

Communication Systems

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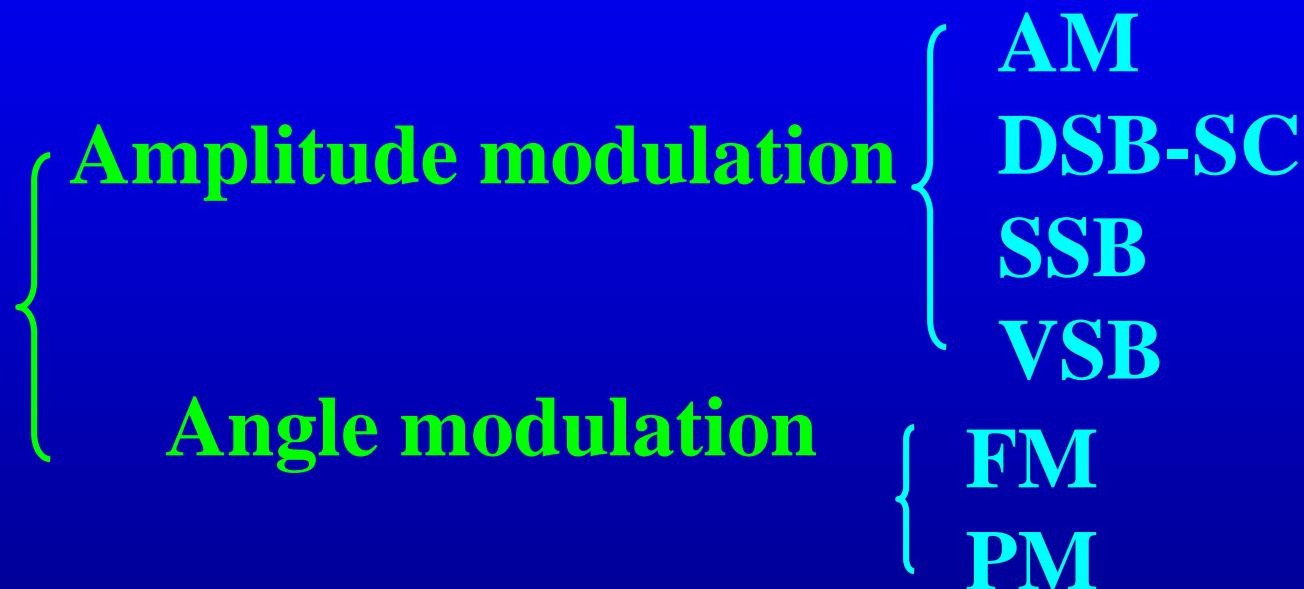
Chapter 2

Continuous-Wave Modulation

连续波调制

contents

- Time-domain and frequency-domain descriptions of continuous-wave modulation



- Noise performance pertaining to modulation schemes

2.1 Introduction

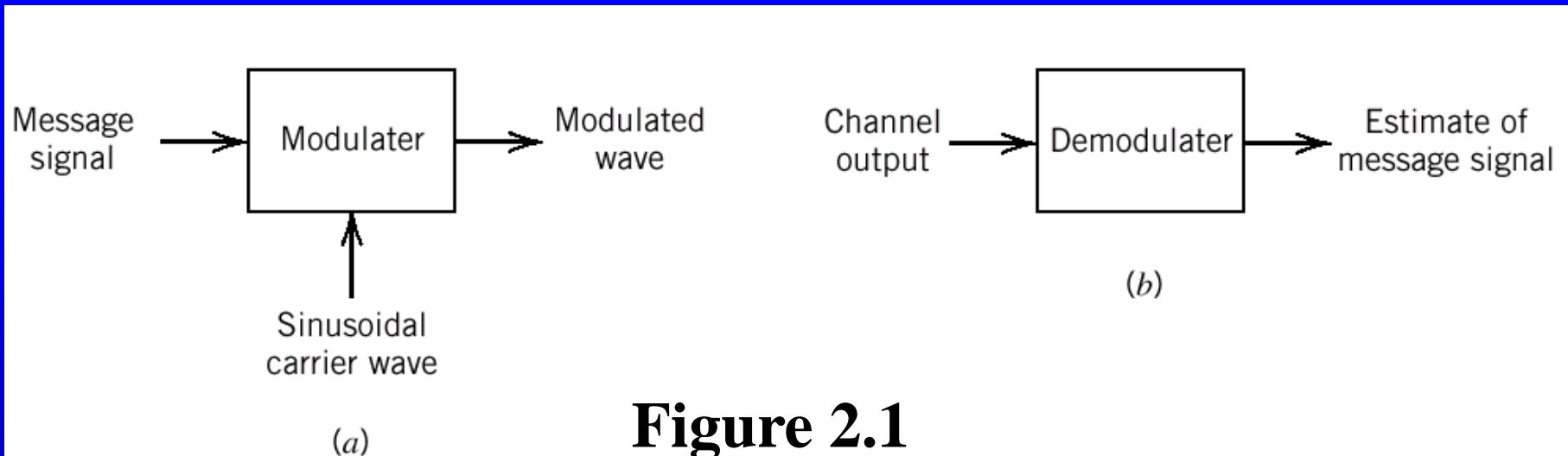


Figure 2.1

Components of a continuous-wave modulation system:
(a) transmitter, and (b) receiver.

In addition to the signal received from the transmitter, the receiver input includes channel noise. The degradation in receiver performance due to channel noise is determined by the type of modulation used. So, it is necessary to study various modulation types and their noise performance.

Basic concepts about modulation

Message signal: 消息信号

Information-bearing signal: 承载信息的信号

Baseband signal: 基带信号

Modulating signal/wave: 调制信号/波

Carrier: 载波, **sinusoidal wave:** 正弦波

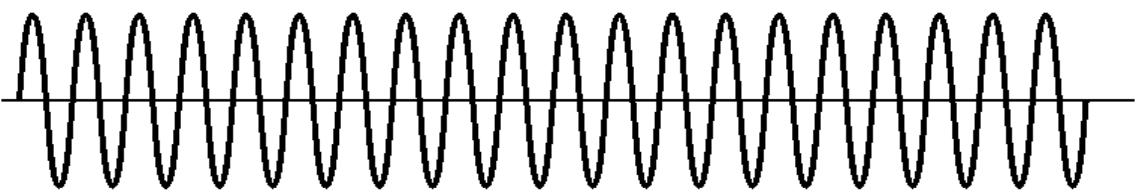
Modulated signal/wave: 已调信号/波

Modulation and Demodulation

- **Modulation:** refers to the process by which some characteristic of a carrier is varied in accordance with a modulating signal. It is also a process of shifting frequency range.
- **Demodulation:** is the reverse of the modulation process.

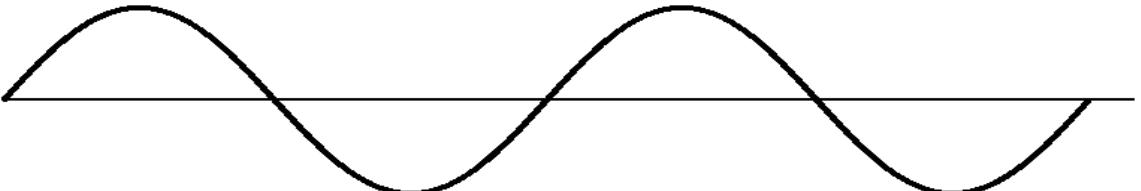
Demo for AM and FM signals

(a) Carrier wave



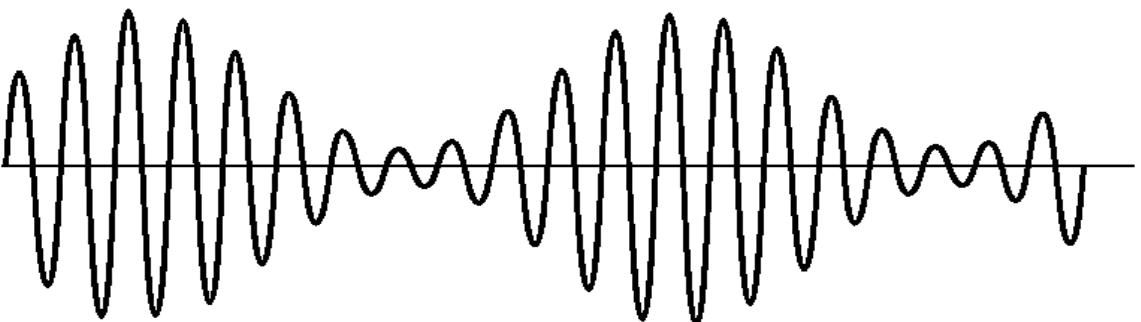
(a)

(b) Sinusoidal modulating signal



(b)

(c) Amplitude-modulated signal



(c)

(d) Frequency-modulated signal



(d)

Time →

2.2 Amplitude Modulation AM

Carrier wave $c(t)$: $c(t) = A_c \cos(2\pi f_c t)$

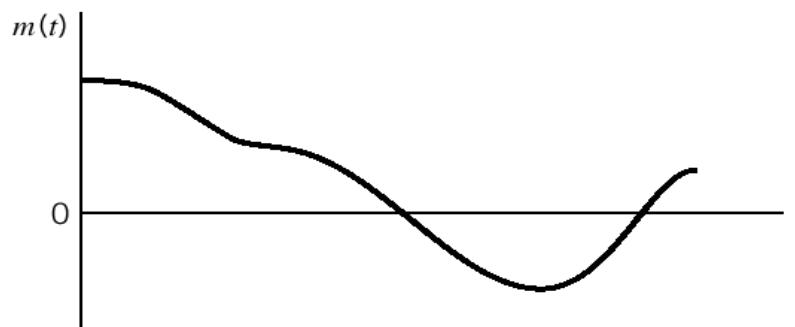
A_c : carrier amplitude f_c : carrier frequency

AM is defined as a process in which the amplitude of the carrier wave $c(t)$ is varied about a mean value, linearly with the baseband signal.

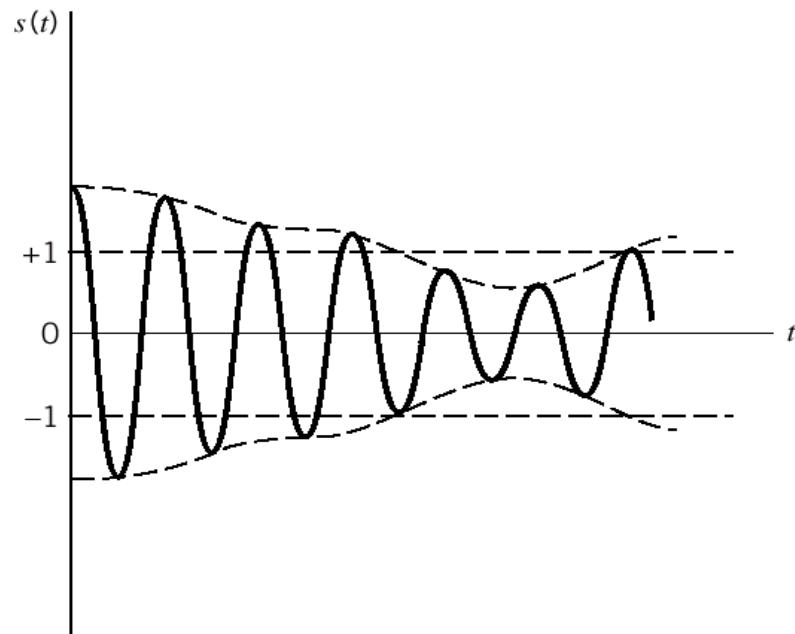
Amplitude-modulated wave $s(t)$

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

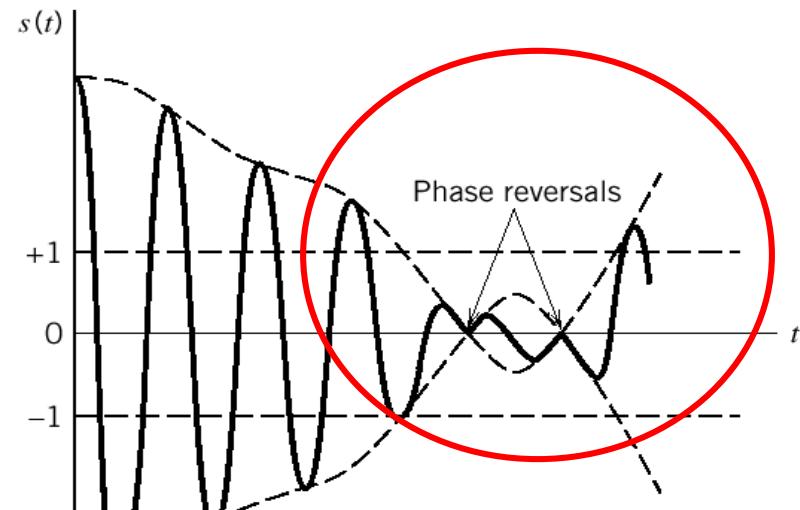
K_a : amplitude sensitivity 调幅灵敏度



Baseband signal $m(t)$



AM wave for $|k_a m(t)| < 1$



AM wave for $|k_a m(t)| > 1$

Figure 2.3

Illustrating the amplitude modulation process.

Observations from figure 2.3

- If $|k_a m(t)| < 1$ for all t, the envelope of modulated signal $s(t)$ is **linear with** the modulating signal $m(t)$. Therefore, we can use **envelope detector** (包络检波器) to recover the message signal in the receiver.
- If $|k_a m(t)| > 1$ for any t, carrier phase reversals (载波相位反转) happen. This is called as **overmodulation** (过调). In this case, the envelope of modulated signal $s(t)$ is **no longer linear with** the modulating signal $m(t)$. Message signal can not be recovered by envelope detector.

In AM, two requirements must be satisfied:

1. $f_c \gg W$, W: message bandwidth, so that the envelope of $s(t)$ can be visualized satisfactorily.
2. $|k_a m(t)| < 1 \dots \dots \dots$ for all t , so that overmodulation can be avoided.

Percentage modulation 调制百分比: $|k_a m(t)| \times 100\%$

调制百分比<1, 正常的AM调制

调制百分比=1, 临界调幅

调制百分比>1, 过度调幅, 过调

Descriptions of AM signal

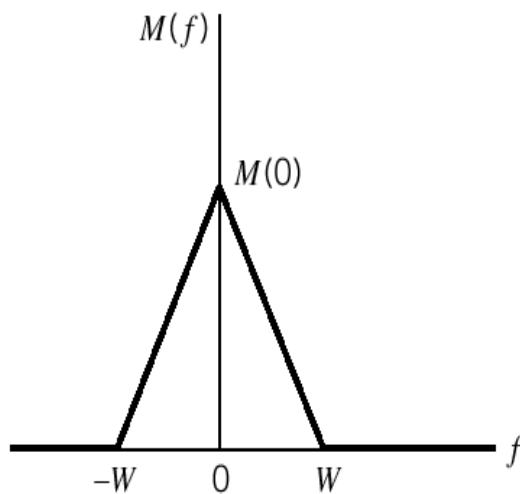
Time-domain description:

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

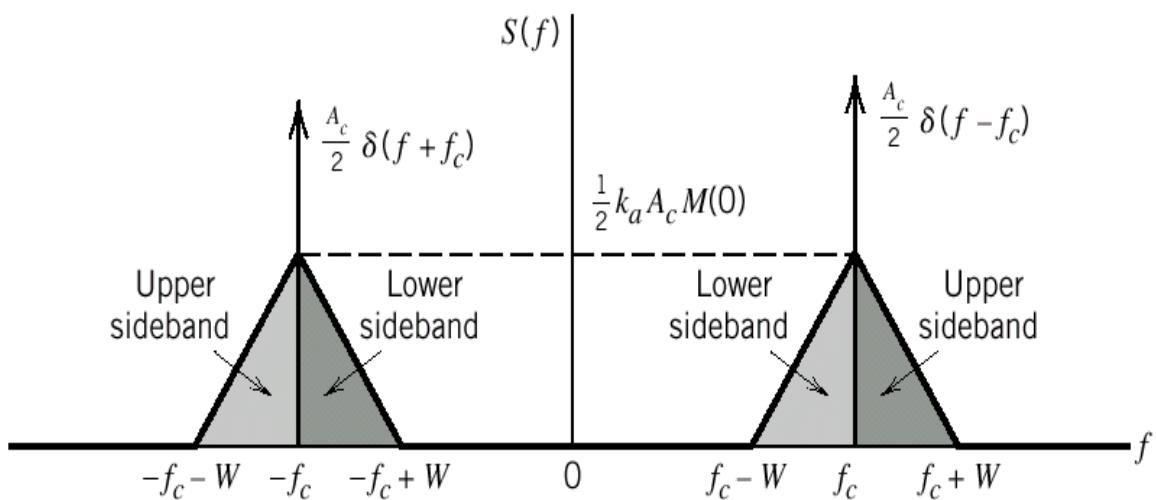


Frequency-domain description:

$$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2} [M(f - f_c) + M(f + f_c)]$$



(a) Spectrum of baseband signal



(b) Spectrum of AM wave

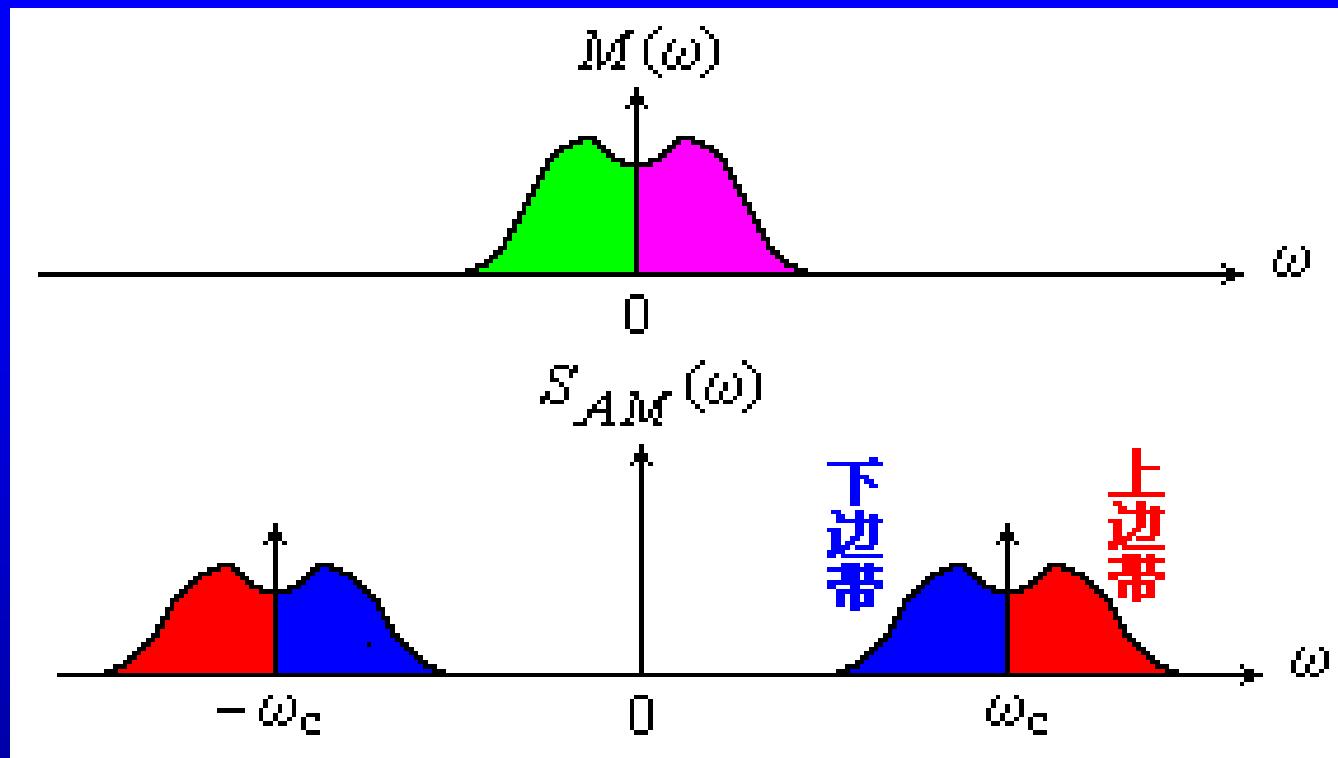
Negative frequency 负频率

Upper band and lower band 上边带 下边带

Transmission bandwidth 传输带宽, 调幅信号带宽

$$B_T = 2W$$

上边带和下边带 (Upper and lower sideband)



- 已调信号频谱中正频率高于 $+\omega_c$ 和负频率低于 $-\omega_c$ 的频谱合称为**上边带 (USB)**，正频率低于 $+\omega_c$ 和负频率高于 $-\omega_c$ 的频谱合称为**下边带 (LSB)**。
- 上、下边带都包含了调制信号的全部信息。

Descriptions of AM signal

Time-domain description:

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

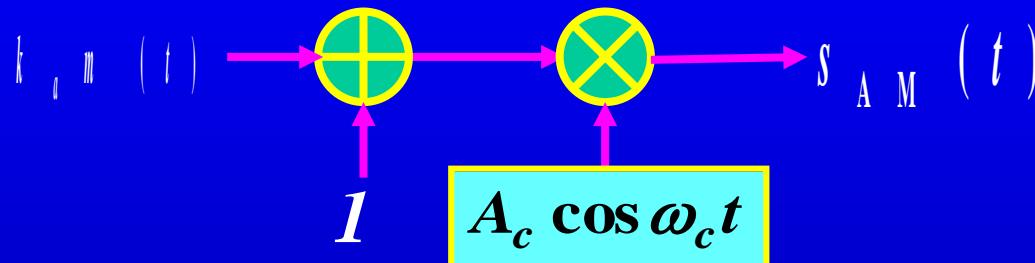
Frequency-domain description:

$$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2} [M(f - f_c) + M(f + f_c)]$$

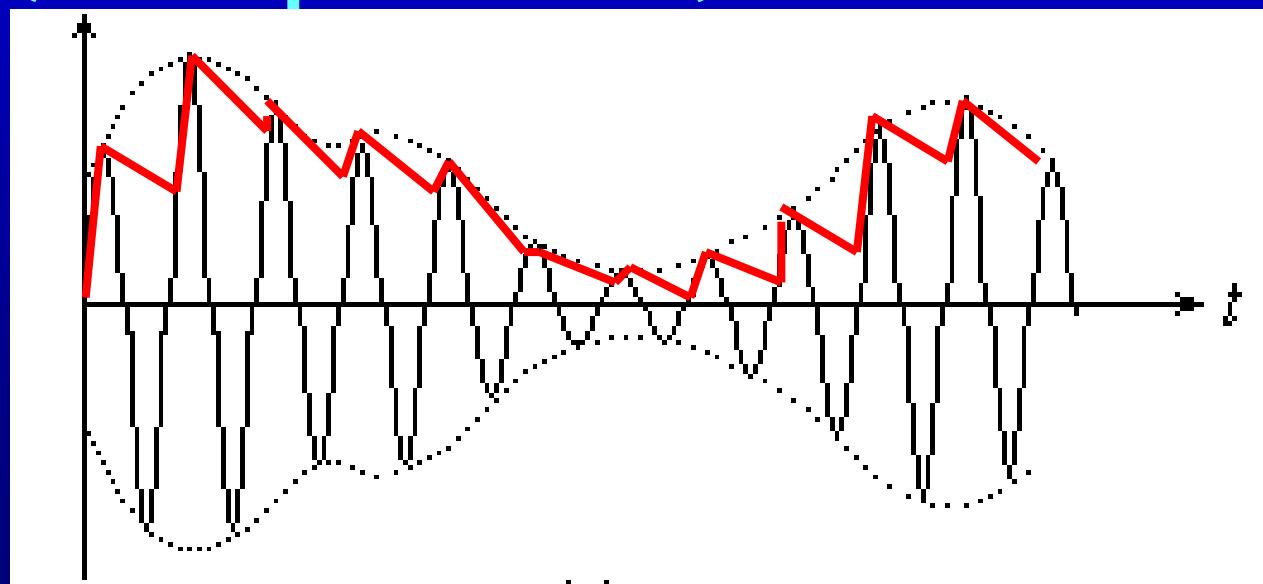
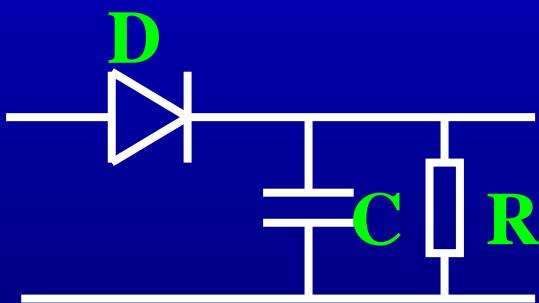
AM Generation and Detection

① AM信号的产生方法

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$



② 包络检测器 (envelope detector) :



Virtues and Limitations of AM 优缺点

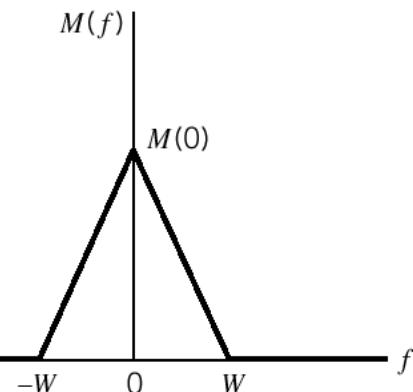
Virtue:

Simplicity of implementation

Limitations:

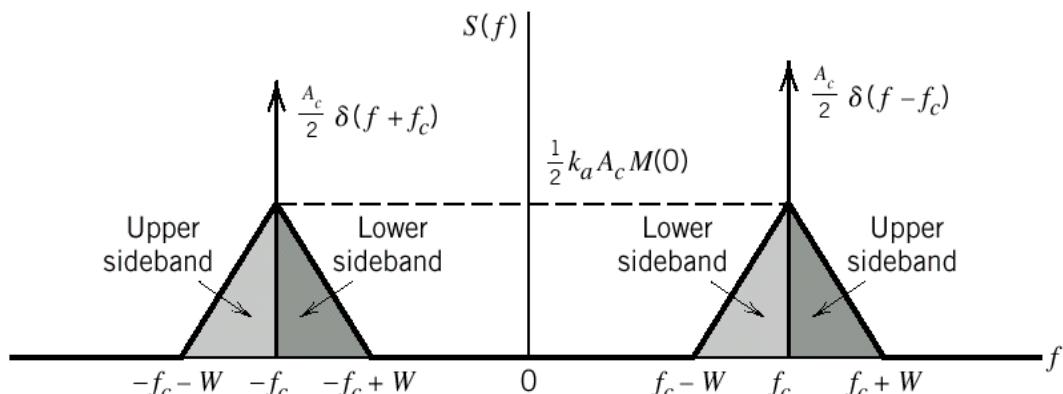
1. AM is wasteful of power
2. AM is wasteful of bandwidth

$$M(f)$$



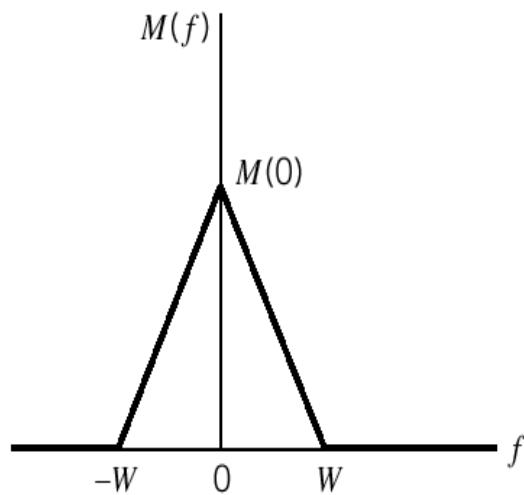
Upper sideband
Lower sideband

$$S(f)$$

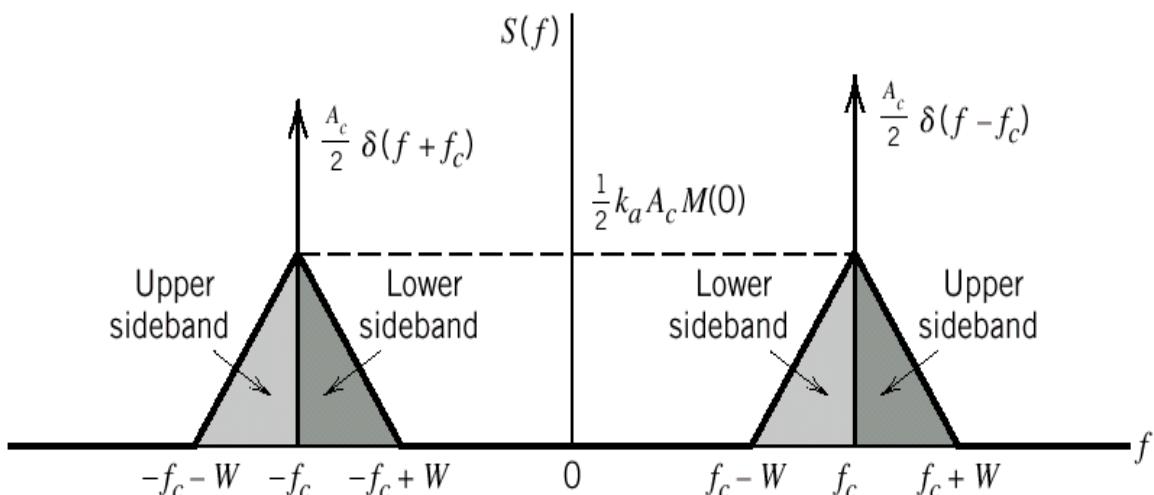


(a) Spectrum of baseband signal

(b) Spectrum of AM wave



(a) Spectrum of baseband signal



(b) Spectrum of AM wave

Upper band and lower band 上边带 下边带

Transmission bandwidth 传输带宽, 调幅信号带宽

$$B_T = 2W$$

How to overcome these limitations ?

Step 1: Suppress the carrier: →DSB-SC

Step 2: Modify the sidebands:→ SSB, VSB

抑制载波双边带调幅 **DSB-SC**: double sideband-suppressed carrier, where only the upper and lower sidebands are transmitted. No carrier frequency component.

单边带调幅 **SSB**: single sideband, where only one sideband (the lower sideband or the upper sideband) is transmitted.

残留边带调幅 **VSB**: vestigial sideband, where only a vestige of one of the sidebands and a corresponding modified version of the other sideband are transmitted.

2.3 linear Modulation Schemes

幅度调制是正弦载波的幅度随调制信号变化的过程，幅度已调信号的频谱为调制信号的频谱的平移及线性变换，一般又称为线性调制。DSB, SSB, VSB都是线性调制。严格意义上讲，AM不属于线性调制，因为AM信号中出现了载波分量。为了分析方便，纳入一起进行讨论。

Linear modulation is defined by: (Narrowband signal)

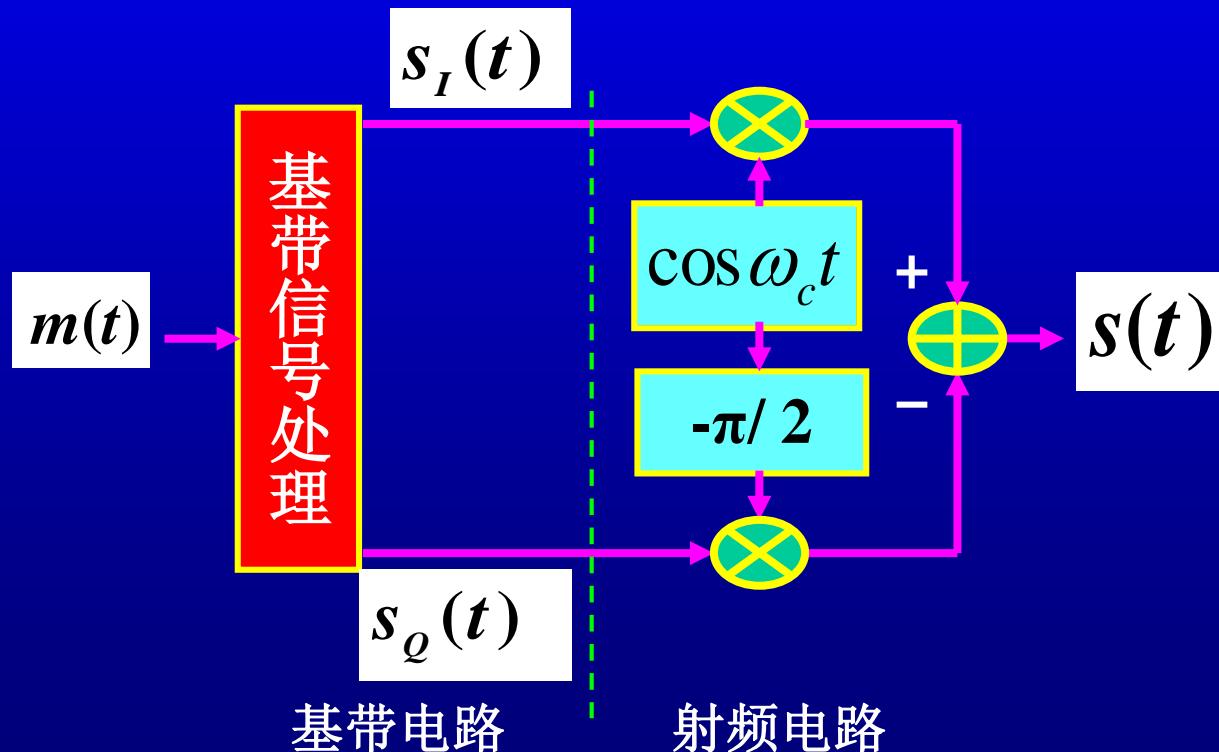
$$s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t)$$

$s_I(t)$: in-phase component of $s(t)$ $s_Q(t)$: quadrature component of $s(t)$

In linear modulation, both $s_I(t)$ and $s_Q(t)$ are low-pass signals that are linearly related to the message signal $m(t)$.

