An Interesting Problem about Dynamic Programming

The original problem is as follow: Given a set S of elements $\{a_i\}$, where $a_i \in \mathbb{Z}$, the goal is to partition S into 2 subsets A and $S - A$, such that minimizing the absolute difference between the sums of the elements in A and $S - A$, i.e. minimizing $|\sum_{a_i \in A} a_i - \sum_{a_i \in S-A} a_i|$. For simplicity, we use the "minimum difference" to denote the above minimized absolute difference.

Now, suppose someone came up with a dynamic programming algorithm to solve this problem. Considering S as a sequence, let $f[i,j]$ be the minimum difference under an optimum assignment on the subsequence S_{ij} of S, such that $S_{ij} = \{a_k\}$, where $i \le k \le j$. Intuitively, we have the transition equation: $f[i,j] = min\,\{|f[i,k] - f[k+1,j])|\}$, where $1 \le i \le k < j \le n$. More concretely, S_{ij} can be partitioned into 2 subsequences S_{ik} and $S_{k+1,j}$ while the minimum difference of S_{ij} can be given by the minimum difference between any such 2 subsequences S_{ik} and $S_{k+1,j}$. For the boundary, $f[i,i] = a_i$, where $1 \le i \le n$. Therefore, we may conclude that the answer to the original problem is $f[1,n]$.

For example, given a set $S = \{1,2,3\}$, we have:

$f[1,1] = 1,\ f[2,2] = 2,\ f[3,3] = 3,$

$f[1,2] = |f[1,1] - f[2,2]| = 1,\ f[2,3] = |f[2,2] - f[3,3]| = 1,$

$f[1,3] = min(|f[1,1] - f[2,3]|, |f[1,2] - f[3,3]|) = 0.$

Therefore, we may conclude that the minimum difference for S is 0. For this specific case, our dynamic programming algorithm gives a correct answer since we can put elements 1 and 2 into a set A and the difference between A and $S - A$ is 0.

The problem for you is to decide and prove whether this dynamic programming is correct or not. Note that you cannot use any unproven theory to prove this problem, however, you may use it as a guidance and reference.