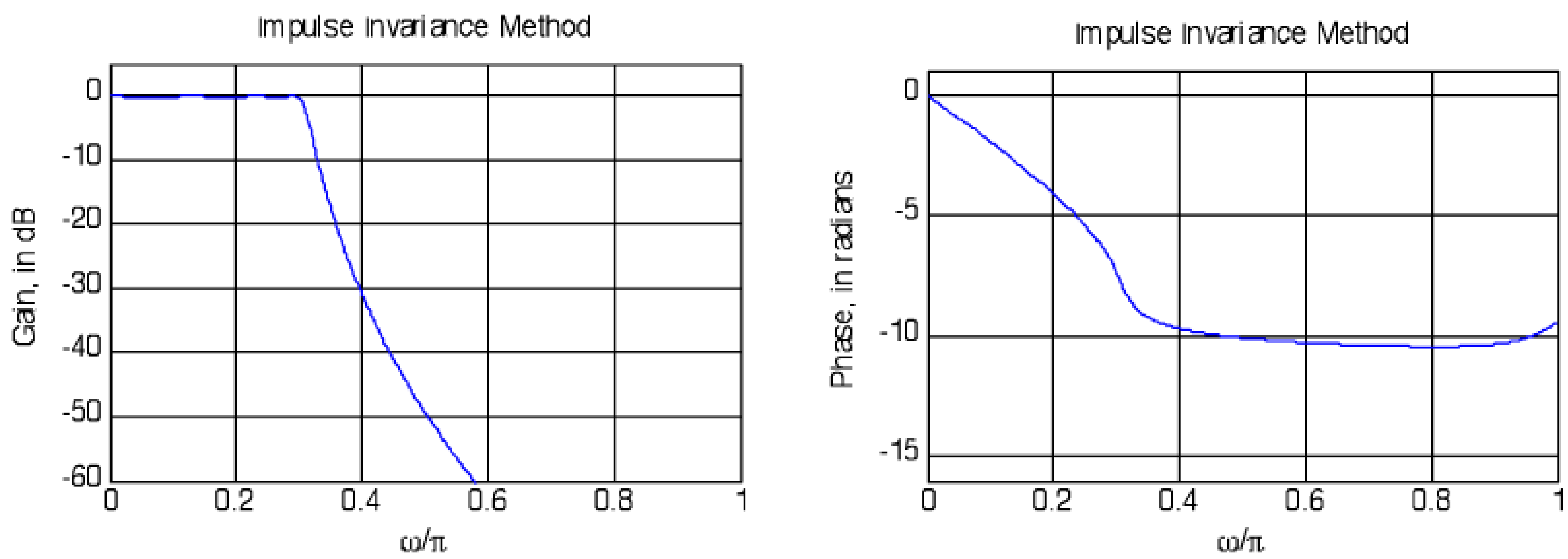
$\varepsilon^2 = 0.0715,\ A^2 = 31623..$ Thus, $\dfrac{1}{k_1} = 1020.62.$

Order of the Type I Chebyshev filter is $N = \dfrac{\cosh^{-1}(1/k_1)}{\cosh^{-1}(1/k)} = 5.554 \rightarrow 6.$

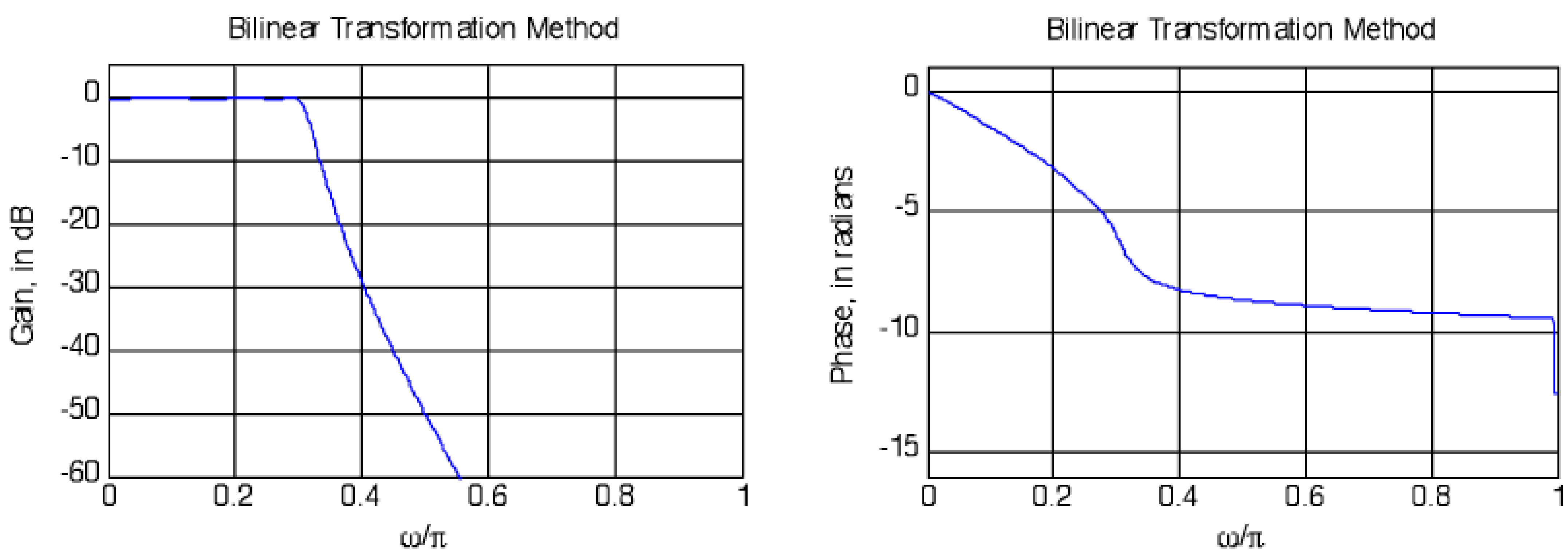
Both designs meet the specifications, while the bilinear transformation method meets with a filter of lower order. MATLAB code is as follows:

```
% Problem #M9.05
% Impulse invariance method
[z, p, k] = cheb1ap(7, 0.3);
[B, A] = zp2tf(z, p, k);
[BT, AT] = lp2lp(B, A, 0.9425);
[num, den] = impinvar(BT, AT, 1);
 [h, w] = freqz(num, den, 512);
figure; plot(w/pi, 20*log10(abs(h))); grid;
axis([0 1 -60 5]);
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('Impulse Invariance Method');
figure; plot(w/pi, unwrap(angle(h))); grid
axis([0 1 -16 1]);
xlabel('\omega/\pi'); ylabel('Phase, in radians');
title('Impulse Invariance Method');


% Bilinear transformation method
[z, p, k] = cheb1ap(6, 0.3);
[B, A] = zp2tf(z, p, k);
[BT, AT] = lp2lp(B, A, 0.5095);
[num, den] = bilinear(BT, AT, 0.5);
[h, w] = freqz(num, den, 512);
figure; plot(w/pi, 20*log10(abs(h))); grid;
axis([0 1 -60 5]);
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('Bilinear Transformation Method');
figure; plot(w/pi, unwrap(angle(h))); grid
axis([0 1 -16 1]);
xlabel('\omega/\pi'); ylabel('Phase, in radians');
title('Bilinear Transformation Method');
```

Impulse Invariance Method:

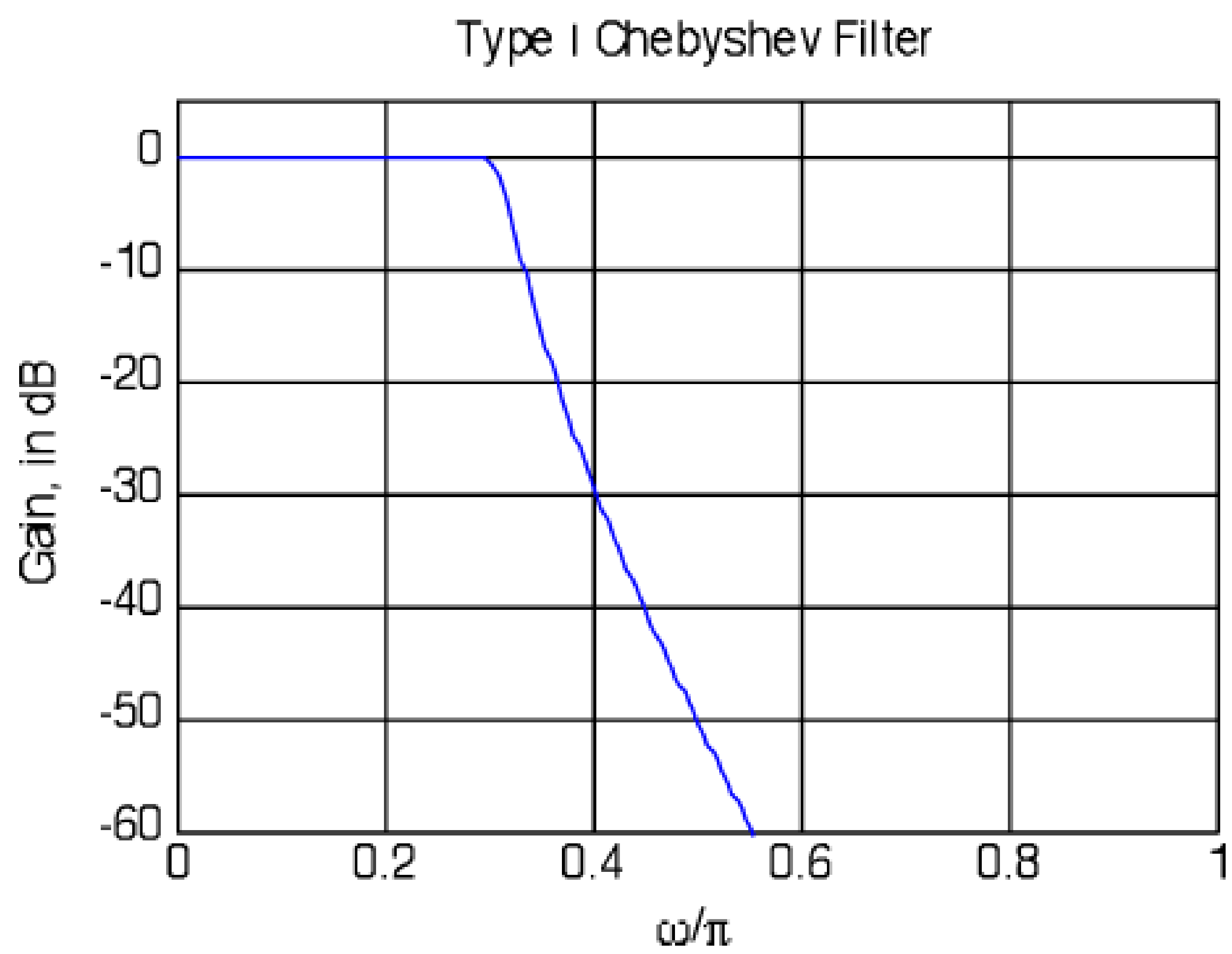


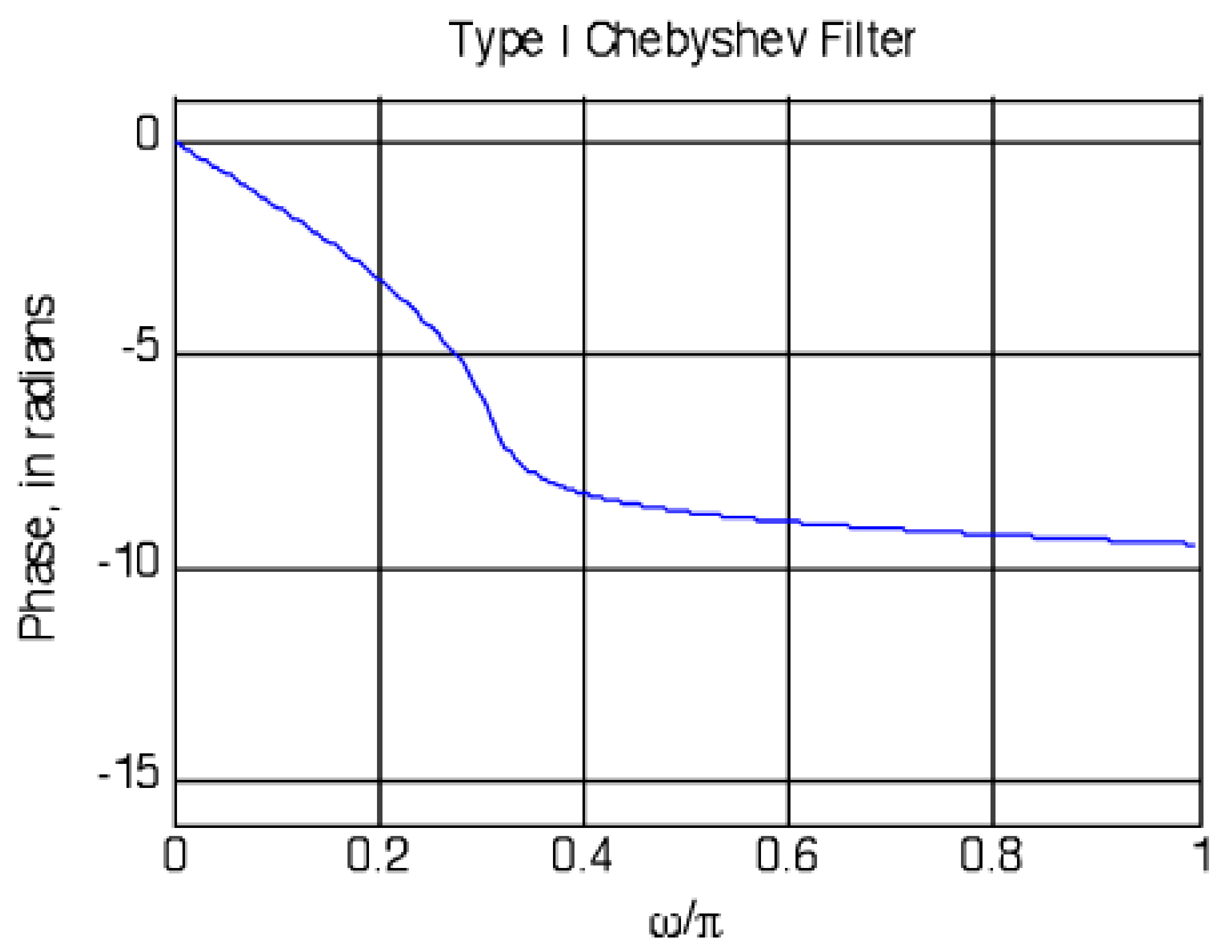
Bilinear Transformation Method:



M9.6

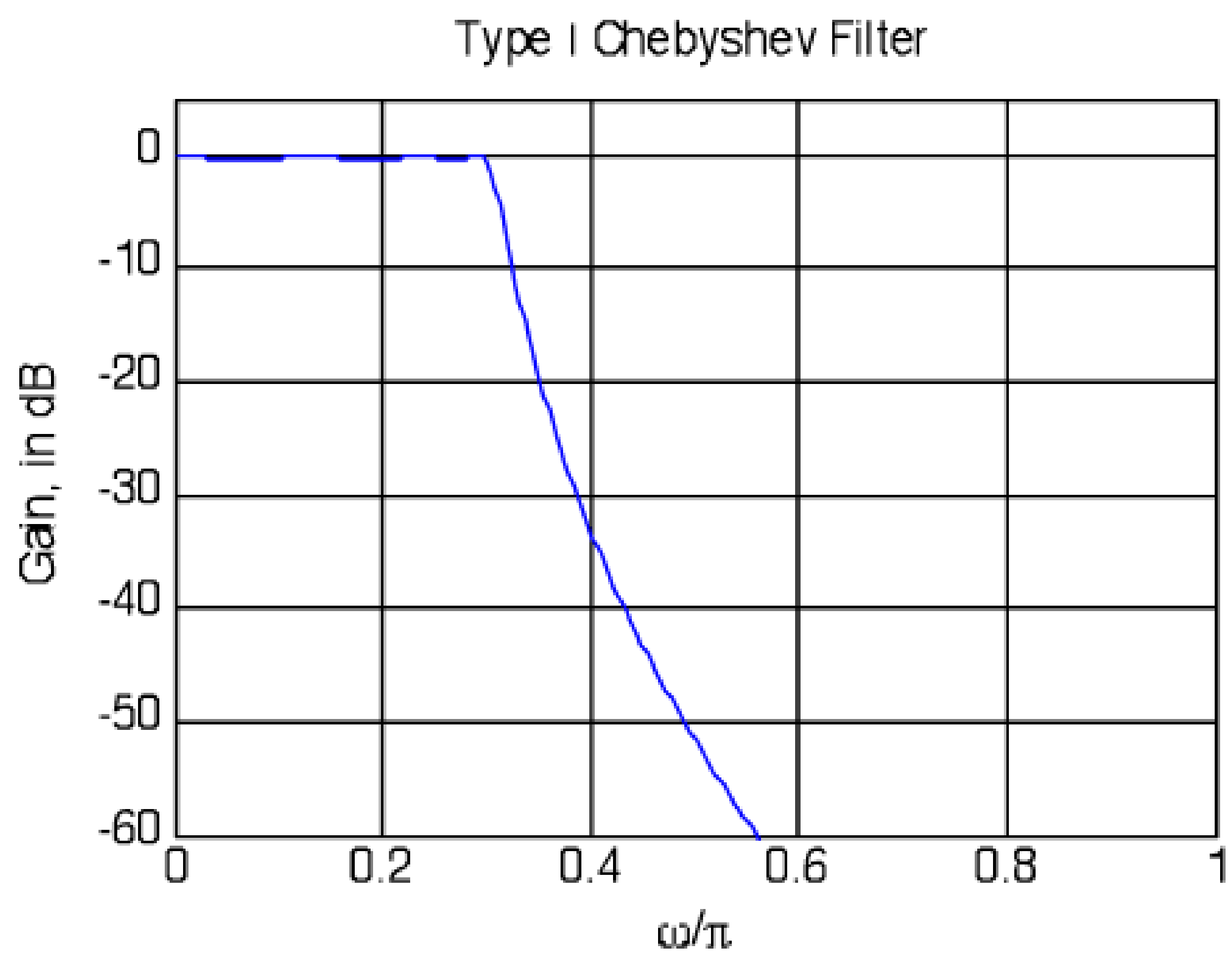
```
% Problem M9.06
Wp = input('Normalized passband edge = ');
Ws = input('Normalized stopband edge = ');
Rp = input('Passband ripple in dB = ');
Rs = input('Minimum stopband attenuation in dB = ');
[N, Wn] = cheb1ord(Wp, Ws, Rp, Rs);
[b, a] = cheby1(N, Rp, Wn);
[h, omega] = freqz(b,a,256);

figure(1);
plot(omega/pi, 20*log10(abs(h)));grid;
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('Type I Chebyshev Filter');
axis([0 1 -60 5]);

figure(2);
plot(omega/pi, unwrap(angle(h))); grid
axis([0 1 -16 1]);
xlabel('\omega/\pi'); ylabel('Phase, in radians');
title('Type I Chebyshev Filter');
```

