~~UNIT-III~~

~~TESTING OF HYPOTHESIS-I~~ UNIT-IV ~~Chapter~~

Estimation

1. ESTIMATION

~~TESTING OF HYPOTHESIS-I~~

1. ESTIMATION

Point Estimation

Interval Estimation

Bayesian Estimation

Estimate:- An estimate is a statement made to find an unknown population parameter.

Estimator:- The method or rule to determine an unknown population parameter is called an estimator.

Ex:- The sample mean ($\bar{x}$) is an estimator of population mean (μ) because sample mean is a method of determining the population mean.

Basically there are two kinds of estimates ~~to determine~~

i) point estimation

ii) Interval estimation

Point estimate:- If an estimate of a population parameter is given by a single value, then the estimate is called point estimate of the parameter.

Interval estimate:- If an estimate of a population parameter is given by two different values between which the parameter may be considered to be, then the estimate is called interval estimate of the parameter.

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Ex:- If the weight of the student is measured as 60kgs, the measurment gives a point estimate.

But if the ~~height~~ weight is given as (60 ± 3) kgs, then the weight lies between 63kgs & 57kgs and the measurement gives an interval estimate.

Note:- The sample mean $\bar{x}$ is a point estimate of population mean μ, sample variance s^2 is a point estimate of population variance σ^2

A point estimate is denoted by θ.

A point estimator is a statistic for estimating the population parameter θ and will be denoted by $\hat{\theta}$.

~~Ex:- ①~~

Unbiased estimator:- Let $\hat{\theta}$ be an estimator of θ, $\hat{\theta}$ is said to be an unbiased estimator of the parameter θ if $E(\hat{\theta}) = \theta$.

ie; E(statistic) = parameter then statistic is said to be an unbiased estimator of the parameter, otherwise the estimation is biased.

Ex:- ①

Prove that the sample mean($\bar{x}$) is an unbiased estimator of population mean(μ)

$(E(\bar{x}) = \mu)$

proof:- Let $x_1, x_2, x_3 \dots x_n$ be a random sample drawn from a given population with mean μ & variance σ^2 then

$$E(\bar{x}) = E\left[\frac{1}{n}\sum_{i=1}^{n} x_i\right]$$

$$= \frac{1}{n} E(x_1 + x_2 + \cdots + x_n)$$

$$= \frac{1}{n}\left[E(x_1) + E(x_2) + \cdots E(x_n)\right]$$

$$= \frac{1}{n}\left[\mu + \mu + \cdots + \mu\right]$$

$$= \frac{1}{\cancel{n}} \times \cancel{n}\mu$$

$$= \mu$$

Hence the sample mean $(\bar{x})$ is an unbiased estimator of the population mean μ.

Ex:- (2) P.T. for a random sample of size n, $X_1, X_2, \ldots X_n$ taken from an infinite population $s^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is not unbiased estimator of the parameter σ^2, but $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is unbiased.

Proof:- Let μ be the population mean.

$E(X_i) = \mu$ & $Var(X_i) = E(X_i - \mu)^2$

If s be the sample S.D. then

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{x})^2 \quad \text{--- (1)}$$

Now $$s^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i^2 - 2X_i\bar{X} + \bar{X}^2)$$

$$= \frac{1}{n}\sum_{i=1}^{n} X_i^2 - 2\bar{X}\frac{\sum_{i=1}^{n} X_i}{n} + \frac{1}{n}\sum \bar{X}^2$$

$$= \frac{\sum X_i^2}{n} - 2\bar{X}\bar{X} + \frac{1}{\cancel{n}}\bar{X}^2 \cdot \cancel{n}$$

$$= \frac{\sum X_i^2}{n} - \bar{X}^2$$

$$s^2 = \frac{\sum (X_i - \mu)^2}{n} - (\bar{X} - \mu)^2 \quad \left(\because \frac{\sum X_i^2}{n} - \bar{X}^2 = \frac{\sum (X_i - \mu)^2}{n} - (\bar{X} - \mu)^2\right.$$

