

# Resource Management Techniques CS635 Unit-2

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#### **MISSION**

**VISION** 

#### **TEXT BOOKS**

Frederick S. Hillier and Gerald J. Lieberman, "Introduction to Operations Research" Tata McGraw Hill, 9th Edition, 2010.

Hamdy A Taha, "Operations Research: An Introduction", 8th Edition, Prentice Hall India, 2012

#### REFERENCE BOOKS

Wayne L. Winston, "Operations Research Applications and Algorithms", Thomson Course Technology, 4th Edition, 2003.

Vohra, "Quantitative Techniques in Management", Tata McGraw Hill, 2006.

AnandSarma, "Operation Research", Himalaya Publishing House, 2014.

Winston, "Operation Research", Thomson Learning, 2003.

JK Sharma, "Operations Research theory and applications", Macmillan India, 2009, Thomson Learning, 4<sup>th</sup> edition, 2009.

## **UNIT-2**

- Simplex method (Big M)
- Duality Theory

# **Simplex Problems**

- Minimization Problems : BIG M Method
- Surplus Variables
- Artificial Variables

#### **BIG M Method**

 $Minimize Z = 2X_1 + 3X_2$ 

$$X_1 + X_2 \ge 6$$
  
 $7X_1 + X_2 \ge 14$   
 $X_1 \text{ and } X_2 \ge 0$ 

# Standard Form Method

Minimize 
$$Z = 2X_1 + 3X_2$$
  
 $X_1 + X_2 - S_1 = 6$ 

 $7X_1 + X_2 - S_2 = 14$ 

 $X_1, X_2, S_1 \text{ and } S_2 \ge 0$ 

Minimize 
$$Z = 2X_1 + 3X_2 + 0S_1 + 0S_2 + MR_1 + MR_2$$
  

$$X_1 + X_2 - S_1 + R_1 = 6$$

$$7X_1 + X_2 - S_2 + R_2 = 14$$

 $X_1, X_2, S_1, S_2, R_1 \text{ and } R_2 \ge 0$ 

$$Minimize Z = 2X_1 + 3X_2$$

$$X_1 + X_2 - S_1 = 6$$
  
 $7X_1 + X_2 - S_2 = 14$   
 $X_1, X_2, S_1 \text{ and } S_2 \ge 0$ 

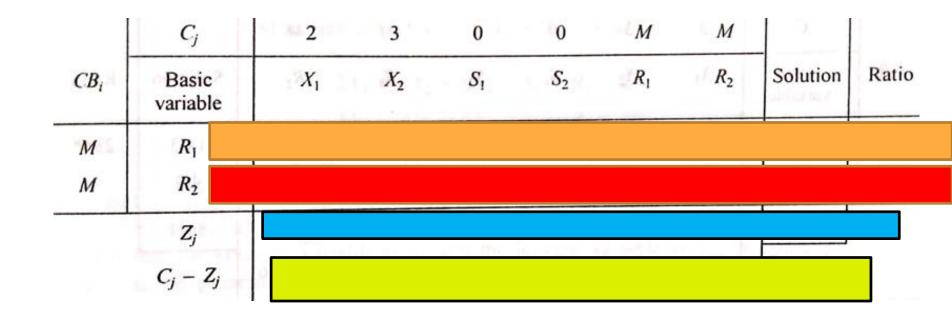
Minimize  $Z = 2X_1 + 3X_2 + 0S_1 + 0S_2 + MR_1 + MR_1$ 