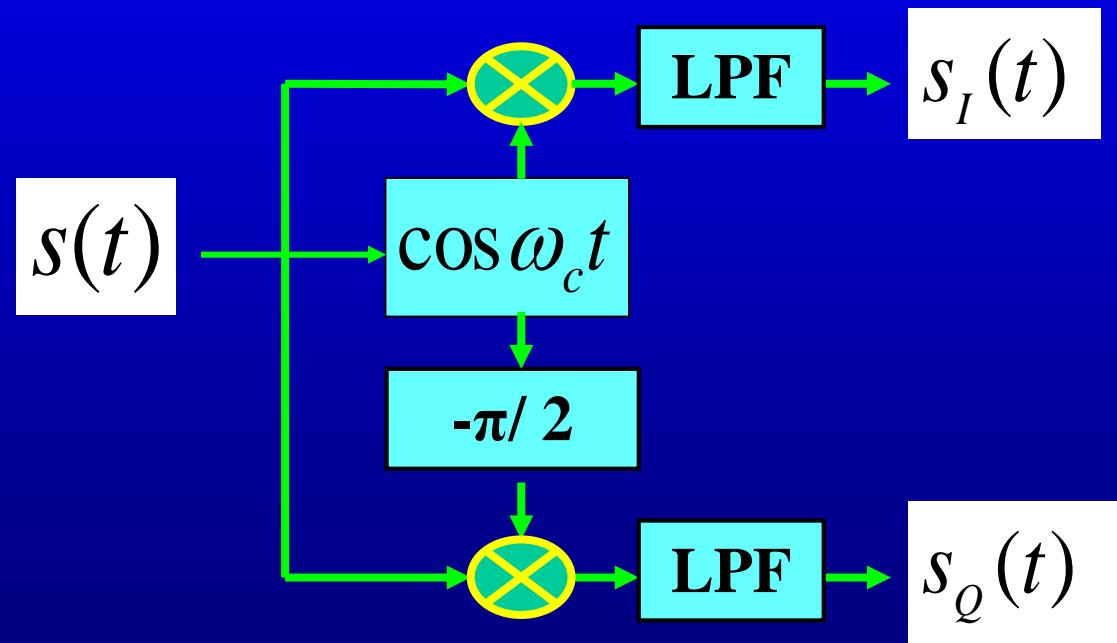
$$s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t)$$

由已调信号的正交表示式可以得到正交调制器的一般模型。



$$s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t)$$

由已调信号的正交表示式可以得到正交相干解调器的一般模型。



Linear modulation is defined by

$$s(t) = s_I(t) \cos(2\pi f_c t) - s_Q(t) \sin(2\pi f_c t)$$

同相分量与正交分量与调制信号具体关系的不同决定了不同的幅度调制类型。

Table 2.1 different forms of linear modulation

Type of modulation		$S_I(t)$	$S_Q(t)$
AM		$1+k_a m(t)$	0
DSB-SC		$m(t)$	0
SSB	USB	$\frac{1}{2} m(t)$	$+\frac{1}{2} \hat{m}(t)$
	LSB	$\frac{1}{2} m(t)$	$-\frac{1}{2} \hat{m}(t)$
VSB	V-LSB	$\frac{1}{2} m(t)$	$+\frac{1}{2} m'(t)$
	V-USB	$\frac{1}{2} m(t)$	$-\frac{1}{2} m'(t)$

$m(t)$ =message signal

$\hat{m}(t)$ =Hilbert transform of $m(t)$

Two important points from table 2.1

1. The in-phase component $S_I(t)$ is solely dependent on the message signal $m(t)$.
2. The quadrature component $S_Q(t)$ is a filtered version of $m(t)$. The spectral modification of the modulated wave $s(t)$ is solely due to $S_Q(t)$.

To be more specific, the role of the quadrature component is merely to interfere with the in-phase component, so as to reduce or eliminate power in one of the sidebands of the modulated signal $s(t)$, depending on how the quadrature component is defined.

Descriptions of AM signals

AM: $s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$

DSB: $s(t) = A_c m(t) \cos(2\pi f_c t)$

SSB: $s(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) \pm \frac{1}{2} A_c \hat{m}(t) \sin(2\pi f_c t)$

- + lower sideband transmitted
- upper sideband transmitted

VSB: $s(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) m(t) \pm \frac{1}{2} A_c m'(t) \cos(2\pi f_c t)$

- + vestige of the upper sideband
- vestige of the lower sideband

DSB-SC Double Sideband-Suppressed Carrier Modulation

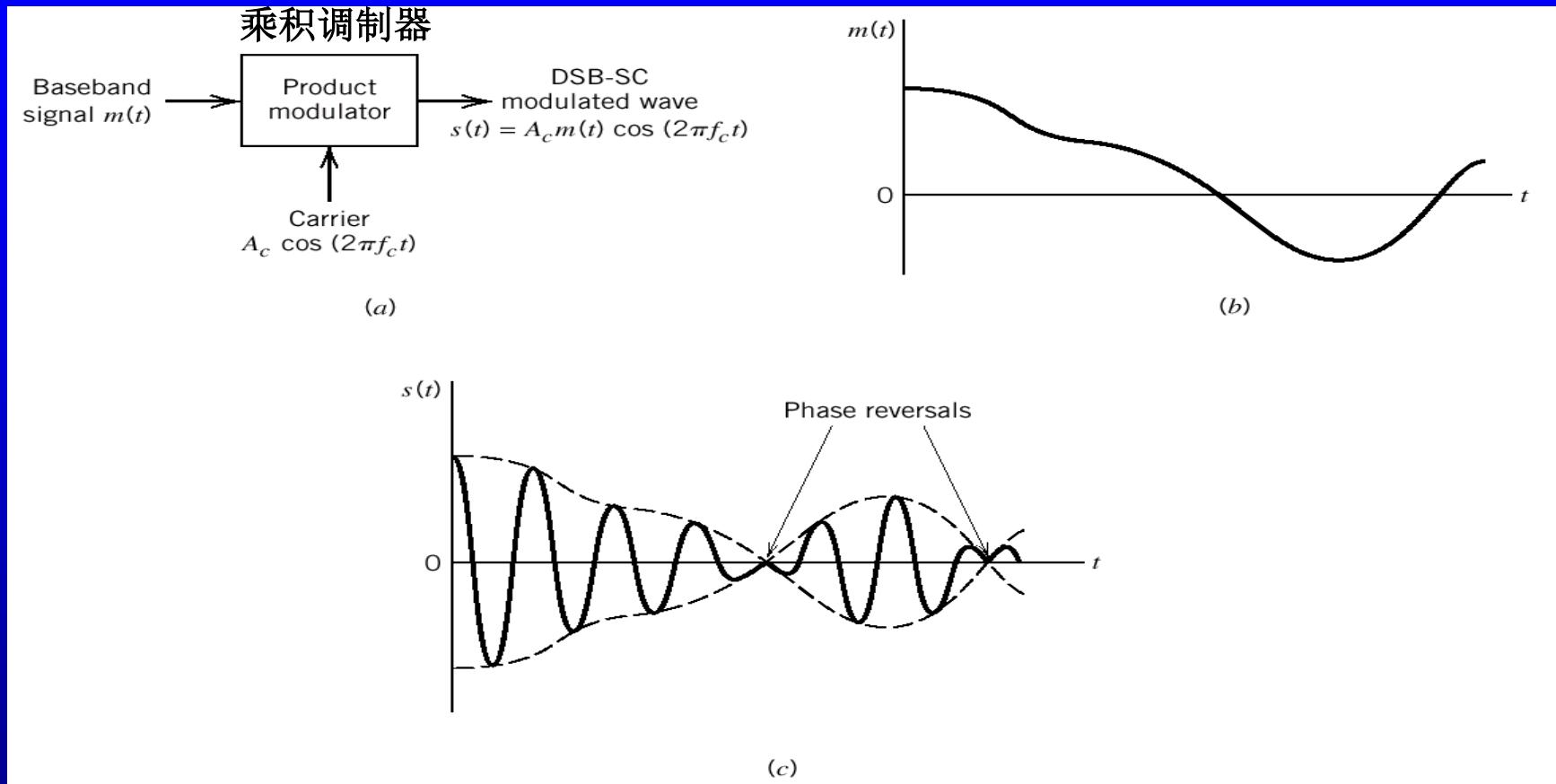
AM:

$$\left\{ \begin{array}{l} s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t) \\ S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2} [M(f - f_c) + M(f + f_c)] \end{array} \right.$$

DSB:

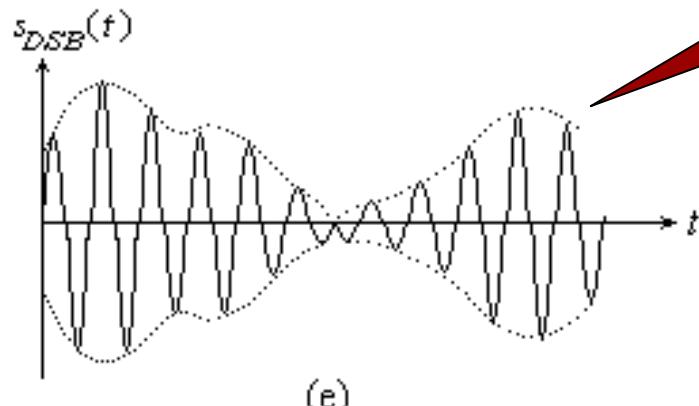
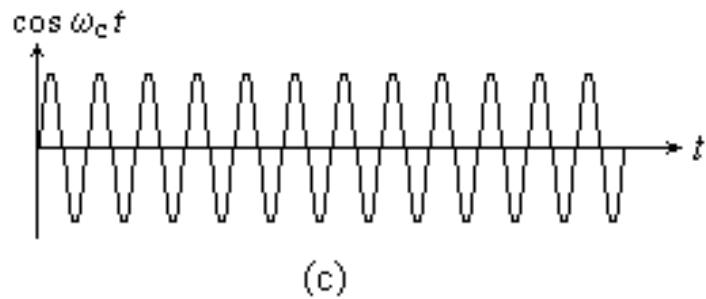
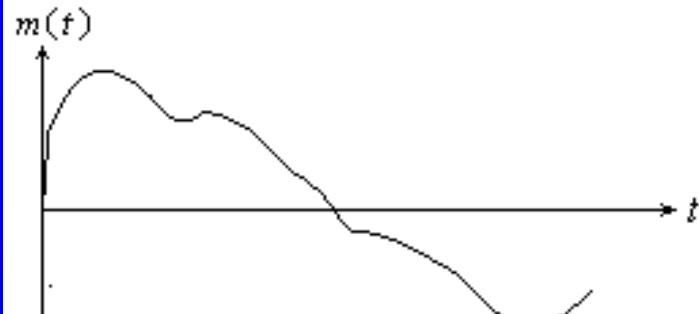
$$\left\{ \begin{array}{l} s(t) = A_c m(t) \cos(2\pi f_c t) \\ S(f) = \frac{1}{2} A_c [M(f - f_c) + M(f + f_c)] \end{array} \right.$$

Figure 2.5 (a) Block diagram of product modulator.
 (b) Baseband signal. (c) DSB-SC modulated wave.



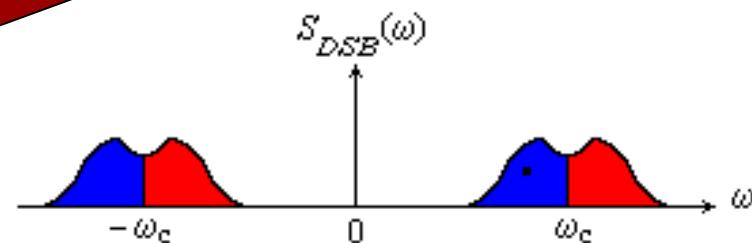
The DSB modulated signal undergoes a **phase reversal** whenever the message signal crosses zero. Consequently, the envelope of DSB signal is different from the message signal. This is unlike the case of AM wave.

DSB信号的波形和频谱示意图：



DSB信号的包络与 $|m(t)|$ 成比例变化，其波形在 $m(t)$ 改变符号时恰好载波也改变符号，出现反相点。

(d)



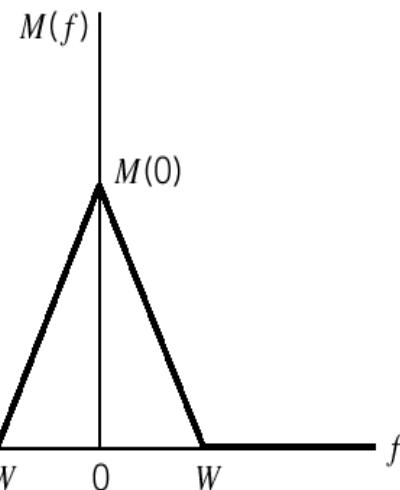
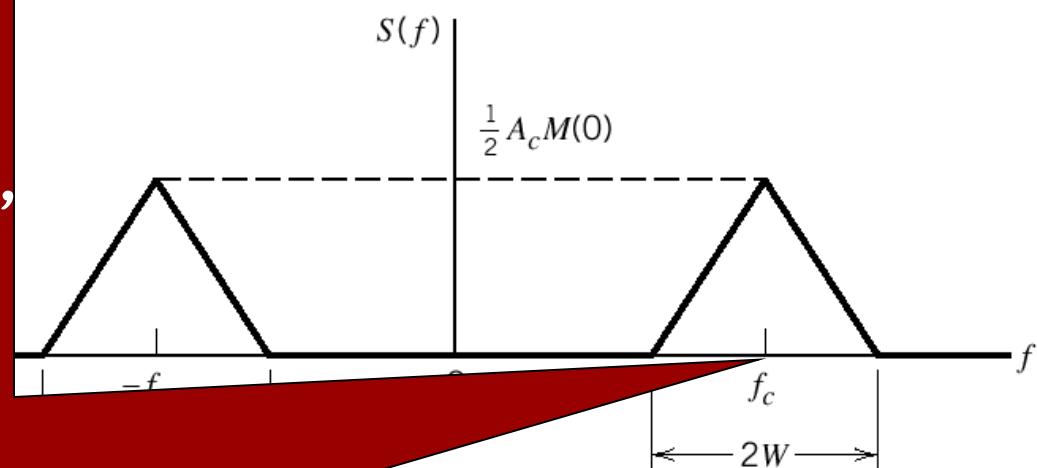
(f)

DSB信号的频谱中不包含载频分量，与调制信号的频谱呈线性对应关系，相当于将基带信号频谱搬移到载频处重现，只是幅度变化。因此，DSB信号的带宽为：

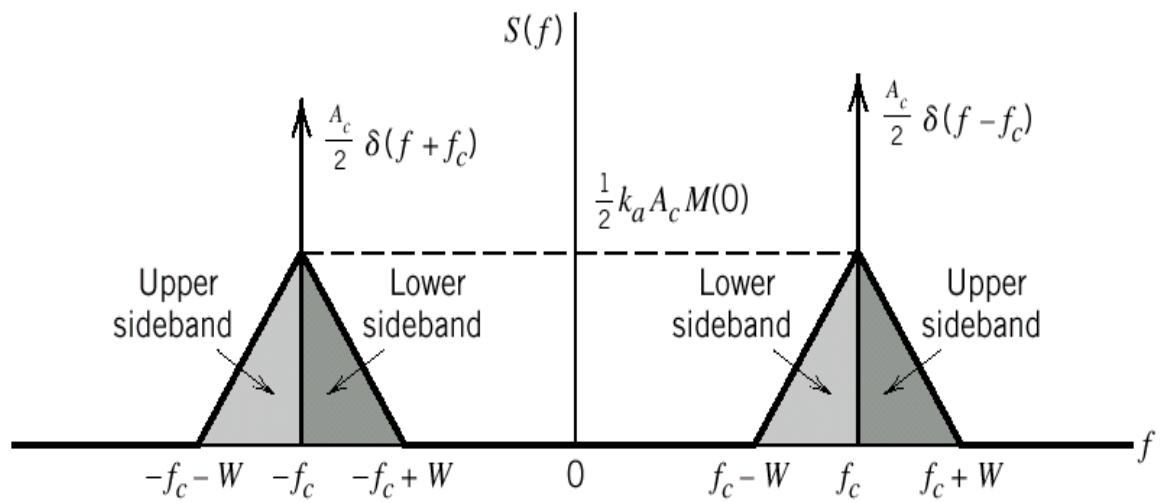


signal

(b) Spectrum of DSB wave



(a) Spectrum of baseband signal



(b) Spectrum of AM wave

Demodulation of DSB

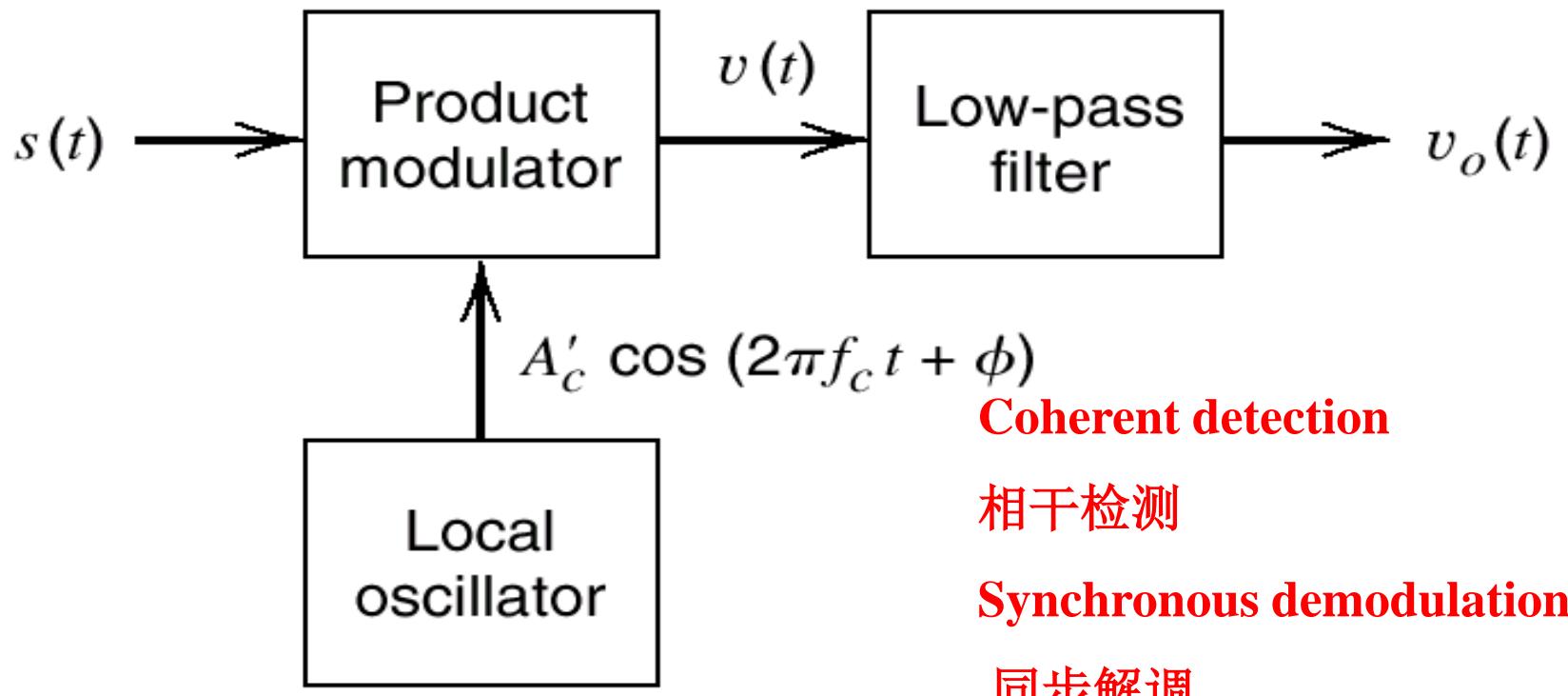
Can we use envelope detector to demodulate DSB signals? Why?

NO. The envelope of DSB signal is no longer linear with modulating signal.

How to demodulate DSB signals?

The baseband signal $m(t)$ can be recovered from a DSB wave by using coherent detection.

Coherent Detection 相干检测/解调



Why is it called Coherent Detection?

Local oscillator signal is exactly synchronized with carrier in **both** frequency and phase.

Coherent Detection Process

the local oscillator signal is supposed as

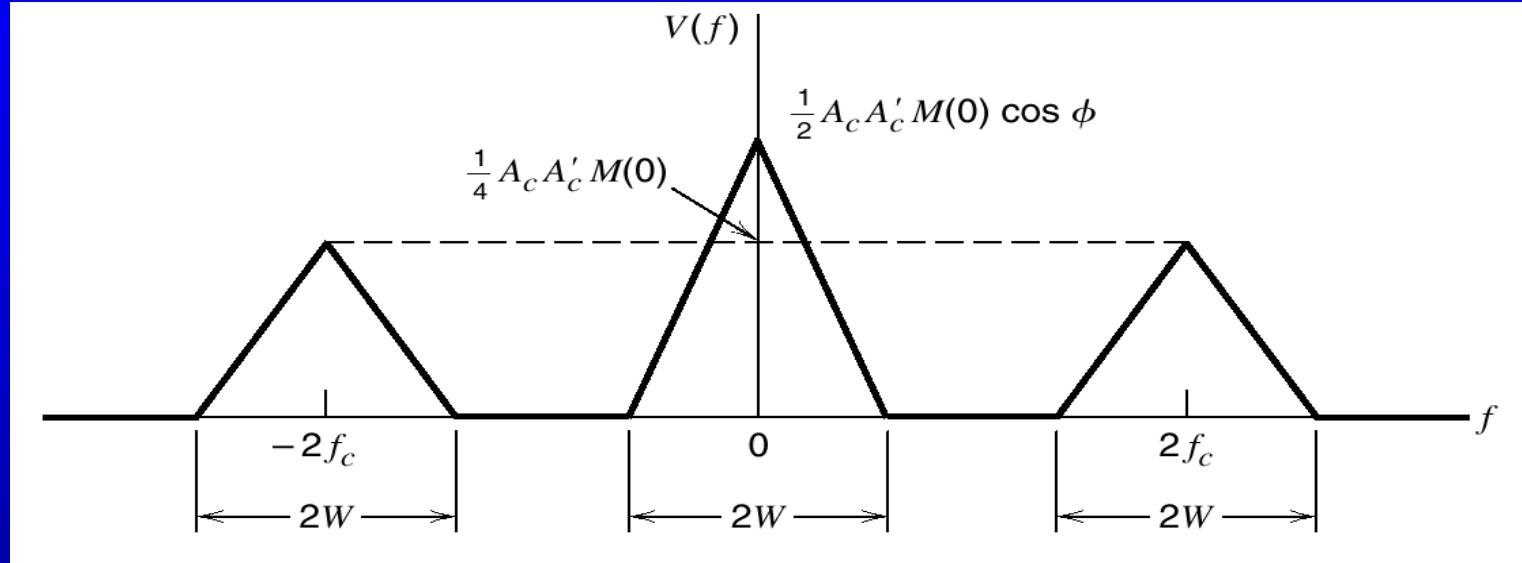
$$A'_c \cos(2\pi f_c t + \phi)$$

DSB signal: $s(t) = A_c m(t) \cos(2\pi f_c t)$

Then, output of product modulator is

$$\begin{aligned} v(t) &= A'_c \cos(2\pi f_c t + \phi) s(t) \\ &= A_c A'_c \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) m(t) \\ &= \frac{1}{2} A_c A'_c \cos(4\pi f_c t + \phi) m(t) + \frac{1}{2} A_c A'_c \cos \phi m(t) \end{aligned}$$

$$v(t) = \frac{1}{2} A_c A'_c \cos(4\pi f_c t + \phi) m(t) + \frac{1}{2} A_c A'_c \cos \phi m(t)$$



Output of low-pass filter

$$v_0(t) = \frac{1}{2} A_c A'_c \cos \phi m(t)$$

Discussion :

Phase difference ϕ

$$v_0(t) = \frac{1}{2} A_c A_c' \cos \phi m(t)$$

Case 1:

$$\phi = \text{constant}, \quad v_0(t) = \frac{1}{2} A_c A_c' k m(t)$$

$m(t)$ can be recovered without any distortion.

$$\phi = 0, \quad v_0(t) = \frac{1}{2} A_c A_c' m(t) \rightarrow \text{Maximum}$$



$$\phi = \pm 90^\circ, \quad v_0(t) = 0 \rightarrow \text{Minimum}$$



Quadrature Null Effect 正交零化效应

Case 2:

In practice, ϕ varies randomly with time. So synchronism must be ensured both in frequency and phase.

How to keep synchronization?

Phase Lock Loop 锁相环:

Square Loop 平方环

Costas Loop 科斯塔斯环, 同相正交环

Quadrature-Carrier Multiplexing

正交载波复用

Quadrature Amplitude Modulation QAM

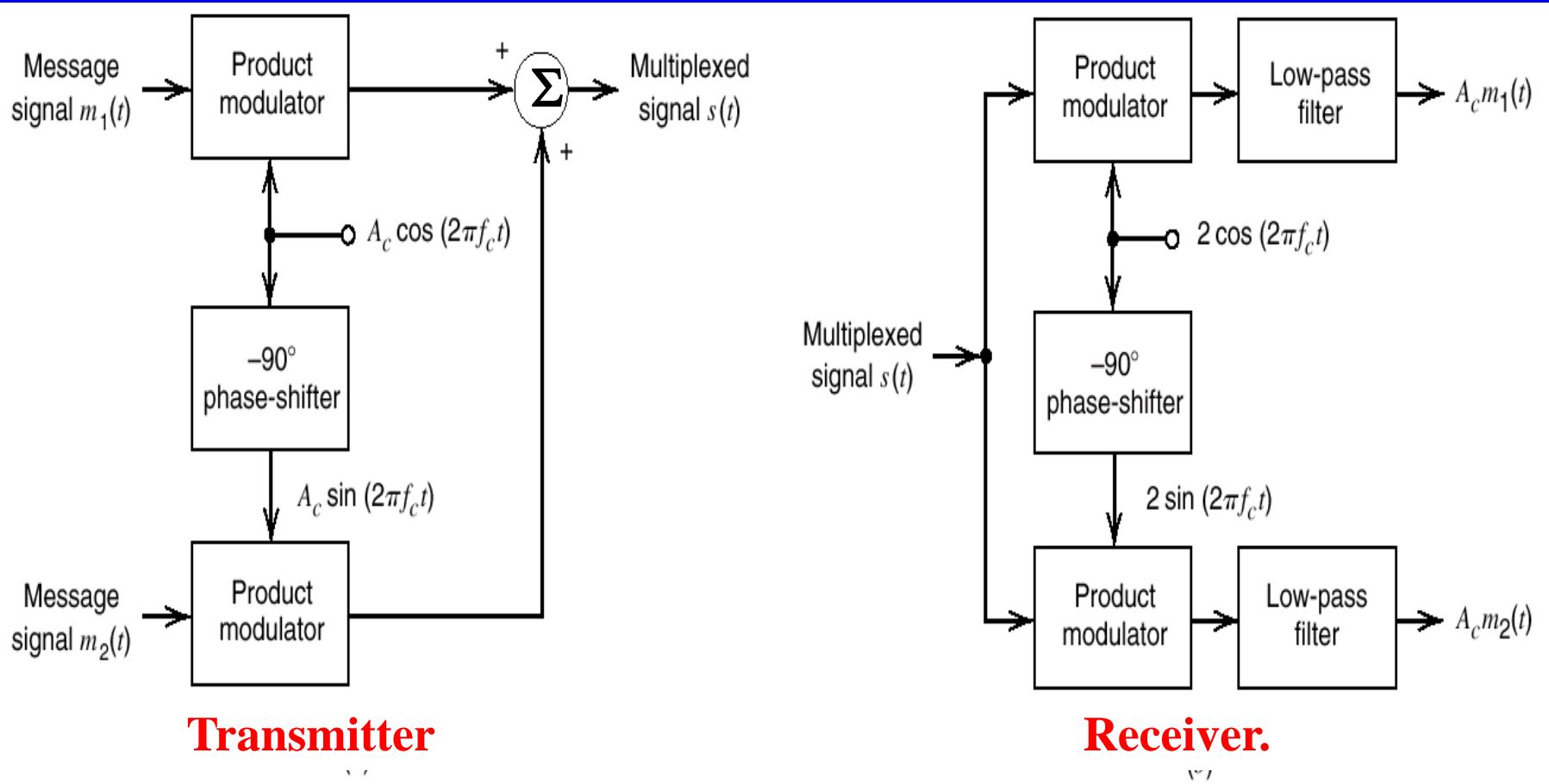
正交调幅

Theory basis: **quadrature null effect** 正交零化效应

The quadrature null effect of the coherent detector may also be **put to good use** in the construction of quadrature-carrier multiplexing.

Figure 2.10

Quadrature-carrier multiplexing system



Modulation and Demodulation process

The **transmitted signal** $s(t)$ consists of sum of two product modulator outputs, as shown by

$$s(t) = A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t)$$

$m_1(t)$ and $m_2(t)$ are two different message signals.

demodulation

$$\left\{ \begin{array}{l} s(t) \cdot 2 \cos(2\pi f_c t) \rightarrow LPF \rightarrow A_c m_1(t) \\ s(t) \cdot 2 \sin(2\pi f_c t) \rightarrow LPF \rightarrow A_c m_2(t) \end{array} \right.$$

QAM is a bandwidth-conservation scheme. why?

This scheme enables two DSB signals to occupy the same channel bandwidth, and yet it allows for the separation of the two message signals at the output. It is therefore a bandwidth-conservation scheme.

Virtues and limitations of DSB

Virtue: saving transmitted power

Limitations:

Complexity

Waste of bandwidth

The resulting system complexity is the price that must be paid for suppressing the carrier wave to save transmitted power.

载波发射功率的节省是以增加系统的复杂度为代价的。

Thinking.....

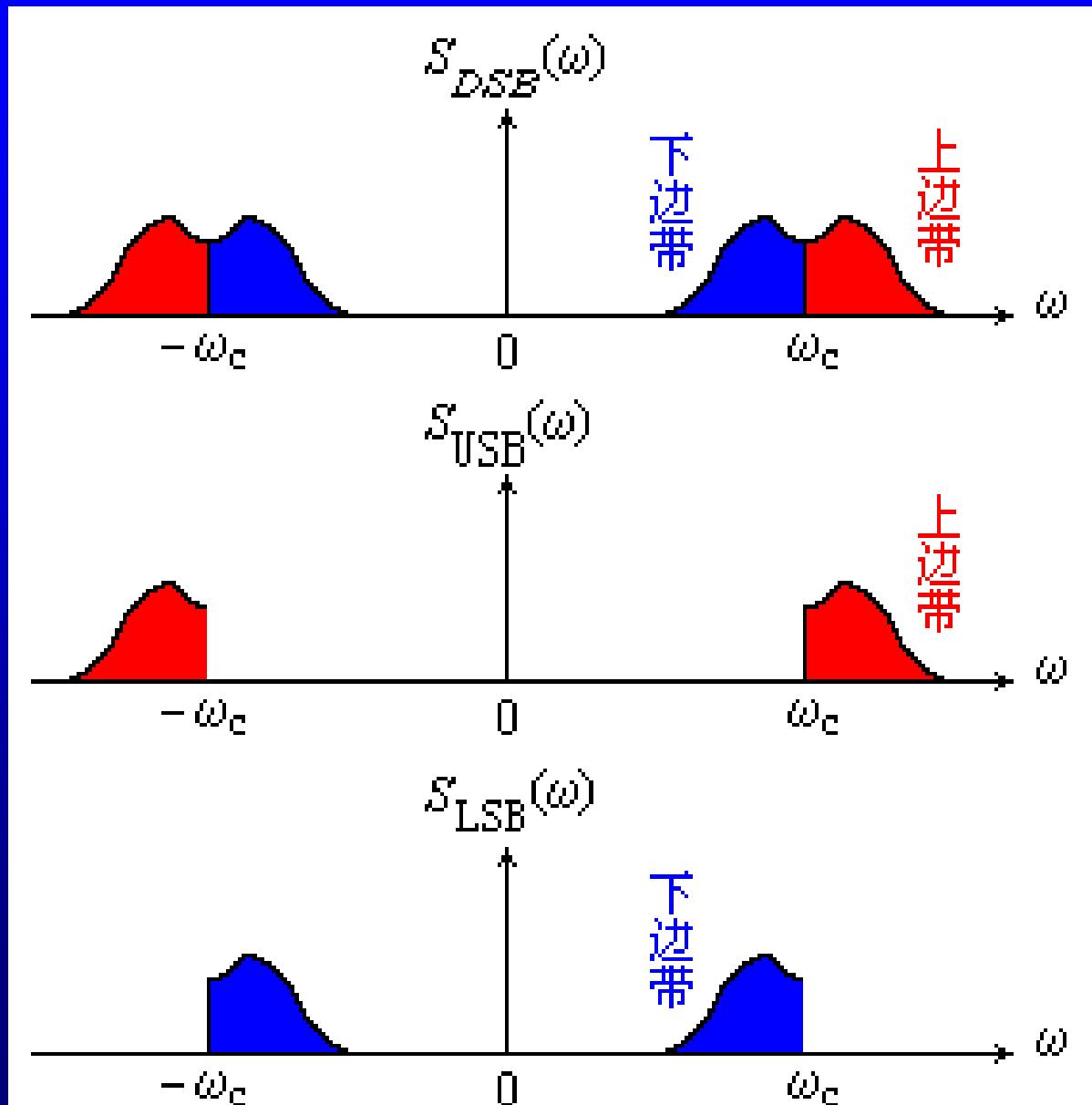
How to improve bandwidth efficiency?