Now

$$E(s^2) = E\left[\frac{\sum (X_i - \mu)^2}{n}\right] - E(\bar{X} - \mu)^2$$

$$= \frac{\sum E(x_i - \mu)^2}{n} - E(\bar{x} - \mu)^2$$

$$= \frac{\sum var(x_i)}{n} - var(\bar{x})$$

$$var\left(\sum_{i=1}^{n} \frac{x_i}{n}\right)$$

$$\sum_{i=1}^{n} \frac{var(x_i)}{n}$$

$$= \frac{\sum \sigma^2}{n} - \frac{\sigma^2}{n} \quad \left(\because var(\bar{x}) = \frac{\sigma^2}{n}\right)$$

$$= \frac{\cancel{n}\sigma^2}{\cancel{n}} - \frac{\sigma^2}{n}$$

$$E(s^2) = \sigma^2\left(\frac{n-1}{n}\right)$$

Thus $E(s^2) \neq \sigma^2$

$\therefore$ s^2 is a biased estimator of σ^2.

Let $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

$$= \frac{n}{n-1} s^2 \quad (\because \text{by } 1)$$

$$E(S^2) = E\left(\frac{n}{n-1} s^2\right)$$

$$= \frac{n}{n-1} E(s^2)$$

$$= \frac{\cancel{n}}{\cancel{n-1}} \times \frac{\cancel{n-1}}{\cancel{n}} \sigma^2$$

$$= \sigma^2$$

$\therefore$ $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$ is an unbiased estimator of σ^2.

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Point estimation :- If from observation in sample, a single value is calculated as an estimate from an unknown population parameter. The procedure to find the parameter is called a point estimation.

Interval estimation :- If θ ($\stackrel{\text{standard of}}{\text{population}}$ parameter) is an interval of the form $\hat{\theta}_L < \theta < \hat{\theta}_u$, where $\hat{\theta}_L$ and $\hat{\theta}_u$ depend on the value of the statistic $\hat{\theta}$ ~~for a~~ ~~paract~~ ~~particular sample and also on the~~ and ~~where~~ $\hat{\theta}_L$ & $\hat{\theta}_u$ are the lower and upper limits of the interval.

Confidence interval :- If $P(\hat{\theta}_L < \theta < \hat{\theta}_u) = 1 - \alpha$ for $0 < \alpha < 1$

then we have a probability of $1-\alpha$ of selecting random sample that will produce an interval containing θ. The fraction $(1-\alpha)100\%$ called confidence interval.

Where $1-\alpha$ is called confidence coefficient & $\hat{\theta}_L$ & $\hat{\theta}_u$ are called the lower & upper confidence limits.

Ex :- When $\alpha = 0.01$ then we have a 99% confidence interval

* [By central limit theorem, the standard normal distribution is

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \qquad \text{Here} \quad S.E. \text{ of } \bar{x} = \frac{\sigma}{\sqrt{n}}$$]

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$\Rightarrow \bar{x} - \mu = z \cdot \sigma/\sqrt{n}$$

$$\Rightarrow \mu = \bar{x} \pm z\, \sigma/\sqrt{n}$$

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Confidence limits for population mean (μ)

i) 95% confidence limits are $\bar{x} \pm 1.96(\text{S.E. of } \bar{x})$

ii) 99% confidence limits are $\bar{x} \pm 2.58(\text{S.E. of } \bar{x})$

iii) 99.73% confidence limits are $\bar{x} \pm 3(\text{S.E. of } \bar{x})$

iv) 90% confidence limits are $\bar{x} \pm 1.64(\text{S.E. of } \bar{x})$

Confidence limits for population proportion (P)

i) 95% confidence limits are $p \pm 1.96(\text{S.E. of } p)$

ii) 99% Confidence limits are $p \pm 2.58(\text{S.E of } p)$

iii) 99.73% Confidence limits are $p \pm 3(\text{S.E. of } p)$

iv) 90% Confidence limits are $p \pm 1.64(\text{S.E. of } p)$

Similarly for $\bar{x}_1 - \bar{x}_2$, $P_1 - P_2$ also same as above

Sample size for estimating population mean

Let E be the sampling error then $E = \bar{x} - \mu$

$\cancel{E = \bar{x}} - \mu$

$$E = z(\text{S.E. of } \bar{x})$$

$$= z\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$E^2 = \frac{z^2\sigma^2}{n}$$

$$\Rightarrow n = \frac{z^2\sigma^2}{E^2}$$

$$\Rightarrow \boxed{n = \left(\frac{Z\sigma}{E}\right)^2}$$

Let $\bar{x}$ be the sample mean drawn from a population having mean μ & S.D σ.