$$X_1 + X_2 - S_1 + R_1 = 6$$

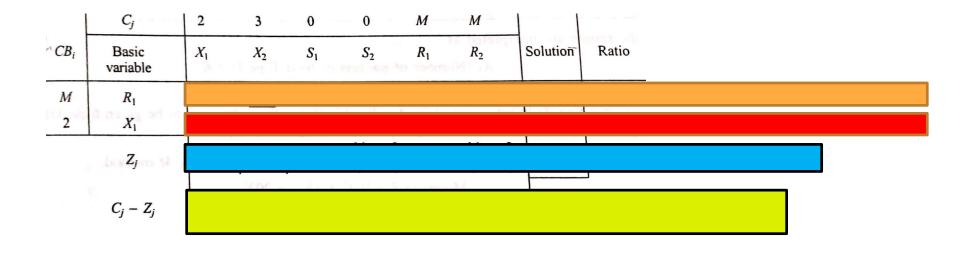
$$7X_1 + X_2 - S_2 + R_2 = 14$$

$$X_1, X_2, S_1, S_2, R_1 \text{ and } R_2 \ge 0$$

### Table 1



# Table-2



# Table -3

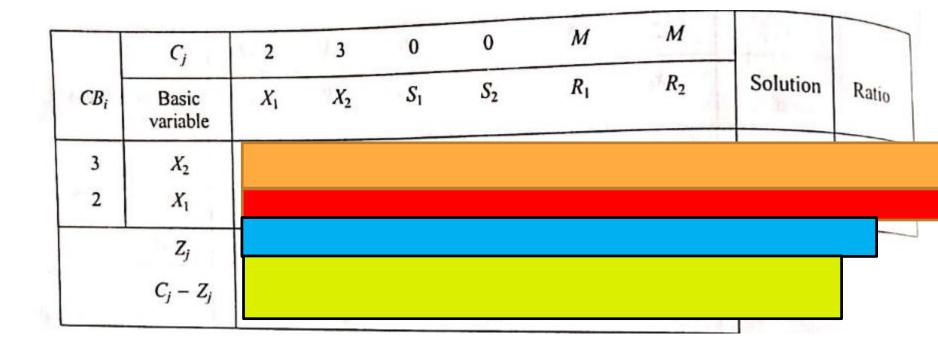


Table-4

7 74	$C_j$	2	3 10	0	0	М	M	111
СВі	Basic variable	<i>X</i> <sub>1</sub>	X <sub>2</sub>	$S_1$	$S_2$	$R_1$	R <sub>2</sub>	Solution
0	$S_2$							
2	$X_1$							
	$Z_j$							
	$C_i - Z_i$							

$$X_1 = 6$$
,  $X_2 = 0$ ,  $S_1 = 0$ ,  $S_2 = 28$ , and  $Z(\text{optimum}) = 12$ 

# **Problem-2: Solve using Big M Method**

Minimize 
$$Z = 10X_1 + 15X_2 + 20X_3$$
  

$$2X_1 + 4X_2 + 6X_3 \ge 24$$

$$3X_1 + 9X_2 + 6X_3 \ge 30$$

$$X_1, X_2, X_3 \ge 0$$

Minimize 
$$Z = 10X_1 + 15X_2 + 20X_3 + MR_1 + MR_2$$
  

$$2X_1 + 4X_2 + 6X_3 - S_1 + R_1 = 24$$

$$3X_1 + 9X_2 + 6X_3 - S_2 + R_2 = 30$$

$$X_1, X_2, X_3, S_1, S_2, R_1 \text{ and } R_2 \ge 0$$

# Table 1: Obtained from initial basic feasible solution

$CB_i$ Basic variable $X_1$ $M$ $R_1$	X <sub>2</sub>	Х3	$S_1$	$S_2$	$R_1$	$R_2$	Solution	Ratio
					•	11.2		
$M \mid R_2 \mid$								
Z <sub>j</sub> C <sub>j</sub> – Z <sub>j</sub>			_	_				
$C_j - Z_j$								

# Table-2

	$C_j$	10	15	20	0	0	M	M		
СВі	Basic variable	<i>X</i> <sub>1</sub>	<i>X</i> <sub>2</sub>	$X_3$	$S_1$	$S_2$	$R_1$	$R_2$	Solution	Ratio
М	$R_1$									
15	<i>X</i> <sub>2</sub>									
	$Z_j$ $C_j - Z_j$								+50	2.5.3