Single-Sideband Modulation SSB

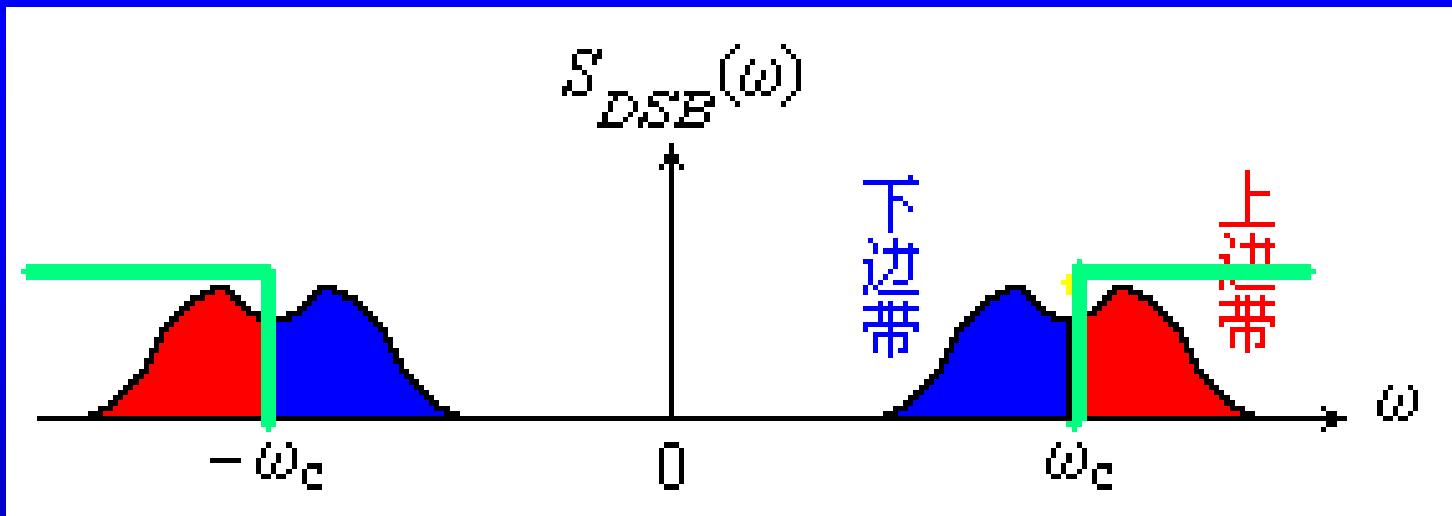
只传送双边带信号中其中一个边带的调制方式，称为单边带调幅（SSB）。

SSB分为
上边带调制(USB)
下边带调制(LSB)。

$$B_{SSB} = W$$



单边带调幅信号的时域和频域表示式:

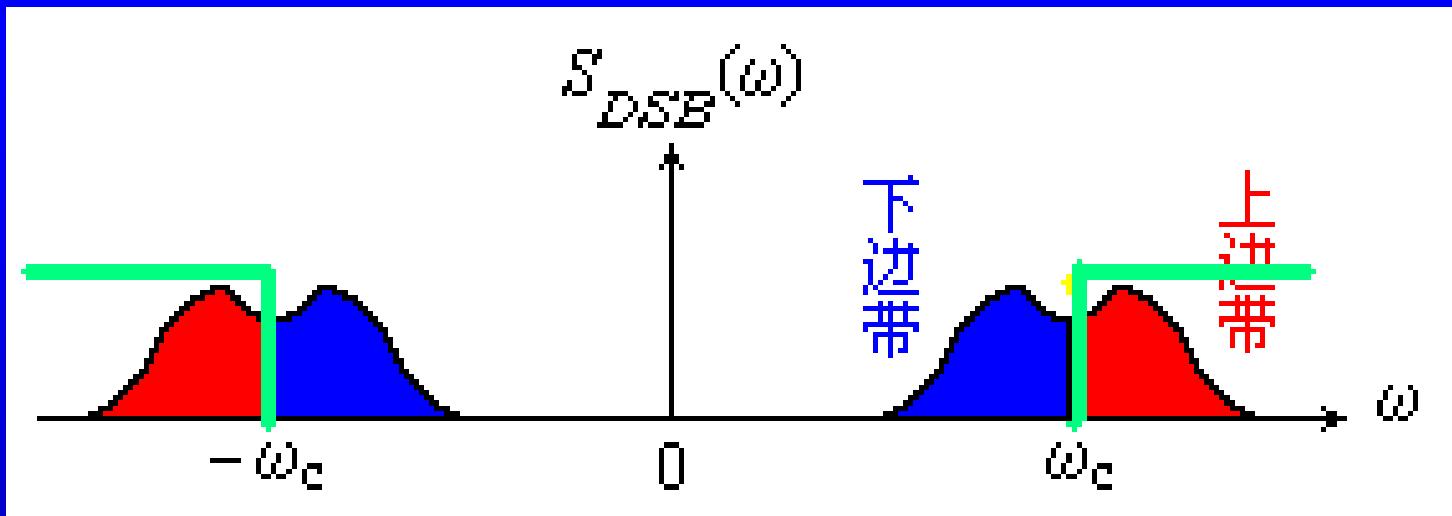


$$S_{\text{USB}}(\omega) = M(\omega + \omega_c) \cdot U[-(\omega + \omega_c)] + M(\omega - \omega_c) \cdot U(\omega - \omega_c)$$

$$S_{\text{LSB}}(\omega) = M(\omega + \omega_c) \cdot U(\omega + \omega_c) + M(\omega - \omega_c) \cdot U[-(\omega - \omega_c)]$$

$$\begin{aligned} s_{\text{SSB}}(t) &= \operatorname{Re}\{[m(t) \pm j\hat{m}(t)] e^{j\omega_c t}\} \\ &= m(t) \cos \omega_c t \mp \hat{m}(t) \sin \omega_c t \end{aligned}$$

单边带调幅信号的时域和频域表示式:



$$S_{\text{USB}}(\omega) = M(\omega + \omega_c) \cdot U[-(\omega + \omega_c)] + M(\omega - \omega_c) \cdot U(\omega - \omega_c)$$

$$S_{\text{LSB}}(\omega) = M(\omega + \omega_c) \cdot U(\omega + \omega_c) + M(\omega - \omega_c) \cdot U[-(\omega - \omega_c)]$$

$$\begin{aligned} s_{\text{SSB}}(t) &= \operatorname{Re}\{[m(t) \pm j\hat{m}(t)] e^{j\omega_c t}\} \\ &= m(t) \cos \omega_c t \mp \hat{m}(t) \sin \omega_c t \end{aligned}$$

证明：

$$S_{\text{USB}}(\omega) = M(\omega + \omega_c) \cdot U[-(\omega + \omega_c)] + M(\omega - \omega_c) \cdot U(\omega - \omega_c)$$

$$= [M(\omega) \cdot U(-\omega)] * \delta(\omega + \omega_c) + [M(\omega) \cdot U(\omega)] * \delta(\omega - \omega_c)$$

对上式进行傅里叶反变换，得：

$$s_{\text{USB}}(t) = \{m(t) * [\frac{1}{2} \delta(t) + \frac{1}{j2\pi t}]\} \cdot e^{-j\omega_c t} + \{m(t) * [\frac{1}{2} \delta(t) - \frac{1}{j2\pi t}]\} \cdot e^{j\omega_c t}$$

$$= m(t) \cdot \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2} - \left[m(t) * \frac{1}{\pi t} \right] \cdot \frac{e^{j\omega_c t} - e^{-j\omega_c t}}{2j}$$

$$= m(t) \cos \omega_c t - \hat{m}(t) \sin \omega_c t$$

Hilbert Transform 希尔伯特变换

Definition : $\hat{x}(t) = x(t) * \frac{1}{\pi t}$

物理意义: $x(t)$ \longrightarrow $h(t) = \frac{1}{\pi t}$ $\longrightarrow \hat{x}(t)$

$$\frac{1}{\pi t} \xleftrightarrow{FT} -j \operatorname{sgn}(\omega) = \begin{cases} -j, & \omega > 0 \\ j, & \omega < 0 \end{cases}$$

对信号进行希尔伯特变换，就是将信号的正频率成分移相-90°，负频率成分移相+90°。
宽带移相网络。

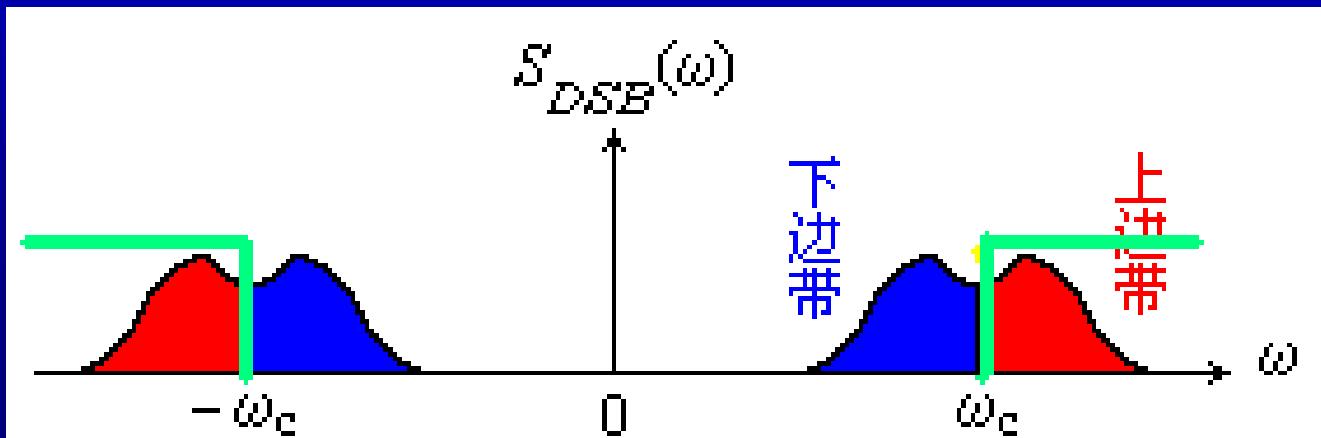
Methods to generate SSB signals

- 1) Frequency-discrimination method to generate SSB 频率分离法, 濾波法
- 2) Phase-shift method to generate SSB 相移法
- 3) Weaver's method to generate SSB 维弗法

1) Frequency-discrimination method to generate SSB 频率分离法，滤波法

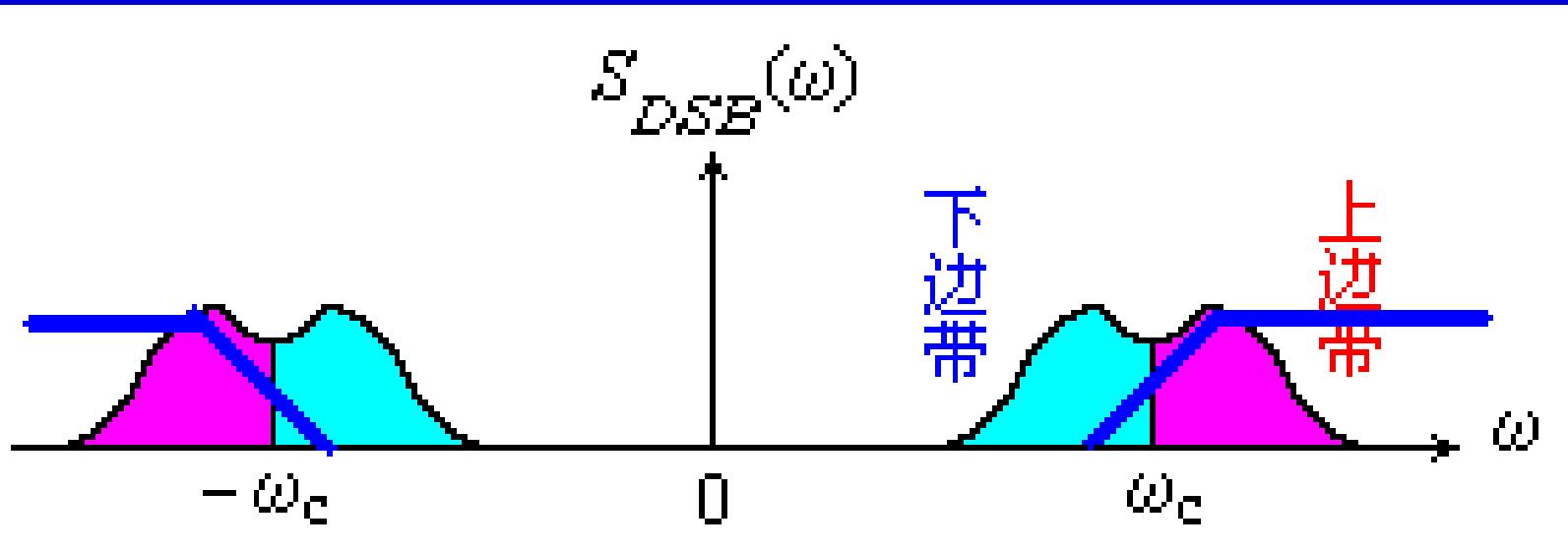
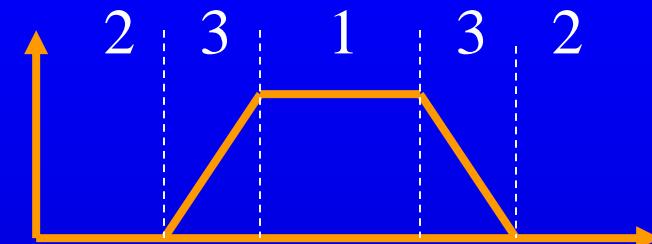
Stage 1: product modulator

Stage 2: band-pass filter



Discussion: Filter

1. Passband 通带
2. Stopband 阻带
3. Transition band 过渡带



When can we use Frequency-discrimination method to generate SSB?



Energy Gap 能量间隙

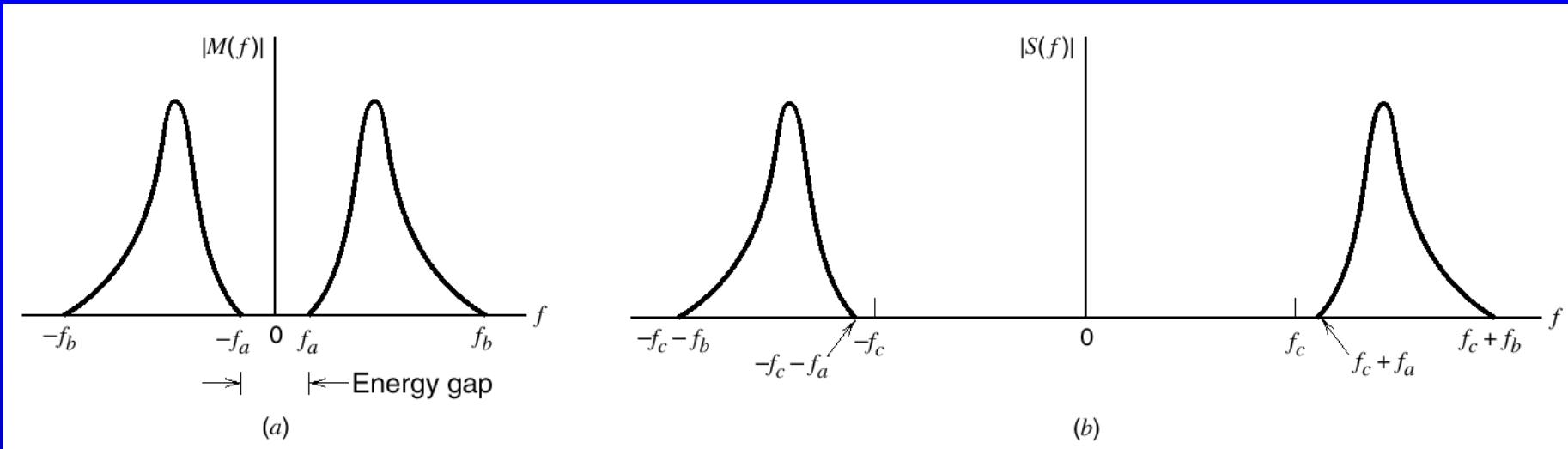


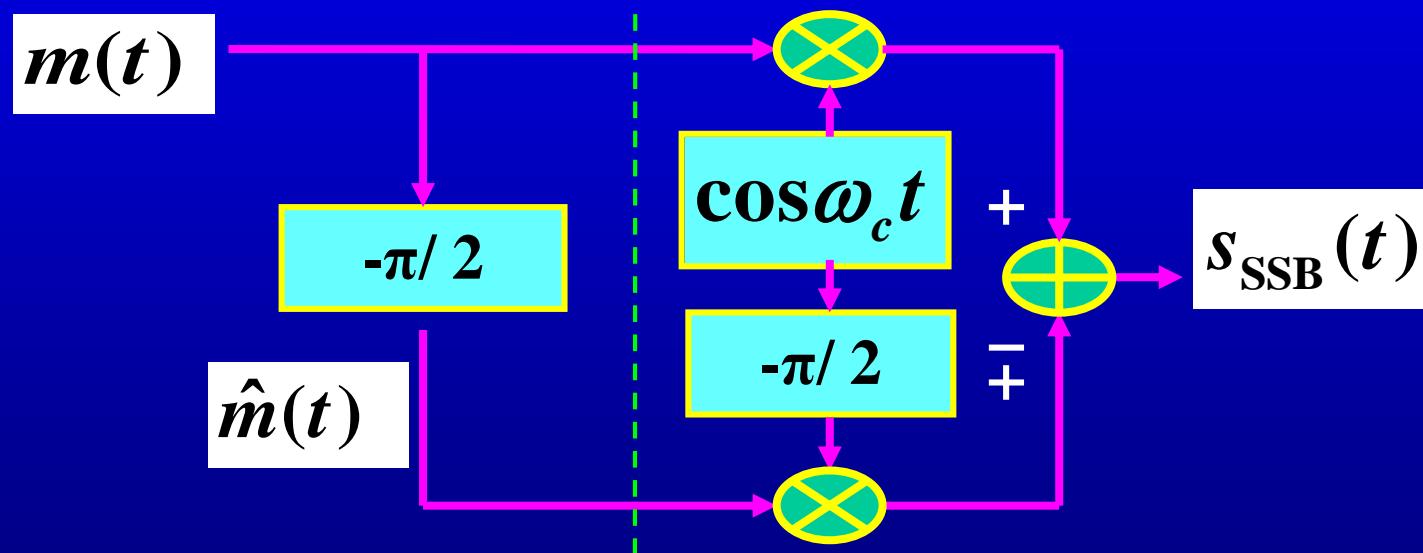
Figure 2.11 (a) Spectrum of a message signal $m(t)$ with an energy gap of width $2f_a$ centered on the origin. (b) Spectrum of corresponding SSB signal containing the upper sideband.

Typical example: telephone voice
 $f \sim (300, 3100\text{Hz})$ energy gap: 600 Hz

2) Phase-shift method to generate SSB 相移法

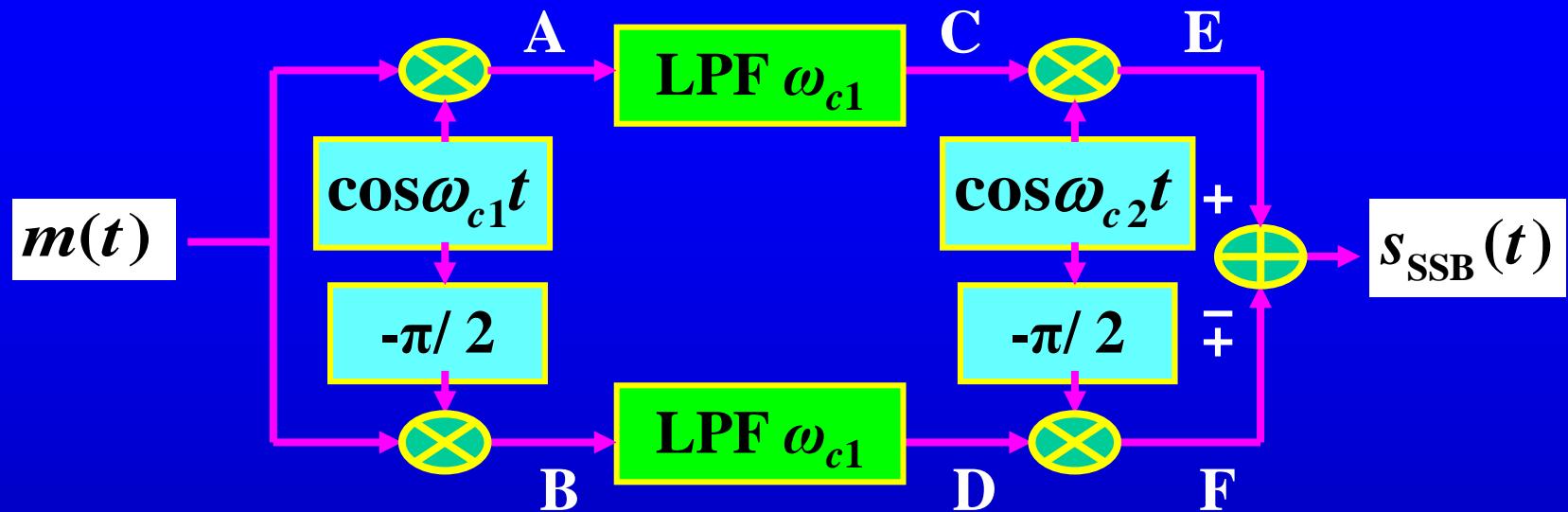
依据单边带信号的时域表示式，可以得到产生单边带信号的相移法。

$$S_{SSB}(t) = \frac{1}{2} m(t) \cos \omega_c t \pm \frac{1}{2} \hat{m}(t) \sin \omega_c t$$



Key points: Hilbert Transform , Phase-shift
宽带移相很难实现

3) Weaver's method to generate SSB 维弗法



- 维弗（Weaver）法，又称为混合法，它是滤波法和相移法的组合，在技术实现上既具有相移法利用正交调制产生单边带信号的优点，又避免了采用宽频带相移 $-\pi/2$ 网络的缺点。
- 采用维弗法，可以产生载频为 $\omega_{c1}+\omega_{c2}$ 的下边带信号，或载频为 $\omega_{c2}-\omega_{c1}$ 的上边带信号。

SSB Coherent Detection

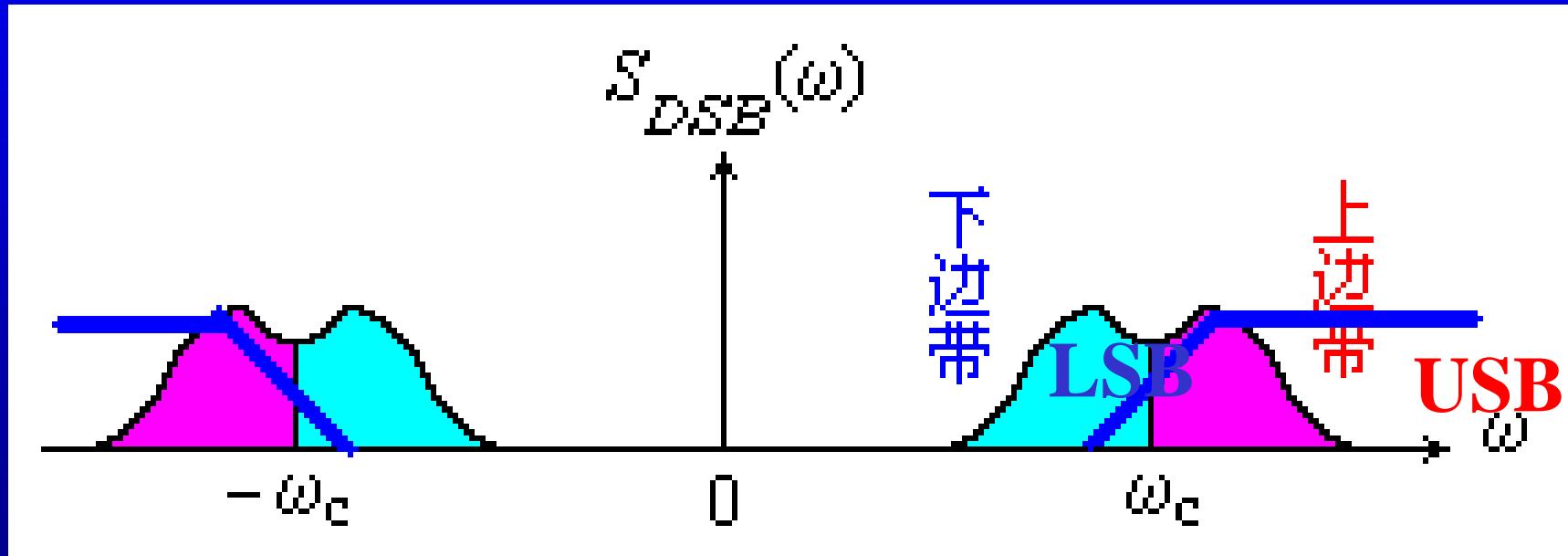


How to keep synchronization between local oscillator and carrier in the transmitter?

1. A *low-power* pilot carrier 导频载波
2. A highly stable oscillator 高稳定性振荡器

Vestigial Sideband Modulation VSB

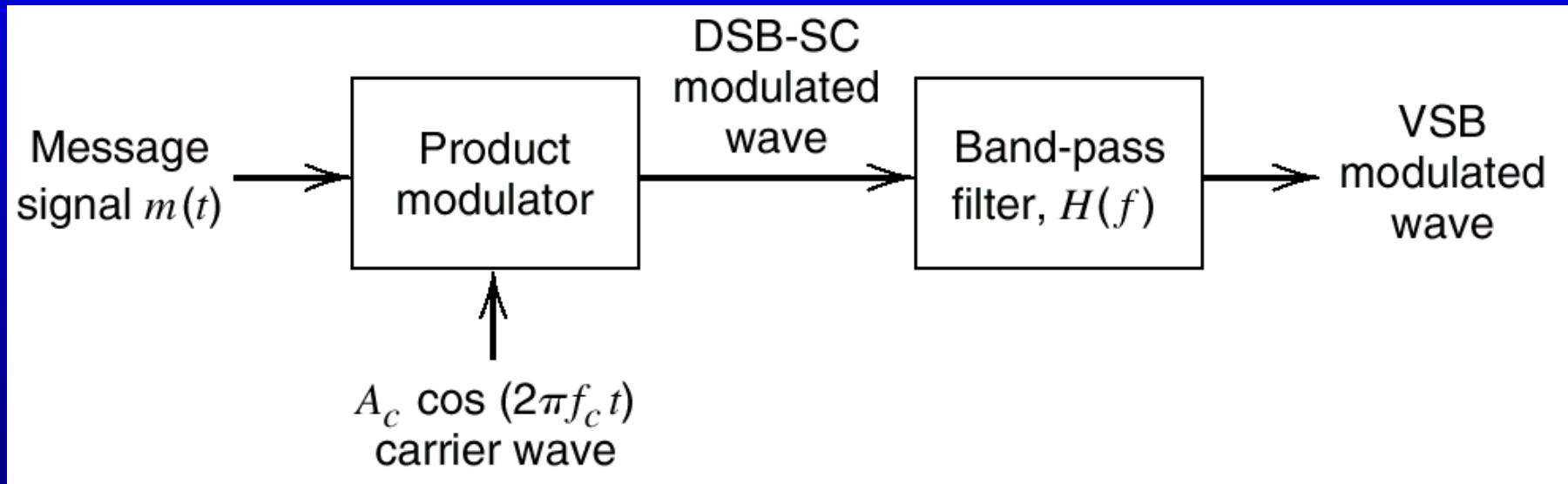
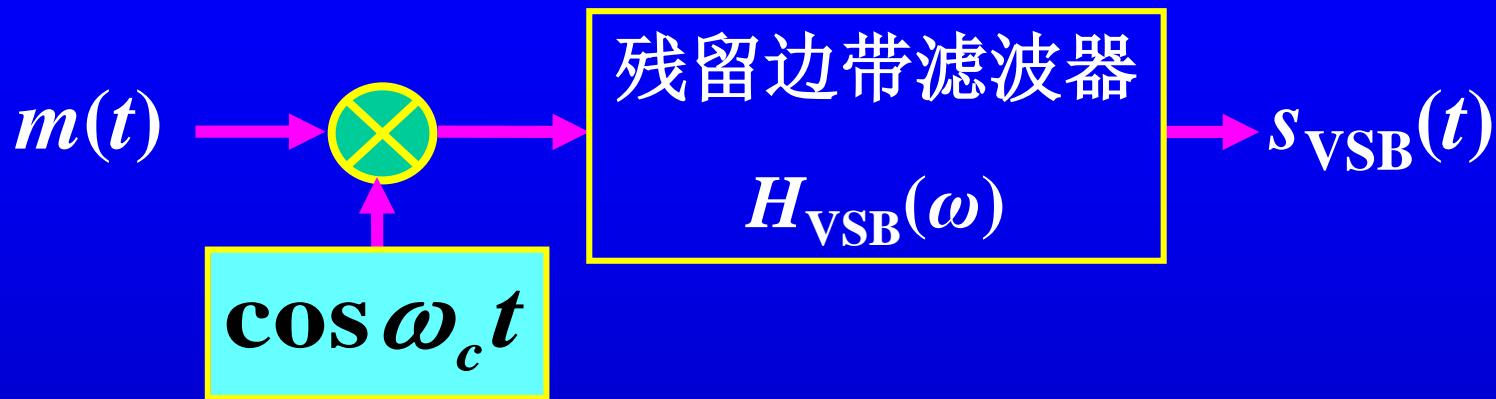
One of the sideband is partially suppressed and a vestige of the other sideband is transmitted to compensate for that suppression.



Vestige of lower sideband 残留下边带调制

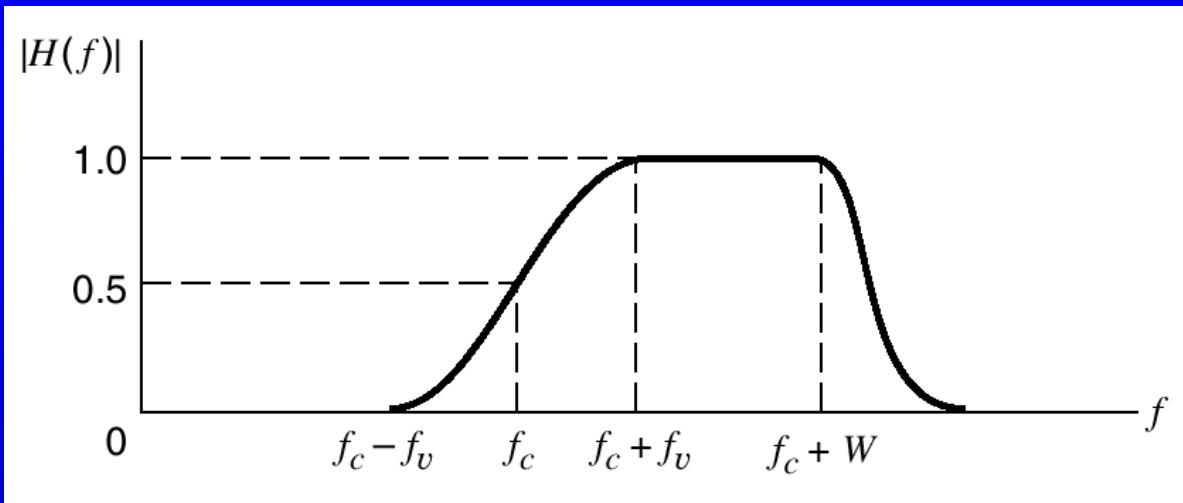
Vestige of upper sideband 残留上边带调制

Frequency Discrimination Method to Generate VSB 频率分离法/滤波法



Key: the design of band-pass filter

Magnitude response of VSB filter



Odd symmetry
around the f_c

$$B_T = W + f_v$$

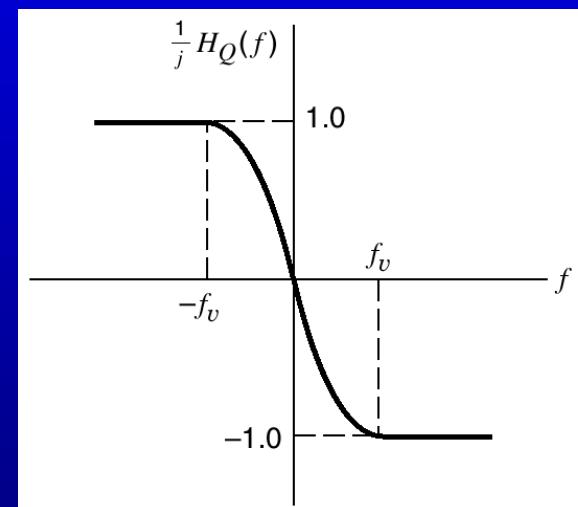
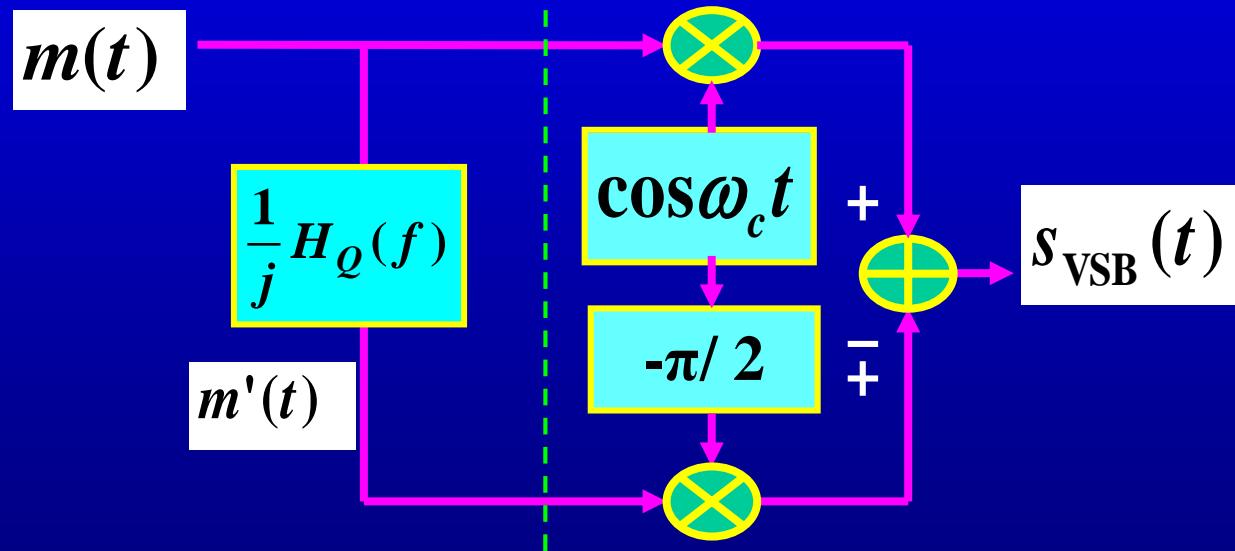
1. The sum of the values of the magnitude response $|H(f)|$ at any two frequencies equally displaced above and below f_c is **unity**.
2. The phase response $\arg(H(f))$ is linear.

$$H(f - f_c) + H(f + f_c) = 1 \dots \dots \dots \text{for } -W \leq f \leq W$$

VSB description in time-domain

$$s(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) \pm \frac{1}{2} A_c m'(t) \sin(2\pi f_c t)$$

- + vestige of the upper sideband
- vestige of the lower sideband



phase –shift method to generate VSB

How to detect a VSB signal?

