

①

Point	X-coordinate	Y-coordinate
P1	0.4005	0.5306
P2	0.2148	0.3854
P3	0.3457	0.3156
P4	0.2652	0.1875
P5	0.0789	0.4139
P6	0.4548	0.3022

a) Single link:- We need to consider the minimum distance b/w the points and choose the minimum distance points as a group or cluster.

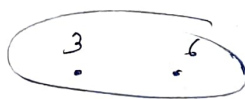
Given Distance Matrix:-

	P1	P2	P3	P4	P5	P6
P1	0.00	0.2357	0.2218	0.3688	0.3421	0.2347
P2	0.2357	0.00	0.1483	0.2042	0.1388	0.2540
P3	0.2218	0.1483	0.00	0.1513	0.2843	0.1100
P4	0.3688	0.2042	0.1513	0.00	0.2932	0.2216
P5	0.3421	0.1388	0.2843	0.2932	0.00	0.3921
P6	0.2347	0.2540	0.1100	0.2216	0.3921	0.00

Smallest distance b/w the points is 0.1100 b/w (P3, P6)

so we need to group/cluster P3, P6 as a single unit.

Dendrogram



Repeating the process by updating the Distance matrix

$$\begin{aligned}\min(\text{dist}(P_3, P_6), P_1) &= \min(\text{dist}(P_1, P_3), \text{dist}(P_1, P_6)) \\ &= \min(0.2218, 0.2347) \\ &= 0.2218\end{aligned}$$

$$\begin{aligned}\min(\text{dist}(P_3, P_6), P_2) &= \min(\text{dist}(P_2, P_3), \text{dist}(P_2, P_6)) \\ &= \min(0.1483, 0.2540) \\ &= 0.1483\end{aligned}$$

$$\begin{aligned}\min(\text{dist}(P_3, P_6), P_4) &= \min(\text{dist}(P_3, P_4), \text{dist}(P_4, P_6)) \\ &= \min(0.1513, 0.2216) \\ &= 0.1513.\end{aligned}$$

$$\begin{aligned}\min(\text{dist}(P_3, P_6), P_5) &= \min(\text{dist}(P_3, P_5), \text{dist}(P_5, P_6)) \\ &= \min(0.2843, 0.3921) \\ &= 0.2843.\end{aligned}$$

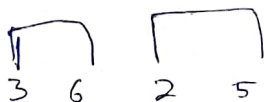
Updating the distance matrix

	P_1	P_2	P_3, P_6	P_4	P_5
P_1	0.00				
P_2	0.2357	0.00			
P_3, P_6	0.2218	0.1483	0.00		
P_4	0.3688	0.2043	0.1513	0.00	
P_5	0.3421	0.1388	0.2843	0.2932	0.00

Smallest distance is 0.1388 b/w (P_2, P_5)

After forming the cluster.

Dendro gram



$$\begin{aligned}\min(\text{dist}(P_2, P_5), P_1) &= \min(\text{dist}(P_1, P_2), \text{dist}(P_1, P_5)) \\ &= \min(0.2357, 0.3421) \\ &= 0.2357\end{aligned}$$

$$\begin{aligned}\min(\text{dist}(P_2, P_5), \text{dist}(P_3, P_6)) &= \min(\text{dist}(P_3, P_6), P_2, \text{dist}(P_3, P_6), P_5) \\ &= \min(0.1483, 0.2843) \\ &= 0.1483\end{aligned}$$

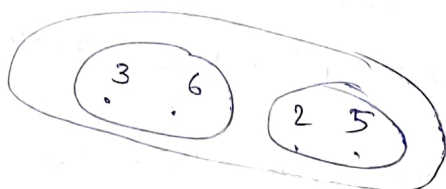
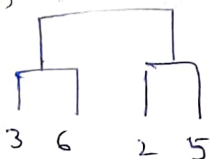
$$\begin{aligned}\min(\text{dist}(P_2, P_5), P_4) &= \min(\text{dist}(P_2, P_4), \text{dist}(P_4, P_5)) \\ &= \min(0.2043, 0.2932) \\ &= 0.2043\end{aligned}$$

Updating distance matrix

	P_1	P_2, P_5	P_3, P_6	P_4
P_1	0.00			
P_2, P_5	0.2357	0.00		
P_3, P_6	0.2218	0.1483	0.00	
P_4	0.3688	0.2043	0.1513	0.00

The smallest distance is 0.1483 b/w the points (P_2, P_5) & (P_3, P_6)
group these two into a single cluster

Dendrogram



Repeating the steps above.