# Table-3

	$C_{j}$	10	15	20	0	. 0	М	М	
СВі	Basic variable	X <sub>1</sub>	X <sub>2</sub>	, X <sub>3</sub>	_ /S <sub>1</sub> +	) L S <sub>2</sub>	$R_1$	$R_2$	Solution
20	<i>X</i> <sub>3</sub>								
15	<i>X</i> <sub>2</sub>								
ofest san ke gorbed keye a	$C_j - Z_j$								82

# **Problem 3: Solve the problem using Simplex.**

Maximize 
$$Z = 20X_1 + 10X_2 + 15X_3$$
  

$$8X_1 + 6X_2 + 2X_3 \le 60$$

$$5X_1 + X_2 + 6X_3 \ge 40$$

$$2X_1 + 6X_2 + 3X_3 \le 30$$

$$X_1, X_2 \text{ and } X_3 \ge 0$$



Maximize  $Z = 20X_1 + 10X_2 + 15X_3 + 0S_1 + 0S_2 + 0S_3 - MR_1$ 

Table: 1

$$8X_1 + 6X_2 + 2X_3 + S_1 = 60$$

$$5X_1 + X_2 + 6X_3 - S_2 + R_1 = 40$$

$$2X_1 + 6X_2 + 3X_3 + S_3 = 30$$

$$X_1, X_2, X_3, S_1, S_2, S_3 \text{ and } R_1 \ge 0$$

	$C_j$	20	10	15	0	0	0	-M		
СВі	Basic variable	<i>X</i> <sub>1</sub>	<i>X</i> <sub>2</sub>	X <sub>3</sub>	$S_1$	S <sub>2</sub>	S <sub>3</sub>	$R_1$	Solution	Ratio
0	$S_1$	8	6	2	1	0	0	0	60	30
-М	$R_1$	5	Z - 1	6	0	-1	0	1 /	40	20/3
0	$S_3$	2	6	3	0	0	1	0	30	10
	$Z_j$	-5M	-М	-6M	0	М	0	-М	-40M	150
	$C_j - Z_j$	20 + 5M	10 + M	15 + 6M	0	-М	0	0		

# Table-2

	$C_j$	20	10	15	0	0	0	-M		
СВі	Basic variable	<i>X</i> <sub>1</sub>	<i>X</i> <sub>2</sub>	<i>X</i> <sub>3</sub>	$S_1$	S <sub>2</sub>	$S_3$	$R_1$	Solution	Ratio
						-				T
0	S <sub>1</sub>	19/3	17/3	0	1	1/3	0	-1/3	140/3	140/19
0 15		19/3	17/3 1/6	0	1 0	1/3	0	-1/3 1/6	140/3 20/3	140/19
	S <sub>1</sub> X <sub>3</sub>	47	277.25				1000	7		
15	$S_1$	5/6	1/6	1 > 12	0	-1/6	0	1/6	20/3	8

# Table-3

	$C_j$	20	10	15	0	0	0	-M		
	Basic variable	<i>X</i> <sub>1</sub>	X <sub>2</sub>	Х3	Sı	S <sub>2</sub>	S <sub>3</sub>	$R_1$	Solution	Ratio
20	X <sub>1</sub>	-1 -	17/19	0	3/19	1/19	0	-1/19	140/19	140
15	X <sub>3</sub>	0	-11/19	1	-5/38	-4/19	0	4/19	10/19	-
0	$S_3$	0	113/19	0	3/38	10/19	1	-10/19	260/19	26
	$Z_j$	20	175/19	15	45/38	-40/19	0	40/19	2950/19	
	$C_j - Z_j$	0	15/19	0	-45/38	40/19	0	-M-40/19	)	