----- Coherent detection



questions

- Why does TV employ VSB instead of SSB?
- Why can TV use envelope detection?
- How does TV work?



Discussion
topic

AM

Virtue: Simplicity of implementation

Limitations: 1. wasteful of power
2. wasteful of bandwidth

DSB

Virtue: saving transmitted power

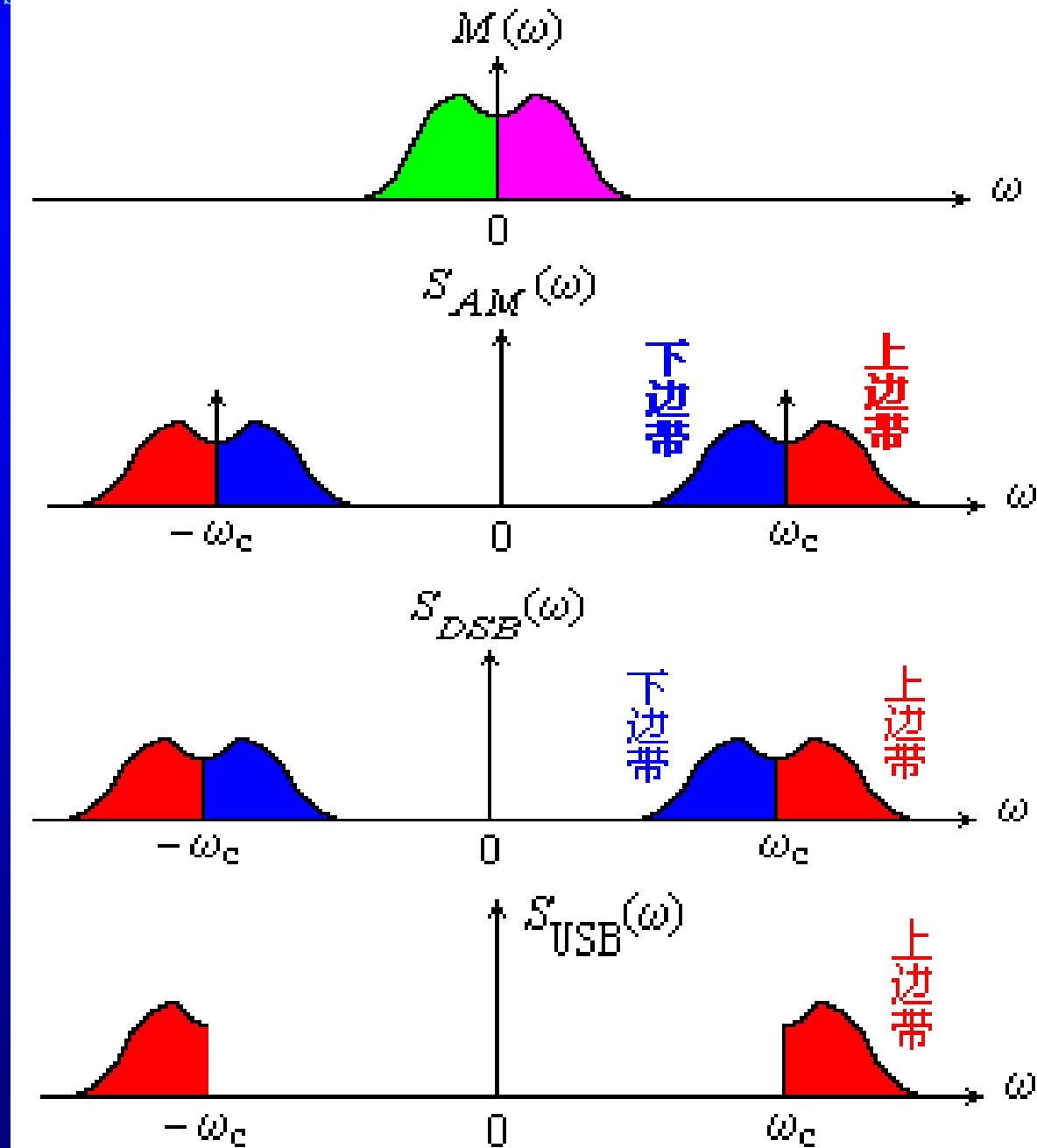
Limitations: 1. Complexity
2. Waste of bandwidth

SSB

Virtue: 1. saving transmitted power
2. saving bandwidth

Limitations: more Complexity

Spectra

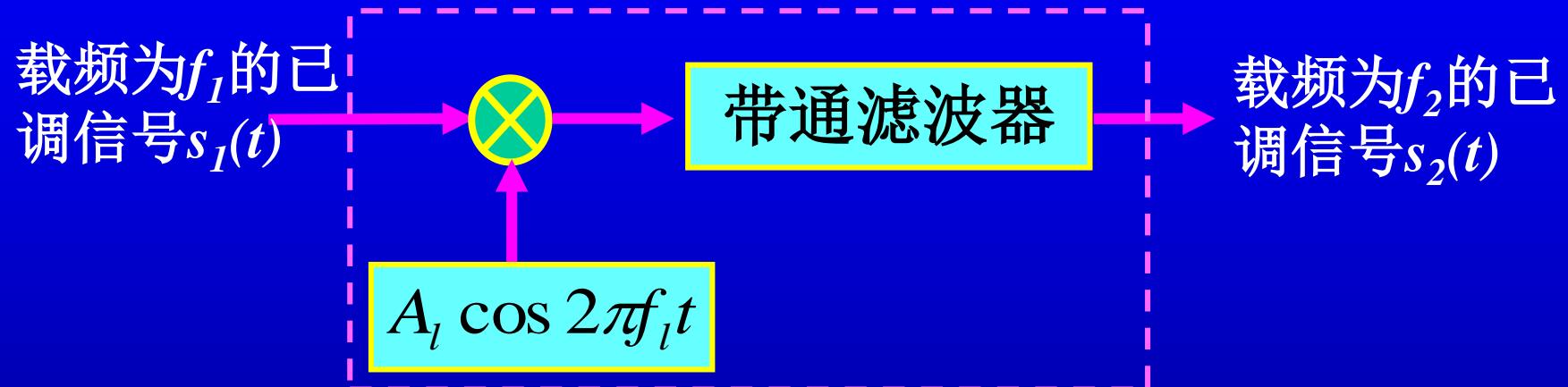


2.4 Frequency Translation 频率搬移

- The basic operation in SSB is in fact a form of frequency translation.
- SSB modulation is also called frequency changing 变频, mixing 混频, or heterodyning 外差.

Mixer 混频器

The mixer is a device that consists of a product modulator followed by a band-pass filter.



$$\cos 2\pi f_1 t \times \cos 2\pi f_l t \quad \left. \begin{array}{l} \text{Sum frequency} \\ f_2 = f_1 + f_l \end{array} \right\} \quad \left. \begin{array}{l} \text{Difference frequency} \\ f_2 = f_1 - f_l \end{array} \right\}$$

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上变频与下变频

Up conversion 上变频

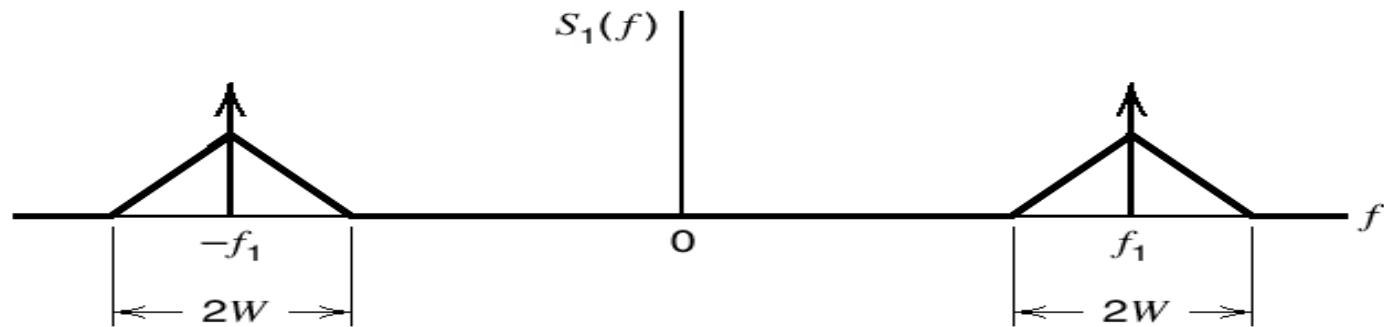
$$f_2 = f_1 + f_l \quad \text{or} \quad f_l = f_2 - f_1$$

Down conversion 下变频

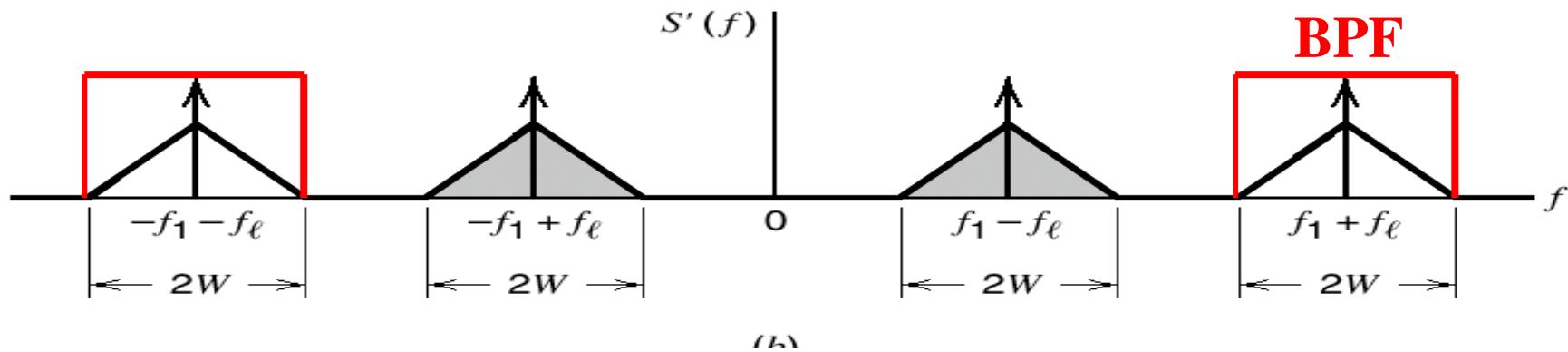
$$f_2 = f_1 - f_l \quad \text{or} \quad f_l = f_1 - f_2$$

Figure 2.17

混频器频谱示意图



Spectrum of modulated signal $s_1(t)$ at the mixer input



Spectrum of the corresponding signal $s'(t)$ at the output of the product modulator in the mixer

2.5 Frequency-Division Multiplexing FDM

频分复用

Multiplexing refers to a number of independent signals are combined into a composite signal suitable for transmission over a common channel.

Types of Multiplexing

- FDM

Separate the signals according to frequency.

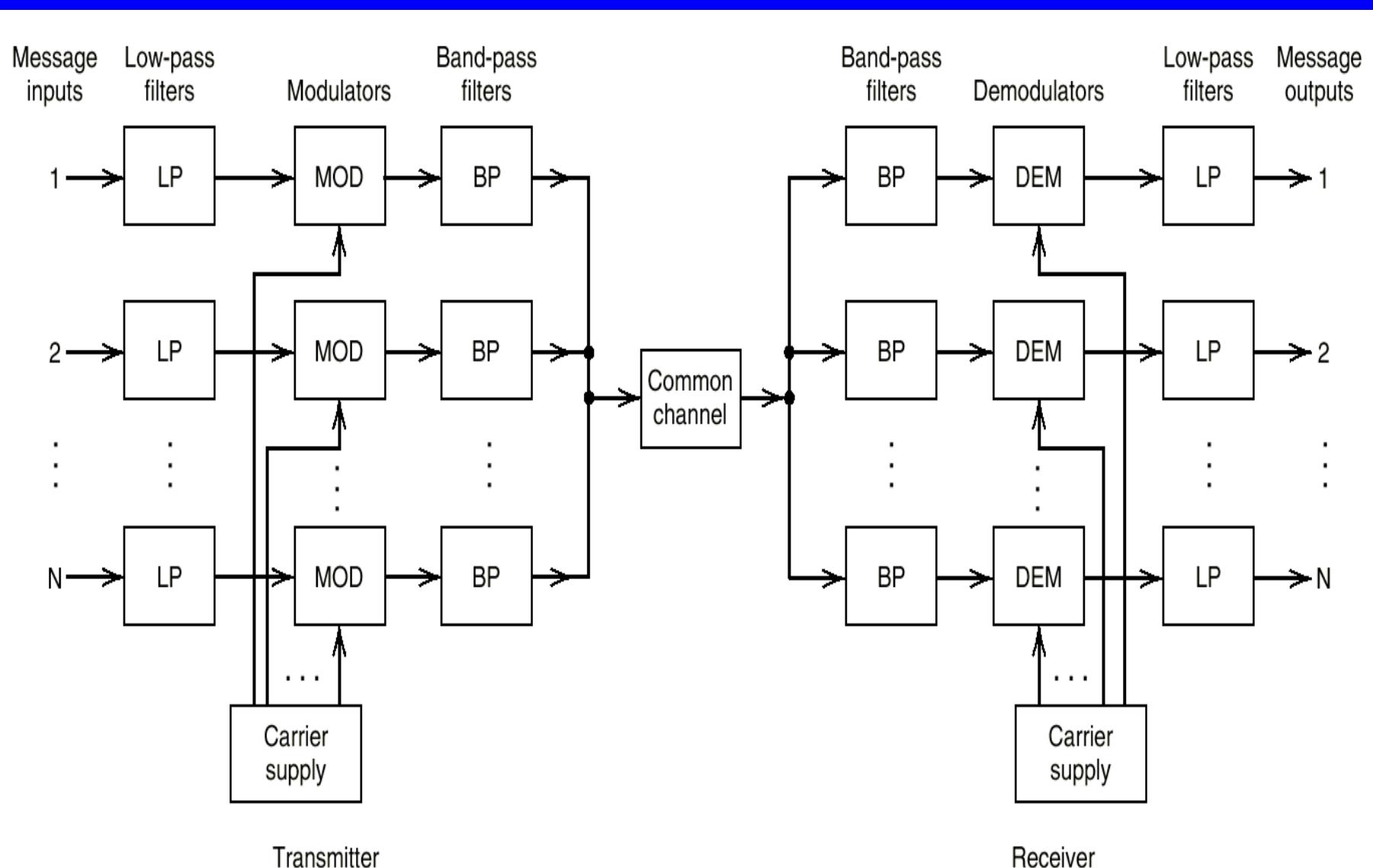
- TDM

Separate the signals according to time.

- CDM

Separate the signals according to code.

Block diagram of FDM system



Example 2.1

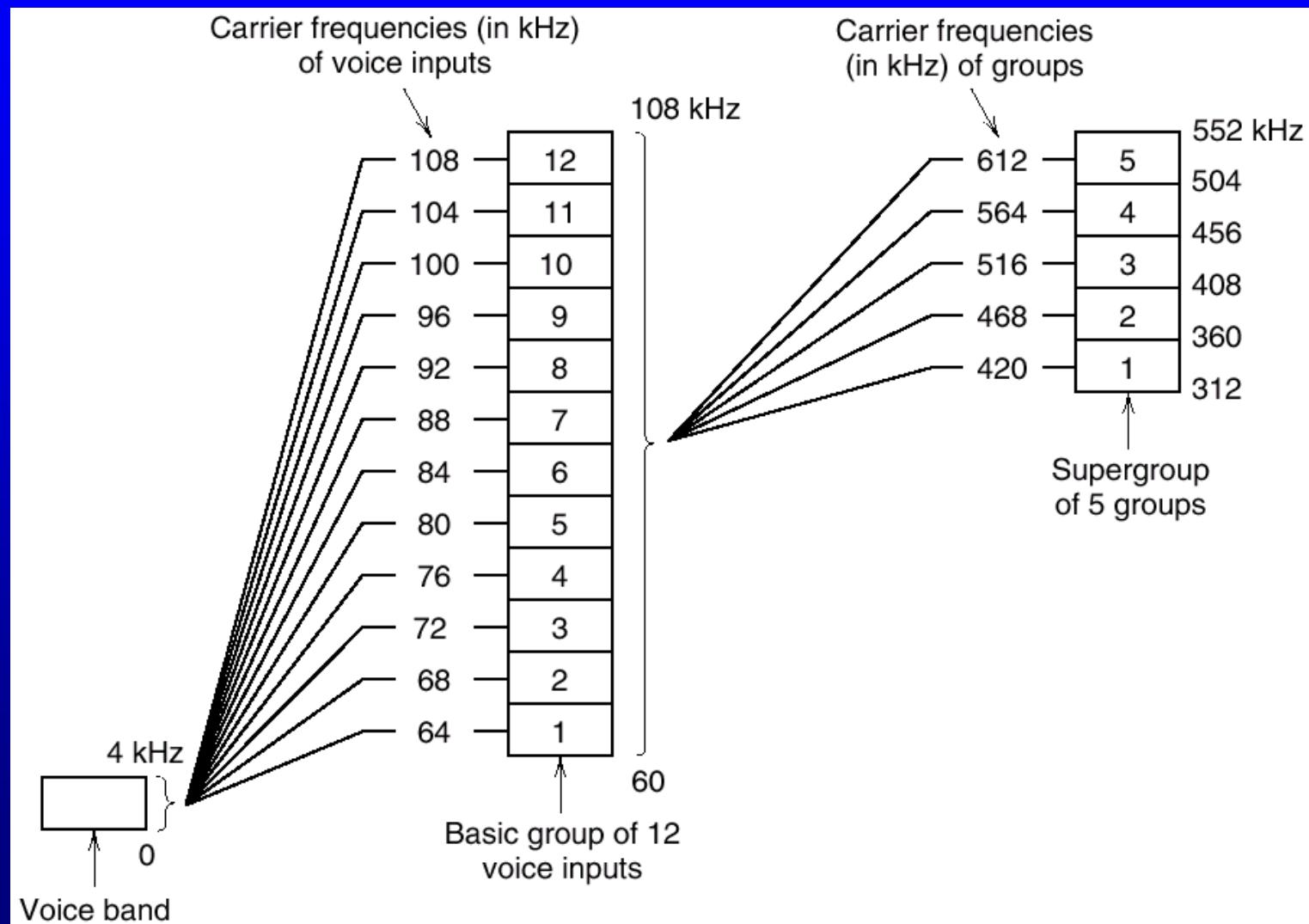
Carrier Telephone System SSB/FDM

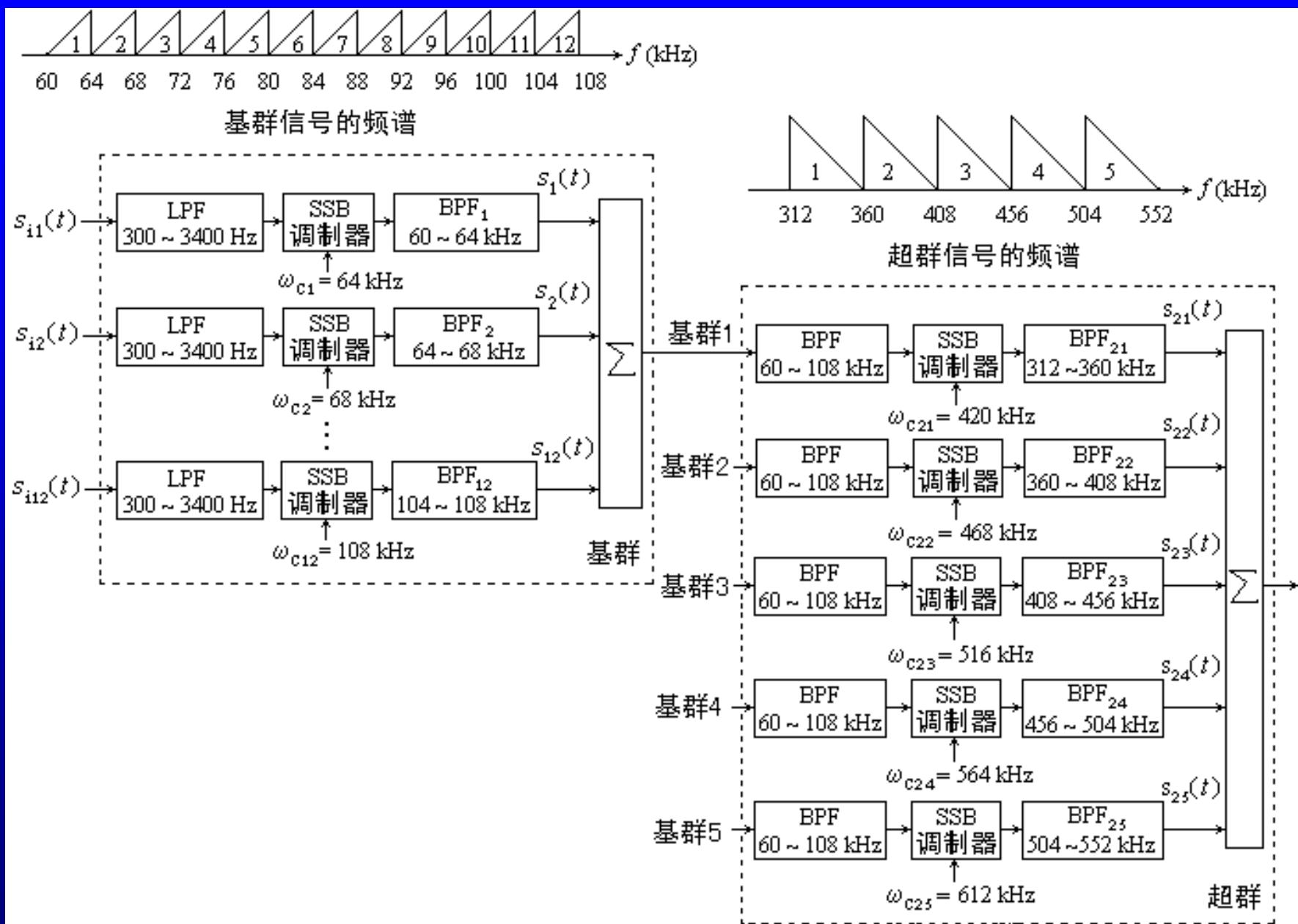
多路载波电话分群等级

分群等级	路 数	带 宽	基 本 频 带
基 群 basic group	12	48kHz	60~108kHz
超 群 super group	$60=5 \times 12$	240kHz	312~552kHz
基本主群 master group	$300=5 \times 60$	1200kHz	812~2044kHz
基本超主群 super master group	$900=3 \times 300$	3600kHz	8516~12388kHz
12MHz 系统	$2700=3 \times 900$	10.8MHz	
60MHz 系统	$10800=12 \times 900$	43.2MHz	

Figure 2.19

Illustrating the modulation steps in an FDM system.





2.6 Angle Modulation 角调制

- **Definition:**
The **angle** of the carrier wave is varied according to the baseband signal. Whereas, the amplitude of the carrier is maintained constant. It consists of PM and FM.
- **An important feature:**
It provides **better** discrimination against noise and interference than amplitude modulation. 抗噪声性能比幅度调制好。

Tradeoff

This improvement in performance is achieved at the expense of increased transmission bandwidth. 抗噪声性能的提高是以增加传输带宽为代价的。

That is, angle modulation provides us with a practical means of exchanging channel bandwidth for improved noise performance.

Such a tradeoff is not possible with amplitude modulation, regardless of its form.

Basic Definition of Angle Modulation

Let $\theta_i(t)$ denote the angle of a modulated carrier, then **angle-modulated wave** can be expressed as

$$s(t) = A_c \cos[\theta_i(t)] \quad \theta_i(t) \propto m(t)$$

If $\theta_i(t)$ increases monotonically with time, the average frequency in Hertz, over an interval from t to $t+\Delta t$, is given by

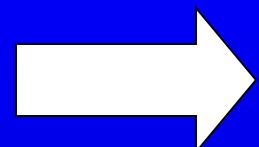
$$f_{\Delta t}(t) = \frac{\theta_i(t + \Delta t) - \theta_i(t)}{2\pi\Delta t} \quad 2.20$$

Instantaneous frequency 瞬时频率

$$f_i(t) = \lim_{\Delta t \rightarrow 0} f_{\Delta t}(t)$$

$$= \lim_{\Delta t \rightarrow 0} \left[\frac{\theta_i(t + \Delta t) - \theta_i(t)}{2\pi\Delta t} \right]$$

$$= \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} \quad 2.21$$



$$\theta_i(t) = 2\pi \int f_i(t) dt$$

In the simple case of an unmodulated carrier, the angle $\theta_i(t)$ is :

$$\theta_i(t) = 2\pi f_c t + \phi_c$$

The constant ϕ_c is the value of $\theta_i(t)$ at $t=0$. Usually it is assumed to be zero for convenience.

Angle modulation is defined as the **angle** of the carrier wave varying with modulating signal .

$$\theta_i(t) \propto m(t)$$

There are an **infinite number of ways** in which the angle may be varied in some manner with the message signal.

However, we shall consider only **two commonly used methods**. They are FM and PM.

Phase Modulation PM

The angle $\theta_i(t)$ is varied linearly with the message signal $m(t)$

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

k_p : phase sensitivity 相位灵敏度

PM signal is described in the time domain by

$$s_{PM}(t) = A_c \cos[\theta_i(t)] = A_c \cos[2\pi f_c t + k_p m(t)] \quad 2.23$$

Frequency Modulation FM

The **instantaneous frequency $f_i(t)$** is varied linearly with the message signal $m(t)$

$$f_i(t) = f_c + k_f m(t) \quad 2.24$$

k_f : frequency sensitivity 频率灵敏度

$$\theta_i(t) = 2\pi \int f_i(t) dt = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \quad 2.25$$

FM signal is described in the time domain by

$$s_{FM}(t) = A_c \cos[\theta_i(t)]$$

$$= A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau] \quad 2.26$$

Relationship between FM and PM

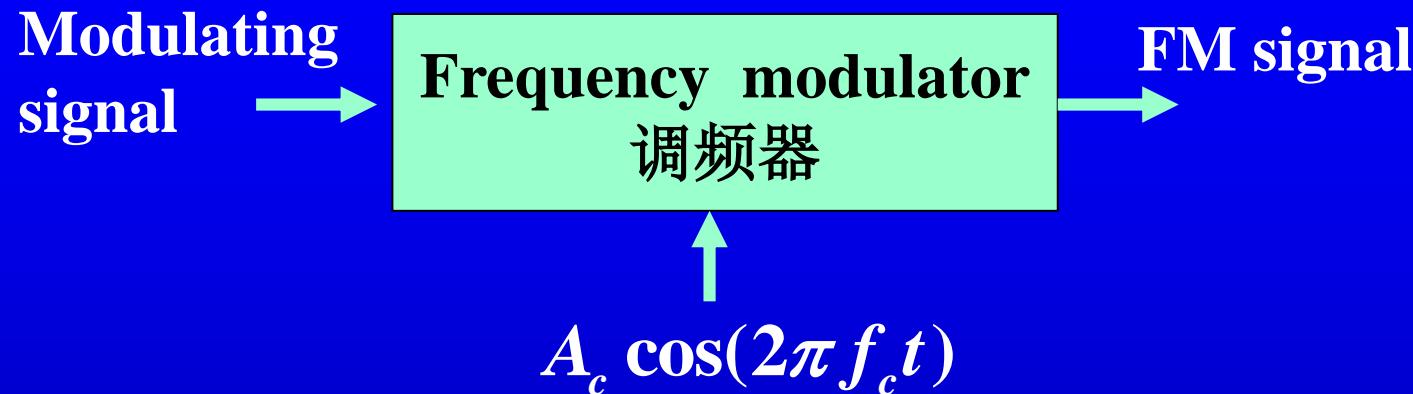
$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)] \quad 2.23$$

$$s_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau] \quad 2.26$$

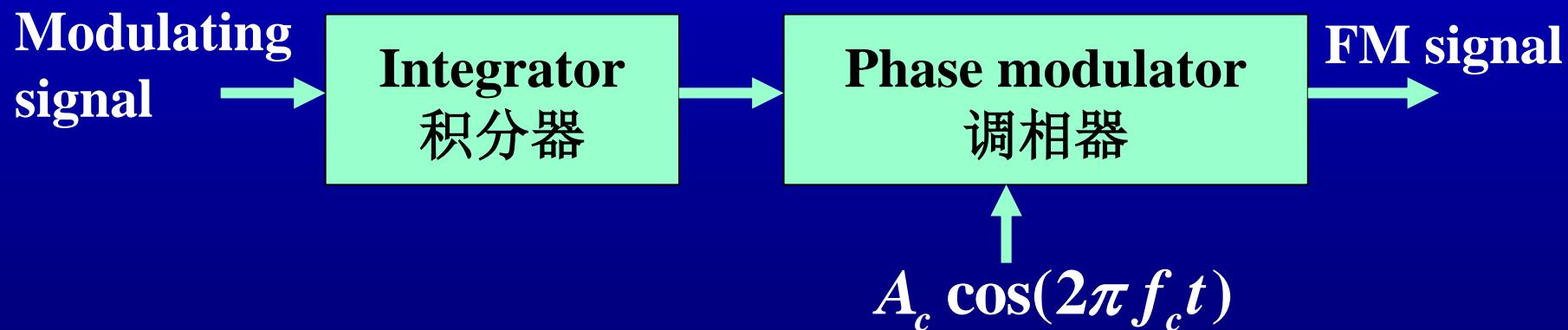
$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} \quad \longleftrightarrow \quad \theta_i(t) = 2\pi \int f_i(t) dt$$

So, we may deduce all the properties of PM signals from those of FM signals and vice versa. Hence, we concentrate our attention on FM signals.

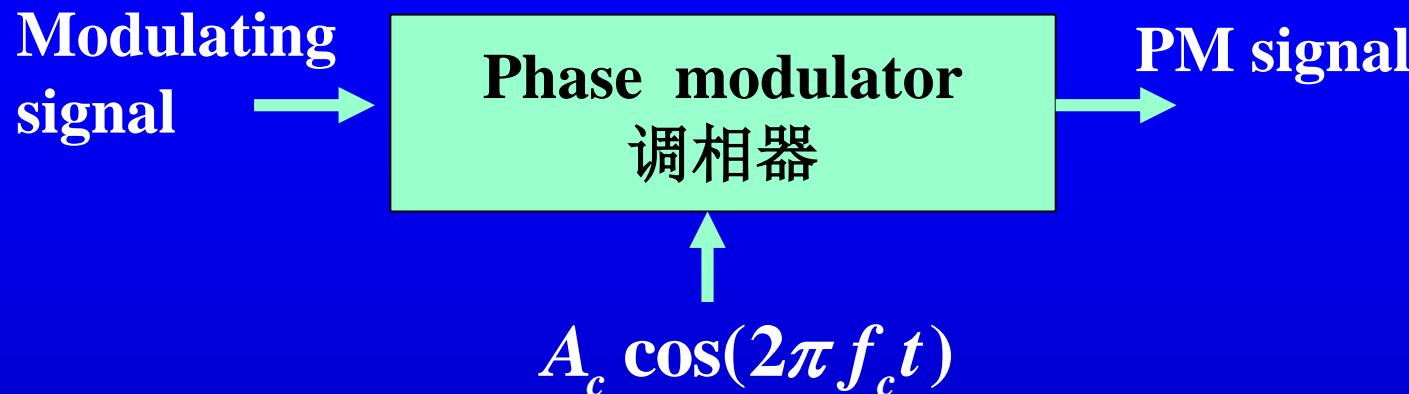
Direct method to generate FM signal 直接法



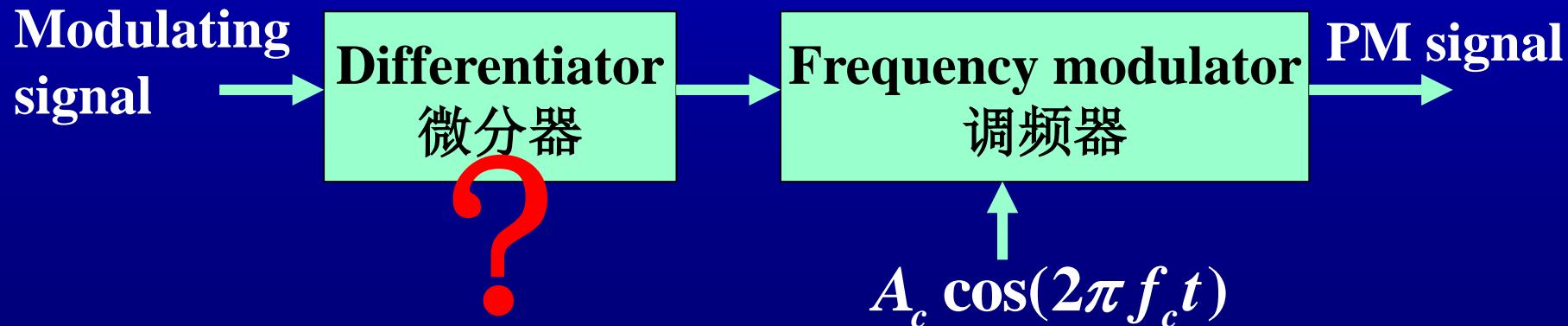
Indirect method to generate FM signal 间接法



Direct method to generate PM signal



Indirect method to generate PM signal



2.7 Frequency Modulation FM

- The FM signal $s(t)$ is a nonlinear function of the modulating signal $m(t)$, which makes the frequency modulation a **nonlinear modulation** process. 非线性
- Consequently, unlike amplitude modulation, the **spectrum** 频谱 of an FM signal is not related in a simple manner to that of the modulating signal; rather, its analysis is much more difficult than that of an AM signal.

How can we tackle the spectral analysis (频谱分析) of an FM signal?