The confidence interval for the population mean μ is $\bar{x} \pm z(\text{S.E. of } \bar{x})$

$= \bar{x} \pm E$

$\bar{x} \pm z\,\sigma/\sqrt{n}$

$\bar{x} \pm z(\text{S.E. of } p)$

$\bar{x} \pm z\sqrt{PQ}$

Sample size for Estimating Population proportion

$E = z(S.E.\ of\ p)$

$= Z\sqrt{\frac{PQ}{n}}$

$E^2 = z^2 \frac{PQ}{n}$

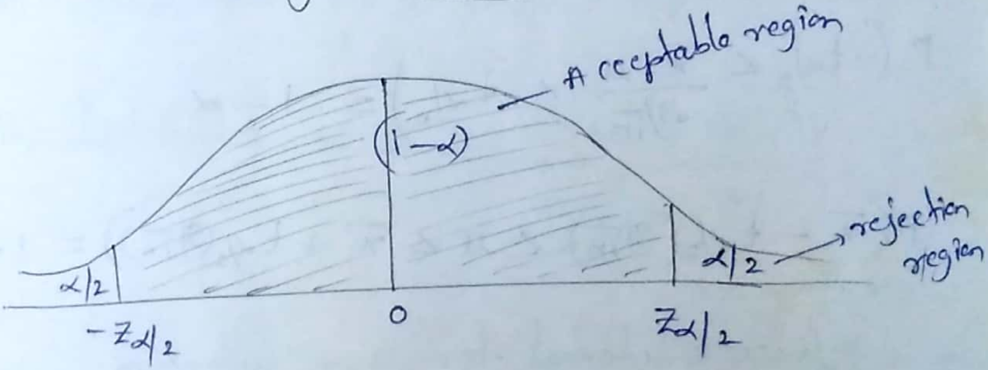
$\Rightarrow \boxed{n = \frac{z^2 PQ}{E^2}}$ where $Q = 1 - P$

Let p be the sample proportion and P be the population proportion. If E be the sampling error then $E = p - P$

The confidence interval for the population proportion P is $p \pm z(S.E\ of\ p)$

$p \pm E$

Maximum error of Estimate for large Samples



$P(-Z_{\alpha/2} < Z < Z_{\alpha/2}) = 1-\alpha$ where $Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

$P\left(-Z_{\alpha/2} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < Z_{\alpha/2}\right) = 1-\alpha$

$P\left[\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right] = 1-\alpha$

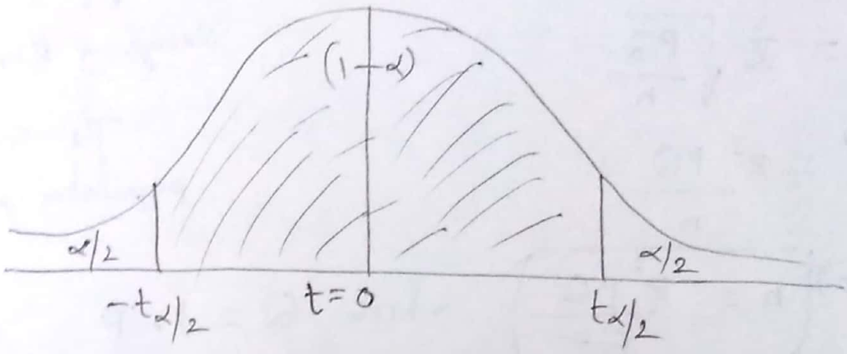
Confidence interval for μ, σ known

If $\bar{x}$ is the mean of a random sample size n from the population with variance σ^2, $(1-\alpha)100\%$. Confidence interval for μ is given by

$$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

The ~~Standard~~ maximum error $\boxed{E = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}$

Maximum error (E) of Estimate (E) for Small samples



$P(-t_{\alpha/2} < t < t_{\alpha/2}) = 1-\alpha$ where $t = \dfrac{\bar{x}-\mu}{s/\sqrt{n}}$

$$P\left(-t_{\alpha/2} < \frac{\bar{x}-\mu}{s/\sqrt{n}} < t_{\alpha/2}\right) = 1-\alpha$$

$$P\left(\bar{x} - t_{\alpha/2}(s/\sqrt{n}) < \mu < \bar{x} + t_{\alpha/2}(s/\sqrt{n})\right) = 1-\alpha$$

Confidence interval for μ, σ unknown

If $\bar{x}$ and s are the mean & S.D. of random sample from a normal population with unknown variance σ^2, $(1-\alpha)100\%$ Confidence interval for μ is

$$\bar{x} - t_{\alpha/2}(s/\sqrt{n}) < \mu < \bar{x} + t_{\alpha/2}(s/\sqrt{n})$$

The maximum error is $\boxed{E = t_{\alpha/2} \cdot s/\sqrt{n}}$

Bayesian Estimation

Bayesian estimation is a method of estimating the mean of a population.