	$C_j$	20	10	15	0	0	0	-M		
	Basic variable	<i>X</i> <sub>1</sub>	X <sub>2</sub>	<i>X</i> <sub>3</sub>	$S_1$	$S_2$	S <sub>3</sub>	$R_1$	Solution	Ratio
20	<i>X</i> <sub>1</sub>	1 -	3/10	0	3/20	0	-1/10	0	6	
15	X <sub>3</sub>	0	9/5	1	-1/10	0	2/5	0	6	
0	$S_2$	0	113/10	0	3/20	1	19/10	-1	26	
	$Z_{j}$	20	33	15	3/2	0	4	0	210	
	$C_j - Z_j$	0	-23	0	-3/2	0	-4	-M		

Rule 1: First we convert the problem into canonical form

- Min >=
- Max <=</p>

Rule 2: Change the objective function of maximization in the primal into minimization in the dual and vice versa

Rule 3: The number of variable in the primal will be the number of constraints in dual and vice versa

# **Rule 3: Examples**

Primal Dual

Number of Variable : m

Number of Constraints: m Number of Contraints: n

Rule 4: Cost coefficient in objective of the primal will be RHS constant of the constraints in dual and vice versa

Rule 5: For formulating constraints we considered the transpose of matrix

## **Summary of Primal – Dual Relationship**

Primal Problem

Minimization

Cannonical Form of Primal

Minimization

Constraints >=

Dual

**Maximization** 

Constraints <=

Primal Problem

Maximization

Cannonical Form of Primal

Maximization

Constraints <=</li>

Dual

**Minimization** 

Constraints >=

# **Summary of Primal – Dual Relationship**

Primal Problem: Maximize Z=x1 + 2x2 + x3

#### Subject to constraints:

$$2x1 + x2 - x3 <= 2$$
  
 $-2x1 + x2 - 5x3 >= -6$   
 $4x1 + x2 + x3 <= 6$  where  $x1$ ,  $x2$ ,  $x3 >= 0$ 

#### **Cannonical Form of Primal**

- Maximization
- Constraints <=</li>

Maximize 
$$Z = x1 + 2x2 + x3$$

#### Subject to constraints:

$$2x1 + x2 - x3 <= 2$$
 $2x1 - x2 + 5x3 <= 6$ 
 $4x1 + x2 + x3 <= 6$  where x1, x2, x3 >= 0

# **Summary of Primal – Dual Relationship**

Primal Problem: Maximize Z=x1 + 2x2 + x3

Cannonical Form : Maximize Z = x1 + 2x2 + x3

Subject to constraints:

$$2x1 + x2 - x3 <= 2$$

$$-2x1 + x2 - 5x3 > = -6$$

$$4x1 + x2 + x3 \le 6$$
 where x1, x2, x3 >=0

Subject to constraints: 2x1 + x2 - x3 <=2 -----y1

$$2x1 - x2 + 5x3 \le 6 ----y2$$
  
 $4x1 + x2 + x3 \le 6 ----y3$ 

where x1, x2, x3 >= 0

**Dual Problem from Primal Problem:** 

Dual

Minimize W = y1 + y2 + y3

Subject to constraints:

2 2

1 -1 1

-**1** 5

>= 1

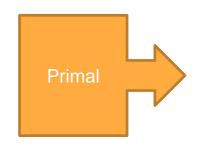
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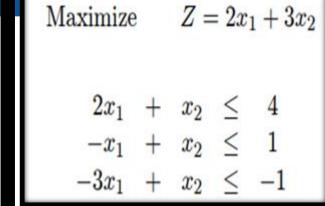
>= 1

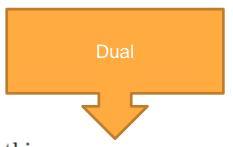
**Example:** Suppose that the primal LP is

Maximize 
$$Z = 2x_1 + 3x_2$$

under constraints







In this case,

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ -3 & 1 \end{bmatrix}, \quad c = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$$

Therefore, the dual is:

Minmize 
$$W = b^T y = 4y_1 + y_2 - y_3$$

under constraints

and  $y_1, y_2, y_3 \geq 0$ .

