Kynch Theory and Application to Wastewater Treatment Plants

1. Overview

Kynch theory, introduced by G.J. Kynch in 1952, describes the sedimentation of concentrated suspensions such as activated sludge in wastewater treatment plants (WWTPs). The theory assumes that the settling velocity of particles depends only on the local solids concentration and not on time or location within the settling zone. It is primarily applied to the zone-settling stage of clarifiers and thickeners, where solids form a sludge blanket that settles as a cohesive mass. Kynch theory forms the foundation of the solids flux method, which is widely used in clarifier and thickener design, as well as in operational assessments.

2. Core Equations

The solids continuity equation for one-dimensional vertical settling is:

$$\frac{\partial C}{\partial t} + \frac{\partial \left( v\_s(C) \cdot C \right)}{\partial z} = 0$$

Where: C is the solids concentration (kg/m³ or g/L), t is time (s), z is depth measured positively downward (m), and v\_s(C) is the hindered settling velocity (m/s) as a function of C. The term v\_s(C)·C represents the solids flux in kg/(m²·s). The first term in the equation represents the local rate of change of solids concentration over time, while the second term represents the net transport of solids due to settling.

The solids flux relationship is defined as:

$$J(C) = v\_s(C) \cdot C$$

Here: J(C) is the solids flux in kg/(m²·s). This is the mass of solids passing through a unit horizontal area per second. For activated sludge systems, typical operating solids flux values range from 0.1 to 0.4 kg/(m²·s) under normal conditions, depending on sludge settleability and concentration.

The velocity of a concentration front is given by:

\frac{dz}{dt} = \frac{d \left( v\_s(C) \cdot C \right)}{dC}

This equation represents the characteristic speed at which a concentration front moves through the settling zone. It is the derivative of the solids flux with respect to solids concentration, describing how changes in concentration affect the movement of settling fronts.

3. Settling Regimes in WWTP Clarifiers

In sedimentation tanks, four distinct settling regimes are commonly observed:  
1. Discrete particle settling – individual particles settle without interaction.  
2. Flocculent settling – particles collide and aggregate to form larger flocs, increasing settling velocity.  
3. Zone (hindered) settling – particles form a blanket that settles as a single mass at a velocity dependent on concentration.  
4. Compression settling – solids at the bottom compress under their own weight, expelling water.  
Kynch theory applies specifically to the zone-settling regime, where the interactions between particles dominate and hinder their settling.

4. Solids Flux Method for Design and Operation

The solids flux method uses the relationship between settling velocity and concentration to determine clarifier capacity. It involves calculating the solids flux for a range of concentrations and finding the minimum value that limits performance. This minimum is known as the limiting solids flux (J\_lim):

J\_{\text{lim}} = \min\_{C} J(C)

Once J\_lim is determined, the minimum required surface area of the clarifier can be calculated:

A\_{\text{min}} = \frac{Q \cdot C\_0}{J\_{\text{lim}}}

Where: A\_min is in m², Q is the influent flow rate (m³/s), C\_0 is the influent solids concentration (kg/m³), and J\_lim is in kg/(m²·s). Additionally, the surface overflow rate (SOR) should be checked to ensure it is less than the settling velocity at the operating concentration:

\mathrm{SOR} = \frac{Q}{A} \;<\; v\_s\!\left(C\_{\text{oper}}\right)

5. Common Settling Velocity Models

Settling velocity functions are typically determined from batch settling tests. Two common empirical models are:

v\_s(C) = v\_0 \cdot e^{-k \cdot C} \quad \text{(Vesilind model)}

Where: v\_0 is the theoretical settling velocity at zero concentration (m/h), and k is the hindered settling coefficient (m³/kg). Typical values for activated sludge are v\_0 = 5–15 m/h and k = 0.05–0.15 m³/kg.

v\_s(C) = v\_{\text{St}} \cdot e^{-r\_h \cdot C} + v\_{\text{floc}} \cdot e^{-r\_f \cdot C} \quad \text{(Tak\'acs model)}

This double-exponential model separates the contributions from hindered settling and flocculent settling. Parameters are determined by fitting experimental data.

6. Practical Steps in WWTP Application

1. Conduct a batch settling test on a representative sludge sample, recording interface height over time.  
2. Convert height-time data to concentration-velocity data.  
3. Fit an appropriate settling velocity model to the data.  
4. Compute solids flux J(C) for a range of concentrations.  
5. Identify J\_lim from the curve.  
6. Calculate A\_min and check SOR for both average and peak flows.  
7. Adjust sludge return and wasting rates to maintain optimal blanket depth.

7. Worked Example

Given: Q = 5,000 m³/d, C\_0 = 3,000 mg/L = 3 kg/m³, and from tests v\_s(3) = 1.0 m/h = 24 m/d.  
Hydraulic check:

A \approx \frac{Q}{v\_s(C\_{\text{oper}})} = \frac{5000}{24} \approx 208 \ \text{m}^2

If J\_lim = 0.12 kg/(m²·s), then:

A\_{\text{min}} = \frac{Q \cdot C\_0}{J\_{\text{lim}}}

Substituting Q = 5000/86400 m³/s and C\_0 = 3 kg/m³ yields approximately 144 m².

8. References

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