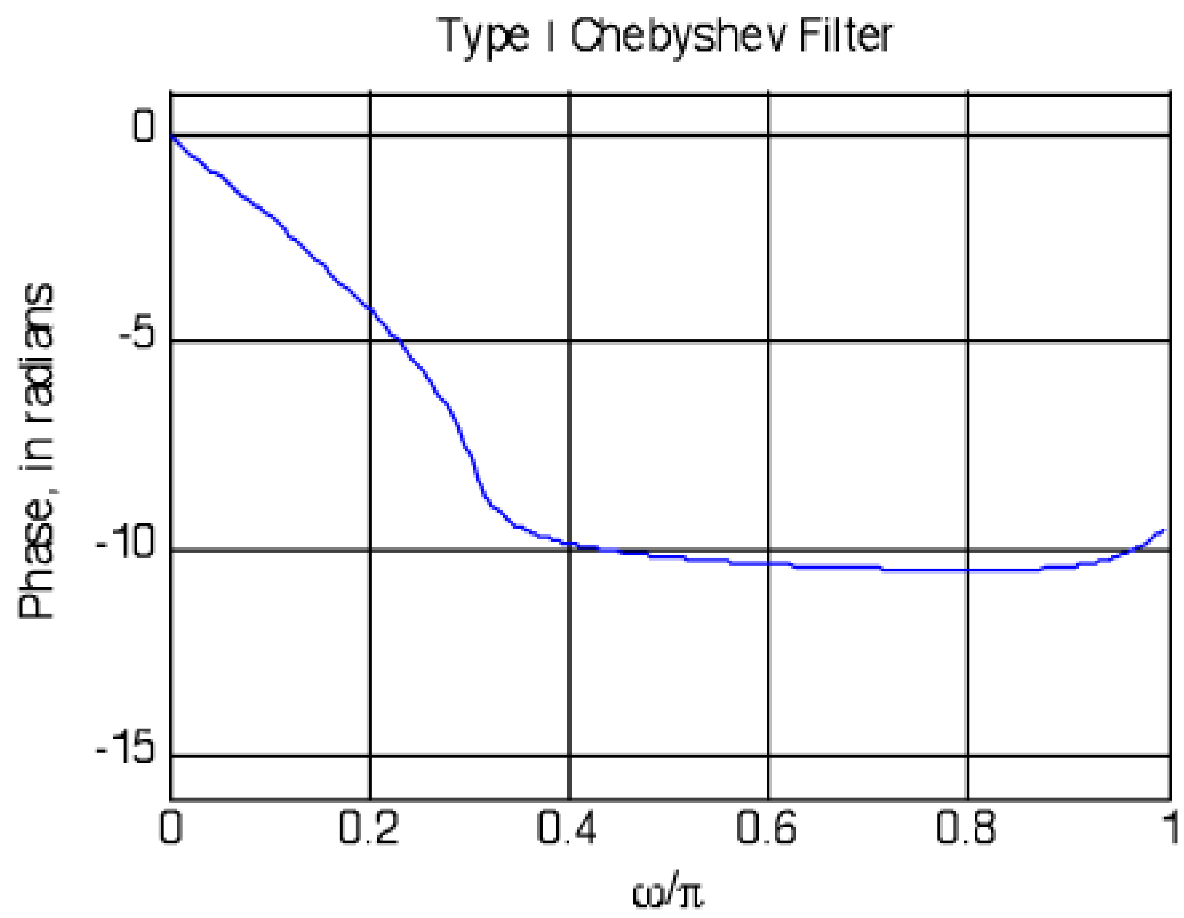




M9.7

```
% Problem M9.07
Wp = input(' Passband edge in radians = ');
Ws = input(' Stopband edge in radians = ');
Rp = input('Passband ripple in dB = ');
Rs = input('Stopband minimum attenuation in dB = ');
[N, Wn] = cheb1ord(Wp,Ws,Rp,Rs, 's');
[B, A] = cheby1(N, 0.5, Wn, 's');
[num, den] = impinvar(B, A, 1);
[h, omega] = freqz(num,den,256);
figure(1);
plot(omega/pi, 20*log10(abs(h)));grid;
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('Type I Chebyshev Filter');
axis([0 1 -60 5]);
figure(2);
plot(omega/pi, unwrap(angle(h))); grid
axis([0 1 -16 1]);
xlabel('\omega/\pi'); ylabel('Phase, in radians');
title('Type I Chebyshev Filter');
```





M9.8 Impulse invariance method:

$\frac{1}{k} = \frac{\Omega_s}{\Omega_p} = 1.6667$, $\frac{1}{k_1} = 665.03$. Hence, $N \approx \frac{2\log_{10}(4/k_1)}{\log_{10}(1/\rho)}$, where

$k' = \sqrt{1-k^2}$, $\rho_0 = \frac{1-\sqrt{k'}}{2\left(1+\sqrt{k'}\right)}$, $\rho = \rho_0 + 2\rho_0^5 + 15\rho_0^9 + 150\rho_0^{13}$. In our case,

$k' = 0.8$, $\rho_0 = 0.02786$, $\rho = 0.02786$. Hence, $N \approx 4.405 \to 5$.

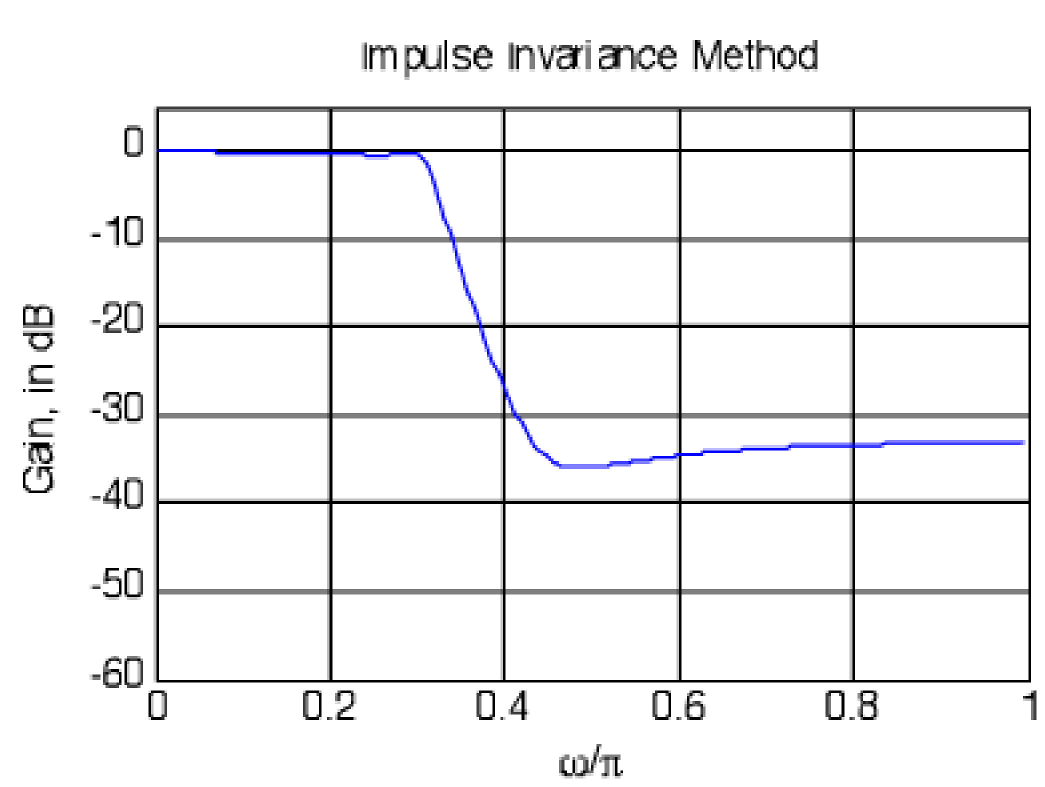
Bilinear transformation method:

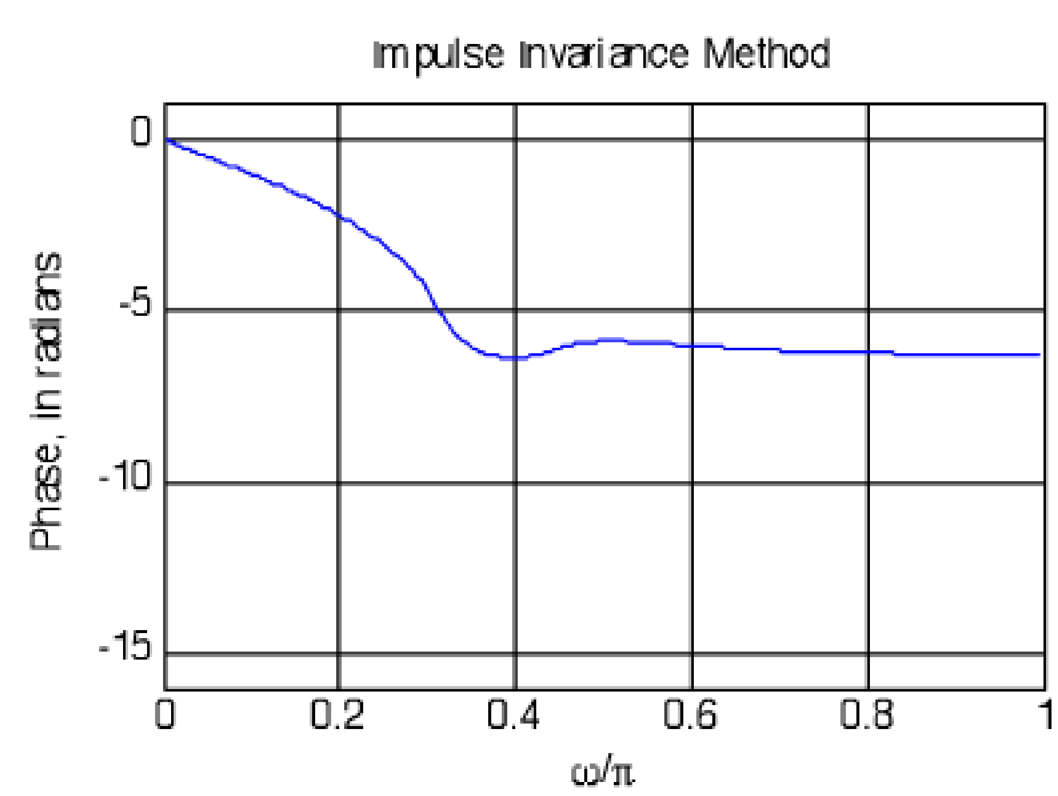
$\frac{1}{k} = \frac{\Omega_s}{\Omega_p} = 1.9627$, $\frac{1}{k_1} = 665.03$. Hence, $N \approx \frac{2\log_{10}(4/k_1)}{\log_{10}(1/\rho)}$, where