We propose to provide an empirical (经验) answer to this question by proceeding in the following manner:

1. First consider the simplest case:
a single-tone modulation, narrowband FM
2. Then consider more general case:
a single-tone modulation, wideband FM

Objective: to establish an empirical formula between the transmission bandwidth of an FM signal and the bandwidth of message signal.

Single-tone frequency modulation 单音频调制

A single tone modulating signal is defined by

$$m(t) = A_m \cos(2\pi f_m t) \quad 2.27$$

f_m : modulation frequency of modulating signal

The instantaneous frequency of the FM signal:

$$\begin{aligned} f_i(t) &= f_c + k_f A_m \cos(2\pi f_m t) \\ &= f_c + \Delta f \cos(2\pi f_m t) \quad 2.28 \end{aligned}$$

Where

$$\Delta f = k_f A_m$$

Δf : Frequency deviation 频偏, 瞬时频率偏移的最大值

因为相位与瞬时频率是互为微分与积分的关系，很容易得到FM信号的相位表示

$$\theta_i(t) = 2\pi \int_0^t f_i(\tau) d\tau = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \quad 2.30$$

We define $\beta = \frac{\Delta f}{f_m}$ **Modulation index** 调制指数
瞬时相位偏移的最大值

$$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$$

FM signal for single-tone modulating signal:

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \quad 2.33$$

Two important concepts in FM

Page 110

Δf : Frequency deviation 频偏

瞬时频率偏移的最大值 (maximum departure of the instantaneous frequency of the FM signal from the carrier frequency f_c). It is proportional to the amplitude of the modulating signal and is independent of the modulation frequency f_m .

Modulation index 调制指数 β

相位偏移的最大值 (Phase deviation, the maximum departure of the angle $\theta_i(t)$ from the angle $2\pi f_c t$ of the unmodulated carrier.)

Narrowband and Wideband FM

Depending on the value of the modulation index β , there are two cases of frequency modulation:

{ **Narrowband FM:** $\beta \ll 1$
Wideband FM: otherwise

2.7.1 Narrowband Frequency Modulation NBFM

FM signal for single-tone $m(t)$:

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

Expanding it:

$$\begin{aligned} s(t) &= A_c \cos(2\pi f_c t) \cos[\beta \sin(2\pi f_m t)] \\ &\quad - A_c \sin(2\pi f_c t) \sin[\beta \sin(2\pi f_m t)] \end{aligned} \quad 2.34$$

When $\beta \ll 1$

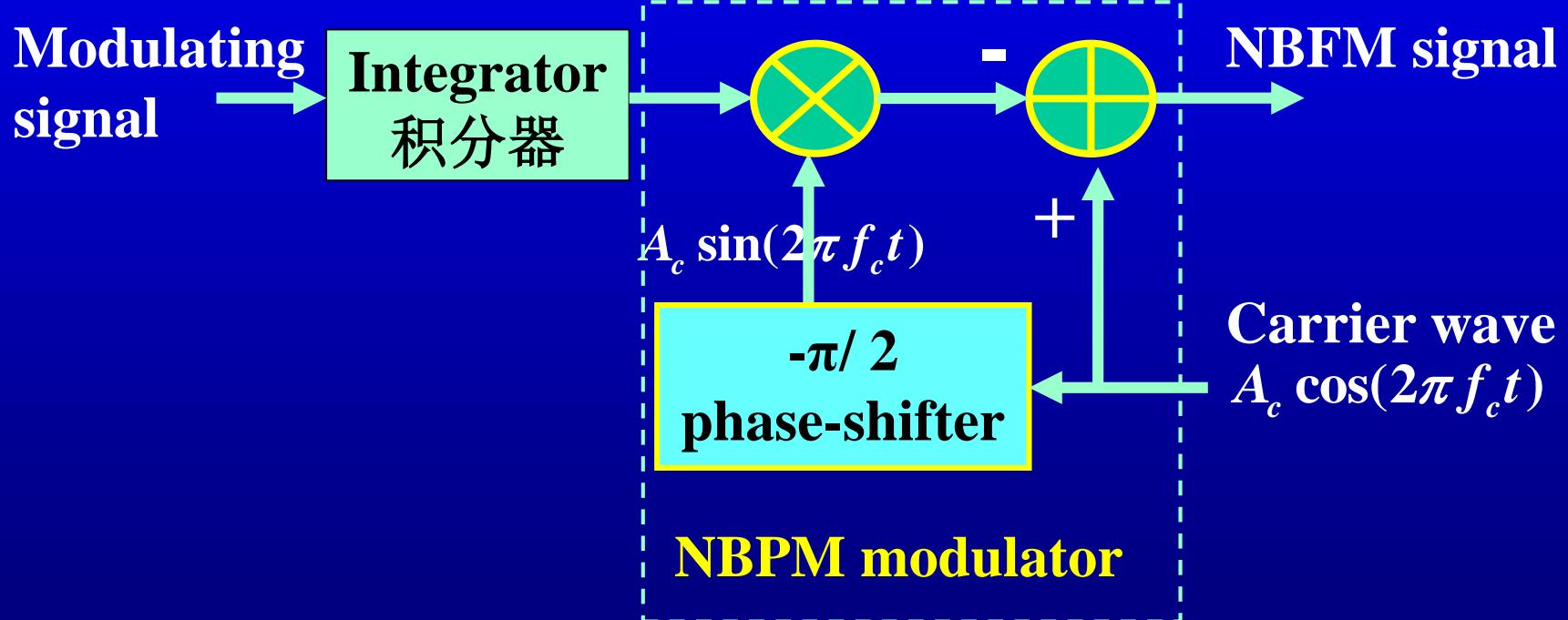
$\cos[\beta \sin(2\pi f_m t)] \approx 1$
$\sin[\beta \sin(2\pi f_m t)] \approx \beta \sin(2\pi f_m t)$

Hence, NBFM signal can be expressed as:

$$s_{NBFM}(t) = s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_m t) \sin(2\pi f_c t)$$

Narrowband Frequency Modulator

$$s_{NBFM}(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_m t) \sin(2\pi f_c t)$$



NBFM is different from ideal FM. Why?

理想的调频信号包络恒定. 但是NBFM却具有:

1. The envelope contains a residual amplitude modulation 残余幅度调制
2. The angle contains harmonic distortion
相位存在谐波失真

If $\beta < 0.3$ radians, the effects of residual AM and harmonic PM are limited to negligible levels.

NBFM is similar to AM.

$$s_{NBFM}(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

$$= A_c \cos(2\pi f_c t) + \frac{1}{2} \beta A_c \{\cos[2\pi(f_c + f_m)t] - \cos[2\pi(f_c - f_m)t]\}$$

2.36

$$s_{AM}(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

$$= A_c [1 + k_a A_m \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

$$= A_c \cos(2\pi f_c t) + \frac{1}{2} k_a A_c \{\cos[2\pi(f_c + f_m)t] + \cos[2\pi(f_c - f_m)t]\}$$

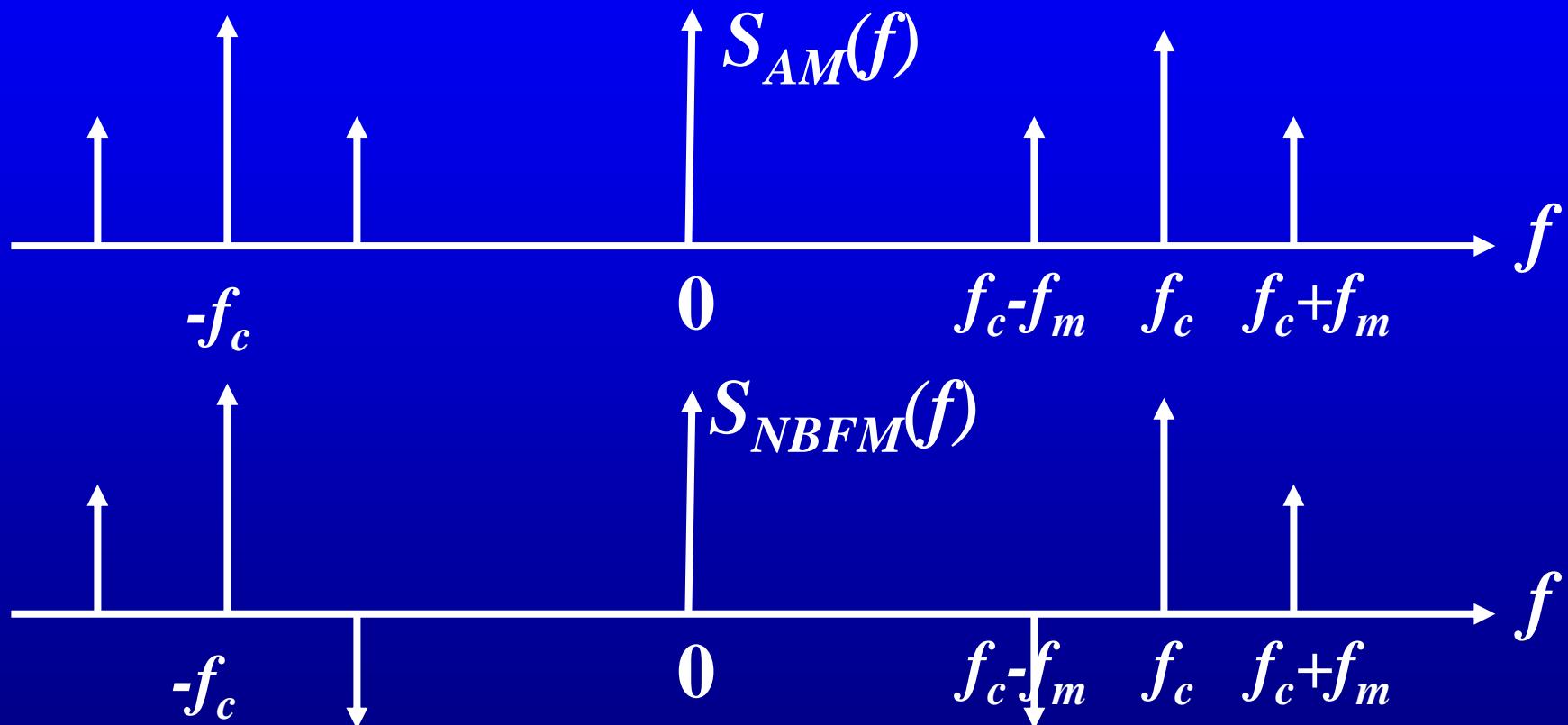
2.37

Basic difference:

the algebraic sign of the lower side frequency

Thinking.....

调制信号为单频余弦信号时的AM与NBFM信号频谱示意图?



NBFM Transmission bandwidth:

$$B_{NBFM} = 2f_m$$

2.7.2 Wideband Frequency Modulation WBFM

任意 β 值的单音频信号调频.

We have known that single-tone FM signal is:

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \quad 2.33$$

For convenience, we use **complex form** 复数形式 to describe band pass signal. It is changed into:

$$\begin{aligned} s(t) &= \operatorname{Re} [A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))] \\ &= \operatorname{Re} [\tilde{s}(t)] \exp(j2\pi f_c t) \quad 2.38 \end{aligned}$$

Complex envelope 复包络

$$\tilde{s}(t) = A_c \exp(j\beta \sin(2\pi f_m t)) \quad 2.39$$

因为 $\tilde{s}(t)$ 是周期的, 傅立叶级数展开:

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_m t) \quad 2.40$$

Where, Fourier coefficient c_n

$$\begin{aligned} c_n &= f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t) \exp(-j2\pi n f_m t) dt \\ &= f_m A_c \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t) \exp[j\beta \sin(2\pi f_m t) - j2\pi n f_m t] dt \end{aligned}$$

定义新的变量:

$$x = 2\pi f_m t$$

Hence, we may rewrite equation 2.41 in new form

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)] dx$$

第一类n阶贝塞尔函数

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)] dx \quad 2.44$$

So, we may reduce Cn

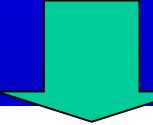
$$c_n = A_c J_n(\beta) \quad 2.45$$

Substituting C_n in $\tilde{s}(t)$, we get

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j2\pi n f_m t)$$

Therefore,

$$s(t) = A_c \bullet \operatorname{Re} \left[\sum_{n=-\infty}^{\infty} J_n(\beta) \exp[j2\pi(f_c + nf_m)t] \right] \quad 2.48$$

 FT

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) [\delta(f - f_c + nf_m) + \delta(f + f_c + nf_m)]$$

Question: How many frequency components does an FM wave have, even though modulating signal only has single frequency? $B_T=?$

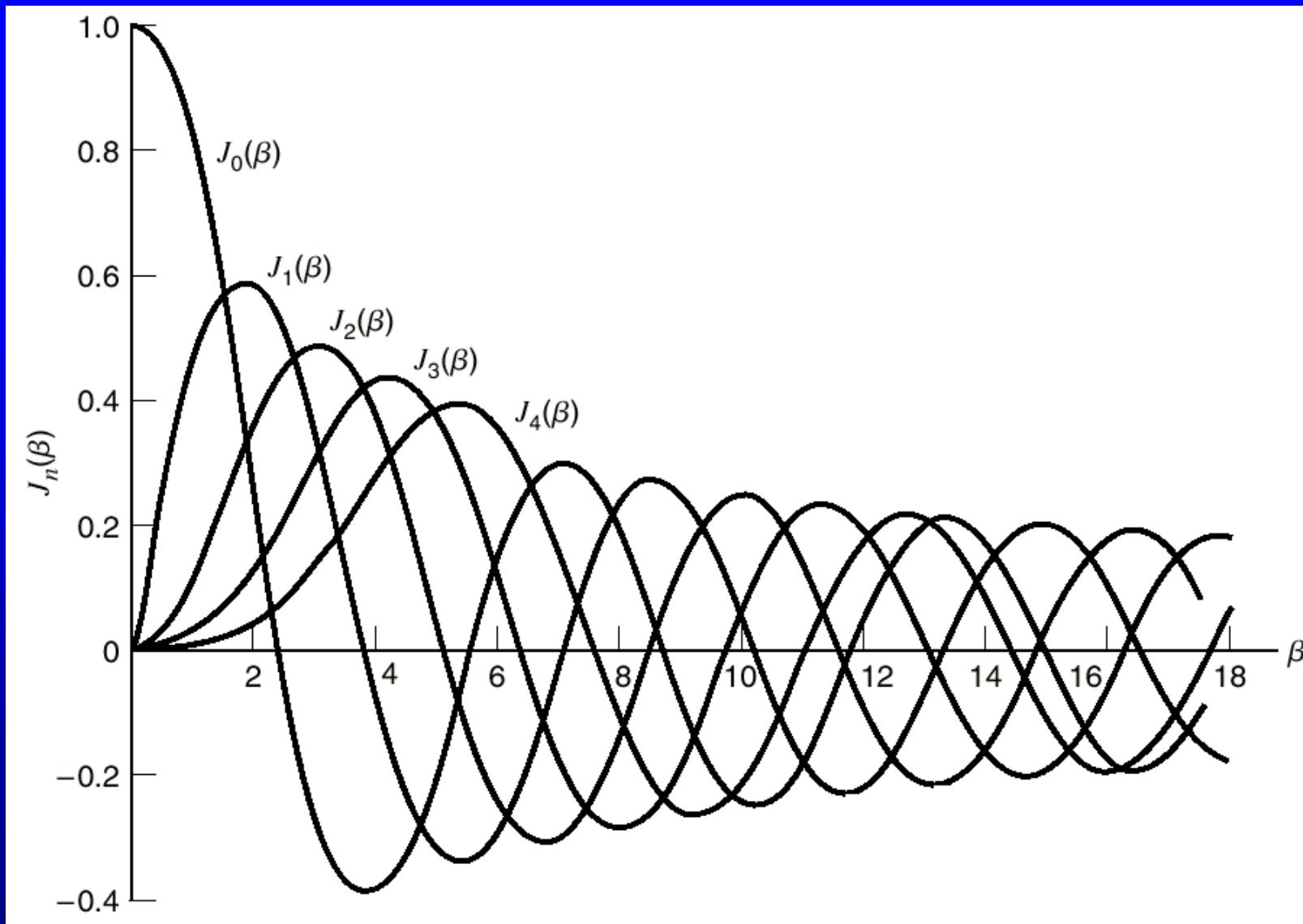


Figure 2.23 Plots of Bessel functions of the first kind for varying order.

Bessel Function Properties

1. $J_n(\beta) = (-1)^n J_{-n}(\beta)$ for all n , both positive and negative

2. For small values of modulation index β ,

$$\begin{cases} J_0(\beta) \approx 1 \\ J_1(\beta) \approx \frac{\beta}{2} \\ J_n(\beta) \approx 0, \quad n > 2 \end{cases} \quad 2.51$$

3.

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1 \quad 2.52$$

Observations About FM

1. The spectrum of an FM signal contains a carrier component and an **infinite** set of side frequencies. 载波分量与无穷多个边频分量
2. If $\beta < 1$, the FM signal is effectively composed of a carrier and a single pair of side frequencies at $f_c \pm fm$. This situation corresponds to **NBFM**.
3. The average power of FM signal is constant.

$$P = \frac{1}{2} A_c^2$$

Review for angle modulation

- **Definition:**

The **angle** of the carrier wave is varied according to the baseband signal.

$$s(t) = A_c \cos[\theta_i(t)]$$

$$\theta_i(t) \propto m(t)$$

- Angle modulation consists of PM and FM.

PM

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$s_{PM}(t) = A_c \cos[\theta_i(t)] = A_c \cos[2\pi f_c t + k_p m(t)] \quad 2.23$$

FM

$$f_i(t) = f_c + k_f m(t) \quad 2.24$$

$$\theta_i(t) = 2\pi \int f_i(t) dt = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \quad 2.25$$

$$s_{FM}(t) = A_c \cos[\theta_i(t)]$$

$$= A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau] \quad 2.26$$

Two important concepts in FM

Page 110

Δf : Frequency deviation 频偏

瞬时频率偏移的最大值 (maximum departure of the instantaneous frequency of the FM signal from the carrier frequency f_c). It is proportional to the amplitude of the modulating signal and is independent of the modulation frequency f_m .

Modulation index 调制指数 β

相位偏移的最大值 (Phase deviation, the maximum departure of the angle $\theta_i(t)$ from the angle $2\pi f_c t$ of the unmodulated carrier.)

Narrowband and Wideband FM

Depending on the value of the modulation index β , there are two cases of frequency modulation:

{ **Narrowband FM:** $\beta \ll 1$
Wideband FM: otherwise

Steps to analyze the spectrum of an FM signal

1. First consider the simplest case:
a single-tone modulation, narrowband FM
2. Then consider more general case:
a single-tone modulation, wideband FM

$$\begin{aligned}
 s_{NBFM}(t) &\approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t) \\
 &= A_c \cos(2\pi f_c t) + \frac{1}{2} \beta A_c \{\cos[2\pi(f_c + f_m)t] - \cos[2\pi(f_c - f_m)t]\}
 \end{aligned}$$

$$s(t) = A_c \bullet \text{Re} \left[\sum_{n=-\infty}^{\infty} J_n(\beta) \exp[j2\pi(f_c + nf_m)t] \right] \quad 2.48$$

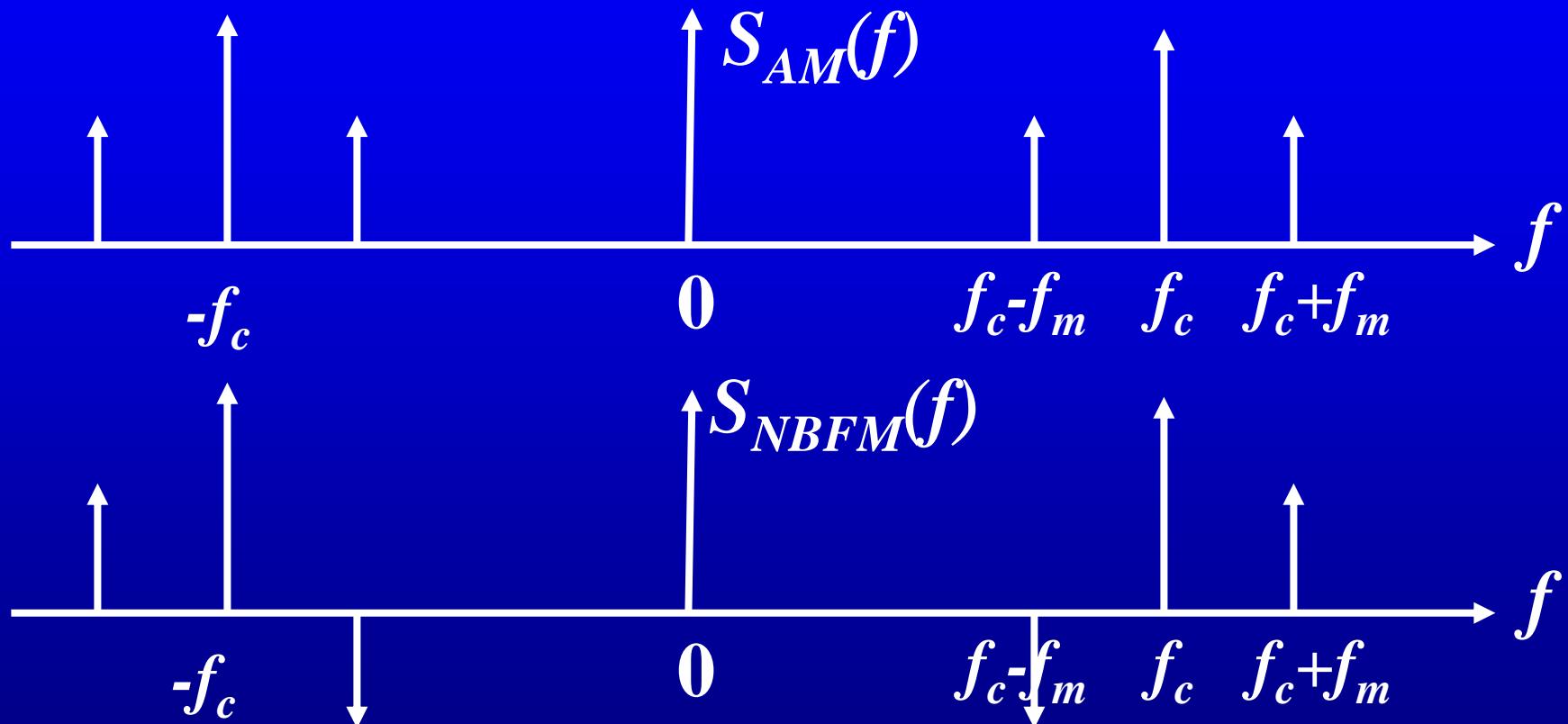
 **FT**

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) [\delta(f - f_c + nf_m) + \delta(f + f_c + nf_m)]$$



Thinking.....

调制信号为单频余弦信号时的AM与NBFM信号频谱示意图?



NBFM Transmission bandwidth:

$$B_{NBFM} = 2f_m$$

Transmission Bandwidth of FM Signals

- In theory, FM 信号包含无穷个边频分量, 所以带宽似乎为无限宽.
- In practice, FM 信号的能量或功率限制在有限的主要边频内.
- 因此, 用有效传输带宽(effective bandwidth) 来表示FM信号所需要的传输带宽.

How to estimate transmission bandwidth of FM signals?

- Percent method 百分比定义法
- Cason's rule 卡森公式

percent method

$$B_T = 2n_{\max}f_m$$

n_{\max} : 幅度大于未调载波幅度的1%的最大边频分量。
给定 β 后，可以通过查表得到 n_{\max} ，也可以查计算FM带宽的一般曲线得到。

Carson's Rule 卡森公式

$$B_T \approx 2\Delta f + 2f_m = 2\Delta f \left(1 + \frac{f_m}{\Delta f}\right) = 2\Delta f \left(1 + \frac{1}{\beta}\right)$$

$$= 2\left(\frac{\Delta f}{f_m} + 1\right)f_m = 2(\beta + 1)f_m \quad 2.55$$

卡森公式估计结果偏小，一般曲线查出的结果偏大，实际中取折中。

单频信号调频→任意信号调频 Page 119

任意信号 $m(t)$, 最高频率 W .

Deviation ratio 偏移率, 频偏比

$$D = \frac{\text{frequency deviation}}{\text{highest modulation frequency}} = \frac{\Delta f}{W}$$

D 类似于调制指数 β .

Carson's rule is modified

$$B_T \approx 2\Delta f + 2W = 2\Delta f \left(1 + \frac{1}{D}\right) = 2(1 + D)W$$

Example 2.3

FM radio broadcasting:

$$W = 15 \text{ kHz}, \Delta f = 75 \text{ kHz}, B_T = ?$$

Deviation ratio is: $D = 75/15 = 5$

1) According to Carson's rule, we get transmission bandwidth:

$$B_T = 2(75 + 15) = 180 \text{ kHz}$$

2) According to percent method, universal curve tells:

$$B_T = 3.2 \times \Delta f = 3.2 \times 75 = 240 \text{ kHz}$$

In practice, a bandwidth of 200 kHz is allocated to each FM transmitter.

Generation of FM signals Page 120

Two basic methods:

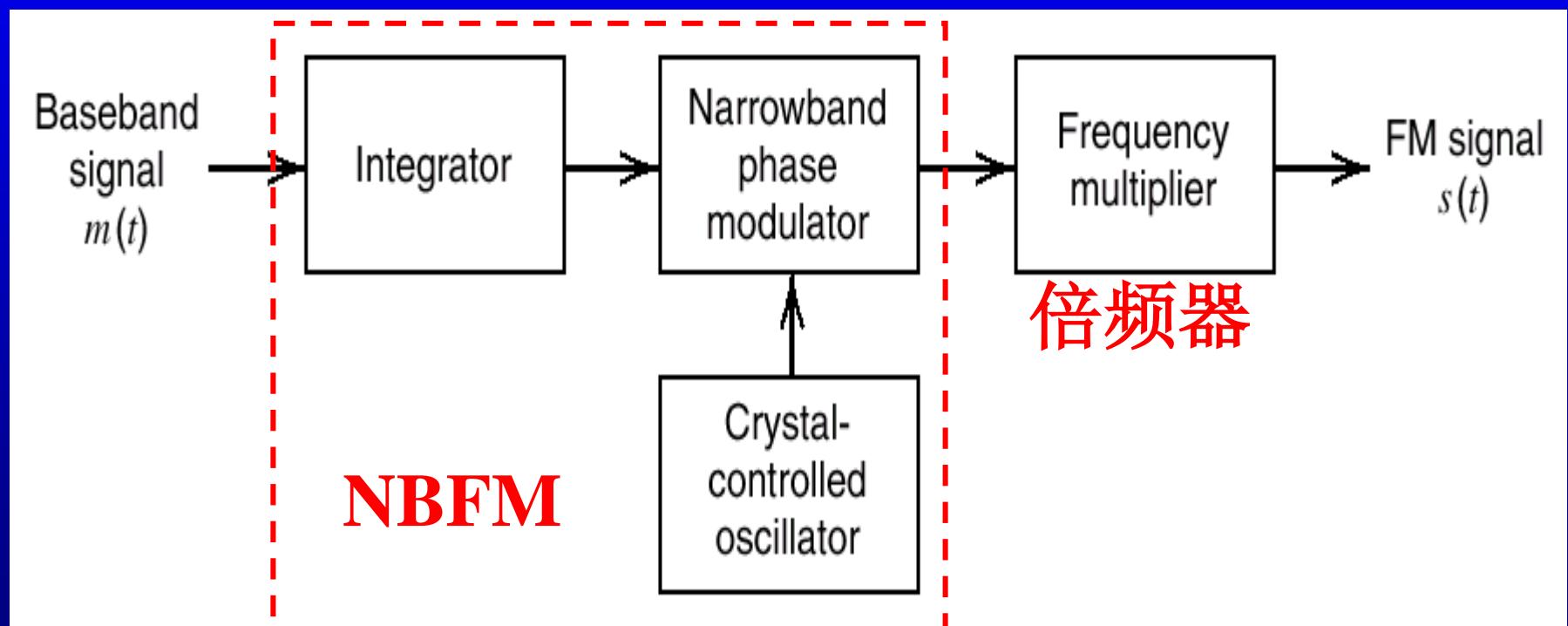
1. Direct FM 直接调频

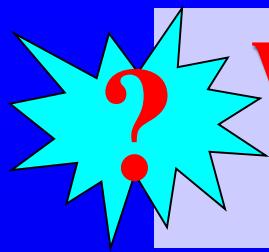


2. Indirect FM 间接调频



Figure 2.27 Block diagram of the indirect method of generating a wideband FM signal





Why can a frequency multiplier change NBFM into WBFM?

倍频器=非线性器件+带通滤波器

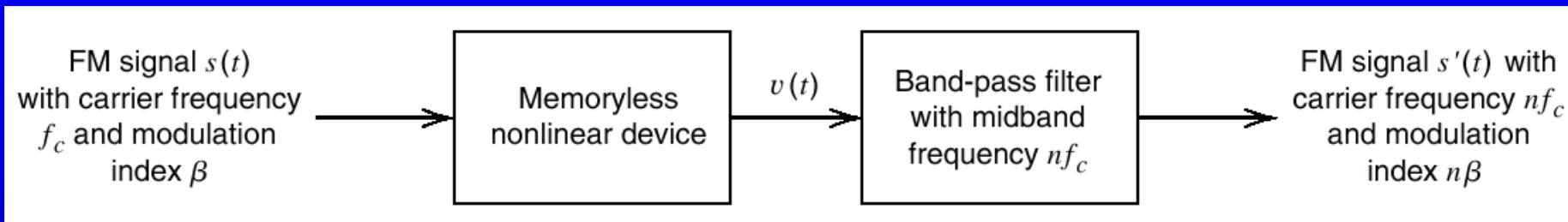


Figure 2.28 Block diagram of frequency multiplier

The input-output relation of a nonlinear device may be expressed in the general form

$$v(t) = a_1 s(t) + a_2 s^2(t) + \cdots + a_n s^n(t)$$

The input is an FM signal defined by

$$s(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]$$

Instantaneous frequency 瞬时频率 is

$$f_i(t) = f_c + k_f m(t)$$

Output signal s'(t) is

$$s'(t) = A'_c \cos \left[2\pi n f_c t + 2\pi n k_f \int_0^t m(\tau) d\tau \right]$$

Instantaneous frequency of output signal s'(t) is

$$f'_i(t) = n f_c + n k_f m(t)$$

So

$$\Delta f' = n \Delta f \quad \beta' = n \beta \quad f_c' = n f_c$$

NBFM → WBFM

Demodulation of FM Signals Page 121

Two basic methods:

1. Indirect method: phase-locked loop 琐相环

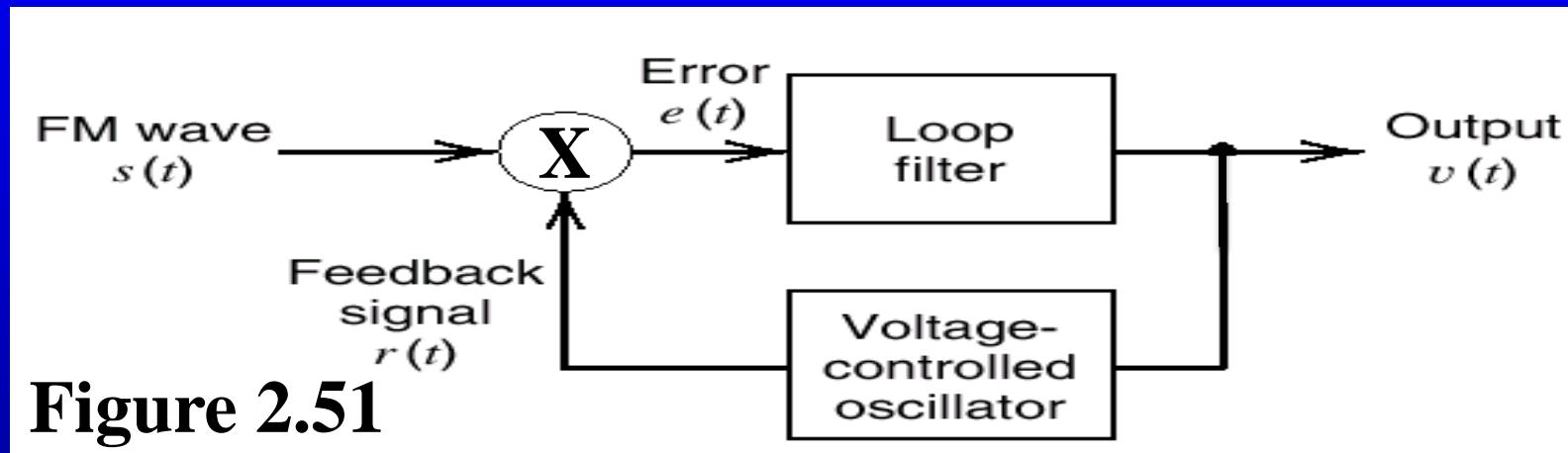


Figure 2.51

2. Direct method: frequency discriminator 鉴频器



鉴频器工作原理:

$$s_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau]$$

$$\frac{d s_{FM}(t)}{d t} = -A_c [2\pi f_c + 2\pi k_f m(t)] \sin[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau]$$

AMFM signal 调幅调频信号

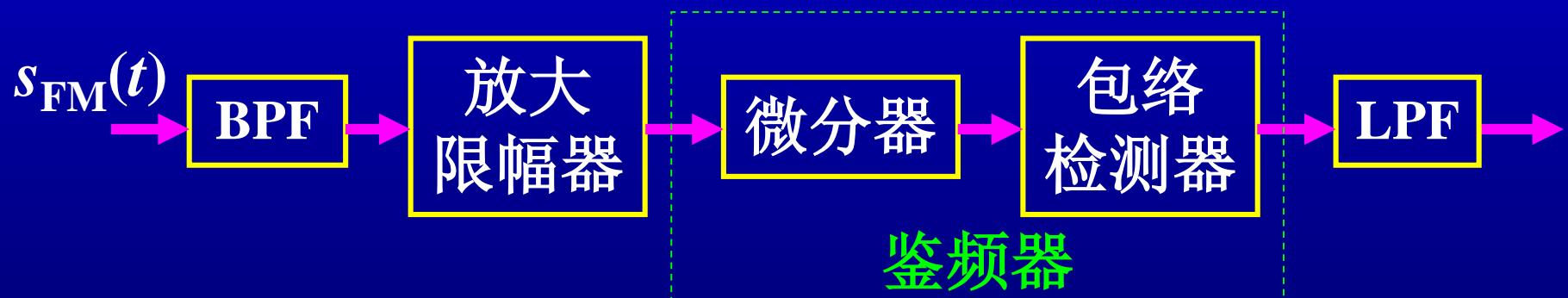
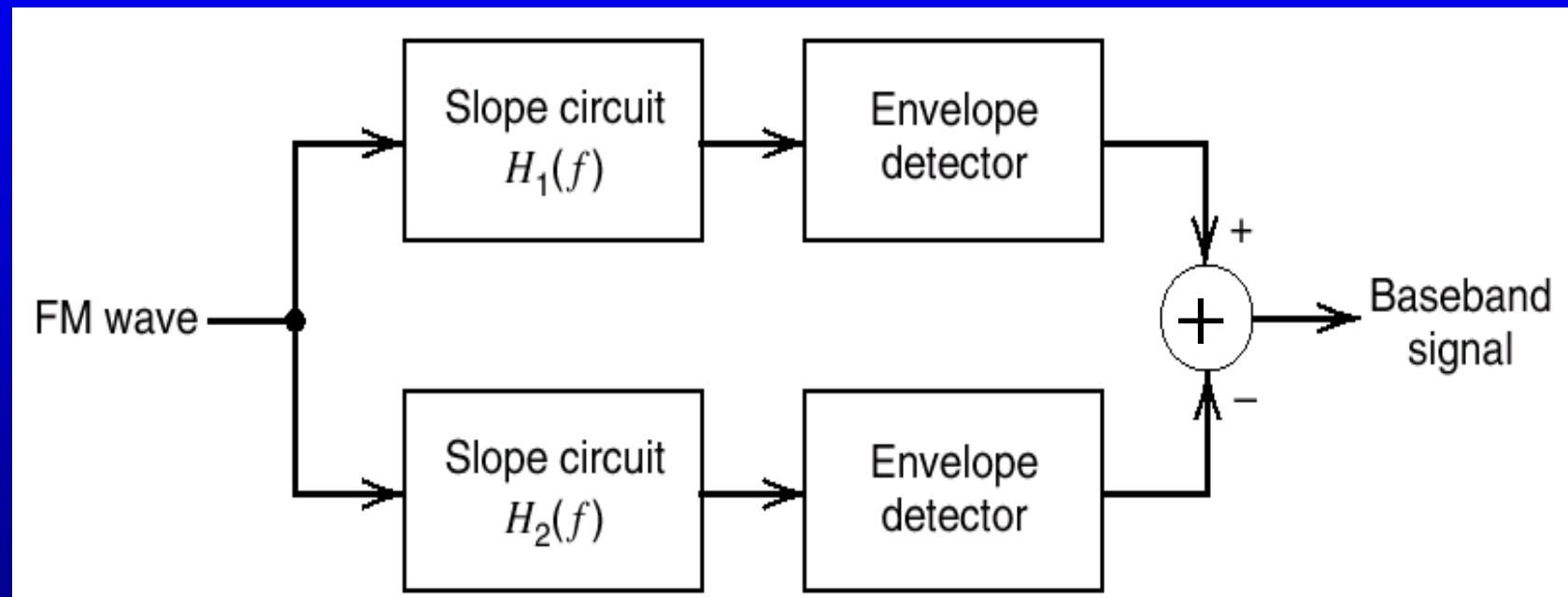


Fig. 2.30 balanced frequency discriminator



FM Stereo Multiplexing



questions

- What is FM stereo?
- What are the factors to influence the FM stereo transmission?
- How does it work?