Suppose prior distribution has a mean μ_0 & S.D σ_0.

In bayesian estimation prior feelings about the possible values of μ combined with direct sample evidence.

This leads to posterior distribution of μ, which under general conditions, can be approximated by a normal distribution with

Mean of posterior distribution is $\mu_1 = \dfrac{n\bar{x}\sigma_0^2 + \mu_0\sigma^2}{n\sigma_0^2 + \sigma^2}$

n = sample size

$\bar{x}$ = sample mean

σ^2 = variance of population.

S.D. of posterior distribution is $\sigma_1 = \sqrt{\dfrac{\sigma^2\sigma_0^2}{n\sigma_0^2 + \sigma^2}}$

Bayesian interval for μ

$(1-\alpha)100\%$ Bayesian interval for μ is given by

$$\mu_1 - Z_{\alpha/2}\cdot\sigma_1 < \mu < \mu_1 + Z_{\alpha/2}\cdot\sigma_1$$

2) If we can asert with 95% that the maximum error is 0.05 and $P = 0.2$, find the size of the sample. (2005, 2010)

Sol:- Given $P = 0.2$, $E = 0.05$

& $Q = 1 - 0.2 = 0.8$

$(1-\alpha)100\% = 95\%$

$(1-\alpha)100 = 95$

$1-\alpha = 0.95$

$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

$Z_{\alpha/2} = 1.96$

Maximum error $E = Z_{\alpha/2}\sqrt{\frac{PQ}{n}}$

$\Rightarrow 0.05 = 1.96\sqrt{\frac{0.2 \times 0.8}{n}}$

$\Rightarrow n = \frac{0.2 \times 0.8 \times (1.96)^2}{(0.05)^2} = 246$

$\therefore$ Sample size $(n) = 246$.

3) Assuming that $\sigma = 20.0$, how large a random sample be taken to assert with probability 0.95 that the sample mean will not differ from the true mean by more than 3.0 points. (2005, 2005, 2010)

Sol:- Given maximum error $E = 3.0$ & $\sigma = 20.0$

$Z_{\alpha/2} = 1.96$

We know that $n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$

$= \left(\frac{1.96 \times 20}{3}\right)^2 = 170.74$

$n \cong 171$

$\therefore$ Sample size $n = 171$.

④ In a study of an automobile insurance a random sample of 80 body repair costs had a mean of Rs. 472.36 and S.D. of Rs. 62.35. If $\bar{x}$ is used as a point estimate to the true average repair costs, with what confidence we can assert that the maximum error doesn't exceed Rs. 10? (2004, 2010)

Sol:- Size of a random sample $n = 80$

The mean of random sample $\bar{x} = $ Rs. 472.36

S.D. $\sigma =$ Rs. 62.35

Maximum error $E =$ Rs. 10.

$\therefore E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$\Rightarrow Z_{\alpha/2} = \frac{E.\sqrt{n}}{\sigma} = \frac{10\sqrt{80}}{62.35}$

$\Rightarrow Z_{\alpha/2} = 0.9236 \longrightarrow$

$\alpha/2 = 0.9236$

$\alpha = 0.9236 \times 2$

$\alpha = 1.8472$

$1-\alpha = 1-1.8472$

$1-\alpha = 0.8472$

$(1-\alpha)100\% = 84.72\%$.

$Z_{\alpha/2} = 1 - \alpha/2 = 0.9236$

$\Rightarrow \alpha/2 = 1 - 0.9236$

$\Rightarrow \alpha = 0.1528$.

The Confidence $= (1-\alpha)\,100\%$.

$= (1-0.1528)100\%$.

$= 84.72\%$.