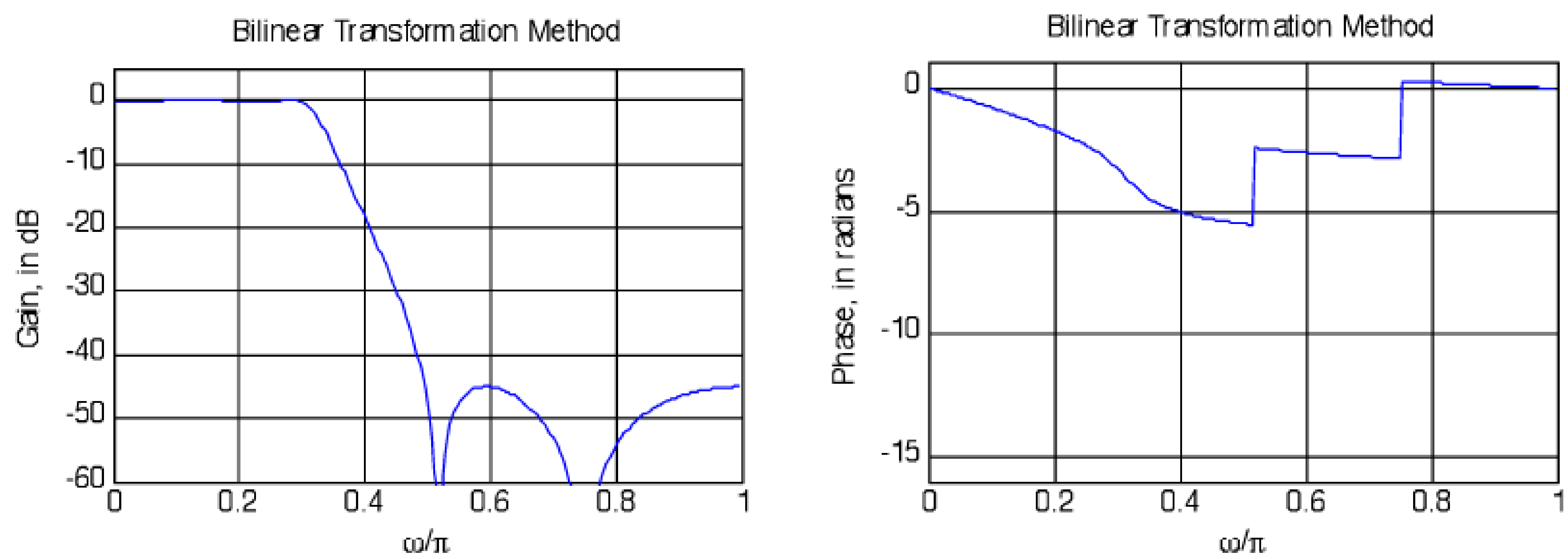
$k' = 0.8605$, $\rho_0 = 0.01878$, $\rho = 0.01878$. Hence, $N \approx 3.97 \to 4$

```
% Problem M9_08
% Impulse Invariance Method
[z, p, k] = ellipap(5, 0.3, 45);
```

```
[B, A] = zp2tf(z, p, k);
[BT, AT] = lp2lp(B, A, 0.9425);
[num, den] = impinvar(BT, AT, 1);
[h, omega] = freqz(num,den,256);
figure(1);
plot(omega/pi, 20*log10(abs(h)));grid;
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('Impulse Invariance Method');
axis([0 1 -60 5]);
figure(2);
plot(omega/pi, unwrap(angle(h))); grid
axis([0 1 -16 1]);
xlabel('\omega/\pi'); ylabel('Phase, in radians');
title('Impulse Invariance Method');

% Bilinear Transformation Method
[z, p, k] = ellipap(4, 0.3, 45);
[B, A] = zp2tf(z, p, k);
[BT, AT] = lp2lp(B, A, 0.5095);
[num, den] = bilinear(BT, AT, 0.5);
 [h, omega] = freqz(num,den,256);
figure(3);
plot(omega/pi, 20*log10(abs(h)));grid;
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('Bilinear Transformation Method');
axis([0 1 -60 5]);
figure(4);
plot(omega/pi, unwrap(angle(h))); grid
axis([0 1 -16 1]);
xlabel('\omega/\pi'); ylabel('Phase, in radians');
title('Bilinear Transformation Method');
```

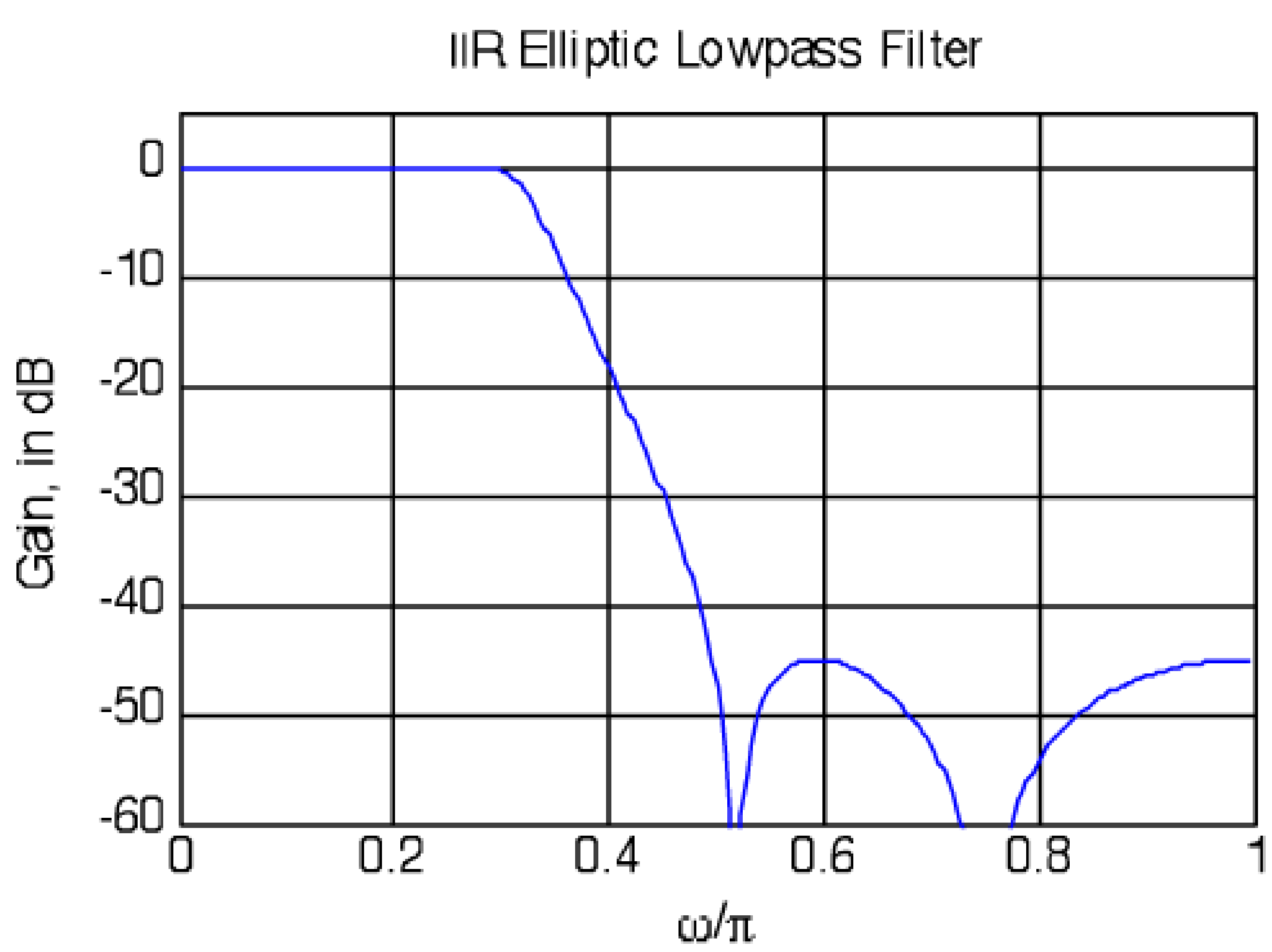






M9.9

```
% Program M9_09
Wp = input('Normalized passband edge = ');
Ws = input('Normalized stopband edge = ');
Rp = input('Passband ripple in dB = ');
Rs = input('Stopband ripple in dB = ');
[N, Wn] = ellipord(Wp, Ws, Rp, Rs);
[b, a] = ellip(N, Rp, Rs, Wn);
[h, omega] = freqz(b, a, 256);
plot(omega/pi, 20*log10(abs(h)));grid;
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('IIR Elliptic Lowpass Filter');
axis([0 1 -60 5]);
```



M9.10 $F_T = 2.0$ MHz, $F_p = 500$ kHz, $F_s = 250$ kHz.

$\alpha_p = 0.5$ dB, $\alpha_s = 50$ dB

(a) $\omega_p = \dfrac{2\pi F_p}{F_T} = 1.5708, \omega_s = \dfrac{2\pi F_s}{F_F} = 0.7854.$

$\hat{\Omega}_p = \tan\left(\dfrac{\omega_p}{2}\right) = 1, \hat{\Omega}_s = \tan\left(\dfrac{\omega_s}{2}\right) = 0.4142.$

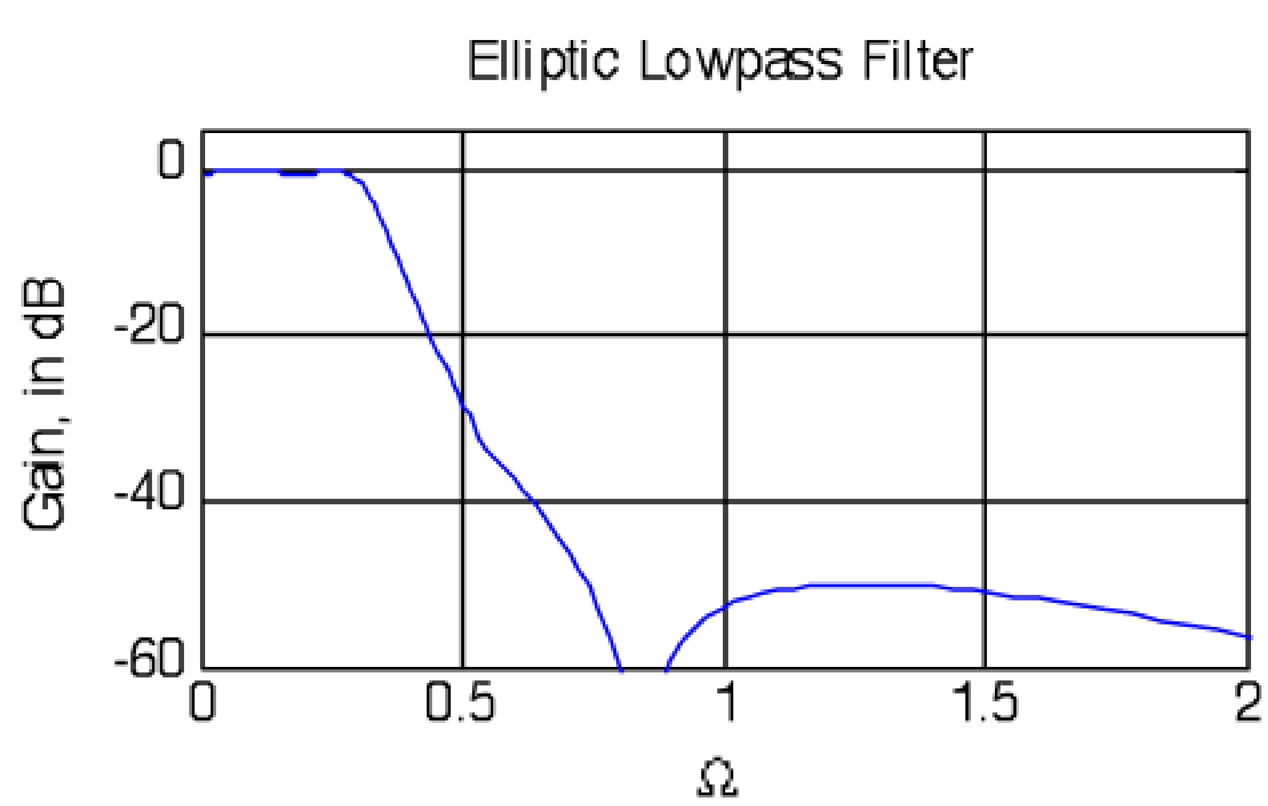
The analog highpass filter specifications are thus:

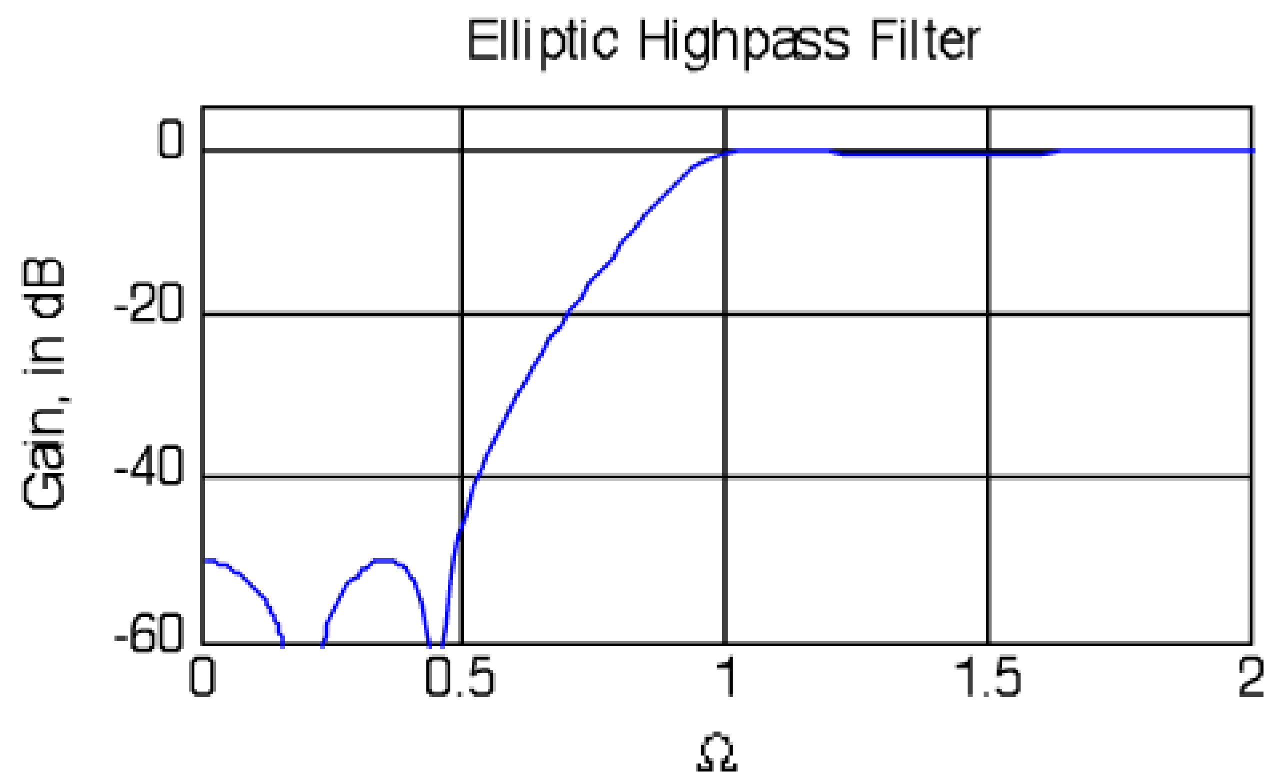
$\hat{\Omega}_p = 1$, $\hat{\Omega}_s = 0.4142$, $\alpha_p = 0.5$ dB, $\alpha_s = 50$ dB

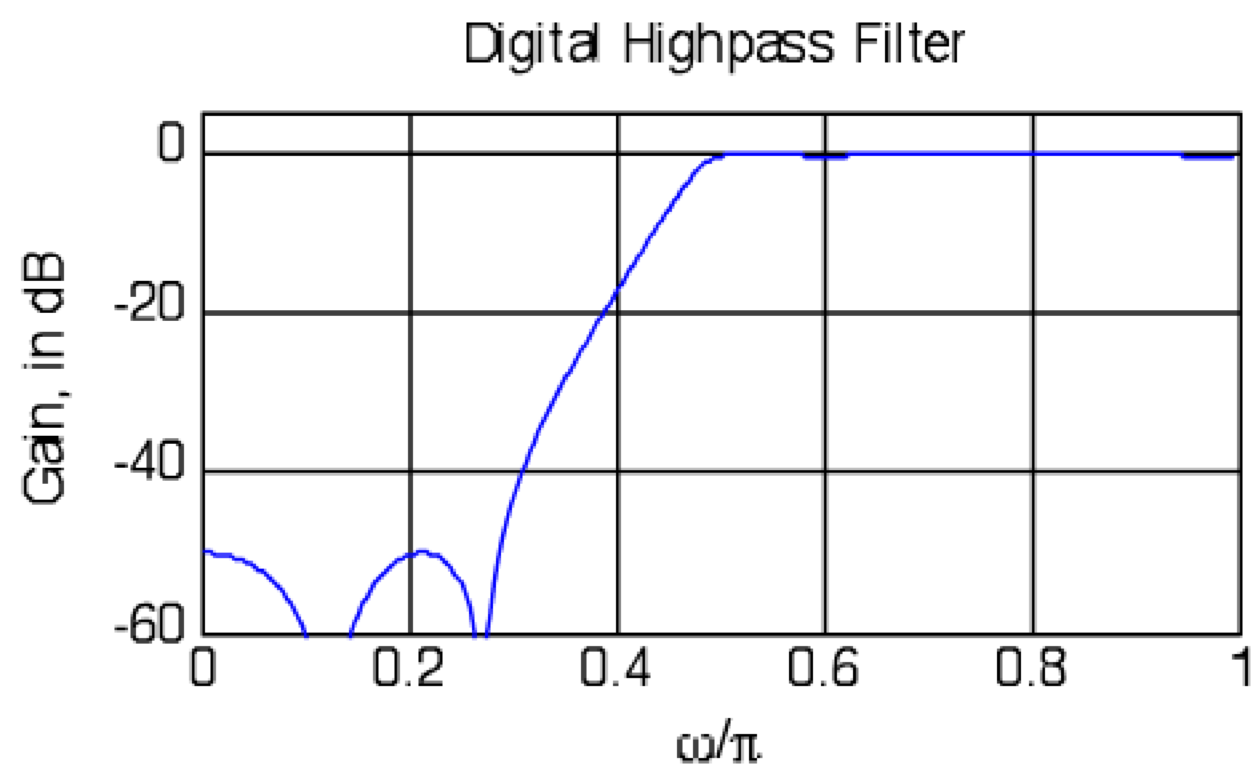
(b) The analog lowpass filter specifications are $\Omega_p = 1, \Omega_s = \dfrac{\hat{\Omega}_p}{\hat{\Omega}_s} = 2.4143$ and

$\alpha_p = 0.5$ dB, $\alpha_s = 50$ dB.

```
% Problem M9.10
close all;
clear;
clc;
[N, Wn] = ellipord(1, 2.4143, 0.5, 50, 's');
[B, A] = ellip(N, 0.5, 50, Wn, 's');
[BT, AT] = lp2hp(B, A, 1.0000);
[num, den] = bilinear(BT, AT, 0.5);

[H, W] = freqs(B, A, 256);
[HT, W] = freqs(BT, AT, 256);
[h, w] = freqz(num, den, 256);

figure; plot(W, 20*log10(abs(H)));grid;
xlabel('\Omega'); ylabel('Gain, in dB');
title('Elliptic Lowpass Filter');
axis([0 2 -60 5]);
figure; plot(W, 20*log10(abs(HT)));grid;
xlabel('\Omega'); ylabel('Gain, in dB');
title('Elliptic Highpass Filter');
axis([0 2 -60 5]);
figure; plot(w/pi, 20*log10(abs(h)));grid;
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('Digital Highpass Filter');
axis([0 1 -60 5]);
```







(c) Analog lowpass transfer function coefficient are obtained by displaying `B` and `A`:

$$H_{LP}(s) = \frac{0.003164\,s^4 + 0.09833 s^2 + 0.4074}{s^4 + 1.1865 s^3 + 1.7375\,s^2 + 1.0609\,s + 0.4316}.$$

Analog highpass transfer function coefficient are obtained by displaying `BT` and `AT`:

$$H_{HP}(s) = \frac{0.6441 s^4 + 0.2279 s^2 + 0.007331}{s^4 + 2.4582 s^3 + 4.0261 s^2 + 2.7495\,s + 2.3172}.$$

Digital highpass transfer function coefficient are obtained by displaying `num` and `den`:

$$G_{HP}(z) = \frac{0.09396 - 0.2985\,z^{-1} + 0.4185 z^{-2} - 0.2985\,z^{-3} + 0.09396 z^{-4}}{1 + 0.4662\,z^{-1} + 0.9442\,z^{-2} + 0.3734\,z^{-3} + 0.1702 z^{-4}}.$$

M9.11 $F_T = 12$ kHz, $F_{p1} = 1.5$ kHz, $F_{p2} = 2.5$ kHz, $F_{s1} = 900$ Hz, $F_{s2} = 4$ kHz, $\alpha_p = 0.7$ dB, $\alpha_s = 42$ dB

(a) $\omega_{p1} = \dfrac{2\pi F_{p1}}{F_T} = 0.7854$, $\omega_{p2} = \dfrac{2\pi F_{p2}}{F_T} = 1.309$, $\omega_{s1} = \dfrac{2\pi F_{s1}}{F_T} = 0.4712$, and

$\omega_{s2} = \dfrac{2\pi F_{s2}}{F_T} = 2.094$, with $\hat{\Omega}_{p1} = \tan\left(\dfrac{\omega_{p1}}{2}\right) = 0.4142$,