Discussion
topic

无线电接收机

一百年前意大利人马可尼和俄罗斯人波波夫发明无线电的时候，他们的接收机利用简单的金属粉末检波器直接将天线接收到的射频信号的幅度检测出来，转换成直流或者低频信号。由于检波器等解调电路要求输入信号达到一定幅度才能正常工作，所以这种射频信号不经放大就进行解调的直接解调式接收机灵敏度很低，只能接收近处的强信号。

1906年人类制成了第一只三极电子管，1947年又制成了第一只晶体管，这些器件都可以放大电信号。利用这些放大器件，人们制成了先将天线收到的微弱射频信号加以放大，再进行解调的无线电接收机。这种带有射频放大级的直接放大式接收机的灵敏度有了显著提高。为了得到足够高的灵敏度，有的收音机具有两级或更多级的射频放大器，20年代的商品收音机就经常采用这种方式。但是射频信号频率高，放大后的信号很容易窜过电极或导线间的分布电容串回放大器的输入段，形成自激振荡，所以多级直接放大式的接收机不但需要良好的屏蔽和各级选频回路的统调，而且射频放大器的总放大倍数和整机稳定性之间往往很难兼顾好，限制了灵敏度的提高。

为了解决射频信号放大量和电路稳定性之间的矛盾，超外差式接收机成为40年代末商品广播及通信接收机的主流。超外差接收机通过一次（或多次）变频电路将射频信号的频率变换为另一个（或多个）中间频率。由于射频信号的总放大量被分配在射频和中频的不同频率的电路中，最终的输出信号即使寄生耦合到最前级的输入端也不致引起自激寄生振荡，大大提高了接收机可以稳定工作的灵敏度。同时，固定的中间频率也简化了各级调谐电路之间的统调，采用适当的中频和谐振回路可以大大改善接收机的选择性。超外差式接收机至今仍是中波、短波和超短波接收机的主要型式。从电影《永不消逝的电波》中的李侠以及《英雄儿女》中王成所使用的电台的接收机，到今天的商品电视机、对讲机和绝大部分广播收音机和专业通信接收机，基本上都是超外差式的。

超外差接收机虽然在灵敏度和选择性方面都有较高的性能，但是它的电路结构比较复杂，尤其是变频过程中会发生镜频干扰、本振谐波寄生响应以及其它的互调干扰，为了消除它们，需要插入各种适当的滤波器。

随着半导体器件制造工艺的进步，现在可以制造出一些增益既高、又可稳定工作的单片射频放大器，以及可以对微弱信号直接解调的器件，因此移动电话的接收机电路又开始摆脱超外差的型式，回到直接解调的型式，以省去消除变频所产生的干扰所需要的滤波器，降低生产成本。

2.9 Superheterodyne Receiver =superhet 超外差接收机

接收机分类:

- { 零中频接收机(直接解调式接收机)
- 超外差接收机

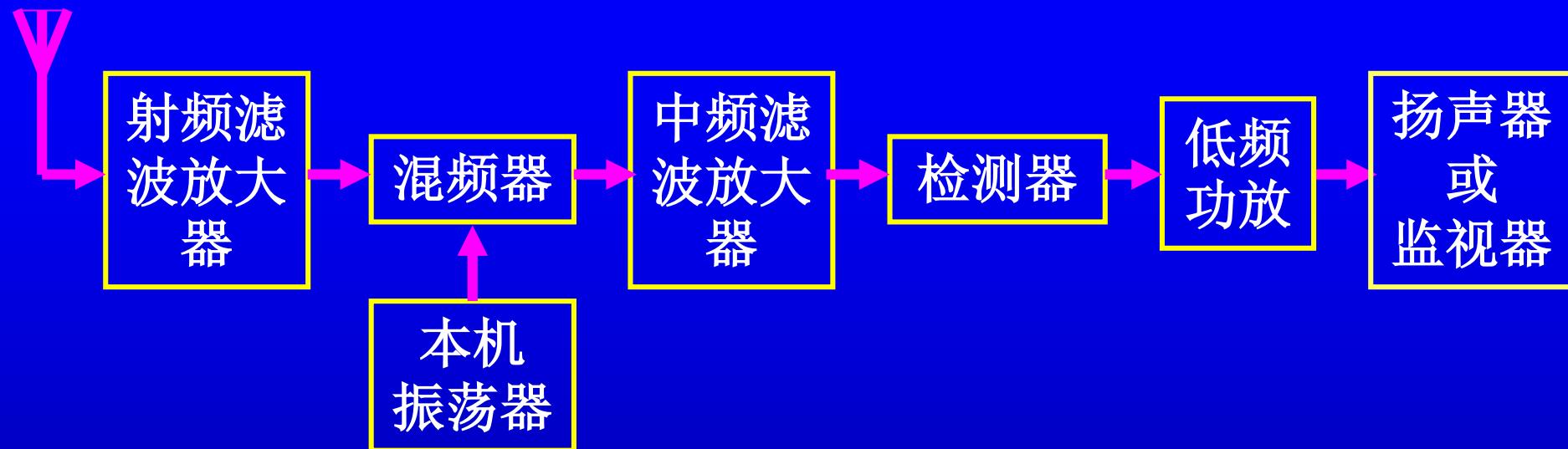
Since Armstrong invented the superheterodyne radio receiver in 1918, almost all radio and TV receivers now being made are of the superhetrodyne type.

Receivers in a broadcasting system performs following functions:

- Carrier-frequency tuning
- Filtering
- amplification

Seperhet is a special type of receiver that fulfills all three functions in an elegant and practical fashion.

超外差接收机的一般模型：



radio-frequency (RF) section 射频放大器

mixer 混频器

local oscillator 本地振荡器

intermediate-frequency (IF) section 中频放大器

demodulator 解调器

power amplifier 功率放大器

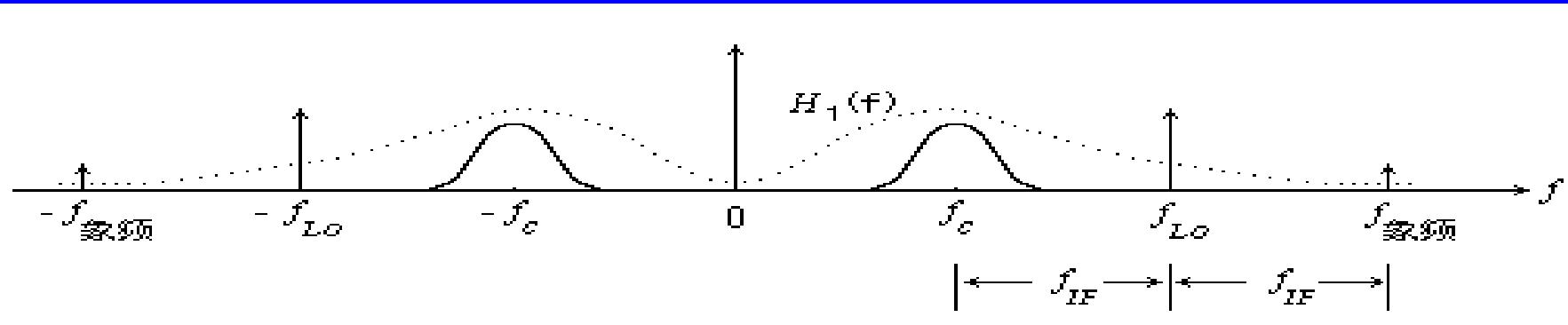


Image Interference 镜像干扰

Image frequency 镜像频率

$$f_{\text{象频}} = \begin{cases} f_c + 2 f_{IF} & \text{如果 } f_{Lo} > f_c \\ f_c - 2 f_{IF} & \text{如果 } f_{Lo} < f_c \end{cases}$$

减小像频干扰的方法是提高射频放大器的选频特性。

AM超外差接收机

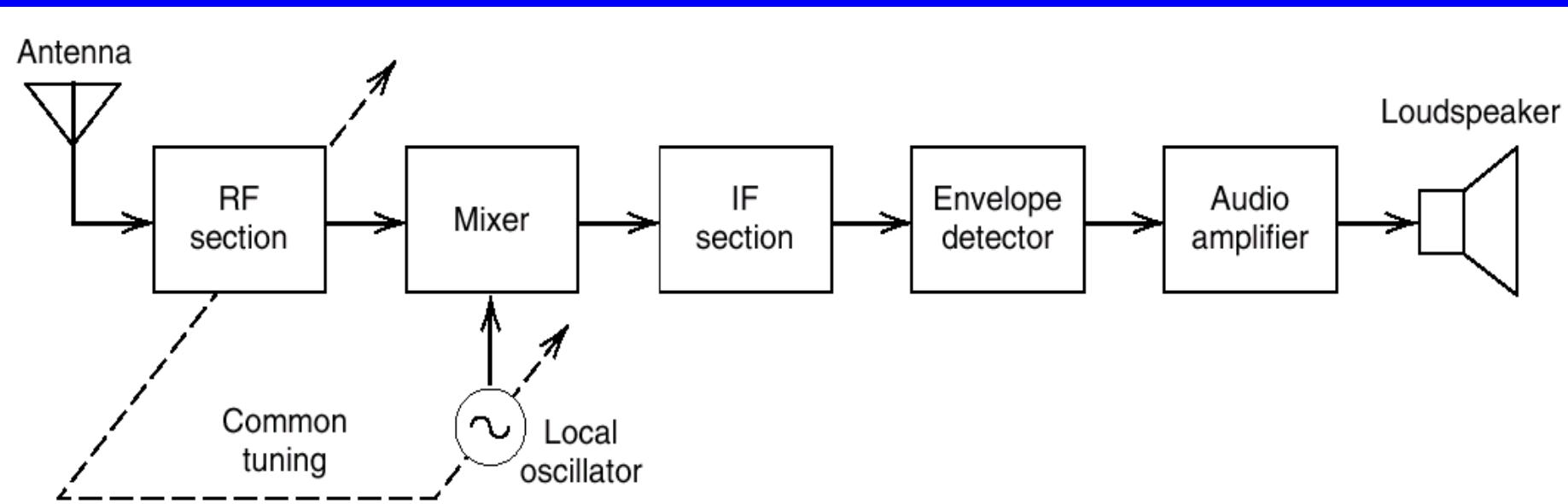


Figure 2.32 Basic elements of an AM radio receiver of the superheterodyne type.

$$f_{IF} = f_{LO} - f_{RF}$$

频率划分

FCC: Federal Communications Commission
美国联邦通信委员会

- 标准AM电台: 535~1605 KHz, 频道间隔为10 KHz, 中频频率 455 KHz.
- 调频 (FM) 广播: 88~108MHz, 频道间隔为200 KHz, 中频频率10.7MHz.
- 广播电视: 视频信号VSB, 音频信号FM, 频道6MHz

How about China?

中国，无线电管理委员会

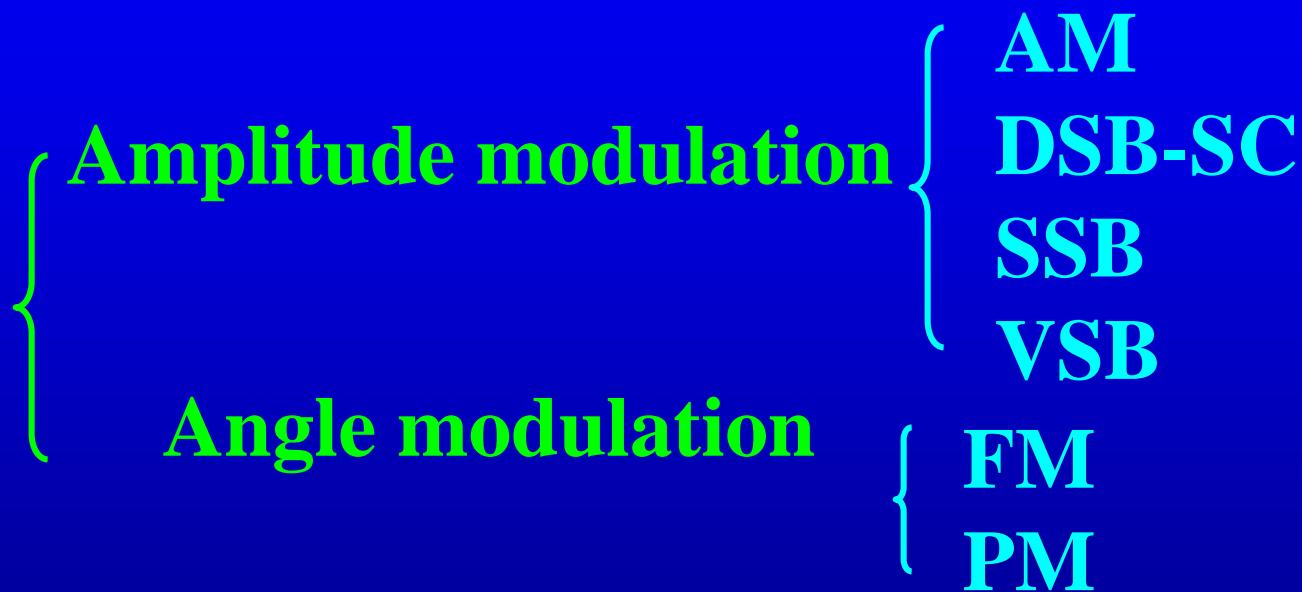
- AM电台: 535~1605KHz, 中频频率465KHz, 频道间隔为9KHz。
- FM广播: 88~108MHz, 中频频率10.7MHz, 每个频道150KHz .
- 电视: 视频信号VSB, 音频信号FM, 频道间隔8MHz

中国无线电频率分配表:

<http://www.mcuhw.com/>

Contents of this chapter

I. continuous-wave modulation schemes



II. Noise performance for different modulation schemes

Descriptions of AM signals

AM: $s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$

DSB: $s(t) = A_c m(t) \cos(2\pi f_c t)$

SSB: $s(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) \pm \frac{1}{2} A_c \hat{m}(t) \sin(2\pi f_c t)$

- + lower sideband transmitted
- upper sideband transmitted

VSB: $s(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) m(t) \pm \frac{1}{2} A_c m'(t) \cos(2\pi f_c t)$

- + vestige of the upper sideband
- vestige of the lower sideband

Description of FM and PM signals

$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)] \quad 2.23$$

$$s_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau] \quad 2.26$$

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} \quad \longleftrightarrow \quad \theta_i(t) = 2\pi \int f_i(t) dt$$

So, we may deduce all the properties of PM signals from those of FM signals and vice versa. Hence, we concentrate our attention on FM signals.

根据解调时是否需要产生与发送载波同频同相的本地载波，将解调方式分为两大类：

- 相干解调
- 非相干解调

AM----包络检波（非相干解调），相干解调

DSB----相干解调

SSB----相干解调

VSB----相干解调

FM----鉴频器（非相干解调）

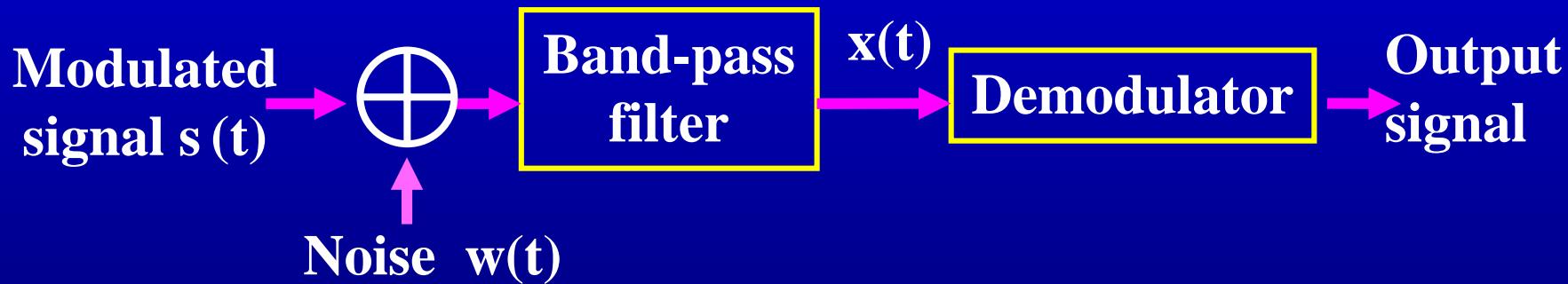
2.10 Noise in CW Modulation Systems

Formulating two models:

1. Channel model

AWGN: additive white Gaussian noise

2. Receiver model



理想带通滤波器的带宽刚好使已调信号无失真地通过,尽可能多地抑制带外噪声.

Signal-to-Noise Ratio: Basic Definitions

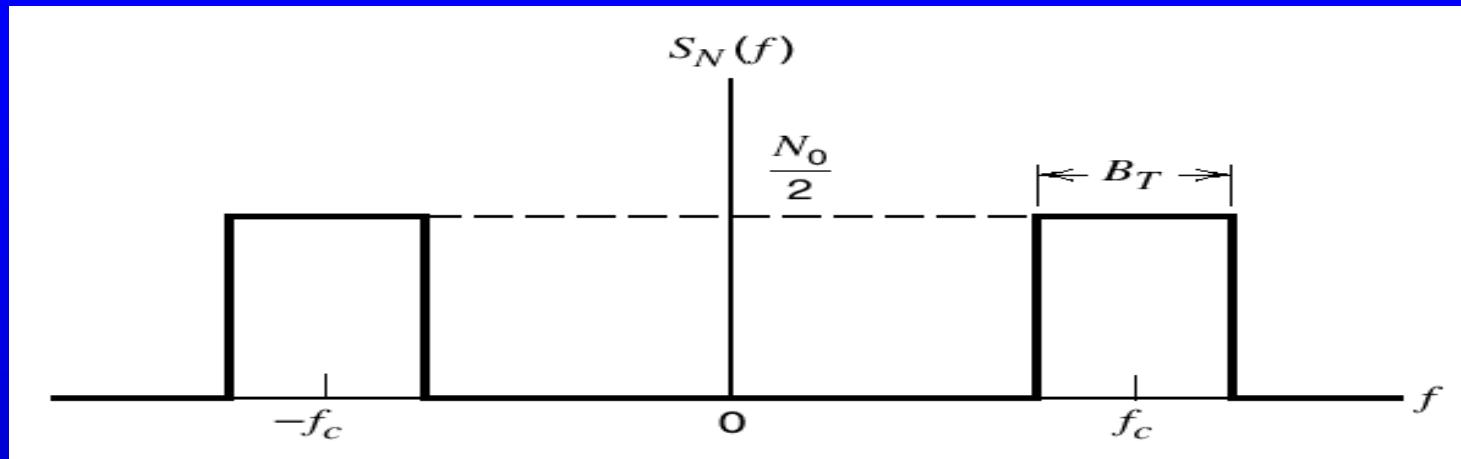
$$SNR = \frac{\text{average power of signal}}{\text{average power of noise}}$$

Power spectral density of White noise $w(t)$:

$$S_W(f) = \frac{N_0}{2} \quad f \in (-\infty, +\infty)$$

N_0 is the average noise power per unit bandwidth.

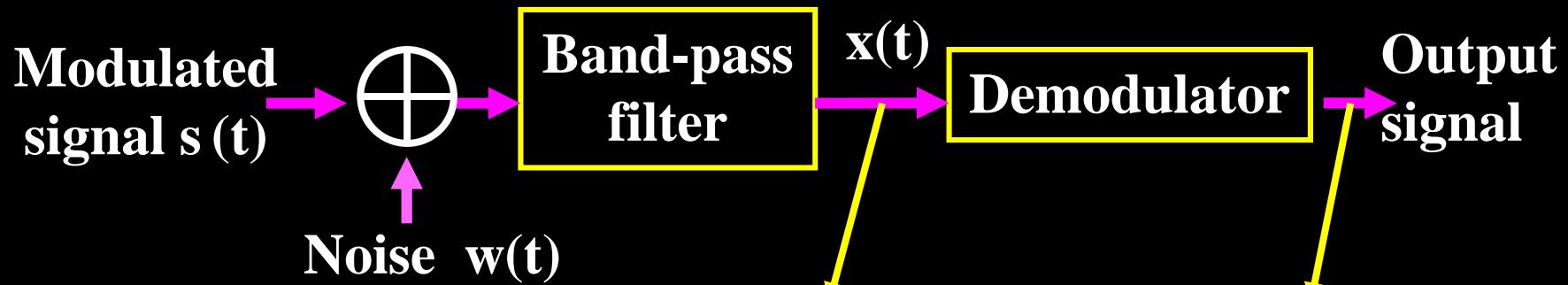
Ideal band-pass filtered noise 带通滤波后的噪声



Average noise power $N_0 B_T$

Furthermore, the output noise of filter can be regarded as **narrowband noise** 窄带带通噪声

$$n(t) = n_I(t) \cos[2\pi f_c t] - n_Q(t) \sin[2\pi f_c t]$$

 SNR_I

输入信噪比

 SNR_O

输出信噪比

The input of the demodulator is

$$x(t) = s(t) + n(t)$$

s(t) is useful modulated signal
 n(t) is narrowband noise

信噪比增益

$$G = \frac{SNR_o}{SNR_I} = \frac{\text{输出信噪比}}{\text{输入信噪比}}$$

How to compare noise performance among different modulation types?

- Compare SNR_o
- Compare G

Fair ?

It's Unfair !!!

Requirements of Fair comparison:

1. The modulated signal $s(t)$ has the same average power.
2. The channel noise $w(t)$ has the same average power measured in the message bandwidth W .

SNR_c 信道信噪比

-----已调信号的平均功率与消息信号带宽内信道噪声的平均功率之比.

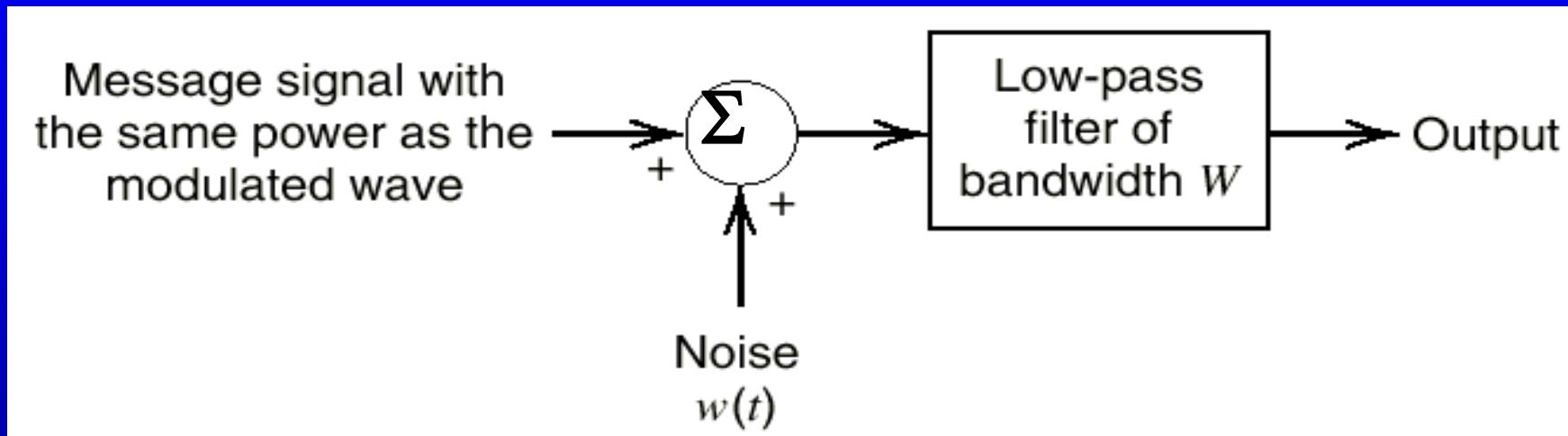


Figure 2.35 The baseband transmission model, assuming a message signal of bandwidth W , used for calculating the channel signal-to-noise ratio.

解调增益

$$\text{Figure of merit} = \frac{(\text{SNR})_O}{(\text{SNR})_C}$$

Two Terms

信噪比增益

$$G = \frac{SNR_o}{SNR_I} = \frac{\text{输出信噪比}}{\text{输入信噪比}}$$

解调增益

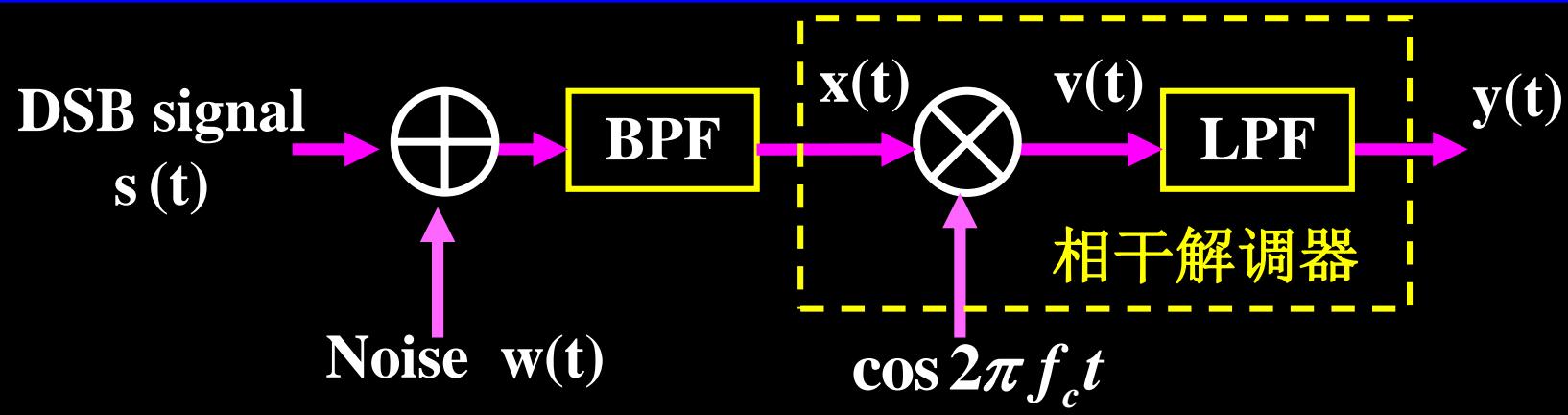
$$\text{Figure of merit} = \frac{(\text{SNR})_o}{(\text{SNR})_c} = \frac{\text{输出信噪比}}{\text{信道信噪比}}$$

2.11 Noise in Linear Receivers Using Coherent Detection 相干检测线性接收机中的噪声

- Linear receiver:
DSB-SC and SSB coherent detector
- Nonlinear receiver:
AM envelope detector

Take DSB for example to analyze noise performance of coherent detection.

Model of DSB-SC receiver using coherent detection



$$s_{DSB}(t) = CA_c m(t) \cos[2\pi f_c t]$$

C: system-dependent scaling factor 系统比例因子

设消息信号 $m(t)$ 的功率谱密度为 $S_M(f)$, 则它的平均功率 P 为

$$P = \int_{-W}^W S_M(f) df$$

DSB信号的平均功率

Average power of DSB

$$S_{DSB} = \frac{C^2 A_c^2 P}{2}$$

消息带宽W内的平均噪声功率

Average noise power

$$N = WN_0$$

信道信噪比

$$(\text{SNR})_{C, DSB} = \frac{C^2 A_c^2 P}{2WN_0} \quad 2.84$$

实际的DSB接收机接收的平均噪声功率

(解调器输入的平均噪声功率)

$$N_i = 2WN_0$$

因此

$$(\text{SNR})_I = \frac{C^2 A_c^2 P}{4WN_0}$$

SNR_O ?

Input of the product-modulator is

$$\begin{aligned}x(t) &= s(t) + n(t) \\&= CA_c \cos(2\pi f_c t)m(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\cos(2\pi f_c t)\end{aligned}$$

The output of the product-modulator is

$$\begin{aligned}v(t) &= x(t)\cos(2\pi f_c t) \\&= \frac{1}{2}CA_c m(t) + \frac{1}{2}n_I(t) \\&\quad + \frac{1}{2}[CA_c m(t) + n_I(t)]\cos(4\pi f_c t) - \frac{1}{2}n_Q(t)\sin(4\pi f_c t)\end{aligned}$$

LPF后的输出

$$y(t) = \frac{1}{2}CA_c m(t) + \frac{1}{2}n_I(t) \quad 2.86$$

The receiver output

$$y(t) = \frac{1}{2}CA_c m(t) + \frac{1}{2}n_I(t)$$

Output message signal:

$$s_o(t) = \frac{1}{2}CA_c m(t)$$

Power of output signal

$$S_o = \frac{1}{4}C^2 A_c^2 P$$

Average Power of filtered noise n(t)

窄带噪声的平均功率

$$N_i = 2N_0W$$

Noise output

$$n_0(t) = \frac{1}{2}n_I(t)$$

Average power of the in-phase noise component $n_I(t)$ is the same as that of the filtered noise $n(t)$. 窄带噪声同相分量的平均功率与窄带噪声的平均功率相等.

Output noise power

$$N_{out} = \left(\frac{1}{2}\right)^2 2WN_0 = \frac{1}{2}WN_0$$

The output SNR for a DSB-SC receiver using coherent detection is therefore

$$(\text{SNR})_{O, \text{DSB-SC}} = \frac{C^2 A_c^2 P / 4}{WN_0 / 2} = \frac{C^2 A_c^2 P}{2WN_0} \quad 2.87$$

Therefore

$$\text{Figure of merit} = \left. \frac{(\text{SNR})_o}{(\text{SNR})_c} \right|_{\text{DSB}} = 1$$

$$G_{\text{DSB}} = \frac{SNR_o}{SNR_I} = 2$$

Similar analysis to SSB demodulator Problem 2.49, we can get

$$\text{Figure of merit} = \left. \frac{(\text{SNR})_o}{(\text{SNR})_c} \right|_{\text{SSB}} = 1$$

$$G_{\text{SSB}} = \frac{SNR_o}{SNR_I} = 1$$

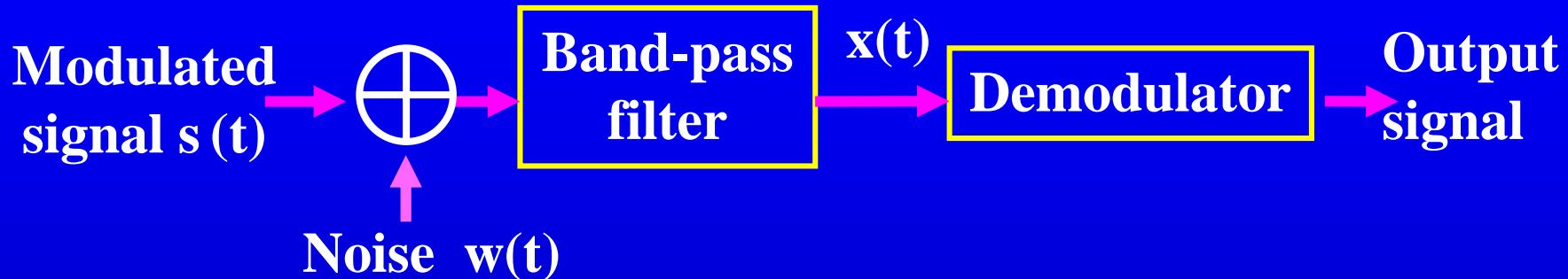
如果单纯根据信噪比增益 $G_{SSB}=1$, $G_{DSB}=2$,
容易得出DSB比SSB抗噪声性能好的错误结论。

SSB has the same figure of merit as DSB.