⑤ A random sample of size 100 has a S.D of 5. What can you say about the maximum error with 95% confidence. (2005, 2012)

Sol:- Given $s = 5 \quad n = 100$

$Z_{\alpha/2} = 1.96$ (95% confidence)

maximum error $(E) = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$= (1.96) \times \frac{5}{\sqrt{100}} = 0.98$

(6) What is the maximum error one can expect to make with probability 0.90 when using the mean of a random sample of size $n = 64$ to estimate the mean of population with $\sigma^2 = 2.56$ (2004, 2009, 2011)

Sol:- Here $n = 64$

The probability $= 0.90$

$\sigma^2 = 2.56 \Rightarrow \sigma = \sqrt{2.56} = 1.6$

Confidence limit $= 90\%$.

$\therefore Z_{\alpha/2} = 1.645$

$\therefore$ Maximum error $E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$= 1.645 \times \frac{1.6}{\sqrt{64}}$

$= 0.329$

(7). It is desired to estimate the mean number of hours of continuous use until a certain computer will first require repairs. If it can be assumed that $\sigma = 48$ hours, how large a sample size be needed so that one will be able to assert with 90% confidence that the sample mean is off by atmost 10 hours.

Sol:- It is given that

maximum error $(E) = 10$ hours

$\sigma = 48$ hours

$Z_{\alpha/2} = 1.645$ (90% Confidence)

$\therefore n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$

$= \left(\frac{1.645 \times 48}{10}\right)^2 = 62.3 \simeq 62$

$\therefore$ The sample size $= 62$.

⑧ Find 95% Confidence limits for the mean of normally distributed population from which the following samples was taken 15, 17, 10, 18, 16, 9, 7, 11, 13, 14. (2008, 2009, 2010, 2010)

Sol:- Given samples are 15, 17, 10, 18, 16, 9, 7, 11, 13, 14.

~~we have~~ Sample mean $(\bar{x}) = \frac{15+17+10+18+16+9+7+11+13+14}{10} = 13$

variance $(s^2) = \frac{\sum(x_i-\bar{x})^2}{n-1}$ (σ unknown)

$= \frac{1}{9}\left[(15-13)^2 + \dots\dots (14-13)^2\right] = \frac{40}{3}$

$t_{\alpha/2} = 2.26$

$\therefore$ Confidence limits $= \left(\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}\right)$

$= 13 \pm 2.26 \cdot \frac{\sqrt{40/3}}{\sqrt{10}}$

$= 13 \pm 2.6$

$= (10.4, 15.6)$

⑨ The mean of random sample is an unbiased estimate of the mean of the population 3, 6, 9, 15, 27.

i) List of all possible samples of size 3 that can be taken without replacement from the finite population.

ii) Calculate the mean of each of the samples listed in (i) and assigning each sample a probability of 1/10. Verify that the mean of these $\bar{x}$ is equal to 12. P.T. $\bar{x}$ is an unbiased estimate of θ.

Sol:- The given samples 3, 6, 9, 15, 27.

i) The possible samples of size 3 taken from 3, 6, 9, 15, 27 without replacement is $^5C_3 = 10$.

The 10 samples are

(3,6,9) (3,6,15) (3,6,27) (6,9,15) (6,9,27) (3,9,15)

(3,9,27) (9,15,27) (6,15,27) (3,15,27)

ii) Mean of the population $\theta = \frac{3+6+9+15+27}{5} = 12$.

Means of the samples are 6, 8, 12, 10, 14, 9, 13, 17, 16, 15.

Probability assigned to each one is $\frac{1}{10}$ each.

$\bar{x}$	6	8	12	10	14	9	13	17	16	15
$P(\bar{x})$	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10

$$E(\bar{x}) = 6 \cdot \frac{1}{10} + 8 \cdot \frac{1}{10} + \cdots \; 15 \times \frac{1}{10}$$

$$= \frac{1}{10} \times 120$$

$$= 12$$

$\therefore E(\bar{x}) = \theta$.

$\therefore \bar{x}$ is an unbiased estimate of θ.

$\therefore$ The mean of a random sample is an unbiased estimator of the mean of the population.

(10) A random sample of 400 items is found to have mean 82 and S.D of 18. Find the maximum error of estimation at 95% confidence interval. Find the confidence limits for the mean if $\bar{x} = 82$.