Minimize 
$$Z_1 = 24X_1 + 30X_2$$
  
 $2X_1 + 3X_2 \ge 10$   
 $4X_1 + 9X_2 \ge 15$   
 $6X_1 + 6X_2 \ge 20$   
 $X_1$  and  $X_2 \ge 0$ 

Maximize 
$$Z = 10Y_1 + 15Y_2 + 20Y_3$$
.  
 $2Y_1 + 4Y_2 + 6Y_3 \le 24$   
 $3Y_1 + 9Y_2 + 6Y_3 \le 30$   
 $Y_1, Y_2 \text{ and } Y_3 \ge 0$ 

Example 2.34 Solve the following linear programming problem using the result of its dual problem.

subject to

Minimize 
$$Z_1 = 24X_1 + 30X_2$$
  
 $2X_1 + 3X_2 \ge 10$   
 $4X_1 + 9X_2 \ge 15$   
 $6X_1 + 6X_2 \ge 20$   
 $X_1$  and  $X_2 \ge 0$ 

**Solution** Let,  $Y_1$ ,  $Y_2$  and  $Y_3$  be the dual variables with respect to the constraints 1, 2 and 3 respectively. Then the corresponding dual problem is:

Maximize 
$$Z = 10Y_1 + 15Y_2 + 20Y_3$$

$$2Y_1 + 4Y_2 + 6Y_3 \le 24$$
  
 $3Y_1 + 9Y_2 + 6Y_3 \le 30$   
 $Y_1, Y_2 \text{ and } Y_3 \ge 0$ 

# Solve the dual of the problem using simplex

	$C_i$	10	15	20	0	0	era their	
СВі	Basic variable	Υ <sub>1</sub>	Y <sub>2</sub>	Υ <sub>3</sub>	$S_1$	$S_2$	Solution	Ratio
0	S <sub>1</sub>	2	4	6	1	0	24	4**
0	$S_2$	3	9	6	0	1	30	5
	$Z_j$	0	0	0	0	0	0	
	$C_i - Z_i$	10	15	20*	0	0		

Maximize	<i>Z</i> =	$10Y_1$	+	$15Y_{2}$	+	$20Y_{3}$

$$2Y_1 + 4Y_2 + 6Y_3 \le 24$$
  
 $3Y_1 + 9Y_2 + 6Y_3 \le 30$   
 $Y_1, Y_2 \text{ and } Y_3 \ge 0$ 



	C <sub>j</sub>	10	15	20	0	0		
CB <sub>i</sub>	Basic variable	Υ <sub>1</sub>	Y <sub>2</sub>	<i>Y</i> <sub>3</sub>	$S_1$	$S_2$	Solution	Ratio
20	Y <sub>3</sub>	1/3	2/3	1	1/6	0	4	12
0	S <sub>2</sub>	1	5	0	-1	1	6	6**
	· Z <sub>j</sub>	20/3	40/3	20	10/3	0	80	
	$C_i - Z_j$	10/3*	5/3	0	-10/3	0		

		0	0	20	15	10	C <sub>j</sub>	
Ratio	Solution	$S_2$	$S_1$	<i>Y</i> <sub>3</sub>	<i>Y</i> <sub>2</sub>	<i>Y</i> <sub>1</sub>	Basic variable	СВ
12	4	0	1/6	1	2/3	1/3	Y <sub>3</sub>	20
6**	6	1	-1	0	5	1	$S_2$	0
	80	0	10/3	20	40/3	20/3	. Z <sub>j</sub>	
		0	-10/3	0	5/3	10/3*	$C_j - Z_j$	



	C <sub>j</sub>	10	15	20	0	0	
СВі	Basic variable	Υ <sub>1</sub>	<i>Y</i> <sub>2</sub>	<i>Y</i> <sub>3</sub>	$S_1$	$S_2$	Solution
20	<i>Y</i> <sub>3</sub>	0	-1	1	1/2	-1/3	2
10	Y <sub>I</sub>	1	5	0	-1	1	6
	$Z_j$	10	30	20	0	10/3	100
	$C_j - Z_j$	0	-15	0	0	-10/3	

#### Final Solution of Primal using Dual:

solution of the primal is inferred as shown.