$\hat{\Omega}_{p2} = \tan\left(\frac{\omega_{p2}}{2}\right) = 0.7673$, $\hat{\Omega}_{s1} = \tan\left(\frac{\omega_{s1}}{2}\right) = 0.2401$, and

$\hat{\Omega}_{s2} = \tan\left(\frac{\omega_{s2}}{2}\right) = 1.731$. $B_\omega = \hat{\Omega}_{p2} - \hat{\Omega}_{p1} = 0.3531$,

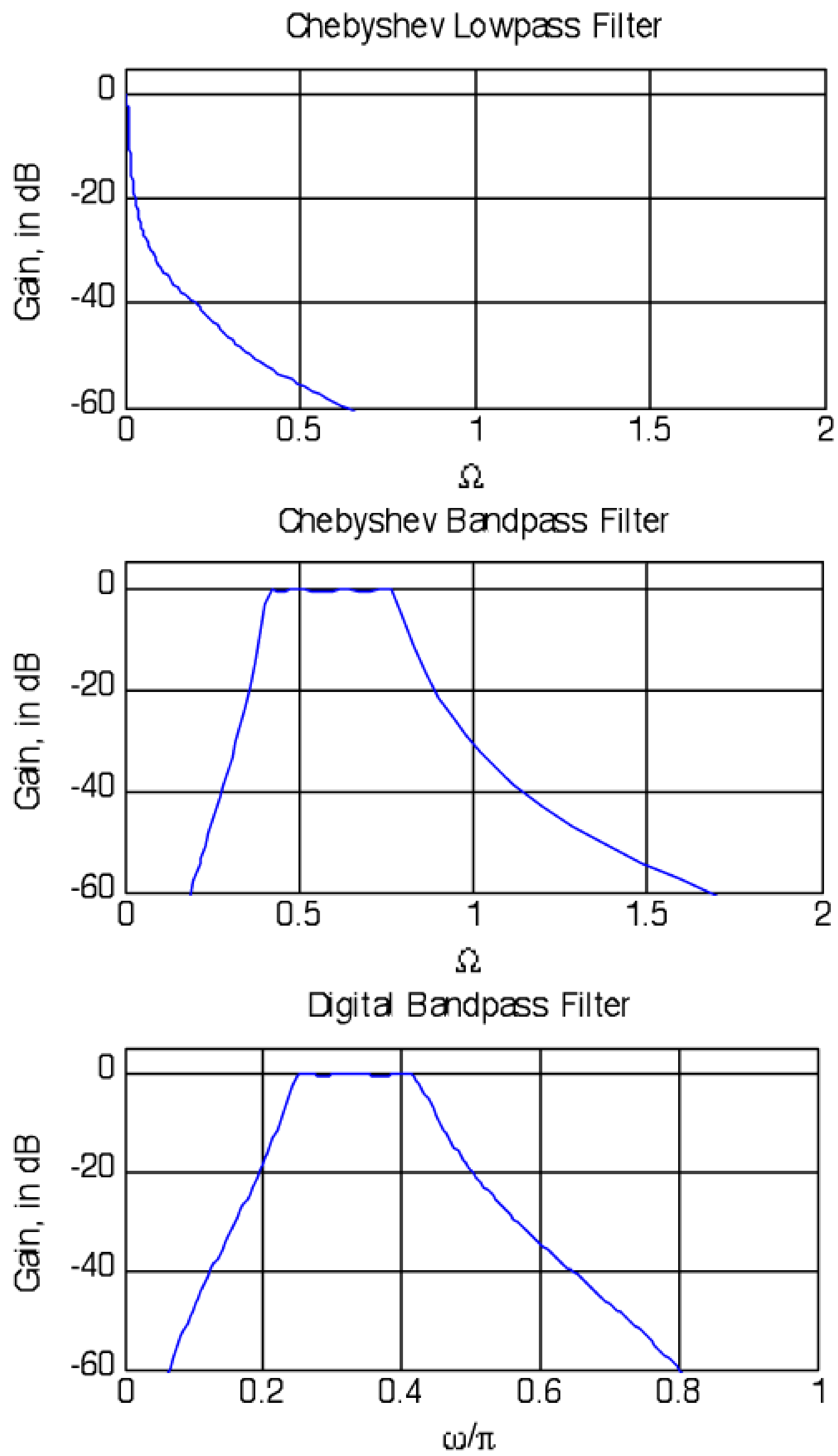
$\hat{\Omega}_0^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2} = 0.3178 \neq \hat{\Omega}_{s1}\hat{\Omega}_{s2} = 0.4156$.

Therefore, we choose to modify the upper stopband edge to: $\hat{\Omega}_{s2} = \frac{\hat{\Omega}_0^2}{\hat{\Omega}_{s1}} = 1.3236$.

The analog bandpass filter specifications are $\hat{\Omega}_{p1} = 0.4142$ rad, $\hat{\Omega}_{p2} = 0.7673$ rad, $\hat{\Omega}_{s1} = 0.2401$ rad, $\hat{\Omega}_{s2} = 1.3236$ rad,
$\alpha_p = 0.7$dB, $\alpha_s = 42$dB.

(b) $\Omega_s = \frac{\hat{\Omega}_0^2 - \hat{\Omega}_{s1}^2}{\hat{\Omega}_{s1} \cdot B_\omega} = 3.0686$, so the analog lowpass specifications are thus:
$\Omega_p = 1$ rad, $\Omega_s = 3.0686$ rad, $\alpha_p = 0.7$dB, $\alpha_s = 42$dB.

```
% Problem M9.11
[N, Wn] = cheb1ord(1, 3.0686, 0.7, 42, 's');
[B, A] = cheby1(N, 0.7, Wn, 's');
[BT, AT] = lp2bp(B, A, sqrt(0.3178), 0.3531);
[num, den] = bilinear(BT, AT, 0.5);

[H, W] = freqs(B, A, 256);
[HT, W] = freqs(BT, AT, 256);
[h, w] = freqz(num, den, 256);

figure; plot(W, 20*log10(abs(H)));grid;
xlabel('\Omega'); ylabel('Gain, in dB');
title('Chebyshev Lowpass Filter');
axis([0 2 -60 5]);
figure; plot(W, 20*log10(abs(HT)));grid;
xlabel('\Omega'); ylabel('Gain, in dB');
title('Chebyshev Bandpass Filter');
axis([0 2 -60 5]);
figure; plot(w/pi, 20*log10(abs(h)));grid;
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('Digital Bandpass Filter');
axis([0 1 -60 5]);
```



(c) Analog lowpass transfer function coefficient are obtained by displaying `B` and `A`:

$$H_{LP}(s) = \frac{0.2989}{s^4 + 1.0776\,s^3 + 1.5806\,s^2 + 0.8799\,s + 0.3240}.$$

Analog bandpass transfer function coefficient are obtained by displaying `BT` and `AT`:

$$H_{BP}(s) = \frac{0.004646\,s^4}{s^8 + 0.3805s^7 + 1.4683s^6 + 0.4015\,s^5 + 0.7363\,s^4 + 0.1276\,s^3 + 0.1483\,s^2 + 0.0122\,s + 0.0102}.$$

Digital bandpass transfer function coefficient are obtained by displaying `den` and `num`:

$$G_{BP}(z) = \frac{0.001084 - 0.004337z^{-2} + 0.006506z^{-4} - 0.004337z^{-6} + 0.001084z^{-8}}{1 - 3.7238z^{-1} + 8.4592z^{-2} - 12.524z^{-3} + 13.761z^{-4} - 10.884z^{-5} + 6.3869z^{-6} - 2.4367z^{-7} + 0.5697z^{-8}}.$$

M9.12 $F_T = 9$ kHz, $F_{p1} = 0.2$ kHz, $F_{p2} = 4$ kHz, $F_{s1} = 2.5$ kHz, $F_{s2} = 3.2$ kHz, $\alpha_p = 1.2$dB, $\alpha_s = 35$ dB

(a) $\omega_{p1} = \frac{2\pi F_{p1}}{F_T} = 0.1396$, $\omega_{p2} = \frac{2\pi F_{p2}}{F_T} = 2.7925$, $\omega_{s1} = \frac{2\pi F_{s1}}{F_T} = 1.7453$, and $\omega_{s2} = \frac{2\pi F_{s2}}{F_T} = 2.234$, with

$\hat{\Omega}_{p1} = \tan\left(\frac{\omega_{p1}}{2}\right) = 0.06991$, $\hat{\Omega}_{p2} = \tan\left(\frac{\omega_{p2}}{2}\right) = 5.3708$, $\hat{\Omega}_{s1} = \tan\left(\frac{\omega_{s1}}{2}\right) = 1.1917$,

and $\hat{\Omega}_{s2} = \tan\left(\frac{\omega_{s2}}{2}\right) = 2.0502$.

$B_\omega = \hat{\Omega}_{s2} - \hat{\Omega}_{s1} = 0.8585$

$\hat{\Omega}_0^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2} = 0.3755 \neq \hat{\Omega}_{s1}\hat{\Omega}_{s2} = 2.4432$

Therefore, we choose to adjust the lower passband:

$\hat{\Omega}_{p1} = \frac{\hat{\Omega}_{s2}\hat{\Omega}_{s1}}{\hat{\Omega}_{p2}} = 0.4549$

The analog bandstop filter specifications are thus:

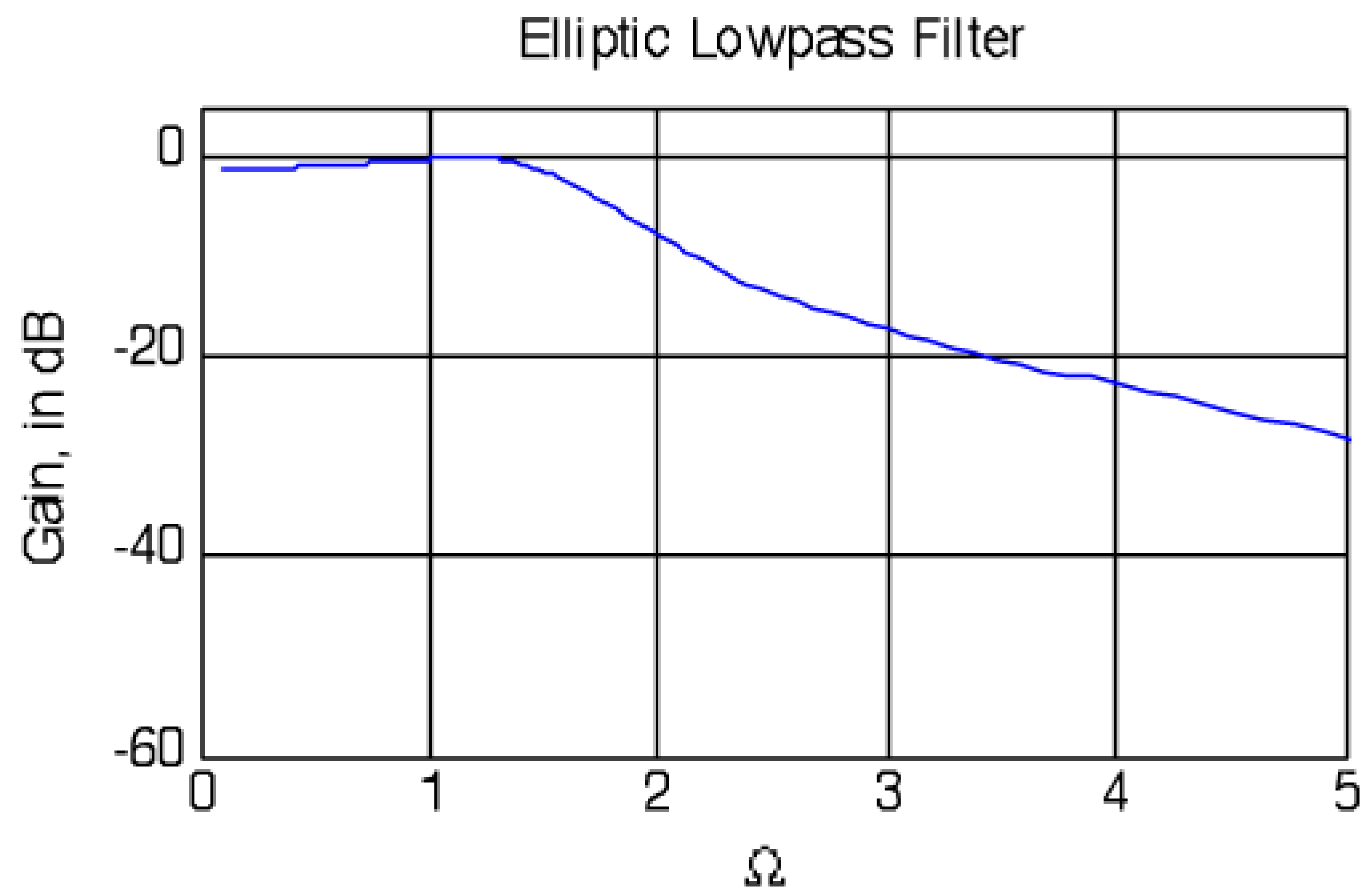
$\hat{\Omega}_{p1} = 0.4549$ rad, $\hat{\Omega}_{p2} = 5.3708$ rad, $\hat{\Omega}_{s1} = 1.1917$ rad, $\hat{\Omega}_{s2} = 2.0502$ rad, $\alpha_p = 1.2$ dB, $\alpha_s = 35$ dB.

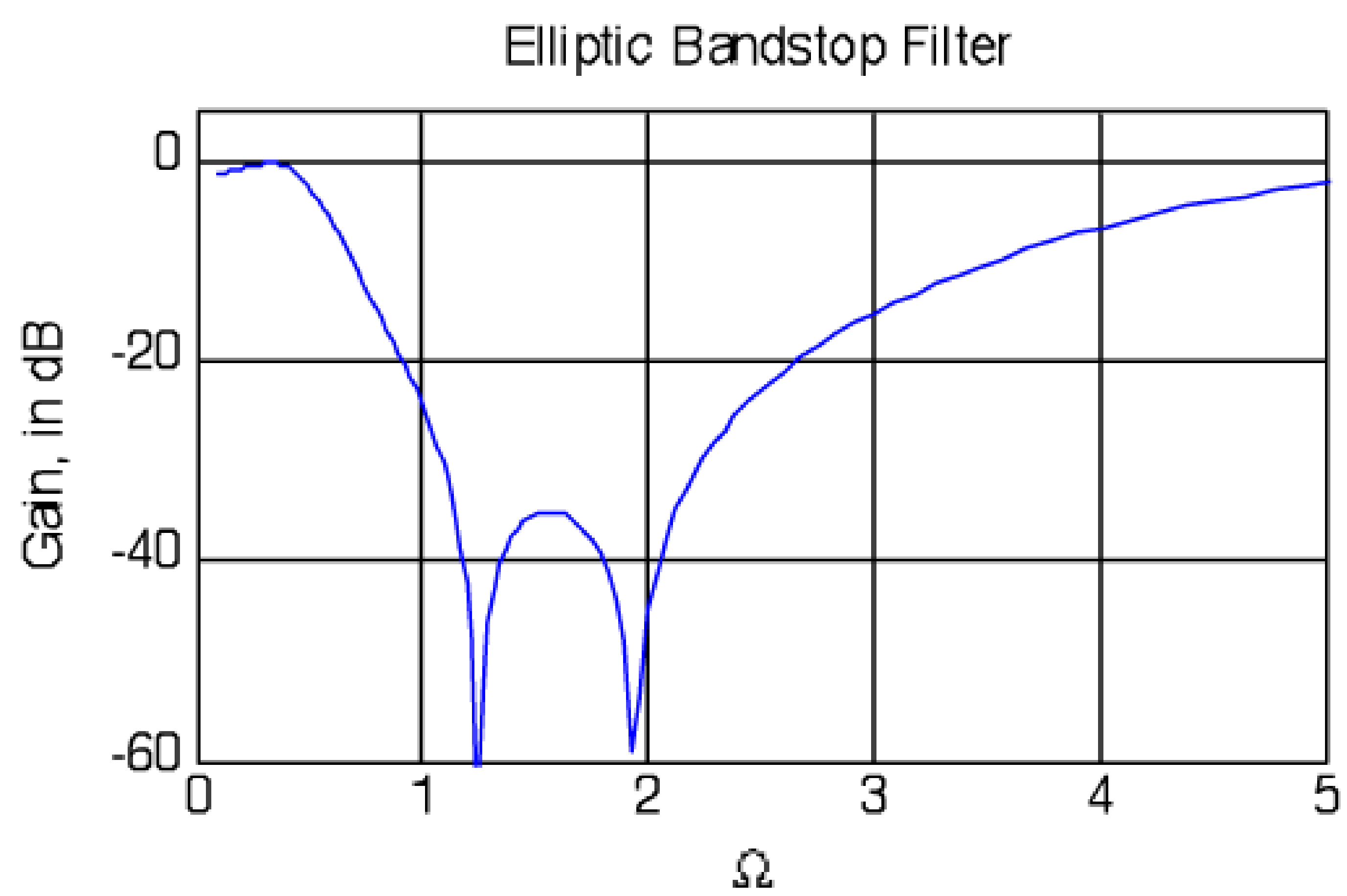
(b) Analog prototype LP filter:

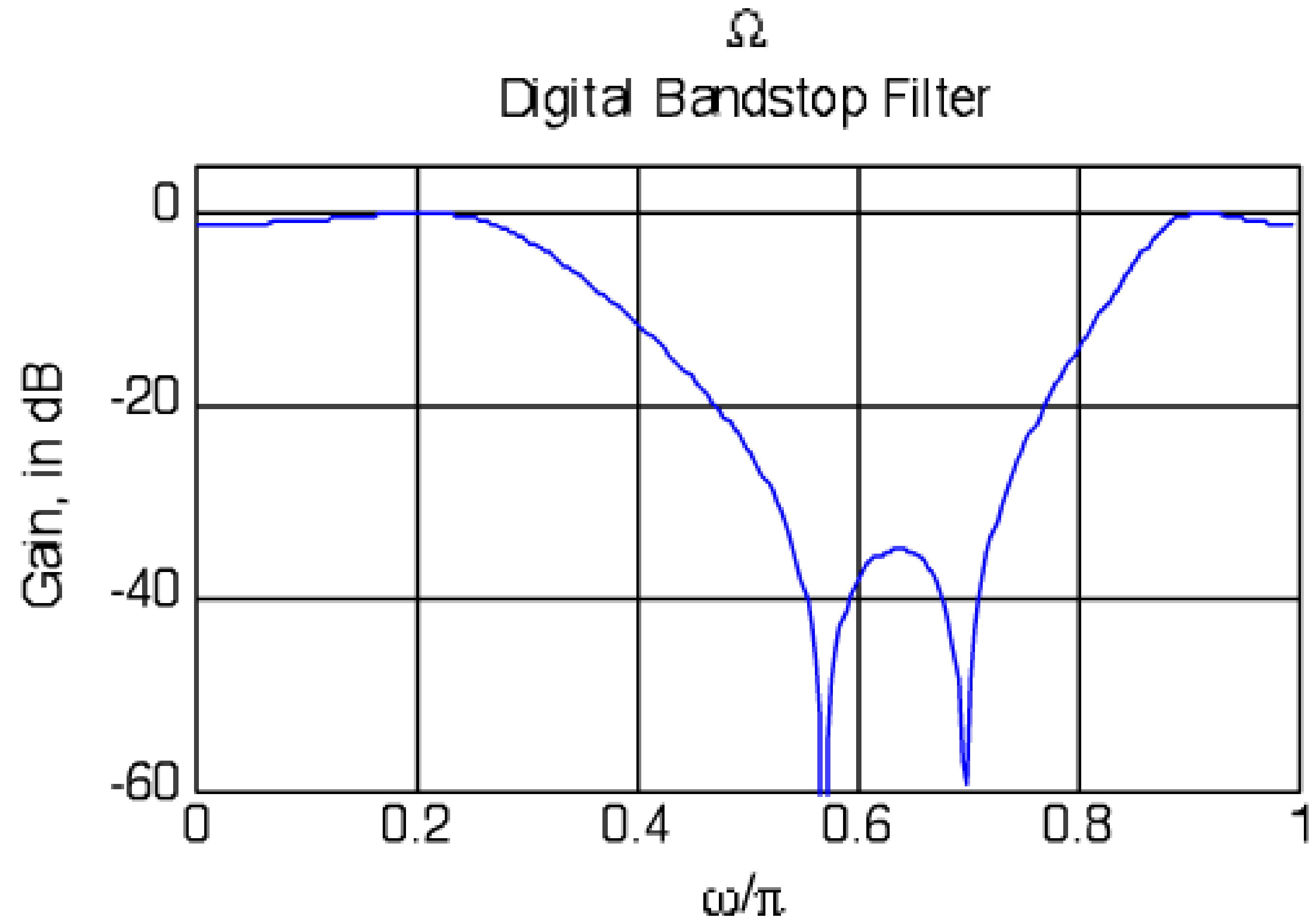
$\Omega_s = 1$ rad, $\Omega_p = \frac{\hat{\Omega}_{p1} \cdot B_\omega}{\hat{\Omega}_0^2 - \hat{\Omega}_{p1}^2} = 0.1746$ rad, $\alpha_p = 1.2$dB, $\alpha_s = 35$dB.

```
% Problem M9.12
close all;
clear;
clc;
[N, Wn] = ellipord(0.1746, 1, 1.2, 35, 's');
[B, A] = ellip(N, 1.2, 35, Wn, 's');
[BT, AT] = lp2bs(B, A, sqrt(2.4332), 0.8585);
[num, den] = bilinear(BT, AT, 0.5);

[H, W] = freqs(B, A, 256);
[HT, W] = freqs(BT, AT, 256);
[h, w] = freqz(num, den, 256);

figure; plot(W, 20*log10(abs(H)));grid;
xlabel('\Omega'); ylabel('Gain, in dB');
title('Elliptic Lowpass Filter');
```

```
axis([0 5 -60 5]);
figure; plot(W, 20*log10(abs(HT)));grid;
xlabel('\Omega'); ylabel('Gain, in dB');
title('Elliptic Bandstop Filter');
axis([0 5 -60 5]);
figure; plot(w/pi, 20*log10(abs(h)));grid;
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('Digital Bandstop Filter');
axis([0 1 -60 5]);
```