(2008, 2011, 2011)

Sol:- Given S.D $(\sigma) = 18$

$n = 400$

$Z_{\alpha/2} = 1.96$ (95% confidence)

Sample mean $(\bar{x}) = 82$

Maximum error$(E) = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = \frac{1.96 \times 18}{20} = 1.764$

$\therefore$ The limits for the confidence are

$$\bar{x} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$\therefore$ $82 - 1.764 < \mu < 82 + 1.764$

$\therefore$ The limits are $(80.236, 83.764)$.

A professor's feelings about the mean mark in the final examination in probability of a large group of students is expressed subjectively by normal distribution with $\mu_0 = 67.2$ and $\sigma_0 = 1.5$

i) Find the professor

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* Ten bearings made by a certain process have a mean diameter of 0.5060 cm with a S.D. of 0.0040 cm. Assuming that the data may be taken as a random sample from a normal distribution. Construct 95% confidence interval for the actual average diameter of the bearings

Ans :-

$n = 10 < 30$

$t_\alpha = 2.262$ with d.o.f $= 10 - 1 = 9$

$$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = \frac{(2.262)(0.004)}{\sqrt{10}} = 0.00286$$

Confidence limits $= \bar{x} \pm t_{\alpha/2} \, s/\sqrt{n}$

$= (0.5031, 0.5089)$

* A random sample of 100 teachers in a large metropolitan area revealed a mean weekly salary of Rs. 487. with a S.D, Rs.48. With what degrees of ~~freedom~~ confidence can we assert that the average weekly salary of all teachers in the metropolitan area is between 472 to 502?

Sol:- Given $\mu = 487$ $\sigma = 48$, $n = 100$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{x} - 487}{48/\sqrt{100}} = \frac{\bar{x} - 487}{4.8}$$

Standard variable corresponding to Rs. 472 is

$$Z_1 = \frac{472 - 487}{4.8} = -3.125$$

Standard variable corresponding to Rs. 502 is

$$Z_2 = \frac{502 - 487}{4.8} = 3.125$$

Let x be the mean salary of teacher, then

$$P(472 < x < 502) = P(-3.125 < Z < 3.125)$$

$$= 0.9982$$

There we can assert with 99.82%. Confidence.

* A population random variable has mean 100 and S.D. is 16. What are the mean and S.D. of the sample mean of the random sample of size 4 drawn with replacement

Sol:- Given $\mu = 100$, $\sigma = 16$, $n = 4$

Since the sampling is done with replacements, the population may be considered as infinite.

We have to find $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$

$\mu_{\bar{x}} = \mu = 100$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{4}} = 8$

mean $(\mu) = 100$

S.D. $\sigma_{\bar{x}} = 8$.

× A sample of size 300 was taken whose variance is 225 and mean 54. Construct 95% confidence interval for the mean.

Sol:- Since the sample size 300 is large (>30), normal distribution is used as the sampling distribution

Here $n = 300$, $\bar{x}$ = sample mean $= 54$

$\sigma = \sqrt{225} = 15$

$\therefore$ S.E. of $\bar{x} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{300}} = 0.866$

95% confidence limits are

$\bar{x} \pm 1.96$ (S.E. of $\bar{x}$) $= 54 \pm 1.96(0.866)$

$= 55.697$ & 52.3

$\therefore$ The required confidence interval is $(52.3, 55.7)$

The mean and S.D of a population are 11,795 and 14,054 respectively. What can one assert with 95% confidence about the maximum error if $\bar{x} = 11,795$ & $n = 50$. Also construct 95% confidence interval for mean

Write down any (5) Test - (2) Write down any 5 questions

1) a) State and prove Lagrange's mean value theorem

b) Verify Rolle's mean value theorem for $f(x) = x^3$ in $[1, 3]$

2) a) Find 'c' by Cauchy's mean value theorem on $[a, b]$ for $f(x) = e^x$ and $g(x) = e^{-x}$

b) Find Taylor's series expansion of $f(x) = x^2 \log x$ about $x = 1$

3) Find ρ of $x = a(t + \sin t)$, $y = a(1 - \cos t)$ in parametric form.

4) Find the centre of curvature and circle of curvature of $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$

5) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ where parameters a & b are connected by $ab = 4$.

6) Show that the radius of curvature for the rectangular hyperbola $xy = c^2$ is $\frac{(x^2+y^2)^{3/2}}{2c^2}$

7) Show ~~that~~ the ~~evaluate~~ evolute of $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$

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