结论：但在相同的噪声环境和相同的发射功率情况下，DSB与SSB的解调增益(Figure of merit) 都为1, 说明DSB和SSB有相同的抗噪声性能。

Review for last class

receiver model



noise performance : DSB, SSB ,AM

$$\text{Figure of merit} = \left. \frac{(\text{SNR})_o}{(\text{SNR})_c} \right|_{DSB} = 1$$

$$\text{Figure of merit} = \left. \frac{(\text{SNR})_o}{(\text{SNR})_c} \right|_{SSB} = 1$$

2.12 Noise in AM receivers Using Envelope Detection

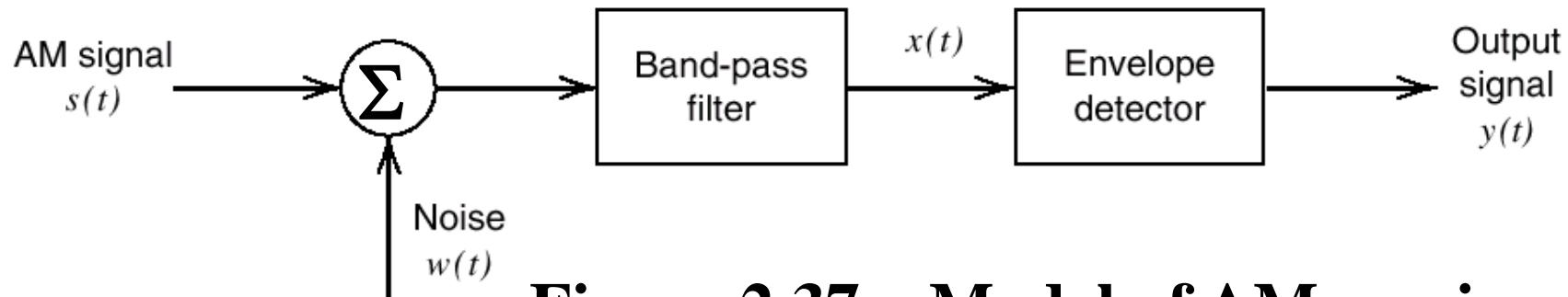


Figure 2.37 Model of AM receiver.

AM signal

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

Power of AM signal

$$S_{AM} = \frac{1}{2} A_c^2 (1 + k_a^2 P)$$

消息带宽内的
Noise power

$$N = WN_0$$

解调器输入端的噪声功率

$$N_I = 2WN_0$$

因为AM调制，所以BPF带宽为2W

输入信噪比

$$(\text{SNR})_{I,AM} = \frac{S_I}{N_I} = \frac{S_{AM}}{N_I} = \frac{A_c^2 (1 + k_a^2 P)}{4WN_0}$$

Channel SNR

$$(\text{SNR})_{C,AM} = \frac{A_c^2 (1 + k_a^2 P)}{2WN_0} \quad 2.90$$

$$\begin{aligned}x(t) &= s(t) + n(t) \\&= [A_c + A_c k_a m(t) + n_I(t)] \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)\end{aligned}\quad 2.91$$

$$\begin{aligned}y(t) &= \text{envelope of } x(t) \\&= \sqrt{[A_c + A_c k_a m(t) + n_I(t)]^2 + n_Q^2(t)}\end{aligned}\quad 2.92$$

It's difficult to get the relationship between signal and noise. So, we just discuss it under different conditions.

1) When Signal>>Noise

$$A_c[1+k_a m(t)] > \sqrt{n_I^2(t) + n_Q^2(t)}$$

$$y(t) = \sqrt{[A_c + A_c k_a m(t) + n_I(t)]^2 + n_Q^2(t)}$$

$$= \sqrt{[A_c + A_c k_a m(t)]^2 + 2[A_c + A_c k_a m(t)] \cdot n_I(t) + n_I^2(t) + n_Q^2(t)}$$

$$\approx \sqrt{[A_c + A_c k_a m(t)]^2 + 2[A_c + A_c k_a m(t)] \cdot n_I(t)}$$

$$= [A_c + A_c k_a m(t)] \sqrt{1 + \frac{2 \cdot n_I(t)}{[A_c + A_c k_a m(t)]}}$$

$$\approx [A_c + A_c k_a m(t)] \left[1 + \frac{n_I(t)}{A_c + A_c k_a m(t)} \right]$$

$$\sqrt{1+x} \approx 1 + \frac{1}{2}x \quad \text{当} |x| \ll 1 \text{ 时}$$

$$y(t) \approx A_c + A_c k_a m(t) + n_I(t) \quad 2.93$$

A_c 可以用隔直电容去掉

$$y(t) = A_c k_a m(t) + n_I(t)$$

$$y(t) = A_c k_a m(t) + n_I(t)$$

输出信号的平均功率

$$S_O = A_c^2 K_a^2 P$$

输出噪声的平均功率

$$N_O = 2WN_0$$

Output SNR

$$(SNR)_{O,AM} \approx \frac{A_c^2 k_a^2 P}{2WN_0} \quad 2.94$$

$$\text{figure of merit} = \left. \frac{(SNR)_O}{(SNR)_C} \right|_{AM} \approx \frac{k_a^2 P}{1 + k_a^2 P} \quad 2.95$$

$$G_{AM} = \frac{SNR_o}{SNR_I} = \frac{2k_a^2 P}{1 + k_a^2 P}$$

Comparison: DSB SSB AM

- the figure of merit of DSB and SSB receivers using coherent detection are always **unity**, the corresponding figure of merit of an AM receiver using envelope detection is always **less than unity**.
- In other words, the noise performance of a full AM receiver is always **inferior(更差)** to that of a DSB or SSB receiver. This is due to the **wastage of transmission power**, which results from transmitting the carrier as a component of the AM wave.

Example 2.4 Single-tone Modulation

Single-tone modulating signal

$$m(t) = A_m \cos(2\pi f_m t)$$

The corresponding AM wave

$$s_{AM}(t) = A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

Modulation factor

$$\mu = k_a A_m$$

Average power of $m(t)$

$$P = \frac{1}{2} A_m^2$$

$$\left. \frac{(\text{SNR})_o}{(\text{SNR})_c} \right|_{AM} = \frac{\frac{1}{2} k_a^2 A_m^2}{1 + \frac{1}{2} k_a^2 A_m^2} = \frac{\mu^2}{2 + \mu^2}$$

discussion

$$\left| \frac{(\text{SNR})_o}{(\text{SNR})_c} \right|_{AM} = \frac{\frac{1}{2} k_a^2 A_m^2}{1 + \frac{1}{2} k_a^2 A_m^2} = \frac{\mu^2}{2 + \mu^2}$$

- When $\mu=1$, it corresponds to 100% modulation, we get a figure of merit equal to 1/3.
- This means that, other factors being equal, an AM system must transmit three times as much as average power as a suppressed-carrier system to achieve the same quality of noise performance.

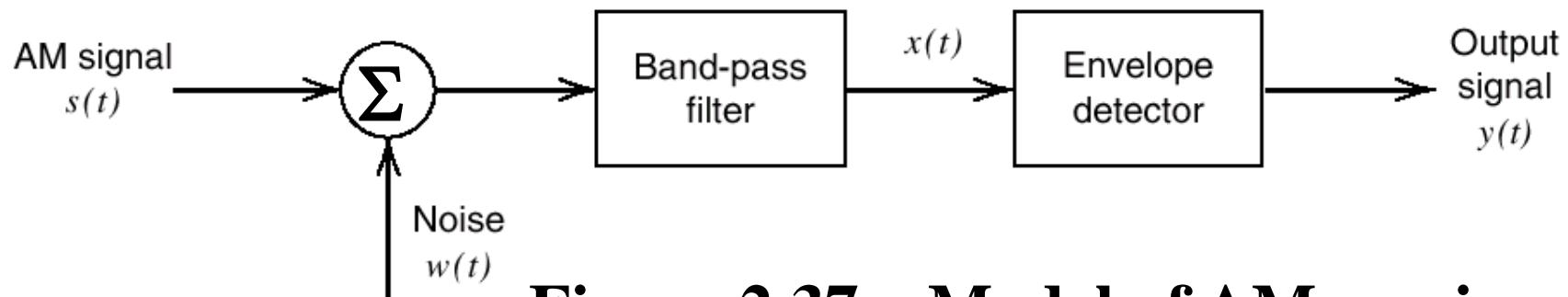


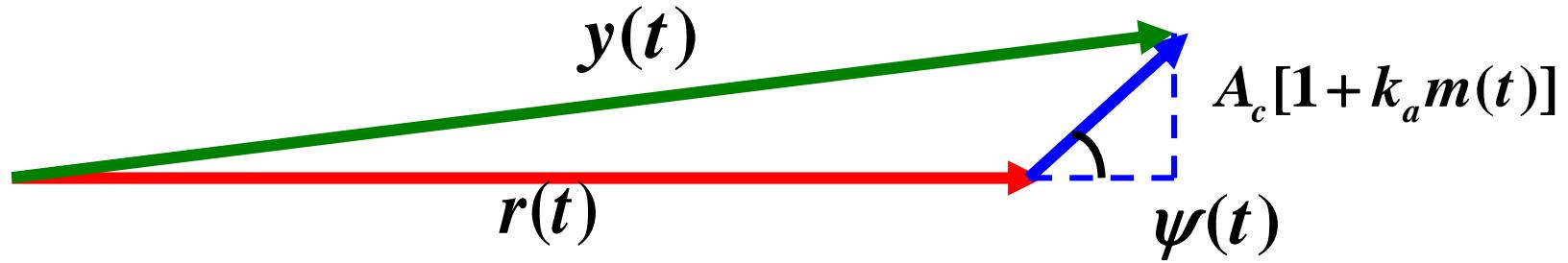
Figure 2.37 Model of AM receiver.

$$\begin{aligned}y(t) &= \text{envelope of } x(t) \\&= \sqrt{\left[A_c + A_c k_a m(t) + n_I(t) \right]^2 + n_Q^2(t)} \quad 2.92\end{aligned}$$

2) When Signal<<Noise

$$A_c[1+k_a m(t)] < \sqrt{n_I^2(t) + n_Q^2(t)} = r(t)$$

$$n(t) = r(t) \cos[2\pi f_c t + \psi(t)]$$



$$y(t) \approx r(t) + A_c \cos[\psi(t)] + A_c k_a m(t) \cos[\psi(t)]$$

In this case, the detector output has no component strictly proportional to the message signal $m(t)$.

Threshold Effect 门限效应

Threshold effect: the loss of a message in an envelope detector that operates at a low SNR is referred to as threshold effect.

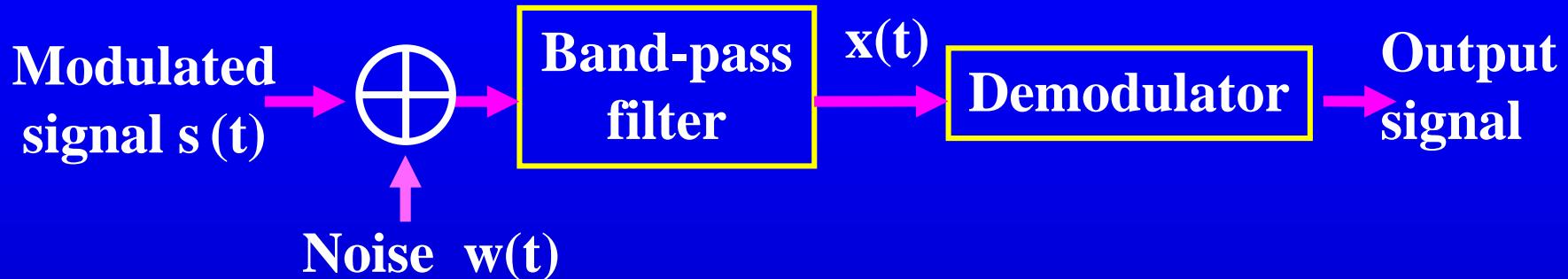
Threshold 门限: we mean a value of the SNR below which the noise performance of a detector deteriorates(恶化,失真) much more rapidly than proportionately to the SNR.

相干检测器不存在门限效应.

非相干检测器存在门限效应.

Review

receiver model



noise performance : DSB, SSB ,AM

$$\text{Figure of merit} = \left. \frac{(\text{SNR})_o}{(\text{SNR})_c} \right|_{DSB} = 1$$

$$\text{Figure of merit} = \left. \frac{(\text{SNR})_o}{(\text{SNR})_c} \right|_{SSB} = 1$$

$$\text{figure of merit} = \left. \frac{(SNR)_o}{(SNR)_c} \right|_{AM} \approx \frac{k_a^2 P}{1 + k_a^2 P}$$

信号远远大于
噪声时

1) When Signal>>Noise

$$A_c[1+k_a m(t)] > \sqrt{n_I^2(t) + n_Q^2(t)}$$

$$y(t) = \sqrt{[A_c + A_c k_a m(t) + n_I(t)]^2 + n_Q^2(t)}$$

$$\approx [A_c + A_c k_a m(t)] \left[1 + \frac{n_I(t)}{A_c + A_c k_a m(t)} \right]$$

$$\sqrt{1+x} \approx 1 + \frac{1}{2}x \quad \text{当} |x| \ll 1 \text{ 时}$$

$$y(t) \approx A_c + A_c k_a m(t) + n_I(t) \quad 2.93$$

A_c 可以用隔直电容去掉

$$y(t) = A_c k_a m(t) + n_I(t)$$

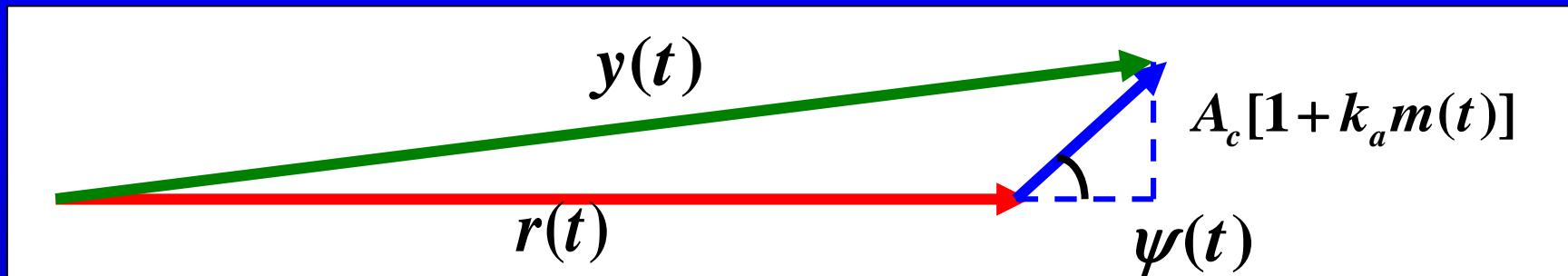
$$\text{figure of merit} = \left. \frac{(SNR)_O}{(SNR)_C} \right|_{AM} \approx \frac{k_a^2 P}{1 + k_a^2 P}$$

The noise performance of a full AM receiver is always inferior(更差) to that of a DSB or SSB receiver.

2) When Signal<<Noise

$$A_c[1+k_a m(t)] < \sqrt{n_I^2(t) + n_Q^2(t)} = r(t)$$

$$n(t) = r(t) \cos[2\pi f_c t + \psi(t)]$$



$$y(t) \approx r(t) + A_c \cos[\psi(t)] + A_c k_a m(t) \cos[\psi(t)]$$

In this case, the detector output has no component strictly proportional to the message signal $m(t)$. When SNR is less than threshold, the useful signal can not be detected, which means threshold effect happens.

Threshold Effect 门限效应

Threshold effect: the loss of a message in an envelope detector that operates at a low SNR is referred to as threshold effect.

Threshold 门限: we mean a value of the SNR below which the noise performance of a detector deteriorates(恶化,失真) much more rapidly than proportionately to the SNR.

相干检测器不存在门限效应.

非相干检测器存在门限效应.

2.13 Noise in FM Receivers

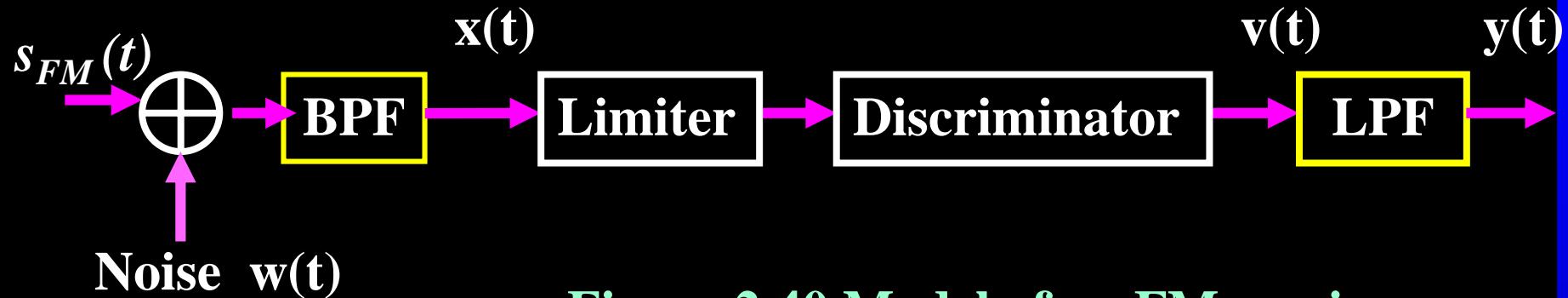
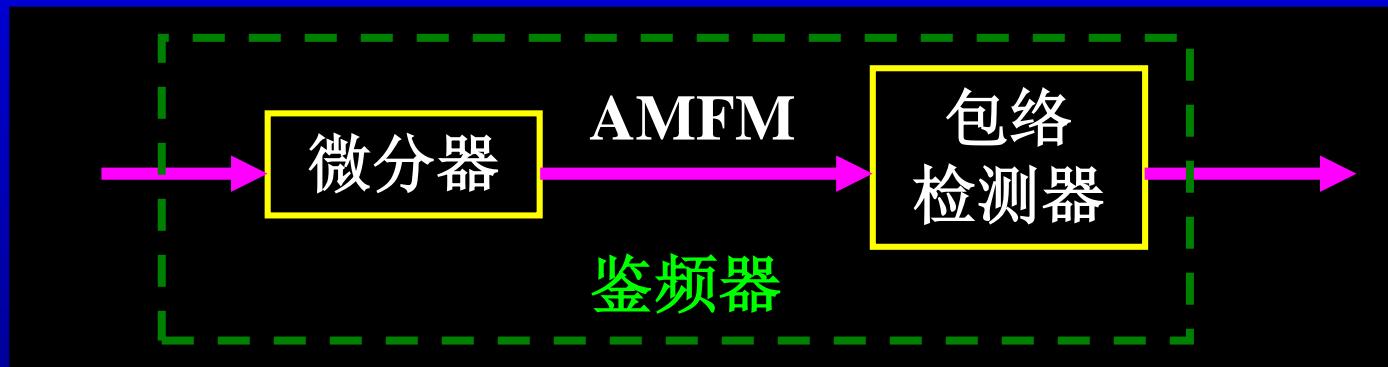


Figure 2.40 Model of an FM receiver.



In theory, discriminator consists two parts;

In practice, these two parts are usually implemented as integral parts as a single physical unit.

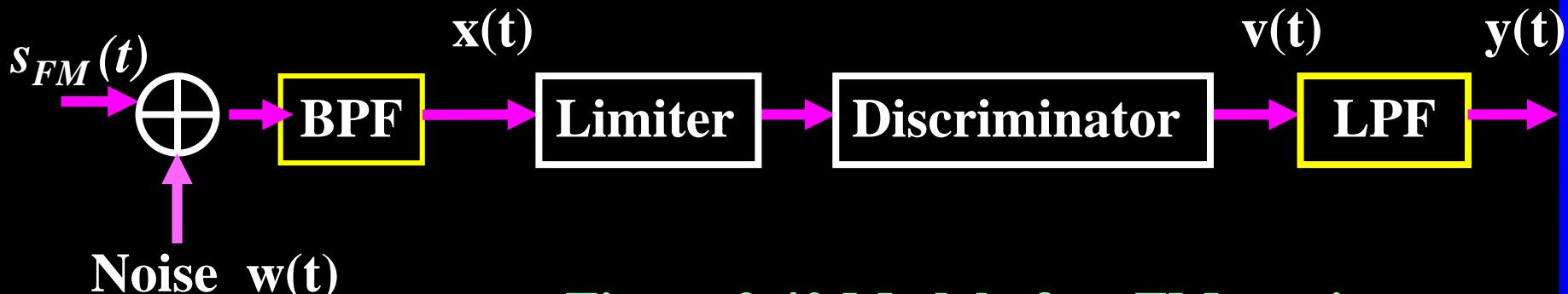


Figure 2.40 Model of an FM receiver.

$$n(t) = n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)$$

In terms of its envelope and phase

$$n(t) = r(t) \cos[(2\pi f_c t) + \psi(t)]$$

$$r(t) = [n_I^2(t) + n_Q^2(t)]^{1/2}$$

$$\psi(t) = \tan^{-1} \left[\frac{n_Q(t)}{n_I(t)} \right]$$

$r(t)$ is Rayleigh distributed, $\psi(t)$ is uniformly distributed.

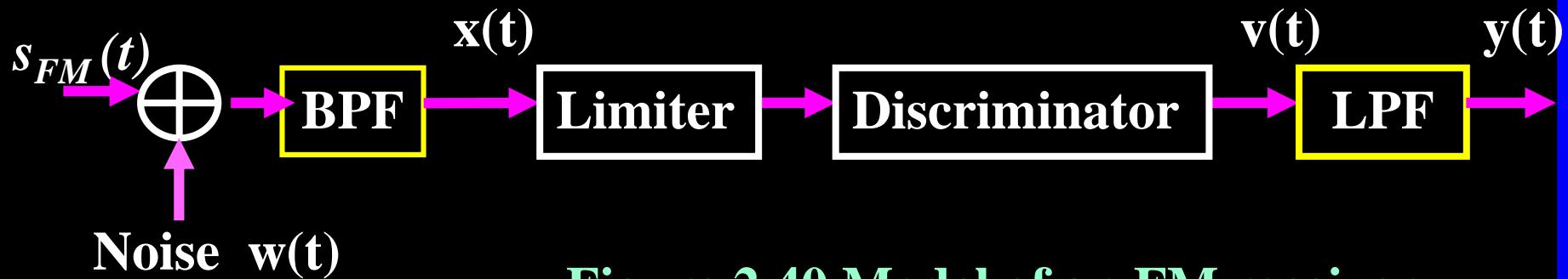


Figure 2.40 Model of an FM receiver.

The incoming
FM signal

$$s(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]$$

We define

$$\phi(t) = 2\pi k_f \int_0^t m(\tau) d\tau$$

thus

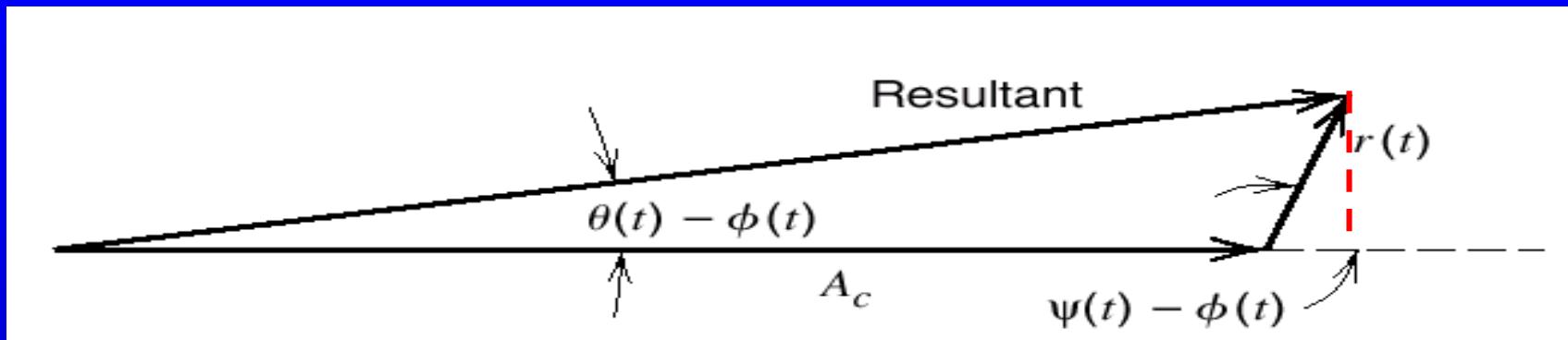
$$s(t) = A_c \cos [2\pi f_c t + \phi(t)]$$

The noisy signal at the band-pass filter output

$$x(t) = s(t) + n(t)$$

$$= A_c \cos [2\pi f_c t + \phi(t)] + r(t) \cos [2\pi f_c t + \psi(t)]$$

Figure 2.41 Phasor diagram for FM wave plus narrowband noise for the case of high carrier-to-noise ratio.



$$\theta(t) = \phi(t) + \tan^{-1} \left\{ \frac{r(t) \sin [\psi(t) - \phi(t)]}{A_c + r(t) \cos [\psi(t) - \phi(t)]} \right\} \quad 2.137$$

The envelope of $x(t)$ is of no interest to us, because any envelope variations at the band-pass filter output are removed by the limiter. So, we only focus on the phase of $x(t)$.

When $\text{CNR} \gg 1$, that is $r(t) \ll A_c$ **CNR: Carrier-to-noise**

$$\begin{aligned}\theta(t) &\approx \phi(t) + \frac{r(t)}{A_c} \sin[\psi(t) - \phi(t)] \\ &= 2\pi k_f \int_0^t m(\tau) d\tau + \frac{r(t)}{A_c} \sin[\psi(t) - \phi(t)]\end{aligned}\quad 2.139$$

The discriminator output

$$v(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = k_f m(t) + n_d(t) \quad 2.140$$

Noise term of $V(t)$

$$n_d(t) = \frac{1}{2\pi A_c} \frac{d \{ r(t) \sin [\psi(t) - \phi(t)] \}}{dt} \quad 2.141$$

Equation 2.140 shows that if CNR is large, the output of the discriminator consists of message signal plus noise.

已知: $\psi(t)$ is uniformly distributed over 2π radians.

推论: If CNR is high, it can be proved that $\psi(t) - \phi(t)$
is also uniformly distributed over 2π radians.

Then, we may simplify Equation 2.141 as:

$$n_d(t) \approx \frac{1}{2\pi A_c} \frac{d}{dt} \{r(t)\sin[\psi(t)]\} \quad 2.142$$

Because

$$n_Q(t) = r(t)\sin[\psi(t)] \quad \text{Thus}$$

$$n_d(t) \approx \frac{1}{2\pi A_c} \frac{dn_Q(t)}{dt} \quad 2.144$$

This means that the additive noise $n_d(t)$ is determined by the carrier amplitude A_c and the quadrature component $n_Q(t)$ of the narrowband noise $n(t)$.

To calculate SNR_0

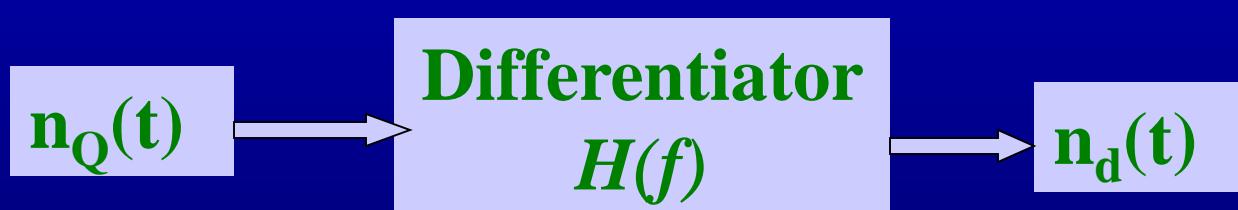
$$(\text{SNR})_0 = \frac{\text{average power of the demodulate d signal}}{\text{average power of the noise}} \quad \text{at the receiver output}$$

Output

$$v(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = k_f m(t) + n_d(t)$$

Output signal= $k_f m(t)$ Average signal power= $k_f^2 P$

P is the power of $m(t)$.



$$H(f) = \frac{j2\pi f}{2\pi A_c} = \frac{jf}{A_c}$$

To calculate output noise power

The power spectral density of $n_d(t)$:

$$S_{N_d}(f) = \frac{f^2}{A_c^2} S_{N_Q}(f)$$

Because

$$S_Y(f) = |H(f)|^2 S_X(f) \quad 1.58$$

$n_Q(t)$ is low-pass filtered noise. Thus

$$S_{N_d}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| \leq \frac{B_T}{2} \\ 0, & \text{otherwise} \end{cases} \quad 2.146$$

Power spectral density of $n_o(t)$ at the receiver out
(after low-pass filter)

$$S_{N_o}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| \leq W \\ 0, & \text{otherwise} \end{cases} \quad 2.147$$

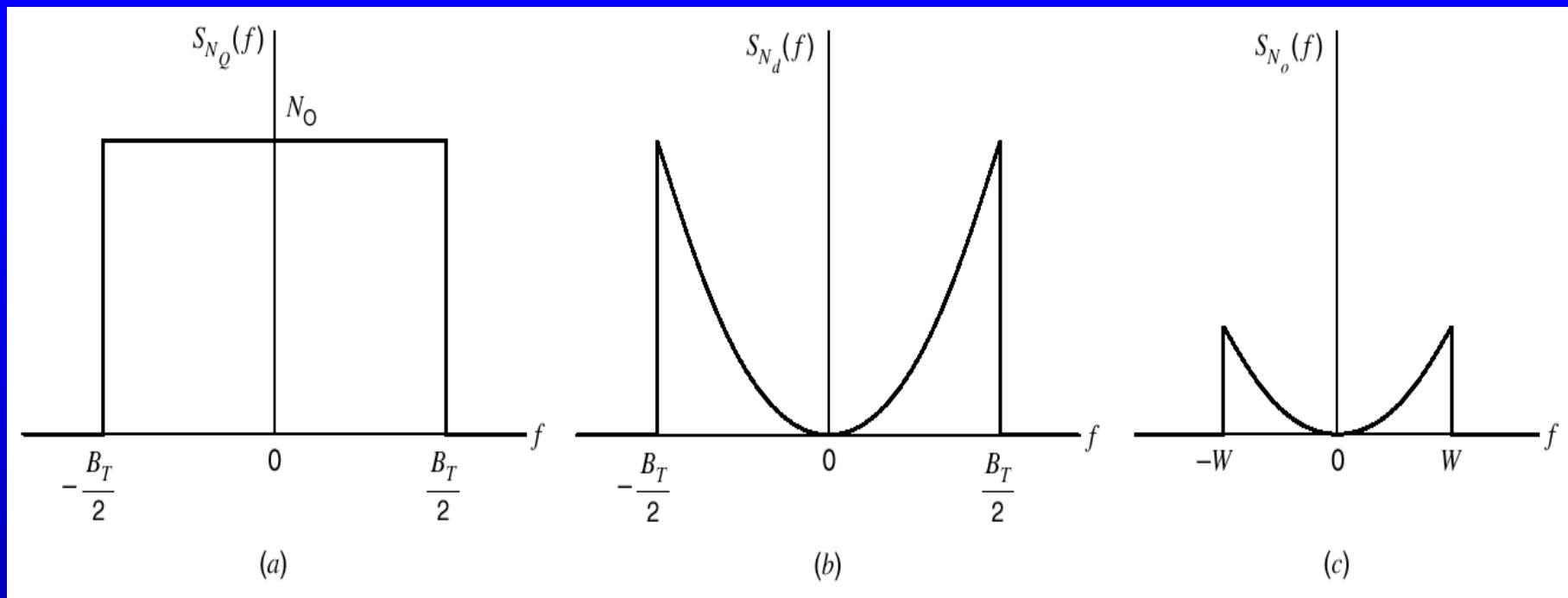


Figure 2.42 Noise analysis of FM receiver.

- (a) Power spectral density of quadrature component $n_Q(t)$ of narrowband noise $n(t)$.
- (b) Power spectral density of noise $n_d(t)$ at the discriminator output.
- (c) Power spectral density of noise $n_o(t)$ at the receiver output.

$$\text{Average power of output noise} = \frac{N_0}{A_c^2} \int_{-W}^W f^2 df = \frac{2N_0 W^3}{3A_c^2} \quad 2.148$$

Average signal power = $k_f^2 P$

Therefore,

$$(SNR)_{O,FM} = \frac{3A_c^2 k_f^2 P}{2N_0 W^3} \quad 2.149$$

FM信号的平均功率
Average power of FM

$$S_{FM} = \frac{A_c^2}{2}$$

Average noise power
消息带宽内平均噪声功率

$$N = W N_0$$

Therefore,
信道信噪比

$$(SNR)_{C,FM} = \frac{A_c^2}{2N_0 W} \quad 2.150$$

$$\left. \begin{aligned} (\text{SNR})_{O,FM} &= \frac{3A_c^2 k_f^2 P}{2N_0 W^3} \\ (\text{SNR})_{C,FM} &= \frac{A_c^2}{2N_0 W} \end{aligned} \right\} \rightarrow \left. \frac{(\text{SNR})_O}{(\text{SNR})_C} \right|_{FM} = \frac{3k_f^2 P}{W^2}$$

2.151

Deviation ratio 偏移率, 频偏比

$$D = \frac{\text{frequency deviation}}{\text{highest modulation frequency}} = \frac{\Delta f}{W}$$

$$B_T \approx 2\Delta f + 2W = 2\Delta f \left(1 + \frac{1}{D}\right) = 2(1+D)W$$

D is similar to modulation index β .