Basic variables in the initial table of dual problem	$S_1$	$S_2$
$-(C_j - Z_j)$ of the final table of dual problem	0	10/3
Corresponding primal variables	$X_1$	$X_2$

# **Exercise:** Form the dual of the following primal problem.

Minimize z = 5x1 + 8x2

Subject to Constraints:

$$4x1 + 9 x2 >= 100$$

$$2x1 + x2 \le 20$$

$$2x1 + 5x2 >= 120$$

$$x1,x2 >= 0$$

#### Cannocial Form:

Minimize z = 5x1 + 8x2

Subject to Constraints:

$$4x1 + 9 x2 >= 100$$

$$-2x1 - x2 > = -20$$

$$2x1 + 5x2 >= 120$$

$$x1,x2 >= 0$$

# Exercise: Form the dual of the following primal problem.

Cannocial Form:

Minimize z = 5x1 + 8x2

Subject to Constraints:

$$4x1 + 9x2 >= 100$$

$$2x1 - x2 > = -20$$

$$2x1 + 5x2 >= 120$$

$$x1,x2 >= 0$$

**Dual Form** 

Maximize

Subject to constraints:

<=

<=

# Q. 1 Solve the following:

```
Construct the dual of the problem Z = 3x_1 - 2x_2 + 4x_3, subject to the constraints 3x_1 + 5x_2 + 4x_3 \ge 7, 6x_1 + x_2 + 3x_3 \ge 4, 7x_1 - 2x_2 - x_3 \le 10, x_1 - 2x_2 + 5x_3 \ge 3, 4x_1 + 7x_2 - 2x_3 \ge 2, x_1, x_2, x_3 \ge 0.
```

#### **Solution:**

```
The dual of the given problem will be maximize W = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5, subject to 3y_1 + 6y_2 - 7y_3 + y_4 + 4y_5 \le 3, 5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \le -2, 4y_1 + 3y_2 + y_3 + 5y_4 - 2y_5 \le 4, y_1, y_2, y_3, y_4, y_5, all \ge 0,
```

where  $y_1$ ,  $y_2$ ,  $y_3$ ,  $y_4$ , and  $y_5$  are the dual variables associated with the first, second, third, four and fifth constraint respectively.

# Q. 2 Solve the following:

big M\_dual\_unrestrictedvat0306202109\_24\_03\CamScann er 03-06-2021 09.24.03.pdf

#### Formulate the Primal Problem:

#### EXAMPLE 2.6-2 (Diet Problem)

A person wants to decide the constituents of a diet which will fulfil his daily requirement of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from fow different types of foods. The yields per unit of these foods are given in table 2.2.

TABLE 2.2

F /-		Cost per unit		
Food type	Proteins	Fats	Carbohydrates	(Rs.)
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum requirement	800	200	700	

Formulate linear programming model for the problem.

#### **Solution:**

#### Formulation of L.P. Model

Let  $x_1, x_2, x_3$  and  $x_4$  denote the number of units of food of type 1, 2, 3 and 4 respectively

Minimize  $Z = Rs. (45x_1 + 40x_2 + 85x_3 + 65x_4).$ 

Constraints are on the fulfilment of the daily requirements of the various constituents.  $3x_1 + 4x_2 + 8x_3 + 6x_4 \ge 800,$ for fats,

 $2x_1 + 2x_2 + 7x_3 + 5x_4 \ge 200,$ for carbohydrates, and  $6x_1 + 4x_2 + 7x_3 + 4x_4 \ge 700,$ 

where  $x_1, x_2, x_3, x_4, \text{ each } \ge 0$ .