$$\begin{aligned} \min(\text{dist}([(P_2, P_5), (P_3, P_6)]), P_1) &= \min(\text{dist}(P_2, P_5), P_1), \text{dist}((P_3, P_6), P_1)) \\ &= \min(0.2357, 0.2218) \\ &= 0.2218 \end{aligned}$$

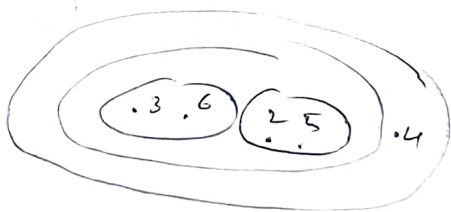
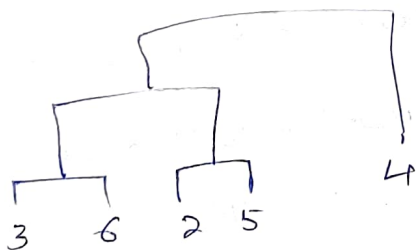
$$\begin{aligned} \min(\text{dist}([(P_2, P_5), (P_3, P_6)]), P_4) &= \min(\text{dist}(P_2, P_5), P_4), \text{dist}((P_3, P_6), P_4)) \\ &= \min(0.2043, 0.1513) \\ &= 0.1513 \end{aligned}$$

updating the distance matrix

	P_1	$(P_2, P_5), (P_3, P_6)$
P_1	0	
$(P_2, P_5), (P_3, P_6)$	0.2218	0
P_4	0.3688	0.1513

The smallest distance is 0.1513 b/w P_4 & $(P_2, P_5), (P_3, P_6)$

Dendrogram

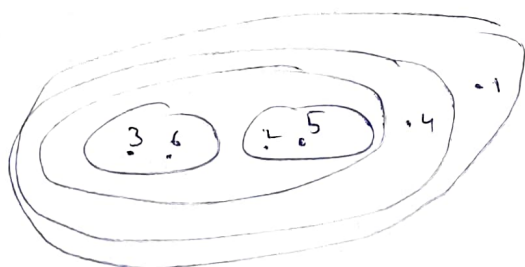
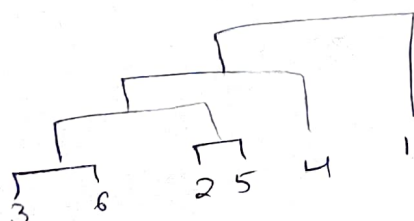


Repeat

$$\begin{aligned} \min(\text{dist}([(P_3, P_6), (P_2, P_5), P_4]), P_1) &= \min(\text{dist}((P_3, P_6), P_1), \text{dist}([(P_2, P_5), P_4]), P_1)) \\ &= \min(0.2218, 0.3688) \\ &= 0.2218 \end{aligned}$$

~~matrix~~ Since this is the last point we need to group into the cluster.

Dendrogram



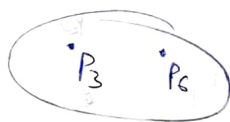
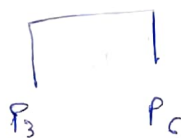
Complete link Proximity function:-

Complete link considers the max distance while ~~update~~ calculating the distance matrix

from the given distance matrix the smallest distance is 0.1100 b/w P_3 & P_6

Dendrogram

cluster



Here we need to find the maximum distance b/w the cluster and the rest of the points.

$$\begin{aligned}\max(\text{dist}(P_3, P_6), P_1) &= \max(\text{dist}(P_1, P_3), \text{dist}(P_1, P_6)) \\ &= \max(0.2218, 0.2347) \\ &= 0.2347\end{aligned}$$

$$\begin{aligned}\max(\text{dist}(P_3, P_6), P_2) &= \max(\text{dist}(P_2, P_3), \text{dist}(P_2, P_6)) \\ &= \max(0.1403, 0.2540) \\ &= 0.2540\end{aligned}$$

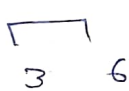
$$\begin{aligned}\max(\text{dist}(P_3, P_6), P_4) &= \max(\text{dist}(P_3, P_4), \text{dist}(P_4, P_6)) \\ &= \max(0.1513, \cancel{0.2921}, 0.2216) \\ &= \cancel{0.2921} \quad 0.2216\end{aligned}$$

$$\begin{aligned}\max(\text{dist}(P_3, P_6), P_5) &= \max(\text{dist}(P_3, P_5), \text{dist}(P_5, P_6)) \\ &= \max(0.2843, 0.3921) \\ &= 0.3921\end{aligned}$$

Distance Matrix

	P_1	P_2	(P_3, P_6)	P_4	P_5
P_1	0				
P_2	0.2357	0			
P_3, P_6	0.2347	0.2540	0		
P_4	0.3688	0.2042	0.2216	0	
P_5	0.3921	0.1388	0.3921	0.2932	0

The smallest distance is 0.1388 b/w P_2 & P_5 forming the dendrogram & cluster.