Since $\Delta f \propto k_f$

Thus $D \propto \frac{k_f}{W}$

So figure merit = $\left. \frac{(\text{SNR})_O}{(\text{SNR})_C} \right|_{FM} \propto 3PD^2$

$$\text{figure merit} = \frac{(SNR)_O}{(SNR)_C} \Big|_{FM} \propto 3PD^2$$



Conclusion

- when carrier-to-noise is high, an increase in the transmission bandwidth B_T provides a corresponding quadratic increase 二次方增长 in the output signal-to-noise ratio of figure of merit.
- FM improves noise performance at the **cost** of transmission bandwidth.

Example 2.5 Single-Tone Modulation

已知：单频信号调频，频率为 f_m ，最大（峰值）频偏为 Δf ，已调信号为

$$s(t) = A_c \cos \left[2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \right]$$

求：调制信号 $m(t)$ 和解调增益

解：根据调频信号的一般时域表示式

$$s(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right]$$

可知调制信号应满足

$$2\pi k_f \int_0^t m(\tau) d\tau = \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

因此，调制信号为

$$m(t) = \frac{\Delta f}{k_f} \cos(2\pi f_m t)$$

$$W = f_m$$

调制信号的平均功率为

$$P = \frac{(\Delta f)^2}{2k_f^2}$$

所以输出信噪比为

$$(SNR)_{O,FM} = \frac{3A_c^2 (\Delta f)^2}{4N_0 W^3} = \frac{3A_c^2 \beta^2}{4N_0 W}$$

调制指数
Modulation index

$$\beta = \frac{\Delta f}{W}$$

解调增益为

$$\left. \frac{(SNR)_O}{(SNR)_C} \right|_{FM} = \frac{3}{2} \left(\frac{\Delta f}{W} \right)^2 = \frac{3}{2} \beta^2$$

Noise performance comparison of AM and FM

From Example 2.4,

$$\left. \frac{(\text{SNR})_o}{(\text{SNR})_c} \right|_{AM} = \frac{1}{3}$$

From Example 2.5,

$$\left. \frac{(\text{SNR})_o}{(\text{SNR})_c} \right|_{FM} = \frac{3}{2} \left(\frac{\Delta f}{W} \right)^2 = \frac{3}{2} \beta^2$$

Which is better, AM or FM ?

- 调制指数小时， FM在抗噪声性能上无优势。但此时所需的传输带宽窄。
- 调制指数大时 ($\beta > 0.471$)， FM比AM的抗噪声性能好，但此时所需的传输带宽很宽。因此 $\beta=0.5$ 可以看成是窄带与宽带FM的分界。
- FM 是以牺牲带宽为代价换取了抗噪声性能的提高。

FM Threshold Effect 门限效应

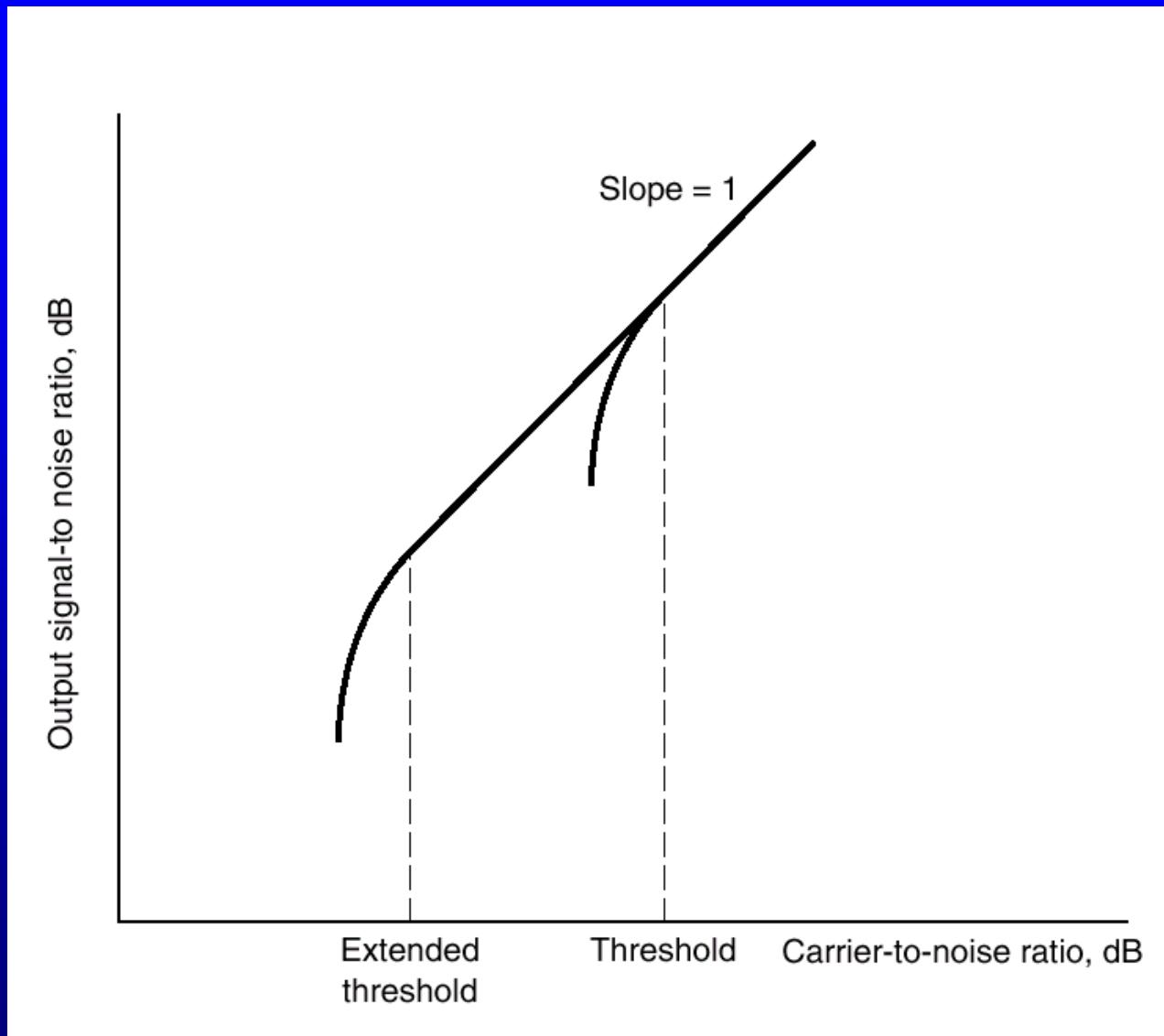
前面的分析都是基于载噪比CNR足够大。而当CNR很小时，即噪声很强，则鉴频器的输出没有纯粹的信号项，计算输出信噪比困难。当载噪比低于某个值后，输出的信噪比急剧下降，性能极度恶化，此时可以认为有用信号完全被噪声淹没了。这种现象被称为门限效应。产生门限效应的临界载噪比值称为门限值。

Noise power $\uparrow \rightarrow$ CNR $\downarrow \rightarrow$ individual clicks

Noise power $\uparrow \uparrow \rightarrow$ CNR $\downarrow \downarrow \rightarrow$ cracking
or sputtering sounds

Threshold effect and threshold

Figure 2.46 FM threshold extension.



FM Threshold Reduction (FM门限的降低)

Threshold reduction in FM receivers may be achieved by using an FM demodulator with negative feedback, or by using a phase-locked loop demodulator.

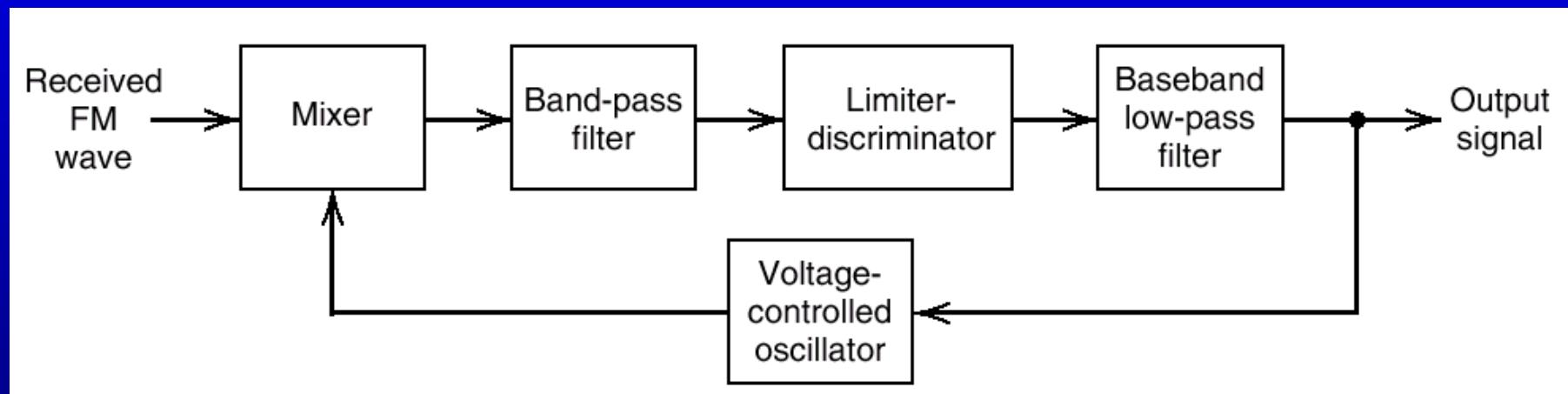


Figure 2.47 FM demodulator with negative feedback.

Pre-Emphasis and De-Emphasis in FM

调频系统的预加重与去加重

Problem

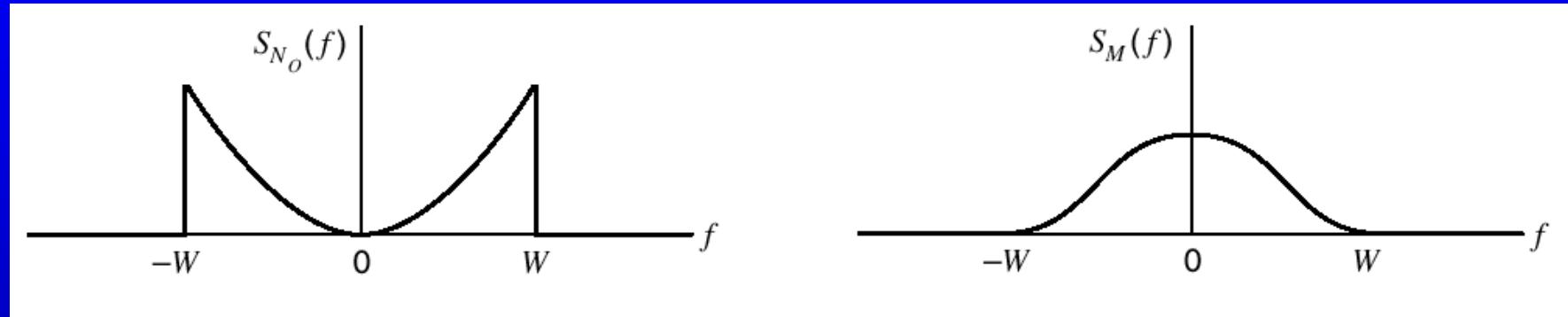


Figure 2.48 (a) Power spectral density of noise at FM receiver output.
 (b) Power spectral density of a typical message signal.

语音和图像信号低频段能量大，高频段信号能量明显小；而鉴频器输出噪声的功率谱密度随频率的平方而增加（低频噪声小，高频噪声大），造成信号的低频信噪比很大，而高频信噪比明显不足，使高频传输困难。

Solution

调频收发技术中，通常采用预加重和去加重技术来解决这一问题。

- 预加重：发送端对输入信号高频分量的提升。
- 去加重：解调后对高频分量的压低。

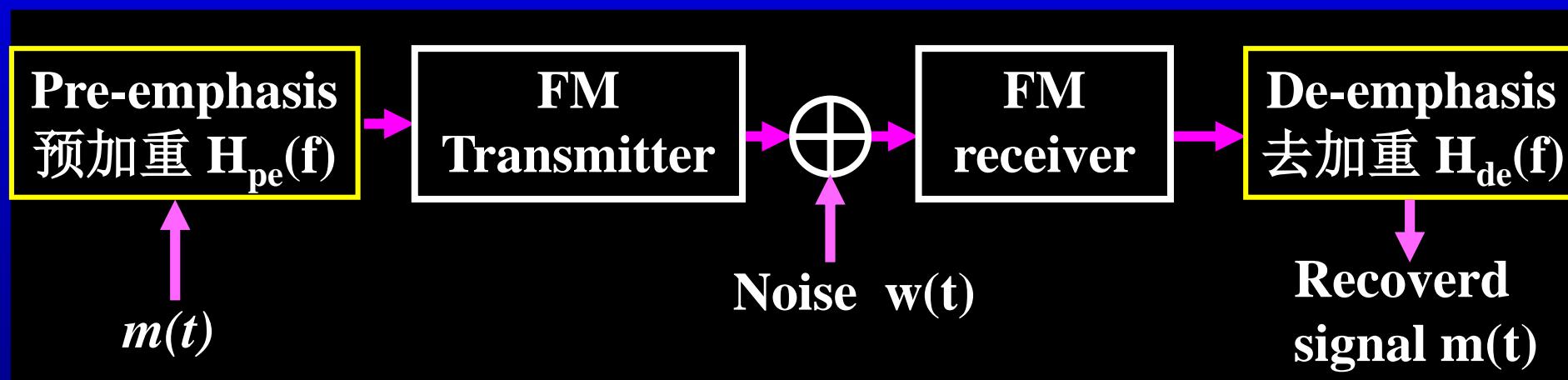


Fig. 2.49 Use of pre-emphasis and de-emphasis in an FM system.

为什么预加重去加重技术可以改善FM系统性能？

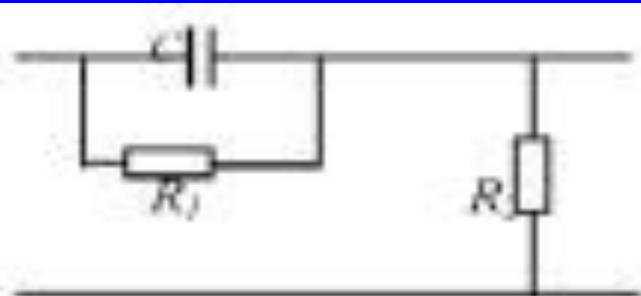
- Pre-emphasize the high-frequency components of the message signal **only** in the transmitter;
- De-emphasize the high-frequency components of the message **signal and noise** in the receiver.
- So effectively increase the output SNR

Frequency response $H_{pe}(f)$ of the pre-emphasis filter and frequency response $H_{de}(f)$ satisfy following form:

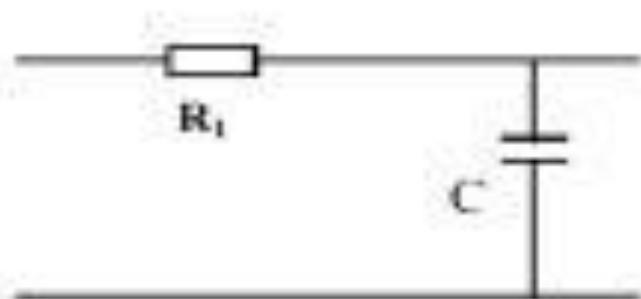
$$H_{de}(f) = \frac{1}{H_{pe}(f)}, \quad -W \leq f \leq W$$

预加重网络传递函数 $H_p(\omega) = j\omega$

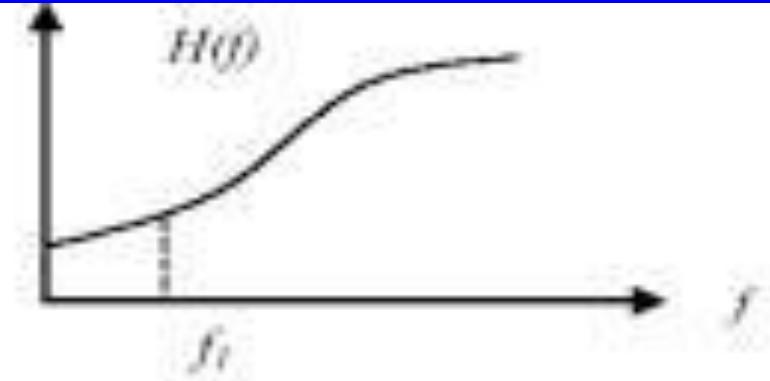
去加重网络传递函数 $H_d(\omega) = 1/H_p(\omega)$



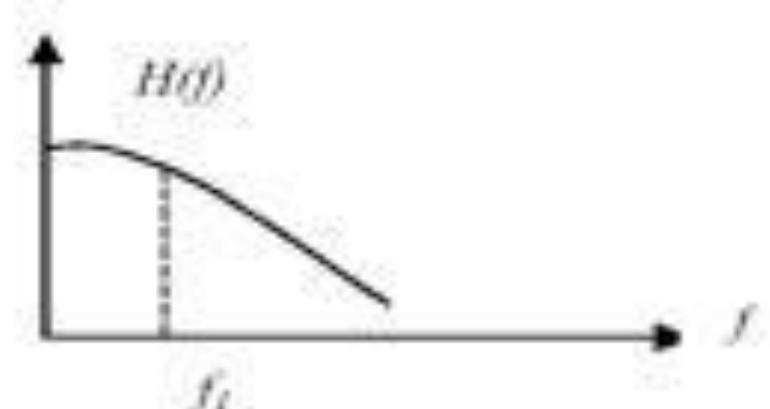
(a)



(c)



(b)



(d)

预加重和去加重网络

没有预-去加重时，输出噪声功率谱

$$S_{N_d}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| \leq \frac{B_T}{2} \\ 0, & \text{otherwise} \end{cases} \quad \text{2.146}$$

采用去加重技术后，输出噪声功率谱

$$|H_{de}(f)|^2 S_{N_d}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2} |H_{de}(f)|^2, & |f| \leq \frac{B_T}{2} \\ 0, & \text{otherwise} \end{cases}$$

After Low Pass filter (-W,W),

Average output noise power with de-emphasis

$$= \frac{N_0 f^2}{A_c^2} \int_{-W}^W f^2 |H_{de}(f)|^2 df$$

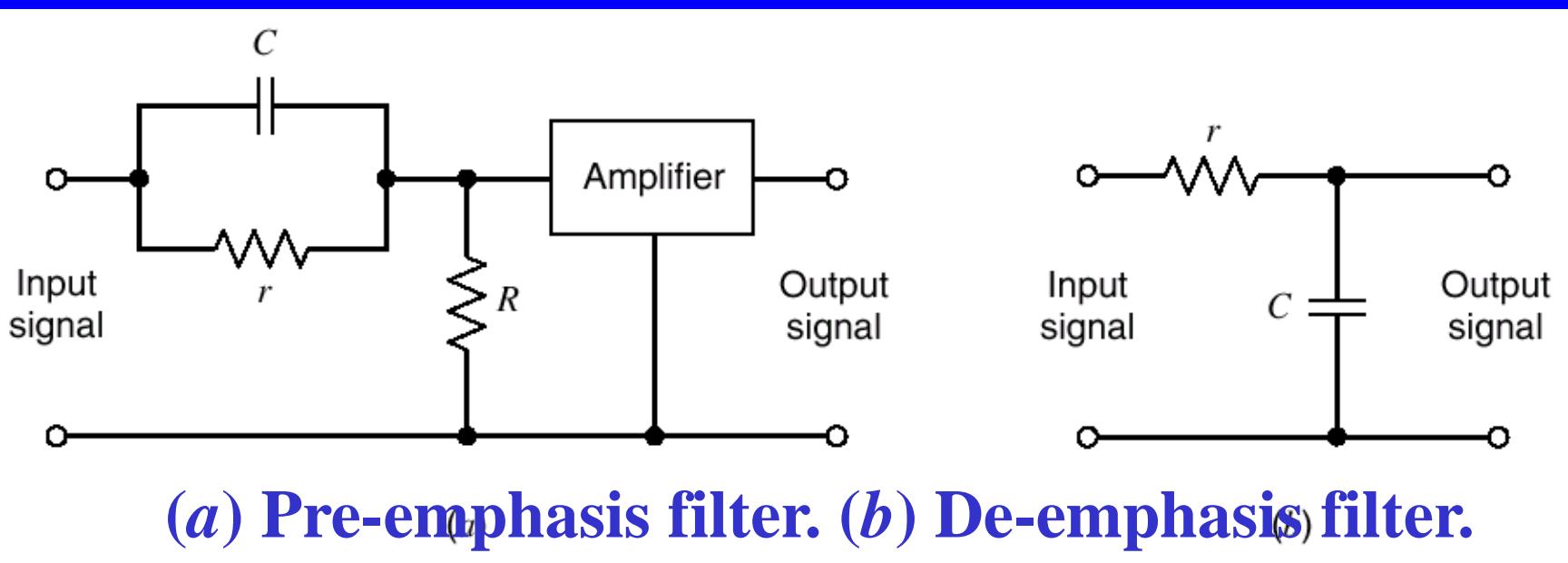
Ideally, 输出信号功率保持不变。

预加重-去加重技术使得输出信噪比改善值为：

$$I = \frac{\text{Average output noise power without pre-emphasis and de-emphasis}}{\text{Average output noise power with pre-emphasis and de-emphasis}}$$

$$I = \frac{2W^2}{3 \int_{-W}^W f^2 |H_{de}(f)|^2 df} \quad 2.160$$

Example 2.6



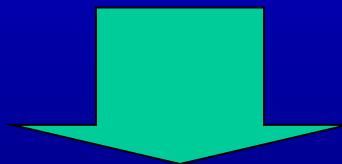
$$H_{pe}(f) = 1 + \frac{jf}{f_0}$$

$$H_{de}(f) = \frac{1}{1 + jf/f_0}$$

$$I = \frac{2W^2}{3 \int_{-W}^W \frac{f^2}{1 + (f/f_0)^2} df} = \frac{(W/f_0)^3}{3[(W/f_0) - \tan^{-1}(W/f_0)]}$$

In commercial FM broadcasting:

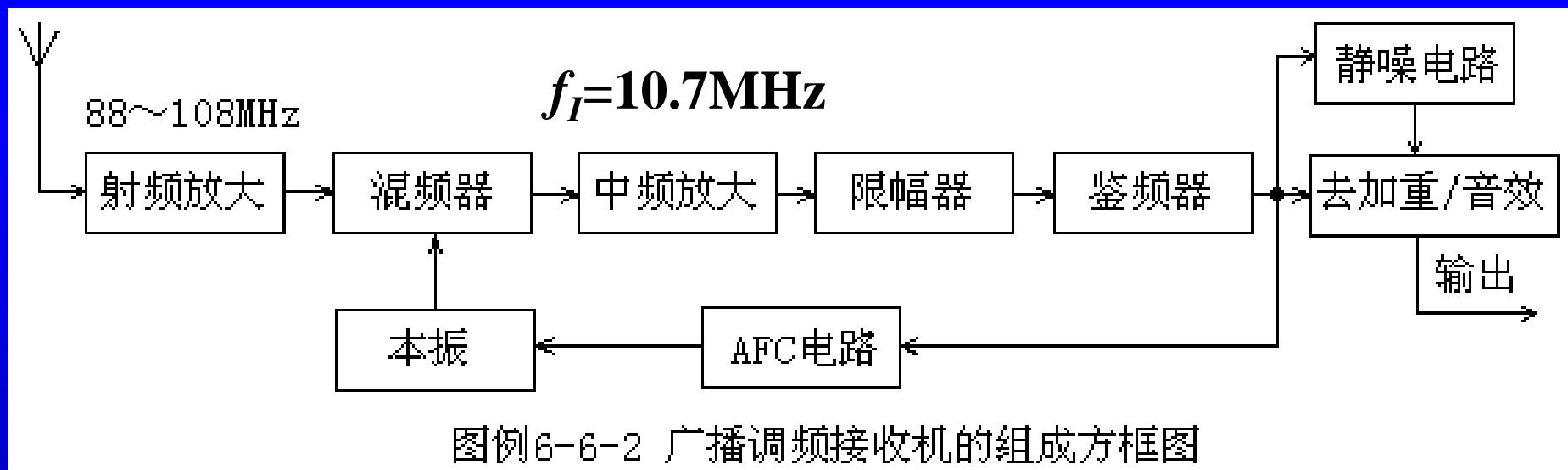
$$f_0 = 2.1 \text{ kHz}, \quad W = 15 \text{ kHz}$$



$$I = 22 \longrightarrow \approx 13 \text{ dB}$$

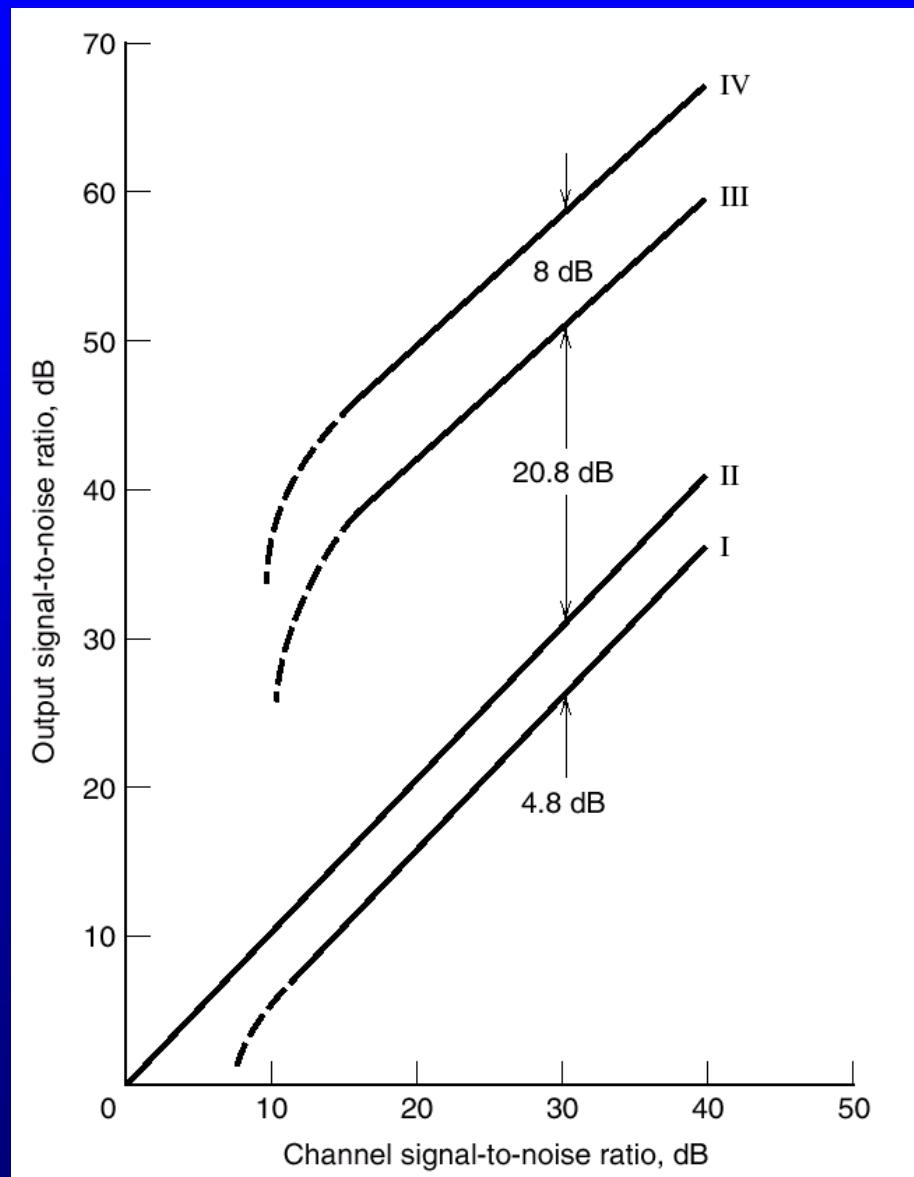
The improvement is Remarkable.

应用电路



Dolby System: 杜比降噪系统. Nonlinear pre-emphasis and de-emphasis techniques have been applied successfully to tape recording. These techniques are known as Dolby-A, Dolby-B, and DBX systems.

2.15 Summary and discussion

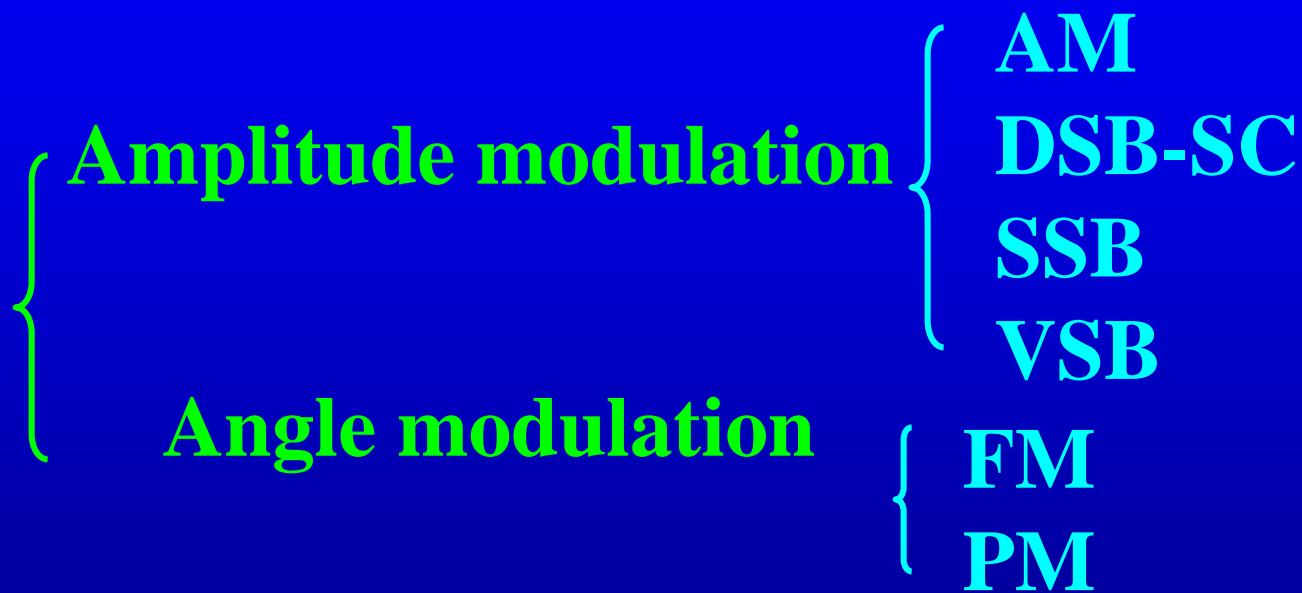


精读并理解记忆该小节！

Figure 2.55
Comparison of the noise performance of various CW modulation systems. Curve I: Full AM, $\mu = 1$. Curve II: DSB-SC, SSB. Curve III: FM, $\beta = 2$. Curve IV: FM, $\beta = 5$. (Curves III and IV include 13-dB pre-emphasis, de-emphasis improvement..)

Summary for this chapter

I. continuous-wave modulation schemes



II. Noise performance for different modulation schemes

Homework

2.8 2.9

2.33 2.36 2.49

补充习题: 1, 2

补充习题 1

若基带信号最高频率分量为 2.7kHz ，先对 5.3kHz 载频进行DSB调制，再进行频率调制，设调频信号的最大频偏为 40kHz ，

- (1) 求接收机输入端带通滤波器的带宽；
- (2) 画出调制与解调系统的原理框图。

补充习题 2:

某角调制信号为

$$s(t) = 10 \cos [2 \times 10^8 \pi t + 4 \sin(4 \times 10^3 \pi t)]$$

- (1) 求该信号的载频、最大相位偏移和最大频偏；
- (2) 如果它是PM调相信号，并且相位灵敏度 $k_p = 2$ ，求调制信号 $m(t)$ ；
- (3) 如果它是FM调频信号，并且频率灵敏度 $k_f = 500\pi$ ，求调制信号 $m(t)$ 。

一、填空

1. 通信的目的是 _____。
2. 通信系统中传输数字信号的通信方式称为 _____。
3. 从信号传输的角度看，通信系统的性能指标主要有_____和_____。
4. 模拟通信系统的传输可靠性用_____来衡量。
5. 数字通信系统的传输有效性用_____来衡量。
6. 无线电通信是利用自由空间传播的_____传递信息的通信方式。
7. 通信网的四要素是_____、_____、_____和通信软件与协议。
8. 多路复用技术主要包括_____、_____和码分复用。
9. 调制是用调制信号的变化规律去改变载波的_____的过程。
10. 连续波模拟调制制式包括_____、_____、_____、_____、_____和_____。

四、判断题（对正确的论断，在括号内划“√”，错误的划“×”）

1. 现代通信限于人与人之间的通信。 ()
2. 通信系统中的信号是运载信息的工具。 ()
3. 无线电通信是利用自由空间传播的电磁波来传递信息的通信方式。
()
4. 电话通信网中并不一定都需要交换设备。 ()
5. 通信系统中的噪声影响甚至破坏正常的通信。 ()

11. 若调制信号为 $f(t)$, 则上边带抑制载波振幅调制信号(USSB-SC)的时域表示式为: $s_{\text{USSB-SC}}(t) = \underline{\hspace{10cm}}$ 。
12. 若基带信号最高频率分量为 2.7kHz, 先对 5.3kHz 载频进行 DSB 调制, 再进行频率调制, 设调频信号的最大频偏为 40kHz, 则接收机输入带通滤波器的带宽应为 $\underline{\hspace{10cm}}$ 。
13. 相干检测器的解调原理等效为一个 $\underline{\hspace{2cm}}$ 后接一个 $\underline{\hspace{2cm}}$ 。
14. 包络检测器从有用信号与噪声之和中检测出 $\underline{\hspace{10cm}}$ 。
15. 调频接收机的鉴频器的解调原理可等效为一个 $\underline{\hspace{2cm}}$ 后接一个 $\underline{\hspace{2cm}}$ 来构成。
16. 收音机上标示的“AM”、“SW”、“FM”和“TV”分别代表 $\underline{\hspace{2cm}}$ 、 $\underline{\hspace{2cm}}$ 、 $\underline{\hspace{2cm}}$ 和 $\underline{\hspace{2cm}}$ 广播。