(c) Analog lowpass transfer function coefficient are obtained by displaying `B` and `A`:

$$H_{LP}(s) = \frac{0.01779\,s^2 + 02729}{s^2 + 0.1758\,s + 0.03133}.$$

Analog bandstop transfer function coefficients are obtained by displaying `BT` and `AT`:

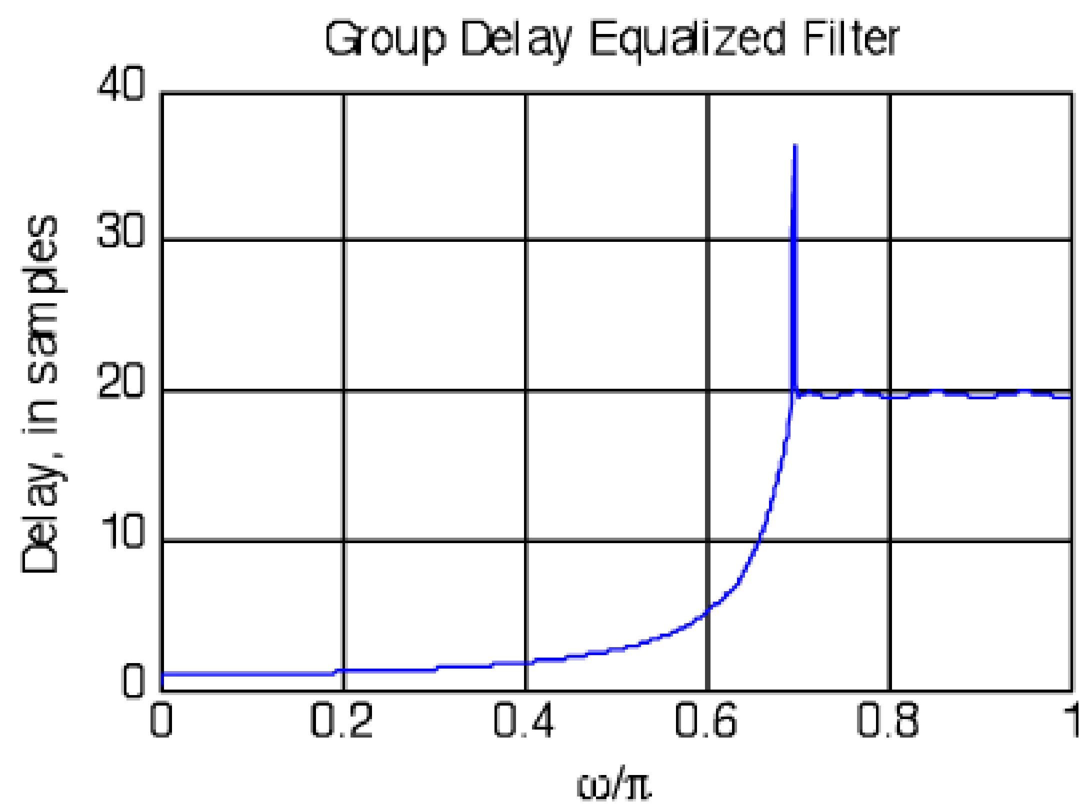
$$H_{BS}(s) = \frac{0.8710\,s^4 + 4.657\,s^2 + 5.1565}{s^4 + 4.8174\,s^3 + 28.392\,s^2 + 11.722\,s + 5.9205}.$$

Digital bandstop transfer function coefficients are obtained by displaying `num` and `den`:

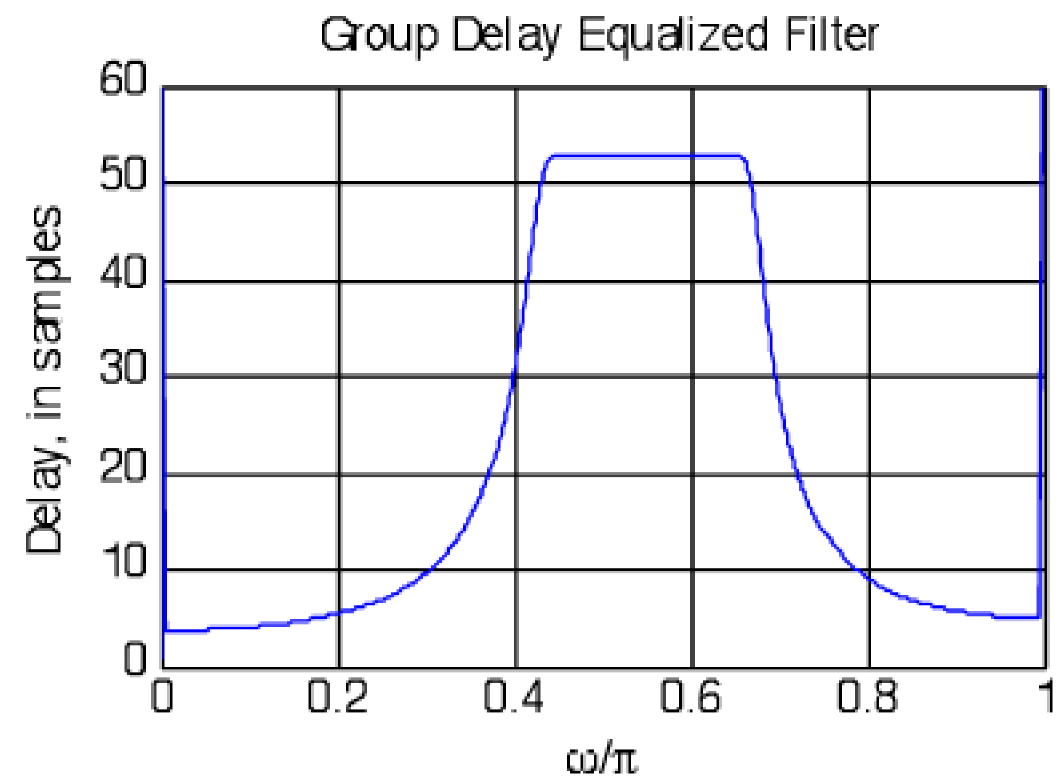
$$G_{BS}(z) = \frac{0.2061 + 0.3306\,z^{-1} + 0.5178\,z^{-2} + 0.3306\,z^{-3} + 0.2061\,z^{-4}}{1 + 0.6459\,z^{-1} - 0.2943\,z^{-2} + 0.1133\,z^{-3} + 0.3621\,z^{-4}}.$$

M9.13

```
% Program M9.13
close all;
clear;
clc;
Wp = 0.7; Ws = 0.5;
Rp = 1; Rs = 32;
[N,Wn] = cheb1ord(Wp,Ws,Rp,Rs);
[b,a] = cheby1(N,Rp,Wn,'high');
[h,omega] = freqz(b,a,256);
plot (omega/pi,20*log10(abs(h)));grid;
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('Type I Chebyshev Highpass Filter');

[GdH,w] = grpdelay(b,a,512);
plot(w/pi,GdH); grid
xlabel('\omega/\pi'); ylabel('Delay, in samples');
title('Original Filter Group Delay');

F = 0.7:0.001:1;
g = grpdelay(b,a,F,2);   % Equalize the passband
Gd = max(g)-g;
% Design the allpass delay equalizer
[num,den,tau] = iirgrpdelay(2*N, F, [0.7 1], Gd);
[GdA,w] = grpdelay(num,den,512);
plot(w/pi,GdH+GdA); grid
xlabel('\omega/\pi');ylabel('Delay, in samples');
title('Group Delay Equalized Filter');
```



M9.14
```
% Program M9.14
close all;
clear;
clc;

Wp = [0.45 0.65]; Ws = [0.3 0.75];
Rp = 1; Rs = 40;
[N,Wn] = buttord(Wp, Ws, Rp, Rs);
[b,a] = butter(N,Wn);
[h,omega] = freqz(b,a,256);
gain = 20*log10(abs(h));
plot (omega/pi,gain);grid;
xlabel('\omega/\pi'); ylabel('Gain, in dB');
title('IIR Butterworth Bandpass Filter');

[GdH,w] = grpdelay(b,a,512);
plot(w/pi,GdH); grid
xlabel('\omega/\pi'); ylabel('Delay, in samples');
title('Original Filter Group Delay');

F = 0.45:0.001:0.65;
g = grpdelay(b,a,F,2);    % Equalize the passband
Gd = max(g)-g;
% Design the allpass delay equalizer
[num,den,tau] = iirgrpdelay(2*N, F, [0.45 0.65], Gd);
[GdA,w] = grpdelay(num,den,512);
plot(w/pi,GdH+GdA); grid
xlabel('\omega/\pi');ylabel('Delay, in samples');
title('Group Delay Equalized Filter');
```


Group Delay Equalized Filter
Delay, in samples
ω/π
60
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
1