Distance ~~next~~ from (P_2, P_5)

$$\begin{aligned} \text{Max}(\text{dist}(P_2, P_5), P_1) &= \max(\text{dist}(P_1, P_2), \text{dist}(P_1, P_5)) \\ &= \max(0.2357, 0.3921) \\ &= 0.3921 \end{aligned}$$

$$\begin{aligned} \text{Max}(\text{dist}(P_2, P_5), (P_3, P_6)) &= \max(\text{dist}((P_3, P_6), P_2), \text{dist}((P_3, P_6), P_5)) \\ &= \max(0.2540, 0.3921) \\ &= 0.3921 \end{aligned}$$

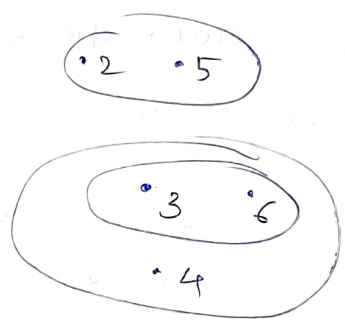
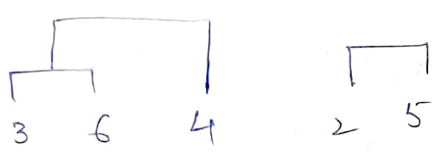
$$\begin{aligned} \text{max}(\text{dist}(P_2, P_5 \cup P_4)) &= \max(\text{dist}(P_2, P_4), \text{dist}(P_4, P_5)) \\ &= \max(0.2042, 0.2932) \\ &= 0.2932 \end{aligned}$$

Distance Matrix:-

	P_1	P_2, P_5	P_3, P_6	P_4
P_1	0			
P_2, P_5	0.3921	0		
P_3, P_6	0.2347	0.3921	0	
P_4	0.3688	0.2932	0.2216	0

Smallest distance is 0.2216 from P_4 to P_3, P_6 cluster.

Dendrogram →



Distance from cluster to all other points

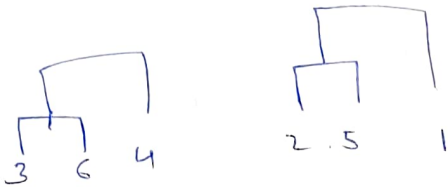
$$\begin{aligned}
 \max(\text{dist}((P_3, P_6), P_4), (P_2, P_5)) &= \max(\text{dist}(P_3, P_6), (P_2, P_5), (P_2, P_5), P_4)) \\
 &= \max(0.3921, 0.2932) \\
 &= 0.3921 \\
 \max(\text{dist}((P_3, P_6), P_4), P_1) &= \max(\text{dist}((P_3, P_6), P_1), (P_4, P_1)) \\
 &= \max(0.2347, 0.3688) \\
 &= 0.3688
 \end{aligned}$$

Updated distance matrix

	P_1	P_2, P_5	$(P_3, P_6), P_4$
P_1	0		
P_2, P_5	0.3921	0	
$(P_3, P_6), P_4$	0.3688	0.3921	0

smallest distance is 0.3421 b/w P_1 & (P_2, P_5)

dendrogram



cluster

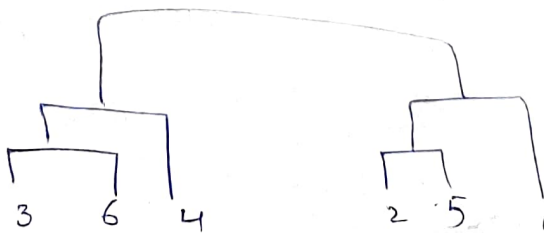


Distance from cluster is.