# **Apply Big M method**

The final product of a firm must weigh exactly 150 kg. It uses two raw materials A and B with costs Rs. 2 and Rs. 8 per unit. At least 14 units of B and no more than 20 units of A must be used. Each unit of A weighs 5 kg and each unit of B weigh 10 kg. How much of each raw material be used per unit of the final product if its cost is to be minimized?

**Excellence and Service** 

# **Apply Big M**

Minimize: 
$$2X_1 + 8X_2$$
  
Subject to:  $5X_1 + 10X_2 = 150$   
 $X_1 \le 20$   
 $X_2 \ge 14$   
 $X_1$ ;  $X_2 \ge 0$ 

Minimize: 
$$2X_1 + 8X_2 + 0S_1 + 0S_2 + 1A_1 + 1A_2$$
  
Subject to:  $5X_1 + 10X_2$   $= 20$   
 $X_1 X_2 - S_2 + A_1 = 14$   
 $X_2 S_1 S_2 S_2 S_1 S_2 S_2 S_2 S_2 S_2 S_2$ 

# **Modified objective function**

Minimize: 
$$2X_1 + 8X_2 + 0S_1 + 0S_2 + 1A_1 + 1A_2$$
  
Subject to:  $5X_1 + 10X_2$   
 $X_1 + S_1$   
 $X_2 - S_2 + A_1 = 150$   
 $X_1 + A_2 = 14$   
 $X_2 + S_1 + S_2 + S_2 = 14$ 

# Big M Method

Minimize: 
$$2X_1 + 8X_2 + 0S_1 + 0S_2 + MA_1 + MA_2$$
  
Subject to:  $5X_1 + 10X_2 + S_1 = 20$   
 $X_1 + S_1 = 20$   
 $X_2 - S_2 + A_2 = 14$   
 $X_1 ; X_2 ; S_1 ; S_2 ; A_1 ; A_2 \ge 0$ 

**Table 3.45** 

			Tal	oleau 1					
	Basis	X <sub>1</sub>	X <sub>2</sub>	$S_1$	$S_2$	Aı	A <sub>2</sub>	X <sub>b</sub>	Min
<u> </u>	Δ.	5	10	0	0	1	0	150	15
ı	S.	1	0	1	0	0	0	20	-
1	A <sub>2</sub>	0	1	0	-1	0	1	14	14
1	C:	2	8	0	0	M	M		
	Z <sub>i</sub>	5M	11M	0	-M	M	М	164M	
	Z,-Ci	5M-2	11M-8	0	-M	0	0		

1.00		77		Tableau 2	Δ.	X <sub>b</sub>	Mir
Basis	$X_1$	$X_2$	$S_1$	32	A <sub>1</sub>		1
-	5	0	0	10	1	10	
A <sub>1</sub>		0	1	0	0	20	_
$S_1$	-	•	^	_1	0	14	-
X <sub>2</sub>	0	1	0	-1			
C	2	8	0	0	M		
Cj	5M	8	0	10M-8	M	10M+112	
$Z_{j}$	5M	0		10M-8	0		

**Table 3.47** 

	4.1		Tableau		C	V.	Min
-	Basis	$X_1$	$X_2$	$S_1$	$S_2$	Λb	
<sup>2</sup> j	Basis	0.5	0	0	1	1	2
)	$S_2$	0.5	0	1	0	20	20
)	$S_1$	1	1	0	0	15	30
	X <sub>2</sub>	0.5	1	0			
,	A <sub>2</sub>	2	8	0	0		
1	Cj	4	8	0	0	90	
	$Z_{j}$	2	0	0	0		

		Ta	ıbleau 4		F and	
$C_j$	Basis	X <sub>1</sub>	$X_2$	$S_1$	S <sub>2</sub>	$X_b$
2	$X_1$	1	0	0	2 -	2
0	$S_1$	0	0	1	-2	18
8	$X_2$	0	1	0	-1	14
	$C_j$	_ 2	8	0	0	
	$Z_{j}$	2	8	0	-4	116
	$Z_i - C_i$	0	0	0	0	