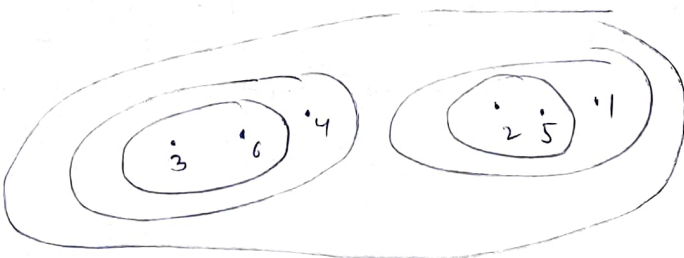
$$\begin{aligned} & \max(\text{dist}(P_2, P_5), P_1), (P_3, P_6), P_4) \\ &= \max(\text{dist}(P_2, P_5), (P_3, P_6), P_4), \text{dist}(P_1, (P_3, P_6), P_4)) \\ &= \max(0.3921, 0.3686) \\ &= 0.3921 \end{aligned}$$

Updated distance matrix is.

	P_1, P_2, P_5	P_3, P_4, P_6
P_1, P_2, P_5	0	
P_3, P_4, P_6	0.3921	0



cluster



Average link:-

Average link proximity function considers the average while calculating the distance matrix

from the given distance matrix the smallest distance is 0.1100 b/w P_3 & P_6

Dendrogram



Cluster



The distance from the Cluster (P_3, P_6) &

$$\begin{aligned} \text{Avg}(\text{dist}(P_3, P_6), P_1) &= \text{dist}((P_3, P_1) + (P_1, P_6)) / 2 \\ &= (0.2218 + 0.2347) / 2 \\ &= 0.22825 \end{aligned}$$

$$\begin{aligned} \text{Avg}(\text{dist}(P_3, P_6), P_2) &= \text{dis}((P_2, P_3) + (P_2, P_6)) / 2 \\ &= (0.1483 + 0.2540) / 2 \\ &= 0.20115 \end{aligned}$$

$$\begin{aligned} \text{Avg}(\text{dist}(P_3, P_6), P_4) &= \text{dist}((P_3, P_4) + (P_4, P_6)) / 2 \\ &= (0.1513 + 0.2216) / 2 \\ &= 0.18645 \end{aligned}$$

$$\begin{aligned} \text{Avg}(\text{dist}(P_3, P_6), P_5) &= (0.2843 + 0.3941) / 2 \\ &= 0.3382 \end{aligned}$$

Updated distance Matrix B.

	P_1	P_2	P_3, P_6	P_4	P_5
P_1	0				
P_2	0.2357	0			
P_3, P_6	0.2825	0.20115	0		
P_4	0.3688	0.2042	0.18645	0	
P_5	0.3421	0.1388	0.3382	0.2932	0

Dendrogram



clusters
=



Distance from the cluster B.

$$\text{Avg}(\text{dist}(P_2, P_5), P_1) = \frac{\text{dist}(0.2357 + 0.3421)}{2} = 0.2889$$

$$\text{Avg}(\text{dist}(P_2, P_5), (P_3, P_6)) = \frac{(0.20115 + 0.3382)}{2} = 0.269675$$

$$\text{Avg}(\text{dist}(P_2, P_5), P_4) = \frac{(0.2042 + 0.2932)}{2} = 0.2487$$

Updated matrix:-

	P_1	P_2, P_5	P_3, P_6	P_4
P_1	0			
P_2, P_5	0.2889	0		
P_3, P_6	0.22825	0.269675	0	
P_4	0.3421	0.2487	0.18645	0

Dendrogram



cluster



Distance from cluster:-

$$\text{Avg}(\text{dist}(P_3, P_6), P_4, P_1)) = (0.22825 + 0.3421) / 2$$

$$= 0.285175$$

$$\text{Avg}(\text{dist}((P_3, P_6), P_4), (P_2, P_5)) = (0.269675 + 0.2487) / 2$$

$$= 0.2591875$$

Updated distance matrix

	P_1	P_2, P_5	$(P_3, P_6), P_4$
P_1	0		
P_2, P_5	0.2889	0	
$(P_3, P_6), P_4$	0.285175	0.2591875	0

Dendrogram



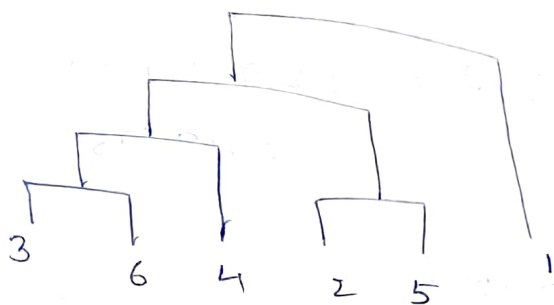
clusters



The distance from clusters is .

$$\begin{aligned} \text{Avg} &= \frac{((B, P_1), A), (P_2, P_5), P_1)}{2} \\ &= (0.285175 + 0.2889) / 2 \\ &= 0.2870375 \end{aligned}$$

Dendrogram



clusters

